Applied Thermodynamics: Software Solutions

Part-III

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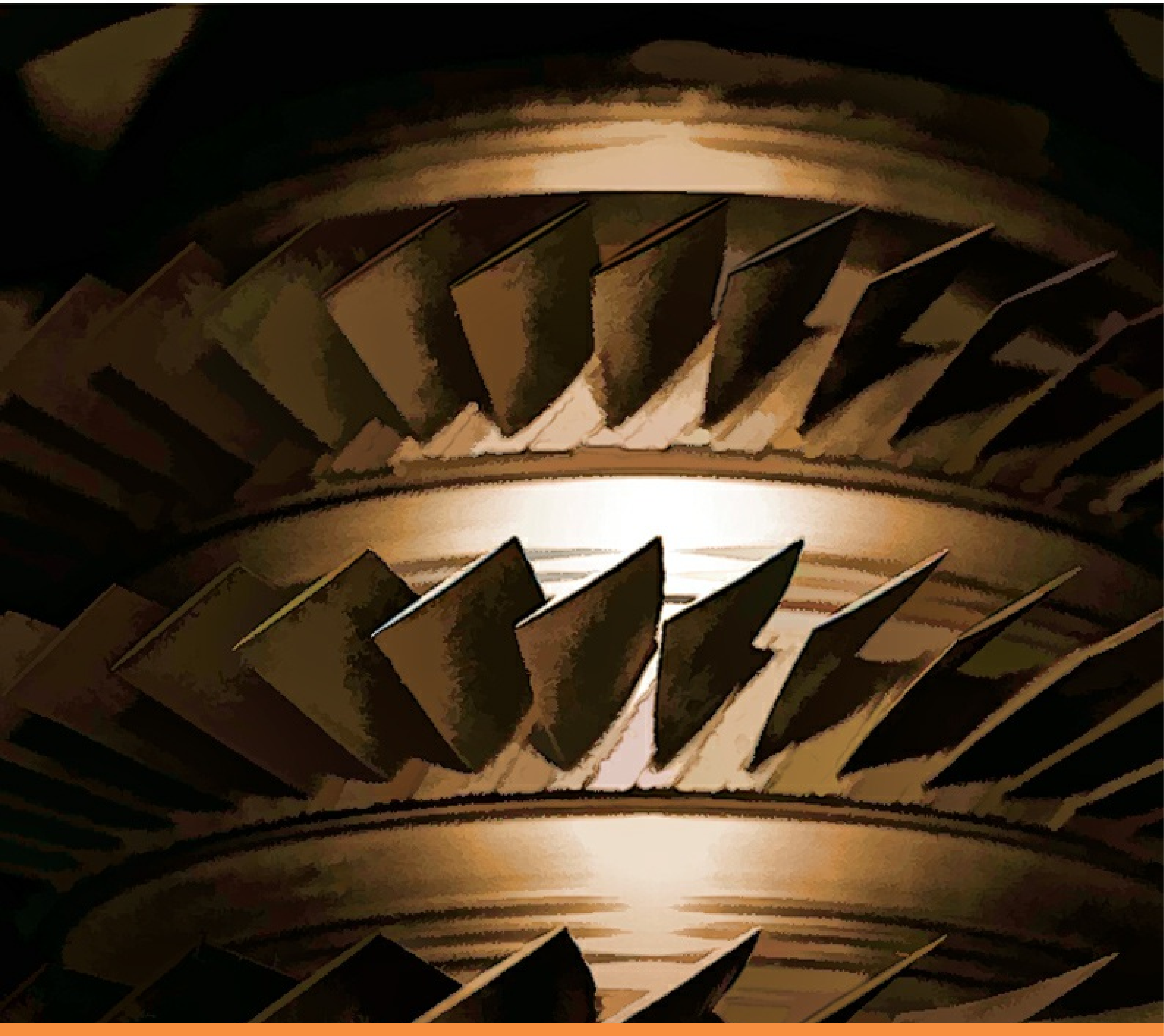

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Dr. M. Thirumaleshwar

Applied Thermodynamics: Software Solutions

Part-III (Refrigeration cycles, Air compressor, Thermodynamic relations)

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Contents

4 Refrigeration Cycles

Learning objectives:

1. In this chapter, 'Refrigeration cycles' are analyzed.
2. Cycles dealt with are: Ideal and actual vapour compression cycle, Ideal and actual reversed Brayton cycle (or, Bell Coleman cycle).
3. Several useful Mathcad Functions are written for properties of Refrigerant-R134a in superheated and two-phase regions, since Mathcad does not have built-in Functions for R134a, and are used in solving problems. Also, useful Mathcad Functions are written to facilitate easy calculations for all these cycles.
4. And, many useful Functions/Procedures are written in EES for different variations of ideal vapour compression refrigeration cycle.
5. Problems from University question papers and standard Text books are solved with Mathcad, EES and TEST.

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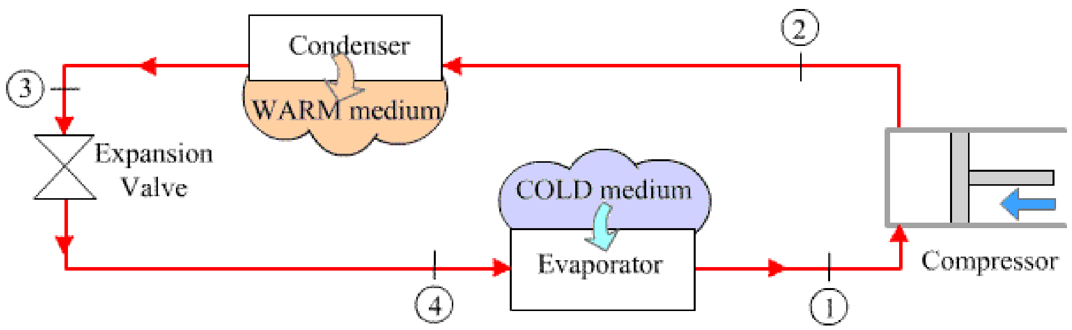
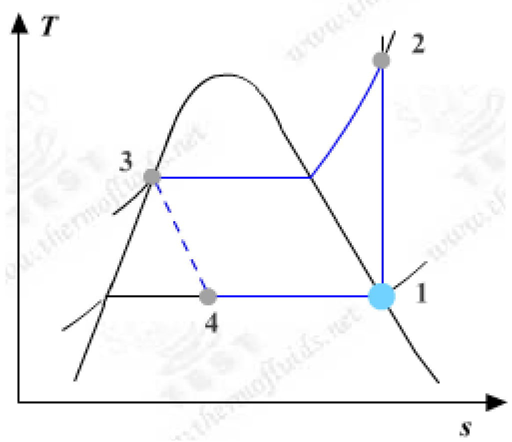
4.1 Definitions, Statements and Formulas used[1-7]:

Note: Figures used in this section are from TEST Software [Ref: 7].

4.1.1 Ideal vapour compression refrigeration cycle:





Schematic diagram and the T-s diagram of the ideal vapour compression cycle are shown above.

1-2: Isentropic compression of sat. refrigerant vapour from the evaporator in compressor

2-3: Cooling and condensing in condenser

3-4: expansion in the expansion valve; this occurs at constant enthalpy.

4-1: supply of refrigeration in evaporator

Note the following:

$w_{comp} = h2(T1) - h1(T1)$ kJ/kg...compressor work

$q_L = h1 - h4$ kJ/kg... refrign. capacity

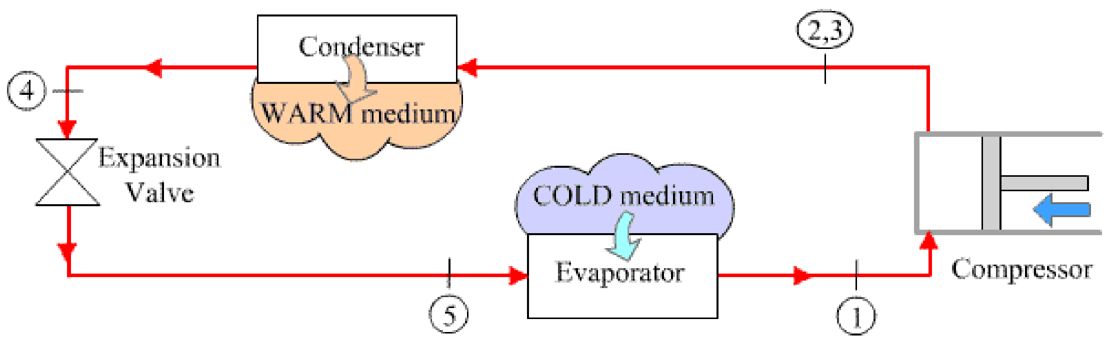
Now, 1 ton of refrigeration = 211 kJ/min.

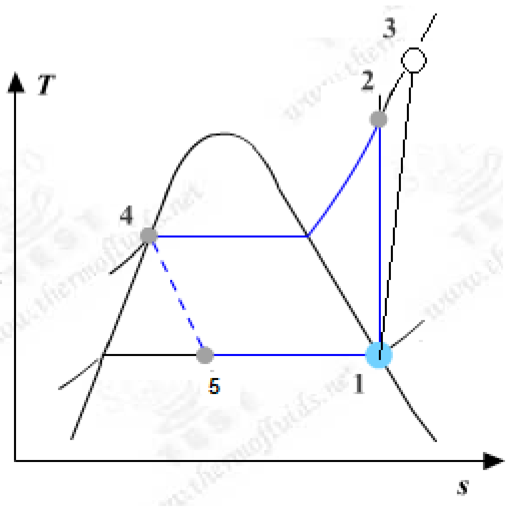
$COP = \frac{q_L}{w_{comp}}$ coeff. of performance .

4.1.2 Actual vapour compression refrigeration cycle:

This takes in to account *the isentropic efficiency* of the compressor.

Schematic diagram of the system and the T-s diagram are shown below:





$w_{comp} = h3 - h1$ **kJ/kg...compressor work**

$\eta_{comp} = \frac{h2 - h1}{h3 - h1}$ **...isentropic effcy of compressor**

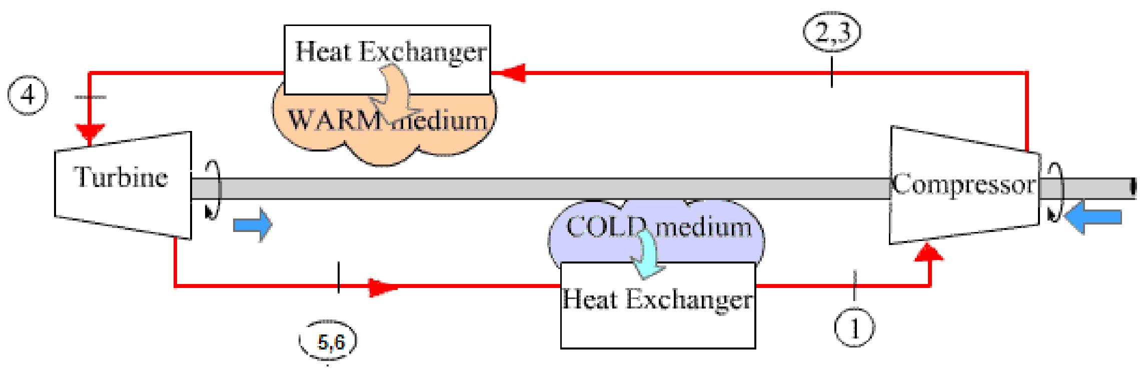
$q_L = h1 - h5$ **kJ/kg... refrign. capacity**

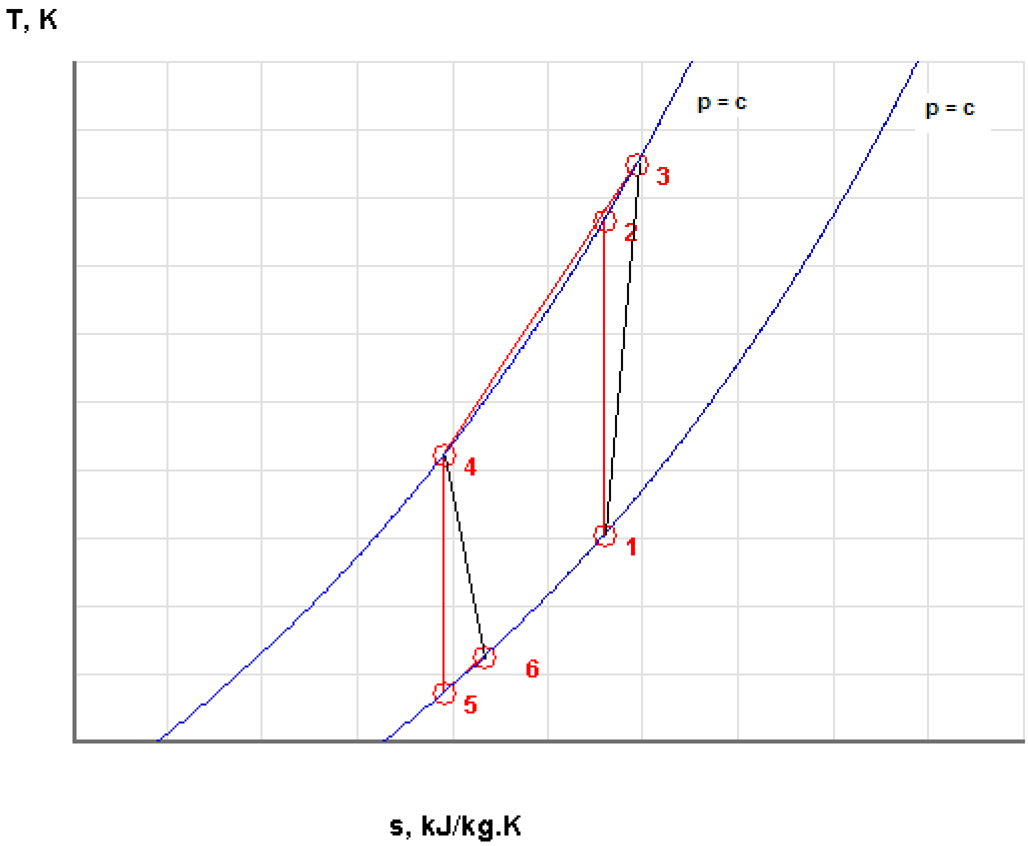
$q_H = h3 - h4$ **kJ/kg..heat transferred in condenser**

$COP = \frac{q_L}{w_{comp}}$ **coeff. of performance**

4.1.3 Reversed Brayton cycle refrigeration (or, Air cycle refrigeration or Bell Coleman cycle):

This is used in aircraft cabin cooling. Schematic diagram of the system and the T-s diagram for an actual reversed Brayton cycle refrigeration cycle are shown below:





1-2: Isentropic compression of air from the cold region in compressor

1-3: actual compression

3-4: cooling of compressed air at constant pressure

4-5: isentropic expansion of air in the turbine

4-6: actual expansion in turbine

Note that the following calculations are done assuming constant sp. heat for air:

$$T2 = T1\cdot\left(\frac{P2}{P1}\right)^{\frac{\gamma-1}{\gamma}}$$ K ... temp. at compressor exit after isentropic compression

$$T3 = T1 + \frac{(T2 - T1)}{\eta_{comp}}$$ K ... temp. at compressor exit after actual compression

$$T5 = \frac{T4}{\left(\frac{P2}{P1}\right)^{\frac{\gamma-1}{\gamma}}}$$ K ... temp. at turbine exit after isentropic expansion

$$T6 = T4 - \eta_{turb}\cdot(T4 - T5)$$ K ... temp. at turbine exit after actual expansion

$$w_{comp} = cp\cdot(T3 - T1)$$ kJ/kg ... compressor work input

$$w_{turb} = cp\cdot(T4 - T6)$$ kJ/kg ... turbine work output

$$w_{net} = w_{comp} - w_{turb}$$ kJ/kg ... net work input

$$q_{in} = cp\cdot(T1 - T6)$$ kJ/kg ... refrigeration effect

$$q_{out} = cp\cdot(T3 - T4)$$ kJ/kg ... heat rejected in HX

$$COP = \frac{q_{in}}{w_{net}}$$ ...coeff. of performance

$$R = cp\cdot\left(\frac{\gamma - 1}{\gamma}\right)$$ kJ/kg.K .... Gas constant for Air

$$spvol1 = \frac{R\cdot T1}{P2\cdot 10^2}$$ m^3/kg ... sp. volume of air at compressor inlet conditions, with P2 in bar

4.2 Problems solved with Mathcad:

Note:

Mathcad does not have built-in functions for Refrigerants. So, generally, while solving problems on vapour compression refrigeration cycles which use refrigerants such as R-12, R-22, R-134a, as working substance, we have to refer to tables often to get properties of refrigerant at various state points.

So, we shall first develop few simple Mathcad Functions for refrigerant R134a, based on published Tables (Ref: TEST software, www.thermofluids.net), and then use them in solving problems. These Functions use the built-in linear interpolation function 'linterp' in Mathcad to get properties from the Tables.

Prob.4.2.1. Write Mathcad programs/Functions for properties of refrigerant R134a:

Mathcad Solution:

Our Mathcad Functions are based on published R134a Tables (Ref:[7]: TEST Software, www.thermofluids.net).

There are separate Tables for Superheated and Saturated R134a

First, for Superheated R134a:

For each pressure, the Table is copied as a matrix in Mathcad, each column is extracted as a vector, and linear interpolation is done for intermediate values.

Functions are written for the following pressures: 0.6, 1.0, 1.4, 1.8, 2, 2.4, 2.8, 3.2, 4, 5, 6, 7, 8, 9, 10, 12, 14, 16 bar.

A sample set of Functions written for a pressure of 5 bar are shown below:

At 5 bar:

T.....deg. C
v....m^3/kg
u, h....kJ/kg;
s...kJ/kg.K.....deg. C

S5 :=

T	v	u	h	s
15.74	0.04086	253.64	256.07	0.9117
20	0.04188	239.4	260.34	0.9264
30	0.04416	248.2	270.28	0.9597
40	0.04633	256.99	280.16	0.9918
50	0.04842	265.83	290.04	1.0229
60	0.05043	274.73	299.95	1.0531
70	0.0524	283.72	309.92	1.0825
80	0.05432	292.8	319.96	1.1114
90	0.0562	302	330.1	1.1397
100	0.05805	311.31	340.33	1.1675
110	0.05988	320.74	350.68	1.1949
120	0.06168	330.3	361.14	1.2218
130	0.06347	339.98	371.72	1.2484
140	0.06524	349.79	382.42	1.2746

$temp5 := S5^{\langle 0 \rangle}$ $\quad$ $length(temp5) = 14$

$spvol5 := S5^{\langle 1 \rangle}$ $\quad$ $enth5 := S5^{\langle 3 \rangle}$ $\quad$ $entrop5 := S5^{\langle 4 \rangle}$

$HR134A5B(T) := linterp(temp5, enth5, T)$ $\quad$ ex: $\quad$ $HR134A5B(40) = 280.16$

$SR134A5B(T) := linterp(temp5, entrop5, T)$ $\quad$ ex: $\quad$ $SR134A5B(40) = 0.992$

Then, all the Functions written for the different pressures are combined into a single program with linear interpolation applied for any desired pressure:

This Function returns enthalpy (h, kJ/kg) and entropy (s, kJ/kg.C) when pressure (P, in bar) and temp (T, in C) are input.

$$
\begin{aligned}
&\text{h\_and\_s\_SuperheatR134a}(P,T) := \\
&\quad \text{return "P should be between 0.6 bar and 16 bar" if } P < 0.6 \lor P > 16 \\
&\quad \text{return "T should be between -37.07 C and 200 C" if } T < -37.07 \lor T > 200 \\
&\quad \text{if } P \ge 0.6 \land P < 1 \\
&\qquad h \leftarrow HR134A06B(T) + \frac{(P-0.6)}{(1-0.6)}\cdot(HR134A1B(T) - HR134A06B(T)) \\
&\qquad s \leftarrow SR134A06B(T) + \frac{(P-0.6)}{(1-0.6)}\cdot(SR134A1B(T) - SR134A06B(T)) \\
&\quad \text{if } P \ge 1 \land P < 1.4 \\
&\qquad h \leftarrow HR134A1B(T) + \frac{(P-1)}{(1.4-1)}\cdot(HR134A014B(T) - HR134A1B(T)) \\
&\qquad s \leftarrow SR134A1B(T) + \frac{(P-1)}{(1.4-1)}\cdot(SR134A014B(T) - SR134A1B(T)) \\
&\quad \text{if } P \ge 1.4 \land P < 1.8 \\
&\qquad h \leftarrow HR134A014B(T) + \frac{(P-1.4)}{(1.8-1.4)}\cdot(HR134A018B(T) - HR134A014B(T)) \\
&\qquad s \leftarrow SR134A014B(T) + \frac{(P-1.4)}{(1.8-1.4)}\cdot(SR134A018B(T) - SR134A014B(T)) \\
&\quad \text{if } P \ge 1.8 \land P < 2 \\
&\qquad h \leftarrow HR134A018B(T) + \frac{(P-1.8)}{(2-1.8)}\cdot(HR134A2B(T) - HR134A018B(T)) \\
&\qquad s \leftarrow SR134A018B(T) + \frac{(P-1.8)}{(2-1.8)}\cdot(SR134A2B(T) - SR134A018B(T))
\end{aligned}
$$

$$
\begin{aligned}
&\quad \text{if } P \ge 2 \land P < 2.4 \\
&\qquad h \leftarrow HR134A2B(T) + \frac{(P-2)}{(2.4-2)}\cdot(HR134A024B(T) - HR134A2B(T)) \\
&\qquad s \leftarrow SR134A2B(T) + \frac{(P-2)}{(2.4-2)}\cdot(SR134A024B(T) - SR134A2B(T)) \\
&\quad \text{if } P \ge 2.4 \land P < 2.8 \\
&\qquad h \leftarrow HR134A024B(T) + \frac{(P-2.4)}{(2.8-2.4)}\cdot(HR134A028B(T) - HR134A024B(T)) \\
&\qquad s \leftarrow SR134A024B(T) + \frac{(P-2.4)}{(2.8-2.4)}\cdot(SR134A028B(T) - SR134A024B(T)) \\
&\quad \text{if } P \ge 2.8 \land P < 3.2 \\
&\qquad h \leftarrow HR134A028B(T) + \frac{(P-2.8)}{(3.2-2.8)}\cdot(HR134A032B(T) - HR134A028B(T)) \\
&\qquad s \leftarrow SR134A028B(T) + \frac{(P-2.8)}{(3.2-2.8)}\cdot(SR134A032B(T) - SR134A028B(T))
\end{aligned}
$$

$$\text{if } P \ge 3.2 \wedge P < 4$$
$$h \leftarrow HR134A032B(T) + \frac{(P-3.2)}{(4-3.2)}\cdot(HR134A4B(T) - HR134A032B(T))$$
$$s \leftarrow SR134A032B(T) + \frac{(P-3.2)}{(4-3.2)}\cdot(SR134A4B(T) - SR134A032B(T))$$
$$\text{if } P \ge 4 \wedge P < 5$$
$$h \leftarrow HR134A4B(T) + \frac{(P-4)}{(5-4)}\cdot(HR134A5B(T) - HR134A4B(T))$$
$$s \leftarrow SR134A4B(T) + \frac{(P-4)}{(5-4)}\cdot(SR134A5B(T) - SR134A4B(T))$$
$$\text{if } P \ge 5 \wedge P < 6$$
$$h \leftarrow HR134A5B(T) + \frac{(P-5)}{(6-5)}\cdot(HR134A6B(T) - HR134A5B(T))$$
$$s \leftarrow SR134A5B(T) + \frac{(P-5)}{(6-5)}\cdot(SR134A6B(T) - SR134A5B(T))$$
$$\text{if } P \ge 6 \wedge P < 7$$
$$h \leftarrow HR134A6B(T) + \frac{(P-6)}{(7-6)}\cdot(HR134A7B(T) - HR134A6B(T))$$
$$s \leftarrow SR134A6B(T) + \frac{(P-6)}{(7-6)}\cdot(SR134A7B(T) - SR134A6B(T))$$
$$\text{if } P \ge 7 \wedge P < 8$$
$$h \leftarrow HR134A7B(T) + \frac{(P-7)}{(8-7)}\cdot(HR134A8B(T) - HR134A7B(T))$$
$$s \leftarrow SR134A7B(T) + \frac{(P-7)}{(8-7)}\cdot(SR134A8B(T) - SR134A7B(T))$$

$$\text{if } P \ge 8 \wedge P < 9$$
$$h \leftarrow HR134A8B(T) + \frac{(P-8)}{(9-8)}\cdot(HR134A9B(T) - HR134A8B(T))$$
$$s \leftarrow SR134A8B(T) + \frac{(P-8)}{(9-8)}\cdot(SR134A9B(T) - SR134A8B(T))$$
$$\text{if } P \ge 9 \wedge P < 10$$
$$h \leftarrow HR134A9B(T) + \frac{(P-9)}{(10-9)}\cdot(HR134A10B(T) - HR134A9B(T))$$
$$s \leftarrow SR134A9B(T) + \frac{(P-9)}{(10-9)}\cdot(SR134A10B(T) - SR134A9B(T))$$

if $P \geq 10 \wedge P < 12$

$$h \leftarrow HR134A10B(T) + \frac{(P-10)}{(12-10)} \cdot (HR134A12B(T) - HR134A10B(T))$$

$$s \leftarrow SR134A10B(T) + \frac{(P-10)}{(12-10)} \cdot (SR134A12B(T) - SR134A10B(T))$$

if $P \geq 12 \wedge P < 14$

$$h \leftarrow HR134A12B(T) + \frac{(P-12)}{(14-12)} \cdot (HR134A14B(T) - HR134A12B(T))$$

$$s \leftarrow SR134A12B(T) + \frac{(P-12)}{(14-12)} \cdot (SR134A14B(T) - SR134A12B(T))$$

if $P \geq 14 \wedge P \leq 16$

$$h \leftarrow HR134A14B(T) + \frac{(P-14)}{(16-14)} \cdot (HR134A16B(T) - HR134A14B(T))$$

$$s \leftarrow SR134A14B(T) + \frac{(P-14)}{(16-14)} \cdot (SR134A16B(T) - SR134A14B(T))$$

$(h \quad s)$

Further, for convenience and uniformity, we write the following programs to get enthalpy and entropy of R134a when P and T are given in bar and deg.C respectively:

```
enthalpy_R134a(P,T) := | return "P should be between 0.6 bar and 16 bar"  if  P < 0.6 ∨ P > 16
                       | return "T should be between -37.07 C and 200 C"  if  T < -37.07 ∨ T > 200
                       | tsat ← TSAT(P)
                       | h ← h_and_s_SuperheatR134a(P,T)0,0  if  T ≥ tsat
                       | (return "State point in two phase region--- use 2 phase Functions")  otherwise
```

```
entropy_R134a(P,T) := | return "P should be between 0.6 bar and 16 bar"  if  P < 0.6 ∨ P > 16
                      | return "T should be between -37.07 C and 200 C"  if  T < -37.07 ∨ T > 200
                      | tsat ← TSAT(P)
                      | s ← h_and_s_SuperheatR134a(P,T)0,1  if  T ≥ tsat
                      | (return "State point in two phase region--- use 2 phase Functions")  otherwise
```

Function to find h when P and s are knpwn:

As a first step, get T when P and s are known:

$P := 8$ bar

$s := 0.934$ kJ/kg.C

$T := 50$ C....guess value

Given

$\text{entropy\_R134a}(P,T) = s$

$\text{Temp\_R134a}(P,s) := \text{Find}(T)$

$\text{Temp\_R134a}(P,s) = 39.043$ C

Now, write the Function to get h:

```
enthalpy_R134a_Ps(P,s) := | return "P should be between 0.6 bar and 16 bar"  if P < 0.6 ∨ P > 16
                          | T ← Temp_R134a(P,s)
                          | h ← enthalpy_R134a(P,T)
```

Ex: P := 9 bar   s := 0.9253 kJ/kg.C

enthalpy_R134a_Ps(P,s) = 272.394 J/kg

Next, we write Functions for properties of R134a in the two-phase region:

Here, the Sat. pressure Table is used.

To write the Functions, we extract each column from the Table as a vector and use them to get interpolated values, in conjunction with the interpolation function 'linterp' in Mathcad.

The different vectors extracted from the Table are shown below:

Properties of Refrigerant R134a:

Sat. properties: (Ref: TEST, Cengel)

Units: psat (bar), tsat(C), vf, vg (m3/kg), hf, hg (kJ/kg), sf,sg (kJ/kg.C)

psat :=	tsat :=	vfsat :=	vgsat :=
0.6	−36.95	0.0007098	0.31121
0.7	−33.87	0.0007144	0.26929
0.8	−31.13	0.0007185	0.23753
0.9	−28.65	0.0007223	0.21263
1	−26.37	0.0007259	0.019254
1.2	−22.32	0.0007324	0.016212
1.4	−18.77	0.0007383	0.14014
1.6	−15.62	0.000744	0.1229
1.8	−12.73	0.000749	0.1098
2	−10.09	0.000753	0.0993
2.4	−5.37	0.000762	0.0834
2.8	−1.23	0.00077	0.0719
3.2	2.48	0.000777	0.0632
3.6	5.84	0.000784	0.0564
4	8.93	0.00079	0.0509
5	15.74	0.000806	0.0409
6	21.58	0.00082	0.0341
7	26.72	0.000833	0.0292
8	31.33	0.000845	0.0255
9	35.53	0.000858	0.0226
10	39.39	0.00087	0.0202
12	46.32	0.000893	0.0166
14	52.43	0.000916	0.014
16	57.92	0.000939	0.0121
18	62.91	0.000963	0.0105
20	67.49	0.000988	0.0093
25	77.59	0.001056	0.0069
30	86.22	0.001142	0.0053

hfsat :=	hgsat :=	sfsat :=	sgsat :=
3.841	227.79	0.01634	0.96441
7.73	229.73	0.03267	0.96042
11.21	231.46	0.04711	0.9571
14.37	233.02	0.06008	0.95427
17.28	234.44	0.07188	0.95183
22.49	236.97	0.09275	0.94779
27.08	239.16	0.11087	0.94456
29.78	241.11	0.1211	0.9295
33.45	242.86	0.1352	0.9273
36.84	244.46	0.1481	0.9253
42.95	247.28	0.171	0.9222
48.39	249.72	0.1911	0.9197
53.31	251.88	0.2089	0.9177
57.82	253.81	0.2251	0.916
62	255.55	0.2399	0.9145
71.33	256.07	0.2723	0.9117
79.48	259.19	0.2999	0.9097
86.78	261.85	0.3242	0.908
93.42	264.15	0.3459	0.9066
99.56	266.18	0.3656	0.9054
105.29	267.97	0.3838	0.9043
115.76	270.99	0.4164	0.9023
125.26	273.4	0.4453	0.9003
134.02	275.33	0.4714	0.8982
142.22	276.83	0.4954	0.8959
149.99	277.94	0.5178	0.8934
168.12	279.17	0.5687	0.8854
185.3	278.01	0.6156	0.8735

Following *very useful* Functions are written to find out enthalpy, entropy, sp. volume of both the sat. liquid and sat. vapor conditions, as functions of sat. temp and sat. pressures.

Note that pressure is in bar in these Functions:

$TSAT(P) := linterp(psat, tsat, P)$

$PSAT(T) := linterp(tsat, psat, T)$

$HFSATP(P) := linterp(psat, hfsat, P)$

$HFSATT(T) := linterp(tsat, hfsat, T)$

$HGSATP(P) := linterp(psat, hgsat, P)$

$HGSATT(T) := linterp(tsat, hgsat, T)$

$HFGSATP(P) := HGSATP(P) - HFSATP(P)$

$HFGSATT(T) := HGSATT(T) - HFSATT(T)$

$SFSATP(P) := linterp(psat, sfsat, P)$

$SFSATT(T) := linterp(tsat, sfsat, T)$

$SGSATP(P) := linterp(psat, sgsat, P)$

$SGSATT(T) := linterp(tsat, sgsat, T)$

$SFGSATP(P) := SGSATP(P) - SFSATP(P)$

$SFGSATT(T) := SGSATT(T) - SFSATT(T)$

$VGSATP(P) := linterp(psat, vgsat, P)$

$VGSATT(T) := linterp(tsat, vgsat, T)$

$VFSATP(P) := linterp(psat, vfsat, P)$

$VFSATT(T) := linterp(tsat, vfsat, T)$

$VFGSATP(P) := VGSATP(P) - VFSATP(P)$

$VFGSATT(T) := VGSATT(T) - VFSATT(T)$

$UGSATP(P) := HGSATP(P) - P \cdot 10^2 \cdot VGSATP(P)$

$UFSATP(P) := HFSATP(P) - P \cdot VFSATP(P) \cdot 10^2$

$UFGSATP(P) := UGSATP(P) - UFSATP(P)$

$UGSATT(T) := HGSATT(T) - PSAT(T) \cdot 10^2 \cdot VGSATT(T)$

$UFSATT(T) := HFSATT(T) - PSAT(T) \cdot 10^2 \cdot VFSATT(T)$

$UFGSATT(T) := UGSATT(T) - UFSATT(T)$

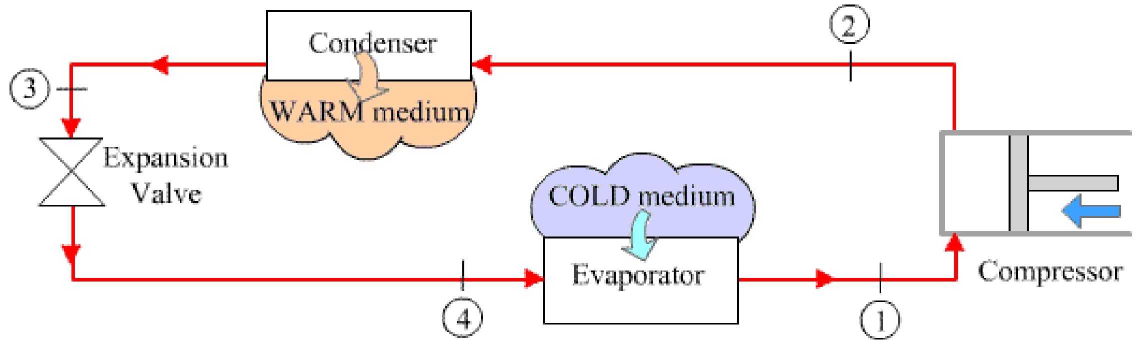
Further, following *additional functions* for finding out the quality in the two-phase region are written. They are very useful in calculations related to vapour compression refrigeration cycle, using R134a.

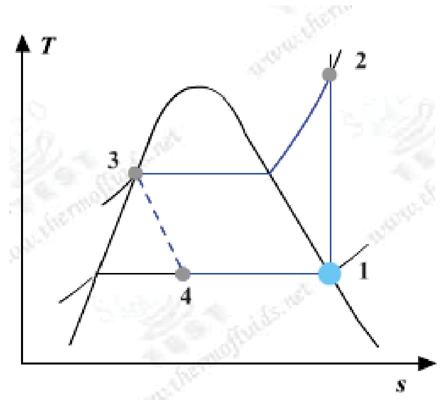
In the following program: psat = sat. pr.(bar), tsat = sat. temp (C), s = entropy (kJ/kg.C), h = enthalpy (kJ/kg), x = quality:

```
quality_Ps(psat, s) := | return "psat should be between 0.6 bar and 30 bar ! "  if  psat < 0.6 ∧ psat > 30
                       | sf ← SFSATP(psat)
                       | sfg ← SFGSATP(psat)
                       | x ← (s − sf)/sfg

quality_Ts(tsat, s) := | return "tsat should be between -36.95 C and 86.22 C ! "  if  tsat < −36.95 ∧ tsat > 86.22
                       | sf ← SFSATT(tsat)
                       | sfg ← SFGSATT(tsat)
                       | x ← (s − sf)/sfg

quality_Th(tsat, h) := | return "tsat should be between -36.95 C and 86.22 C ! "  if  tsat < −36.95 ∧ tsat > 86.22
                       | hf ← HFSATT(tsat)
                       | hfg ← HFGSATT(tsat)
                       | x ← (h − hf)/hfg

quality_Ph(psat, h) := | return "psat should be between 0.6 bar and 30 bar ! "  if  tsat < 0.6 ∧ tsat > 30
                       | hf ← HFSATP(psat)
                       | hfg ← HFGSATP(psat)
                       | x ← (h − hf)/hfg
```

Prob.4.2.2. In an ideal vapour compression refrigeration system, R134a is the refrigerant. Cold space is at -10 C. Condenser pressure is 9 bar. Find, for a flow rate of 1 kg/s: (i) the compressor power in kW, (ii) refrigeration capacity in tons, and (iii) the coeff. of performance (COP).

(b) Plot COP and refrigeration capacity vs evaporator temp (T1) as evaporator temp varies from -30 C to -10 C:





Mathcad Solution:

Essentially, we have to find the enthalpies at various state points.

And, this is done very easily with the Mathcad Functions written above:

Data:

$T1 := -10$ C $\quad P2 := 9$ bar $\quad P3 := P2 \quad T4 := T1$

$x1 := 1$ ...quality at point 1 $\quad x3 := 0$ ...quality at point 3

Calculations:

Write the relevant quantities as functions of T1 since we have to plot the graphs later:

$P1(T1) := PSAT(T1)$ i.e. $P1(T1) = 2.008$ bar

Then $P4(T1) := P1(T1)$

Enthalpies at various state points:

State point 1:

$h1(T1) := \text{enthalpy\_2phase\_Tx}(T1, x1)$

i.e. $h1(T1) = 244.514$ kJ/kg

$s1(T1) := \text{entropy\_2phase\_Tx}(T1, x1)$

i.e. $s1(T1) = 0.925$ kJ/kg.C

State point 2:

$s2(T1) := s1(T1)$ ...for isentropic compression

$h2(T1) := \text{enthalpy\_R134a\_Ps}(P2, s2(T1))$

i.e. $h2(T1) = 272.375$ kJ/kg

State point 3:

$h3 := \text{enthalpy\_2phase\_Px}(P3, x3)$

i.e. $h3 = 99.56$ **kJ/kg**

State point 4:

$T4 := T1$

$h4 := h3$ ...since expansion in the expansion valve is isenthalpic

i.e. $h4 = 99.56$ **kJ/kg**

and: $x4(T1) := \text{quality\_Th}(T1, h4)$

i.e. $x4(T1) = 0.302$ **...quality of fluid at exit of expn. valve .... Ans.**

Now, make the other calculations:

$w_{comp}(T1) := h2(T1) - h1(T1)$

i.e. $w_{comp}(T1) = 27.861$ **kJW...compressor power...Ans.**

$q_L(T1) := h1(T1) - h4$ i.e. $q_L(T1) = 144.954$ **kJ/s... refrign. capacity**

Now, 1 ton = 211 kJ/min.

Therefore, $\text{Refrign\_capacity}(T1) := \dfrac{q_L(T1) \cdot 60}{211}$

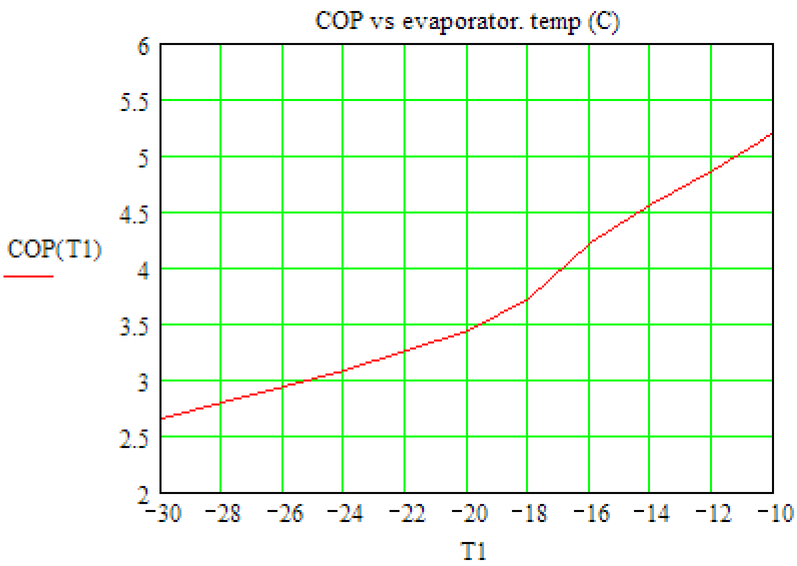
i.e. $\text{Refrign\_capacity}(T1) = 41.219$ **tons of refrigeration ... Ans.**

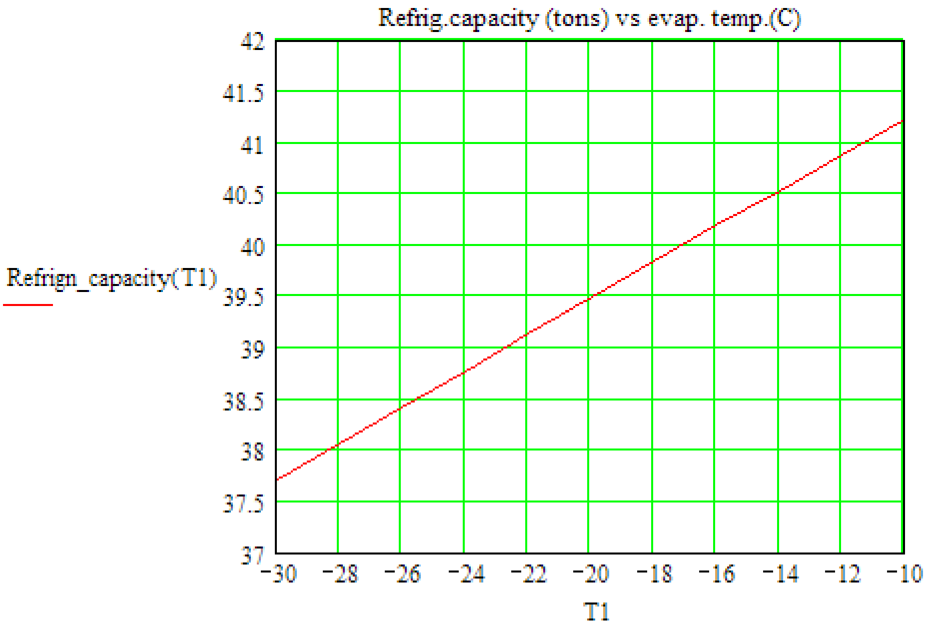
$COP(T1) := \dfrac{q_L(T1)}{w_{comp}(T1)}$ i.e. $COP(T1) = 5.203$ **coeff. of performance ... Ans.**

(b) Plot COP and refrigeration capacity vs evaporator temp (T1) as evaporator temp varies from -30 C to -10 C:

$T1 := -30, -28 .. -10$ C...define a range variable

T1 =	COP(T1) =	Refrign_capacity(T1)
-30	2.657	37.709
-28	2.792	38.066
-26	2.935	38.42
-24	3.088	38.776
-22	3.255	39.13
-20	3.431	39.481
-18	3.715	39.832
-16	4.227	40.184
-14	4.562	40.53
-12	4.864	40.875
-10	5.203	41.219




Refrig.capacity (tons) vs evap. temp.(C)
Refrign_capacity(T1)
42
41.5
41
40.5
40
39.5
39
38.5
38
37.5
37
−30
−28
−26
−24
−22
−20
−18
−16
−14
−12
−10
T1

==

Prob.4.2.3. In prob. 4.2.2, if the isentropic efficiency of compressor is 80%, determine the values for compressor work, refrigeration capacity, heat exchange in condenser and the COP.

(b) Plot compressor work, heat transfer in condenser and the COP as compressor efficiency varies from 60% to 100%.

(c) Also, plot the variation of these quantities as condenser pressure varies from 4 bar to 13 bar, for compressor efficiencies of 0.8, 0.9 and 1, other parameters remaining constant.

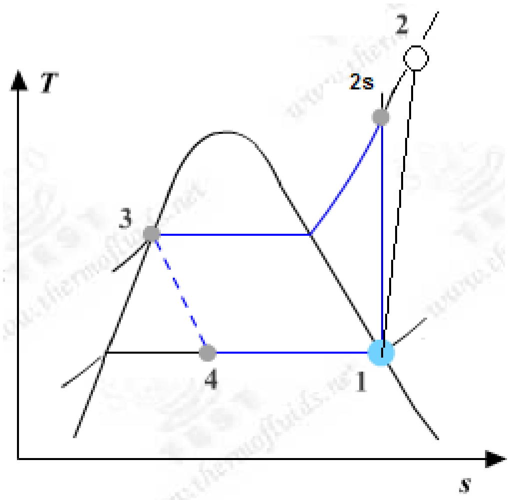


Fig.Prob.4.2.3 T-s diagram for actual vapour compression cycle

Mathcad Solution:

As in the previous case, we have to find the enthalpies at various state points.

And, this is done very easily with the Mathcad Functions written above.

Data:

$T1 := -10$ C $\quad P2 := 9$ bar $\quad P3 := P2 \quad T4 := T1 \quad \eta_{comp} := 0.8$

$x1 := 1$ ...quality at point 1 $\quad x3 := 0$ ...quality at point 3

Calculations:

Write the relevant quantities as functions of η_{comp} since we have to plot the graphs later:

$P1(T1) := PSAT(T1)$ i.e. $P1(T1) = 2.008$ bar... evaporator pressure

Then $P4(T1) := P1(T1)$

Enthalpies at various state points:

State point 1:

$h1(T1) := enthalpy\_2phase\_Tx(T1, x1)$

i.e. $h1(T1) = 244.514$ **kJ/kg**

$s1(T1) := entropy\_2phase\_Tx(T1, x1)$

i.e. $s1(T1) = 0.925$ **kJ/kg.C**

State point 2:

$s2s(T1) := s1(T1)$ **...for isentropic compression 1-2s**

$h2s(T1, P2) := enthalpy\_R134a\_Ps(P2, s2(T1))$

i.e. $h2s(T1, P2) = 272.375$ **kJ/kg... after isentropic compression**

For actual compression 1-2:

$\eta_{comp} = \frac{h2s - h1}{h2 - h1}$ ..isentropic effcy of compressor

Then: $h2 = h1 + \frac{h2s - h1}{\eta_{comp}}$

i.e. $h2(T1, P2, \eta_{comp}) := h1(T1) + \frac{h2s(T1, P2) - h1(T1)}{\eta_{comp}}$

i.e. $h2(T1, P2, \eta_{comp}) = 279.341$ **kJ/kg... after actual compression**

State point 3:

$P3 = P2$

$h3(P2) := enthalpy\_2phase\_Px(P2, x3)$

i.e. $h3(P2) = 99.56$ **kJ/kg**

State point 4:

$T4 := T1$

$h4(P2) := h3(P2)$ ...since expansion in the expansion valve is isenthalpic

i.e. $h4(P2) = 99.56$ **kJ/kg**

and: $x4(T1,P2) := \text{quality\_Th}(T1, h4(P2))$

i.e. $x4(T1,P2) = 0.302$ **...quality of fluid at exit of expn. valve .... Ans.**

Now, make the other calculations:

$$w_{comp}(T1,P2,\eta_{comp}) := h2(T1,P2,\eta_{comp}) - h1(T1)$$

i.e. $w_{comp}(T1,P2,\eta_{comp}) = 34.827$ **kW...compressor power...Ans.**

$q_L(T1,P2) := h1(T1) - h4(P2)$ i.e. $q_L(T1,P2) = 144.954$ **kJ/s... refrign. capacity**

Now, 1 ton = 211 kJ/min.

Therefore, $Refrign\_capacity(T1,P2) := \frac{q_L(T1,P2)\cdot 60}{211}$

i.e. $Refrign\_capacity(T1,P2) = 41.219$ **tons of refrigeration ... Ans.**

$q_H(T1,P2,\eta_{comp}) := h2(T1,P2,\eta_{comp}) - h3(P2)$

i.e. $q_H(T1,P2,\eta_{comp}) = 179.781$ **kW..heat transferred in condenser**

$$COP(T1,P2,\eta_{comp}) := \frac{q_L(T1,P2)}{w_{comp}(T1,P2,\eta_{comp})}$$

i.e. $COP(T1,P2,\eta_{comp}) = 4.162$ **coeff. of performance ... Ans.**

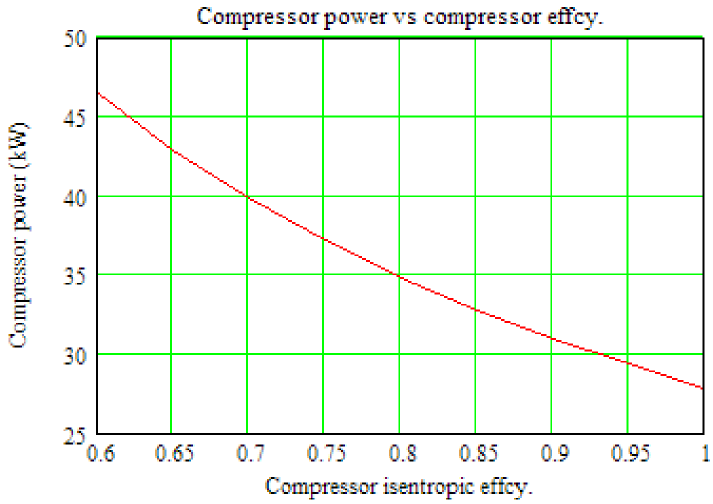
Compare these results with those for the previous problem, where compressor effcy. was 100%.

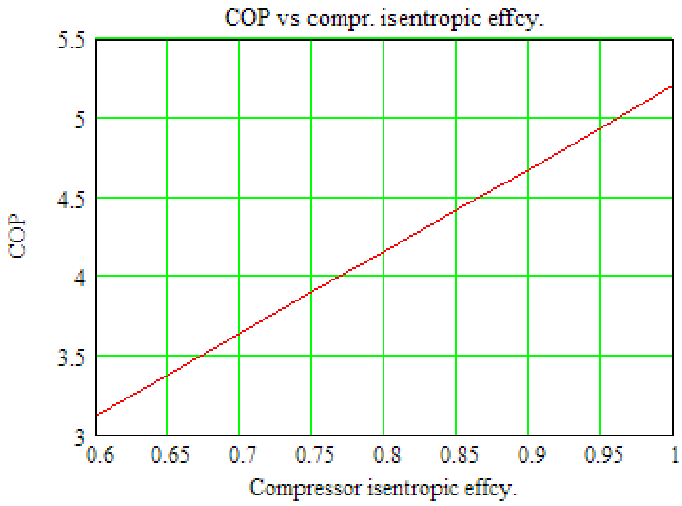
(b) Plot compressor work, heat tr. in condenser, and the COP as compressor efficiency varies from 60% to 100%.

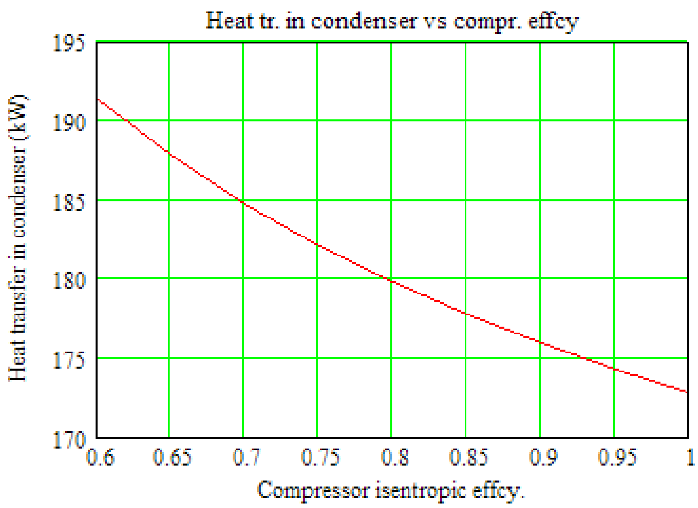
Note that the Mathcad Functions defined in this problem for q_L, q_H, w_{comp}, COP etc are very versatile, and we can plot graphs for variation of any one or more of them together. This is illustrated below:

$\eta_{comp} := 0.6, 0.65..1$ ...define a range variable

η_{comp} =	$w_{comp}(T1,P2,\eta_{comp})$	$COP(T1,P2,\eta_{comp})$	$q_H(T1,P2,\eta_{comp})$
0.6	46.436	3.122	191.389
0.65	42.864	3.382	187.817
0.7	39.802	3.642	184.756
0.75	37.149	3.902	182.102
0.8	34.827	4.162	179.781
0.85	32.778	4.422	177.732
0.9	30.957	4.682	175.911
0.95	29.328	4.943	174.282
1	27.861	5.203	172.815


Compressor power vs compressor effcy.
Compressor power (kW)
50
45
40
35
30
25
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
Compressor isentropic effcy.


COP vs compr. isentropic effcy.
COP
5.5
5
4.5
4
3.5
3
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
Compressor isentropic effcy.


Heat tr. in condenser vs compr. effcy
Heat transfer in condenser (kW)
195
190
185
180
175
170
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
Compressor isentropic effcy.

(c) Also, plot the variation of these quantities as condenser pressure varies from 4 bar to 13 bar, for compressor efficiencies of 0.8, 0.9 and 1, other parameters remaining constant.

We have:

Data:

$T1 := -10$ C $\quad P2 := 9$ bar $\quad P3 := P2 \quad T4 := T1 \quad \eta_{comp} := 0.8$

$x1 := 1$ ...quality at point 1 $\quad x3 := 0$ ...quality at point 3

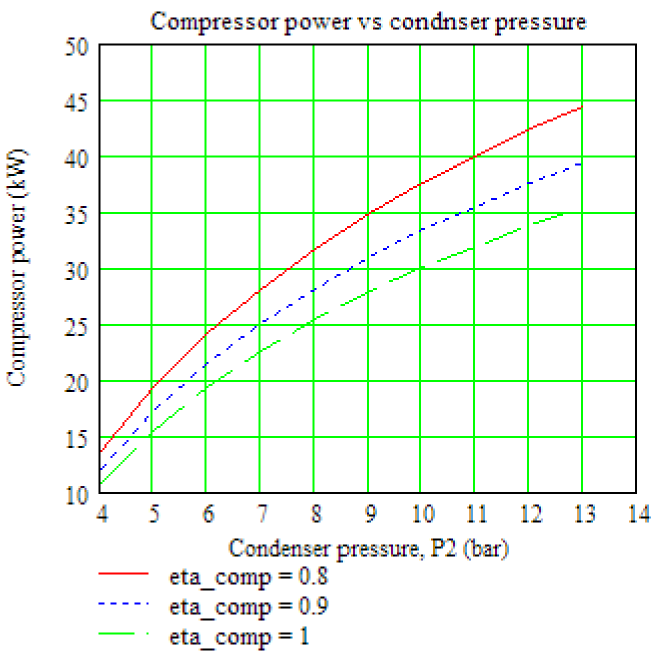
Now:

$P2 := 4, 5 .. 13$ ....define a range variable

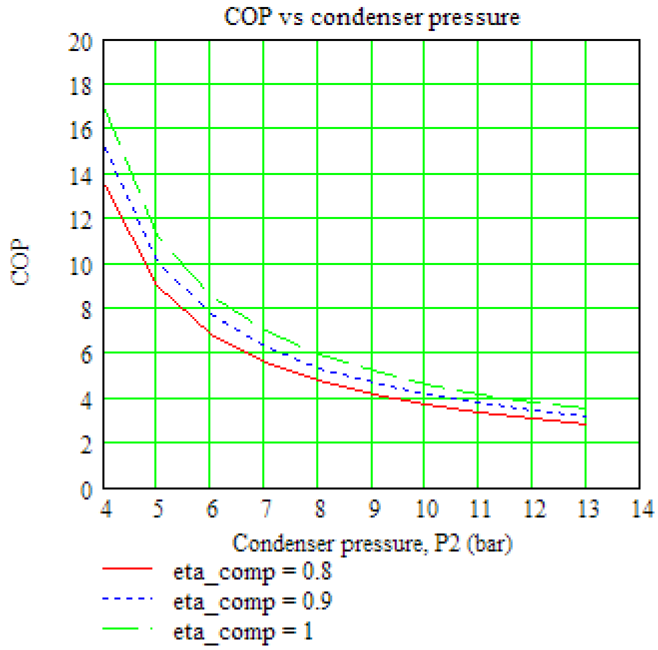
And, vary η_{comp} as 0.8, 0.9 and 1:

We get:

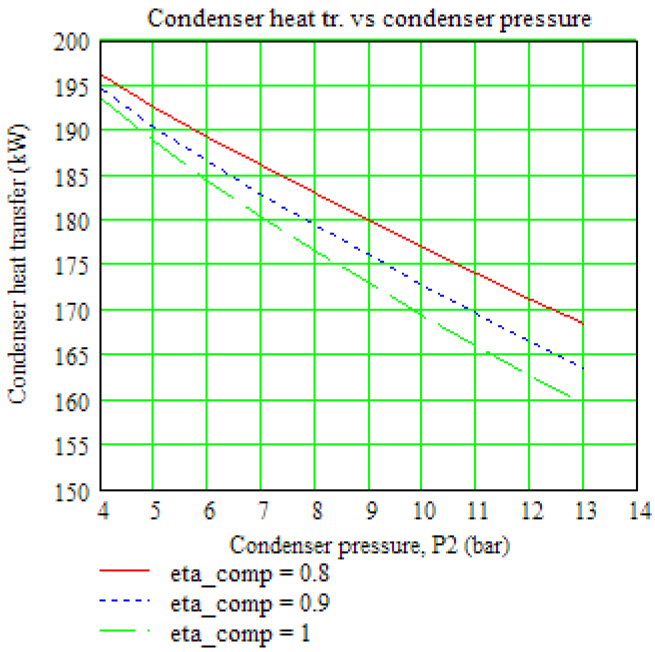
P2 =	$w_{comp}(T1, P2, 0.8)$	$w_{comp}(T1, P2, 0.9)$	$w_{comp}(T1, P2, 1)$
4	13.585	12.076	10.868
5	19.362	17.211	15.49
6	24.153	21.469	19.322
7	28.209	25.075	22.567
8	31.74	28.213	25.392
9	34.827	30.957	27.861
10	37.617	33.437	30.093
11	39.935	35.498	31.948
12	42.387	37.677	33.909
13	44.379	39.448	35.503



P2 =	COP(T1, P2, 0.8)	COP(T1, P2, 0.9)	COP(T1, P2, 1)
4	13.435	15.114	16.793
5	8.945	10.063	11.181
6	6.833	7.687	8.541
7	5.592	6.291	6.99
8	4.76	5.355	5.95
9	4.162	4.682	5.203
10	3.701	4.164	4.626
11	3.355	3.775	4.194
12	3.038	3.417	3.797
13	2.794	3.144	3.493



P2 =	$q_H(T1,P2,0.8)$	$q_H(T1,P2,0.9)$	$q_H(T1,P2,1)$ =
4	196.099	194.589	193.382
5	192.546	190.394	188.673
6	189.187	186.503	184.356
7	185.943	182.808	180.301
8	182.834	179.307	176.486
9	179.781	175.911	172.815
10	176.84	172.661	169.317
11	173.923	169.486	165.937
12	171.14	166.431	162.663
13	168.382	163.451	159.507



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Prob.4.2.4. Arefrigerator uses R134a as working fluid and operates on the ideal vapour compression refrigeration cycle except for the compression process. The refrigerant enters the evaporator at 129 kPa with a quality of 30% and leaves the compressor at 60 C. If the compressor consumes 450 W of power, determine: (i) mass flow rate of refrigerant, (ii) condenser pressure, and (iii) the COP. [Ref: 1]

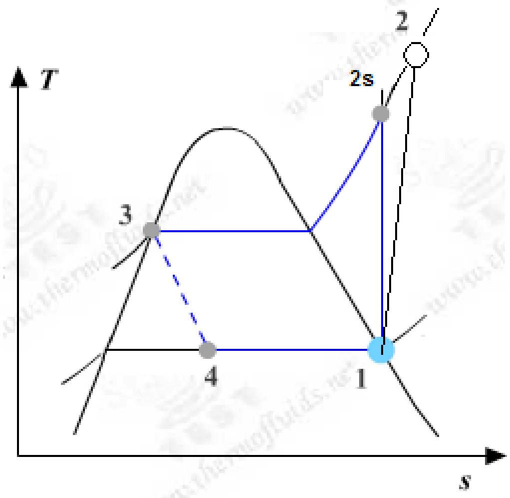


Fig.Prob.4.2.4 T-s diagram for actual vapour compression cycle

Mathcad Solution:

Essentially, we have to find the enthalpies at various state points.

And, this is done very easily with the Mathcad Functions written above:

Data:

$P1 := 1.2$ bar $T2 := 60$ C $P4 := P1$ $x4 := 0.3$...quality at point 4, i.e. entry to evaporator

$x1 := 1$ ...quality at point 1 $x3 := 0$ $P_{comp} := 0.450$ kW.... compr. power

Calculations:

$h4 := enthalpy\_2phase\_Px(P4, x4)$ i.e. $h4 = 86.834$ kJ/kg

Now: $h3 := h4$ ....for isenthalpic process 3-4 in expn. valve

To find P3: Use 'Solve block' of Mathcad:

$P3 := 6$ bar....guess value

Given

$h3 = enthalpy\_2phase\_Px(P3, x3)$

$P3 := Find(P3)$

$P3 = 7.008$ **bar... pressure in condenser = compressor exit pressure... Ans.**

And: $P2 := P3$

$s1 := entropy\_2phase\_Px(P1, x1)$ i.e. $s1 = 0.948$

$h1 := enthalpy\_2phase\_Px(P1, x1)$ i.e. $h1 = 236.97$ **kJ/kg**

Then: $s2s := s1$ ...for isentropic compression

$h2s := enthalpy\_R134a\_Ps(P2, s2s)$ i.e. $h2s = 274.066$ kJ/kg....after isentr. comprn.

Recollect:

```
enthalpy_R134a(P,T) := | return "P should be between 0.6 bar and 16 bar"  if P < 0.6 ∨ P > 16
                       | return "T should be between -37.07 C and 200 C"  if T < −37.07 ∨ T > 200
                       | tsat ← TSAT(P)
                       | h ← h_and_s_SuperheatR134a(P,T)_{0,0}  if T ≥ tsat
                       | (return "State point in two phase region--- use 2 phase Functions")  otherwise
```

Therefore: $h2 := \text{enthalpy\_R134a}(P2, T2)$

i.e. $h2 = 296.676$ **kJ/kg**

Therefore: compressor isentropic efficiency:

$\eta_{comp} := \dfrac{h2s - h1}{h2 - h1}$ i.e. $\eta_{comp} = 0.621$ **...compressor isentr. effcy.**

$w_{comp} := h2 - h1$ i.e. $w_{comp} = 59.706$ **kJ/kg**

Therefore: mass flow rate of R134a:

$$mass := \frac{P_{comp}}{w_{comp}} \quad \text{kg/s}$$

i.e. $mass = 7.537 \times 10^{-3}$ **kg/s....Ans.**

$q_L := h1 - h4$ i.e. $q_L = 150.136$ **kJ/kg.... refrig. effect**

Therefore:

$$COP := \frac{q_L}{w_{comp}}$$

i.e. $COP = 2.515$ **..COP....Ans.**

==

Prob.4.2.5. In a vapour compression cycle using R134a as refrigerant, evaporator pressure is 1.2 bar, condenser pressure is 8 bar,and compressor isentropic effcy. is 65%. If there is 5 C superheating at entry to compressor and 5 C subcooling before entry to expansion valve, calculate the values for refrig. effect (kJ/kg), compressor work (kJ/kg) and COP.

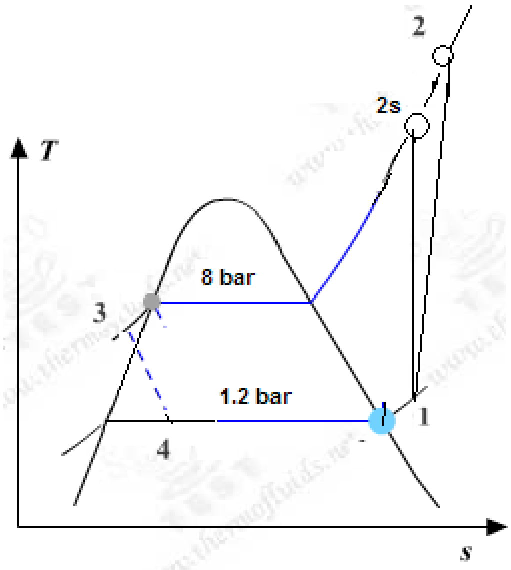


Fig.Prob.4.2.5 T-s diagram for actual vapour compression cycle, with superheating and subcooling

Mathcad Solution:

Finding out the enthalpies at various points is done very easily with the Mathcad Functions written above:

Data:

$P1 := 1.2$ bar $P2 := 8$ bar $P3 := P2$ $P4 := P1$ $\eta_{comp} := 0.65$

$x3 := 0$ ..since h3 is calculated as sat. liq. enthalpy at T3

To calculate enthalpies at various state points:

State point 1:

$T1 := TSAT(P1) + 5$ i.e. $T1 = -17.32$ C

$h1 := enthalpy\_R134a(P1, T1)$ i.e. $h1 = 238.002$ kJ/kg

$s1 := entropy\_R134a(P1, T1)$ i.e. $s1 = 0.953$ kJ/kg.C

State point 2:

$s2s := s1$ ...for isentropic process 1-2s

$h2s := enthalpy\_R134a\_Ps(P, s)$ i.e. $h2s = 272.394$ kJ/kg

Therefore:

$$h2 := h1 + \frac{h2s - h1}{\eta_{comp}}$$ i.e. $h2 = 290.913$ kJ/kg

State point 3:

$T3 := TSAT(P3) - 5$ i.e. $T3 = 26.33$ C

Therefore: $h3 := HFSATT(T3)$ i.e. $h3 = 86.226$ kJ/kg

State point 4:

$h4 := h3$ ....for isenthalpic expn. in the expansion valve

Then, we have:

Refrig. effect:

$q_L := h1 - h4$ i.e. $q_L = 151.776$ **kJ/kg....Ans.**

Compressor work:

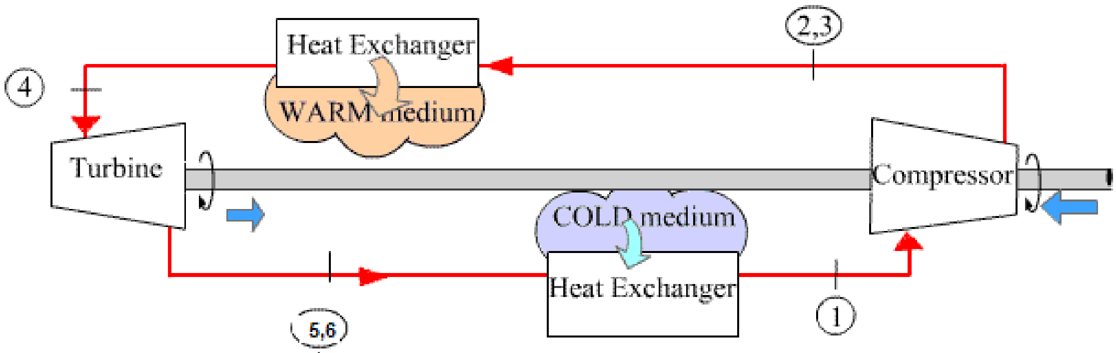
$w_{comp} := h2 - h1$ i.e. $w_{comp} = 52.91$ kJ/kg....Ans.

Coeff. of Performance:

$COP := \frac{q_L}{w_{comp}}$ i.e. $COP = 2.869$ ....COP...Ans.

==

Prob.4.2.6. Write Mathcad Functions for reversed Brayton cycle refrigeration cycle (i.e. Air cycle refrigeration or Bell – Coleman cycle) to find out COP etc:



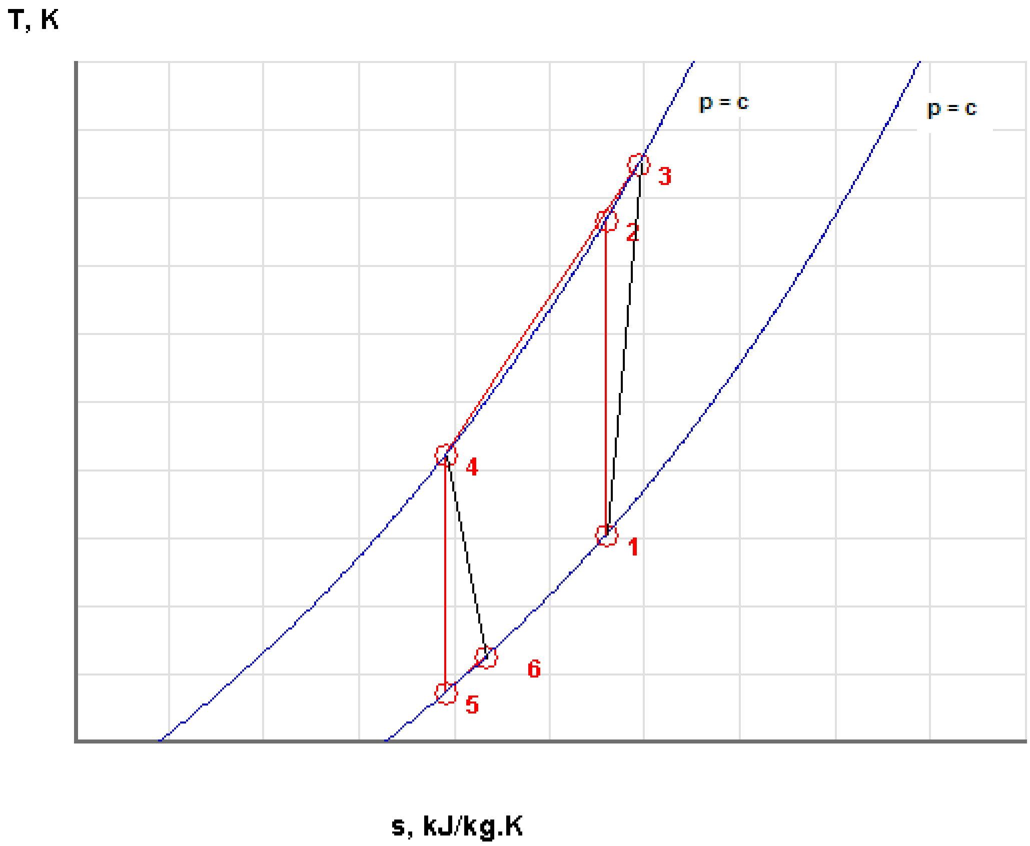


Fig.Prob.4.2.6 Reversed Brayton cycle refirigeration cycle and its T-s diagram

Mathcad Solution:

We write Mathcad Function for air as working substance with constant sp. heats, including the isentropic efficiencies of compressor and turbine:

The Mathcad Program is shown below:

In this program the LHS gives the name of the Function, and the inputs inside brackets; (here, it is shown as the first line, and rest of the program is shown below it, to conserve space).

Inputs are:

P1, P2 …compressor inlet and exit pressures, in bar

T1, T4…compressor and turbine inlet temps, in K

η_{comp}, η_{turb} …. compressor and turbine isentropic efficiencies

γ, cp … ratio of sp. heats (cp/cv) and sp. heat at const. pressure (kJ/kg.K), for air

$$\text{Reversed\_Brayton\_Air}(P1, P2, T1, T4, \eta_{comp}, \eta_{turb}, \gamma, cp) :=$$

$$T2 \leftarrow T1 \cdot \left(\frac{P2}{P1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T3 \leftarrow T1 + \frac{(T2 - T1)}{\eta_{comp}}$$

$$T5 \leftarrow \frac{T4}{\left(\frac{P2}{P1}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$T6 \leftarrow T4 - \eta_{turb} \cdot (T4 - T5)$$

$$w_{comp} \leftarrow cp \cdot (T3 - T1)$$

$$w_{turb} \leftarrow cp \cdot (T4 - T6)$$

$$w_{net} \leftarrow w_{comp} - w_{turb}$$

$$q_{in} \leftarrow cp \cdot (T1 - T6)$$

$$q_{out} \leftarrow cp \cdot (T3 - T4)$$

$$COP \leftarrow \frac{q_{in}}{w_{net}}$$

$$R \leftarrow 0.287$$

$$spvol1 \leftarrow \frac{R \cdot T1}{P2 \cdot 10^2}$$

$$\begin{pmatrix} \text{"T3(K)"} & \text{"T6(K)"} & \text{"w\_comp(kJ/kg)"} & \text{"w\_turb(kJ/kg)"} & \text{"q\_in(kJ/kg)"} & \text{"COP"} & \text{"spvol\_1(m\^3/kg)"} \\ T3 & T6 & w_{comp} & w_{turb} & q_{in} & COP & spvol1 \end{pmatrix}$$

Outputs are:

T3 (K) …. exit temp of compressor after actual compression

T6 (K) …. exit temp of turbine after actual expansion

w_comp …. compressor work in kJ/kg

w_turb …, turbine work in kJ/kg

w_net …. net work requirement for refrigerator = (w_comp – w_turb), kJ/kg q_in … refrigeration effect, kJ/kg

COP … coeff. of performance = (q_in / w_net)

spvol1 … specific volume of air at compressor inlet, m^3/kg. (This is given since many times, while solving problems, volume of air handled by compressor is required to be calculated.)

Now, let us solve a problem to illustrate the use of this Function:

==

Prob.4.2.7. Air enters the compressor of an ideal Brayton cycle refrigerator at 100 kPa, 270 K. Compressor exit pressure is 300 kPa. Temp at turbine inlet is 310 K. Determine: (i) net work input in kJ/kg (ii) refrigeration capacity in kJ/kg, and (iii) the COP

(b) Plot the above quantities as compressor exit pressure varies from 2 to 6.

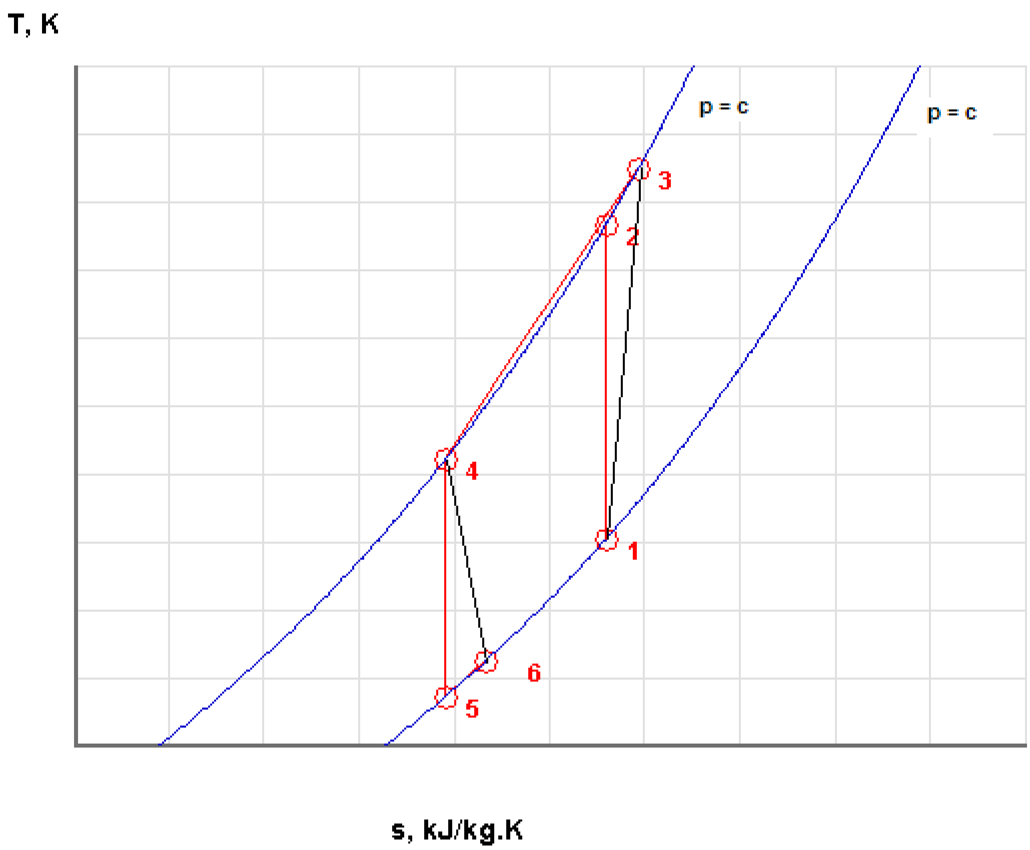


Fig.Prob.4.2.7 Brayton cycle refrigeration – T-s diagram

Mathcad Solution:

The problem is solved easily by using the Mathcad Function written above.

Data:

$cp := 1.005$ kJ/kg.K $\gamma := 1.4$ ...ratio of sp. heats

$P1 := 1$ bar $P2 := 3$ bar $\eta_{comp} := 1$ $\eta_{turb} := 1$

$T1 := 270$ K.... compressor inlet temp

$T4 := 310$ K.... turbine inlet temp

Now, use the Function written above:

$Reversed\_Brayton\_Air(P1, P2, T1, T4, \eta_{comp}, \eta_{turb}, \gamma, cp) =$

"T3(K)"	"T6(K)"	"w_comp(kJ/kg)"	"w_turb(kJ/kg)"	"q_in(kJ/kg)"	"COP"	"spvol_1(m^3/kg)"
369.559	226.486	100.057	83.932	43.732	2.712	0.258

Therefore:

Net work input = (w_comp - w_turb)

i.e. $w_{net} := 100.057 - 83.932$

i.e. $w_{net} = 16.125$ **kJ/kg .... Ans.**

Refrigeration capacity, q_in:

We see that: $q_{in} = 43.732$ **kJ/kg .... Ans.**

Coeff. of performance, COP:

We see that: $COP = 2.712$ **....Ans.**

To plot w_{net}, q_{in} and COP against P2:

$P2 := 2, 2.5 .. 6$ ...define a range variable

Also, define:

$w_{net}(P2) := Reversed\_Brayton\_Air(P1, P2, T1, T4, \eta_{comp}, \eta_{turb}, \gamma, cp)_{1,2} - Reversed\_Brayton\_Air(P1, P2, T1, T4, \eta_{comp}, \eta_{turb}, \gamma, cp)_{1,3}$

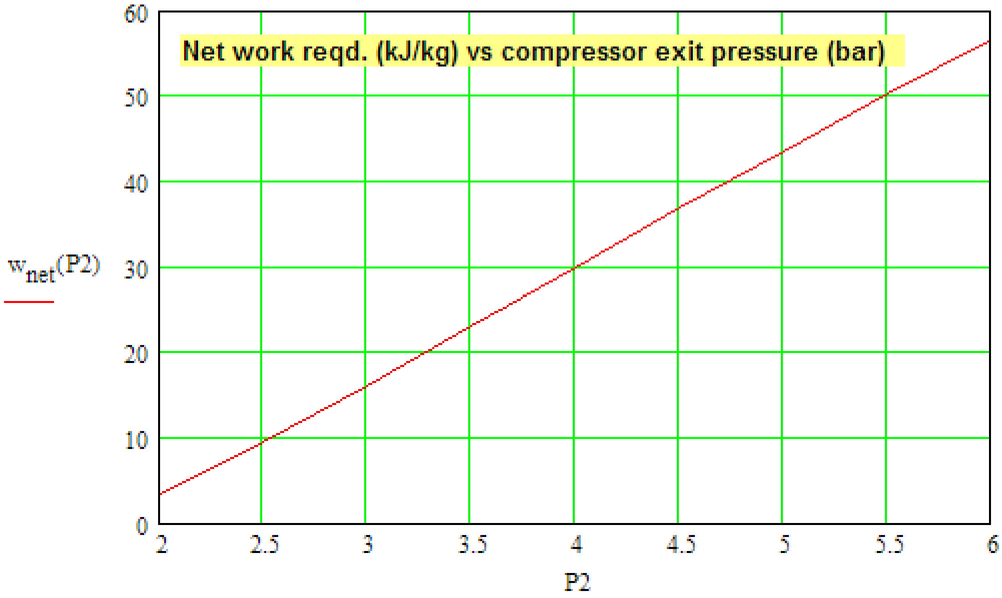
$q_{in}(P2) := Reversed\_Brayton\_Air(P1, P2, T1, T4, \eta_{comp}, \eta_{turb}, \gamma, cp)_{1,4}$

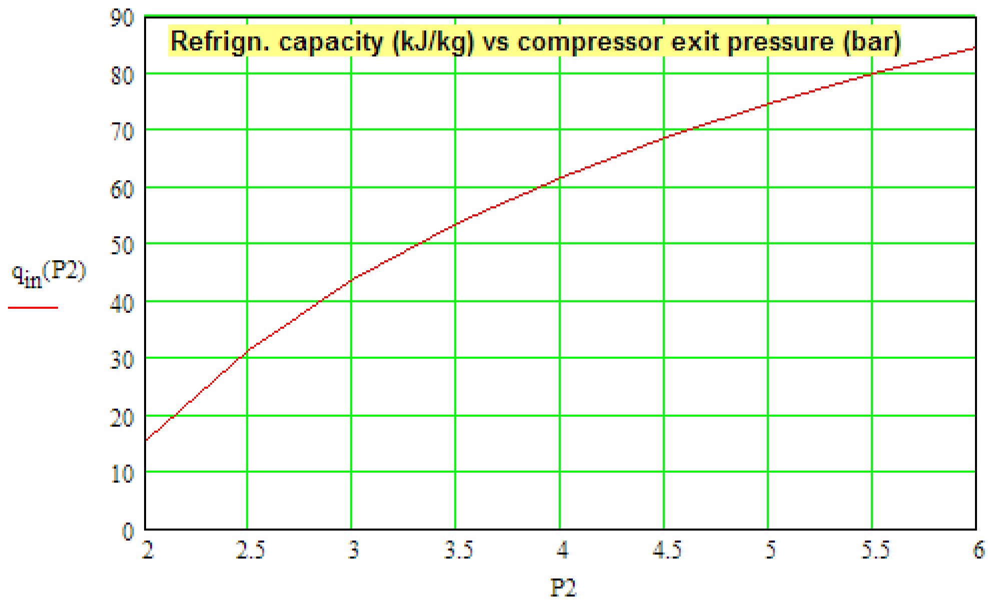
$COP(P2) := Reversed\_Brayton\_Air(P1, P2, T1, T4, \eta_{comp}, \eta_{turb}, \gamma, cp)_{1,5}$

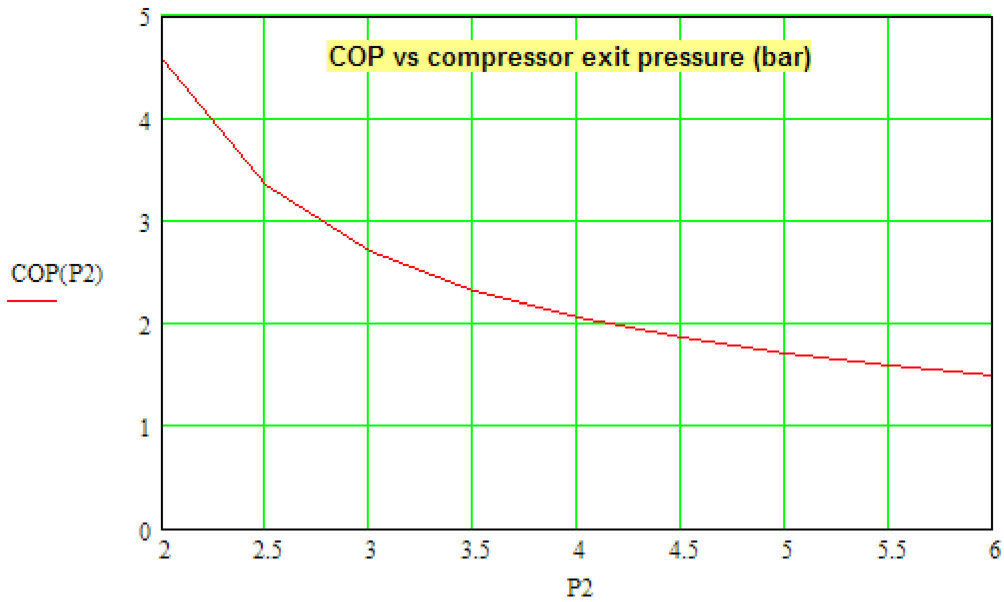
Then, we have:

P2 =	$w_{net}(P2)$ =	$q_{in}(P2)$ =	COP(P2) =
2	3.455	15.775	4.566
2.5	9.445	31.56	3.342
3	16.126	43.732	2.712
3.5	23.042	53.539	2.324
4	29.982	61.692	2.058
4.5	36.844	68.63	1.863
5	43.577	74.642	1.713
5.5	50.157	79.926	1.594
6	56.574	84.627	1.496

Now, plot the results:







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Prob.4.2.8. In Prob.4.2.7, if the compressor and turbine isentropic efficiencies are 80% and 90% respectively, determine the new values for net work, refrigeration effect and COP.

(b) Plot the variation of these quantities for equal compressor and turbine efficiencies of 80%, 90% and 95%.

Mathcad Solution:

Data:

$cp := 1.005$ kJ/kg.K $\gamma := 1.4$ ...ratio of sp. heats

$P1 := 1$ bar $P2 := 3$ bar $\eta_{comp} := 0.8$ $\eta_{turb} := 0.9$

$T1 := 270$ K.... compressor inlet temp

$T4 := 310$ K.... turbine inlet temp

Now, use the Function written above:

$\text{Reversed\_Brayton\_Air}(P1, P2, T1, T4, \eta_{comp}, \eta_{turb}, \gamma, cp) =$

"T3(K)"	"T6(K)"	"w_comp(kJ/kg)"	"w_turb(kJ/kg)"	"q_in(kJ/kg)"	"COP"	"spvol_1(m^3/kg)"
394.449	234.837	125.071	75.538	35.338	0.713	0.258

Therefore:

Net work input = (w_comp - w_turb)

i.e. $w_{net} := 125.071 - 75.538$

i.e. $w_{net} = 49.533$ **kJ/kg .... Ans.**

Refrigeration capacity, q_in:

We see that: $q_{in} = 35.338$ **kJ/kg .... Ans.**

Coeff. of performance, COP:

We see that: $COP = 0.713$ **....Ans.**

To plot w_{net}, q_{in} and COP against P2 for different values of compressor and turbine efficiencies:

$P2 := 2, 2.5 .. 6$ ...define a range variable

Also, define the above quantities as functions of P2, η_{comp} and η_{turb}, so that we can easily generate the plots:

$$w_{net}(P2, \eta_{comp}, \eta_{turb}) := \text{Reversed\_Brayton\_Air}(P1, P2, T1, T4, \eta_{comp}, \eta_{turb}, \gamma, cp)_{1,2} - \text{Reversed\_Brayton\_Air}(P1, P2, T1, T4, \eta_{comp}, \eta_{turb}, \gamma, cp)_{1,3}$$

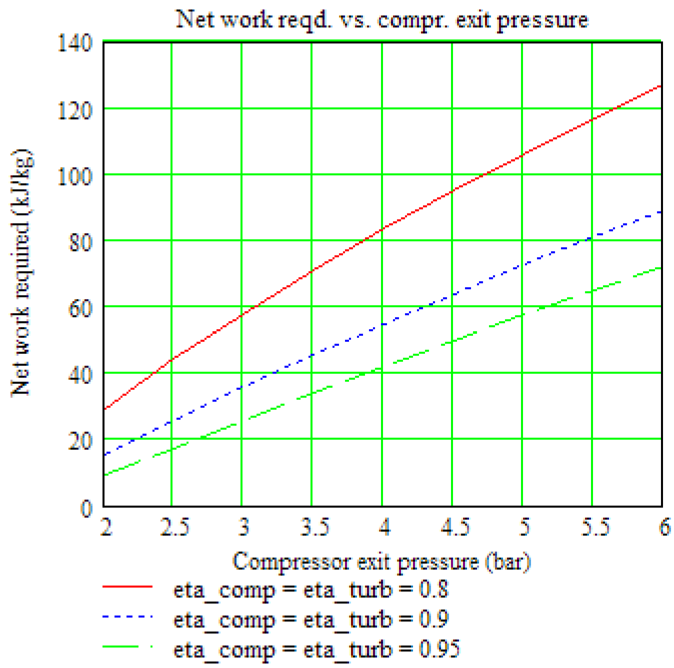
$$q_{in}(P2,\eta_{comp},\eta_{turb}) := \text{Reversed\_Brayton\_Air}(P1,P2,T1,T4,\eta_{comp},\eta_{turb},\gamma,cp)_{1,4}$$

$$COP(P2,\eta_{comp},\eta_{turb}) := \text{Reversed\_Brayton\_Air}(P1,P2,T1,T4,\eta_{comp},\eta_{turb},\gamma,cp)_{1,5}$$

Then, for net work, we get:

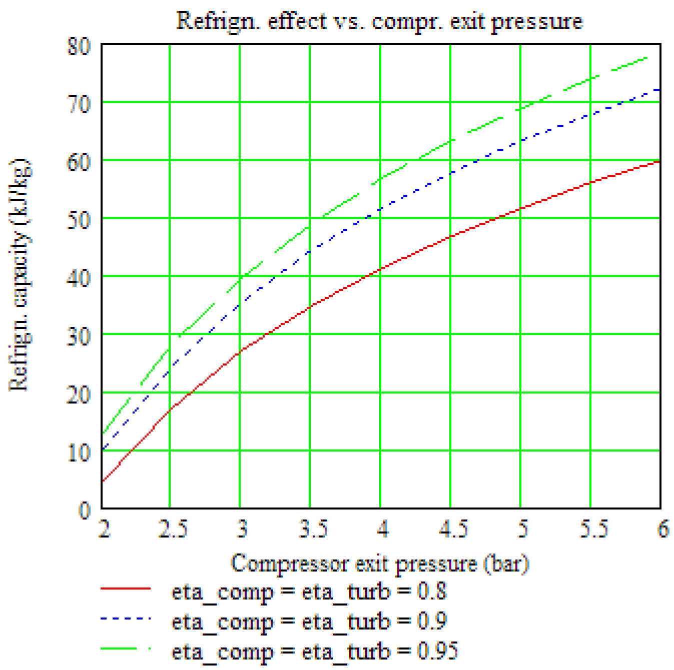
P2 =	$w_{net}(P2,0.8,0.8)$	$w_{net}(P2,0.9,0.9)$	$w_{net}(P2,0.95,0.95)$
2	29.507	15.656	9.381
2.5	44.098	25.644	17.307
3	57.926	35.636	25.588
3.5	70.984	45.391	33.875
4	83.329	54.824	42.018
4.5	95.029	63.914	49.953
5	106.151	72.664	57.657
5.5	116.753	81.09	65.126
6	126.889	89.212	72.362

And:



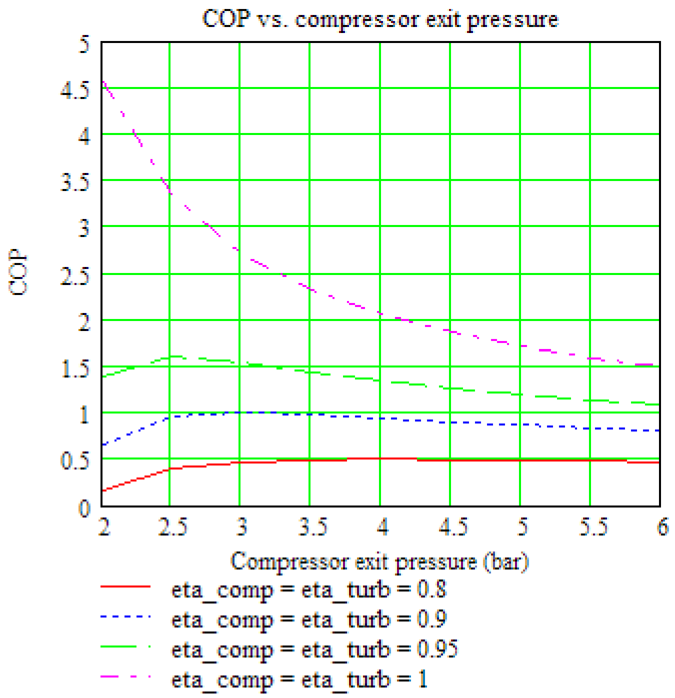
And, for refrigeration effect, we get:

P2 =	$q_{in}(P2, 0.8, 0.8)$	$q_{in}(P2, 0.9, 0.9)$	$q_{in}(P2, 0.95, 0.95)$
2	4.58	10.177	12.976
2.5	17.208	24.384	27.972
3	26.945	35.338	39.535
3.5	34.791	44.165	48.852
4	41.314	51.503	56.598
4.5	46.864	57.747	63.189
5	51.674	63.158	68.9
5.5	55.901	67.914	73.92
6	59.661	72.144	78.385



And, for COP, we get:

P2 =	COP(P2, 0.8, 0.8)	COP(P2, 0.9, 0.9)	COP(P2, 0.95, 0.95)	COP(P2, 1, 1)
2	0.155	0.65	1.383	4.566
2.5	0.39	0.951	1.616	3.342
3	0.465	0.992	1.545	2.712
3.5	0.49	0.973	1.442	2.324
4	0.496	0.939	1.347	2.058
4.5	0.493	0.904	1.265	1.863
5	0.487	0.869	1.195	1.713
5.5	0.479	0.838	1.135	1.594
6	0.47	0.809	1.083	1.496



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Prob.4.2.9. A dense air refrigeration machine operating on Bell-Coleman cycle operates between 3.4 bar and 17 bar. Temp. of air after the cooler is 15 C and after the refrigerator is 6 C. For a refrigeration capacity of 6 Tons, find: (i) Temp after compression and expansion, (ii) Air circulation requied per sec., (iii) Work of compression and expansion, and (iv) COP … [M.U.]

(b) Plot the variation of mass of air, compressor work and COP for a refrign. capacity of 6 tons, as compressor and turbine efficiencies vary together from 70% to 100%

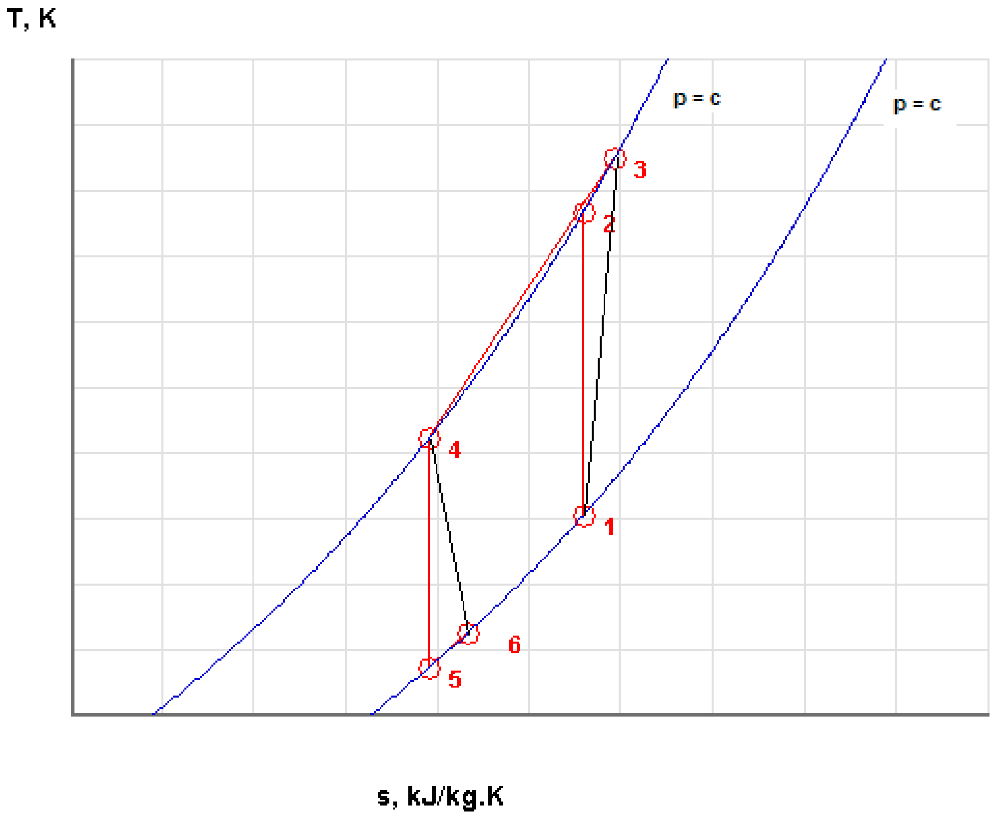


Fig.Prob.4.2.9 T-s diagram of Reversed Brayton cycle refrigerator

Mathcad Solution:

Data:

$cp := 1.005$ kJ/kg.K $\gamma := 1.4$ ...ratio of sp. heats

$P1 := 3.4$ bar $P2 := 17$ bar $\eta_{comp} := 1$ $\eta_{turb} := 1$

$T1 := 279$ K.... compressor inlet temp

$T4 := 288$ K.... turbine inlet temp

Now, use the Function written above:

$Reversed\_Brayton\_Air(P1, P2, T1, T4, \eta_{comp}, \eta_{turb}, \gamma, cp) =$

"T3(K)"	"T6(K)"	"w_comp(kJ/kg)"	"w_turb(kJ/kg)"	"q_in(kJ/kg)"	"COP"	"spvol_1(m^3/kg)"
441.886	181.839	163.7	106.692	97.647	1.713	0.047

Therefore:

$T3 := 441.886$ **K....temp after compresson .... Ans.**

$T6 := 181.839$ **K....temp after expansion...Ans.**

$q_{in} := 97.647$ **kJ/kg .... refrign. capacity...Ans.**

$w_{comp} := 163.7$ kJ/kg .... compressor work

$w_{turb} := 106.692$ kJ/kg ... turbine work

Air circulation rate required:

Refrigeration capacity, q_in:

We see that:

1 Ton is equivalent to 211 kJ/min.

Therefore, mass flow rate of air for 6 Tons capacity:

$$mass_{air} := \frac{6 \cdot 211}{60 q_{in}} \quad \text{kg/s}$$

i.e. $mass_{air} = 0.216$ **kg/s ... Ans.**

Compressor power:

$W_C := w_{comp} \cdot mass_{air}$

i.e. $W_C = 35.373$ **kW .... actual compressor power ... Ans.**

Turbine power:

$W_T := w_{turb} \cdot mass_{air}$

i.e. $W_T = 23.054$ **kW .... actual turbine power ... Ans.**

Net work input = (w_comp - w_turb)*mass_air

i.e. $W_{net} := (163.7 - 106.692) \cdot mass_{air}$

i.e. $W_{net} = 12.319$ **kW .... net work required..Ans.**

Coeff. of performance, COP:

We see that: $COP = 1.713$ **....Ans.**

(b) Plot the variation of mass of air, compressor work and COP for a refrign. capacity of 6 tons, as compressor and turbine efficiencies vary together from 70% to 100%:

Now, define:

$q_{in}(\eta_{comp}, \eta_{turb}) := Reversed\_Brayton\_Air(P1, P2, T1, T4, \eta_{comp}, \eta_{turb}, \gamma, cp)_{1,4}$

$mass_{air}(\eta_{comp}, \eta_{turb}) := \dfrac{6 \cdot 211}{60\, q_{in}(\eta_{comp}, \eta_{turb})}$

$W_C(\eta_{comp}, \eta_{turb}) := Reversed\_Brayton\_Air(P1, P2, T1, T4, \eta_{comp}, \eta_{turb}, \gamma, cp)_{1,2} \cdot mass_{air}(\eta_{comp}, \eta_{turb})$

$COP(\eta_{comp}, \eta_{turb}) := Reversed\_Brayton\_Air(P1, P2, T1, T4, \eta_{comp}, \eta_{turb}, \gamma, cp)_{1,5}$

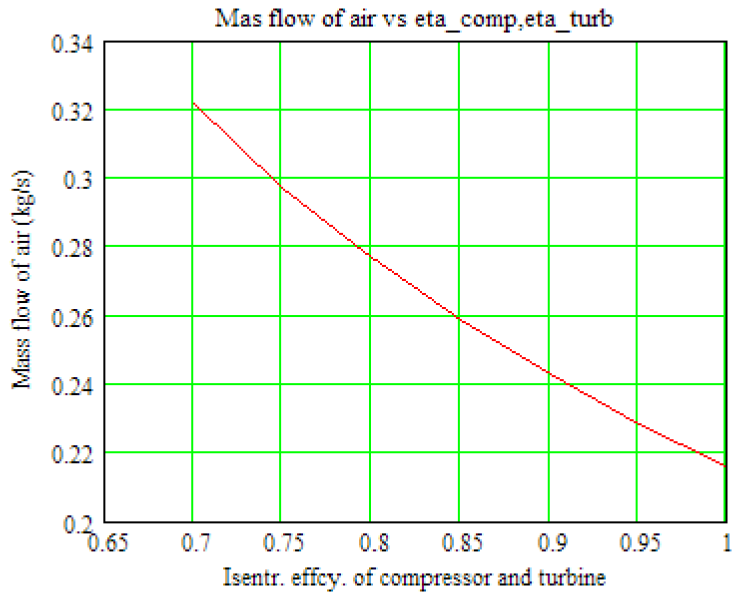
$\eta_{comp} := 0.7, 0.75 .. 1$ ...define a range variable

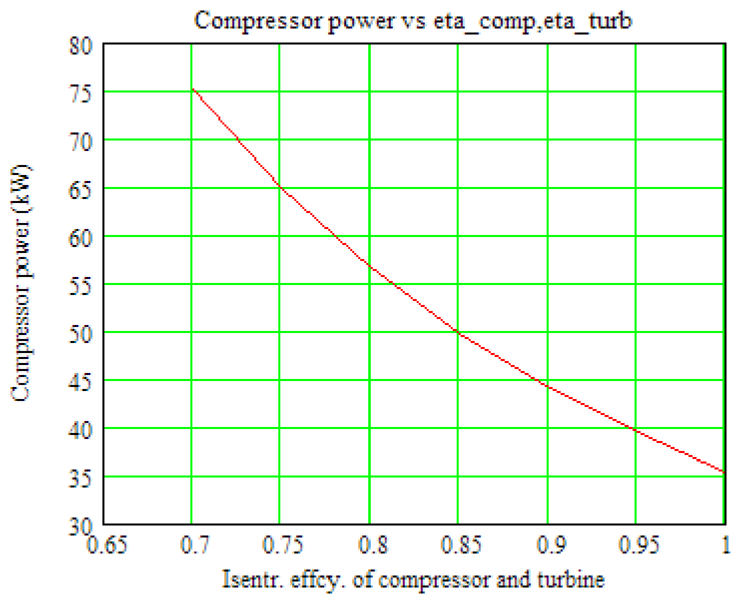
$\eta_{turb} = \eta_{comp}$

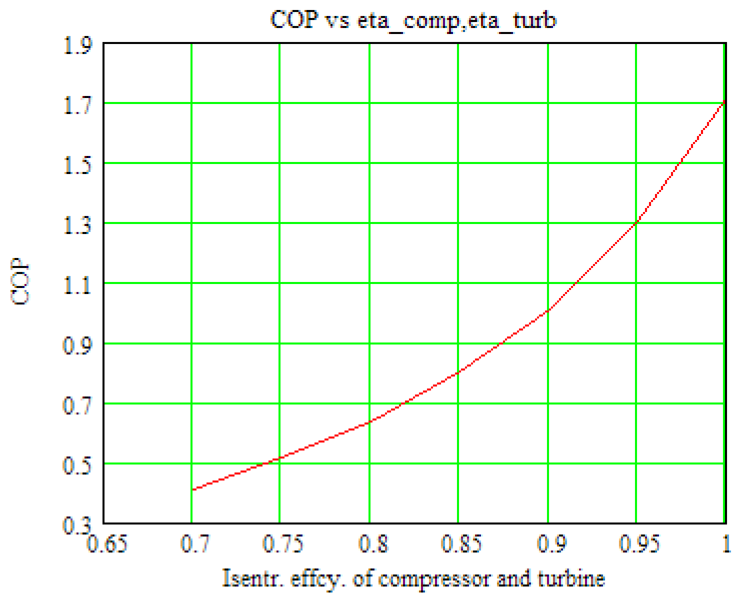
Then:

η_{comp} =	$mass_{air}(\eta_{comp}, \eta_{comp})$	$W_C(\eta_{comp}, \eta_{comp})$	$COP(\eta_{comp}, \eta_{comp})$
0.7	0.321	75.174	0.412
0.75	0.297	64.889	0.513
0.8	0.277	56.581	0.64
0.85	0.258	49.773	0.801
0.9	0.243	44.125	1.013
0.95	0.229	39.387	1.301
1	0.216	35.373	1.713

Now, plot the graphs:







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4.3 Problems solved with DUPREX (free software from DUPONT) [8]:

About the software:

'DUPONT Refrigeration Expert' or DUPREX is a free software supplied by M/s DUPONT.

(Ref: http://www2.dupont.com/Refrigerants/en_US/products/DUPREX/DUPREX_registration.html)

It is a very versatile software, extremely useful to solve many practical problems. It handles a large number of refrigerants produced by DUPONT.

It has a choice of four cycles: (i) single stage vapour compression cycle (ii) single stage vapour compression cycle with internal heat exchanger, (iii) Two stage compression cycle with a cascade, and (iv) Single stage Heat pump cycle.

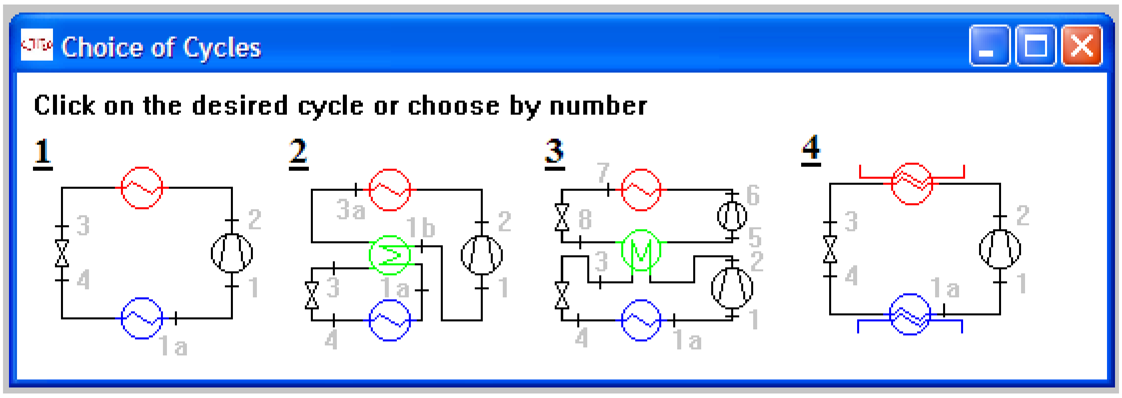
It can also give properties of refrigerants at a single point, and can also give in tabular form for a given range and increment. You can copy and paste the results to MS Word or EXCEL.

There is also a window giving detailed information on Ozone depletion potential, Global warming potential, Safety classification etc of various refrigerants produced by DUPONT.

See the manual of the software for more information.

Following are the steps:

1. After installing DUPREX, as you click o the DuPont emblem, an opening welcome screen appears, click OK, then a disclaimer appears, and click OK on it too, and following screen appears for choice of cycles:



In the above:

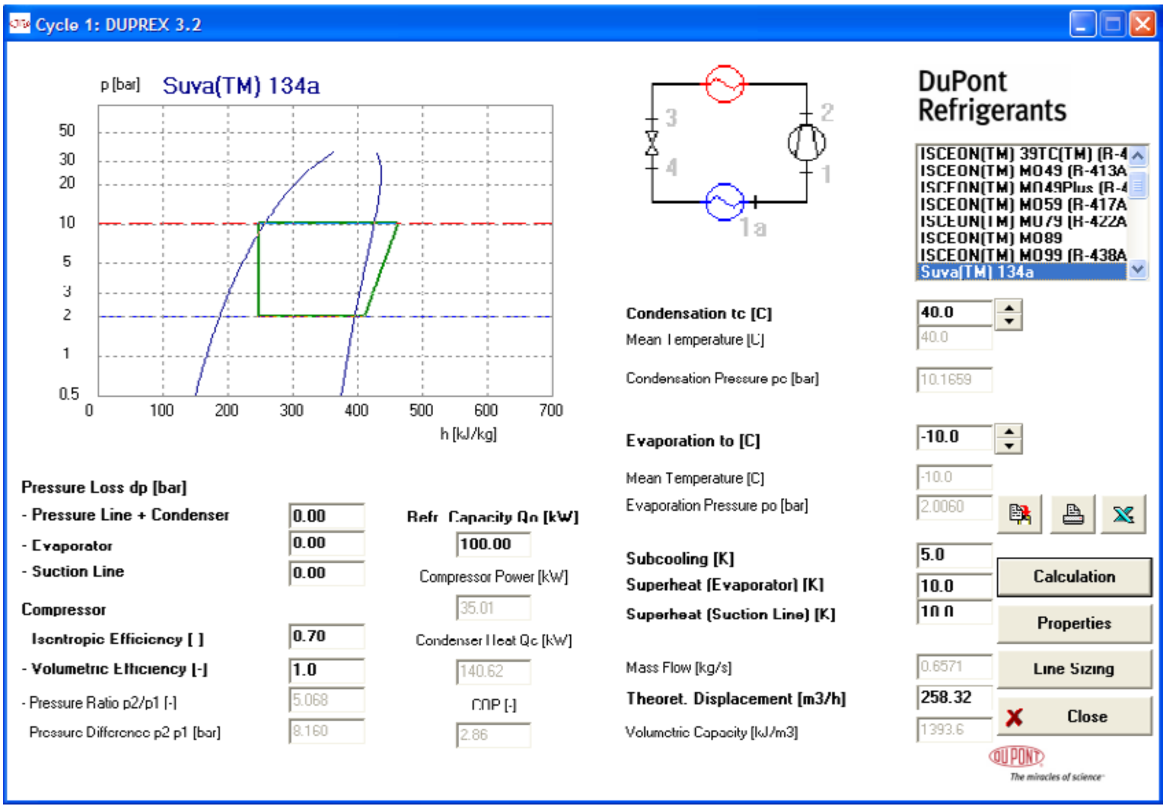
Cycle 1: Single stage compression cycle

Cycle 2: Single stage compression cycle, with internal heat exchanger

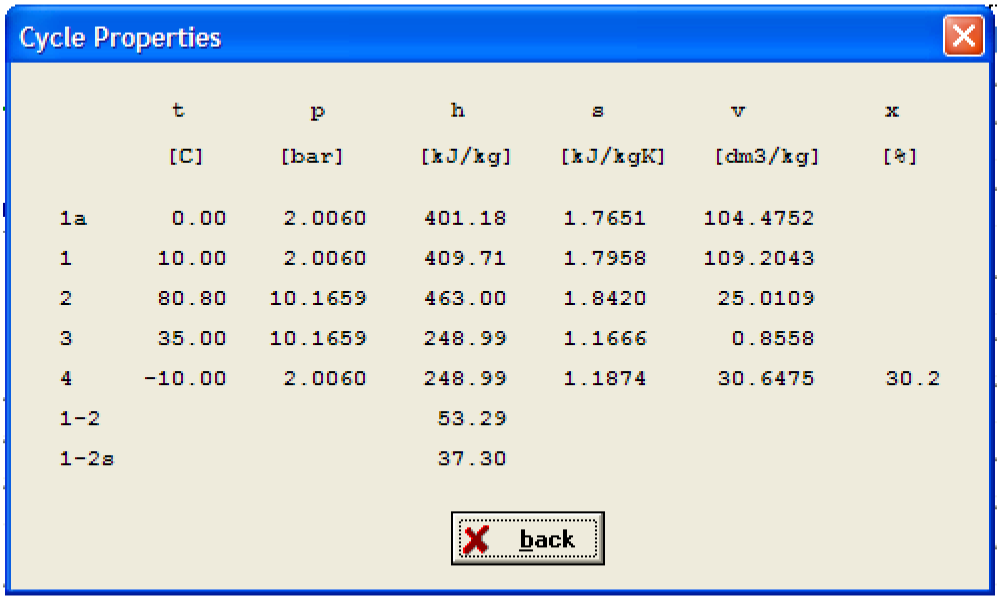
Cycle 3: Two stage compression cycle with a cascade, and

Cycle 4: Single stage Heat pump cycle.

2. Click on Cycle 1. We get:



3. In the above screen, numbers shown bold are the ones which we can change, depending on the data of our problem. Numbers shown in light are the results. Refrigerant R134a is chosen by default (see the top right hand corner).
4. Here, by default, calculations are made ***for a refrigeration effect of 100 kW***. For the given condenser and evaporator temps, and given superheating, subcooling, compressor isentropic and volumetric efficiencies, and pressure drops in different components and lines,following quantities are calculated: the mass flow rate (kg/s) and theoretical compressor displacement (m^3/h), Volumetric capacity (kJ/m^3), Refrig. capacity (100 kW), compressor power (kW), condenser heat (kW), COP, pressure ratio (P2/P1) and pressure difference (P2 – P1, bar).
5. Clicking on 'Properties' on the right hand bottom corner, gives a table of properties at salient points, a shown below:



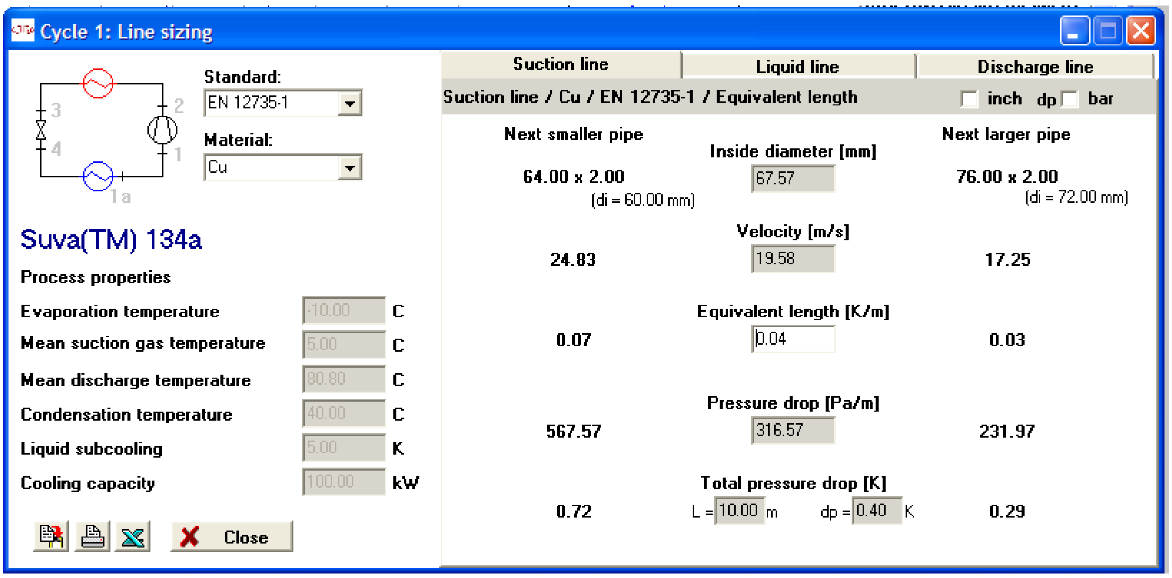
Cycle Properties

	t [C]	p [bar]	h [kJ/kg]	s [kJ/kgK]	v [dm3/kg]	x [%]
1a	0.00	2.0060	401.18	1.7651	104.4752	
1	10.00	2.0060	409.71	1.7958	109.2043	
2	80.80	10.1659	463.00	1.8420	25.0109	
3	35.00	10.1659	248.99	1.1666	0.8558	
4	-10.00	2.0060	248.99	1.1874	30.6475	30.2
1-2			53.29			
1-2s			37.30			

back

In the above, remember that reference values for enthalpy and entropy are as per IIR: h = 200 kJ/kg and s = 1 kJ/kg.C for sat. liquid at 0 C.

6. Clicking on 'Line sizing', gives another window a shown below:



Here, we can use the tube material and Standard on the left, top corner.

On the top horizontal line, there are tabs for Suction line, Liquid line and Discharge line and we can change the required line parameters in the respective windows.

Thus, it is a very useful software to make calculations for practical cycles, with a choice of very large number of refrigerants.

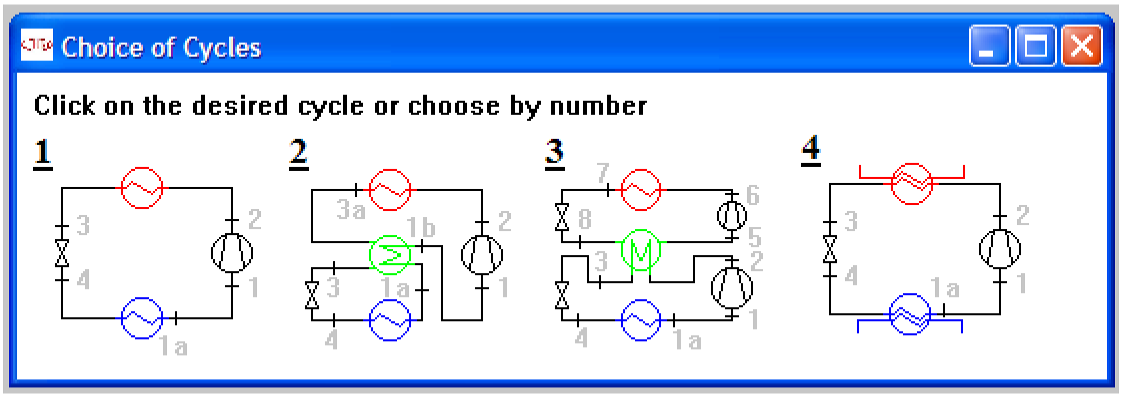
Now, let us solve a few problems with DUPREX:

Prob. 4.3.1. Arefrigerator uses R134a as working fluid and operates on the ideal vapour compression refrigeration cycle. Evaporator temp is 0 C and condenser temp is 30 C. For a refrigerant flow rate of 0.08 kg/s, calculate: (i) compressor power in kW, (ii) refrigeration capacity in tons, (iii) condenser heat transfer, and (iii) the COP.

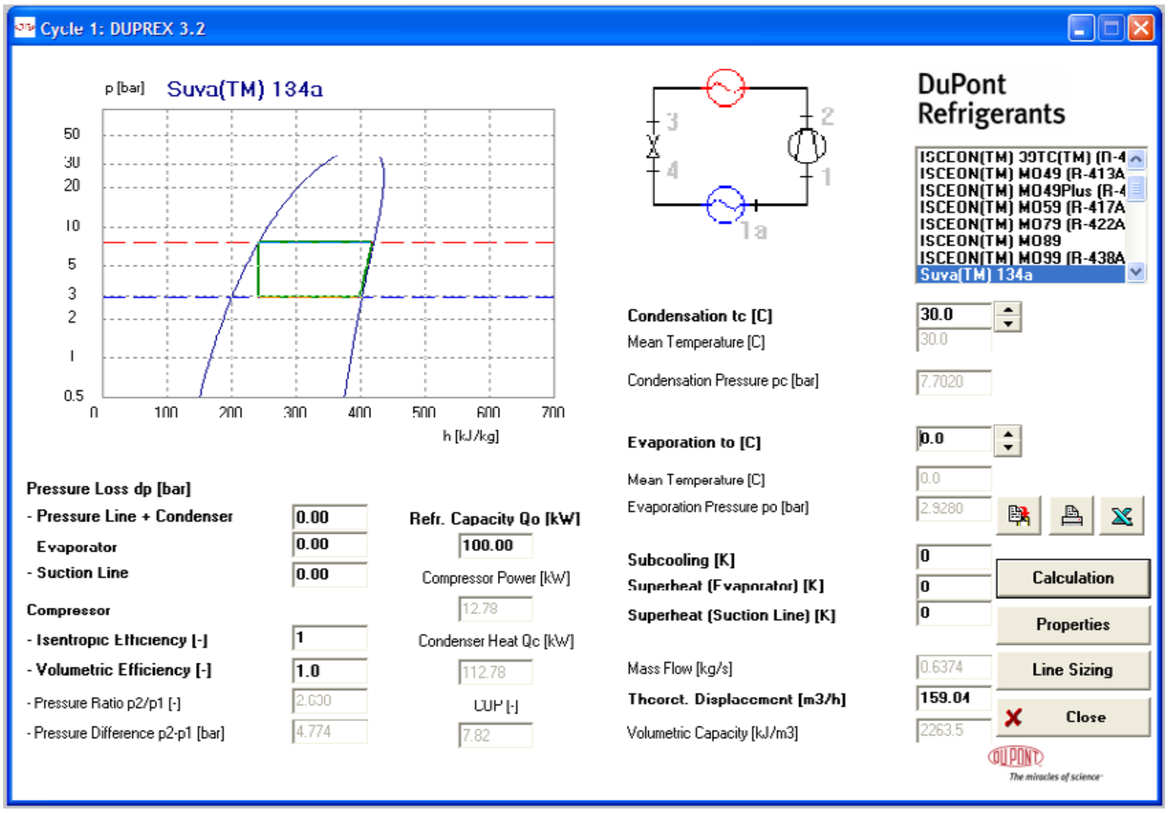
Solution:Let us use DUPREX software.

Following are the steps:

1. After installing DUPREX, as you click o the DuPont emblem, an opening welcome screen appears, click OK, then a disclaimer appears, and click OK on it too, and following screen appears for choice of cycles:



2. Click on Cycle 1. We get the screen with default values; change the numbers in the bold to required ones for this problem:



3. Note in the above screen that: presuure loses are set to zero, compressor effcy = 100%, condenser temp = 30 C, evaporator temp = 0 C, subcooling and superheatings are zero.

4. Results (shown in light font)are for a refrign. Capacity of 100 kW. We see that the mass flow rate of refrigerant is 0.6374 kg/s. Therefore, the refrign. Capacity for a mass flow rate of 1 kg/s can easily be calculated. To convert refrign. Capacity to tons, remember that 1 ton = 211 kJ/min. Similarly for compressor power etc.
5. Then, final results for a refrigerant flow rate of 0.08 kg/s are:

With eta_comp = 1

$q_L := \frac{100}{0.6374} \cdot 0.08$ i.e. $q_L = 12.551$ **kJ/s**

$\text{Refrign\_capacity} := \frac{q_L \cdot 60}{211}$ i.e. $\text{Refrign\_capacity} = 3.569$ **tons ... Ans.**

$w_{comp} := \frac{12.78 \cdot 0.08}{0.6374}$ i.e. $w_{comp} = 1.604$ **kW**

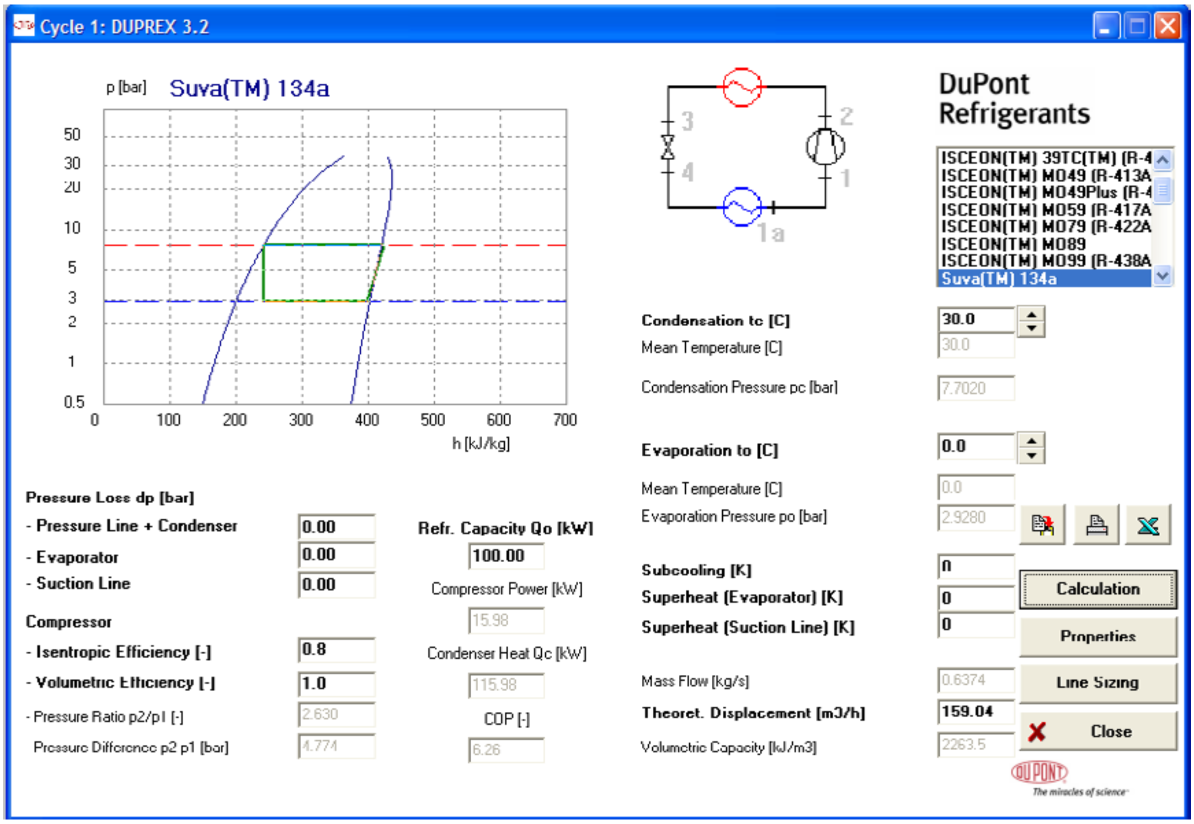
$q_{cond} := \frac{112.78 \cdot 0.08}{0.6374}$ i.e. $q_{cond} = 14.155$ **kW**

$COP := 9.24$ **....COP...Ans.**

Prob.4.3.2. Now, if in the actual cycle, the compressor isentropic effcy is 80%, what will be the new values of compressor work, condenser heat loss and the COP?

Solution:

In the previous problem, on the same screen, change the compressor isentropic effcy to 0.8, and hit Return. We get following results:



Note that now refrigerator mass flow rate remains the same, but compressor power, condenser heat and COP change.

Again, for a refrigerant mass flow rate of 0.08 kg/s, we have:

With eta_comp = 0.8

$q_L := \frac{100}{0.6374} \cdot 0.08$ i.e. $q_L = 12.551$ **kJ/s**

$Refrign\_capacity := \frac{q_L \cdot 60}{211}$ i.e. $Refrign\_capacity = 3.569$ **tons ... Ans.**

$w_{comp} := \frac{15.98 \cdot 0.08}{0.6374}$ $w_{comp} = 2.006$ **kW**

$q_{cond} := \frac{115.98 \cdot 0.08}{0.6374}$ $q_{cond} = 14.557$ **kW**

$COP := 6.26$ **....COP....Ans.**

Clicking on 'Properties' in the above screen gives properties at various state points:

Cycle Properties

	t [C]	p [bar]	h [kJ/kg]	s [kJ/kgK]	v [dm3/kg]	x [%]
1a	0.00	2.9280	398.60	1.7271	69.3087	
1	0.00	2.9280	398.60	1.7271	69.3087	
2	38.47	7.7020	423.67	1.7433	28.0577	
3	30.00	7.7020	241.72	1.1435	0.8421	
4	0.00	2.9280	241.72	1.1527	15.1704	21.0
1-2			25.07			
1-2s			20.06			

back

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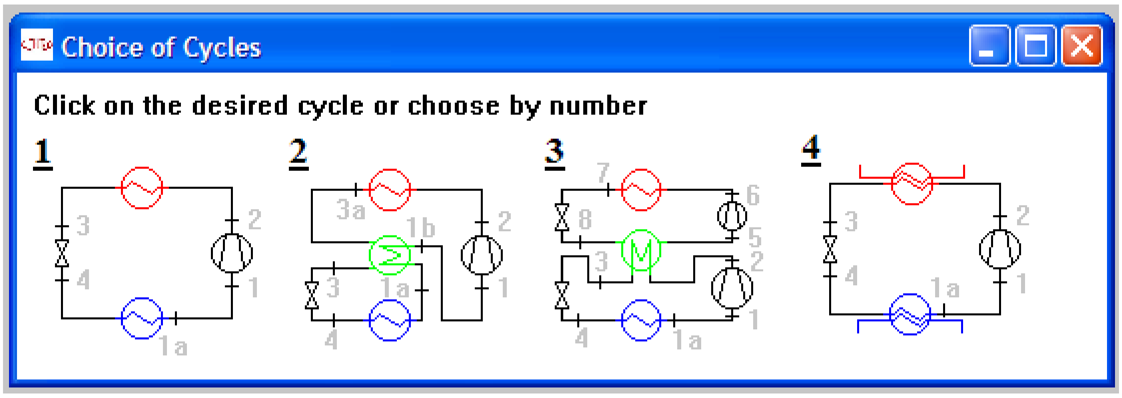
Prob.4.3.3.A vapour compression refrigerator of 10 tons capacity using Freon-12 as refrigerant has an evaporator temp of -10 C and a condenser temp of 30 C. Assuming simple refrigeration cycle, determine: (i) mass flow rate of refrigerant, (ii) Power input, (iii) COP. [VTU-ATD-Jan./Feb. 2006 & Dec. 2009–Jan. 2010]

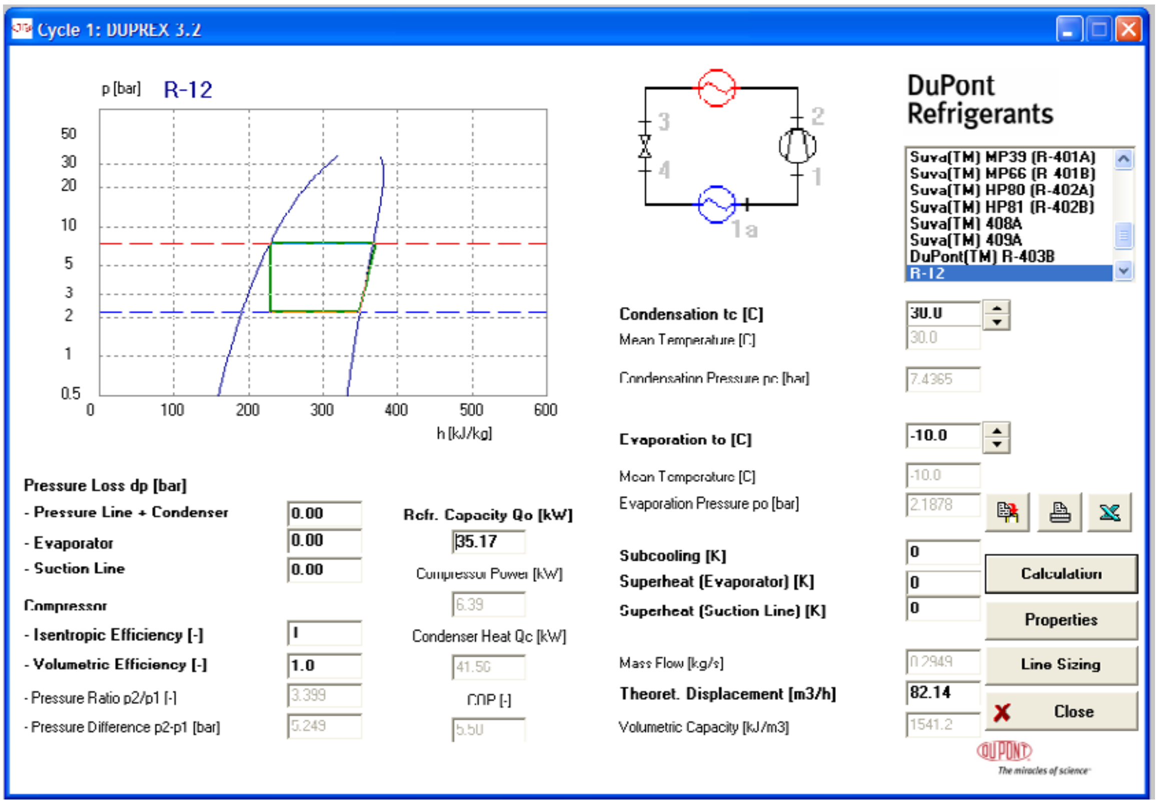
Solution with DUPREX:

Following are the steps:

1. After installing DUPREX, as you click o the DuPont emblem, an opening welcome screen appears, click OK, then a disclaimer appears, and click OK on it too, and following screen appears for choice of cycles:



2. Click on Cycle 1. We get the screen with default values; change the Refrigerant to R-12, and the numbers in the bold to required ones for this problem. Note that we have changed the Refrign. Capacity Q0 to 35.17 since 10 tons of refrigeration = 10 * 211 / 60 = 35.167 kJ/s (=kW). We get:



Thus, we get:

Refrigerant flow rate = 0.2949 kg/s = 1.7694 kg/min. … Ans.

Power input to compressor = 6.39 kW … Ans.

COP = 5.5 … Ans.

Condenser heat transfer = 41.56 kW …. Ans.

Clicking on 'Properties' gives following screen:

Cycle Properties

	t [C]	p [bar]	h [kJ/kg]	s [kJ/kgK]	v [dm3/kg]	x [%]
1a	-10.00	2.1878	348.29	1.5644	77.3701	
1	-10.00	2.1878	348.29	1.5644	77.3701	
2	36.53	7.4365	369.96	1.5644	24.6375	
3	30.00	7.4365	229.04	1.0997	0.7736	
4	-10.00	2.1878	229.04	1.1113	19.3476	24.3
1-2			21.68			
1-2s			21.68			

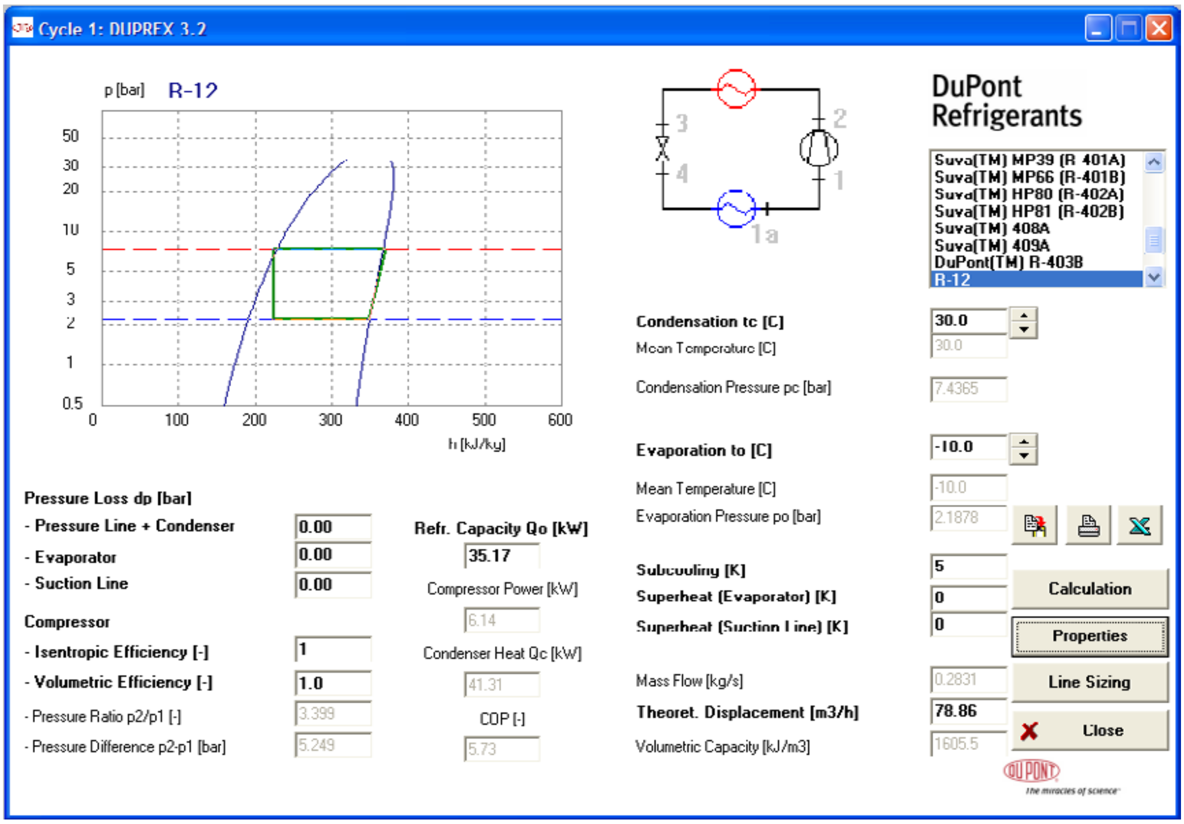
back

===

Prob.4.3.4. What will be the changes in the above quantities if the liquid leaving the condenser is subcooled by 5 deg. C? [VTU-ATD-Jan.–Feb. 2006]

Solution:

In the above calculation screen, just make this change to show subcooling by 5 C, and click 'Calculations' (or, hit Enter). We get:



We note that:

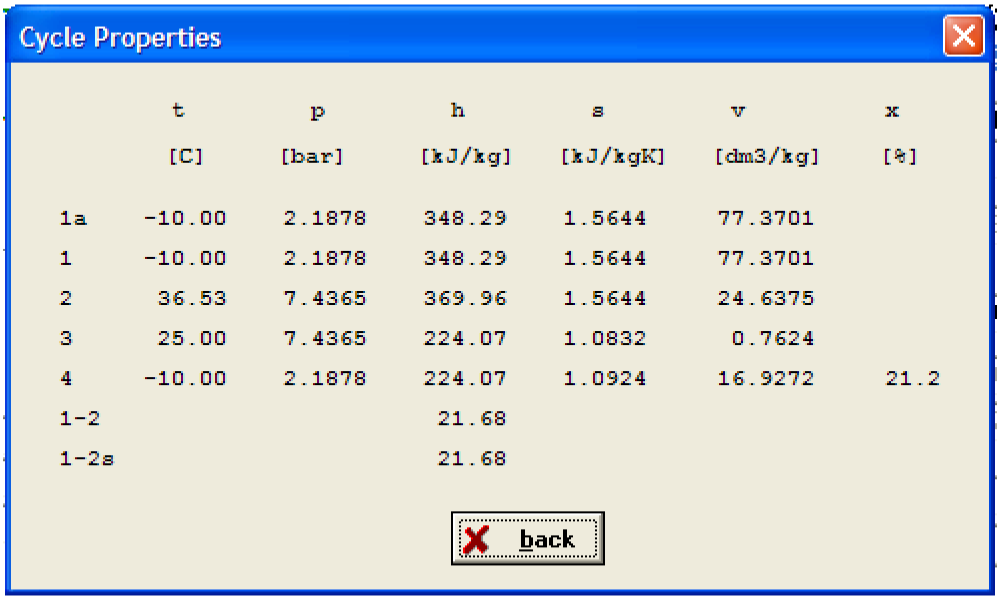
Refrigerant flow rate = 0.2831 kg/s = 1.6986 kg/min. … Ans.

Power input to compressor = 6.14 kW … Ans.

COP = 5.73 … Ans.

Condenser heat transfer = 41.31 kW …. Ans.

And, Clicking on 'Properties' gives following screen:



Cycle Properties

	t	p	h	s	v	x
	[C]	[bar]	[kJ/kg]	[kJ/kgK]	[dm3/kg]	[%]
1a	-10.00	2.1878	348.29	1.5644	77.3701	
1	-10.00	2.1878	348.29	1.5644	77.3701	
2	36.53	7.4365	369.96	1.5644	24.6375	
3	25.00	7.4365	224.07	1.0832	0.7624	
4	-10.00	2.1878	224.07	1.0924	16.9272	21.2
1-2			21.68			
1-2s			21.68			

back

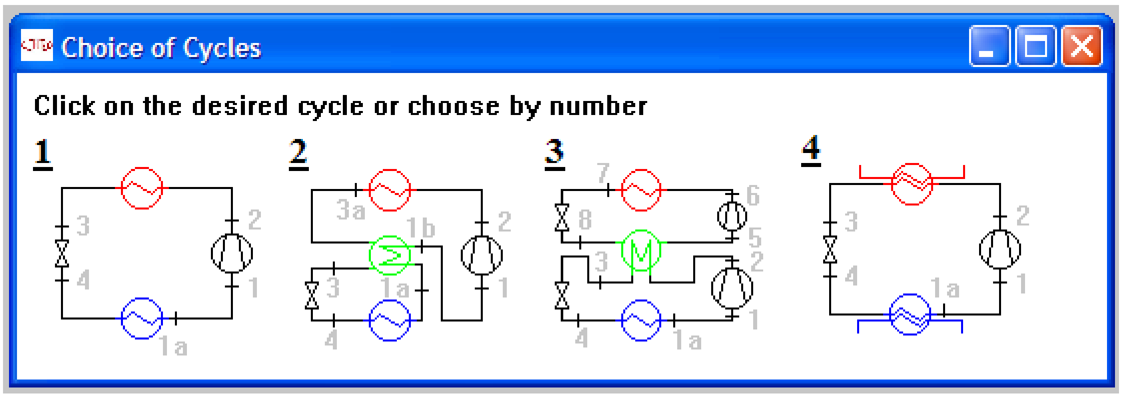
==

Prob.4.3.5. A vapour compression refrigerator of 5 tons capacity using Freon-12 as refrigerant has an evaporator temp of -10 C and a condenser temp of 40 C. Assuming simple refrigeration cycle, determine: (i) mass flow rate of refrigerant in kg/s, (ii) volume flow rate handled by the compressor, in m^3/s, (iii) compressor discharge temp., (iv) the pressure ratio, (v) heat rejected to the condenser in kW, (vi) COP, and (vii) Power inputto compressor. [VTU-ATD-July2007]

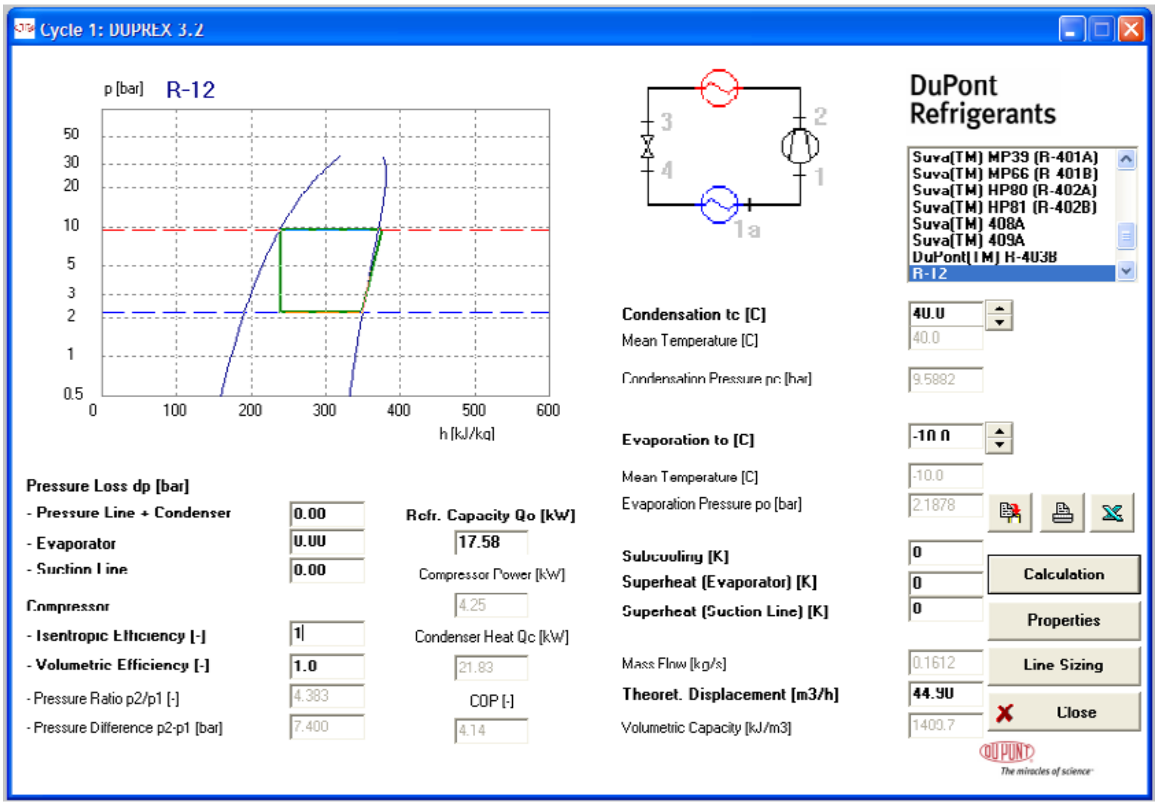
Solution with DUPREX:

Following are the steps:

1. After installing DUPREX, as you click o the DuPont emblem, an opening welcome screen appears, click OK, then a disclaimer appears, and click OK on it too, and following screen appears for choice of cycles:



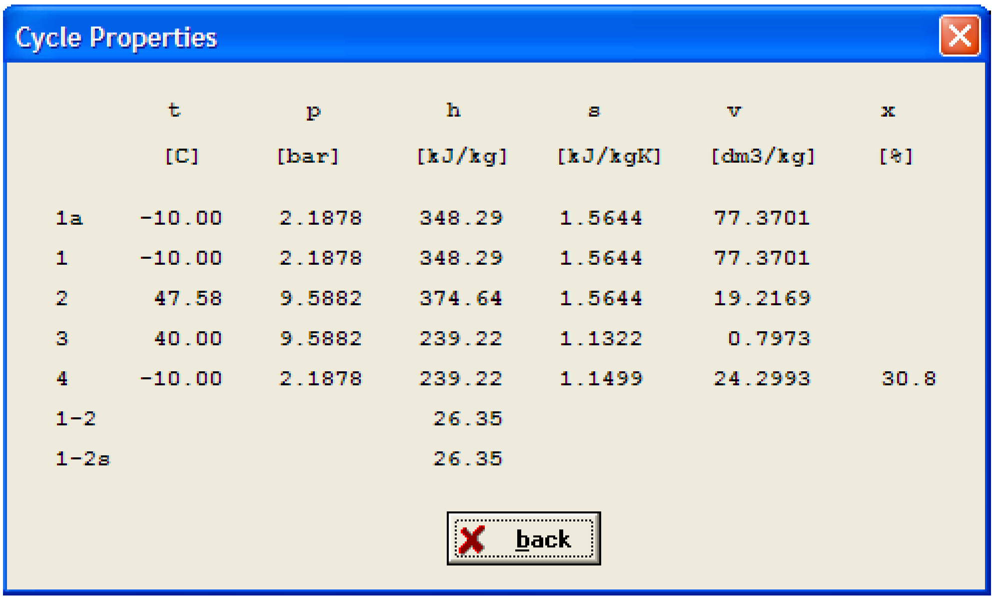
2. Click on Cycle 1. We get the screen with default values; change the Refrigerant to R-12, and the numbers in the bold to required ones for this problem. Note that we have changed the Refrign. Capacity Q0 to 17.583 since 5 tons of refrigeration = 5 * 211 / 60 = 17.583 kJ/s (=kW). We get:



Thus:

1. **mass flow rate of refrigerant in kg/s = 0.1612 kg/s …. Ans.**
2. **volume flow rate handled by the compressor = 44.9 m^3/h = 0.0125 m^3/s … Ans.**
3. **compressor discharge temp.= T2 = 47.58 C (from Properties tab) … Ans.**
4. **the pressure ratio = P2/P1 = 4.383… Ans.**
5. **heat rejected to the condenser in kW = 21.83 kW … Ans.**
6. **COP = 4.14 …Ans.**
7. **Power input to compressor = 4.25 kW … Ans.**

Clicking on Properties tab gives:



	t [C]	p [bar]	h [kJ/kg]	s [kJ/kgK]	v [dm3/kg]	x [%]
1a	-10.00	2.1878	348.29	1.5644	77.3701	
1	-10.00	2.1878	348.29	1.5644	77.3701	
2	47.58	9.5882	374.64	1.5644	19.2169	
3	40.00	9.5882	239.22	1.1322	0.7973	
4	-10.00	2.1878	239.22	1.1499	24.2993	30.8
1-2			26.35			
1-2s			26.35			

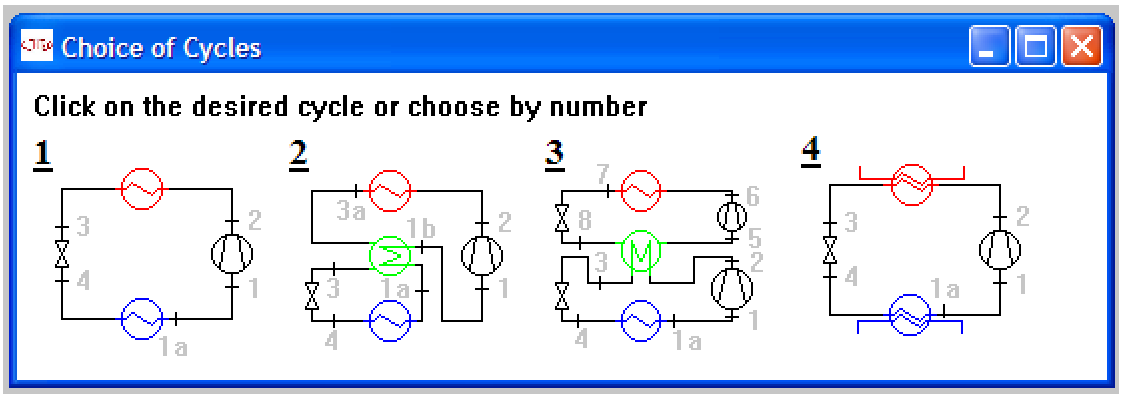
==

Prob.4.3.6. A A food storage chamber requires a refrign. System of 10 T capacity with evaporator temp of -10 C and condenser temp of 30 C. The refrigerant F-12 is subcooled by 5 C before entering the throttle valve and the vapour is superheated by 6 C before entering the compressor. Determine: (i) refrig. capacity per kg (ii) mass of refrigerant circulated in kg/s, and (iii) COP. [VTU-ATD-Jan.–Feb. 2003]

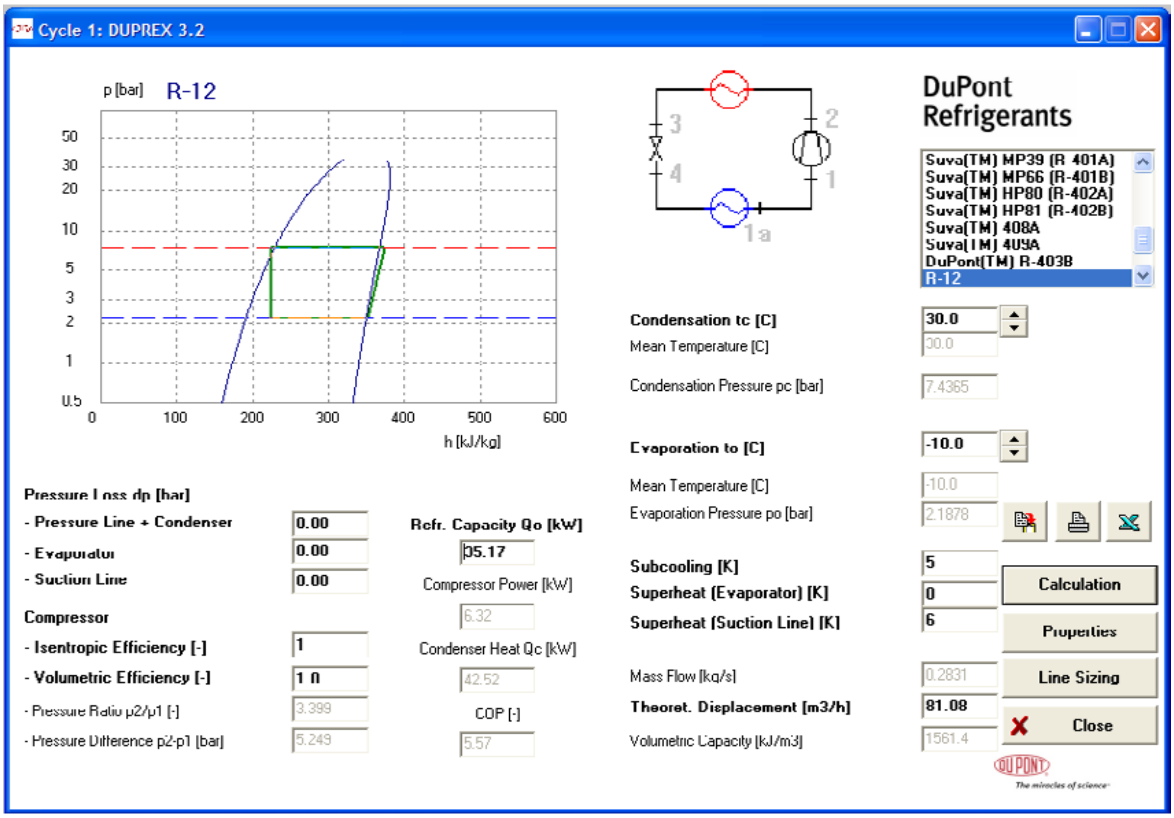
Solution with DUPREX:

Following are the steps:

1. After installing DUPREX, as you click o the DuPont emblem, an opening welcome screen appears, click OK, then a disclaimer appears, and click OK on it too, and following screen appears for choice of cycles:



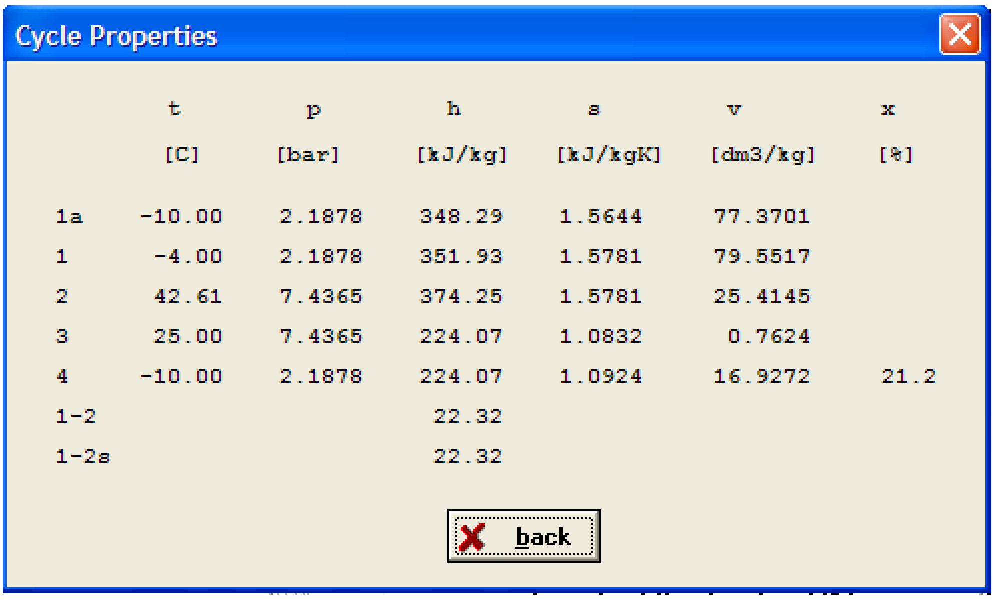
2. Click on Cycle 1. We get the screen with default values; change the Refrigerant to R-12, and the numbers in the bold to required ones for this problem. Note that we have changed the Refrign. capacity Q0 to 35.167 since 10 tons of refrigeration = 10 * 211 / 60 = 35.167 kJ/s (=kW). We get:



Thus:

1. **mass flow rate of refrigerant in kg/s = 0.2831 kg/s …. Ans.**
2. **Refrign. capacity per kg = 35.17/0.2831= 124.232 kJ/kg … Ans.**
3. **COP = 5.57 …Ans.**
4. **Power input to compressor = 6.32 kW … Ans.**

Clicking on Properties tab gives:



	t [C]	p [bar]	h [kJ/kg]	s [kJ/kgK]	v [dm3/kg]	x [%]
1a	-10.00	2.1878	348.29	1.5644	77.3701	
1	-4.00	2.1878	351.93	1.5781	79.5517	
2	42.61	7.4365	374.25	1.5781	25.4145	
3	25.00	7.4365	224.07	1.0832	0.7624	
4	-10.00	2.1878	224.07	1.0924	16.9272	21.2
1-2			22.32			
1-2s			22.32			

==

4.4 Problems solved with EES:

Prob.4.4.1 Write an EES Procedure to calculate COP etc of an actual vapour compression refrigeration cycle i.e. including the subcooling before entry to expansion valve and superheating before entry to compressor and the isentropic efficiency of the compressor.

EES Solution:

We shall write an EES Procedure which can be used for any refrigerant for which properties are available as built-in functions in EES.

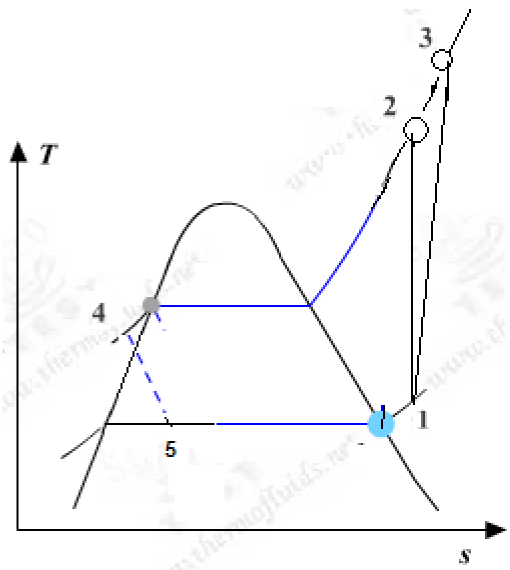


Fig.Prob.4.4.1. T-s diagram for actual vapour compression refrig. cycle

```
$UnitSystem bar C kJ

PROCEDURE Vap_Comp_Refrign_cycle_actual( Fluid$,T[1],T[4],eta_comp,DELTAT_subcool,
DELTAT_superheat : w_comp_isentr,w_comp_act,q_L,q_cond,P[1],P[2],x[5],T[3],COP)

{Vap_Comp_Refrign_cycle_actual .... finds COP etc for an actual, vap. comprn. refrign. cycle.

Pressures in bar, Temps in C, Work in kJ/kg

Inputs: T[1] ... evaporator temp (C), T[4] .... condenser temp (C), eta_comp .... isentr. effcy of compressor,
,DELTAT_subcool (C),DELTAT_superheat (C)

Outputs: w_comp_isentr,w_comp_act, q_L, q_cond, P[1], P[2], x[5], COP

w_comp_isentr .... compressor isentropic work, kJ/kg
w_comp_act .... compressor actual work, kJ/kg
```

```
q_L , q_cond.... refrign. effect and heat tr in condenser, kJ/kg

P[1], P[2]] ..evaporator pressure, and condenser pressure, bar
x[5] ... quality after expn in expansion valve
T[3] …temp at exit of compressor after actual compression, C
 COP...coefff. of performance = q_L / w_comp_act
}

x[1] = 1 "...quality at entry to compressor"

x[4]:=0 "...quality at entry to expn. valve"

P[1] :=P_sat(Fluid$,T=T[1]) "...sat.pressure in evaporator"

P[4]:=P_sat(Fluid$,T=T[4])"...sat.pressure in condenser"

P[2]:= P[4]

P[3]:= P[4]

IF (DELTAT_superheat > 0) THEN

   s[1]:= Entropy(Fluid$,T=T[1] + DELTAT_superheat,P=P[1])

   h[1]:= Enthalpy(Fluid$,P=P[1],s=s[1])

   s[2]:=s[1]

   h[2]:=Enthalpy(Fluid$,P=P[2],s=s[2])

ELSE

   s[1]:=Entropy(Fluid$,T=T[1],x =x[1])"...entropy at entry to compressor"

   h[1]:= Enthalpy(Fluid$,T=T[1],x =x[1])"...enthalpy at entry to compressor"

   s[2]:= s[1] "...for isentropic compression"

   h[2]:=Enthalpy(Fluid$,P=P[2],s=s[2])"...enthalpy after isentropic comprn."
```

```
ENDIF

h[3]:= h[1] + (h[2] – h[1])/eta_comp"...enthalpy after actual comprn."

T[2] := Temperature(Fluid$,P=P[2],s=s[2])"...temp after isentropic comprn."

T[3]:= Temperature(Fluid$,P=P[3],h=h[3]) "...temp after actual comprn."

P[5]:=P[1]

IF (DELTAT_subcool > 0) THEN

    h[4]:=Enthalpy(Fluid$,x=x[4],T=T[4] – DELTAT_subcool)

ELSE

    h[4]:=Enthalpy(Fluid$,P=P[4],x=x[4])
```

```
ENDIF

h[5]:=h[4]

T[5] := T[1]

x[5]=Quality(Fluid$,T=T[5],h=h[5])"...quality after expn. in expansion valve"

w_comp_isentr := h[2] – h[1] "kJ/kg ... isentr. compressor work"

w_comp_act := h[3] – h[1] "kJ/kg ... actual compressor work"

q_L := h[1] – h[5] "kJ/kg .... refrig. effect"

q_cond := h[3] – h[4]"kJ/kg ....condenser heat transfer"

COP = q_L / w_comp_act "...coeff. of performance"

END

"=================================================================="
```

Now, use the above EES Procedure to solve the following problem:

Prob.4.4.2. A 10 ton Ammonia Ice plant operates between an evaporator temp of -15 C and a condenser temp of 35 C. Ammonia enters the compressor as dry saturated liquid. Assuming isentropic compression, determine: (i) mass flow rate of ammonia, (ii) COP, and (iii) compressor power input in kW. [VTU – ATD – July 2006]

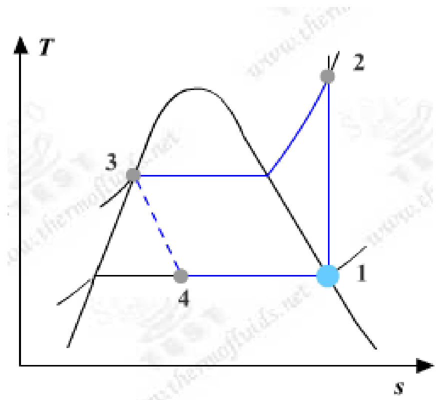


Fig.Prob.4.4.2 T-s diagram for ideal vap. compression cycle

EES Solution:

We will first write the data required as inputs for the above EES Procedure, and then call that Procedure:

Data:

Fluid$ = 'Ammonia'

T[1] = -15 "C…. evap. Temp."

T[4] = 35 "C … condenser temp."

DELTAT_subcool = 0"C … subcooling"

DELTAT_superheat = 0 "C … superheat"

eta_comp = 1"…isentr. Effcy. of compressor"

CALL Vap_Comp_Refrign_cycle_actual(Fluid$,T[1],T[4],eta_comp,DELTAT_subcool,DELTAT_superheat: w_comp_isentr,w_comp_act,q_L,q_cond,P[1],P[2],x[5],T[3],COP)

"Now, 1 ton is equivalent to 211 kJ/min. of refrigeration.

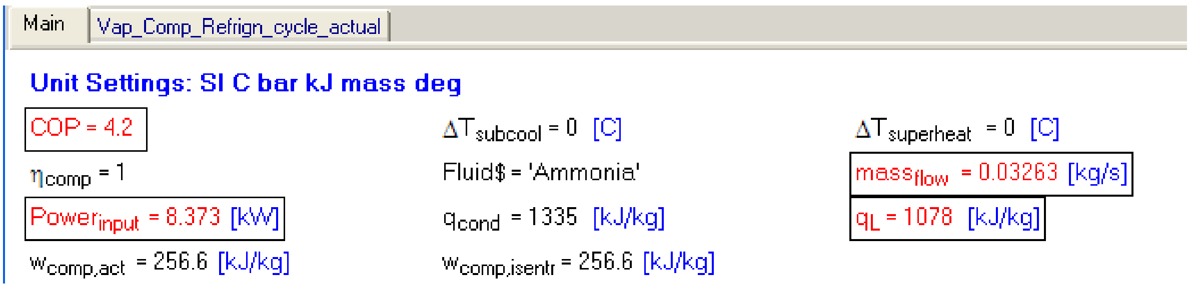
Therefore, 10 tons of refrigeration is equiv. to (10 * 211 / 60) kJ/s. And q_L i the refrig. effect for a flow rate of 1 kg/s.

So, we have, for mass flow rate of refrigerant required:"

mass_flow = (10 * 211/60)/q_L "kg/s"

Power_input = mass_flow * w_comp_act "kW"

Results:



Thus:

Mass flow rate of Ammonia = 0.03263 kg/s … Ans.

Compressor power = 8.373 kW .. Ans.

COP = 4.2 …. Ans.

Other results of Procedure:

Main | Vap_Comp_Refrign_cycle_actual

Local variables in Procedure Vap_Comp_Refrign_cycle_actual (1 call, 0.88 sec)

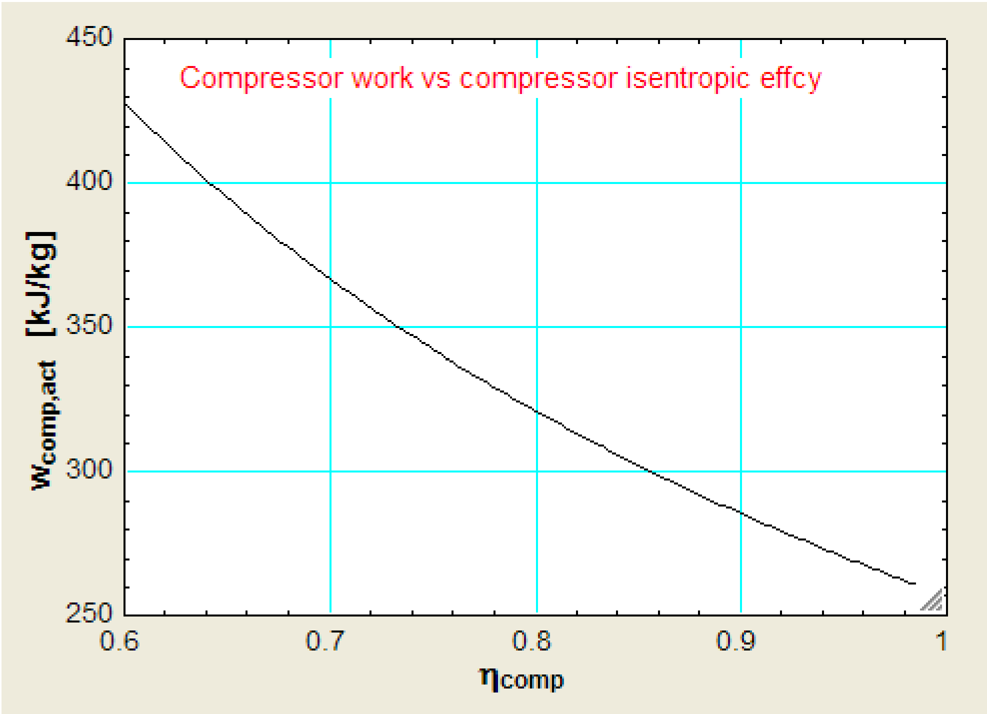
COP=4.2	$\Delta T_{subcool}$=0 [C]	$\Delta T_{superheat}$ =0 [C]
η_{comp}=1	Fluid$='Ammonia'	h_1=1444 [kJ/kg]
h_2=1701 [kJ/kg]	h_3=1701 [kJ/kg]	h_4=366 [kJ/kg]
h_5=366 [kJ/kg]	P_1=2.362 [bar]	P_2=13.51 [bar]
P_3=13.51 [bar]	P_4=13.51 [bar]	P_5=2.362 [bar]
q_{cond} =1335 [kJ/kg]	q_L=1078 [kJ/kg]	s_1=5.827 [kJ/kg-C]
s_2=5.827 [kJ/kg-C]	T_1=-15 [C]	T_2=111.1 [C]
T_3=111.1 [C]	T_4=35 [C]	T_5=-15 [C]
$w_{comp,act}$ =256.6 [kJ/kg]	$w_{comp,isentr}$=256.6 [kJ/kg]	x_1 =1
x_4 =0	x_5 =0.1788	

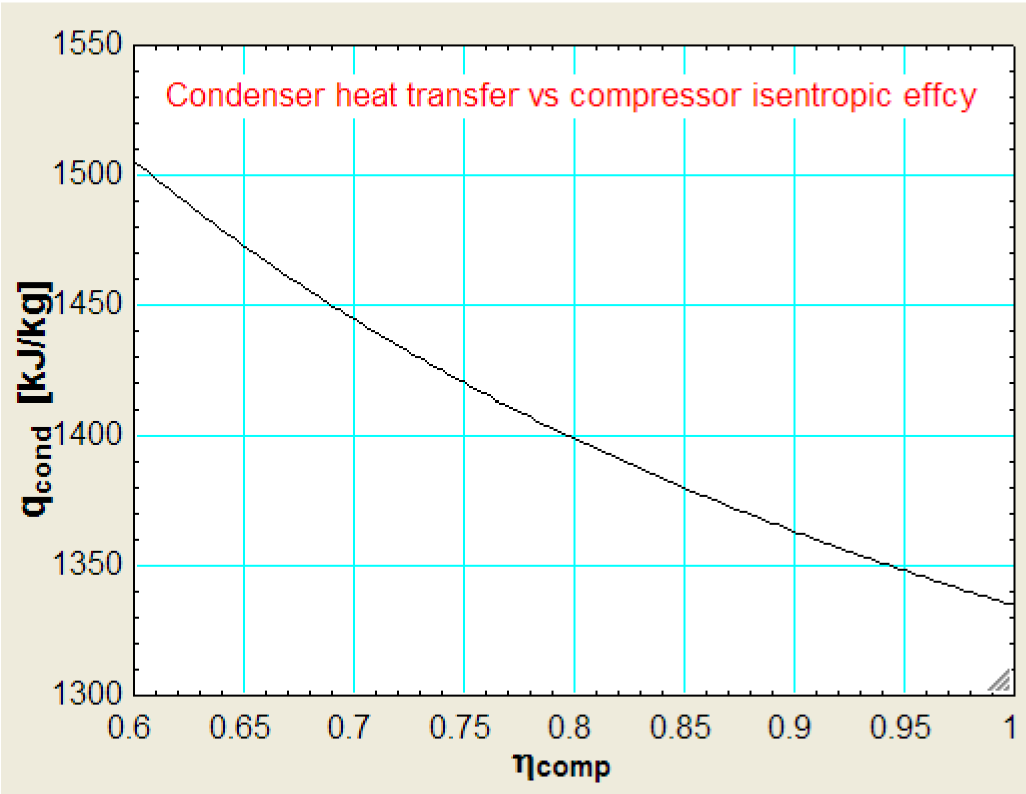
(b) Plot the variation of compressor work (kJ/kg), condenser heat transfer (kJ/kg) and COP as the isentropic effcy. of compressor varies from 0.6 to 1:

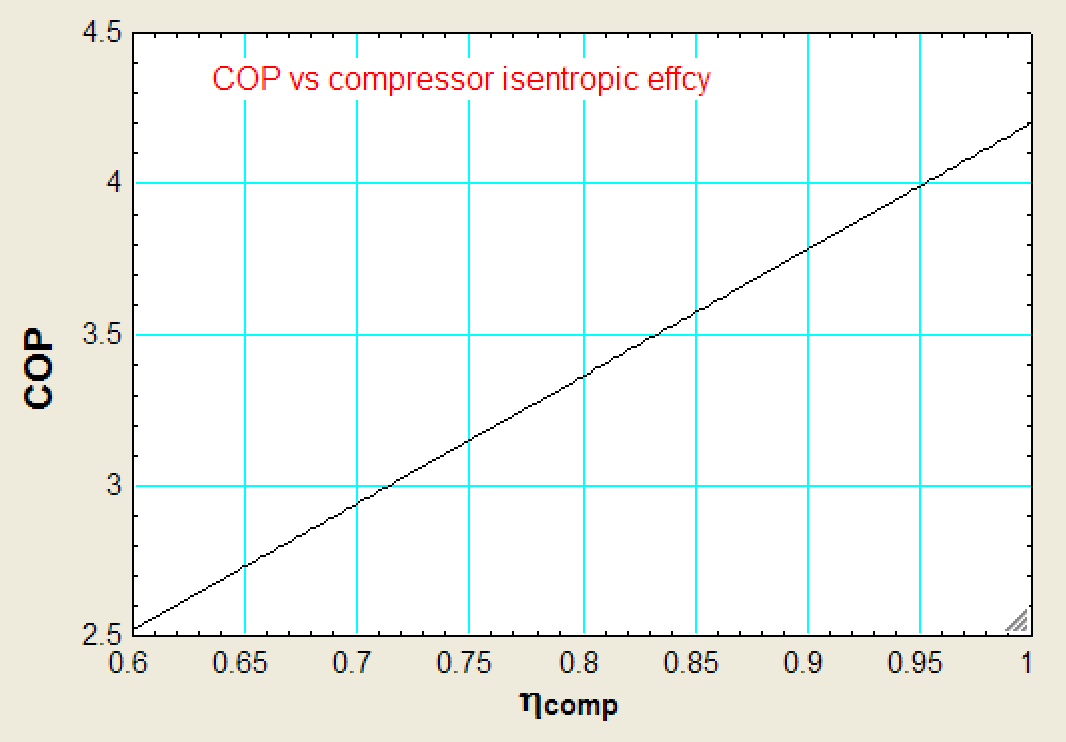
First, compute the Parametric Table:

1..9	η_{comp}	Fluid$	q_{cond} [kJ/kg]	$w_{comp,act}$ [kJ/kg]	COP
Run 1	0.6	Ammonia	1506	427.7	2.52
Run 2	0.65	Ammonia	1473	394.8	2.73
Run 3	0.7	Ammonia	1445	366.6	2.94
Run 4	0.75	Ammonia	1420	342.2	3.15
Run 5	0.8	Ammonia	1399	320.8	3.36
Run 6	0.85	Ammonia	1380	301.9	3.57
Run 7	0.9	Ammonia	1363	285.2	3.78
Run 8	0.95	Ammonia	1348	270.1	3.99
Run 9	1	Ammonia	1335	256.6	4.2

Now, plot the results:






COP vs compressor isentropic effcy
COP
4.5
4
3.5
3
2.5
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
ηcomp

==

Prob. 4.4.3. A food storage chamber requires a refrigeration system of 10 T capacity with an evaporator temp. of -10 C and condenser temp. of 30 C. The refrigerant R-12 is subcooled by 5 deg. C before entering the throttle valve and the vapour is superheated by 6 deg. C before entering the compressor. The specific heats of liquid and vapour are 1.235 and 0.7327 kJ/kg.K respectively. Determine: (i) The refrigerating capacity per kg (ii) Mass of refrigerant circulated per minute, and (iii) COP. [VTU-ATD-Jan. 2003]

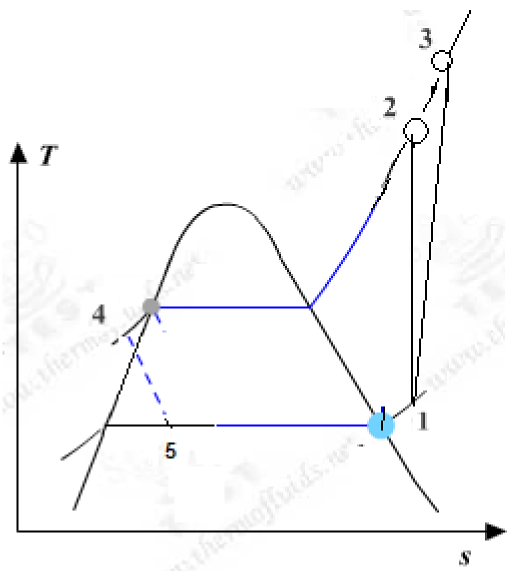


Fig.Prob.4.4.3 T-s diagram for actual vap. compression cycle, with subcooling and superheat

EES Solution:

"Data:"

Fluid$ = 'R12'

T[1] = -10 "C"

T[4] = 30 "C"

DELTAT_subcool = 5 "C"

DELTAT_superheat = 6 "C"

eta_comp = 1

"Calculations:"

CALL Vap_Comp_Refrign_cycle_actual(
Fluid$,T[1],T[4],eta_comp,DELTAT_subcool,DELTAT_superheat:
w_comp_isentr,w_comp_act,q_L,q_cond,P[1],P[2],x[5],T[3],COP)

mass_flow = (10 * 211/60)/q_L "kg/s"

Power_input = mass_flow * w_comp_act "kW"

Results:

Main | Vap_Comp_Refrign_cycle_actual

Unit Settings: SI C bar kJ mass deg

COP = 5.733	$\Delta T_{subcool}$ = 5 [C]	$\Delta T_{superheat}$ = 6 [C]
η_{comp} = 1	Fluid$ = 'R12'	$mass_{flow}$ = 0.2764 [kg/s]
$Power_{input}$ = 6.134 [kW]	q_{cond} = 149.4 [kJ/kg]	q_L = 127.2 [kJ/kg]
$w_{comp,act}$ = 22.19 [kJ/kg]	$w_{comp,isentr}$ = 22.19 [kJ/kg]	

Main | Vap_Comp_Refrign_cycle_actual

Local variables in Procedure Vap_Comp_Refrign_cycle_actual (1 call, 0.03 sec)

COP=5.733	$\Delta T_{subcool}$=5 [C]	$\Delta T_{superheat}$=6 [C]
η_{comp}=1	Fluid$='R12'	h_1=186.9 [kJ/kg]
h_2=209.1 [kJ/kg]	h_3=209.1 [kJ/kg]	h_4=59.69 [kJ/kg]
h_5=59.69 [kJ/kg]	P_1=2.189 [bar]	P_2=7.443 [bar]
P_3=7.443 [bar]	P_4=7.443 [bar]	P_5=2.189 [bar]
q_{cond}=149.4 [kJ/kg]	q_L=127.2 [kJ/kg]	s_1=0.716 [kJ/kg-C]
s_2=0.716 [kJ/kg-C]	T_1=-10 [C]	T_2=42.9 [C]
T_3=42.9 [C]	T_4=30 [C]	T_5=-10 [C]
$w_{comp,act}$=22.19 [kJ/kg]	$w_{comp,isentr}$=22.19 [kJ/kg]	x_1=1
x_4=0	x_5=0.21	

Thus:

Mass flow rate of R-12 = 0.2764 kg/s … Ans.

Refrig. capacity = 127.2 kJ/kg …. Ans.

Compressor power = 6.134 kW .. Ans.

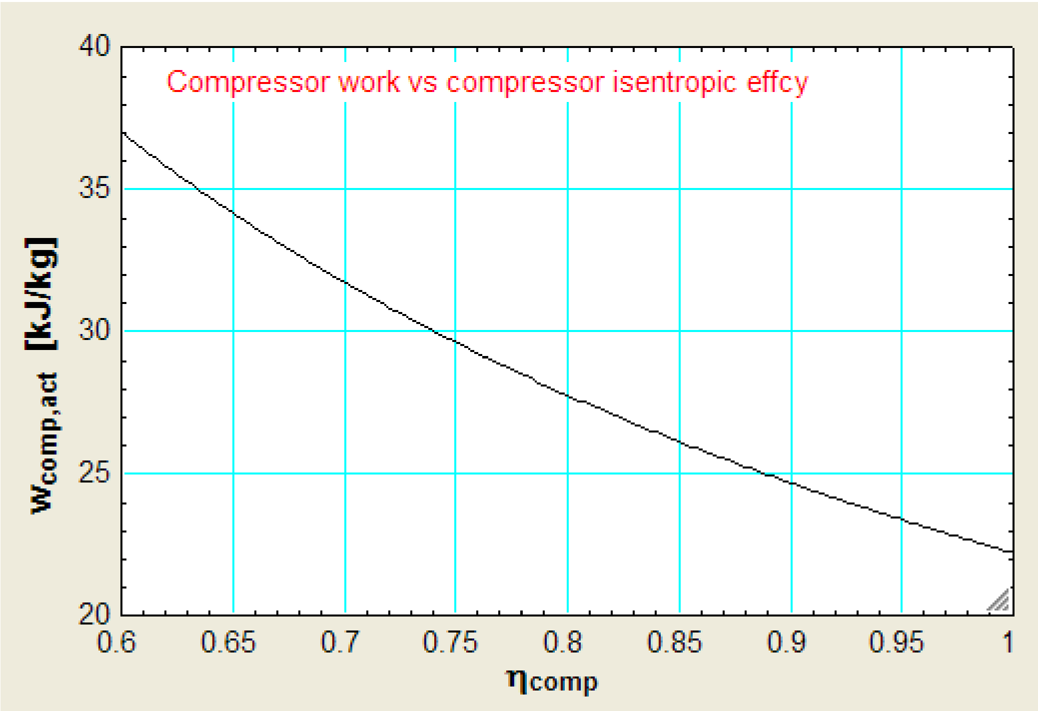
COP = 5.733 …. Ans.

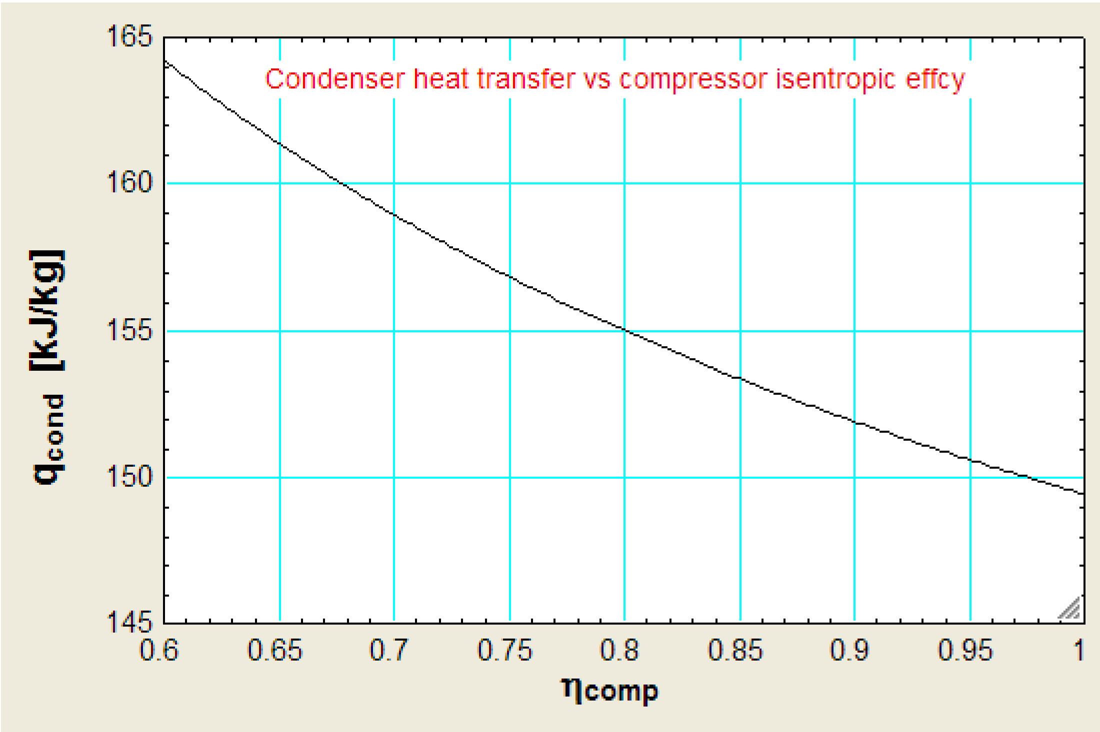
(b) Plot the variation of compressor work (kJ/kg), condenser heat transfer (kJ/kg) and COP as the isentropic effcy. of compressor varies from 0.6 to 1:

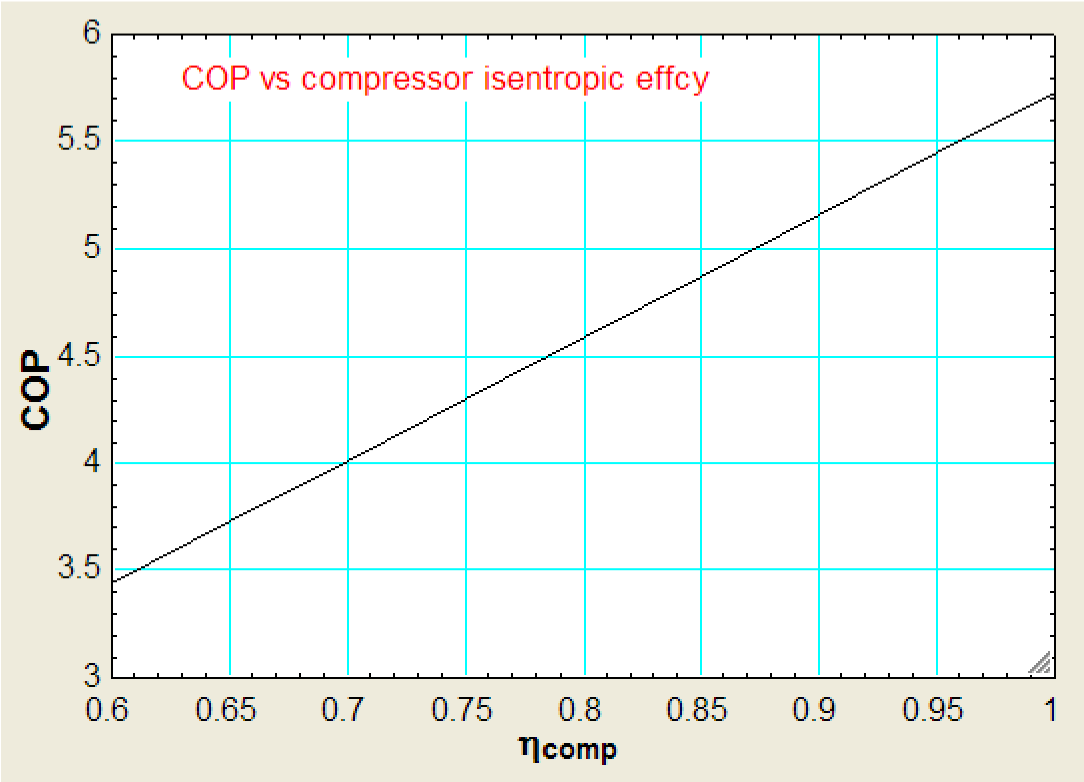
First, compute the Parametric Table:

1..9	η_{comp}	$w_{comp,act}$ [kJ/kg]	q_{cond} [kJ/kg]	COP
Run 1	0.6	36.99	164.2	3.44
Run 2	0.65	34.14	161.4	3.726
Run 3	0.7	31.71	158.9	4.013
Run 4	0.75	29.59	156.8	4.299
Run 5	0.8	27.74	155	4.586
Run 6	0.85	26.11	153.3	4.873
Run 7	0.9	24.66	151.9	5.159
Run 8	0.95	23.36	150.6	5.446
Run 9	1	22.19	149.4	5.733

Now, plot the results:







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"**Prob.4.4.4.** A food storage chamber requires a refrigeration system of 5 kW capacity with an evaporator temp. of -15 C and condenser temp. of 20 C. The refrigerant used is R-12. Determine: (i) The refrigerating capacity per kg (ii) Mass of refrigerant circulated per minute, and (iii) COP. [Ref:2]"

EES Solution:

The EES Procedure written above is quite versatile and powerful and useful, since we can analyse an actual vapour compression cycle by varying various parameters such as compressor efficiency, subcooling and superheat, and evaporator and condenser temperatures, and the refrigerant.

In this problem, let us use the EES facility to input the variables from the Diagram Window:

First, write the EES program as usual, but later, after making entries in the Diagram window, comment out the inlet parameters in the equation window, since we are going to input them from the Diagram window. See below:

"Data:"

{

T[1] = -15 [C]

T[4] = 20 [C]

DELTAT_subcool = 0"C"

DELTAT_superheat = 0"C"

eta_comp = 1

}

"Calculations:"

CALL Vap_Comp_Refrign_cycle_actual
(Fluid$,T[1],T[4],eta_comp,DELTAT_subcool,DELTAT_superheat:
w_comp_isentr,w_comp_act,q_L,q_cond,P[1],P[2],x[5],T[3],COP)

mass_flow = 5/q_L "kg/s …. Mass flow rate for a refrign. capacity of 5 kW"

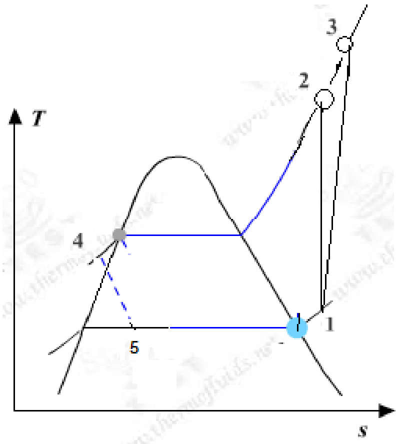
Power_input = mass_flow * w_comp_act "kW"

The procedure of having the input and calculations done from the diagram window was explained in detail in Prob. 3.3.3.

In the diagram window, the the refrigerant desired (Fluid$) can also be changed with a 'drop down' menu. We have given the following options of refrigerants: Ammonia, R134a, R12, R22, R13, R502, since they are commonly used. However, we can easily add more from the list of refrigerants handled by EES, if required.

For the above case, after entering the inputs and clicking on 'Calculate' button, the results are:

Actual vapour compression cycle with subcooling and superheat



Calculate

INPUTS:

T_1 = -15 [C]

Fluid$= R12

T_4 = 20 [C]

η_{comp} = 1

$\Delta T_{subcool}$ = 0 [C]

$\Delta T_{superheat}$ = 0 [C]

OUTPUTS:

P_1 = 1.824 [bar]

P_2 = 5.668 [bar]

q_L = 126.1 [kJ/kg]

q_{cond} = 145.9 [kJ/kg]

$w_{comp,act}$ = 19.77 [kJ/kg]

x_5 = 0.2051

See the OUTPUTS above for results.

Also, from Results tab:

Main | Vap_Comp_Refrign_cycle_actual

Unit Settings: SI C bar kJ mass deg

COP = 6.378	$\Delta T_{subcool}$ = 0 [C]	$\Delta T_{superheat}$ = 0 [C]
η_{comp} = 1	Fluid$ = 'R12'	$mass_{flow}$ = 0.03965 [kg/s]
$Power_{input}$ = 0.7839 [kW]	q_{cond} = 145.9 [kJ/kg]	q_L = 126.1 [kJ/kg]
$w_{comp,act}$ = 19.77 [kJ/kg]	$w_{comp,isentr}$ = 19.77 [kJ/kg]	

And:

Main | Vap_Comp_Refrign_cycle_actual

Local variables in Procedure Vap_Comp_Refrign_cycle_actual (1 call, 0.02 sec)

COP=6.378	$\Delta T_{subcool}$=0 [C]	$\Delta T_{superheat}$=0 [C]	η_{comp}=1
h_1=181 [kJ/kg]	h_2=200.7 [kJ/kg]	h_3=200.7 [kJ/kg]	h_4=54.86 [kJ/kg]
P_1=1.824 [bar]	P_2=5.668 [bar]	P_3=5.668 [bar]	P_4=5.668 [bar]
q_{cond}=145.9 [kJ/kg]	q_L=126.1 [kJ/kg]	s_1=0.7051 [kJ/kg-C]	s_2=0.7051 [kJ/kg-C]
T_2=27.03 [C]	T_3=27.03 [C]	T_4=20 [C]	T_5=-15
$w_{comp,isentr}$=19.77 [kJ/kg]	x_1=1	x_4=0	x_5=0.2051

Thus:

Refrig. capacity per kg = 126.1 kJ/kg …. Ans.

Mass flow rate of refrigerant = 0.03965 kg/s … Ans.

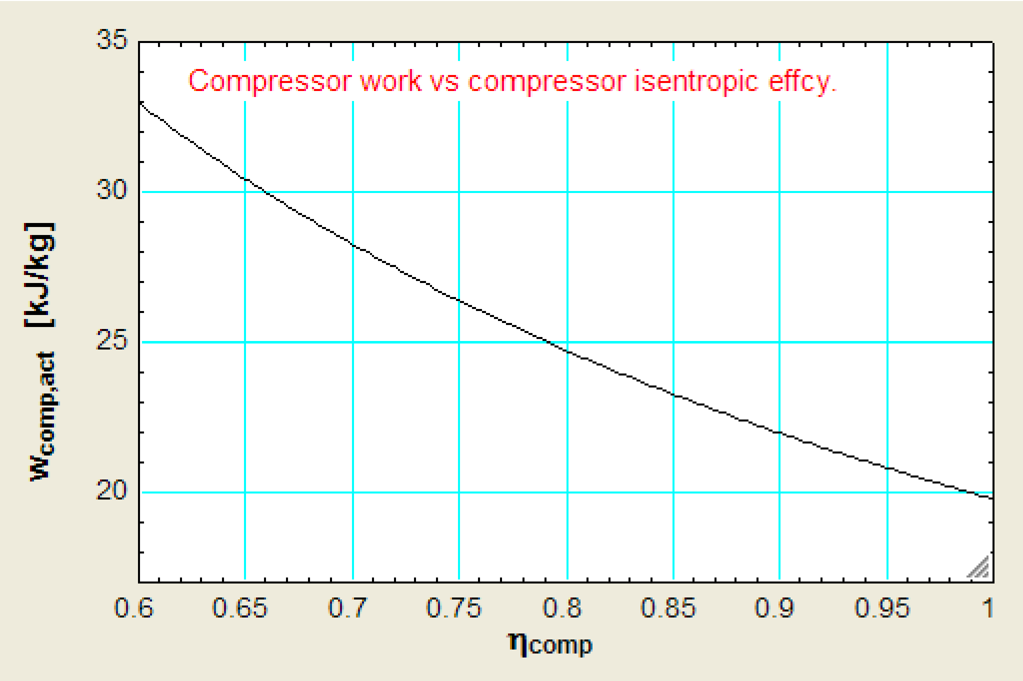
COP = 6.378 … Ans.

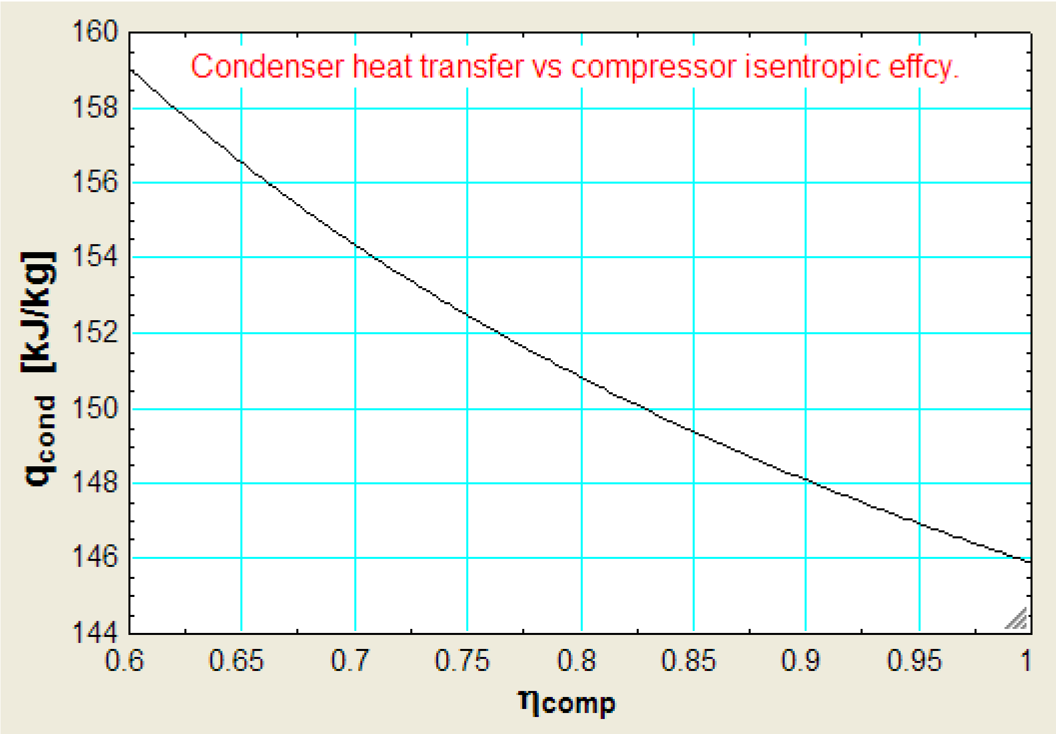
(b) Plot the variation of compressor work, condenser heat transfer, COP and temp(T3) at the exit of compressor after actual compression, as the compressor effcy. varies from 0.6 to 1:

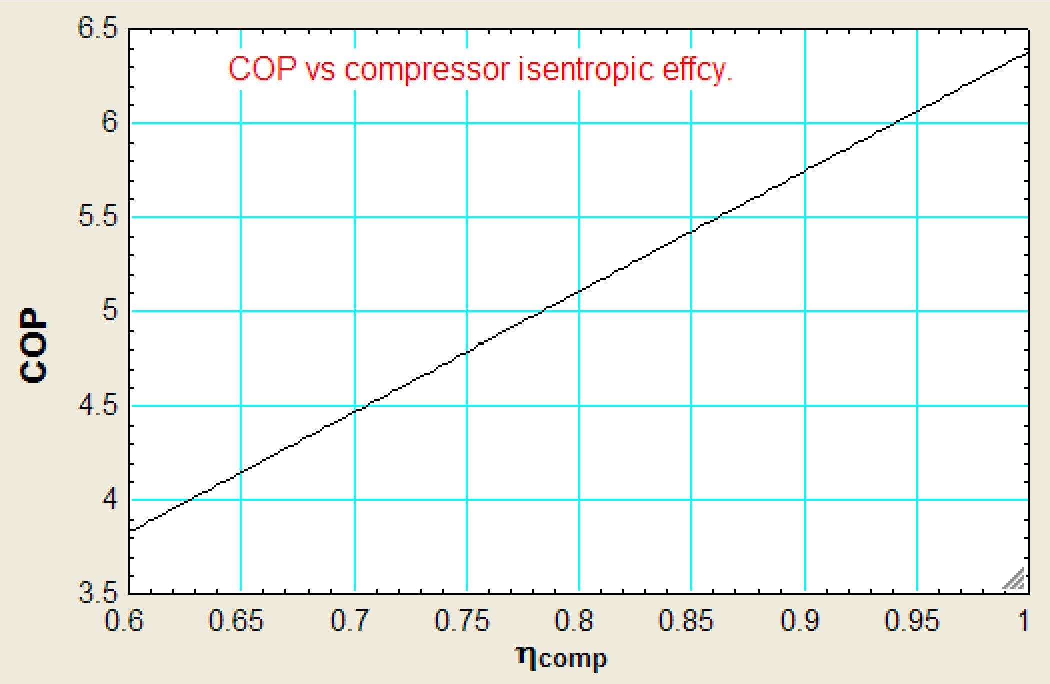
First, compute the Parametric Table:

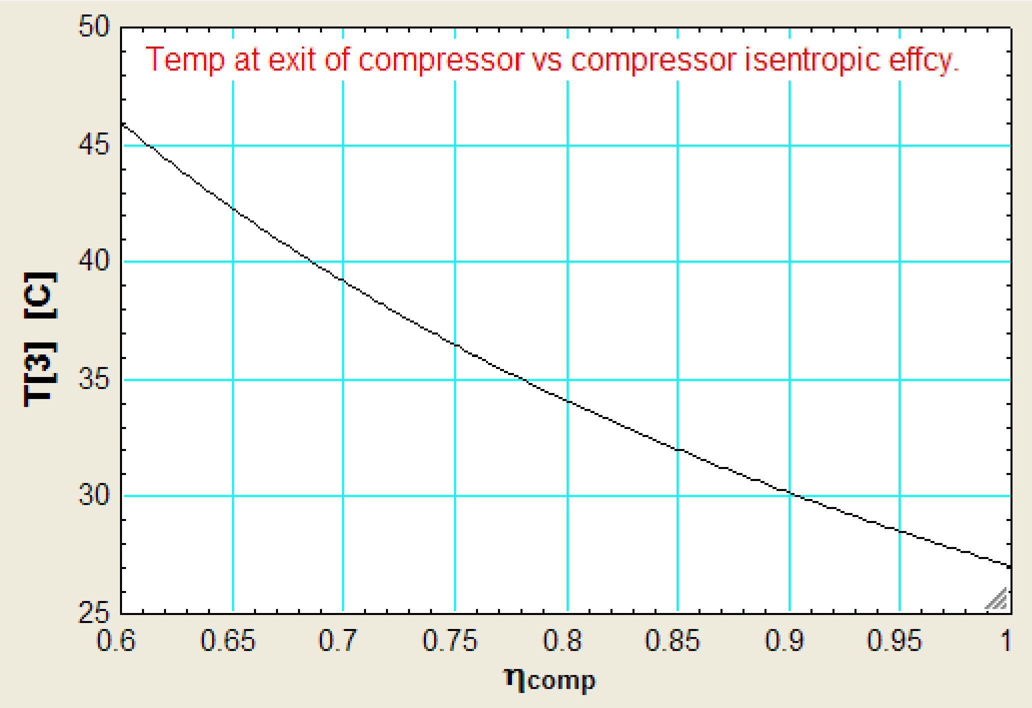
1..9	η_{comp}	$w_{comp,act}$ [kJ/kg]	q_{cond} [kJ/kg]	COP	T_3 [C]
Run 1	0.6	32.95	159.1	3.827	45.93
Run 2	0.65	30.42	156.5	4.146	42.28
Run 3	0.7	28.24	154.4	4.465	39.16
Run 4	0.75	26.36	152.5	4.784	36.45
Run 5	0.8	24.71	150.8	5.103	34.09
Run 6	0.85	23.26	149.4	5.422	32.01
Run 7	0.9	21.97	148.1	5.741	30.16
Run 8	0.95	20.81	146.9	6.06	28.51
Run 9	1	19.77	145.9	6.378	27.03

Now, plot the results:






COP vs compressor isentropic effcy.
COP
ηcomp
6.5
6
5.5
5
4.5
4
3.5
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1


Temp at exit of compressor vs compressor isentropic effcy.
T[3] [C]
ηcomp
50
45
40
35
30
25
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1

==

Reversed Brayton cycle refrigerator:

"**Prob.4.4.5.** Air enters the compressor of a Brayton cycle refrigerator at 7 C and 35 kPa, and the turbine at 37 C and 160 kPa.Determine, per kg of air, (i) refrign. effect, (ii) net work input, and (iii) the COP. Take the efficiencies of compressor and turbine as 80% and 85%.

(b) Plot these quantities as the both the efficiencies vary together from 70% to 100%."

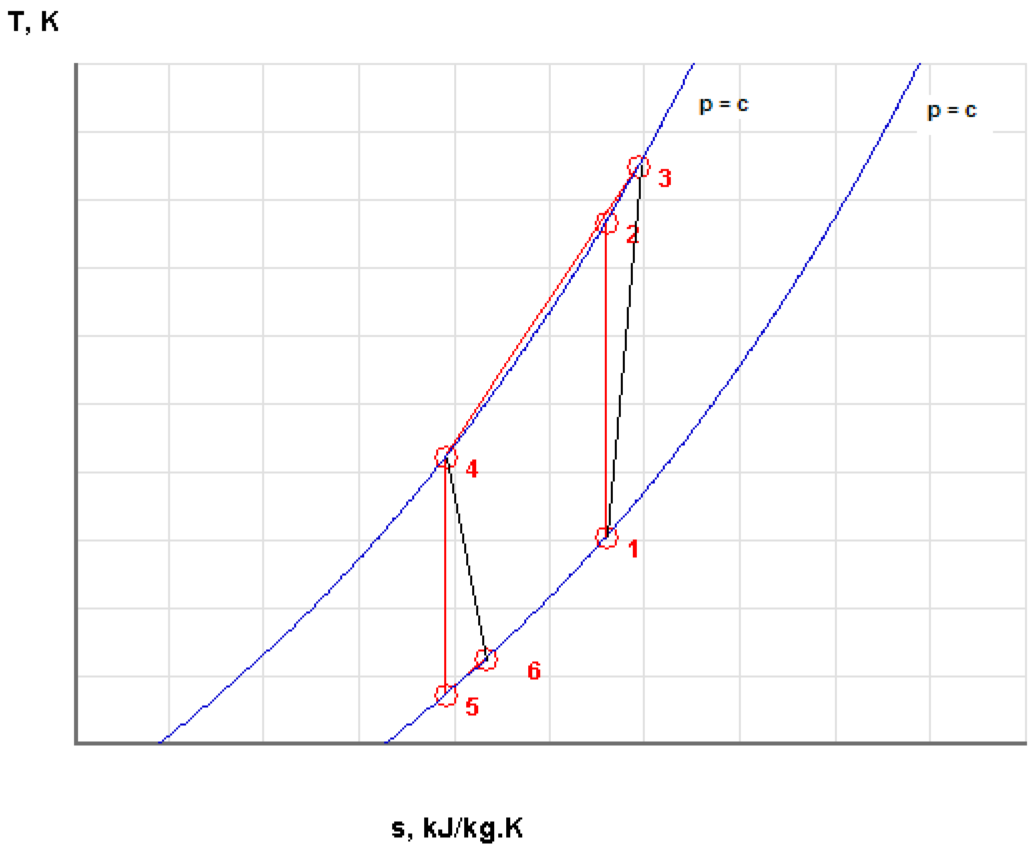


Fig.Prob.4.4.5 T-s diagram for Brayton cycle refrigeration

EES Solution:

"Refer to the schematic diagram in diagram window:"

"Data:"

gamma = 1.4
cp = 1.005 "kJ/kg-C"
P1 = 35 "kPa"
T1 = 7 + 273 "k"
P2 = 160 "kPa"

T4 = 37 + 273 “k”

eta_comp = 0.8“...compressor isentropic effcy.”

eta_turb = 0.85“..turbine isentropic effcy.”

P3 = P2

P4 = P2

P6 = P1

P5 = P1

“Calculations:”

“Find temperatures at various State points:”

“State 2:”

T2/T1 = (P2 / P1)^ ((gamma – 1) / gamma)” ....finds T2 (K)”

“State 3:”

T3 = T1 + (T2 – T1) / eta_comp“...finds T3 (K)”

"State 5:"

T4/T5 = (P4 / P5)^((gamma – 1) / gamma)" ....finds T5 (K)"

"State 6:"

T6 = T4 – (T4 – T5) * eta_turb"...finds T6 (K)"

"Isentropic compressor work:"

w_comp_id = cp * (T2 – T1) "kJ/kg"

"Actual compressor work:"

w_comp_act = cp * (T3 – T1) "kJ/kg"

"Isentropic turbine work:"

w_turb_id = cp * (T4 – T5) "kJ/kg"

"Actual turbine work:"

w_turb_act = cp * (T4 – T6) "kJ/kg"

"Net work required:"

w_net = w_comp_act – w_turb_act "kJ/kg"

"Refrign. effect:"

q_in = cp * (T1 – T6) "kJ/kg"

"Coeff. of Performance:"

COP = q_in / w_net

Results:

Unit Settings: SI K kPa kJ mass deg

COP = 0.6442	cp = 1.005 [kJ/kg-K]	η_{comp} = 0.8
η_{turb} = 0.85	γ = 1.4	P1 = 35 [kPa]
P2 = 160 [kPa]	P3 = 160 [kPa]	P4 = 160 [kPa]
P5 = 35 [kPa]	P6 = 35 [kPa]	q_{in} = 63.13 [kJ/kg]
T1 = 280 [K]	T2 = 432.3 [K]	T3 = 470.3 [K]
T4 = 310 [K]	T5 = 200.8 [K]	T6 = 217.2 [K]
$w_{comp,act}$ = 191.3 [kJ/kg]	$w_{comp,id}$ = 153 [kJ/kg]	w_{net} = 98 [kJ/kg]
$w_{turb,act}$ = 93.28 [kJ/kg]	$w_{turb,id}$ = 109.7 [kJ/kg]	

Thus:

Net work input = w_net = 98 kJ/kg …. Ans.

Refrig. effect = q_in = 63.13 kJ/kg …. Ans.

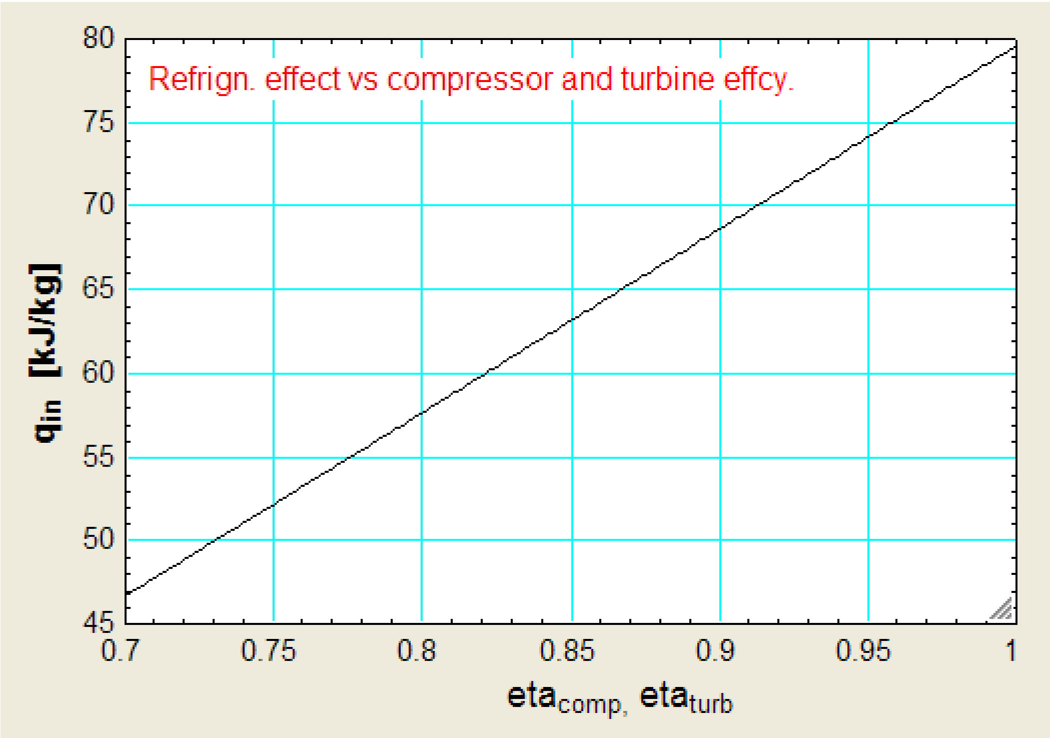
COP = 0.6442 …Ans.

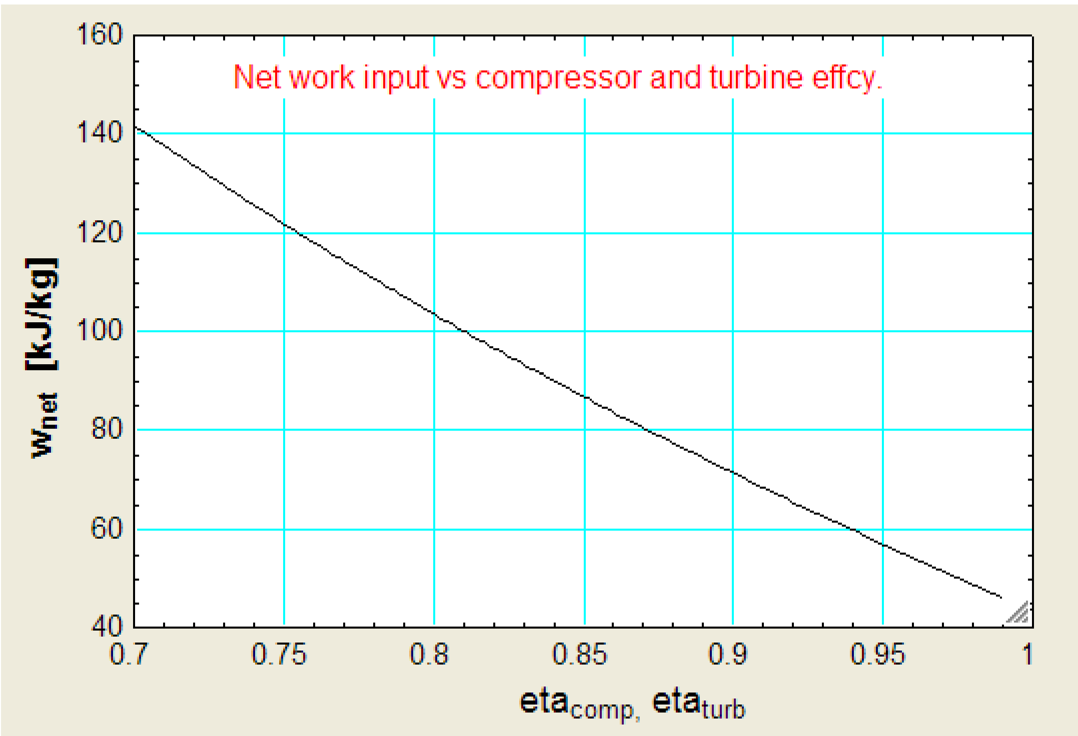
(b) Plot these quantities as the both the efficiencies vary together from 70% to 100%:

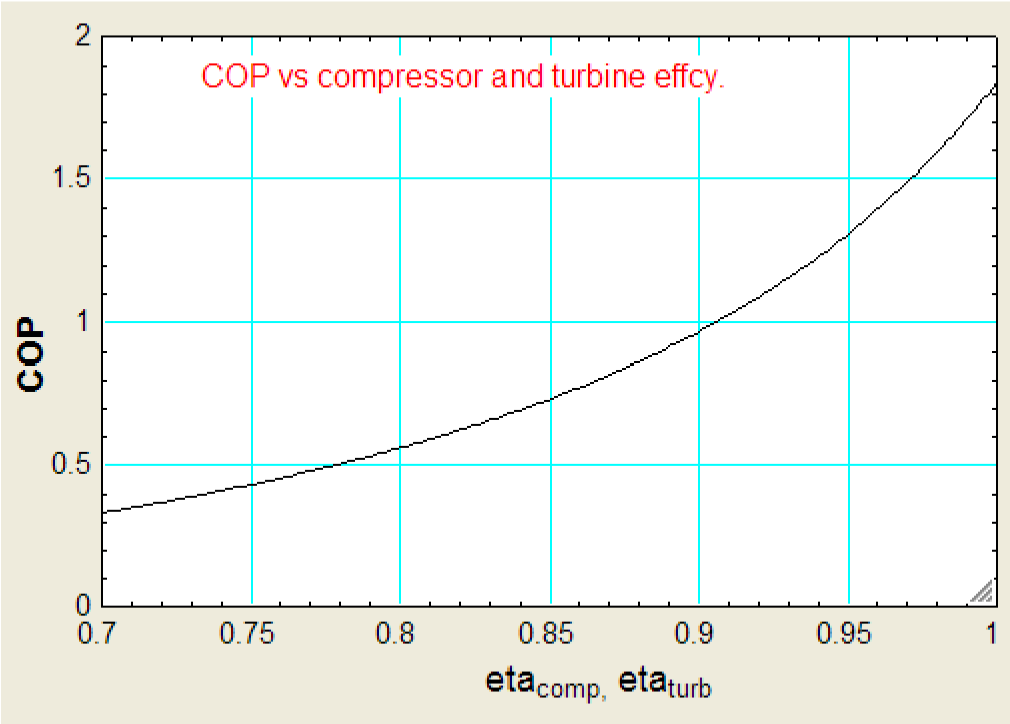
First, compute the parametric Table:

1..7	η_{comp}	η_{turb}	q_{in} [kJ/kg]	w_{net} [kJ/kg]	COP
Run 1	0.7	0.7	46.67	141.8	0.3292
Run 2	0.75	0.75	52.16	121.7	0.4285
Run 3	0.8	0.8	57.64	103.5	0.557
Run 4	0.85	0.85	63.13	86.74	0.7278
Run 5	0.9	0.9	68.62	71.26	0.963
Run 6	0.95	0.95	74.1	56.82	1.304
Run 7	1	1	79.59	43.28	1.839

Now, plot the results:






COP vs compressor and turbine effcy.
COP
2
1.5
1
0.5
0
0.7
0.75
0.8
0.85
0.9
0.95
1
eta$_{comp}$, eta$_{turb}$

===

4.5 Problems solved with TEST:

Prob.4.5.1 Refrigeration capacity of a R-12 vapour compression system is 300 kJ/min. The refrigerant enters the compressor as sat. vapour at 140 kPa and is compressed to 800 kPa. Enthalpy of vapour after compression is 215 kJ/kg. Show the cycle on T-s and P-h diagrams. Determine: (i) quality of refrigerant after throttling, (ii) COP, and (iii) power input to compressor. [VTU-ATD-Feb.2004]

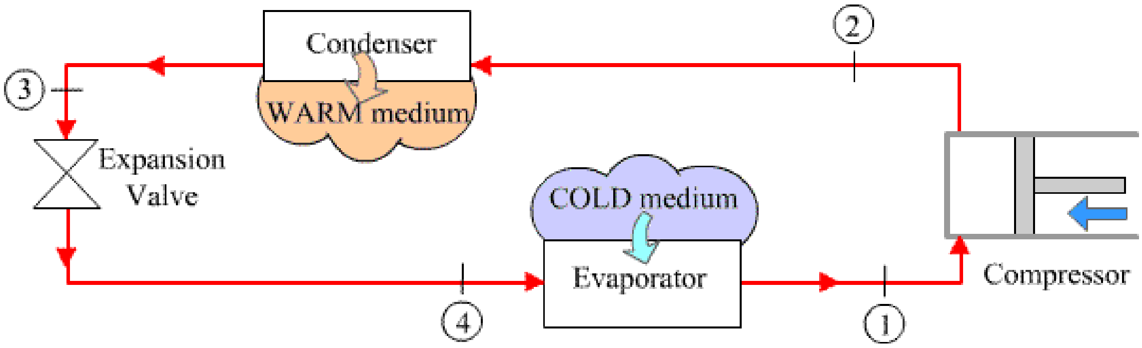
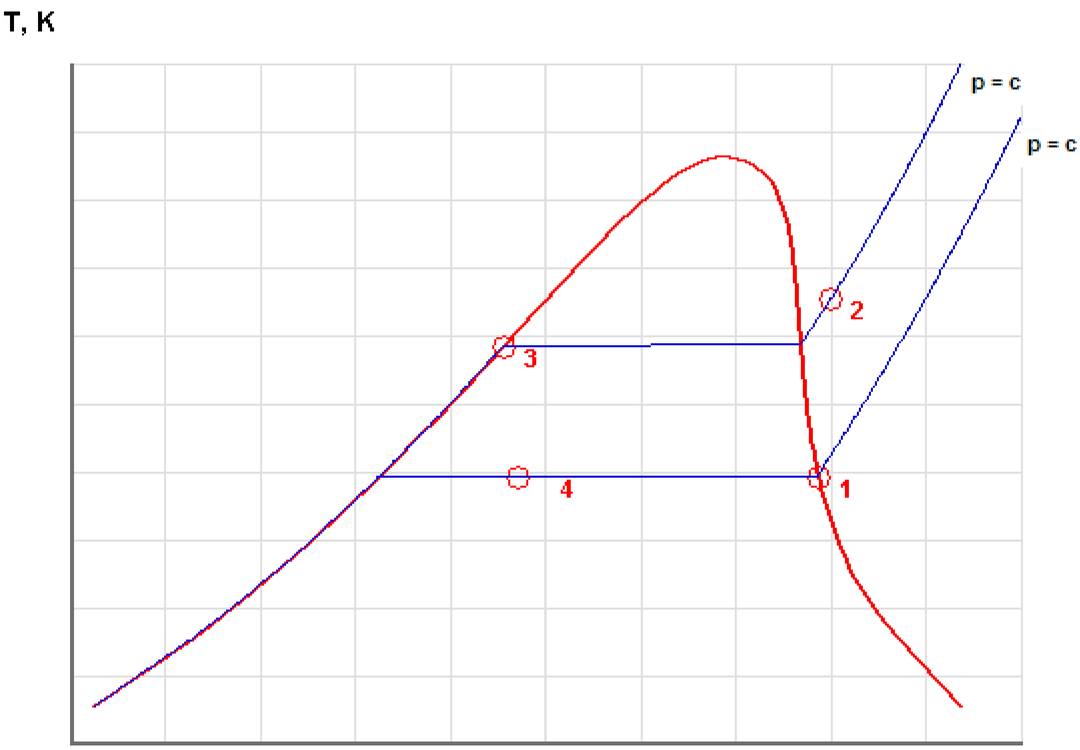


Fig.Prob.4.5.1. Vapour compression refrigeration system



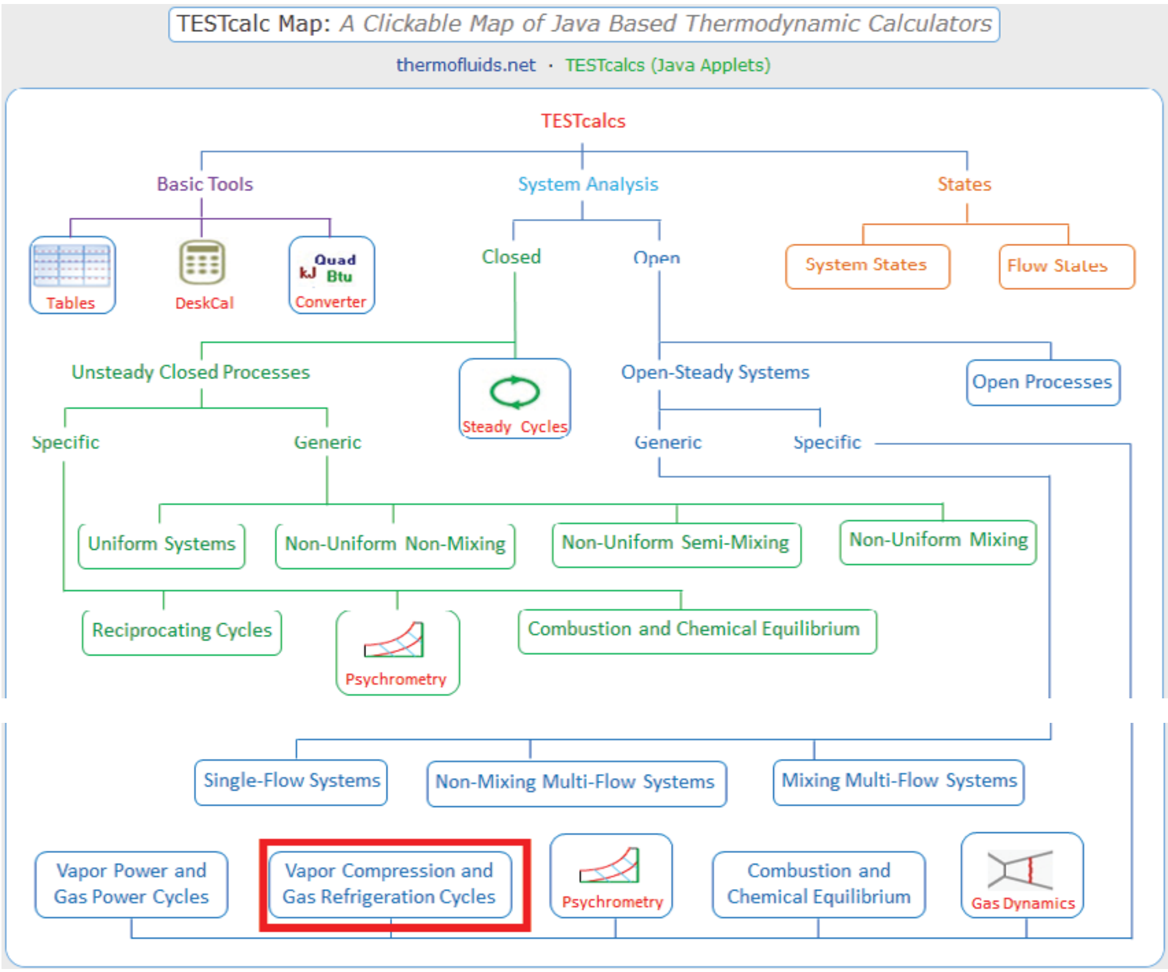
Note: Actual T-s and P-h diagrams are shown later in the solution.

TEST Solution:

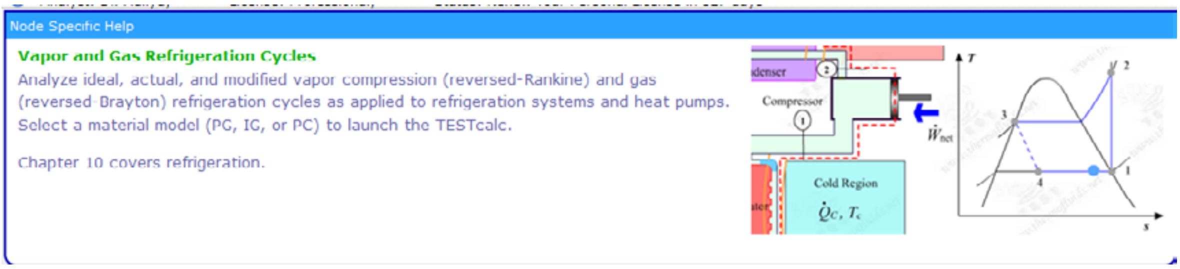
Note: We shall assume the mass flow rate of refrigerant as 1 kg/s to start with, and calculate refrig. effect, compressor work etc. Then, for a refrig. effect of 300 kJ/min, the flow rate required can easily be calculated. Then, find out the compressor work for that flow rate.

Following are the steps:

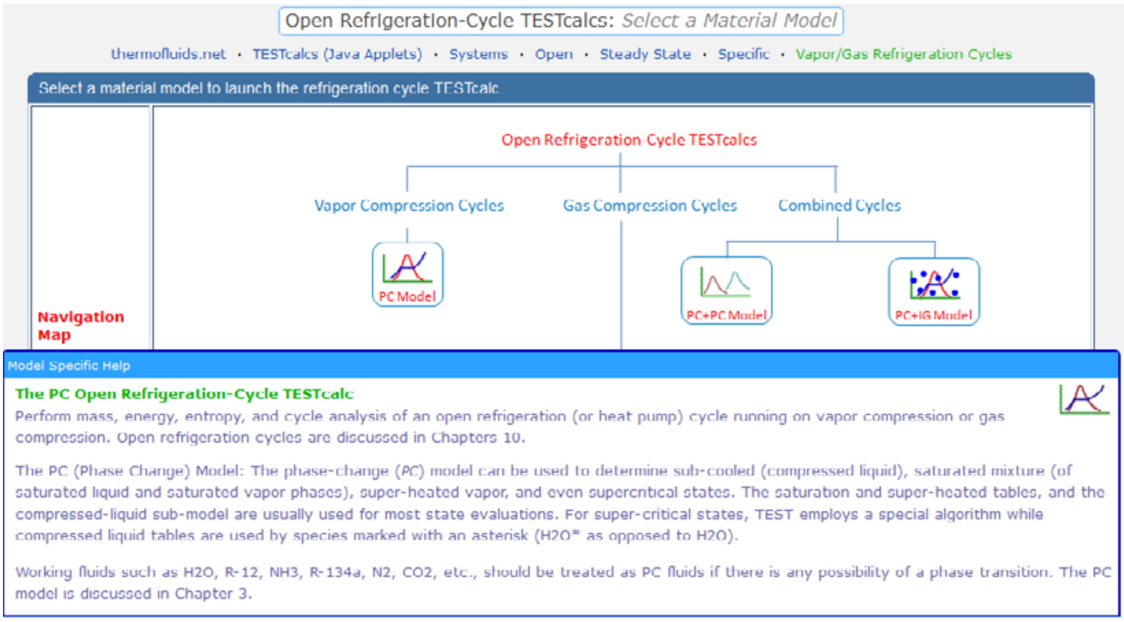
1. Go to www.thermofluids.net, enter your e-mail ID and pass word, and get the opening welcome screen. (It is assumed that you have already done the free registration at this site). Click on the TESTcalc tab at the bottom of the window to get the 'TESTcalc Map', shown below:



Hovering the mouse pointer on 'Vapor Compression and Gas Refrigeration Cycles' gives the following explanatory pop-up:



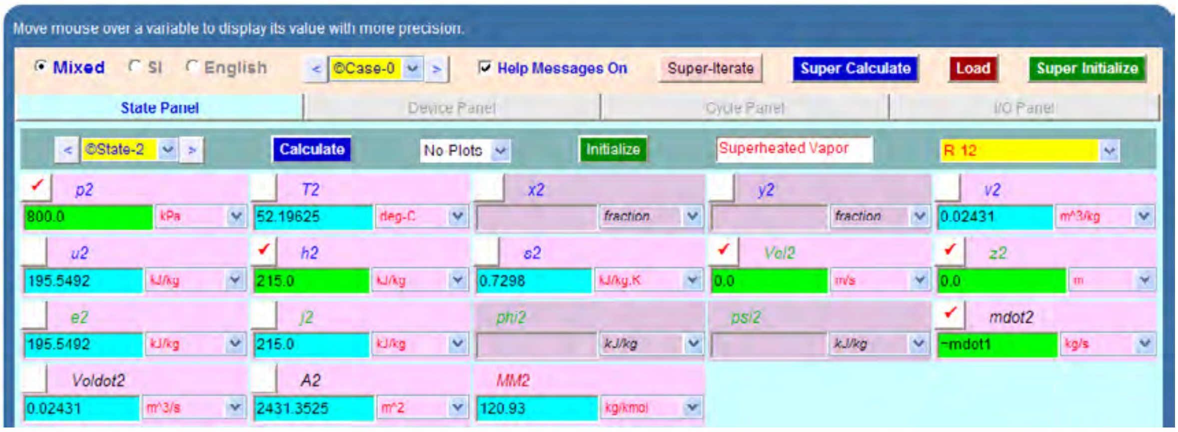
2. Click on 'Vapor Compression and Gas Refrigeration Cycles', choose PC model for 'material model' as shown below:



3. Choose R-12 as working substance and fill up the known parameters for State 1, i.e. P1, x1 and mdot1 = 1 kg/s. Hit Enter. We get:

Note that all parameters such as h1, T1, s1 etc are calculated.

4. State 2: Enter P2, h2, and mdot2 = mdot1. Hit Enter. We get:



Here again, T2, s2 etc are calculated.

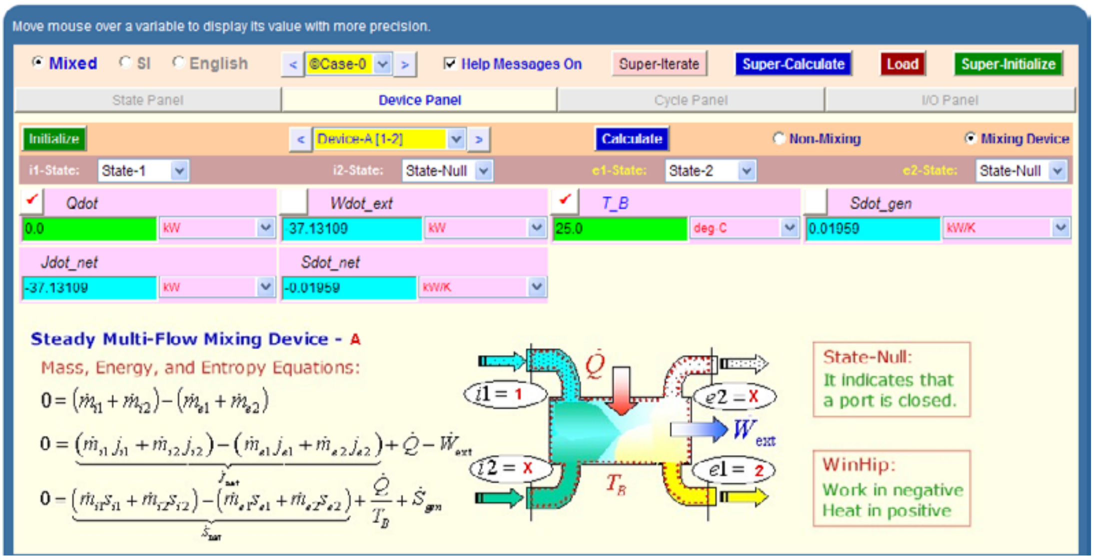
5. For State 3: Enter p3 = p2, x3 = 0, mdot3 = mdot1. Hit Enter. We get:

Note that h3, s3 etc are calculated.

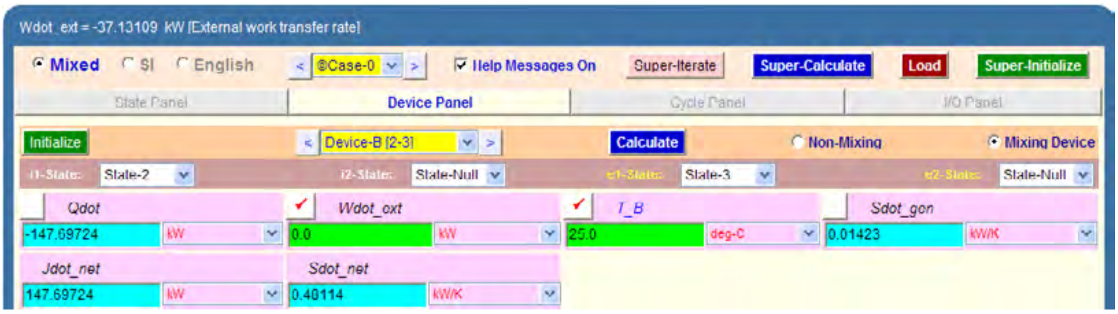
6. For State 4: Enter p4 = p1, h4 = h3, mdot4 = mdot1. Hit Enter. We get:

Note that h4, T4, s4, and x4 etc are calculated. Note that x4 = 0.317

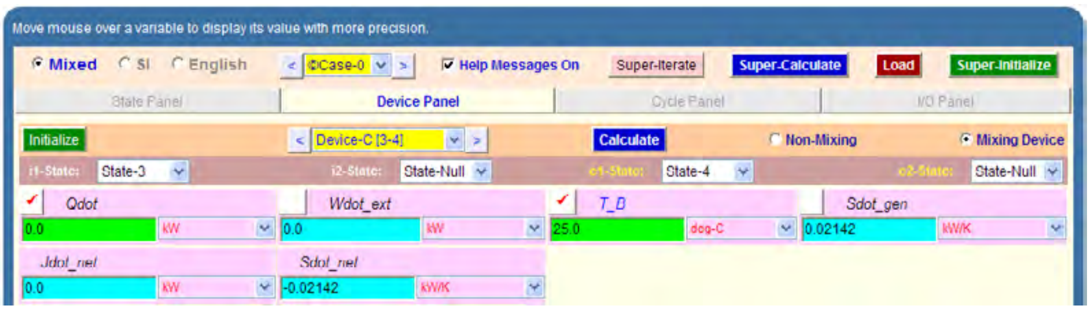
7. Now, go to Device panel. For Device A, fill up State 1 and State 2 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. And, Qdot = 0. Hit Enter. We get:



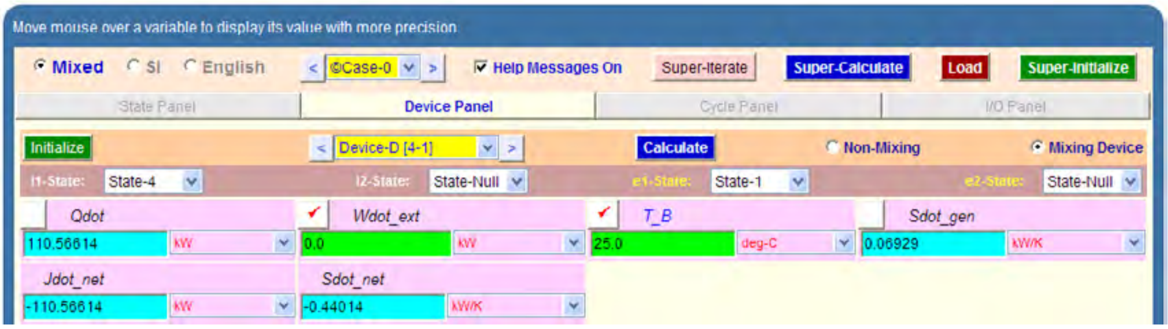
8. For Device B:fill up State 2 and State 3 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Wdot_ext = 0, since there is no external work in process 2-3. Hit Enter. We get:



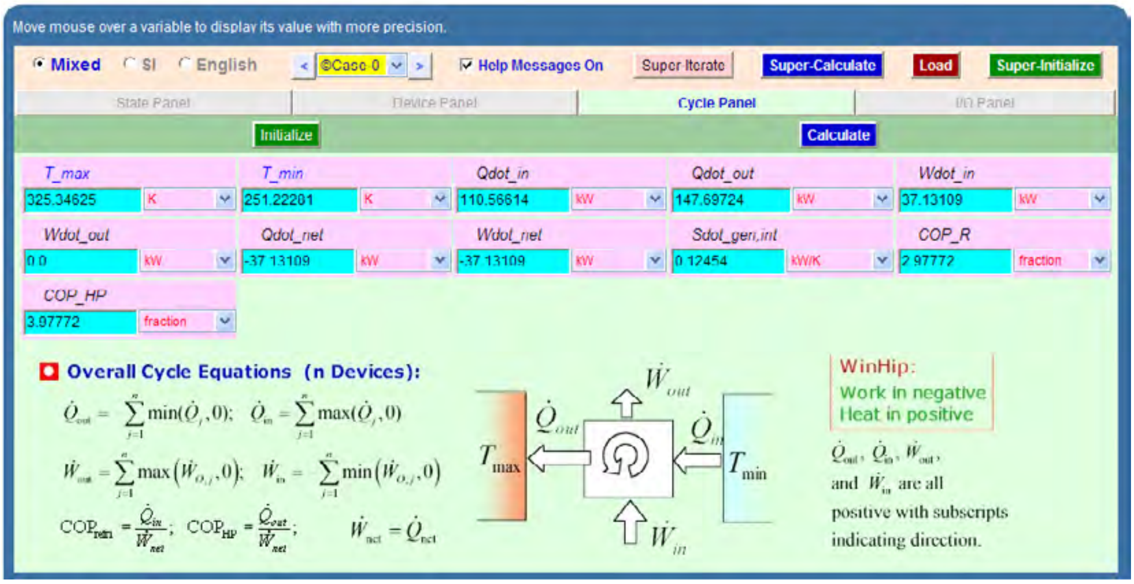
9. For Device C: fill up State 3 and State 4 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Qdot = 0, since there is no heat transfer in process 3-4. Hit Enter. We get:



10. For Device D: fill up State 4 and State 1 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Wdot_ext = 0, since there is no work transfer in process 4-1. Hit Enter. And click on SuperCalculate.We get:



11. Now go to Cycle panel. All important cycle parameters are available here:



Thus:

Refrign. effect = Qdot_in = h1-h4 = 110.566 kJ/kg, for a refrigerant mass flow rate of 1 kg/s

And, compressor power = Wdot_in = h2- h1 = 37.13 kW, for a refrigerant mass flow rate of 1 kg/s

Therefore, mass flow rate of R-12 required for a refrign. effect of 300 kJ/min is:

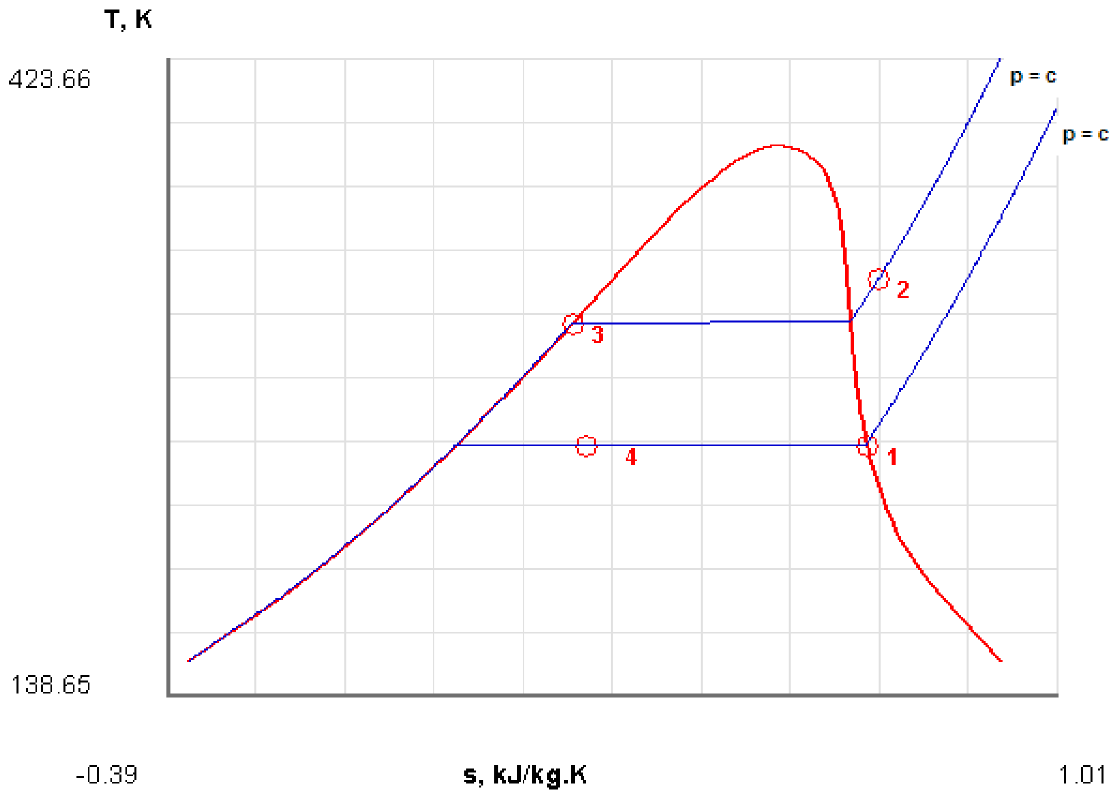
Mass flow rate = (300/60)/(h1-h4) = 0.04522 kg/s … Ans.

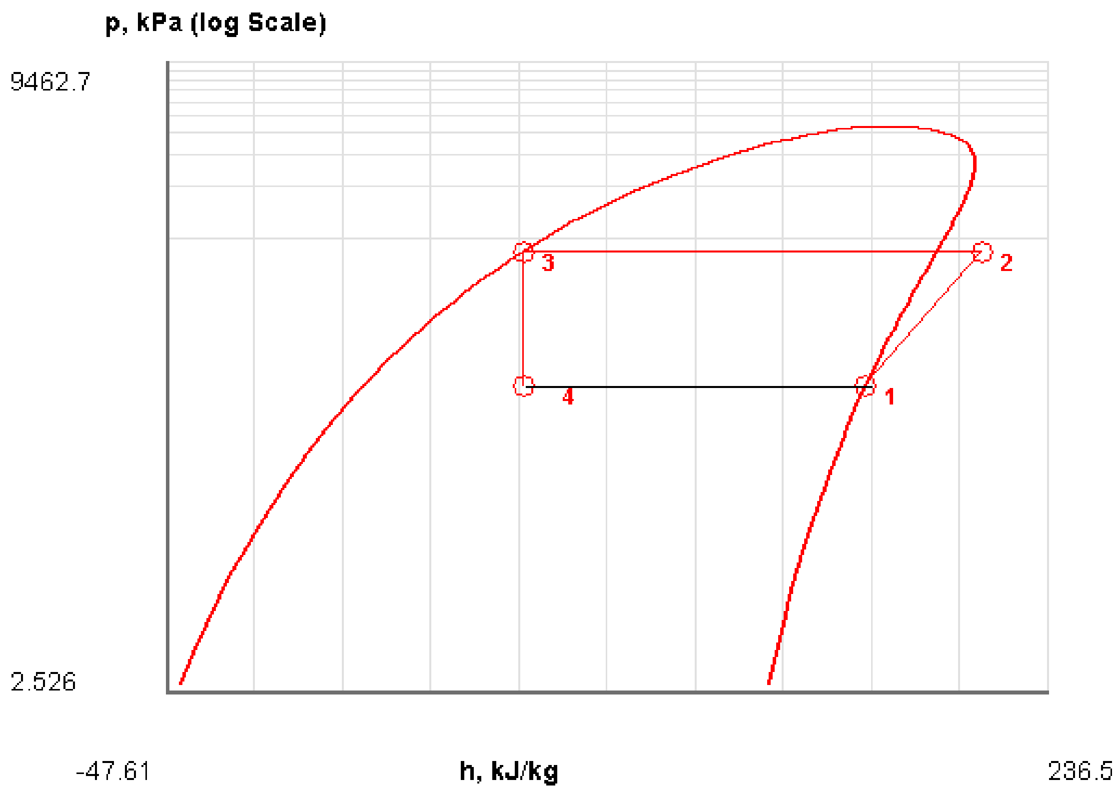
And, compressor power = 0.04522*(h2-h1) = 1.679 kW.. Ans.

COP of refrigerator = COP_R = 2.98 … Ans.

Note that quality of refrigerant after throttling, x4 = 0.317 … Ans.

12. **From the Plots widget, first get the T-s plot, and then get h-s plot:**





13. The I/O panel gives the TEST code etc:

TEST-code: To save the solution, copy the codes generated below into a text file. To reproduce the solution at a later time, launch the daemon (TESTcalc) (see path name below), paste the saved TEST-code at the bottom of this I/O panel, and click the Load button.

Daemon (TESTcalc) Path: Systems>Open>SteadyState>Specific>RefrigCycle>PC-Model; v-10.cd03

#--------------------Start of TEST-code ---

States {

State-1: R-12;

Given: { p1= 140.0 kPa; x1= 1.0 fraction; Vel1= 0.0 m/s; z1= 0.0 m; mdot1= 1.0 kg/s; }

State-2: R-12;

Given: { p2= 800.0 kPa; h2= 215.0 kJ/kg; Vel2= 0.0 m/s; z2= 0.0 m; mdot2= "mdot1" kg/s; }

State-3: R-12;

Given: { p3= "p2" kPa; x3= 0.0 fraction; Vel3= 0.0 m/s; z3= 0.0 m; mdot3= "mdot1" kg/s; }

State-4: R-12;

Given: { p4= "p1" kPa; h4= "h3" kJ/kg; Vel4= 0.0 m/s; z4= 0.0 m; mdot4= "mdot1" kg/s; }

}

Analysis {

Device-A: i-State = State-1; e-State = State-2; Mixing: true;

Given: { Qdot= 0.0 kW; T_B= 25.0 deg-C; }

Device-B: i-State = State-2; e-State = State-3; Mixing: true;

Given: { Wdot_ext= 0.0 kW; T_B= 25.0 deg-C; }

Device-C: i-State = State-3; e-State = State-4; Mixing: true;

Given: { Qdot= 0.0 kW; T_B= 25.0 deg-C; }

Device-D: i-State = State-4; e-State = State-1; Mixing: true;

Given: { Wdot_ext= 0.0 kW; T_B= 25.0 deg-C; }

}

#----------------------End of TEST-code ---

#--------Property spreadsheet starts:

# State	p(kPa)	T(K)	x	v(m3/kg)	u(kJ/kg)	h(kJ/kg)	s(kJ/kg.K)
# 01	140.0	251.2	1.0	0.117	161.52	177.87	0.71
# 02	800.0	325.3		0.0243	195.55	215.0	0.73
# 03	800.0	305.9	0.0	8.0E-4	66.68	67.3	0.249
# 04	140.0	251.2	0.3	0.0375	62.06	67.3	0.27

 # Cycle Analysis Results:

Calculated: T_max= 325.34625 K; T_min= 251.22281 K; **Qdot_in= 110.56614 kW**;

Qdot_out= 147.69724 kW; **Wdot_in= 37.13109 kW**; Wdot_out= 0.0 kW;

Qdot_net= -37.13109 kW; Wdot_net= -37.13109 kW; Sdot_gen,int= 0.12454 kW/K;

COP_R= 2.97772 fraction; COP_HP= 3.97772 fraction; BWR= Infinity %;

#******CALCULATE VARIABLES: Type in an expression starting with an '=' sign ('= mdot1*(h2-h1)', '= sqrt(4*A1/PI)', etc.) and press the Enter key)*********

##Refrign. effect:

h1-h4 = 110.5661392211914 kJ/kg

#mass flow rate for a refrign. capacity of 300 kJ/min:

(300/60)/(h1-h4) = 0.045221801495639875 kg/s

#compressor power:

0.04522*(h2-h1) = 1.6790678109741215 kW

==

Prob.4.5.2. An ammonia vapour compression refrigeration plant operates between evaporator pressure of 1.907 bar and condenser pressure of 15.57 bar. The vapour has a dryness fraction of 0.8642 at entry to the compressor. Determine (i) COP, and (ii) refrigeration effect produced for a work input of 1 kW. [VTU-ATD-July–Aug.2005]

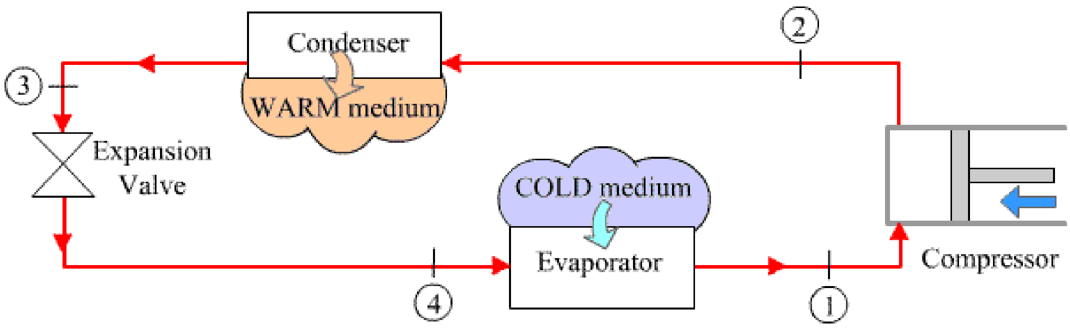


Fig.Prob.4.5.2. Vapour compression refrigeration system

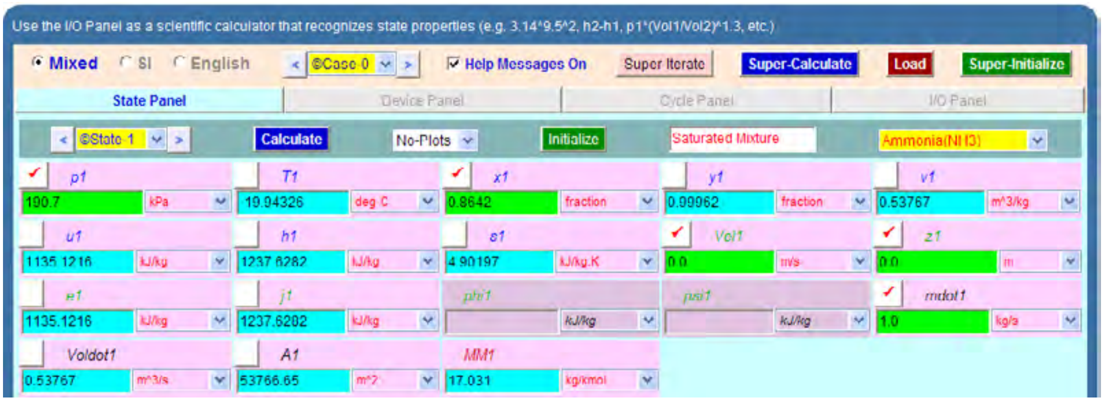
TEST Solution:

Note: We shall assume the mass flow rate of refrigerant as 1 kg/s to start with, and calculate refrig. effect, compressor work etc. Then, for a compressor work of 1 kW, the flow rate required can easily be calculated. Then, find out the refrigeration effect for that flow rate.

Following are the steps:

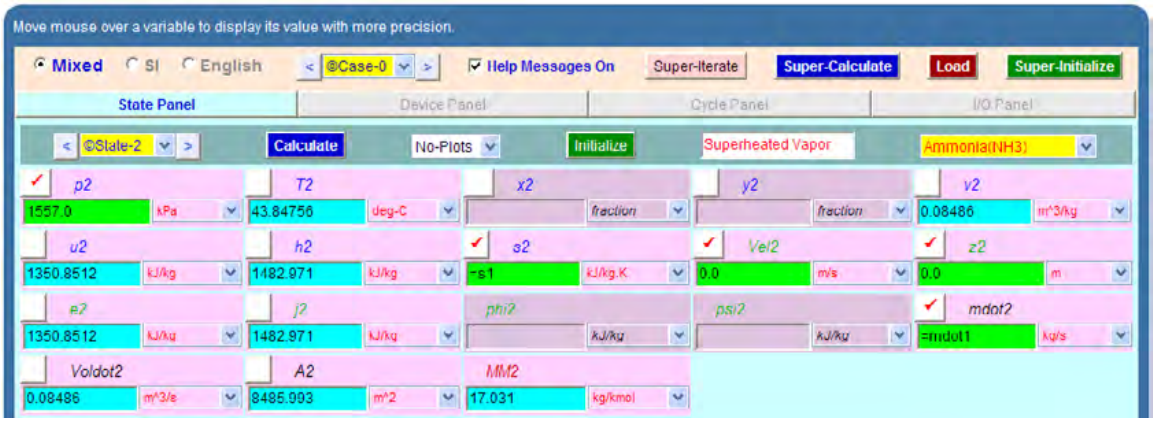
Steps 1 and 2 are the same as for previous problem. But, now the working fluid is Ammonia (NH3).

3. Choose NH3 as working substance and fill up the known parameters for State 1, i.e. P1, x1 and mdot1 = 1 kg/s. Hit Enter. We get:



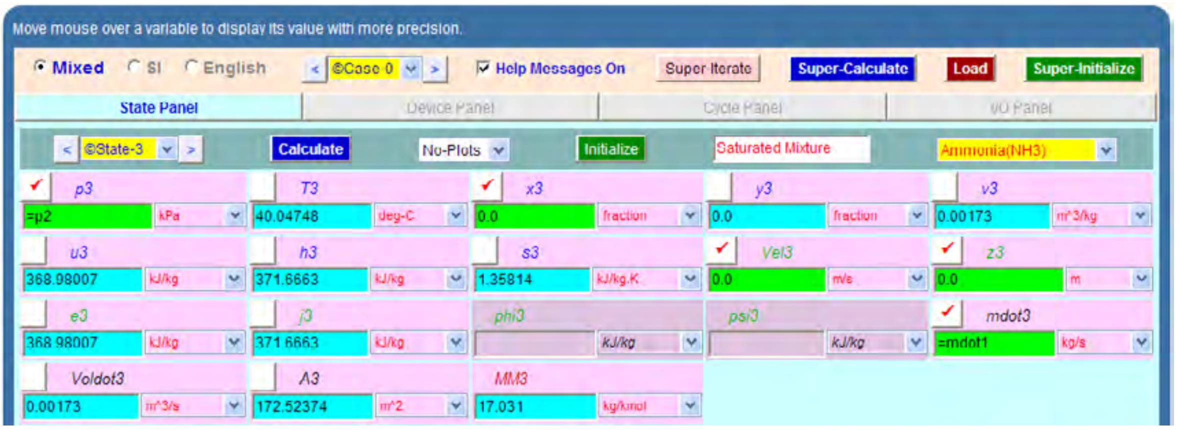
Note that all parameters such as h1, T1, s1 etc are calculated.

4. State 2: Enter P2, s2 = s1, and mdot2 = mdot1. Hit Enter. We get:



Here again, T2, h2 etc are calculated.

5. For State 3: Enter p3 = p2, x3 = 0, mdot3 = mdot1. Hit Enter. We get:

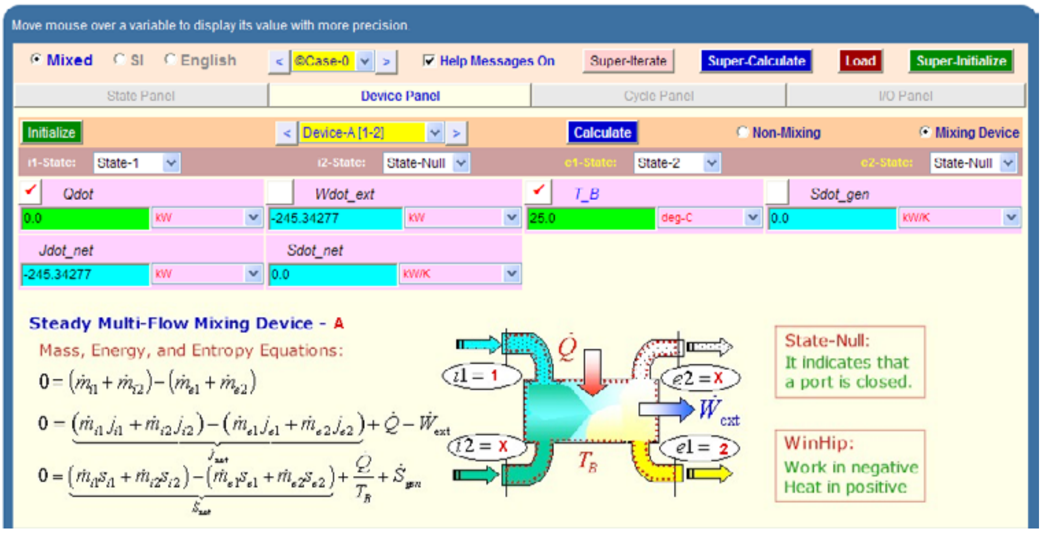


Note that h3, s3 etc are calculated.

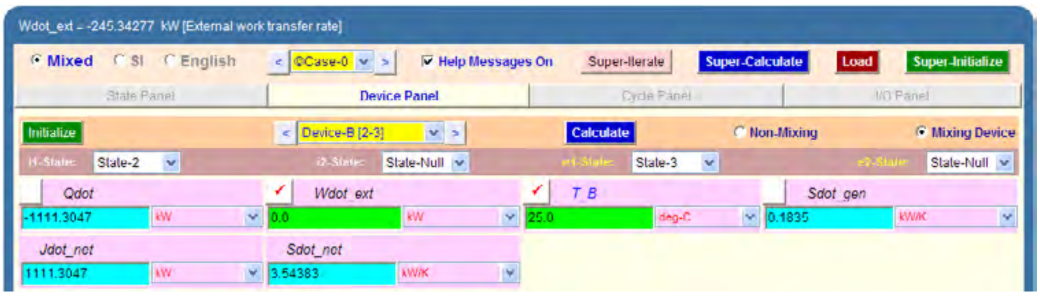
6. For State 4: Enter p4 = p1, h4 = h3, mdot4 = mdot1. Hit Enter. We get:

Note that h4, T4, s4, and x4 etc are calculated. Note that x4 = 0.2125

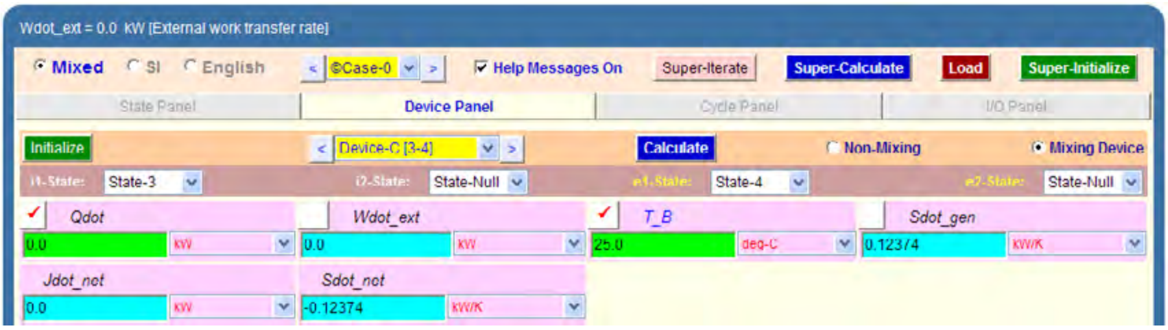
7. Now, go to Device panel. For Device A, fill up State 1 and State 2 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. And, Qdot = 0. Hit Enter. We get:



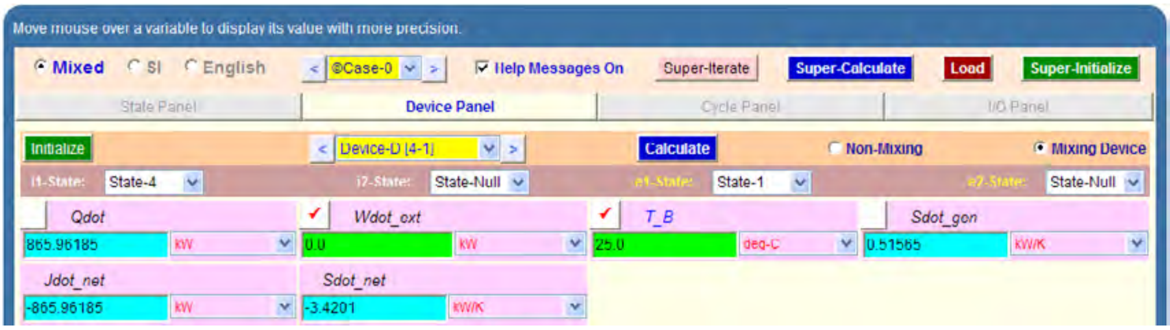
8. For Device B: fill up State 2 and State 3 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Wdot_ext = 0, since there is no external work in process 2-3. Hit Enter. We get:



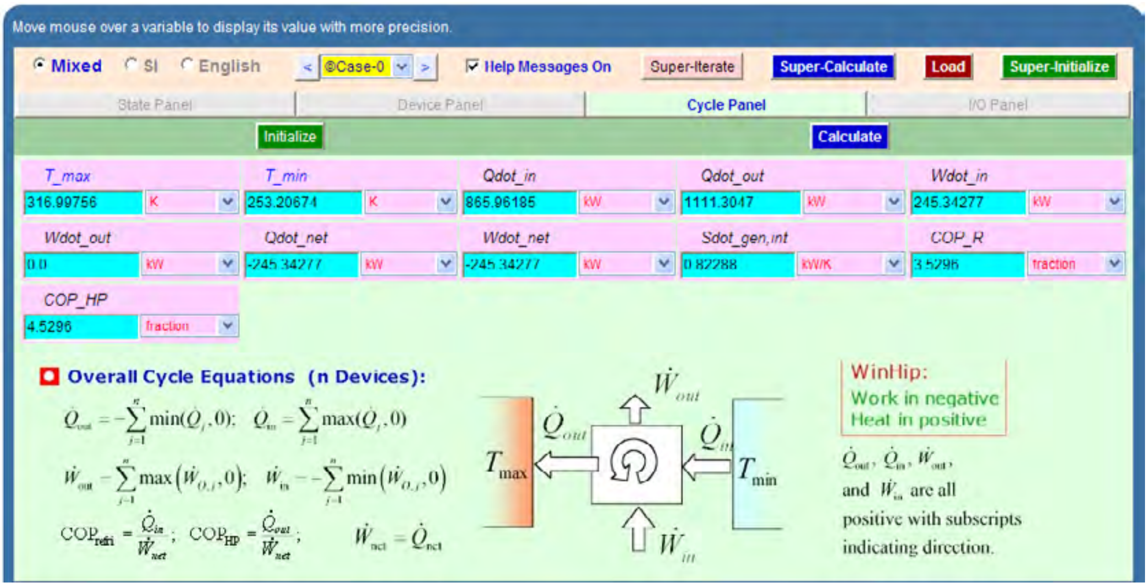
9. For Device C: fill up State 3 and State 4 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Qdot = 0, since there is no heat transfer in process 3-4. Hit Enter. We get:



10. For Device D: fill up State 4 and State 1 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Wdot_ext = 0, since there is no work transfer in process 4-1. Hit Enter. And click on SuperCalculate. We get:



11. Now go to Cycle panel. All important cycle parameters are available here:



Thus:

Refrign. effect = Qdot_in = h1-h4 = 865.96 kJ/kg, for a refrigerant mass flow rate of 1 kg/s

And, compressor power = Wdot_in = h2- h1 = 245.34 kW, for a refrigerant mass flow rate of 1 kg/s

Therefore, mass flow rate of NH3 required for a compressor power of 1 kW is:

Mass flow rate = 1/245.34277 = 0.0040759 kg/s… Ans.

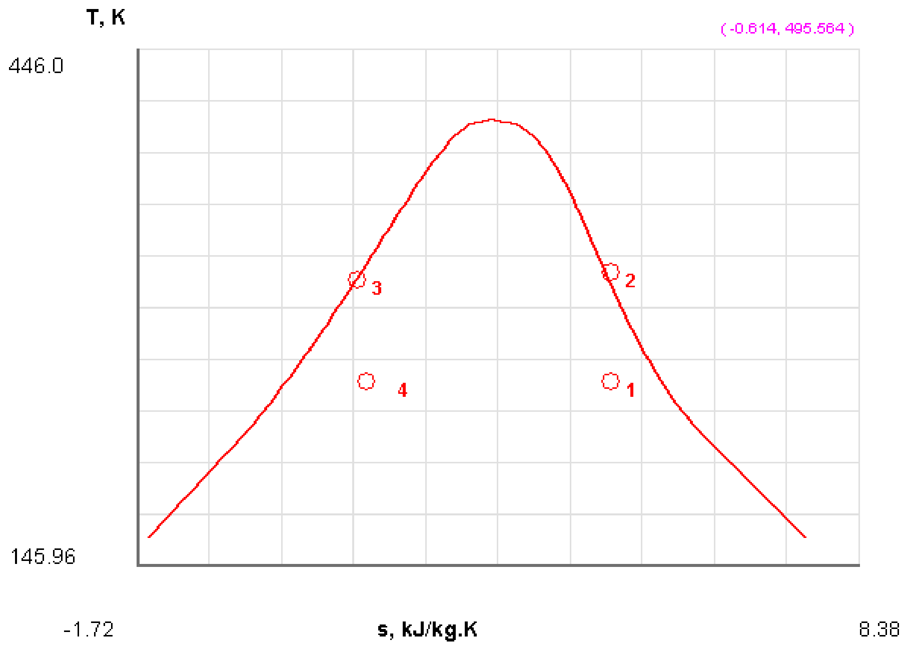
And, actual refrig. effect for this flow rate = 0.0040759*(h1-h4)= 3.53 kW.. Ans.

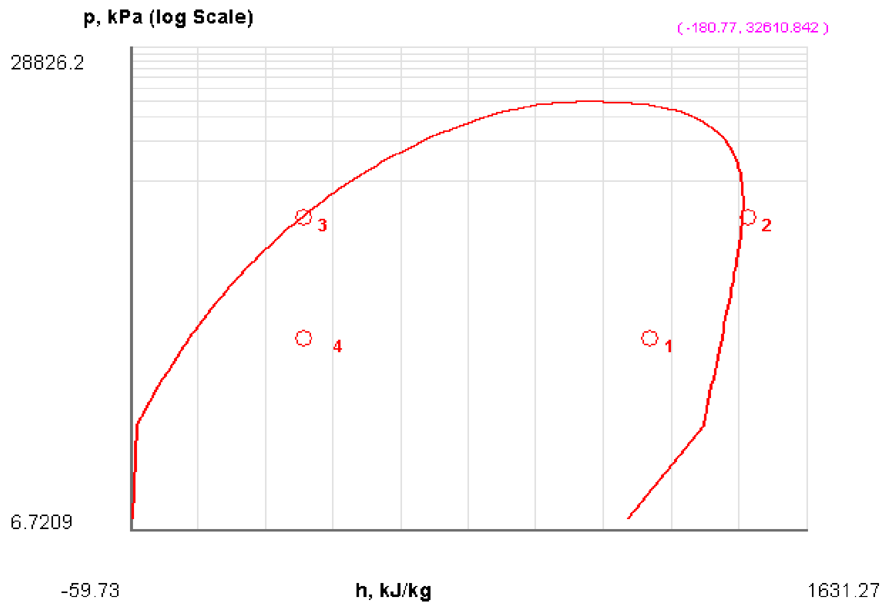
COP of refrigerator = COP_R = 3.53 … Ans.

Note that quality of refrigerant after throttling, x4 = 0.2125 … Ans.

---

12. From the Plots widget, first get the T-s plot, and then get h-s plot:





13. The I/O panel gives the TEST code etc:

#~~~~~~~~~~~~~~~~~~~~~OUTPUT OF SUPER-CALCULATE

#

Daemon (TESTcalc) Path: Systems>Open>SteadyState>Specific>RefrigCycle>PC-Model; v-10.cd03

#-------------------Start of TEST-code ---

States {

State-1: Ammonia(NH3);

Given: { p1= 190.7 kPa; x1= 0.8642 fraction; Vel1= 0.0 m/s; z1= 0.0 m; mdot1= 1.0 kg/s; }

State-2: Ammonia(NH3);

Given: { p2= 1557.0 kPa; s2= "s1" kJ/kg.K; Vel2= 0.0 m/s; z2= 0.0 m; mdot2= "mdot1" kg/s; }

State-3: Ammonia(NH3);

Given: { p3= "p2" kPa; x3= 0.0 fraction; Vel3= 0.0 m/s; z3= 0.0 m; mdot3= "mdot1" kg/s; }

State-4: Ammonia(NH3);

Given: { p4= "p1" kPa; h4= "h3" kJ/kg; Vel4= 0.0 m/s; z4= 0.0 m; mdot4= "mdot1" kg/s; }

}

Analysis {

Device-A: i-State = State-1; e-State = State-2; Mixing: true;

Given: { Qdot= 0.0 kW; T_B= 25.0 deg-C; }

Device-B: i-State = State-2; e-State = State-3; Mixing: true;

Given: { Wdot_ext= 0.0 kW; T_B= 25.0 deg-C; }

Device-C: i-State = State-3; e-State = State-4; Mixing: true;

Given: { Qdot= 0.0 kW; T_B= 25.0 deg-C; }

Device-D: i-State = State-4; e-State = State-1; Mixing: true;

Given: { Wdot_ext= 0.0 kW; T_B= 25.0 deg-C; }

}

#----------------------End of TEST-code ---

#--------Property spreadsheet starts:

# State	p(kPa)	T(K)	x	v(m3/kg)	u(kJ/kg)	h(kJ/kg)	s(kJ/kg.K)
# 01	190.7	253.2	0.9	0.5377	1135.12	1237.63	4.902
# 02	1557.0	317.0		0.0849	1350.85	1482.97	4.902
# 03	1557.0	313.2	0.0	0.0017	368.98	371.67	1.358
# 04	190.7	253.2	0.2	0.1333	346.24	371.67	1.482

 # Cycle Analysis Results:

Calculated: T_max= 316.99756 K; T_min= 253.20674 K; **Qdot_in= 865.96185 kW**;

Qdot_out= 1111.3047 kW; **Wdot_in= 245.34277 kW**; Wdot_out= 0.0 kW;

Qdot_net= -245.34277 kW; Wdot_net= -245.34277 kW; Sdot_gen,int= 0.82288 kW/K;

COP_R= 3.5296 fraction; COP_HP= 4.5296 fraction; BWR= Infinity %;

#

#******CALCULATE VARIABLES: Type in an expression starting with an '=' sign ('= mdot1*(h2-h1)', '= sqrt(4*A1/PI)', etc.) and press the Enter key)*********

#Refrign per kg flow of NH3:

=h1-h4 = 865.9618835449219 kJ/kg

#compr. work per kg flow of NH3:

=h2-h1 = 245.3427734375 kJ/kg

#Mass flow for 1 kW compr. power:

=1/245.34277 = **0.0040759301771965805 kg/s**

#Then, refrig. effect for this flow rate:

= 0.0040759301771965805*(h1-h4) = **3.5296001734427382 kW**

==

Prob.4.5.3. An ammonia vapour compression refrigeration plant operates between an evaporator pressure of 1.2 bar and condenser pressure of 12 bar. The refrigerant leaves the evaporator at -20 C and leaves the condenser at 20 C. Determine the COP of the system and the power required per ton of refrigeration. Determine also the bore and stroke of the compressor cylinder if the speed is 200 rpm, volumetric efficiency is 0.8 and stroke is 1.5 times the bore. [VTU-ATD-June–July 2008]

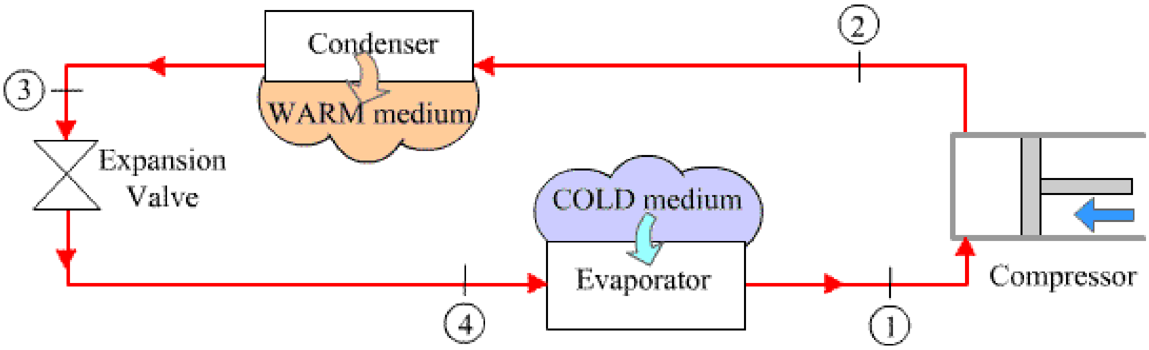


Fig.Prob.4.5.3. Vapour compression refrigeration system

TEST Solution:

Note: We shall assume the mass flow rate of refrigerant as 1 kg/s to start with, and calculate refrig. effect, compressor work etc. Knowing the refrgn. effect and compressor power, refrign. effect per ton of refrigeration is found out. Also, for a refrigeration of 1 ton (= 211 kJ/min), the flow rate required can easily be calculated. Then, find out the volume of refrigerant at the inlet to compressor, and knowing the volumetric effcy and the stroke to bore ratio, bore dia is calculated, and the the stroke is calculated.

Following are the steps:

Steps 1 and 2 are the same as for Prob.4.5.1. And, the working fluid is Ammonia (NH3).

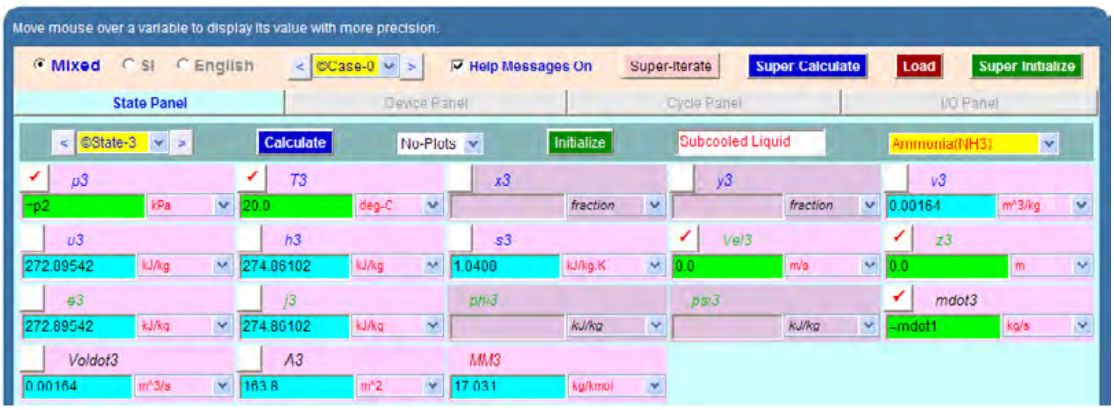
1. Choose NH3 as working substance and fill up the known parameters for State 1, i.e. P1, T1 and mdot1 = 1 kg/s. Hit Enter. We get:

Note that all parameters such as h1, T1, s1 etc are calculated.

2. State 2: Enter P2, s2 = s1, and mdot2 = mdot1. Hit Enter. We get:

Here again, T2, h2 etc are calculated.

3. For State 3: Enter p3 = p2, T3 = 20 C, mdot3 = mdot1. Hit Enter. We get:

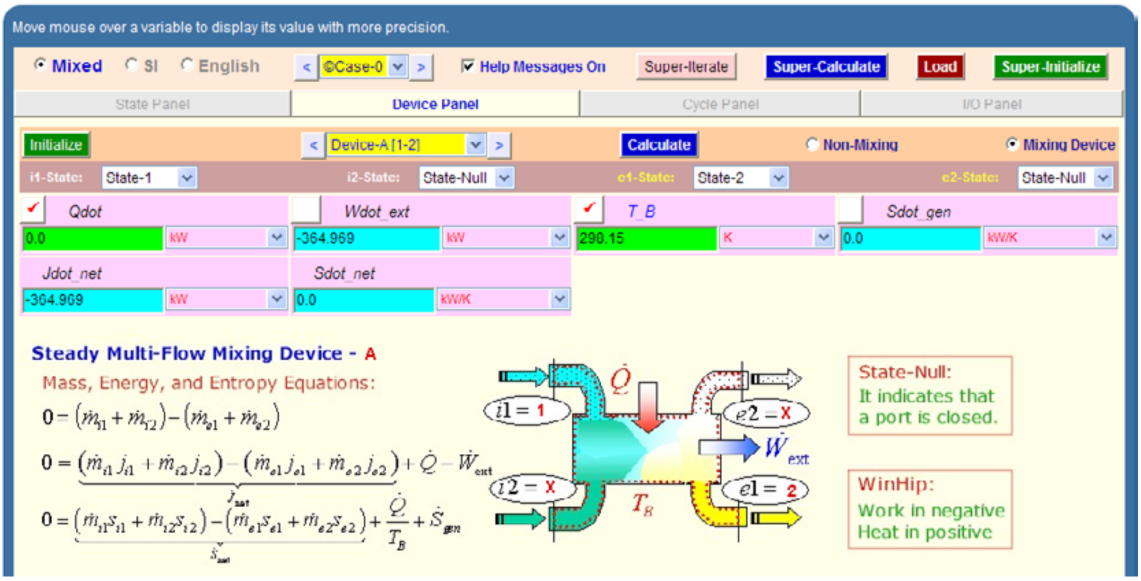


Note that h3, s3 etc are calculated.

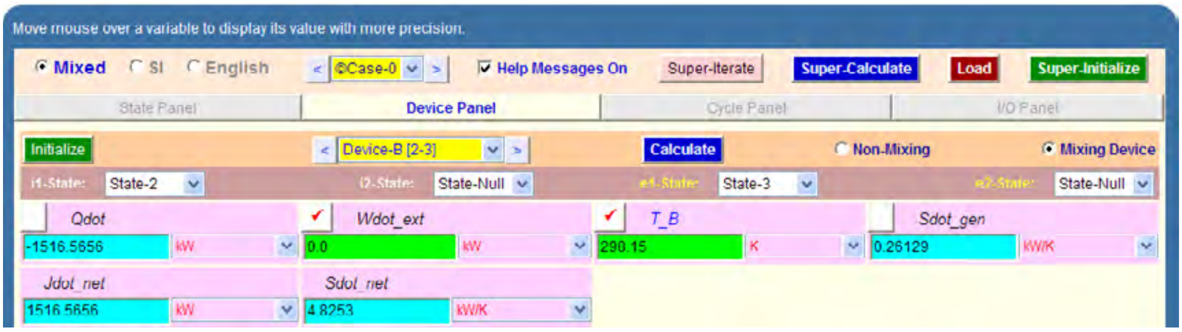
4. For State 4: Enter p4 = p1, h4 = h3, mdot4 = mdot1. Hit Enter. We get:

Note that h4, T4, s4, and x4 etc are calculated. Note that x4 = 0.169

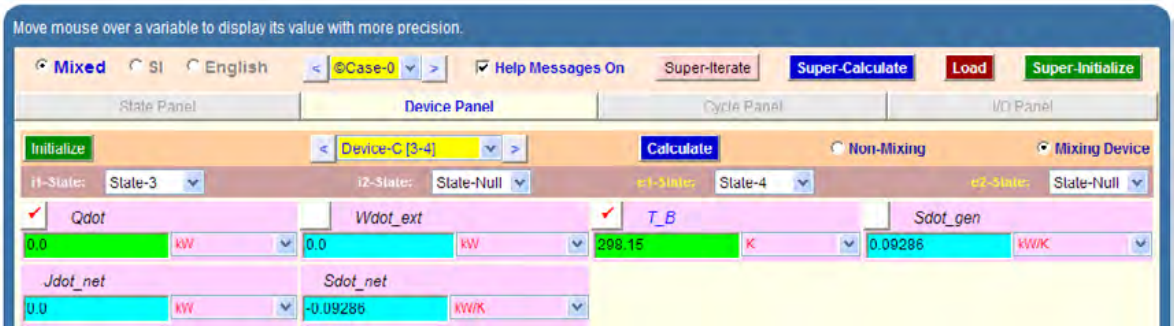
5. Now, go to Device panel. For Device A, fill up State 1 and State 2 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. And, Qdot = 0. Hit Enter. We get:



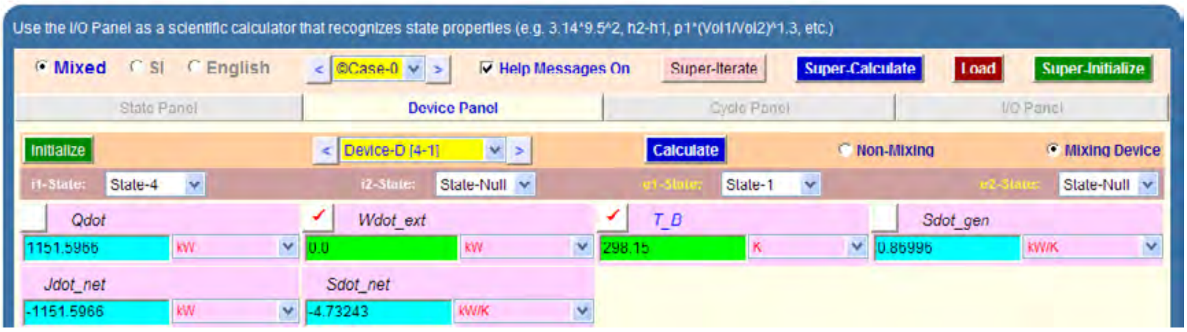
6. For Device B: fill up State 2 and State 3 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Wdot_ext = 0, since there is no external work in process 2-3. Hit Enter. We get:



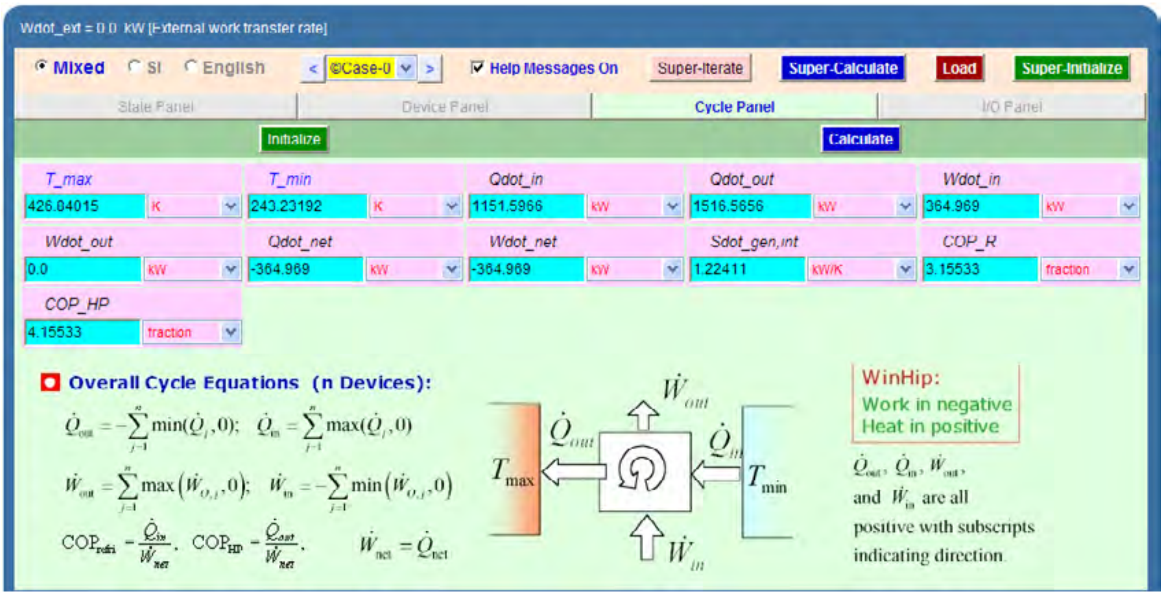
7. For Device C: fill up State 3 and State 4 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Qdot = 0, since there is no heat transfer in process 3-4. Hit Enter. We get:



8. For Device D: fill up State 4 and State 1 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Wdot_ext = 0, since there is no work transfer in process 4-1. Hit Enter. And click on SuperCalculate.We get:



9. Now go to Cycle panel. All important cycle parameters are available here:



Thus:

#Power reqd./ton of refrign:

1 ton = 211 kJ/min

#Power = Wdot_in/(Qdot_in/1400)

=364.969/(1151.5966*60/211)= **1.1145 kW/ton of refrigeration. ... Ans.**

 # Let Mass flow rate of NH3 per ton of refrign. = w kg/s

 # Then, w = 1/(1151.5966*60/211)=0.003054 kg/s

 # volume of flow: w * v1 where v1 is the sp. vol. in State 1

#Note from State 1 that v1 = 1.00389 m^3/kg

Now, w * v1 = (pi/4)* D^2 * L *(N /60) * eta_vol where eta_vol = vol. effcy = 0.8, by data

Then, we have:w * v1 = (pi/4) * D^2 * 1.5 * D * (N/60) * 0.8

Therefore: D = ((w * v1) / ((pi/4) * 1.5 * (N/60) * 0.8))^ (1/3) m

#i.e D = ((0.00305373 * v1) / ((pi/4) * 1.5 * (200/60) * 0.8))^ (1/3)

 # or, D = 0.099187 m ... dia of cylinder .... Ans.

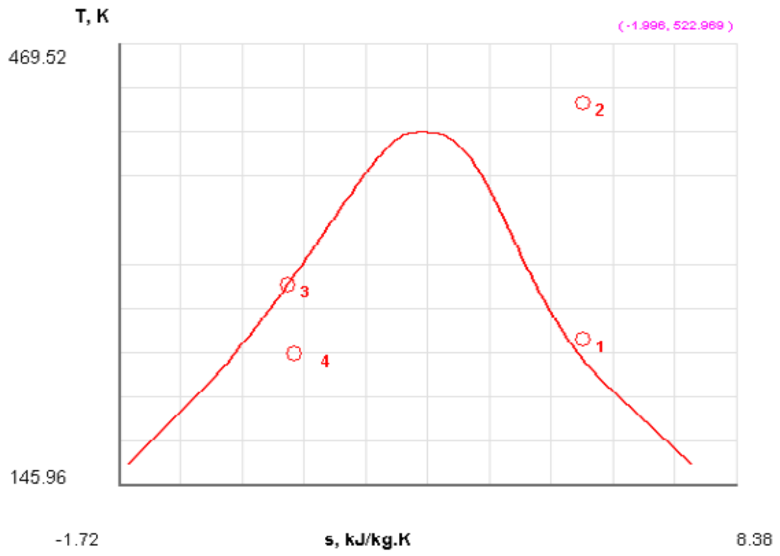
#And: L = 1.5 * D, by data

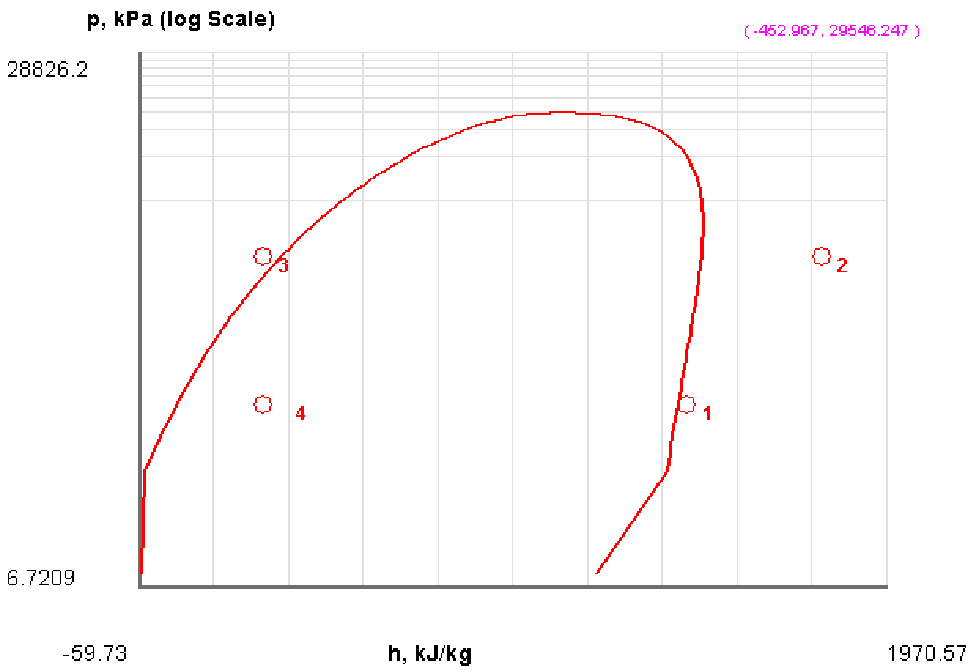
 # Therefore, D =1.5 * 0.099187= 0.14878 m .... length of cyl.....Ans.

#And, COP_R = COP of refrigerator = 3.15533… Ans.

---

10. From the Plots widget, first get the T-s plot, and then get h-s plot:





11. The I/O panel gives the TEST code etc:

#

#~~~~~~~~~~~~~~~~~~~~~~OUTPUT OF SUPER-CALCULATE

 # Daemon (TESTcalc) Path: Systems>Open>SteadyState>Specific>RefrigCycle>PC-Model; v-10.cd03

#-------------------Start of TEST-code --

States {

State-1: Ammonia(NH3);

Given: { p1= 120.0 kPa; T1= -20.0 deg-C; Vel1= 0.0 m/s; z1= 0.0 m; mdot1= 1.0 kg/s; }

State-2: Ammonia(NH3);

Given: { p2= 1200.0 kPa; s2= “s1” kJ/kg.K; Vel2= 0.0 m/s; z2= 0.0 m; mdot2= “mdot1” kg/s; }

State-3: Ammonia(NH3);

Given: { p3= “p2” kPa; T3= 20.0 deg-C; Vel3= 0.0 m/s; z3= 0.0 m; mdot3= “mdot1” kg/s; }

State-4: Ammonia(NH3);

Given: { p4= "p1" kPa; h4= "h3" kJ/kg; Vel4= 0.0 m/s; z4= 0.0 m; mdot4= "mdot1" kg/s; }

}

Analysis {

Device-A: i-State = State-1; e-State = State-2; Mixing: true;

Given: { Qdot= 0.0 kW; T_B= 298.15 K; }

Device-B: i-State = State-2; e-State = State-3; Mixing: true;

Given: { Wdot_ext= 0.0 kW; T_B= 298.15 K; }

Device-C: i-State = State-3; e-State = State-4; Mixing: true;

Given: { Qdot= 0.0 kW; T_B= 298.15 K; }

Device-D: i-State = State-4; e-State = State-1; Mixing: true;

Given: { Wdot_ext= 0.0 kW; T_B= 298.15 K; }

}

#----------------------End of TEST-code ---

#--------Property spreadsheet starts:

# State	p(kPa)	T(K)	x	v(m3/kg)	u(kJ/kg)	h(kJ/kg)	s(kJ/kg.K)
# 01	120.0	253.2		1.0039	1305.99	1426.46	5.866
# 02	1200.0	426.8		0.1679	1589.95	1791.43	5.866
# 03	1200.0	293.2		0.0016	272.9	274.86	1.041
# 04	120.0	243.2	0.2	0.1638	255.21	274.86	1.134

#

 # Cycle Analysis Results:

Calculated: T_max= 426.84015 K; T_min= 243.23192 K; **Qdot_in= 1151.5966 kW;**

Qdot_out= 1516.5656 kW; **Wdot_in= 364.969 kW**; Wdot_out= 0.0 kW;

Qdot_net= -364.969 kW; Wdot_net= -364.969 kW; Sdot_gen,int= 1.22411 kW/K;

COP_R= 3.15533 fraction; COP_HP= 4.15533 fraction; BWR= Infinity %;

#

#******CALCULATE VARIABLES: Type in an expression starting with an '=' sign ('= mdot1*(h2-h1)', '= sqrt(4*A1/PI)', etc.) and press the Enter key)*********

#Power reqd./ton of refrign:

1 ton = 211 kJ/min

#Power = Wdot_in/(Qdot_in*60/211)

=364.969/(1151.5966*60/211)

= 1.1145172855378929 kW/ton of refrigeration. ... Ans.

#Mass flow rate of NH3 per ton of refrign. = w

=1/(1151.5966*60/211)

= 0.0030537313731793464 kg/s

 # volume of flow: w * v1

#w * v1 = (pi/4)* D^2 * L *(N /60) * eta_vol

w * v1 = (pi/4) * D^2 * 1.5 * D * (N/60) * 0.8

Therefore: D = ((w * v1) / ((pi/4) * 1.5 * (N/60) * 0.8))^ (1/3)

= ((0.00305373 * v1) / ((pi/4) * 1.5 * (200/60) * 0.8))^ (1/3)

= 0.09918727579026226 m ... dia of cylinder .... Ans.

#And: L = 1.5 * D

=1.5 * 0.099187

= 0.14878050000000004 m .... length of cyl.....Ans.

==

Prob.4.5.4. A food storage chamber requires a refrigeration system of 10 T capacity with an evaporator temp. of -10 C and condenser temp. of 30 C. The refrigerant R-12 is subcooled by 5 deg. C before entering the throttle valve and the vapour is superheated by 6 deg. C before entering the compressor. The specific heats of liquid and vapour are 1.235 and 0.7327 kJ/kg.K respectively. Determine: (i) The refrigerating capacity per kg (ii) Mass of refrigerant circulated per minute, and (iii) COP. [VTU-ATD-Jan. 2003]

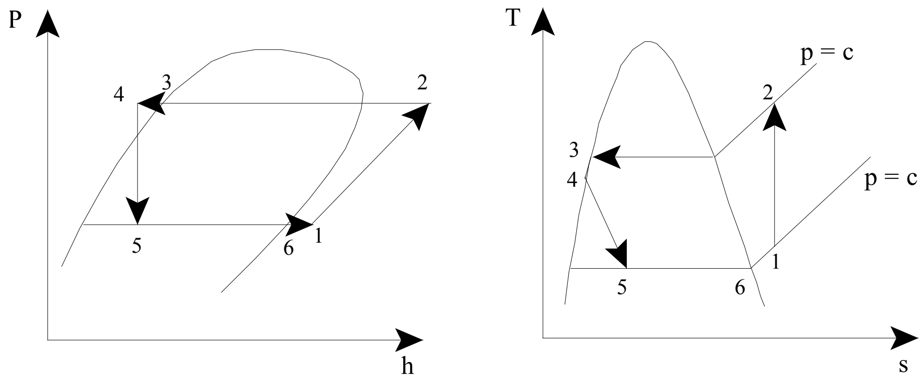


Fig. Vap. Comprn. Refrig. Cycle with subcooling and superheating

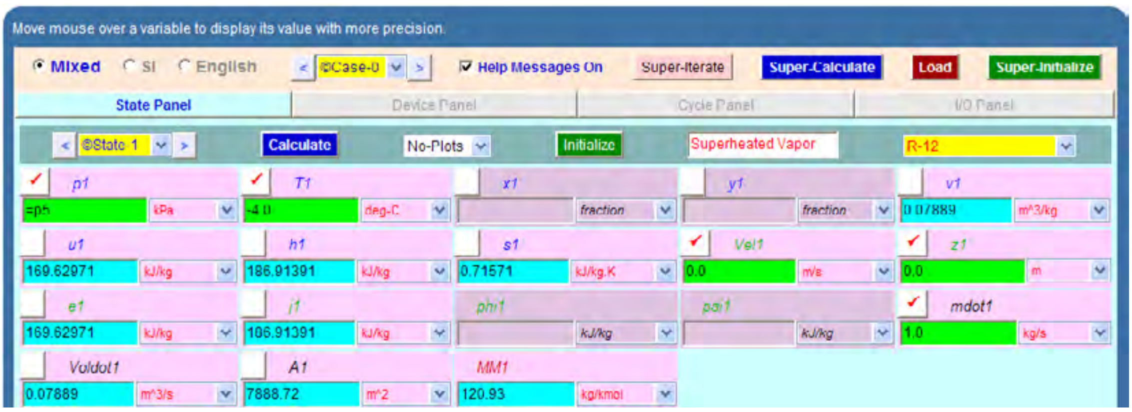
TEST Solution:

Note: We shall assume the mass flow rate of refrigerant as 1 kg/s to start with, and calculate refrig. effect, compressor work etc. Then for refrig. capacity of 10 tons, (1 ton = 211 kJ/min), the flow rate required can easily be calculated.

Following are the steps:

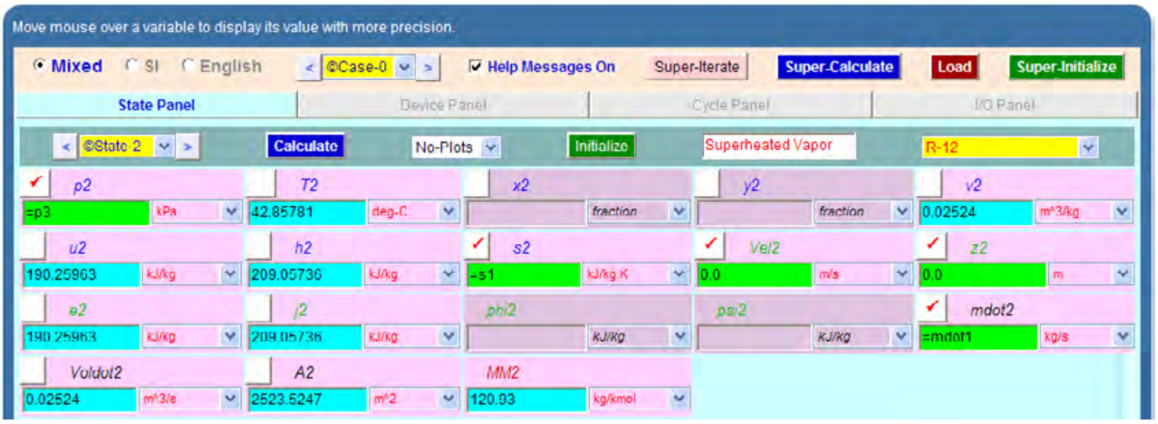
Steps 1 and 2 are the same as for Prob.4.5.1. But, now the working fluid is R-12.

1. Choose R-12as working substance and fill up the known parameters for State 1, i.e. P1= P5, T1 = (-10+ 6)= -4 C and mdot1 = 1 kg/s. Hit Enter. We get:



Note that all parameters such as h1, T1, s1 etc are calculated, *at the end after SuperCalculation*.

2. State 2: Enter P2, s2 = s1, and mdot2 = mdot1. Hit Enter. We get:

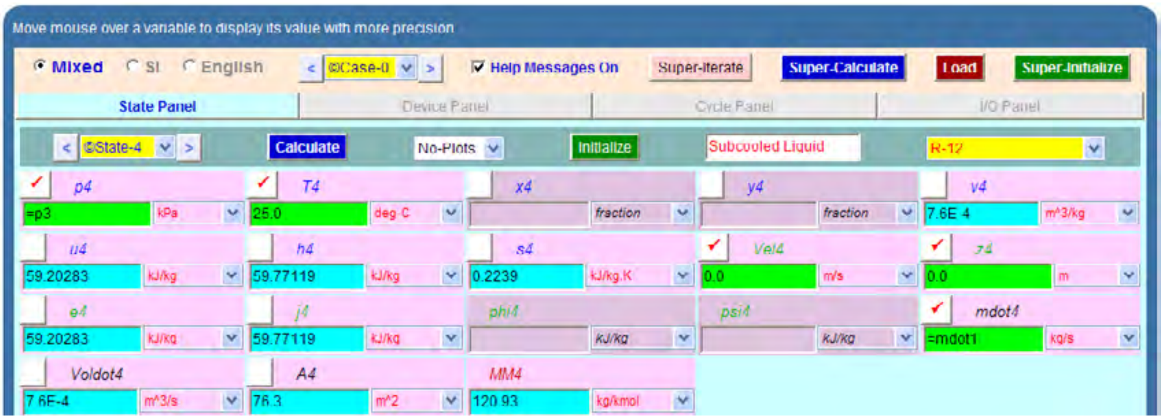


Here again, T2, h2 etc are calculated later, afer SuperCalculate.

3. For State 3: Enter x3 = 0, T3 = 30 C, mdot3 = mdot1. Hit Enter. We get:

Note that p3, h3, s3 etc are calculated.

4. For State 4: Enter p4 = p3, T4 = 25 C, mdot4 = mdot1. Hit Enter. We get:

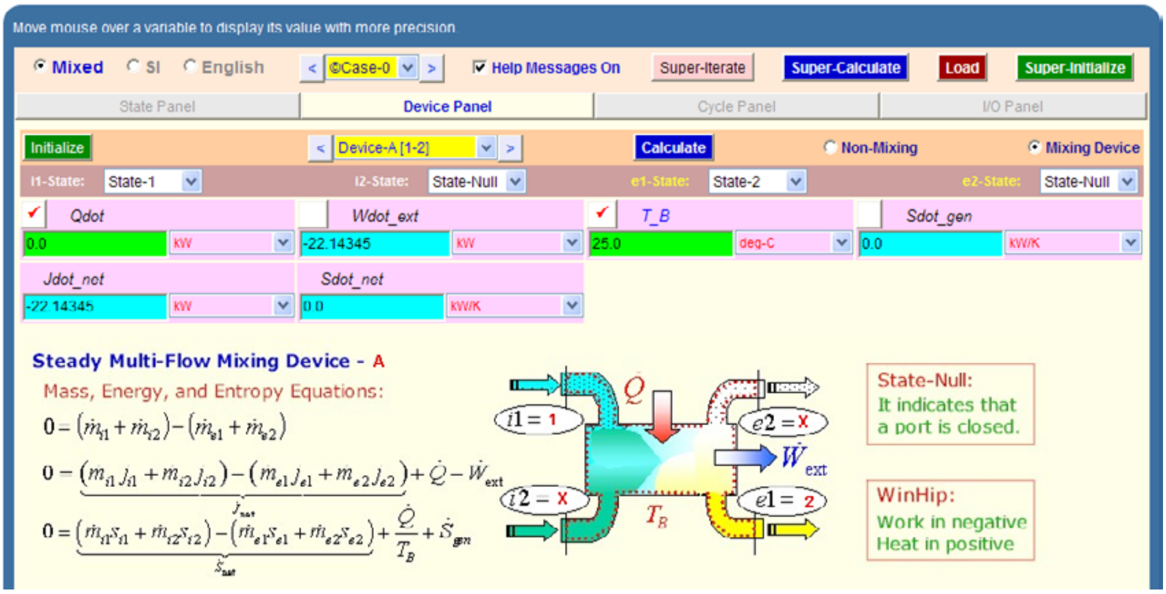


Note that h4, T4, s4 etc are calculated.

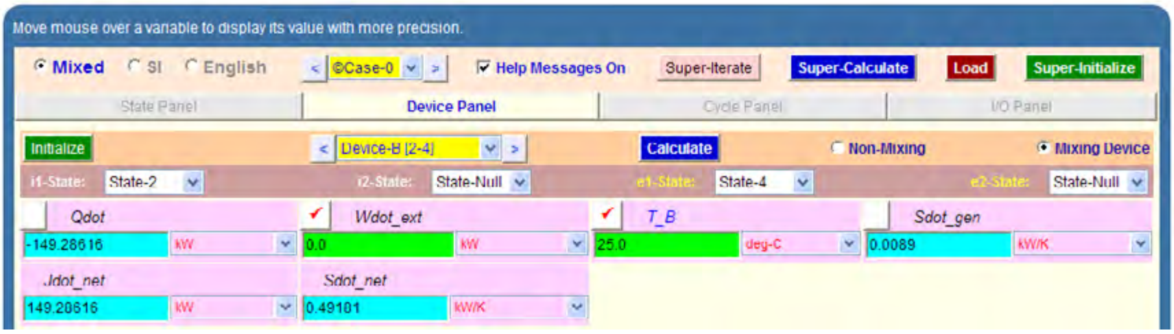
5. For State 5: Enter p5 = p6, h5 = h4, mdot5 = mdot1. Hit Enter. We get:

6. For State 6: Enter T6 = -10 C, x6= 1, mdot6 = mdot1. Hit Enter. We get:

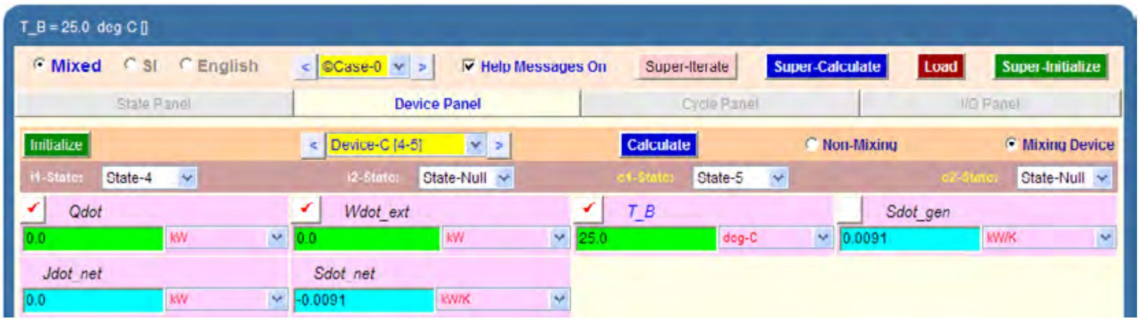
7. Now, go to Device panel. For Device A, fill up State 1 and State 2 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. And, Qdot = 0. Hit Enter. We get:



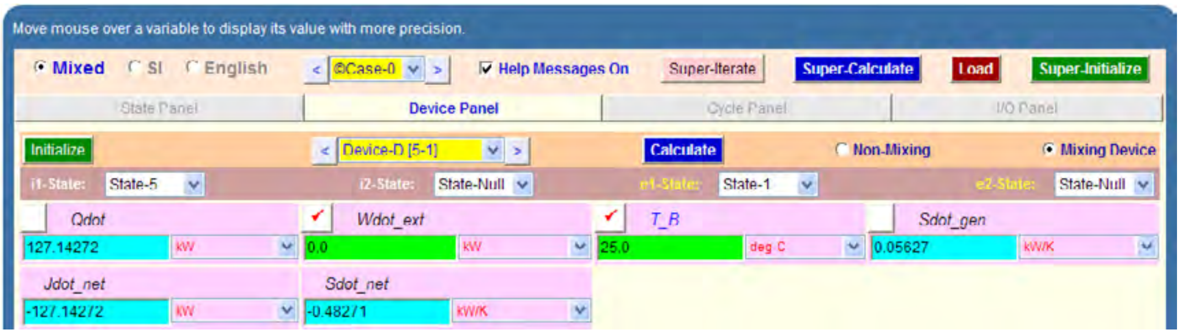
8. For Device B: fill up State 2 and State 4 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Wdot_ext = 0, since there is no external work in process 1-2. Hit Enter. We get:



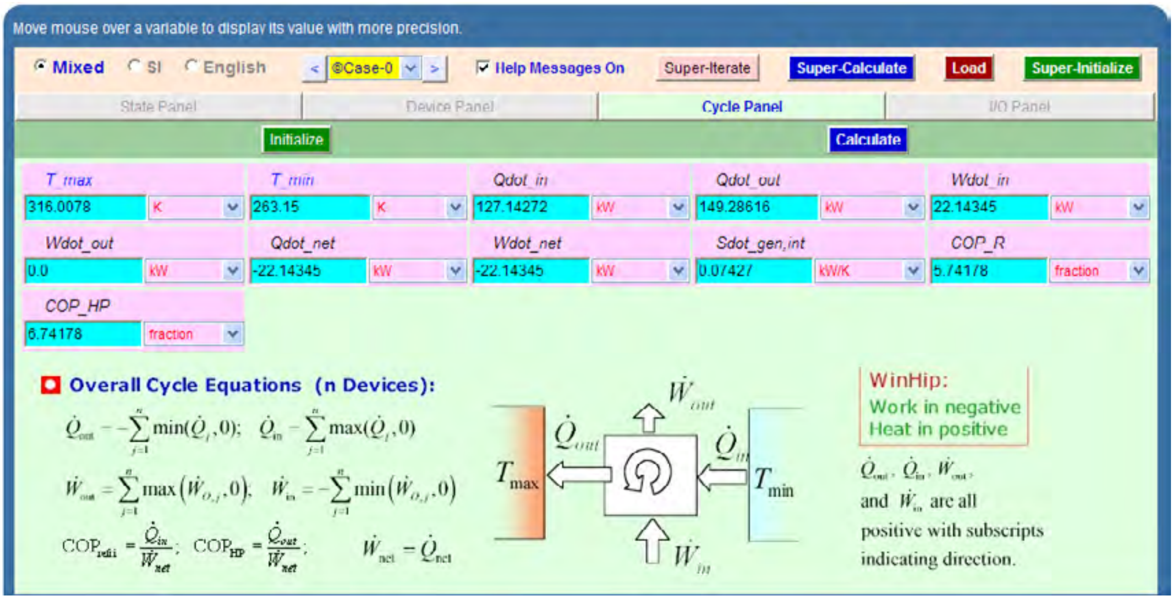
9. For Device C: fill up State 4 and State 5 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Qdot = 0. Hit Enter. We get:



10. For Device D: fill up State 5 and State 1 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Wdot_ext = 0.
Hit Enter. And click on SuperCalculate. We get:



11. Now go to Cycle panel. All important cycle parameters are available here:



Thus:

Refrign. capacity per kg = (h1 – h5) = Qdot_in = 127.143 kJ/kg

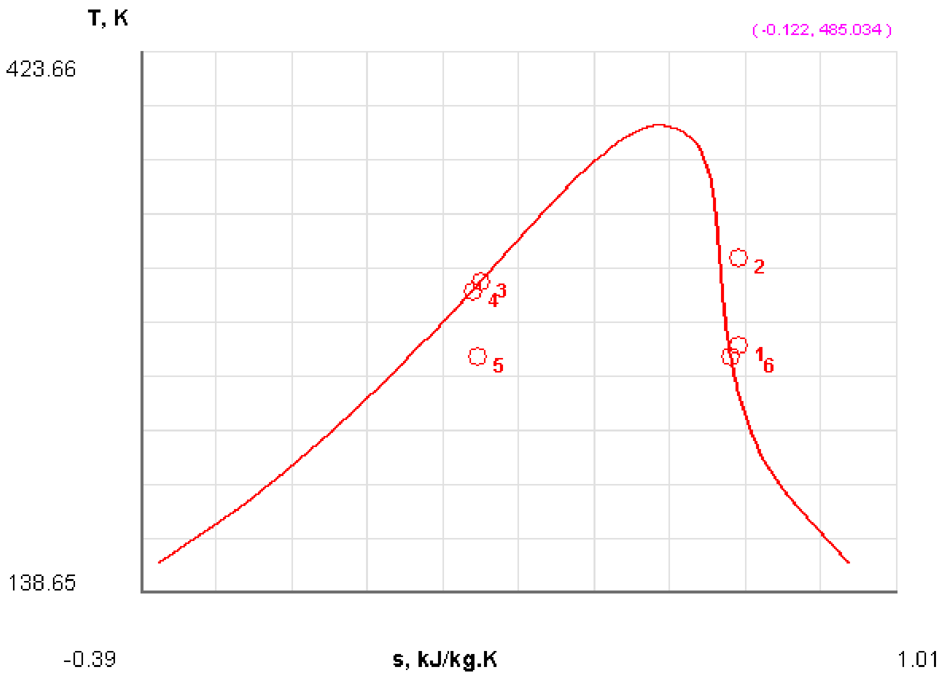
#Then, mass flow rate for a refrign. capacity of 10 Tons:

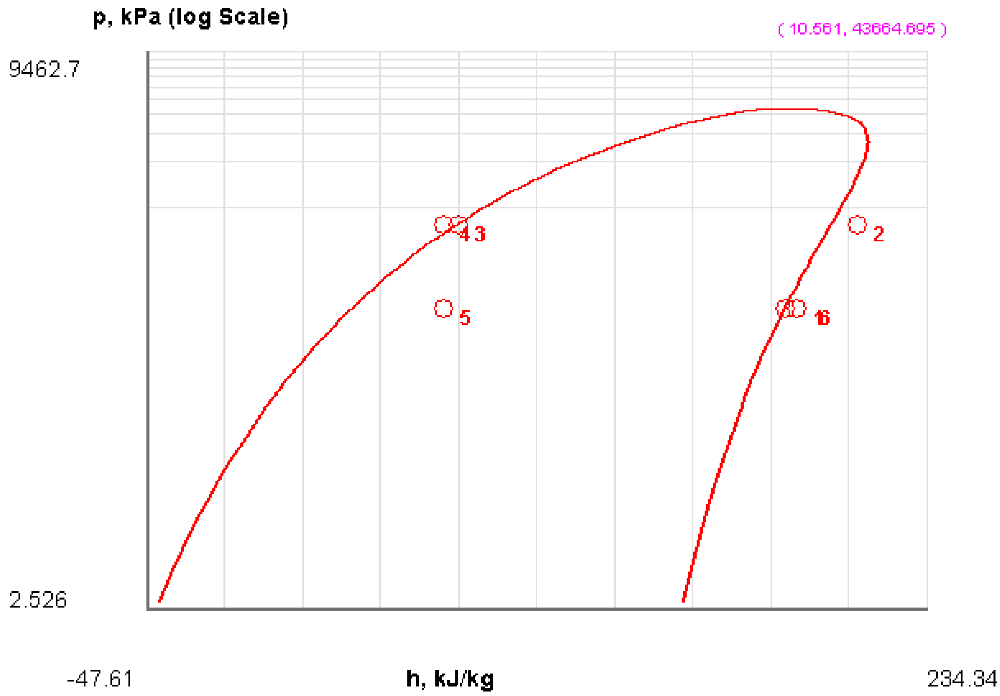
=10*211/((h1-h5) *60)= 0.2766 kg/s

#And, COP_R = COP of refrigerator = 5.742… Ans.

---

12. From the Plots widget, first get the T-s plot, and then get h-s plot:





13. The I/O panel gives the TEST code etc:

#~~~~~~~~~~~~~~~~~~~~~OUTPUT OF SUPER-CALCULATE :

Daemon (TESTcalc) Path: Systems>Open>SteadyState>Specific>RefrigCycle>PC-Model; v-10.cd03

#-------------------Start of TEST-code ---

States {

State-1: R-12;

Given: { p1= "p5" kPa; T1= -4.0 deg-C; Vel1= 0.0 m/s; z1= 0.0 m; mdot1= 1.0 kg/s; }

State-2: R-12;

Given: { p2= "p3" kPa; s2= "s1" kJ/kg.K; Vel2= 0.0 m/s; z2= 0.0 m; mdot2= "mdot1" kg/s; }

State-3: R-12;

Given: { T3= 30.0 deg-C; x3= 0.0 fraction; Vel3= 0.0 m/s; z3= 0.0 m; mdot3= "mdot1" kg/s; }

State-4: R-12;

Given: { p4= “p3” kPa; T4= 25.0 deg-C; Vel4= 0.0 m/s; z4= 0.0 m; mdot4= “mdot1” kg/s; }

State-5: R-12;

Given: { p5= “p6” kPa; h5= “h4” kJ/kg; Vel5= 0.0 m/s; z5= 0.0 m; mdot5= “mdot1” kg/s; }

State-6: R-12;

Given: { T6= -10.0 deg-C; x6= 1.0 fraction; Vel6= 0.0 m/s; z6= 0.0 m; mdot6= “mdot1” kg/s; }

}

Analysis {

Device-A: i-State = State-1; e-State = State-2; Mixing: true;

Given: { Qdot= 0.0 kW; T_B= 25.0 deg-C; }

Device-B: i-State = State-2; e-State = State-4; Mixing: true;

Given: { Wdot_ext= 0.0 kW; T_B= 25.0 deg-C; }

Device-C: i-State = State-4; e-State = State-5; Mixing: true;

Given: { Qdot= 0.0 kW; Wdot_ext= 0.0 kW; T_B= 25.0 deg-C; }

Device-D: i-State = State-5; e-State = State-1; Mixing: true;

Given: { Wdot_ext= 0.0 kW; T_B= 25.0 deg-C; }

}

#----------------------End of TEST-code --

#--------Property spreadsheet starts: #

# State	p(kPa)	T(K)	x	v(m3/kg)	u(kJ/kg)	h(kJ/kg)	s(kJ/kg.K)
# 01	219.1	269.2		0.0789	169.63	186.91	0.716
# 02	744.9	316.0		0.0252	190.26	209.06	0.716
# 03	744.9	303.2	0.0	8.0E-4	64.01	64.59	0.24
# 04	744.9	298.2		8.0E-4	59.2	59.77	0.224
# 05	219.1	263.1	0.2	0.0167	56.12	59.77	0.233
# 06	219.1	263.2	1.0	0.0767	166.4	183.19	0.702

 # Cycle Analysis Results:

Calculated: T_max= 316.0078 K; T_min= 263.15 K; Qdot_in= 127.14272 kW;

Qdot_out= 149.28616 kW; Wdot_in= 22.14345 kW; Wdot_out= 0.0 kW;

Qdot_net= -22.14345 kW; Wdot_net= -22.14345 kW; Sdot_gen,int= 0.07427 kW/K;

COP_R= 5.74178 fraction; COP_HP= 6.74178 fraction; BWR= Infinity %;

#******CALCULATE VARIABLES: Type in an expression starting with an '=' sign ('= mdot1*(h2-h1)', '= sqrt(4*A1/PI)', etc.) and press the Enter key)*********

Mass of refrig. circulated per min, to give 10 TR cooling capacity:

=10*211/((h1-h5) *60)= 0.27659205909515694 kg/s

===

Prob.4.5.5. A vapour compression refrigeration system with R134a as the refrigerant, operates between an evaporator temp of -10 C and condenser temp of 20 C. The refrigerant leaves the evaporator at -10 C as sat. vapour and the isentropic effcy of compressor is 80%. Determine the refrigeration effect, compressor power required and the COP for a flow rate of 1 kg/s of refrigerant.

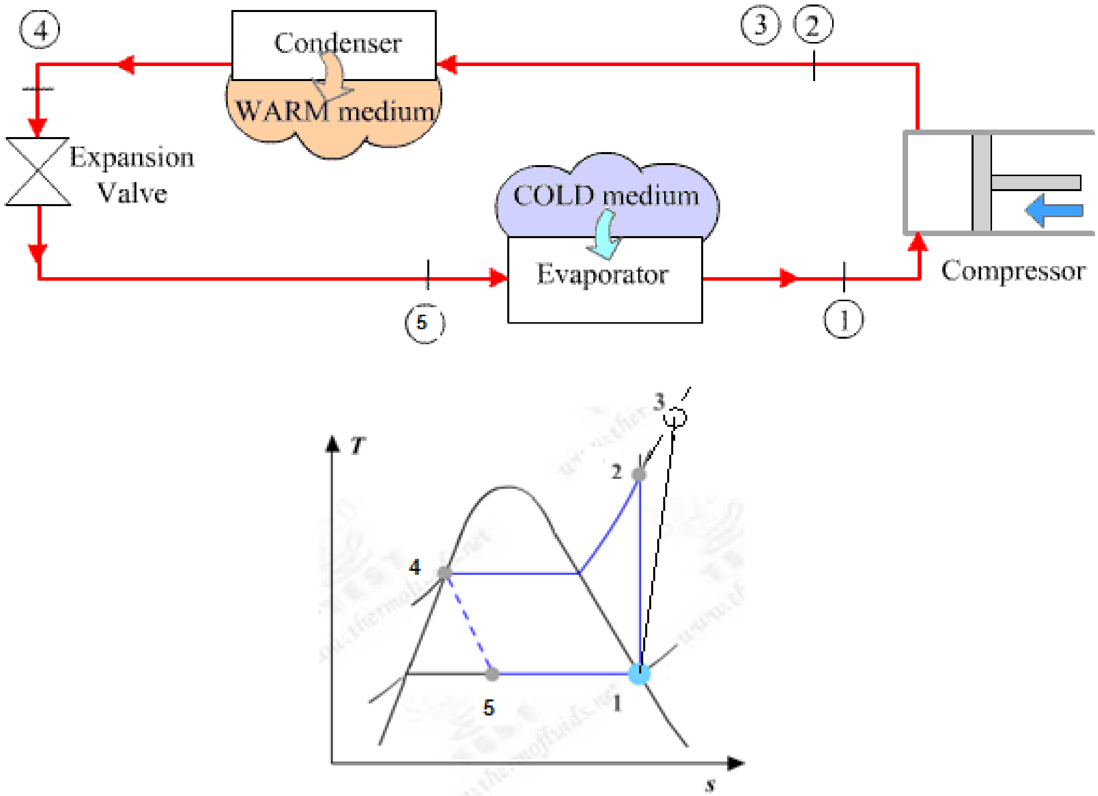


Fig.Prob.4.5.5. Vapour compression refrigeration system and (b) T-s diagram

TEST Solution:

Following are the steps:

Steps 1 and 2 are the same as for Prob.4.5.1. But, now the working fluid is R-134a.

1. Choose R-134a as working substance and fill up the known parameters for State 1, i.e. T1 = -10 C, x1 = 1, and mdot1 = 1 kg/s. Hit Enter. We get:

Note that all parameters such as h1, T1, s1 etc are calculated.

2. State 2: Enter p2= p4, s2 = s1, and mdot2 = mdot1. Hit Enter. We get:

Later, after entering properties for State 4, and 'calculate' and 'SuperCalculate' we get:

Here again, p2, T2, h2 etc are calculated.

3. For State 3: Enter p3 = p4, h3 = h1 + (h2-h1)/0.8 where 0.8 is the isentropic effcy of compressor, and mdot3 = mdot1. Hit Enter. We get:

Later, after entering properties for State 4, and 'calculate' and 'SuperCalculate' we get:

Note that p3, T3, h3, s3 etc are calculated.

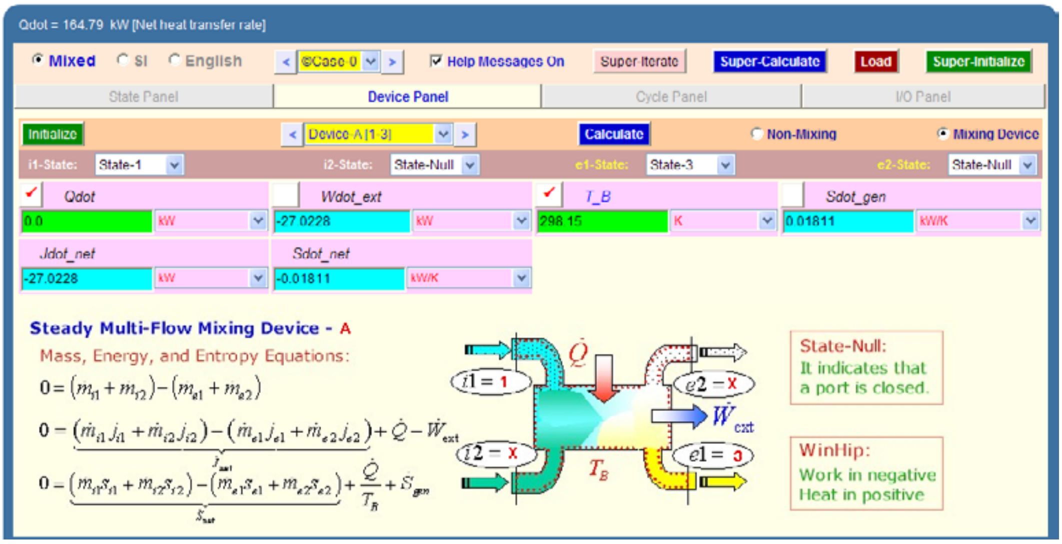
4. For State 4: Enter T4 = 20 C, x4 = 0, mdot4 = mdot1. Hit Enter. We get:

Note that p4, h4, s4 etc are calculated.

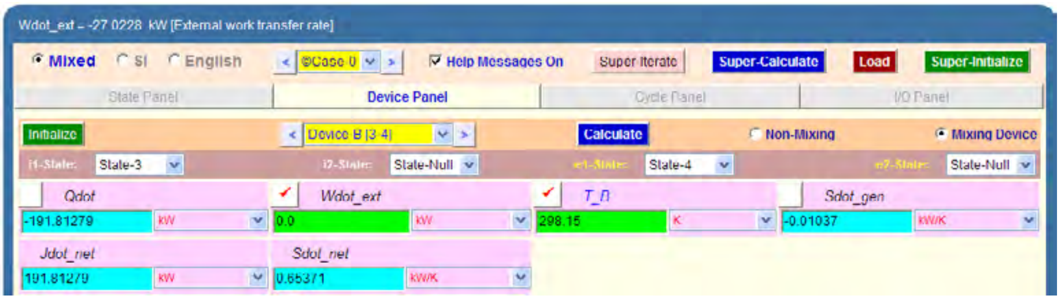
5. For State 5: Enter p5 = p1, h5 = h4, mdot5 = mdot1. Hit Enter. We get:

Note that T5, x5 etc are calculated.

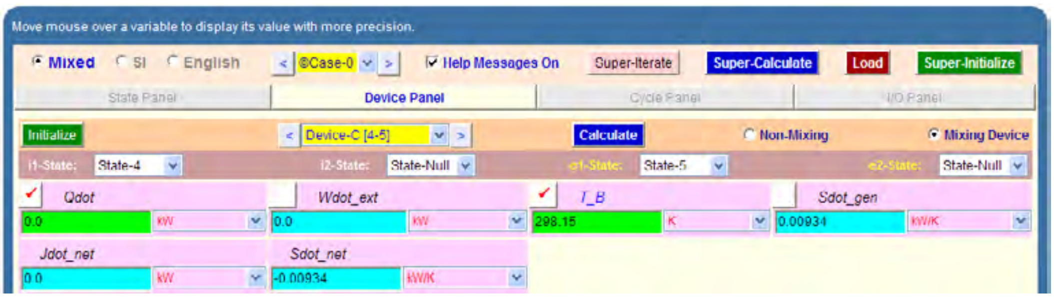
6. Now, go to Device panel. For Device A, fill up State 1 and State 3 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. And, Qdot = 0. Hit Enter. We get:



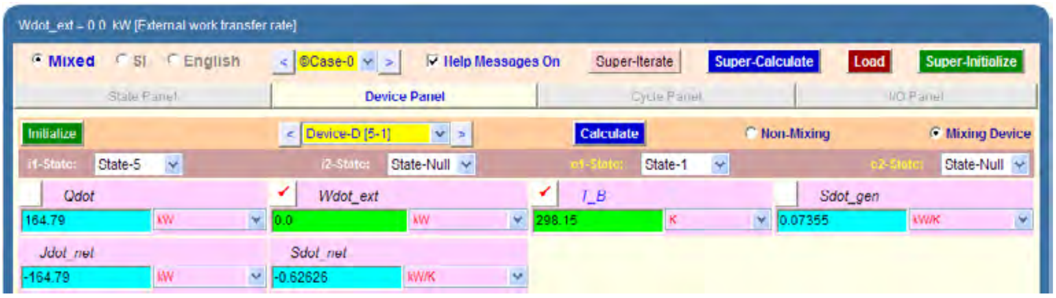
7. For Device B: fill up State 3 and State 4 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Wdot_ext = 0, since there is no external work in process 1-2. Hit Enter. We get:



8. For Device C: fill up State 4 and State 5 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Qdot = 0. Hit Enter. We get:



9. For Device D: fill up State 5 and State 1 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Wdot_ext = 0. Hit Enter. And **click on SuperCalculate**.We get:



Note: Now, you can go back to State 2 and State 3 and verify that all calculations are updated.

10. Now go to Cycle panel. All important cycle parameters are available here:

Thus:

Refrign. capacity per kg = (h1 – h5) = Qdot_in = 164.79kW

Compressor power = h3-h1 = Wdot_in = 27.0228 kW … Ans.

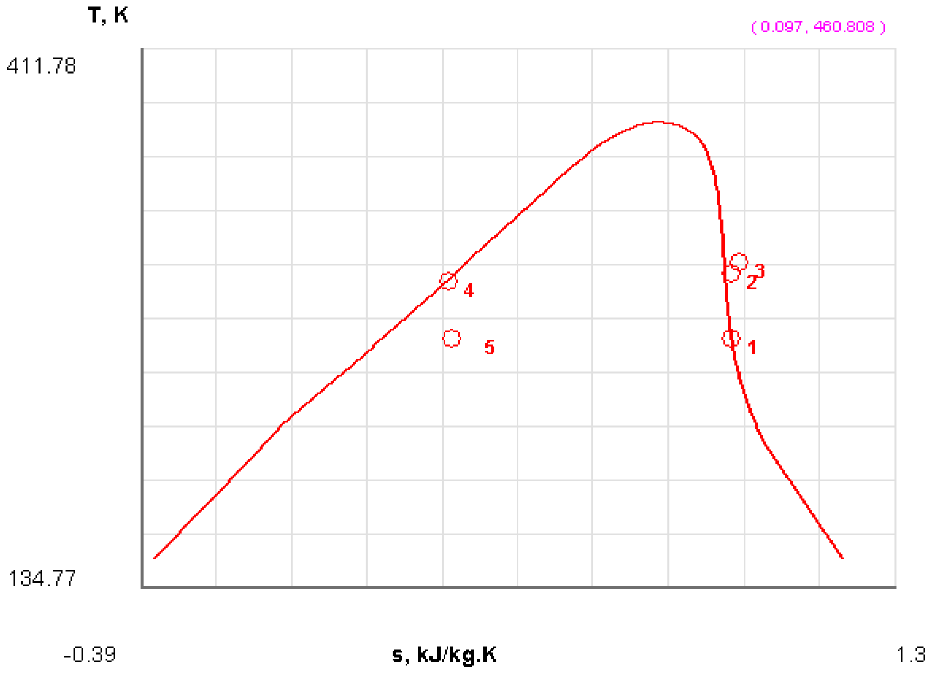
Condenser heat transfer = h3 – h4 = Qsot_out = 191.81279 kW … Ans.

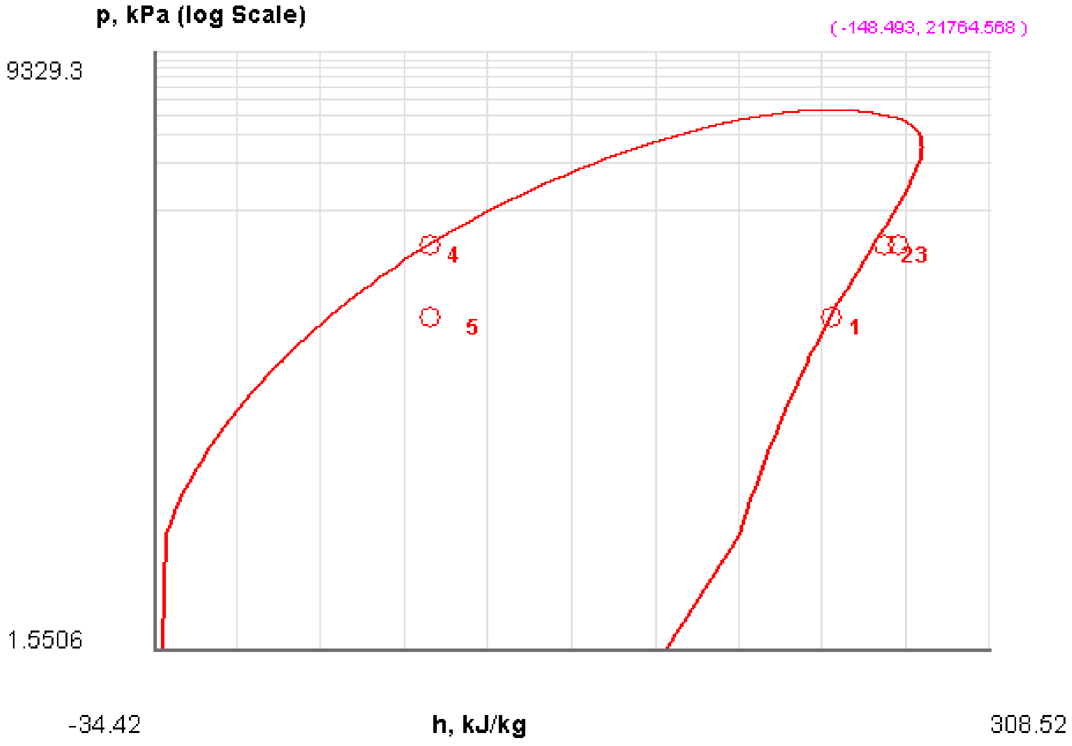
COP_R = COP of refrigerator = 6.098… Ans.

Quality at exit of expn. valve = x5 = 0.19834 (from State 5) … Ans.

---

11. From the Plots widget, first get the T-s plot, and then get h-s plot:





12. The I/O panel gives the TEST code etc:

#~~~~~~~~~~~~~~~~~~~~~~OUTPUT OF SUPER-CALCULATE (

 # Daemon (TESTcalc) Path: Systems>Open>SteadyState>Specific>RefrigCycle>PC-Model; v-10.cd03

#-------------------Start of TEST-code ---

States {

State-1: R-134a;

Given: { T1= -10.0 deg-C; x1= 1.0 fraction; Vel1= 0.0 m/s; z1= 0.0 m; mdot1= 1.0 kg/s; }

State-2: R-134a;

Given: { p2= “p4” kPa; s2= “s1” kJ/kg.K; Vel2= 0.0 m/s; z2= 0.0 m; mdot2= “mdot1” kg/s; }

State-3: R-134a;

Given: { p3= "p4" kPa; h3= "h1+(h2-h1)/0.8" kJ/kg; Vel3= 0.0 m/s; z3= 0.0 m; mdot3= "mdot1" kg/s; }

State-4: R-134a;

Given: { T4= 20.0 deg-C; x4= 0.0 fraction; Vel4= 0.0 m/s; z4= 0.0 m; mdot4= "mdot1" kg/s; }

State-5: R-134a;

Given: { p5= "p1" kPa; h5= "h4" kJ/kg; Vel5= 0.0 m/s; z5= 0.0 m; mdot5= "mdot1" kg/s; }

}

Analysis {

Device-A: i-State = State-1; e-State = State-3; Mixing: true;

Given: { Qdot= 0.0 kW; T_B= 298.15 K; }

Device-B: i-State = State-3; e-State = State-4; Mixing: true;

Given: { Wdot_ext= 0.0 kW; T_B= 298.15 K; }

Device-C: i-State = State-4; e-State = State-5; Mixing: true;

Given: { Qdot= 0.0 kW; T_B= 298.15 K; }

Device-D: i-State = State-5; e-State = State-1; Mixing: true;

Given: { Wdot_ext= 0.0 kW; T_B= 298.15 K; }

}

#----------------------End of TEST-code ---

#--------Property spreadsheet starts:

# State	p(kPa)	T(K)	x	v(m3/kg)	u(kJ/kg)	h(kJ/kg)	s(kJ/kg.K)
# 01	201.7	263.2	1.0	0.0992	223.29	243.3	0.933
# 02	572.8	297.3	0.0369	243.79	264.92	0.933	
# 03	572.8	302.8	0.038	248.56	270.32	0.951	
# 04	572.8	293.2	0.0	8.0E-4	78.04	78.51	0.297
# 05	201.7	263.1	0.2	0.0203	74.42	78.51	0.307

 # Cycle Analysis Results:

Calculated: T_max= 302.7867 K; T_min= 263.15 K; **Qdot_in= 164.79 kW**;

Qdot_out= 191.81279 kW; **Wdot_in= 27.0228 kW**; Wdot_out= 0.0 kW;

Qdot_net= -27.0228 kW; Wdot_net= -27.0228 kW; Sdot_gen,int= 0.09063 kW/K;

COP_R= 6.09818 fraction; COP_HP= 7.09818 fraction; BWR= Infinity %;

(b) Plot refrign. effect, compressor work, heat transfer in condenser, COP, and quality at exit of expn. valve as the condenser temp varies from 15 C to 35 C:

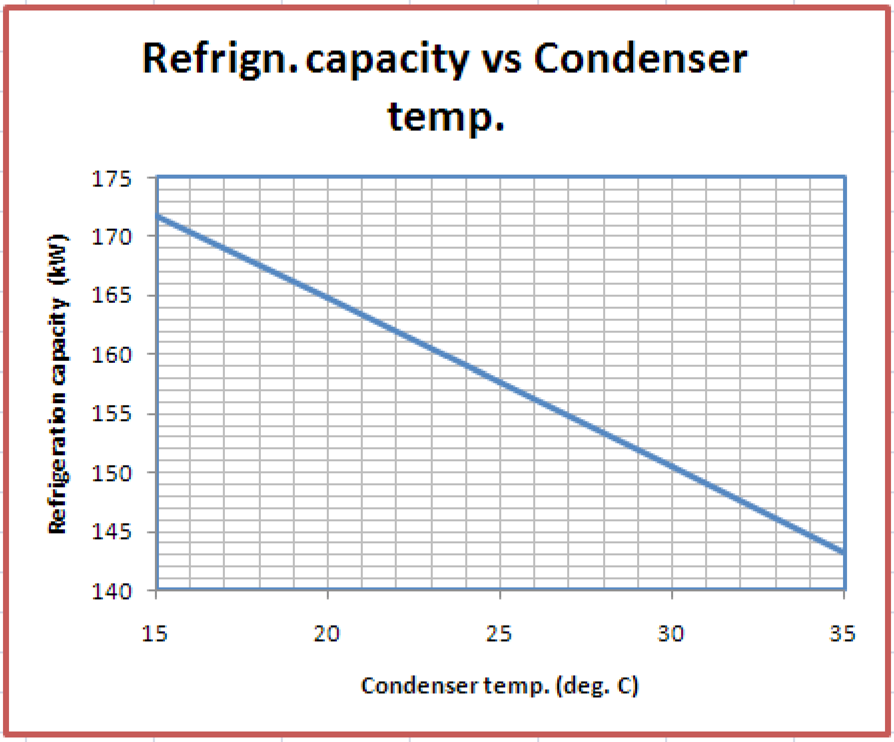
It is now very easy to get the desired parameters as condenser temp T4 is varied:

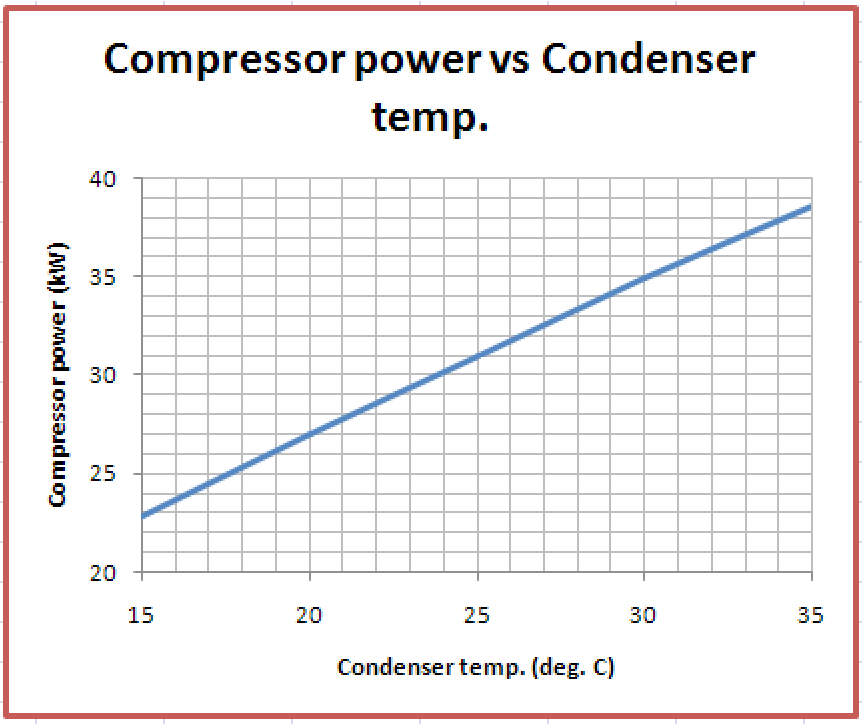
Following are the steps:

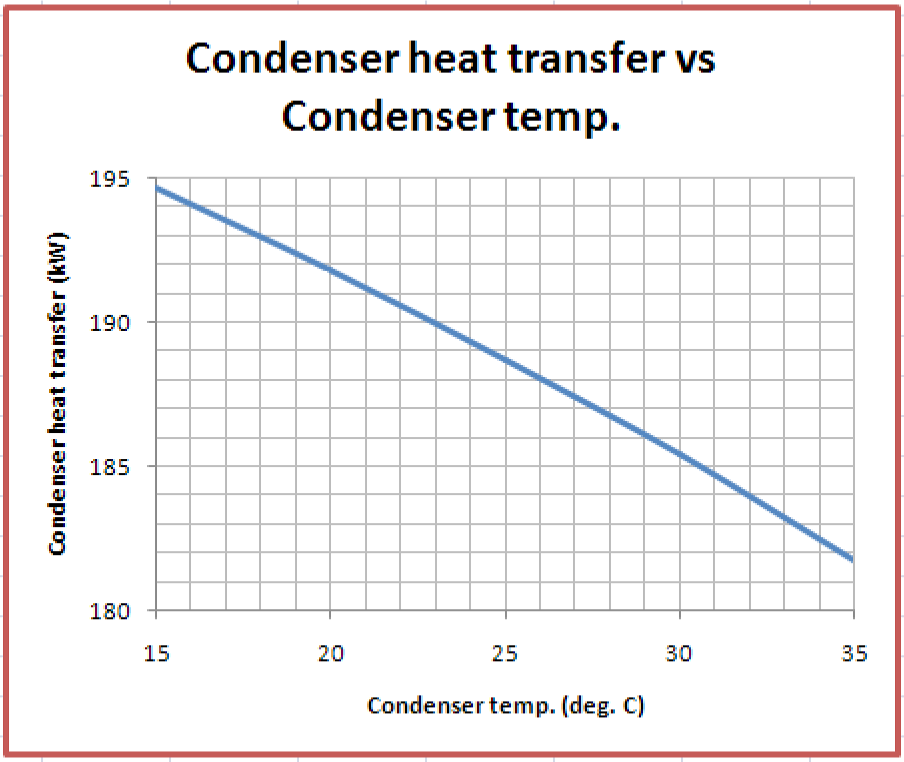
1. Go to State 4 panel, change T4 to the desired value, Hit Enter, and click on SuperCCalculate to update all results.
2. Go to State 5 panel, read the value of quality, x5
3. Go to Cycle panel, read the values of Qdot_in, Wdot_in, Qdot_out and COP.
4. Repeat this procedure for all desired values of T4.
5. Tabulate as shown below.
6. Transfer this Table to EXCEL and plot the results.

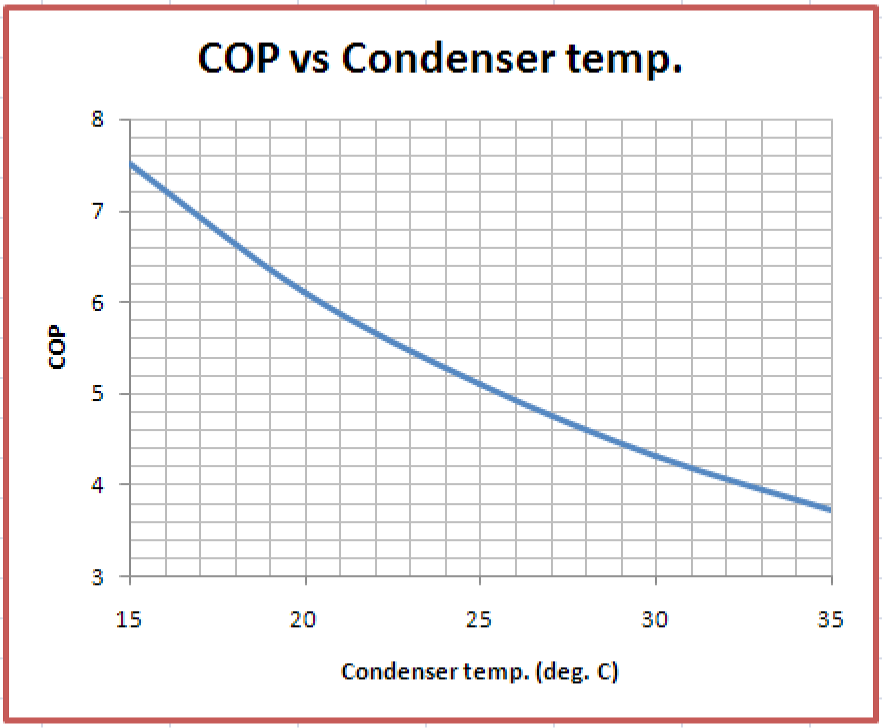
T4 (C)	Qdot_in (kW)	Wdot_in (kW)	Qdot_out (kW)	COP	X5
15	171.79	22.84	194.63	7.52	0.164
20	164.79	27.02	191.81	6.098	0.19834
25	157.69	30.99	188.68	5.098	0.233
30	150.49	34.89	185.38	4.314	0.2679
35	143.18	38.58	181.76	3.71	0.303

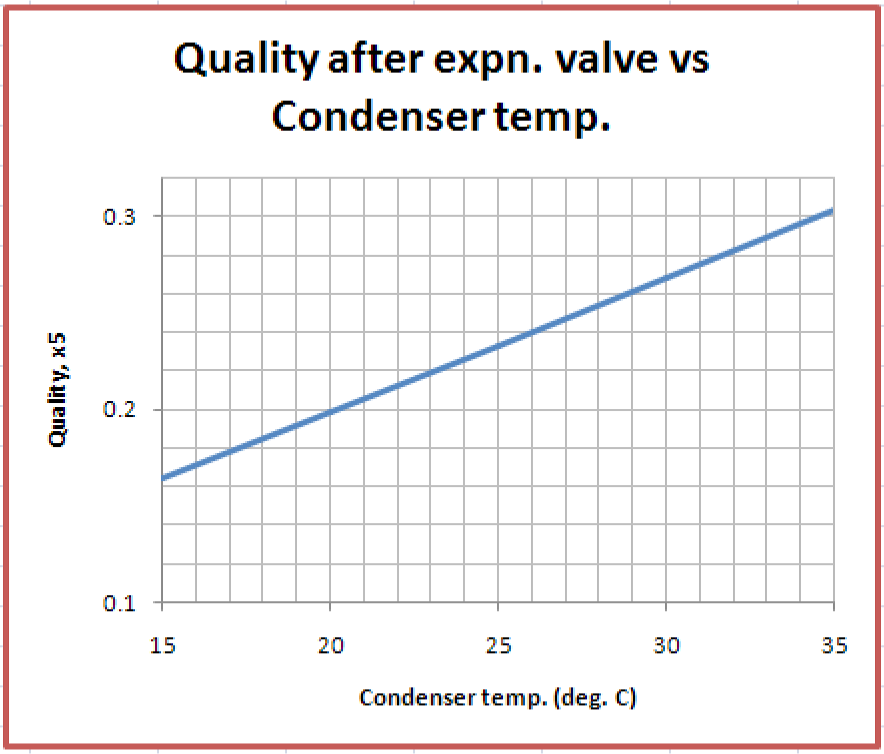
Now, plot the results in EXCEL:




Compressor power vs Condenser temp.
Compressor power (kW)
40
35
30
25
20
15
20
25
30
35
Condenser temp. (deg. C)


Condenser heat transfer vs Condenser temp.
Condenser heat transfer (kW)
195
190
185
180
15
20
25
30
35
Condenser temp. (deg. C)


COP vs Condenser temp.
COP
8
7
6
5
4
3
15
20
25
30
35
Condenser temp. (deg. C)



==

Air cycle refrigeration: Reversed Brayton cycle or Bell Coleman cycle:

Prob.4.5.6. An air refrigeration system is to be designed according to the following specifications:

Pressure of air at compressor inlet = 101 kPa, Pressure of air at compressor exit = 404 kPa,

Temperature ofair at compressor inlet = -6^{0}C, Temperature of air at turbine inlet = 27^{0}C,

Isentropic efficiency of compressor = 85%, Isentropic efficiency of turbine = 85%,

Determine i) C.O.P of the cycle. (ii) Power required for producing 1ton of refrigeration, and

(iii) Mass flow rate of air required for 1ton of refrigeration.[VTU-ATD-July–Aug. 2004]

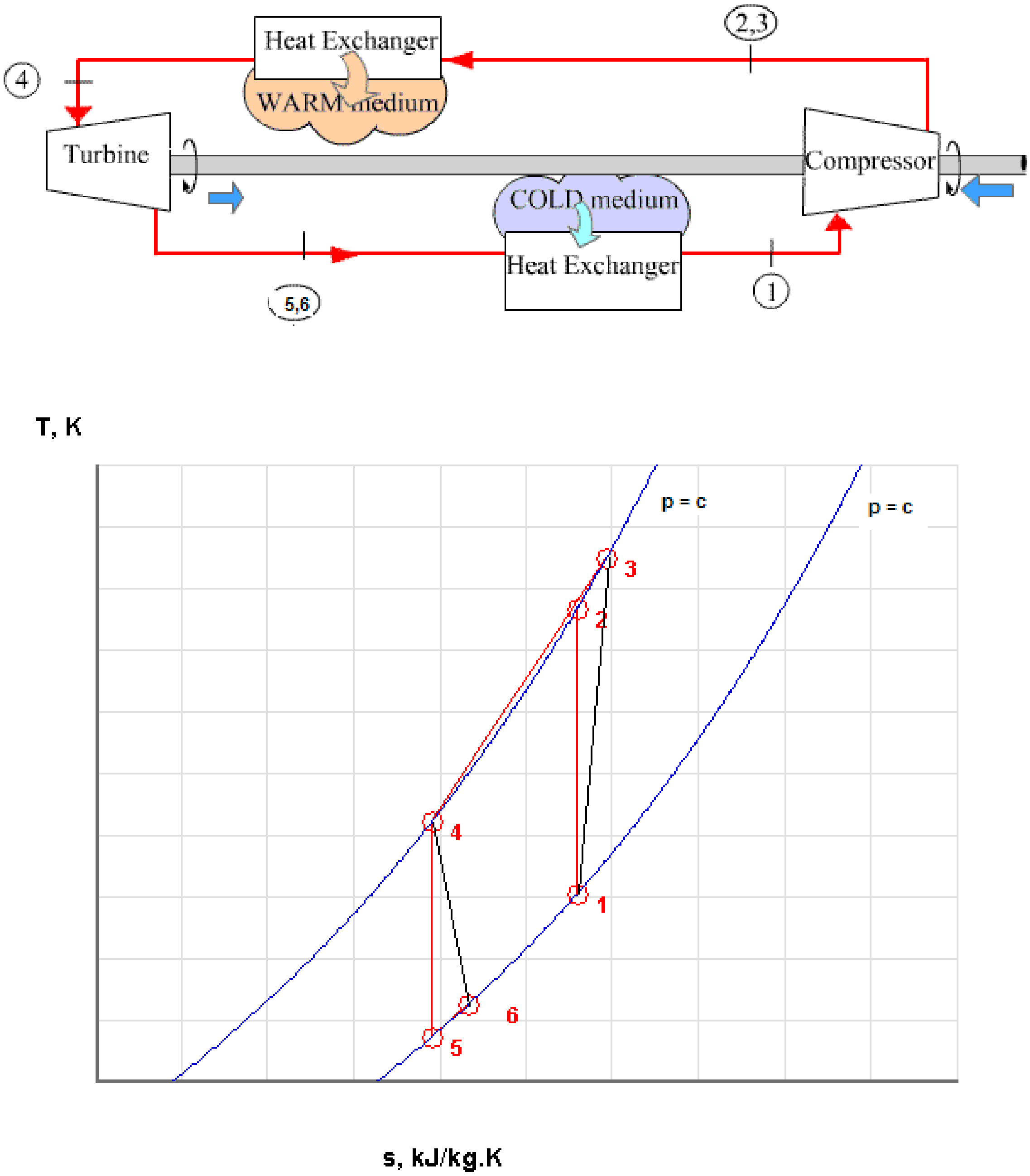


Fig.Prob.4.5.6 Reversed Brayton cycle and its T-s diagram

TEST Solution:

Following are the steps:

Steps 1 and 2 are the same as for Prob.4.5.1. But, now choose for material model, the PG model, and for working substance, Air.

1. Choose Airas working substance and fill up the known parameters for State 1, i.e. T1 = -6 C, P1 = 101 kPa, and mdot1 = 1 kg/s. Hit Enter. We get:

Note that all parameters such as h1, s1 etc are calculated.

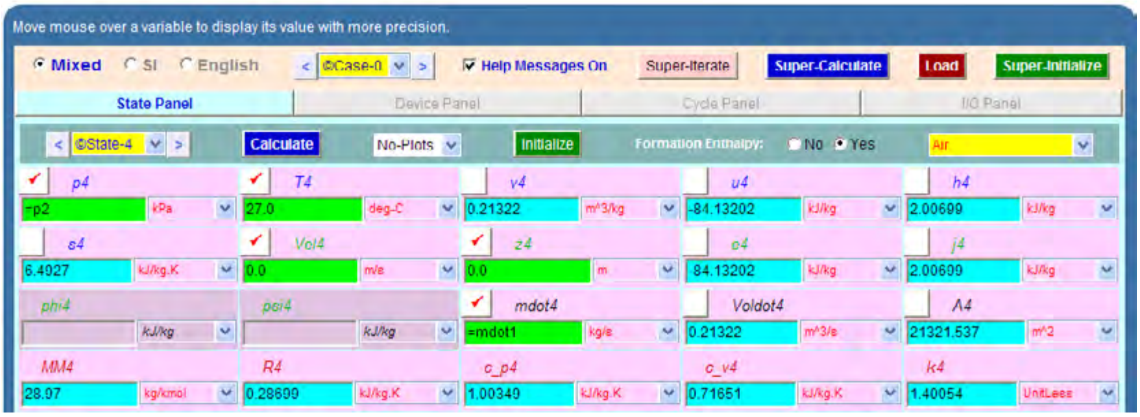
2. State 2: Enter p2= 404 kPa, s2 = s1, and mdot2 = mdot1. Hit Enter. We get:

Note that h2, T2 etc are calculated.

3. For State 3: Enter p3 = p2, h3 = h1 + (h2-h1)/0.85 where 0.85 is the isentropic effcy of compressor, and mdot3 = mdot1. Hit Enter. We get:

Note that T3, s3 etc are calculated.

4. For State 4: Enter T4 = 27 C, p4 = p2, mdot4 = mdot1. Hit Enter. We get:



Note that h4, s4 etc are calculated.

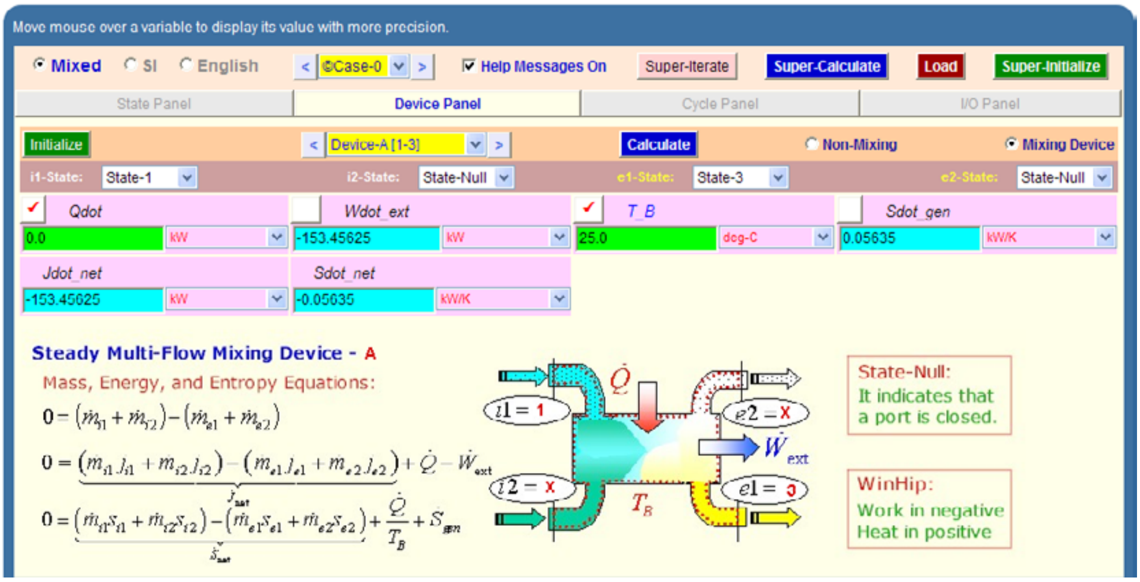
5. For State 5: Enter p5 = p1, s5 = s4, mdot5 = mdot1. Hit Enter. We get:

Note that T5, h5 etc are calculated.

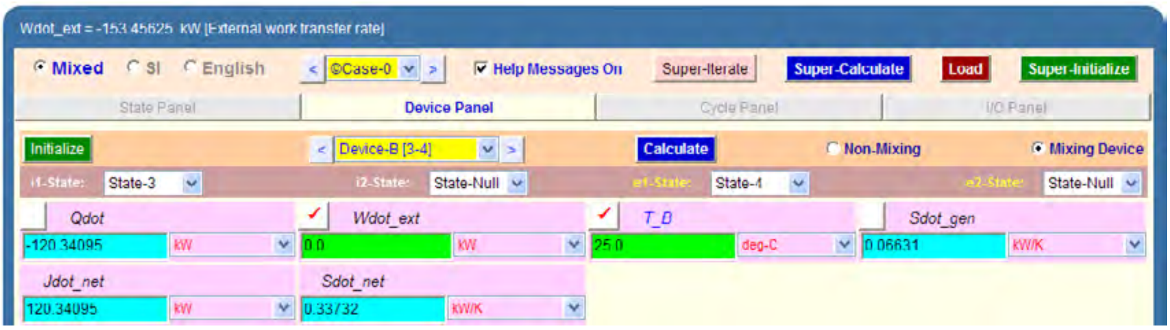
6. For State 6: Enter p6 = p5, h6 = h4 – 0.85 * (h4 – h5) where 0.85 is the isentropic effcy of the turbine, mdot5 = mdot1. Hit Enter. We get:

Note that T6, s6 etc are calculated.

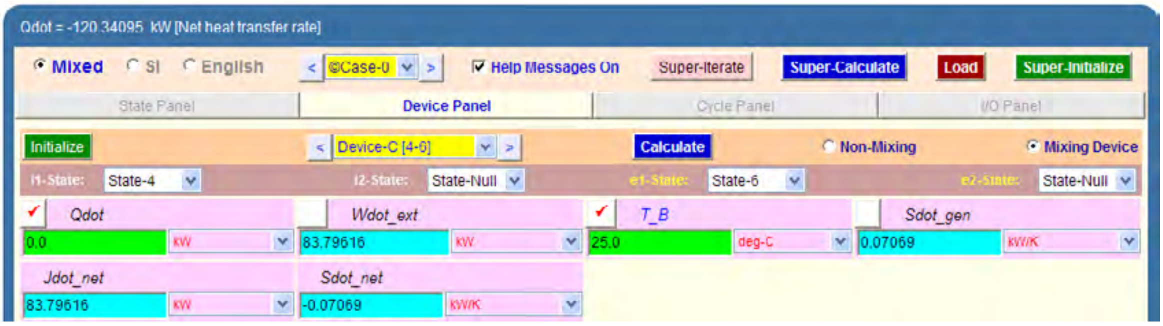
7. Now, go to Device panel. For Device A, fill up State 1 and State 3 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. And, Qdot = 0. Hit Enter. We get:



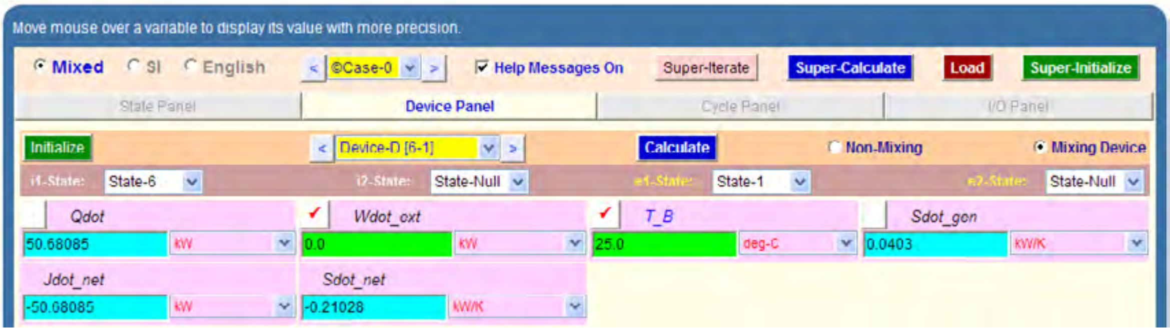
8. For Device B: fill up State 3 and State 4 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Wdot_ext = 0, since there is no external work in process 1-2. Hit Enter. We get:



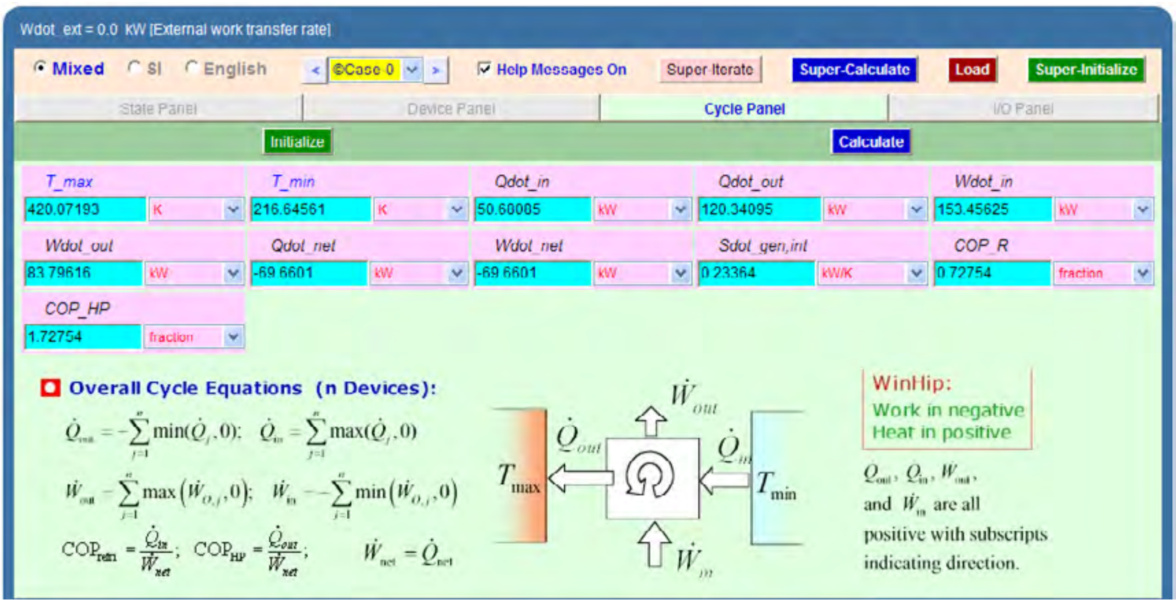
9. For Device C: fill up State 4 and State 6 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Qdot = 0. Hit Enter. We get:



10. For Device D: fill up State 6 and State 1 for i1 state and e1 state respectively. For i2 state and e2 states, fill up Null State as there is no second stream of flow. Also, Wdot_ext = 0. Hit Enter. And **click on SuperCalculate**.We get:



11. Now go to Cycle panel. All important cycle parameters are available here:



Thus:

COP_R = COP of refrigerator = 0.72754… Ans.

#For 1 kg/s circulation---refrig. effect is Qdot_in = 50.681 kW

#1 Ton is equiv. to: 211 kJ/min

=211/60 = 3.5167 kW

#Therefore, mass circulation rate reqd. to produce 1 Ton of refrigeration.:

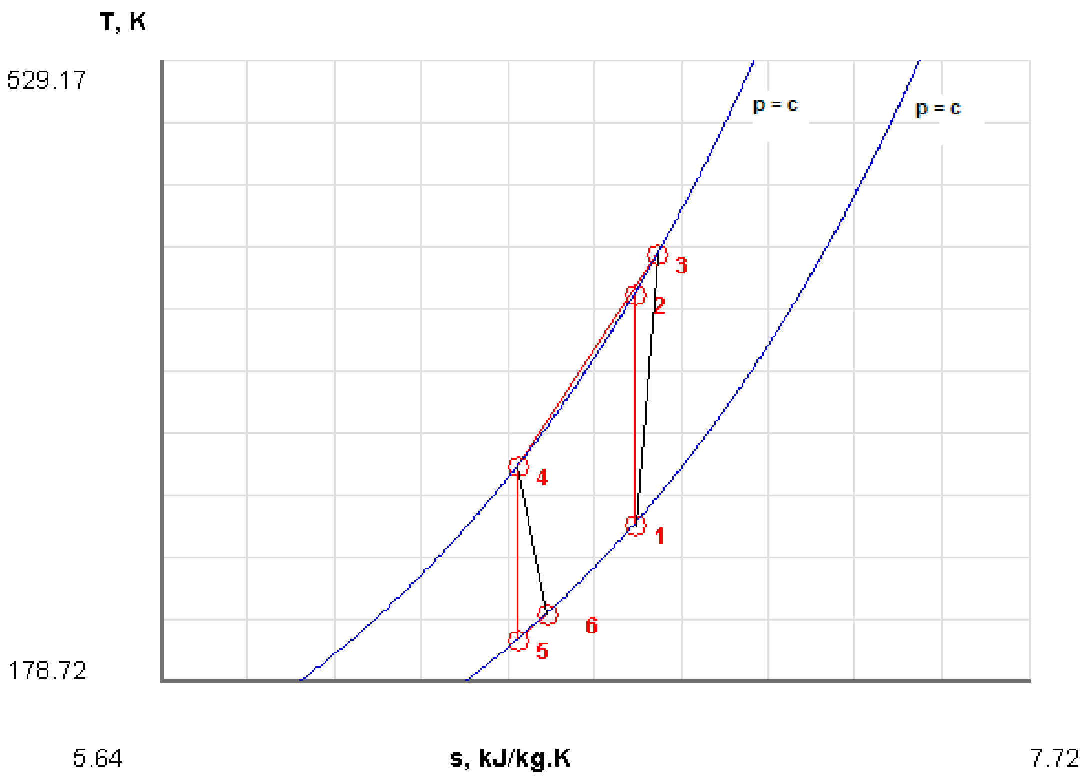
=3.517/50.681 = 0.069395 kg/s .... Ans.

#Power reqd. to produce 1 TR:

#Power reqd. with 1 kg/s circulation = Wdot_in = 153.456 kW. Therefore, power reqd. with 0.069395 kg/s circulation (or, 1 TR):

=153.45625*0.069395 = 10.649kW...Ans.

12. From the Plots widget, get the T-s plot:



13. The I/O panel gives the TEST code etc:

#~~~~~~~~~~~~~~~~~~~~~~OUTPUT OF SUPER-CALCULATE

 # TESTcalc Path: Systems>Open>SteadyState>Specific>RefrigCycle>PG-Model; v-10.ce02

#-------------------Start of TEST-code ---

States {

State-1: Air;

Given: { p1= 101.0 kPa; T1= -6.0 deg-C; Vel1= 0.0 m/s; z1= 0.0 m; mdot1= 1.0 kg/s; }

State-2: Air;

Given: { p2= 404.0 kPa; s2= "s1" kJ/kg.K; Vel2= 0.0 m/s; z2= 0.0 m; mdot2= "mdot1" kg/s; }

State-3: Air;

Given: { p3= "p2" kPa; h3= "h1+(h2-h1)/0.85" kJ/kg; Vel3= 0.0 m/s; z3= 0.0 m; mdot3= "mdot1" kg/s; }

State-4: Air;

Given: { p4= “p2” kPa; T4= 27.0 deg-C; Vel4= 0.0 m/s; z4= 0.0 m; mdot4= “mdot1” kg/s; }

State-5: Air;

Given: { p5= “p1” kPa; s5= “s4” kJ/kg.K; Vel5= 0.0 m/s; z5= 0.0 m; mdot5= “mdot1” kg/s; }

State-6: Air;

Given: { p6= “p5” kPa; h6= “h4-0.85*(h4-h5)” kJ/kg; Vel6= 0.0 m/s; z6= 0.0 m; mdot6= “mdot1” kg/s; }

}

Analysis {

Device-A: i-State = State-1; e-State = State-3; Mixing: true;

Given: { Qdot= 0.0 kW; T_B= 25.0 deg-C; }

Device-B: i-State = State-3; e-State = State-4; Mixing: true;

Given: { Wdot_ext= 0.0 kW; T_B= 25.0 deg-C; }

Device-C: i-State = State-4; e-State = State-6; Mixing: true;

Given: { Qdot= 0.0 kW; T_B= 25.0 deg-C; }

Device-D: i-State = State-6; e-State = State-1; Mixing: true;

Given: { Wdot_ext= 0.0 kW; T_B= 25.0 deg-C; }

}

#----------------------End of TEST-code --

#--------Property spreadsheet starts:

#	State	p(kPa)	T(K)	v(m^3/kg)	u(kJ/kg)	h(kJ/kg)	s(kJ/kg.K)
#	1	101.0	267.2	0.7591	-107.78	-31.11	6.774
#	2	404.0	397.1	0.2821	-14.64	99.33	6.774
#	3	404.0	420.1	0.2984	1.79	122.35	6.83
#	4	404.0	300.2	0.2132	-84.13	2.01	6.493
#	5	101.0	201.9	0.5737	-154.52	-96.58	6.493
#	6	101.0	216.6	0.6156	-143.96	-81.79	6.563

#--------Property spreadsheet ends---

 # Cycle Analysis Results:

Calculated: T_max= 420.07193 K; T_min= 216.64561 K; **Qdot_in= 50.68085 kW**;

Qdot_out= 120.34095 kW; **Wdot_in= 153.45625 kW**; Wdot_out= 83.79616 kW;

Qdot_net= -69.6601 kW; Wdot_net= -69.6601 kW; Sdot_gen,int= 0.23364 kW/K;

COP_R= 0.72754 fraction; COP_HP= 1.72754 fraction; BWR= 183.13042 %;

#

#******CALCULATE VARIABLES: Type in an expression starting with an '=' sign ('= mdot1*(h2-h1)', '= sqrt(4*A1/PI)', etc.) and press the Enter key)*********

#

#For 1 kg/s circulation---refrig. effect is Qdot_in = 50.681 kW

#1 Ton is equiv. to: 211 kJ/min

=211/60 = 3.5167 kW

#Therefore, mass circulation rate reqd. to produce 1 Ton of refrigeration.:

=3.517/50.681 = 0.069395 kg/s .... Ans.

#Power reqd. to produce 1 TR:

#Power reqd. with 1 kg/s circulation = Wdot_in = 153.456 kW. Therefore, power reqd. with 0.069395 kg/s circulation (or, 1 TR):

=153.45625*0.069395 = 10.64901kW...Ans.

==

4.6 References:

1. *Yunus A. Cengel & Michael A. Boles*, Thermodynamics, An Engineering Approach, 7th Ed. McGraw Hill, 2011.
2. *Sonntag, Borgnakke & Van Wylen*, Fundamentals of Thermodynamics, 6th Ed. John Wiley & Sons, 2005.
3. *Michel J. Moran & Howard N. Shapiro*, Fundamentals of Engineering Thermodynamics, 4th Ed. John Wiley & Sons, 2000.
4. *P.K. Nag*, Engineering Thermodynamics, 2nd Ed. Tata McGraw Hill Publishing Co., 1995.
5. *R.K. Rajput,* A Text Book of Engineering Thermodynamics, Laxmi Publications, New Delhi, 1998.
6. *Domkunndwar et al,* A course in Thermal Engineering, Dhanpat Rai & Co., New Delhi, 2000
7. TEST Software: www.thermofluids.net
8. DUPREX free Software from M/s DUPONT… Refer to following web address: http://www2.dupont.com/Refrigerants/en_US/products/DUPREX/DUPREX_registration.html

5 Air compressors

Learning objectives:

9. In this chapter, 'Air compressors' are dealt with.
10. While solving problems, quantities of interest are: volumetric efficiency, work required for actual compression without clearance volume and with clearance volume, isothermal efficiency, minimum work required for two stage (or multistage) compression with perfect intercooling, heat transferred to the intercooler, determining the cylinder diameter and stroke etc.
11. Formulas to calculate the above quantities are summarized.
12. Problems from University question papers and standard Text books are solved with Mathcad and EES.

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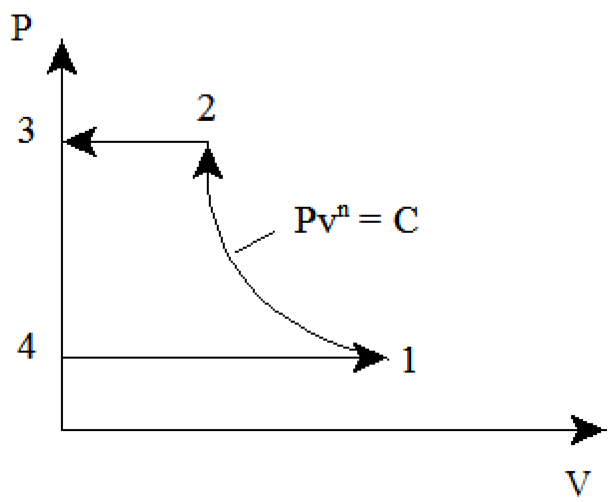
5.1 Definitions, Statements and Formulas used[1-6]:

5.1.1 Work done per kg of air compressed:

(a) Without clearance:



4-1: suction of air at pressure P1

1-2: polytropic compression

2-3: discharge of air to the receiver at pressure P2.

Area 4-1-2-3-4 in the P-V diagram represents the work done per cycle.

Work done per cycle:

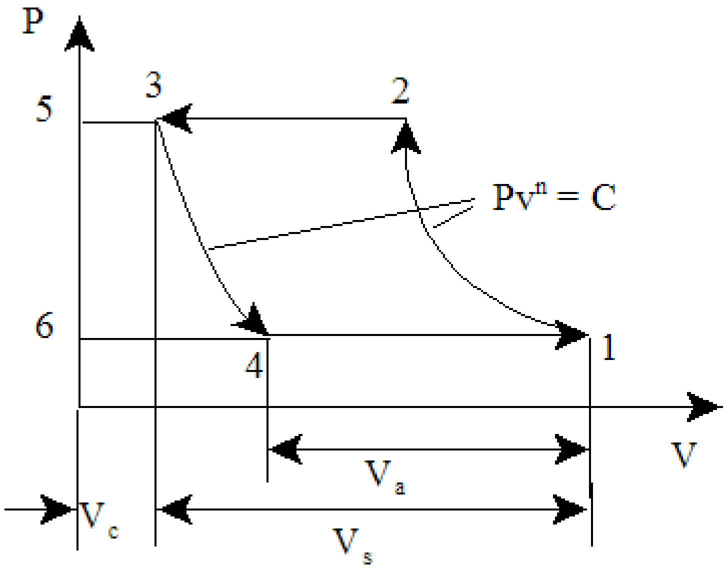
$$W_c = \frac{n}{n-1} \cdot P1 \cdot V1 \cdot \left[\left(\frac{P2}{P1} \right)^{\frac{n-1}{n}} - 1 \right] \quad \text{...J/cycle}$$

i.e. $$W_c = \frac{n}{n-1} \cdot m \cdot R \cdot T1 \cdot \left[\left(\frac{P2}{P1} \right)^{\frac{n-1}{n}} - 1 \right]$$ ...J/cycle, where m is the mass delivered per cycle

Work done per kg of air delivered:

$$W_c = \frac{n}{n-1} \cdot R \cdot T1 \cdot \left[\left(\frac{P2}{P1} \right)^{\frac{n-1}{n}} - 1 \right] \quad \text{J/kg.}$$

(b) With clearance volume:



Area 4-1-2-3-4 in the P-V diagram represents the work done per cycle, assuming that compression and expansion follow the same law.

Work done per cycle:

$$W_c = \frac{n}{n-1} \cdot P1 \cdot V_a \cdot \left[\left(\frac{P2}{P1} \right)^{\frac{n-1}{n}} - 1 \right] \quad \text{J/cycle}$$

i.e. $$W_c = \frac{n}{n-1} \cdot m_1 \cdot R \cdot T1 \cdot \left[\left(\frac{P2}{P1} \right)^{\frac{n-1}{n}} - 1 \right] \quad \text{J/cycle}$$

Work done per kg of air delivered:

$$W_c = \frac{n}{n-1} \cdot R \cdot T1 \cdot \left[\left(\frac{P2}{P1} \right)^{\frac{n-1}{n}} - 1 \right] \quad \text{J/kg}$$

Note: clearance volume does not affect the work of compression per kg of air.

5.1.2 Volumetric efficiency: See the above fig.

$$\eta_{vol} = \frac{V1 - V4}{V1 - V3} = \frac{V_a}{V_s} = \frac{actual\_volume}{stroke\_volume}$$

Also:

$$\eta_{vol} = 1 + C - C \cdot \left(\frac{P2}{P1}\right)^{\frac{1}{n}}$$ .... where C = clearance ratio = Vc/Vs

Volumetric efficiency referred to ambient conditions:

η_v = volume of air sucked referred to ambient conditions divided by swept volume

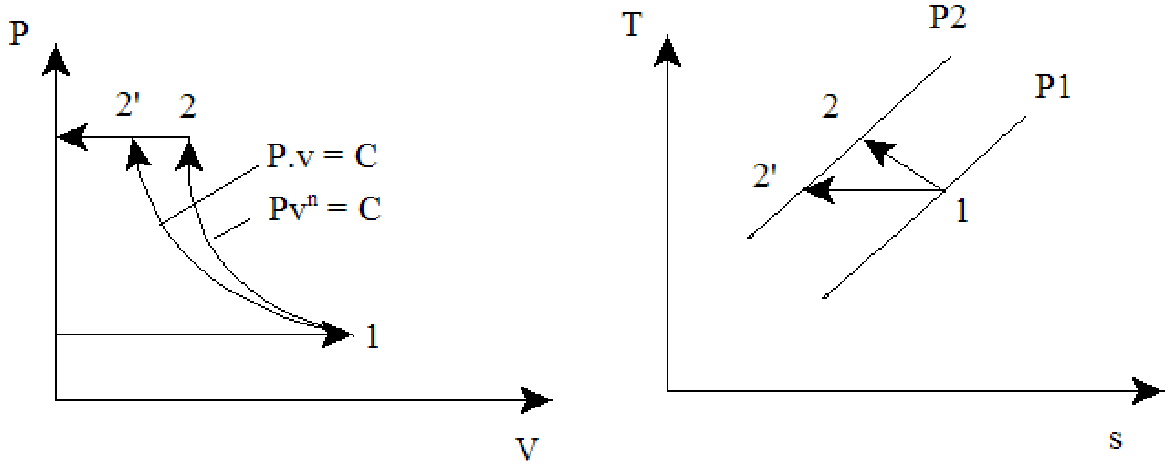
We get:

$$\eta_v = \frac{P1 \cdot T0}{P0 \cdot T1} \cdot \frac{(V1 - V4)}{V_s}$$

i.e. $$\eta_v = \frac{P1 \cdot T0}{P0 \cdot T1} \cdot \left[1 + C - C \cdot \left(\frac{P2}{P1}\right)^{\frac{1}{n}}\right]$$

Note: To find out the cylinder dimensions, use the volumetric effcy. at suction conditions only.

5.1.3 Isothermal efficiency:



Isothermal effcy. is defined as the ratio of isothermal work to actual work:

And, we have for Isothermal work:

$$W_{iso} = R \cdot T1 \cdot \ln\left(\frac{P2}{P1}\right) \quad \text{J/kg}$$

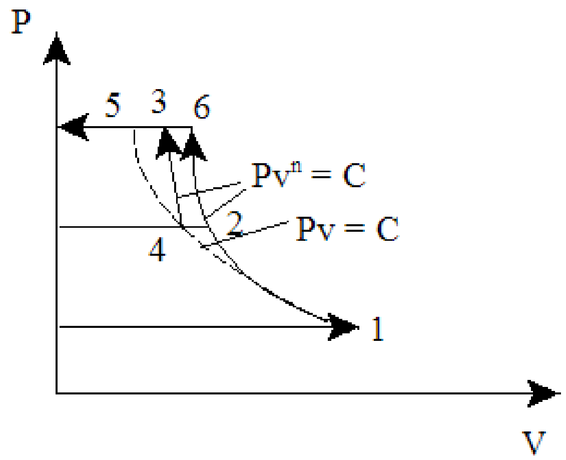
And, actual work:

$$W_c = \frac{n}{n-1} \cdot R \cdot T1 \cdot \left[\left(\frac{P2}{P1}\right)^{\frac{n-1}{n}} - 1\right] \quad \text{J/kg}$$

And:

$$\eta_{iso} = \frac{W_{iso}}{W_c} \quad \text{....Isothermal effcy.}$$

5.1.4 Two stage compression with 'perfect intercooling' (with no clearance):



1-2: polytropic compression in first stage compressor from P1 to P2

2-4: 'perfect intercooling' in intercooler (i.e. T4 = T1)

4-3: polytropic compression in second stage compressor from P2 to P3

1-4: isothermal compression from P1 to P2 (….for reference)

4-5: isothermal compression from P2 to P3 (….for reference)

With 'perfect intercooling', *condition for minimum work required per kg of air delivered* is:

$$\frac{P2}{P1} = \frac{P3}{P2} \quad \text{...for two stage compressor}$$

i.e. pressure ratio in each stage is same.

For N stage compressor:

$P1/P2 = P3/P2 = \ldots\ldots = P_{N+1}/P_N = k$, say

Then:

$k = (P_{N+1}/P1)\text{^}(1/N)$

Then, work done in each stage is same.

So, total work for two stages is:

$$W_{tot} = 2 \cdot \frac{n}{(n-1)} \cdot R \cdot T1 \cdot \left[\left(\frac{P2}{P1} \right)^{\frac{n-1}{n}} - 1 \right] \quad \text{J/kg}$$

For N stages, total work is:

$$W_{tot} = N \cdot \frac{n}{(n-1)} \cdot R \cdot T1 \cdot \left[\left(\frac{P2}{P1} \right)^{\frac{n-1}{n}} - 1 \right] \quad \text{J/kg}$$

Remember again that above two equations are valid for the conditions:

1. perfect intercooling, and
2. index of compression and expansion are same.

5.1.5 To find the cylinder dimensions:

Use the condition that mass of air passing through each cylinder per stroke must be the same in steady flow.

$$v_{a1} \cdot \rho_1 = v_{a2} \cdot \rho_2 = v_{a3} \cdot \rho_3 = \text{const} \quad \text{...for 3 stage compressor}$$

i.e. $$v_{a1} \cdot \frac{P1}{R \cdot T1} = v_{a2} \cdot \frac{P2}{R \cdot T2} = v_{a3} \cdot \frac{P3}{R \cdot T3}$$

But, with perfect intercooling, T1 = T2 = T3

Then: $$v_{a1} \cdot P1 = v_{a2} \cdot P2 = v_{a3} \cdot P3$$

i.e. $$v_{s1} \cdot \eta_{v1} \cdot P1 = v_{s2} \cdot \eta_{v2} \cdot P2 = v_{s3} \cdot \eta_{v3} \cdot P3$$

And, stroke volume in each case is calculated as: $v_s = \pi \cdot \frac{D^2}{4} \cdot L$

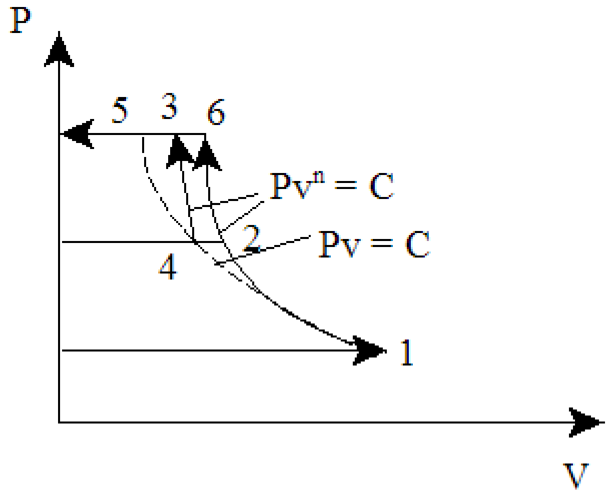
If stroke and vol. effcy. are same for each stage, then, we have:

$$D_1^{\,2} \cdot P1 = D_2^{\,2} \cdot P2 = D_3^{\,2} \cdot P3$$

Generally, L/D ratio is given.

Thus, both D and L are calculated.

5.1.6 Heat transferred in intercooler:



$Q_{intercooler} = m_{air} \cdot cp \cdot (T2 - T4) = m_{air} \cdot cp \cdot (T2 - T1)$ W ... since T4 = T1 for perfect intercooling, m_{air} = mass flow rate of air, kg/s

==

5.2 Problems solved with Mathcad:

Prob.5.2.1 Plot the effects of pressure ratio, discharge pressure and polytropic index, n on Vol. effcy:

Mathcad Solution:

Let:

$$C = clearance\_ratio = \frac{clearance\_volume}{Stroke\_volume} = \text{"4\% to 10 \%, generally."}$$

$P1 = inlet\_pressure$ $P2 = discharge\_pressure$

$n = index\_of\_compression$

Then:

Volumetric effcy. = actual volume at suction conditions / swept volume

$$\eta_{vol}(C, P1, P2, n) := 1 + C - C\cdot\left(\frac{P2}{P1}\right)^{\frac{1}{n}}$$ ....vol. effcy. defined as a Mathcad Function

Ex: $n := 1.3$ $P1 := 1$ $P2 := 4$ bar $C := 0.04$

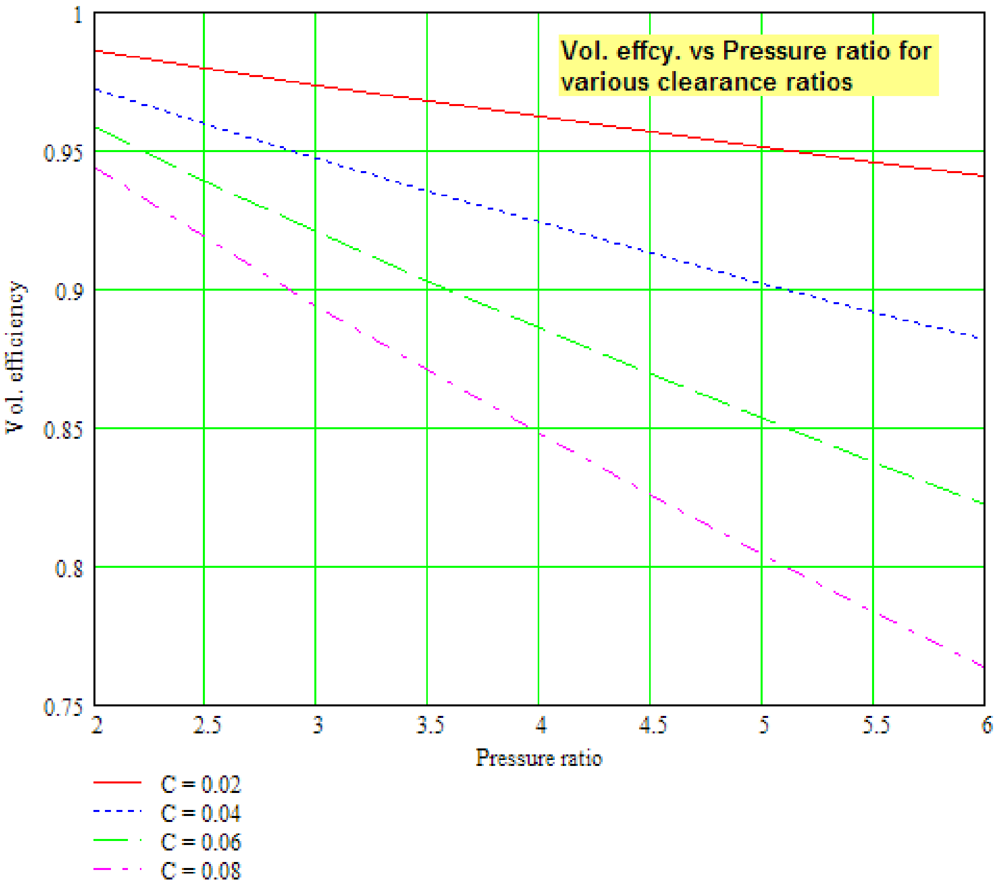
$\eta_{vol}(C, P1, P2, n) = 0.924$ ....vol. effcy.

1. **Now, plot the effect of clearance volume for different pressure ratios:**

$$\eta_v(C, Pr\_ratio, n) := 1 + C - C\cdot(Pr\_ratio)^{\frac{1}{n}}$$ ...define the Mathcad Function again.

$Pr\_ratio := 2, 2.5 .. 6$ ...define a range variable

Pr_ratio	$\eta_v(0.02, Pr\_ratio, n)$	$\eta_v(0.04, Pr\_ratio, n)$	$\eta_v(0.06, Pr\_ratio, n)$	$\eta_v(0.08, Pr\_ratio, n)$
2	0.986	0.972	0.958	0.944
2.5	0.98	0.959	0.939	0.918
3	0.973	0.947	0.92	0.894
3.5	0.968	0.935	0.903	0.87
4	0.962	0.924	0.886	0.848
4.5	0.956	0.913	0.869	0.826
5	0.951	0.902	0.853	0.804
5.5	0.946	0.892	0.837	0.783
6	0.941	0.881	0.822	0.763

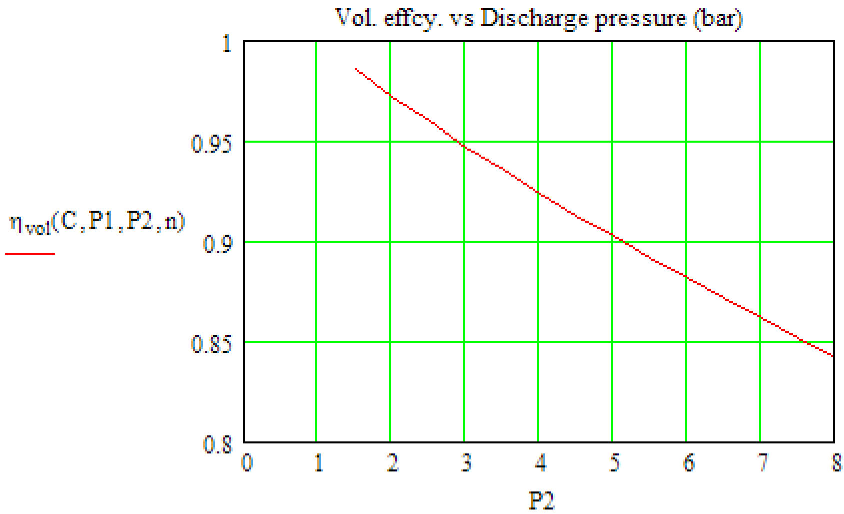


2. **Now, plot the effect of discharge pressure P2 on vol. effcy.:**

P2 := 1.5, 2.. 8 ....define a range variable

C = 0.04 n = 1.3

P2 =	$\eta_{vol}(C, P1, P2, n)$
1.5	0.985
2	0.972
2.5	0.959
3	0.947
3.5	0.935
4	0.924
4.5	0.913
5	0.902
5.5	0.892
6	0.881
6.5	0.871
7	0.861
7.5	0.852
8	0.842

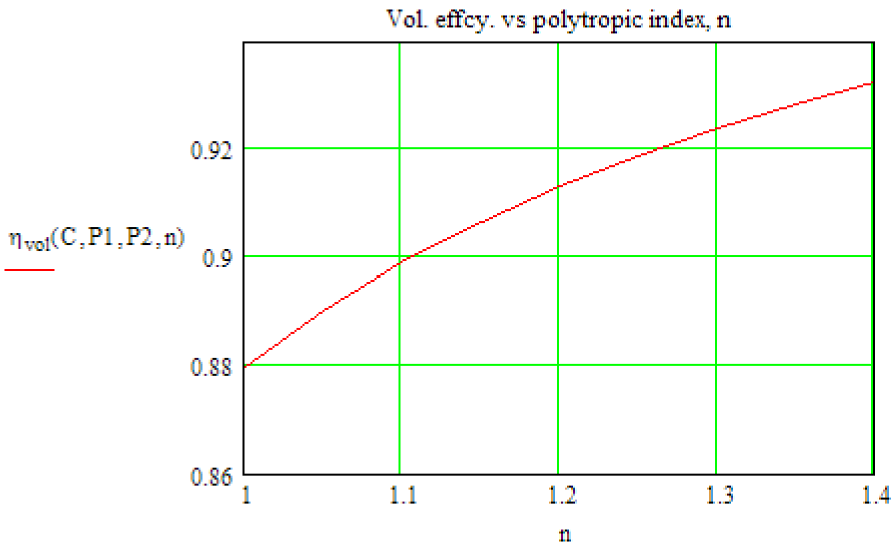

Vol. effcy. vs Discharge pressure (bar)
$\eta_{vol}(C, P1, P2, n)$
1
0.95
0.9
0.85
0.8
0
1
2
3
4
5
6
7
8
P2

3. **And, plot the effect of polytropic index, n on vol. effcy.:**

$P1 := 1 \quad P2 := 4$ bar $\quad C := 0.04$

$n := 1, 1.05 .. 1.4$ ....define a rage variable

n =	$\eta_{vol}(C,P1,P2,n)$
1	0.88
1.05	0.89
1.1	0.899
1.15	0.906
1.2	0.913
1.25	0.919
1.3	0.924
1.35	0.928
1.4	0.932



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Prob.5.2.2 Write Mathcad Functions for compressor work per stage.

Mathcad Solution:

1. When compression is ploytropic:

$$W_{polytr}(n, P1, P2, T1, R_{air}) := \frac{n \cdot R_{air} \cdot T1}{n-1} \cdot \left[\left(\frac{P2}{P1}\right)^{\frac{n-1}{n}} - 1\right] \quad \text{...kJ/kg}$$

where $n \equiv comprn\_index$ P1,P2 = inlet and exit pressures in bar or kPa

$T1 \equiv inlet\_temp\_K$

$m \equiv mass\_kgpersec$ $R_{air} \equiv 0.287$ kJ/kg.K

Ex: $P1 := 1$ $P2 := 4$ bar $T1 := 300$ K $R_{air} := 0.287$ kJ/kg.K

$n := 1.3$

$W_{polytr}(n, P1, P2, T1, R_{air}) = 140.662$ kJ/kg

---

2. When compression is isothermal:

$$W_{isoth}(R_{air}, T1, P1, P2) := R_{air} \cdot T1 \cdot \ln\left(\frac{P2}{P1}\right)$$ **kJ/kg ... Isothermal work**

where, T1 (K) , P1, P2 (bar or kPa), R_air = 0.287 kJ/kg.K

Ex: $P1 := 1$ $P2 := 4$ bar $T1 := 300$ K $R_{air} := 0.287$ kJ/kg.K

$W_{isoth}(R_{air}, T1, P1, P2) = 119.36$ kJ/kg

---

3. When compression is isentropic:

$$W_{isentr}(\gamma, P1, P2, T1, R_{air}) := \frac{\gamma \cdot R_{air} \cdot T1}{n-1} \cdot \left[\left(\frac{P2}{P1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$ ...kJ/kg

where $\gamma \equiv 1.4$ for air P1,P2 = inlet and exit pressures in bar or kPa

$T1 \equiv inlet\_temp\_K$ $R_{air} \equiv 0.287$ kJ/kg.K

Ex: $P1 := 1$ $P2 := 4$ bar $T1 := 300$ K $R_{air} := 0.287$ kJ/kg.K

$\gamma := 1.4$

$W_{isentr}(\gamma, P1, P2, T1, R_{air}) = 195.273$ kJ/kg

Note: Thus, we see that when the compression is isentropic, max. work input is required, and when it is isothermal, min. work input is required; when the compression is polytropic, the work requirement is in between that required for the other two cases.

==

Prob.5.2.3. A single cylinder, double acting air compressor is reqd. to compress 10 m^3 of free air per min. The free air conditions are 1 bar and 27 C. The delivery pressure is 16 bar. Determine the power of the motor required and the cylinder dimensions, if the following data is given:

Speed of compressor = N = 350 rpm; Clearance vol. = 5% of stroke vol.; Stroke to bore ratio = 1.3; Mech. effcy. = 80%; n = 1.3; The suction pressure = 0.95 bar; suction temp. = 35 C; The compressor is single stage. [M.U.]

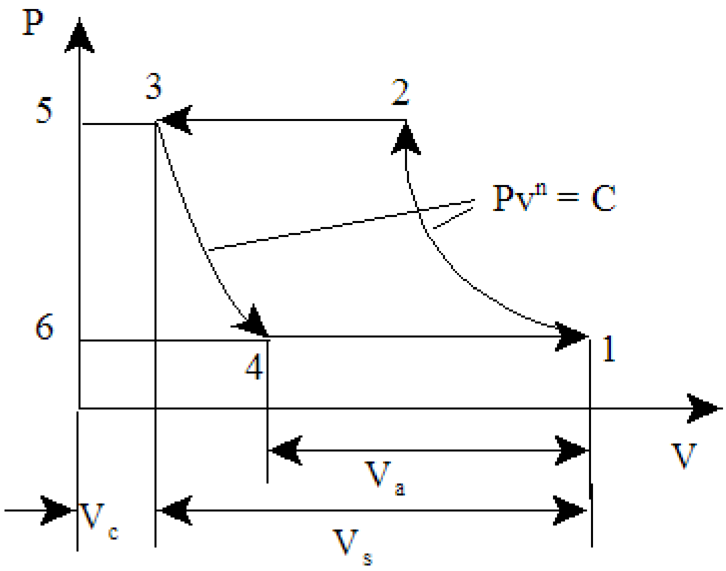

Fig.Prob.5.2.3 Single stage air compressor with clearance

Mathcad Solution:

Data:

$P1 := 0.95 \cdot 10^5$ Pa $\quad P2 := 16 \cdot 10^5$ Pa $\quad T1 := 273 + 35$ K $\quad N := 350$ rpm

$n := 1.3 \quad R := 287$ kJ/kg.K $\quad C := 0.05$ ...clearance ratio $\quad \eta_{mech} := 0.80$

$V_f := 10$ m^3/min $\quad T_f := 27 + 273$ K $\quad P_f := 1 \cdot 10^5$ Pa

Calculations:

$m_a := \frac{P_f \cdot V_f}{R \cdot T_f \cdot 60}$ i.e. $m_a = 0.194$ kg/s

Let V_a be the actual vol. of air inhaled at suction conditions; V_s, the stroke vol.; η_v the vol. effcy

Then, we have:

$\eta_v := 1 + C - C \cdot \left(\frac{P2}{P1}\right)^{\frac{1}{n}}$ i.e. $\eta_v = 0.611$ ....vol. effcy. ref. to suction conditions

To find swept volume, V_s: Use the 'Solve block' of Mathcad:

$V_s := 0.1$ m^3.....Trial value

Given

$$m_a = \frac{2 \cdot \eta_v \cdot V_s \cdot P1}{R \cdot T1} \cdot \frac{N}{60}$$ factor 2....for double acting compr.

$\mathrm{Find}(V_s) = 0.02526$ m^3......Swept vol

i.e. $V_s := 0.02526$ m^3.........This is equal to stroke * area = (π*D^2/4) * (1.3 D)

Therefore:

$$D := \left(\frac{V_s \cdot 4}{1.3 \cdot \pi}\right)^{\frac{1}{3}}$$ i.e. $D = 0.291$ **m....Ans.**

And: $L := 1.3 \cdot D$ i.e. $L = 0.379$ **m....Ans.**

Compressor power input:

$$W_c := \frac{n \cdot R \cdot T1}{n-1} \cdot \left[\left(\frac{P2}{P1}\right)^{\frac{n-1}{n}} - 1\right] \cdot 10^{-3} m_a$$ i.e. $W_c = 68.122$ kW

Therefore:

$$P := \frac{W_c}{\eta_{mech}}$$ $P = 85.153$ **kW.....Motor power required... Ans.**

===

Prob.5.2.4. A single acting reciprocating air compressor has cylinder bore 15 cm, stroke 25 cm, C = 0.05. N = 500 rpm. Air is taken in at 1 bar and 27 C and delivered at 11 bar. Assume n = 1.25. Find: (i) vol. effcy (ii) Power reqd. to drive the compressor, if mech. effcy = 0.8 [M.U.]

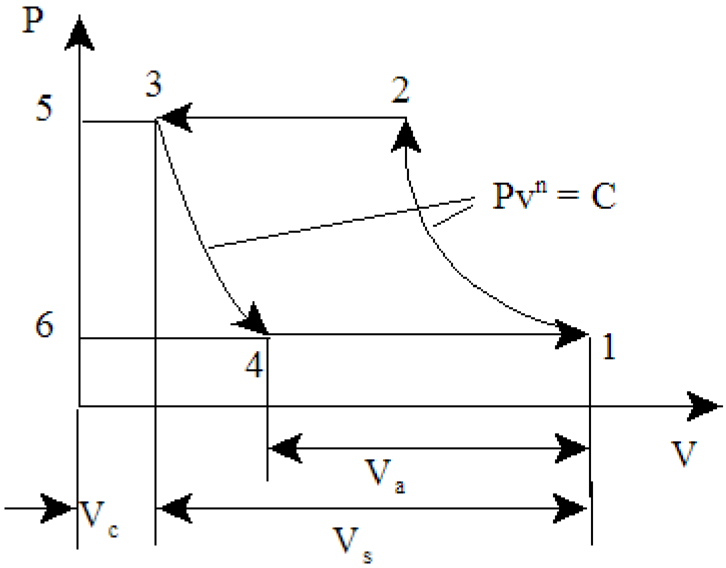

Fig.Prob.5.2.4 Single stage air compressor with clearance

Mathcad Solution:

Data:

$P1 := 1.0 \cdot 10^5$ Pa $\quad P2 := 11 \cdot 10^5$ Pa $\quad N := 500$ rpm

$C := 0.05 \quad R := 287$ J/kg.K $\quad n := 1.25 \quad \eta_{mech} := 0.8 \quad T1 := 300$ K

$d := 0.15$ m $\quad stroke := 0.25$ m

Calculations:

$V_s := \frac{\pi \cdot d^2}{4} \cdot stroke$ i.e. $V_s = 4.418 \times 10^{-3}$ **m^3,...Piston Displ.**

$\eta_v := 1 + C - C \cdot \left(\frac{P2}{P1}\right)^{\frac{1}{n}}$ i.e. $\eta_v = 0.71$ **....vol. effcy....Ans.**

$m_a := \frac{V_s \cdot \eta_v \cdot P1}{R \cdot T1} \cdot N$ i.e. $m_a = 1.82$ **kg/min....mass flow rate of air based on stroke vol. filled at suction conditions**

Power reqd.:

$$W_c := \frac{n \cdot R \cdot T1}{n-1} \cdot \left[\left(\frac{P2}{P1}\right)^{\frac{n-1}{n}} - 1\right] \cdot 10^{-3} \cdot \frac{m_a}{60} \cdot \frac{1}{\eta_{mech}}$$

i.e. $W_c = 10.047$ **kW....Ans.**

===

Prob.5.2.5. A single stage, double acting air compressor requires 62.5 kW indicated power at 120 r.p.m. It takes air in at 1 bar and delivers at 10 bar. The compression and expansion follow pV^1.35 = C. Taking the following data, find the dia and stroke of compressor: Piston speed = 200 m/min. Vol. effcy. = 90%. Also find the clearance vol. as a percentage of stroke volume. [M.U.]

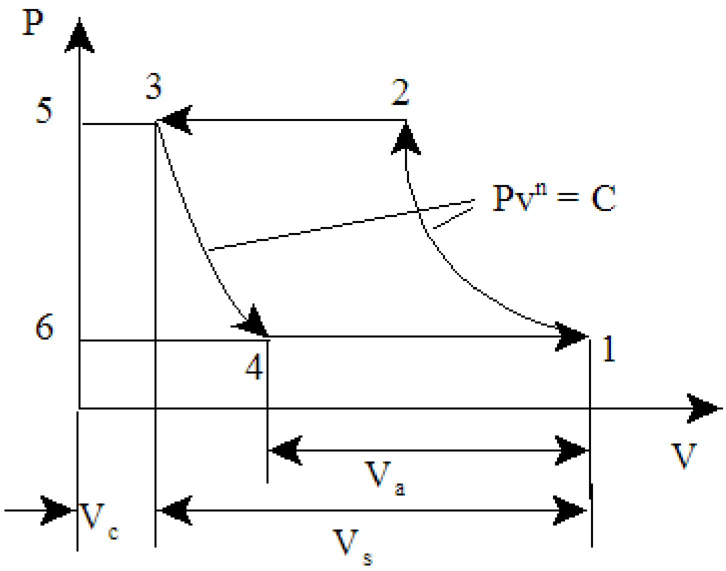

Fig.Prob.5.2.5 Single stage air compressor with clearance

Mathcad Solution:

Data:

$P1 := 1.0 \cdot 10^5$ Pa $\quad P2 := 10.0 \cdot 10^5$ Pa $\quad N := 120$ rpm $\quad W_c := 62.5$ kW

$\eta_v := 0.9 \quad R := 287$ J/kg.K $\quad V := 200$ m/min $\quad n := 1.35$

Calculations:

2*L*N = V.....piston speed

$L := \frac{V}{2 \cdot N}$ i.e. $L = 0.833$ **m....stroke.....Ans.**

$C := 0.05$ --- trial value

Given

$$\eta_v = 1 + C - C \cdot \left(\frac{P2}{P1}\right)^{\frac{1}{n}}$$

$\text{Find}(C) = 0.022$

i.e. $C := 0.022$ **....clearance ratio.....Ans.**

$V_s := 0.5$ m^3...trial value

Given

$$W_c = \frac{n \cdot P1 \cdot \eta_v \cdot V_s}{n-1} \cdot \left[\left(\frac{P2}{P1} \right)^{\frac{n-1}{n}} - 1 \right] \cdot 10^{-3} \cdot 2 \cdot \frac{N}{60}$$ ....2..since double acting

$\text{Find}(V_s) = 0.05512$

i.e. $V_s := 0.05512$ m^3.....stroke vol

Therefore, $V_c := C \cdot V_s$ i.e. $V_c = 1.21264 \times 10^{-3}$ **m^3....clearance vol.**

And: $D := \sqrt{\frac{4 \cdot V_s}{\pi \cdot L}}$ i.e. $D = 0.29$ **m.....cyl.dia....Ans.**

===

Prob.5.2.6. A single acting 12 cm * 10 cm reciprocating air compressor having 4% clearance gives the following data from a performance test:

suction pressure = 0 bar gauge; suction temp. = 20 C; Barometer reading = 76 cm; Discharge pressure = 5 bar gauge; Disch. temp. = 180 C; Speed = 1200 rpm; Shaft power = 6.247 kW; Mass of air delivered = 1.7 kg/min. Calculate: (i) actual vol. effcy.; (ii) Indicated power (iii) Mech. effcy. (iv) Isothermal effcy. [M.U.]

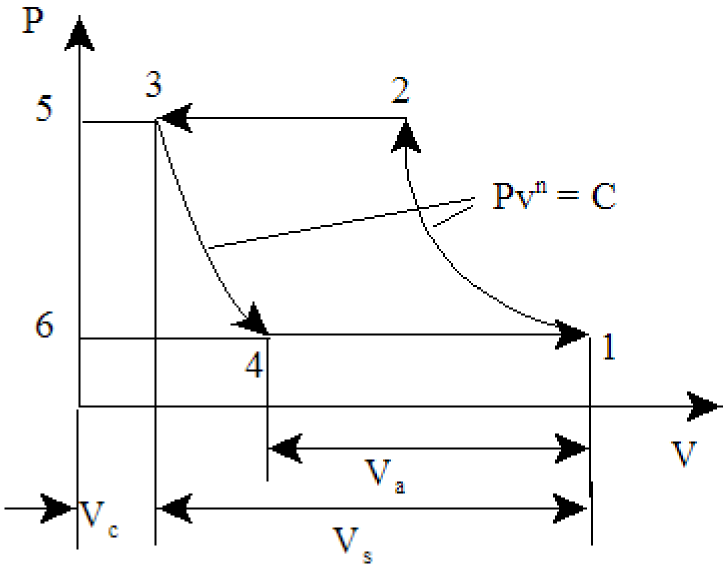

Fig.Prob.5.2.6 Single stage air compressor with clearance

Mathcad Solution:

Data:

By data, barometric pr. = 76 cm; this is equal to 1 atm. = 1.013 bar

$P1 := 1.013 \cdot 10^5$ Pa $\quad P2 := 6.013 \cdot 10^5$ Pa $\quad N := 1200$ rpm $\quad P := 6.247$ kW

$C := 0.04 \quad R := 287$ J/kg.K $\quad T1 := 293$ K $\quad T2 := 180 + 273$ K $\quad m_a := 1.7$ kg/min

$d := 0.1 \quad stroke := 0.12$ m

Calculations:

$V_s := \frac{\pi \cdot d^2}{4} \cdot stroke$ i.e. $V_s = 9.425 \times 10^{-4}$ m^3,...Piston Displ.

$m_s := \frac{V_s \cdot P1}{R \cdot T1} \cdot N$ i.e. $m_s = 1.362$ kg/min....based on stroke vol. filled at suction conditions

To find n:

$n := 1.2$ Trial value

Given

$$\frac{T2}{T1} = \left(\frac{P2}{P1}\right)^{\frac{n-1}{n}}$$

$\text{Find}(n) = 1.324$ i.e. $n := 1.324$

Vol. effcy.:

$$\eta_v := 1 + C - C\cdot\left(\frac{P2}{P1}\right)^{\frac{1}{n}}$$ i.e. $\eta_v = 0.886$ **....vol. effcy..... Ans.**

Indicated power:

$$W_c := \frac{n\cdot R\cdot T1}{n-1}\cdot\left[\left(\frac{P2}{P1}\right)^{\frac{n-1}{n}} - 1\right]\cdot 10^{-3}\cdot\frac{m_a}{60}$$

i.e. $W_c = 5.318$ **kW.....indicated power.... Ans.**

And:

$\eta_{mech} := \frac{W_c}{P}$ i.e. $\eta_{mech} = 0.851$ **..mech. effcy.... Ans.**

Isothermal power:

$$W_{iso} := R\cdot T1\cdot\ln\left(\frac{P2}{P1}\right)\cdot 10^{-3}\cdot\frac{m_a}{60}$$ i.e. $W_{iso} = 4.243$ **kW.... isothermal power ...Ans.**

And:

$\eta_{iso} := \frac{W_{iso}}{P}$ i.e. $\eta_{iso} = 0.679$ **..Isothermal effcy.....Ans.**

==

Prob.5.2.7. The LP cylinder of 2 stage, single acting air compressor running at 120 rpm has 50 cm dia and 75 cm stroke. It draws in air at a pressure of 1 bar and temp 20 C and compresses it polytropically with n=1.3 to 3 bar. The air is then delivered to the intercooler and cooled at const. pressure to 35 C. The air is further compressed polytropically with index = 1.3 to 10 bar in the HP cylinder. Determine the required power of motor if mech. effcy. is 80%. Find also heat transfer in LP and HP compression. [M.U.]

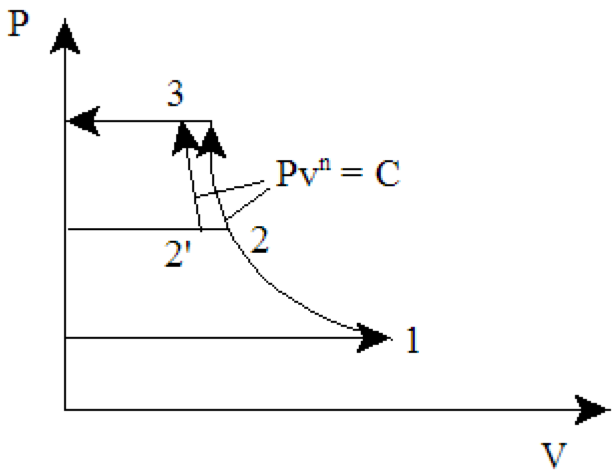

Fig.Prob.5.2.7 Single stage air compressor without clearance

Mathcad Solution:

Data:

$P1 := 1 \cdot 10^5$ Pa $P2 := 3 \cdot 10^5$ Pa $N := 120$ rpm $\eta_m := 0.8$

$P2' := 3.0 \cdot 10^5$ Pa $P3 := 10 \cdot 10^5$ Pa

$n := 1.3$ $R := 287$ J/kg.K $T1 := 293$ K $cp := 1005$ J/kg.K $T2' := 35 + 273$ K

$d_{lp} := 0.50$ m $stroke_{lp} := 0.75$ m

Calculations:

$PD_{lp} := \frac{\pi \cdot d_{lp}^2}{4} \cdot stroke_{lp}$ i.e. $PD_{lp} = 0.147$ **m^3,...Piston Displ. of LP cyl**

$v1 := \frac{R \cdot T1}{P1}$ i.e. $v1 = 0.841$ **m^3/kg...sp. vol. at inlet to LP stage**

$m := \frac{PD_{lp}}{v1}$ i.e. $m = 0.175$ **kg/cycle, for single acting**

$T2 := T1 \cdot \left(\frac{P2}{P1}\right)^{\frac{n-1}{n}}$ i.e. $T2 = 377.548$ **K.. temp at end of first stage compression**

Heat rejected in intercooler:

$Q := m \cdot \frac{N}{60} \cdot cp \cdot (T2 - T2') \cdot 10^{-3}$ i.e. $Q = 24.481$ **kW....Ans.**

Work done in First Stage:

$$W_{c1} := \frac{n \cdot R \cdot T1}{n-1} \cdot \left[\left(\frac{P2}{P1}\right)^{\frac{n-1}{n}} - 1\right] \cdot 10^{-3} \cdot m \cdot \frac{N}{60}$$

i.e. $W_{c1} = 36.828$ **kW.... first stage work**

Work done in Second Stage:

$$W_{c2} := \frac{n \cdot R \cdot T2'}{n-1} \cdot \left[\left(\frac{P3}{P2}\right)^{\frac{n-1}{n}} - 1\right] \cdot 10^{-3} \cdot m \cdot \frac{N}{60}$$

i.e. $W_{c2} = 42.968$ kW... second stage work

Motor Power reqd.:

$$P := \frac{W_{c1} + W_{c2}}{\eta_m}$$ i.e. $P = 99.746$ kW...Ans.

Heat transferred during compression in First & Second stages:

$$T3 := T2' \cdot \left(\frac{P3}{P2}\right)^{\frac{n-1}{n}}$$ i.e. $T3 = 406.645$ K...temp. at the end of 2nd stage

$$Q1 := m \cdot \frac{N}{60} \cdot cp \cdot (T1 - T2) \cdot 10^{-3} + W_{c1}$$ i.e. $Q1 = 7.068$ kW...heat rej. in 1st stage...Ans.

$$Q2 := m \cdot \frac{N}{60} \cdot cp \cdot (T2' - T3) \cdot 10^{-3} + W_{c2}$$ i.e. $Q2 = 8.246$ kW...heat rej. in 2nd stage....Ans.

And:

$$m \cdot \frac{N}{60} \cdot cp \cdot (T2 - T2') \cdot 10^{-3} = 24.481$$ kW,...Heat tr. in Intercooler....Ans.

==

Prob.5.2.8. A single acting air compressor is required to deliver air at 70 bar from suction pressure of 1 bar at the rate of 2.3 m^3/min. measured at free conditions 1.013 bar and temp. 15 C. The temp. at the end of suction is 32 C. Calculate the indicated power if compression is carried out in two stages with ideal intermediate pressure and complete intercooling. The index of compression is 1.25. Also, find saving in power over single stage compressor. Neglect clearance volume. [M.U.]

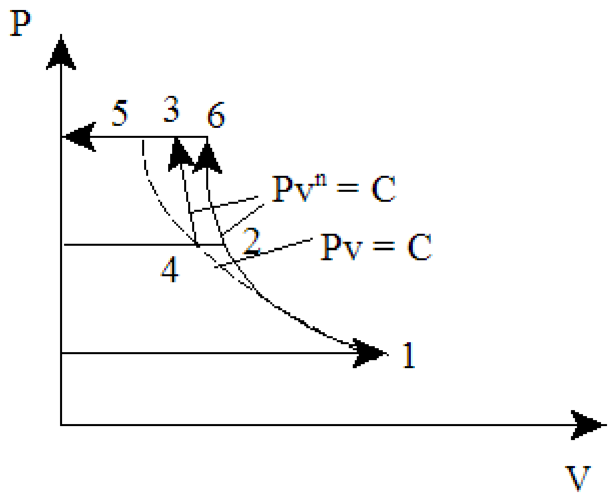

Fig.Prob.5.2.8 Single stage air compressor without clearance

Mathcad Solution:

Data:

$P1 := 1 \cdot 10^5$ Pa $\quad P3 := 70 \cdot 10^5$ Pa

$P2 := \sqrt{P3 \cdot P1}$ i.e. $P2 = 8.367 \times 10^5$ Pa....intermediate pressure

$n := 1.25 \quad R := 287$ J/kg.K $\quad T1 := 273 + 32$ K

$V := \frac{2.3}{60}$ m^3/s i.e. $V = 0.038$ m^3/s ... vol. flow rate at 1.013 bar, 15 C

Calculations:

$m := \frac{1.013 \cdot 10^5 \cdot V}{R \cdot 288}$ i.e. $m = 0.047$ kg/s.... mass flow rate

Power reqd. per stage:

$W_c := \frac{n \cdot R \cdot T1}{n-1} \cdot \left[\left(\frac{P2}{P1} \right)^{\frac{n-1}{n}} - 1 \right] \cdot 10^{-3} \cdot m$ i.e. $W_c = 10.885$ kW..Power per stage

Therefore, Power reqd. for 2 stages:

$P := 2 \cdot W_c \quad P = 21.769$ kW...total power required....Ans.

Power for single stage compressor: (between P1 and P3)

$$W_c := \frac{n \cdot R \cdot T1}{n-1} \cdot \left[\left(\frac{P3}{P1} \right)^{\frac{n-1}{n}} - 1 \right] \cdot 10^{-3} \cdot m$$

i.e. $W_c = 27.531$ kW... .. for single stage compressor.... Ans.

Therefore, saving in power required:

$$saving\_in\_powrer := W_c - P$$

i.e. $saving\_in\_powrer = 5.762$ kW .... Ans.

==

Prob.5.2.9. A 2 stage reciprocating air compressor delivering air at 17.25 bar has its clearance vol. 4% of its swept volume for the low pressure cylinder. At the start of compression, pressure in L.P. cylinder is 0.98 bar. Atm. conditions are 1 bar and 25 C. Temp. at the start of compression in each stage is 35 C and the intercooler pressure is 4 bar. Index of compression and expansion in both the stages are 1.25. Determine: (i) vol. effcy. referred to free air conditions (ii) work input per kg of air delivered if mech. effcy. is 0.75 (iii) Isothermal effcy. of the compressor. Take R = 287 J/kg.K. [M.U.]

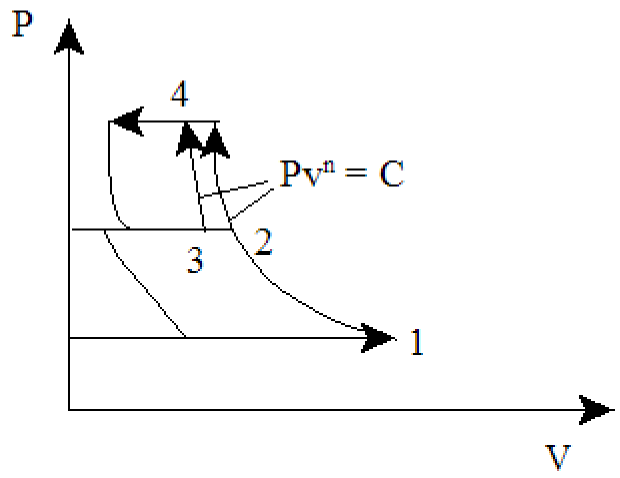

Fig.Prob.5.2.9 Two stage air compressor with clearance

Mathcad Solution:

Data:

$P1 := 0.98 \cdot 10^5$ Pa $P2 := 4 \cdot 10^5$ Pa $P3 := 17.25 \cdot 10^5$ Pa

$T1 := 273 + 35$ i.e. $T3 := T1$ K

$n := 1.25$ $R := 287$ J/kg.K $C := 0.04$ $\eta_{mech} := 0.75$

$T_f := 25 + 273$ K $P_f := 1 \cdot 10^5$ Pa

Calculations:

$$\eta_v := \frac{P1 \cdot T_f}{T1 \cdot P_f} \cdot \left[1 + C - C \cdot \left(\frac{P2}{P1} \right)^{\frac{1}{n}} \right]$$ ....vol. effcy.

i.e. $\eta_v = 0.869$ **Vol. effcy. ref. to ambient condition...... Ans.**

$$W_c := \frac{n \cdot R \cdot T1}{n-1} \cdot \left[\left(\frac{P2}{P1}\right)^{\frac{n-1}{n}} + \left(\frac{P3}{P2}\right)^{\frac{n-1}{n}} - 2 \right] \cdot 10^{-3} \quad \text{kW}$$

i.e. $W_c = 293.634$ **kW .... work required per kg of air**

$P := \frac{W_c}{\eta_{mech}}$ i.e. $P = 391.512$ **kW..Motor Power required... Ans.**

$W_{iso} := R \cdot T1 \cdot \ln\left(\frac{P3}{P1}\right) \cdot 10^{-3}$ i.e. $W_{iso} = 253.521$ **kW per kg of air... Isothermal work**

$\eta_{iso} := \frac{W_{iso}}{P}$ i.e. $\eta_{iso} = 0.648$ **= 64.8 %....Isothermal effcy.....Ans.**

==

Prob.5.2.10. A single acting, 2 stage compressor with perfect intercooling delivers 5 kg/min. of air at 15 bar pressure. The entry condition of air is at 1 bar, 288 K. Compression and expansion follow the law pV^1.3 = C. Calculate the power required to run the compressor at 420 rpm. Assume the clearance of LP and HP cylinders to be 5% and 6% of respective cylinder swept volumes. Also, find out the clearance volume for each cylinder [M.U.]

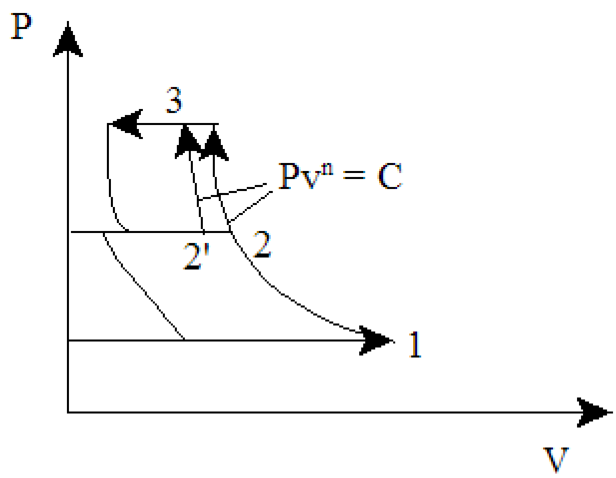

Fig.Prob.5.2.10 Two stage air compressor with clearance

Mathcad Solution:

Data:

$P1 := 1 \cdot 10^5$ Pa $P3 := 15 \cdot 10^5$ Pa $N := 420$ rpm

And: $P2 := \sqrt{P3 \cdot P1}$ $P2 = 3.873 \times 10^5$ Pa.... intermediate pressure, perfect intercooling

$n := 1.3$ $R := 287$ J/kg.K $T1 := 288$ K $C1 := 0.05$ $C2 := 0.06$

$m := \frac{5}{60}$ kg/s

Calculations:

$\eta_{v1} := 1 + C1 - C1 \cdot \left(\frac{P2}{P1}\right)^{\frac{1}{n}}$ i.e. $\eta_{v1} = 0.908$ ...vol. effcy. of LP stage

$\eta_{v2} := 1 + C2 - C2 \cdot \left(\frac{P3}{P2}\right)^{\frac{1}{n}}$ i.e. $\eta_{v2} = 0.89$ ...vol. effcy. of HP stage

Power reqd. per stage:

$$W_c := \frac{n \cdot R \cdot T1}{n-1} \cdot \left[\left(\frac{P2}{P1}\right)^{\frac{n-1}{n}} - 1\right] \cdot 10^{-3} \cdot m \qquad \text{i.e.} \qquad W_c = 10.948$$

kW... power per stage

Therefore, Power reqd. for 2 stages:

$P := 2 \cdot W_c \qquad P = 21.896$ **kW...power for two stages....Ans.**

Isothermal effcy.:

$$W_{iso} := R \cdot T1 \cdot \ln\left(\frac{P3}{P1}\right) \cdot 10^{-3} \cdot m \qquad \text{i.e.} \qquad W_{iso} = 18.653$$

kW....Isothermal work

$$\eta_{iso} := \frac{W_{iso}}{P} \qquad \text{i.e.} \qquad \eta_{iso} = 0.852$$

= 85.2 %....Isothermal effcy.....Ans.

Clearance vol. for each cyl.:

LP cylinder:

$V_{s1} := 0.1$ m^3.....Trial value

Given

$$m = \frac{\eta_{v1} \cdot V_{s1} \cdot P1}{R \cdot T1} \cdot \frac{N}{60}$$

$\text{Find}(V_{s1}) = 0.01083$ m^3......Stroke vol

i.e. $V_{s1} := 0.01083$ **m^3.........stroke vol. of LP cyl**

And: $V_{c1} := C1 \cdot V_{s1}$

i.e. $V_{c1} = 5.415 \times 10^{-4}$ **m^3....clearance vol. of LP.....Ans.**

HP cylinder:

$V_{s2} := 0.1$ m^3.....Trial value

Given

$$m = \frac{\eta_{v2} \cdot V_{s2} \cdot P2}{R \cdot T1} \cdot \frac{N}{60}$$ ....entry to HP is at T1, since perfect intercooling.

$Find(V_{s2}) = 2.85475 \times 10^{-3}$ ^3......Stroke vol

i.e. $V_{s2} := 0.00285$ m^3.........stroke vol. of HP cyl

And: $V_{c2} := C2 \cdot V_{s2}$

i.e. $V_{c2} = 1.71 \times 10^{-4}$ m^3....clearance vol. of HP...Ans.

==

Prob.5.2.11. A two stage, double acting Air compressor operating at 220 rpm takes in air at 1 bar, 27 C. Size of LP cylinder is 360 mm * 400 mm. Stroke of HP cylinder is same as that of LP cyl = 400 mm. Clearance in both cylinders is 4%. LP cylinder discharges at a pressure of 4 bar. Air passes through the intercooler and enters HP cylinder at 3.8 bar, 27 C. Finally, discharged from the compressor at 15.2 bar. Value of n in both cylinders is 1.3. Take cp = 1.0035 kJ/kg.K and R=0.287 kJ/kg. Calculate:

1. heat rejected by air in intercooler
2. dia of HP cyl
3. power required to drive the HP cylinder. [M.U.]

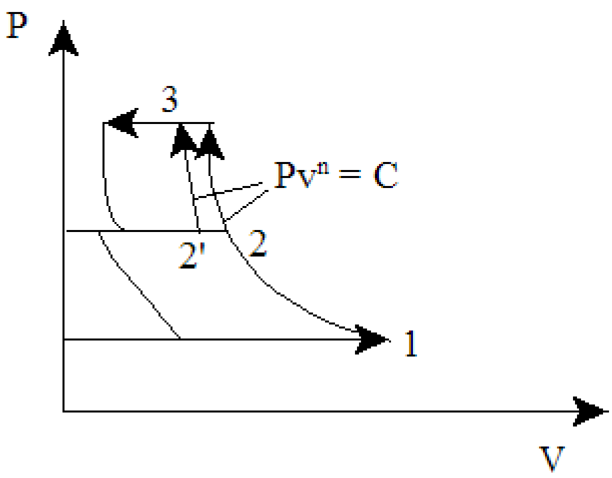

Fig.Prob.5.2.11 Two stage air compressor with clearance

Mathcad Solution:

Data:

$P1 := 1 \cdot 10^5$ Pa $P2 := 4 \cdot 10^5$ Pa $N := 220$ rpm

$P2' := 3.8 \cdot 10^5$ Pa $P3 := 15.2 \cdot 10^5$ Pa

$n := 1.3$ $C := 0.04$ clearance $R := 287$ J/kg.K $T1 := 300$ K $cp := 1003.5$ J/kg.K

$d_{lp} := 0.36$ m...dia of LP cylinder

$stroke_{lp} := 0.4$ m...stroke of LP cylinder

Calculations:

$PD_{lp} := \frac{\pi \cdot d_{lp}^2}{4} \cdot stroke_{lp}$ i.e. $PD_{lp} = 0.041$ **m^3,...Piston Displ. of LP cyl**

$\eta_{vol} := 1 + C - C \cdot \left(\frac{P2}{P1}\right)^{\frac{1}{n}}$ i.e. $\eta_{vol} = 0.924$ **vol. effcy. of LP stage**

$v1 := \frac{R \cdot T1}{P1}$ i.e. $v1 = 0.861$ **m^3/kg...sp. vol. at inlet to LP stage**

$m := \frac{PD_{lp} \cdot \eta_{vol}}{v1} \cdot 2$ i.e. $m = 0.087$ **kg/cycle, for double acting**

$T2 := T1 \cdot \left(\frac{P2}{P1}\right)^{\frac{n-1}{n}}$ i.e. $T2 = 413.103$ **K... temp after compression in first stage**

Heat rejected in intercooler:

$Q := m \cdot \frac{N}{60} \cdot cp \cdot (T2 - T1) \cdot 10^{-3}$ i.e. $Q = 36.36$ **kW..heat rejected in intercooler...Ans.**

Dia of HP cyl:

$\eta_{vol2} := 1 + C - C \cdot \left(\frac{P3}{P2'}\right)^{\frac{1}{n}}$ i.e. $\eta_{vol2} = 0.924$ **Vol. effcy. of HP stage**

$v2' := \frac{R \cdot T1}{P2'}$ i.e. $v2' = 0.227$ **m^3/kg .... sp.vol at entry to HP cyl**

$stroke_{hp} := stroke_{lp}$ ...equal strokes for two cylinders

Diameter of HP cylinder:

$D_{hp} := 0.3$ Trial value

Given

$$\frac{\pi \cdot D_{hp}^{2}}{4} \cdot stroke_{hp} \cdot \eta_{vol2} \cdot \frac{1}{v2'} \cdot 2 = m$$ **...double acting, so a multuplied by 2**

$d_{hp} := \text{Find}(D_{hp})$

i.e. $d_{hp} = 0.185$ **m,....dia of HP cyl....Ans.**

Power reqd. to drive the HP cyl:

$$T3 := T1 \cdot \left(\frac{P3}{P2'}\right)^{\frac{n-1}{n}}$$ i.e. $T3 = 413.103$ **K... temp at discharge of HP cylinder**

$$W_c := \frac{n \cdot R \cdot T1}{n-1} \cdot \left[\left(\frac{P3}{P2'}\right)^{\frac{n-1}{n}} - 1\right] \cdot 10^{-3} \cdot m \cdot \frac{N}{60} \quad \text{kW}$$

i.e. $W_c = 45.062$ **kW ......Ans.**

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5.3 Problems solved with EES:

"**Prob.5.3.1.** A single stage, single acting compressor delivers 15 m^3 of free air per min. from 1 bar to 8 bar. Speed of compressor = 300 RPM. Assume that compression and expansion follow the law P.V^1.3 = const. and clearance is 1/16th of swept vol. Temp. and pressure of air at suction are the same as atmospheric air.. Take L/D=1.5,and find the diam. and stroke of the compressor."

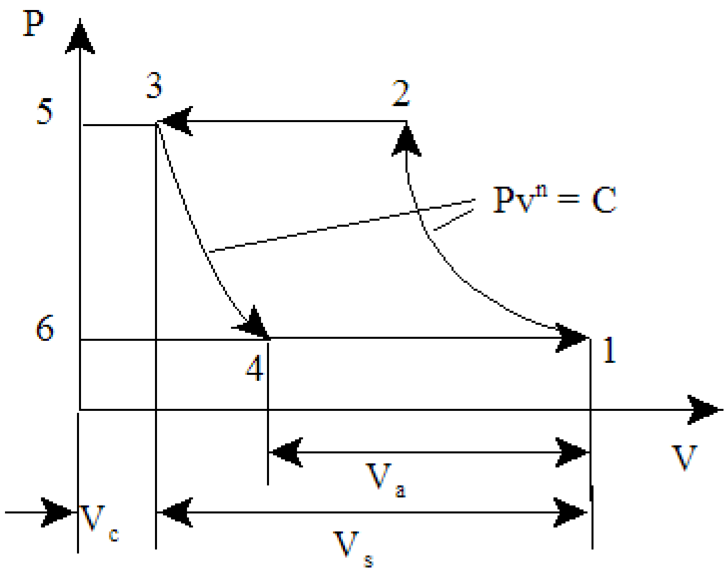


Fig.Prob.5.3.1 Two stage air compressor with clearance

EES Solution:

"Data:"

C=1/16"Clearance ratio=Vc/Vs"

P2=800"kPa"

P1=100"kPa"

n=1.3

V_a =15"m3/min"

Speed=300"RPM"

"Calculations:"

"Vol. effcy:"

eta_vol=1+C – C * (P2/P1)^(1/n)

"Free air:"

V_s * eta_vol * Speed = V_a "...finds Vs"

V_s=(pi/4) * (D^2) * L"Stroke vol."

L=1.5 * D

"Indicated Power of Compressor:"

IP=(n/(n-1)) * P1 * (V_a/60) * ((P2/P1)^((n-1)/n)-1)"kW"

Results:

Unit Settings: SI K kPa kJ mass deg

C = 0.0625	D = 0.3834 [m]	η_{vol} = 0.7531	IP = 66.72 [kW]
L = 0.5751 [m]	n = 1.3	P1 = 100 [kPa]	P2 = 800 [kPa]
Speed = 300 [rpm]	V_a = 15 [m^3/min]	V_s = 0.0664 [m^3]	

Thus:

Dia of cylinder = D = 0.3834 m ….Ans.

Stroke = L = 0.5751 m …. Ans.

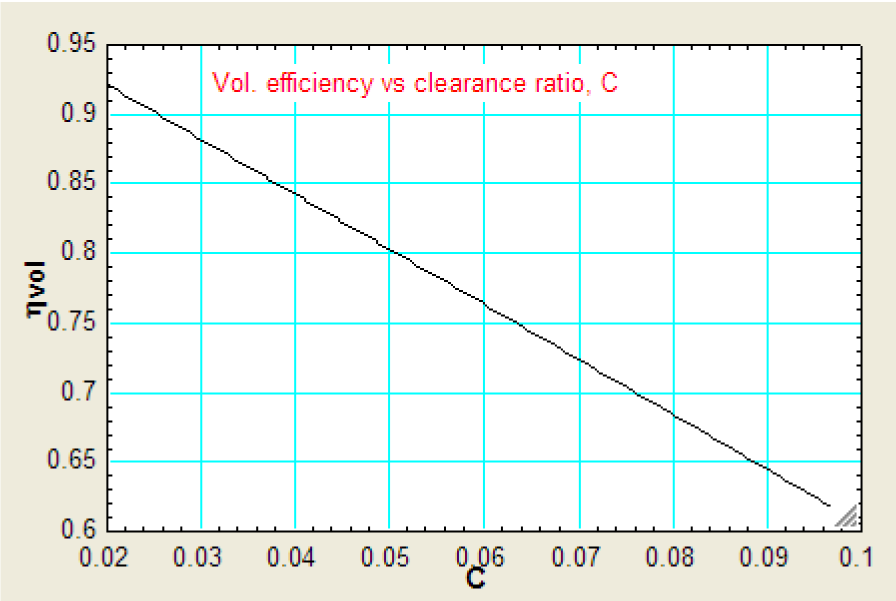
Compressor power required = IP = 66.72 kW … Ans.

(b) Plot the vol. efficiency against Clearance ratio (C varying from 0.02 to 0.1):

First, compute the Parametric Table:

1..9	C	η_{vol}
Run 1	0.02	0.921
Run 2	0.03	0.8815
Run 3	0.04	0.842
Run 4	0.05	0.8025
Run 5	0.06	0.7629
Run 6	0.07	0.7234
Run 7	0.08	0.6839
Run 8	0.09	0.6444
Run 9	0.1	0.6049

Now, plot the results:

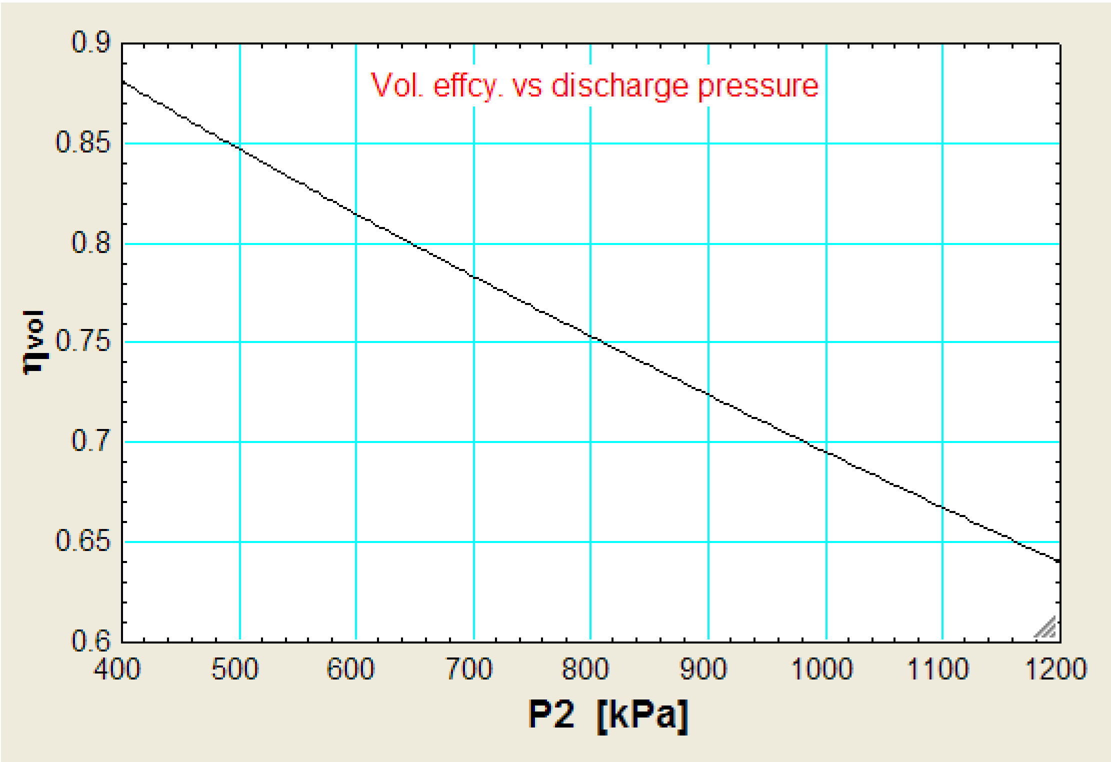


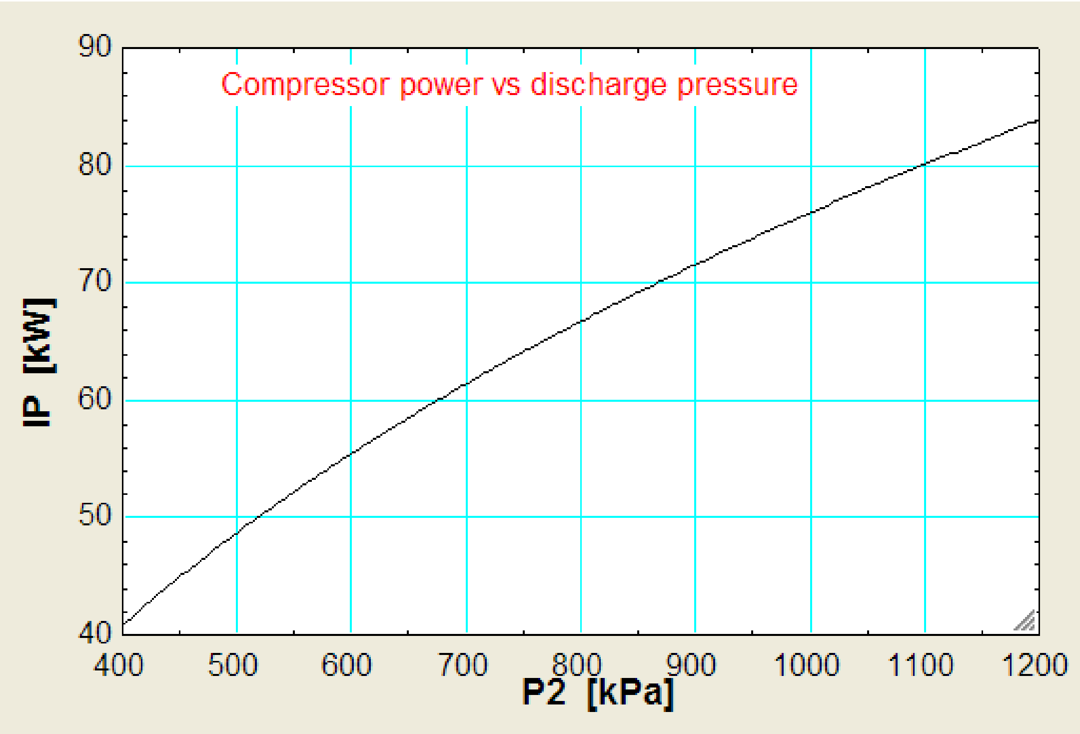
(c) Plot the vol. efficiency and Compressor power against discharge pressure, P2, other data remaining the same (C = 1/16):

First, compute the Parametric Table:

1..9	P2 [kPa]	η_{vol}	IP [kW]
Run 1	400	0.8809	40.84
Run 2	500	0.8469	48.73
Run 3	600	0.8145	55.47
Run 4	700	0.7833	61.41
Run 5	800	0.7531	66.72
Run 6	900	0.7237	71.54
Run 7	1000	0.6951	75.97
Run 8	1100	0.6672	80.07
Run 9	1200	0.6398	83.89

Now, plot the results:





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Prob.5.3.2. A single stage reciprocating air compressor has a cylinder of 15 cm bore and 15 cm stroke. The clearance is 5%. Air is sucked into the compressor at 1 bar, 27 C. The discharge pressure is 5 bar. The polytropic exponent of compression and expansion is 1.3. (i) Sketch the ideal indicator diagram and find the air handling capacity of the compressor in m3/min. (measured at suction conditions), given that the speed of the compressor is 720 rpm. (ii) Find also the ideal vol. effcy. (iii) compressor power in kW [VTU-ATD-2004]"

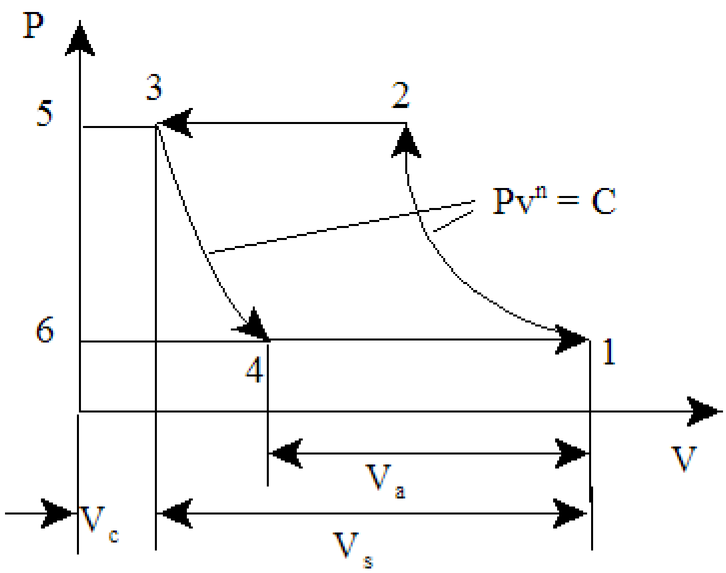


Fig.Prob.5.3.2 Two stage air compressor with clearance

EES Solution:

"Data:"

D=0.15"[m]"
L=0.15"[m]"
C=0.05
P1=100"[kPa]"
T1=27+273"[K]"
P2=500"[kPa]"
n=1.3
Speed=720"[RPM]"

"Calculations:"

eta_v=1+C – C * (P2/P1)^(1/n)"vol. effcy."

V_s=pi * ((D^2)/4) * L"stroke vol."

V_a=V_s * eta_v"actual vol. sucked"

Volumepermin=V_a * Speed"m3/min"

IP=(n/(n-1)) * P1* V_a * ((P2/P1)^((n-1)/n)-1) * Speed/60 "Indicated power of compressor, kW"

Results:

Unit Settings: SI K kPa kJ mass deg

C = 0.05	D = 0.15 [m]	η_v = 0.8776
IP = 5.441 [kW]	L = 0.15 [m]	n = 1.3
P1 = 100 [kPa]	P2 = 500 [kPa]	Speed = 720 [RPM]
T1 = 300 [K]	Volumepermin = 1.675 [m^3/min]	V_a = 0.002326 [m^3]
V_s = 0.002651 [m^3]		

Thus:

Air handling capacity = 1.675 m^3/min….. Ans.

Vol. efficiency = 0.8776 …. Ans.

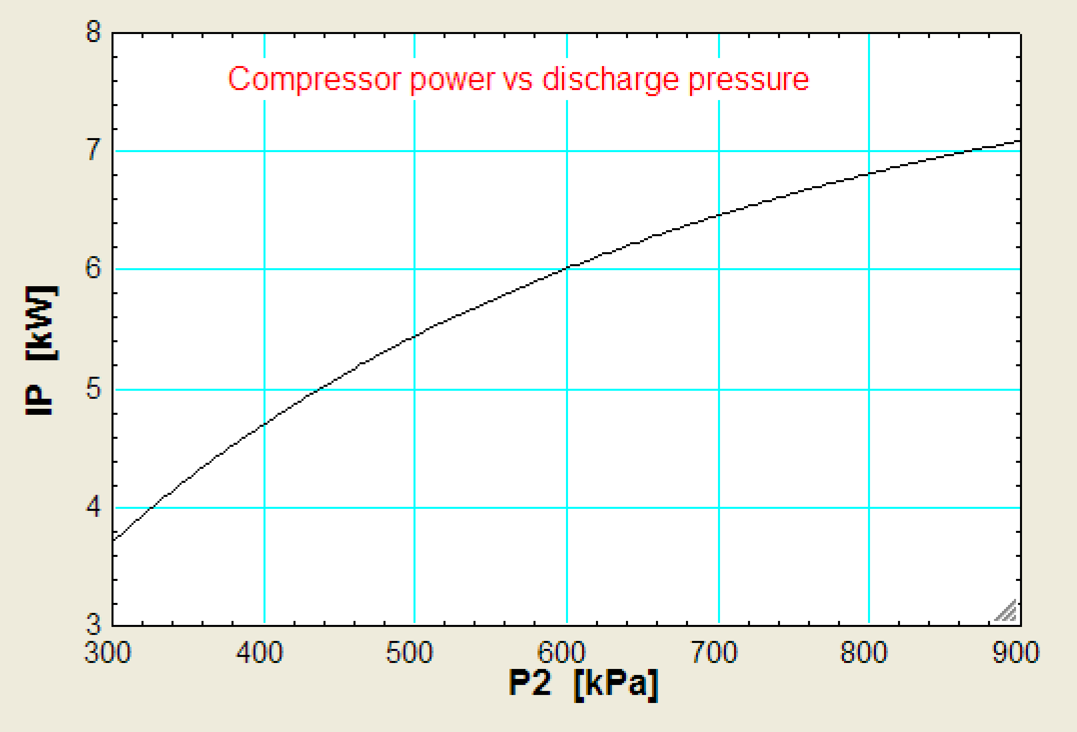
Compressor power = IP = 5.441 kW … Ans.

(b) Plot compressor power for discharge pressures varying from 3 to 9 bar:

First, produce the Parametric Table:

1..7	P2 [kPa]	IP [kW]
Run 1	300	3.713
Run 2	400	4.702
Run 3	500	5.441
Run 4	600	6.011
Run 5	700	6.458
Run 6	800	6.812
Run 7	900	7.091

Now, plot the results:



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" **Prob.5.3.3.** Air at 1 bar and 27 C is compressed to 7 bar by a single stage reciprocating compressor according to the law: PV^1.3 = C. The free air delivered was 1 m^3/min. Speed of compressor = 300 RPM. Stroke to bore ratio = 1.5. Mech. effcy = 85% and motor transmission effcy = 90%. Determine: (i) Indicated power and isoth. effcy. (ii) cylinder dimensions and power of the motor required to drive the compressor. [VTU-ATD-2005]"

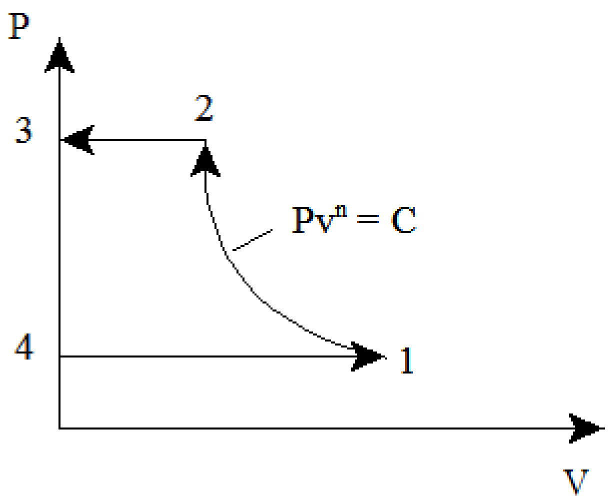

Fig.Prob.5.3.3 Two stage air compressor with clearance

EES Solution:

"Data:"

"Free air conditions:"

```
P_f=101.325"kPa"
T_f=15+273"k"

V_f=1.0/60"m3/s"
P1=100"kPa"
T1=27+273"k"
P2=700"kPa"
n=1.3
R=0.287"kJ/kg.K"
Speed = 300 "RPM"
eta_mech=0.85
eta_trans=0.9
LbyD=1.5
```

"Calculations:"

"mass compressed:"

```
m=(P_f * V_f)/(R * T_f)"kg/s"

IP=(n/(n-1)) * m * R * T1* ((P2/P1)^((n-1)/n)-1)"kW"

W_iso=m * R * T1* ln(P2/P1)"kW...Isothermal work reqd."

eta_iso=W_iso/IP

MotorPower=IP/(eta_mech * eta_trans)
```

"Cylinder dimensions:"

```
m=(P1* V1) * (Speed/60)/(R * T1)"...finds V1, vol. at suction conditions"

V1=(pi/4) * (D^2) * (LbyD * D)"..finds D"

L=LbyD * D"..finds L"
```

Results:

Unit Settings: SI K kPa kJ mass deg

D = 0.144 [m]	η_{iso} = 0.7922	η_{mech} = 0.85
η_{trans} = 0.9	IP = 4.321 [kW]	L = 0.216 [m]
LbyD = 1.5	m = 0.02043 [kg/s]	MotorPower = 5.648 [kW]
n = 1.3	P1 = 100 [kPa]	P2 = 700 [kPa]
P_f = 101.3 [kPa]	R = 0.287 [kJ/kg.K]	Speed = 300 [RPM]
T1 = 300 [K]	T_f = 288 [K]	V1 = 0.003518 [m3/s]
V_f = 0.01667 [m3/s]	W_{iso} = 3.423 [kW]	

Thus:

Indicated power = IP = 4.321 kW …. Ans.

Isothermal effcy. = eta_iso = 0.7922 = 79.22% …. Ans.

Motor power required = 5.648 kW … Ans.

Cylinder dia = D = 0.144 m …. Ans.

Cylinder stroke = L = 0.216 m … Ans.

===

"**Prob.5.3.4.** A multistage compressor has a suction pressure of 1 bar and final discharge pressure is 130 bar, such that stage pressure ratio should not exceed 4.2. Assuming perfect intercooling, determine: (i) no. of stages (ii) exact pressure ratio (iii) intermediate pressures, (iv) min. power required to compress 17 m3/min of free air. Take n = 1.32 [VTU-ATD-2005]"

EES Solution:

"Data:"

P1=1"bar"

{k=4.2 "pressure ratio per stage = P2/P1 = P3/P2=....etc."}

Ph=130"bar"

"Calculations:"

"Let x be the no. of stages; Then:"

Ph/P1=k^x

"Then, we get : x =3.392; No. of stages should be an integer figure. So, we take x=4"

x=4

"Then, intermediate pressures:"

P2=k*P1"bar"

P3=k*P2 "bar"

P4=k*P3 "bar"

P5=k * P4"...This P5 should be equal to Ph"

"min. Power reqd. to compress 17 m3/min of free air:"

n=1.32"Index of compression"

P = 4 * (n/(n-1)) * P1 * 100*(17/60) * ((P2/P1)^((n-1)/n)-1)"kW....Note that there are 4 stages."

Results:

Unit Settings: SI K bar kJ mass deg

k = 3.377	n = 1.32	P = 160.4 [kW]	P1 = 1 [bar]	P2 = 3.377 [bar]
P3 = 11.4 [bar]	P4 = 38.5 [bar]	P5 = 130 [bar]	Ph = 130 [bar]	x = 4

Thus:

No. of stages = x = 4 …Ans.

Exact pressure ratio for each stage = k = 3.377 …. Ans.

Intermediate pressures: P2 = 3.377 bar, P3 = 11.4 bar, P4 = 38.5 bar ….Ans.

Compressor power = P = 160.4 kW …. Ans.

==

"**Prob.5.3.5.** Following data refer to a two stage, single acting reciprocating compressor: Air compressed and delivered = 4 kg/min., Pressure rise from 100 kPa to 2.5 MPa, LP cylinder dia = 15 cm, HP cylinder dia = 7.5 cm, stroke length in each stage = 20 cm, Index of compression and expansion in each stage = 1.2, Temp of air at inlet = 25 C, clearance volume = 4% of stroke vol. in each cylinder, intercooling is perfect but, condition for minimum work input is not satisfied. Determine: (i) intermediate pressure, P2 (ii) power required to drive LP and HP pistons if the mech. effcy. is 75% (iii) speed of crankshaft driving the compressor in RPM, and (iv) energy rejected in the intercooler in kJ/min. [VTU-ATD-2006]"

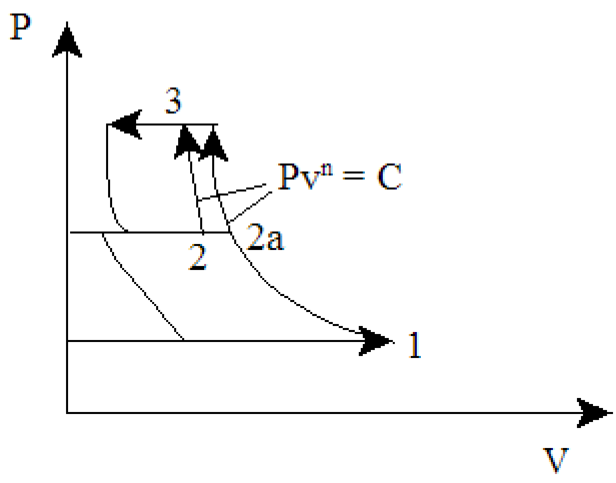

Fig.Prob.5.3.5 Two stage air compressor with clearance

EES Solution:

"Data:"

P1=100"kPa"
T1=25+273"k"
P3=2500"kPa"
n=1.2
cp=1.003"kJ/kg.K"
R=0.287"kJ/kg.K"
"mass compressed:"
m=4/60"kg/s"
D1=0.15"m...dia of LP cyl."
D2=0.075"m...dia of HP cyl."
L=0.2"m...stroke for each stage"
C=0.04"..clearance ratio for each stage"
eta_mech=0.75"...mechanical effcy of each stage"
"Intercooling is perfect (i.e. T2=T1), but condition for min. work is not satisfied...i.e. pressure ratio in each stage is NOT the same"
"-----------------------------------"

"Calculations:"

eta_v1=1+C – C * (P2/P1)^(1/n)"...vol. effcy of first stage"

eta_v2=1+C – C * (P3/P2)^(1/n)"...vol. effcy of second stage"

V_s1=(pi * D1^2/4) * L"m3...swept vol. of LP cylinder"

V_s2=(pi * D2^2/4) * L"m3...swept vol. of HP cylinder"

V_a1=V_s1* eta_v1"m3...actual vol. sucked in LP cyl"

V_a2=V_s2 * eta_v2"m3...actual vol. sucked in HP cyl"

T2=T1"..perfect intercooling"

"Apply the condition that mass flow rate through both the stages is the same:"

P1* V_a1/(R * T1)=P2 * V_a2/(R * T2)"...finds P2...kPa" m=P1* V_a1* (RPM/60)/(R * T1)"...finds RPM"

"Work reqd.: is calculated for each stage:"

W_c1=(n/(n-1)) * m * R * T1* ((P2/P1)^((n-1)/n)-1)/eta_mech"kW...for 1st stage"

W_c2=(n/(n-1)) * m * R * T2 * ((P3/P2)^((n-1)/n)-1)/eta_mech"kW...for 2nd stage"

W_total=W_c1+W_c2"kW...Total motor power reqd."

"Heat carried away by the Intercooler:"

T2a/T1=(P2/P1)^((n-1)/n)"k...actual temp. at the end of first stage polytr. comprn."

Q_intercooler=m * cp * (T2a-T1)"kW"

Results:

Unit Settings: SI K kPa kJ mass deg

C = 0.04	cp = 1.003 [J/kg-K]	D1 = 0.15 [m]
D2 = 0.075 [m]	η_{mech} = 0.75	η_{v1} = 0.9076
η_{v2} = 0.8633	L = 0.2 [m]	m = 0.06667 [kg/s]
n = 1.2	P1 = 100 [kPa]	P2 = 420.5 [kPa]
P3 = 2500 [kPa]	$Q_{intercooler}$ = 5.389 [kW]	R = 0.287 [kJ/kg-K]
RPM = 1066 [rpm]	T1 = 298 [K]	T2 = 298 [K]
T2a = 378.6 [K]	V_{a1} = 0.003208 [m^3]	V_{a2} = 0.0007628 [m^3]
V_{s1} = 0.003534 [m^3]	V_{s2} = 0.0008836 [m^3]	W_{c1} = 12.34 [kW]
W_{c2} = 15.78 [kW]	W_{total} = 28.12 [kW]	

Thus:

Intermediate pressure, P2 = 420.5 kPa … Ans.

Power required for LP stage = Wc1 = 12.34 kW …. Ans.

Power required for HP stage = Wc2 = 15.78 kW … Ans.

Speed = RPM = 1066 rpm … Ans.

Heat rejected in intercooler = Q_intercooler = 5.389 kW … Ans.

===

" **Prob.5.3.6.** A two stage air compressor with perfect intercooling takes in air at 1 bar, 27 C. The law of compression in both stages is P.V^1.3 = constant. The compressed air is delivered at 9 bar. Calculate for unit mass flow rate of air, the min. work done and the heat rejected in the intercooler. Compare the values if the compression is carried out in a single stage compressor with aftercooler. [VTU-ATD-2007]"

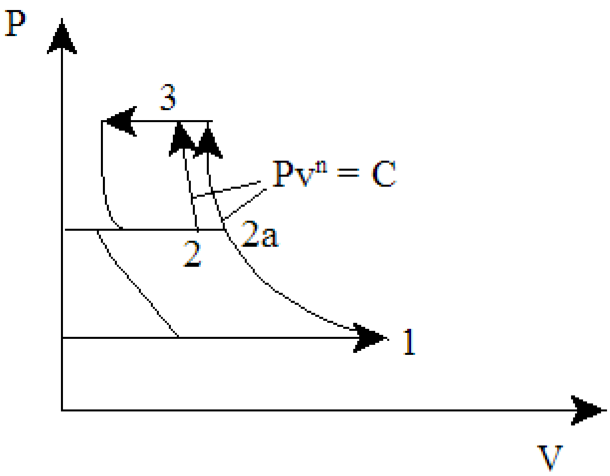

Fig.Prob.5.3.6 Two stage air compressor with clearance

EES Solution:

"Data:"

P1=100"kPa"
T1=27+273"k"
P2=300"kPa, since P2/P1 = P3/P2 for min. work "
P3=900"kPa"
n=1.3
cp=1.003"kJ/kg.K"
R=0.287"kJ/kg.K"
m=1"kg/s"

"Calculations:"

"Work reqd.: is the same for each stage, for perfect intercooling"

W_c_2stage = 2 * (n/(n-1)) * m * R * T1* ((P2/P1)^((n-1)/n)-1)"kW...for 2 stages"

"Heat carried away by the Intercooler:"

Q_intercooler=m *cp * (T2a-T1)"kW"

T2a/T1=(P2/P1)^((n-1)/n)"...gives T2a, temp. at the end of polytr. comprn."

"If comprn. is carried out in Single stage:"

W_c_singlestage=(n/(n-1)) * m*R * T1* ((P3/P1)^((n-1)/n)-1)"kW...for single stage comprn."

T2b/T1=(P3/P1)^((n-1)/n)"...gives T2b, temp. at the end of polytr. comprn. in a single stage compressor, compressing from P1 to P3"

"Heat carried away by the aftercooler:"

Q_aftercooler =m *cp * (T2b-T1)"kW"

Results:

Unit Settings: SI C kPa kJ mass deg

cp = 1.003 [kJ/kg-K]	m = 1 [kg/s]	n = 1.3
P1 = 100 [kPa]	P2 = 300 [kPa]	P3 = 900 [kPa]
$Q_{aftercooler}$ = 198.7 [kW]	$Q_{intercooler}$ = 86.83 [kW]	R = 0.287 [kJ/kg-K]
T1 = 300 [K]	T2a = 386.6 [K]	T2b = 498.1 [K]
$W_{c,2stage}$ = 215.3 [kW]	$W_{c,singlestage}$ = 246.4 [kW]	

Thus:

For two stage compressor:

Min. work done for two stage compressor = 215.3 kW … Ans.

Heat rejected in intercooler = 86.83 kW … Ans.

For single stage compressor:

Work done for single stage compressor = 246.4 kW … Ans.

Heat rejected in aftercooler = 198.7 kW … Ans.

==

"**Prob.5.3.7**. A multistage air compressor compresses air from 1 bar to 40 bar. The max. temp. of air is not to exceed 400 K in any stage. If the law of compression is P.V^1.3 = constant, find the number of stages for minimum power input. Also, find the actual intermediate pressures and temperatures. What will be the min. power input (kW) required to compress and deliver 10 kg/min of air and the rate of heat rejection in each intercooler? Assume ambient temp = 27 C and perfect intercooling in between stages. [VTU-ATD-2006]"

EES Solution:

"Data:"

P_f=4000"kPa....final pressure"
T_max=400"k....max. temp in any stage"
P1=100"kPa"
T1=27+273"k"

n=1.3
cp=1.003"kJ/kg.K"
R=0.287"kJ/kg.K"
"mass compressed:"
m=10/60"kg/s"
"---------------------------------"

"Calculations:"

{
T_max/T1=(P2/P1)^((n-1)/n)"....finds P2"

P3/P2=P2/P1"..finds P3"

P4/P3=P3/P2"...finds P4"

"From the above eqns. we get: P1 = 100 kPa, P2 = 347.9 kPa, P3 = 1210 kPa, P4 = 4209 kPa > Pf"

"Therefore, 3 stages are required."
}

"Then, pr.ratio for each stage:"

k=(P_f/P1)^(1/3)"....pressure ratio in each stage"

P2/P1=k"...finds P2"

P3/P2=k"...finds P3"

P4/P3=k"...finds P4"
T2/T1=k^((n-1)/n)"..finds T2"

T3=T2

T4=T3

"Work reqd.: is the same for each stage, for perfect intercooling and same pressure ratio in each stage"

W_c=3 * (n/(n-1)) * m * R * T1* ((P2/P1)^((n-1)/n)-1)"kW...for 3 stages"

"Heat carried away by each Intercooler:"

Q_intercooler=m * cp * (T2-T1)"kW"

Results:

Unit Settings: SI C kPa kJ mass deg

cp = 1.003 [kJ/kg-K]	k = 3.42	m = 0.1667 [kg/s]
n = 1.3	P1 = 100 [kPa]	P2 = 342 [kPa]
P3 = 1170 [kPa]	P4 = 4000 [kPa]	P_f = 4000 [kPa]
$Q_{intercooler}$ = 16.45 [kW]	R = 0.287 [kJ/kg-K]	T1 = 300 [K]
T2 = 398.4 [K]	T3 = 398.4 [K]	T4 = 398.4 [K]
T_{max} = 400 [K]	W_c = 61.21 [kW]	

Thus:

No. of stages required = 3 …. Ans.

Intermediate pressures and temps: P2 = 342 kPa, P3 = 1170 kPa, T2 = T3 = 398.4 K … Ans.

Min. power input to compress 10 kg/min = Wc = 61.21 kW … Ans.

Heat rejected in each intercooler = Q_intercooler = 16.45 kW … Ans.

===

"**Prob.5.3.8.** A two stage air compressor delivers 1.5 m^3 of free air per min. The delivery pressure is 14 bar. The suction pressure and temp. are 1 bar and 20 C. The index of compression is 1.25 for both the stages. The intermediate pressure is optimum and intercooling is complete. Calculate the power required to drive the compressor and the heat carried away by the intercooler. For air, cp = 1003 J/kg.K and R= 287 J/kg.K. [VTU-ATD-2004]"

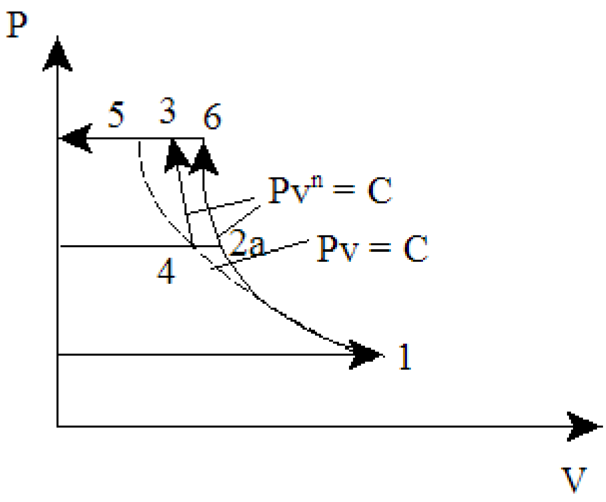

Fig.Prob.5.3.8 Two stage air compressor without clearance

EES Solution:

"Data:"

"Free air conditions:"

P_f=101.325"kPa"
T_f=15+273"k"
V_f=1.5/60"m3/s"

P1=100"kPa"
T1=20+273"k"
P3=1400"kPa"
n=1.25
cp=1.003"kJ/kg.K"
R=0.287"kJ/kg.K"

"Calculations:"

"mass compressed:"

m=(P_f * V_f)/(R * T_f)"kg/s"

P2=(P1 * P3)^0.5"Optimum intermediate pressure"

"Work reqd.: is the same for each stage, for perfect intercooling"

W_c=2 * (n/(n-1)) * m * R * T1 * ((P2/P1)^((n-1)/n)-1)"kW...for 2 stages"

"Heat carried away by the Intercooler:"

Q_intercooler=m * cp * (T2a-T1)"kW"

T2a/T1=(P2/P1)^((n-1)/n)"...gives T2a, temp. at the end of polytr. comprn."

Results:

Unit Settings: SI K kPa kJ mass deg

cp = 1.003 [kJ/kg.K]	m = 0.03065 [kg/s]	n = 1.25
P1 = 100 [kPa]	P2 = 374.2 [kPa]	P3 = 1400 [kPa]
P_f = 101.3 [kPa]	$Q_{intercooler}$ = 2.72 [kW]	R = 0.287 [kJ/kg.K]
T1 = 293 [K]	T2a = 381.5 [K]	T_f = 288 [K]
V_f = 0.025 [m3/s]	W_c = 7.783 [kW]	

Thus:

Optimum intermediate pressure = P2 = 374.2 kPa … Ans.

Compressor power required = Wc = 7.783 kW … Ans.

Heat transferred in intercooler = 2.72 kW … Ans.

==

"**Prob.5.3.9.** Following data refer to a single stage air compressor. Atmospheric conditions: 1 bar and 25 C. Receiver pressure is 10 bar, cylinder dia = 12 cm, stroke to bore ratio is unity. Clearance volume is 1/25 th of stroke vol. Index for both compression and expansion = 1.25. Mech. effcy = 80%. If the receiver capacity is 600 litres and it takes 8 min to fill the receiver till its pressure is 10 bar starting from 1 bar, determine: (i) actual vol. effcy. (ii) Mass of air compressed per second (iii) Speed of the compressor (iv) Power input.

Assume the receiver temp. to remain at 25 C throughout the filling process. [VTU-ATD-2004]"

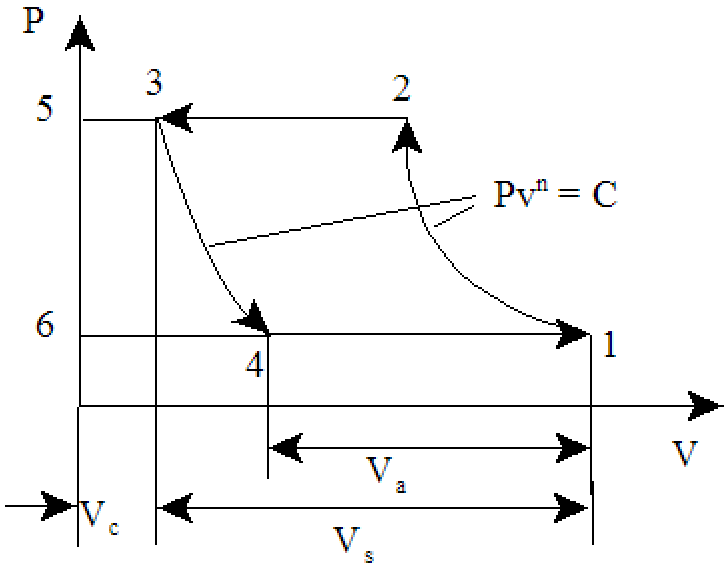

Fig.Prob.5.3.9 Single stage air compressor with clearance

EES Solution:

"Data:"

"mass flow rate:"

Vol=0.6"m3...Receiver capacity"

P_i=100"kPa...initial pressure"

P_f=1000"kPa...final pressure"

time=8 * 60"s....time to fill the receiver"

T_r=25+273"k...Receiver temp."

R=0.287"kJ/kg.K"

n=1.25

"Calculations:"

m=(Vol * (P_f – P_i)/(R * T_r))/time"kg/s....mass flow rate"

C=1/25"Clearance ratio"

D=0.12"m"

L=D

eta_mech=0.8

P1=100"kPa"

T1=25+273"k"

eta_vol=1+C – C * (P_f/P1)^(1/n"vol. effcy."

V_s=(pi/4) * (D^2) * L"Stroke vol."

m=(RPM/60) * (eta_vol * V_s) * P1/(R * T1)"..finds the speed, RPM"

IP=(n/(n-1)) * m * R * T1 * ((P_f/P1)^((n-1)/n)-1)"kW...Indicated Power"

Power=IP/eta_mech"kW...Power input"

Results:

Unit Settings: SI K kPa kJ mass deg

C = 0.04	D = 0.12 [m]	η_{mech} = 0.8	η_{vol} = 0.7876
IP = 3.29 [kW]	L = 0.12 [m]	m = 0.01315 [kg/s]	n = 1.25
P1 = 100 [kPa]	Power = 4.113 [kW]	P_f = 1000 [kPa]	P_i = 100 [kPa]
R = 0.287 [kJ/kg.K]	RPM = 631.5	T1 = 298 [K]	time = 480 [s]
T_r = 298 [K]	Vol = 0.6 [m^3]	V_s = 0.001357 [m^3]	

Thus:

Mass flow rate = m = 0.01315 kg/s …. Ans.

Vol. efficiency = eta_vol = 0.7876 …. Ans.

Speed of compressor = 631.5 RPM … Ans.

Compressor power required = Power = 4.113 kW … Ans.

===

5.4 References:

1. *Yunus A. Cengel & Michael A. Boles*, Thermodynamics, An Engineering Approach, 7th Ed. McGraw Hill, 2011.
2. *Sonntag, Borgnakke & Van Wylen*, Fundamentals of Thermodynamics, 6th Ed. John Wiley & Sons, 2005.
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6 Thermodynamic relations

Learning objectives:

10. In this chapter, 'Thermodynamic relations' are dealt with.
11. Here, relations are developed to calculate 'Thermodynamic properties' which are impossible or difficult to measure (such as: entropy, internal energy, enthalpy, Helmholtz function and Gibbs function), in terms of measurable quantities such as pressure, volume and temperature.
12. We deal with Maxwell's equations, TdS equations, heat capacity relations, energy equations, Joule – Kelvin effect, Clausius – Clapeyron equation etc. which are practically important.
13. These relations are extremely useful to solve problems involving the immeasurable quantities and in constructing 'Property tables' from the experimental data.
14. Summary of important Thermodynamic relations is given at the beginning of this chapter.
15. Problems from University question papers and standard Text books are solved with Mathcad, EES and TEST.

==

6.1 Summary of Thermodynamic relations [1–6]:

6.1.1 Important mathematical relations:

(1) Exactness criteria:

If F = F(x,y), then:

$$dF = M\,dx + N\,dy$$

And, exactness criterion is:

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

(2) Cyclic relation:

If a relation exists among the variables x, y and z, then:

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

This will be applied to variables P, V and T later.

(3) Also, we have:

$$\left(\frac{\partial z}{\partial x}\right)_y = \left(\frac{\partial z}{\partial w}\right)_y \left(\frac{\partial w}{\partial x}\right)_y$$

$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{\left(\frac{\partial y}{\partial x}\right)_z}$$

6.1.2 Maxwell's relations:

These are derived from the relations for changes in internal energy (dU), enthalpy (dH), Helmholtz free energy (dF) and Gibb's Function (dG), using the exactness criteria mentioned above.

We have:

From I Law:

dU = dQ – PdV

But, from II Law:

dQ = TdS

Therefore, combining them, we get Gibb's equation:

dU = TdS– PdV ……. (a)

Now, by definition: Enthalpy is: H = U + P.V

Differentiating:

dH = dU + PdV + VdP

But, dU+PdV = dQ = TdS … from combined I Law and II Law

Therefore:

dH = TdS + VdP ……….(b)

Now, Helmholtz Function is:

F = U – TS

Differentiating:

dF = dU – TdS – SdT

i.e. dF = -PdV – SdT …….(c)

And, Gibbs Function is:

G = H – TS

Differentiating:

dG = dH – TdS – SdT

i.e. dG = VdP– SdT …….(d)

Now, apply the 'Exactness criteria' to the equations (a), (b), (c) and (d) given above, and we get the **four important Maxwell's equations:**

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial P}{\partial s}\right)_v$$

$$\left(\frac{\partial T}{\partial P}\right)_s = \left(\frac{\partial v}{\partial s}\right)_P$$

$$\left(\frac{\partial P}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$$

$$\left(\frac{\partial v}{\partial T}\right)_P = -\left(\frac{\partial s}{\partial P}\right)_T$$

Above equations relating entropy (S), (which is not measurable quantity) are particularly useful.

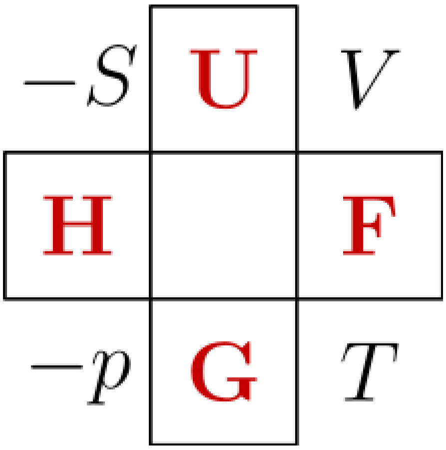
6.1.3 Mnemonic 'Thermodynamic square' to remember Maxwell's relations:

Since the Maxwell's equations are very useful, several mnemonic diagrams have been devised to remember them.

One such 'Thermodynamic diagram' known as 'Guggenheim scheme' or 'Born square' is given below: [Ref: 7]

It is very useful to:

1. get differentials of the thermodynamic potentials U, F, G and H, and
2. get Maxwell's equations



In the above, **thermodynamic square,** thepotentials highlighted in red.

Following description is quoted from the Ref.[7], viz. Wikipedia.

"It is a mnemonic diagram attributed to Max Born and used to help determine thermodynamic relations. The corners represent common conjugate variables while the sides represent thermodynamic potentials. The placement and relation among the variables serves as a key to recall the relations they constitute.A mnemonic used by students to remember the Maxwell relations is "**G**ood **P**hysicists **H**ave **S**tudied **U**nder **V**ery **F**ine **T**eachers", which helps them remember the order of the variables in the square, in clockwise direction.

How to use?

The Thermodynamic square is mostly used **to compute the derivative of any thermodynamic potential** of interest.

Suppose for example one desires to compute the derivative of the Internal energy *U*. The following procedure should be considered:

1. Place yourself in the thermodynamic potential of interest, namely (*G*, *H*,*U*, *F*). In our example, that would be *U*.
2. The two opposite corners of the potential of interest represent the coefficients of the overall result. If the coefficient lies on the left hand side of the square, a negative sign should be added. In our example, an intermediate result would be:
$dU = -p[\text{Differential}] + T[\text{Differential}]$.
3. In the opposite corner of each coefficient, you will find the associated differential. In our example, the opposite corner to *P* would be *V* (Volume) and the opposite corner for *T* would be *S* (Entropy). In our example, an interim result would be:
$dU = -pdV + TdS$.

Notice that the *sign convention will affect only the coefficients* and NOT the differentials.

The Thermodynamic square can **also be used to find the Maxwell Relations**:

Looking at the four corners of the square and making a ⊔ shape, one can find

$$\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p.$$

By rotating the ⊔ shape (randomly, for example by 90 degrees counterclockwise into a ⊐ shape) other relations such as:

$$\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

can be found.

Finally, the potential at the center of each side is a natural function of the variables at the corner of that side.

So, G is a natural function of p and T, and U is a natural function of S and V".

6.1.4 TdS relations:

We have the following definitions for Cv, Cp, Volume expansivity, β, and isothermal compressibility,κ :

$$\left(\frac{\partial U}{\partial T}\right)_V = C_V$$

$$\left(\frac{\partial H}{\partial T}\right)_P = C_P$$

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

$$\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

1. **First TdS equation, obtained by considering S as a function of T and V, i.e. S = S(T,V):**

$$dS = \frac{C_V}{T}dT + \left(\frac{\partial P}{\partial T}\right)_V dV$$

For a Van der Waal's gas:

$$dS = \frac{C_V}{T}dT - \frac{R}{V-b}dV$$

2. **Second TdS equation, obtained by considering S as a function of T and P, i.e. S = S(T,P):**

$$dS = \frac{C_P}{T}dT - \left(\frac{\partial V}{\partial T}\right)_P dP$$

6.1.5 Relations for specific heats:

Equating the First and Second TdS equations, and simplifying, and using the cyclic relation among P, V and T, we get:

$$C_P = T\left(\frac{\partial P}{\partial T}\right)_S\left(\frac{\partial V}{\partial T}\right)_P$$

$$C_V = -T\left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial V}{\partial T}\right)_S$$

Sp. heat difference:

$$C_P - C_V = -T\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P^2$$

Note from the above that:

1. (Cp-Cv) is always positive, i.e. Cp > Cv
2. As T tends to zero (i.e. T goes to Absolute zero temp), cp tends to Cv
3. For an Ideal gas, i.e. PV = RT, it can easily be shown that (Cp-Cv) = R

Also:

$$C_P - C_V = T\frac{V\beta^2}{\kappa}$$

Sp. heat ratio: (Cp/Cv = γ)

$$\frac{C_P}{C_V} = \frac{(\partial P/\partial V)_S}{(\partial P/\partial V)_T}$$

In the above, since γ > 1, numerator on RHS > denominator; therefore, we have:

Slope of an isentrope is greater than that of an isotherm on the P-V digram.

And, variations of sp. heats:

$$\left(\frac{\partial C_P}{\partial P}\right)_T = -T\left(\frac{\partial^2 V}{\partial T^2}\right)_P$$

$$\left(\frac{\partial C_V}{\partial V}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V$$

6.1.6 Relations for Energy:

(i) For Internal energy, we have:

dU = TdS – PdV

Substituting in the first TdS equation:

$$dU = C_V dT + \left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right] dV$$

Writing U as a function of T and V, and comparing the coefficients of dT and dV, we get:

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

This is known as ***Energy equation.***

For an Ideal gas (i.e. PV = RT), and we get:

$$\left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right] = 0$$

Thus, dU = Cv. dT for an Ideal gas.

For a van der Waal's gas, we get:

$$dU = C_V dT + \frac{a}{V^2} dV$$

(ii) Similarly, for Enthalpy, we have:

dH = TdS + V dP

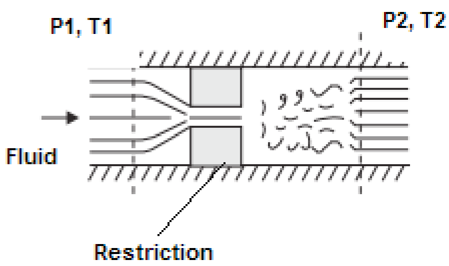
Substituting in the second TdS equation:

$$dH = C_P dT + \left[V - T\left(\frac{\partial V}{\partial T}\right)_P\right] dP$$

Writing H as a function of T and P, and comparing the coefficients of dT and dP, we get:

$$\left(\frac{\partial H}{\partial P}\right)_T = V - T\left(\frac{\partial V}{\partial T}\right)_P$$

6.1.7 Joule-Thomson (J-T) effect:



This refers to a Throttling process (i.e. an isenthalpic process), where ΔH = 0

Important practical applications are in refrigerating systems and gas liquefaction systems.

J – T coefficient is defined as:

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_H$$

i.e. J-T coeff. is the change in temperature with pressure at constant enthalpy.

From dH = TdS + VdP, and the second TdS equation, we get:

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_H = \frac{T(\partial V/\partial T)_P - V}{C_P}$$

Thus, for an Ideal gas (i.e. PV = RT), we get:

$\mu_J = 0$ i.e. for an ideal gas, there is no temperature change during throttling.

Note that:

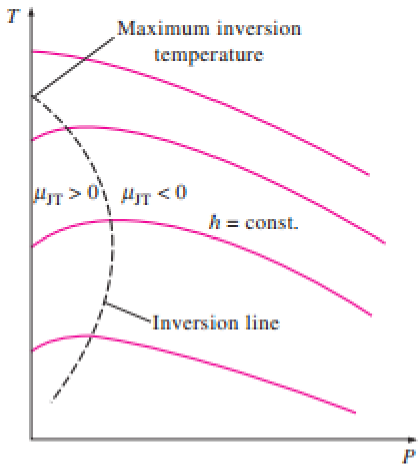
If $\mu_J < 0$ …. Temp increases when pressure decreases

If $\mu_J > 0$ …. Temp decreases when pressure decreases

If $\mu_J = 0$ …. No change in temp when pressure decreases

Inversion line:

Inversion line is the line that passes through all the points with $\mu_J = 0$ in P-T diagram, as shown below (Ref: Cengel):



Max. Inversion temp is the temp where the inversion line cuts the Temp axis.

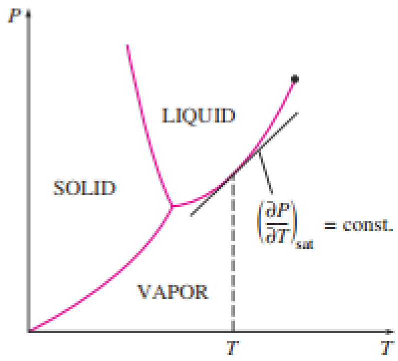
No cooling can occur if the temp before throttling is above the max. inversion temp.

To the left of the inversion line, we have the *cooling zone*, and to the right of the inversion line, *heating will occur* on throttling from a high pressure to a low pressure.

6.1.8 Clapeyron equation (Ref: Cengel):

It is applicable for any phase change process which occurs at constant temperature and pressure.

It gives the enthalpy of vaporization if the slope of the vaporization line in the P-T diagram and the sp. volumes of sat. liquid and sat. vapor are known.



$$\left(\frac{dP}{dT}\right)_{sat} = \frac{h_{fg}}{T \cdot V_{fg}}$$

6.1.9 Clausius-Clapeyron equation (Ref: Cengel):

Following approximations can be made for the solid-vapor and liquid-vapor phase changes:

1. $V_g >> V_f$
2. Treat the vapor as an ideal gas. i.e. $V_g = RT/P$
3. For small temp changes, treat h_{fg} as a constant

Then, Clapeyron equation becomes:

$$\left(\frac{dP}{dT}\right)_{sat} = \frac{P \cdot h_{fg}}{R \cdot T^2}$$

i.e.

$$\left(\frac{dP}{P}\right)_{sat} = \frac{h_{fg}}{R} \cdot \left(\frac{dT}{T^2}\right)_{sat}$$

Integrating this equation, between two states 1 and 2:

$$\ln\left(\frac{P2}{P1}\right) = \frac{h_{fg}}{R} \cdot \left(\frac{1}{T1} - \frac{1}{T2}\right) \quad \text{...under sat. conditions}$$

This is the **Clausius-Clapeyron equation.**

It is used to determine the variation of saturation pressure with temp.

This eqn.can also be used in the solid-vapor region by replacing the enthalpy of vaporization with the enthalpy of sublimation.

6.1.10 Clausius-Clapeyron equation in conjunction with Trouton's rule (Ref: 4):

Trouton's rule states that

$$\frac{h_{fg}}{T_B} = 88 \quad \text{kJ/kg mol K}$$

where h_{fg} is the latent heat of vaporization in kJ/kg mol and T_B is the boiling point at 1.013 bar.

Substituting this in Clausius Clapeyron eqn:

$$\frac{dP}{dT} = \frac{88 \cdot T}{R_u \cdot T^2} \cdot P$$

Integrating from 1.01325 bar to desired pressure P, temp. T:

$$\int_{101.325}^{P} \frac{dP}{P} = \frac{88 \cdot T_B}{R_u} \cdot \int_{T_B}^{T} \frac{dT}{T^2}$$

i.e. $$\ln\left(\frac{P}{101.325}\right) = \frac{-88 \cdot T_B}{R_u} \cdot \left(\frac{1}{T} - \frac{1}{T_B}\right)$$

i.e. $$P = 101.325 \cdot \exp\left[\frac{88}{R_u} \cdot \left(1 - \frac{T_B}{T}\right)\right]$$

Above equation gives vapor pressure P in kPa at any temperature T.

(Note: R_u is Universal Gas Const = 8.3143 kJ/kg mol K.)

===

6.2 Problems solved with Mathcad:

Prob.6.2.1 Verify the 4th Maxwell relation for steam at 300 C and 4 bar.

Mathcad Solution:

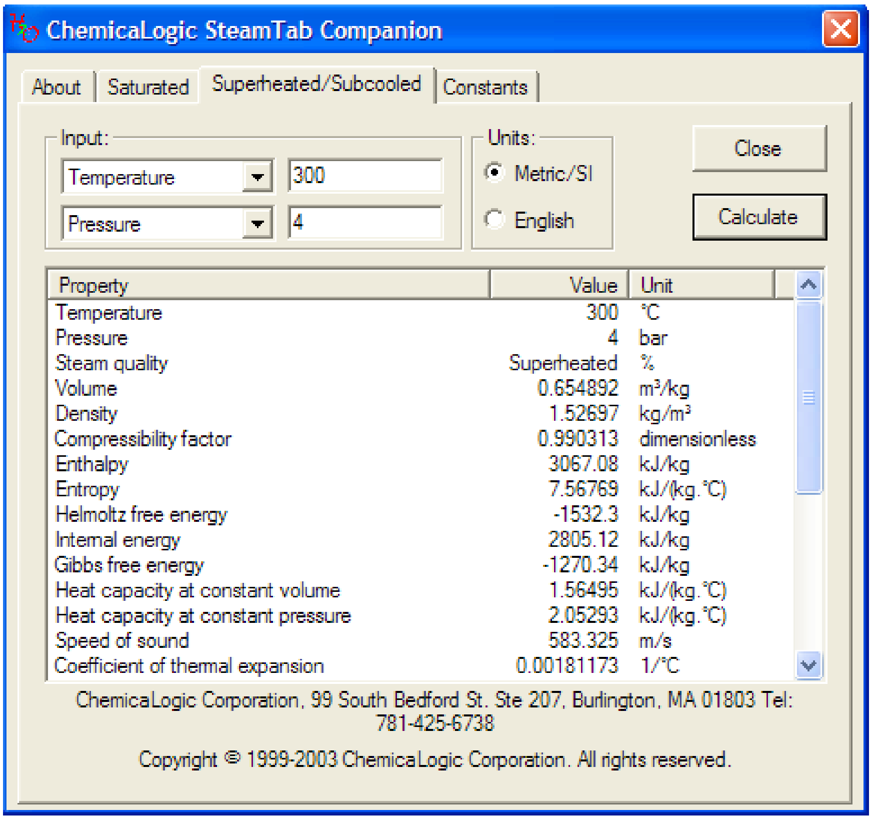
4th Maxwell equation is:

$$\left(\frac{\partial v}{\partial T}\right)_P = -\left(\frac{\partial s}{\partial P}\right)_T$$

We will replace the differential quantities in this equation by corresponding differences, obtained from Stem Tables. In our case, instead of Steam Tables, we shall use the free software 'SteamTab' from ChemicaLogic:

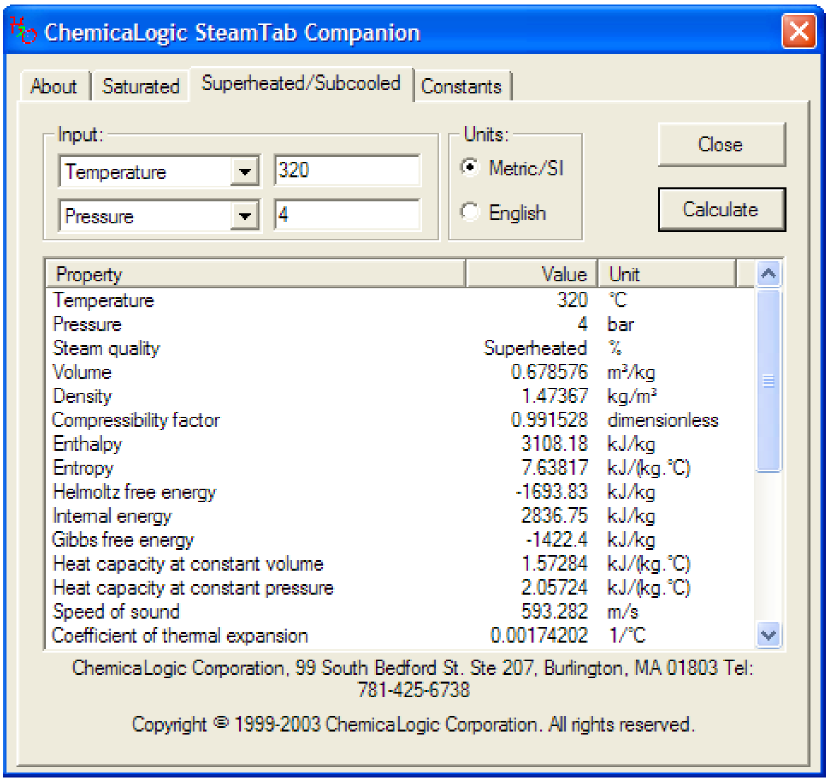
To find the term in the LHS of above eqn:

At T1 = 300 C, P1 = 4 bar:



We get:v1 = 0.654892 m3/kg

At T2 = 320 C, 4 bar:



We get: v2 = 0.678576 m3/kg

Then:

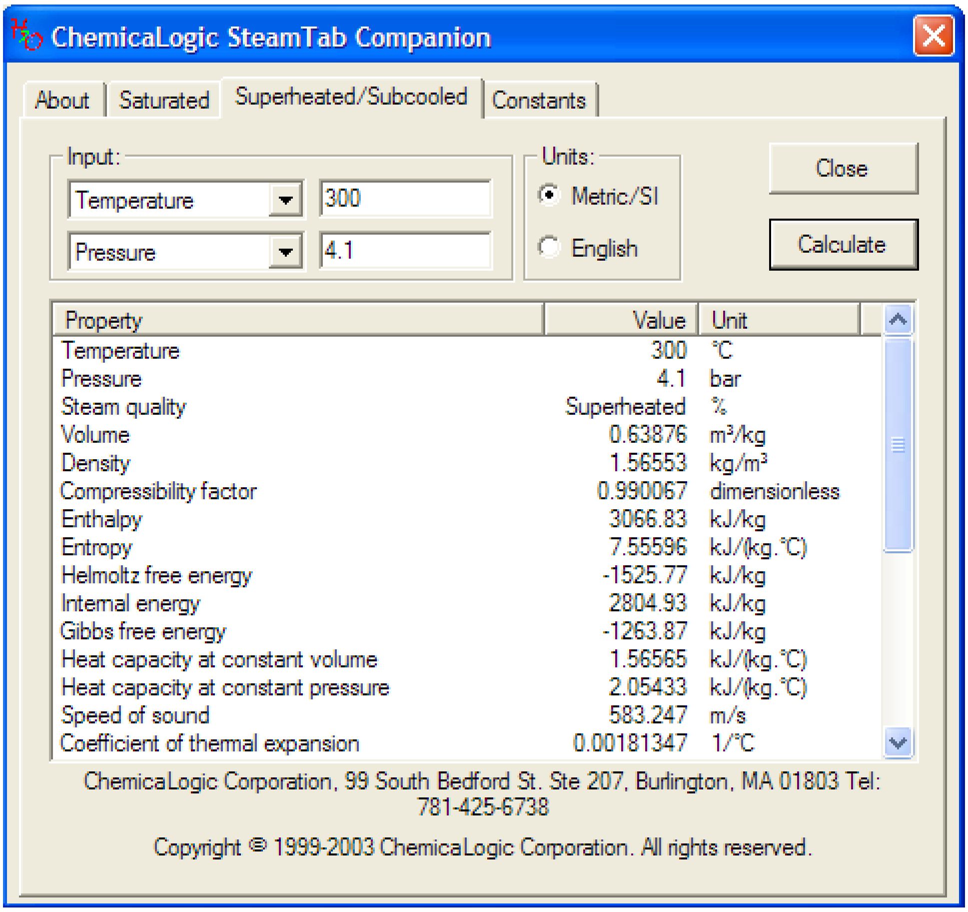
(v2 – v1) / (T2 – T1) at 4 bar =

$$\frac{0.678576 - 0.654892}{320 - 300} = 1.184 \times 10^{-3} \quad \text{m\^{}3/kg.K}$$

Now, to find the term in the RHS of Maxwell's 4th eqn:

At T1 = 300 C, P1 = 4 bar: s1 = 7.56769 kJ/kg.C

At T1 = 300 C, P2 = 4.1 bar: s2 = 7.55596 kJ/kg.C …. See below:



Then:

-(s2 – s1) / (P2 – P1) at 300 C =

$$-\frac{7.55596 - 7.56769}{410 - 400} = 1.173 \times 10^{-3} \quad \text{m^3/kg.K}$$

Note that pressure should be entered in kPa since kJ = kPa.m^3, and temp differences are the same in Kelvin or deg.C.

The difference in the values of LHS and RHS is:

$$\frac{1.184 \times 10^{-3} - 1.173 \times 10^{-3}}{1.184 \times 10^{-3}} \cdot 100 = 0.929 \quad \%$$

This is within a difference of 1%:

Therefore, 4th Maxwell eqn is verified….. Ans.

==

Prob.6.2.2 Use Clapeyron equation to find enthalpy of vaporization of R134a at 15 C and compare it with the tabulated value.

Mathcad Solution:

We have, from Clapeyron eqn:

$$h_{fg} = T \cdot v_{fg} \cdot \left(\frac{d}{dT}P\right)_{sat}$$

Using the Mathcad Functions written earlier for R134a (see Prob.4.2.1), we have:

$$v_{fg} := \text{VFGSATT}(15)$$

i.e. $v_{fg} = 0.041$ m^3/kg…at 15 C

And:

$$\left(\frac{\Delta P}{\Delta T}\right)_{sat,\,15C} = \frac{P_{satat20C} - P_{satat10C}}{20 - 10}$$

i.e. $LHS := \frac{(PSAT(20) - PSAT(10)) \cdot 100}{20 - 10}$

i.e. $LHS = 15.723$ kPa/K

Therefore:

$T := 273 + 15$ K

$h_{fg} := T \cdot v_{fg} \cdot LHS$

i.e. $h_{fg} = 186.487$ kJ/kg.... calculated from Clapeyron eqn.... Ans.

Compare with result from Tables:

From Tables: we get the h_{fg} as: $HFGSATT(15) = 185.697$ kJ/kg

Therefore, difference =

$$\frac{186.487 - 185.697}{186.487} \cdot 100 = 0.424$$ **% .... this is quite small.... verified.**

==

Prob.6.2.3 Given that boiling point of Benzene at 1 atm is 353 K, estimate its vapor pressure at 290 K.

Mathcad Solution:

We use Clausius Clapeyron eqn along with Trouton's rule:

Data:

$R_u := 8.3143$ kJ/kg mol. K $\quad T_B := 353$ K $\quad T := 290$ K

We have:

$$P := 101.325 \cdot \exp\left[\frac{88}{R_u} \cdot \left(1 - \frac{T_B}{T}\right)\right]$$

i.e. $P = 10.166$ **kPa....Ans....Vapor pressure of Benzene at 290 K**

==

Prob.6.2.4 Vapor pressure of Mercury at 399 K and 401 K is found to be 0.988 mm and 1.084 mm of Hg respectively. Calculate the latent heat of vaporization of liquid Hg at 400 K. [4]

Mathcad Solution:

Data:

$P1 := 0.988$ mm of Hg $\quad T1 := 399$ K $\quad P2 := 1.084$ mm of Hg $\quad T2 := 401$ K

$P := \frac{P1 + P2}{2}$ i.e. $P = 1.036$ mm hg .... average pressure

$T := 400$ K ... avg. temp.

Calculations:

$$dPdT := \frac{P2 - P1}{T2 - T1} \quad \text{i.e.} \quad dPdT = 0.048 \quad \text{mm Hg/ K}$$

Then, using Clausius Clapeyron eqn:

$$h_{fg} := \frac{R_u \cdot T^2}{P} \cdot dPdT$$

i.e. $h_{fg} = 6.163 \times 10^4$ **kJ/kg mol .....latent heat of vap. of Hg.... Ans.**

==

Prob.6.2.5 In the vicinity of the triple point, vapor pressure of liquid ammonia (in atm.) is represented by: ln(P) = 15.16 – 3063/T. This is the eqn of the liquid-vapor boundary curve in the P-T diagram. Similarly, the vapor pressure of solid ammonia is: ln(P) = 18.70 – 3754/T.

1) what is the temp and pressure at the triple point?
2) what are the latent heats of sublimation and vaporization?
3) what is the latent heat of fusion at the triple point? [4]

Mathcad Solution:

Data:

$R_u := 8.314$ kJ/kg mol. K

$M_{NH3} := 17$ ..Mol. wt. of NH3

Note that at the triple point, the sat. vapor line and the sublimation lines meet.

So, we solve the two equations for these lines simultaneously, using the 'Solve block' of Mathcad:

Start with the guess values for T and P:

$T := 100$ K $P := 100$ atm ...guess values

Given

$$\ln(P) = 15.16 - \frac{3063}{T}$$

$$\ln(P) = 18.7 - \frac{3754}{T}$$

$$\mathrm{Find}(T,P) = \begin{pmatrix} 195.198 \\ 0.588 \end{pmatrix}$$

i.e. $T := 195.18$ **K...triple point temp.. Ans.**

$P := 0.588$ **atm...triple point pressure....Ans.**

To find the latent heats:

We have, from Clausius - Clapeyron eqn:

$$h_{fg} = \frac{R_u \cdot T^2}{P} \cdot \frac{dP}{dT} \qquad \text{....eqn. (A)}$$

In the above, for the prsent case, find dP/dT from the eqn for vapor pressure:

For liquid NH3:

$$\ln(P) = 15.16 - \frac{3063}{T}$$ ...eqn. for vapor pressure

Differentiating w.r.t. T, we get:

$$\frac{1}{P} \cdot \frac{dP}{dT} = \frac{3063}{T^2}$$

i.e. $$\frac{dP}{dT} = \frac{3063 \cdot P}{T^2}$$

Substituting this in eqn. (A):

$$h_{fg} := \frac{R_u}{M_{NH3}} \cdot 3063$$

i.e. $h_{fg} = 1.498 \times 10^3$ **kJ/kg....latent heat of vaporization .... Ans.**

Similarly, for solid NH3:

$$\ln(P) = 18.7 - \frac{3754}{T}$$ ...eqn. for vapor pressure

Differentiating w.r.t. T, we get:

$$\frac{1}{P} \cdot \frac{dP}{dT} = \frac{3754}{T^2}$$

i.e. $$\frac{dP}{dT} = \frac{3754 \cdot P}{T^2}$$

Substituting this in eqn. (A):

$$h_{fg} := \frac{R_u}{M_{NH3}} \cdot 3754$$

i.e. $h_{fg} = 1.836 \times 10^3$ **kJ/kg....latent heat of sublimation .... Ans.**

And, latent heat of fusion:

Latent heat of fusion = latent heat of sublimation - latent heat of vaporization

Therefore:

$l_{fusion} := 1836 - 1498$ kJ/kg

i.e. $l_{fusion} = 338$ **kJ/kg....latent heat of fusion .... Ans.**

===

Prob.6.2.6. Pressure on a block of copper of 1 kg is increased from 20 bar to 800 bar at a constant temp of 20 C. Determine the following:

(i) work done on the copper block, (ii)change in entropy, (iii) heat transfer, (iv) change in internal energy, and (v) (cp – cv) for this change of state

Given: $\beta = 5 * 10^{-5}$ 1/K, $\kappa_T = 8.6 * 10^{-12}$ m²/N and v = 0.114 m³/kg

Mathcad Solution:

Data:

$\beta := 5 \cdot 10^{-5}$ 1/K ... volume expansivity

$\kappa_T := 8.6 \cdot 10^{-12}$ m^2/N ...isothermal compressibility

$v := 0.114 \cdot 10^{-3}$ m^3/kg .... sp. volume

$p1 := 20 \cdot 10^5$ Pa $p2 := 800 \cdot 10^5$ Pa $T := 20 + 273$ K

Calculations:

(i) Work done in isothermal compression:

$$W = \int_1^2 p\, dv$$

Now, by definition, κ_T is:

$$\kappa_T = \frac{-1}{v} \cdot \left(\frac{\partial}{\partial T} v\right)_T$$

i.e. $dv = -\kappa_T \cdot (v \cdot dp)_T$

Therefore: $$W = -\int_1^2 p\cdot\kappa_T\cdot v\,dP = -v\cdot\kappa_T\cdot\int_1^2 p\,dp$$

i.e. $$W := \frac{-v\cdot\kappa_T}{2}\cdot\left(p2^2 - p1^2\right) \quad \text{J/kg}$$

i.e. $W = -3.135$ **J/kg....isothermal work done ....Ans.**

Note: Work is done on the copper block...so, negative.

(ii) Change in entropy:

From Maxwell's relation:

$$\left(\frac{\partial}{\partial p}s\right)_T = -\left(\frac{\partial}{\partial T}v\right)_p = \frac{-v}{v}\cdot\left(\frac{\partial}{\partial T}v\right)_p = -v\cdot\beta$$

Therefore: $ds_T = -v\cdot\beta\cdot dp_T$

Integrating the above, assuming v and β to be constants, we get:

$$\Delta s := -v\cdot\beta\cdot(p2 - p1)$$

i.e. $\Delta s = -0.445$ **J/kg.K .... change in entropy ... Ans.**

(iii) Heat transfer, Q:

$$Q := T\cdot\Delta s$$

i.e. $Q = -130.268$ **J/kg ... heat transfer .... Ans.**

Note: negative sign indicates that heat ***flows out*** of the copper block during isothermal compression.

(iv) Change in internal energy, dU:

$$dU := Q - W$$

i.e. $dU = -127.132$ J/kg ...change in internal energy.... Ans.

(iv) Find (cp-cv):

We have:

$$cp - cv = \frac{T \cdot v \cdot \beta^2}{\kappa_T} = 9.71$$ J/kg.K ..... Ans.

==

6.3 Problems solved with EES:

"**Prob.6.3.1** Refrigerant NH3 at 15 bar and 20 C is expanded in an expansion valve.. Find out the temp drop and the J-T coeff. for a final pressure of 2 bar.

(b) Then plot these quantities as the final pressure varies from 2 bar to 8 bar, other conditions remaining the same."

EES Solution:

Fluid$ = 'Ammonia'

P1 = 1500"kPa"

P2 = 200 "kPa"

T1 = 20"C"

DELTAP = P1 – P2"kPa"

h1 = Enthalpy(Fluid$,T=T1,P=P1)"kJ/kg"

T2 = Temperature(Fluid$,P=P2,h=h1) "C"

DELTAT = T1 – T2

mu_JT = DELTAT/DELTAP

Results:

Unit Settings: SI C kPa kJ mass deg

ΔP = 1300 [kPa]	ΔT = 38.85 [C]	Fluid$ = 'Ammonia'
h1 = 294.1 [kJ/kg]	μ_{JT} = 0.02988 [C/kPa]	P1 = 1500 [kPa]
P2 = 200 [kPa]	T1 = 20.000 [C]	T2 = -18.850 [C]

Thus:

Temp. drop = ΔT = 38.85 C …. Ans.

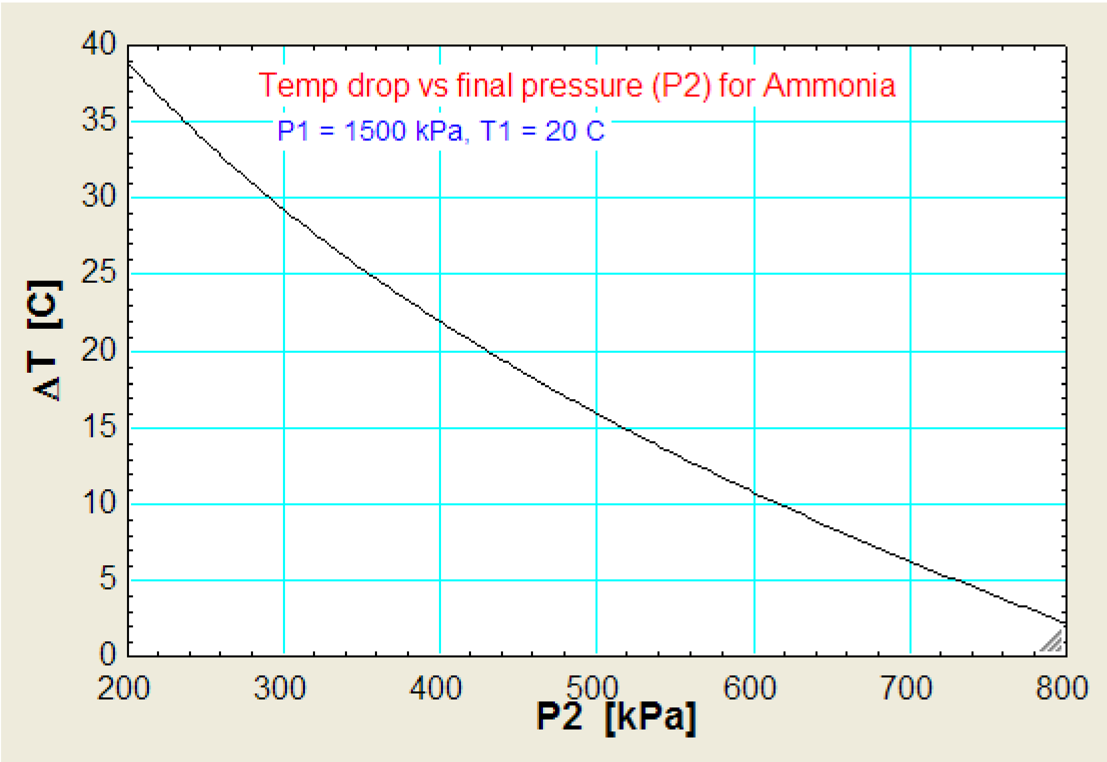
J-T coeff. = 0.02988 C/kPa…. Ans.

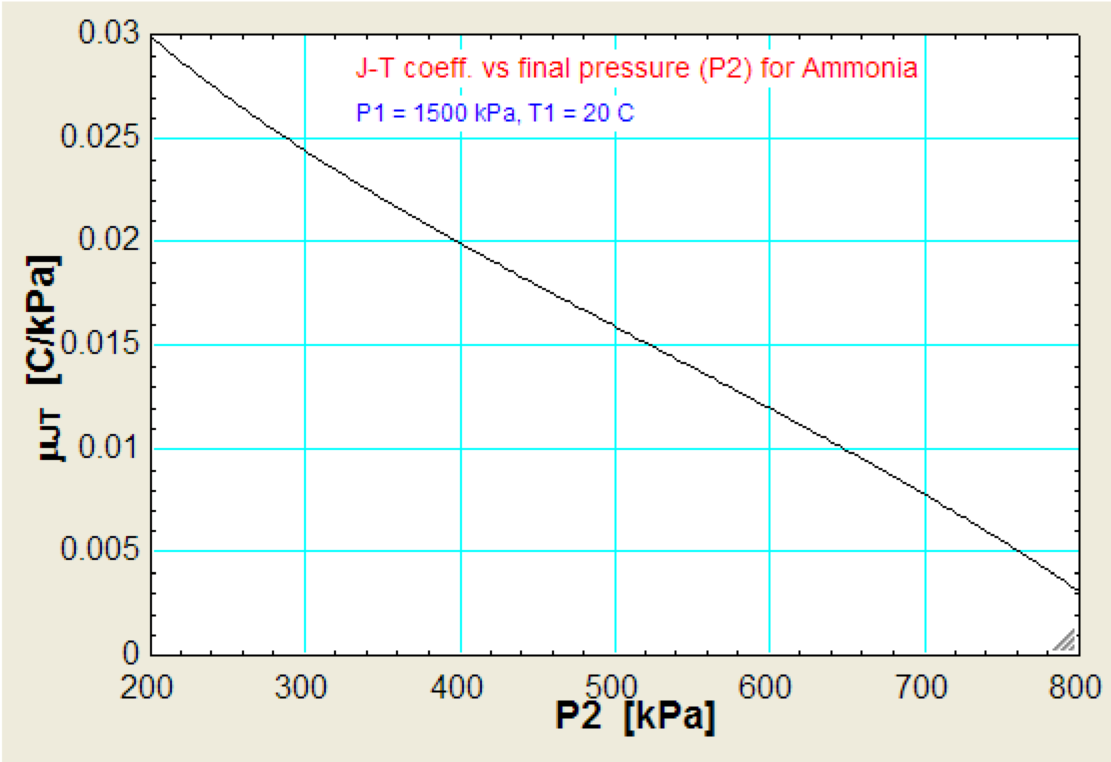
(b) Plot these quantities as the final pressure varies from 2 bar to 8 bar, other conditions remaining the same:

First, compute the Parametric Table:

1..7	P2 [kPa]	ΔT [C]	ΔP [kPa]	μ_{JT} [C/kPa]
Run 1	200	38.85	1300	0.02988
Run 2	300	29.23	1200	0.02436
Run 3	400	21.89	1100	0.0199
Run 4	500	15.87	1000	0.01587
Run 5	600	10.73	900	0.01192
Run 6	700	6.209	800	0.007761
Run 7	800	2.163	700	0.003089

Now, plot the results:





==

"**Prob.6.3.2** Refrigerant R134a at 13 bar and 20 C is expanded in an expansion valve. Find out the temp drop and the J-T coeff. for a final pressure of 1 bar. Then plot these quantities as the final pressure varies from 1 bar to 5 bar, other conditions remaining the same."

EES Solution:

The EES program is similar to the previous one written for NH3.

```
Fluid$ = 'R134a'

P1 = 1300"kPa"
P2 = 100 "kPa"
T1 = 20"C"
DELTAP = P1 – P2"kPa"
h1 = Enthalpy(Fluid$,T=T1,P=P1)"kJ/kg"

T2 = Temperature(Fluid$,P=P2,h=h1) "C"

DELTAT = T1 – T2

mu_JT = DELTAT/DELTAP
```

Results:

Unit Settings: SI C kPa kJ mass deg

ΔP = 1200 [kPa]	ΔT = 46.37 [C]	Fluid$ = 'R134a'
h1 = 79.41 [kJ/kg]	μ_{JT} = 0.03865 [C/kPa]	P1 = 1300 [kPa]
P2 = 100 [kPa]	T1 = 20 [C]	T2 = -26.37 [C]

Thus:

Temp drop = ΔT = 46.37 C … Ans.

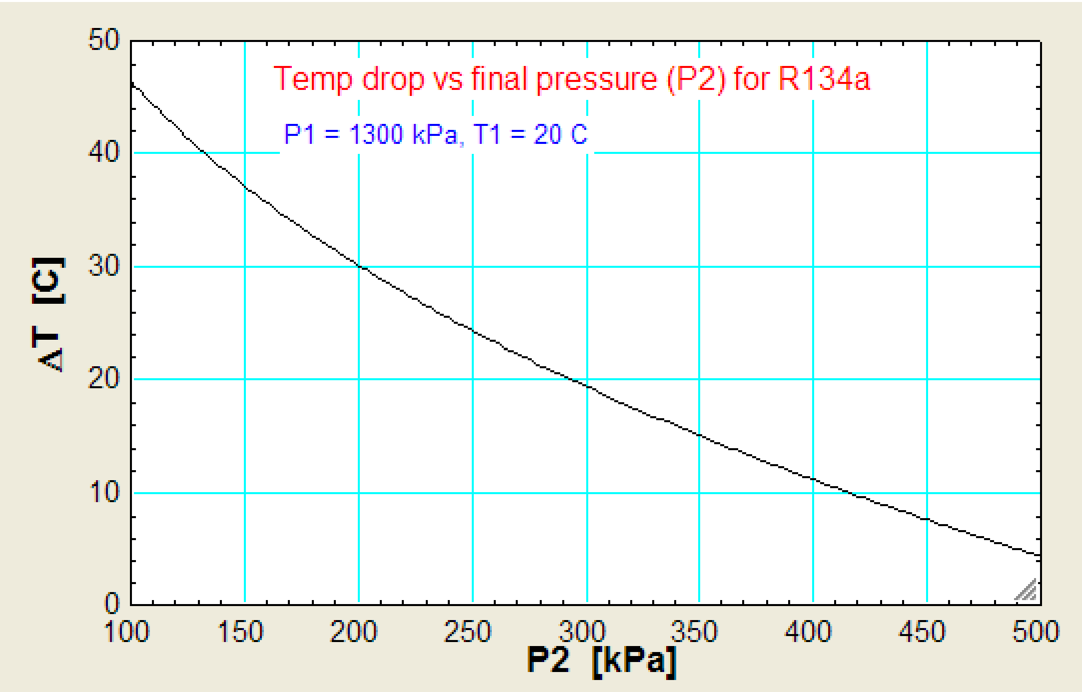
J-T coeff. = 0.03865 C/kPa … Ans.

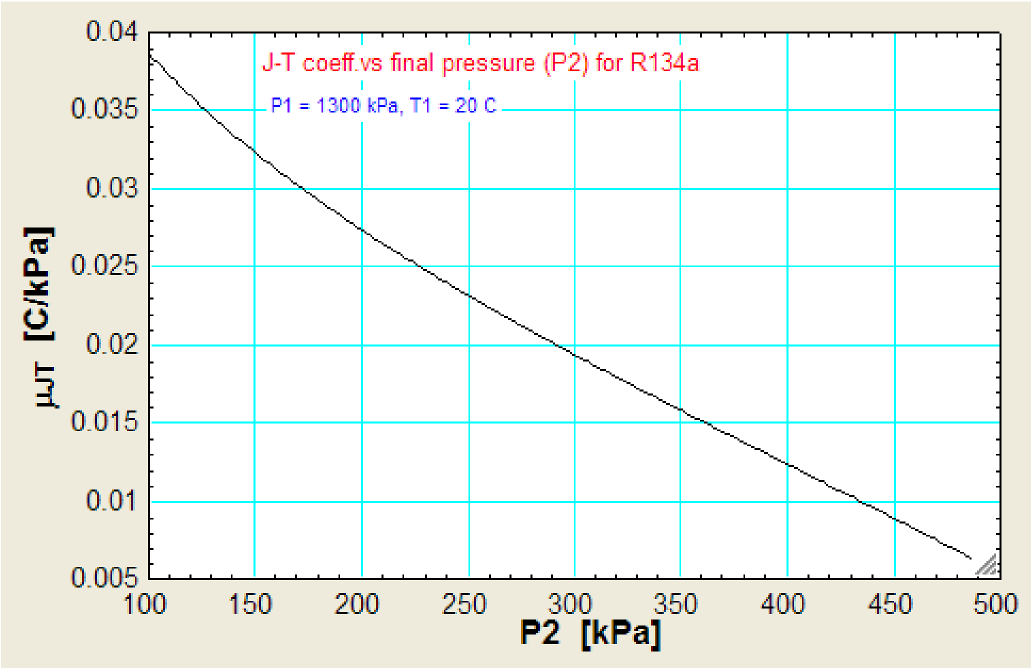
(b) Plot these quantities as the final pressure varies from 1 bar to 5 bar, other conditions remaining the same:

First, compute the Parametric Table:

1..11	P2 [kPa]	ΔT [C]	ΔP [kPa]	μ_{JT} [C/kPa]
Run 1	100	46.37	1200	0.03865
Run 2	140	38.77	1160	0.03343
Run 3	180	32.73	1120	0.02922
Run 4	220	27.66	1080	0.02561
Run 5	260	23.25	1040	0.02236
Run 6	300	19.35	1000	0.01935
Run 7	340	15.82	960	0.01648
Run 8	380	12.6	920	0.0137
Run 9	420	9.633	880	0.01095
Run 10	460	6.871	840	0.00818
Run 11	500	4.286	800	0.005357

Now, plot the results:





==

"**Prob.6.3.3** It is found that a certain liquid boils at a temp of 95 C at the top of a hill, and it boils at a temp of 105 C at the bottom of hill. The latent heat is 4187 kJ/kg.mol. What is the approximate height of the hill? [4]"

EES Solution:

"Data:"

T1 = 105+273"k ... at the bottom of hill"

T2 = 95+273"k.... at the top of hill"

h_fg = 4187 "kJ/kg.mol"

R_u = 8.314 "kJ/kg.mol"

R_air = 287 "J/kg.K"

T_amb = 300 "k .... assumed"

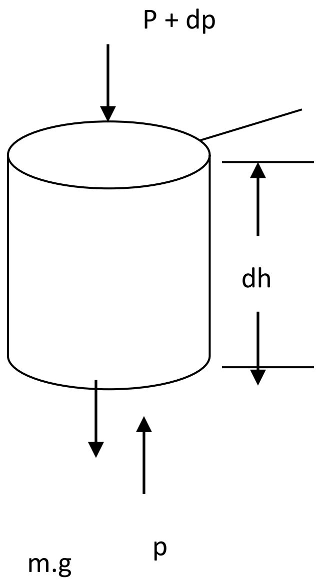
"Applying the Clausius Clapeyron equation:"

ln(P2byP1) = (h_fg/R_u)* (1/T1 – 1/T2)

"Now: we need another relation linking the pressure to the height of the hill.

So, consider a small volume element of the atmosphere and make a force balance:

In the following analysis, an isothermal atmosphere is assumed. i.e. $p.v = p_{amb}.v_{amb} = R_{air}.Tamb$"



By making a force balance:

$$A \cdot (p + dp) + m \cdot g = P \cdot A$$

i.e. $$A \cdot (p + dp) + A \cdot dh \cdot \rho \cdot g = p \cdot A$$

i.e. $$dp = -\rho \cdot g \cdot dh = \frac{-g \cdot dh \cdot p}{p_{amb} \cdot v_{amb}}$$

Integrating:

$$\int_{p1}^{p2} \frac{dp}{p} = -\int \frac{g \cdot dh}{p_{amb} \cdot v_{amb}}$$

i.e. $$\ln\left(\frac{p2}{p1}\right) = \frac{-g \cdot h}{p_{amb} \cdot v_{amb}} = \frac{-g \cdot h}{R_{air} \cdot T_{amb}}$$

"Add the following to the code:"

ln(P2byP1) = -9.81 * h / (R_air * T_amb)

Results:

Unit Settings: SI C kPa kJ mass deg

h = 317.8 [m]	h_{fg} = 4187 [kJ/kg-mole-K]	P2byP1 = 0.9644
R_{air} = 287 [J/kg-K]	R_u = 8.314 [kJ/kg-mole-K]	T1 = 378 [K]
T2 = 368 [K]	T_{amb} = 300 [K]	

Thus:

Approx. height of the hill = h = 317.8 m …. Ans.

==

"**Prob.6.3.4** A pressure cooker works at 2 bar. Given that water boils at 100 C at a pressure of 1 bar, and the latent heat of vaporization of water is 2257 kJ/kg, estimate the boiling point of water in the pressure cooker."

EES Solution:

"Data:"

P1 = 1 "bar"

P2 = 2 "bar"

T1 = 100 + 273 "k"

h_fg = 2257 "kJ/kg"

M_H2O = 18 "....mol. wt. of water"

R_H2O = 8.314/M_H2O

"Calculations:"

"From Clausius – Clapeyron equation:"

ln(P2/P1) = (h_fg/R_H2O) * (1/T1 – 1/T2)

Results:

Unit Settings: SI C kPa kJ mass deg

h_{fg} = 2257 [kJ/kg]	M_{H2O} = 18	P1 = 1 [kPa]
P2 = 2 [kPa]	R_{H2O} = 0.4619 [kJ/kg-K]	T1 = 373 [K]
T2 = 393.8 [K]		

Thus:

Boiling temp of water at 2 bar = T2 = 393.8 K = 120.8 C.... Ans.

==

"**Prob.6.3.5** For mercury, following relation exists between sat. pressure and sat. temp:

log(p) = 7.0323 – 3276.6/T – 0.652 log(T)

Calculate the sp. volume v_g at 0.1 bar. Given: latent heat of vaporization at 0.1 bar = 294.54 kJ/kg.

Neglect the sp. volume of sat. liquid. [5]"

EES Solution:

"Data:"

h_fg = 294.54 "kJ/kg"

p = 0.1 "bar"

"We have: from Clausius – Clapeyron eqn:

dp/dT = h_fg /(v_fg . T) = h_fg / ((v_g – v_f). T)

Neglecting v_f: dp/dT = h_fg / (v_g . T)"

"Differentiating the vap. pressure eqn:

(1/(2.302 * p))* dp/dT = 3276.6/T^2 – 0.652/(2.302 * T)"

"Therefore:"

dpdT = 2.302 * 3276.6 * p*100 / T^2 – 0.652 * p*100/T "..pressure converted to kPa since h_fg is in kJ/kg"

dpdT = h_fg / (v_g * T)

log10(p) = 7.0323 – 3276.6/T – 0.652* log10(T)

Results:

Unit Settings: SI C kPa kJ mass deg

dpdT = 0.2628 [kPa/K] h_{fg} = 294.5 [kJ/kg] p = 0.1 [bar]

T = 523.5 [K] v_g = 2.141 [m^3/kg]

Thus:

Sat. temp = T = 523.5 K …. Ans.

Sp. vol. of sat. mercury vapor = v_g = 2.141 m^3/kg … Ans.

==

6.4 Problems solved with TEST [Ref: 8]:

Prob.6.4.1 Verify the validity of 4th Maxwell eqn for steam at 300 C and 300 kPa.

TEST Solution:

4th Maxwell eqn is:

$$\left(\frac{\partial v}{\partial T}\right)_P = -\left(\frac{\partial s}{\partial P}\right)_T$$

First, fix the State 1 with p1 = 300 kpa and T1 = 300 C.

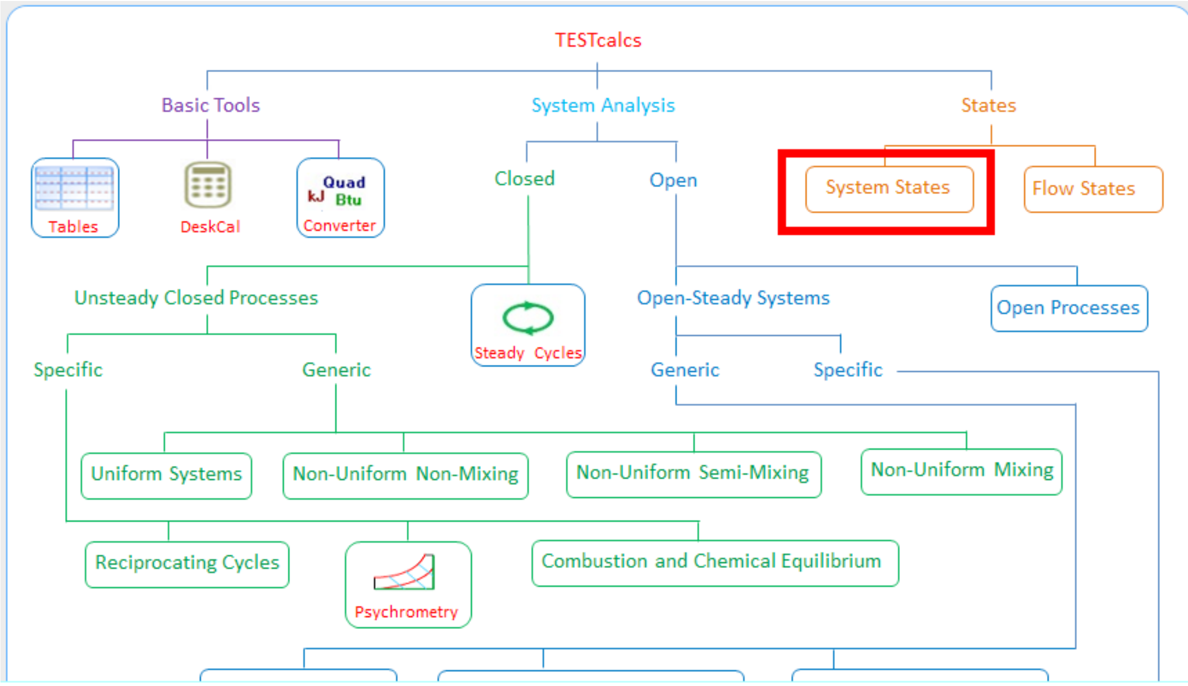
Then, to calculate the RHS of above Maxwell eqn, keeping T1 same, give a perturbation of 1% of p1 on its either side, (i.e. p2 = p1 – 0.01 * p1, and p3 = p1 + 0.01 * p1) and compute those States as State 2 and State 3. Then, RHS is calculated as RHS = – (s3 – s2) / (p3 – p2).

Similarly, to calculate the LHS of above Maxwell eqn, keeping p1 same, give a perturbation of 1% of T1 on its either side, (i.e. T4 = T1 + 0.01 * T1, and T5 = T1 – 0.01 * T1) and compute those States as State 4 and State 5. Then, LHS is calculated as LHS = (v4 – v5) / (T4 – T5).

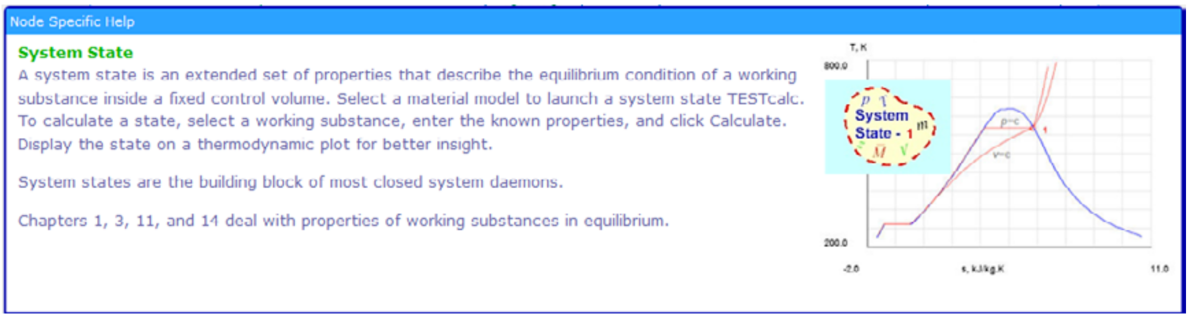
Then, calculate their difference as a percentage of LHS.

Following are the steps:

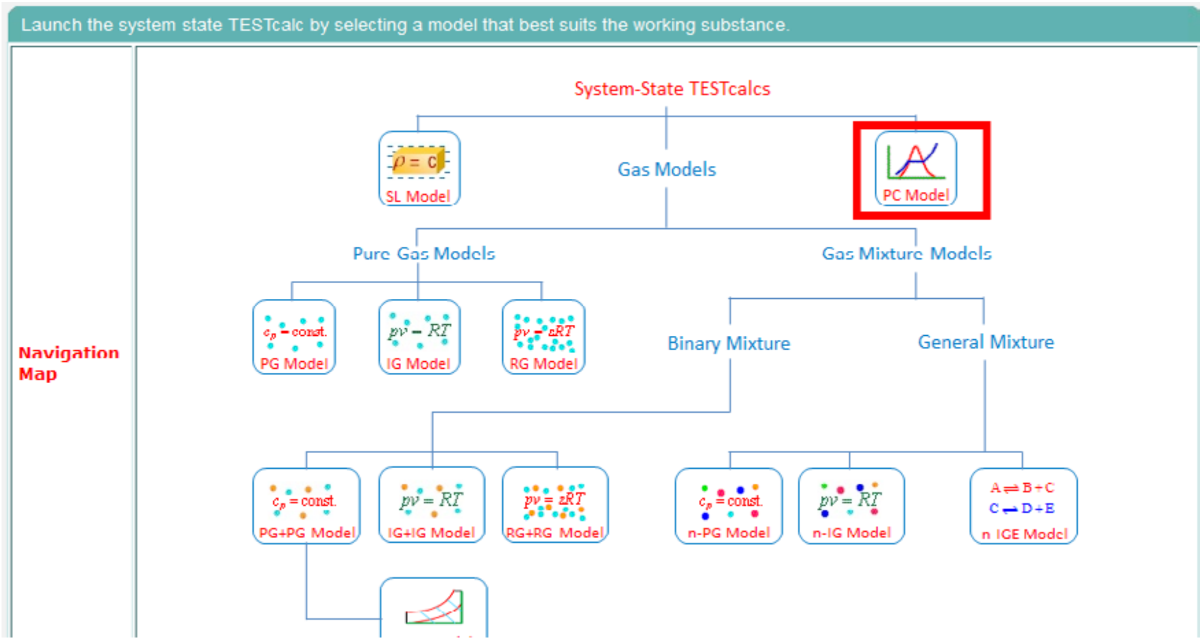
1. From the daemon tree, choose 'System States':



Hovering the mouse pointer on 'System States' brings up the following explanatory window:



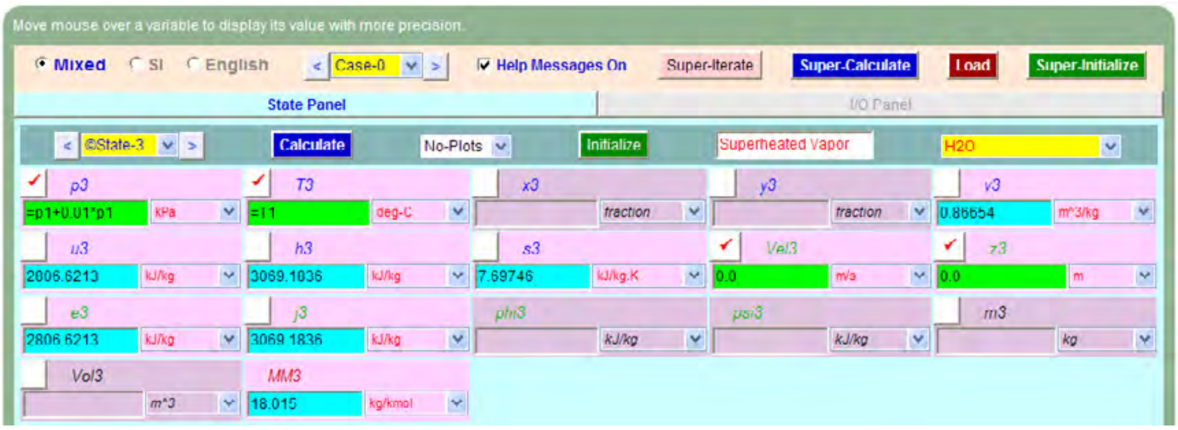
2. Clicking on 'System States' takes us to the material model selection:



3. Click on PC Model, since we are dealing with H2O. Observe that H2O is selected by default. Enter for State 1, p1 = 300 kPa, T1 = 300 C, and hit Enter We get:

4. For State 2: Enter p2 = p1 – 0.01*p1, T2 = T1, hit Enter. We get:

5. For State 3: Enter p3 = p1 + 0.01 * p1, T3 = T1, hit Enter. We get:



Therefore:

-(Δs/Δp) at T = 300 C = – (s3-s2)/(p3-p2) = 0.0015710989634195964

Now, to calculate the LHS, i.e. at const. p1:

6. For State 4: Enter p4 = p1, T4 = T1 + 0.01 * T1, hit Enter. We get:

7. For State 5: Enter p5 = p1, T5 = T1 – 0.01 * T1, hit Enter. We get:

Therefore:

(Δv/ΔT) at p = 300 kPa =(v4-v5)/(T4-T5) = 0.0015704333782196045

See how they match:

Difference = (LHS – RHS)* 100/LHS =

(0.0015704333782196045-0.0015710989634195964)*100/0.0015704333782196045= -0.04238%

Thus, the LHS and RHS match very well.

And, the 4^{th} Maxwell eqn is verified…. Ans.

8. I/O panel gives the TEST code etc:

#*******TEST-code:

.# Daemon (TESTcalc) Path: States>System>PC-Model; v-10.cd03

#--------------------Start of TEST-code---

States {

State-1: H2O;

Given: { p1= 300.0 kPa; T1= 300.0 deg-C; Vel1= 0.0 m/s; z1= 0.0 m; }

State-2: H2O;

Given: { p2= "p1-0.01*p1" kPa; T2= "T1" deg-C; Vel2= 0.0 m/s; z2= 0.0 m; }

State-3: H2O;

Given: { p3= "p1+0.01*p1" kPa; T3= "T1" deg-C; Vel3= 0.0 m/s; z3= 0.0 m; }

State-4: H2O;

Given: { p4= "p1" kPa; T4= "T1+0.01*T1" deg-C; Vel4= 0.0 m/s; z4= 0.0 m; }

State-5: H2O;

Given: { p5= "p1" kPa; T5= "T1-0.01*T1" deg-C; Vel5= 0.0 m/s; z5= 0.0 m; }

}

#----------------------End of TEST-code--

#******DETAILED OUTPUT:

 # Evaluated States:

#	State-1: H2O > Superheated Vapor;

#		Given: p1= 300.0 kPa; T1= 300.0 deg-C; Vel1= 0.0 m/s;

#			z1= 0.0 m;

#		Calculated: v1= 0.8753 m^3/kg; u1= 2806.6775 kJ/kg; h1= 3069.2598 kJ/kg;

#			s1= 7.7022 kJ/kg.K; e1= 2806.6775 kJ/kg; j1= 3069.2598 kJ/kg;

#			MM1= 18.015 kg/kmol;

#	State-2: H2O > Superheated Vapor;

#		Given: p2= "p1-0.01*p1" kPa; T2= "T1" deg-C; Vel2= 0.0 m/s;

#			z2= 0.0 m;

#		Calculated: v2= 0.8842 m^3/kg; u2= 2806.7334 kJ/kg; h2= 3069.335 kJ/kg;

#			s2= 7.7069 kJ/kg.K; e2= 2806.7334 kJ/kg; j2= 3069.335 kJ/kg;

MM2= 18.015 kg/kmol;

#

State-3: H2O > Superheated Vapor;

Given: p3= “p1+0.01*p1” kPa; T3= “T1” deg-C; Vel3= 0.0 m/s;

z3= 0.0 m;

Calculated: v3= 0.8665 m^3/kg; u3= 2806.6213 kJ/kg; h3= 3069.1836 kJ/kg;

s3= 7.6975 kJ/kg.K; e3= 2806.6213 kJ/kg; j3= 3069.1836 kJ/kg;

MM3= 18.015 kg/kmol;

State-4: H2O > Superheated Vapor;

Given: p4= “p1” kPa; T4= “T1+0.01*T1” deg-C; Vel4= 0.0 m/s;

z4= 0.0 m;

Calculated: v4= 0.88 m^3/kg; u4= 2811.4421 kJ/kg; h4= 3075.4304 kJ/kg;

s4= 7.7121 kJ/kg.K; e4= 2811.4421 kJ/kg; j4= 3075.4304 kJ/kg;

MM4= 18.015 kg/kmol;

State-5: H2O > Superheated Vapor;

Given: p5= “p1” kPa; T5= “T1-0.01*T1” deg-C; Vel5= 0.0 m/s;

z5= 0.0 m;

Calculated: v5= 0.8705 m^3/kg; u5= 2801.9968 kJ/kg; h5= 3063.1584 kJ/kg;

s5= 7.691 kJ/kg.K; e5= 2801.9968 kJ/kg; j5= 3063.1584 kJ/kg;

MM5= 18.015 kg/kmol;

#--------Property spreadsheet starts:

# State	p(kPa)	T(K)	x	v(m3/kg)	u(kJ/kg)	h(kJ/kg)s	(kJ/kg.K)
# 01	300.0	573.2		0.8753	2806.68	3069.26	7.702
# 02	297.0	573.2		0.8842	2806.73	3069.33	7.707
# 03	303.0	573.2		0.8665	2806.62	3069.18	7.697
# 04	300.0	576.2		0.88	2811.44	3075.43	7.712
# 05	300.0	570.2		0.8705	2802.0	3063.16	7.691

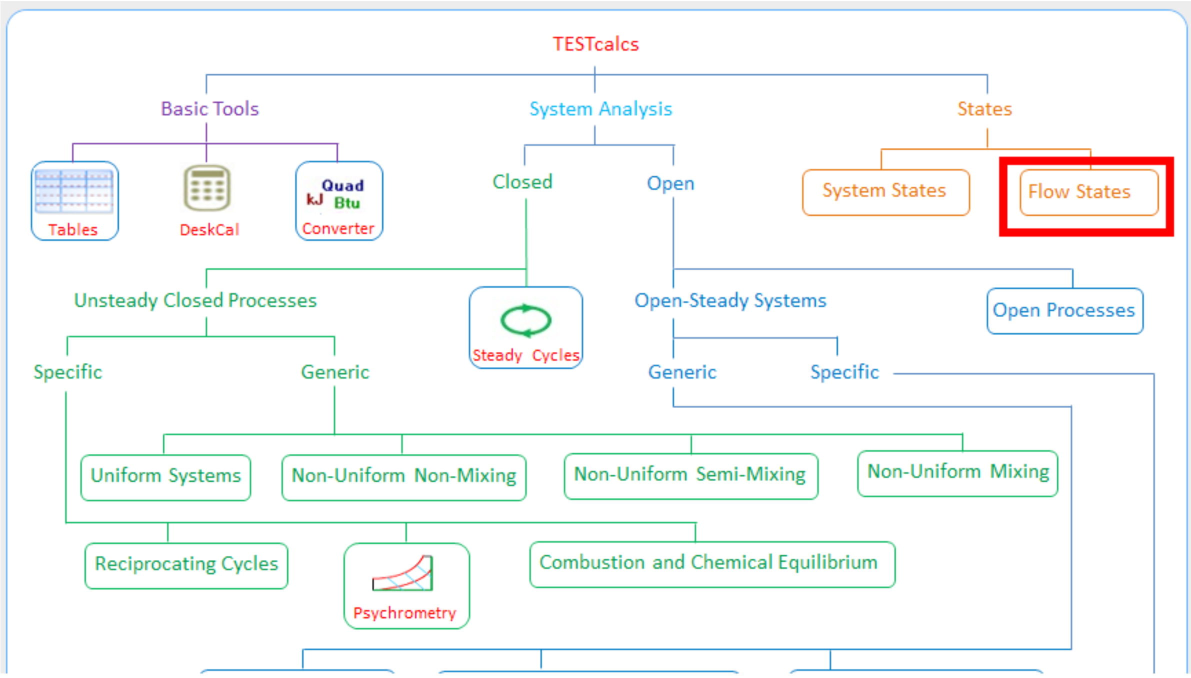
===

Prob.6.4.2 Sat. Refrigerant R22 vapor at 28 C is expanded in an expansion valve. Find out the temp drop and the J-T coeff. for a final pressure of 3 bar. Then plot these quantities as the final pressure varies from 1 bar to 5 bar, other conditions remaining the same.

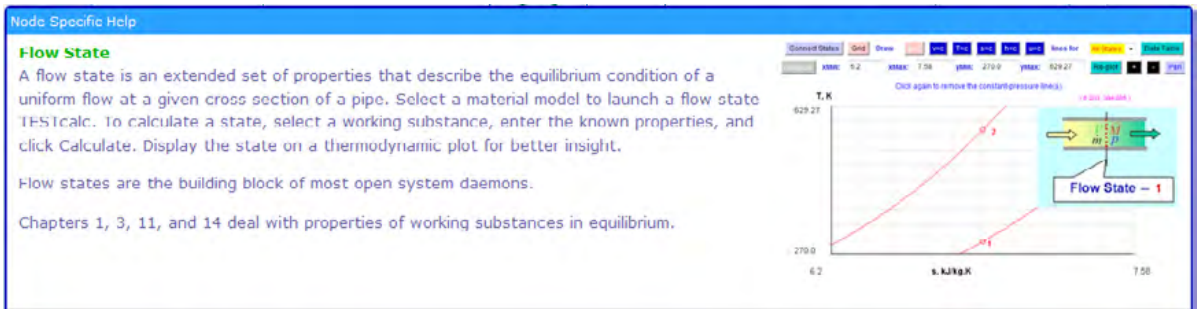
TEST Solution:

Following are the steps:

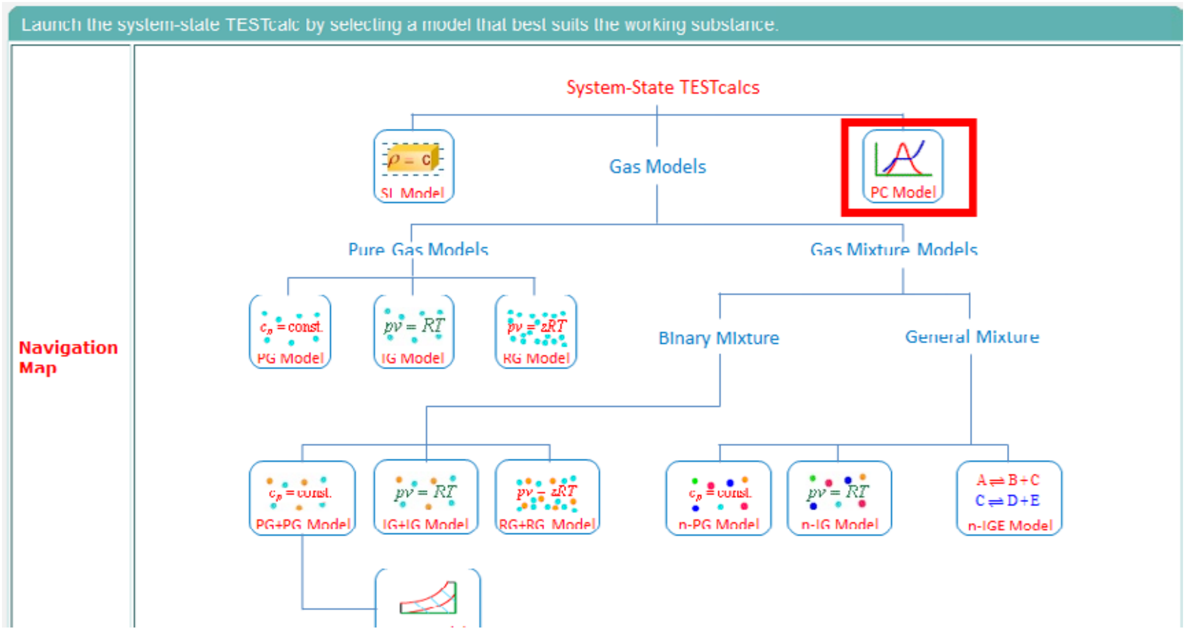
1. From the daemon tree, choose 'Flow States':



Hovering the mouse pointer on 'Flow States' brings up the following explanatory window:



2. Clicking on 'Flow States' takes us to the material model selection:



3. Click on PC Model, since we are dealing with R22. Choose R22 as shown below. Enter for State 1, T1 = 28 C, x1 = 1 for sat. vapor, and hit Enter We get:

4. For State 2: Enter p2 = 300 kPa, h2 = h1 since expansion in a J-T valve is isenthalpic, and hit Enter. We get:

5. For State 3: **State 3 is chosen to get dummy variables**, wherein we can insert the calculated values of temp. drop = ΔT = T1 – T2, and the J-T coeff. μ_{JT} = (T1 – T2) / (p1 – p2). We can choose any variable as the dummy variable. Note that here we have chosen to put Δt under mdot3 and therein we enter (T1 – T2), and under Voldot3, we enter μ_{JT} as: (T1 – T2) / (p1 – p2).

We get for ΔT = T1 – T2:

Hovering the mouse pointer over **mdot3**, we see on the top of window, the result as:

T1 – T2 = 21.297674 C.

Similarly, see below the result for μ_{JT}, under **Voldot3**:

Hovering the mouse pointer over **Voldot3**, we see on the top of window, the result as:

μ_{JT} = (T1 – T2) / (p1 – p2) = 0.025646692 C/kPa.

6. Click on SuperCalculate, and the I/O panel gives the TEST code etc:

```
#~~~~~~~~~~~~~~~~~~~~~~~OUTPUT OF SUPER-CALCULATE

#        Daemon (TESTcalc) Path: States>Flow>PC-Model; v-10.cd03

#--------------------Start of TEST-code-----------------------------------------------------------------------

 States  {

        State-1: R-22;

        Given: { T1= 28.0 deg-C; x1= 1.0 fraction; Vel1= 0.0 m/s; z1= 0.0 m; }

        State-2: R-22;

        Given: { p2= 300.0 kPa; h2= "h1" kJ/kg; Vel2= 0.0 m/s; z2= 0.0 m; }

        State-3: R-22;

        Given: { Vel3= 0.0 m/s; z3= 0.0 m; mdot3= "T1-T2" kg/s; Voldot3= "(T1-T2)/(p1-p2)" m^3/s;
}

        }

#----------------------End of TEST-code----------------------------------------------------------------------

#******DETAILED OUTPUT:

# Evaluated States:

#        State-1: R-22 > Saturated Mixture;

#                Given: T1= 28.0 deg-C; x1= 1.0 fraction; Vel1= 0.0 m/s;

#                        z1= 0.0 m;

#                Calculated: p1= 1130.4258 kPa; y1= 1.0 fraction; v1= 0.0208 m^3/kg;

#                        u1= 235.0604 kJ/kg; h1= 258.624 kJ/kg; s1= 0.8896 kJ/kg.K;
```

```
#                   e1= 235.0604 kJ/kg; j1= 258.624 kJ/kg; MM1= 86.476 kg/kmol;
#       State-2: R-22 > Superheated Vapor;
#               Given: p2= 300.0 kPa; h2= "h1" kJ/kg; Vel2= 0.0 m/s;
#                   z2= 0.0 m;
#               Calculated: T2= 6.7023 deg-C; v2= 0.0846 m^3/kg; u2= 233.2422 kJ/kg;
#                   s2= 1.0031 kJ/kg.K; e2= 233.2422 kJ/kg; j2= 258.624 kJ/kg;
#                   MM2= 86.476 kg/kmol;
#       State-3: R-22 > Unknown Phase;
#               Given: Vel3= 0.0 m/s; z3= 0.0 m; mdot3= "T1-T2" kg/s;
#                   Voldot3= "(T1-T2)/(p1-p2)" m^3/s;
```

Calculated: v3= 0.0012 m^3/kg; A3= 2564.6692 m^2; MM3= 86.476 kg/kmol;

#--------Property spreadsheet starts:

# State	p(kPa)	T(K)	x	v(m3/kg)	u(kJ/kg)	h(kJ/kg)	s(kJ/kg.K)
# 01	1130.43	301.2	1.0	0.0208	235.06	258.62	0.89
# 02	300.0	279.9	0.0846	233.24	258.62	1.003	
# 03							

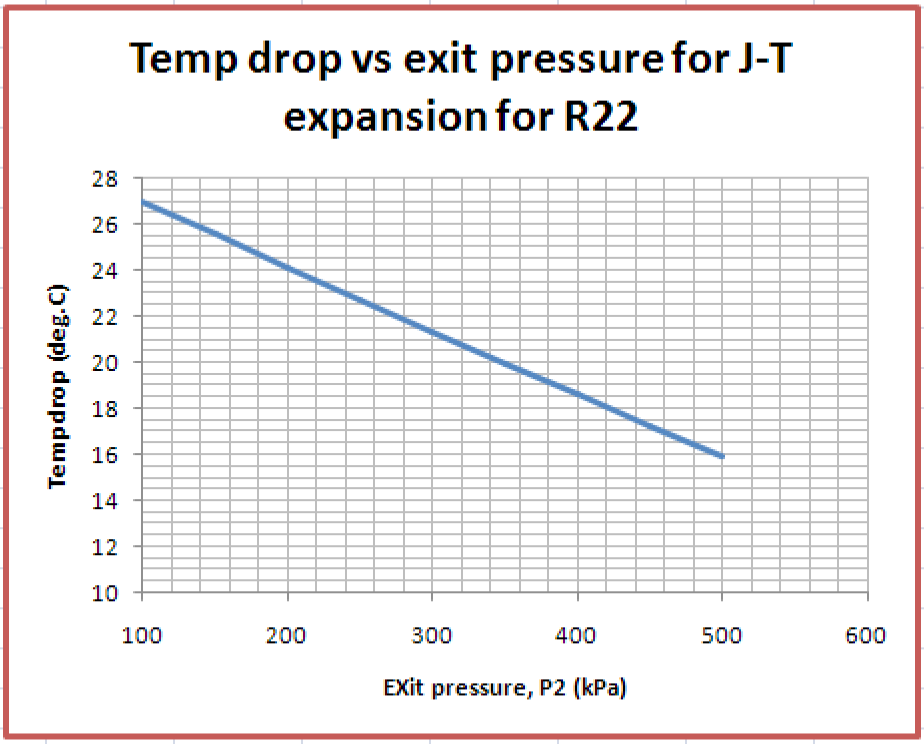
(b) Plot ΔT and μ_{JT} as the final pressure varies from 1 bar to 5 bar, other conditions remaining the same:

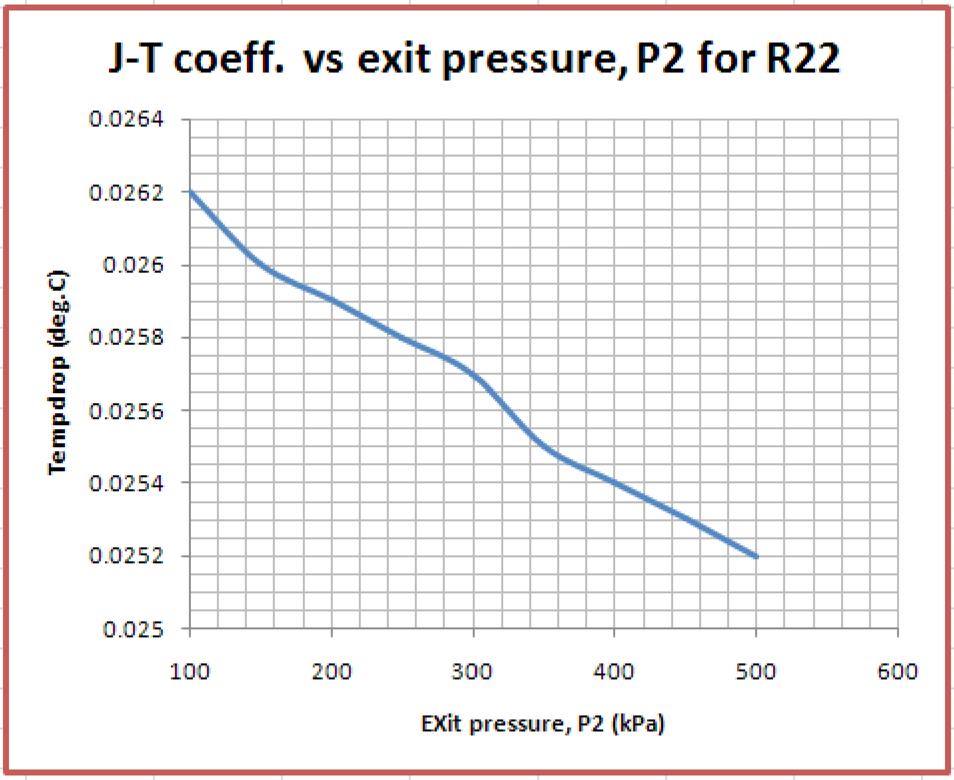
The procedure is quite simple:

1. Go to State 2 and change the pressure p2 to desired value, hit Enter
2. Click on SuperCalculate to update all calculations in other States too
3. Go to State 3 and note the values of ΔT and μ_{JT} and tabulate
4. Now, go to State 2, change the value of P2, hit Enter, and repeat steps 2 and 3
5. Prepare a Table as shown below:

P2 (kPa)	ΔT (deg.C)	μ_{JT} (C/kPa)
100	26.95	0.0262
150	25.52	0.026
200	24.10	0.0259
250	22.7	0.0258
300	21.3	0.0257
350	19.91	0.0255
400	18.56	0.0254
450	17.2	0.0253
500	15.88	0.0252

Now, plot the results in EXCEL:





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6.5 References:

1. *Yunus A. Cengel & Michael A. Boles*, Thermodynamics, An Engineering Approach, 7^{th} Ed. McGraw Hill, 2011.
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