Applied Thermodynamics: Software Solutions

Part-V

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Part-V (Compressible flow: Isentropic flow – Normal shocks – Fanno flow – Rayleigh flow, and Engine trials)

Applied Thermodynamics: Software Solutions: Part-V (Compressible flow: Isentropic flow – Normal shocks – Fanno flow – Rayleigh flow, and Engine trials)
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Contents

9 Compressible flow

Learning objectives:

1. In this chapter, **'Compressible flow'** is dealt with.
2. Formulas for stagnation temp, stagnation pressure, property variations in isentropic flow are compiled first.
3. Then, formulas for property changes during isentropic flow through convergent as well as Convergent-Divergent (C-D) nozzles, normal shocks, Fanno flow (i.e. adiabatic flow with friction) and frictionless flow through ducts with heat transfer (i.e. Rayleigh flow) are enumerated.
4. Property tables for these cases, which should be useful in calculations, are also given.
5. **Many useful Functions are written in Mathcad and EES to calculate the property variations for different cases mentioned above.** ***These Functions make the calculations very easy. Using these Functions all the property variations are tabulated and also the plots are drawn.***
6. A large number of Problems from University question papers as well as from standard Text books are solved to demonstrate the use of the Functions written in Mathcad and EES.
7. Convenience of visual solutions using TEST is demonstrated by solving problems on isentropic flow through nozzles and for normal shocks.
8. **An Appendix on Engine trials is included**, since this topic forms part of the syllabus in some universities.

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9.1 Definitions, Statements and Formulas used [1, 2]:

9.1.1 Stagnation properties:

We have: Enthalpy = Internal energy + flow energy

i.e. $h = u + P \cdot v$ kJ/kg

When P.E. and K.E. are negligible, Enthalpy represents the 'total energy'.

However, for high speed flows, as in the case of nozzles or jet engines, K.E. is not negligible, and then the enthalpy and K.E. are combined in to a single term called **'stagnation (or total) enthalpty'** as follows:

$$h_0 = h + \frac{V^2}{2} \quad \text{kJ/kg ... stagnation enthalpy}$$

where V is the velocity of fluid.

For isentropic flow through a duct such as a nozzle, with no change in P.E., we have:

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

i.e. $h_{01} = h_{02}$

i.e. stagnation enthalpy remains constant.

Stagnation enthalpy represents the enthalpy of a fluid when it is brought to rest adiabatically**.**

For an ideal gas:

$$h_0 = h + \frac{V^2}{2}$$

i.e. $cp \cdot T_0 = cp \cdot T + \frac{V^2}{2}$

i.e. $T_0 = T + \frac{V^2}{2 \cdot cp}$

Stagnation pressure (P_0): It is the pressure a fluid attains when brought to rest isentropically.

For an ideal gas with constant sp. heats, P_0 is related to static pressure by:

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{k}{k-1}}$$

And, ratio of stagnation density to static density is given by:

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{k-1}}$$

So, energy balance for a steady flow device becomes:

$$q_{in} + w_{in} + (h_{01} + g \cdot z_1) = q_{out} + w_{out} + (h_{02} + g \cdot z_2)$$

And, for an ideal gas:

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = cp \cdot (T_{02} - T_{01}) + g \cdot (z_2 - z_1)$$

9.1.2 Speed of sound and Mach Number:

Speed of sound, c is given by:

$$c^2 = \left(\frac{\partial}{\partial\rho}P\right)_s$$

From this, we get:

i.e. $c = \sqrt{k \cdot R \cdot T}$ m/s .... speed of sound, R is gas constant

Thus, for a given ideal gas, speed of sound is a function of temp alone.

Mach Number (Ma): It is the ratio of actual velocity of the fluid to the speed of sound in the same fluid at the same state.

i.e. $Ma = \frac{V}{c}$

Flow is called 'sonic' when Ma = 1, 'subsonic' when Ma < 1, 'supersonic' when Ma > 1, and 'hypersonic' when Ma >> 1.

9.1.3 One dimensional isentropic flow:

Flow through nozzles, diffusers and turbine blade passages can be approximated as one dimensional isentropic flow and low parameters vary in the direction of flow only.

Variation of fluid velocity with flow area: Starting with continuity equation and applying the energy balance, and using the definition of Mach Number, we get:

$$\frac{dA}{A} = \frac{-dV}{V} \cdot \left(1 - Ma^2\right)$$

In the above, since A and V are positive, we conclude:

For subsonic flow (Ma < 1):

$$\frac{dA}{dV} < 0$$

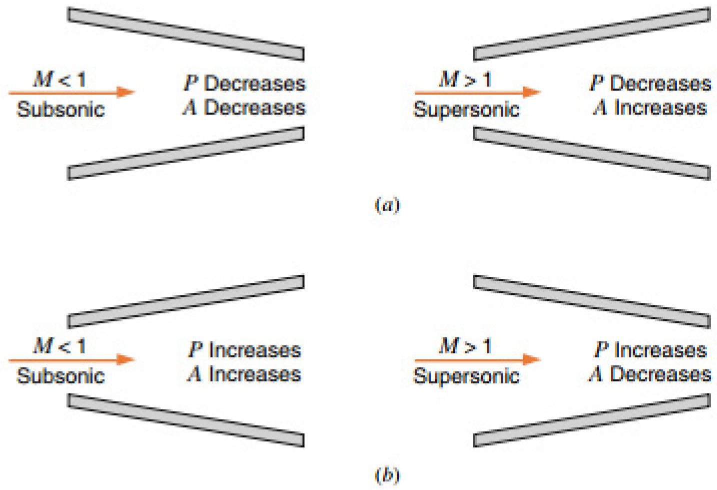
For supersonic flow (Ma > 1):

$$\frac{dA}{dV} > 0$$

For sonic flow (Ma = 1):

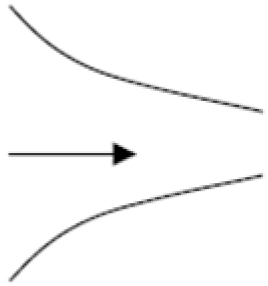
$$\frac{dA}{dV} = 0$$

Fig. below summarizes these observations [2]:

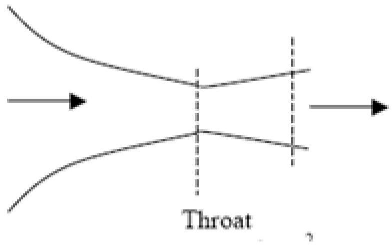


Thus, to accelerate a fluid we should have a convergent nozzle at subsonic velocities and a diverging nozzle at supersonic velocities.

Highest velocity we can achieve with a converging nozzle is sonic velocity, which occurs at the exit:



***To get supersonic velocities*, we should use a convergent – divergent (C-D) nozzle:**



9.1.4 One dimensional isentropic flow of Ideal gases [1]:

Following are the property relations for isentropic flow of ideal gases:

We have the stagnation temp:

$$T_0 = T + \frac{V^2}{2 \cdot cp}$$

Then, ratio of stagnation to static temp is given by:

$$\text{i.e.} \quad \frac{T_0}{T} = 1 + \frac{V^2}{2 \cdot cp \cdot T}$$

Simplifying, we get:

$$\frac{T_0}{T} = 1 + \left(\frac{k-1}{2}\right) \cdot Ma^2$$

Ratio of stagnation to static pressure is given by:

$$\frac{P_0}{P} = \left[1 + \left(\frac{k-1}{2}\right) \cdot Ma^2\right]^{\frac{k}{k-1}}$$

Ratio of stagnation to static density is given by:

$$\frac{\rho_0}{\rho} = \left[1 + \left(\frac{k-1}{2}\right) \cdot Ma^2\right]^{\frac{1}{k-1}}$$

Properties of fluid where Mach No. is 1 (i.e. at the throat) are called **'critical properties'** and are denoted by asterisk (or star). So, setting Ma = 1 in the above relations, we get the **critical ratios**:

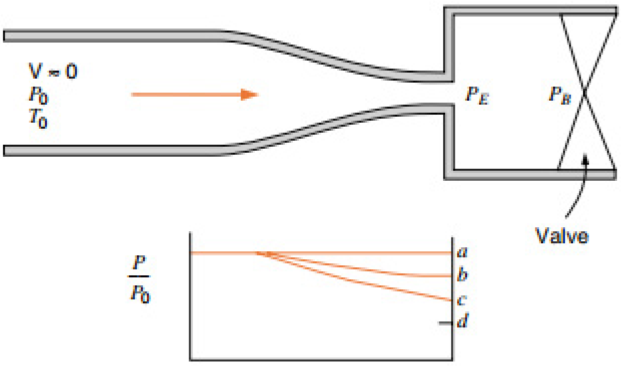
$$\frac{Tstar}{T_0} = \frac{2}{k+1} \qquad \frac{Pstar}{P_0} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \qquad \frac{\rho star}{\rho_0} = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}$$

It is convenient to have following values of critical properties readily available:

	Superheated steam: k = 1.3	Hot products of combustion: k = 1.33	Air: k = 1.4	Mono-atomic gases: k = 1.667
$\frac{Pstar}{P_0}$	0.5457	0.5404	0.5283	0.4871
$\frac{Tstar}{T_0}$	0.8696	0.8584	0.8333	0.7499
$\frac{\rho star}{\rho_0}$	0.6276	0.6295	0.6340	0.6495

9.1.5 Effect of back pressure on exit velocity, mass flow rate and pressure distribution [1]:

For convergent nozzle: See fig. below[2]:



Since inlet velocity is almost zero, stagnation pressure, P_0 and temp T0 are equal to the inlet pressure and temp.

P_B is the back pressure, P_E is the pressure at the exit plane of the nozzle.

In the above fig:

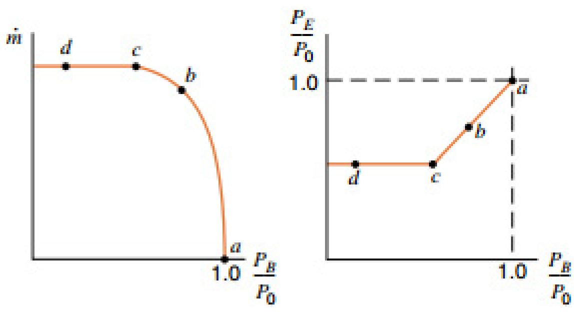
For curve designated by 'a': $P_B = P_0$, therefore, there is no flow.

For curve designated by 'b': $P_B < P_0$, but $P_B > P_{crit}$. Now, $P_E = P_B$ and there is subsonic flow, and at the exit, Mach No. is less than 1.

For curve designated by 'c': $P_B < P_0$, and $P_B = P_{crit}$. Now, $P_E = P_B$ and there is subsonic flow in the nozzle and at the exit, flow is sonic.

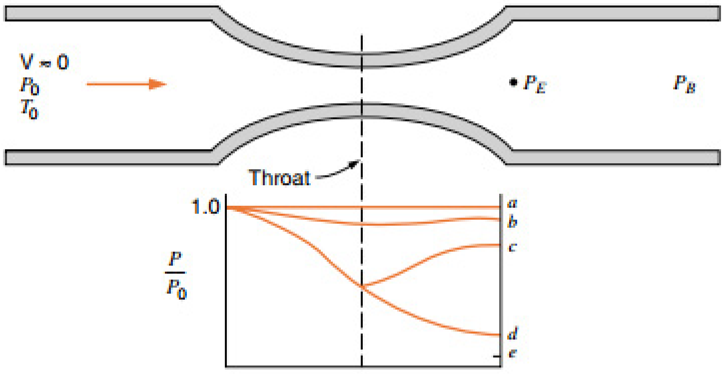
For curve designated by 'd': $P_B < P_{crit}$, Now, P_E remains P_{crit} and there is subsonic flow in the nozzle and at the exit, flow is still sonic only. Pressure falls from PE to PB outside the exit. **Now, the nozzle is said to be 'choked'.**

Mass flow rate and pressure ratios for the above scheme are shown below:



Note that after point c is reached (i.e. after sonic velocity is reached at the exit), mass flow rate remains constant.

For Convergent-Divergent (C-D) nozzle: See fig. below[2]:



In the above fig:

For curve designated by 'a': $P_B = P_0$, therefore, there is no flow.

For curve designated by 'b': $P_B < P_0$, but $P_B > P_{crit}$. Now, $P_E = P_B$ and velocity increases in the convergent section, but Ma < 1 at the throat, and the diverging section acts as a subsonic diffuser, i.e. pressure increases and velocity decreases.

For curve designated by 'c': Now, the P_B is such that Ma = 1 at the throat, but the diverging section still acts as a subsonic diffuser, in which the pressure increases and velocity decreases..

For curve designated by 'd': Now the PB is such that there is isentropic flow throughout and the divergent section acts as a supersonic nozzle, with a decrease in pressure and increase in velocity.

Between the back pressures corresponding to points designated by c and d, isentropic solution is not possible, and **shock waves will be present in the divergent section.**

For P_B less than that for the point designated by d, exit pressure P_E remains constant, and the drop in pressure from P_E to P_B occurs outside the nozzle. This is designated by point e.

9.1.6 Mass flow rate and area ratio [1]:

For steady flow conditions, mass flow rate through the nozzle can be expressed as:

$$m\_dot = \rho \cdot A \cdot V = \left(\frac{P}{R \cdot T}\right) \cdot A \cdot \left(Ma \cdot \sqrt{k \cdot R \cdot T}\right) = P \cdot A \cdot Ma \cdot \sqrt{\frac{k}{R \cdot T}}$$

Now, writing P and T in terms of stagnation pressure and stagnation temp:

$$m\_dot = \frac{A \cdot Ma \cdot P_0 \cdot \sqrt{\dfrac{k}{R \cdot T_0}}}{\left[1 + \dfrac{(k-1) \cdot Ma^2}{2}\right]^{\frac{k+1}{2 \cdot (k-1)}}} \qquad \text{....eqn. A}$$

Eqn. (A) is valid at any cross-section of nozzle along the length of nozzle.

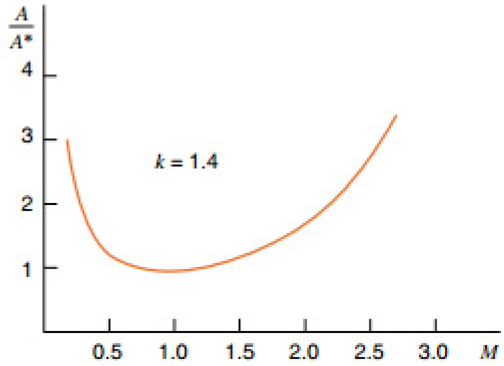
Max. mass flow rate occurs when the Mach No. is equal to 1, and this occurs at the throat. Denoting the area at the throat by Astar, and substituting Ma = 1 in eqn. (A), we get:

$$m\_dot_{max} = Astar \cdot P_0 \cdot \sqrt{\frac{k}{R \cdot T_0}} \cdot \left(\frac{2}{k+1}\right)^{\frac{k+1}{2 \cdot (k-1)}} \qquad \text{....eqn. B}$$

From eqn. (A) by eqn. (B), we get:

$$\frac{A}{Astar} = \frac{1}{Ma}\cdot\left[\frac{2}{k+1}\cdot\left(1+\frac{k-1}{2}\cdot Ma^2\right)\right]^{\frac{k+1}{2\cdot(k-1)}} \quad \text{...eqn. C}$$

Area ratio (A/Astar) is the ratio of the area at the point where Mach No. is Ma to the throat area, and (A/Astar) as a function of Mach No. is plotted below[2]:



Note that for a given (A/Astar) there are two values of Ma, one for the subsonic region and the other for the supersonic region.

Also:

From eqn.(B), for an ideal gas with k = 1.4, we get:

$$m\_dot_{max} = 0.0404\cdot Astar\cdot\frac{P_0}{\sqrt{T_0}} \quad \text{...kg/s.}$$

....when P0 in Pa, T0 in K, Astar in m^2

9.1.7 Impulse Function (F) and A*P ratio[10]:

Both the quantities P.A and r.A.V^2 occur frequently in compressible flow calculations, and both have units of Force. So, they are conveniently expressed together as an important gas dynamic parameter, called **Impulse Function**, or **the wall force function**. It is defined as:

$$F = P \cdot A + \rho \cdot A \cdot V^2$$

i.e. $$F = P \cdot A + k \cdot P \cdot A \cdot M^2$$

i.e. $$F = P \cdot A \cdot \left(1 + k \cdot M^2\right)$$

Now, at M = 1, we have: F = Fstar.

And, the non-dimensional Impulse Function is:

$$\frac{F}{Fstar} = \frac{1 + k \cdot M^2}{M \cdot \sqrt{2 \cdot (1 + k) \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right)}}$$

Second function that occurs frequently in compressible flow calculations is:

$$\frac{A}{Astar} \cdot \frac{P}{P_0}$$

Simplified expression for this function is:

$$\frac{A}{Astar} \cdot \frac{P}{P_0} = \frac{\left(\frac{2}{k+1}\right)^{\frac{k+1}{2 \cdot (k-1)}}}{M \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right)^{0.5}}$$

9.1.8 Table of Isentropic compressible flow functions [2]:

Following Table gives Isentropic flow functions discussed above for different Mach Nos.

TABLE A.12

One-Dimensional Isentropic Compressible-Flow Functions for an Ideal Gas with Constant Specific Heat and Molecular Weight and k = 1.4

M	M^*	A/A^*	P/P_0	ρ/ρ_0	T/T_0
0.0	0.00000	∞	1.00000	1.00000	1.00000
0.1	0.10944	5.82183	0.99303	0.99502	0.99800
0.2	0.21822	2.96352	0.97250	0.98028	0.99206
0.3	0.32572	2.03506	0.93947	0.95638	0.98232
0.4	0.43133	1.59014	0.89561	0.92427	0.96899
0.5	0.53452	1.33984	0.84302	0.88517	0.95238
0.6	0.63481	1.18820	0.78400	0.84045	0.93284
0.7	0.73179	1.09437	0.72093	0.79158	0.91075
0.8	0.82514	1.03823	0.65602	0.73999	0.88652
0.9	0.91460	1.00886	0.59126	0.68704	0.86059
1.0	1.0000	1.00000	0.52828	0.63394	0.83333
1.1	1.0812	1.00793	0.46835	0.58170	0.80515
1.2	1.1583	1.03044	0.41238	0.53114	0.77640
1.3	1.2311	1.06630	0.36091	0.48290	0.74738
1.4	1.2999	1.11493	0.31424	0.43742	0.71839
1.5	1.3646	1.17617	0.27240	0.39498	0.68966
1.6	1.4254	1.25023	0.23527	0.35573	0.66138
1.7	1.4825	1.33761	0.20259	0.31969	0.63371
1.8	1.5360	1.43898	0.17404	0.28682	0.60680
1.9	1.5861	1.55526	0.14924	0.25699	0.58072
2.0	1.6330	1.68750	0.12780	0.23005	0.55556
2.1	1.6769	1.83694	0.10935	0.20580	0.53135
2.2	1.7179	2.00497	0.93522E-01	0.18405	0.50813
2.3	1.7563	2.19313	0.79973E-01	0.16458	0.48591
2.4	1.7922	2.40310	0.68399E-01	0.14720	0.46468
2.5	1.8257	2.63672	0.58528E-01	0.13169	0.44444
2.6	1.8571	2.89598	0.50115E-01	0.11787	0.42517
2.7	1.8865	3.18301	0.42950E-01	0.10557	0.40683
2.8	1.9140	3.50012	0.36848E-01	0.94626E-01	0.38941
2.9	1.9398	3.84977	0.31651E-01	0.84889E-01	0.37286
3.0	1.9640	4.23457	0.27224E-01	0.76226E-01	0.35714
3.5	2.0642	6.78962	0.13111E-01	0.45233E-01	0.28986
4.0	2.1381	10.7188	0.65861E-02	0.27662E-01	0.23810
4.5	2.1936	16.5622	0.34553E-02	0.17449E-01	0.19802
5.0	2.2361	25.0000	0.18900E-02	0.11340E-01	0.16667
6.0	2.2953	53.1798	0.63336E-03	0.51936E-02	0.12195
7.0	2.3333	104.143	0.24156E-03	0.26088E-02	0.09259
8.0	2.3591	190.109	0.10243E-03	0.14135E-02	0.07246
9.0	2.3772	327.189	0.47386E-04	0.81504E-03	0.05814
10.0	2.3905	535.938	0.23563E-04	0.49482E-03	0.04762
∞	2.4495	∞	0.0	0.0	0.0

Note: See Prob. 9.3.1 for Mathcad Functions, Tables and plots of Isentropic flow functions.

9.1.9 Normal shocks [2]:

Normal shock occurs in a plane normal to the flow direction. Also, shock occurs only in the divergent portion of the C-D nozzle, since before the shock, the velocity should be supersonic (i.e. Ma > 1). Flow through the shock is highly irreversible, and therefore, can not be approximated as isentropic. Property changes across the shock are of special interest, and are, therefore, tabulated for easy reference.

Denoting the properties upstream of the shock by subscript x and properties downstream of the shock by y, we have:

Conservation of mass:

$$\rho_x \cdot A \cdot V_x = \rho_y \cdot A \cdot V_y$$

i.e. $\rho_x \cdot V_x = \rho \cdot V_y$

Conservation of energy:

$$h_x + \frac{V_x^{\ 2}}{2} = h_y + \frac{V_y^{\ 2}}{2}$$

i.e. $h_{0x} = h_{0y}$

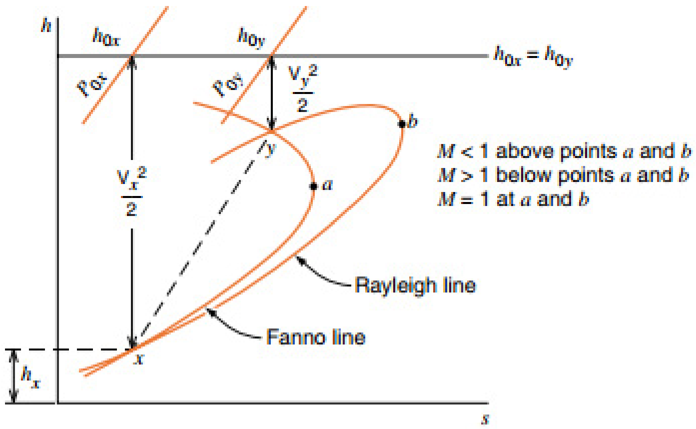
Conservation of momentum:

$$A \cdot (P_x - P_y) = m\_dot \cdot (V_y - V_x)$$

Increase of entropy:

$$(s_y - s_x) \geq 0$$

Combining the conservation of mass and energy equations and plotting on h-s diagram, gives the **Fanno line**. And, combining the conservation of mass and momentum equations and plotting on h-s diagram, gives the **Rayleigh line**. Max. entropy points in these lines (i.e. points a and b in the following fig.) correspond to Ma = 1. Upper part of each curve represents subsonic states and the lower part, supersonic.



In the above fig., at the two points of intersection of Fanno and Rayleigh lines, i.e. at x and y, all the three equations are satisfied; x is in the supersonic region and y is in the subsonic region.

Since $(s_y - s_x) > 0$, normal shock proceeds from x to y. Thus, velocity changes from supersonic before the shock to subsonic after the shock.

9.1.10 Equations governing normal shocks [2]:

From the conservation of energy principle, stagnation enthalpy remains constant across the shock,

i.e. $T_{0x} = T_{0y}$.

We have:

$$\frac{T_{0x}}{T_x} = 1 + \left(\frac{k-1}{2}\right) \cdot Ma_x^{\ 2} \qquad \text{and,}$$

$$\frac{T_{0y}}{T_y} = 1 + \left(\frac{k-1}{2}\right) \cdot Ma_y^{\ 2}$$

Therefore, dividing these two equations, we get:

$$\frac{T_y}{T_x} = \frac{1 + \left(\frac{k-1}{2}\right) \cdot Ma_x^{\ 2}}{1 + \left(\frac{k-1}{2}\right) \cdot Ma_y^{\ 2}} \qquad \text{....eqn.(a)}$$

From continuity equation:

$$\rho_x \cdot V_x = \rho_y \cdot V_y$$

In the above, using the equation of state and definition of Mach No., and eqn for sonic velocity c, we get:

$$\frac{T_y}{T_x} = \left(\frac{P_y}{P_x}\right)^2 \cdot \left(\frac{M_y}{M_x}\right)^2 \quad \text{....eqn.(b)}$$

Combining eqns (a) and (b), i.e. combining the energy and continuity eqns, we get the **eqn for the Fanno line:**

$$\frac{P_y}{P_x} = \frac{M_x \cdot \sqrt{1 + \left(\frac{k-1}{2}\right) \cdot Ma_x^2}}{M_y \cdot \sqrt{1 + \left(\frac{k-1}{2}\right) \cdot Ma_y^2}} \quad \text{....eqn.(c)}$$

Similarly, combining the momentum and continuity equations, we get the **eqn for the Rayleigh line:**

$$\frac{P_y}{P_x} = \frac{1 + k \cdot M_x^2}{1 + k \cdot M_y^2} \quad \text{....eqn.(d)}$$

Combining equations (c) and (d), we get the following **eqn relating M_x and M_y**:

$$M_y^2 = \frac{M_x^2 + \frac{2}{k-1}}{\left(\frac{2 \cdot k}{k-1}\right) \cdot M_x^2 - 1} \quad \text{....eqn.(e)}$$

Properties across a normal shock change as follows [1]:

After the shock, we have:

P → increases

P0 → decreases

V → decreases

M → decreases

T → increases

T0 → remains const.

ρ → increases

s → increases

Ratio of stagnation pressures across a shock (P0y/P0x) is often useful [3]:

$$\frac{P_{0y}}{P_{0x}} = \frac{M_x}{M_y} \cdot \left(\frac{1 + \frac{k-1}{2} \cdot M_y^{2}}{1 + \frac{k-1}{2} \cdot M_x^{2}} \right)^{\frac{k+1}{2 \cdot (k-1)}} \qquad \text{....eqn.(f)}$$

Since there is no area change across a shock, we get from eqn C (for A/Astar) and eqn. (f) above:

$$\frac{A_{xstar}}{A_{ystar}} = \frac{P_{0y}}{P_{0x}} \qquad \text{....eqn.(g)}$$

9.1.11 Table below gives the Normal shock functions for an ideal gas with k = 1.4: [Ref:2]

TABLE A.13

One-Dimensional Normal Shock Functions for an Ideal Gas with Constant Specific Heat and Molecular Weight and k = 1.4

M_x	M_y	P_y/P_x	ρ_y/ρ_x	T_y/T_x	P_{0y}/P_{0x}	P_{0y}/P_x
1.00	1.00000	1.0000	1.0000	1.0000	1.00000	1.8929
1.05	0.95313	1.1196	1.0840	1.0328	0.99985	2.0083
1.10	0.91177	1.2450	1.1691	1.0649	0.99893	2.1328
1.15	0.87502	1.3763	1.2550	1.0966	0.99669	2.2661
1.20	0.84217	1.5133	1.3416	1.1280	0.99280	2.4075
1.25	0.81264	1.6563	1.4286	1.1594	0.98706	2.5568
1.30	0.78596	1.8050	1.5157	1.1909	0.97937	2.7136
1.35	0.76175	1.9596	1.6028	1.2226	0.96974	2.8778
1.40	0.73971	2.1200	1.6897	1.2547	0.95819	3.0492
1.45	0.71956	2.2863	1.7761	1.2872	0.94484	3.2278
1.50	0.70109	2.4583	1.8621	1.3202	0.92979	3.4133
1.55	0.68410	2.6362	1.9473	1.3538	0.91319	3.6057
1.60	0.66844	2.8200	2.0317	1.3880	0.89520	3.8050
1.65	0.65396	3.0096	2.1152	1.4228	0.87599	4.0110
1.70	0.64054	3.2050	2.1977	1.4583	0.85572	4.2238
1.75	0.62809	3.4063	2.2791	1.4946	0.83457	4.4433
1.80	0.61650	3.6133	2.3592	1.5316	0.81268	4.6695
1.85	0.60570	3.8263	2.4381	1.5693	0.79023	4.9023
1.90	0.59562	4.0450	2.5157	1.6079	0.76736	5.1418
1.95	0.58618	4.2696	2.5919	1.6473	0.74420	5.3878
2.00	0.57735	4.5000	2.6667	1.6875	0.72087	5.6404
2.05	0.56906	4.7362	2.7400	1.7285	0.69751	5.8996
2.10	0.56128	4.9783	2.8119	1.7705	0.67420	6.1654
2.15	0.55395	5.2263	2.8823	1.8132	0.65105	6.4377
2.20	0.54706	5.4800	2.9512	1.8569	0.62814	6.7165
2.25	0.54055	5.7396	3.0186	1.9014	0.60553	7.0018
2.30	0.53441	6.0050	3.0845	1.9468	0.58329	7.2937
2.35	0.52861	6.2762	3.1490	1.9931	0.56148	7.5920
2.40	0.52312	6.5533	3.2119	2.0403	0.54014	7.8969
2.45	0.51792	6.8363	3.2733	2.0885	0.51931	8.2083
2.50	0.51299	7.1250	3.3333	2.1375	0.49901	8.5261
2.55	0.50831	7.4196	3.3919	2.1875	0.47928	8.8505
2.60	0.50387	7.7200	3.4490	2.2383	0.46012	9.1813
2.70	0.49563	8.3383	3.5590	2.3429	0.42359	9.8624
2.80	0.48817	8.9800	3.6636	2.4512	0.38946	10.569
2.90	0.48138	9.6450	3.7629	2.5632	0.35773	11.302
3.00	0.47519	10.333	3.8571	2.6790	0.32834	12.061
4.00	0.43496	18.500	4.5714	4.0469	0.13876	21.068
5.00	0.41523	29.000	5.0000	5.8000	0.06172	32.653
10.00	0.38758	116.50	5.7143	20.387	0.00304	129.22

Note: See Prob. 9.3.2 for Mathcad Functions, Tables and plots of Normal shock functions.

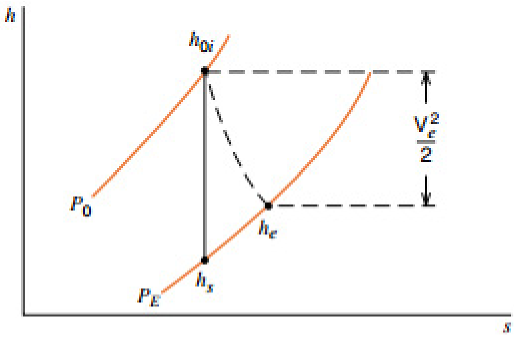
9.1.12 Nozzle and diffuser coefficients [2]:

Nozzle efficiency (η_N): is defined as:

$$\eta_N = \frac{Actual\_KE\_at\_nozzle\_exit}{KE\_at\_exit\_for\_isentropic\_flow\_to\_same\_exit\_pressure}$$

i.e. $$\eta_N = \frac{h_{0i} - h_e}{h_{0i} - h_s}$$

See the following fig:



Velocity coeff. of Nozzle (C_V):

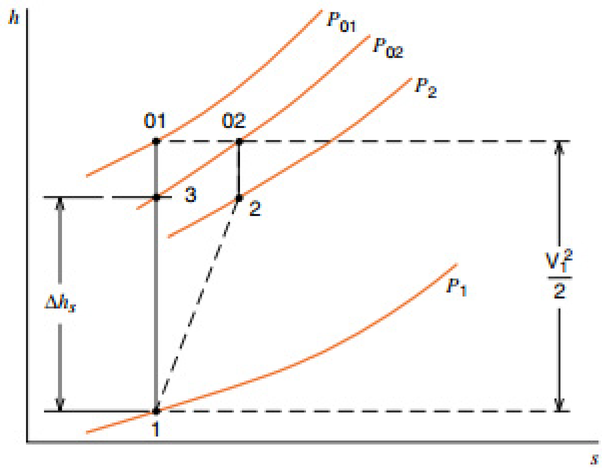
$$C_V = \frac{Actual\_velocty\_at\_nozzle\_exit}{Velocity\_at\_exit\_for\_isentropic\_flow\_to\_same\_exit\_pressure}$$

And: $$C_V = \sqrt{\eta_N}$$

Coeff. of discharge for a Nozzle (C_D):

$$C_D = \frac{Actual\_mass\_rate\_of\_flow}{Mass\_rate\_of\_flow\_with\_isentropic\_flow}$$

Diffuser efficiency (η_D): is defined with respect to the following fig:



In the above fig: 1 and 01 are the actual and stagnation states of fluid entering the diffuser, and 2 and 02 are the actual and stagnation states of fluid leaving the diffuser. Then, diffuser efficiency is defined as:

$$\eta_D = \frac{\Delta h_s}{\frac{V_1^{\,2}}{2}} = \frac{h_3 - h_1}{h_{01} - h_1} = \frac{h_3 - h_1}{h_{02} - h_1}$$

After some manipulation, we get:

$$\eta_D = \frac{\left(1 + \frac{k-1}{2}\cdot M_1^2\right)\cdot\left(\frac{P_{02}}{P_{01}}\right)^{\frac{k-1}{k}} - 1}{\left(\frac{k-1}{2}\right)\cdot M_1^2}$$

9.1.13 Flow in constant area ducts with Friction – Fanno flow[10]:

Flow in a constant area duct with friction in the absence of work and heat transfer is known as Fanno flow. Flow in gas ducts of aircraft engines, air conditioning systems etc are examples of Fanno flow.

Fanno Flow is specified by: (i) continuity eqn. (ii) Energy eqn. and (iii) const. area, no work and no heat transfer. Also, all sonic properties are constant… p*, rho*, V*, A* etc. Stagnation properties at the sonic state are also const.

Fanno flow is represented as **Fanno line** in a h-s diagram as explained earlier.

Variation of flow properties in Fanno flow are given by following equations:

Velocity:

$$\frac{V}{Vstar} = M\cdot\sqrt{\frac{k+1}{2+(k-1)\cdot M^2}}$$

Density:

$$\frac{\rho}{\rho star} = \frac{1}{M}\cdot\left[\frac{2\cdot\left(1+\frac{k-1}{2}\cdot M^2\right)}{k+1}\right]^{0.5}$$

Pressure:

$$\frac{P}{Pstar} = \frac{1}{M}\cdot\sqrt{\frac{k+1}{2\cdot\left(1+\frac{k-1}{2}\cdot M^2\right)}}$$

Temperature:

$$\frac{T}{Tstar} = \frac{k+1}{2\cdot\left(1+\frac{k-1}{2}\cdot M^2\right)}$$

Also:

$$\frac{T}{Tstar} = \frac{P}{Pstar} \cdot \frac{V}{Vstar}$$

Stagnation pressure:

$$\frac{P_0}{P_{0star}} = \frac{1}{M} \cdot \left[\frac{2 \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right)}{k+1} \right]^{\frac{k+1}{2 \cdot (k-1)}}$$

Impulse Function:

$$\frac{F}{Fstar} = \frac{1 + k \cdot M^2}{M \cdot \left[2 \cdot (k+1) \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right) \right]^{0.5}}$$

Change of entropy:

$$\frac{s - s_{star}}{R} = -\ln\left(\frac{P_0}{P_{0star}}\right)$$

Also: $$\frac{s_2 - s_1}{R} = \ln\left(\frac{P_{01}}{P_{02}}\right)$$

i.e. $$\frac{s_2 - s_1}{R} = \ln\left[\frac{M_2}{M1} \cdot \left[\frac{1 + \left(\frac{k-1}{2}\right) \cdot {M_1}^2}{1 + \left(\frac{k-1}{2}\right) \cdot {M_2}^2} \right]^{\frac{k+1}{2 \cdot (k-1)}} \right]$$

Variation of Mach No. with duct length:

$$\frac{4 \cdot f \cdot L_{max}}{D} = \frac{1 - M^2}{k \cdot M^2} + \frac{k+1}{2 \cdot k} \cdot \ln\left[\frac{(k+1) \cdot M^2}{2 \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right)} \right]$$

9.1.14 Table below gives the Fanno flow functions for an ideal gas with k = 1.4:

Note: See Prob.9.3.4 for Mathcad Functions, Tables and plots of Fanno flow parameters.

For M < 1:

M	P/Pstar	V/Vstar	T/Tstar	P0/P0star	F/Fstar	4.f.Lmax/D
0.05	21.903	0.055	1.199	11.591	9.158	280.02
0.1	10.944	0.109	1.198	5.822	4.624	66.922
0.15	7.287	0.164	1.195	3.91	3.132	27.932
0.2	5.455	0.218	1.19	2.964	2.4	14.533
0.25	4.355	0.272	1.185	2.403	1.973	8.483
0.3	3.619	0.326	1.179	2.035	1.698	5.299
0.35	3.092	0.379	1.171	1.778	1.509	3.452
0.4	2.696	0.431	1.163	1.59	1.375	2.308
0.45	2.386	0.483	1.153	1.449	1.276	1.566
0.5	2.138	0.535	1.143	1.34	1.203	1.069
0.55	1.934	0.585	1.132	1.255	1.147	0.728
0.6	1.763	0.635	1.119	1.188	1.105	0.491
0.65	1.618	0.684	1.107	1.136	1.073	0.325
0.7	1.493	0.732	1.093	1.094	1.049	0.208
0.75	1.385	0.779	1.079	1.062	1.031	0.127
0.8	1.289	0.825	1.064	1.038	1.019	0.072
0.85	1.205	0.87	1.048	1.021	1.01	0.036
0.9	1.129	0.915	1.033	1.009	1.004	0.015
0.95	1.061	0.958	1.017	1.002	1.001	$3.278 \cdot 10^{-3}$
1	1	1	1	1	1	0

For M > 1:

M	P/Pstar	V/Vstar	T/Tstar	P0/P0star	F/Fstar	4.f.Lmax/D
1	1	1	1	1	1	0
1.2	0.804	1.158	0.932	1.03	1.011	0.034
1.4	0.663	1.3	0.862	1.115	1.035	0.1
1.6	0.557	1.425	0.794	1.25	1.063	0.172
1.8	0.474	1.536	0.728	1.439	1.094	0.242
2	0.408	1.633	0.667	1.687	1.123	0.305
2.2	0.355	1.718	0.61	2.005	1.15	0.361
2.4	0.311	1.792	0.558	2.403	1.175	0.41
2.6	0.275	1.857	0.51	2.896	1.198	0.453
2.8	0.244	1.914	0.467	3.5	1.218	0.49
3	0.218	1.964	0.429	4.235	1.237	0.522
3.2	0.196	2.008	0.394	5.121	1.253	0.55
3.4	0.177	2.047	0.362	6.184	1.268	0.575
3.6	0.161	2.081	0.334	7.45	1.281	0.597
3.8	0.146	2.111	0.309	8.951	1.292	0.616
4	0.134	2.138	0.286	10.719	1.303	0.633
4.2	0.123	2.162	0.265	12.792	1.312	0.648
4.4	0.113	2.184	0.246	15.21	1.321	0.661
4.6	0.104	2.203	0.229	18.018	1.328	0.673
4.8	0.096	2.22	0.214	21.264	1.335	0.684
5	0.089	2.236	0.2	25	1.342	0.694

Changes in M, V, P, T, rho and s, with increasing distance are summarized below, for Fanno flow:

	dM	dV	dP	dT	dρ	ds	dP_0	dρ_0
M < 1	+	+	-	-	-	+	-	-
M > 1	-	-	+	+	+	+	-	-

Note: + means: increase; – means: decrease.

Note: From the above Table, observe that ds always increases, and dP_0 and dρ_0 always decrease.

9.1.15 Flow with heat transfer and negligible friction – Rayleigh flow [1]:

Considering the one dimensional flow in a constant area duct, with heat transfer and no friction:

Mass equation:

$$\rho_1 \cdot V_1 = \rho_2 \cdot V_2 \quad \text{for constant area of duct ...eqn.(a)}$$

x-Momentum equation:

$$P_1 \cdot A - P_2 \cdot A = m\_dot \cdot V_2 - m\_dot \cdot V_1$$

i.e. $$(P_1 - P_2) = (\rho_2 \cdot V_2) \cdot V_2 - (\rho_1 \cdot V_1) \cdot V_1$$

i.e. $$P_1 + \rho_1 \cdot V_1^2 = P_2 + P_2 \cdot V_2^2 \quad \text{....eqn.(b)}$$

Energy equation:

$$q + h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \quad \text{....eqn.(c)}$$

For an ideal gas with const. sp. heats:

$$q = cp \cdot (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} \quad \text{....eqn.(d)}$$

or: $$q = h_{02} - h_{01} = cp \cdot (T_{02} - T_{01}) \quad \text{....eqn.(e)}$$

Entropy change: No friction - therefore, entropy changes by heat transfer only.

$$s_2 - s_1 = cp \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{P_2}{P_1}\right) \quad \text{....eqn.(f)}$$

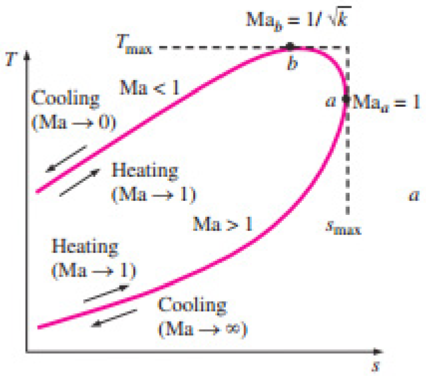
Equation of State:

$$P = \rho \cdot R \cdot T$$

or: $$\frac{P_1}{\rho_1 \cdot T_1} = \frac{P_2}{\rho_2 \cdot T_2} \quad \text{....eqn.(g)}$$

For a given gas, with specified inlet state 1, exit properties P2, T2, r2, V2 and s2 can be calculated from the above equations for a specified heat transfer q.

Plot of possible exit states 2 for specified inlet state 1, on a T-s diagram, is shown below[1]. The resulting line is known as **Rayleigh line:**



9.1.16 Property functions for Rayleigh flow [1]:

Property relations for Rayleigh flow are summarized below:

$$\frac{P_2}{P_1} = \frac{1 + k \cdot Ma_1^2}{1 + k \cdot Ma_2^2}$$

$$\frac{T_2}{T_1} = \left[\frac{Ma_2 \cdot \left(1 + k \cdot Ma_1^2\right)}{Ma_1 \cdot \left(1 + k \cdot Ma_2^2\right)}\right]^2$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{Ma_1^2 \cdot \left(1 + k \cdot Ma_2^2\right)}{Ma_2^2 \cdot \left(1 + k \cdot Ma_1^2\right)}$$

Denoting sonic condition (at exit, state 2, i.e. $Ma_2 = 1$) by star:

$$\frac{P}{Pstar} = \frac{1+k}{1+k\cdot Ma^2}$$

$$\frac{T}{Tstar} = \left[\frac{Ma\cdot(1+k)}{1+k\cdot Ma^2}\right]^2$$

$$\frac{V}{Vstar} = \frac{\rho star}{\rho} = \frac{(1+k)\cdot Ma^2}{1+k\cdot Ma^2}$$

Dimensionless stagnation temp:

$$\frac{T_0}{T_{0star}} = \frac{(k+1)\cdot Ma^2\cdot\left[2+(k-1)\cdot Ma^2\right]}{\left(1+k\cdot Ma^2\right)^2}$$

Dimensionless stagnation pressure:

$$\frac{P_0}{P_{0star}} = \frac{k+1}{1+k\cdot Ma^2}\cdot\left[\frac{2+(k-1)\cdot Ma^2}{k+1}\right]^{\frac{k}{k-1}}$$

9.1.17 Rayleigh flow functions for an ideal gas with k = 1.4 are tabulated below

Note: See Prob. 9.3.6 for Mathcad Functions, Tables and plots of Rayleigh flow parameters.

M	T0/T0star	P0/P0star	T/Tstar	P/Pstar	V/Vstar
0	0	1.268	0	2.4	0
0.1	0.047	1.259	0.056	2.367	0.024
0.2	0.174	1.235	0.207	2.273	0.091
0.3	0.347	1.199	0.409	2.131	0.192
0.4	0.529	1.157	0.615	1.961	0.314
0.5	0.691	1.114	0.79	1.778	0.444
0.6	0.819	1.075	0.917	1.596	0.574
0.7	0.908	1.043	0.993	1.423	0.698
0.8	0.964	1.019	1.025	1.266	0.81
0.9	0.992	1.005	1.025	1.125	0.911
1	1	1	1	1	1
1.1	0.994	1.005	0.96	0.891	1.078
1.2	0.979	1.019	0.912	0.796	1.146
1.3	0.958	1.044	0.859	0.713	1.205
1.4	0.934	1.078	0.805	0.641	1.256
1.5	0.909	1.122	0.753	0.578	1.301

M	T0/T0star	P0/P0star	T/Tstar	P/Pstar	V/Vstar
1.6	0.884	1.176	0.702	0.524	1.34
1.7	0.86	1.24	0.654	0.476	1.375
1.8	0.836	1.316	0.609	0.434	1.405
1.9	0.814	1.403	0.567	0.396	1.431
2	0.793	1.503	0.529	0.364	1.455
2.1	0.774	1.616	0.494	0.335	1.475
2.2	0.756	1.743	0.461	0.309	1.494
2.3	0.74	1.886	0.431	0.286	1.51
2.4	0.724	2.045	0.404	0.265	1.525
2.5	0.71	2.222	0.379	0.246	1.538
2.6	0.697	2.418	0.356	0.229	1.55
2.7	0.685	2.634	0.334	0.214	1.561
2.8	0.674	2.873	0.315	0.2	1.571
2.9	0.664	3.136	0.297	0.188	1.58
3	0.654	3.424	0.28	0.176	1.588

Note: There is a limit on the heat addition in this flow process. Max. possible heat transfer occurs when the end state corresponds to (T0 / T0_star) = 1.

For inlet Mach No. M, max. possible heat transfer is given by:

$$Q_{max} = Qstar = \frac{\left(1 - M^2\right)^2}{2 \cdot (1 + k) \cdot M^2} \cdot cp \cdot T1$$

At M = 0: $Q_{max} = \infty$

Alternate expression for heat transfer:

$$Q = cp \cdot \left(T_{02} - T_{01}\right)$$

i.e. $$\frac{Q}{cp \cdot T_{0star}} = \frac{T_{02}}{T_{0star}} - \frac{T_{01}}{T_{0star}}$$

Above expression can be used to find heat transfer required to change the Mach No. from M1 to M2, since (T_{02}/T_{0star}) and (T_{01}/T_{0star}) are functions of M2 and M1.

9.2 Two *free* software to calculate compressible flow functions [8, 9]:

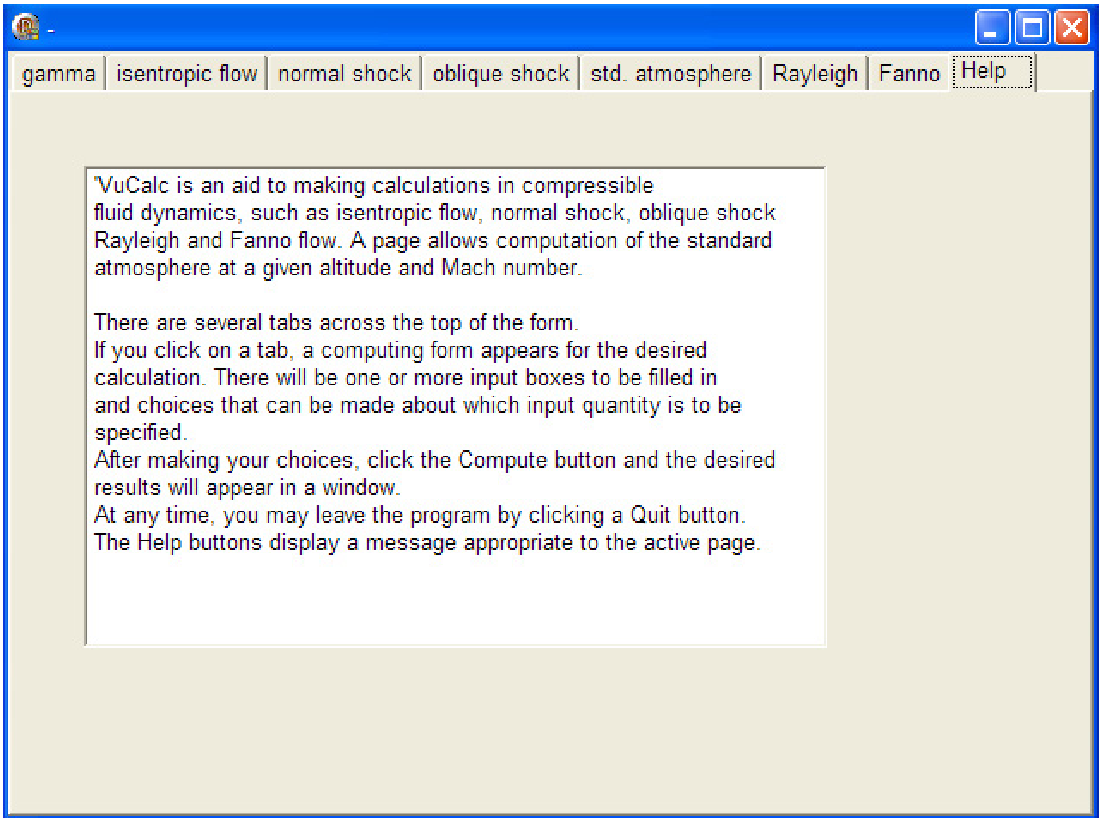
There are many calculators on-line to calculate compressible flow functions described in earlier sections.

Here, we mention two very useful calculators: (i) VUCALC, a window based calculator, and (ii) a browser based compressible aerodynamic calculator.

9.2.1 VUCALC [8]: VuCalc is based on a program of the same name written by Tom Benson of NASA

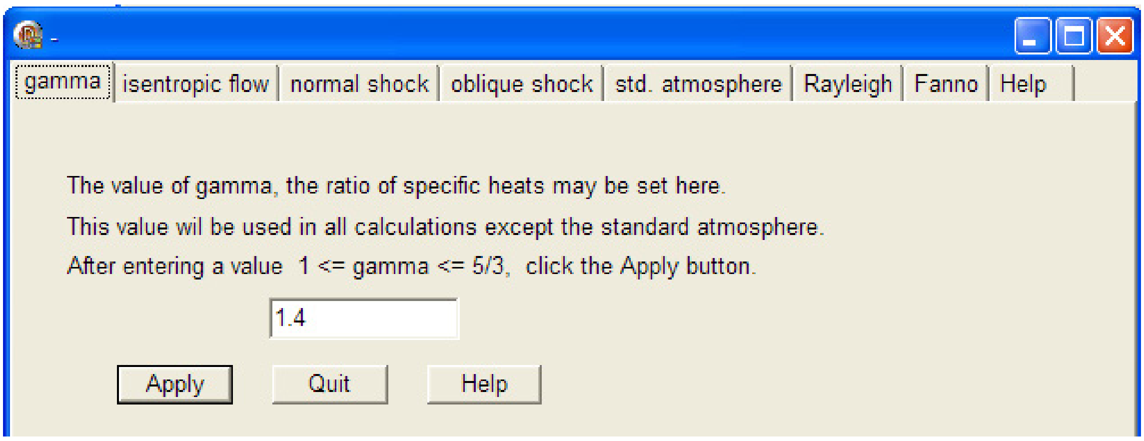
This is a stand-alone, window based calculator. It does not need installation, i.e. it works from the folder in which it is located.

Opening screen starts with the **'Help'** tab, and looks as follows:

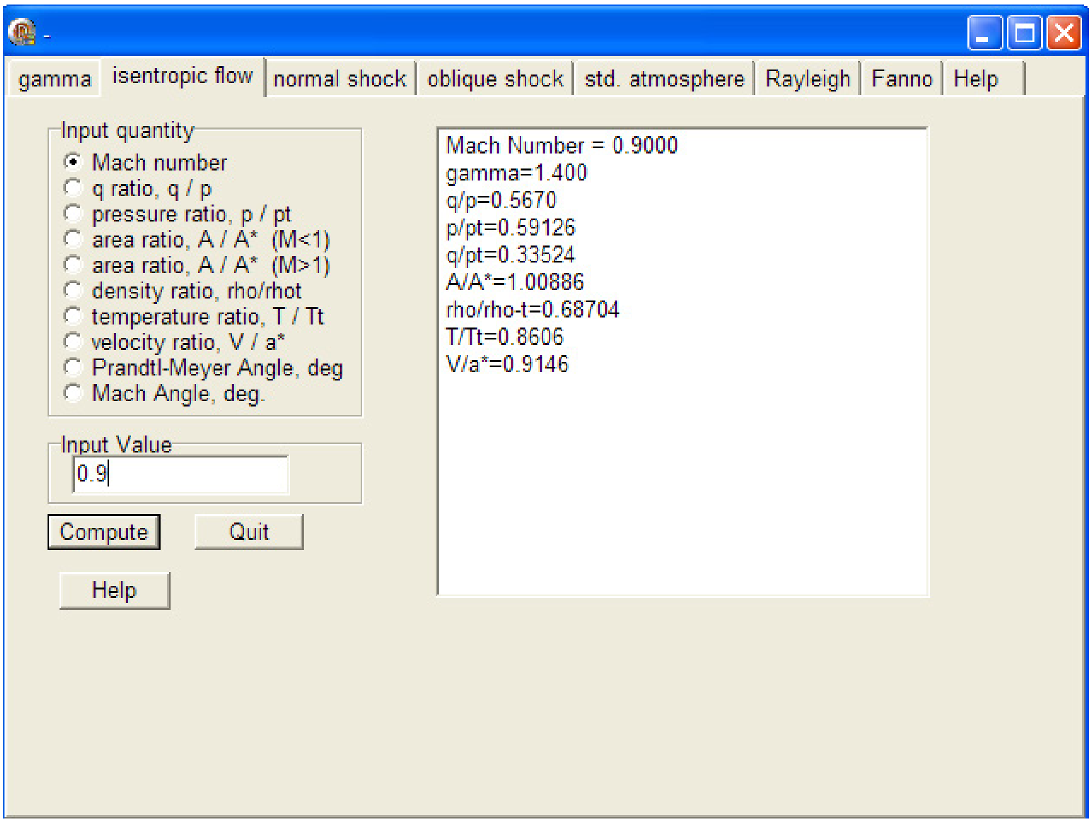


Note that there are 8 tabs on the top: gamma, isentropic flow, normal shock, oblique shock, std. atmosphere, Rayleigh (flow), Fanno (flow) and Help.

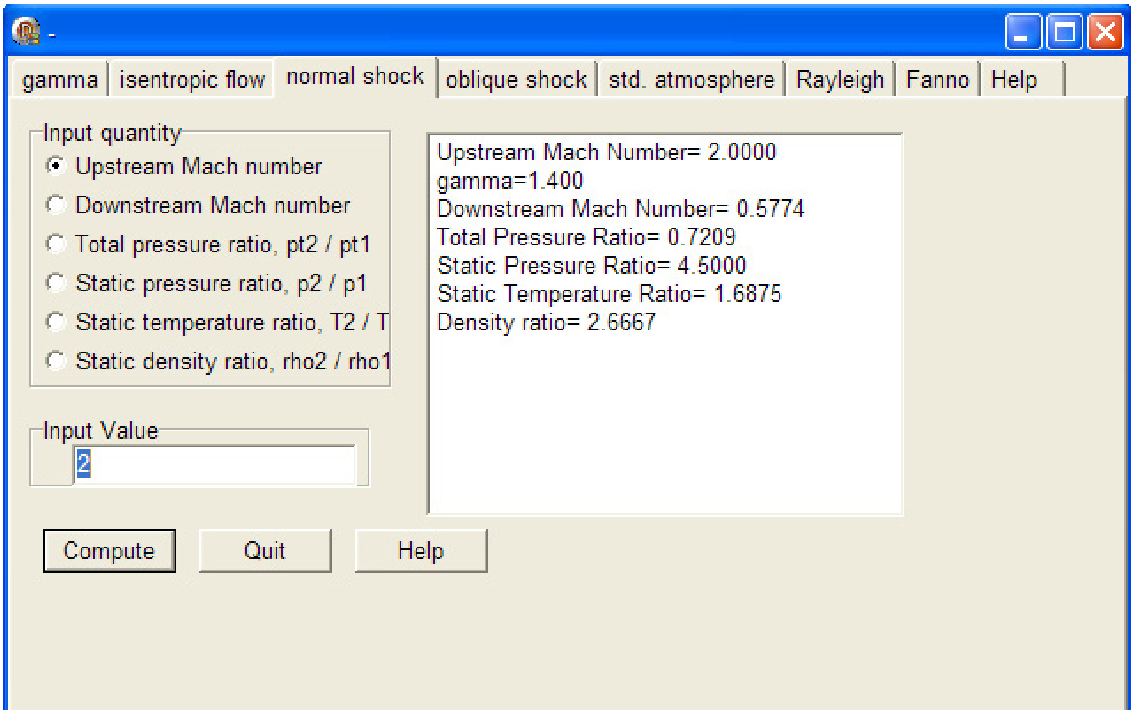
- **'gamma' tab: allows you to change the value of gamma (or, k in the notation used by us)**



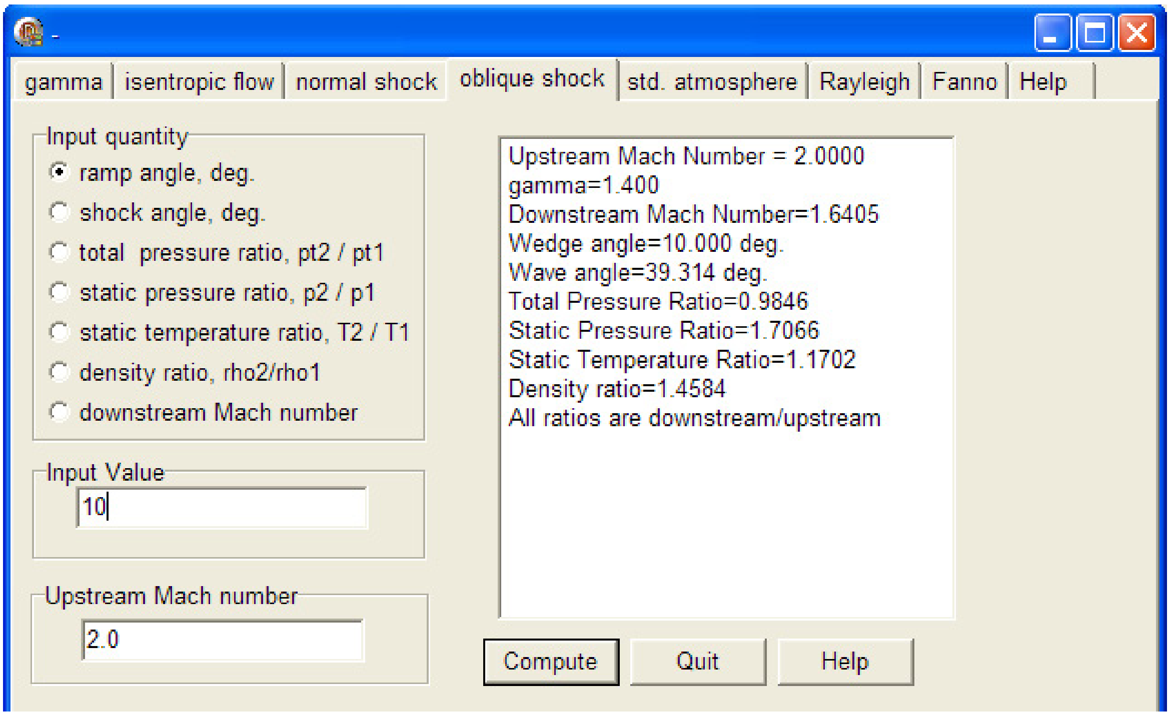
- **'isentropic flow' tab – for calculations of isentropic flow functions: you can select *any one* parameter with the radio button as shown below, and click on 'compute' to get results:**



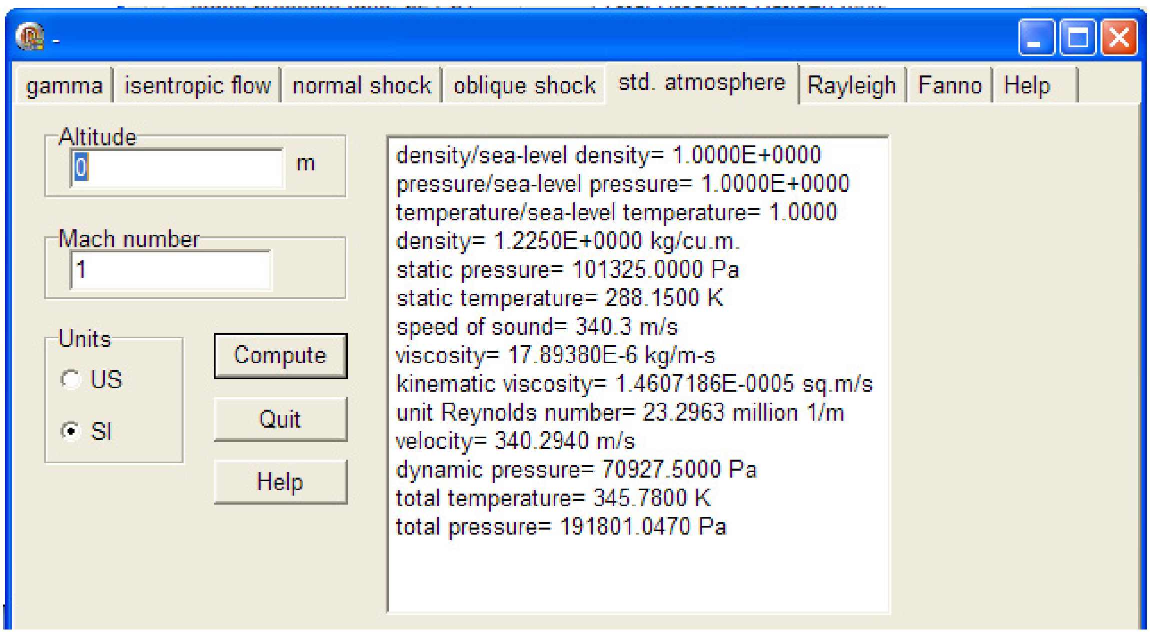
- **'normal shock' tab – for calculations of normal shock functions: you can select *any one* parameter as shown below, and click on 'compute' to get results:**



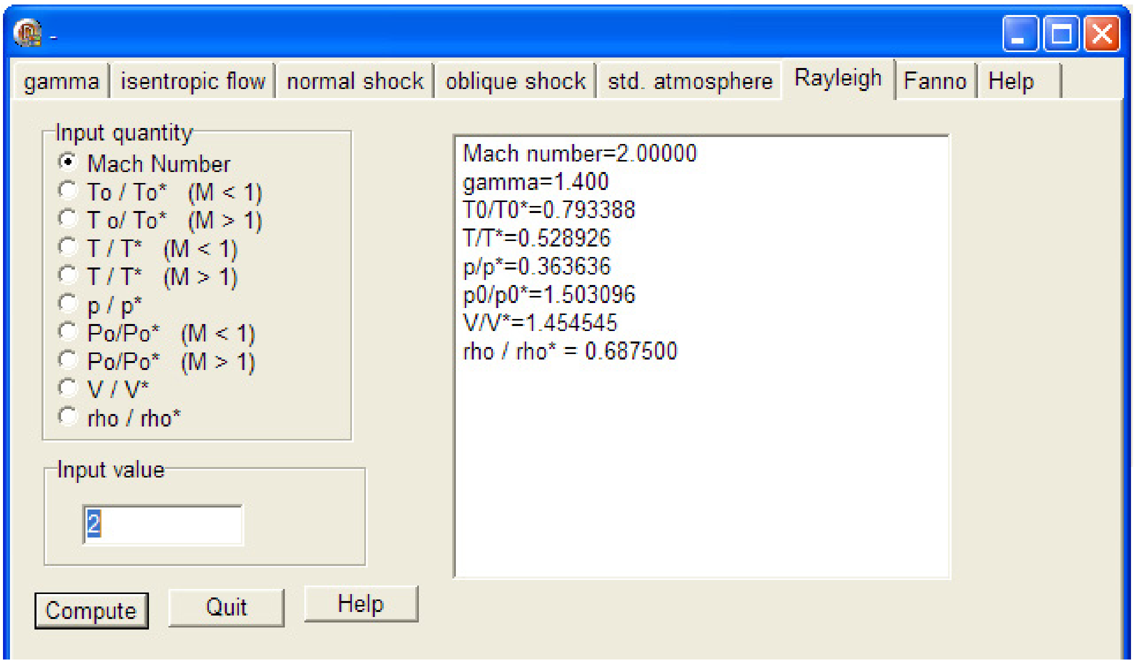
- **'oblique shock' tab – for calculations of oblique shock functions: here, there are *two inputs*: upstream Mach No. is the *necessary* input, and the other input may be chosen with the radio button as shown below. Click on 'compute' to get results:**



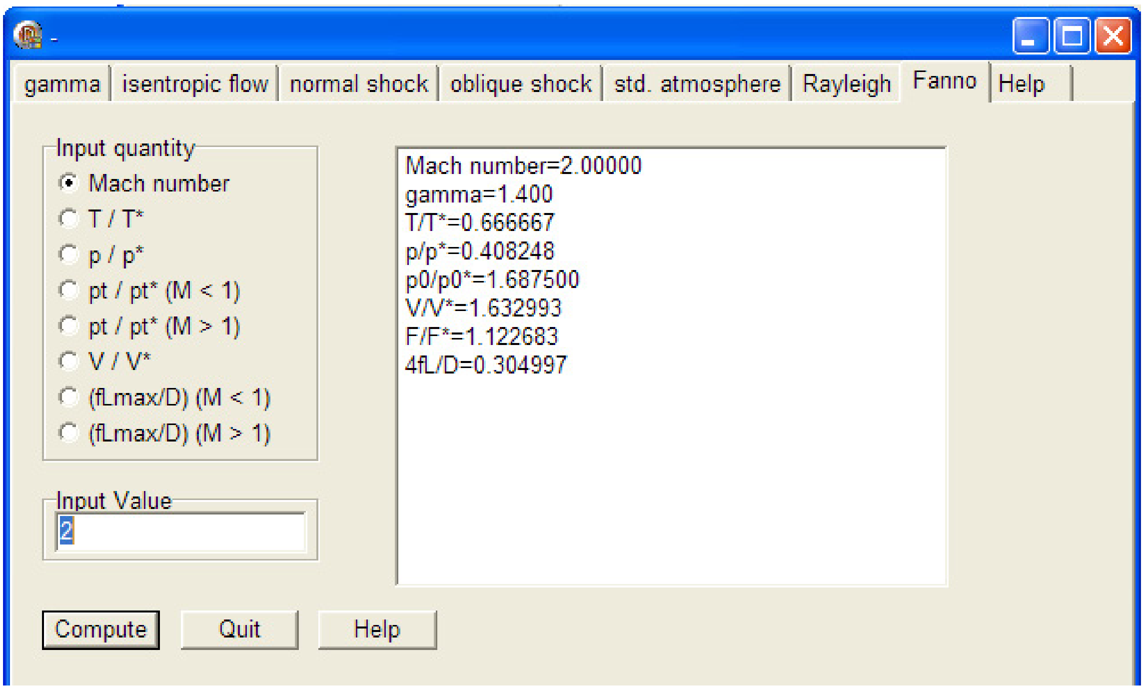
- **'standard atmosphere' tab – computes various parameters for given altitude and Mach No. as shown below:**



- 'Rayleigh' tab – computes various parameters for *any one* input, chosen with radio button, as shown below:



- 'Fanno' tab – computes various parameters for *any one* input, chosen with radio button, as shown below:



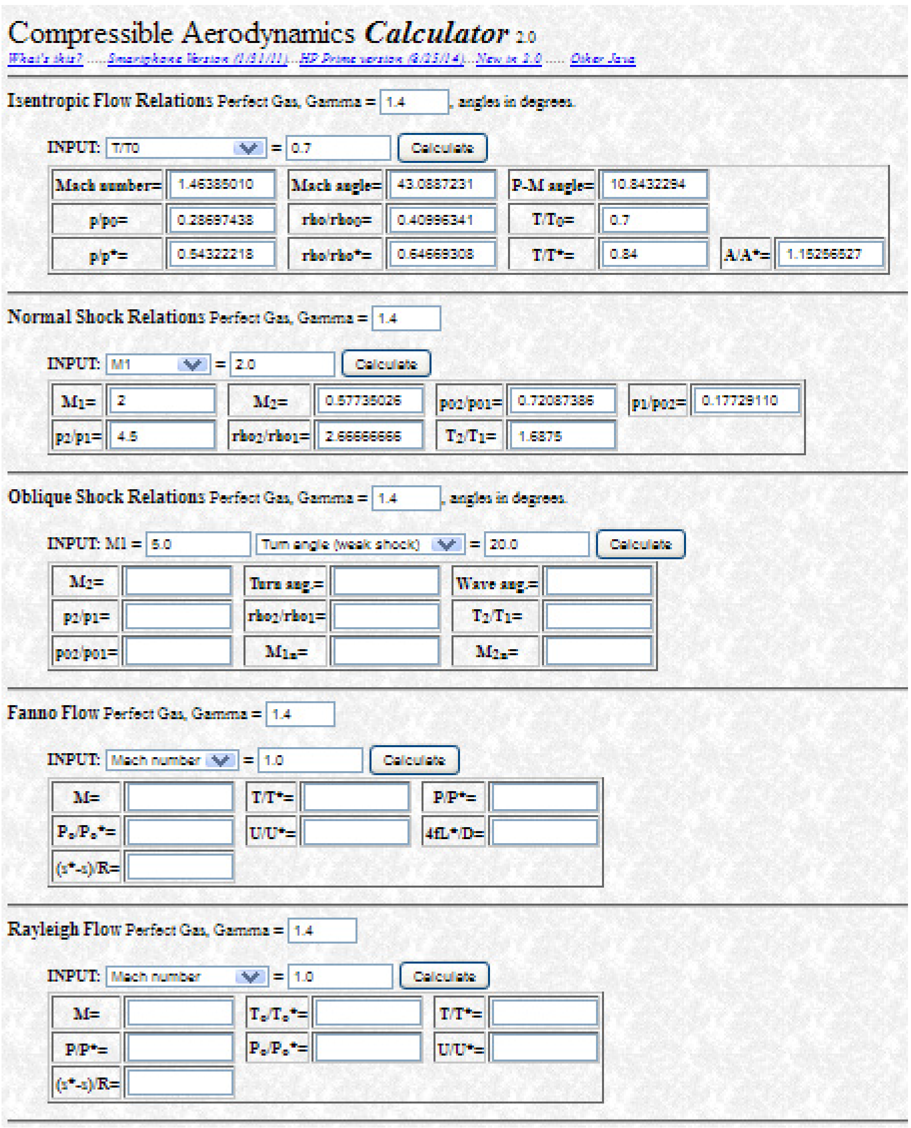
9.2.2 A compressible Aerodynamic calculator based on program by Devenport:

This is a browser based calculator. i.e. you have to save the page once from Internet. Afterwards, you can use it with the web browser, without being connected to Internet.

Here the different calculators are the following:

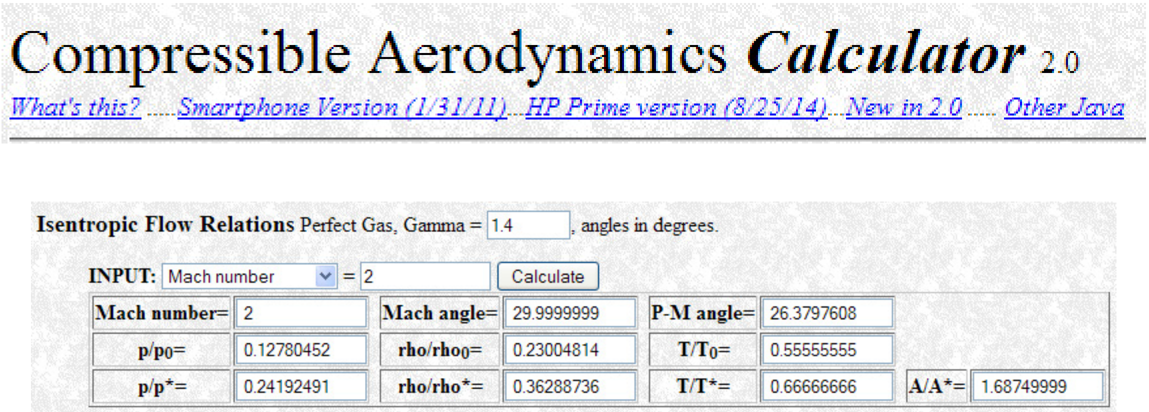
- Isentropic flow calculator
- Normal shock calculator
- Oblique shock calculator
- Fanno flow calculator, and
- Rayleigh flow calculator

All the calculators are on one page, and it looks like this:

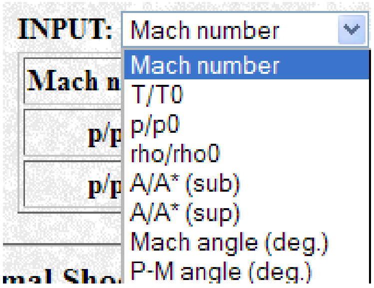


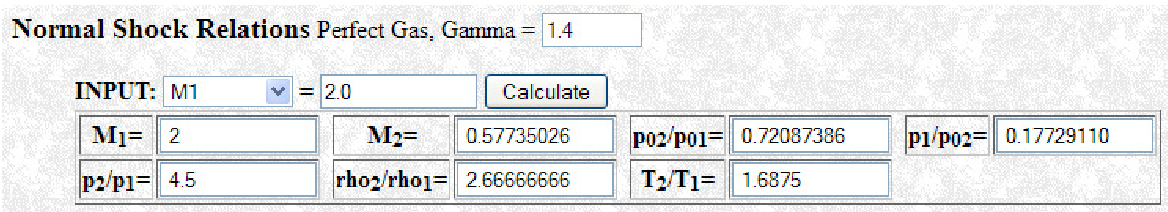
Now, we will explain each of the calculators:

In the following screens, **INPUT** should be filled in, and then press **Calculate** to get the results:

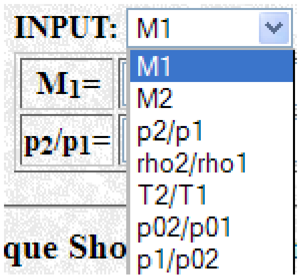


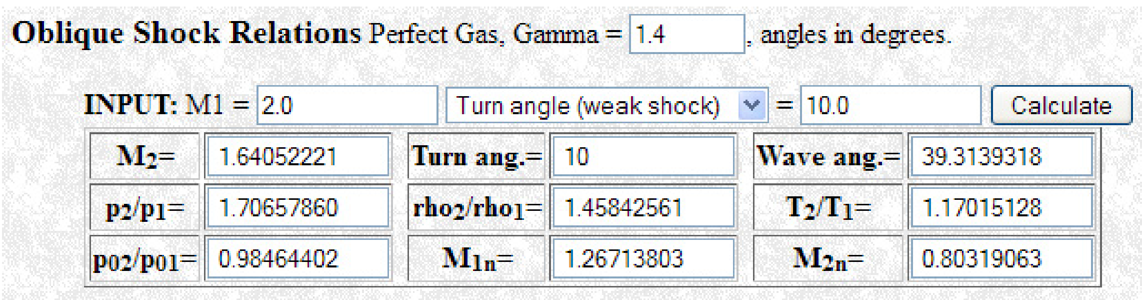
In the above, possible Inputs are:



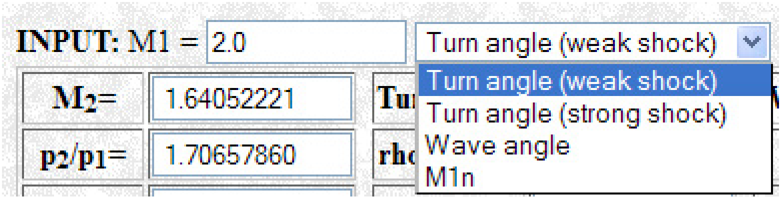


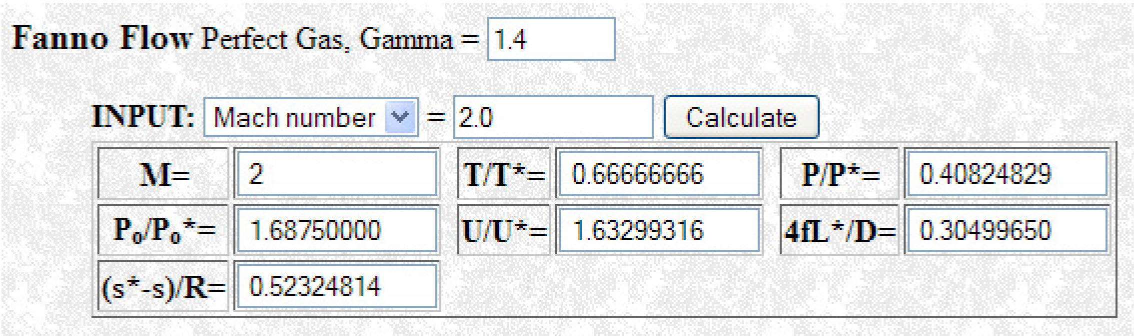
In the above, again, possible Inputs are:



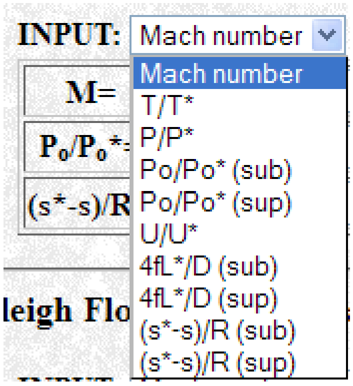


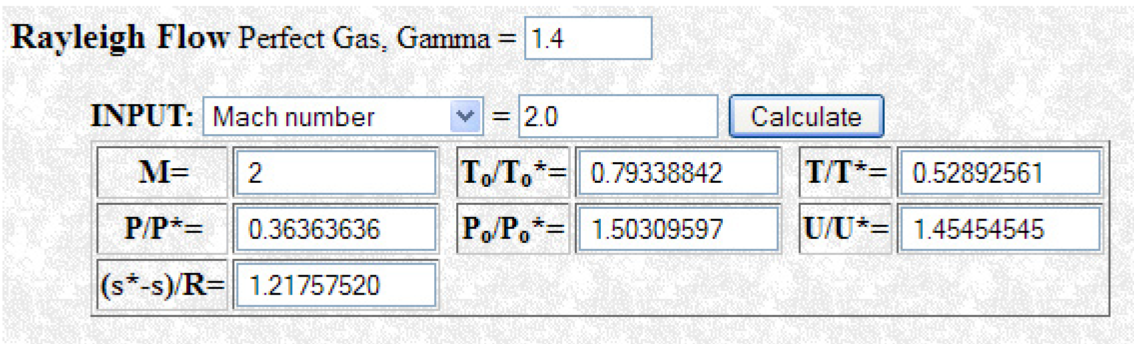
In the above, possible Inputs are:



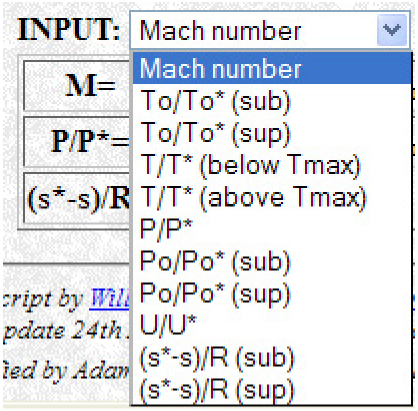


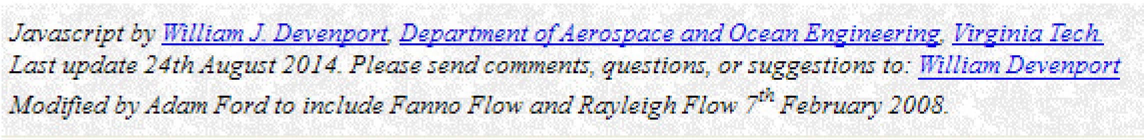
In the above, possible Inputs are:





In the above, possible Inputs are:





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9.3 Problems solved with Mathcad:

Prob.9.3.1 Write Mathcad Functions for one dimensional isentropic flow functions for an ideal gas with

k = 1.4. Also plot these functions against M.

Mathcad Solution:

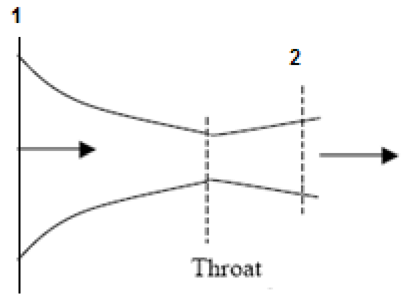


Fig.Prob.9.3.1 Isentropic flow in a C-D nozzle

$$MSTAR(M,k) := M\cdot\sqrt{\frac{k+1}{2+(k-1)\cdot M^2}}$$

MSTAR is the ratio of local velocity to the velocity of sound at the throat... M* = V/C*

$$ABYASTAR(M,k) := \frac{1}{M}\cdot\left[\left(\frac{2}{k+1}\right)\cdot\left(1+\frac{k-1}{2}\cdot M^2\right)\right]^{\frac{k+1}{2\cdot(k-1)}}$$

Area ratio; .. ASTAR is throat area

$$PBYP0(M,k) := \left(1+\frac{k-1}{2}\cdot M^2\right)^{-\frac{k}{k-1}}$$

Pressure ratio ...P0 is the stagnation pressure

$$RHOBYRHO0(M,k) := \left(1+\frac{k-1}{2}\cdot M^2\right)^{-\frac{1}{k-1}}$$

Density ratio.... RHO0 is the stagnation density

$$TBYT0(M,k) := \left(1+\frac{k-1}{2}\cdot M^2\right)^{-1}$$

Temperature ratio.... T0 is the stagnation Temp.

$$FBYFSTAR(M,k) := \frac{1+k\cdot M^2}{M\cdot\sqrt{2\cdot(1+k)\cdot\left(1+\frac{k-1}{2}\cdot M^2\right)}}$$

F is Impulse function = p*A + ρ*A*V^2

$$APRATIO(M,k) := \frac{\left(\frac{2}{k+1}\right)^{\frac{k+1}{2\cdot(k-1)}}}{M\cdot\left(1+\frac{k-1}{2}\cdot M^2\right)^{0.5}}$$ =(A*p)/(Astar*p0)

Tables and Plots:

For Air (k = 1.4), with Mach No. less than 1:

$k := 1.4 \quad M := 0.1, 0.2 .. 1.0$

M =	MSTAR(M,k)	ABYASTAR(M,k)	PBYP0(M,k)	RHOBYRHO0(M,k)	TBYT0(M,k)
0.1	0.109	5.822	0.993	0.995	0.998
0.2	0.218	2.964	0.972	0.98	0.992
0.3	0.326	2.035	0.939	0.956	0.982
0.4	0.431	1.59	0.896	0.924	0.969
0.5	0.535	1.34	0.843	0.885	0.952
0.6	0.635	1.188	0.784	0.84	0.933
0.7	0.732	1.094	0.721	0.792	0.911
0.8	0.825	1.038	0.656	0.74	0.887
0.9	0.915	1.009	0.591	0.687	0.861
1	1	1	0.528	0.634	0.833

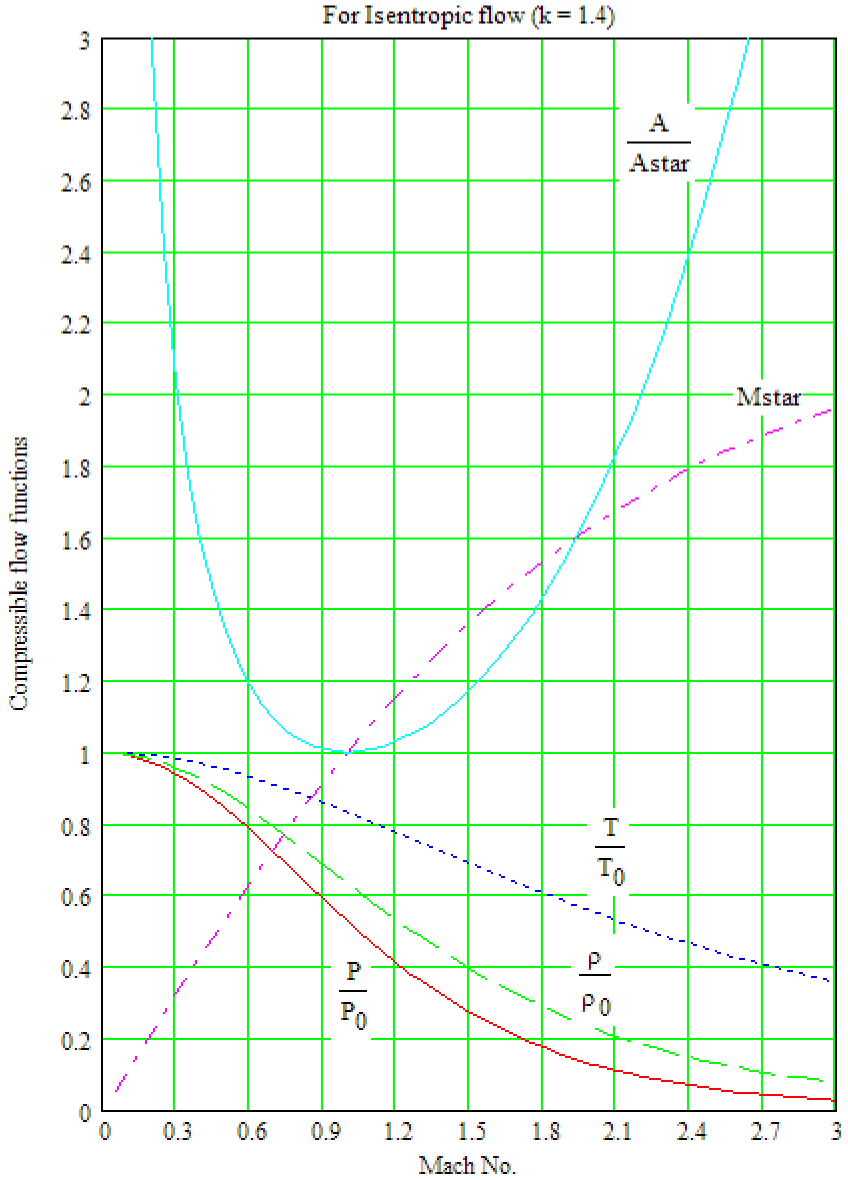
M =	FBYFSTAR(M,k)	APRATIO(M,k)
0.1	4.624	5.781
0.2	2.4	2.882
0.3	1.698	1.912
0.4	1.375	1.424
0.5	1.203	1.13
0.6	1.105	0.932
0.7	1.049	0.789
0.8	1.019	0.681
0.9	1.004	0.597
1	1	0.528

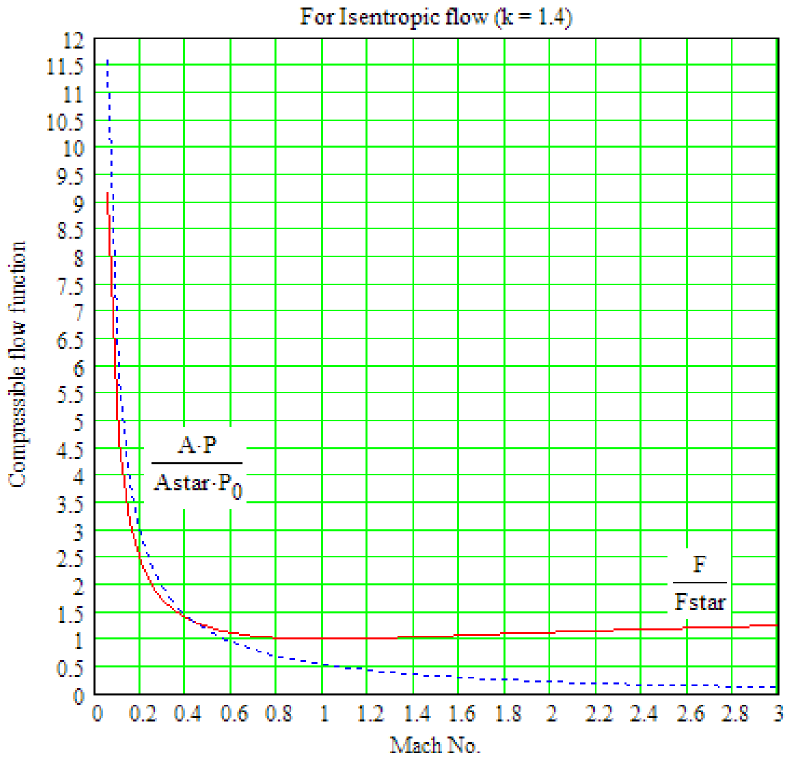
For Air (k = 1.4), with Mach No. more than 1:

$k := 1.4 \qquad M := 1.0, 2.0 .. 10$

M =	MSTAR(M,k)	ABYASTAR(M,k)	PBYP0(M,k) =	RHOBYRHO0(M,k)	TBYT0(M,k)
1	1	1	0.528	0.634	0.833
2	1.633	1.688	0.128	0.23	0.556
3	1.964	4.235	0.027	0.076	0.357
4	2.138	10.719	$6.586 \cdot 10^{-3}$	0.028	0.238
5	2.236	25	$1.89 \cdot 10^{-3}$	0.011	0.167
6	2.295	53.18	$6.334 \cdot 10^{-4}$	$5.194 \cdot 10^{-3}$	0.122
7	2.333	104.143	$2.416 \cdot 10^{-4}$	$2.609 \cdot 10^{-3}$	0.093
8	2.359	190.109	$1.024 \cdot 10^{-4}$	$1.414 \cdot 10^{-3}$	0.072
9	2.377	327.189	$4.739 \cdot 10^{-5}$	$8.15 \cdot 10^{-4}$	0.058
10	2.39	535.938	$2.356 \cdot 10^{-5}$	$4.948 \cdot 10^{-4}$	0.048

M =	FBYFSTAR(M,k)	APRATIO(M,k)
1	1	0.528
2	1.123	0.216
3	1.237	0.115
4	1.303	0.071
5	1.342	0.047
6	1.365	0.034
7	1.381	0.025
8	1.391	0.019
9	1.399	0.016
10	1.404	0.013





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Prob.9.3.2 Write Mathcad Functions for Normal shock functions for an ideal gas with

k = 1.4. Also plot these functions against M.

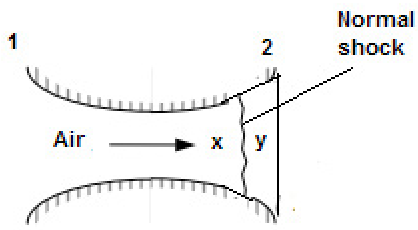


Fig.Prob.9.3.2 Normal shock in a C-D nozzle

Mathcad Solution:

NORMAL SHOCKS: Note the following:

subscript x…before the shock
subscript y…after the shock
stagnation enthalpy remains same…h0x = h0y; Therefore, stagnation temp Tox = Toy
Velocity and stagnation pressure…*decrease* after the shock

Static pressure Py, temp.Ty, and density rhoy…*increase* after the shock

Mach No. My is *always less than 1* after the shock

Particularly, the *increase of temp. after the shock* is of major concern to the Aerospace Engineer.

Increase in entropy after the shock: (sy-sx) = cp.ln(Ty/Tx) – R*ln(Py/Px)

Normal shock Functions:

$$\mathrm{Machy}(M_x,k) := \sqrt{\frac{M_x^{\,2} + \frac{2}{(k-1)}}{\left(\frac{2\cdot M_x^{\,2}\cdot k}{k-1}\right) - 1}} \qquad \text{Mach No. after the shock...}M_y$$

$$\mathrm{PYBYPX}(M_x,k) := \frac{1 + k\cdot M_x^{\,2}}{1 + k\cdot \mathrm{Machy}(M_x,k)^2} \qquad \text{Static pressure ratio,.... Py/Px}$$

$$\mathrm{TYBYTX}(M_x,k) := \frac{1 + M_x^{\,2}\cdot\frac{k-1}{2}}{1 + \mathrm{Machy}(M_x,k)^2\cdot\frac{k-1}{2}} \qquad \text{Static temp. ratio,... Ty/Tx}$$

$$RHOYBYRHOX(M_x, k) := \frac{PYBYPX(M_x, k)}{TYBYTX(M_x, k)}$$ Static density ratio... rhoy/rhox

$$P0YBYP0X(M_x, k) := \frac{M_x}{Machy(M_x, k)} \cdot \left(\frac{1 + Machy(M_x, k)^2 \cdot \frac{k-1}{2}}{1 + M_x^2 \cdot \frac{k-1}{2}} \right)^{\frac{k+1}{2\cdot(k-1)}}$$ Stagnation pressure ratio...P0y/P0x

$$P0YBYPX(M_x, k) := \frac{\left(1 + k \cdot M_x^2\right) \cdot \left(1 + Machy(M_x, k)^2 \cdot \frac{k-1}{2}\right)^{\frac{k}{k-1}}}{1 + k \cdot Machy(M_x, k)^2}$$ Ratio of stagn. pr. after shock to static pr. before shock ...P0y/Px

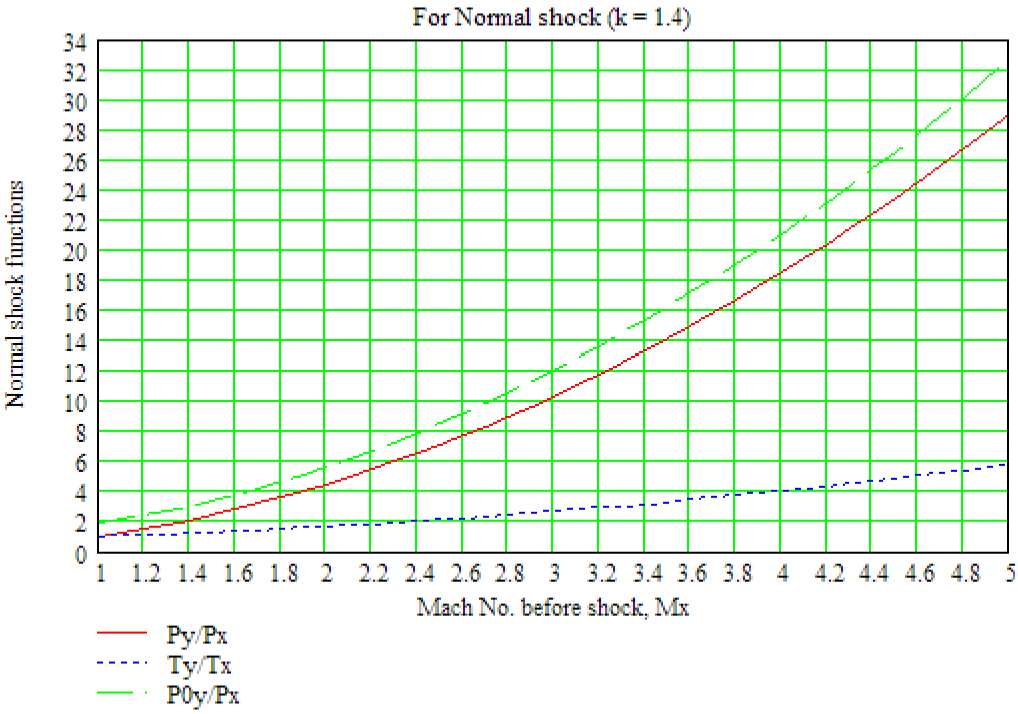
NORMAL SHOCK TABLES FOR AIR (k = 1.4)……using above relations:

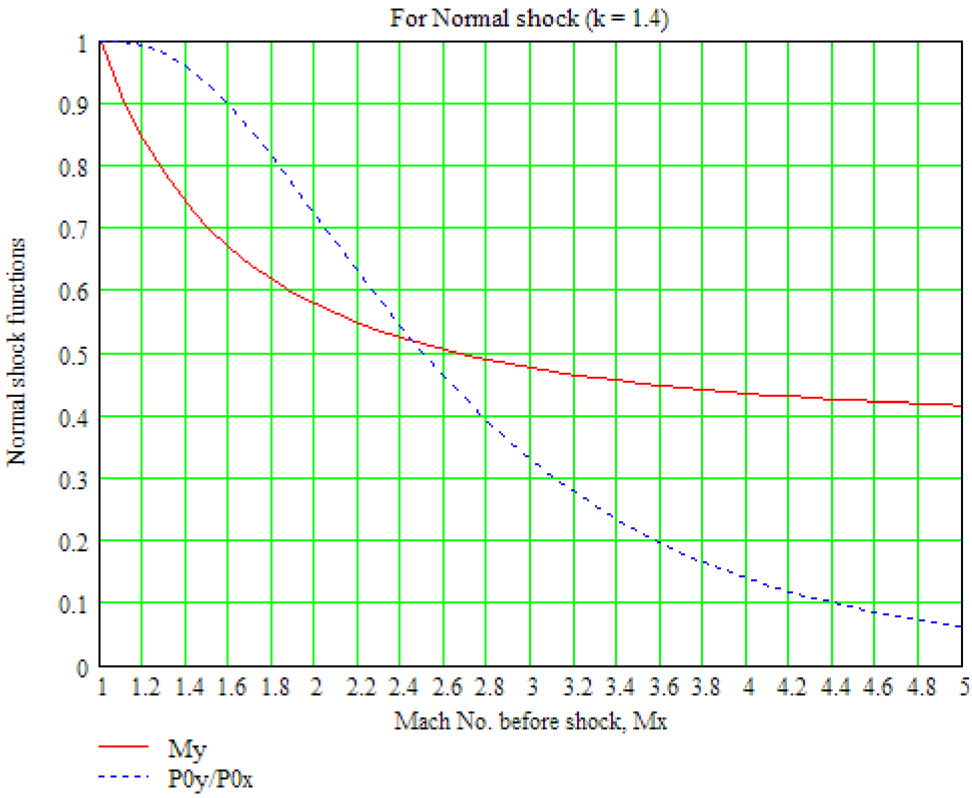
$k := 1.4$

$M_x := 1, 1.2 .. 5$

M_x =	Machy(M_x,k)	PYBYPX(M_x,k)	TYBYTX(M_x,k)	P0YBYP0X(M_x,k)	P0YBYPX(M_x,k)
1	1	1	1	1	1.893
1.2	0.842	1.513	1.128	0.993	2.408
1.4	0.74	2.12	1.255	0.958	3.049
1.6	0.668	2.82	1.388	0.895	3.805
1.8	0.617	3.613	1.532	0.813	4.67
2	0.577	4.5	1.687	0.721	5.64
2.2	0.547	5.48	1.857	0.628	6.716
2.4	0.523	6.553	2.04	0.54	7.897
2.6	0.504	7.72	2.238	0.46	9.181
2.8	0.488	8.98	2.451	0.389	10.569
3	0.475	10.333	2.679	0.328	12.061
3.2	0.464	11.78	2.922	0.276	13.656
3.4	0.455	13.32	3.18	0.232	15.354
3.6	0.447	14.953	3.454	0.195	17.156
3.8	0.441	16.68	3.743	0.164	19.06
4	0.435	18.5	4.047	0.139	21.068
4.2	0.43	20.413	4.367	0.117	23.179
4.4	0.426	22.42	4.702	0.099	25.393
4.6	0.422	24.52	5.052	0.085	27.71
4.8	0.418	26.713	5.418	0.072	30.13
5	0.415	29	5.8	0.062	32.653

Plots:


For Normal shock (k = 1.4)
Normal shock functions
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
1 1.2 1.4 1.6 1.8 2 2.2 2.4 2.6 2.8 3 3.2 3.4 3.6 3.8 4 4.2 4.4 4.6 4.8 5
Mach No. before shock, Mx
Py/Px
Ty/Tx
P0y/Px


For Normal shock (k = 1.4)
Normal shock functions
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1 1.2 1.4 1.6 1.8 2 2.2 2.4 2.6 2.8 3 3.2 3.4 3.6 3.8 4 4.2 4.4 4.6 4.8 5
Mach No. before shock, Mx
My
P0y/P0x

==

Prob. 9.3.3. Plot the area ratio (A / Astar) against Mach No. M. Write a Mathcad program to find the two values of M, one in the subsonic region, and the other in the supersonic region for a given (A/star).

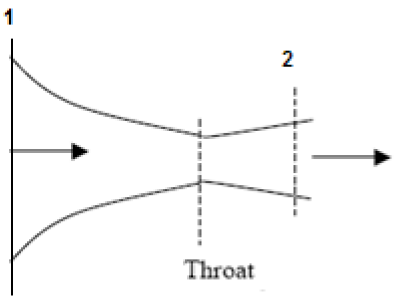


Fig.Prob.9.3.3 Isentropic flow in a C-D nozzle

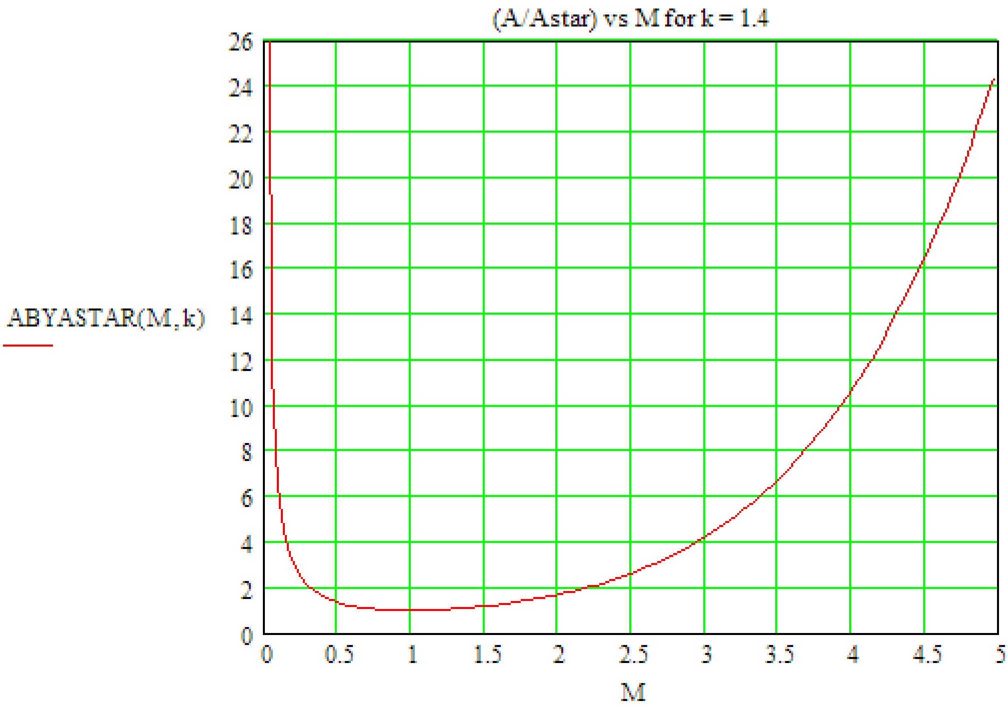
Mathcad Solution:

We have:

$$ABYASTAR(M,k) := \frac{1}{M}\cdot\left[\left(\frac{2}{k+1}\right)\cdot\left(1+\frac{k-1}{2}\cdot M^2\right)\right]^{\frac{k+1}{2\cdot(k-1)}}$$

k := 1.4

M := 0.01, 0.05 .. 5 ...define a range variable



Note from the above graph that that Area goes on decreasing up to an *increase* in Mach No. of 1 at the throat. i.e. *it is the convergent portion* of C-D nozzle.

For getting Mach No. greater than 1 i.e. *for supersonic velocities*, area should go on increasing from the throat, i.e. *it is the divergent portion* of the C-D nozzle.

To find out two values of M when A/A* is given:

We shall use the Solve Block of Mathcad.

Guess value determines whether the output is subsonic M or suopersonic M:

Given

ABYASTAR(Mguess, k) = abyastar

MABYASTAR(abyastar, k, Mguess) := Find(Mguess)   Function to find out 2 values of M for given A/A*

Now, use the above Function MABYASTAR to get two values of M in a single Function:

$$\text{MACH\_ABYASTAR}(abyastar, k) := \left| \begin{array}{l} M1 \leftarrow \text{MABYASTAR}(abyastar, k, 0.1) \\ M2 \leftarrow \text{MABYASTAR}(abyastar, k, 1.1) \\ \begin{pmatrix} \text{"A/Astar"} & \text{"Subsonic M"} & \text{"Supersonic M"} \\ abyastar & M1 & M2 \end{pmatrix} \end{array} \right.$$

Here, the **inputs are**: abyastar and k.

Outputs: two values of M, one subsonic, and the other supersonic.

Example:

$abyastar := 4 \quad k := 1.4$

$$\text{MACH\_ABYASTAR}(abyastar, k) = \begin{pmatrix} \text{"A/Astar"} & \text{"Subsonic M"} & \text{"Supersonic M"} \\ 4 & 0.147 & 2.94 \end{pmatrix}$$

These results can easily be verified from the curve drawn above.

==

Prob.9.3.4 Write Mathcad Functions for Fanno flow parameters for an ideal gas with k = 1.4.

Also plot these functions against M.

Mathcad Solution:

Fanno Flow is specified by: (i) continuity eqn. (ii) Energy eqn. and (iii) constant area, no work and no heat transfer. Also, all sonic properties are constant… p*, rho*, V*, A* etc. Stagnation properties at the sonic state are also constant.

Fanno flow is represented by the Fanno line in a h-s diagram as described earlier.

Effect of friction is to drive the flow to sonic velocity in the subsonic region (i.e. upper part of Fanno line) as well as in the supersonic region (i.e. lower part of Fanno line).

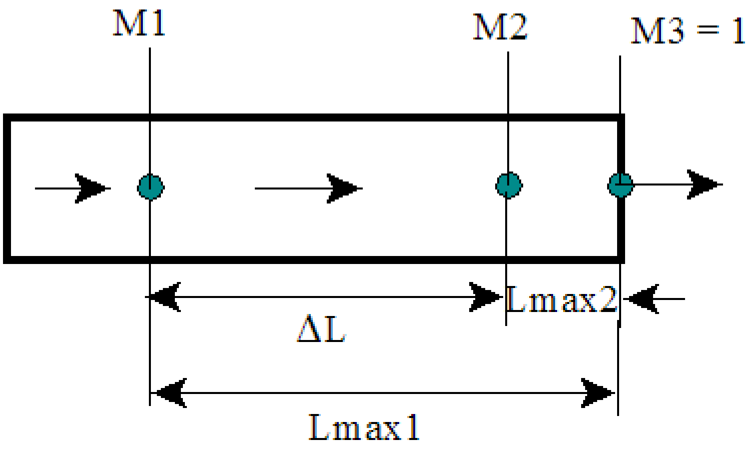


Fig.Prob.9.3.4 Fanno flow (i.e. adiabatic flow with friction).

Following are the Mathcad functions for property calculations:

$$PBYPSTAR(M,k) := \frac{1}{M}\cdot\sqrt{\frac{k+1}{2\cdot\left(1+\frac{k-1}{2}\cdot M^2\right)}}$$

..pressure P, at any state x on a Fanno line is related to the sonic pressure, P*, by this eqn.

Then, between any two states x and y, we can write: Py/Px = (Py/P*).(P*/Px)

$$VBYVSTAR(M,k) := M\cdot\sqrt{\frac{k+1}{2+(k-1)\cdot M^2}}$$

This is also equal to M* defined earlier for isentropic flow = V/C* = rho*/rho..

$$RHOBYRHOSTAR(M,k) := \frac{1}{M}\cdot\left[\frac{2\cdot\left(1+\frac{k-1}{2}\cdot M^2\right)}{k+1}\right]^{0.5}$$

....Density ratio

$$FBYFSTAR(M,k) := \frac{1+k\cdot M^2}{M\cdot\left[2\cdot(k+1)\cdot\left(1+\frac{k-1}{2}\cdot M^2\right)\right]^{0.5}}$$

...Impulse Function ratio

$$TBYTSTAR(M,k) := PBYPSTAR(M,k)\cdot VBYVSTAR(M,k)$$

...Temp ratio

$$P0BYP0STAR(M,k) := \frac{1}{M}\cdot\left[\frac{2\cdot\left(1+\frac{k-1}{2}\cdot M^2\right)}{k+1}\right]^{\frac{k+1}{2\cdot(k-1)}}$$

...stagnation pressure ratio

$$FOURFLMAXBYD(M,k) := \frac{1-M^2}{k\cdot M^2}+\frac{k+1}{2\cdot k}\cdot\ln\left[\frac{(k+1)\cdot M^2}{2\cdot\left(1+\frac{k-1}{2}\cdot M^2\right)}\right]$$

=(4fLmax)/D where f = Fanning friction factor, Lmax = length, D = dia

(*Remember*: Darcy friction factor, f_D = 4 * fanning friction factor f_f)

Table of Fanno flow parameters obtained using the above Mathcad Functions:

For M < 1:

$k := 1.4$

$M := 0.05, 0.1 .. 1$

M	P/Pstar	V/Vstar	T/Tstar	P0/P0star	F/Fstar	4.f.Lmax/D
0.05	21.903	0.055	1.199	11.591	9.158	280.02
0.1	10.944	0.109	1.198	5.822	4.624	66.922
0.15	7.287	0.164	1.195	3.91	3.132	27.932
0.2	5.455	0.218	1.19	2.964	2.4	14.533
0.25	4.355	0.272	1.185	2.403	1.973	8.483
0.3	3.619	0.326	1.179	2.035	1.698	5.299
0.35	3.092	0.379	1.171	1.778	1.509	3.452
0.4	2.696	0.431	1.163	1.59	1.375	2.308
0.45	2.386	0.483	1.153	1.449	1.276	1.566
0.5	2.138	0.535	1.143	1.34	1.203	1.069
0.55	1.934	0.585	1.132	1.255	1.147	0.728
0.6	1.763	0.635	1.119	1.188	1.105	0.491
0.65	1.618	0.684	1.107	1.136	1.073	0.325
0.7	1.493	0.732	1.093	1.094	1.049	0.208
0.75	1.385	0.779	1.079	1.062	1.031	0.127
0.8	1.289	0.825	1.064	1.038	1.019	0.072
0.85	1.205	0.87	1.048	1.021	1.01	0.036
0.9	1.129	0.915	1.033	1.009	1.004	0.015
0.95	1.061	0.958	1.017	1.002	1.001	$3.278 \cdot 10^{-3}$
1	1	1	1	1	1	0

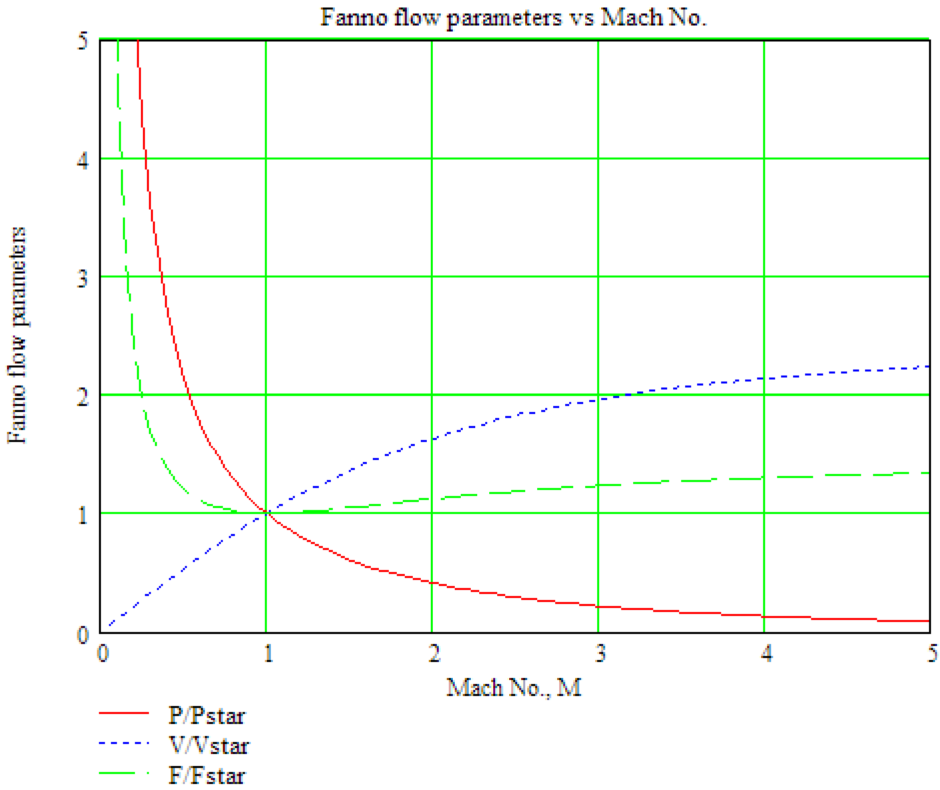
For M > 1:

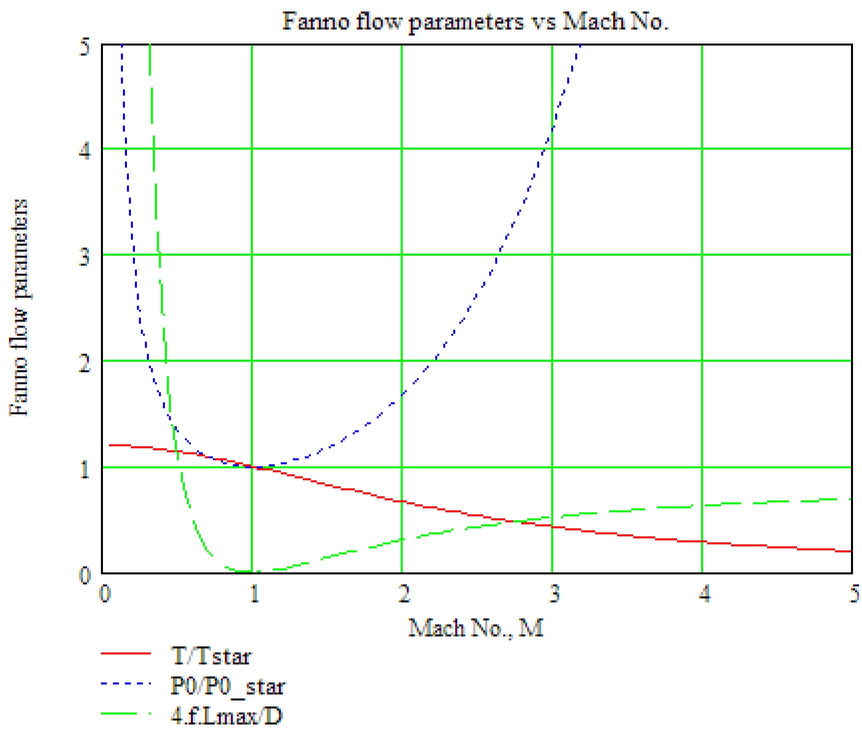
$k := 1.4$

$M := 1, 1.2 .. 5$

M	P/Pstar	V/Vstar	T/Tstar	P0/P0star	F/Fstar	4.f.Lmax/D
1	1	1	1	1	1	0
1.2	0.804	1.158	0.932	1.03	1.011	0.034
1.4	0.663	1.3	0.862	1.115	1.035	0.1
1.6	0.557	1.425	0.794	1.25	1.063	0.172
1.8	0.474	1.536	0.728	1.439	1.094	0.242
2	0.408	1.633	0.667	1.687	1.123	0.305
2.2	0.355	1.718	0.61	2.005	1.15	0.361
2.4	0.311	1.792	0.558	2.403	1.175	0.41
2.6	0.275	1.857	0.51	2.896	1.198	0.453
2.8	0.244	1.914	0.467	3.5	1.218	0.49
3	0.218	1.964	0.429	4.235	1.237	0.522
3.2	0.196	2.008	0.394	5.121	1.253	0.55
3.4	0.177	2.047	0.362	6.184	1.268	0.575
3.6	0.161	2.081	0.334	7.45	1.281	0.597
3.8	0.146	2.111	0.309	8.951	1.292	0.616
4	0.134	2.138	0.286	10.719	1.303	0.633
4.2	0.123	2.162	0.265	12.792	1.312	0.648
4.4	0.113	2.184	0.246	15.21	1.321	0.661
4.6	0.104	2.203	0.229	18.018	1.328	0.673
4.8	0.096	2.22	0.214	21.264	1.335	0.684
5	0.089	2.236	0.2	25	1.342	0.694

Now, plot the Fanno flow parameters:





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Prob.9.3.5 Determine the length of 15 cm ID commercial steel pipe required to change the flow of air from M = 0.2 to M = 0.4 in Fanno flow.

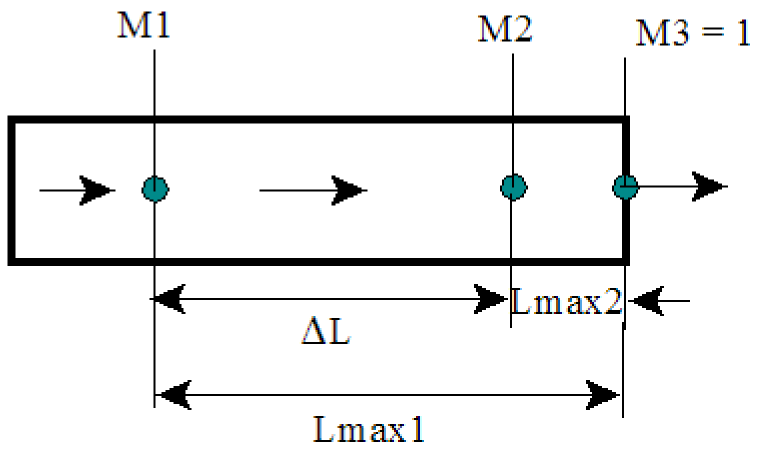


Fig.Prob.9.3.5 Fanno flow (i.e. adiabatic flow with friction).

Mathcad Solution:

From Moody's chart, taking into account the roughness factor, Darcy friction factor, $f_D = 0.015$

$f_D := 0.015 \quad d := 0.15\ m$

At M=0.2:

$M := 0.2 \qquad k := 1.4$

$A := FOURFLMAXBYD(M,k) \qquad A = 14.533$ .... Value of $f_D.L/d$ at M=0.2

At M=0.4:

$M := 0.4$

$B := FOURFLMAXBYD(M,k) \qquad B = 2.308$ ..... Value of $f_D.L/d$ at M=0.4

Therefore, $(A - B) = (L2 - L1).(d/f_D)$

$(A - B)\cdot\frac{d}{f_D} = 122.248$ **m.... length of pipe reqd. to change M from 0.2 to 0.4.....Ans.**

==

Prob.9.3.6 Write Mathcad Functions to calculate Rayleigh Flow functions.
Plot these functions against Mach No.

Rayleigh flow is frictionless flow with heat transfer.

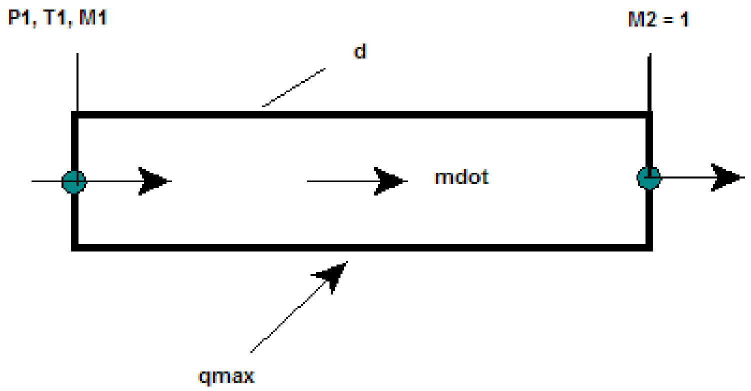


Fig.Prob.9.3.6 Rayleigh flow

Mathcad Solution:

We have:

$$RAYLEIGH\_PBYPSTAR(M,k) := \frac{1+k}{1+k\cdot M^2}$$ ...for P/Pstar

$$RAYLEIGH\_TBYTSTAR(M,k) := \left[\frac{M\cdot(1+k)}{1+k\cdot M^2}\right]^2$$ ...for T/Tstar

$$RAYLEIGH\_VBYVSTAR(M,k) := \frac{(1+k)\cdot M^2}{1+k\cdot M^2}$$ ...for V/Vstar

$$RAYLEIGH\_T0BYT0STAR(M,k) := \frac{(k+1)\cdot M^2\cdot\left[2+(k-1)\cdot M^2\right]}{\left(1+k\cdot M^2\right)^2}$$ ...for T_0/T_0star

$$RAYLEIGH\_P0BYP0STAR(M,k) := \frac{k+1}{1+k\cdot M^2}\cdot\left[\frac{2+(k-1)\cdot M^2}{k+1}\right]^{\frac{k}{k-1}}$$ ...for P_0/P_0star

Function to find heat transfer, Q when M1 and M2 are known in Rayleigh flow:

Q in J/kg, cp in J/kg.K, T1 in K

$$\text{RAYLEIGH\_Q}(M1, M2, T1, cp, k) := \left| \begin{array}{l} AA \leftarrow M2^2 - M1^2 \\ BB \leftarrow 2 - 2 \cdot k \cdot M1^2 \cdot M2^2 + (k-1) \cdot \left(M2^2 + M1^2\right) \\ CC \leftarrow 2 \cdot M1^2 \cdot \left(1 + k \cdot M2^2\right)^2 \\ \left(\dfrac{AA \cdot BB}{CC}\right) \cdot cp \cdot T1 \end{array} \right.$$

Ex: $cp := 1005$ $\quad k := 1.4$ $\quad T1 := 450$ $\quad M1 := 0.4447$ $\quad M2 := 1$

$$\text{RAYLEIGH\_Q}(M1, M2, T1, cp, k) = 3.06629 \times 10^5 \quad \text{J/kg}$$

Function to find Entropy change, DELTAS, when M1 and M2 are known in Rayleigh flow:

Entropy change in J/kg.K, when T (K), R (J/kg.K)

$$\text{RAYLEIGH\_DELTAS}(M1, M2, R, k) := \left| \begin{array}{l} AA \leftarrow \left(\dfrac{M2}{M1}\right)^{\frac{2 \cdot k}{k-1}} \\ BB \leftarrow \dfrac{1 + k \cdot M1^2}{1 + k \cdot M2^2} \\ CC \leftarrow \dfrac{k+1}{k-1} \\ R \cdot \ln\left(AA \cdot BB^{CC}\right) \end{array} \right.$$

Ex: $R := 287$ $\quad k := 1.4$ $\quad M1 := 0.4447$ $\quad M2 := 1$

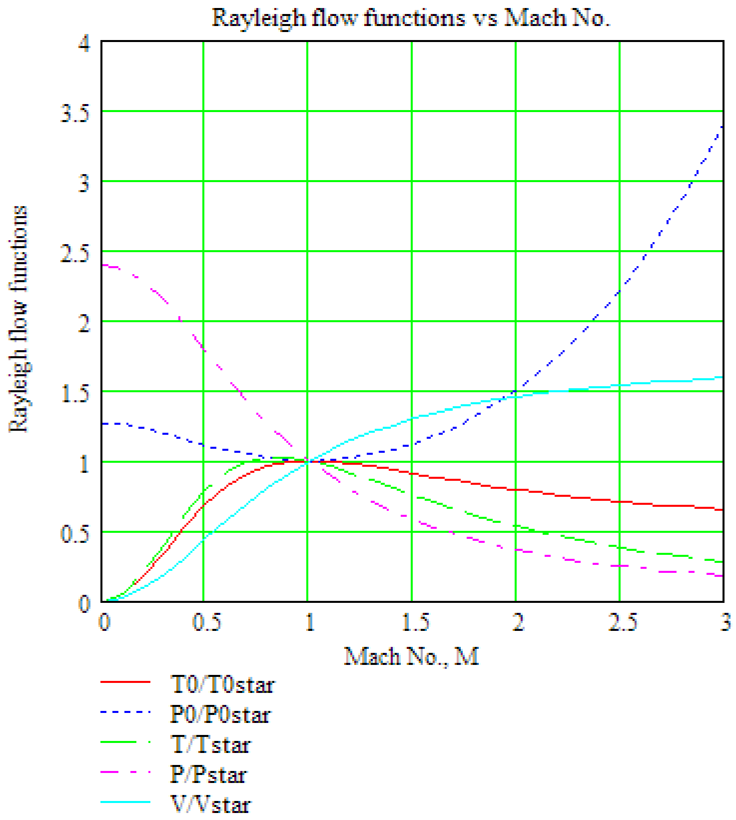
$$\text{RAYLEIGH\_DELTAS}(M1, M2, R, k) = 541.312 \quad \text{J/kg.K}$$

Table of results obtained using the above Mathcad Functions:

M	T0/T0star	P0/P0star	T/Tstar	P/Pstar	V/Vstar
0	0	1.268	0	2.4	0
0.1	0.047	1.259	0.056	2.367	0.024
0.2	0.174	1.235	0.207	2.273	0.091
0.3	0.347	1.199	0.409	2.131	0.192
0.4	0.529	1.157	0.615	1.961	0.314
0.5	0.691	1.114	0.79	1.778	0.444
0.6	0.819	1.075	0.917	1.596	0.574
0.7	0.908	1.043	0.993	1.423	0.698
0.8	0.964	1.019	1.025	1.266	0.81
0.9	0.992	1.005	1.025	1.125	0.911
1	1	1	1	1	1
1.1	0.994	1.005	0.96	0.891	1.078
1.2	0.979	1.019	0.912	0.796	1.146
1.3	0.958	1.044	0.859	0.713	1.205
1.4	0.934	1.078	0.805	0.641	1.256
1.5	0.909	1.122	0.753	0.578	1.301

M	T0/T0star	P0/P0star	T/Tstar	P/Pstar	V/Vstar
1.6	0.884	1.176	0.702	0.524	1.34
1.7	0.86	1.24	0.654	0.476	1.375
1.8	0.836	1.316	0.609	0.434	1.405
1.9	0.814	1.403	0.567	0.396	1.431
2	0.793	1.503	0.529	0.364	1.455
2.1	0.774	1.616	0.494	0.335	1.475
2.2	0.756	1.743	0.461	0.309	1.494
2.3	0.74	1.886	0.431	0.286	1.51
2.4	0.724	2.045	0.404	0.265	1.525
2.5	0.71	2.222	0.379	0.246	1.538
2.6	0.697	2.418	0.356	0.229	1.55
2.7	0.685	2.634	0.334	0.214	1.561
2.8	0.674	2.873	0.315	0.2	1.571
2.9	0.664	3.136	0.297	0.188	1.58
3	0.654	3.424	0.28	0.176	1.588

Plot of results:



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Prob.9.3.7 Steam flows through a device at 800 kPa, 400 C with a velocity of 275 m/s. Determine the Mach No. assuming ideal gas behavior, with k = 1.3. Also, plot the Mach No. vs temp as temp varies from 200 C to 400 C.

Mathcad Solution:

Data:

$k := 1.3$ $\quad R := \dfrac{8314}{18}$ $\quad R = 461.889$ J/kg.K .... gas constant for steam

$T := 400$ C $\quad V := 275$ m/s

Calculations:

$C(T) := \sqrt{k \cdot R \cdot (T + 273)}$ m/s....velocity of steam written as a function of T

i.e. $C(T) = 635.694$ m/s

Therefore: $M(T) := \dfrac{V}{C(T)}$ ...Mach No. as a function of T

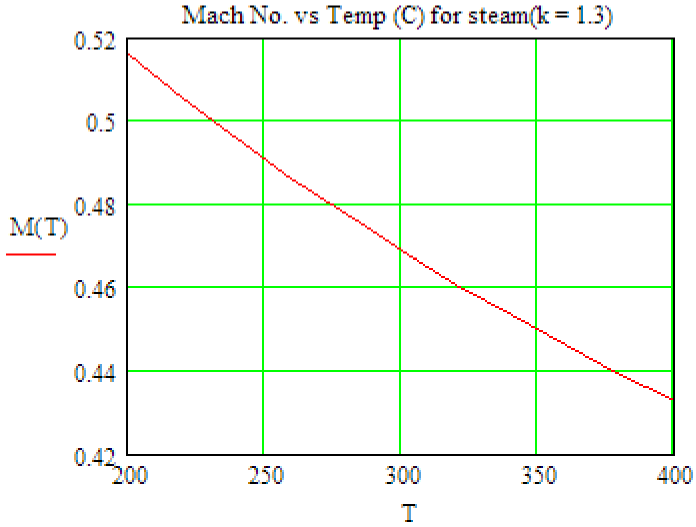
i.e. $M(T) = 0.433$ **...Mach No.... Ans.**

Plot Mach no. vs temp:

$T := 200, 220 .. 400$ ....define a range variable

T =	M(T) =
200	0.516
220	0.505
240	0.495
260	0.486
280	0.477
300	0.469
320	0.461
340	0.453
360	0.446
380	0.439
400	0.433

Now, plot the results:



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Prob. 9.3.8 Air at 310 K stagnation temp. and 40 kPa pressure flows through a duct of 10 cm dia at a rate of 1 kg/s. Calculate the velocity, Mach No. and stagnation pressure at that section.[M.U.]

Mathcad Solution:

Data:

$T0 := 310 \cdot K \quad P := 40 \cdot 10^{3} \cdot Pa \quad k := 1.4 \quad R := 287 \cdot \frac{J}{kg \cdot K} \quad cp := 1005 \cdot \frac{J}{kg \cdot K}$

$D := 0.1 \cdot m \quad flow := 1 \cdot \frac{kg}{sec}$

Calculations:

$A := \frac{\pi \cdot D^{2}}{4}$ i.e. $A = 7.854 \times 10^{-3} \, m^{2}$ ...area of cross-section

To find temp of air: Use the Solve block of Mathcad:

$T := 300 \cdot K$ ...Trial value

Given

$T0 = T + \left(\frac{flow \cdot R \cdot T}{P \cdot A}\right)^{2} \cdot \frac{1}{2 \cdot cp}$ ...by definition of stagnation temp

$Find(T) = 277.928 \, K$

Therefore: $T := 277.928 \cdot K$

$\rho := \frac{P}{R \cdot T}$ i.e. $\rho = 0.501 \frac{kg}{m^3}$ ...density

$V := \frac{flow}{\rho \cdot A}$ i.e. $V = 253.901 \frac{m}{s}$ **...velocity .... Ans.**

$C := \sqrt{k \cdot R \cdot T}$ i.e. $C = 334.173 \frac{m}{s}$ ..velocity of sound

$M := \frac{V}{C}$ i.e. $M = 0.76$ **...Mach No. ..... Ans.**

$P0 := \frac{P}{\left(\frac{T}{T0}\right)^{\frac{k}{k-1}}}$ i.e. $P0 = 5.862 \times 10^4 Pa$ **...Stagn. pressure. ..... Ans.**

===

Prob. 9.3.9 Air at 10 bar, 127 C flows in a convergent nozzle with a velocity of 150 m/s. Cross-sectional area at throat is 6.5 cm^2. Assuming the flow to be isentropic, compute the mass rate of flow for a back pressure of (i) 8 bar, (ii) critical pressure, (iii) 3 bar.[M.U.]

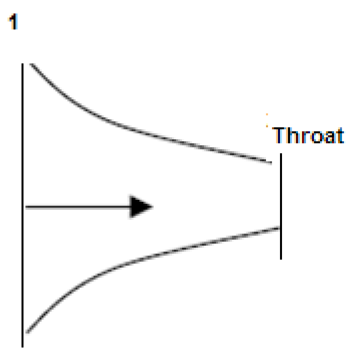


Fig.Prob.9.3.9 Isentropic flow through a Convergent nozzle

Mathcad Solution:

Data:

$P1 := 10$ bar $T1 := 127 + 273$ K $V1 := 150$ m/s

$A_t := 6.5 \cdot 10^{-4}$ m^2 .... throat area $k := 1.4$ $R := 287$ J/kg.K for air

$cp := 1005$ J/kg.K for air

Calculations:

$C1 := \sqrt{k \cdot R \cdot T1}$ i.e. $C1 = 400.899$ m/s .... bvelocity of sound at inlet

$M1 := \frac{V1}{C1}$ i.e. $M1 = 0.374$ ...Mach No. at inlet

Always, find the critical pressure first.

Remember: for air, Pstar/P0 = 0.5283, and Tstar/T0 = 0.8333

Therefore:

$T_0 := \frac{T1}{TBYT0(M1,k)}$ i.e. $T_0 = 411.2$ K...stagn. temp at inlet

$P_0 := \frac{P1}{PBYP0(M1,k)}$ i.e. $P_0 = 11.015$ bar ... stagn. pressure at inlet

$Pstar := P_0 \cdot 0.5283$ i.e. $Pstar = 5.819$ bar....critical pressure

Case (i): Note that critical pressure is < back pressure of 8 bar for case (i):

$P_t := 8$ bar..throat pressure, by data

T0 remains constant for isentropic flow, from energy eqn. Therefore, at the throat:

$$\frac{T_t}{T_0} = \left(\frac{P_t}{P_0}\right)^{\frac{k-1}{k}}$$ ...for isentropic flow

i.e. $T_t := T_0 \cdot \left(\frac{P_t}{P_0}\right)^{\frac{k-1}{k}}$ i.e. $T_t = 375.294$ K....temp at throat

Therefore: $\rho_t := \frac{P_t \cdot 10^5}{R \cdot T_t}$ i.e. $\rho_t = 7.427$ kg/m^3... density of air at throat

And: $V_t := \sqrt{2 \cdot cp \cdot (T_0 - T_t)}$ i.e. $V_t = 268.646$ m/s ... velocity at throat

Then, mass flow rate:

$mdot_1 := \rho_t \cdot A_t \cdot V_t$ i.e. $mdot_1 = 1.297$ **kg/s ... mass flow rate ... Ans.**

Case (ii): Now, throat pressure is critical pressure =5.819 bar :

Then, we have:

$$Tstar := T_0 \cdot \left(\frac{Pstar}{P_0}\right)^{\frac{k-1}{k}}$$ i.e. $Tstar = 342.67$ K....temp at throat

Therefore: $$\rho star := \frac{Pstar \cdot 10^5}{R \cdot Tstar}$$ i.e. $\rho star = 5.917$ kg/m^3... density of air at throat

And: $$Vstar := \sqrt{2 \cdot cp \cdot (T_0 - Tstar)}$$ i.e. $Vstar = 371.14$ m/s ... velocity at throat

Verify: $$C2 := \sqrt{k \cdot R \cdot Tstar}$$ i.e. $C2 = 371.059$ **...sonic velocity....verified**

Then, mass flow rate:

$$mdot_2 := \rho star \cdot A_t \cdot Vstar$$ i.e. $mdot_2 = 1.427$ **kg/s ... mass flow rate ... Ans.**

Case (iii): Now, throat pressure is critical pressure =3 bar :

This throat pressure is < critical pressure.

Therefore, it has no effect on mass flow rate, since flow is choked when back pressure is equal to or below critical pressure.....Ans.

(b) Plot the mass flow rate vs back pressure:

Write a Mathcad program as shown below:

Here, the Inputs are: P1 (bar), T1 K), V1(m/s), P_t (bar), At (m^2), k (=cp/cv), cp (J/kg.K) and R (J/kg.K).

Output is : mdot (kg/s)

$$\mathrm{MDOT}(P1, T1, V1, P_t, A_t, k, cp, R) :=$$

$$C1 \leftarrow \sqrt{k \cdot R \cdot T1}$$

$$M1 \leftarrow \frac{V1}{C1}$$

$$T0 \leftarrow \frac{T1}{\mathrm{TBYT0}(M1, k)}$$

$$P0 \leftarrow \frac{P1}{\mathrm{PBYP0}(M1, k)}$$

$$Pstar \leftarrow P_0 \cdot \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

$$T_t \leftarrow T_0 \cdot \left(\frac{P_t}{P_0}\right)^{\frac{k-1}{k}} \quad \text{if } P_t \geq Pstar$$

$$T_t \leftarrow T_0 \cdot \left(\frac{Pstar}{P_0}\right)^{\frac{k-1}{k}} \quad \text{otherwise}$$

$$\rho_t \leftarrow \frac{P_t \cdot 10^5}{R \cdot T_t} \quad \text{if } P_t \geq Pstar$$

$$\rho_t \leftarrow \frac{Pstar \cdot 10^5}{R \cdot T_t} \quad \text{otherwise}$$

$$V_t \leftarrow \sqrt{2 \cdot cp \cdot (T_0 - T_t)} \quad \text{if } P_t \geq Pstar$$

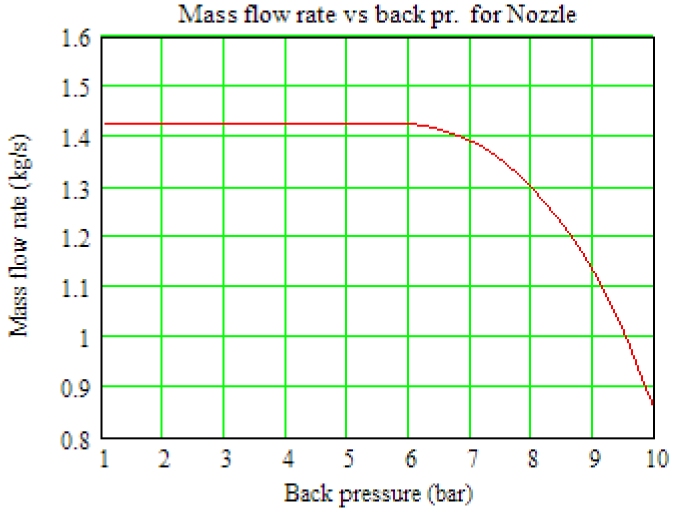
$$V_t \leftarrow \sqrt{k \cdot R \cdot T_t} \quad \text{otherwise}$$

$$mdot \leftarrow \rho_t \cdot A_t \cdot V_t$$

Then, mdot vs P_t table is:

$P_t =$	$MDOT(P1, T1, V1, P_t, A_t, k, cp, R)$	$P_t =$	$MDOT(P1, T1, V1, P_t, A_t, k, cp, R)$
10	0.85	5.4	1.427
9.8	0.919	5.2	1.427
9.6	0.981	5	1.427
9.4	1.036	4.8	1.427
9.2	1.086	4.6	1.427
9	1.131	4.4	1.427
8.8	1.171	4.2	1.427
8.6	1.208	4	1.427
8.4	1.241	3.8	1.427
8.2	1.27	3.6	1.427
8	1.297	3.4	1.427
7.8	1.321	3.2	1.427
7.6	1.342	3	1.427
7.4	1.361	2.8	1.427
7.2	1.377	2.6	1.427
7	1.391	2.4	1.427
6.8	1.402	2.2	1.427
6.6	1.411	2	1.427
6.4	1.419	1.8	1.427
6.2	1.424	1.6	1.427
6	1.427	1.4	1.427
5.8	1.427	1.2	1.427
5.6	1.427	1	1.427

And, plot the results:



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Prob. 9.3.10 Air at a pressure of 15 bar, temp. 150 C, and velocity 50 m/s, expands in a nozzle isentropically to 1.4 bar pressure. The mass flow rate is 240 kg/min. Determine: (i) cross-sectional areas at inlet, throat and exit (ii) velocity, temp and Mach Nos. at throat and exit. [M.U.]

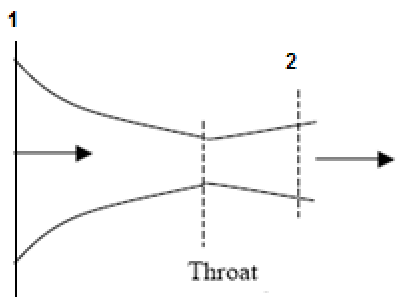


Fig.Prob.9.3.10 Isentropic flow through a C-D nozzle

Mathcad Solution:

Data:

$T_1 := 150 + 273$ $\quad V_1 := 50$ m/s $\quad P_1 := 15$ bar $\quad P_2 := 1.4$ bar

$k := 1.4$ $\quad R := 287$ J/kg.K $\quad cp := 1005$ J/kg.K

$m := \frac{240}{60}$ i.e. $m = 4$ kg/s

Calculations:

$T_0 := T_1 + \frac{V_1^2}{2 \cdot cp}$ i.e. $T_0 = 424.244$ K....stagn. temp.

$P_0 := P_1 \cdot \left(\frac{T_0}{T_1}\right)^{\frac{k}{k-1}}$ $\quad P_0 = 15.155$ bar... stagn. pressure

First find the critical pressure, to ascertain if choking occurs:

$PSTAR := P_0 \cdot 0.5283$ i.e. $PSTAR = 8.006$ **bar...critical pressure**

But, by data, expansion is upto 1.4 bar. So, a C-D nozzle is reqd.

$\rho_1 := \frac{P_1 \cdot 10^5}{R \cdot T_1}$ i.e. $\rho_1 = 12.356$ kg/m^3 ... density at inlet

Therefore:

$A_1 := \frac{m}{\rho_1 \cdot V_1} \cdot 10^4$ i.e. $A_1 = 64.747$ cm^2....area at inlet...Ans.

$C_1 := \sqrt{k \cdot R \cdot T_1}$ i.e. $C_1 = 412.264$ m/s....sonic velocity at inlet

$M_1 := \frac{V_1}{C_1}$ i.e. $M_1 = 0.121$ **Mach No. at inlet**

At Throat:

$abyastar := ABYASTAR(M_1, k)$ i.e. $abyastar = 4.814$

$Astar := \frac{A_1}{abyastar}$ i.e. $Astar = 13.45$ cm^2....Throat area... Ans.

Or, calculate Astar by continuity eqn. at throat:

$$TSTAR := T_0 \cdot \left(\frac{PSTAR}{P_0}\right)^{\frac{k-1}{k}} \quad \text{i.e.} \quad TSTAR = 353.54$$ K...temp. at throat.. Ans.

$$VSTAR := \sqrt{k \cdot R \cdot TSTAR} \quad \text{i.e.} \quad VSTAR = 376.898$$ m/s...vel. at throat... Ans.

$$ROSTAR := \frac{PSTAR \cdot 10^5}{R \cdot TSTAR} \quad \text{i.e.} \quad ROSTAR = 7.891$$ kg/m^3.... density at throat

$$ASTAR := \frac{m}{ROSTAR \cdot VSTAR} \cdot 10^4 \quad \text{i.e.} \quad ASTAR = 13.45$$ cm^2....throat area

At exit:

$$T_2 := T_1 \cdot \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \quad \text{i.e.} \quad T_2 = 214.815$$ K.....temp. at exit... Ans.

$$\rho_2 := \frac{P_2}{R \cdot T_2} \cdot 10^5 \quad \text{i.e.} \quad \rho_2 = 2.271$$ kg/m^3.... density at exit

$$V_2 := \sqrt{2 \cdot cp \cdot (T_1 - T_2) + V_1^2} \quad \text{i.e.} \quad V_2 = 648.808$$ m/s..vel. at exit... Ans.

$$A_2 := \frac{m}{V_2 \cdot \rho_2} \cdot 10^4 \quad \text{i.e.} \quad A_2 = 27.15$$ cm^2...area at exit.... Ans.

$$C_2 := \sqrt{k \cdot R \cdot T_2} \quad \text{i.e.} \quad C_2 = 293.79$$ m/s....vel. of sound at exit

$$M_2 := \frac{V_2}{C_2} \quad \text{i.e.} \quad M_2 = 2.208$$ Mach No. at exit... Ans.

===

Prob.9.3.11 Air flows in a C-D nozzle having exit area ratio of 2. Stagnation pressure and temp. at entry are 1 MPa and 800 K respectively. Find out the back pressure necessary for a normal shock to appear just at the exit plane of the nozzle. Also find out temp., velocity just upstream of the shock wave.

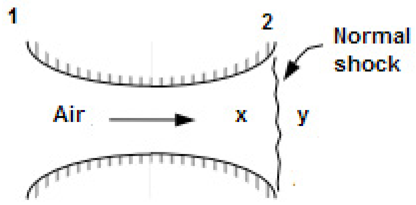


Fig.Prob.9.3.11 Normal shock in a C-D nozzle

Mathcad Solution:

Data:

$P_0 := 1$ MPa $\quad T_0 := 800$ K $\quad k := 1.4 \quad R := 287$ J/kg.K

Flow throughout the nozzle is isentropic. Normal shock stands just at the exit. So, refer to the isentropic table and look for value of M at (A/ASTAR) = 2. This is M2 = Mx.

Using the Mathcad Function:

$abyastar := 2 \quad k := 1.4$

$Mguess := 4$

$M_2 := MABYASTAR(abyastar, k, Mguess)$

$M_2 = 2.197$

i.e. $M_x := M_2$ ..Mach No. at exit, before the shock

Now, we have:

$pbyp0 := PBYP0(M_x, k)$ i.e. $pbyp0 = 0.094$

$P_2 := P_0 \cdot pbyp0$ i.e. $P_2 = 0.094$ MPa ... pressure at exit, before the shock

And: $P_x := P_2$

$P_x = 0.094$ **MPa... pressure before the shock ... Ans.**

And: $tbyt0 := TBYT0(M_x, k)$ i.e. $tbyt0 = 0.509$

Then: $T := tbyt0 \cdot T_0$

i.e. $T = 407.014$ **K.....temp. before the shock.... Ans.**

And: $T_x := T$

$C_x := \sqrt{k \cdot R \cdot T_x}$ i.e. $C_x = 404.398$ m/s....Sonic velocity

$V_x := C_x \cdot M_x$ i.e. $V_x = 888.543$ **m/s....velocity before the shock .... Ans.**

Then, from shock tables: (We use the Mathcad Functions written earlier)

$M_y := \text{Machy}(M_x, k)$ i.e. $M_y = 0.547$ **...Mach No. after the shock**

$\text{pybypx} := \text{PYBYPX}(M_x, k)$ i.e. $\text{pybypx} = 5.466$

$P_y := \text{pybypx} \cdot P_x$ i.e. $P_y = 0.513$ **MPa....Pr. after the shock....back pressure necessary for shock at nozzle exit .... Ans.**

Note: Compare the results from Mathcad Functions with those in the Isentropic/Normal shock Tables. Then, you will appreciate the convenience (and accuracy) in using the Mathcad Functions.

===

Prob. 9.3.12 In a C-D nozzle, inlet conditions of air are 1000 kPa, 800 K, the velocity being zero. It is required to produce a supersonic flow at 800 m/s, mass flow rate being 5 kg/s. Find throat and exit areas, Mach no. at exit, P and T at throat and exit.[M.U.]

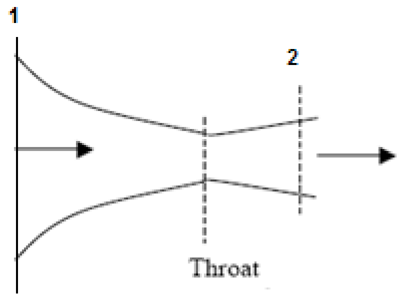


Fig.Prob.9.3.12 Isentropic flow in a C-D nozzle

Mathcad Solution:

Data:

At Inlet: $T_1 := 800$ K $\quad P_1 := 10$ bar

$V_1 := 0$ **...Therefore, T1 is the stagnation temp too.**

$R := 287$ J/kg.K $\quad m := 5$ kg/s $\quad cp := 1005$ J/kg.K $\quad k := 1.4$

$$\rho_1 := \frac{P_1 \cdot 10^5}{R \cdot T_1}$$ i.e. $\rho_1 = 4.355$ kg/m^3

At exit:

$V_2 := 800$ m/s

$$T_2 := T_1 - \frac{V_2^2}{2 \cdot cp}$$ **...since stagnation temp is constant for nozzle isentropic flow**

i.e. $T_2 = 481.592$ **K.... temp at exit .... Ans.**

Calculations:

$$P_2 := P_1 \cdot \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}$$ i.e. $P_2 = 1.693$ **bar ... pressure at exit ... Ans.**

$\rho_2 := \frac{P_2 \cdot 10^5}{R \cdot T_2}$ i.e. $\rho_2 = 1.225$ kg/m^3

$A_2 := \frac{m}{\rho_2 \cdot V_2}$ i.e. $A_2 = 5.10362 \times 10^{-3}$ **m^2... area at exit .... Ans.**

$C_2 := \sqrt{k \cdot R \cdot T_2}$ i.e. $C_2 = 439.891$ m/s....sonic vel. at exit

$M_2 := \frac{V_2}{C_2}$ i.e. $M_2 = 1.819$ **Mach No. at exit ... Ans.**

At Throat:

$PSTAR := P_1 \cdot 0.528$ i.e. $PSTAR = 5.28$ **bar.....pressure at the throat ... Ans.**

$TSTAR := T_1 \cdot \left(\frac{PSTAR}{P_1}\right)^{\frac{k-1}{k}}$ i.e. $TSTAR = 666.565$ **K...temp. at throat.... Ans.**

$RHOSTAR := \frac{PSTAR \cdot 10^5}{R \cdot TSTAR}$ i.e. $RHOSTAR = 2.76$ **kg/m^3.... density at throat**

$VSTAR := \sqrt{k \cdot R \cdot TSTAR}$ i.e. $VSTAR = 517.519$ **m/s...velocity at throat**

$ASTAR := \frac{m}{RHOSTAR \cdot VSTAR}$ i.e. $ASTAR = 3.50053 \times 10^{-3}$ **m^2.... throat area ... Ans.**

===

Prob.9.3.13 In a C-D nozzle, air enters with stagnation pressure of 12 bar and temp. 627 C. Normal shock occurs at a section where M = 1.8. Exit area ratio is 2.5. Throat area is 500 mm^2. Find P. T, M, C, V, stagnation pressure loss just downstream of the shock and exit.[M.U.]

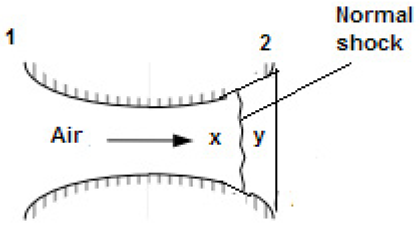


Fig.Prob.9.3.13 Normal shock in a C-D nozzle

Mathcad Function:

Data:

$P_0 := 12$ bar $\quad T_0 := 900$ K $\quad A_t := 500 \cdot 10^{-6}$ m^2 $\quad a2byastar := 2.5 \quad k := 1.4$

$A_2 := 1250 \cdot 10^{-6}$ mm^2 .... since exit area ratio is given as 2.5

$M_x := 1.8$ ....Mach No. before shock

Calculations:

For the given value of Mx, now refer to **Normal shock tables** and get:
(In our case, we use the Mathcad Functions written earlier)

$My := Machy(M_x, k)$ i.e. $My = 0.617$ .... Mach No. after shock

$pybypx := PYBYPX(M_x, k)$ i.e. $pybypx = 3.613$

$tybytx := TYBYTX(M_x, k)$ i.e. $tybytx = 1.532$

$p0ybyp0x := P0YBYP0X(M_x, k)$ i.e. $p0ybyp0x = 0.813$

$p0ybypx := P0YBYPX(M_x, k)$ i.e. $p0ybypx = 4.67$

$axbyastar := ABYASTAR(M_x, k)$ i.e. $axbyastar = 1.439$ ..see isentropic flow tables

$A_x := A_t \cdot axbyastar$ i.e. $A_x = 7.195 \times 10^{-4}$ m^2

$A_y := A_x$ ...since area does not change at section where shock occurs

When M = 1.8, from Isentropic table, get P/Po and T/To:

$tbyt0 := TBYT0(M_x, k)$ i.e. $tbyt0 = 0.607$

$pbyp0 := PBYP0(M_x, k)$ i.e. $pbyp0 = 0.174$

$P_x := P_0 \cdot pbyp0$ i.e. $P_x = 2.088$ **bar...pressure before shock**

$T_x := T_0 \cdot tbyt0$ i.e. $T_x = 546.117$ **K...temp. before the shock**

$P_y := P_x \cdot pybypx$ i.e. $P_y = 7.546$ **bar...pressure after the shock... Ans.**

$T_y := T_x \cdot tybytx$ i.e. $T_y = 836.42$ **K...temp. after the shock .... Ans.**

$P0y := P_x \cdot p0ybypx$ i.e. $P0y = 9.752$ **bar...stagn. pressure after the shock**

$C_y := \sqrt{k \cdot R \cdot T_y}$ i.e. $C_y = 579.718$ **m/s .... velocity of sound, after the shock .... Ans.**

$V_y := My \cdot C_y$ i.e. $V_y = 357.397$ **m/s,...velocity just after the shock .... Ans.**

$rhoy := \frac{P_y \cdot 10^5}{R \cdot T_y}$ i.e. $rhoy = 3.144$ kg/m^3 .... density after shock

$P0x := P_0$

$P0x - P0y = 2.248$ **bar.....stagn. pressure loss after the shock .... Ans.**

$deltas := -R \cdot \ln\left(\frac{P0y}{P0x}\right)$ i.e. $deltas = 59.528$ **J/kg.K.....Entropy change across the shock**

For My = 0.6165:

$aybyastar := ABYASTAR(My, k)$ i.e. $aybyastar = 1.169$

$a2byastar := \frac{A_2}{A_y} \cdot aybyastar$ i.e. $a2byastar = 2.032$

Now, for this value of A2/Astar, refer to Isentropic tables: (We use the Mathcad Functions written earlier):

$M_2 := MABYASTAR(a2byastar, k, 0.1)$ i.e. $M_2 = 0.301$ ...**Mach no. at exit,** corresp. to A2/Astar = 2.032; Note that after the shock, at exit M should be less than 1.

$p2byp0y := PBYP0(M_2, k)$ i.e. $p2byp0y = 0.939$

$t2byt0y := TBYT0(M_2, k)$ i.e. $t2byt0y = 0.982$

$P_2 := P0y \cdot p2byp0y$ i.e. $P_2 = 9.16$ **bar,....Pressure at exit... Ans.**

$T0y := T_0$ **...by energy eqn.**

$T_2 := t2byt0y \cdot T0y$ i.e. $T_2 = 884.029$ **K,...Temp. at exit ... Ans.**

$C_2 := \sqrt{k \cdot R \cdot T_2}$ i.e. $C_2 = 595.989$ **m/s ....velocity of sound at exit ... Ans.,**

$V_2 := C_2 \cdot M_2$ i.e. $V_2 = 179.127$ **m/s,..... velocity at exit .... Ans.**

==

Prob.9.3.14 Hot gases having cp = 1005 J/kg.K and k = 1.36 flow through a CD nozzle at a rate of 45 kg/s. The P, T and V of gas entering into the nozzle are: 105 kPa, 1100 K, and 180 m/s respectively. The discharge pressure is 35 kPa. Assuming the nozzle effcy. of 0.88, determine the throat and exit areas and exit temp. of gases.

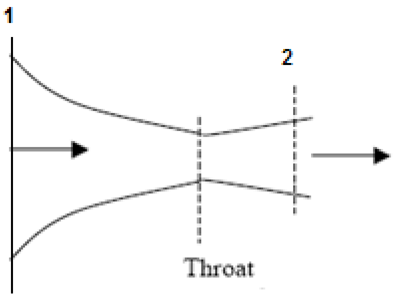


Fig.Prob.9.3.14 Flow in a C-D nozzle

Mathcad Solution:

Data:

$cp := 1005$ J/kg.K $k := 1.36$ $flow := 45$ kg/s $P1 := 105 \cdot 10^3$ Pa $P2 := 35 \cdot 10^3$ Pa

$T1 := 1100$ K $V1 := 180$ m/s $\eta := 0.88$ $Mt := 1$ ...Mach No. at throat

Calculations:

$R := cp \cdot \frac{k-1}{k}$ i.e. $R = 266.029$ J/kg.K

$\rho 1 := \frac{P1}{R \cdot T1}$ i.e. $\rho 1 = 0.359$ kg/m^3 ... density at inlet

$A1 := \frac{flow}{\rho 1 \cdot V1}$ i.e. $A1 = 0.697$ m^2 .... inlet area

$T0 := T1 + \frac{V1^2}{2 \cdot cp}$ i.e. $T0 = 1.116 \times 10^3$ K.... stagn. temp at inlet

$P0 := P1 \cdot \left(\frac{T0}{T1}\right)^{\frac{k}{k-1}}$ i.e. $P0 = 1.109 \times 10^5$ Pa .... stagn. pressure at inlet

$Tthroat := T0 \cdot TBYT0(Mt, k)$ i.e. $Tthroat = 945.864$ K.... temp at throat

$Pthroat := P0 \cdot PBYP0(Mt, k)$ i.e. $Pthroat = 5.936 \times 10^4$ Pa ... pressure at throat

$Vthroat := Mt \cdot \sqrt{k \cdot R \cdot Tthroat}$ i.e. $Vthroat = 584.99$ m/s ..... velocity at throat

$\rho t := \frac{Pthroat}{R \cdot Tthroat}$ i.e. $\rho t = 0.236$ kg/m^3 .... density at throat

$At := \frac{flow}{\rho t \cdot Vthroat}$ i.e. $At = 0.326$ **m^2 .... area at throat ... Ans.**

$T2 := T1 \cdot \left(\frac{P2}{P1}\right)^{\frac{k-1}{k}}$ i.e. $T2 = 822.424$ **K .... isentropic exit temp .... Ans.**

$\rho 2 := \frac{P2}{R \cdot T2}$ i.e. $\rho 2 = 0.16$ kg/m^3 ... isentr. density at exit

$V2_{isentr} := \sqrt{(T0 - T2) \cdot 2 \cdot cp}$ i.e. $V2_{isentr} = 768.328$ m/s .... isentropic exit vel.

$V2_{act} := \sqrt{\eta \cdot V2_{isentr}^2}$ i.e. $V2_{act} = 720.755$ m/s .... actual exit velocity

$T2_{act} := T0 - \frac{V2_{act}^2}{2 \cdot cp}$ i.e. $T2_{act} = 857.668$ K.... actual exit temp.... Ans.

$C2 := \sqrt{k \cdot R \cdot T2_{act}}$ i.e. $C2 = 557.05$ m/s .... velocity of sound at exit

$M2 := \frac{V2_{act}}{C2}$ i.e. $M2 = 1.294$ ... Mach No. at exit

$\rho2_{act} := \frac{P2}{R \cdot T2_{act}}$ i.e. $\rho2_{act} = 0.153$ kg/m^3 .... actual density at exit

$A2 := \frac{flow}{\rho2_{act} \cdot V2_{act}}$ i.e. $A2 = 0.407$ m^2 .... area at exit .... Ans.

===

Prob. 9.3.15 A C-D duct with a throat area 0.35 times the exit area is supplied with air at a stagnation pressure of 1.5 bar. It discharges into atmosphere with a static pressure of 100 kPa. Assuming that there is normal shock in the divergent part, find: Mach Nos., pressure just upstream and downstream of the shock, loss in stagnation pressure and area ratio at the section where shock occurs. [M.U.]

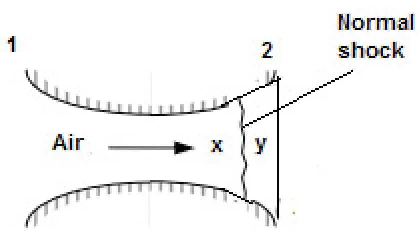


Fig.Prob.9.3.15 Normal shock in a C-D nozzle

Mathcad Solution:

Data:

$P2 := 1$ bar $P01 := 1.5$ bar $R := 287$ J/kg.K

At/A2 = 0.35; A1star = At = Atstar = Axstar; A2star = Aystar

i.e. A2/At = 1/0.35 = 2.857

Calculations:

Flow is isentropic from section 1 to x and then from y to 2.

Since there is a normal shock, flow is choked i.e. sonic velocity at throat.

Then: m *sqrt(T0)/(Astar*P0) = const. i.e. for const. m and T0, Astar*P0 = const.

i.e. A1star*P01 = A2star * P02 or, A1star/A2star = P02/P01

Also, P01 = P0t = P0x and P0y = P02

We can write: A2/At = (A2/A2star) * (A2star/At)

But, (At/A2star) = (At/A1star) * (A1star/A2star) = 1 * (P02/P01)

So, (A2/At) = (A2/A2star) * (P01/P02)

i.e. 2.857 = (A2/A2star) * (1.5/P02) ….(eqn. A)

Let apratio = (A2*P2)/(A2star*P02) …..(eqn. B)

Remembering that P2 = 100 kPa = 1 bar, we write, from eqns (A) and (B):

apratio = (2.857 * P2)/1.5

So, continuing in Mathcad:

$$apratio := \frac{2.857 \cdot 1}{1.5} \qquad \text{i.e.} \qquad apratio = 1.905$$

Then from isentropic tables, find M corresponding to this apratio.

We use Mathcad Function:

$$APRATIO(M,k) := \frac{\left(\frac{2}{k+1}\right)^{\frac{k+1}{2\cdot(k-1)}}}{M\cdot\left(1+\frac{k-1}{2}\cdot M^2\right)^{0.5}}$$

M2trial := 0.8   Trial value

Given

apratio = APRATIO(M2trial, k)

M2 := Find(M2trial)

i.e.   M2 = 0.303   **...Value of M2 ... Mach No. at exit**

Therefore:

$$\frac{P02}{P01} = 0.709$$ **...This is also equal to P0y/P0x**

Also: P0y := P02   and,   P0x := P01

Therefore:

P0x − P0y = 0.436   **..bar,....Loss in stagn. pr. in shock .... Ans.**

Now from Normal shock tables, for P0y/P0x = 0.709 find Mx and My:

We use Mathcad Function as shown below:

$Mxtrial := 1.1$ Trial value

Given

$$\frac{P02}{P01} = P0YBYP0X(Mxtrial, k)$$

$Mx := Find(Mxtrial)$

i.e. $Mx = 2.008$ ...Mach No. before shock .... Ans.

$My := Machy(Mx, k)$ i.e. $My = 0.57$ ....Mach No. after shock .... Ans.

Now, we have:

$pybypx := PYBYPX(Mx, k)$ i.e. $pybypx = 4.495$

And: $p0ybypx := P0YBYPX(Mx, k)$ i.e. $p0ybypx = 5.574$

Therefore:

$Px := \frac{P0y}{p0ybypx}$ i.e. $Px = 0.191$ bar.. pressure before shock ... Ans.

$Py := pybypx \cdot Px$ i.e. $Py = 0.858$ bar ..... pressure after shock ... Ans.

$Aratio := ABYASTAR(Mx, k)$ ...area ratio

$Aratio = 1.732$ ... Area ratio at the location of shock = Ax/Astar .... Ans.

$Dels := -R \cdot \ln\left(\frac{P0y}{P0x}\right)$ i.e. $Dels = 98.632$ J/kg.K.....Entropy change across shock

==

Prob.9.3.16 Air expands adiabatically in a nozzle from initial state of 6 bar, 527 C to a pressure of 2.5 bar. Calculate: (i) mass flow rate if throat dia is 2 cms (ii) velocity at the throat (iii) exit dia [M.U.]

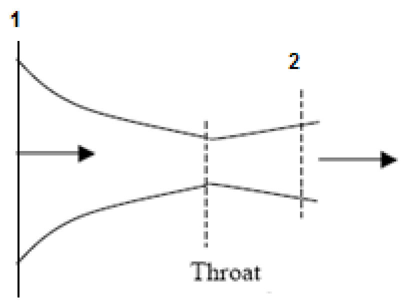


Fig.Prob.9.3.16 Flow in a C-D nozzle

Mathcad Solution:

Data:

$P_0 := 6$ bar $T_0 := 527 + 273$ K $P2 := 2.5$ bar $d := 0.02$ m

$k := 1.4$ $R := 287$ J/kg.K

Calculations:

First find out the critical pressure to see if the flow is choked:

$\frac{P2}{P_0} = 0.417$ **..this is less than 0.528; therefore, C-D nozzle is reqd.**

Therefore:

$pstar := P_0 \cdot 0.528$ i.e. $pstar = 3.168$ **...bar, throat pressure**

$athroat := \pi \cdot \frac{d^2}{4}$ i.e. $athroat = 3.142 \times 10^{-4}$ **m^2 .... throat area**

$tstar := 0.8333 \cdot T_0$ i.e. $tstar = 666.64$ **K, temp. at throat**

$vstar := \sqrt{k \cdot R \cdot tstar}$ i.e. $vstar = 517.548$ **m/s ...vel. at throat (sonic)...Ans.**

$rhostar := \dfrac{pstar \cdot 10^5}{R \cdot tstar}$ i.e. $rhostar = 1.656$ **kg/m^3 .... density at throat**

$m := rhostar \cdot athroat \cdot vstar$ i.e. $m = 0.269$ **kg/s ...mass flow rate...Ans.**

$T2 := T_0 \cdot \left(\dfrac{P2}{P_0}\right)^{\frac{k-1}{k}}$ i.e. $T2 = 622.957$ **K....temp. at exit**

$rho2 := \dfrac{P2 \cdot 10^5}{R \cdot T2}$ i.e. $rho2 = 1.398$ **kg/m^3 ... density at exit**

For V2, velocity at exit, apply Energy eqn. between sections 1 & 2:

$cp := \dfrac{k \cdot R}{k - 1}$ i.e. $cp = 1.005 \times 10^3$ J/kg.K ... sp. heat

$V2 := \sqrt{2 \cdot cp \cdot (T_0 - T2)}$ i.e. $V2 = 596.388$ **m/s....vel. at exit**

$A2 := \dfrac{m}{rho2 \cdot V2}$ i.e. $A2 = 3.228 \times 10^{-4}$ **m^2....exit area**

$d2 := \sqrt{\dfrac{A2 \cdot 4}{\pi}} \cdot 100$ i.e. $d2 = 2.027$ **cm,....exit diameter.....Ans.**

==

Prob.9.3.17 Consider a C-D nozzle of exit area ratio 3. Air at 5 bar, 127 C flows through the nozzle with a velocity of 200 m/s. If a normal shock stands at the point where Mach No. is 2, find fluid properties at a section just immediately after the shock.[M.U.]

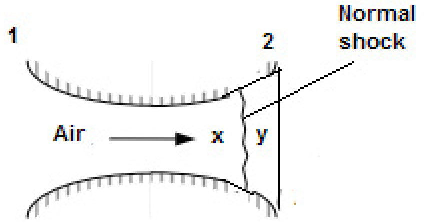


Fig.Prob.9.3.17 Normal shock in a C-D nozzle

Mathcad Solution:

Data:

$P1 := 5$ bar $T1 := 400$ K $V1 := 200$ m/s $a2byastar := 3$ $k := 1.4$ $R := 287$ J/kg.K

$M_x := 2$ ...Mach No. before shock

Calculations:

For Mx = 2, now refer to Normal shock tables. We, of course, use Mathcad Functions written earlier, and get:

$My := Machy(M_x, k)$ i.e. $My = 0.577$ **Mach No. after shock .... Ans.**

$pybypx := PYBYPX(M_x, k)$ i.e. $pybypx = 4.5$

$tybytx := TYBYTX(M_x, k)$ i.e. $tybytx = 1.687$

$p0ybyp0x := P0YBYP0X(M_x, k)$ i.e. $p0ybyp0x = 0.721$

$p0ybypx := P0YBYPX(M_x, k)$ i.e. $p0ybypx = 5.64$

When M=2, from Isentropic table, get P/Po and T/To:

$tbyt0 := TBYT0(M_x, k)$ i.e. $tbyt0 = 0.556$

$pbyp0 := PBYP0(M_x, k)$ i.e. $pbyp0 = 0.128$

To calculate T0, apply Energy eqn.:

$T0 := T1 + \frac{V1^2}{2 \cdot cp}$ i.e. $T0 = 419.91$ **K.... stagnation temp.**

$P0 := P1 \cdot \left(\frac{T0}{T1}\right)^{\frac{k}{k-1}}$ i.e. $P0 = 5.927$ **bar ... stagnation pressure**

$P_x := P0 \cdot pbyp0$ i.e. $P_x = 0.757$ **bar...pressure before shock ... Ans.**

$T_x := T0 \cdot tbyt0$ i.e. $T_x = 233.284$ **K...temp. before the shock .... Ans.**

$P_y := P_x \cdot pybypx$ i.e. $P_y = 3.409$ **bar...pressure after the shock .... Ans.**

$T_y := T_x \cdot tybytx$ i.e. $T_y = 393.666$ **K...temp. after the shock .... Ans.**

$P0y := P_x \cdot p0ybypx$ i.e. $P0y = 4.272$ **bar...stagn. pr. after the shock .... Ans.**

$rhoy := \frac{P_y \cdot 10^5}{R \cdot T_y}$ i.e. $rhoy = 3.017$ **kg/m^3 ... density after shock ... Ans.**

$P0x := P0$ **...stagn. pressure before shock**

$P0x - P0y = 1.654$ **bar.....stagn. pr. loss after the shock .... Ans.**

$deltas := -R \cdot \ln\left(\frac{P0y}{P0x}\right)$ i.e. $deltas = 93.933$ **J/kg.K.....Entropy change across the shock**

(b) Plot the static and stagn. pressure, temp. before the shock and after the shock against M_x, as M_x varies from 1 to 2:

First write the relevant quantities as functions of M_x:

$$My(M_x, k) := Machy(M_x, k)$$

$$T_x(M_x, k) := T0 \cdot TBYT0(M_x, k)$$

$$P_x(M_x, k) := P0 \cdot PBYP0(M_x, k)$$

$$T_y(M_x, k) := T_x(M_x, k) \cdot TYBYTX(M_x, k)$$

$$P_y(M_x, k) := P_x(M_x, k) \cdot PYBYPX(M_x, k)$$

$$P0_y(M_x, k) := P_x(M_x, k) \cdot P0YBYPX(M_x, k)$$

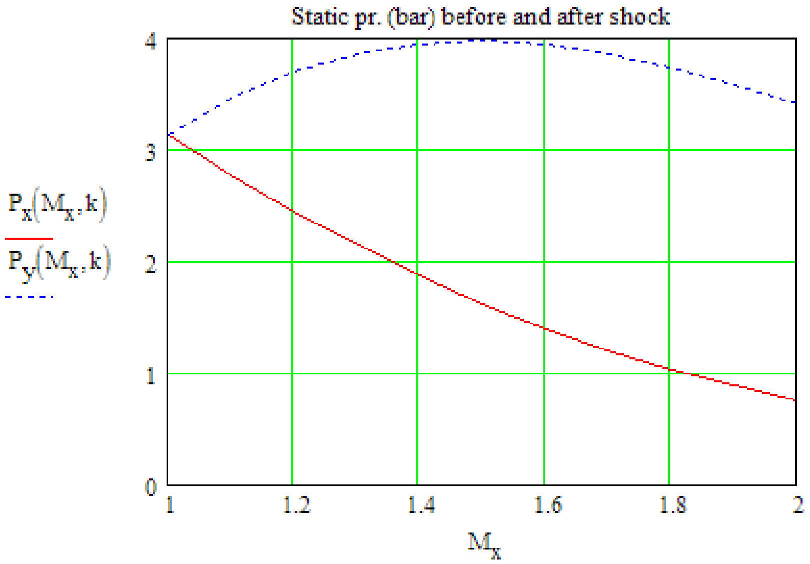
$$DELTAS(M_x, k) := -R \cdot \ln\left(\frac{P0_y(M_x, k)}{P0x}\right)$$

Now, plot the results:

1. **Static pressures before and after shock:**

$M_x := 1, 1.1..2$ ...define a range variable

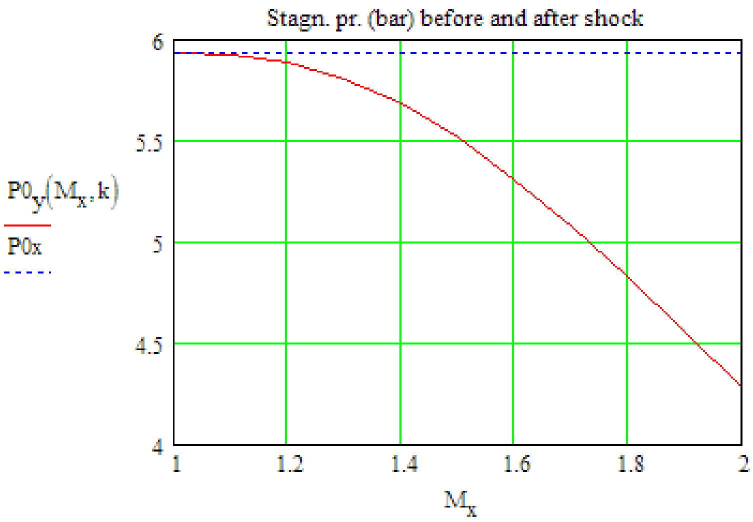
$M_x =$	$P_x(M_x, k) =$	$P_y(M_x, k) =$
1	3.131	3.131
1.1	2.776	3.456
1.2	2.444	3.699
1.3	2.139	3.861
1.4	1.862	3.948
1.5	1.614	3.969
1.6	1.394	3.932
1.7	1.201	3.848
1.8	1.031	3.727
1.9	0.884	3.578
2	0.757	3.409



2. **Stagnation pressures before and after shock:**

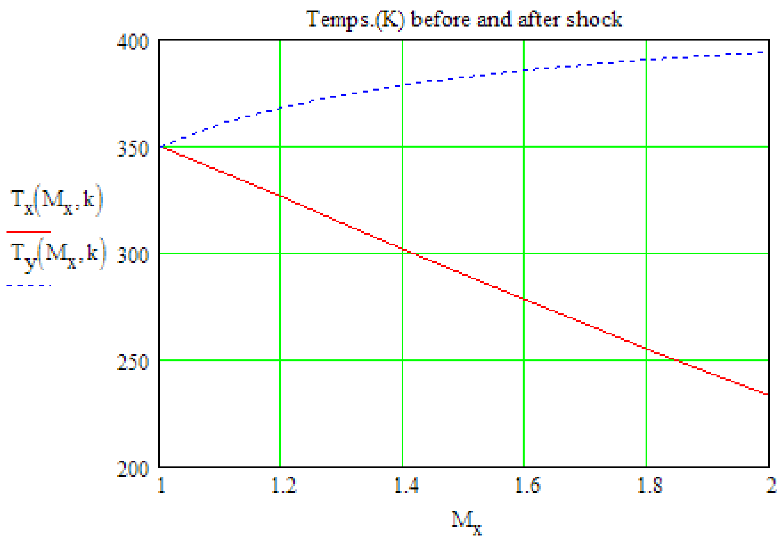
Note: Stagn. Pressure before shock = P0 is constant.

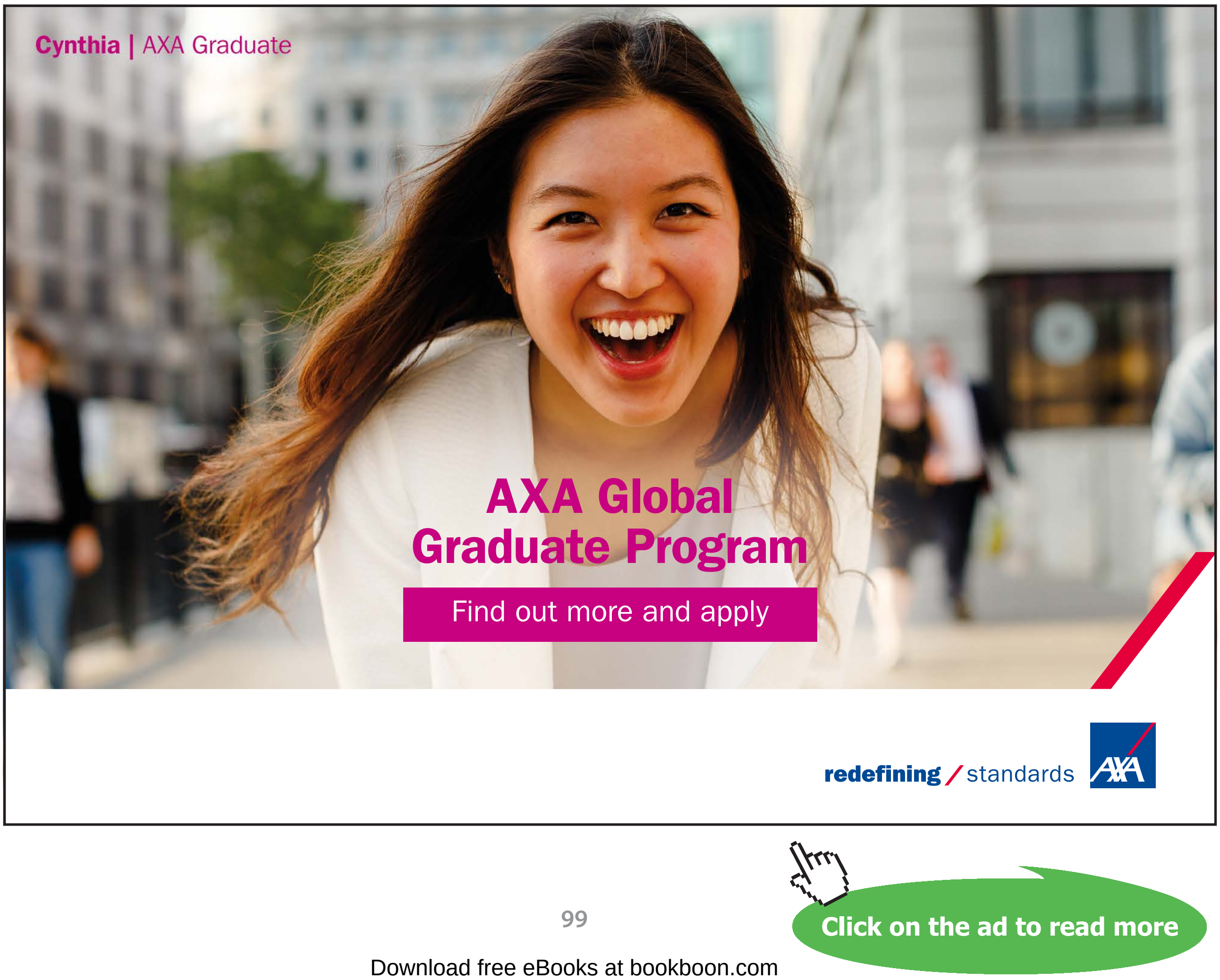
M_x =	$P0_y(M_x, k)$ =
1	5.927
1.1	5.92
1.2	5.884
1.3	5.804
1.4	5.679
1.5	5.511
1.6	5.306
1.7	5.072
1.8	4.816
1.9	4.548
2	4.272



3. **Static temps. before and after shock:**

M_x =	$T_x(M_x, k)$ =	$T_y(M_x, k)$ =
1	349.925	349.925
1.1	338.092	360.047
1.2	326.017	367.746
1.3	313.834	373.737
1.4	301.66	378.491
1.5	289.593	382.326
1.6	277.719	385.465
1.7	266.103	388.066
1.8	254.8	390.246
1.9	243.85	392.091
2	233.284	393.666

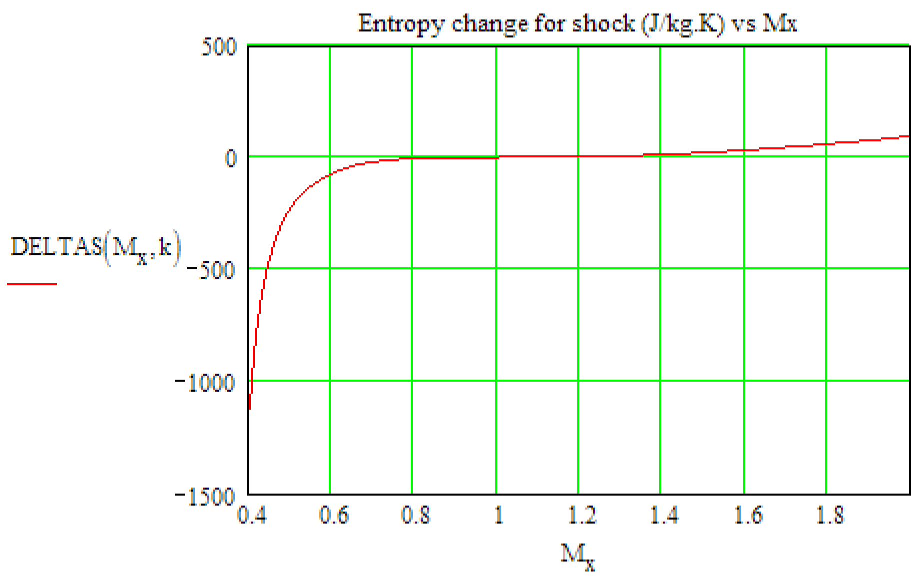

Temps.(K) before and after shock
400
350
300
250
200
1
1.2
1.4
1.6
1.8
2
Tx(Mx,k)
Ty(Mx,k)
Mx



(c) Plot entropy change across the shock against M_x, as M_x varies from 0.4 to 2. What is the inference from the plot?

$M_x := 0.4, 0.5 .. 2$ ....define a range variable

$M_x =$	$DELTAS(M_x, k) =$
0.4	$-1.118 \cdot 10^{3}$
0.5	-233.599
0.6	-72.217
0.7	-21.196
0.8	-4.7
0.9	-0.46
1	$3.186 \cdot 10^{-14}$
1.1	0.308
1.2	2.074
1.3	5.982
1.4	12.256
1.5	20.894
1.6	31.773
1.7	44.718
1.8	59.528
1.9	75.999
2	93.933



Note: We know that Normal shock is highly irreversible, and therefore, *entropy must increase* across the shock.

From the plot above, we observe that:

Up to Mach No. = 1, entropy change is –ve, which is impossible. Therefore, for a normal shock to occur, Mach No. before the shock (Mx) **must be** more than 1. i.e. Normal shock can occur only in a supersonic flow, i.e. in the divergent part of a C-D nozzle.

==

Prob.9.3.18 In a C-D nozzle, air enters with stagnation pressure of 10 bar and temp. 360 K. Exit area ratio is 2.0. Throat area is 500 mm^2. Find stagnation exit pressure, temp, Mach No., stagnation pressure loss just downstream of the shock and exit, when (i) Normal shock occurs at a section where M = 1.5.

(ii) Normal shock stands at the exit plane of nozzle. [M.U.]

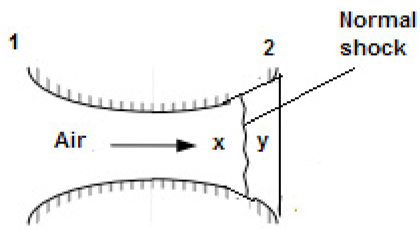


Fig.Prob.9.3.18 Normal shock in a C-D nozzle

Mathcad Solution:

Data:

$P_0 := 10$ bar $T_0 := 360$ K a2byastar := 2.0 $k := 1.4$ $R := 287$ J/kg.K

$A_t := 500 \cdot 10^{-6}$ m^2 ... throat area

$A_2 := 1000 \cdot 10^{-6}$ mm^2 ... exit area

$M_x := 1.5$ ...Mach No. before shock

Calculations:

For Mx = 1.5, now refer to Normal shock tables.

Of course, we use Mathcad Functions written earlier. We get:

$My := Machy(M_x, k)$ i.e. $My = 0.701$ **Mach No. after shock .... Ans.**

$pybypx := PYBYPX(M_x, k)$ i.e. $pybypx = 2.458$

$tybytx := TYBYTX(M_x, k)$ i.e. $tybytx = 1.32$

$p0ybyp0x := P0YBYP0X(M_x, k)$ i.e. $p0ybyp0x = 0.93$

$p0ybypx := P0YBYPX(M_x, k)$ i.e. $p0ybypx = 3.413$

$axbyastar := ABYASTAR(M_x, k)$ i.e. $axbyastar = 1.176$

$A_x := A_t \cdot axbyastar$ i.e. $A_x = 5.881 \times 10^{-4}$ m^2

$A_y := A_x$ ...since shock occurs at a thin section

When M=1.5, from Isentropic table (or, using Mathcad Functions), get P/Po and T/To:

$tbyt0 := TBYT0(M_x, k)$ i.e. $tbyt0 = 0.69$

$pbyp0 := PBYP0(M_x, k)$ i.e. $pbyp0 = 0.272$

$P_x := P_0 \cdot pbyp0$ i.e. $P_x = 2.724$ bar...pressure before shock

$T_x := T_0 \cdot tbyt0$ i.e. $T_x = 248.276$ K...temp. before the shock

$P_y := P_x \cdot pybypx$ i.e. $P_y = 6.697$ bar...pressure after the shock

$T_y := T_x \cdot tybytx$ i.e. $T_y = 327.778$ K...temp. after the shock .... Ans.

$P0y := P_x \cdot p0ybypx$ i.e. $P0y = 9.298$ bar...stagn. pr. after the shock .... Ans.

$C_y := \sqrt{k \cdot R \cdot T_y}$ i.e. $C_y = 362.906$ m/s ...sonic velocity

$V_y := My \cdot C_y$ i.e. $V_y = 254.43$ m/s,...vel. just after the shock

$rhoy := \frac{P_y \cdot 10^5}{R \cdot T_y}$ i.e. $rhoy = 7.119$ kg/m^3 ... density after shock

$P0x := P_0$ ...since stagn. pressure before shock is equal to that at inlet

$P0x - P0y = 0.702$ bar.....stagn. pressure loss after the shock ... Ans.

Now: $deltas := -R \cdot \ln\left(\frac{P0y}{P0x}\right)$

i.e. $deltas = 20.894$ J/kg.K.....Entropy change across the shock .... Ans.

For My = 0.701: $aybyastar := ABYASTAR(My, k)$

i.e. $aybyastar = 1.094$

Therefore:

$a2byastar := \frac{A_2}{A_y} \cdot aybyastar$ i.e. $a2byastar = 1.86$

Now, for this value of A2/Astar, refer to Isentropic tables (or, use Mathcad Functions):

$M_2 := MABYASTAR(a2byastar, k, 0.1)$

i.e. $M_2 = 0.332$ **...Mach no. at exit, corresp. to A2/Astar = 1.86; Note that after the shock, at exit M should be less than 1.**

$p2byp0y := PBYP0(M_2, k)$ i.e. $p2byp0y = 0.926$

$t2byt0y := TBYT0(M_2, k)$ i.e. $t2byt0y = 0.978$

$P_2 := P0y \cdot p2byp0y$ i.e. $P_2 = 8.614$ **bar,....Pressure at exit**

$T0y := T_0$ **...by energy eqn.**

Then:

$T_2 := t2byt0y \cdot T0y$ i.e. $T_2 = 352.222$ **K,...Temp. at exit ... Ans.**

$C_2 := \sqrt{k \cdot R \cdot T_2}$ i.e. $C_2 = 376.195$ **m/s ...sonic velocity at exit**

$V_2 := C_2 \cdot M_2$ i.e. $V_2 = 125.001$ **m/s,..... velocity at exit .. Ans.**

(ii) When shock is at exit plane, i.e. A2/Astar = 2:

$a2byastar := 2$

Now, for this value of A2/Astar, refer to Isentropic tables, or use Mathcad Functions:

$M_x := MABYASTAR(a2byastar, k, 2)$ i.e. $M_x = 2.197$ ...more than 1

$M_x := 2.197$ **...For this Mx, now refer to Normal shock tables and get:**

$My := Machy(M_x, k)$ i.e. $My = 0.547$ **Mach No. after shock ... Ans.**

$pybypx := PYBYPX(M_x, k)$ i.e. $pybypx = 5.465$

$tybytx := TYBYTX(M_x, k)$ i.e. $tybytx = 1.854$

$p0ybyp0x := P0YBYP0X(M_x, k)$ i.e. $p0ybyp0x = 0.63$

$p0ybypx := P0YBYPX(M_x, k)$ i.e. $p0ybypx = 6.7$

When M=2.197, from Isentropic table, get P/Po and T/To:

$tbyt0 := TBYT0(M_x, k)$ i.e. $tbyt0 = 0.509$

$pbyp0 := PBYP0(M_x, k)$ i.e. $pbyp0 = 0.094$

$P_x := P_0 \cdot pbyp0$ i.e. $P_x = 0.94$ **bar...pressure before shock**

$T_x := T_0 \cdot tbyt0$ i.e. $T_x = 183.172$ K...temp. before the shock

$P_y := P_x \cdot pybypx$ i.e. $P_y = 5.135$ bar...pressure after the shock .. Ans.

$T_y := T_x \cdot tybytx$ i.e. $T_y = 339.641$ K...temp. after the shock ... Ans.

$P0y := P_x \cdot p0ybypx$ i.e. $P0y = 6.295$ bar...stagn. pr. after the shock

$C_y := \sqrt{k \cdot R \cdot T_y}$ i.e. $C_y = 369.416$ m/s .... sonic vel.

$V_y := My \cdot C_y$ i.e. $V_y = 202.24$ m/s,...vel. just after the shock ... Ans.

$rhoy := \dfrac{P_y \cdot 10^5}{R \cdot T_y}$ i.e. $rhoy = 5.268$ kg/m^3 .... density after shock

$P0x := P_0$

$P0x - P0y = 3.705$ bar.....stagn. pressure loss after the shock ... Ans.

And: $deltas := -R \cdot \ln\left(\dfrac{P0y}{P0x}\right)$

i.e. $deltas = 132.831$ J/kg.K.....Entropy change across the shock ... Ans.

===

Prob.9.3.19 A C-D nozzle has cross-section of 15 cm at the throat and 20 cm at exit; air leaves the nozzle at 1 bar and 27 C and M = 1.8. At inlet to the nozzle, the stagnation pressure and temp. are 7 bar and 210 C respectively. Calculate: (i) Discharge coeff. (ii) Efficiency of nozzle

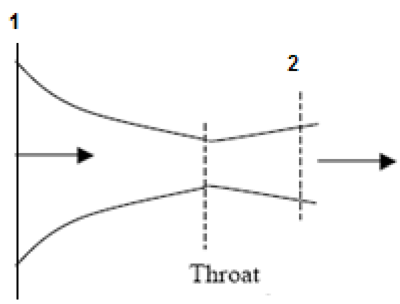


Fig.Prob.9.3.19 Flow in a C-D nozzle

Mathcad Solution:

Data:

$P_0 := 7$ bar $\quad T_0 := 210 + 273$ K

$P_2 := 1$ bar $\quad T_2 := 27 + 273$ K $\quad M_2 := 1.8$ ...Mach No. at exit

$D_2 := 0.2$ m $\quad D_t := 0.15$ m $\quad A2 := \pi \cdot \frac{D_2^{\,2}}{4}$ i.e. $A2 = 0.031$ m^2

$R = 287$ J/kg.K $\quad k := 1.4$

Calculations:

$a2byastar := \left(\frac{D_2}{D_t}\right)^2$ i.e. $a2byastar = 1.778$

Mach No. in the diverging part of C-D nozzle, corresponding to this area ratio:

$MABYASTAR(a2byastar, k, 4) = 2.062$ **....Mach No. at exit for isentropic flow**

$C2 := \sqrt{k \cdot R \cdot T_2}$ i.e. $C2 = 347.189$ m/s .... vel. of sound

$V2 := M_2 \cdot C2$ i.e. $V2 = 624.94$ m/s .... vel. at exit

$\rho 2 := \frac{P_2 \cdot 10^5}{R \cdot T_2}$ i.e. $\rho 2 = 1.161$ kg/m^3 .... density at exit

$m2 := \rho 2 \cdot A2 \cdot V2$ i.e. $m2 = 22.803$ **kg/s.....actual flow rate**

(i) Dicharge coeff, C_D:

CD is defined as: $\quad C_D = \frac{Actual\_mass\_rate\_of\_flow}{Mass\_rate\_pf\_flow\_for\_isentropic\_flow}$

Now, mass flow remains practically the same, i.e. a maximum, as long as the velocity at the throat is sonic.

Therefore: $C_D = 1$ **in this case,. .... Ans.**

(ii) Nozzle effcy η_N:

When M2 = 2.062, from Isentropic table (or, using Mathcad Functions), get P/Po and T/To:

$M2 := 2.062$ ...at exit, for isentropic flow

Then, we have:

$tbyt0 := TBYT0(M2, k)$ i.e. $tbyt0 = 0.54$

$pbyp0 := PBYP0(M2, k)$ i.e. $pbyp0 = 0.116$

Therefore:

$T2_{isentr} := T_0 \cdot tbyt0$ i.e. $T2_{isentr} = 261.029$ K

$P2_{isentr} := P_0 \cdot pbyp0$ i.e. $P2_{isentr} = 0.812$ bar

$C2_{isentr} := \sqrt{k \cdot R \cdot T2_{isentr}}$ i.e. $C2 = 347.189$ m/s...sonic vel. at exitt

$V2_{isentr} := M2 \cdot C2_{isentr}$ i.e. $V2_{isentr} = 667.787$ **m/s...vel. at exit, isentr. flow**

Nozzle effcy = actual KE/ Isentr. KE

$\eta_N := \dfrac{V2^2}{V2_{isentr}^{\ 2}}$ i.e. $\eta_N = 0.876$ **...=87.6%, Nozzle efficiency ... Ans.**

==

Prob. 9.3.20 Air is flowing through a pipe of 20 mm dia, 40 m length. Conditions at the exit of the pipe are: M = 0.5, pressure = 1 bar, temp. = 270 K. Assuming adiabatic, one dimensional flow with coeff. of friction of 0.005, calculate Mach No., static pressure and temp. at the entrance of pipe. What is the max. length to get choked condition ? [M.U.]

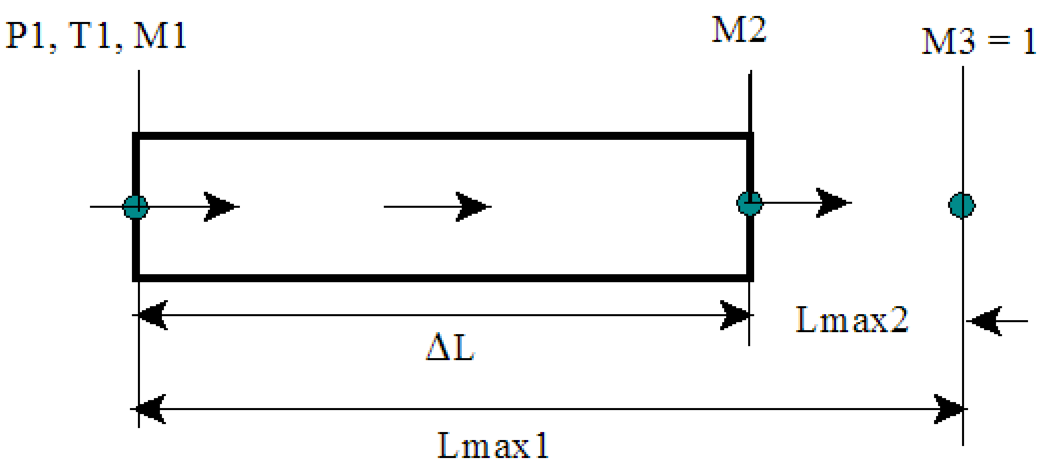


Fig.Prob.9.3.20 Fanno flow

Mathcad Solution:

This s adiabatic flow, with friction. i.e. it is Fanno flow.

Therefore, we use Mathcad Functions for Fanno flow, written earlier.

Data:

$D := 0.02$ m $P_2 := 1$ bar $M_2 := 0.5$ ...Mach No. at exit

$L := 40$ m $T_2 := 270$ K

$f := 0.005$ ...Darcy friction factor = 4. ff where ff is fanning friction factor.

At M2 = 0.5:

$p2bypstar := PBYPSTAR(M_2, k)$ i.e. $p2bypstar = 2.138$

$v2byvstar := VBYVSTAR(M_2, k)$ i.e. $v2byvstar = 0.535$ ..This is also equal to: rhostar/rho

$t2bytstar := p2bypstar \cdot v2byvstar$ i.e. $t2bytstar = 1.143$

$fourflmaxbyd := FOURFLMAXBYD(M_2, k)$ i.e. $fourflmaxbyd = 1.069$

Then: $Lmax := fourflmaxbyd \cdot \frac{D}{4 \cdot f}$ i.e. $Lmax = 1.069$ m ... distance downstream from exit, where M would be equal to 1

Now, to find Mach No. M1 at inlet:

Now, Lmax is 41.069 m from entrance. And 4 f Lmax / D = 41.069 = A , say.

Then: $A := 41.069$

Corresponding to this value of A, find out M1:

$M := 0.5$ Trial value

Given

$FOURFLMAXBYD(M, k) = A$

$M_1 := Find(M)$

$M_1 = 0.126$ **...Mach No. at entrance....Ans.**

Corresponding to this M1, find p/pstar, etc.

$p1bypstar := PBYPSTAR(M_1, k)$ $p1bypstar = 8.699$

$v1byvstar := VBYVSTAR(M_1, k)$ $v1byvstar = 0.138$

$t1bytstar := TBYTSTAR(M_1, k)$ $t1bytstar = 1.196$

Threrefore: $P_1 := \frac{P_2 \cdot p1bypstar}{p2bypstar}$ $P_1 = 4.069$ **bar...static pressure at entrance...Ans.**

$T_1 := \frac{T_2 \cdot t1bytstar}{t2bytstar}$ $T_1 = 282.606$ **K...static temp. at entrance....Ans.**

Max. length to get choked condition:

At choked condition, M = 1

Therefore: **Lmax = 41.069 m ...Ans.**

===

Prob.9.3.21 Air enters a duct of constant area with pressure of 5 bar, temp. 150 C and velocity 100 m/s. Heat transfer to air takes place at constant rate, and pressure at the outlet is 4 bar. There is no work transfer. Assume frictionless flow and calculate exit temp and heat transfer per unit mass flow.[M.U.]

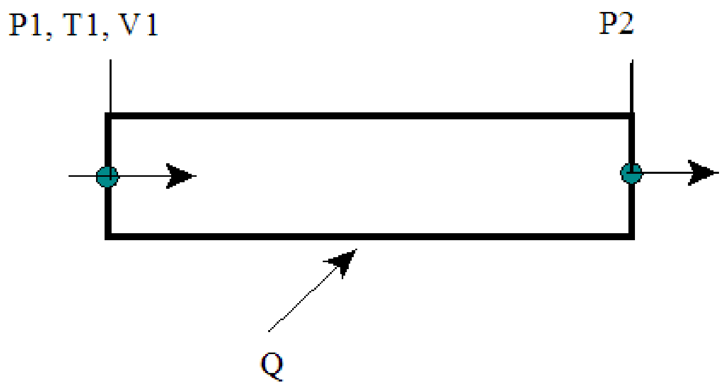


Fig.Prob.9.3.21 Rayleigh flow

Mathcad Solution:

This is Rayleigh flow. i.e. heat transfer with no friction.

Governing eqns. are: mass flow and Momentum.

Data:

$P1 := 5 \cdot 10^5$ Pa $\quad P2 := 4 \cdot 10^5$ Pa $\quad cp := 1005$ J/kg.K $\quad R := 287$ J/kg.K

$T1 := 150 + 273$ K $\quad V1 := 100$ m/s $\quad \rho1 := \frac{P1}{R \cdot T1}$ i.e. $\rho1 = 4.119$ kg/m^3

Calculations:

Solve the eqns. of continuity, momentum and energy along with eqn. of state.

Use Solve block of Mathcad.

Trial values: $T2 := 100 \quad \rho2 := 1 \quad V2 := 100 \quad Q := 100$

Given

$$\rho2 = \frac{P2}{R \cdot T2}$$ ...Eqn. of State

$$\rho1 \cdot V1 = \rho2 \cdot V2$$ ...Continuity eqn.

$$P1 + \rho1 \cdot V1^2 = P2 + \rho2 \cdot V2^2$$ ...Momentum eqn.

$$cp \cdot T1 + \frac{V1^2}{2} + Q = cp \cdot T2 + \frac{V2^2}{2}$$ ...Energy eqn.

$A := \text{Find}(\rho2, V2, T2, Q)$

$\rho2 := A_0$ i.e. $\rho2 = 1.201$ kg/m^3 .... density

$V2 := A_1$ i.e. $V2 = 342.802$ **m/s .... velocity at exit**

$T2 := A_2$ i.e. $T2 = 1.16 \times 10^3$ **K .... exit temp....Ans.**

$Q := A_3$ i.e. $Q = 7.945 \times 10^5$ **W .... heat transfer per unit mass flow .... Ans.**

==

Prob.9.3.22 Air flows with negligible friction through a 10 cm dia duct at a rate of 2.3 kg/s. Temp and pressure at inlet are: T1 = 450 K and P1 = 200 kPa. Mach No. at exit is M2 = 1. Determine the rate of heat transfer and the pressure drop for this section of duct. [Ref: 1]

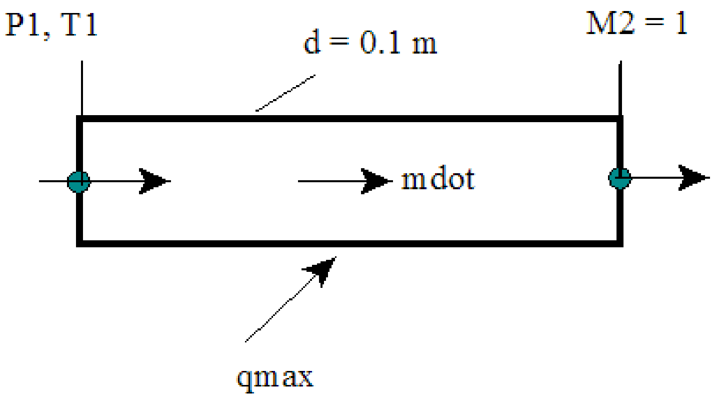


Fig.Prob.9.3.22 Rayleigh Flow

Mathcad Solution:

This is Rayleigh flow. i.e. heat transfer with no friction.

Data:

$P1 := 200 \cdot 10^3$ Pa $T1 := 450$ K $cp := 1005$ J/kg.K $R := 287$ J/kg.K

$M2 := 1$ $d := 0.1$ m...dia of pipe $mdot := 2.3$ kg/s.... mass flow rare of air

Calculations:

$A := \frac{\pi \cdot d^2}{4}$ i.e. $A = 7.854 \times 10^{-3}$ m^2....cross-sectional area of pipe

$\rho1 := \frac{P1}{R \cdot T1}$ i.e. $\rho1 = 1.549$ kg/m^3 ... density at inlet

Find velocity V1 at inlet:

$V1 := \frac{mdot}{\rho1 \cdot A}$ i.e. $V1 = 189.105$ m/s .... inlet velocity

And, sonic velocity at inlet: $C1 := \sqrt{k \cdot R \cdot T1}$ i.e. $C1 = 425.218$ m/s

Therefore, Stagn. temp T01 and Mach No. M1 at inlet:

$T01 := T1 + \frac{V1^2}{2 \cdot cp}$ i.e. $T01 = 467.791$ **K .... stagn. temp at inlet**

$M1 := \frac{V1}{C1}$ i.e. $M1 = 0.445$ **...Mach No. at inlet**

We have:

$\frac{T01}{T0star} = \text{RAYLEIGH\_T0BYT9STAR}(M1, k)$

i.e. $T0star := \frac{T01}{\text{RAYLEIGH\_T0BYT0STAR}(M1, k)}$

i.e. $T0star = 772.839$ K.... stagn. temp at M = 1.

Further, since we have M1 and M2, we can use Rayleigh flow functions to get exit parameters as follows:

Ex: (V2/V1) = (V2/Vstar) / (V1/Vstar) etc.

$$V2 := V1 \cdot \frac{RAYLEIGH\_VBYVSTAR(M2,k)}{RAYLEIGH\_VBYVSTAR(M1,k)} \quad \text{i.e.} \quad V2 = 508.701$$ m/s ...exit vel. ... Ans.

Similarly:

$$P2 := P1 \cdot \frac{RAYLEIGH\_PBYPSTAR(M2,k)}{RAYLEIGH\_PBYPSTAR(M1,k)} \quad \text{i.e.} \quad P2 = 1.064 \times 10^{5}$$ Pa...exit pressure. ... Ans.

$$T2 := T1 \cdot \frac{RAYLEIGH\_TBYTSTAR(M2,k)}{RAYLEIGH\_TBYTSTAR(M1,k)} \quad \text{i.e.} \quad T2 = 644.045$$ K ....exit temp. ... Ans.

Max. possible heat transfer, qmax occurs when M2 = 1:

Then: T02 = T0star

Therefore:

Max. heat transfer, qmax:

$$q_{max} := cp \cdot (T0star - T01) \quad \text{i.e.} \quad q_{max} = 3.0657 \times 10^{5}$$ J/kg .... heat transfer .... Ans.

Verify with the Mathcad Function for Q written earlier:

$$qmax := RAYLEIGH\_Q(M1,M2,T1,cp,k) \quad \text{i.e.} \quad qmax = 3.0658 \times 10^{5}$$ J/kg ... verified.

Pressure drop:

$$\Delta P := P1 - P2 \quad \text{i.e.} \quad \Delta P = 9.359 \times 10^{4}$$ Pa.... pressure drop ...Ans.

Entropy change, ΔS:

$$\Delta s := RAYLEIGH\_DELTAS(M1,M2,R,k) \quad \text{i.e.} \quad \Delta s = 541.242$$ J/kg.K ... Ans.

==

9.4 Problems solved with EES:

$UnitSystem SI Pa K J

"**Prob.9.4.1** Write EES Functions for property variations of an ideal gas in an isentropic flow."

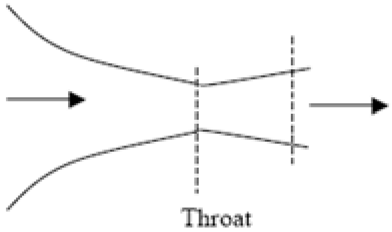


Fig.Prob.9.4.1 Isentropic flow

"EES Functions for Isentropic flow:"

"For stagnation temp:"

FUNCTION STAGNATION_T0(T, V, cp)

"Stagnation temp (K)"

"Inputs: T – static temp, K, V – velocity, m/s, cp – sp. heat J/kg.K"

"Outputs: Stagn. temp, T0 (K)"

STAGNATION_T0:= T + V^2 / (2 * cp)

END

"==="

"For T/T0:"

FUNCTION TBYT0(M,k)

"Inputs: M – Mach No., k -- ratio of sp. heats (= 1.4 for air)"

"Outputs: T/T0"

TBYT0 := (1 + ((k – 1) / 2) * M^2)^(-1)

END

"==="

“For P/P0:”

FUNCTION PBYP0(M,k)

“Inputs: M – Mach No., k -- ratio of sp. heats (= 1.4 for air)”

“Outputs: P/P0”

PBYP0 := (1 + ((k – 1) / 2) * M^2)^(-k/(k – 1))

END

“==”

“For rho/rho0:”

FUNCTION RHOBYRHO0(M,k)

“Inputs: M – Mach No., k -- ratio of sp. heats (= 1.4 for air)”

“Outputs: rho/rho0”

```
RHOBYRHO0 := (1 + ((k – 1) / 2) * M^2)^(-1/(k – 1))

END

"=========================================================================="

"For Mstar:"

FUNCTION MSTAR(M,k)

"Inputs: M – Mach No., k -- ratio of sp. heats (= 1.4 for air)"

"Outputs: Mstar"

MSTAR := M * sqrt((k + 1) / ( 2 + (k – 1) * m^2))

END

"=========================================================================="

"For A/Astar:"

FUNCTION ABYASTAR(M,k)

"Inputs: M – Mach No., k -- ratio of sp. heats (= 1.4 for air)"

"Outputs: A / Astar"

ABYASTAR := (1/M) * ((2 / (k + 1)) + (k – 1) * M^2/ (k + 1))^ ((k + 1) / (2 * (k – 1)))

END

"=========================================================================="

"For F/Fstar:"

FUNCTION FBYFSTAR(M,k)

"Inputs: M – Mach No., k -- ratio of sp. heats (= 1.4 for air)"

"Outputs: F / Fstar"
```

```
AA := 1 + k * M^2

BB:= 1 + ((k – 1)/2) * M^2

FBYFSTAR := (1/M) * AA / sqrt(2 * (1 + k) * BB)

END

"=========================================================================="

"For F/Fstar:"
FUNCTION APRATIO(M,k)

"Inputs: M – Mach No., k -- ratio of sp. heats (= 1.4 for air)"

"Outputs: (A * P) / ( Astar * P0)"

AA := (1/M) * ( 2 / (k + 1)) ^ ((k + 1) / (2 * (k – 1)))

BB:= sqrt(1 + ((k – 1)/2) * M^2)

APRATIO :=AA / BB

END

"=========================================================================="
```

"**Prob.9.4.2** Using the EES Functions for property variations of an ideal gas in an isentropic flow, written above, plot the property variations against Mach No."

EES Solution:

```
{M = 1}

 k = 1.4

Mstar = MSTAR(M,k)

abyastar = ABYASTAR(M,k)
```

tbyt0 = TBYT0(M,k)

pbyp0 = PBYP0(M,k)

rhobyrho0 = RHOBYRHO0(M,k)

fbyfstar = FBYFSTAR(M,k)

apratio = APRATIO(M,k)

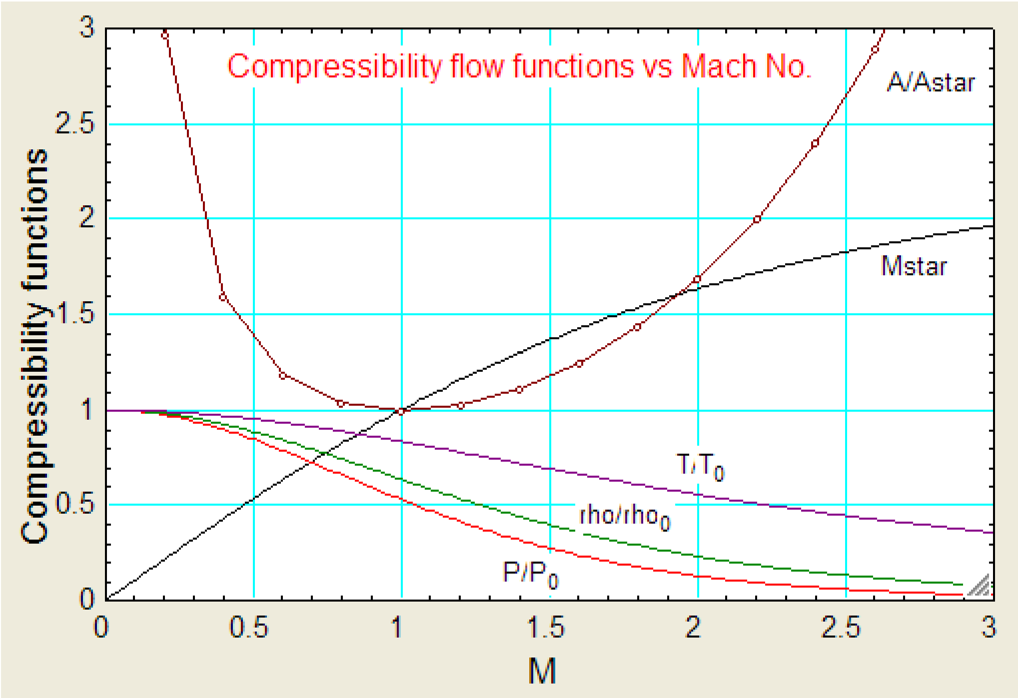
Parametric Table:

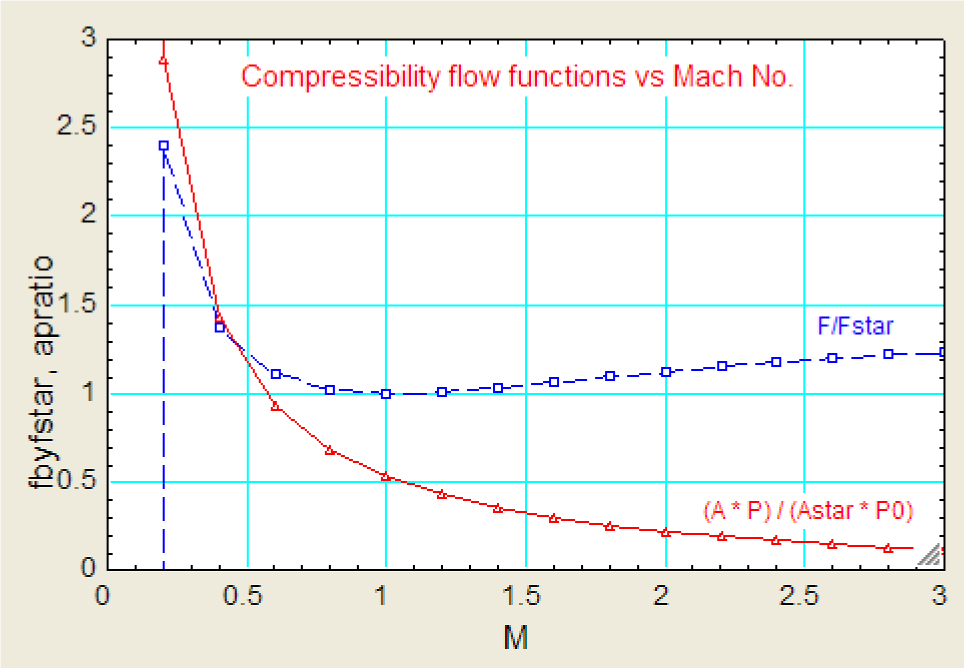
1..16	M	Mstar	abyastar	pbyp0	rhobyrho0	tbyt0
Run 1	1.000E-10	1.095E-10	5.787E+09	1	1	1
Run 2	0.2	0.2182	2.964	0.9725	0.9803	0.9921
Run 3	0.4	0.4313	1.59	0.8956	0.9243	0.969
Run 4	0.6	0.6348	1.188	0.784	0.8405	0.9328
Run 5	0.8	0.8251	1.038	0.656	0.74	0.8865
Run 6	1	1	1	0.5283	0.6339	0.8333
Run 7	1.2	1.158	1.03	0.4124	0.5311	0.7764
Run 8	1.4	1.3	1.115	0.3142	0.4374	0.7184
Run 9	1.6	1.425	1.25	0.2353	0.3557	0.6614
Run 10	1.8	1.536	1.439	0.174	0.2868	0.6068
Run 11	2	1.633	1.688	0.1278	0.23	0.5556
Run 12	2.2	1.718	2.005	0.09352	0.1841	0.5081
Run 13	2.4	1.792	2.403	0.0684	0.1472	0.4647
Run 14	2.6	1.857	2.896	0.05012	0.1179	0.4252
Run 15	2.8	1.914	3.5	0.03685	0.09463	0.3894
Run 16	3	1.964	4.235	0.02722	0.07623	0.3571

1..16	M	fbyfstar	apratio
Run 1	1.000E-10	4.564E+09	5.787E+09
Run 2	0.2	2.4	2.882
Run 3	0.4	1.375	1.424
Run 4	0.6	1.105	0.9316
Run 5	0.8	1.019	0.6811
Run 6	1	1	0.5283
Run 7	1.2	1.011	0.4249
Run 8	1.4	1.035	0.3504
Run 9	1.6	1.063	0.2941
Run 10	1.8	1.094	0.2504
Run 11	2	1.123	0.2157
Run 12	2.2	1.15	0.1875
Run 13	2.4	1.175	0.1644
Run 14	2.6	1.198	0.1451
Run 15	2.8	1.218	0.129
Run 16	3	1.237	0.1153

Note: In the above Tables, minimum value for M is shown as 1E-10, since M = 0 will give a 'Division by zero' error.

Plots:





==

"**Prob.9.4.3** Write EES Functions for property variations across a normal shock for an ideal gas."

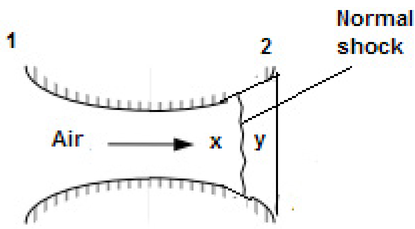


Fig.Prob.9.4.3 Normal shock

EES Functions:

"For My, Mach No. after shock:"

FUNCTION My(Mx,k)

"Inputs: Mx – Mach No. before shock, k -- ratio of sp. heats (= 1.4 for air)"

"Outputs: My"

AA := (2 / (k – 1)) + Mx^2

BB:= ((2 * k)/(k-1))*Mx^2 -1

```
My := sqrt(AA / BB)

END

"======================================================================="

"For Py/Px, static pressure ratio:"

FUNCTION PYBYPX(Mx,k)

"Inputs: Mx – Mach No. before shock, k -- ratio of sp. heats (= 1.4 for air)"

"Outputs: Py/Px"

PYBYPX := ((2*k) / (k + 1)) * Mx^2 – (k – 1)/(k + 1)

END

"======================================================================="

"For Ty/Tx, static temp ratio:"

FUNCTION TYBYTX(Mx,k)

"Inputs: Mx – Mach No. before shock, k -- ratio of sp. heats (= 1.4 for air)"

"Outputs: Ty/Tx"

AA := 1 + ((k – 1)/2) * Mx^2

BB:= ((2 * k) / (k – 1)) * Mx^2 – 1

CC := (1/2) * (k + 1)^2 * Mx^2 / (k – 1)

TYBYTX := AA * BB / CC

END

"======================================================================="
```

"For rhoy/rhox, densityratio:"

FUNCTION RHOYBYRHOX(Mx,k)

"Inputs: Mx – Mach No. before shock, k -- ratio of sp. heats (= 1.4 for air)"

"Outputs: rhoy/rhox"

RHOYBYRHOX := PYBYPX(Mx, k) / TYBYTX(Mx, k)

END

"=="

"For P0y/P0x, stagn. pressure ratio:"

FUNCTION P0YBYP0X(Mx,k)

"Inputs: Mx – Mach No. before shock, k -- ratio of sp. heats (= 1.4 for air)"

```
"Outputs: P0y/P0x"

AA := Mx / My(Mx,k)

BB := 1 + (My(Mx,k))^2 * (k – 1)/2

CC:= 1 + Mx^2 * (k – 1)/2

DD := ( k + 1)/ (2 * (k -1))

P0YBYP0X := AA * (BB / CC)^DD

END

"=========================================================================="
```

"For P0y/Px:"

```
FUNCTION P0YBYPX(Mx,k)

"Inputs: Mx – Mach No. before shock, k -- ratio of sp. heats (= 1.4 for air)"

"Outputs: P0y/Px"

AA := 1 + k * Mx^2

BB := 1 + k * (My(Mx,k))^2

CC:= 1 + (My(Mx,k))^2 * (k – 1)/2

DD := k /(k – 1)

P0YBYPX := (AA/BB) * CC^DD

END

"=========================================================================="
```

"For entropy change, DELTAS across the shock:"

FUNCTION DELTAS(Mx,R, k)

"Inputs: Mx – Mach No. before shock, k -- ratio of sp. heats (= 1.4 for air)"

"Outputs: DELTAS (J/kg.K) = -R * ln (P0y/P0x).
We write: (P0y/P0x) = (P0y/Px) * (Px/P0x)
Now, (Px/P0x) is obtained from Isentropic relation since flow from inlet to shock is isentropic

Another relation for DELTAS is DELTS = cp . ln(Ty/Tx) – R . ln(Py/Px)"

AA := P0YBYPX(Mx,k)

BB := PBYP0(Mx,k)

DELTAS := -R * ln(AA * BB)

END

"=="

"**Prob.9.4.4** Using the EES Functions for property variations across a normal shock in an ideal gas, written above, plot the property variations against Mach No."

EES Solution:

{Mx = 2}

k = 1.4

My = My(Mx, k)

pybypx = PYBYPX(Mx, k)

rhoybyrhox = RHOYBYRHOX(Mx, k)

tybytx = TYBYTX(Mx, k)

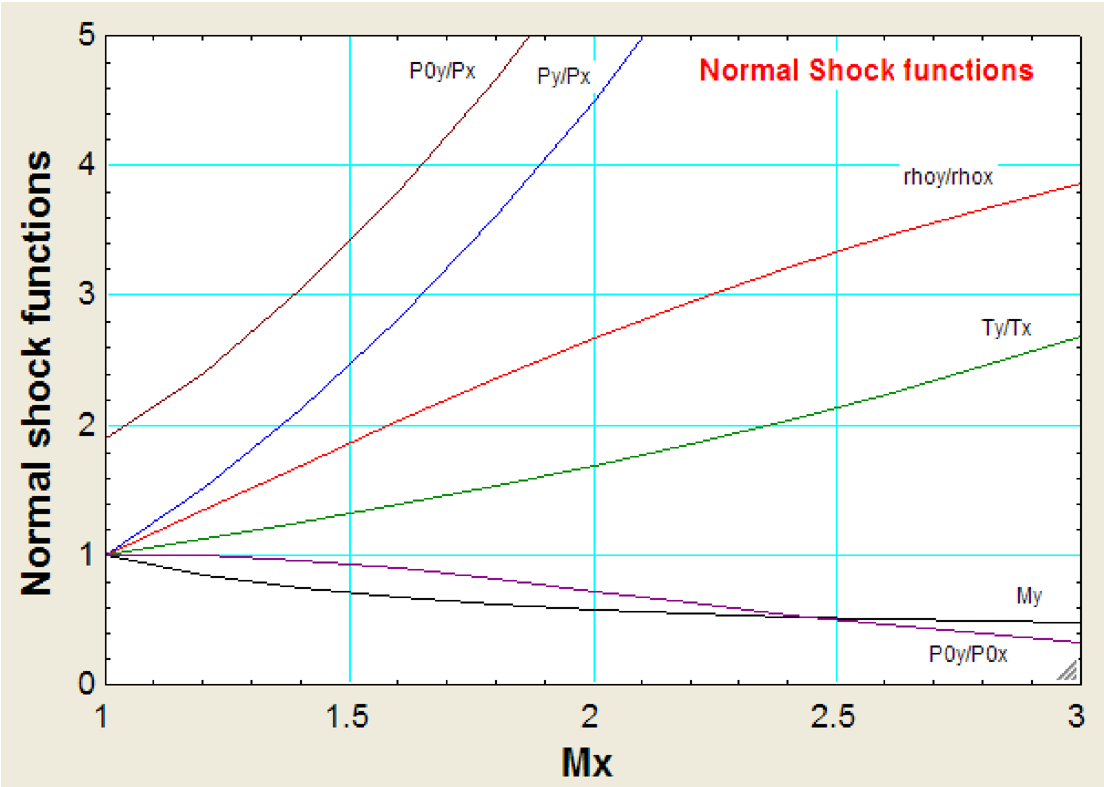
p0ybyp0x = P0YBYP0X(Mx, k)

p0ybypx = P0YBYPX(Mx, k)

Parametric Table:

1..11	Mx	My	pybypx	rhoybyrhox	tybytx	p0ybyp0x	p0ybypx
Run 1	1	1	1	1	1	1	1.893
Run 2	1.2	0.8422	1.513	1.342	1.128	0.9928	2.408
Run 3	1.4	0.7397	2.12	1.69	1.255	0.9582	3.049
Run 4	1.6	0.6684	2.82	2.032	1.388	0.8952	3.805
Run 5	1.8	0.6165	3.613	2.359	1.532	0.8127	4.67
Run 6	2	0.5774	4.5	2.667	1.688	0.7209	5.64
Run 7	2.2	0.5471	5.48	2.951	1.857	0.6281	6.716
Run 8	2.4	0.5231	6.553	3.212	2.04	0.5401	7.897
Run 9	2.6	0.5039	7.72	3.449	2.238	0.4601	9.181
Run 10	2.8	0.4882	8.98	3.664	2.451	0.3895	10.57
Run 11	3	0.4752	10.33	3.857	2.679	0.3283	12.06

Now, plot the results:



==

"**Prob.9.4.5** Write EES Functions for property variations of an ideal gas in Fanno flow (i.e. adiabatic flow with friction in constant area ducts)."

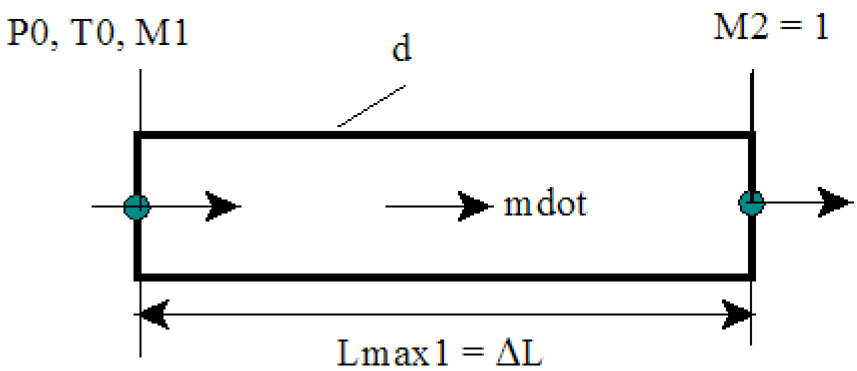


Fig.Prob.9.4.5 Fanno flow

EES Functions:

Here, functions are non-dimensionalised with respect to values at M = 1, denoted by star.
Thus, to find velocity ratio between sections 1 and 2 of pipe, we write:

V1/V2 = (V1/Vstar) * (Vstar/V2)

```
"For V/Vstar : Also, this is equal to: rhostar/rho"

FUNCTION FANNO_VBYVSTAR(M, k)

"Inputs: M – Mach No. k -- ratio of sp. heats (= 1.4 for air)"

"Outputs: V/Vstar"

AA := 2 * (1 + ((k – 1)/2) * M^2)

FANNO_VBYVSTAR := M * sqrt((k + 1) / AA)

END

"=================================================================="

"For P/Pstar:"

FUNCTION FANNO_PBYPSTAR(M, k)

"Inputs: M – Mach No. k -- ratio of sp. heats (= 1.4 for air)"

"Outputs: P/Pstar"

AA := 2 * (1 + ((k – 1)/2) * M^2)

FANNO_PBYPSTAR := (1/M) * sqrt((k + 1) / AA)

END

"=================================================================="
```

"For T/Tstar:"

FUNCTION FANNO_TBYTSTAR(M, k)

"Inputs: M – Mach No. k -- ratio of sp. heats (= 1.4 for air)"

"Outputs: T/Tstar"

AA := 2 * (1 + ((k – 1)/2) * M^2)

FANNO_TBYTSTAR := (k + 1) / AA

END

"=="

"For P0/P0star:"

FUNCTION FANNO_P0BYP0STAR(M, k)

"Inputs: M – Mach No. ..k -- ratio of sp. heats (= 1.4 for air)"

```
"Outputs: P0/P0star"

AA := 2 * (1 + ((k – 1)/2) * M^2)

BB := (k + 1) / ( 2 * (k – 1))

FANNO_P0BYP0STAR := (1/M) * (AA / (k + 1))^BB

END

"=================================================================="

"For F/Fstar : "

FUNCTION FANNO_FBYFSTAR(M, k)

"Inputs: M – Mach No. k -- ratio of sp. heats (= 1.4 for air)"

"Outputs: F/Fstar"

AA := sqrt (2 * ( k+ 1) * (1 + ((k – 1)/2) * M^2))

BB :=(1 + k * M^2) / M

FANNO_FBYFSTAR := BB / AA

END

"=================================================================="

"For 4.f.Lmax/D:"

FUNCTION FANNO_FOURFLMAXBYD(M, k)

"Inputs: M – Mach No. k -- ratio of sp. heats (= 1.4 for air)"

"Outputs: 4.f.Lmax/D"

AA :=(1 – M^2) / (k * M^2)
```

BB := (k + 1) / (2 * k)

CC := (k + 1) * M^2

DD := (2 * (1 + ((k – 1)/2) * M^2))

FANNO_FOURFLMAXBYD := AA + BB * ln(CC/DD)

END

"==="

"**Prob.9.4.6** Plot property variations of an ideal gas with k = 1.4,in Fanno flow against Mach No."

EES Solution:

{M = 2}

k = 1.4

vbyvstar = FANNO_VBYVSTAR(M, k)

pbypstar = FANNO_PBYPSTAR(M, k)

tbytstar = FANNO_TBYTSTAR(M, k)

p0byp0star = FANNO_P0BYP0STAR(M, k)

fbyfstar = FANNO_FBYFSTAR(M, k)

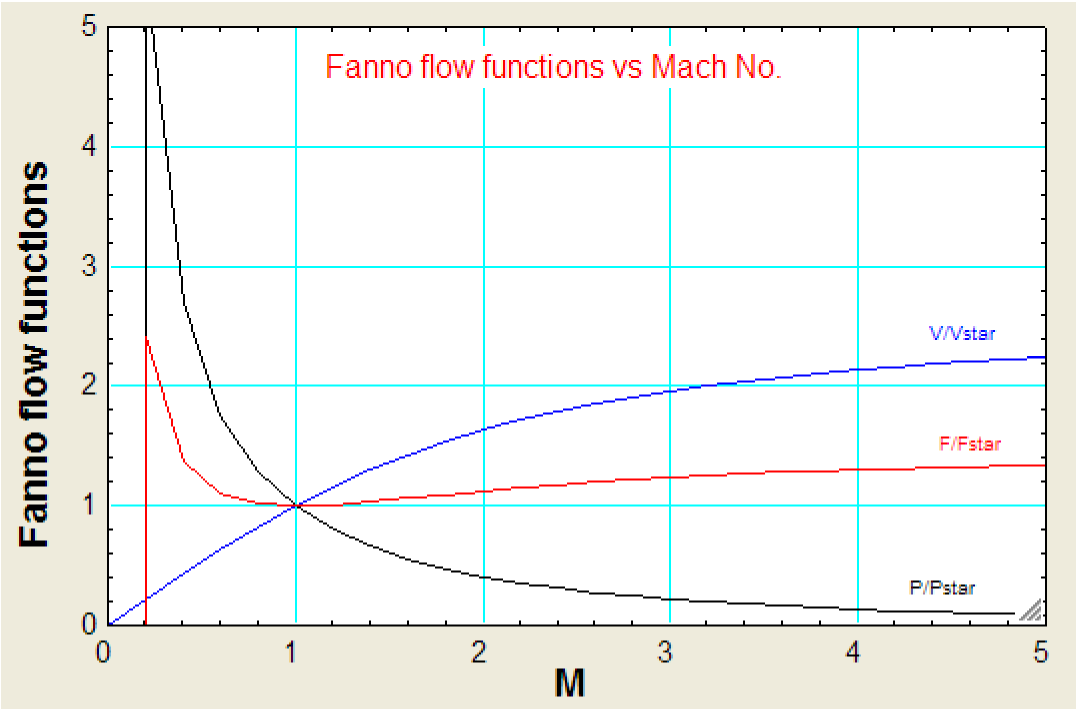
fourflmaxbyd = FANNO_FOURFLMAXBYD(M, k)

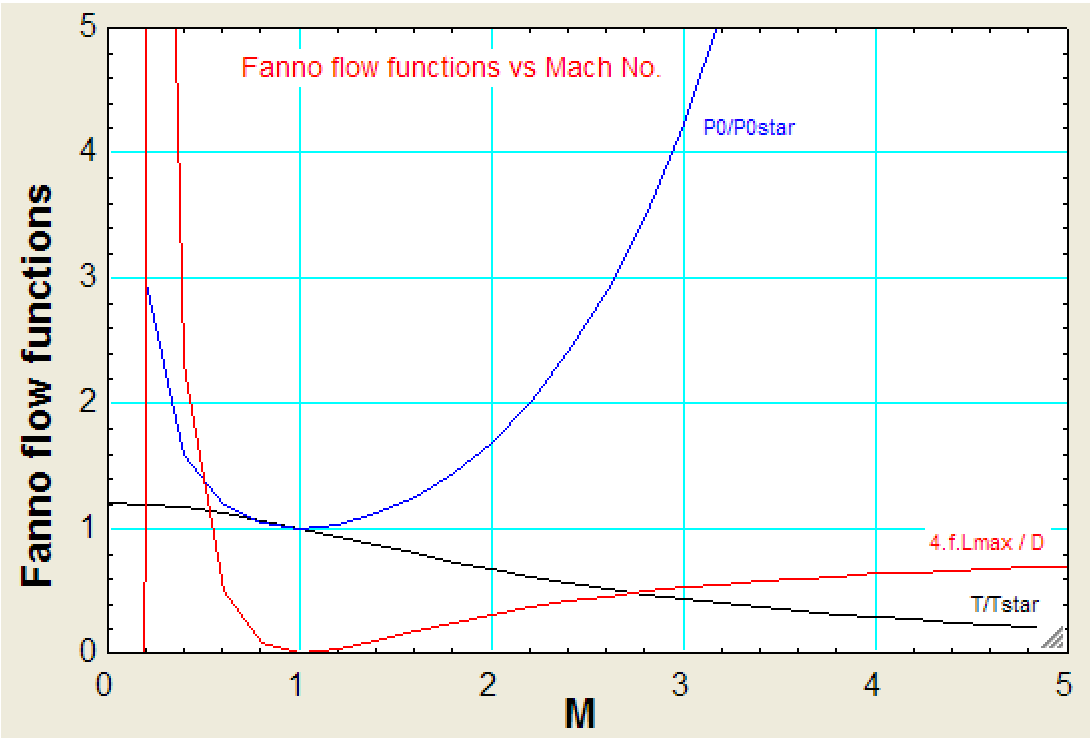
Parametric Table:

1..26	M	pbypstar	vbyvstar	tbytstar	p0byp0star	fbyfstar	fourflmaxbyd
Run 1	1.000E-10	1.095E+10	1.095E-10	1.2	5.787E+09	4.564E+09	7.143E+19
Run 2	0.2	5.455	0.2182	1.19	2.964	2.4	14.53
Run 3	0.4	2.696	0.4313	1.163	1.59	1.375	2.308
Run 4	0.6	1.763	0.6348	1.119	1.188	1.105	0.4908
Run 5	0.8	1.289	0.8251	1.064	1.038	1.019	0.07229
Run 6	1	1	1	1	1	1	0
Run 7	1.2	0.8044	1.158	0.9317	1.03	1.011	0.03364
Run 8	1.4	0.6632	1.3	0.8621	1.115	1.035	0.09974
Run 9	1.6	0.5568	1.425	0.7937	1.25	1.063	0.1724
Run 10	1.8	0.4741	1.536	0.7282	1.439	1.094	0.2419
Run 11	2	0.4082	1.633	0.6667	1.688	1.123	0.305
Run 12	2.2	0.3549	1.718	0.6098	2.005	1.15	0.3609
Run 13	2.4	0.3111	1.792	0.5576	2.403	1.175	0.4099
Run 14	2.6	0.2747	1.857	0.5102	2.896	1.198	0.4526
Run 15	2.8	0.2441	1.914	0.4673	3.5	1.218	0.4898
Run 16	3	0.2182	1.964	0.4286	4.235	1.237	0.5222
Run 17	3.2	0.1961	2.008	0.3937	5.121	1.253	0.5504
Run 18	3.4	0.177	2.047	0.3623	6.184	1.268	0.5752
Run 19	3.6	0.1606	2.081	0.3341	7.45	1.281	0.597
Run 20	3.8	0.1462	2.111	0.3086	8.951	1.292	0.6161
Run 21	4	0.1336	2.138	0.2857	10.72	1.303	0.6331
Run 22	4.2	0.1226	2.162	0.265	12.79	1.312	0.6481
Run 23	4.4	0.1128	2.184	0.2463	15.21	1.321	0.6615
Run 24	4.6	0.1041	2.203	0.2294	18.02	1.328	0.6734
Run 25	4.8	0.09637	2.22	0.214	21.26	1.335	0.6842
Run 26	5	0.08944	2.236	0.2	25	1.342	0.6938

Note: In the above Table, note that in Run 1, we have put M = 1E-10 (i.e. a very small no.) instead of M = 0, to avoid 'Divide by zero' error.

Now, plot the results:





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"**Prob.9.4.7** Write EES Functions for property variations of an ideal gas in Rayleigh flow, i.e. frictionless flow in a constant area duct, with heat transfer"

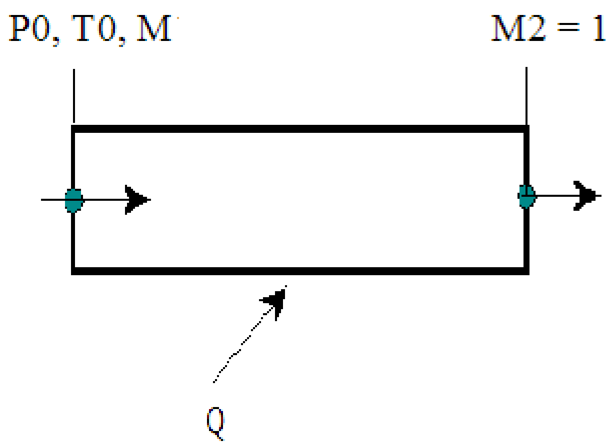


Fig.Prob.9.4.7 Rayleigh flow

"EES Functions for Rayleigh flow:"

"For T0 / T0star"

FUNCTION RAYLEIGH_T0BYT0STAR(M, k)

"Inputs: M … Mach No., k …. ratio of sp. heats"

"Outputs: T0/T0star"

AA := (k + 1) * M^2 * (2 + (k – 1) * M^2)

BB := (1 + k * M^2)^2

RAYLEIGH_T0BYT0STAR := AA / BB

END

"=="

"For P0 / P0star"

FUNCTION RAYLEIGH_P0BYP0STAR(M, k)

"Inputs: M … Mach No., k …. ratio of sp. heats"

"Outputs: P0/P0star"

AA := (k + 1) / (1 + k * M^2)

BB := ((2 + (k – 1) * M^2) / (k + 1))^(k / (k – 1))

RAYLEIGH_P0BYP0STAR := AA * BB

END

"==="

"For T / Tstar"

FUNCTION RAYLEIGH_TBYTSTAR(M, k)

"Inputs: M … Mach No., k …. ratio of sp. heats"

"Outputs: T/Tstar"

```
RAYLEIGH_TBYTSTAR := (M * ( 1 + k) / ( 1 + k * M^2))^2

END

"=========================================================================="

"For P / Pstar"

FUNCTION RAYLEIGH_PBYPSTAR(M, k)

"Inputs: M … Mach No., k …. ratio of sp. heats"

"Outputs: P / Pstar"

RAYLEIGH_PBYPSTAR := ( 1 + k) / ( 1 + k * M^2)

END

"=========================================================================="

"For V / Vstar"

FUNCTION RAYLEIGH_VBYVSTAR(M, k)

"Inputs: M … Mach No., k …. ratio of sp. heats"

"Outputs: V / Vstar. Also, note: V/Vstar = rhostar/rho"

RAYLEIGH_VBYVSTAR := ( 1+ k) * M^2/ ( 1 + k * M^2)

END

"=========================================================================="

"For Entropy change: (s2 – s1)"

FUNCTION RAYLEIGH_DELTAS(M1, M2, R, k)

"Inputs: M1, M2 … Mach Nos at inlet and exit of duct., R = gas constant (J/kg.K), k …. ratio of sp. heats"
```

```
"Outputs: (s2 – s1), J/kg.K "

AA = (M2/ M1)^((2 * k)/(k – 1))

BB = (1+ k * M1^2)/ (1 + k * M2^2)

CC = (k + 1) / (k – 1)

RAYLEIGH_DELTAS := R * ln(AA * BB^CC)

END

"====================================================================="

"For heat transfer, Q"

FUNCTION RAYLEIGH_Q(M1, M2, cp, T1, k)

"Inputs: M1, M2 … Mach Nos at inlet and exit of duct., T1 = inlet temp (K), cp = sp.heat at const. pressure (J/kg.K), k …. ratio of sp. heats"

"Outputs: Q, J/kg "

AA = M2^2 – M1^2

BB = 2 – 2 * k * M1^2 * M2^2

CC = (k – 1) * (M2^2 + M1^2)

DD = 2 * M1^2 * (1 + k * M2^2)^2

RAYLEIGH_Q := cp * T1 * AA * (BB + CC) / DD

END

"====================================================================="
```

"**Prob.9.4.8** Using the EES Functions for property variations of an ideal gas in Rayleigh flow, plot the Rayleigh flow functions against Mach No."

EES Solution:

{M = 2}

k = 1.4

t0byt0star = RAYLEIGH_T0BYT0STAR(M, k)

p0byp0star = RAYLEIGH_P0BYP0STAR(M, k)

tbytstar = RAYLEIGH_TBYTSTAR(M, k)

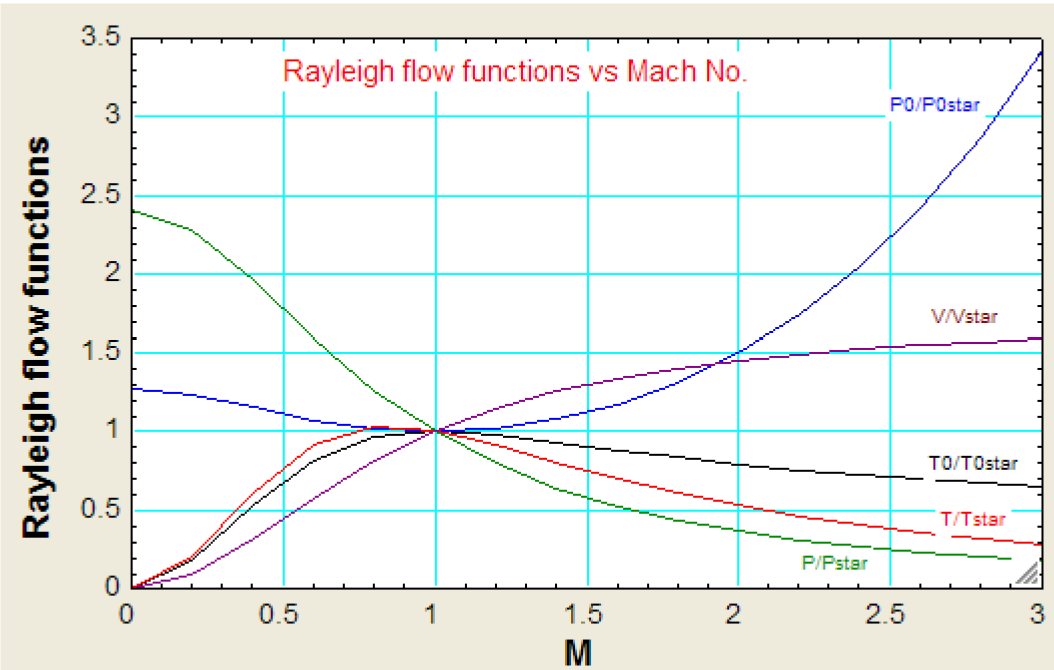
pbypstar = RAYLEIGH_PBYPSTAR(M, k)

vbyvstar = RAYLEIGH_VBYVSTAR(M, k)

Parametric Table:

1..16	M	t0byt0star	p0byp0star	tbytstar	pbypstar	vbyvstar
Run -1	M	t0byt0star	p0byp0star	tbytstar	pbypstar	vbyvstar
1..16	M	t0byt0star	p0byp0star	tbytstar	pbypstar	vbyvstar
Run 1	0	0	1.268	0	2.4	0
Run 2	0.2	0.1736	1.235	0.2066	2.273	0.09091
Run 3	0.4	0.529	1.157	0.6151	1.961	0.3137
Run 4	0.6	0.8189	1.075	0.9167	1.596	0.5745
Run 5	0.8	0.9639	1.019	1.025	1.266	0.8101
Run 6	1	1	1	1	1	1
Run 7	1.2	0.9787	1.019	0.9118	0.7958	1.146
Run 8	1.4	0.9343	1.078	0.8054	0.641	1.256
Run 9	1.6	0.8842	1.176	0.7017	0.5236	1.34
Run 10	1.8	0.8363	1.316	0.6089	0.4335	1.405
Run 11	2	0.7934	1.503	0.5289	0.3636	1.455
Run 12	2.2	0.7561	1.743	0.4611	0.3086	1.494
Run 13	2.4	0.7242	2.045	0.4038	0.2648	1.525
Run 14	2.6	0.697	2.418	0.3556	0.2294	1.55
Run 15	2.8	0.6738	2.873	0.3149	0.2004	1.571
Run 16	3	0.654	3.424	0.2803	0.1765	1.588

Now, plot the results:



==

"**Prob.9.4.9** Air at 330 K and 200 kPa enters a duct with a varying flow area. If the Mach No. at the entrance is 0.25, determine the temp, pressure and Mach No. at a location where flow area has reached to 60% of the entrance area. Assume the flow to be steady and isentropic and oxygen to be an ideal gas."

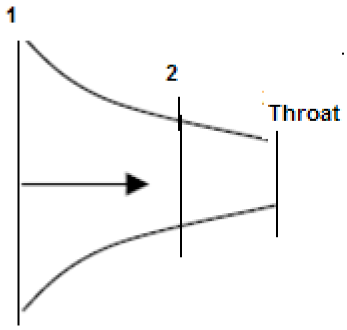


Fig.Prob.9.4.9 Isentropic flow in a duct of varying area

EES Solution:

"Data:"

"Let 1 refer to entrance and 2 to the required state."

k = 1.4 "for air"

M1 = 0.25

T1 = 330 "K"

P1 = 200 "kPa"

"Calculations:"

"Use EES functions for property variations in isentropic flow written above:"

"At M1 = 0.25, we have:"

a1byastar = ABYASTAR(M1, k) "…gives A1/Astar"

T1 / T0 = TBYT0(M1, k) "… gives T0"

P1 / P0 = PBYP0(M1, k) "… gives P0"

"Now, at the desired location: A2 = 0.6 * A1"

a2bya1 = 0.6 "…by data: A2/A1 = 0.6"

"Therefore: (A2/Astar) = (A2/A1) * (A1 / Astar). Then, we have:"

a2byastar = a2bya1 * a1byastar "…. gives A2/Astar"

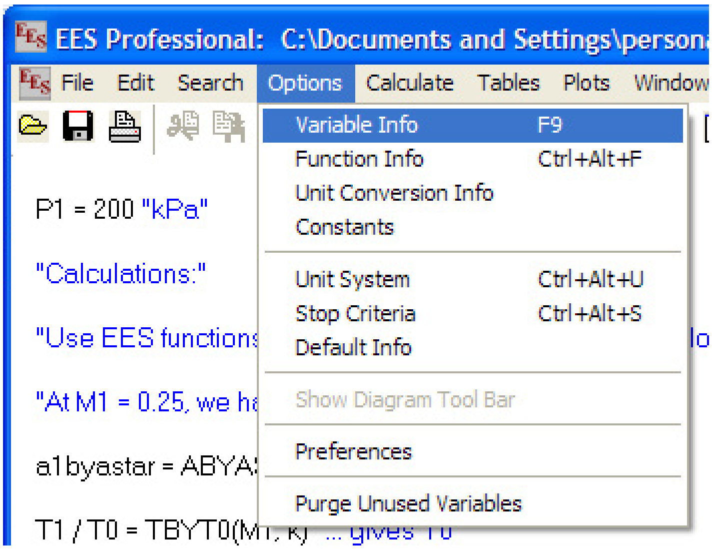
"Now, for this a2byastar, from isentropic flow relations, we get:"

a2byastar = ABYASTAR(M2,k) "….finds M2 "

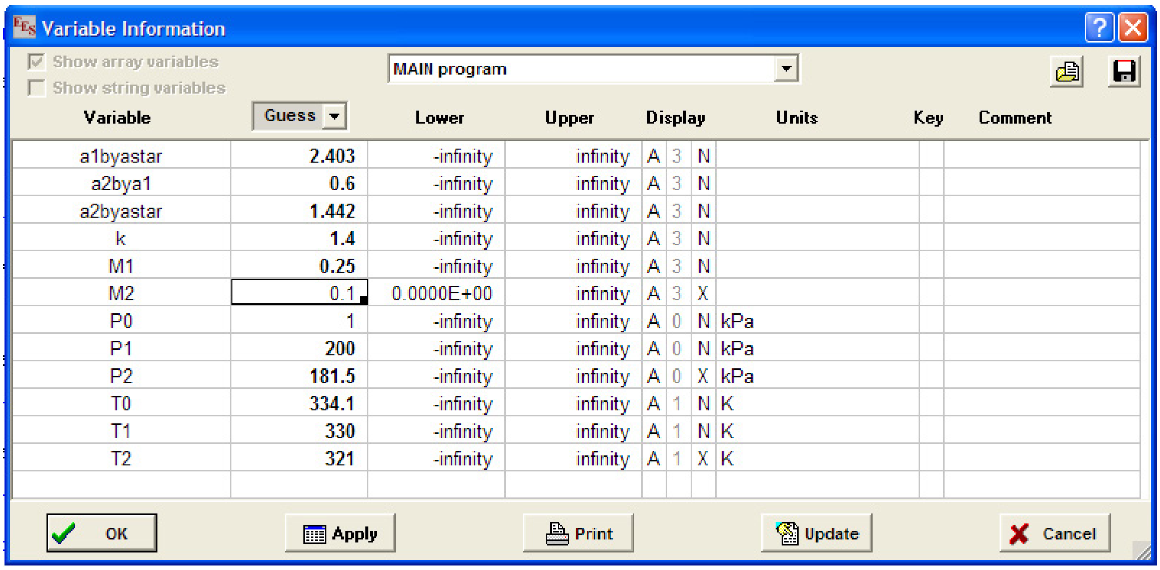
"While finding M2, remember that it should be subsonic, since at the entry, flow is subsonic, and it is a convergent nozzle since area is decreasing.

So, select the guess value as less than 1 for M2, as explained below:"

“Select Options – Variable Info from the EES menu:



Click on Variable Info. And, for M2, change the guess value to 0.1:



Click on OK. And, proceed to calculate in EES:”

“For this M2, get T2/T0 and P2/P0:”

T2 / T0 = TBYT0(M2, k) “…gives T2”

P2 / P0 = PBYP0(M2, k) “…gives P2”

Results:

Unit Settings: SI K Pa J mass deg

a1byastar = 2.403	a2bya1 = 0.6	a2byastar = 1.442	k = 1.4
M1 = 0.25	M2 = 0.4529	P0 = 208.9 [kPa]	P1 = 200 [kPa]
P2 = 181.5 [kPa]	T0 = 334.1 [K]	T1 = 330 [K]	T2 = 321 [K]

Thus:

Mach No. at section 2 = M2 = 0.4529 …. Ans.

Pressure at section 2 = P2 = 181.5 kPa … Ans.

Temp. at section 2 = T2 = 321 K … Ans.

===

"**Prob.9.4.10** A conical diffuser has entry and exit dia of 15 cm and 30 cm respectively. P, T and V of air at entry are 0.69 bar, 340 K and 180 m/s respectively. Determine: (i) exit pressure (ii) exit velocity (iii) force exerted on the diffuser walls. Assume isentropic flow, k = 1.4, cp = 1000 J/kg.K"

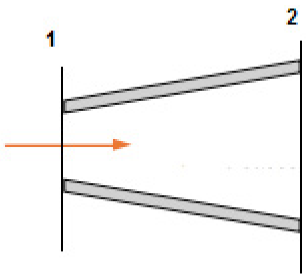


Fig.Prob.9.4.10 Isentropic flow in a diffuser

EES Solution:

"**Data:**"

D1 = 0.15 "m"

D2 = 0.3 "m"

P1 = 0.69 "bar"

T1 = 340 "K"

V1 = 180 “m/s”

k = 1.4

cp = 1000 “J/kg.K”

R = 287 “J/kg.K”

“Calculations:”

A1 = pi * D1^2 /4 “m^2 …. inlet area”

A2 = pi * D2^2 /4 “m^2… exit area”

T01 = T1 + V1^2 / (2 * cp) “K… stagn. temp at inlet”

T02 = T01 “..for isentropic flow”

rho1 = (P1 * 10^5) / (R * T1) “m^3 /kg … density at inlet”

“Therefore,T1/T01 can be found out, and for this T1/T01, find M1 etc from isentropic flow functions:”

T1/T01 = TBYT0(M1, k) “…finds M1”

P1/P01 = PBYP0(M1, k) “..finds P01”

P02 = P01 “…for isentropic flow”

A1/A1star = ABYASTAR(M1, k) “…finds A1star”

A2star = A1star”…for isentropic flow”

F1/F1star = FBYFSTAR(M1, k) “…finds F1 for M1”

“When M = 1: pressure is Pstar, Temp is Tstar:”

P1star/P01 = PBYP0(1, k) “…finds P1star”

P2star = P1star

T1star/T01 = TBYT0(1, k)"…finds T1star"

F1star = (P1star * 10^5) * A1star * (1 + k) "N"

F2star = F1star

"At the exit: Find (A2/A2star) and then find M2 and other functions corresponding to this (A2/A2star) from isentropic functions:"

A2/A2star = ABYASTAR(M2, k) "… finds M2"

P2/P02 = PBYP0(M2, k) "..finds P2"

F2/F2star = FBYFSTAR(M2, k) "…finds F2 for M2"

T2/T02 = TBYT0(M2, k) "…finds T2"

rho2 = P2 * 10^5 / (R * T2) "..kg/m^3…density at exit"

mass_flow = rho1 * A1 * V1 "kg/s"

mass_flow = rho2 * A2 * V2 "..finds V2, vel. at exit"

"Force exerted on Diffuser walls = Thrust of the flow in backward direction"

"i.e. Thrust = F2 – F1"

Thrust_on_walls = F2 – F1 "N"

Resuts:

Unit Settings: SI K Pa J mass deg

A1 = 0.01767 [m^2]	A1star = 0.01296 [m^2]	A2 = 0.07069 [m^2]
A2star = 0.01296 [m^2]	cp = 1000 [J/kg-K]	D1 = 0.15 [m]
D2 = 0.3 [m]	F1 = 1626 [N]	F1star = 1335 [N]
F2 = 5786 [N]	F2star = 1335 [N]	k = 1.4
M1 = 0.4881	M2 = 0.1068	$mass_{flow}$ = 2.249 [kg/s]
P01 = 0.8121 [bar]	P02 = 0.8121 [bar]	P1 = 0.69 [bar]
P1star = 0.429 [bar]	P2 = 0.8056 [bar]	P2star = 0.429 [bar]
R = 287 [J/kg-K]	rho1 = 0.7071 [kg/m^3]	rho2 = 0.7899 [kg/m^3]
T01 = 356.2 [K]	T02 = 356.2 [K]	T1 = 340 [K]
T1star = 296.8 [K]	T2 = 355.4 [K]	$Thrust_{on,walls}$ = 4160 [N]
V1 = 180 [m/s]	V2 = 40.29 [m/s]	

Thus:

Exit pressure = P2 = 0.8056 bar .. Ans.

Exit velocity = V2 = 40.29 m/s …. Ans.

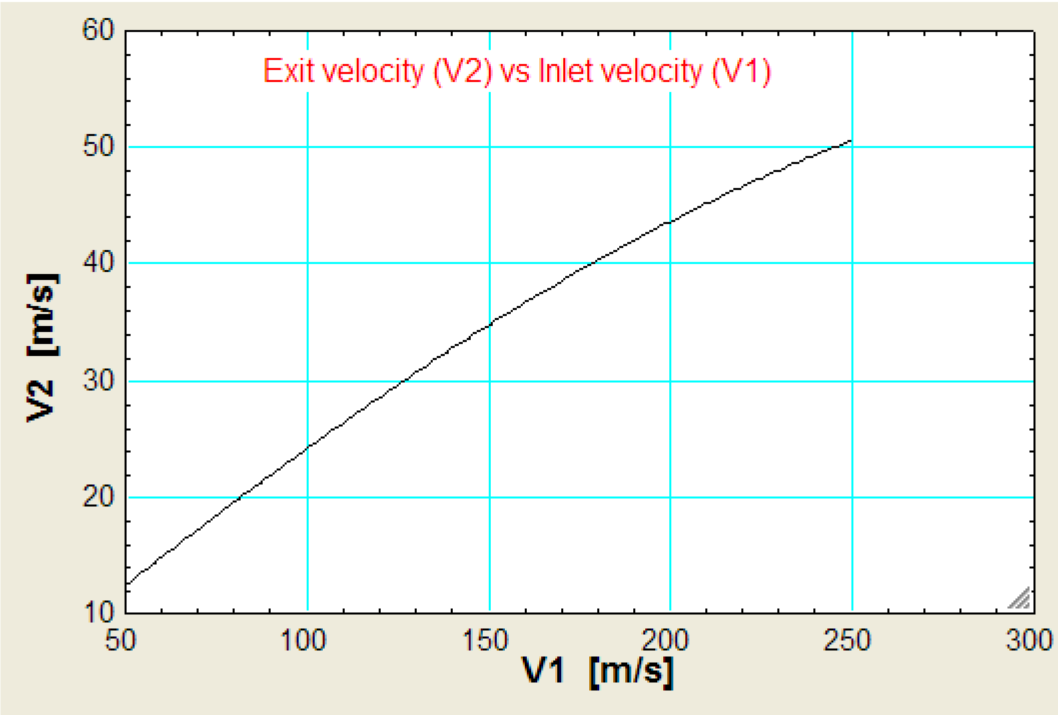
Force exerted on diffuser walls = 4160 N … Ans.

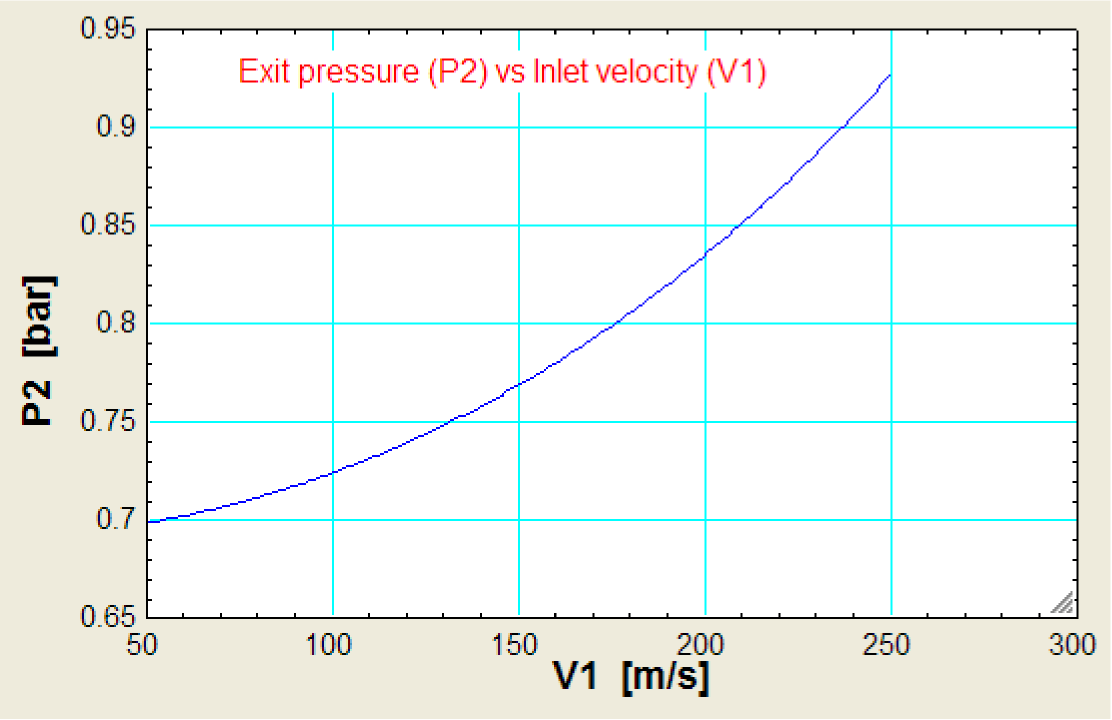
(b) Plot the variation of V2, P2 and Force on walls as the inlet velocity V1 varies from 50 to 250 m/s:

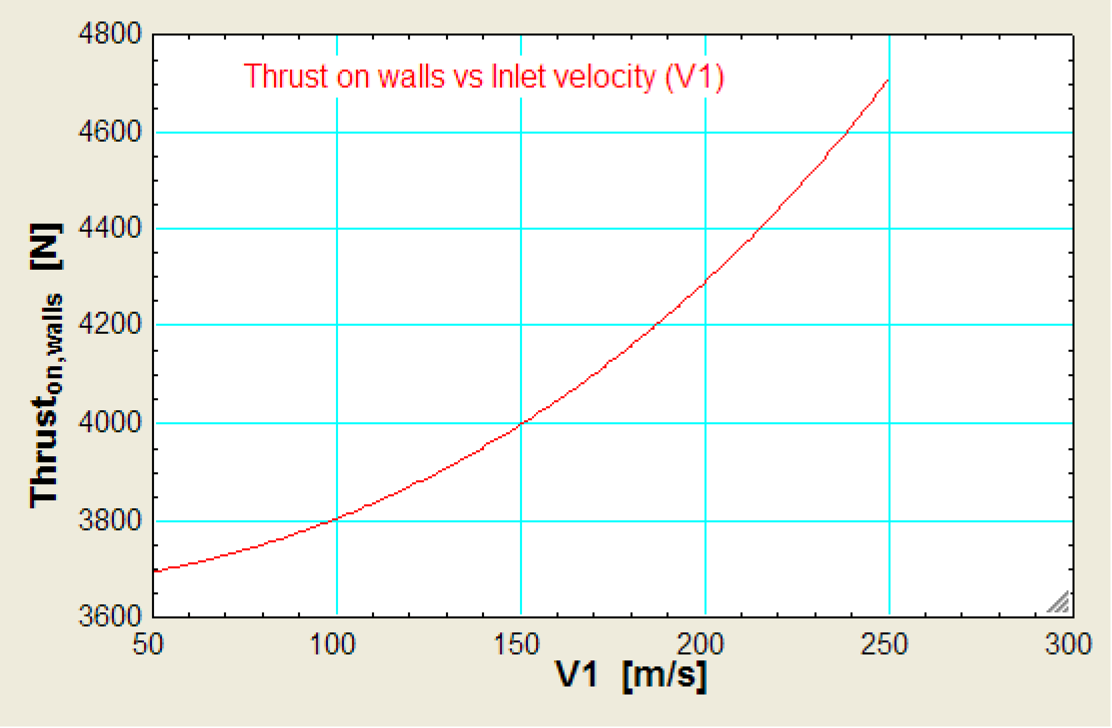
First, compute the parametric Table:

1..21	V1 [m/s]	V2 [m/s]	P2 [bar]	$Thrust_{on,walls}$ [N]
Run 1	50	12.39	0.6984	3694
Run 2	60	14.82	0.7021	3709
Run 3	70	17.21	0.7065	3728
Run 4	80	19.56	0.7116	3750
Run 5	90	21.88	0.7174	3775
Run 6	100	24.16	0.724	3803
Run 7	110	26.38	0.7314	3835
Run 8	120	28.55	0.7395	3870
Run 9	130	30.67	0.7484	3908
Run 10	140	32.72	0.7581	3951
Run 11	150	34.72	0.7686	3997
Run 12	160	36.64	0.7801	4047
Run 13	170	38.5	0.7924	4101
Run 14	180	40.29	0.8056	4160
Run 15	190	42	0.8198	4223
Run 16	200	43.63	0.835	4291
Run 17	210	45.19	0.8513	4364
Run 18	220	46.66	0.8686	4442
Run 19	230	48.06	0.887	4525
Run 20	240	49.37	0.9066	4615
Run 21	250	50.6	0.9274	4710

Now, plot the results:




Exit pressure (P2) vs Inlet velocity (V1)
P2 [bar]
V1 [m/s]
0.95
0.9
0.85
0.8
0.75
0.7
0.65
50
100
150
200
250
300


Thrust on walls vs Inlet velocity (V1)
Thrust on,walls [N]
V1 [m/s]
4800
4600
4400
4200
4000
3800
3600
50
100
150
200
250
300

==

"**Prob.9.4.11** Find the dimensions for an ideal nozzle that will allow a mass flow rate of 3 kg/s of air from a large tank at 10 bar, 300 K to a discharge region at 1.1 bar."

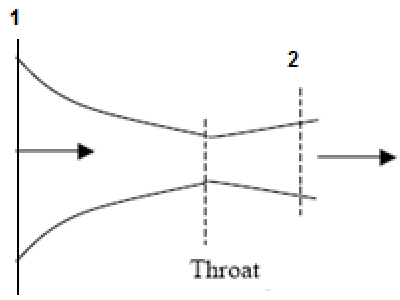


Fig.Prob.9.4.11 Isentropic flow in a C-D nozzle

EES Solution:

"Data:"

"Note that inlet velocity is negligible; so, inlet pressure and temp are the stagnation values."

P0 = 10 "bar"
T0 = 300 "K"
P2 = 1.1 "bar"
R = 287 "J/kg.K"
k = 1.4

Mass_flow = 3 "kg/s"

"Calculations:"

"First find out the critical pressure and determine if the flow is choked:

Critical pressure when k = 1.4 (air) is:"

P_crit = 0.5283 * P0 "bar"

"P_crit = 5.283 bar, and the exit pressure is given as 1.1 bar.

Therefore, it is choked flow, i.e. a C-D nozzle is required, and the pressure at throat is sonic or at throat, M = 1"

"Then, from isentropic relations:"

P_star = P0 * PBYP0(1, k) "bar…pressure at throat"

T_star = T0 * TBYT0(1, k) "K…temp at throat"

rho_star = rho0 * RHOBYRHO0(1, k) "kg/m^3….density at throat"

rho0 = P0 * 10^5 / (R * T0) "kg/m^3 … density at inlet"

V_star = sqrt(k * R * T_star) "m/s …. vel. at throat = sonic vel."

Mass_flow = rho_star * A_star * V_star "m^2…finds area at throat, A_star"

"At the exit:"

P2/P0 = PBYP0(M2,k) "..finds M2 for known P2/P0..

Remember here to have the guess value for M2 as more than 1, since it is a C-D nozzle with exit pressure less than critical pressure, and the flow will accelerate in the divergent section and M2 will be more than 1… see Prob. 9.4.9"

"With this M2, find other properties at exit:"

T2/T0 = TBYT0(M2,k) "..finds T2"

A2/A_star = ABYASTAR(M2,k) "…finds A2"

rho2/rho0 = RHOBYRHO0(M2,k) "…finds rho2"

V2 = M2 * sqrt(k * R * T2) "m/s …. finds V2"

Results:

Unit Settings: SI K Pa J mass deg

A2 = 0.002354 [m^2]	A_{star} = 0.001286 [m^2]	k = 1.4	M2 = 2.096
$Mass_{flow}$ = 3 [kg/s]	P0 = 10 [bar]	P2 = 1.1 [bar]	P_{crit} = 5.283 [bar]
P_{star} = 5.283 [bar]	R = 287 [J/kg-K]	rho0 = 11.61 [kg/m^3]	rho2 = 2.4 [kg/m^3]
ρ_{star} = 7.363 [kg/m^3]	T0 = 300 [K]	T2 = 159.7 [K]	T_{star} = 250 [K]
V2 = 531 [m/s]	V_{star} = 316.9 [m/s]		

Thus:

Exit area = A2 = 23.54 cm^2 … Ans.

Throat area = A_star = 12.86 cm^2 …. Ans.

Mach No. at exit = M2 = 2.096 … Ans.

Pressure at exit = P2 = 1.1 bar … Ans.

Temp. at exit = T2 = 159.7 K … Ans.

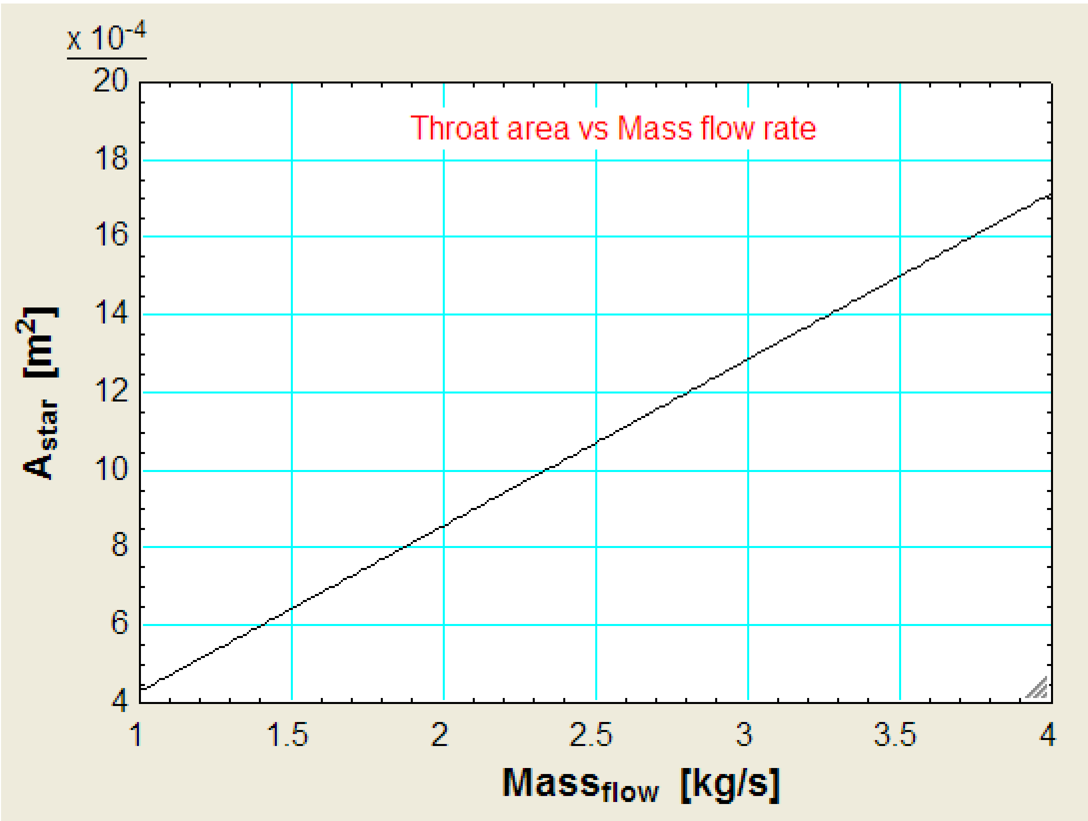
Velocity at exit = V2 = 531 m/s … Ans.

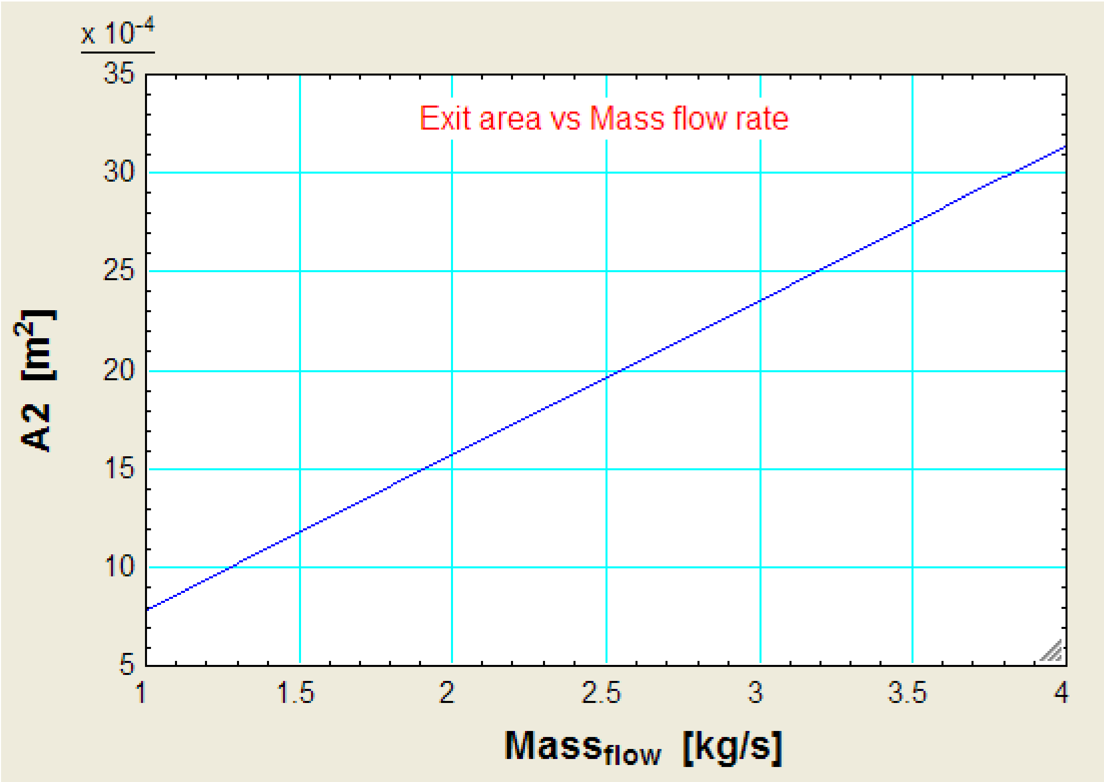
(b) Plot the variation of Throat area (Astar) and Exit area (A2), as the mass flow rate varies from 1 kg/s to 4 kg/s:

First, compute the Parametric Table:

1..16	$Mass_{flow}$ [kg/s]	A_{star} [m2]	A2 [m2]
Run 1	1	0.0004285	0.0007846
Run 2	1.2	0.0005142	0.0009416
Run 3	1.4	0.0005999	0.001098
Run 4	1.6	0.0006856	0.001255
Run 5	1.8	0.0007714	0.001412
Run 6	2	0.0008571	0.001569
Run 7	2.2	0.0009428	0.001726
Run 8	2.4	0.001028	0.001883
Run 9	2.6	0.001114	0.00204
Run 10	2.8	0.0012	0.002197
Run 11	3	0.001286	0.002354
Run 12	3.2	0.001371	0.002511
Run 13	3.4	0.001457	0.002668
Run 14	3.6	0.001543	0.002825
Run 15	3.8	0.001628	0.002982
Run 16	4	0.001714	0.003139

Now, plot the results:




Exit area vs Mass flow rate
x 10^-4
A2 [m^2]
Mass_flow [kg/s]

==

"**Prob.9.4.12** An impact tube is in a supersonic air stream where the static pressure as measured in a wall-tap is 0.25 bar. A normal shock occurs before the tip of the tube and the velocity is then reduced isentropically to the stagnation value, and the pressure measured in the impact tube is 1.41 bar. What was the initial Mach No.? What is the entropy change?"

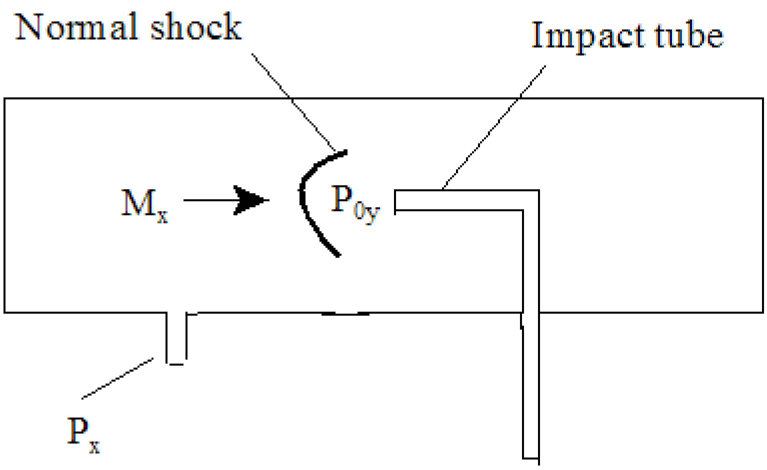


Fig.Prob.9.4.12 Impact tube – normal shock

EES Solution:

"Data:"

"Note that impact tube reads the stagnation pressure after the normal shock, i.e. P0y. Wall tap reads the static pressure before the shock, i.e. Px."

Px = 0.25"bar"

P0y = 1.41"bar"

k = 1.4

R = 287 "J/kg.K"

"Calculations:"

"P0y/Px is calculated directly once the Mach No. before the shock, i.e. Mx is known. Alternatively, knowing P0y/Px, we can immediately get the value of Mx:"

p0ybypx = P0YBYPX(Mx, k)"…finds Mx"

p0ybypx = P0y/Px

Results:

Unit Settings: SI K Pa J mass deg

ΔS = 93.92 [J/kg-K]	k = 1.4	Mx = 2
P0y = 1.41 [bar]	p0ybypx = 5.64	Px = 0.25 [bar]
R = 287 [J/kg-K]		

Thus:

Mach No. before shock = Mx = 2 …. Ans.

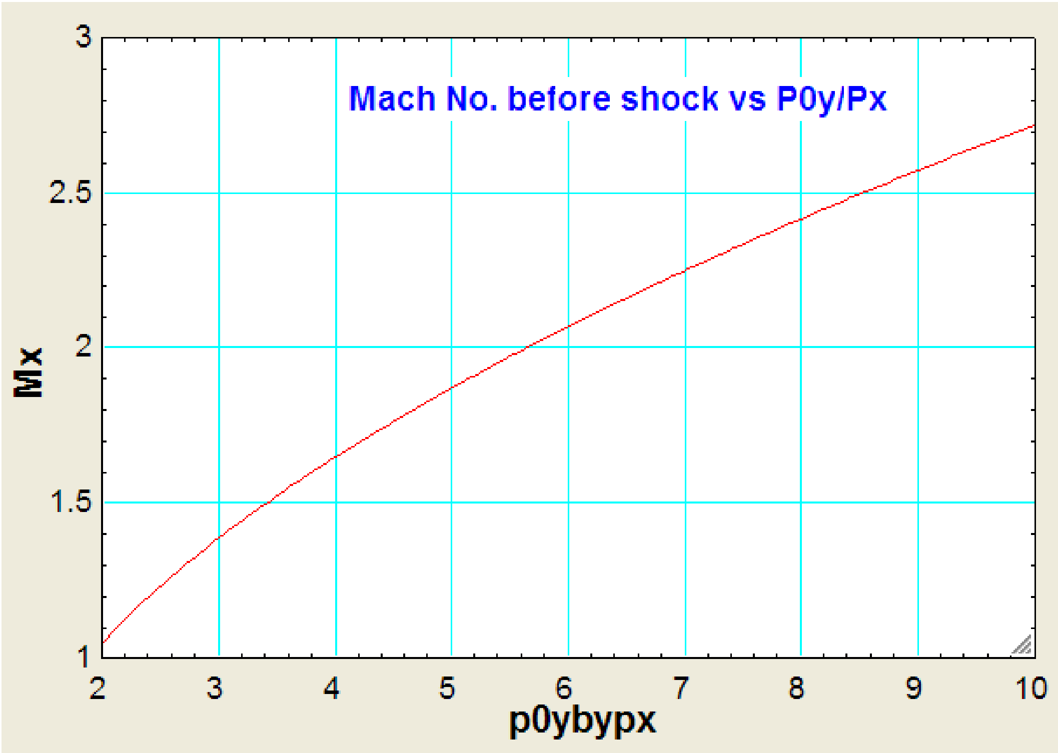
Entropy change = ΔS = 93.92 J/kg.K … Ans.

(b) Prepare a plot of Mx vs P0y/Px:

First. Compute the Parametric Table:

1..17	p0ybypx	Mx
Run 1	2	1.047
Run 2	2.5	1.231
Run 3	3	1.386
Run 4	3.5	1.523
Run 5	4	1.647
Run 6	4.5	1.763
Run 7	5	1.871
Run 8	5.5	1.972
Run 9	6	2.069
Run 10	6.5	2.161
Run 11	7	2.25
Run 12	7.5	2.335
Run 13	8	2.417
Run 14	8.5	2.496
Run 15	9	2.573
Run 16	9.5	2.647
Run 17	10	2.72

Now, plot the graph:



Note: Above method is very convenient to find the Mach No. in Supersonic flow.

Position the Impact tube parallel to flow as shown, to ensure a normal shock, measure static and stagnation pressures Px and P0y, and then use the above graph to read the value of Mach No. Mx.

==

"**Prob.9.4.13** Air enters a nozzle at a pressure of 3.5 MN/m^2 and a temp of 500 C. It leaves at a pressure of 0.7 MN/m^2. air flow rate is 1.3 kg/s and may be considered as isentropic. Determine: (i) throat area (ii) exit area (iii) Mach No. at exit. Take k = 1.4, R = 287 J/kg.K"

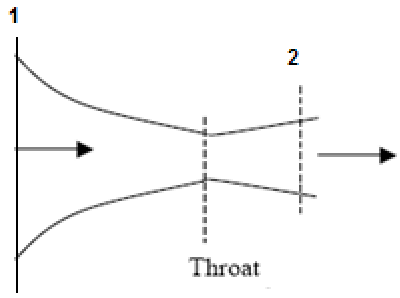


Fig.Prob.9.4.13 Isentropic flow in a C-D nozzle

EES Solution:

"Data:"

P0 = 35 "bar … since inlet velocity is not given, taken as negligible"
T0 = 773 "K"
P2 = 7"bar"
R = 287 "J/kg.K"
k = 1.4
Mass_flow = 1.3 "kg/s"

"Calculations:"

"First find out the critical pressure and determine if the flow is choked:

Critical pressure when k = 1.4 (air) is:"

P_crit = 0.5283 * P0 "bar"

"P_crit = 18.49 bar, and the exit pressure is given as 7 bar.

Therefore, it is choked flow, i.e. a C-D nozzle is required, and the pressure at throat is sonic or at throat, M = 1"

"Then, from isentropic relations:"

P_star = P0 * PBYP0(1, k) "bar…pressure at throat"

T_star = T0 * TBYT0(1, k) "K…temp at throat"

rho_star = rho0 * RHOBYRHO0(1, k)"kg/m^3….density at throat"

rho0 = P0 * 10^5 / (R * T0) "kg/m^3 … density at inlet"

V_star = sqrt(k * R * T_star) "m/s …. vel. at throat = sonic vel."

Mass_flow = rho_star * A_star * V_star "m^2…finds area at throat, A_star"

"At the exit:"

P2/P0 = PBYP0(M2,k) "..finds M2 for known P2/P0"

"With this M2, find other properties at exit:"

T2/T0 = TBYT0(M2,k)"..finds T2"

A2/A_star = ABYASTAR(M2,k)"…finds A2"

rho2/rho0 = RHOBYRHO0(M2,k) "…finds rho2"

V2 = M2 * sqrt(k * R * T2)"m/s …. finds V2"

Results:

Unit Settings: SI K Pa J mass deg

A2 = 0.0003438 [m^2]	A_{star} = 0.0002555 [m^2]	k = 1.4
M2 = 1.709	$Mass_{flow}$ = 1.3 [kg/s]	P0 = 35 [bar]
P2 = 7 [bar]	P_{crit} = 18.49 [bar]	P_{star} = 18.49 [bar]
R = 287 [J/kg-K]	rho0 = 15.78 [kg/m^3]	rho2 = 4.997 [kg/m^3]
ρ_{star} = 10 [kg/m^3]	T0 = 773 [K]	T2 = 488.1 [K]
T_{star} = 644.2 [K]	V2 = 756.6 [m/s]	V_{star} = 508.7 [m/s]

Thus:

Throat area = Astar = 2.555 cm^2 …. Ans.

Exit area = A2 = 3.438 cm^2 …. Ans.

Exit Mach No. = M2 = 1.709 ….Ans.

Exit velocity = V2 = 756.6 m/s … Ans.

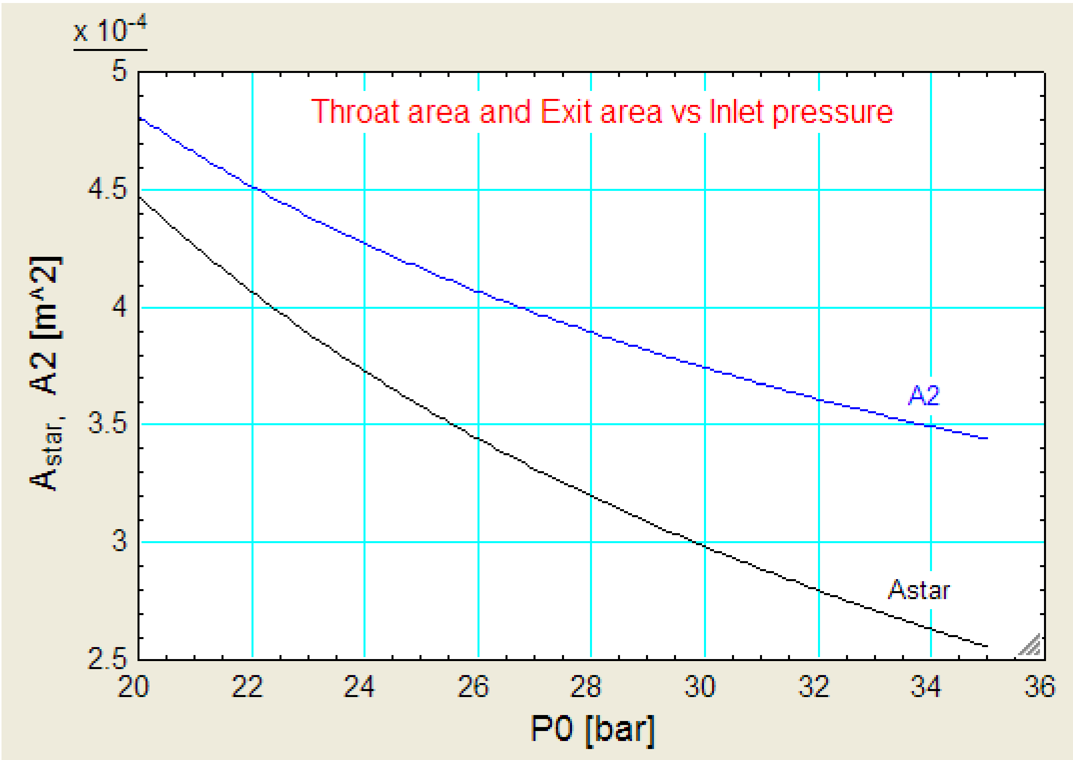
Exit temp = T2 = 488.1 K …. Ans.

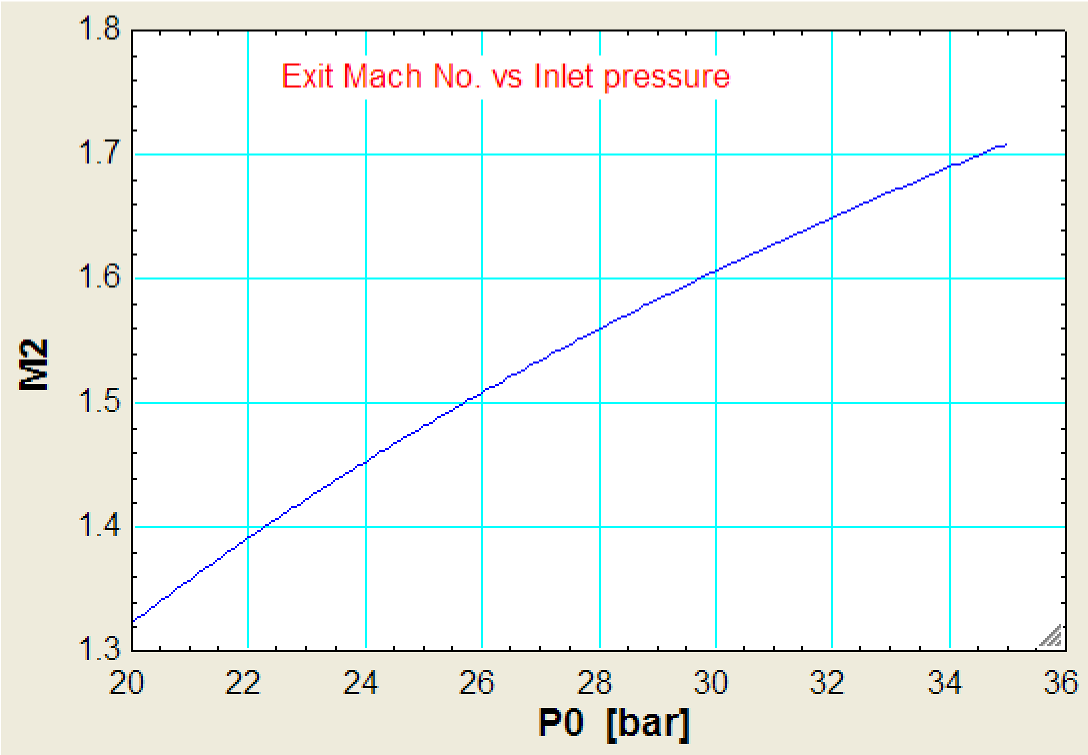
(b) Plot the throat area, exit area and exit Mach No. as inlet pressure varies from 20 bar to 35 bar, other parameters remaining the same:

First, compute the Parametric Table:

1..16	P0 [bar]	P_{star} [bar]	A_{star} [m2]	A2 [m2]	V2 [m/s]	M2
Run 1	20	10.57	0.0004471	0.0004812	634.4	1.322
Run 2	21	11.09	0.0004258	0.0004654	646.8	1.358
Run 3	22	11.62	0.0004065	0.0004512	658.3	1.391
Run 4	23	12.15	0.0003888	0.0004384	668.9	1.423
Run 5	24	12.68	0.0003726	0.0004268	678.9	1.453
Run 6	25	13.21	0.0003577	0.0004162	688.1	1.481
Run 7	26	13.74	0.0003439	0.0004064	696.8	1.508
Run 8	27	14.26	0.0003312	0.0003974	705	1.534
Run 9	28	14.79	0.0003194	0.000389	712.7	1.559
Run 10	29	15.32	0.0003084	0.0003813	719.9	1.583
Run 11	30	15.85	0.0002981	0.000374	726.8	1.606
Run 12	31	16.38	0.0002885	0.0003672	733.4	1.628
Run 13	32	16.91	0.0002794	0.0003608	739.6	1.649
Run 14	33	17.43	0.000271	0.0003548	745.5	1.669
Run 15	34	17.96	0.000263	0.0003492	751.2	1.689
Run 16	35	18.49	0.0002555	0.0003438	756.6	1.709

Now, plot the results:





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"**Prob.9.4.14** Consider a CD nozzle with throat area = 20 cm^2, exit area = 33.75 cm^2, inlet pressure = 10 bar, inlet temp = 800 K, negligible inlet velocity. Take k = 1.4, R = 287 J/kg.K for air. Determine: mass flow rate, exit pressure and exit Mach No. for following cases:

i. Isentropic flow with M = 0.7 at the throat,
ii. Isentropic flow with M = 1 at the throat and diverging portion acting as diffuser,
iii. Isentropic flow with M = 1 at the throat and diverging portion acting as nozzle,
iv. Isentropic flow through nozzle with a normal shock standing at the exit,
v. A normal shock stands in the diverging section where the area is 25 cm^2; elsewhere in the nozzle, the flow is isentropic."

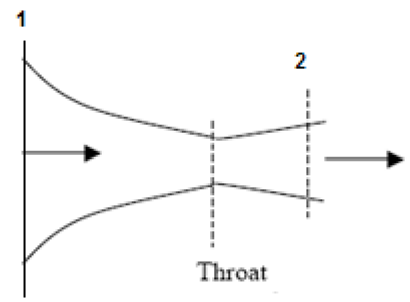


Fig.Prob.9.4.14 Isentropic flow in a C-D nozzle

EES Solution:

"Data:"

P0 = 10 "bar … since inlet velocity is not given, taken as negligible"

T0 = 800 "K"

A_throat = 20E-04"m^2 … throat area"

A2 = 33.75E-04 "m^2 … exit area"

R = 287 "J/kg.K"

k = 1.4

"Case(i): M = 0.7 at throat"

"Now, flow through the entire nozzle is subsonic"

"At M = 0.7 find A_throat/Astar:"

M = 0.7

A_throat/Astar = ABYASTAR(M,k)"….finds Astar when M_throat = 0.7"

"Therefore:"

A2/Astar = ABYASTAR(M2,k)"…finds exit Mach No. M2. This should be less than 1; so, choose the guess value for M2 as less than 1, see Prob. 9.4.9"

"Now, for this value of M2, get T2, P2, rho2 etc:"

T2/T0 = TBYT0(M2,k)"…finds T2"

P2/P0 = PBYP0(M2,k)"..finds P2"

V2 = M2 * sqrt(k * R * T2)"m/s … exit velociy"

rho2 = P2 *10^5 / (R * T2)"kg/m^3 …. density at exit"

Mass_flow_case1 = rho2 * A2 * V2"kg/s"

Results for case (i):

Unit Settings: SI K Pa J mass deg

A2 = 0.003375 [m^2]	Astar = 0.001828 [m^2]	A_{throat} = 0.002 [m^2]
k = 1.4	M = 0.7	M2 = 0.3349
$Mass_{flow,case1}$ = 2.612 [kg/s]	P0 = 10 [bar]	P2 = 9.253 [bar]
R = 287 [J/kg-K]	rho2 = 4.12 [kg/m^3]	T0 = 800 [K]
T2 = 782.4 [K]	V2 = 187.8 [m/s]	

Thus, for case (i):

Mass flow rate = 2.612 kg/s … Ans; Exit pressure = P2 = 9.253 bar … Ans.

Exit Mach No. = M2 = 0.3349 … Ans.

"………"

"Case(ii): M = 1 at throat, diverging portion acting as diffuser"

"Now, flow through the diverging portion decelerates and at exit it is subsonic, and M2 < 1"

Astar = A_throat "…at the throat when M = 1"

A2/Astar = ABYASTAR(M2,k)"….finds M2 when M_throat =1"

"Therefore:"

"Now, for this value of M2, get T2, P2, rho2 etc:"

T2/T0 = TBYT0(M2,k)"…finds T2"

P2/P0 = PBYP0(M2,k)"..finds P2"

V2 = M2 * sqrt(k * R * T2)"m/s … exit velociy"

rho2 = P2 *10^5 / (R * T2)"kg/m^3 …. density at exit"

Mass_flow_case1 = rho2 * A2 * V2"kg/s"

Results for case (ii):

Unit Settings: SI K Pa J mass deg

A2 = 0.003375 [m^2]	Astar = 0.002 [m^2]	A_{throat} = 0.002 [m^2]
k = 1.4	M2 = 0.3722	$Mass_{flow,case2}$ = 2.858 [kg/s]
P0 = 10 [bar]	P2 = 9.088 [bar]	R = 287 [J/kg-K]
rho2 = 4.068 [kg/m^3]	T0 = 800 [K]	T2 = 778.4 [K]
V2 = 208.2 [m/s]		

Thus, for case (ii):

Mass flow rate = 2.858 kg/s … Ans.

Exit pressure = P2 = 9.088 bar .. Ans.

Exit Mach No. = M2 = 0.3722 … Ans.

Note: This is the max. possible mass flow, as mentioned earlier.

"Case(iii): M = 1 at throat, diverging portion acting as nozzle"

"Now, flow through the diverging portion accelerates and at exit it is supersonic, and M2 > 1"

"So, choose M2 > 1 for guess value of M2….see Prob.9.4.9"

```
Astar = A_throat "…at the throat since M = 1 at throat"

A2/Astar = ABYASTAR(M2,k)"….finds M2 when Mthroat =1"

"Therefore:"

"Now, for this value of M2, get T2, P2, rho2 etc:"

T2/T0 = TBYT0(M2,k)"…finds T2"

P2/P0 = PBYP0(M2,k)"..finds P2"

V2 = M2 * sqrt(k * R * T2)"m/s … exit velociy"

rho2 = P2 *10^5 / ( R * T2)"kg/m^3 …. density at exit"

Mass_flow_case3 = rho2 * A2 * V2"kg/s"
```

Results for case (iii):

Unit Settings: SI K Pa J mass deg

A2 = 0.003375 [m^2]	Astar = 0.002 [m^2]	A_{throat} = 0.002 [m^2]
k = 1.4	M2 = 2	$Mass_{flow,case3}$ = 2.858 [kg/s]
P0 = 10 [bar]	P2 = 1.278 [bar]	R = 287 [J/kg-K]
rho2 = 1.002 [kg/m^3]	T0 = 800 [K]	T2 = 444.4 [K]
V2 = 845.2 [m/s]		

Thus, for case (iii):

Mass flow rate = 2.858 kg/s … Ans.

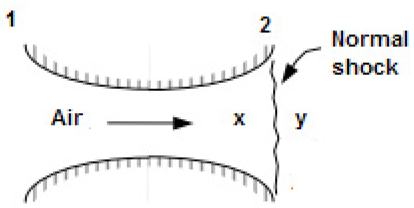
Exit pressure = P2 = 1.278 bar .. Ans.

Exit Mach No. = M2 = 2 … Ans.

Note: This is the max. possible mass flow for given geometry and stagn. Pressure and temp.

" .... "

"Case(iv): M = 1 at throat, Normal shock at exit:"



"So, just before the shock, we have Mx = M2 = 2, obtained from case(iii)"

Astar = A_throat "…at the throat since M = 1 at throat"

A2/Astar = ABYASTAR(M2,k)"….finds M2 when Mtroat =1"

Mx = M2

Px = 1.278 "bar = P2 from previous case"

Tx = 444.4 "K = T2 from previous case"

"Therefore:"

"Now, for this value of Mx, get property values from normal shock Tables (i.e. EES functions written earlier):"

My = MY(Mx,k)"…finds Mach No. after the shock, My"

Py/Px = PYBYPX(Mx,k)"…finds static pressure after the shock, Py"

Ty/Tx = TYBYTX(Mx,k)"…finds static temp after the shock, Ty"

Mass_flow_case4 = 2.858"kg/s …. max. flow, from previous case"

Results for case (iv):

Unit Settings: SI K Pa J mass deg

A2 = 0.003375 [m^2]	Astar = 0.002 [m^2]	A_{throat} = 0.002 [m^2]
k = 1.4	M2 = 2	$Mass_{flow,case4}$ = 2.858 [kg/s]
Mx = 2	My = 0.5774	P0 = 10 [bar]
P0x = 10	P0y = 7.209 [bar]	Px = 1.278 [bar]
Py = 5.751 [bar]	R = 287 [J/kg-K]	rhox = 1.002 [kg/m^3]
rhoy = 2.672 [kg/m^2]	T0 = 800 [K]	Tx = 444.4 [K]
Ty = 749.9 [K]		

Thus, for case (iv):

Mass flow rate = 2.858 kg/s … Ans.

Pressure after shock = Py = 5.751 bar .. Ans.

Stagn. pressure after shock = P0y = 7.209 bar … Ans.

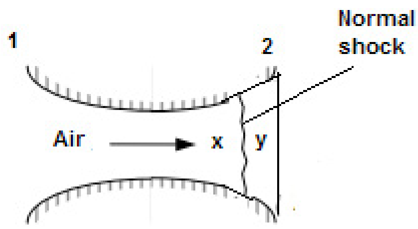
Therefore. Loss in stagn. Pressure because of shock = 10 – 7.209 = 2.79 bar … Ans.

Temp. after shock = Ty = 749.9 K … Ans.

Mach No. after shock = My = 0.5774 … Ans.

"………………………………………………………………………………………………………"

"Case(v): M = 1 at throat, Normal shock at at a section where Ax = 25 cm^2:"



"So, Ax / Astar known. For this value of Ax/Astar, get Mx etc from isentropic functions:"

Astar = A_throat "…at the throat… since M = 1 at throat"

Ax = 25E-04"m^2"

Ay = Ax

Ax/Astar = ABYASTAR(Mx,k)"….finds Mx when Ax/Astar is known"

Px = P0 * PBYP0(Mx,k) "bar … finds Px"

Tx = T0 * TBYT0(Mx,k) "K … finds Tx "

"Therefore:"

My = MY(Mx,k)"…finds Mach No. after the shock, My"

Py/Px = PYBYPX(Mx,k)"…finds static pressure after the shock, Py"

Ty/Tx = TYBYTX(Mx,k)"…finds static temp after the shock, Ty"

rhoy = Py * 10^5 / (R * Ty) "…kg/m^3… density after shock"

rhoy/rhox = RHOYBYRHOX(Mx,k)"…finds rhox, kg/m^3"

P0x = P0 "..isentr. flow up to shock"

P0y/P0x = P0YBYP0x(Mx,k) "..finds P0y"

Mass_flow_case5 = 2.858"kg/s …. max. flow, from previous case"

"After the shock, flow is isentropic up to the exit.

Therefore, use the isentropic functions again:"

AybyAstar = ABYASTAR(My,k)

"Then:"

A2/Aystar = (A2/Ay) * (AybyAstar)

A2/Aystar = ABYASTAR(M2,k)"…finds M2 at exit"

"Now, corresponding to this M2, we have:"

Ty/T0y = TBYT0(My,k)"…finds T0y"

P2/P0y = PBYP0(M2,k)"..finds P2"

T2/T0y = TBYT0(M2,k)"…finds T2"

Stagn_Press_loss = P0x – P0y "bar"

V2 = M2 * sqrt(k * R * T2)"m/s … exit velociy"

"Entropy increase in shock:"

DELTAS = DELTAS(Mx,R,k)"J/kg.K"

Results for case (v):

Unit Settings: SI K Pa J mass deg

A2 = 0.003375 [m^2]	Astar = 0.002 [m^2]	Ax = 0.0025 [m^2]
Ay = 0.0025 [m^2]	AybyAstar = 1.119	Aystar = 0.002234
A_{throat} = 0.002 [m^2]	ΔS = 31.74 [J/kg-K]	k = 1.4
M2 = 0.4263	$Mass_{flow,case5}$ = 2.858 [kg/s]	Mx = 1.6
My = 0.6685	P0 = 10 [bar]	P0x = 10 [bar]
P0y = 8.953 [bar]	P2 = 7.901 [bar]	Px = 2.354 [bar]
Py = 6.635 [bar]	R = 287 [J/kg-K]	rhox = 1.55 [kg/m^3]
rhoy = 3.148 [kg/m^3]	$Stagn_{Press,loss}$ = 1.047 [bar]	T0 = 800 [K]
T0y = 800 [K]	T2 = 771.9 [K]	Tx = 529.2 [K]
Ty = 734.4 [K]	V2 = 237.4 [m/s]	

Thus, for case (v):

Mass flow rate = 2.858 kg/s … Ans.

Pressure after shock = Py = 6.635 bar .. Ans.

Stagn pressure after shock = P0y = 8.953 bar … Ans.

Therefore. Loss in stagn. pressure because of shock = (P0x – P0y) = 1.074 bar … Ans.

Temp. after shock = Ty = 734.4 K … Ans.

Mach No. after shock = My = 0.6685 … Ans.

Entropy change across shock = ΔS = 31.74 J/kg.K … Ans.

Exit pressure = P2 = 7.901 bar … Ans.

Exit temp = T2 = 771.9 K …. Ans.

==

"**Prob.9.4.15** Air flows subsonically in an adiabatic 2 cm dia duct. Average friction factor is 0.006. What length of duct is necessary to accelerate the flow from M1 = 0.1 to M2 = 0.5? What additional length will accelerate it to M3 = 1? Assume k = 1.4"

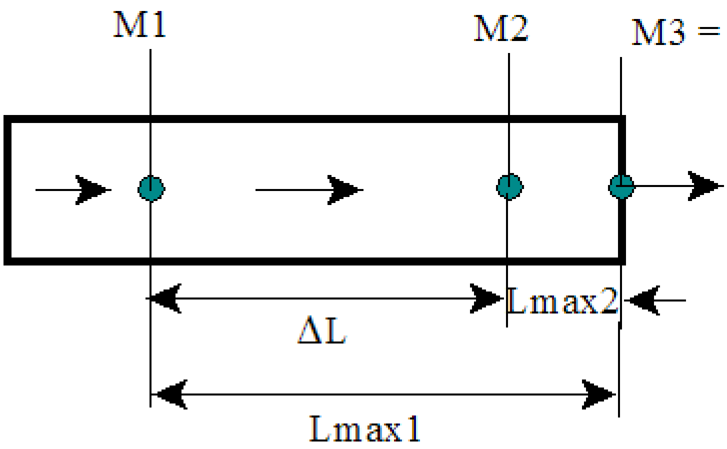


Fig.Prob.9.4.15 Fanno flow

EES Solution:

"Data:"

d = 0.02"m"

f = 0.006

M1 = 0.1

M2 = 0.5

M3 = 1

k = 1.4

"Calculations:"

"We use the EES Functions written earlier for Fanno flow:"

"At M1 = 0.1, find Lmax1:"

4 * f * Lmax1 /d = FANNO_FOURFLMAXBYD(M1,k) "…finds Lmax1 from the point where M1 = 0.1to the location where M = 1"

"At M2 = 0.5, find Lmax2:"

4 * f * Lmax2 /d = FANNO_FOURFLMAXBYD(M2,k) "…finds Lmax2 from the point where M = 0.5 to the location where M = 1"

"Therefore:"

DELTAL = Lmax1 – Lmax2"m… distance between locations where Mach Nos. are M1 and M2"

Results:

Unit Settings: SI K Pa J mass deg

d = 0.02 [m]	ΔL = 54.88 [m]	f = 0.006	k = 1.4
Lmax1 = 55.77 [m]	Lmax2 = 0.8909 [m]	M1 = 0.1	M2 = 0.5
M3 = 1			

Thus:

Distance between locations where Mach Nos. are M1 and M2 = dL = 54.88 m …Ans.

Distance between locations where Mach Nos. are M2 and M3 = Lmax2 = 0.8909 m … Ans.

Note: It takes 54.88 m to accelerate from M1 = 0.1 to M2 = 0.5, but takes only 0.89 m to reach sonic velocity (i.e. M = 1) from the location of M2 = 0.5

==

"**Prob.9.4.16** In the above problem, assume that at M1 = 0.1, the pressure and temp are: P1 = 600 kPa and T1 = 450 K respectively. At section 2 further downstream, M2 = 0.5. Compute P2, T2, V2 and P02."

EES Solution:

"Data:"

M1 = 0.1

M2 = 0.5

P1 = 600"kPa"

T1 = 450"K"

k = 1.4

R = 287"J/kg.K"

"Calculations:"

"Find V1 and P01 at section1:"

V1 = M1 * sqrt(k * R * T1)

"From Isentropic functions, at M1 = 0.1:"

P1/P01 = PBYP0(M1,k)"…finds P01, kPa"

"Now, use Fanno flow Functions to compute all properties downstream:"

P2/P1 = FANNO_PBYPSTAR(M2,k) / FANNO_PBYPSTAR(M1,k)"…finds P2, kPa"

T2/T1 = FANNO_TBYTSTAR(M2,k) / FANNO_TBYTSTAR(M1,k)"…finds T2, K"

V2/V1 = FANNO_VBYVSTAR(M2,k) / FANNO_VBYVSTAR(M1,k)"…finds V2, m/s"

P02/P01 = FANNO_P0BYP0STAR(M2,k) / FANNO_P0BYP0STAR(M1,k)"…finds P02, kPa"

Results:

Unit Settings: SI K Pa J mass deg

k = 1.4	M1 = 0.1	M2 = 0.5	P01 = 604.2 [kPa]
P02 = 139.1 [kPa]	P1 = 600 [kPa]	P2 = 117.2 [kPa]	R = 287 [J/kg-K]
T1 = 450 [K]	T2 = 429.4 [K]	V1 = 42.52 [m/s]	V2 = 207.7 [m/s]

Thus, at section 2, where M2 = 0.5:

P2 = 117.2 kPa … Ans.

T2 = 429.4 K ….Ans.

V2 = 207.7 m/s … Ans.

P02 = Stagn. pressure = 139.1 kPa … Ans.

Stagn. pressure loss due to friction = (P01 – P02) = 465.1 kPa … Ans.

==

"**Prob.9.4.17** A circular duct passes 8.25 kg/s of air at an exit Mach No. of 0.5. The entry pressure and temp are 3.45 bar and 38 C. Coeff. of friction is 0.0005. If the Mach No. at entry is 0.15, determine: (i) dia of duct (ii) length of duct (iii) pressure and temp at exit (iv) stagnation pressure loss (v) verify exit Mach No. through exit velocity and temp."

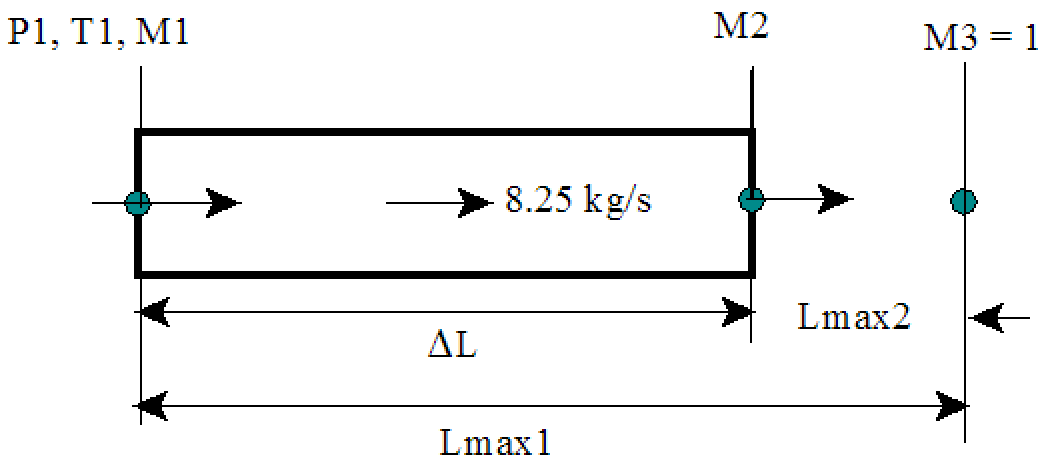


Fig.Prob.9.4.17 Fanno flow

EES Solution:

"Data:"

P1 = 3.45"bar"
T1 = 38+273"K"
M1 = 0.15
mdot = 8.25"kg/s"
f = 0.005

M2 = 0.5

k = 1.4

R = 287"J/kg.K"

"Calculations:"

rho1 = P1*1E05/(R * T1)"kg/m^3….density at entry"

C1 = sqrt(k * R * T1) "m/s …. sonic velocity at entry"

V1 = M1 * C1 "m/s…velocity at entry"

mdot = rho1 * A1 * V1 "m^2 …. finds area at entry"

A1 = pi * d^2 / 4 "m….finds dia of duct"

"To find P01:"

"From Isentropic Functions:"

P1/P01 = PBYP0(M1,k)"….finds P01, bar"

"At M1 = 0.15, find Lmax1:"

4 * f * Lmax1 /d = FANNO_FOURFLMAXBYD(M1,k) "…finds Lmax1 from the point where M1 = 0.1to the location where M = 1"

"At M2 = 0.5, find Lmax2:"

4 * f * Lmax2 /d = FANNO_FOURFLMAXBYD(M2,k) "…finds Lmax2 from the point where M = 0.5 to the location where M = 1"

"Therefore:"

DELTAL = Lmax1 – Lmax2"m… distance between locations where Mach Nos. are M1 and M2"

"Now, find properties from Fanno flow Functions:"

P2/P1 = FANNO_PBYPSTAR(M2,k) / FANNO_PBYPSTAR(M1,k)"…finds P2, bar"

T2/T1 = FANNO_TBYTSTAR(M2,k) / FANNO_TBYTSTAR(M1,k)"…finds T2, K"

V2/V1 = FANNO_VBYVSTAR(M2,k) / FANNO_VBYVSTAR(M1,k)"…finds V2, m/s"

P02/P01 = FANNO_P0BYP0STAR(M2,k) / FANNO_P0BYP0STAR(M1,k)"…finds P02, kPa"

"To verify M2:"

C2 = sqrt(k * R * T2)"m/s …. sonic vel. at exit"

"Therefore, M2_verify:"

M2_verify = V2/C2 "…should be equal to M2"

Results:

Unit Settings: SI K Pa J mass deg

A1 = 0.04025 [m^2]	C1 = 353.5 [m/s]	C2 = 345.8 [m/s]	d = 0.2264 [m]
ΔL = 304.1 [m]	f = 0.005	k = 1.4	Lmax1 = 316.2 [m]
Lmax2 = 12.1 [m]	M1 = 0.15	M2 = 0.5	$M2_{verify}$ = 0.5
mdot = 8.25 [kg/s]	P01 = 3.505 [bar]	P02 = 1.201 [bar]	P1 = 3.45 [bar]
P2 = 1.012 [bar]	R = 287 [J/kg-K]	rho1 = 3.865 [kg/m^3]	T1 = 311 [K]
T2 = 297.5 [K]	V1 = 53.02 [m/s]	V2 = 172.9 [m/s]	

Thus:

Dia of duct = d = 0.2264 m … Ans.

Length of duct = ΔL = 304.1 m … Ans.

Pressure at exit = P2 = 1.012 bar … Ans.

Temp at exit = T2 = 297.5 K … Ans.

Stagn. pressure loss = (P01 – P02) = 2.304 bar … Ans.

M2 = M2_verify ….verified…Ans.

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"**Prob.9.4.18** Air at P0 = 10 bar, T0 = 400 K is supplied to a 50 mm dia pipe. Friction factor = 0.002. If Mach No. changes from 3 at entry to 1 at exit, determine: (i) the length of pipe (ii) mass flow rate. (b) Plot the variation of ΔL and mdot as M1 varies from 1.5 to 3.5, all other parameters remaining the same."

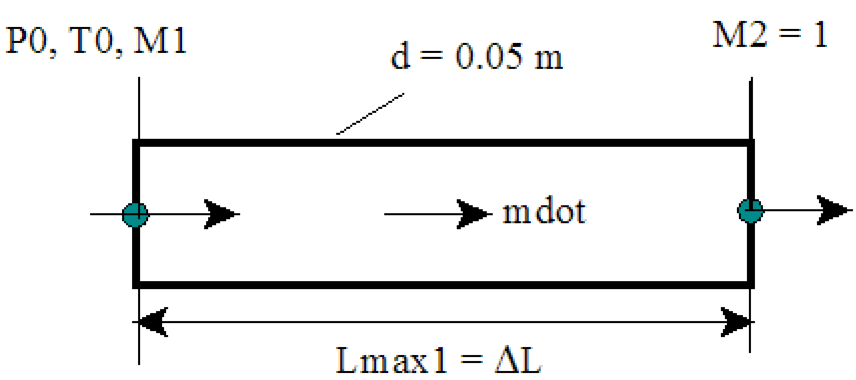


Fig.Prob.9.4.18 Fanno flow

EES Solution:

"Data:"

P0 = 10"bar"
T0 = 400"K"
M1 = 3 "….Mach No. at entry"
M2 = 1 "….Mach No. at exit"
d = 0.05"m…dia of pipe"
f = 0.002
k = 1.4
R = 287"J/kg.K"

"Calculations:"

A = pi * d^2 / 4 "m^2 …. cross-sectional area of pipe"

"We shall use EES functions for Fanno flow, written earlier:"

"At M1 =3, find Lmax1:"

4 * f * Lmax1 /d = FANNO_FOURFLMAXBYD(M1,k) "…finds Lmax1 from the point where M1 = 3 to the location where M = 1, i.e. the exit, in the present case"

"Therefore:"

DELTAL = Lmax1"m… Length of pipe, i.e. distance between locations where Mach Nos. are M1 and M2"

"To find mass flow rate, mdot: We have: mdot = rho1 * A * V1:"

"Now, use the Isentropic Functions:"

"At M1 = 3:"

P1/P0 = PBYP0(M1,k)"…finds pressure at inlet, P1(bar)"

T1/T0 = TBYT0(M1,k)"…finds temp at inlet, T1 (K)"

"And:"

rho1 = P1*1E05 / (R * T1)"kg/m^3 …. density at inlet"

C1 = sqrt(k * R * T1)"m/s …. sonic velocity at inlet"

"Therefore:"

V1 = M1 * C1 "m/s…. velocity at inlet"

"And:"

mdot = rho1 * A * V1"kg/s …. mass flow rate"

Results:

Unit Settings: SI K Pa J mass deg

A = 0.001963	C1 = 239.6 [m/s]	d = 0.05 [m]
ΔL = 3.263 [m]	f = 0.002	k = 1.4
Lmax1 = 3.263 [m]	M1 = 3	M2 = 1
mdot = 0.9371 [kg/s]	P0 = 10 [bar]	P1 = 0.2722 [bar]
R = 287 [J/kg-K]	rho1 = 0.664 [kg/m^3]	T0 = 400 [K]
T1 = 142.9 [K]	V1 = 718.7 [m/s]	

Thus:

Length of pipe = ΔL = 3.263 m … Ans.

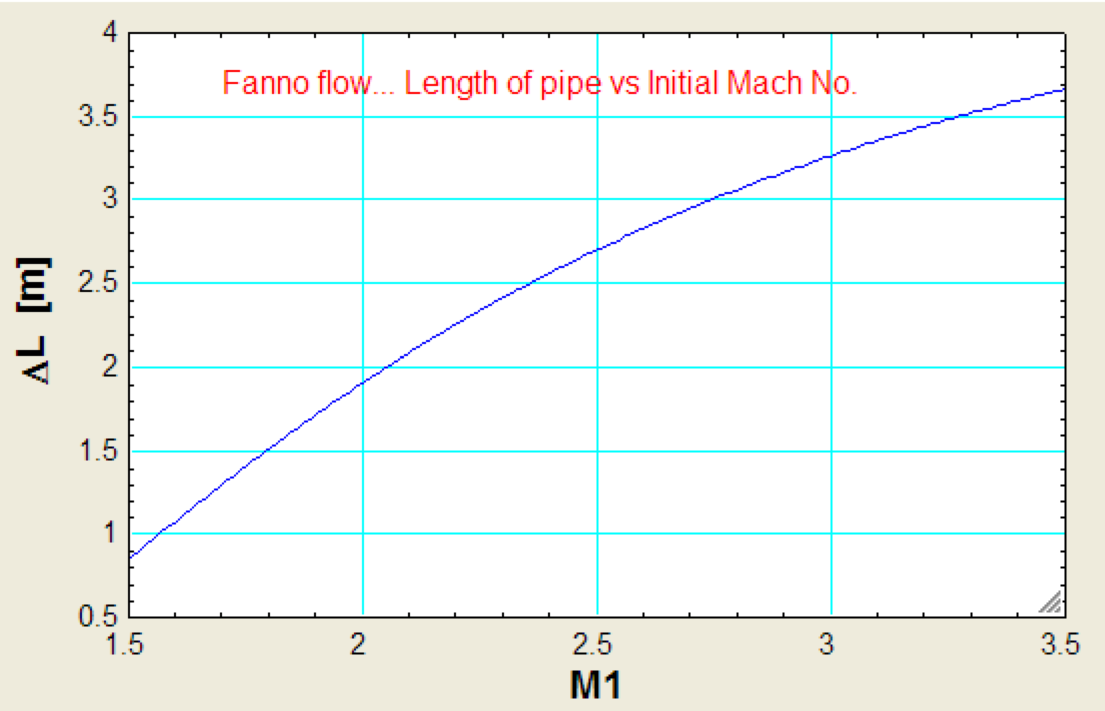
Mass flow rate = mdot = 0.9371 kg/s … Ans.

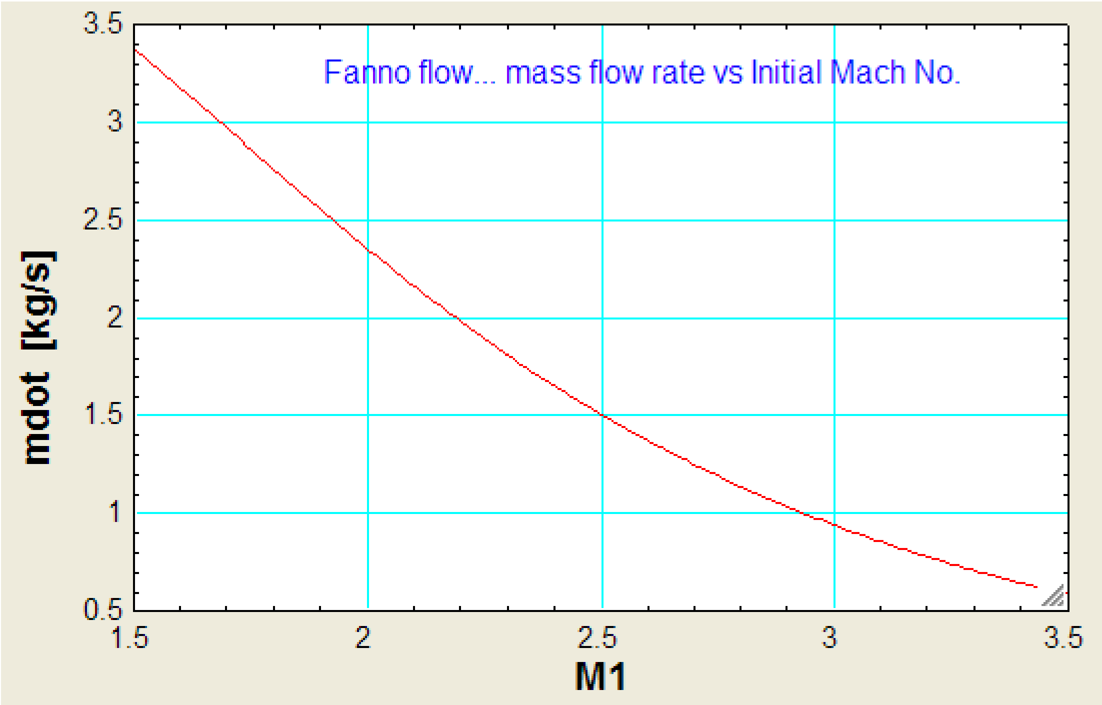
(b) Plot the variation of ΔL and mdot as M1 varies from 1.5 to 3.5, all other parameters remaining the same:

First, compute the Parametric Table:

1..11	M1	ΔL [m]	mdot [kg/s]
Run 1	1.5	0.8503	3.374
Run 2	1.7	1.299	2.967
Run 3	1.9	1.715	2.551
Run 4	2.1	2.087	2.16
Run 5	2.3	2.414	1.809
Run 6	2.5	2.7	1.505
Run 7	2.7	2.949	1.247
Run 8	2.9	3.166	1.031
Run 9	3.1	3.355	0.852
Run 10	3.3	3.52	0.705
Run 11	3.5	3.665	0.5844

Now, plot the results:





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"**Prob.9.4.19** A fuel-air mixture, approximated as air with k = 1.4, enters a duct combustion chamber at V1 = 75 m/s, P1 = 150 kPa, T1 = 300 K. The heat addition by combustion is 900 kJ/kg. Compute: (i) the exit properties V2, P2, T2, and (ii) the total heat addition which would cause a sonic exit flow."

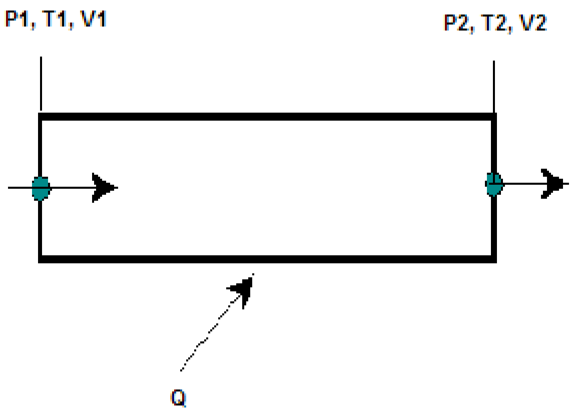


Fig.Prob.9.4.19 Rayleigh flow

EES Solution:

This is Rayleigh flow:

"Data:"

k = 1.4

cp = 1005 "J/kg.K"

R = 287"J/kg.K"

V1 = 75"m/s"

P1 = 150"kPa"

T1 = 300"K"

q = 9E05"J/kg"

"Calculations:"

T01 = T1 + V1^2/(2 * cp)"K….finds stagn. temp T01 at 1"

"Therefore:"

q = cp * (T02 – T01)"..finds T02(K)"

"Now:"

C1 = sqrt(k * R * T1)"m/s …. sonic velocity at 1"

"Then:"

M1 = V1 / C1"….Mach No. at 1"

"For this M1, find Rayleigh flow parameters using the EES functions for Rayleigh flow written earlier:"

T01/T0star = RAYLEIGH_T0BYT0STAR(M1,k)"….finds T0star"

"At section 2:"

T02/T0star = RAYLEIGH_T0BYT0STAR(M2,k)"…finds M2"

"Now that we have M1 and M2, we can use Rayleigh flow functions to get exit parameters as follows:"

"Ex: (V2/V1) = (V2/Vstar) / (V1/Vstar)"

V2/V1 = RAYLEIGH_VBYVSTAR(M2,k) / RAYLEIGH_VBYVSTAR(M1,k) "…finds V2 (m/s)"

P2/P1 = RAYLEIGH_PBYPSTAR(M2,k) / RAYLEIGH_PBYPSTAR(M1,k) "…finds P2 (kPa)"

T2/T1 = RAYLEIGH_TBYTSTAR(M2,k) / RAYLEIGH_TBYTSTAR(M1,k) "…finds T2 (K)"

"Max. possible heat transfer, q_max:"

"Now, Mach No. will be 1, and T02 = T0star"

"Therefore:"

q_max = cp * (T0star – T01)"J/kg"

Results:

Unit Settings: SI K Pa J mass deg

C1 = 347.2 [m/s]	cp = 1005 [J/kg-K]	k = 1.4
M1 = 0.216	M2 = 0.5731	P1 = 150 [kPa]
P2 = 109.5 [kPa]	q = 900000 [J/kg]	q_{max} = 1.223E+06 [J/kg]
R = 287 [J/kg-K]	T01 = 302.8 [K]	T02 = 1198 [K]
T0star = 1520 [K]	T1 = 300 [K]	T2 = 1124 [K]
V1 = 75 [m/s]	V2 = 385.2 [m/s]	

Thus:

Exit Mach No. = M2 = 0.5731 … Ans.

Exit pressure = P2 = 109.5 kPa … Ans.

Exit temp = T2 = 1124 K …. Ans.

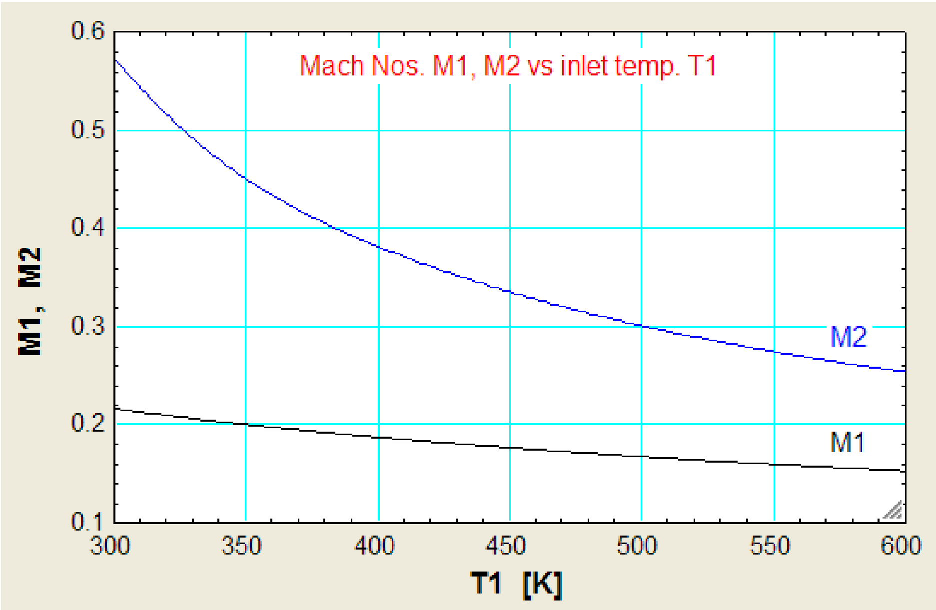
Exit velocity = V2 = 385.2 m/s …. Ans.

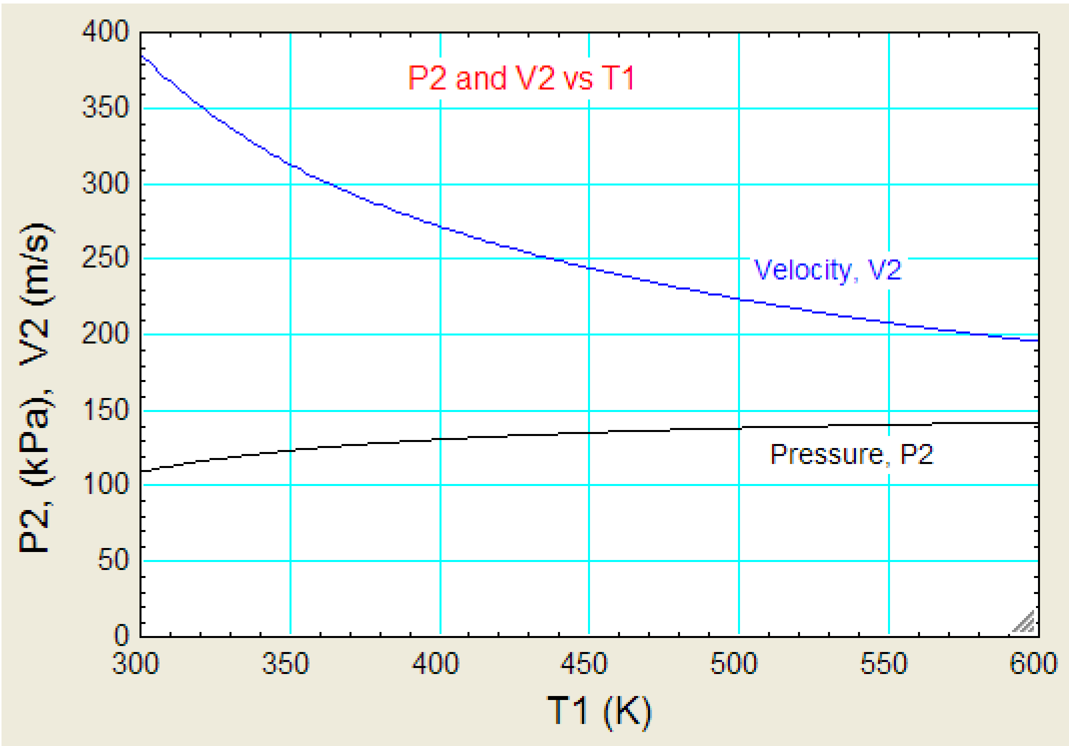
(b) If T1 varies from 300 K to 600 K, how do other properties, including the max. possible heat transfer change?

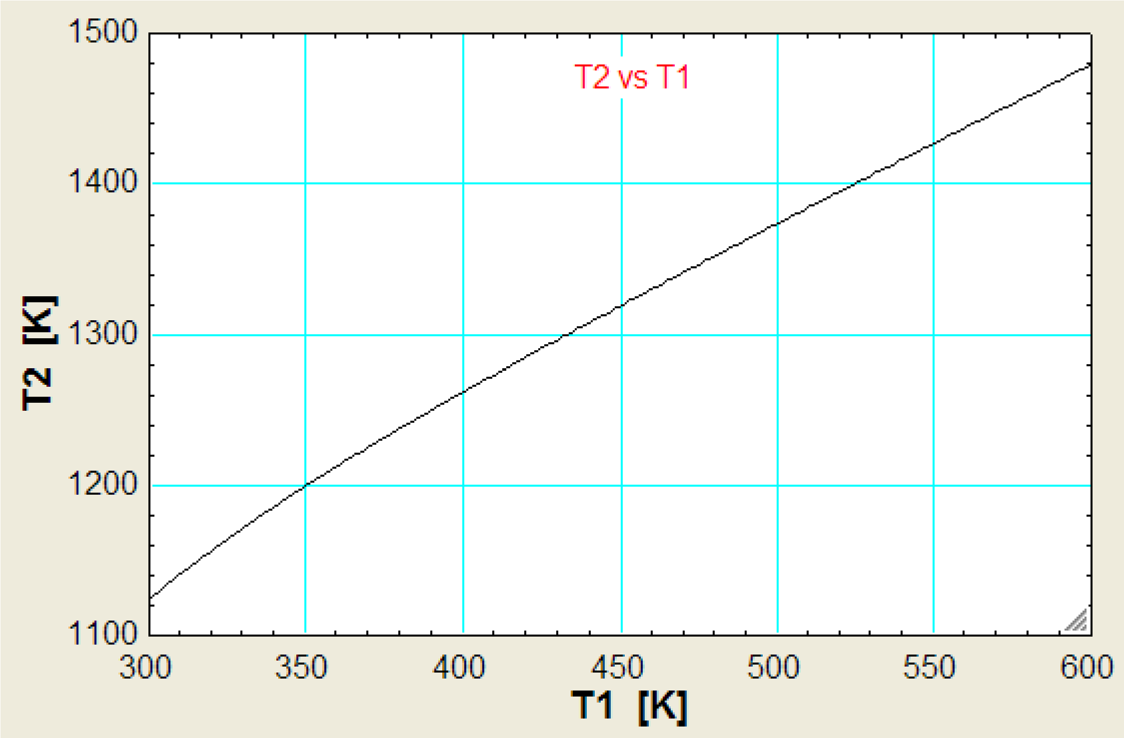
First, compute the Parametric Table:

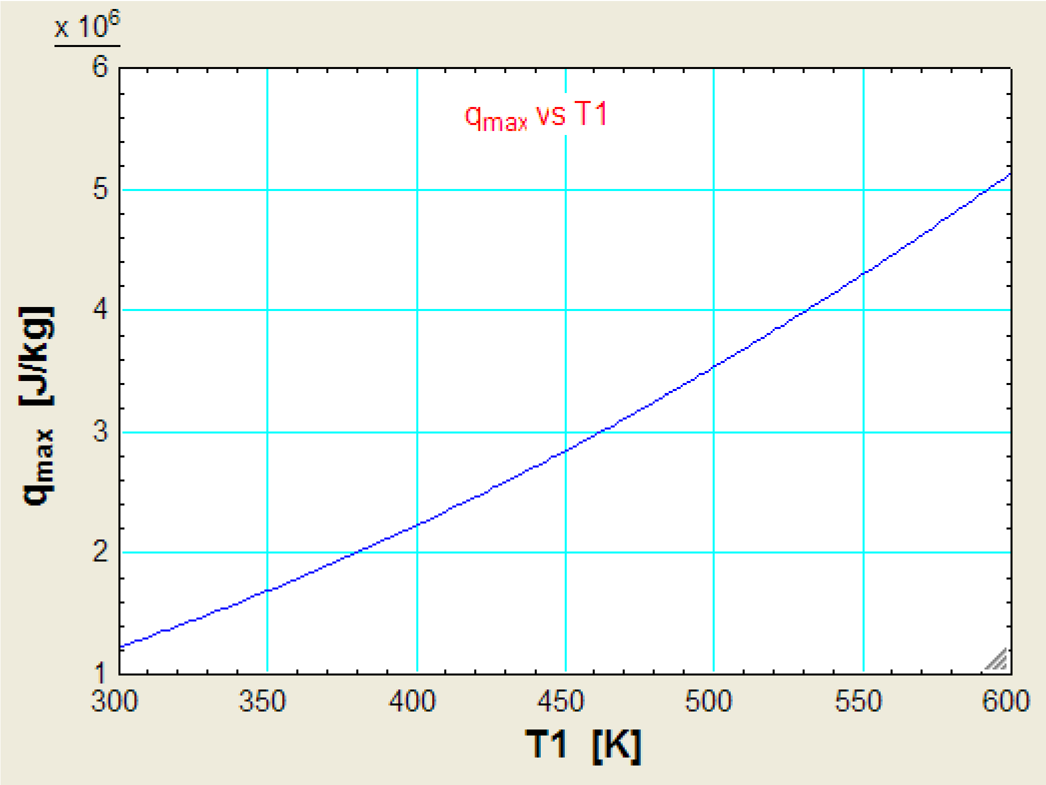
1..7	T1 [K]	M1	M2	P2 [kPa]	V2 [m/s]	T2 [K]	q_{max} [J/kg]
Run 1	300	0.216	0.5731	109.5	385.2	1124	1.223E+06
Run 2	350	0.2	0.4501	123.4	312.5	1200	1.688E+06
Run 3	400	0.1871	0.3812	130.8	271.4	1262	2.228E+06
Run 4	450	0.1764	0.3347	135.3	243.7	1319	2.843E+06
Run 5	500	0.1673	0.3006	138.4	223.3	1373	3.533E+06
Run 6	550	0.1595	0.2743	140.5	207.7	1427	4.297E+06
Run 7	600	0.1527	0.2531	142.2	195.1	1479	5.136E+06

Now, plot the results:






T2 vs T1
T2 [K]
T1 [K]
1500
1400
1300
1200
1100
300
350
400
450
500
550
600


x 10^6
q_{max} vs T1
q_{max} [J/kg]
T1 [K]
6
5
4
3
2
1
300
350
400
450
500
550
600

==

"**Prob.9.4.20** Air flows with negligible friction through a 10 cm dia duct at a rate of 2.3 kg/s. Temp and pressure at inlet are: T1 = 450 K and P1 = 200 kPa. Mach No. at exit is M2 = 1. Determine the rate of heat transfer and the pressure drop for this section of duct. [Ref: 1]"

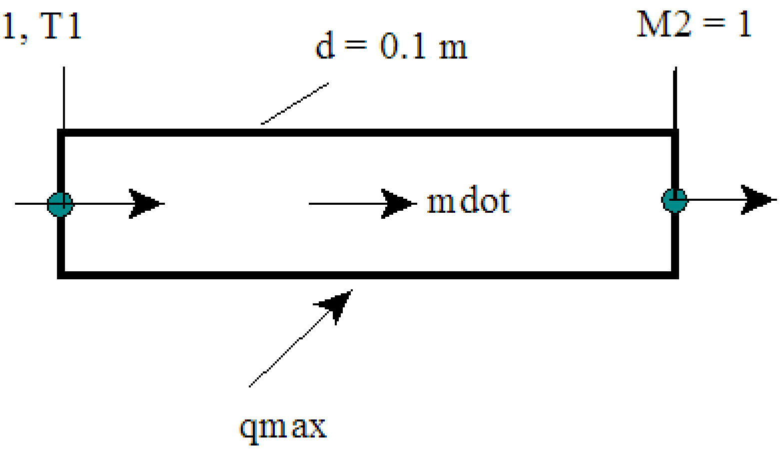


Fig.Prob.9.4.20 Rayleigh flow

Note: This problem is the same as Prob.9.3.22 which was solved with Mathcad.

Now, we shall solve it with EES.

EES Solution:

This is Rayleigh flow.

"Data:"

k = 1.4

cp = 1005 "J/kg.K"

R = 287"J/kg.K"

d = 0.1"m…dia of duct"

P1 = 200"kPa"

T1 = 450"K"

mdot = 2.3"kg/s"

M2 = 1"..Mach No. at exit"

"Calculations:"

A = pi * d^2 / 4 "m^2 …. cross-sectional area of duct"

rho1 = P1*1000 / (R * T1)"kg/m^3…density at inlet"

mdot = rho1 * A * V1"…finds inlet velocity, V1 (m/s)"

C1 = sqrt(k * R * T1)"m/s …. sonic velocity at inlet"

T01 = T1 + V1^2/(2 * cp)"K….finds stagn. temp T01 at 1"

"Therefore:"

M1 = V1 / C1 "…Mach No. at inlet"

"And:"

T01/T0star = RAYLEIGH_T0BYT0STAR(M1,k)"….finds T0star"

"Now that we have M1 and M2, we can use Rayleigh flow functions to get exit parameters as follows:"

"Ex: (V2/V1) = (V2/Vstar) / (V1/Vstar) … etc."

V2/V1 = RAYLEIGH_VBYVSTAR(M2,k) / RAYLEIGH_VBYVSTAR(M1,k) "…finds V2 (m/s)"

P2/P1 = RAYLEIGH_PBYPSTAR(M2,k) / RAYLEIGH_PBYPSTAR(M1,k) "…finds P2 (kPa)"

T2/T1 = RAYLEIGH_TBYTSTAR(M2,k) / RAYLEIGH_TBYTSTAR(M1,k) "…finds T2 (K)"

"Max. possible heat transfer, qmax when M2 = 1:"

"Then: T02 = T0star"

"Therefore:"

q_max = cp * (T0star – T01)"J/kg"

"Pressure drop:"

DELTAP = (P1 – P2) "kPa"

"Entropy change:"

DELTAS = RAYLEIGH_DELTAS(M1, M2, R, k) "..using the EES Function written earlier"

"Verify the heat transfer qmax using the EES Function for Q written earlier:"

qmax2 = RAYLEIGH_Q(M1, M2, cp, T1, k)

Results:

A = 0.007854 [m^2]	C1 = 425.2 [m/s]	cp = 1005 [J/kg-K]
d = 0.1 [m]	ΔP = 93.59 [kPa]	ΔS = 541.2 [J/kg-K]
k = 1.4	M1 = 0.4447	M2 = 1
mdot = 2.3 [kg/s]	P1 = 200 [kPa]	P2 = 106.4 [kPa]
qmax2 = 306579 [J/kg]	q_{max} = 306572 [J/kg]	R = 287 [J/kg-K]
rho1 = 1.549 [kg/m^3]	T01 = 467.8 [K]	T0star = 772.8 [K]
T1 = 450 [K]	T2 = 644 [K]	V1 = 189.1 [m/s]
V2 = 508.7 [m/s]		

Thus:

At the inlet: M1 = 0.4447 … Ans.

At the exit: M2 = 1, P2 = 106.4 kPa, V2 = 508.7 m/s, T2 = 644 K … Ans.

Pressure drop = ΔP = 93.59 kPa … Ans.

Heat transfer = qmax = 306572 J/kg … Ans.

Entropy change = Δs = 541.2 J/kg.K … Ans.

qmax2 = 306579 J/kg … using the EES Function written earlier…should be equal to qmax ,… verified.

===

"**Prob.9.4.21** Heat is added to air flowing in a duct until it is choked, and the amount of heat added is 600 kJ/kg. The exit temp is 1000 K. Calculate the temp and Mach No. at entry. [Ref: 11]"

EES Solution:

This is Rayleigh flow.

"Data:"

k = 1.4

cp = 1005 "J/kg.K"

R = 287"J/kg.K"

M2 = 1"..Mach No. at exit .. since choked"

T2 = 1000"K … temp at exit"

q = 600*1E03 "J/kg … heat added"

"Calculations:"

"Find stagn. temp T02 and then find T01 using q = cp * (T02 – T01)"

T2/T02 = TBYT0(M2,k) "…finds T02..using isentropic function written earlier"

q = cp * (T02 – T01)"…finds T01"

"At exit, since M2 = 1:"

T0star = T02

"Now, find M1 at entry:"

T01/T0star = RAYLEIGH_T0BYT0STAR(M1,k)"…finds M1, using the EES Function written earlier"

"Then, entrance temp T1:"

T1/T01 = TBYT0(M1,k)"…finds T1"

Results:

Unit Settings: SI K Pa J mass deg

cp = 1005 [J/kg-K]	k = 1.4	M1 = 0.385	M2 = 1
q = 600000 [J/kg]	R = 287 [J/kg-K]	T01 = 603 [K]	T02 = 1200 [K]
T0star = 1200 [K]	T1 = 585.6 [K]	T2 = 1000 [K]	

Thus:

Temp at entry = T1 = 585.6 K …. Ans.

Mach No. at entry = M1 = 0.385 … Ans.

Note: See the ease with which these complicated eqns are solved with EES Functions. No need to refer to Rayleigh Tables and interpolate.

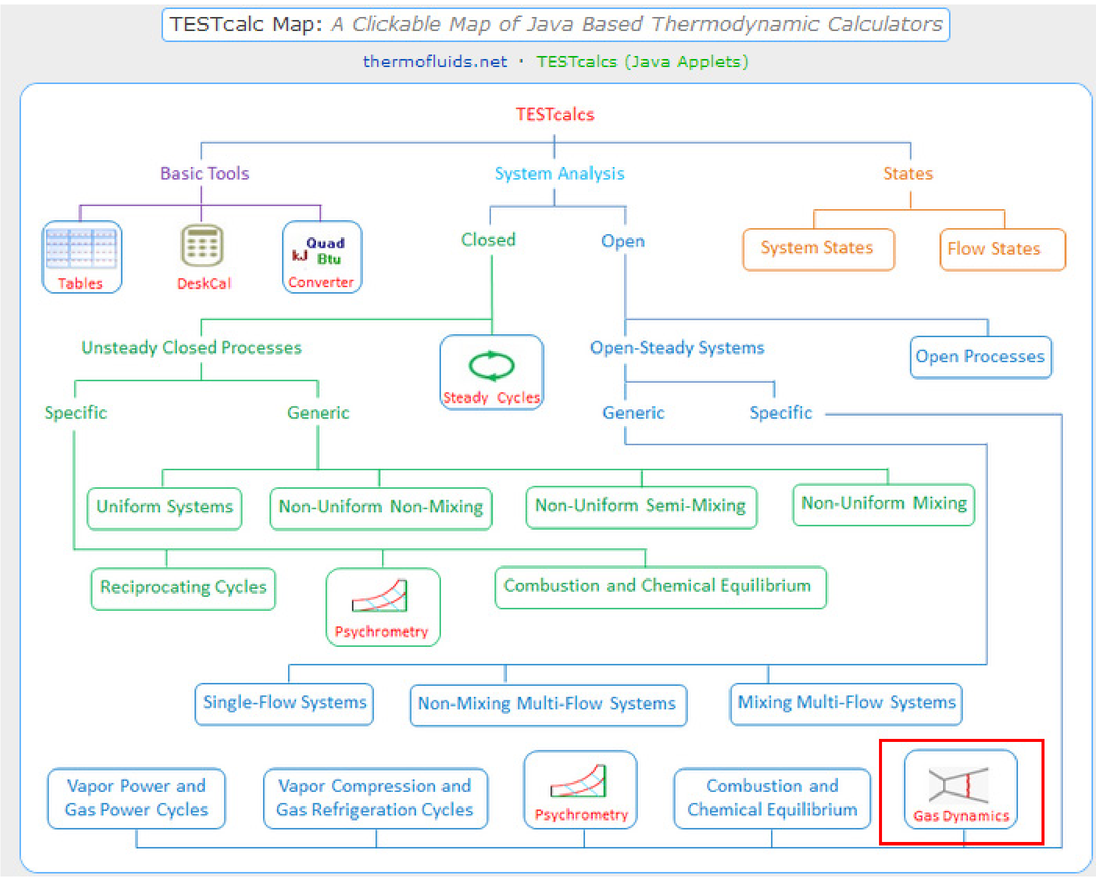
===

9.5 Problems solved with TEST:

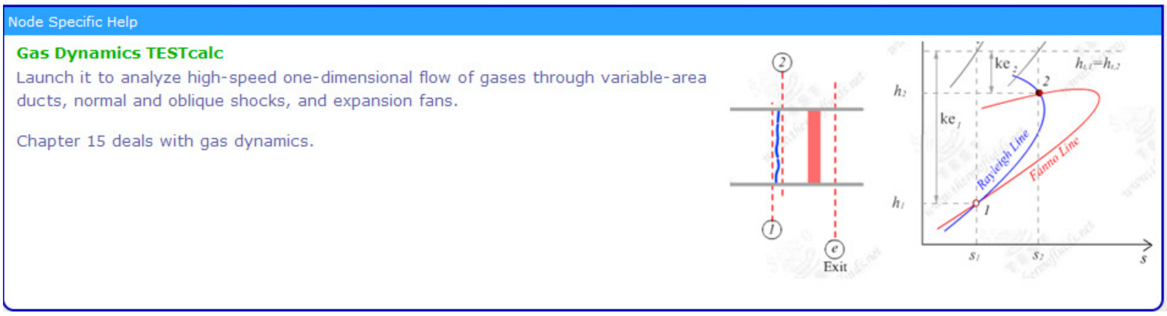
Like other calculators (or 'daemons') in TEST, Gas Dynamics calculator is very easy and convenient to use.

It is assumed that you have already done free registration at the web-site www.thermofluids.net.

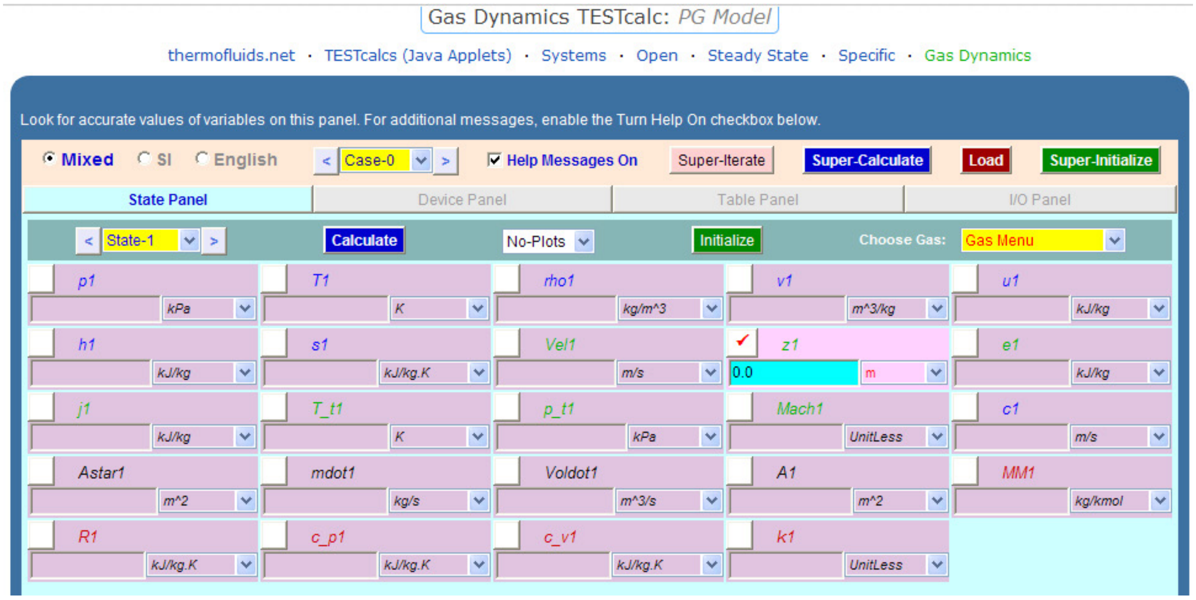
Now, as you go to www.thermofluids.net, you are asked to enter your email address and the password. On supplying that information, the opening screen with a personal greeting appears. Click on 'TESTCalcs' on the Menu bar at the bottom of the window. We get the TESTCalcs tree, from which you can choose the **'Gas Dynamics' calculator**, at the right hand bottom corner, as shown below:



Hovering the mouse pointer over the 'Gas Dynamics' gives the following explanatory window:

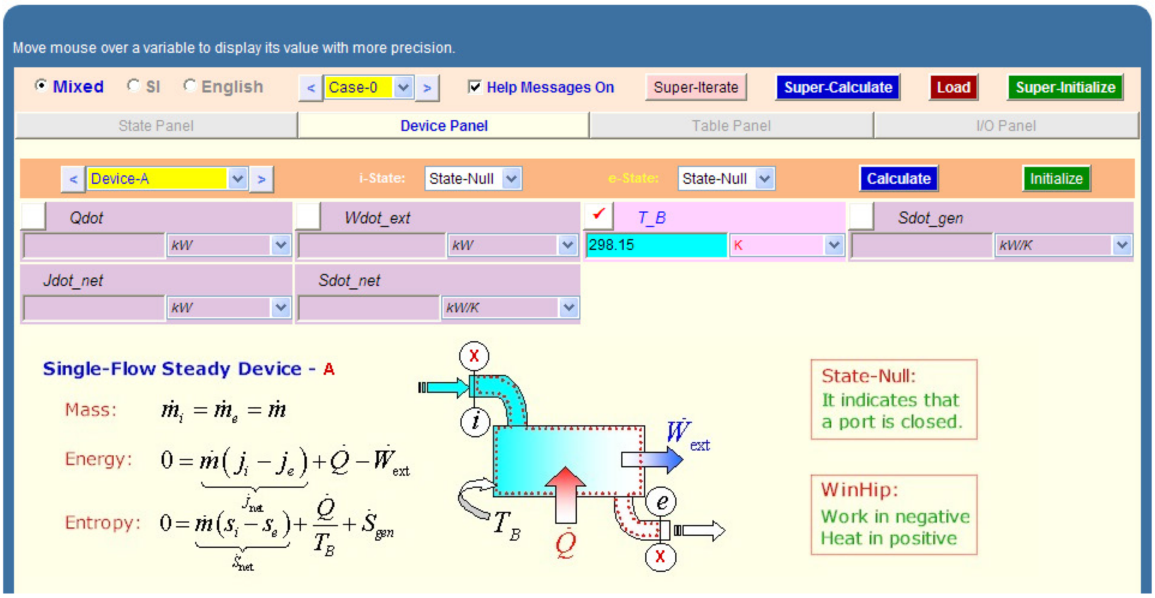


Click on 'Gas Dynamics', and the following window appears:

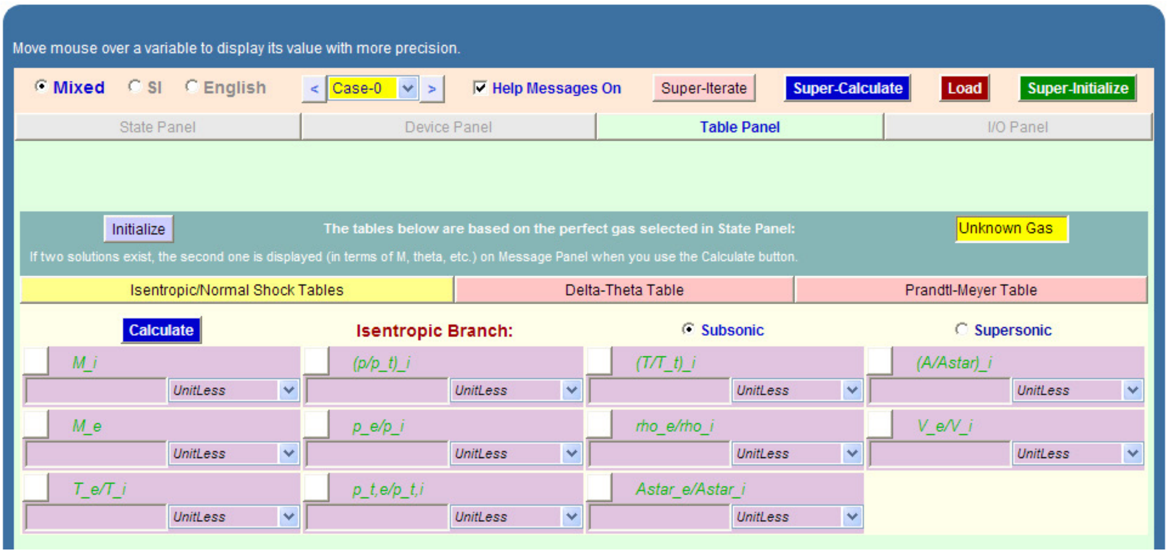


Note from the above screen print that there are 4 tabs: State Panel, Device Panel, Table Panel and I/O Panel. By default, State Panel is selected.

Click on 'Device Panel' tab, and we get:

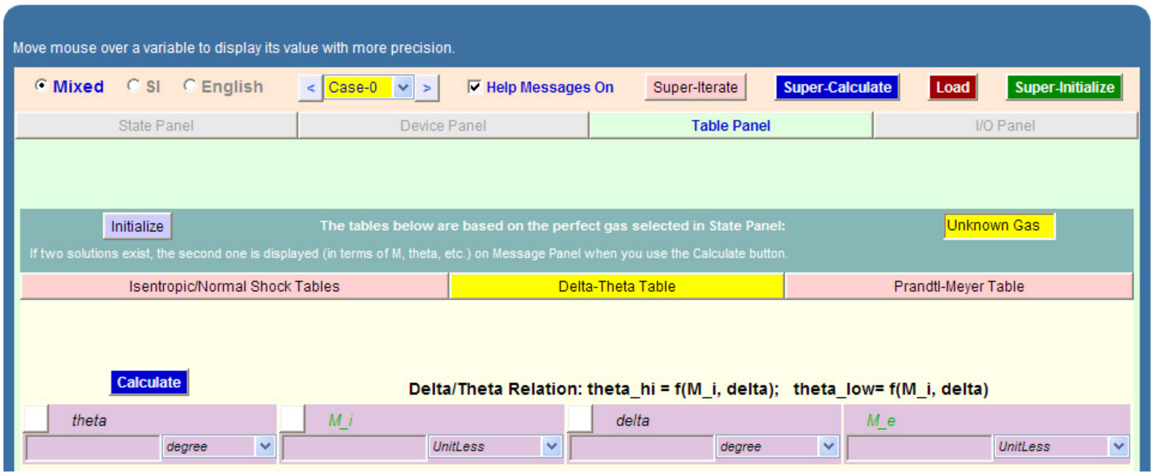


Clicking on 'Table Panel; gives the following:

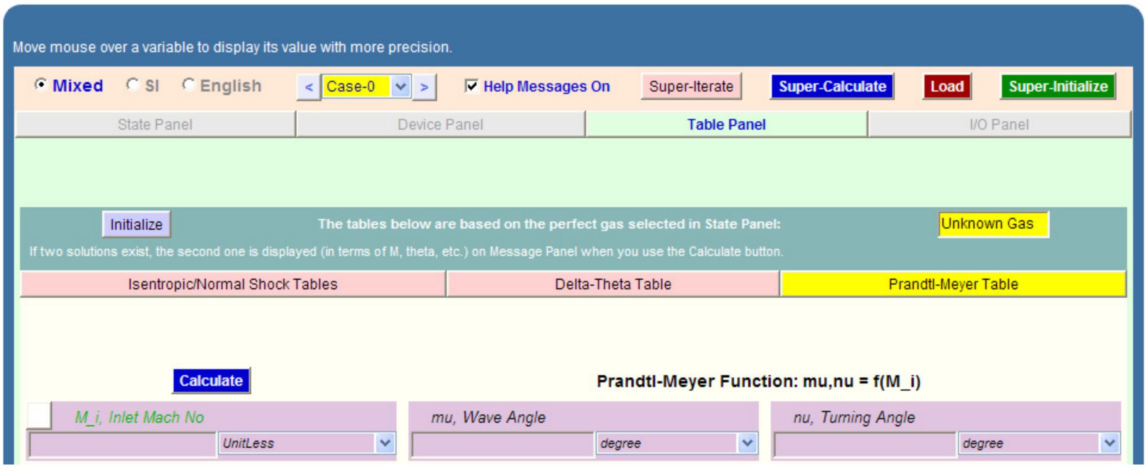


Note from the above Table Panel that Isentropic/Normal Shock Tables is selected by default. First line in this panel gives the Isentropic functions. And the second and third lines give the Normal Shock functions.

Clicking on 'Delta-Theta' tab gives:



And, clicking on Prandtl – Mayer Table gives:



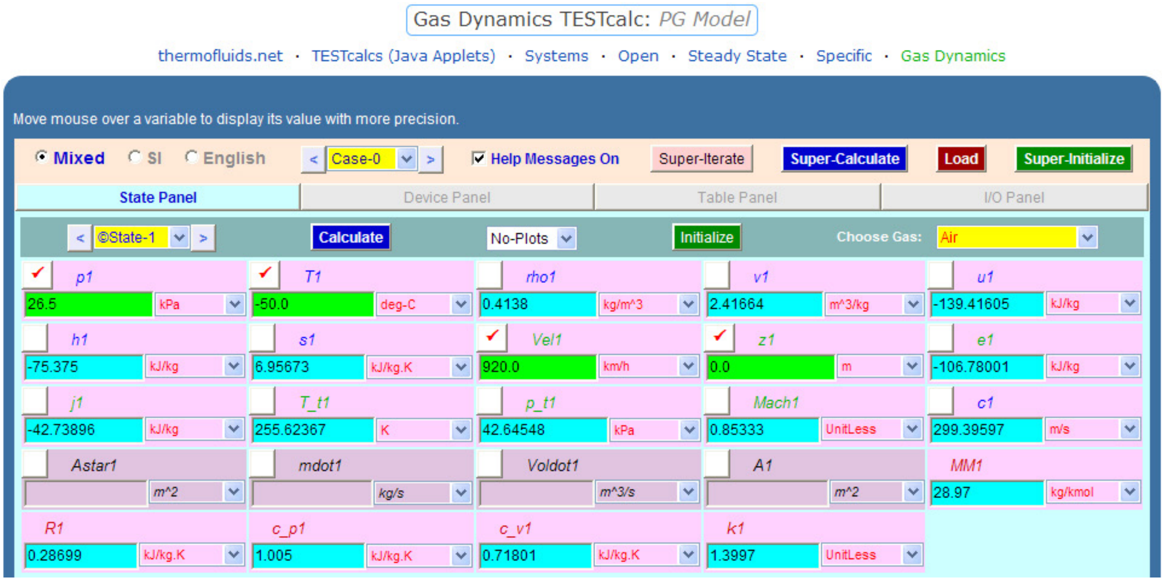
With this brief introduction to Gas Dynamics calculator, let us solve a few problems in TEST:

Prob.9.5.1 An airplane is flying at a speed of 920 km/h at an altitude of 10 km where the temp is -50 C. Determine if the speed of this airplane is subsonic or supersonic. [Ref: 1]

TEST Solution:

From the Thermodynamic Tables in Text books, pressure at an altitude of 10 km is read as: 26.5 kPa.

Go to Gas Dynamics calculator (as already explained). Choose air as the substance. In the State Panel, for State 1: enter p1, T1 and Vel1 as shown. Hit Enter (or click on Calculate). We get:



Thus:

Mach No. = Mach1 = 0.8533.

Therefore, speed of the airplane is subsonic…. Ans.

Note that other properties such as density (rho1), sp. volume (v1), enthalpy (h1), stagnation (or total) temp (T_t1), stagnation pressure (p_t1) etc are also available in the same panel.

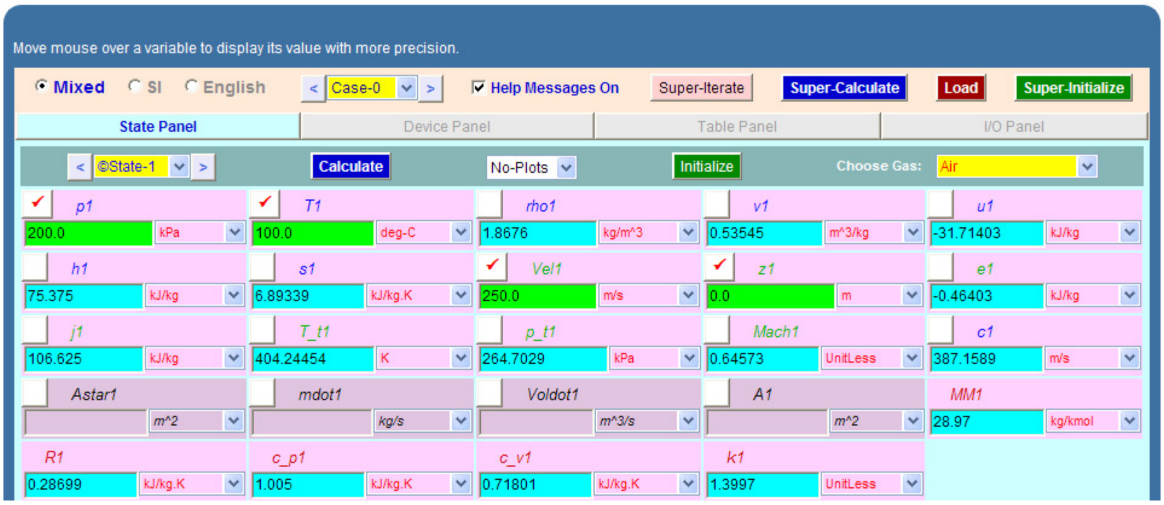
===

Prob.9.5.2 Calculate the critical temp, pressure, and density of (i) air at 200 kPa, 100 C, and 250 m/s, and (ii) helium at 200 kPa, 40 C and 300 m/s. [Ref: 1]

TEST Solution:

i. **For Air:**

Go to Gas Dynamics calculator. In the State Panel, select Air as working substance. Enter values of p1, T1 and Vel1 as shown. Hit Enter. We get:



Now, critical properties refer to M = 1. So, go to State 2, and for Isentropicflow, enter: p_t2 = p_t1, T_t2 = T_t1, and M = 1, and hit Enter. We get:

Thus:

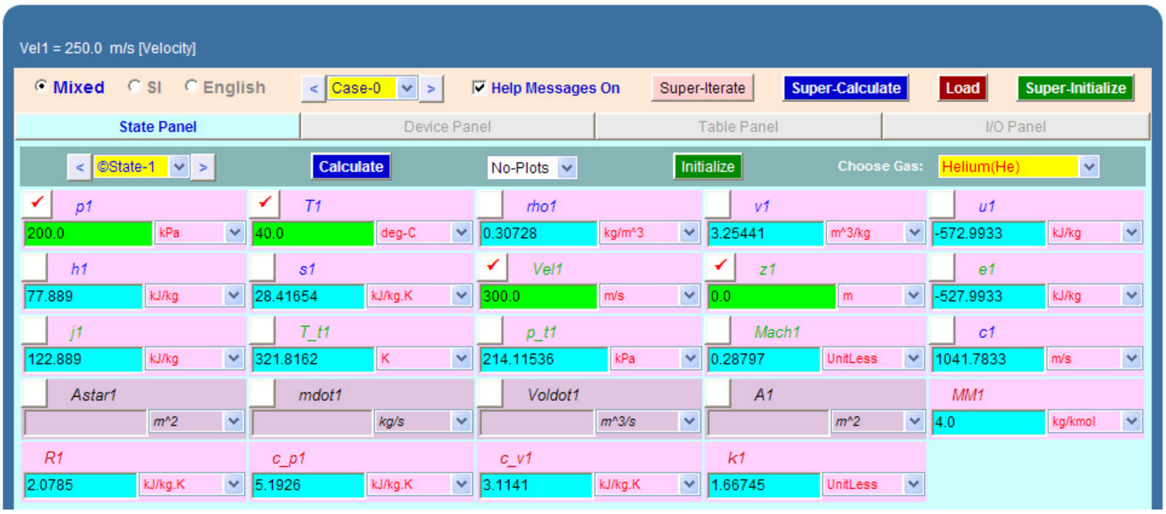
#critical pressure = p2 = 139.85 kPa … Ans.

#critical temp = 2 = 336.91 K .. Ans.

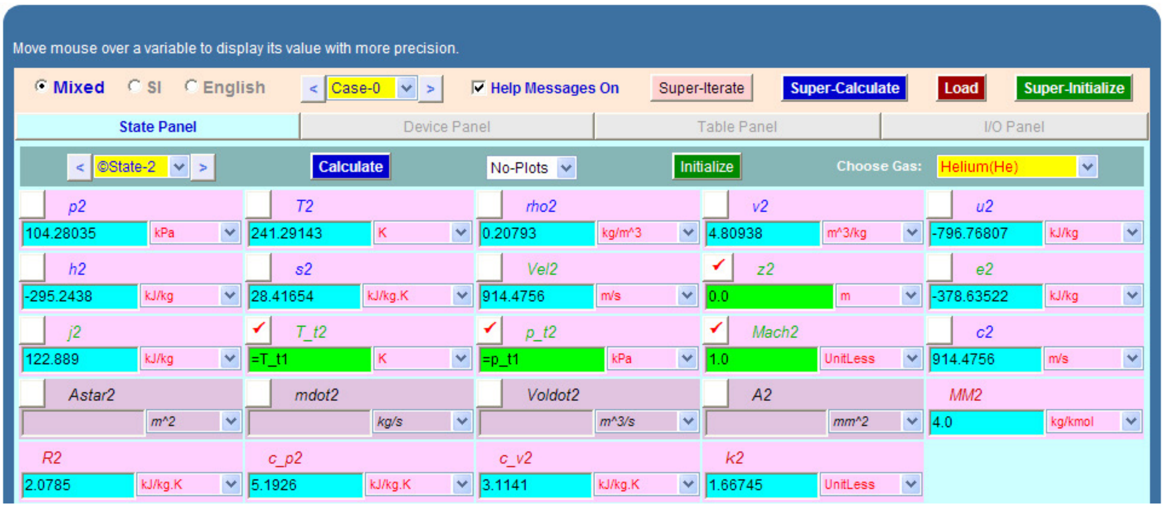
#critical density = rho2 = 1.4464 kg/m^3 … Ans.

ii. **For helium:**

In the State Panel, now select helium as working substance. Enter values of p1, T1 and Vel1. Hit Enter. We get:



Now, critical properties refer to M = 1. So, go to State 2, and for Isentropicflow, enter: p_t2 = p_t1, T_t2 = T_t1, and M = 1, and hit Enter. We get:



Thus:

#critical pressure = p2 = 104.28 kPa … Ans.

#critical temp = T2 = 241.29 K .. Ans.

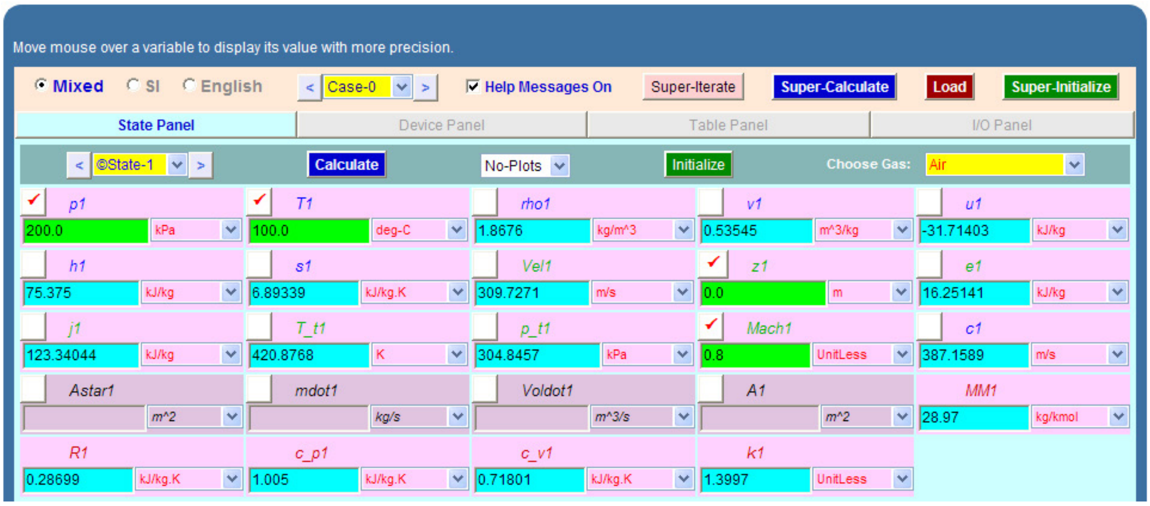
#critical density: = rho2 = 0.208 kg/m^3 … Ans.

===

Prob.9.5.3 Air at 200 kPa, 100 C and Mach No. M1 = 0.8 flows through a duct. Calculate the velocity, and stagnation pressure, temp and density of air. [Ref: 1]

TEST Solution:

Go to Gas Dynamics calculator. In the State Panel, select Air as working substance. Enter values of p1, T1 and Mach1 as shown. Hit Enter. We get:



We note the results:

Velocity, Vel1 = 309.727 m/s … Ans.

Stagn. pressure = p_t1 = 304.85 kPa … Ans.

Stagn. temp = T_t1 = 420.88 K … Ans.

Density = rho1 = 1.8676 kg/m^3 … Ans.

==

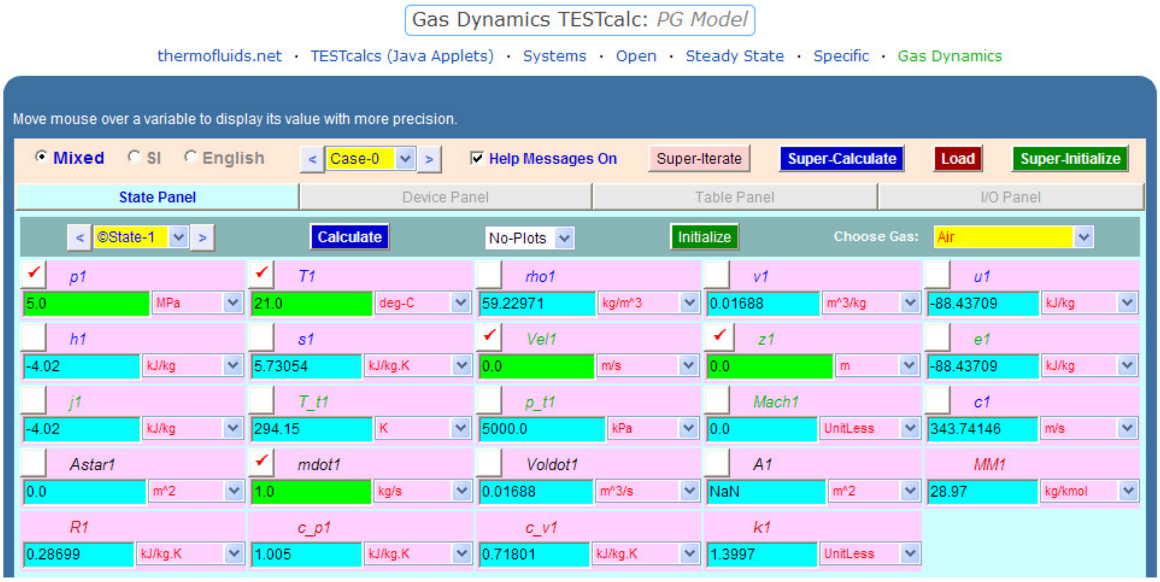
Prob.9.5.4 Air is contained in a reservoir at 5 MPa, 21 C and flows out at a rate of 1 kg/s through a tube. At a particular location the static pressure is measured as 3 MPa. Neglect the velocity at the reservoir and assuming isentropic flow, calculate the Mach No., velocity and area at that location. [Ref: 11]

TEST Solution:

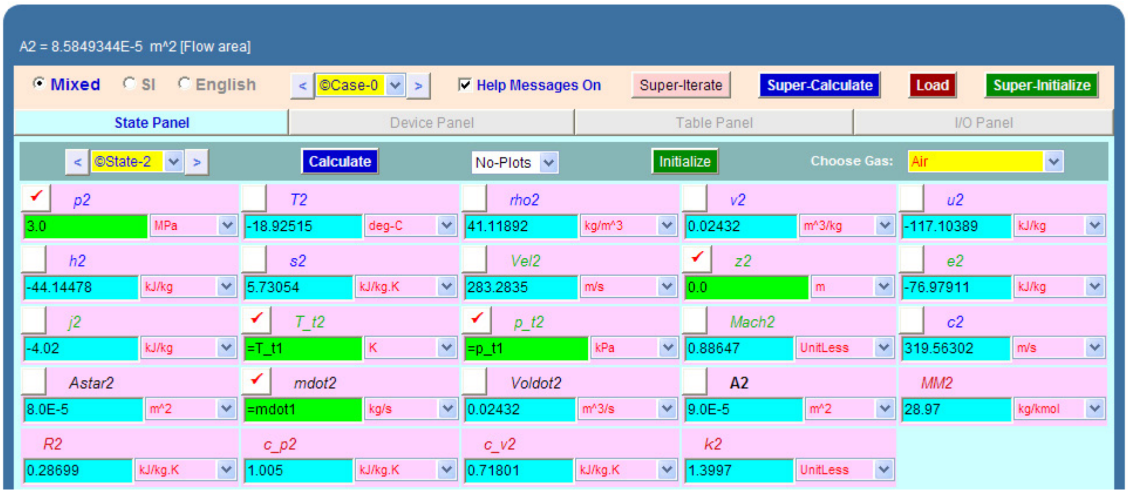
Since velocity is negligible at reservoir, P1 and T1 are: stagn. pressure and stagn. temp. Let the reservoir be designated by state 1.

At the given location, say state 2, P2 is known. For isentropic flow, use the condition that stagn. pressure and stagn. temp remain constant, i.e. p_t1 = p_t2 , and T_t1 = T_t2.

For State 1: Enter p1 = 5 MPa, T1 = 21 C, Vel1 = 0 and mdot1 = 1 kg/s. Hit Enter. We get:



Now, for State 2: For isentropic flow, Enter p2 = 3 MPa, p_t2 = p_t1, T_t2 = T_t1, mdot2 = mdot1. Hit Enter. We get:



From the above, read:

Mach No. at location 2 = Mach2 = 0.886 …Ans.

Velocity = Vel2 = 283.28 m/s …. Ans. (see the top of above screen shot)

Area at location 2 = A2 = 8.5849E-5 m^2 = 85.85 mm^2 …. Ans.

Click on SuperCalculate: And, get the TEST code etc from the I/O panel:

#~~~~~~~~~~~~~~~~~~~~~OUTPUT OF SUPER-CALCULATE

#*******TEST-code: To save the solution, copy the codes generated below into a text file. To reproduce the solution at a later time, launch the TESTcalc (see path name below), paste the saved TEST-code at the bottom of this I/O panel, and click the Load button.

Daemon Path: Systems>Open>SteadyState>Specific>GasDynamics; v-10.ce02

```
#----------Start of TEST-code ------------------------------------

 States {

     State-1: Air;

     Given: { p1= 5.0 MPa; T1= 21.0 deg-C; Vel1= 0.0 m/s; z1= 0.0 m; mdot1= 1.0 kg/s; }

     State-2: Air;

     Given: { p2= 3.0 MPa; z2= 0.0 m; T_t2= "T_t1" K; p_t2= "p_t1" kPa; mdot2= "mdot1" kg/s; }

 }

#-----------End of TEST-code ------------------------------------
```

#----Property spreadsheet starts:

#State	MachNo	Vel(m/s)	p(kPa)	p_t(kPa)	T(K)	T_t(K)	Astar(m2)	v(m3/kg)	u(kJ/kg)
# 1	0.0	0.0	5000.0	5000.0	294.2	294.2	0.0	0.0169	-88.44
# 2	0.89	283.28	3000.0	5000.0	254.2	294.2	0.0	0.0243	-117.1

==

Prob.9.5.5 A convergent nozzle has an exit area of 500 mm^2. Air enters the nozzle with a stagnation pressure of 1000 kPa and a stagnation temp of 360 K. Determine the mass rate of flow for back pressures of 800 kPa, 528 kPa, and 300 kPa, assuming isentropic flow. [Ref: 2]

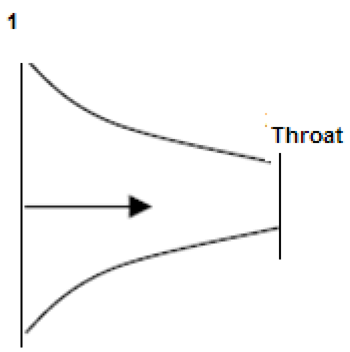


Fig.Prob.9.5.5 Isentropic flow in a convergent nozzle

TEST Solution:

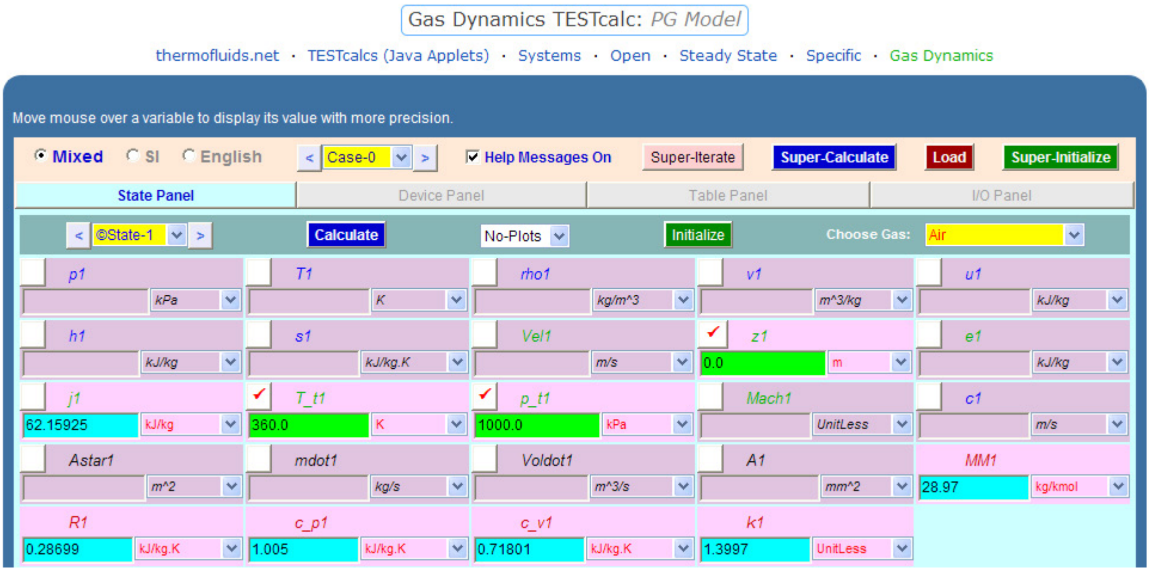
Remember that in a convergent nozzle, maximum possible Mach No. is 1, and it occurs at the throat.

At that time, M = 1, and the pressure is critical pressure Pstar, and the temp is critical temp Tstar, and the mass flow rate is maximum. If the back pressure is further decreased below the critical pressure, it has no effect on mass flow, and it remains constant.

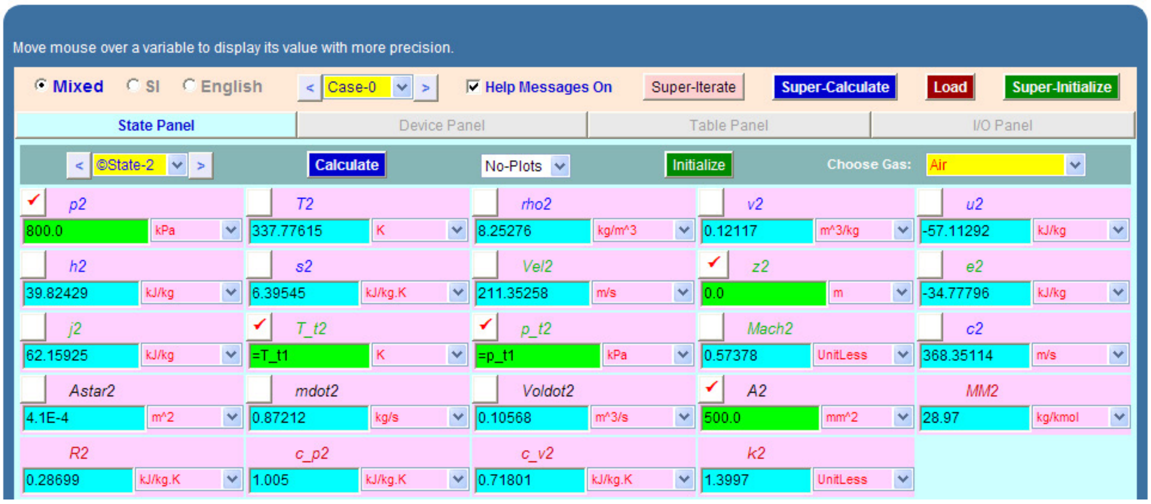
Also, we have for air (k = 1.4), Tstar/T0 = 0.833, Pstar/P0 = 0.5283

Go to Gas Dynamics calculator. In the State Panel, select Air as working substance.

1. **For State 1:** Enter p_t1 = 1000 kPa, T_t1 = 360 K, and hit Enter. We get:



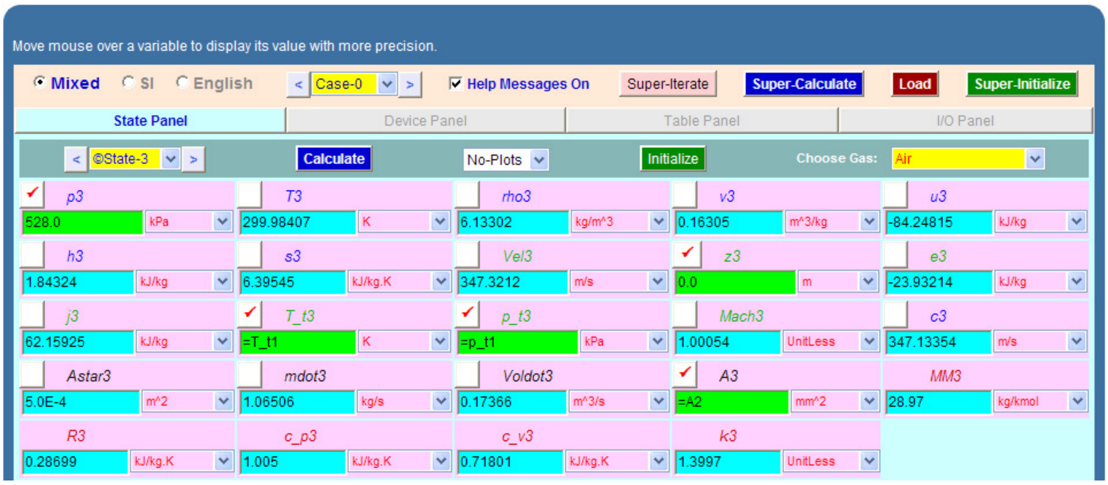
2. **Case (i):** At the throat, let the state be 2. Then, for isentropic flow, use the condition that stagnation pressure and stagn. temp remain constant, i.e. p_t1 = p_t2 , and T_t1 = T_t2. In addition, for state 2, also enter p2 = 800 kPa, and hit Enter. We get:



Thus: Mach No. at throat = Mach2 = 0.57378, and Mass flow rate = mdot2 = 0.872 kg/s … Ans.

3. **Case(ii): Back pressure = 538 kPa. Let this be designated by State 3.**

For State 3, we enter: p3 = 528 kPa, T_t3 = T_t1, p_t3 = p_t1, A3 = A2, and hit Enter. We get:



We see that: Mach3 = 1 and, mass flow = mdot3 = 1.065 kg/s. ….Ans.

Note that this is the max. possible flow rate for this convergent nozzle.

4. **Case (iii): Back pressure = 300 kPa. This is less than the critical pressure of 528 kPa;**

So, there is no effect on mass flow and it remains constant at 1.065 kg/s … Ans.

===

Prob.9.5.6 A C_D nozzle has an exit area to throat area ratio of 2. Air enters the nozzle with a stagnation pressure of 1000 kPa and stagnation temp of 360 K. The throat area is 500 mm^2. Determine the mass rate of flow, exit pressure, exit temp, exit Mach No, and exit velocity for the following conditions: (i) sonic velocity at the throat, diverging section acting as a nozzle (ii) sonic velocity at the throat, diverging section acting as a diffuser. [Ref: 2]

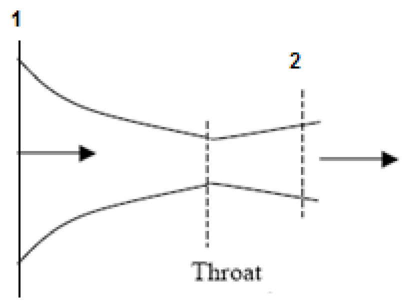


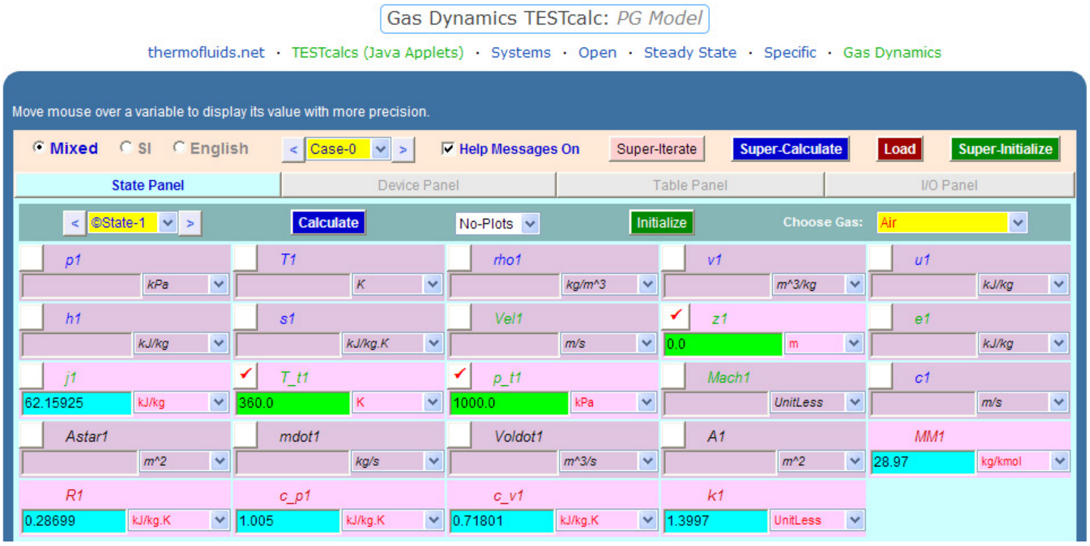
Fig.Prob.9.5.6 Isentropic flow in a C-D nozzle

TEST Solution:

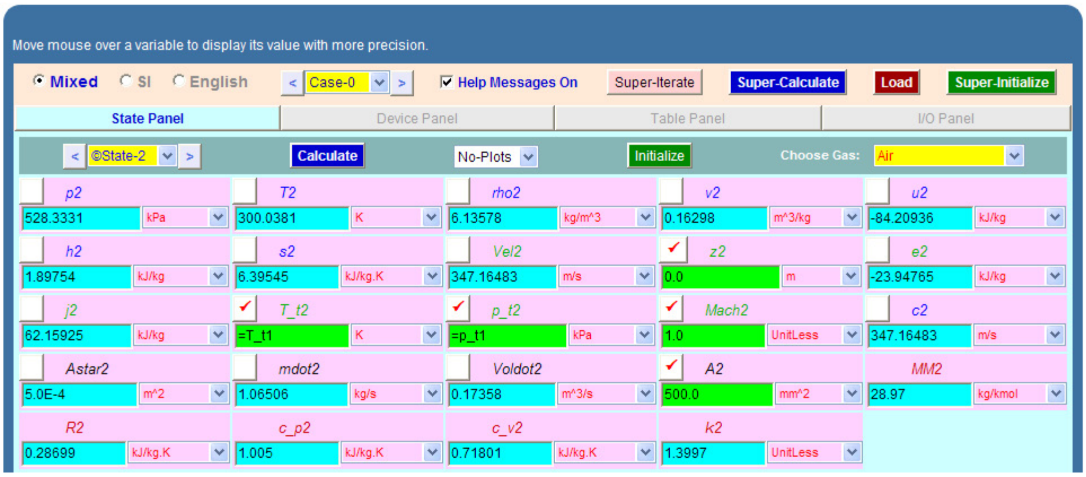
Remember that in a C-D nozzle: when the back pressure is equal to or less than the 'critical pressure' (i.e. pressure corresponding to M = 1 at throat), the flow is maximum. Supersonic velocity occurs only in the divergent section of C-D nozzle, which acts as a nozzle accelerating the flow. If the back pressure is less than the critical pressure, divergent section acts as a diffuser and the flow decelerates, i.e. Mach No. reduces.

Go to Gas Dynamics calculator. In the State Panel, select Air as working substance.

1. **For State 1:** Enter p_t1 = 1000 kPa, T_t1 = 360 K, and hit Enter. We get:



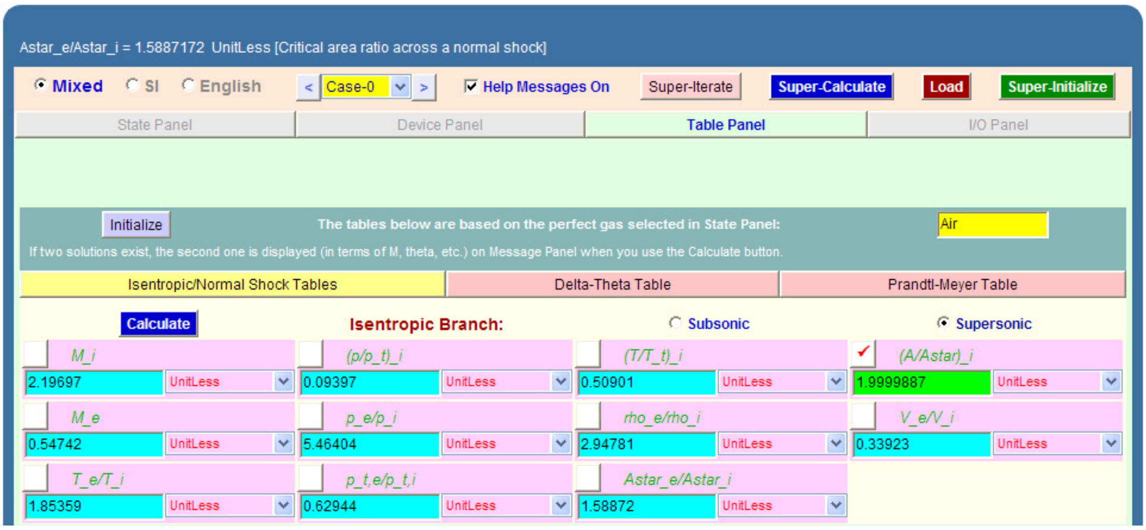
2. **State 2: this at the throat.** For Isentropic flow, p_t2 = p_t1, T_t2 = T_t1. Also fill in A2 = 500 mm^2. Fill in Mach2 = 1, so that we get critical pressure p2 and critical temp. T2 at the throat. Hit Enter. We get:



Thus: critical pressure, p2 = 528.331 kPa, critical temp. T2 = 300.04 K.

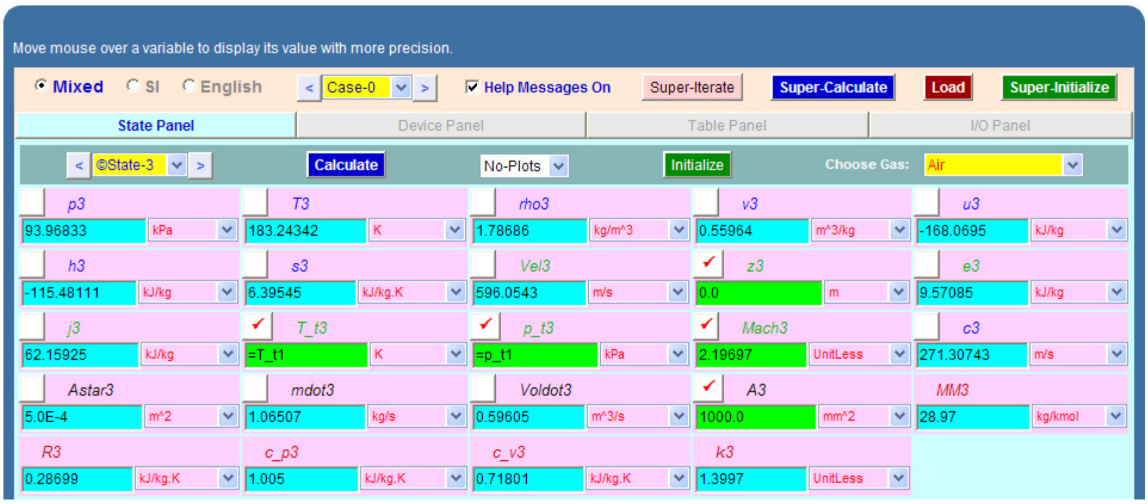
3. **Case (i): When diverging portion acts as a nozzle: i.e. flow accelerates, and is supersonic at exit.**

First, let us get the Mach No. at exit. Go to Tables Panel, and first row gives Isentropic Functions. Now that we know (A2/Astar) from data, fill in (A/Astar) in the Table, Choose *Supersonic radio button*, since we know that flow is supersonic. (***This is important.***) And hit Enter. We get:



We see from the above screen shot that Mach No. at exit is M_i = 2.197.

Now, go to State 3. Fill in this Mach No as Mach3 = 2.19697, A3 = 1000 mm^2 (since area ratio is 2), and p_t3 = p_t1 and T_t3 = T_t1 as shown. Hit Enter. We get:



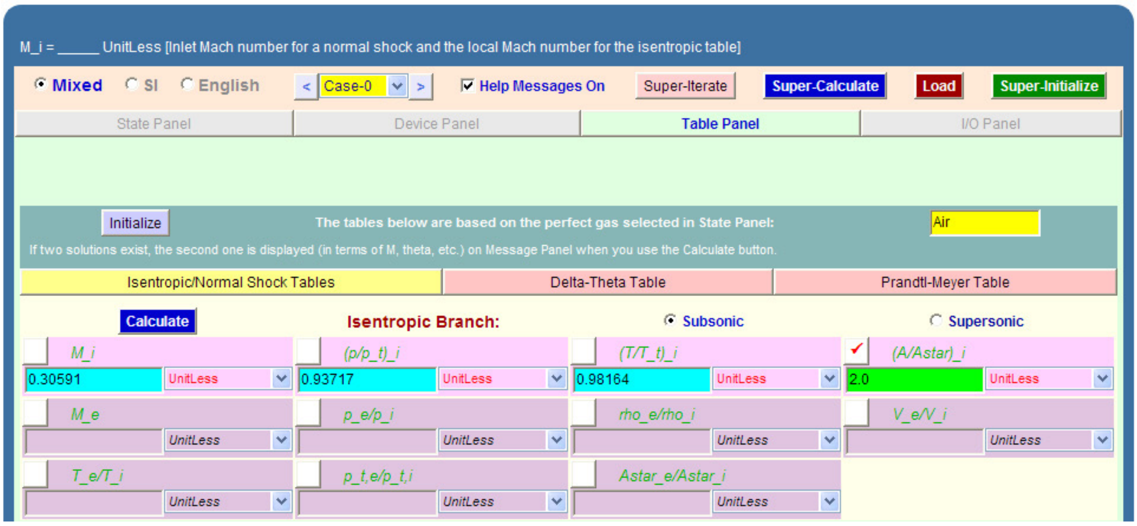
Thus:

At exit: Mach3 = 2.19697, p3 = 93.968 kPa, T3 = 183.24 K, Velocity = Vel3 = 596.05 m/s … Ans.

And, mass flow rate = mdot3 = 1.065 kg/s … Ans.

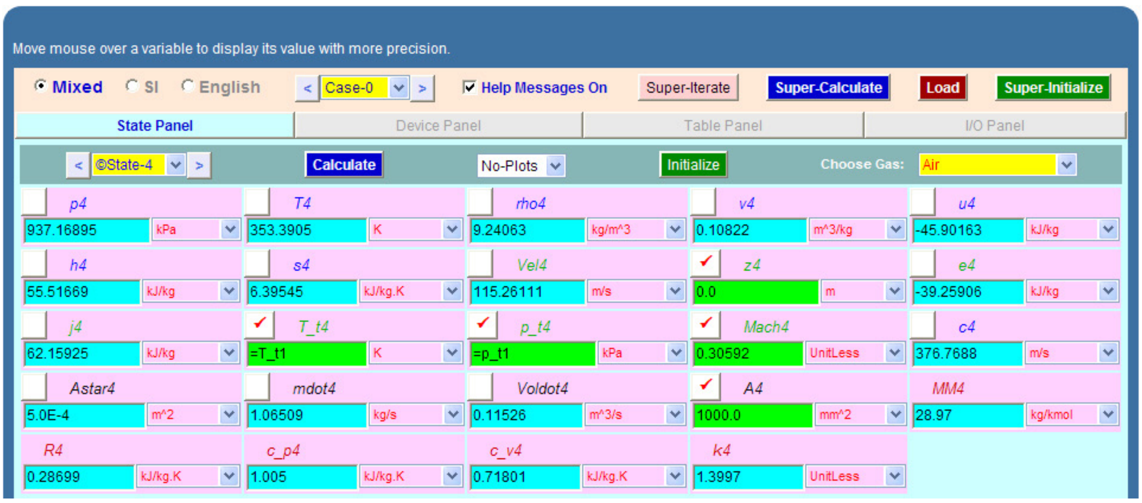
4. **Case (ii): When diverging portion acts as diffuser: i.e. flow decelerates, and is subsonic at exit.**

Now, the Mach No. at exit will be less than 1 since flow will be subsonic.To get the Mach No. at exit, again go to Tables Panel where the first row gives Isentropic Functions. fill in (A/Astar) in the Table, choose *Subsonic radio button*, since we know that flow is subsonic. (***This is important.***) And hit Enter. We get:



We see from the above that Mach No. at exit is M_i = 0.30591.

Now, go to State 4. Fill in this Mach No as Mach4 = 0.30591, A4 = 1000 mm^2 (since area ratio is 2), and p_t4 = p_t1 and T_t4 = T_t1 as shown. Hit Enter. We get:



Thus:

At exit: Mach4 = 0.30592, p4 = 937.17 kPa, T4 = 353.39 K, Velocity = Vel4 = 115.26 m/s … Ans.

And, mass flow rate = mdot4 = 1.065 kg/s … Ans.

5. **Click on SuperCalculate and get the TEST code etc from the I/O Panel:**

```
#~~~~~~~~~~~~~~~~~~~~~~~~OUTPUT OF SUPER-CALCULATE

#          Daemon Path: Systems>Open>SteadyState>Specific>GasDynamics; v-10.ce02

#

#----------Start of TEST-code ------------------------------------

 States   {

           State-1: Air;

           Given: { z1= 0.0 m; T_t1= 360.0 K; p_t1= 1000.0 kPa; }
```

State-2: Air;

Given: { z2= 0.0 m; T_t2= "T_t1" K; p_t2= "p_t1" kPa; Mach2= 1.0 UnitLess; A2= 500.0 mm^2; }

State-3: Air;

Given: { z3= 0.0 m; T_t3= "T_t1" K; p_t3= "p_t1" kPa; Mach3= 2.19697 UnitLess; A3= 1000.0 mm^2; }

State-4: Air;

Given: { z4= 0.0 m; T_t4= "T_t1" K; p_t4= "p_t1" kPa; Mach4= 0.30592 UnitLess; A4= 1000.0 mm^2; }

}

#-----------End of TEST-code ----------------------------------

==

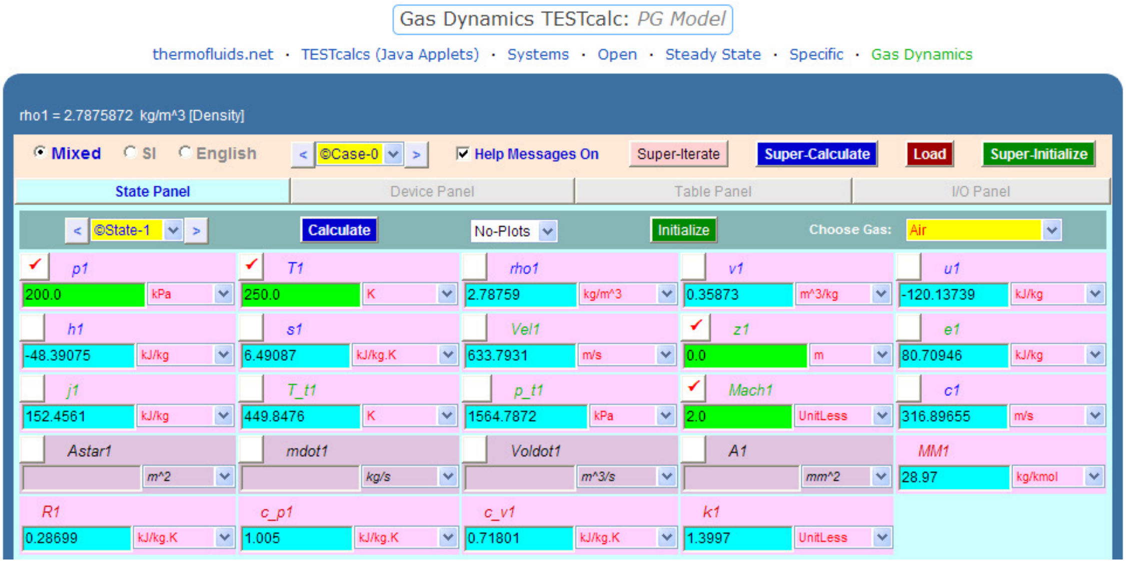
Prob.9.5.7 At a certain point A in a tube, the Mach No. is 2, static temp is 250 K and static pressure is 200 kPa. Find out the Mach No. and static temp at a point B downstream where static pressure is 150 kPa. Assume isentropic flow and the fluid is air. [Ref: 11]

TEST Solution:

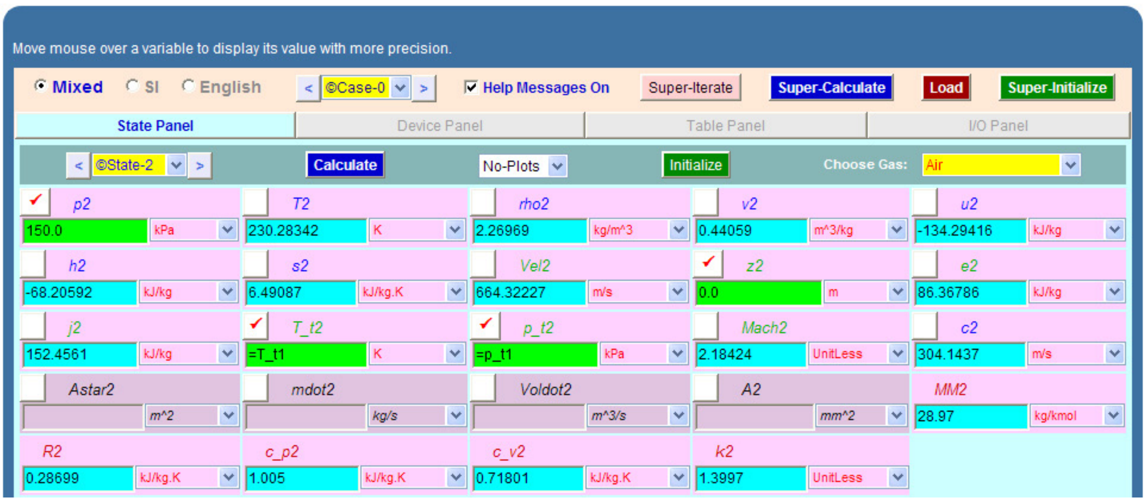
Remember that in isentropic flow, stagnation temp and stagnation pressure remain constant.

Go to Gas Dynamics calculator. In the State Panel, select Air as working substance.

1. **For State 1:** Enter p1 = 200 kPa, T1 = 250 K, Mach1 = 2, and hit Enter. We get:



2. **Now, for State 2:** Enter p2 = 150 kPa, p_t2 = p_t1, T_t2 = T_t1, and hit Enter. We get:



Then, we see that:

Mach No. at State 2 = Mach2 = 2.184 …Ans.

Static temp. at State 2 = T2 = 230.283 K … Ans.

3. Click on SuperCalculate and get the TEST code etc from the I/O Panel:

#~~~~~~~~~~~~~~~~~~~~~~OUTPUT OF SUPER-CALCULATE:

#

Daemon Path: Systems>Open>SteadyState>Specific>GasDynamics; v-10.ce02

#----------Start of TEST-code -----------------------------------

States {

State-1: Air;

Given: { p1= 200.0 kPa; T1= 250.0 K; z1= 0.0 m; Mach1= 2.0 UnitLess; }

State-2: Air;

Given: { p2= 150.0 kPa; z2= 0.0 m; T_t2= "T_t1" K; p_t2= "p_t1" kPa; }

}

#-----------End of TEST-code ----------------------------------

==

Prob.9.5.8 Air at 900 kPa, 400 K enters a converging nozzle with negligible velocity. Throat area of nozzle is 10 cm^2. For isentropic flow, calculate and plot the exit pressure, exit velocity and mass flow rate as the back pressure varies from 900 kPa to 100 kPa. [Ref: 1]

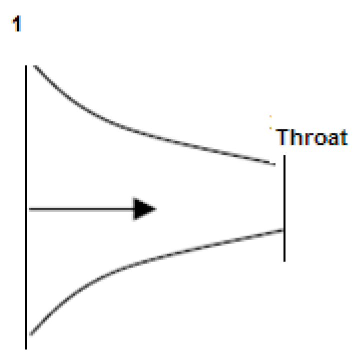


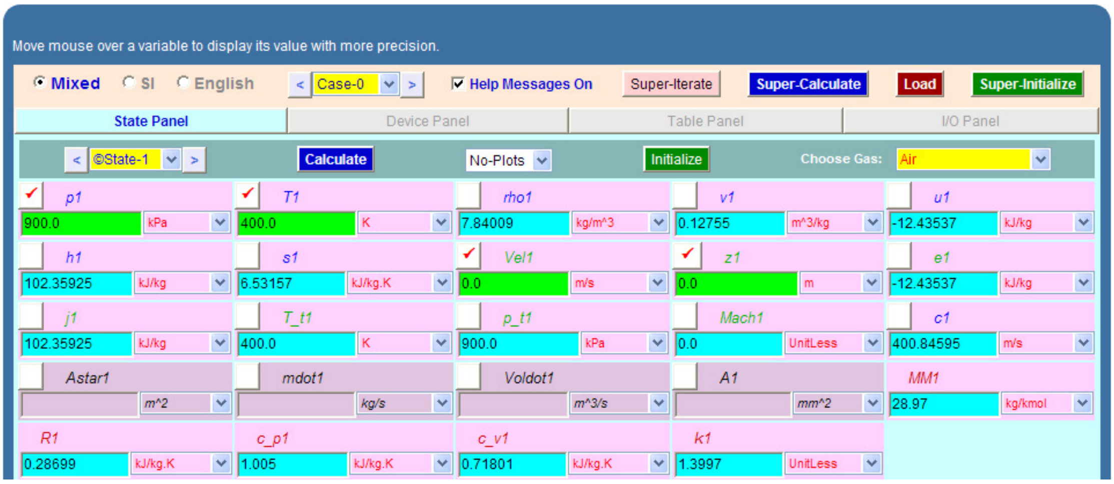
Fig.Prob.9.5.8 Isentropic flow in a convergent nozzle

TEST Solution:

Again, remember that in isentropic flow, stagnation temp and stagnation pressure remain constant.

Go to Gas Dynamics calculator. In the State Panel, select Air as working substance.

1. **For State 1:** Enter p1 = 900 kPa, T1 = 400 K, Vel1 = 0, and hit Enter. We get:



Observe from the above that stagn. pressure p_t1 and stagn. temp T_t1 are calculated.

4. **Now, for State 2:** When the pressure is equal to p1 (i.e. 900 kPa), obviously, there is no flow. Now, put p2 = 800 kPa, p_t2 = p_t1, T_t2 = T_t1, and hit Enter. We get:

From the above, we see that for an exit pressure p2 = 800 kPa, we have: exit velocity, Vel2 = 163.07 m/s, and mass flow rate = mdot2 = 1.1753 kg/s.

5. Now, for this converging nozzle, max. velocity will occur at the throat only, when the back pressure is equal to the critical pressure, and the max. Mach No. possible is 1. Also, mass flow will be max. at that time. So, let us get the parameters when throat Mach No. is 1. Again, designate the throat condition as State 2, enter Mach2 = 1, and p_t2 = p_t1, T_t2 = T_t1 (since isentropic flow), A2 = 10 cm^2, and hit Enter. We get:

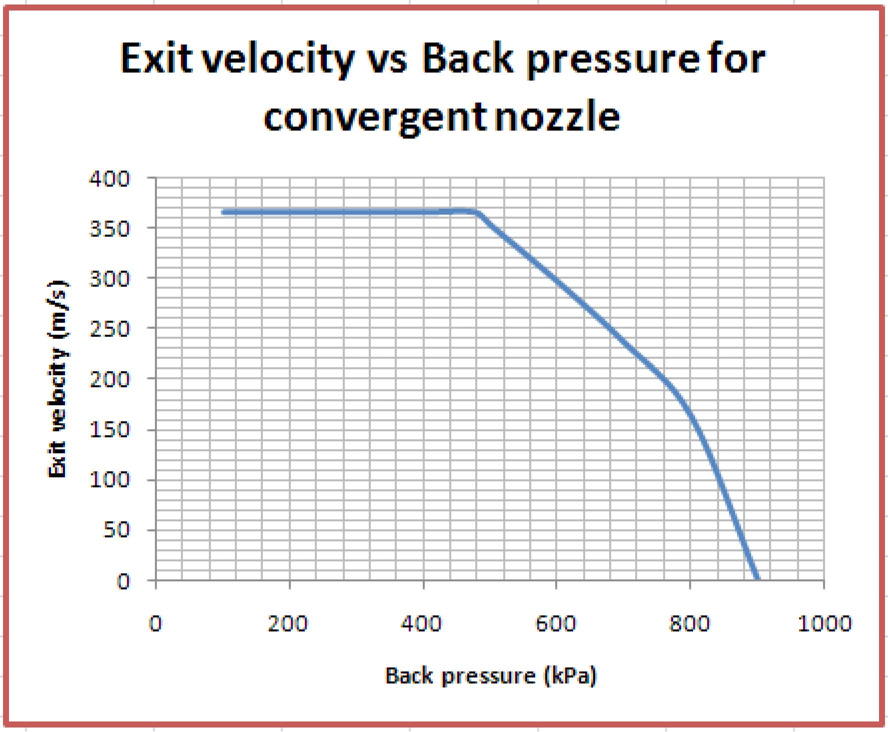
Thus, the critical pressure is p2 = 475.5 kPa, the max. possible velocity is Vel2 = 365.944 m/s and max. mass flow rate is mdot2 = 1.81873 kg/s.

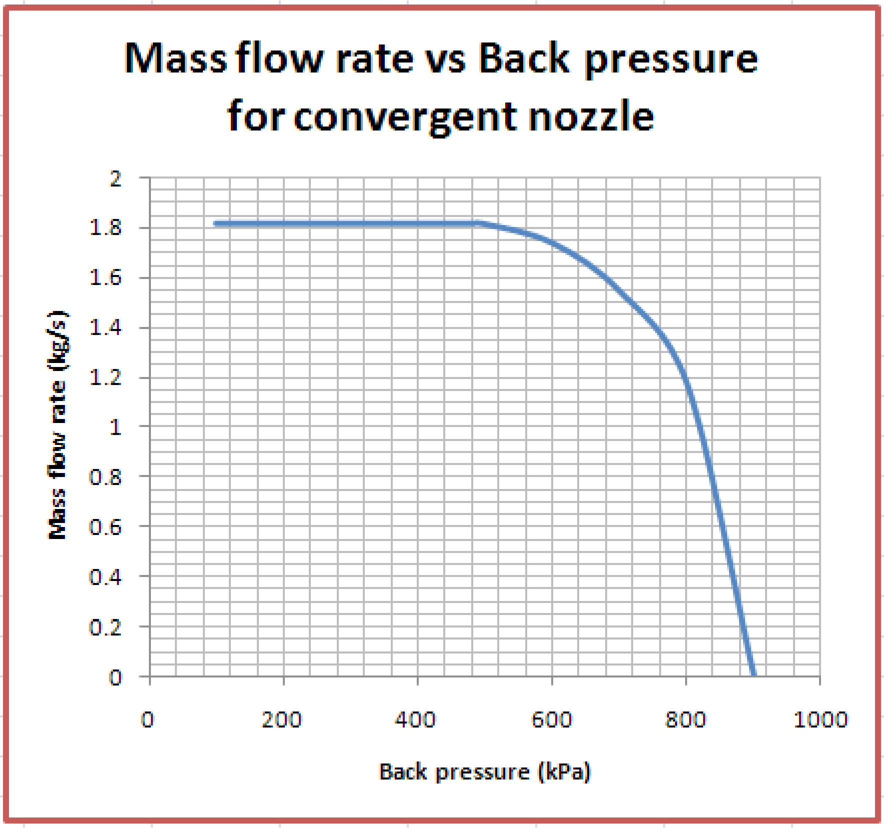
As the back pressure is decreased below the critical pressure of 475.5 kPa, exit pressure remains constant at the critical pressure, and the exit velocity and mass flow rate also remain constant.

6. **To plot Exit velocity and mass flow rate against the back pressure:** It is quite easy. All that we have to do is go to State 2, and change the value of p2 to the desired value, and click on Calculate (or, hit Enter). Immediately, all other calculations for State 2 are updated. Note the results in a Tabular form for different values of p2 as shown below:

Back pressure (= P2), kPa	Mach No.	Exit Velocity (m/s)	Mass flow rate (kg/s)
900	0	0	0
800	0.414	163.07	1.175
700	0.61	235.96	1.546
600	0.784	296.48	1.74
500	0.956	352.47	1.816
475.5	**1.0**	**365.944**	**1.8187**
400	1.0	365.944	1.8187
300	1.0	365.944	1.8187
200	1.0	365.944	1.8187
100	1.0	365.944	1.8187

Now, plot the results in EXCEL:




Mass flow rate vs Back pressure
for convergent nozzle
Mass flow rate (kg/s)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
200
400
600
800
1000
Back pressure (kPa)

7. Click on SuperCalculate and get the TEST code etc from the I/O Panel:

~~~~~~~~~~~~~~~~~~~~~OUTPUT OF SUPER-CALCULATE :

Daemon Path: Systems>Open>SteadyState>Specific>GasDynamics; v-10.ce02

#––––––––––Start of TEST-code –––––––––––––––––––––––––––––––––––––

States {

State-1: Air;

Given: { p1= 900.0 kPa; T1= 400.0 K; Vel1= 0.0 m/s; z1= 0.0 m; }

State-2: Air;

Given: { p2= 800.0 kPa; z2= 0.0 m; T_t2= "T_t1" K; p_t2= "p_t1" kPa; A2= 10.0 cm^2; }

}

#–––––––––––End of TEST-code ––––––––––––––––––––––––––––––––––––

==

Prob.9.5.9 Air enters a normal shock at 18 kPa, 205 K and 740 m/s. Calculate the stagnation pressure and Mach No. upstream of the shock, as well as the pressure, temp, velocity, Mach No. and stagnation pressure downstream of the shock. Also, find the entropy change across the shock. [Ref: 1]

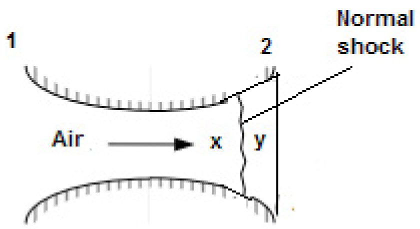
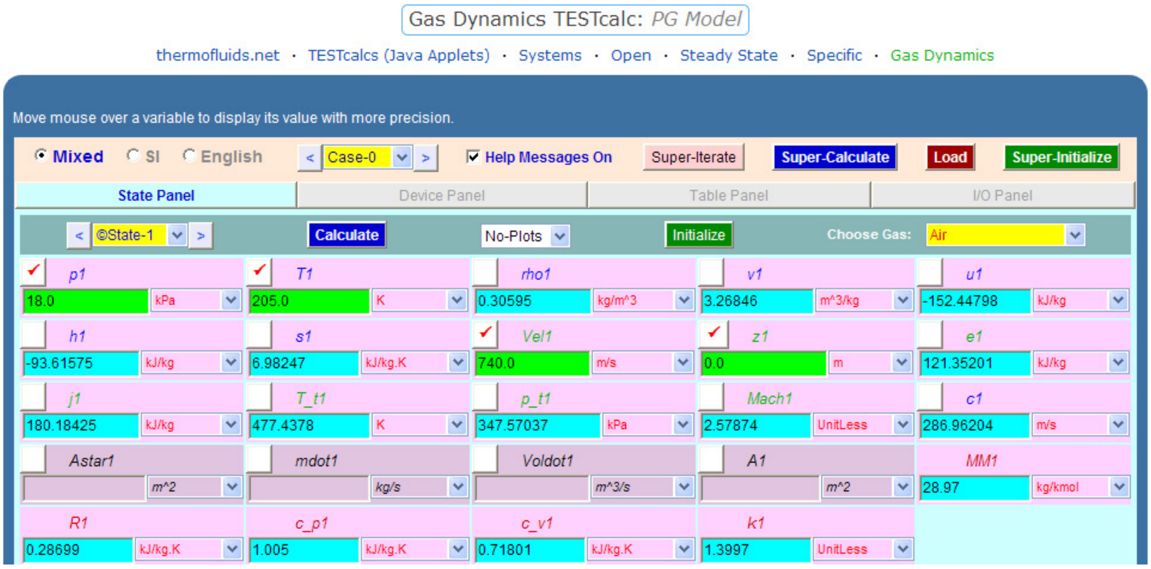


Fig.Prob.9.5.9 Normal shock

TEST Solution:

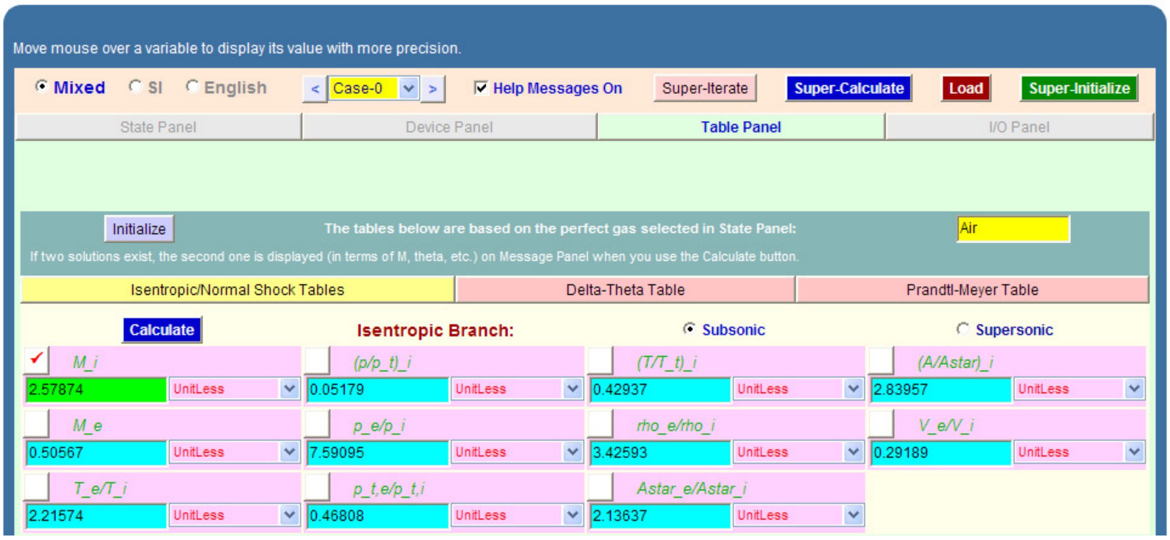
Again, remember that up to the shock, the flow is isentropic, and after the shock also, flow is isentropic. Stagnation temp remains constant across the shock, but stagn. pressure drops.
~~~~~~~~~~~~~~~~~~~~~

1. Go to Gas Dynamics calculator. In the State Panel, select Air as working substance. Enter the parameters before the shock, viz. p1, T1 and Vel1 as shown. Hit Enter. We get:



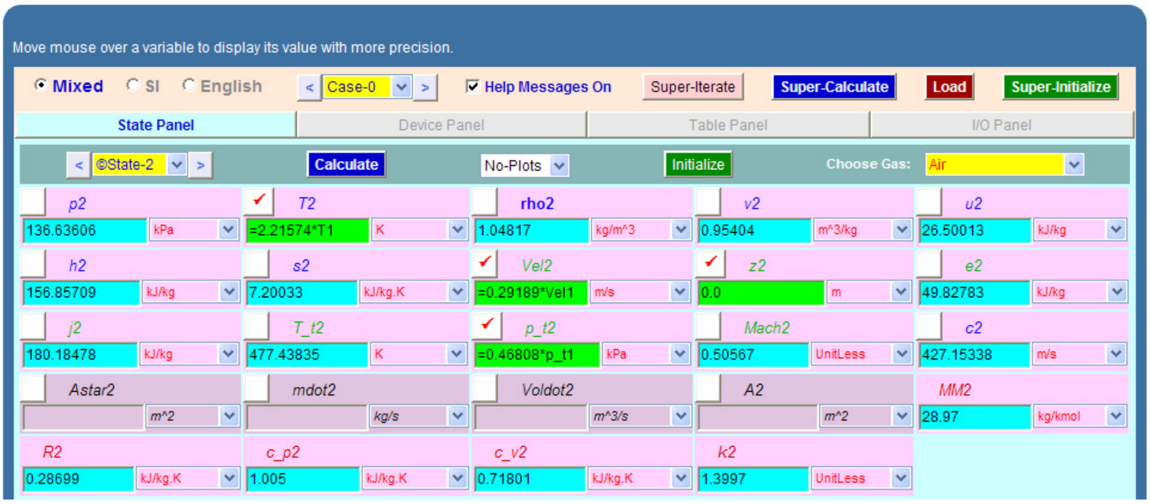
Note that stagn. pressure p_t1 = 347.57 kPa, Mach1 = 2.5787, entropy before shock = s1 = 6.98247 kJ/kg.K

2. Now, Mach1 = 2.5787 is the Mach No. before the shock. To get properties after the shock, go to Table Panel. Enter M_i = 2.5787 and hit Enter. We get:



Note from the above that first row gives isentropic functions and second and third rows give Normal shock functions. So, Mach No. after shock = M_e = 0.50567 and other property ratios are also available in lines 2 and 3. We use these ratios to get properties after shock.

3. **Properties after shock are designated by State 2:** Enter T2 = 2.21574 * T1, Vel2 = 0.29189 * Vel1 and p_t2 = 0.46808 * p_t1, using the property ratios obtained in the Shock Table above. Hit Enter. We get:



Thus:

Vel2 = 215.999 m/s, p_t2 = 162.69 kPa, T2 = 454.23 K, Mach2 = 0.50567, entropy after shock = s2 = 7.20033 kJ/kg.K … Ans.

And, entropy change across the shock = (s2 – s1) = 0.21786 kJ/kg.K = 217.86 J/kg.K .. Ans.

4. Click on SuperCalculate and get TEST code etc from the I/O Panel:

#~~~~~~~~~~~~~~~~~~~~~~~~OUTPUT OF SUPER-CALCULATE:

Daemon Path: Systems>Open>SteadyState>Specific>GasDynamics; v-10.ce02

#––––––––––Start of TEST-code ––––––––––––––––––––––––––––––––––––-

States {

State-1: Air;

Given: { p1= 18.0 kPa; T1= 205.0 K; Vel1= 740.0 m/s; z1= 0.0 m; }

State-2: Air;

Given: { T2= "2.21574*T1" K; Vel2= "0.29189*Vel1" m/s; z2= 0.0 m; p_t2= "0.46808*p_t1" kPa;
}

}

#–––––––––––End of TEST-code ––––––––––––––––––––––––––––––––––––

==

Prob.9.5.10. Air enters a C-D nozzle of supersonic wind tunnel at 1 MPa, 300 K with a low velocity. If a normal shock wave occurs at the exit plane of the nozzle at M = 2.4, determine the P, T, Mach No., Vel, and stagnation pressure after the shock. [Ref: 1]

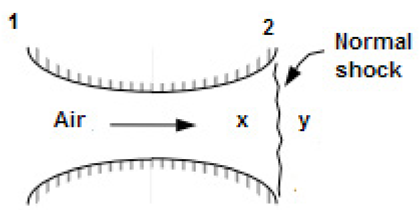
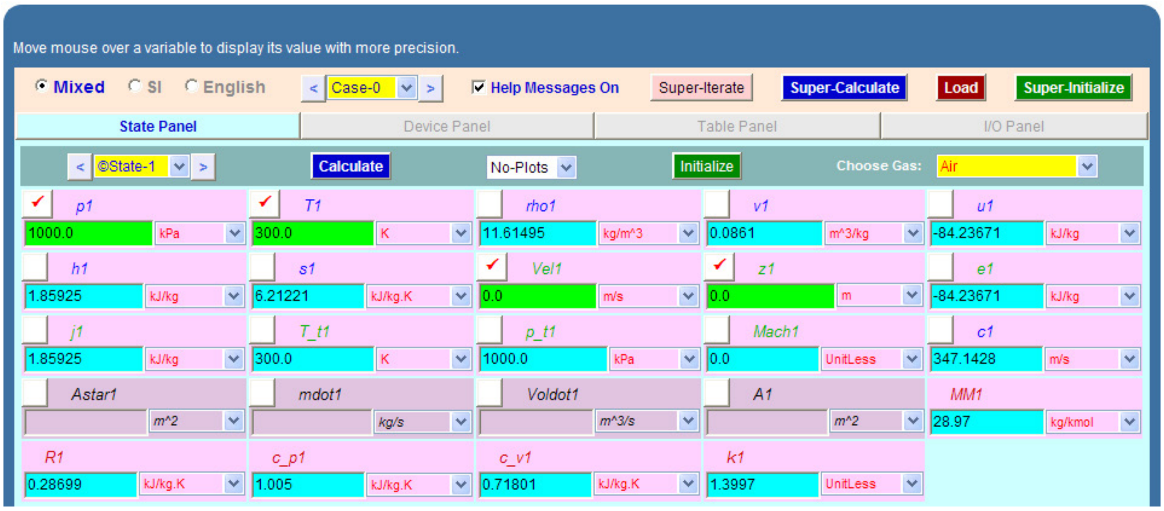


Fig.Prob.9.5.10 Normal shock

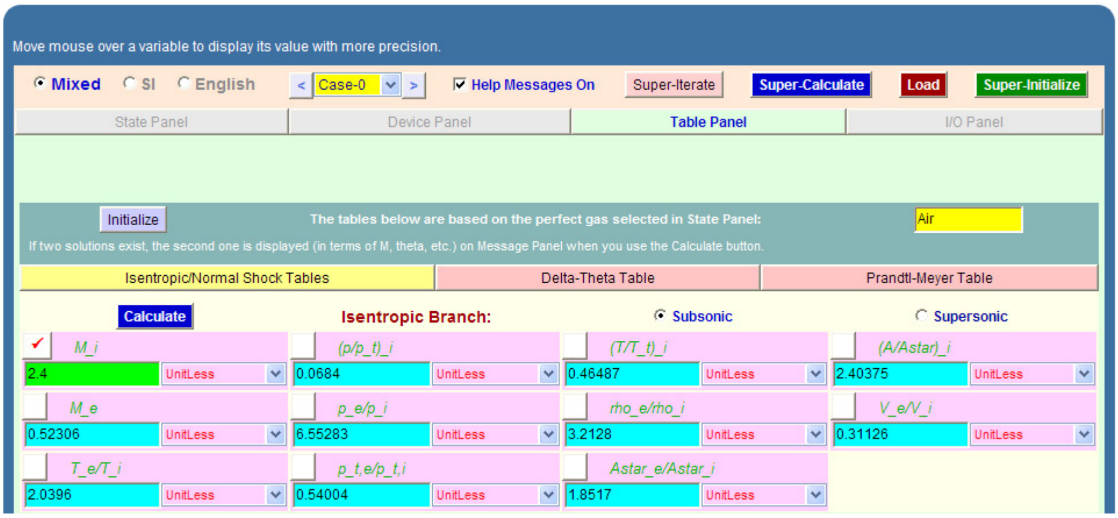
TEST Solution:

Again, remember that up to the shock, the flow is isentropic, and after the shock also, flow is isentropic. Stagnation temp remains constant across the shock, but stagn. pressure drops.

1. Go to Gas Dynamics calculator. In the State Panel, select Air as working substance. Enter the parameters before the shock, viz. p1, T1 and Vel1 as shown. Hit Enter. We get:

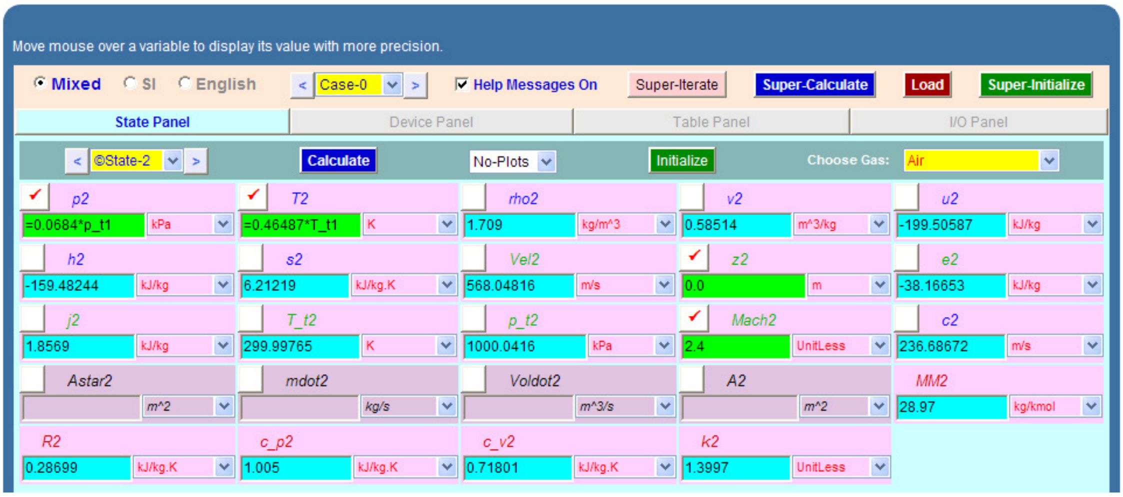


2. Go to the Table Panel to get properties before and after the shock. Enter M_i = 2.4 and hit Enter. We get:

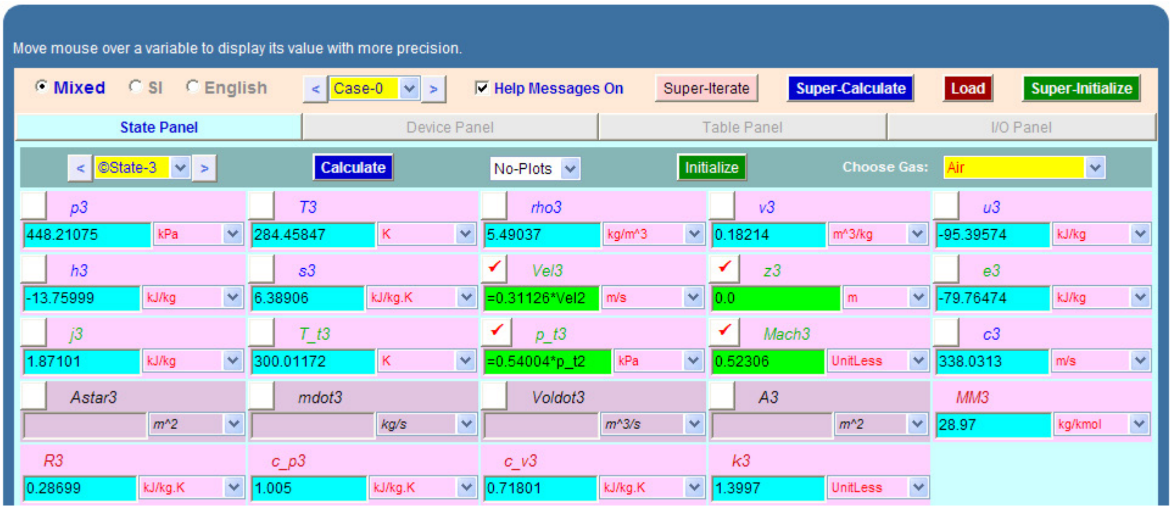


In the above Table, in the first row, we have property ratios for *isentropic conditions*. They are used below to get properties before shock:

3. And, designating the State as 2 as the state before the shock, enter p2, T2 and Mach2 as shown. Hit Enter. We get:



4. Properties after shock are designated as State 3, and we use the property ratios in second and third rows of the above Table. Using these ratios, enter p_t3, Vel3 and Mach3 as shown below. Hit Enter. We get:



Thus: After the shock:

Pressure, p3 = 448.21 kPa, Temp. T3 = 284.46 K, Mach3 = 0.523, Velocity, Vel3 = 176.8 m/s, and stagn. pressure p_t3 = 540.06 kPa …Ans.

5. Click on SuperCalculate and get TEST code etc from the I/O Panel:

#~~~~~~~~~~~~~~~~~~~~~~OUTPUT OF SUPER-CALCULATE:

Daemon Path: Systems>Open>SteadyState>Specific>GasDynamics; v-10.ce02

#––––––––––Start of TEST-code ––––––––––––––––––––––––––––––––––––

States {

State-1: Air;

Given: { p1= 1000.0 kPa; T1= 300.0 K; Vel1= 0.0 m/s; z1= 0.0 m; }

State-2: Air;

Given: { p2= "0.0684*p_t1" kPa; T2= "0.46487*T_t1" K; z2= 0.0 m; Mach2= 2.4 UnitLess; }

State-3: Air;

Given: { Vel3= "0.31126*Vel2" m/s; z3= 0.0 m; p_t3= "0.54004*p_t2" kPa; Mach3= 0.52306 UnitLess; }

}

#-----------End of TEST-code ----------------------------------

===

Prob.9.5.11. Air enters a C-D nozzle at low velocity at 2 MPa and 100 C. If the exit area of nozzle is 3.5 times the throat area, what must be the back pressure to produce a normal shock at the exit plane of the nozzle? [Ref: 1]

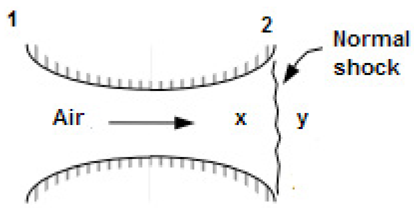
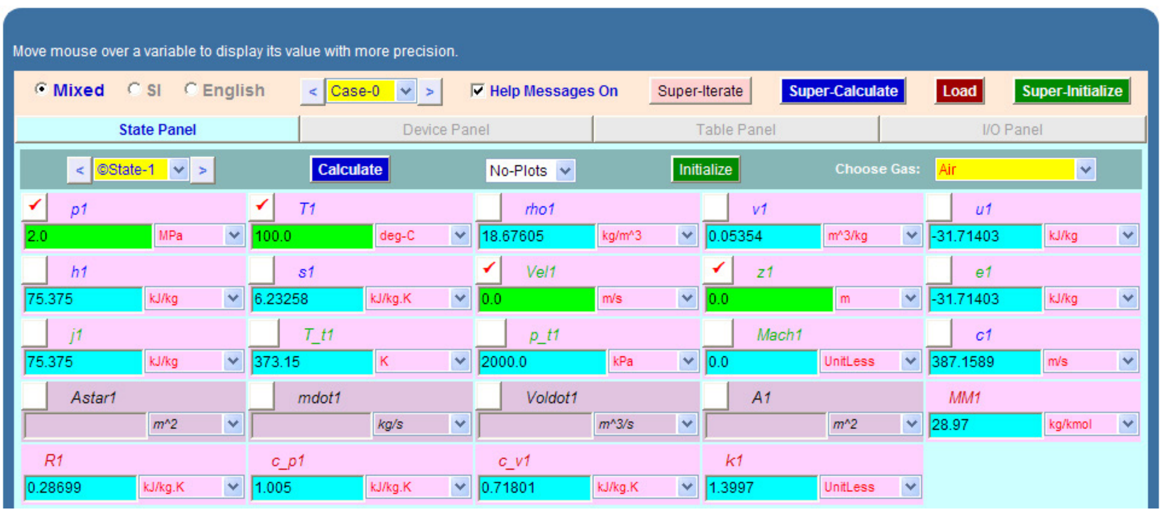


Fig.Prob.9.5.11 Normal shock

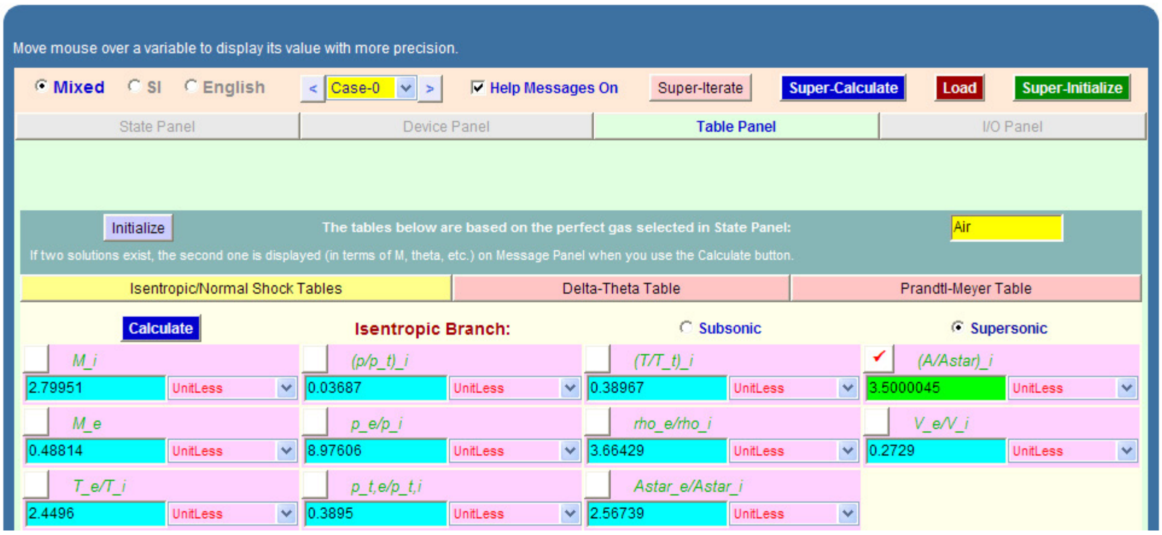
TEST Solution:

Following are the steps:

1. Go to Gas Dynamics calculator. In the State Panel, select Air as working substance. Enter the parameters of flow, viz. p1, T1 and Vel1=0, as shown. Hit Enter. We get:



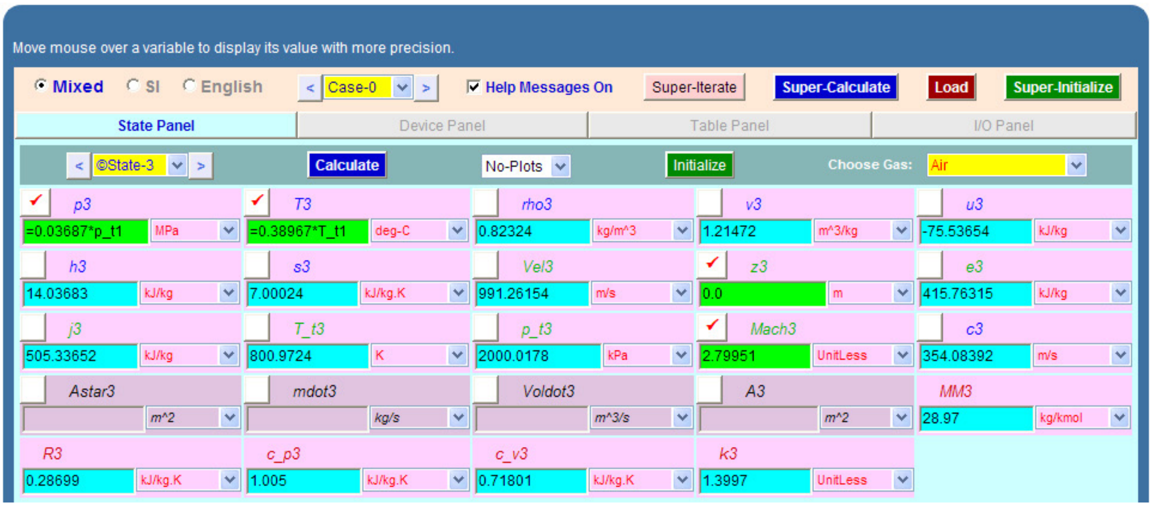
2. Go to the Table Panel to get properties before and after the shock. Enter A/Astar = 3.5 and hit Enter. We get:



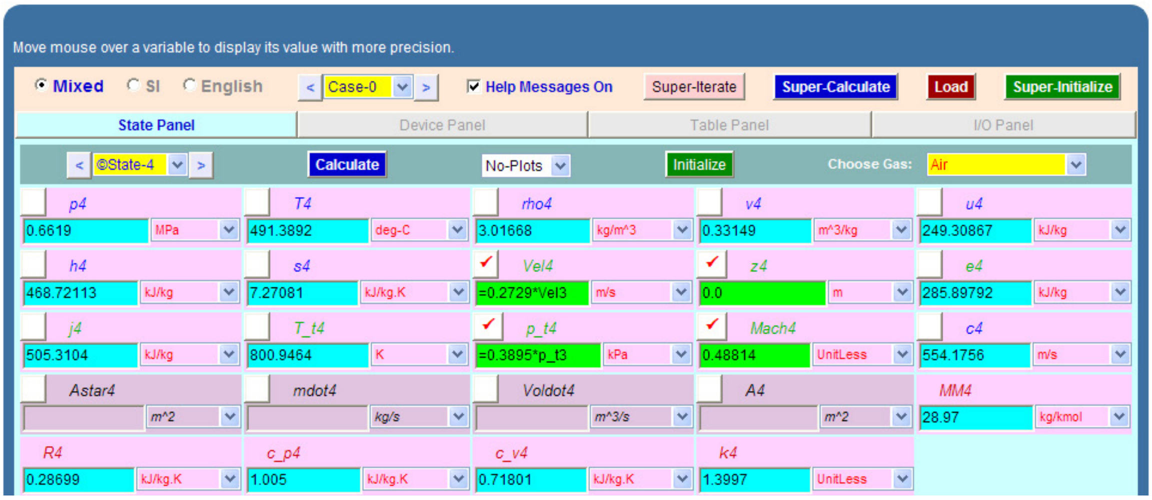
In the above Table, in the first row, we have property ratios for *isentropic conditions*. Second and third rows give property ratios for normal shock.

3. **At throat,** designated by State 2 below, Mach No. must be equal to 1, so that there will be supersonic flow in the divergent section such that a shock can occur. Since the flow is isentropic, from inlet up to the shock, we fill in Mach2 = 1, p_t2 = p_t1 and T_t2 = T_t1. Hit Enter. We get:

4. **At exit, before shock:** Let it be designated as State 3. Remember that up to the shock, flow is isentropic. So, using the property ratios from the above Isentropic/Shock Table, we enter: p3 = 0.03687 * p_t1, T3 = 0.38967 * T_t1, and Mach3 = 2.79951. Hit Enter. We get:



5. **At exit, after shock:** Let it be designated as State 4. Using the property ratios from the above Isentropic/Shock Table, we enter: p_t4 = 0.3895 * p_t3, Vel4 = 0.0.2729 * Vel3, and Mach4 = M_e = 0.48814. Hit Enter. We get:



Note from the above that p4 = 0.6619 MPa.

i.e. back pressure required to produce a normal shock at exit = p4 = 0.6619 MPa…Ans.

6. Click on SuperCalculate and get TEST code etc from the I/O Panel:

#~~~~~~~~~~~~~~~~~~~~~OUTPUT OF SUPER-CALCULATE

Daemon Path: Systems>Open>SteadyState>Specific>GasDynamics; v-10.ce02

#----------Start of TEST-code -----------------------------------

States {

State-1: Air;

Given: { p1= 2.0 MPa; T1= 100.0 deg-C; Vel1= 0.0 m/s; z1= 0.0 m; }

State-2: Air;

Given: { z2= 0.0 m; T_t2= "T_t1" K; p_t2= "p_t1" kPa; Mach2= 1.0 UnitLess; }

State-3: Air;

Given: { p3= “0.03687*p_t1” MPa; T3= “0.38967*T_t1” deg-C; z3= 0.0 m; Mach3= 2.79951 UnitLess; }

State-4: Air;

Given: { Vel4= “0.2729*Vel3” m/s; z4= 0.0 m; p_t4= “0.3895*p_t3” kPa; Mach4= 0.48814 UnitLess; }

}

#-----------End of TEST-code -----------------------------------

==

Prob.9.5.12. In Prob.9.5.10, what is the back pressure required for a normal shock to occur at a location where the cross-sectional area is twice the throat area (i.e. A/Astar = 2)? [Ref: 1]

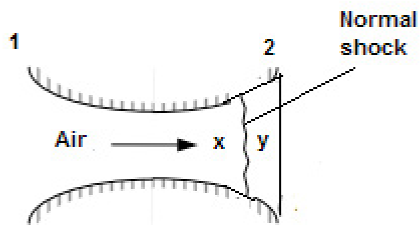
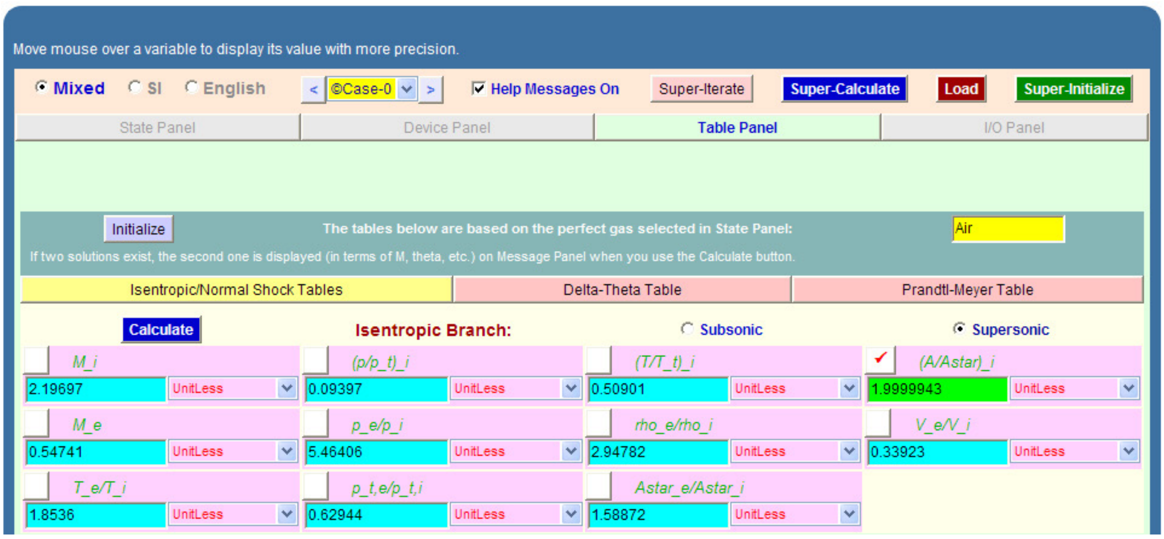


Fig.Prob.9.5.12 Normal shock

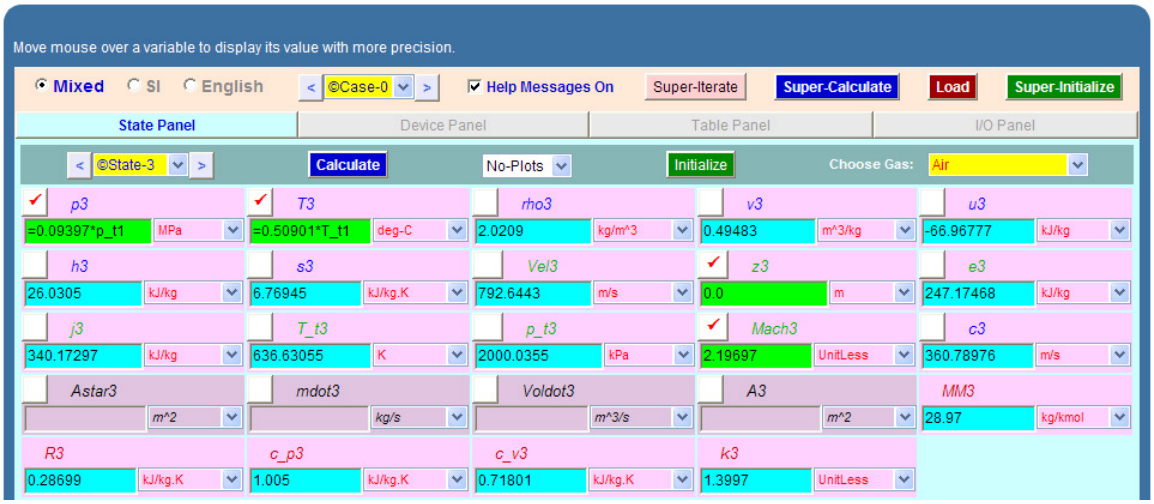
TEST Solution:

1. For (A/Asta)r = 2, there will be two values of M, one subsonic and the other supersonic. We need supersonic value; so, select the Supersonic radio button. We get, from Table Panel (i.e. Isentropic / Shock Tables):



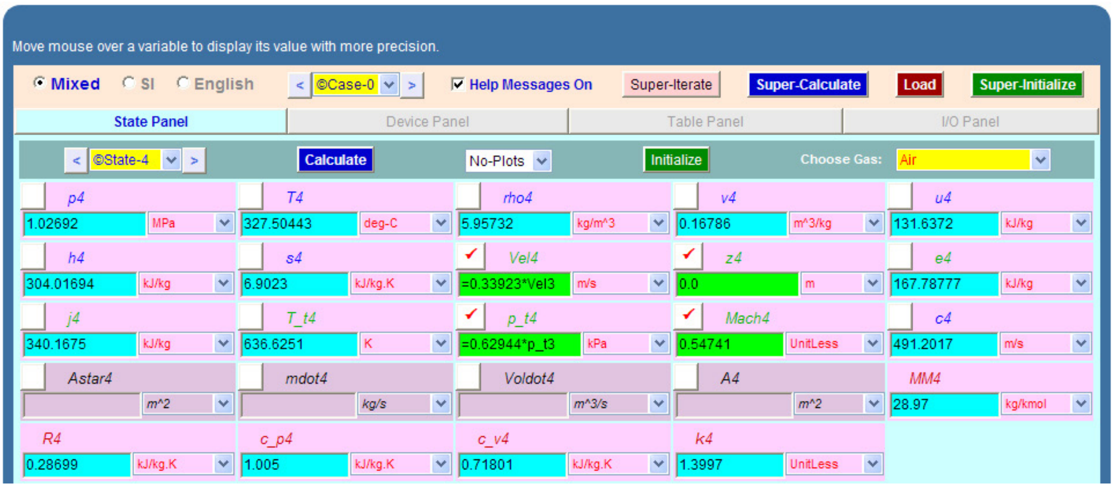
Note that Mach No. (before the shock), M_i = 2.19697.

2. Now, go to State 3, before the shock, and change Mach3 = 2.19697. Also, using the Isentropic flow functions in the above Table, enter p3 = 0.09397 * p_t1 and T3 = 0.50901 * T_t3, and hit Enter. We get:



Above screen shot gives properties before shock.

3. **And, after the shock:** Let the state be designated as State 4. Enter p_t4, Vel4 and Mach4 (= M_e) from the Isentropic/Shock Table. Hit Enter. We get:



We see that p4 = 1.02692 MPa.

i.e. back pressure for this condition = p4 = 1.0269 MPa.

===

Prob.9.5.13. Air flowing steadily in a nozzle experiences a normal shock at a Mach No. M = 3.2. If the pressure and temp of air are 58 kPa and 270 K respectively upstream of the shock, calculate the P, T, Vel, Mach No. and stagnation pressure downstream of the shock. Also, find the entropy change across the normal shock. Compare these results with those for helium undergoing a normal shock under the same conditions. [Ref: 1]

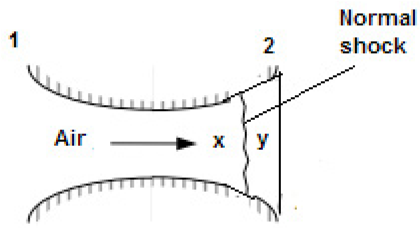
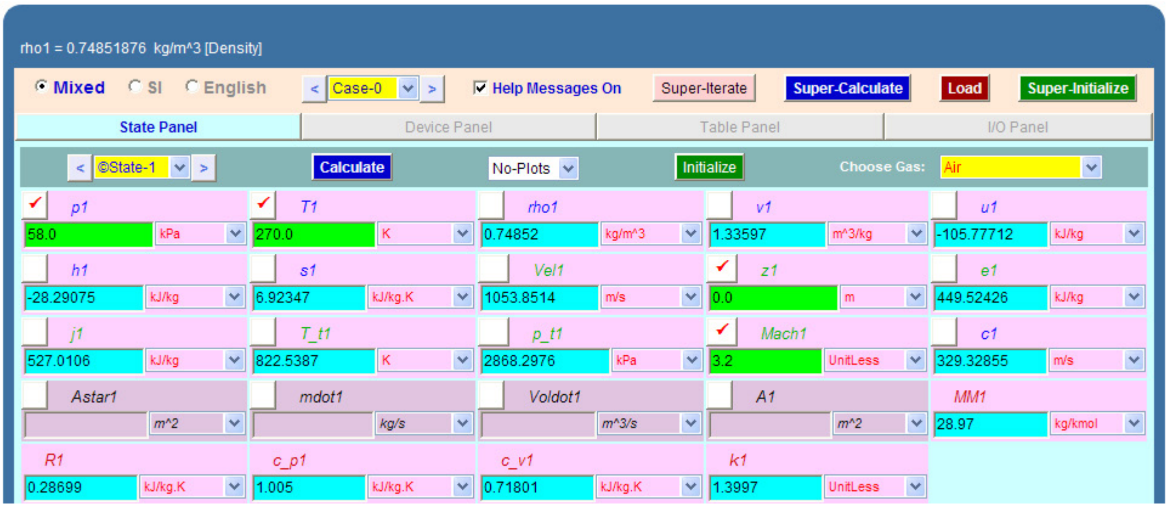


Fig.Prob.9.5.13 Normal shock

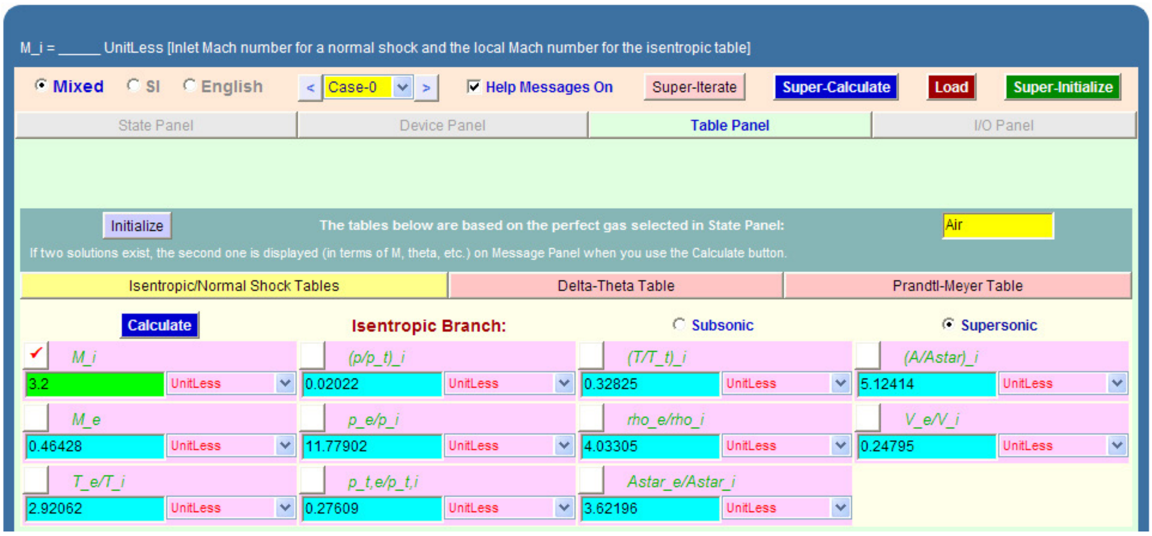
TEST Solution:

Following are the steps:

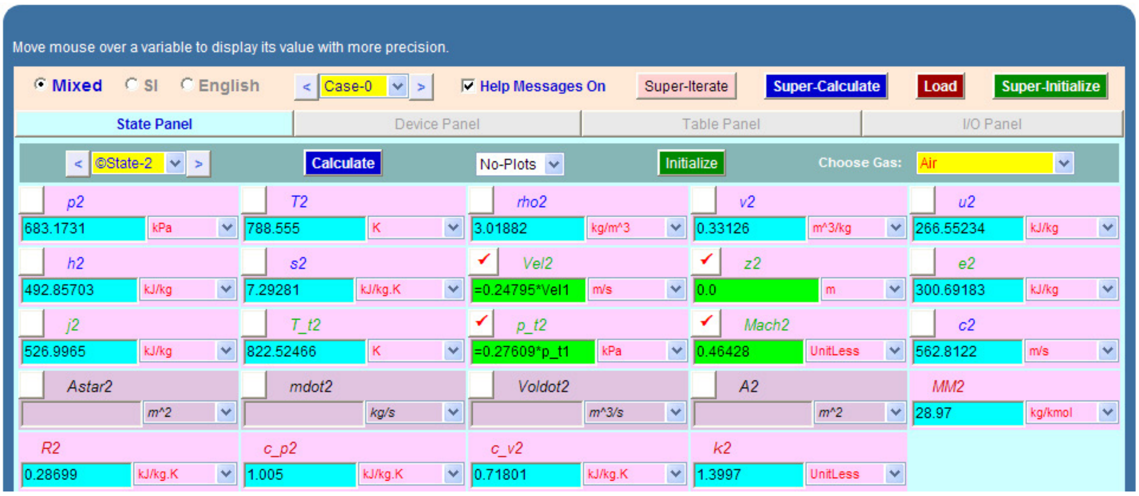
1. Go to Gas Dynamics calculator. In the State Panel, select Air as working substance. Enter the parameters of flow, viz. p1, T1 and Mach1= 3.2, as shown. Hit Enter. We get:



2. Go to Table Panel to get Isentropic/Normal shock functions. Enter M_i = 3.2 and hit Return. We get:



3. **After the shock: This state is designated as State 2.** Use the property ratios in the above Table to get properties after the shock. Enter Vel2, p_t2 and Mach2 as shown below, and hit Enter. We get:



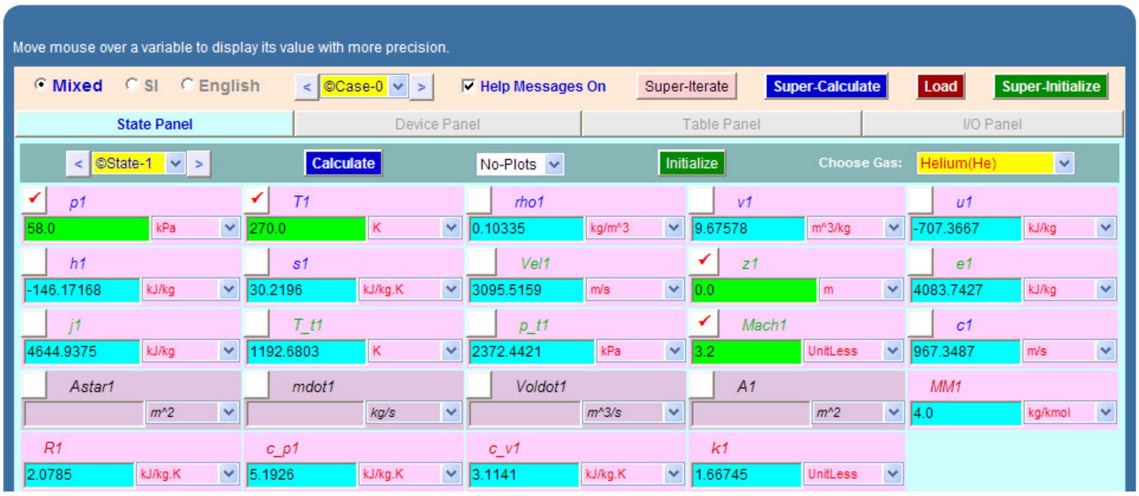
Thus:

After the shock: p2 = 683.17 kPa, T2 = 788.56 K, Vel2 = 261.3 m/s, Mach2 = 0.46428, and stagn. pressure p_t2 = 791.91 kPa …. Ans.

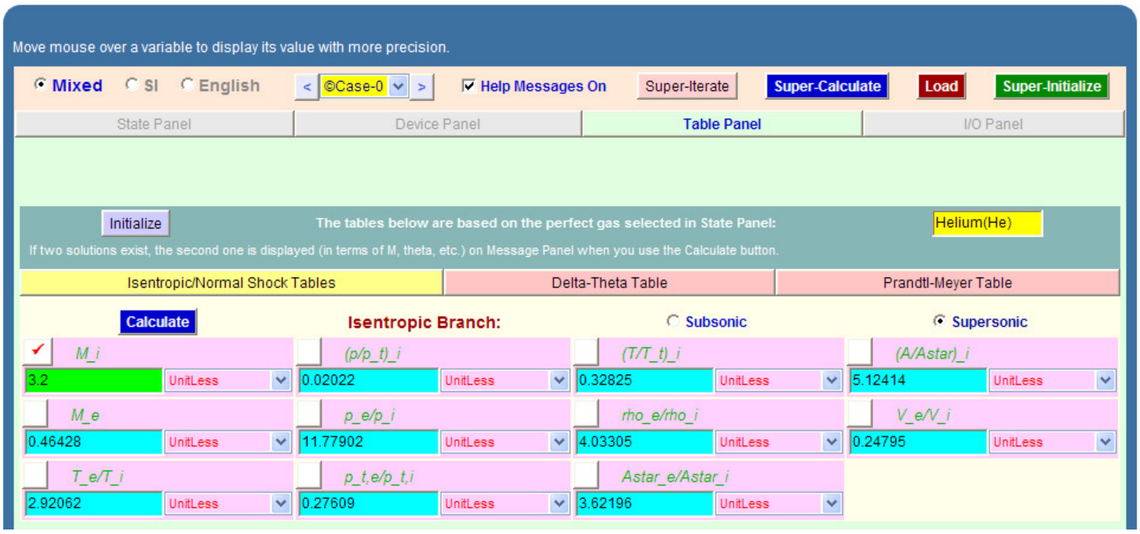
Entropy change across the shock = (s2 – s1) = (7.29281 – 6.92347) = 0.36934 kJ/kg.K = 369.34 J/kg.K …. Ans.

(b) For Helium: Go to State 1, change the substance to helium, click on SuperCalculate. All calculations get updated. We get:

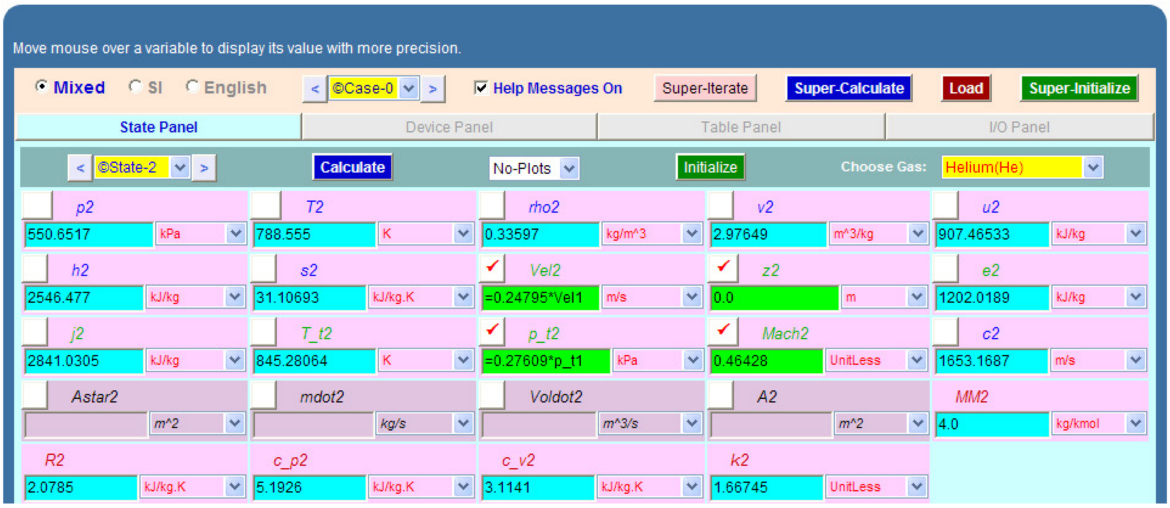
1. **Before shock:**



2. **Table Panel:**



3. **And, after the shock, we have:**



Thus, for helium:

After the shock: p2 = 550.65 kPa, T2 = 788.56 K, Vel2 = 767.53 m/s, Mach2 = 0.46428, and stagn. pressure p_t2 = 655 kPa …. Ans.

Entropy change across the shock = (s2 – s1) = (31.10693 – 30.2196) = 0.88733 kJ/kg.K = 887.33 J/kg.K …. Ans.

4. Click on SuperCalculate and get TEST code etc from the I/O Panel:

#~~~~~~~~~~~~~~~~~~~~~~OUTPUT OF SUPER-CALCULATE

Daemon Path: Systems>Open>SteadyState>Specific>GasDynamics; v-10.ce02

#-----------Start of TEST-code ---

States {

State-1: Helium(He);

Given: { p1= 58.0 kPa; T1= 270.0 K; z1= 0.0 m; Mach1= 3.2 UnitLess; }

State-2: Helium(He);

Given: { Vel2= "0.24795*Vel1" m/s; z2= 0.0 m; p_t2= "0.27609*p_t1" kPa; Mach2= 0.46428 UnitLess; }

}

#-----------End of TEST-code ---

#******DETAILED OUTPUT:

 # Evaluated States:

State-1: Helium(He) > PG-Model;
Given: p1= 58.0 kPa; T1= 270.0 K; z1= 0.0 m;
Mach1= 3.2 UnitLess;
Calculated: rho1= 0.1034 kg/m^3; v1= 9.6758 m^3/kg; u1= -707.3667 kJ/kg;
h1= -146.1717 kJ/kg; s1= 30.2196 kJ/kg.K; Vel1= 3095.5159 m/s;
e1= 4083.7427 kJ/kg; j1= 4644.9375 kJ/kg; T_t1= 1192.6803 K;
p_t1= 2372.4421 kPa; c1= 967.3487 m/s; MM1= 4.0 kg/kmol;
R1= 2.0785 kJ/kg.K; c_p1= 5.1926 kJ/kg.K; c_v1= 3.1141 kJ/kg.K;
k1= 1.6674 UnitLess;
State-2: Helium(He) > PG-Model;
Given: Vel2= "0.24795*Vel1" m/s; z2= 0.0 m; p_t2= "0.27609*p_t1" kPa;
Mach2= 0.46428 UnitLess;
Calculated: p2= 550.6517 kPa; T2= 788.555 K; rho2= 0.336 kg/m^3;
v2= 2.9765 m^3/kg; u2= 907.4654 kJ/kg; h2= 2546.4768 kJ/kg;
s2= 31.1069 kJ/kg.K; e2= 1202.0189 kJ/kg; j2= 2841.0305 kJ/kg;
T_t2= 845.2807 K; c2= 1653.1687 m/s; MM2= 4.0 kg/kmol;
R2= 2.0785 kJ/kg.K; c_p2= 5.1926 kJ/kg.K; c_v2= 3.1141 kJ/kg.K;
k2= 1.6674 UnitLess;

#––––Property spreadsheet:

#

#State	MachNo	Vel(m/s)	p(kPa)	p_t(kPa)	T(K)	T_t(K)	Astar(m2)	v(m3/kg)	u(kJ/kg)
# 1	3.2	3095.52	58.0	2372.44	270.0	1192.7		9.6758	-707.37
# 2	0.46	767.53	550.65	655.01	788.6	845.3		2.9765	907.47

..

===

9.6 References:

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11. http://www.potto.org/gasDynamics/gasDynamics.php: Free Text Book on 'Fundamentals of compressible flow Mechanics' by Genick Bar- Meir

==

Appendix Engine trials

A.1 Formulas used:

Indicated Power (ip):

$$ip = p_{mi} \cdot L \cdot A \cdot n \cdot k \quad \text{... kW}$$

...where p_{mi} = indicated mean effective pressure (kPa)

L = stoke length (m), A = cross-sectional area of piston (m^2),

n = no. of working strokes per sec.

Also, n= N/2 for 4 stroke engine, = N for 2 stroke engine,

And, n = 2 * N for a double acting 2 stroke engine

where N = no. of revolutions per sec.

k = no. of cylinders

Mean effective pressure (m.e.p):

It is the average pressure inside the cylinder based on calculated or measured power input.

Indicated mep: based on calculated power:

$$p_{mi} = \frac{\text{Area\_of\_indicator\_diagram}(cm)^2}{\text{Length\_of\_diagram}(cm)} \cdot \text{Spring\_const}\left(\frac{kPa}{cm}\right) \quad \text{...kPa}$$

$$p_{mi} = \frac{ip}{L \cdot A \cdot n \cdot k} \quad \text{...kPa}$$

Brake mep: based on measured power:

$$p_{mb} = \frac{bp}{L \cdot A \cdot n \cdot k} \quad \text{...kPa}$$

Brake Power (bp):

$$bp = 2 \cdot \pi \cdot N \cdot T$$ W... where N = no. of rev. per sec, T = Torque, N.m

and, $T = g \cdot (M - m_s) \cdot \frac{(D + d)}{2}$ **Nm...Torque for a rope brake** where, M = Load mass (kg), m_s = spring balance reading (kg) , D = dia of fly wheel, d = dia of rope, g = 9.81 m/s^2

Torque for a Prony brake:

$T = g \cdot M \cdot r$ W...where M = mass on hanger (kg), r = dist. from centre of fly wheel to hanger (m), g = 9.81 m/s^2

Torque for a hydraulic dynamometer:

$T = g \cdot M \cdot r$ W...where M = mass on hanger (kg), r = dist. from centre of dynamometerl to hanger (m), g = 9.81 m/s^2

Torque for an Electrical dynamometer:

$T = g \cdot M \cdot r$ W...where M = mass on hanger (kg), r = dist. from centre of dynamometer to hanger (m), g = 9.81 m/s^2

Friction power (fp):

$$fp = ip - bp$$

Mechanical efficiency:

$$\eta_{mech} = \frac{bp}{ip}$$

Indicated thermal efficiency:

$$\eta_{ith} = \frac{ip}{m_f \cdot CV}$$

where, ip = Indicated power (kW), mf = mass rate of fuel (kg/s),
CV = calorific value of fuel (kJ/kg)

Brake thermal efficiency:

$$\eta_{bth} = \frac{bp}{m_f \cdot CV}$$

where, bp = brake power (kW), mf = mass rate of fuel (kg/s),
CV = calorific value of fuel (kJ/kg)

Thus, we can also write: $\eta_{mech} = \frac{\eta_{bth}}{\eta_{ith}}$

Relative efficiency or efficiency ratio:

$$\eta_{rel} = \frac{Actual\_thermal\_effcy}{Air\_std\_effcy}$$

Air standard efficiency:

For Otto cycle (i.e. Petrol engines):

$$\eta_{airstd\_otto} = 1 - \frac{1}{r^{\gamma-1}}$$ where r = compression ratio = (Vs + Vc) / Vc, γ = 1.4 for air

For Diesel cycle (i.e. Diesel engines):

$$\eta_{airstd\_diesel} = 1 - \frac{1}{r^{\gamma-1}} \cdot \left[\frac{r_k^{\gamma} - 1}{\gamma \cdot (r_k - 1)} \right]$$ where, r = comprn. ratio, rk = cut off ratio, γ = 1.4 for air

Specific fulel consumption (sfc):

It may be based on ip or bp:

Indicated specific fuel consumption (isfc) = Fuel used in (kg/h) / ip ..kg/kWh

Brake specific fuel consumption (bsfc) = Fuel used in (kg/h) / bp ..kg/kWh

Volumetric efficiency (ηv):

$$\eta_v = \frac{mass\_of\_charge\_actually\_inhaled}{mass\_of\_charge\_as\_per\_swept\_vol\_at\_ambient\_temp\_and\_pressure}$$

Also:

$$\eta_v = \frac{vol\_of\_charge\_aspirated\_per\_stroke\_at\_ambient\_conditions}{swept\_volume}$$

Fuel-Air ratio:

$$\frac{F}{A} = \frac{m_f}{m_a}$$ f .... fuel, a .... air

Air-Fuel ratio:

$$\frac{A}{F} = \frac{m_a}{m_f}$$

Measurement of air consumption:

Generally, this is done by **Air box method**. Here, a sufficiently large air box is connected to the engine, and air enters the air box through an orifice. Pressure difference between the ambient and air box is measured with a water manometer. This is the pressure differential across the orifice.

Let:

A = area of orifice (m^2), d = dia of orifice (cm)

h_w = head of water in manometer (cm)

Cd = coeff. of discharge for orifice

Therefore: head in meters of air is given by:

$$h \cdot \rho_a = \frac{h_w}{100} \cdot \rho_w$$ where ρ_a = density of air, ρ_w = density of water

Then, **velocity of air** through orifice is:

$$C_a = \sqrt{2 \cdot g \cdot h} \quad \text{m/s}$$

and, volume of air through the orifice is:

$$V_a = C_d \cdot A \cdot C_a = C_d \cdot A \cdot \sqrt{2 \cdot g \cdot h}$$

i.e. $$V_a = 840 \cdot A \cdot C_d \cdot \sqrt{\frac{h_w}{\rho_a}} \quad \text{m^3/s}$$

And, mass of air through the orifice is:

$$m_a = V_a \cdot \rho_a = 0.0011 \cdot C_d \cdot d^2 \cdot \sqrt{h_w \cdot \rho_a} \quad \text{kg/s}$$

Indicated power by Morse test:

Applicable to multicylinder petrol/oil engines only.

To make an estimate of indicated power (ip) in the absence of an engine indicator.

Procedure:

First, determine the total power output(i.e. bp), b, when all the cylinders are firing, by coupling the engine to a suitable brake. Note the speed (RPM).

Now, cut out the first cylinder. This is achieved by shorting out the spark plug of first cylinder in case of a petrol engine; and, for an oil engine, fuel supply to the first cyl is interrupted.

Now, the engine speed will drop since one cylinder is cut out. Load is now adjusted to restore the speed to the original value. Determine the bp under this new condition. Let it be b1.

Now, restore the cylinder 1, and cut out the cylinder 2. Again, engine speed will fall, and correct it to original value by adjusting the load. And find the bp under this condition, when cyl 2 is cut out. Let it be b2.

Adopt this procedure to each cylinder of this multicylinder engine.

Then, ip of each cylinder is given as: (taking the example of a 4 cyl machine):

i1 = b - b1

i2 = b - b2

i3 = b - b3, and

i4 = b - b4

And:

Total ip = i1 + i2 + i3 + i4

==

A.2 Problems solved with Mathcad:

Prob.A.2.1 For a 4 cyl, 4 stroke petrol engine, air flow was measured by a 75 mm dia sharp edged orifice, Cd = 0.65. Following data obtained during a test: bore = 110 mm, stroke = 130 mm, speed = 2500 rpm, b.p. = 40 kW, fuel consumption = 11 kg/h, CV of fuel = 42000 kJ/kg, pressure drop across orifice = 4.1 cm of water, atm. temp and pressure are 15 C and 1.013 bar. Calculate: brake thermal effcy., bmep and vol. effcy. based on free air conditions.

Mathcad Soution:

Data:

$g := 9.81$ m/s^2 $C_d := 0.65$ $\rho_w := 1000$ kg/m^3... density of water $R := 287$ J/kg.K

$h_w := 4.1 \cdot 10^{-2}$ m of water... head across orifice

$d := 0.075$ m ... dia of orifice

$L := 0.13$ m.... stroke $D := 0.11$ m ... bore $N := 2500$ RPM $k := 4$ ..no. of cyl.

$P := 1.013 \cdot 10^5$ Pa $T := 15 + 273$ K

Calculations:

$A := \frac{\pi \cdot d^2}{4}$ i.e. $A = 4.418 \times 10^{-3}$ m^2.... area of orifice

Air flow: $\rho_a := \frac{P}{R \cdot T}$

i.e. $\rho_a = 1.226$ kg/m^3... density of air

$V_a := C_d \cdot A \cdot \sqrt{\frac{2 \cdot g \cdot h_w \cdot \rho_w}{\rho_a}} \cdot 60$ i.e. $V_a = 4.414$ m^3/min.... vol. of air through orifice

Swept volume: $V_s := \frac{\pi \cdot D^2}{4} \cdot L \cdot \frac{N}{2} \cdot k$ ...N/2 since it is a 4 stroke engine

i.e. $V_s = 6.177$ m^3/min

Vol. effcy.: $\eta_{vol} := \frac{V_a}{V_s}$ i.e. $\eta_{vol} = 0.715$ **= 71.5 %...Ans.**

Brake thermal effcy: $bp := 40 \cdot 10^3$ W

$Q_{supp} := \frac{11}{3600} \cdot 42000 \cdot 10^3$ J/s.... heat supplied in fuel

Therefore: $\eta_{bth} := \frac{bp}{Q_{supp}}$

i.e. $\eta_{bth} = 0.312$ **= 31.2% brake thermal effcy......Ans**

bmep: $bmep := \frac{bp \cdot 60}{L \cdot A \cdot \frac{N}{2} \cdot k}$

i.e. $bmep = 8.358 \times 10^{5}$ **Pa = 8.358 bar....Ans.**

===

Prob.A.2.2 During a trial on a 4 stroke S.I. engine, following observations were made: Duration of trial = 45 min., fuel consumption = 5 litres, sp. gravity of fuel = 0.84, heating value of fuel = 40 MJ/kg, net area of indicator diagram = 8.75 cm^2, length of indicator diagram = 8.5 cm, spring constant = 6 bar/cm, speed = 1000 rpm, cyl. dia = 150 mm, stroke = 200 mm. Calculate: ip, sp. fuel consumption and thermal efficiency.

Mathcad Solution:

Data:

$Duration := 45$ min $V_{fuel} := 5 \cdot 10^{-3}$ m^3 $sp\_gr := 0.84$

$HV := 40 \cdot 10^{3}$ kJ/kg $Area_{ind} := 8.75$ cm^2 $Length_{ind} := 8.5$ cm

$spring\_const := 600 \frac{kPa}{cm}$ $d := 0.15$ m....dia $L := 0.2$ m ... stroke

$N := 1000$ rpm $n := \frac{N}{2 \cdot 60}$ per sec...divided by 2, since 4 stroke

$k := 1$ ...no. of cyl

Calculations:

$A := \frac{\pi \cdot d^{2}}{4}$ i.e. $A = 0.018$ m^2.... cross-sectional area of cyl

Mean effective pressure:

$p_{mi} := \frac{Area_{ind}}{Length_{ind}} \cdot spring\_const$ i.e. $p_{mi} = 617.647$ kPa...indicated mep

Therefore, Indicated power:

$ip := p_{mi} \cdot L \cdot A \cdot n \cdot k$ i.e. $ip = 18.191$ **kW.... indicated power ... Ans.**

Sp. fuel consumption:

$mdot := \frac{V_{fuel} \cdot sp\_gr \cdot 1000 \cdot 60}{Duration}$ i.e. $mdot = 5.6$ kg/h ... fuel cons.

Therefore:

$sfc := \frac{mdot}{ip}$ i.e. $sfc = 0.308$ **kg/kWh .... isfc ... Ans.**

Thermal efficiency:

$\eta_{ith} := \frac{ip \cdot 3600}{mdot \cdot HV}$

i.e. $\eta_{ith} = 0.292$ **29.2 % ... indicated thermal effcy... Ans.**

===

Prob.A.2.3 A 6 cyl, 4 stroke S.I. engine develops 40 kW. During a Morse test at 2000 rpm, the power output with each cylinder made inoperative turn by turn was 32.2, 32.0, 32.5, 32.4, 32.1 and 32.3 kW respectively. Estimate the mech. Efficiency, air standard efficiency, when bore = 100 mm, stroke = 125 mm, clearance vol. = 1.23 * 10^{-4} m^3. Also, calculate the thermal efficiency when fuel consumption is 9 kg/h and HV of fuel = 40 MJ/kg and the relative efficiency.

Mathcad Solution:

Data:

$N := 2000$ rpm $n := \frac{N}{2 \cdot 60}$ ..per sec; divided by 2 since 4 stroke

$k := 6$ ...no. of cyl. $d := 0.1$ m....dia $L := 0.125$ m ... stroke

$V_c := 1.23 \cdot 10^{-4}$ m^3 .... clearance vol.

$m_f := 9$ kg/h .... fuel cons. $HV := 40 \cdot 10^3$ kJ/kg $\gamma := 1.4$ ...for air

$bp := 40$ kW.... total power developed when all the 6 cyl are firing

$bp1 := 32.2$ kW.... when cyl1 is cut out

$bp2 := 32.0$ kW.... when cyl2 is cut out

$bp3 := 32.5$ kW.... when cyl3 is cut out

$bp4 := 32.4$ kW.... when cyl4 is cut out

$bp5 := 32.1$ kW.... when cyl5 is cut out

$bp6 := 32.3$ kW.... when cyl6 is cut out

Calculations:

$A := \frac{\pi \cdot d^2}{4}$ i.e. $A = 7.854 \times 10^{-3}$ m^2.... cross-sectional area of cyl

Total indicated power:

$$ip := (bp - bp1) + (bp - bp2) + (bp - bp3) + (bp - bp4) + (bp - bp5) + (bp - bp6)$$

i.e. $ip = 46.5$ **kW...total ip ... Ans.**

Mech. effcy.:

$\eta_{mech} := \frac{bp}{ip}$ i.e. $\eta_{mech} = 0.86$ **=86 % .... mech. efficiency ... Ans.**

Brake mep:

$bmep := \frac{bp}{L \cdot A \cdot n \cdot k}$ i.e. $bmep = 407.437$ **kPa = 4.074 bar ... Ans.**

Air standard effcy:

$V_s := \frac{\pi \cdot d^2}{4} \cdot L$ i.e. $V_s = 9.817 \times 10^{-4}$ m^3 ... stroke vol.

$r := \frac{V_s + V_c}{V_c}$ i.e. $r = 8.982$ ...compression ratio

Therefore, for Otto cycle of S.I. engine, air standard effcy:

$\eta_{air\_std} := 1 - \frac{1}{r^{\gamma-1}}$ i.e. $\eta_{air\_std} = 0.584$ **= 58.4 %**

Brake thermal effcy.:

$\eta_{bth} := \frac{bp}{\frac{m_f}{3600} \cdot HV}$ i.e. $\eta_{bth} = 0.4$ **= 40 % ... brake thermal effcy ... Ans.**

Relative thermal effcy.:

$\eta_{rel} := \frac{\eta_{bth}}{\eta_{air\_std}}$ i.e. $\eta_{rel} = 0.684$ **= 68.4 % ...relative effcy... Ans.**

===

Prob.A.2.4 Following observations were recorded in a test of 1 hour duration on a single cylinder, 4 stroke oil engine: Bore = 220 mm, stroke = 300 mm, fuel used = 4 kg, CV of fuel = 42000 kJ/kg, shaft speed = 300 rpm, no. of explosions/min = 148, mep = 5 bar, load on brake drum = 60 kg, spring balance reading = 30 N, dia of brake drum = 1.4 m, quantity of cooling water circulated = 500 kg, increase in temp of cooling water = 20 C, AF ratio = 16, exhaust gas temp = 410 C, sp. heat of exh. gases = 1.1 kJ/kg.K, ambient temp = 30 C. Determine: ip, bp, mech. efficiency, brake thermal effcy., sfc. Draw the heat balance in kJ/min. [VTU]

Mathcad Solution:

Data:

$P1 := 101.3$ kPa .... assumed $N := 300$ rpm $mep := 500$ kPa

$Duration := 60$ min. $d := 0.22$ m ...bore $L := 0.3$ m.... stroke

$m_f := 4$ kg $CV := 42000$ kJ/kg $n := 148$ ...no. of explosions per min.

$g := 9.81$ m/s^2 ... accn. due to gravity $M := 60$ kg...load on brake drum

$S := 30$ N... spring force $R_{drum} := 0.7$ m.... rad. of brake drum

$m_{cw} := 500$ kg....cooling water $\Delta T_{cw} := 20$ C $AF := 16$ ...Air/Fuel ratio

$T_{exh} := 410$ C... temp of exh. gases $cp_{exh} := 1.1$ kJ/kg.K $T_{amb} := 30$ C

$k := 1$ ....no. of cyl. $cp_{cw} := 4.18$ kJ/kg.K .... sp. heat of cooling water

Calculations:

$m_a := m_f \cdot AF$ i.e. $m_a = 64$ kg/h .... mass of air

$m_{exh} := m_f + m_a$ i.e. $m_{exh} = 68$ kg/h .... mass of exhaust gases

Indicated power:

$ip := mep \cdot L \cdot \left(\frac{\pi \cdot d^2}{4}\right) \cdot n \cdot k$ i.e. $ip = 843.895$ **kJ/min ... Ans.**

Working on per min. basis:

$Q_{supp} := \frac{m_f}{60} \cdot CV$ i.e. $Q_{supp} = 2.8 \times 10^3$ **kJ/min .... heat supplied in fuel**

$bp := \frac{2 \cdot \pi \cdot N \cdot (M \cdot g - S) \cdot R_{drum}}{1000}$ i.e. $bp = 737.055$ **kJ/min ... brake power .... Ans.**

$$\eta_{mech} := \frac{bp}{ip} \quad \text{i.e.} \quad \eta_{mech} = 0.873$$ = 87.3 % ....mech. effcy. .... Ans.

$$\eta_{bth} := \frac{bp}{Q_{supp}} \quad \text{i.e.} \quad \eta_{bth} = 0.263$$ = 26.3 % ....brake thermal effcy. .... Ans.

$$sfc := \frac{m_f}{\frac{bp}{60}} \quad \text{i.e.} \quad sfc = 0.326$$ kg/kWh ... sp. fuel consumption ... Ans.

$$Q_{cw} := \frac{m_{cw}}{60} \cdot cp_{cw} \cdot \Delta T_{cw} \quad \text{i.e.} \quad Q_{cw} = 696.667$$ kJ/min. ...heat to cooling water ... Ans.

$$Q_{exh} := \frac{m_{exh}}{60} \cdot cp_{exh} \cdot \left(T_{exh} - T_{amb}\right) \quad \text{i.e.} \quad Q_{exh} = 473.733$$ kJ/min... heat to exh gases .. Ans.

Percent losses:

$Q_{bp\_percent} := \frac{bp}{Q_{supp}} \cdot 100$ i.e. $Q_{bp\_percent} = 26.323$ %... percent bp ... Ans.

$Q_{cw\_percent} := \frac{Q_{cw}}{Q_{supp}} \cdot 100$ i.e. $Q_{cw\_percent} = 24.881$ %... percent cooling water ... Ans.

$Q_{exh\_percent} := \frac{Q_{exh}}{Q_{supp}} \cdot 100$ i.e. $Q_{exh\_percent} = 16.919$ %... percent exh. gases ... Ans.

$Q_{unacc} := Q_{supp} - (bp + Q_{cw} + Q_{exh})$ i.e. $Q_{unacc} = 892.545$ kJ/min..... unaccounted losses

$Q_{unacc\_percent} := \frac{Q_{unacc}}{Q_{supp}} \cdot 100$ i.e. $Q_{unacc\_percent} = 31.877$ %... percent unaccounted losses ... Ans.

Table:

Details	kJ/min	%
Heat supplied: Q_supp	**2800**	**100**
Heat to bp: Q_bp	737.055	26.323
Heat to cooling water: Q_cw	696.667	24.881
Heat to exh. Gases: Q_exh	473.733	16.919
Heat unaccounted: Q_unacc	892.545	31.877

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Prob.A.2.5 A test on a single cylinder, 4 stroke oil engine having a bore = 180 mm, stroke = 360 mm, gave the following results: speed = 290 rpm, brake torque = 392 Nm, indicated mep = 7.2 bar, oil consumption = 3.5 kg/h, cooling water flow rate = 270 kg/h, cooling water ΔT = 36 C, Air/Fuel ratio by weight = 25, exhaust gas temp = 415 C, barometric pressure = 1.013 bar, room temp = 21 C, CV of fuel = 45200 kJ/kg. Fuel contains 15 % H2 by weight. Calculate: (i) indicated thermal efficiency (ii) volumetric efficiency based on atmospheric conditions, and (iii) draw up a heat balance sheet in terms of kJ/min. Take R = 0.287 kJ/kg.K, cp for exhaust gases = 1.0035 kJ/kg.K, cp for superheated steam = 2.093 kJ/kg.K.

Mathcad Solution:

Data:

$p_m := 7.2 \cdot 10^5$ Pa ... mep $L := 0.36$ m ... stroke $d := 0.18$ m... bore

$N := 290$ rpm $n := \frac{N}{2}$ ...since 4 stroke engine $k := 1$ ....no. of cyl.

$m_f := 3.5$ kg/h .... fuel consumption $CV := 45200$ kJ/kg $T := 392$ Nm ... torque

$m_{cw} := 270$ kg/h ... cooling water flow rate $\Delta T := 36$ C .... cooling water temp rise

$AF := 25$ ...air-fuel ratio, by weight $T_{exh} := 415$ C .... exh. gas temp

$P_{amb} := 1.013$ bar $T_{amb} := 21$ C $R := 287$ J/kg.K

$cp_{dry\_exh} := 1.0035$ kJ/kg.K $cp_{steam} := 2.093$ kJ/kg.K $cp_{cw} := 4.18$ kJ/kg.K

Fuel contains 15 % H2 by weight.

Calculations:

$A := \frac{\pi \cdot d^2}{4}$ i.e. $A = 0.025$ m^2 .... cross-sectional area of cyl

$ip := \frac{p_m \cdot L \cdot A \cdot n \cdot k}{60 \cdot 1000}$ i.e. $ip = 15.94$ **kW....indicated power... Ans.**

Heat in fuel: $q_{fuel} := m_f \cdot CV$ i.e. $q_{fuel} = 1.582 \times 10^5$ kJ/h

$\eta_{ith} := \frac{ip \cdot 3600}{q_{fuel}}$ i.e. $\eta_{ith} = 0.363$ **= 36.3 % .... ind. thermal effcy. ... Ans.**

Volumetric effcy:

Air inhaled: $m_a := AF \cdot \frac{m_f}{60}$ i.e. $m_a = 1.458$ kg/min.

Volume of air: $V_a := \frac{m_a \cdot R \cdot (T_{amb} + 273)}{P_{amb} \cdot 10^5}$

i.e. $V_a = 1.215$ m^3/min

Swept volume: $V_s := \frac{\pi \cdot d^2}{4} \cdot L \cdot n$ i.e. $V_s = 1.328$ m^3/min

Therefore: Vol. effcy: $\eta_{vol} := \frac{V_a}{V_s}$ i.e. $\eta_{vol} = 0.914$ **= 91.4 % .... Vol. effcy.... Ans.**

Heat balance on a 'per minute' basis:

$Q_{supp} := \frac{m_f \cdot CV}{60}$ i.e. $Q_{supp} = 2.637 \times 10^3$ **kJ/min ... heat supplied in fuel**

$bp := \frac{2 \cdot \pi \cdot N \cdot T}{60 \cdot 1000}$ i.e. $bp = 11.905$ **kW = 714.3 kJ/min**

$Q_{cw} := \frac{m_{cw}}{60} \cdot cp_{cw} \cdot \Delta T$ i.e. $Q_{cw} = 677.16$ **kJ/min ... heat to cooling water**

Mass of exh. gas: $m_{exh} := m_a + \frac{m_f}{60}$ i.e. $m_{exh} = 1.517$ kg/min

Now, we have the combustion eqn for H2:

$H2 + O = H2O$ i.e. 2 kg H2 + 16 kg O2 = 18 kg H2O

i.e. 1 kg of H2 produces 9 kg of H2O on complete combustion.

Therefore, 15% H2 contained in 3.5/60 kg of fuel (per min.) produces:

$m_w := 0.15 \cdot \frac{m_f}{60} \cdot 9$ kg of water i.e. $m_w = 0.079$ kg of water

Therefore, amount of dry exh. gas:

$m_{dry\_exh} := m_{exh} - m_w$ i.e. $m_{dry\_exh} = 1.438$ kg/min

Therefore: heat carried away by dry exh. gas:

$$Q_{dry\_exh} := m_{dry\_exh} \cdot cp_{dry\_exh} \cdot (T_{exh} - T_{amb})$$

i.e. $Q_{dry\_exh} = 568.522$ **kJ/min.**

And: heat carried away by water vapor in exh. gas:

From steam tables: enthalpy of superheated steam, hv at 415 C = 3310 kJ/kg, and therefore:

$Q_{water\_vap} := m_w \cdot 3310$ i.e. $Q_{water\_vap} = 260.662$ **kJ/min.**

Table:

Details	kJ/min	%
Heat supplied: Q_supp	**2637**	**100**
Heat to bp: Q_bp	714.3	27.09
Heat to cooling water: Q_cw	677.16	25.68
Heat to dry exh. gases: Q_dry_exh	568.52	21.56
Heat to water vap.: Q_water_vap	260.66	9.89
Heat unaccounted: Q_unacc	416.356	15.79
Total:	2637	100

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Prob.A.2.6 During a trial of 60 min. on a single cylinder oil engine, working on a two stroke cycle, following data were obtained. Determine: IP, BP and mechanical efficiency. Draw up a heat balance sheet on a 'per minute' basis. [VTU]

Mathcad Solution:

Data:

$Duration := 60$ min. $d := 0.3$ m ...bore $L := 0.45$ m.... stroke

$V_f := 9.6$ lit $CV := 45000$ kJ/kg $sp\_gr := 0.8$ ...sp. gr. of oil

$N_{tot} := 12624$ ..total no. of revolutions $N := \frac{N_{tot}}{60}$ i.e. $N = 210.4$ rpm

$n := N$ ...since two stroke engine

$p_{m\_gross} := 7.24$ bar .... gross mep $p_{m\_pump} := 0.34$ bar...pumping mep

$g := 9.81$ m/s^2 ... accn. due to gravity $F := 3150$ N...Net load on brake

$D_{drum} := 1.78$ m.... dia. of brake drum $D_{rope} := 0.04$ m ... dia of rope

$m_{cw} := 545$ kg....cooling water $\Delta T_{cw} := 25$ C

$k := 1$ ....no. of cyl. $cp_{cw} := 4.18$ kJ/kg.K .... sp. heat of cooling water

Qexh = 15 % of heat supplied. ..... heat to exh. gases

Calculations:

$p_m := (p_{m\_gross} - p_{m\_pump}) \cdot 10^2$ i.e. $p_m = 690$ **kPa .... mep.**

$m_f := \dfrac{V_f \cdot sp\_gr}{60}$ i.e. $m_f = 0.128$ kg/min

$A := \dfrac{\pi \cdot d^2}{4}$ i.e. $A = 0.071$ m^2 .... cross-sectional area of cyl

$ip := \dfrac{p_m \cdot L \cdot A \cdot n \cdot k}{60}$ i.e. $ip = 76.964$ **kW....indicated power... Ans.**

Heat in fuel: $q_{fuel} := m_f \cdot CV$ i.e. $q_{fuel} = 5.76 \times 10^3$ **kJ/min**

$\eta_{ith} := \dfrac{ip \cdot 60}{q_{fuel}}$ i.e. $\eta_{ith} = 0.802$ **= 80.2 % .... ind. thermal effcy. ... Ans.**

Brake Torque: $T := F \cdot \frac{(D_{drum} + D_{rope})}{2}$ i.e. $T = 2.866 \times 10^3$ Nm

$bp := \frac{2 \cdot \pi \cdot N \cdot T}{60 \cdot 1000}$ i.e. $bp = 63.158$ **kW ... brake power .... Ans.**

$\eta_{mech} := \frac{bp}{ip}$ i.e. $\eta_{mech} = 0.821$ **= 82.1 % ....mech. effcy. .... Ans.**

Heat balance on a 'per minute' basis:

$Q_{supp} := m_f \cdot CV$ i.e. $Q_{supp} = 5.76 \times 10^3$ **kJ/min ... heat supplied in fuel**

$bp := \frac{2 \cdot \pi \cdot N \cdot T}{60 \cdot 1000}$ i.e. $bp = 63.158$ **kW = 3789 kJ/min**

$Q_{cw} := \frac{m_{cw}}{60} \cdot cp_{cw} \cdot \Delta T_{cw}$ i.e. $Q_{cw} = 949.208$ **kJ/min ... heat to cooling water**

$Q_{exh} := 0.15 \cdot Q_{supp}$ i.e. $Q_{exh} = 864$ **kJ/min ... heat to exh. gases**

$Q_{unacc} := Q_{supp} - bp \cdot 60 - Q_{cw} - Q_{exh}$

i.e. $Q_{unacc} = 157.33$ **kJ/min ... heat unaccounted for**

Table:

Details	kJ/min	%
Heat supplied: Q_supp	**5760**	**100**
Heat to bp: Q_bp	3789	65.78
Heat to cooling water: Q_cw	949.21	16.48
Heat to exh. gases: Q_exh	864	15
Heat unaccounted: Q_unacc	157.33	2.73
Total:	5760	100

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A.3 References:

1. *Rayner Joel*, Basic Engineering Thermodynamics, 5th Ed. Addison Wesley Longman Inc. 1996.
2. *R.K. Rajput,* Thermal Engineering, Laxmi Publications, New Delhi, 2001
3. *Domkunndwar et al,* A course in Thermal Engineering, Dhanpat Rai & Co., New Delhi, 2000

Postscript

Dear Student,

Thank you for walking with me up to this point.

I hope that we had an interesting exploration of the subject of 'Thermodynamics', together.

In this series of 10 books, (i.e. 5 on Basic Thermodynamics and 5 on Applied Thermodynamics), we solved problems using 3 softwares: Mathcad, Engineering Equation Solver (EES) and The Expert System for Thermodynamics (TEST). In the beginning of each chapter, for quick reference, we gave a summary of Definitions, Statements and Formulas used. But, our emphasis was on problem solving.

In addition to solving problems on Basic and Applied Thermodynamics, these books will also help you learn the use of these softwares.

I would like to state again: simply reading these books will not be enough. To derive full benefit from these books, using the solutions given here as a guide, *you should solve the problems yourself*. I am sure that the large number of examples showing parametric analysis and graphical presentations, given in these books, will enthuse you to work further.

Finally, I would like to say that I greatly enjoyed solving these problems with these three softwares. And I hope that you will also appreciate their utility in problem solving, parametric analysis and graphical presentation of results.

Good luck!

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