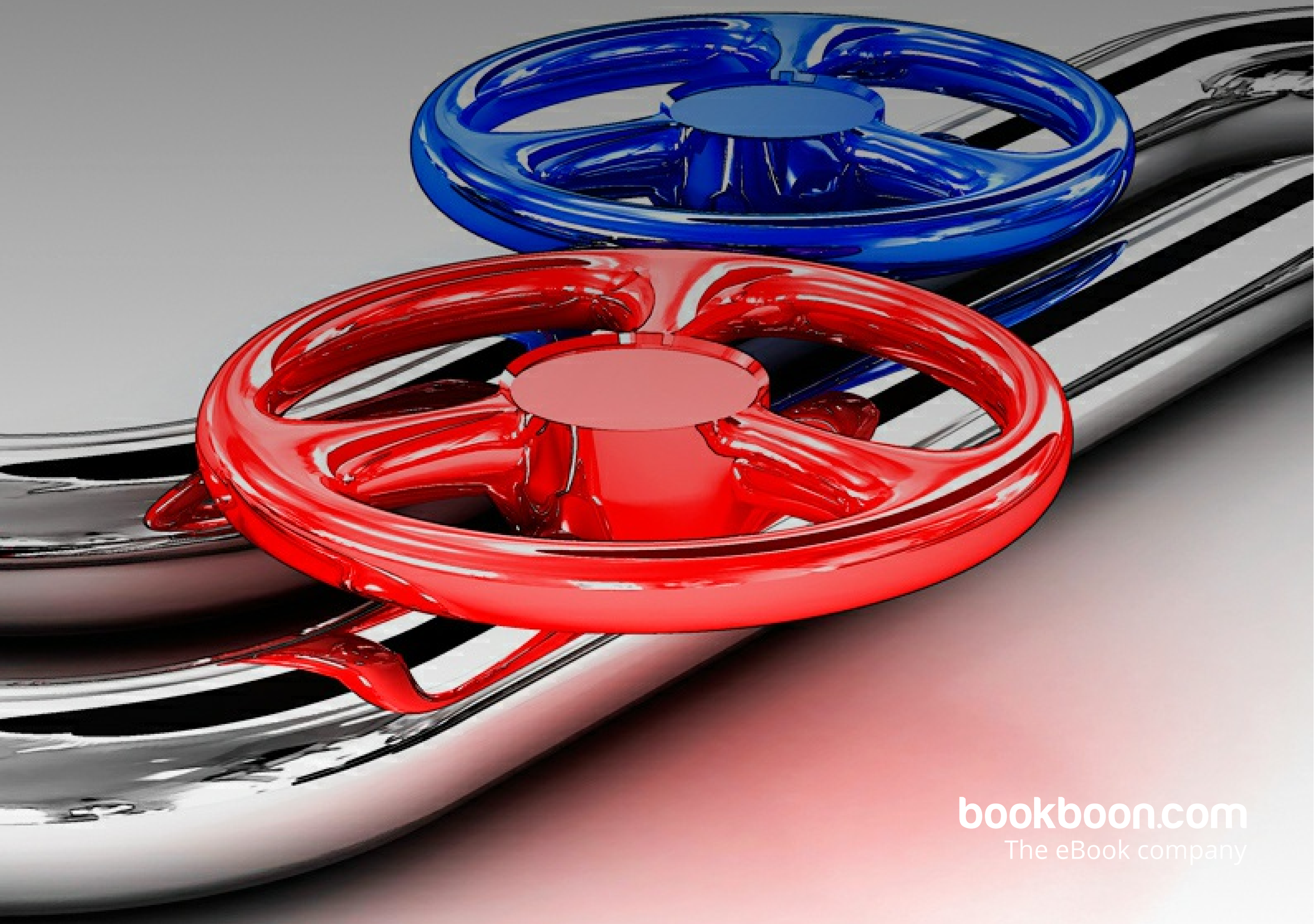
Daniel Micallef

Fundamentals of refrigeration thermodynamics



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Contents

List of Figures

List of Tables

Preface

The aim of the book is to provide first degree engineering students with the foundations on refrigeration engineering. It covers the basic principles in refrigeration and is not intended to provide specialized knowledge on the subject. Students who further their studies on refrigeration shall still find this book as a good reference. Refrigeration courses could be part of a mechanical, chemical or for instance energy engineering courses. The philosophy employed throughout the entire book is to provide a modern approach to the subject. The later chapters provide fundamental knowledge of refrigeration system simulations. This can be considered as a more advanced topic which is usually more useful to the Masters degree student. Nonetheless, the material presented is comprehensive and will give students a good background of modelling refrigeration systems.

The book is structured into eight chapters. Chapters 1–3 provide some background thermodynamic knowledge required for refrigeration systems engineering. Chapters 4 and 5 are the core chapters of the book which focus on the various refrigeration cycles which are most commonly used. Chapter 6 is an introduction to chapters 7 and 8 which are focused on numerical modelling of vapour compression and vapour absorption systems. The software used for the implementation of the mathematical models is MathWorks®MATLAB. The following is an overview of the chapters in the book:

1. Fundamentals of thermodynamics – The basic thermodynamic principles are presented here. The topics covered are sufficient for what will be used throughout the text when analyzing refrigeration systems. Since in most cases, students taking courses in refrigeration have a good thermodynamics background, this chapter is aimed to provide a revision of the salient points in thermodynamics.
2. Heat engines and reversed heat engines – This chapter aims at complementing chapter 1 and exposes the fundamental principles of heat engines and reversed heat engines. Thermodynamic cycles are also introduced which will also be very important to describe refrigeration units.
3. Working fluids – The working fluids and their thermodynamic characteristics are described in this chapter. The properties of refrigerants and environmental issues associated with them are discussed.
4. Vapour compression cycles – The basic vapour compression refrigeration cycle is here presented in detail. More advanced multi-stage cycles are also introduced. The inclusion of a description on heat pumps is inevitable even though the book is only dedicated to refrigeration.
5. Vapour absorption and other cycles – Vapour absorption cycles as well as other, less common refrigeration methods are discussed to give the student a broad overview of refrigeration systems other than vapour compression cycles.
6. Principles of numerical modelling and experimentation – This chapter serves as an introduction to the final chapters of the book. Such topics are becoming ever so important with the widespread use of computer programs to analyze thermodynamic systems.
7. Vapour compression system modelling – In this chapter, mathematical modelling of the vapour compression systems is discussed. The possibility to analyze a large number of test cases is emphasized and the behaviour of these systems with respect to various parameters is considered.
8. Vapour absorption system modelling – This chapter follows the same organization as chapter 7 but this time focussed on vapour absorption cycles.

Acknowledgements

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Last but not least, special thanks go to my family for their love and support throughout the writing of this book.

Daniel Micallef, PhD

1 Fundamentals of thermodynamics

1.1 Systems and energy

1.1.1 Systems

The term system can take various meanings depending on the context. In subjects such as sociology, engineering and information technology, systems have different characteristics. There are however certain common features for all types of systems. A definition which holds for any type of system is the following:

> **Definition**
>
> *A system is a set of interacting elements which operate in a holistic manner to perform a particular function Bac[illegible]00).*

A thermodynamic system involves a collection of matter which is distinct from its surroundings by means of a boundary. Figure 1.1 depicts the system and its separation from the surroundings by means of a boundary. The boundary can either be fixed or moving. Thermodynamic systems have varying functions such as converting heat energy into work or converting work into heat energy. The elements making up such as system are known as sub-systems. In a closed system, no matter is allowed to enter or leave the system and hence no matter is allowed to transfer across the system boundaries (refer to Figure 1.2). On the contrary, open systems allow matter to be transferred across the system boundaries (refer to Figure 1.3).

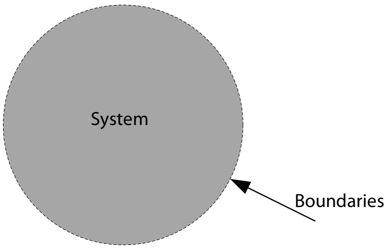


Figure 1.1: System, surroundings and system boundaries.

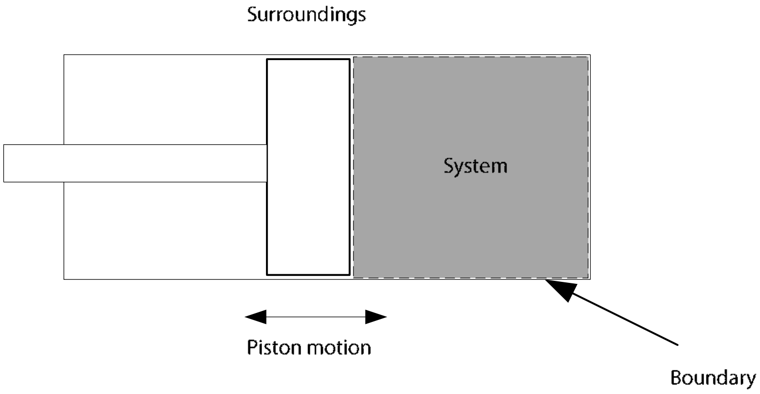


Figure 1.2: Piston work input on the system. System boundaries change dimension depending on the piston motion but no mass enters or leaves the system (closed system).

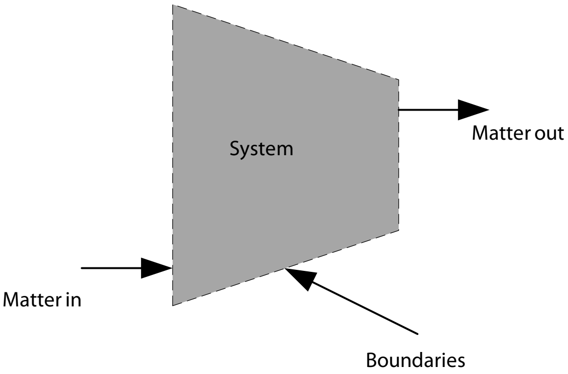


Figure 1.3: Compressor work input. System boundaries remain fixed. Mass enters and leaves the system (open system).

1.1.2 Energy

In its simplest definition, energy is the ability of a system to do work. In thermodynamics, the system may do work on the surroundings or the surroundings may do work on the system. The S.I. unit of energy is the Joule (J) and the unit for the rate of energy transfer (or power) is the Watt (W). In the case of a piston compressing a gas, the surroundings do work on the system (in this case the gas). On the other hand, when the gas expands, work is done by the system on the surroundings. During the compression and expansion processes, energy is converted from one form to another. In doing so, the total energy of the entire system remains unchanged. In other words, energy can neither be created not destroyed. This is a fundamental principle in physics which will be treated further in another section under the first law of thermodynamics. The science of thermodynamics is related to the understanding of how heat, work and other forms of energy within a system interact and change.

The definition of work is the scalar product of a force and a displacement vector at the point of application of the force. Work has the same units as energy (Joules). Using an elemental convention of work the following equation may be written:

$$\delta W = \mathrm{F} \cdot \delta \mathrm{x} \tag{1.1}$$

Where F is the force vector acting on the system and x is the displacement vector. Integrating yields:

$$W = \int_{x_1}^{x_2} F dx \tag{1.2}$$

where the integration is carried out between two points x_1 and x_2. The force and displacement vectors along a path are illustrated in Figure 1.4.

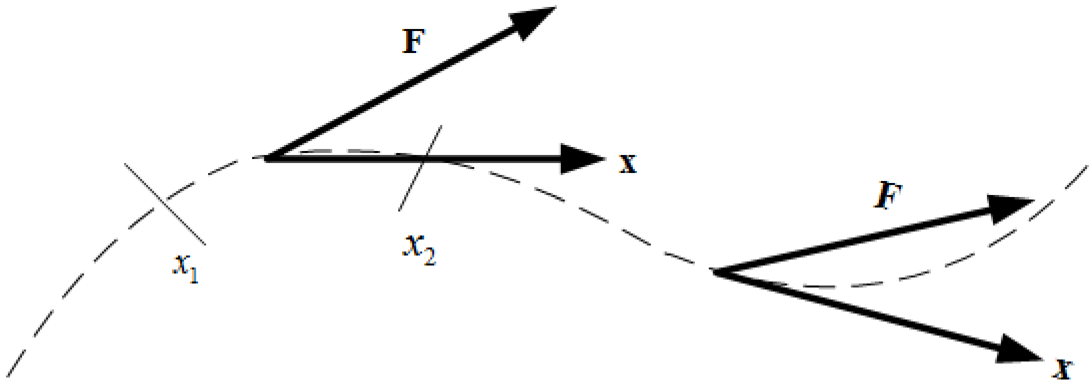


Figure 1.4: Concept of work done between two points.

When the force acts over an area, a pressure results and the work expression of eqn. 1.2 can also be written in terms of a pressure (a negative sign is introduced to show that work is done by the system on the surroundings ($x_2 > x_1$). For a system of constant area, eqn. 1.2 can be re-written as:

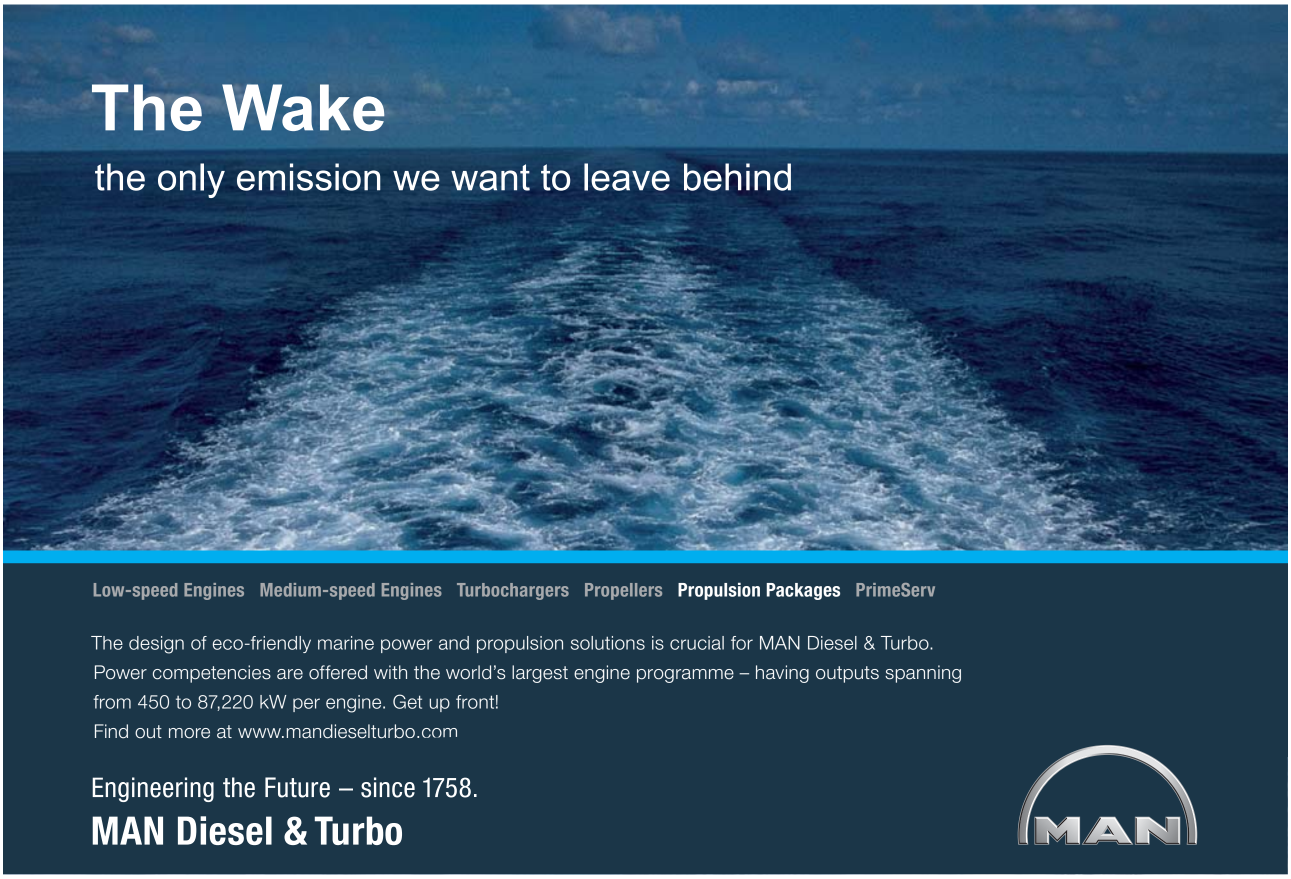
$$W = -\int_{x_1}^{x_2} p \times A dx \tag{1.3}$$

$$W = -\int_{V_1}^{V_2} p dV \tag{1.4}$$

Heat energy is transferred between two bodies as a result of a temperature difference between the bodies. It is not the scope of this textbook to describe the heat transfer mechanisms of conduction, convection and radiation (refer to incropera et al. (2011) for a complete description). In thermodynamics, heat energy plays a fundamental role and there are many situations where heat is either rejected to the surroundings or is input into the system. In other situations, due to the system characteristics such as for instance the use of lagging material around the system, the heat energy transferred in or out of the system can be considered negligibly small. In such cases the system is said to be *adiabatic*.

In the case of open systems were fluid enters or leaves the system (known as the working fluid), kinetic ergies are also important. The kinetic energy per unit time is defined as:

$$\dot{E}_k = \frac{1}{2}\dot{m}c^2 \tag{1.5}$$



where $\dot{m}$ is the mass flow rate and c is the speed of the fluid. The potential energy is defined as:

$$\dot{E}_p = \dot{m}gz \tag{1.6}$$

where g is the acceleration due to gravity and z is the height from some reference location.

1.1.3 The energy convention

Throughout the rest of this book, the energy convention used is that energy output to the surroundings will be denoted as negative while energy input into the system will be denoted as positive. This is clearly depicted in Figure 1.5 It is important to note that this is used only to denote whether energy is being input or output. Energy is a scalar quantity which has only magnitude and does not change with different coordinate systems used.

1.2 Properties of working fluids

The energy transfers in and out of a thermodynamic system are only possible by means of a working fluid. A simple definition is given in Eastop and McConkey (1993) as follows:

Definition

A working fluid is the matter contained within a system boundary.

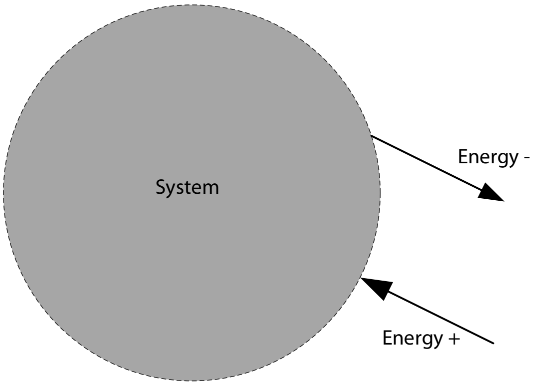


Figure 1.5: Energy convention used throughout the text book.

The transfer of energies across the system boundaries highly depends on the physical properties of the working fluid. Before considering thermodynamic processes, it is essential to understand the most important thermodynamic properties of the working fluid. When at least two thermodynamic properties (three thermodynamic properties in the case of a wet vapour which will be discussed later) are known, the state of the working fluid is said to be known since it is then straightforward to find any other unknown properties. The discussion on the property of *entropy* will be postponed to another section once the *second law of thermodynamics* is introduced.

1.2.1 Specific volume

Definition

The volume occupied by a fluid per unit mass is defined as the specific volume and is denoted by v*.*

The units of specific volume are m^3/kg. The expression for specific volume is therefore:

$$v = \frac{V}{m} = \frac{1}{\rho} \tag{1.7}$$

1.2.2 Pressure

Definition

The pressure experienced by a surface of area A *is defined as the normal force exerted on the surface per unit surface area.*

The units of pressure are either N/m^2 or the *Pascal* (*Pa*). Another commonly used unit in engineering is the *bar* where $1 bar = 1 \times 10^5 Pa$. In fluids, pressure is defined at particular points. At first, this may seem to be counter-intuitive since pressure requires some sort of finite area. The more generic definition of pressure is therefore a limit where the area A approaches zero:

$$p = \lim_{\delta A \to 0} -\frac{\delta(F \cdot n)}{\delta A} = -\frac{d(F \cdot n)}{dA} \tag{1.8}$$

where $F \cdot n$ is the normal force (pointing out of the element) and A represents the area. The negative sign is due to the fact that the normal force is taken to act into the area while the normal vector n is directed out of the area.

When referring to pressures in fluids a clear distinction is made between *absolute* and *gauge* pressure. The absolute pressure is the total pressure as defined in eqn. 1.8. The gauge pressure on the other hand refers to the pressure above (or below) the atmospheric pressure.

1.2.3 Internal energy

Definition

The internal energy of a thermodynamic system is the energy that is intrinsic to the system matter. In principle it is the sum of the total energy of the atoms and molecules making up the working fluid.

The total energy of the atoms and molecules is divided into two primary components being their kinetic and potential energies. The symbol used for internal energy is U and its S.I. units are the Joules. Taking the example of a gas, the kinetic energy would be the associated with the random motion of the molecules while the potential energy would be associated with the electrical static energy.

The specific internal energy is defined as the ratio of internal energy to the mass:

$$u = \frac{U}{m} \tag{1.9}$$

A more detailed treatment of internal energy can be found in Rogers and Mayhew (1962).

1.2.4 Enthalpy

Definition

The sum of the internal energy and work required for a fluid element to enter or exit a system is known as the enthalpy of the fluid.

In mathematical terms:

$$H = U + pV \tag{1.10}$$

The S.I. unit of enthalpy is hence also the Joule. The specific internal energy is the enthalpy per unit mass of fluid:

$$h = \frac{H}{m} \tag{1.11}$$

1.3 Reversibility and irreversibility

Thermodynamic states of systems can easily be represented by means of indicator diagrams where two fluid properties are plotted on an x and y axis. One of the most commonly used is the $p-V$ (pressure-volume) diagram.

Definition

When the thermodynamic system undergoes a change from state A to state B then the system is said to undergo a thermodynamic process.

In order to carry out its function, the system may perform a number of consecutive processes. A *thermodynamic cycle* is defined as follows:

Definition

When the thermodynamic system undergoes a series of processes starting from state A and ending again at state A then the system is said to have performed a thermodynamic cycle.

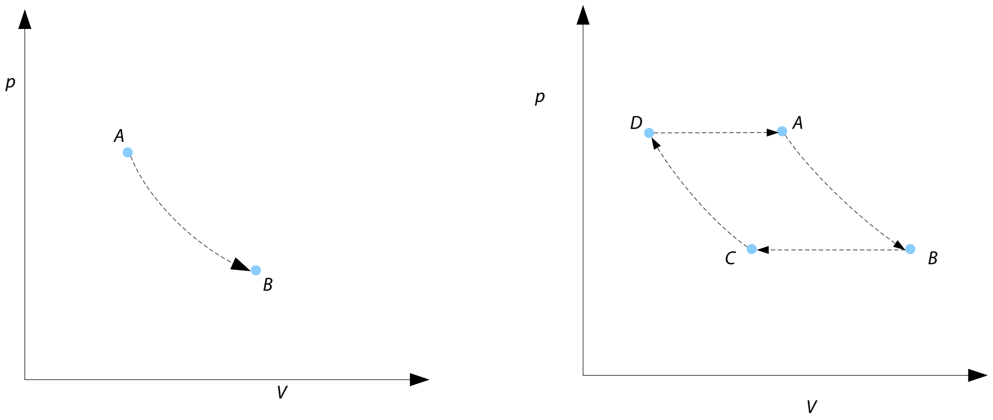
Figure 1.6 shows an example of a thermodynamic process and a cycle on a $p-V$ diagram.

Thermodynamic processes can either be *reversible* or *irreversible*.

Definition

A reversible process from state A to state B is one which can be reversed from state B to state A through the same path since the system is in continuous thermodynamic equilibrium. In an irreversible process, the path taken by the process from state A to state B cannot be reversed from state B back to state A.

In practice, reversible processes are ideal. In theory certain processes can be assumed to be reversible. This assumption is important particularly when establishing the behaviour of such systems. To illustrate further, consider the case of expanding a volume of gas. In the case of a controlled expansion such as that performed by a piston, the intermediate states between the start and finish of the expansion can easily be replicated upon compression if the compression conditions are exactly the same as the expansion (but in the opposite direction). Such a process can be assumed reversible. On the other hand, if the expansion of the gas is performed by means of an opening in the piston, the gas would expand freely to fill in the rest of the container. In such a case the intermediate states between the situation where the gas is contained within a small volume and the state where the gas occupies a larger volume is impossible to replicate in reverse. Such a process is highly irreversible. Reversibility is related to the property of entropy and will be explored in another section. The processes shown by dotted lines in Figure 1.6 are irreversible. For reversible processes, the lines should be sketched as full lines to show that the path can be replicated if the process where to be reversed.



(a) Process (b) Cycle

Figure 1.6: Processes and cycles

1.4 The first law of thermodynamics

The principle of conservation of energy states that energy can neither be created nor destroyed. This is perhaps one of the (if not the) most important laws in physics. The same principle not only applies to energy but also to mass. These laws are the basis of many subjects in the physical sciences and are essential to the study of thermodynamics.

The first law of thermodynamics is a statement of the principle of conservation of energy applied to thermodynamics systems.

Definition

In a complete thermodynamic cycle, the sum of the net work and the net heat transferred from the surroundings to the system should equate to zero.

Mathematically:

$$\sum_C W + \sum_C Q = 0 \tag{1.12}$$

where C denotes that the sum is carried out over the entire thermodynamic cycle.

1.4.1 Non-flow processes

The first law of thermodynamics can also be extended to the case where there is no complete thermodynamic cycle. In a system where there is no flow in or out of the system boundaries (open system), the sum of elemental changes in work and heat energy will result in a change in the internal energy of the system. for this reason, the non-flow energy equation may be written as follows:

$$dW + dQ = dU \tag{1.13}$$

Integrating between any two states yields:

$$\Delta W + \Delta Q = \Delta U \tag{1.14}$$

1.4.2 Steady flow processes

In the case where matter flows in and out of the system boundaries, the energies associated with that matter need to be included when equating the energy inputs and energy outputs. Figure 1.7 depicts all the energy inputs and energy outputs for the system. For the purpose of this chapter, only the steady flow case will be treated. This means that all time derivative terms are equal to zero.

From Figure 1.7, the energy inputs and outputs associated with the inlet and outlet are as follows:

1) pV associated with the work done to push an element of fluid into and out of the system to overcome the opposing pressure.
2) The internal energy of the fluid U
3) The kinetic energy of the fluid $1/2mc^2$
4) The potential energy of the fluid mgz

In addition, work and heat are input into the system.

[illegible]ass conservation equation for the inlet and outlet:

$$\dot{m}_1 = \dot{m}_2 \tag{1.15}$$

or for N inputs/outputs of mass flow rate

$$\sum_{i=1}^{N} m_i = 0 \tag{1.16}$$

Writing the energy inputs on the left hand side and the energy outputs on the right hand side:

$$U_1 + p_1 V_1 + mgz_1 + \frac{1}{2} mc_1^2 + Q + W = U_2 + p_2 V_2 + mgz_2 + \frac{1}{2} mc_2^2 \tag{1.17}$$

or in terms of power:

$$\dot{m}u_1 + p_1 \dot{V}_1 + \dot{m}gz_1 + \frac{1}{2}\dot{m}c_1^2 + \dot{Q} + \dot{W} = \dot{m}u_2 + p_2 \dot{V}_2 + \dot{m}gz_2 + \frac{1}{2}\dot{m}c_2^2 \tag{1.18}$$

Using eqn. 1.10, the enthalpy term can be introduced:

$$\dot{m}h_1 + \dot{m}gz_1 + \frac{1}{2}\dot{m}c_1^2 + \dot{Q} + \dot{W} = \dot{m}h_2 + \dot{m}gz_2 + \frac{1}{2}\dot{m}c_2^2$$

$$\dot{m}\left(h_1 + gz_1 + \frac{1}{2}c_1^2\right) + \dot{Q} + \dot{W} = \dot{m}\left(h_2 + gz_2 + \frac{1}{2}c_2^2\right) \tag{1.19}$$

in general, for N inlets/outlets:

$$\sum_{i=1}^{N}\left[\dot{m}_i\left(h_i + gz_i + \frac{1}{2}c_i^2\right)\right] + \dot{Q} + \dot{W} = 0 \tag{1.20}$$

This is known as the steady flow energy equation.

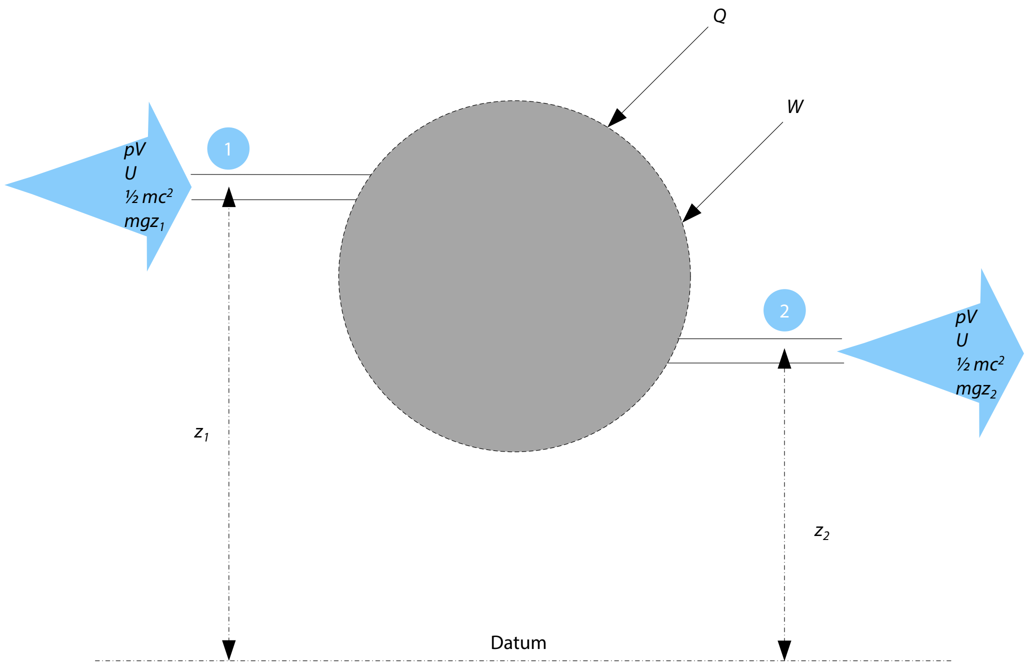


Figure 1.7: Steady flow thermodynamic system showing energy inputs and energy outputs at the inlet (1) and outlet (2).

1.5 The second law of thermodynamics

While the first law of thermodynamics describes the amount of energy which is input and/or output during a thermodynamic system in relation to its surroundings, the second law of thermodynamics describes the natural direction of energy. Heat energy flows from a high to a low temperature. For this to be reversed, work input must be performed by the surroundings on the system. A definition of the second law of thermodynamics can be as follows:

Definition

If heat energy is extracted by a system from a high temperature reservoir, that energy cannot be entirely converted into work and some heat energy must be rejected to a lower temperature reservoir.

The second law of thermodynamics also hints towards the *quality*. In this context, *quality* refers to how easily energy can be converted. It is impossible to convert a certain amount of heat energy into work. On the other hand it is possible to convert a certain amount of work entirely into heat. The definition is graphically represented in Figure 1.8 where heat is extracted from a high temperature reservoir in order to produce work. Heat energy is shown to be transferred to the low temperature reservoir. The second law of thermodynamics states that $Q_2 > 0$. If a system efficiency parameter is defined as the work output divided by the heat energy input, then

$$\eta = \frac{W}{Q_1} \times 100\% \tag{1.21}$$

From the first law of thermodynamics, $W = Q_1 - Q_2$ and hence $Q_1 > Q_2$,

$$\eta = \frac{Q_1 - Q_2}{Q_1} \times 100\% = \left(1 - \frac{Q_2}{Q_1}\right) \times 100\% \tag{1.22}$$

This means that the second law states that no system can be 100% efficient:

$$\eta < 100\% \tag{1.23}$$

1.5.1 Entropy

The *thermodynamic scale* is defined on the basis of the ratio of the heat extracted and heat rejected by a thermodynamic system from a high to a low temperature reservoir. If the temperature of the high temperature reservoir is T_1 and that of the low temperature reservoir is T_2, $T_1 = T_2$ when $Q_1 = Q_2$ and,

$$T_2 = T_1 \frac{Q_2}{Q_1} \tag{1.24}$$

or

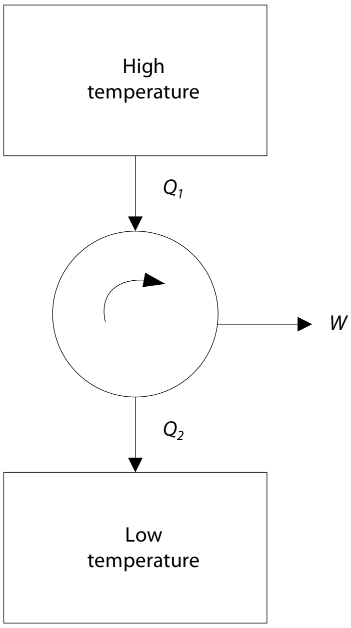
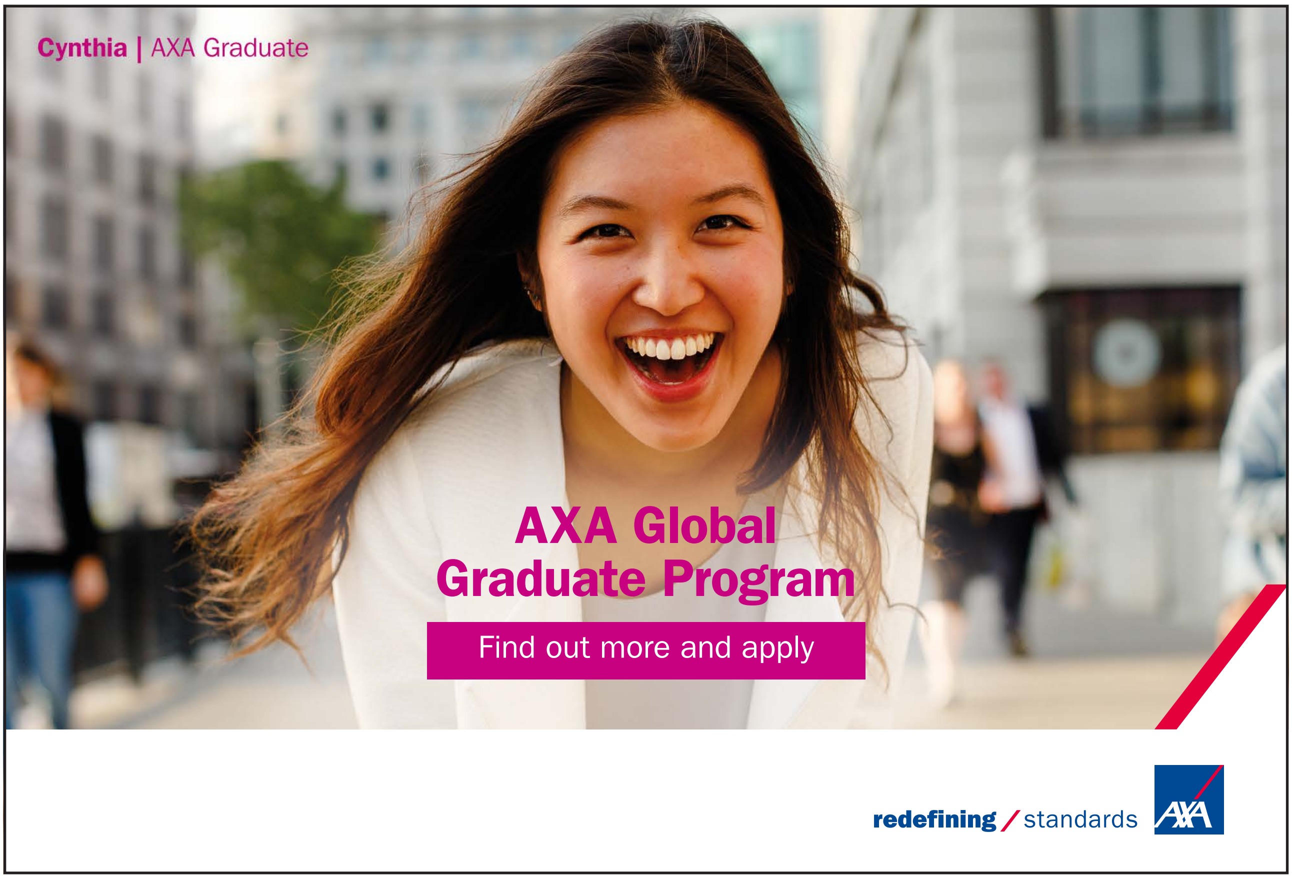


Figure 1.8: System extracting heat energy and producing work on the surroundings while rejecting heat to a low temperature reservoir.

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \tag{1.25}$$



This thermodynamic scale defines the absolute zero and since Q_2 can never be zero by the second law of thermodynamics, the absolute zero is a theoretical limit which can never be reached.

Suppose that a thermodynamic system operates between a large number of reservoirs as shown in Figure 1.9. The temperature reservoirs are at $T_{1i} > T_{2i'} > T_3$. Heat energy is transferred from T_3 to T_{1i} via a series of reversible heat pumps (represented by a single system) and heat energy is transferred from $T_{2i'}$ to T_3 via a series of reversible heat engines (again represented by a single system). The subscripts i and i' are labels for the constant temperature reservoirs. All heat transfers are labelled by Q. The first subscript denotes from or to which reservoir from 1, 2 or 3 the heat is being transferred. The second subscript denotes the heat pump number to or from which the heat energy is transferred.

From the second law of thermodynamics the net work should be input into the whole system for heat to be transferred from a low to a high temperature reservoir. Alternatively the net work may also be zero. Hence,

$$\Sigma W_X + \Sigma W + \Sigma W_Y \geq 0$$

$$-\Sigma Q_X - \Sigma Q - \Sigma Q_Y \geq 0$$

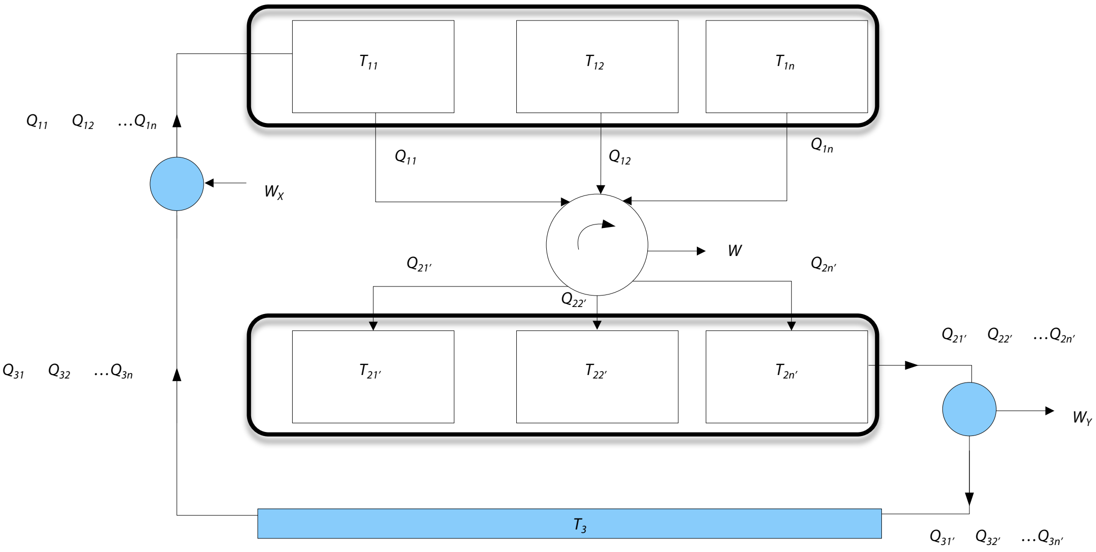


Figure 1.9: System operating between a finite number of reservoirs.

$$\begin{aligned}
\sum W_X + \sum W + \sum W_Y &\leq 0 \\
-\sum Q_X - \sum Q - \sum Q_Y &\geq 0 \\
\sum Q_X + \sum Q + \sum Q_Y &\leq 0 \\
\sum_{i=1}^{n}(Q_{3i} - Q_{1i}) + \sum_{i=1}^{n} Q_{1i} - \sum_{i'=1}^{n'} Q_{2i'} + \sum_{i'=1}^{n'}(Q_{2i'} - Q_{3i'}) &\leq 0 \\
\sum_{i=1}^{n} Q_{3i} - \sum_{i'=1}^{n'} Q_{3i'} &\leq 0 \qquad (1.26)
\end{aligned}$$

Using 1.24, similar relationships can be written for all reservoirs:

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_3}{T_3} \tag{1.27}$$

Inserting in 1.26 we have,

$$\sum_{i=1}^{n} T_{3i} \frac{Q_{1i}}{T_{1i}} - \sum_{i'=1}^{n'} T_{3i} \frac{Q_{2i'}}{T_{2i'}} \leq 0$$

$$\sum_{i=1}^{n} \frac{Q_{1i}}{T_{1i}} - \sum_{i'=1}^{n'} \frac{Q_{2i'}}{T_{2i'}} \leq 0 \tag{1.28}$$

Summing over the entire cycle:

$$\sum_{C} \frac{Q}{T} \leq 0 \tag{1.29}$$

For a continuous distribution of constant temperature reservoirs, this may be written in integral form:

$$\oint \frac{dQ}{T} \leq 0 \tag{1.30}$$

This is known as the *Cluasius inequality*. Further details on the corollaries resulting from the second law of thermodynamics can be found in Rogers and Mayhew (1962) and also in which the authors give a comprehensive summary of the original theories of Clausius and Hirst (1867). The quantity $\frac{dQ}{T}$ is a thermodynamic property and is known as the *entropy* and has units of Joules/Kelvin (J/K):

$$dS = \frac{dQ}{T} \tag{1.31}$$

If we consider the sum of such quantities between only two states we have:

$$\Delta S = S_2 - S_1 = \int_1^2 \frac{dQ}{T} \tag{1.32}$$

This equation only applies for a reversible process. The specific entropy is hereby defined as the entropy per unit mass:

$$s = \frac{S}{m} \tag{1.33}$$

The scope of this section of the book is to give the reader the necessary understanding for refrigeration thermodynamics. Therefore, further mathematical details will not be considered. Readers who wish to delve deeper into the details of the second law of thermodynamics should consult Fermi (1956) or Morris (2008).

1.6 Thermodynamic processes

1.6.1 Isobaric process

In an isobaric process, there is no change in pressure between one state and another ($\Delta p = 0$). The process is shown on a p-V diagram in Figure 1.10.

From eqn. 1.4, since pressure is constant, we have

$$W = -p(V_2 - V_1) \tag{1.34}$$

For a reversible process, the work done during process 1 to 2 is equal to the area under the line shown in Figure 1.10.

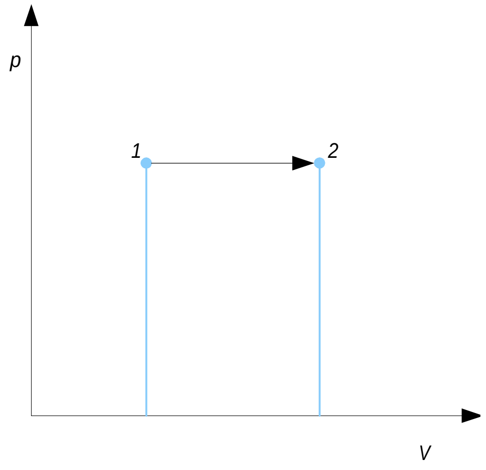


Figure 1.10: Isobaric process on a p-V diagram.

From the first law of thermodynamics,

$$Q - p(V_2 - V_1) = U_2 - U_1 \tag{1.35}$$

1.6.2 Constant volume process

In a constant volume process as shown on the p-V diagram of Figure 1.11, the volume remains the same throughout the process. During the process, the pressure and temperature change. An example is the heating of a closed container filled with gas.

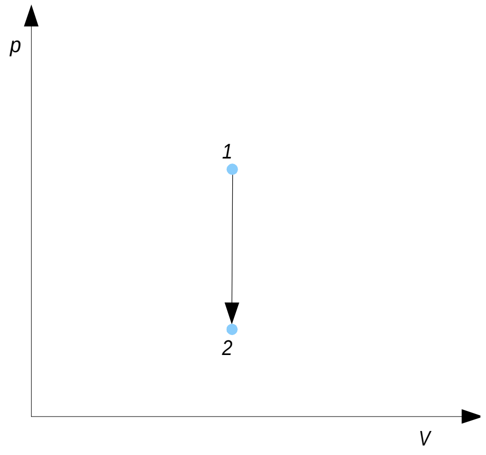


Figure 1.11: Constant volume process on a p-V diagram.

From eqn. 1.4 it can easily be deduced that there is no work done:

$$W = 0 \tag{1.36}$$

$$Q = U_2 - U_1 \tag{1.37}$$

1.6.3 Polytropic process

In a polytropic process, the process follows the follows the following equation:

$$\boxed{pV^n = K} \tag{1.38}$$

where n is known as the polytropic index and K is a constant. By definition, the polytropic process is a reversible process. The process is also represented in 1.12. As indicated in the figure, the two limits as $n \to 0$ and $n \to \infty$ correspond to an isobaric and a constant volume process respectively. Therefore the previous processes are special cases of a polytropic processes. For the two states shown in Figure 1.12 we have:

$$p_1V_1^n = p_2V_2^n \tag{1.39}$$

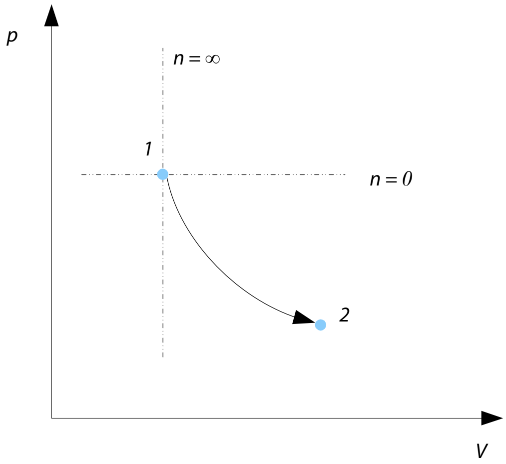


Figure 1.12: Polytropic process on a p-V diagram.

The work done can be estimated from eqn. 1.4 and using eqn. 1.39:

$$W = -\int_{V_1}^{V_2} \frac{K}{V^n} dV$$

$$W = -K\int_{V_1}^{V_2} V^{-n} dV$$

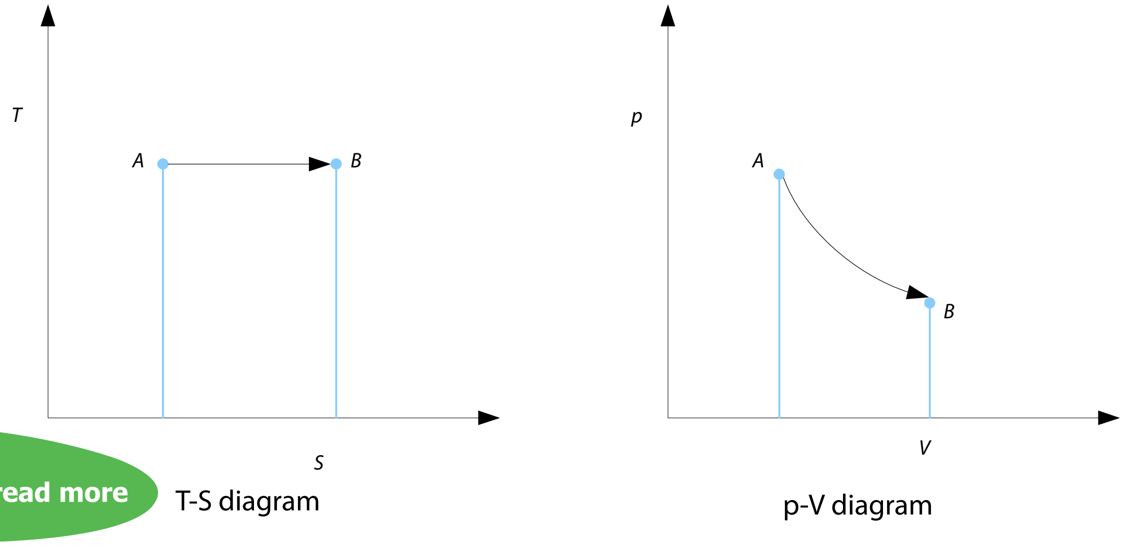
$$W = \frac{K(V_2^{1-n} - V_1^{1-n})}{n-1} \tag{1.40}$$

After some mathematical manipulation, this can be reduced to

$$\boxed{W = \frac{p_2V_2 - p_1V_1}{n-1}} \tag{1.41}$$

1.6.4 Isothermal process

The isothermal process can be visualized using a temperature-entropy (T-S) diagram since the temperature remains constant throughout the process. This is shown in Figure 1.13a. On the other hand, Figure 1.13b shows the process on a p-V diagram. On such a diagram, the process line is located on an isothermal line (a line on which temperature does not vary).



T-S diagram

p-V diagram

Figure 1.13: Isothermal process shown on a T-S and p-V diagram

Using eqn. 1.32, it can be easily shown that the area under the process line in a T-S diagram is equal to the heat energy:

$$Q = \int TdS \tag{1.42}$$

It is important to note that this only holds for a reversible process. For an isothermal case T = constant and hence,

$$Q = T(S_2 - S_1) \tag{1.43}$$

1.6.5 Adiabatic and isentropic process

An adiabatic process is one where no heat energy is transferred to or from the system. Such a process may be both reversible or irreversable. From the second law of thermodynamics, for a reversible process:

$$dQ = TdS = 0 \tag{1.44}$$

This means that the entropy for a reversible adiabatic process remains unchanged $dS = 0$. A reversible adiabatic process is therefore also an isentropic process (the entropy remains unchanged). The adiabatic reversible (and hence isentropic) process is depicted on a T-S and p-V diagram in 1.14a and 1.14b respectively

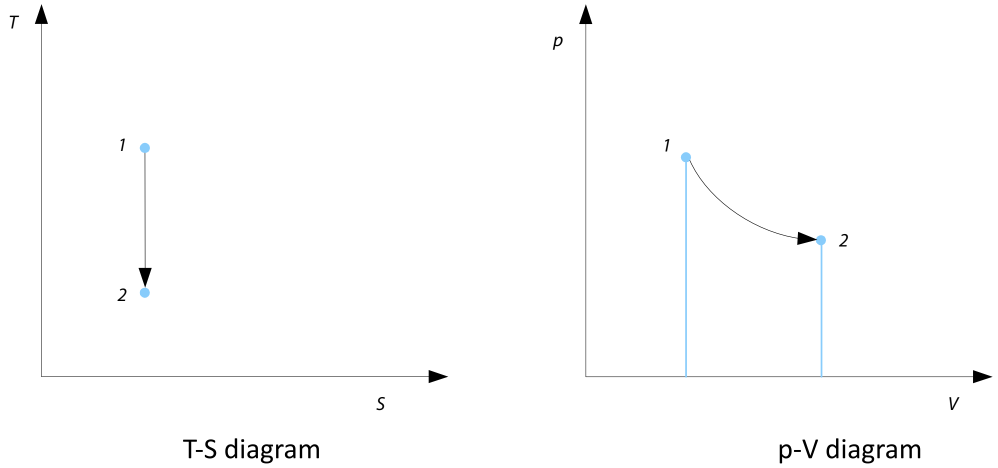


Figure 1.14: Isentropic process shown on a T-S and p-V diagram

For an irreversible adiabatic process, for no heat transfer, the entropy must increase (refer to eqn. 1.30):

$$dQ < TdS$$

$$0 < TdS$$

$$0 < dS \tag{1.45}$$

From the first law of thermodynamics, some useful relationships may be introduced:

$$\delta W = \delta U$$

$$-\int_1^2 pdV = U_2 - U_1 \tag{1.46}$$

In using the differential form of the non-flow version of the first law of thermodynamics we may also write (in terms of specific or per unit mass quantities):

$$dQ + dW = du$$

$$0 = du + pdv \tag{1.47}$$

Using eqn. 1.10 we may write:

$$h = u + pv$$

$$dh = du + pdv + vdp \tag{1.48}$$

$$dh = vdp \tag{1.49}$$

1.7 Conclusions

Student checklist

1. Understand the meaning of a thermodynamic system and the concept of energy transfer to, from and within a thermodynamic system.
2. The convention of energy flow in this textbook should be understood as an input when positive and an output when negative.
3. The quantities of specific volume, pressure, internal energy and enthalpy should be well noted as these will be used in the next chapters.
4. The term *specific* is used to denote per unit mass quantities of volume, internal energy and enthalpy
5. The student is to understand the difference between reversible and irreversible process and how these are represented on a thermodynamic diagram of quantities.
6. The first law of thermodynamics is concerned with the conservation of energy principle applied to thermodynamic systems. This has been described in the context of non-flow and steady flow processes.
7. The second law of thermodynamics is on the other hand concerned with the *quality* of energy and is therefore a statement on the efficiency of thermodynamic systems.
8. The existence of a thermodynamic quantity known as *entropy* is a direct consequence of the second law of thermodynamics.
9. Various practical processes are carried out at constant volume, pressure, temperature or following the relationship $pV^n = K$. On the other hand, a process may be carried out with no heat transfer which is called an adiabatic process.
10. A reversible adiabatic process is also called an *isentropic* process because the entropy change is zero.
11. For an irreversible adiabatic process, the entropy will always increase

2 Heat engines and reversed heat engines

2.1 Heat engines

Definition

A heat engine is a system which converts heat energy from a high temperature source into useful work on the surroundings while at the same time rejecting heat to a low temperature reservoir in accordance with the second law of thermodynamics.

In Figure 1.8 reference was already made to the heat engine. Heat engines operate on thermodynamic cycles which satisfy the first law of thermodynamics. The high temperature source can be for instance obtained from the burning of fossil fuels. The heat engine which produces work could be composed of valves, pistons, heat exchangers and so on. The low temperature reservoir might be the atmosphere or f le a pond of water at a uniform temperature.

It is not the aim of this book to discuss further heat engine cycles but the reader is advised to read various thermodynamic text books such as Eastop and McConkey (1993) which give a comprehensive insight on various heat engines such as the Otto cycle and the Brayton cycle.

The efficiency of a heat engine has already been defined by eqn. 1.21 in chapter 1 and it was also emphasized that the second law of thermodynamics implies that $\eta < 100\%$. It can also be proven that the best possible thermodynamic efficiency of a heat engine is that of an *ideal cycle* known as the *Carnot cycle*.

It is important at this stage to underline what is really meant by the word *ideal* in thermodynamic terms. The thermal models and analysis which will be presented in this textbook for various types of refrigeration systems will be considered ideal in order to simplify the analysis involved. The efficiencies of idealized cycles, apart from being limited from the results of the second law of thermodynamics, are also limited due to their differences from the Carnot cycle. In practice however, if such cycles are built and operated in a real environment, their efficiency will also be limited due to the irreversibility of the cycle. Çengel and Turner (2005) provide further detail on this topic and should be helpful to the interested reader.

A T-S diagram of the Carnot cycle for a heat engine is shown in Figure 2.1. The processes involved in a Carnot cycle require a heat supply at a constant temperature (1-2), an isentropic expansion (2-3), heat rejection at constant temperature (3-4) and an isentropic compression (4-1). All processes are represented by full lines and hence are reversible processes. In practice, such a system is particularly difficult to build due to the need for heat to be transferred at a constant temperature while maintaining the process reversible. This can be approximately satisfied if the heat transfer area is made as large as possible and the heat transfer is carried out very slowly. Both of these methods can be either prohibitively expensive or even impossible to carry out.

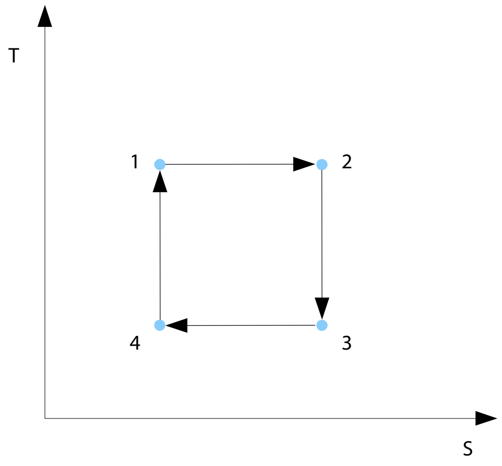


Figure 2.1: Carnot cycle T-S diagram for a heat engine.

For engineering purposes, the Carnot cycle would seem at first to be of purely academic interest. It is nonetheless worthwhile to have a benchmark on which to base heat engine efficiency. Seen from this point of view, the Carnot cycle becomes a handy reference on which engineers may base design decisions or perform practical checks on their calculations. The efficiency of the Carnot cycle can be found from eqn. 1.22. The heat supplied and heat rejected can be found from the areas under processes 1-2 and 3-4 respectively. Hence:

$$Q_1 = T_1 \ (S_2 - S_1) \tag{2.1}$$

and the heat rejected is equal to

$$Q_2 = T_3 \ (S_3 - S_4) \tag{2.2}$$

where

$$S_2 - S_1 = S_3 - S_4 \tag{2.3}$$

implying that

$$Q_2 = T_3 \ (S_2 - S_1) \tag{2.4}$$

From eqn. 1.22:

$$\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100\%$$

$$\eta = \left(1 - \frac{T_3 \ (S_1 - S_2)}{T_1 \ (S_1 - S_2)}\right) \times 100\%$$

$$\eta = \left(1 - \frac{T_3}{T_1}\right) \times 100\% \tag{2.5}$$

Eqn. 2.5 states that the efficiency of a Carnot cycle increases, as the difference between the high temperature and low temperature reservoirs increases. This is another important message to the practising engineer.

2.2 Reversed heat engines

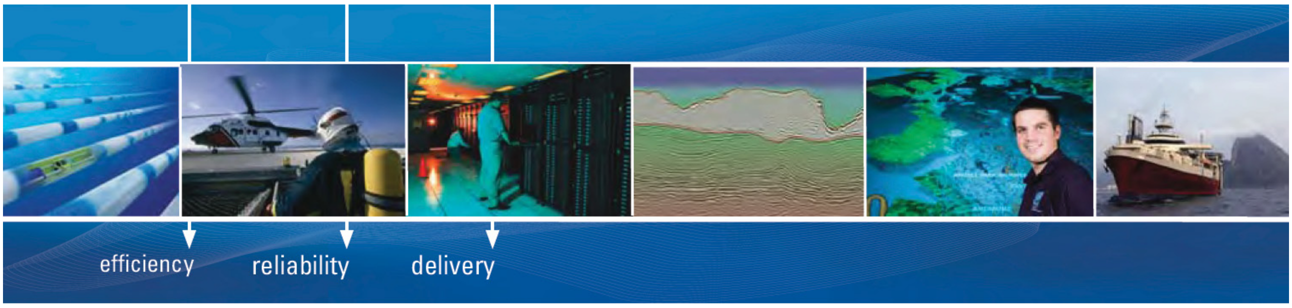
We have already seen that heat energy has a natural tendency to flow from the hot to the cold body. A heat engine will enable the heat energy from a high temperature reservoir to be partially converted into useful work and in the process, rejecting heat to the low temperature reservoir.

Definition

A reversed heat engine enables heat to be transferred against its natural gradient. For this to happen, energy must be input since otherwise the second law of thermodynamics would be violated.

The reversed heat engine is shown in Figure 2.2 where it is shown that for heat energy Q_1 to be transferred from the low temperature reservoir, work W needs to be supplied to the system. Heat energy Q_2 is then rejected to the high temperature reservoir.

Whereas the quantity of interest in a heat engine is the work (which is considered to be a high grade form of energy since it can entirely be converted into heat), in the case of a reversed heat engine there are two quantities of interest. Depending on the application of the system the quantity of interest might be the heat extracted from the low temperature reservoir (to provide a cooling effect). On the other hand, the heat energy which is transferred to the high temperature reservoir (to provide heating) might be required. Although in a reversed heat engine cycle there are two quantities of particular interest, in most applications it is only one of these which is sought after. If heat needs to be extracted from the low temperature reservoir for refrigeration purposes the engine would be called a refrigerator. If heat needs to be supplied to a high temperature reservoir then the engine is known as a heat pump. The purpose of this book is refrigerator operation. Some consideration will also however be given to the heat pump since as described here a distinction between refrigerators and heat pumps is just based on the application required.



In principle, some net work might also be extracted from a reversed heat engine. Compared to the other quantities this is very small and in most situations no attempt is made to extract this minute amount of energy. The principle of operation of a reversed cycle is shown in Figure 2.3. Heat Q_1 is transferred to the evaporator. The working fluid is then compressed to a high pressure and transferred to the condenser where heat energy Q_2 is rejected. Work can be extracted from the turbine and the pressure drops back to the evaporator pressure.

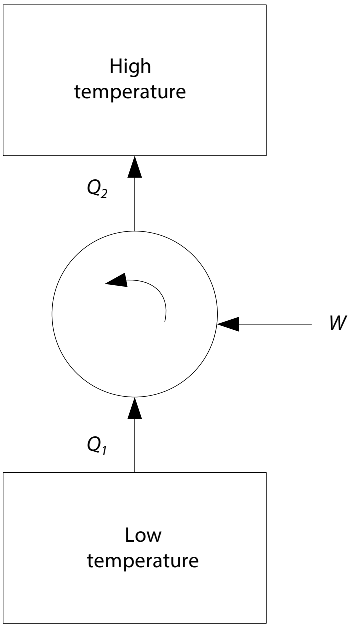


Figure 2.2: The reversed heat engine.

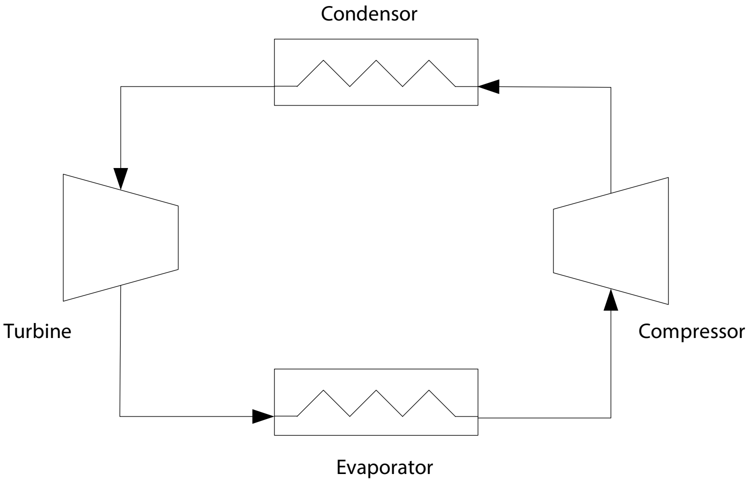


Figure 2.3: Reversed heat engine components.

In reversed heat engines we use the term Coefficient of Performance (COP) in order to have a *measure* of efficiency. The COP is defined as follows:

$$COP = \frac{Q}{\sum W} \tag{2.6}$$

For a refrigerator, the quantity of interest is $Q1$ and hence eqn. 2.6 becomes:

$$\boxed{COP_{ref} = \frac{Q_1}{\sum W}} \tag{2.7}$$

For a heat pump, the quantity of interest is $Q2$ and hence eqn. 2.6 becomes:

$$\boxed{COP_{hp} = -\frac{Q_2}{\sum W}} \tag{2.8}$$

A useful relationship between the coefficient of performance of a refrigerator and a heat pump can be derived from the first law of thermodynamics. Using eqn. 1.12:

$$\begin{gathered} Q_1 + W - Q_2 = 0 \\ \frac{Q_1}{W} + 1 - \frac{Q_2}{W} = 0 \end{gathered} \tag{2.9}$$

Thus using eqns. 2.7 and 2.8,

$$\boxed{COP_{hp} = COP_{ref} + 1} \tag{2.10}$$

This means that the COP of heat pumps is always higher than that of a refrigerator by a value of unity. This relationship is applicable in all cases since it is derived from the first law of thermodynamics. Also, COP_{hp} is always more than 1.

2.3 Carnot cycle for a reversed heat engine

As with the case of heat engines, a Carnot cycle for a reversed heat engine is the most efficient cycle possible. This means that the maximum possible COP for a reversed heat engine will always be less than this theoretical maximum, both for refrigerators as well as heat pumps. Again, such a system is not practical to build due to the reasons mentioned for the heat engine. However it may still be used as a benchmark or reference in the design of more practical systems.

The T-S diagram of the reversed Carnot cycle is shown in Figure 2.4. During process 1-2 the working fluid is compressed from a low to a high pressure and temperature. Heat is then rejected at a constant temperature in process 2-3. The pressure and temperature drops in process 3-4 where some work can be extracted. The fluid is then heated again at constant temperature during 4-1.

The heat input into the system is given by:

$$Q_1 = T_1 (S_2 - S_1) \tag{2.11}$$

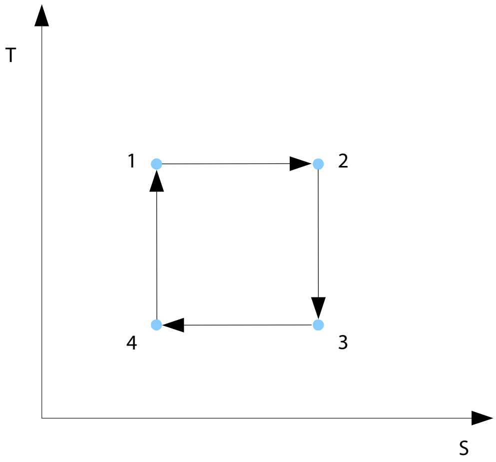


Figure 2.4: Carnot cycle T-S diagram for a reversed heat engine.

ected is given by:

$$Q_2 = T_3(S_3 - S_4) \tag{2.12}$$

where

$$S_2 - S_1 = S_3 - S_4 \tag{2.13}$$

which gives

$$Q_2 = T_3 (S_2 - S_1) \tag{2.14}$$

For the refrigerator, eqn. 2.6 can be simplified to

$$COP_{ref} = \frac{Q_1}{-(Q_1 - Q_2)}$$

$$COP_{ref} = \frac{T_1(S_2 - S_1)}{-(T_1(S_2 - S_1) - T_3(S_2 - S_1))} \tag{2.15}$$

Upon simplifying,

$$COP_{ref} = \frac{T_1}{(T_3 - T_1)} \tag{2.16}$$

, the derivation of the COP is similar:

$$COP_{ref} = \frac{Q_2}{-(Q_1 - Q_2)}$$

$$COP_{ref} = \frac{T_3(S_2 - S_1)}{-(T_1(S_2 - S_1) - T_3(S_2 - S_1))} \tag{2.17}$$

Simplifying

$$\boxed{COP_{hp} = \frac{T_3}{T_3 - T_1}} \tag{2.18}$$

As can be seen both definitions of the COP for a Carnot cycle are dependent on the low and high temperature reservoirs. This was also the case for a heat engine.

Example

Example 1. *A reversed Carnot cycle operates between temperatures of 5°C and 60°C. Calculate the refrigeration COP and the heat rejected in the condenser for a work input of 300kJ/kg.*

Answer.

The refrigeration COP can be found using eqn. :

$$COP_{ref} = \frac{T_1}{T_3 - T_1}$$

Note that the temperature in the numerator must be in Kelvin. Since the denominator involves a temperature difference this is not really required but would make the calculation clearer. Thus:

$$T_1 = 5 + 273 = 278K$$

$$T_1 = 60 + 273 = 333K$$

$$COP_{ref} = \frac{278}{333 - 278} = 5.05$$

Using eqn. 2.10, the heat pump COP is given by

$$COP_{hp} = 1 + COP_{ref} = 1 + 5.05 = 6.05$$

and using 2.8

$$Q_2 = -COP_{hp}.\sum W$$

$$Q_2 = -6.05 \times 300 = -1815kJ/kg$$

2.4 Conclusions

Student checklist

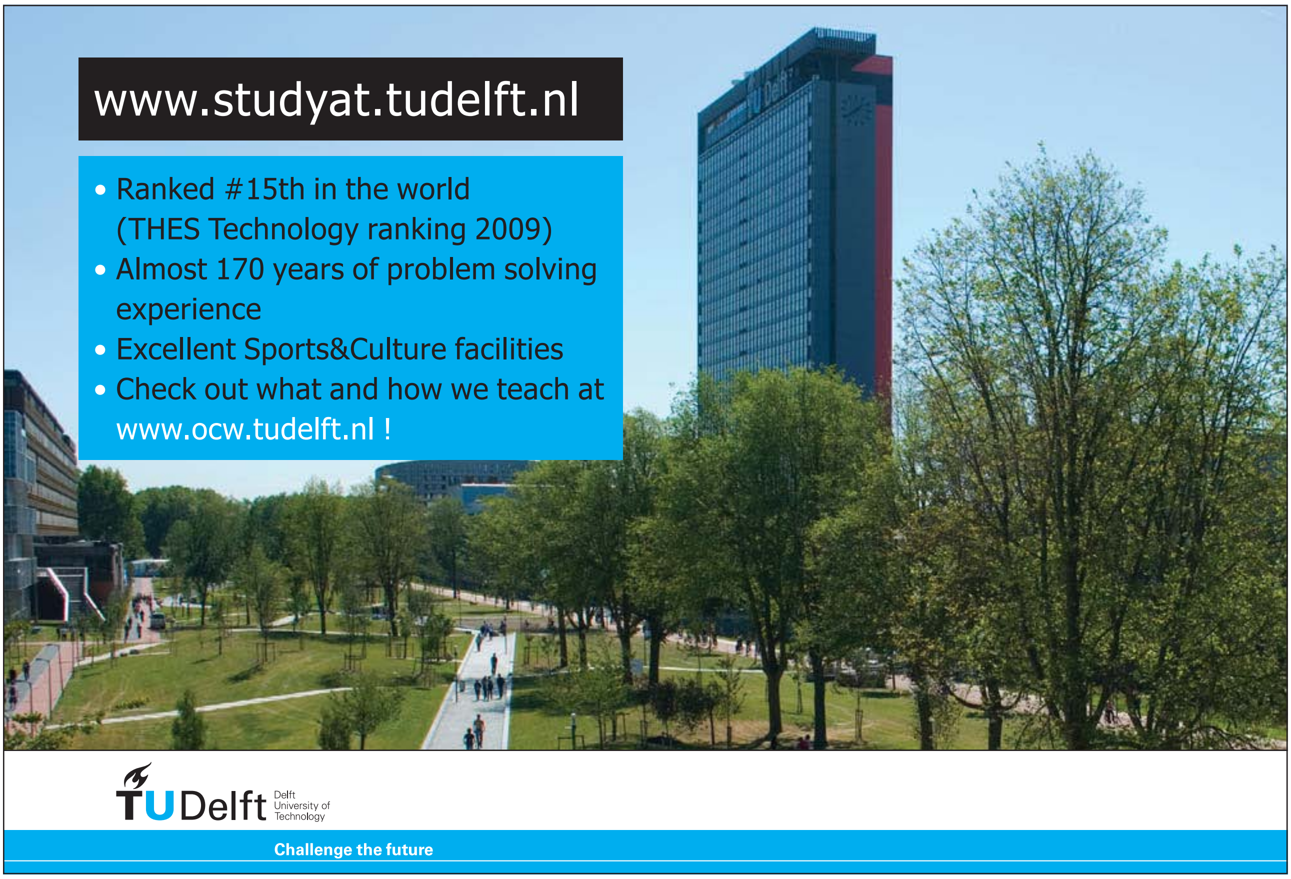
1. The heat engine concept should be fully grasped.
2. Understand what is meant by a system idealisation.
3. The Carnot cycle for a heat engine has been discussed. Its usefulness to engineering practice should be appreciated.
4. The T-S diagram for the Carnot cycle of a heat engine has been described. The student is to understand how the heat transfers may be quantified and should be able to apply the first law of thermodynamics to derive the efficiency of a Carnot cycle.
5. The principles of a reversed heat engine should be understood along with the refrigerator and heat pump modes of operation.
6. The COP of a refrigerator and a heat pump have been defined and should be well familiar to the student.
7. The COP of a heat pump is always a value of unity higher than that of a refrigerator.
8. The Carnot cycle for a reversed heat engine has been presented. The COPs for both refrigerator and heat pump operation have been derived and are seen to be entirely dependent on the temperatures of the low and high temperature reservoirs.

3 Working fluids

3.1 Vapours

Thermodynamic systems operate on working fluids which can either be in the liquid or gaseous state. Consider a liquid being heated at a constant pressure. Once the liquid reaches a certain temperature, boiling starts. This is usually observed as bubbles forming in the bulk of the liquid. This process continues until complete vaporization of the liquid occurs. It can be observed that for boiling to start, the pressure of the liquid must be increased with increasing specific volume. On the other hand, for complete vaporisation, the higher the specific volume, the lower the pressure required. The condition when boiling starts and full vaporization occurs can be drawn by means of a line on a p-v diagram. The start of boiling line is commonly known as the saturated liquid line. All along such a line the liquid just starts boiling. On the other hand, the line showing complete vaporization is known as the saturated vapour line. At higher pressures, these two lines intersect and form what is known as the *critical point*. At such a point, there is no real distinction between a liquid and a vapour. This is ch terized by:

$$\frac{\partial p}{\partial v}=\frac{\partial^2 p}{\partial v^2}=0 \tag{3.1}$$



This holds true at a unique value of temperature which depends on the fluid. The saturated liquid and saturated vapour lines as well as the critical point are depicted in Figure 3.1.

To the left of the saturated liquid line, the fluid is a pure liquid. To the right of the saturated vapour line, the fluid is a known as a *dry* vapour and no liquid phase exists.

Definition

Below the curve, the fluid is a mixture of a liquid and a vapour. This is known as a wet vapour. The dryness fraction is the mass of vapour in kg per kg of mixture and is denoted by x. The wetness fraction is thus $1 - x$.

Consider another thought experiment where a liquid is heated by some means. The temperature rises at first. Once the saturated liquid line is reached, boiling starts. Heat continues to be supplied to the water but no increase in temperature will result. The heat energy is being used to change the phase of the liquid to a vapour. Once the saturated vapour line is reached, the temperature will again start to increase. The heat supplied during the change of phase where no temperature change occurs is known as the *latent heat of vaporisation*. With increasing pressure, the latent heat of vaporisation reduces. At the critical point it becomes zero.

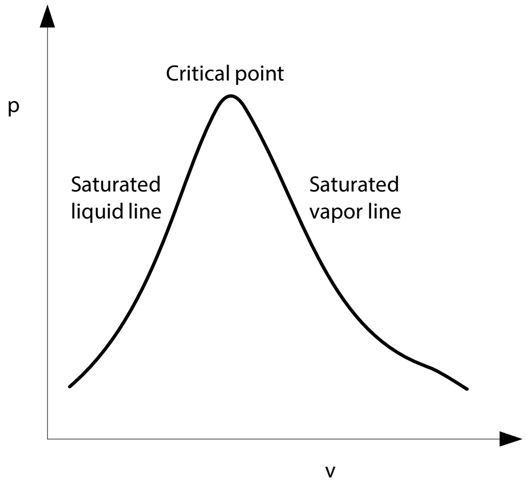


Figure 3.1: p-v diagram showing saturated liquid and vapour lines.

3.1.1 Finding working fluid properties

The thermodynamic properties for each state can be determined in three ways:

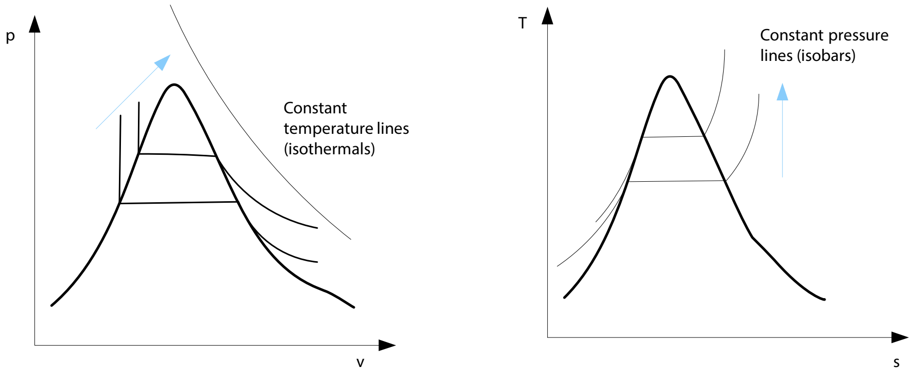
1. By extracting the thermodynamic properties directly from a chart
2. By using a table of properties
3. By using interpolating functions of the form:

$$\Psi = f(\phi_1, \phi_2, \ldots, \phi_n) \qquad (3.2)$$

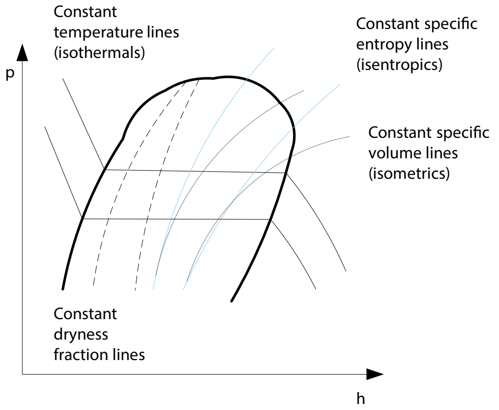
Thermodynamic cycles can be sketched on various thermodynamic diagrams. The most commonly used in refrigeration are the T-s diagram and the p-h diagram. Figure 3.2 illustrates these diagrams and their important features. Such diagrams exist for different working fluids and properties can be read off directly from the chart.

The interpolation function of eqn. 3.2 can usually be obtained from the tables mentioned in the second method. If such a function is not available, interpolation between different values in a table may be performed. Table 3.1 shows an example of the format of a thermodynamic table of properties. Two quantities with subscripts 1 and 2 are shown for illustration purposes. Subscript f stands for a saturated liquid quantity, subscript g stands for a dry saturated vapour quantity. Moreover, a parameter with subscript fg is simply defined as $\phi_{fg} = \phi_g - \phi_f$.

For a linear interpolation to find ϕ_i between two values ϕ_1 and ϕ_2 at pressure p_i between p_1 and p_2 the following expression may be used:



(a) p-v diagram (b) T-s diagram



(c) p-h diagram

Figure 3.2: Important features of p-v, T-s and p-h diagrams. Arrows point towards increasing quantities.

$$\boxed{\frac{\phi_i-\phi_1}{\phi_2-\phi_1}=\frac{p_i-p_1}{(p_2-p_1)}} \tag{3.3}$$

The properties of a wet vapour of dryness fraction x can be found by the addition of properties of the saturated liquid fraction and the saturated vapour fraction as follows:

$$\begin{aligned}\phi_x &= \phi_f(1-x)+\phi_g x\\ \phi_x &= \phi_f - x\phi_f + \phi_g x\phi_x\\ \phi_x &= x\phi_g + \phi_f - x\phi_f\end{aligned} \tag{3.4}$$

$$\boxed{\phi_x = \phi_f + x\phi_{fg}} \tag{3.5}$$

p bar	T degC	v_g m^3/kg	u_f kJ/kg	u_g kJ/kg	h_f kJ/kg	h_{fg} kJ/kg	h_g kJ/kg	s_f kJ/kgK	s_{fg} kJ/kgK	s_g kJ/kgK
p_1	T_1	v_{g1}	u_{f1}	u_{g1}	h_{f1}	h_{fg1}	h_{g1}	s_{f1}	s_{fg1}	s_{g1}
p_2	T_2	v_{g2}	u_{f2}	u_{g2}	h_{f2}	h_{fg2}	h_{g2}	s_{f2}	s_{fg2}	s_{g2}

Table [illegible] Typical table of thermodynamic properties

where ϕ_x is the property required for the fluid having dryness fraction x, ϕ_f is the value of the required property when it is in saturated liquid form and ϕ_g is the property required when it is in fully saturated vapour form.

Temperature	Absolute pressure	Density	Volume	hf	hg	sf	sg
24	0.97262	604.5	0.13204	313.11	1482.49	1.3937	5.329
26	1.0345	601.5	0.12434	322.73	1483.72	1.4257	5.3066
28	1.0993	598.4	0.11717	332.39	1484.87	1.4575	5.2844
30	1.1671	595.4	0.11048	342.08	1485.93	1.4892	5.2623

Table 3.2: Sample properties of ammonia. Source: ASHRAE Fundamentals Handbook ASHRAE (2009) (Refer to the appendix for the full data.)

Example

Example 1. *Using the properties provided in table 3.2 for ammonia calculate:*

1. *the enthalpy of vaporization at a temperature of 27^oC.*
2. *the entropy of wet vapour having a dryness fraction of 0.8 at a temperature of 28^oC.*

Answer.

1. *The enthalpy h_g at 27^oC can be obtained by linear interpolation as follows:*

$$h_{g,27^oC} = h_{g,26^oC} + \frac{27-26}{28-26}(h_{g,28^oC} - h_{g,26^oC})$$

$$h_{g,27^oC} = 1483.72 + \frac{27-26}{28-26}(1484.87 - 1483.72) = 1484.295 kJ/kg$$

2. *Given that $x = 0.8$ and using eqn.3.5:*

$$s_{0.8} = 1.4575 + 0.8(5.2844 - 1.4575) = 4.519 kJ/kgK$$

3.1.2 Working fluids in the superheat region

Consider a fluid which is just dry saturated. Further heating will cause the temperature of the fluid state to move to the right of the saturated vapour line of Figure 3.1. This region is known as the superheat region. Each point in the superheat region can be defined from the temperature and pressure of the fluid. With such quantities, the required thermodynamic properties can be established.

Definition

The degree of superheat is the difference between the temperature of the superheated vapour and its saturation temperature at the same pressure.

Further details on the interpretation of thermodynamic properties from tables can be found in specialised thermodynamic textbooks such as Eastop and McConkey (1993).

3.1.3 Specific heat capacity at constant volume

For a process carried out at constant volume, eqn. 1.37 applies:

$$\Delta u = \Delta Q \tag{3.6}$$

The increase of specific internal energy per unit rise in temperature is therefore equivalent to the rise in heat energy per unit mass supplied per unit rise in temperature:

$$\frac{\Delta u}{\Delta T} = \frac{\Delta Q}{\Delta T} \tag{3.7}$$

When the rise in temperature is made to approach zero, the specific heat capacity at constant volume may be defined as:

$$c_v = \frac{\partial u}{\partial T} | v = constant \tag{3.8}$$

Using eqn. 3.8, the heat transfer per unit mass is given by:

$$\begin{gathered} c_v = \frac{\partial Q}{\partial T} \\ Q = c_v \Delta T \end{gathered} \tag{3.9}$$

3.1.4 Specific heat capacity at constant pressure

For a process carried out at constant pressure, the first law can be written down in terms of enthalpy (refer to eqn. 1.35):

$$\begin{gathered} \Delta u = \boldsymbol{\Delta} Q - \Delta W \\ \Delta u = \Delta Q - p\boldsymbol{\Delta} v \\ \Delta h = \Delta Q \end{gathered} \tag{3.10}$$

Where Q and W represent heat energy and work per unit mass. Similar to the constant volume case, the increase of specific enthalpy per unit rise in temperature is hence equal to the rise in heat energy per unit mass supplied per unit rise in temperature:

$$\frac{\Delta h}{\Delta T} = \frac{\Delta Q}{\Delta T} \tag{3.11}$$

When the rise in temperature is made to approach zero, the specific heat capacity at constant pressure is:

$$c_p = \frac{\partial h}{\partial T} | p = constant \tag{3.12}$$

Using eqn. 3.10, the heat transfer per unit mass is given by:

$$\begin{gathered} c_p = \frac{\partial Q}{\partial T} \\ Q = c_p \Delta T \end{gathered} \tag{3.13}$$

3.2 Perfect gases

In heating vapours to very high temperatures above the critical point or when having very low pressures, gases start exhibiting a behaviour which tends to follow the equation:

$$\frac{pv}{T} = \frac{p}{\rho T} = R \tag{3.14}$$

where R is known as the specific gas constant. Note that the temperature T must be in Kelvin. Such gases are known *as perfect* or *ideal gases*. Eqn. 3.14 is known as the *equation of state*. Between any two states we may write:

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \tag{3.15}$$

Using the definition of molar mass as:

$$\tilde{m} = \frac{m}{n} \tag{3.16}$$

where m is the mass of substance in kg and n is the amount of substance in moles then we may write

tate as:

$$pV = mRT$$

$$pV = \tilde{m}nRT \tag{3.17}$$

But, by definition, the term $\tilde{m}R$ is known as the *molar* or *universal gas constant* and is symbolized by *R*. The value of $\widetilde{R}$ is known to be equal to 8314.462 J/kmolK. Thus:

$$pV = n\widetilde{R}T \tag{3.18}$$

Example

Example 2. *A compression process is carried out on 1kg of air. The process' polytropic index is 1.2. If initially the air is at atmospheric conditions and the work input is 40kJ, what is the final pressure of the air if the volume compression ratio is 10:1. Find the pressure of the air after compression. Take the atmospheric conditions to be 1.013bar and 15°C. For air, R = 287J/kgK.*

Answer.

For a polytropic process:

$$W = \frac{p_2V_2 - p_1V_1}{n-1}$$

$$\frac{W(n-1)}{V_2} + p_1\frac{V_1}{V_2} = p_2$$

The density of air at the given atmospheric conditions can be established using the ideal gas law

$$\rho_1 = \frac{p_1}{RT_1}$$

$$\rho_1 = \frac{1.013e5}{287.288} = 1.22kg/m^3$$

$$\rho_1 = \frac{m}{V_1}$$

$$V_1 = \frac{m}{\rho_1} = \frac{1}{1.22} = 0.816m^3$$

Since the compression ratio is 10, then

$$V_2 = \frac{0.816}{10} = 0.0816m^3$$

The pressure at state 2 is therefore

$$p_2 = \frac{40e3 \times (1.2-1)}{0.0816} + 1.013e5 \times 10 = 1.108MPa$$

3.2.1 Joule's law

Definition

Joule's law states that the internal energy of a perfect gas is a function of absolute temperature T only

Considering an isovolumetric process the first law states that (refer to eqns. 1.37 and 3.9):

$$dQ = du = c_v dT \tag{3.19}$$

Integrating:

$$u = \int c_v dT = c_v \int dT = c_v T + C \tag{3.20}$$

where C is the constant of integration. As can be seen, the internal energy varies linearly with temperature. Choosing $u = 0$ at $T = 0$ as reference then $C = 0$.

Consider now a constant pressure process. Then from the first law of thermodynamics (refer to eqns. 1.35 and 3.9):

$$\Delta Q + \Delta W = mc_v \delta T$$

$$\Delta Q - p\Delta V = mc_v \delta T$$

$$\Delta Q - mR\Delta T = mc_v \delta T$$

$$\Delta Q = m\Delta T(R+c_v) \tag{3.21}$$

For the constant pressure process it was already shown that $\Delta Q = mc_p \Delta T$ (refer to eqn. 3.13). Hence,

$$mc_p \, \Delta T = m\Delta T(R + c_v) \tag{3.22}$$

Thus

$$R = c_p - c_v \tag{3.23}$$

Another important relationship for specific enthalpy follows since $h = u + pv$. From the ideal gas equation:

$$h = c_v T + RT = T(c_v + R) \tag{3.24}$$

Thus

$$h = c_p T \tag{3.25}$$

It must be emphasized that eqn. 3.25 holds only for an ideal gas.

3.3 Fluids in refrigeration

The heat transfers required in refrigeration cycles are brought about by means of the conversion of heat into internal energy. The fluid by which this takes place is known as a refrigerant. When such changes in internal energy occur, the refrigerant may change phase from liquid to vapour.

3.3.1 Desirable refrigerant properties

There are many properties of refrigerants which are sought after. Many of these requirements may indeed be conflicting and this creates some problems when coming to select an appropriate refrigerant for the cycle. Some of these requirements are listed next:

- Environmentally friendly
- Economically viable
- Safe (in terms of say toxicity and flammability)
- Compatibility
- Corrosiveness
- latent heat of vaporization

3.3.2 Types of refrigerants

Definition

Halocarbons consist of one or more atoms of bromine, chlorine or fluorine

Chlorofluorocarbons (CFCs) (see Badr et al. (1990)) are a type of halocarbons consisting of chlorine, fluorine and carbon. Examples include refrigerants such as R11 and R12. These are known to cause ozone depletion. When CFCs leak from systems, these diffuse upwards towards the atmosphere into the stratosphere. The molecules then breakdown and liberate Carbon as well as the halogens Chlorine and Fluorine as shown in Figure 3.3. In this case it is Chlorine which is responsible for ozone depletion.

Halons are on the other hand compounds containing bromine, fluorine and carbon. the consequences of molecular breakdown for these are even more problematic. As a partial solution to the problems with CFCs and halons, one or more of the halogens in the molecule can be replaced by a hydrogen atom.

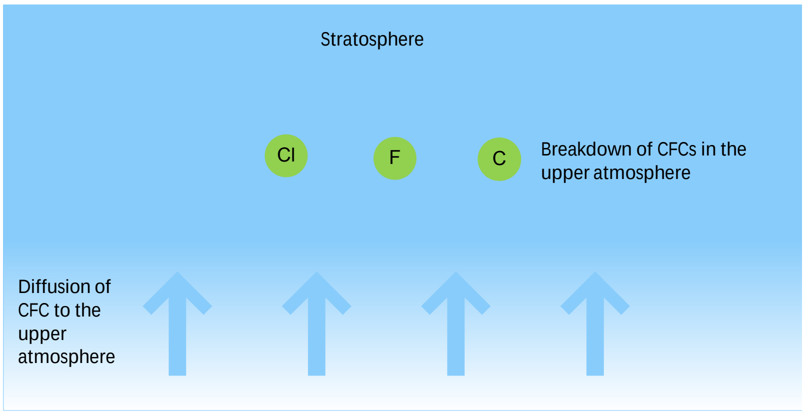


Figure 3.3: CFC diffusion into the upper atmosphere. Breakdown occurs and ozone layer depletion ensues.

Definition

Hydrocarbons are compounds that include carbon and hydrogen.

A suitable replacement to CFCs are the HCFCs or *hydrochlorofluorocarbons*. These result in less environmental damage.

Inorganic compounds can also be used for refrigeration purposes Dincer and Kanoglu (2010). Examples include the following:

- Ammonia (R-717) (refer to Lorentzen (1988) for further information on this refrigerant)
- Carbon Dioxide (R-744)
- Water (R-718)
- Helium (R-704)
- Neon(R-720)
- Air(R-729)

Definition

A mixture of two substances which individually have different properties but together behave as a single substance is known as an azeotropic mixture.

Azeotropic refrigerants such as for example R-502 provide better COP. In the case of R-502 the mixture is composed of 48.8% R-22 and 51.2% R-115. A characteristic of azeotropic refrigerants is that the individual refrigerants making up the mixture cannot be separated by a distillation process.

Definition

Zeotropic mixtures "Refers to blends comprising multiple components of different volatility that, when used in refrigeration cycles, change volumetric composition and saturation temperatures as they evaporate (boil) or condense at constant pressure. The word is derived from the Greek words zein (to boil) and tropos (to change)"ASHRAE (2010)

Zeotropic mixtures provide various advantages including better refrigeration/heat pump performance as well as modifying the composition to suit various temperature needs. An example of a zeotropic mixture is R-407c which is composed of R-32, R-125 and R-134a.

3.3.3 The Montreal Protocol

On the 16th of September, 1987 the Montreal Protocol was signed by the European Economic Community (nowadays known as the European Union) and another 24 nations. Its aim was to limit the production of any substance which caused ozone depletion including CFCs and Halons (for more information on ozone depletion refer to Rowland (2006)). Its implementation was conducted from 1st of January 1989. The target was to limit down the production of such substances to 50% the levels of the year 1986. In the phase out management plan, the target is to halt production of HCFCs by 2013 and also reduce its consumption by 2015. A complete halt of HCFC use is required by 2030. An overview of some of the most important years for the Montreal protocol is shown in Table 3.3.

Year	Event
1987	Montreal protocol signed
1989	Actual implementation
1990	London amendment of original protocol
1992	Copenhagen mendment
1994	No further Halons production
1996	No further CFC production
1996	HCFC production frozen to a 1989 reference level
1997	The Montreal amendment
1999	The Beijing amendment
2007	Accelerated phase-out, Montreal adjustment
2009	Montreal protocol Universal ratification (196 nations)
2010	Production CFCs and halons halted
2013	Halt to HCFC production and increased usage
2015	Reduction in HCFC usage

Table 3.3: Chronology of the Montreal Protocol.

In order to set a measure on the effect of chemicals on the ozone layer, the Ozone Depletion Potential (ODP) is defined as follows:

Definition

The ODP is the amount of ozone depletion of a unit mass of chemical relative to the depletion caused by unit mass of R-11 (trichlorofluoromethane)

Such a definition of the ODP does not consider the dynamics of the checmical process with time. A more accurate definition would hence include the ozone depletion over a particular time-span. The calculation of the ODP of a chemical can usually be found experimentally. Since it is a relative quantity (with respect to R-11), the experimental determination of the ODP can be quite accurate. Figure 3.4 shows the time-dependent ODPs as obtained from Solomon and Albritton (1992).

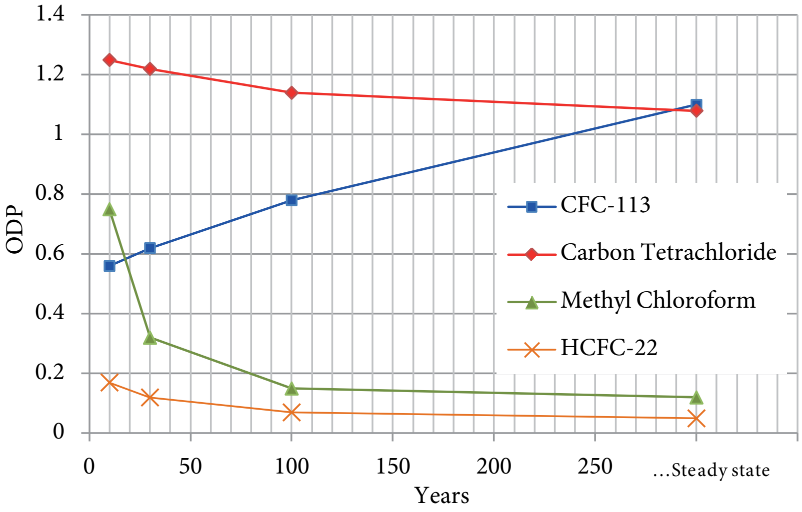


Figure 3.4: Unsteady Ozone Depletion Potential (ODP) for various substances.
Source information: Solomon and Albritton (1992)

3.3.4 Refrigerant names

Refrigerant designations and safety classifications are presented in the ASHRAE (2010)/ASHRAE Standard 34-2010. The need to label refrigerants with a shortened name is to avoid the lengthy chemical formula associated with many refrigerants. The system provides a concise way of referring to these refrigerants. The standard describes how to assign refrigerant numbers from right to left. Four digits are employed as follows:

Definition

1. *The first digit from the right represents the number of Flourine (F) atoms.*
2. *The second digit from the right represents the number of Hydrogen (H) atoms plus one.*
3. *The third digit from the right represents the number of Carbon (C) atoms minus one. If there are no Carbon atoms, then this digit is ignored.*
4. *The fourth digit from the right represents the number of Carbon-Carbon (C) bonds in the compound. If this is not applicable, then this digit is ignored.*

When Bromine (Br) is present, then following the designation of the compound, the letter *B* must be included at the end followed by the number of Bromine atoms.

Two examples are shown here to demonstrate the system mentioned in this section.

Example

Example 3. *Give the refrigerant names for pentafluoroethane (C_2ClF_5) and Chlorodifluoromethane ($CHClF_2$) using the ANSI/ASHRAE Standard 34-2010.*
Answer.
pentafluoroethane: There are 5 F atoms, 0 H atoms and 2 C atoms. Therefore, using the standard designation, the refrigerant is R115.
Chlorodifluoromethane: There are 2 F atoms, 1 H atom, 1 C atom. Again, using the standard designation, the refrigerant is R22

3.4 Conclusions

Student checklist

1. The thermodynamic characteristics of vapours has been described.
2. Readers should become familiar with the interpretation of thermodynamic data from various sources.
3. The concept of a perfect gas has been introduced.
4. Various types of fluids used in refrigeration have been introduced including halocarbons, hydrocarbons, inorganic compounds, azeotropic and zeotropic mixtures.
5. International regulation of refrigerant fluids has been discussed as well as the environmental impact that certain refrigerants have.
6. The nomenclature of refrigerants has been described along with an example.

4 Vapour compression cycles

4.1 Limitations of the Carnot cycle

While the Carnot cycle would be the most efficient cycle applied for the purpose of refrigeration/heat pumping, in practice there are a number of difficulties which hinder the application of such a cycle. A reconsideration of the Carnot cycle of Figure 2.4 enables the identification of these inherent problems. The cycle is shown again here in Figure 4.1 with the saturation lines included.

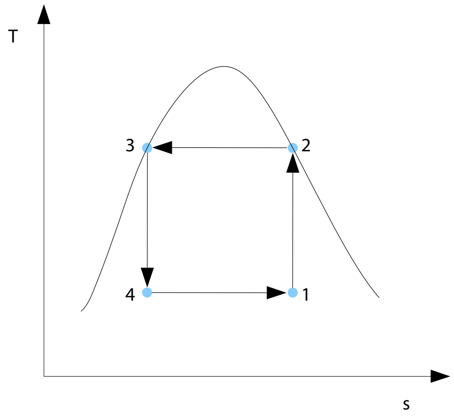


Figure 4.1: T-s diagram for a Carnot cycle with the saturation lines shown.

At state 1, the refrigerant is partly in the liquid phase and partly in the vapour phase. This wet vapour needs to be compressed to a higher pressure. In practice, a compressor can only compress a pure vapour. If a liquid is present at the inlet of the compressor, then the compressor components might be damaged due to the fact that a liquid is practically incompressible (this is valid only for positive displacement machines with very small clearances).

During the isentropic compression from 1 to 2, the refrigerant heats up and its pressure increases to the condenser pressure. Due to this, process 2 to 3 is not carried out at a constant temperature. This presents another deviation and as a result, the efficiency of the cycle would be expected to decrease.

From process 3 to 4, work can be extracted by the system. In practice, this work is rather small and cannot justify an investment for a turbine to extract this energy. Another way must therefore be found to lower the pressure of the refrigerant from the condenser pressure to the evaporator pressure.

All of these difficulties in building a practical Carnot cycle lead the engineer to find a better alternative, one which is both practical and which possibly has the highest COP. The modified Carnot cycle is known as the *ideal vapour compression cycle*.

4.2 Modifications to the Carnot cycle

4.2.1 Compressor inlet

At exit from the evaporator, the refrigerant should either be dry saturated or superheated such that no liquid is present at the compressor inlet. This enables protection of the compressor which would be designed to operate on a vapour. In practice it is very difficult to control the state of the refrigerant such that it is just dry saturated at the compressor inlet. Hence at state 1, the refrigerant is usually a superheated vapour. The modified cycle is shown in Figure 4.2. Since the work input increases, the COP will decrease.

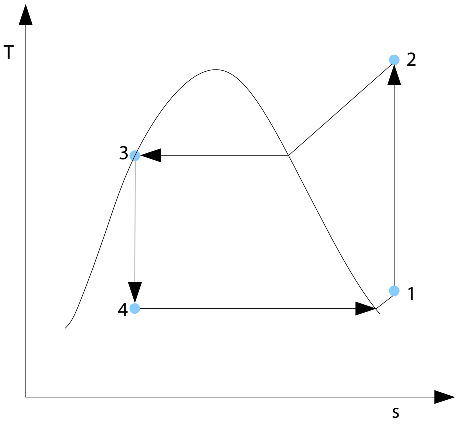


Figure 4.2: Modified Carnot cycle with superheated vapour at compressor inlet.

4.2.2 Condenser outlet

In the condenser, the refrigerant cools and gradually the quantity of liquid increases until the refrigerant is entirely in the liquid phase at state 3. The cooling of the liquid, causes state 3 shown in Figure 4.1 to move further to the left. This effect is positive for a refrigerator since the cooling process 4 to 1 produces a larger cooling effect (the area and hence heat energy absorbed by the refrigerant is increased). This prolonged cooling in the condenser is known as under-cooling. The downside of this is that it requires a larger condenser which may not be economical. The cycle is shown in Figure 4.3. The heat input in the evaporator increases and hence the COP will increase.

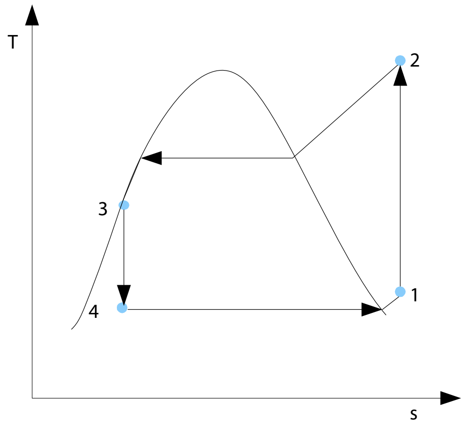


Figure 4.3: Modified Carnot cycle with under-cooled liquid at the condenser exit.

4.2.3 Replacement of the turbine

The expensive turbine can be replaced by a *throttling valve*. The thermodynamic characteristics of the throttling valve will be discussed in section 4.3. The throttling process is highly irreversible and will inevitably lead to an increase in entropy. This causes state point 4 to move to the right. As a result, the area under process 4 to 1 will reduce resulting in a lower cooling effect and hence a lower COP. The components of the cycle are shown in Figure 4.4 while the T-s diagram for the vapour compression cycle is shown in Figure 4.5.

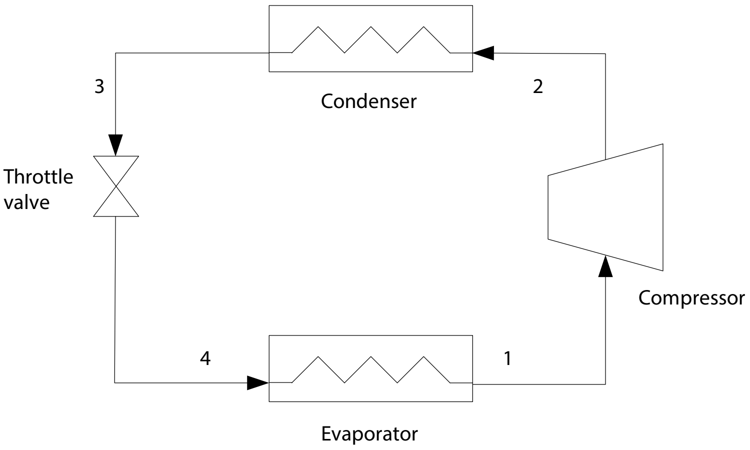


Figure 4.4: Components of a vapour compression cycle.

The vapour compression cycle p-h diagram is shown in Figure 4.6. Such a diagram is somewhat simpler than the T-s diagram. The constant pressure lines 4-1 and 2-3 represent the constant pressures in the evaporator and condenser respectively. It is interesting to note that the throttling process 3-4 occurs at a constant enthalpy. This will also be explained in section 4.3.

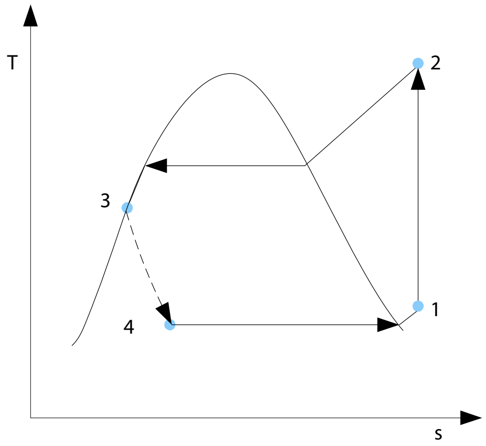


Figure 4.5: Modified Carnot cycle with the turbine replaced by a throttling valve.

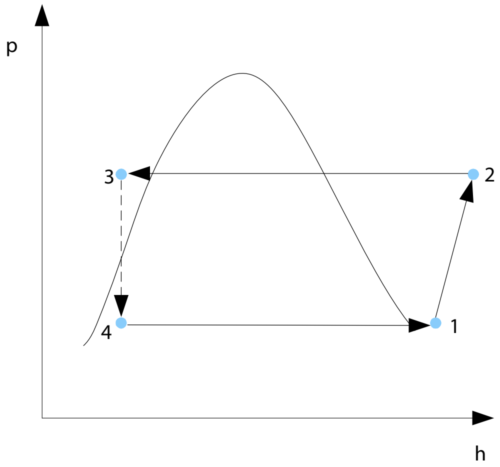


Figure 4.6: p-h diagram for a vapour compression cycle.

4.3 Thermodynamic analysis of the vapour compression cycle

4.3.1 The steady flow energy equation applied to the system components

The steady flow energy equation will be used to estimate the heat or work input/output from of each of the vapour compression system components. It will be shown that the thermodynamic property which determines the evaluation of these energies is the enthalpy (or specific enthalpy) at the various state points. These quantities can be obtained from the knowledge of the evaporator and condenser pressures (the low and high system pressures respectively) which can be used along with any one of the methods for finding the thermodynamic properties of fluids shown in section 3.1.

Consider the evaporating process 4-1. Both the height differences between 4 and 1 as well as the change in refrigerant speed across the evaporator will be assumed negligible. There is no work done by or on the evaporator. Using the steady flow energy equation on the evaporator system of Figure 6.3:

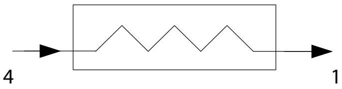
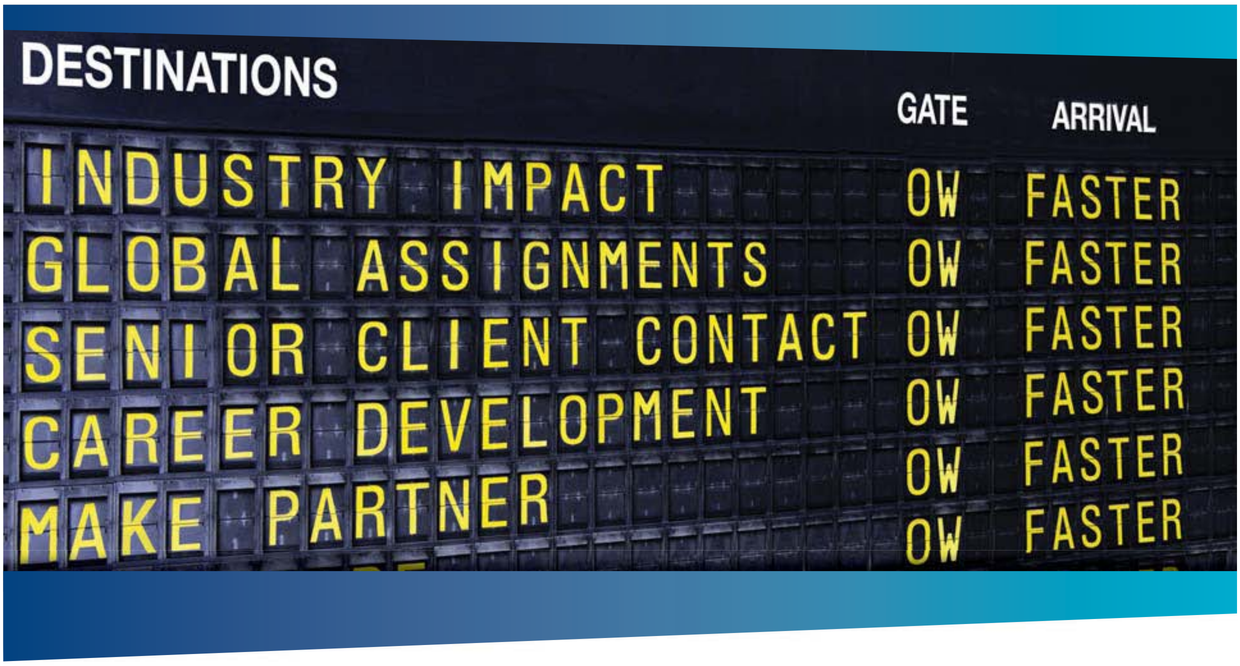


Figure 4.7: Evaporator component.

$$\dot{m}h_4 + \dot{Q} = \dot{m}h_1$$

$$\dot{Q} = \dot{m}(h_1 - h_4) \qquad (4.1)$$



Process 1-2 is a compression process. Height changes from inlet to outlet as well as velocity differences are ignored. Using the same approachas with the evaporator but assuming that the compressor (shown in 4.8) is perfectly insulated (no heat transfer) then

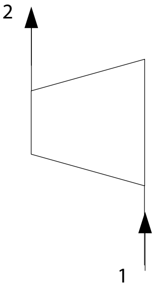


Figure 4.8: Compressor component.

$$\begin{aligned} \dot{m}h_1 + \dot{W} &= \dot{m}h_2 \\ \dot{W} &= \dot{m}(h_2 - h_1) \end{aligned} \tag{4.2}$$

Process 2–3 involves the condenser shown in Figure 6.4. The resulting steady flow energy equation is very similar to the case of the evaporator since no work is done on or by the condenser.

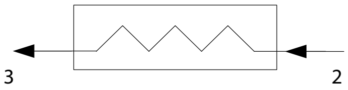


Figure 4.9: Condenser component.

$$\begin{aligned} \dot{m}h_2 + \dot{Q} &= \dot{m}h_3 \\ \dot{Q} &= \dot{m}(h_3 - h_2) \end{aligned} \tag{4.3}$$

In a throttling process shown in Figure 4.10 there is neither work nor heat transfer across the system boundaries. Height changes will be considered again negligible. The throttling valve is essentially an obstacle to the flow resulting in high levels of turbulence generation and separated flow. If however the state points are considered far enough for the valve opening, the flow will again have uniform streamlines. Across these points, the velocity differences are also negligible. While the flow streamlines are restored some distance downstream, the entropy of the system increases and the process is highly irreversible. The resulting energy equation is as follows:

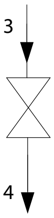


Figure 4.10: Throttle valve component.

$$\begin{aligned} \dot{m}h_3 &= \dot{m}h_4 \\ h_3 &= h_4 \end{aligned} \tag{4.4}$$

4.3.2 COP of a vapour compression cycle

From the energy analysis of subsection 4.3.1 the COP for both a refrigerator and a heat pump can be defined as follows:

$$COP_{ref} = \frac{Q_1}{W} = \frac{h_1 - h_4}{h_2 - h_1} \tag{4.5}$$

$$COP_{hp} = \frac{-Q_2}{W} = \frac{-(h_3 - h_2)}{h_2 - h_1} \tag{4.6}$$

Example

Example 1. *A vapour compression refrigeration cycle uses R134a as a refrigerant and is used to keep the temperature inside a small chamber at 4^oC. The system condenser pressure is kept at 16bar. Before throttling, the refrigerant is under-cooled by 5^oC. The refrigerant mass flow is maintained at 2.4kg/s. Find:*

1. *The power input required by the compressor*
2. *The cycle COP*
3. *The percentage reduction in COP when compared to an idealized system operating with a Carnot cycle*

Answer.

**Note: The thermodynamic properties for refrigerant R134a are taken from the ASHRAE Fundamentals Handbook ASHRAE (2009). Using a different source may result in certain differences in the following calculations due to slight differences in the thermodynamic data for the refrigerant. Also note that the enthalpy values can have a different reference temperature. Enthalpy values at different states will therefore differ in magnitude. Differences in enthalpies are not affected.*

The T-s diagram for the refrigeration cycle is shown in Figure 4.5. The evaporator is at 4^oC while the condenser is at a pressure of 16bar. Hence at exit from the evaporator, assuming that the vapour is just dry saturated:

$$h_1 = 401kJ/kg$$

also,

$$s_1 = 1.7252kJ/kgK$$

Assuming a perfect isentropic process in the compressor:

$$s_2 = s_1 = 1.7252kJ/kgK$$

hence at the inlet of the condenser (state 2), the R134a vapour is superheated with a pressure of 16bar. Interpolating using the superheated tables for R134a:

$$T_2 = 60 + \frac{1.7252 - 1.7124}{1.7493 - 1.7124}(70 - 60)$$

$$T_2 = 63.5^oC$$

$$h_2 = 428.99 + \frac{63.5 - 60}{70 - 60}(441.47 - 428.99)$$

$$h_2 = 433.4kJ/kg$$

Since the refrigerant at condenser exit is under-cooled by 5°C then the exit temperature is the saturation temperature at 16bar minus the under-cooling temperature.

$$T_3 = 58 - 5 = 53^oC$$

Assuming that the enthalpy of the refrigerant liquid is the same as the saturated value at this temperature:

$$h_3 = h_{f,53^oC} = 274.71 + \frac{53 - 52}{54 - 52}(277.86 - 274.71) = 276.29kJ/kg$$

After the throttling process,

$$h_4 = h_3 = 276.29kJ/kg$$

1. *The power input into the cycle is given by:*

$$W = \dot{m}(h_2 - h_1) = 2.4(433.4 - 401)$$

$$W = 77kW$$

2. *The refrigeration capacity is calculated as follows:*

$$Q_1 = \dot{m}(h_1 - h_4) = 2.4(401 - 276.29)$$

$$Q_1 = 299.3kW$$

The COP is therefore

$$COP_{ref} = Q_1/W = 299.3/77 = 3.887$$

3. *For a Carnot cycle operating between* 4^oC *and* 58^oC*, the COP is given by*

$$COP_{ref} = \frac{T_1}{T_{sat,2} - T_1} = 277/54 = 5.130$$

The percentage loss in COP for a practical cycle is

$$\frac{3.887 - 5.13}{5.13} \times 100 = -24.22\%$$

4.4 Multistage vapour compression cycles

4.4.1 Flash chamber use in multistage vapour compression cycles

During the throttling process shown on the p-h diagram of Figure 4.6, as the refrigerant liquid passes through the throttle valve, a fraction suddenly *flashes* into a vapour (process 3-4). At entry to the evaporator, there is therefore a mixture of a liquid and a dry vapour. The dry vapour fraction cannot contribute to the cooling effect. For this reason, it would be ideal to divert the vapour component such that a pure liquid enters the evaporator in order to decrease the compressor work. This can be done by the use of a flash chamber operated at a pressure between the evaporator and the condenser pressures. The process is shown in Figure 4.11 and its p-h diagram in Figure 4.12. As can be seen, the process uses an additional throttle valve and in practice, the refrigerant entering the evaporator will not be a pure liquid.

It is also advantageous to make use of two small compressors rather than a large one. This improves the operating efficiency. State point 9 is the result of a mixing process of the vapours at state points 2 and 3. The mixing process is assumed to be carried out adiabatically Another important note is that the mass of vapour per unit kg of refrigerant at 3 is $x6$ or simply the dryness fraction. The mass of vapour per unit mass of refrigerant through the evaporator is therefore $1 - x6$. The enthalpy at state point 9 can be evaluated again using the steady flow energy equation. This can be done by ignoring kinetic and potential energy changes. These is no work done and for adiabatic mixing no heat transfer. Hence:

$$\dot{m}(1-x_6)h_2 + \dot{m}x_6h_3 = \dot{m}h_9$$

$$(1-x_6)h_2 + x_6h_3 = h_9 \tag{4.7}$$

Note that h_3 is the enthalpy of dry saturated vapour at the pressure of the flash intercooler. At state 4, the mass flow rate is again m.

The refrigerating effect is given by:

$$Q_1 = \dot{m}(1-x_6)(h_1 - h_8) \tag{4.8}$$

The heat rejected in the condenser is:

$$Q_1 = \dot{m}(h_5 - h_4) \tag{4.9}$$

The refrigerating COP is hence given by:

$$COP_{ref} = \frac{Q_1}{W_1 + W_1} = \frac{\dot{m}(1-x_6)(h_1 - h_8)}{\dot{m}((1-x_6)(h_2 - h_1) + (h_4 - h_9))} \tag{4.10}$$

while the heat pump COP is given by:

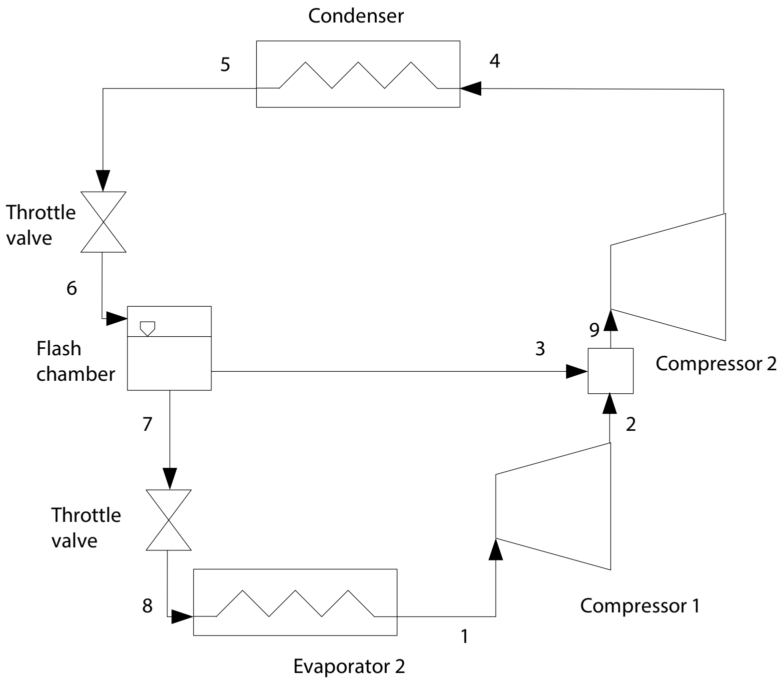


Figure 4.11: The multistage vapour compression cycle.

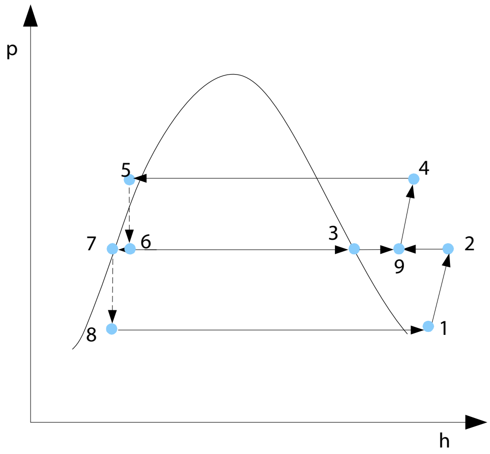


Figure 4.12: p-h diagram for a multistage vapour compression cycle.

$$COP_{hp} = \frac{Q_2}{W_1 + W_2} = \frac{\dot{m}(h_5 - h_4)}{\dot{m}((1 - x_6)(h_2 - h_1) + (h_4 - h_9))} \tag{4.11}$$

tages used in such a cycle highly depends on a balance between factors such as simplicity irements and efficiency. The larger the number of stages the better the efficiency but clearly the complexity and cost of the system will increase.

4.4.2 Cascade vapour compression cycles

When requiring particularly low evaporator temperatures or high condenser temperatures, two or more vapour compression cycles can be operated in series as shown in Figure 4.15. In this manner the refrigerants of the individual cycles need not necessarily be the same. The evaporator of the top system acts as a cooler to the condenser of the lower system. The evaporator of the lower system is on the other hand responsible for the cooling effect. As with the case of a flash chamber multistage cycle, two compressors are used along with throttling valves for the individual systems. The cycle is shown in Figure 4.16 on a p-h diagram. The shaded regions show an increase in the refrigerating effect (area below process 4-1) as well as a decrease in the work input that is required by the two compressors. These two factors increase the COP of the cascaded refrigeration cycle as compared to the single stage cycle.

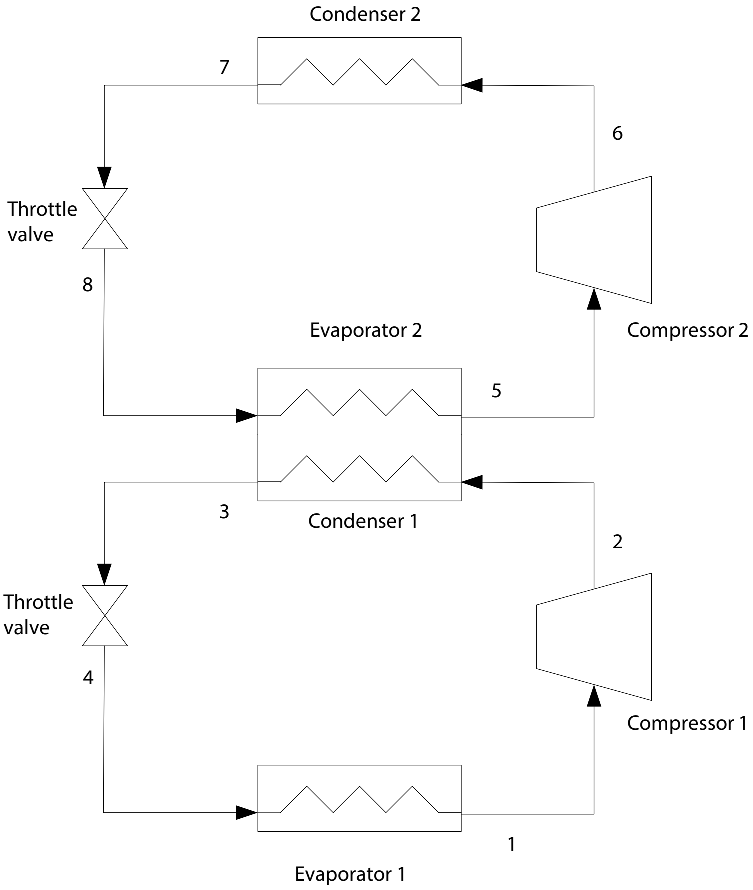


Figure 4.13: Vapour compression cycle using two evaporators.

The refrigerating COP is hence given by:

$$COP_{hp} = \frac{Q_2}{W_1 + W_2} = \frac{\dot{m}(h_5 - h_4)}{\dot{m}((1 - x_6)(h_2 - h_1) + (h_4 - h_9))} \tag{4.12}$$

while the heat pump COP is given by:

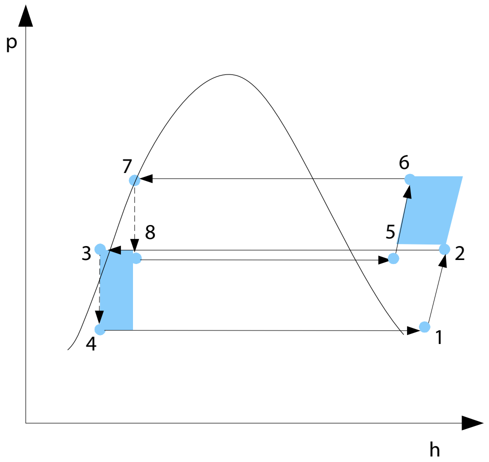


Figure 4.14: p-h diagram for vapour compression cycle using two evaporators.

$$COP_{hp} = \frac{Q_2}{W_1 + W_2} = \frac{-\dot{m}_H (h_7 - h_6)}{\dot{m}_L (h_2 - h_1) + \dot{m}_H (h_6 - h_5)} \tag{4.13}$$

where $\dot{m}_L$ is the mass flow rate of the low pressure system and $\dot{m}_H$ is the mass flow rate of the high pressure system. The system mass flow rates maybe optimized to give maximum COP.

4.4.3 Refrigeration at various temperatures

In certain applications, the cooling effect of a system is required at various temperatures. A common example of such an application is the fridge-freezer. A single compressor is used along with two evaporators instead of one. This is shown in Figure 7.10. The cycle is also shown in Figure 4.14 using a p-h diagram. As can be noticed there are two different evaporator pressures (and hence temperatures). The refrigerant is throttled at various stages. In the definition of the COP for refrigeration, all evaporator energy transfers must be included. Hence:

$$COP_{ref} = \frac{\sum_{j=1}^{N} Q_j}{W} \tag{4.14}$$

where N is the number of evaporators used.

4.5 Heat pumps

Heat pumps transfer heat from a low temperature reservoir to a high temperature reservoir (analogous to a pump which transfers a fluid from a low to a high pressure). Heat pumps are a very efficient method of heating.

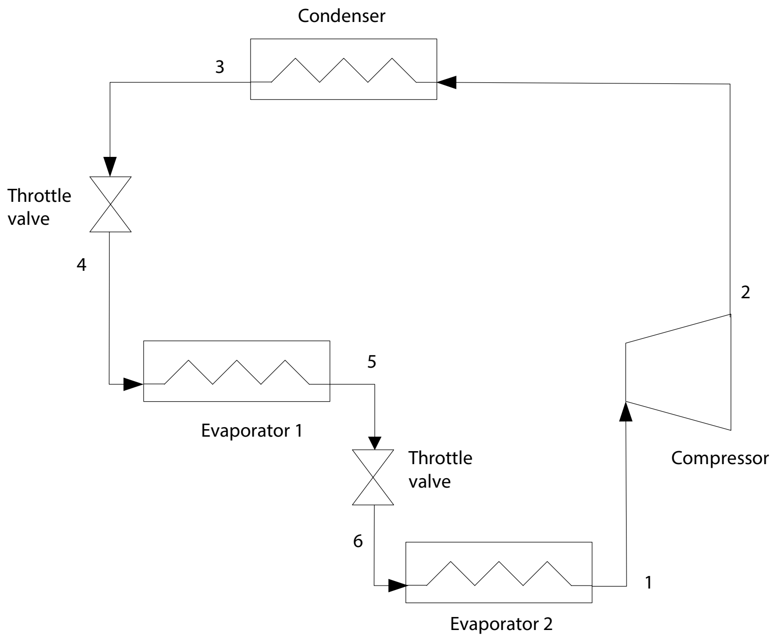


Figure 4.15: Two cascaded vapour compression cycles.

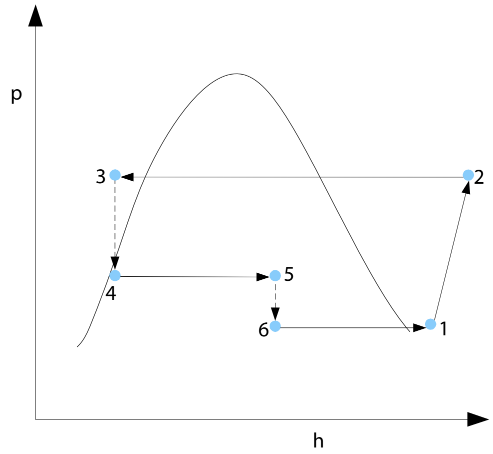


Figure 4.16: p-h diagram for two cascaded vapour compression cycles.

Such systems however may require a substantial capital investment. The cold temperature source must be at a uniform temperature. The evaporator coils are therefore placed in the low temperature source and heat is transferred to the refrigerant flowing in the coils. Clearly for heat transfer to be present, the refrigerant temperature at the evaporator must be below the temperature of the source. Another important factor to consider in order to enhance the heat transfer is the icing problem. Ice formation on the surface of the coil causes an increase in the resistance to heat transfer thus reducing the system COP. The condenser coils would be located in the space which needs to be heated. Again for effective heat transfer, the refrigerant flowing through the condenser must have a slightly higher temperature than the space being heated.

4.5.1 Heat pump performance

The COP as defined in eqn. 4.6 is the most commonly used measure of the performance of heat pumps. Heat pump COPs can range from around 2 to 4. The use of variable speed electric motors (for the compressor) can increase the energy efficiency substantially.

Heat pumps may use for instance renewable energy sources to power the compressor rather than electrical energy. For this reason, the COP is no longer truly indicative of the energy performance of the heat pump. A better way to assess heat pump performance would be to incorporate the primary energy input rather than the work done by the compressor. A better definition of the heat pump performance is therefore as follows:

Definition

The Performance Energy Ratio (PER) is defined as the heat output from the condenser to the primary energy input:

$$PER = -\frac{Q_2}{E} \tag{4.15}$$

where E is the primary energy supplied.

4.5.2 Heat pump types and heat sources

Heat pumps can use natural sources of heat such as water, air or the ground. All of these contain a certain amount of heat but the usefulness of such heat energy is dependent on the temperature of these sources. The heat source must ideally be of a uniform temperature, easily accessible and should have thermophysical properties which are favourable for the cost effectiveness of the system. Air source heat pumps for instance, uses a widely available source but is prone to various issues such as frosting of the outdoor coils once the air temperature reaches temperatures approaching around 3°C This is usually resolved through a defrosting process which can ultimately reduce the operating efficiency of the heat pump. Another problem is that when space heating is mostly required in buildings, the outdoor temperature is usually low.

Water source heat pumps can be a good alternative since it provides a more uniform temperature than air. River or groundwater can for instance be used. Piping and pumping is generally required especially for groundwater heat pumps which may lead to additional costs. Moreover, the quality of the ground water can result in the reduction of heat exchange performance due to fouling. River or lake water can also be prone to freezing.

Geothermal heat pumps use pipes buried horizontally in the soil or boreholes drilled at vertical depths of around 100m to 200m. The latter types are generally used for large scale or industrial applications.

4.5.3 Heat pump utilization

Since the heat energy which is extracted from air, water or the ground is free and renewable and since the heat energy supplied on the condenser side is higher than the energy input via the compressor heat pumps are considered as a green alternative to more wasteful processes such as gas heating. The energy savings by heat pumps installed from 2005-2011 and projected till 2020 is given in Table 4.1.

	2005–2010	2011	2005–2011	Total 2005–2020
RES contribution	29.17	5.72	34.89	130.64
Green House Gas (GHG) emmision savings	6.80	1.33	8.13	30.10
Final energy saved	36.63	7.32	43.95	164.22
Primary energy saved	15.06	3.37	18.44	73.87
Number of units sold	3798670	771469	4570139	17575934

Table 4.1: Savings achieved by heat pumps from 2005-2020 (projected). Source: EHPA (European Heat Pump Association)

4.6 Compressors

4.6.1 Compressor technologies

Compressors are designed to increase the pressure of a gas by means of a work input. In the case of refrigerators or heat pumps, the compressor operates on the refrigerant in order to increase the pressure from that of the evaporator to that of the condenser. There are various types of compressors. The choice depends particularly on the pressure ratio required as well as the flow rates of the gas. Rotary type compressors are used for high mass flow rates and low pressure ratios. Reciprocating type compressors are used for low mass flow rate and high pressure ratios (see Eastop and McConkey (1993)). In the rotary type compressors, the flow is steady whereas in the reciprocating type compressors, the flow is pulsating. Figure 4.17 shows a reciprocating type compressor. As the piston moves towards the left, the fluid inside the cylinder is compressed and exits through the delivery port. When the piston moves to the right, fluid enters the cylinder from the intake port. This motion is repeated at a particular frequency or speed of the reciprocating piston.

In the rotary type compressor shown in figure 4.18, an eccentric rotor having a number of vanes rotates in [illegible] anes seal off different segments. The gas is introduced in a segment and is compressed during [illegible] the rotor. Vane type compressors are quite common in domestic refrigerators, freezers and air-conditioners. Other rotary compressors include screw type compressors, scroll compressors and turbo compressors.(Dincer and Kanoglu (2010)).

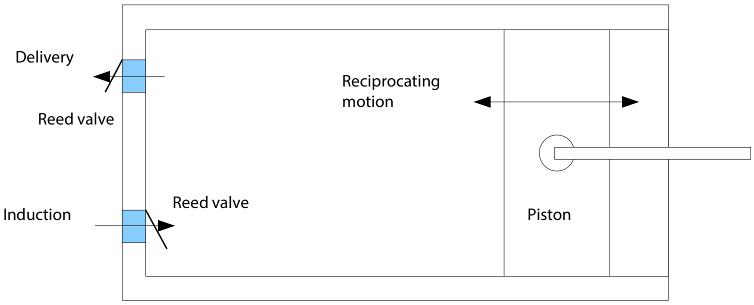


Figure 4.17: A reciprocating type compressor cylinder.

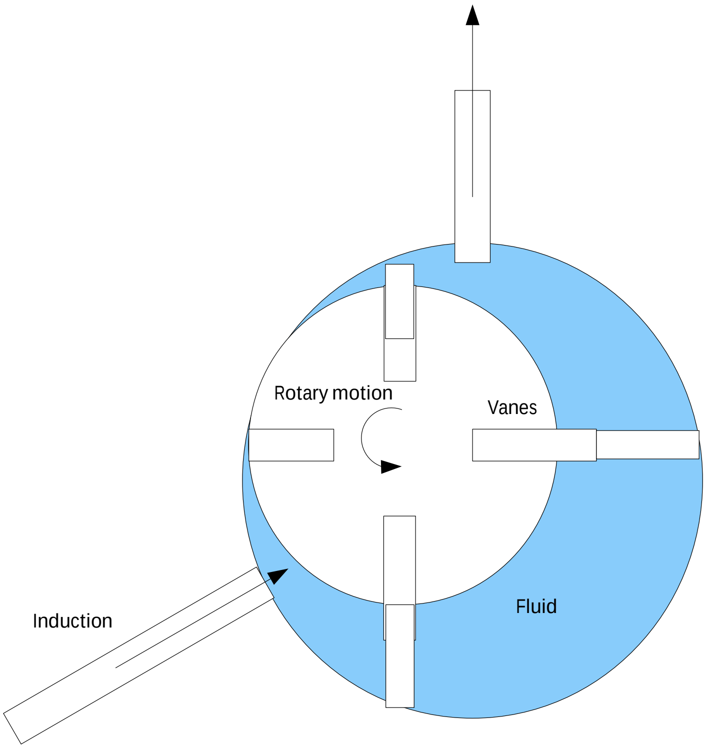


Figure 4.18: A rotary vane type compressor.

4.6.2 Compressor efficiency

Volumetric efficiency

In reciprocating compressors, the swept volume by the piston is not necessarily the same as the volume of the cylinder. In practice there exists a clearance volume. This reduces the performance of the compressor. The volumetric efficiency can be written as follows:

$$\eta_v = 1 - \frac{V_c}{V_s}\left(\frac{p_2^{1/n}}{p_1} - 1\right) \tag{4.16}$$

Where V_c is the clearance volume, V_s is the compressor volume, p_2 is the pressure at the outlet, p_1 is the pressure at the inlet and n is the polytropic index of the compression process.

Isentropic efficiency

A compression process is in practice irreversible. Hence the process is not really performed at a constant entropy. The entropy of the end state after compression is higher than the initial state, in line with the second law of thermodynamics. The vapour compression T-s diagram is shown again here with the compressor irreversibilities included. The dotted line shows the actual process undergone in the compressor. The bold line from 1 to 2s shows the compression process if the compressor compression process was isentropic.

The isentropic efficiency is defined as follows:

$$\eta_{isen} = \frac{-W_{12s}}{-W_{12}} \tag{4.17}$$

where W_{12} is the actual work done and W_{12s} is the work done if the compression is isentropic.

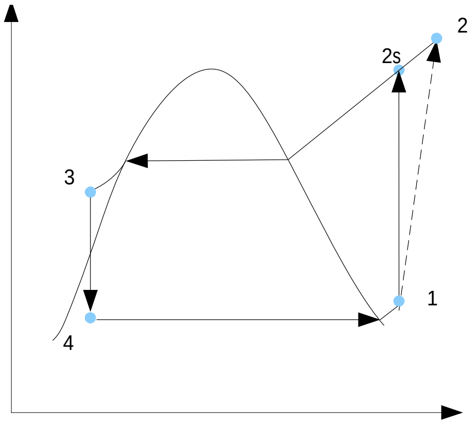


Figure 4.19: T-s diagram for the vapour compression refrigeration cycle including the isentropic efficiency of the compressor.

4.7 Conclusions

Student checklist

1. A number of limitations have been identified for a practical implementation of the Carnot Cycle to refrigeration.
2. Modifications to the Carnot cycle have been proposed. This leads to the vapour compression cycle.
3. The T-s and p-h diagram for the vapour compression cycle has been discussed.
4. A thermodynamic analysis of each of the vapour compression cycle components has been performed and a COP for both refrigeration and heat pumping defined in terms of the enthalpies at various state points.

5. In certain applications where added system complexity (and cost) is acceptable for the benefit of efficiency, multistage vapour compression cycles are the solution.
6. Flash chambers can be used at an intermediate pressure between the evaporator and compressor pressures which benefits the cycle efficiency. Compression is carried out in two stages. Dry saturated vapour is diverted by the flash chamber and mixed with the vapour refrigerant from the low pressure compressor.
7. Cascaded systems make use of two or more vapour compression cycles in series. Compression is also carried out in two stages and the work required is reduced. Also, the cooling effect is increased.
8. When refrigeration is required at various temperatures, a number of evaporators can be used. This requires an equal amount of throttling valves.
9. Heat pumps transfer heat energy from a cold to a hotter source.
10. The cold source in heat pumps may be any space which has a uniform temperature and may be for instance air, water or the ground.
11. The definition of Performance Energy Ratio (PER) has been introduced in order to account for primary energy input rather than the actual work done by the compressor.

rious compressor technologies have been introduced. The student is encouraged to research further essor technologies.

Two types of compressor efficiencies have been described - Volumetric efficiency and isentropic efficiency.

5 Vapour absorption and other cycles

5.1 Principles of vapour absorption

5.1.1 Components of vapour absorption cycles

Vapour compression cycles require a compressor to do work on the refrigerant. Vapour absorption cycles replace this component with various other components in order to use heat energy rather than work as an energy input. The components unique to the vapour absorption cycle are the following:

- Absorber
- Pump(s)
- Generator (or Desorber)

The compressor is replaced by an absorber, pump and generator (and an optional heat exchanger) which is generally known as the solution heat exchanger (SHX). Another important difference from vapour compression cycles is that this system, apart from the refrigerant, makes use of an absorbent fluid.

Figure 5.1 depicts a vapour absorption cycle schematic. The absorber-generator part replaces the compressor. As can be noted in the figure, there are two additional heat transfers. In the absorber, the absorbent fluid absorbs within it the refrigerant vapour, with the consequence of a heat release.

Definition

The absorption process is exothermic. This means that during the process, heat is rejected to the surroundings.

The refrigerant-absorbent mixture is then supplied to the generator via a pump. In this unit, heat is transferred to the mixture in order to drive off the refrigerant from the mixture, leaving behind a weak solution (low percentage of refrigerant) which is transferred back to the absorber via a throttle valve. As the refrigerant-absorbent mixture is pumped to the generator some heat may be gained from the weak solution leaving the generator by means of using a solution heat exchanger. This greatly enhances the system performance.

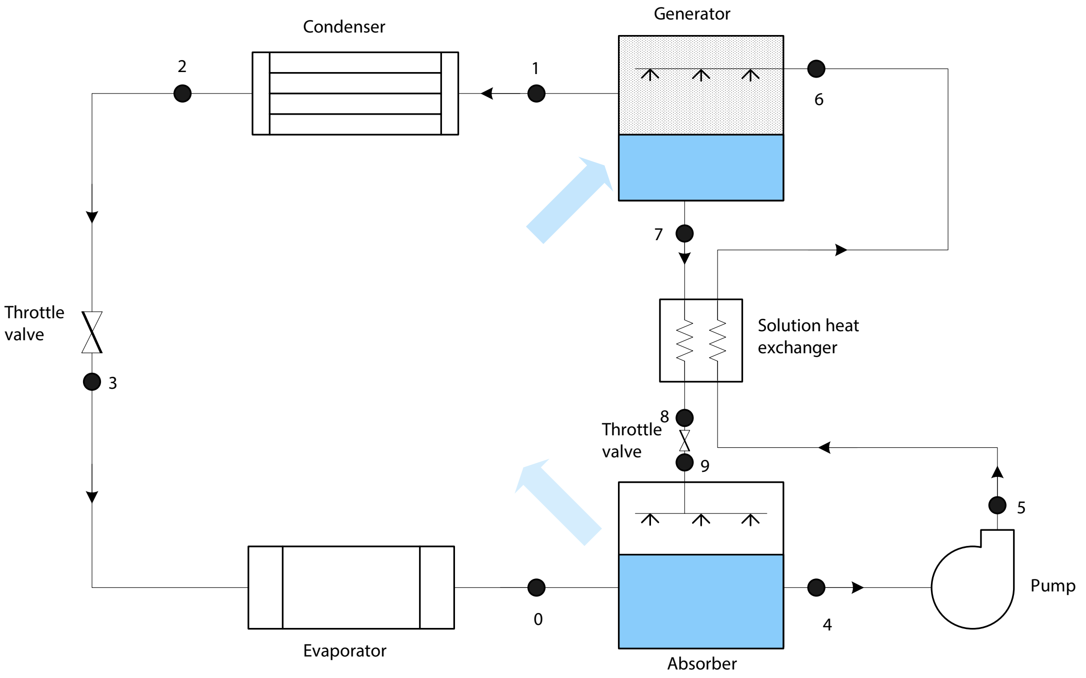


Figure 5.1: Vapour absorption cycle diagram.

The pumps required in this system consume a minute amount of power compared to compressors. In vapour compression the energy input is of a high quality (shaft power), in vapour absorption on the other hand the energy input is in the form of heat from a source at a suitable temperature. It is therefore possible to use any source provided the temperature is high enough. These sources of heat maybe the burning of natural gas, waste heat from a particular process, solar energy or even geothermal energy.

5.1.2 Refrigerants and absorbents in vapour absorption cycles

In an absorption system the working fluids used are a refrigerant and an absorbent. At particular points in the cycle we find a pure refrigerant while in others we find a solution involving a refrigerant and an absorbent. There are a number of properties required by such solutions. The most important of these are given here:

- No Crystallization – The cycle should operate between concentration limits which do not go beyond the crystallization line as otherwise a solid phase forms which would be problematic on the equipment operation.
- Volatility – The refrigerant should be much more volatile than the absorbent for easier separation in a generator.
- Affinity – The absorbent should have a strong affinity for the refrigerant under the conditions of absorption. The stronger this affinity the less the mass flow rate required for a given refrigerating effect.

- Pressures – The operating pressures should neither be too high (as this requires thicker wall structures) nor too low
- Chemical stability – High stability is needed to avoid formation of unwanted substances which may also lead to corrosiveness.
- Corrosiveness – Absorption fluids are corrosive and corrosion inhibitors are necessary.
- Safety – Certain standards are used to ensure safety under conditions of say toxicity flammability or even high pressure operation.
- Transport properties – Viscosity, surface tension, thermal and mass diffusivity are all important properties as these may affect heat transfer as well as pumping requirements.
- Latent Heat of vaporization – Again the refrigerant's latent heat of vaporization should be as high as possible such that flow rates of both the refrigerant and the solution are minimized.
- Environmental concerns – Due to rising global warming concerns the use of certain refrigerants and absorbents may be prohibited (refer to ASHRAE (2009))

Most of these requirements are met by two particular pairs:

- um Bromide as the absorbent and water as the refrigerant ($LiBr - H_2O$)
- Water as the absorbent and ammonia as the refrigerant ($H_2O - NH_3$)

There are advantages of using one pair relative to the other. Table 5.1 summarizes some of these advantages.

	LiBr -H2O	*H2O-NH3*
Refrigerant latent heat	Excellent	Very good
Toxicity	Non-toxic	Toxic refrigerant
Operating pressures	Low	High
Flammability	Non-flammable	Refrigerant flammable
Corrosiveness	Corrosive	Corrosive
Refrigerating temperature possible	Quite high	Low
Absorbent non-volatility	Excellent	Poor

Table 5.1: Comparison of $LiBr - H_2O$ and $H_2O - NH_3$.

5.1.3 Enthalpy-concentration charts

For the analysis of absorption systems, information regarding the refrigerant-absorbent pair is necessary. This may be obtained by the use of enthalpy-concentration charts. In the case of $LiBr - H_2O$ pairs, the system operates at very low pressures and the LiBr remains in its liquid phase throughout the entire cycle. Hence, when heat is supplied in the generator, the resulting vapour is composed of 100 per cent refrigerant. One problem with LiBr is that it crystallizes at moderate to high concentrations, a condition which must be avoided at all costs if the equipment is not to fail. Figure 5.2 shows the fundamental features of a $LiBr - H_2O$ enthalpy-concentration chart.

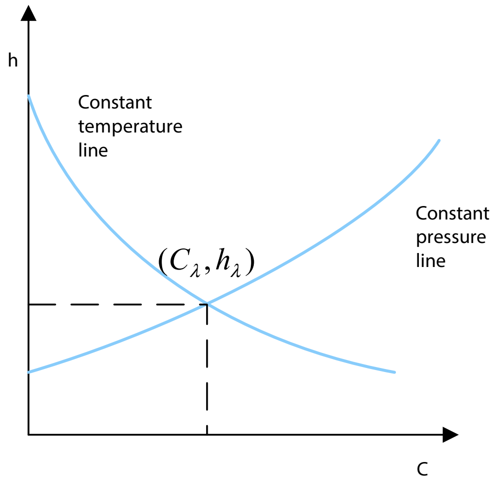


Figure 5.2: Extracting information from enthalpy concentration chart.

Such a chart is based on the fundamental assumption that the solution is saturated at a given temperature and pressure. The fixed point (C_λ, h_λ) may be established from the intersection of an isobar and an isothermal for a fixed temperature and pressure. The symbol C will be used to denote concentration of absrobent in the refrigerant. Hence, knowledge of the temperature and pressure in the absorber/generator would enable us to establish a value for the concentration of absorbent in the refrigerant-absorbent solution. Apart from this, we are also able to find the specific enthalpy of the mixture. The enthalpy concentration charts are shown in Figure A.10 in the Appendix section.

5.1.4 Thermodynamic analysis

Let the associated temperature and pressure for the generator be denoted by T_g and p_g respectively. Similarly, for the absorber, the temperature and pressure will be denoted by T_a and p_a respectively. On the other hand the condenser and evaporator quantities will be denoted *by* T_E, p_E, T_C and p_C respectively. Then, by considering the *LiBr -*H_2O absorption cycle given in Figure 5.1, we can deduce the following by neglecting all pressure drops:

$$p_g = p_C \tag{5.1}$$

$$p_a = p_E \tag{5.2}$$

By knowing the temperatures in the absorber and generator it is hence possible to establish the concentration of LiBr and enthalpy of the solution at exit from the absorber (C_4, h_4) and the generator (C_7, h_7). From the pressures and temperatures we can also find the enthalpies at every state point for the part of the cycle where we have 100 percent refrigerant (states 0 to 3). Therefore, for the required heat transfer for cooling, Q_1, the refrigerant mass flow rate is given by,

$$\dot{m} = \frac{Q_1}{h_0 - h_3} \tag{5.3}$$

Where

$$\dot{m} = \dot{m}_0 = \dot{m}_1 = \dot{m}_2 = \dot{m}_3 \tag{5.4}$$

The heat rejected from the condenser, which is important in the case of heat pump operation or in the design of the cooling unit for the system, is given by:

$$\dot{Q}_2 = \dot{m}(h_2 - h_1) \tag{5.5}$$

In order to determine the cooling requirements of the absorber, a mass and energy balance are required. Figure 5.3 depicts the mass terms going in and out of the absorber. The overall mass balance is given in eqn. 5.6. Also, considering only the mass balance of the LiBr, eqn. 5.7 may be written.

$$\dot{m}_4 = \dot{m}_0 + \dot{m}_9 \tag{5.6}$$

$$C_4\dot{m}_4 = C_9\dot{m}_9 \tag{5.7}$$

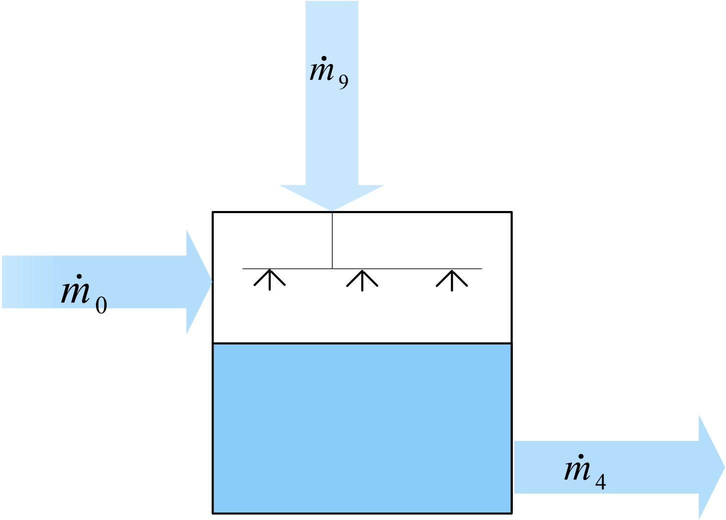


Figure 5.3: Absorber mass transfer.

The energy transfers in the absorber are shown in Figure 5.4. Applying the steady flow energy equation and ignoring kinetic and potential energy changes eqn. 5.8 maybe written ($\dot{Q}_a$ is negative since it

$$-\dot{Q}_a + \dot{m}_4 h_4 = \dot{m}_0 h_0 + \dot{m}_9 h_9 \qquad (5.8)$$

For the generator, the same principles may be applied. The mass and energy transfers are shown in Figure 5.5 and Figure 5.6 respectively. The overall mass balance, LiBr mass balance and energy balances are given in the following equations (note that $\dot{Q}_a$ is negative since it is supplied):

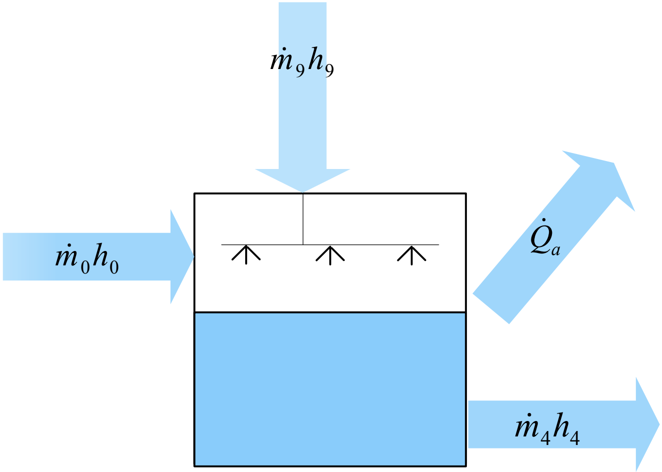


Figure 5.4: Absorber energy transfer.

$$\dot{m}_6 = \dot{m}_1 + \dot{m}_7 \tag{5.9}$$

$$C_6\dot{m}_6 = C_7\dot{m}_7 \tag{5.10}$$

$$\dot{Q}_g + \dot{m}_6 h_6 = \dot{m}_1 h_1 + \dot{m}_7 h_7 \tag{5.11}$$

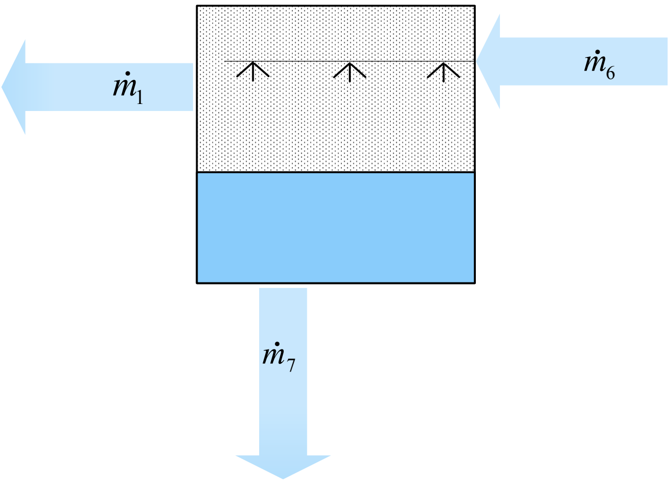


Figure 5.5: Generator mass transfer.

In many cases, in order to determine the cooling requirement of the absorber or the heating requirement of the generator, it is sufficient to perform an analysis on one of these sub-systems. The reason for this is that by knowing the cooling capacity and the condenser heat rejected the unknown energy quantity can be obtained by applying the first law of thermodynamics:

The COP for refrigeration and for heat pumping is given by:

$$\dot{Q}_1+\dot{Q}_2+\dot{Q}_a+\dot{Q}_g=0 \tag{5.12}$$

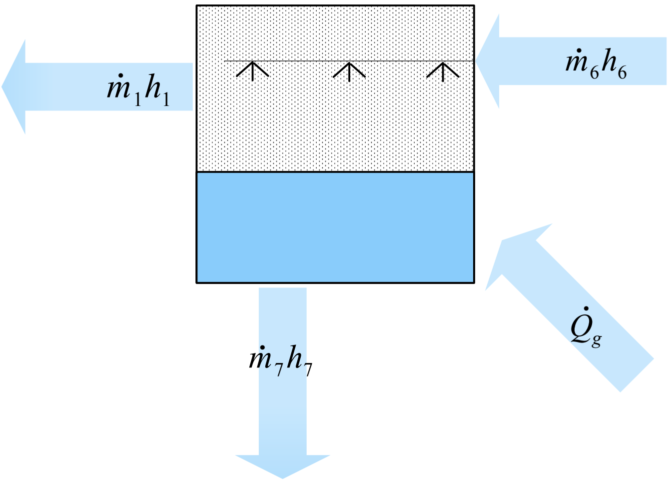


Figure 5.6: Generator energy transfer.

$$COP_{ref}=\frac{Q_1}{Q_g} \tag{5.13}$$

$$COP_{hp}=\frac{Q_2}{Q_g} \tag{5.14}$$

For an exergy analysis of LiBr-H_2O system the reader may refer to Palacios-Bereche et al. (2012).

Example

Example 1. *A solar cooling chiller using a lithium bromide - water solution is used for the fermentation of white wine. An array of solar panels using concentrators provide heat to the generator. The temperature of the wine must be kept to around 7^oC. The refrigeration capacity required is 30kW. The absorber temperature is at 25^oC. The generator is kept at a temperature of 100^oC. The generator pressure is at 0.2bar. No solution heat exchanger is used. For a refrigerant mass flow rate of 0.0134kg/s Calculate:*

1. *The enthalpy and concentration of the solution leaving the generator to the absorber*
2. *The enthalpy and concentration of the solution leaving the absorber to the generator*
3. *The mass flow rates in the low and high H_2O concentration lines*
4. *The heat input required in the generator*
5. *The system COP*

Answer.

Note: The reader can find the specific enthalpy-concentration chart for the LiBr-H_2O solution. The state point labels used here correspond to those shown in the system diagram of Figure 5.8.

1. *The absorber is at the same pressure as the evaporator at 0.01bar and a temperature of 25^oC. At these conditions, from the LiBr-H_2O chart: $C_4 = 0.48$, $h_4 = -173kJ/kg$. Note: In order to have a cooling effect, there must be a temperature difference between the wine and the evaporator temperature which is hence less than 7^oC*

2. *The generator is at the same pressure as the condenser at 0.2bar and 100^oC. At these conditions, from the LiBr-H_2O chart: $C_6 = 0.57$, $h_6 = -42kJ/kg$.*

3. *Mass balance of LiBr in absorber:*

$$C_4 m_4 = C_7 m_7$$

Total mass balance in absorber:

$$m_0 = m_4 - m_7$$

Substituting for m_7 using the first equation:

$$m_0 = \frac{C_7}{C_4} m_7 - m_7$$

$$m_7 = \frac{m_0}{\frac{C_7}{C_4} - 1}$$

$$m_7 = \frac{0.0134}{0.1875} = 0.0715kg/s$$

hence,

$$m_4 = \frac{C_7}{C_4} m_7 = 0.0849kg/s$$

4. *The generator heat input can be found using an energy balance in the generator:*

$$Q_g = m_1h_1 + m_6h_6 - m_5h_5$$

The enthalpy of the superheated steam at exit from the generator is taken at the generator exit temperature and the generator pressure (100^oC and 0.2bar respectively). The enthalpy at state point 1 is therefore $h_1 = 2675kJ/kg$:

$$Q_g = 0.0134 \times 2675 + 0.0715 \times (-42) - 0.0849 \times (-173)$$

$$Q_g = 35.85 - 3.003 + 14.69$$

$$Q_g = 47.54kW$$

5. *The COP for refrigeration can be found as follows:*

$$COP = \frac{Q_1}{Q_g} = \frac{30}{47.54} = 0.63$$

5.2 Dühring charts

An important chart for the analysis of vapour absorption systems is the equilibrium chart. A Dühring plot shows the cycle which the solution undergoes on the equilibrium chart. For the $LiBr - H_2O$ pair, the chart plots water saturation pressure (or corresponding saturation pressure) against the solution temperature. The concentration of LiBr in solution is also represented on such a chart. These are shown as constant concentration lines. With increasing solution temperature the liquid LiBr transforms into solid crystals. Figure 5.7 shows an example of Dühring plot for the cycle shown in Figure 5.8. The Dühring chart for $LiBr–H_2O$ solutions is shown in Figure A.9 in the Appendix section.

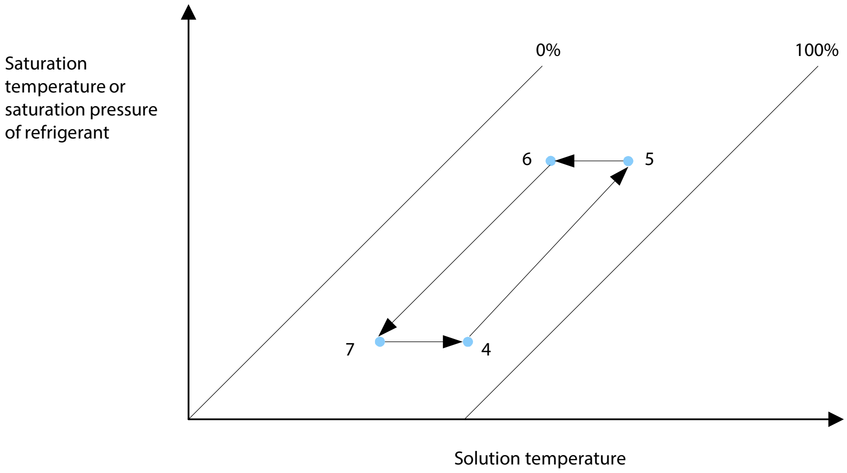


Figure 5.7: Dühring plot for a LiBr-water vapour absorption system without SHX.

5.3 Ammonia – water absorption systems

The requirements of refrigerant-absorbent pairs include the need that the refrigerant must have a much higher volatility than the absorbent for easy separation in the generator. The absorbent in water-ammonia systems is the water which has a volatility which is quite close to that of ammonia. Moreover, a lower latent heat of vaporization as well as higher operating pressures make $H_2O - NH_3$ systems more complicated than systems employing $LiBr - H_2O$. Higher operating pressures also means that the system must be more robust, employing thicker shell sections.

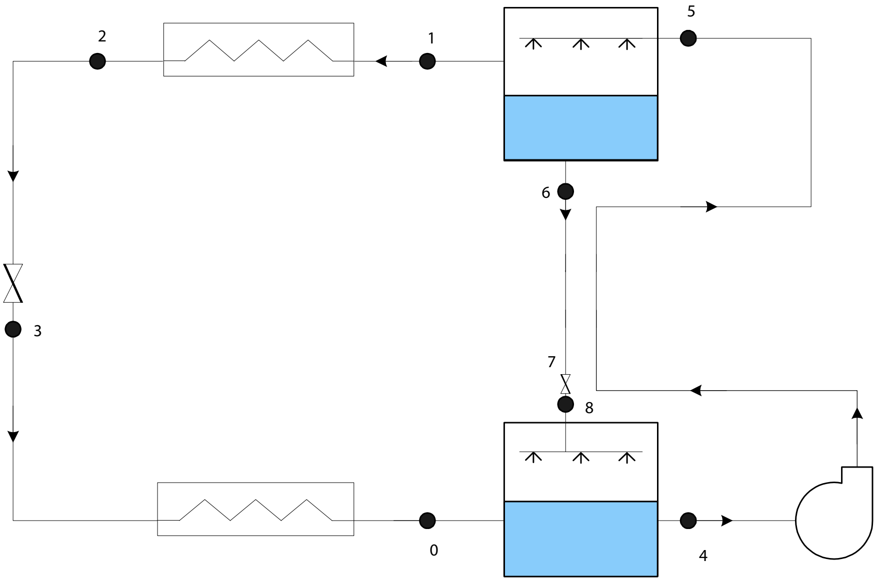


Figure 5.8: LiBr-water vapour absorption system without SHX.

This can result in increased costs. Since the refrigerant in $LiBr - H_2O$ systems is water, evaporator temperatures cannot approach and operate the freezing point of water. The ammonia in $H_2O - NH_3$ is the refrigerant and hence when evaporator temperatures below *0°C are* required, such a system can provide an alternative to the use of $LiBr - H_2O$ which cannot be used near the freezing point of water.

A refrigerant heat exchanger (RHX) or sub-cooler maybe used to improve system performance. The advantages of under-cooling the refrigerant were already introduced. Heat is hence transferred from the refrigerant exiting the condenser to the stream exiting the evaporator. This increases the cycle COP at the expense of increasing the heat rejected in the absorber. The choice between counter flow, parallel flow or cross-flow heat exchangers is rather important for cycle performance optimization. Another addition to the cycle is the *solution cooled rectifier* (SCR) or simply the *rectifier.* This is used to avoid water transferring to the condenser during the desorption process in the generator. With the SCR, this water is hence transferred back to the generator whereas the ammonia is transferred to the condenser.

A diagram of an ammonia-water vapour absorption cycle is shown in Figure 5.9. The RHX and the rectifier are also shown.

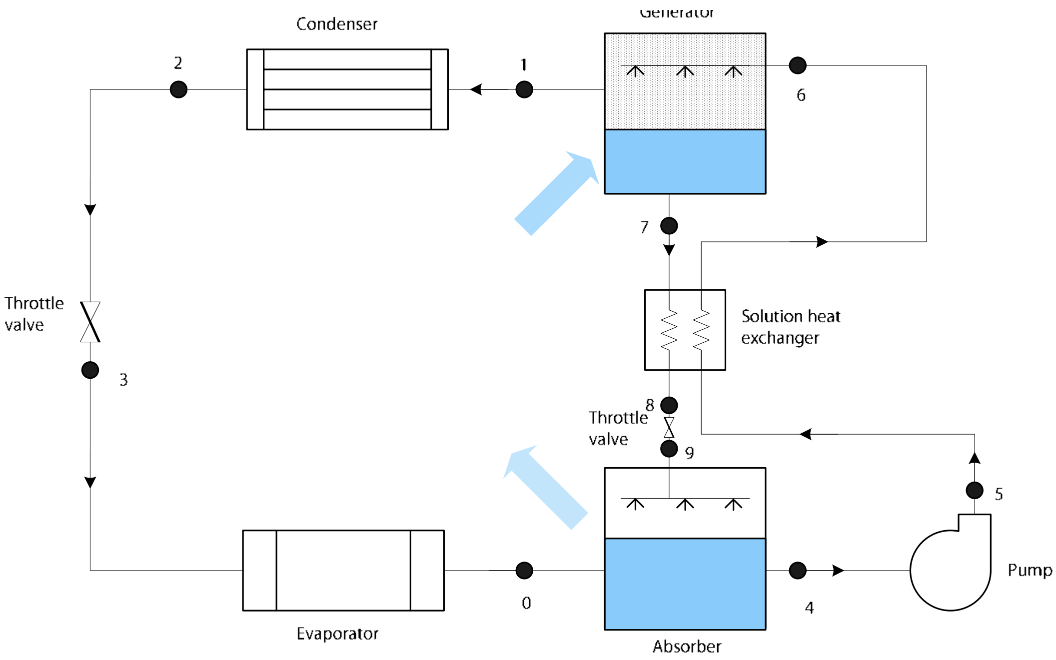


Figure 5.9: Ammonia-water system using a refrigerant heat exchanger and a rectifier.

5.4 Absorption chillers and heat pumps

Absorption heat pumps maybe employed in space heating applications while absorption chillers (refrigerators) maybe used in space cooling. The practical construction maybe quite different from that which maybe suggested in Figure 5.1. The construction of single stage heat pump systems is the same as in absorption chillers but the location of components would be different depending on which region requires the necessary heat transfer. It is most often convenient to engineer a system in such a way that the necessary cooling is obtained in a certain period of the year while heating is possible during the rest of the year. Figure 5.10 shows the heat pump or chiller system, this time using a diagrammatic representation with low and high pressure shells which contain the evaporator/absorber and the condenser/generator respectively.

5.5 Heat transformers

Heat at a low temperature is most often considered as waste heat or low grade energy. Absorption systems may be used to increase (or boost) the temperature of a fluid. Such systems are known as heat transformers. Most heat transformers are single effect and use a $LiBr - H_2O$ solution.

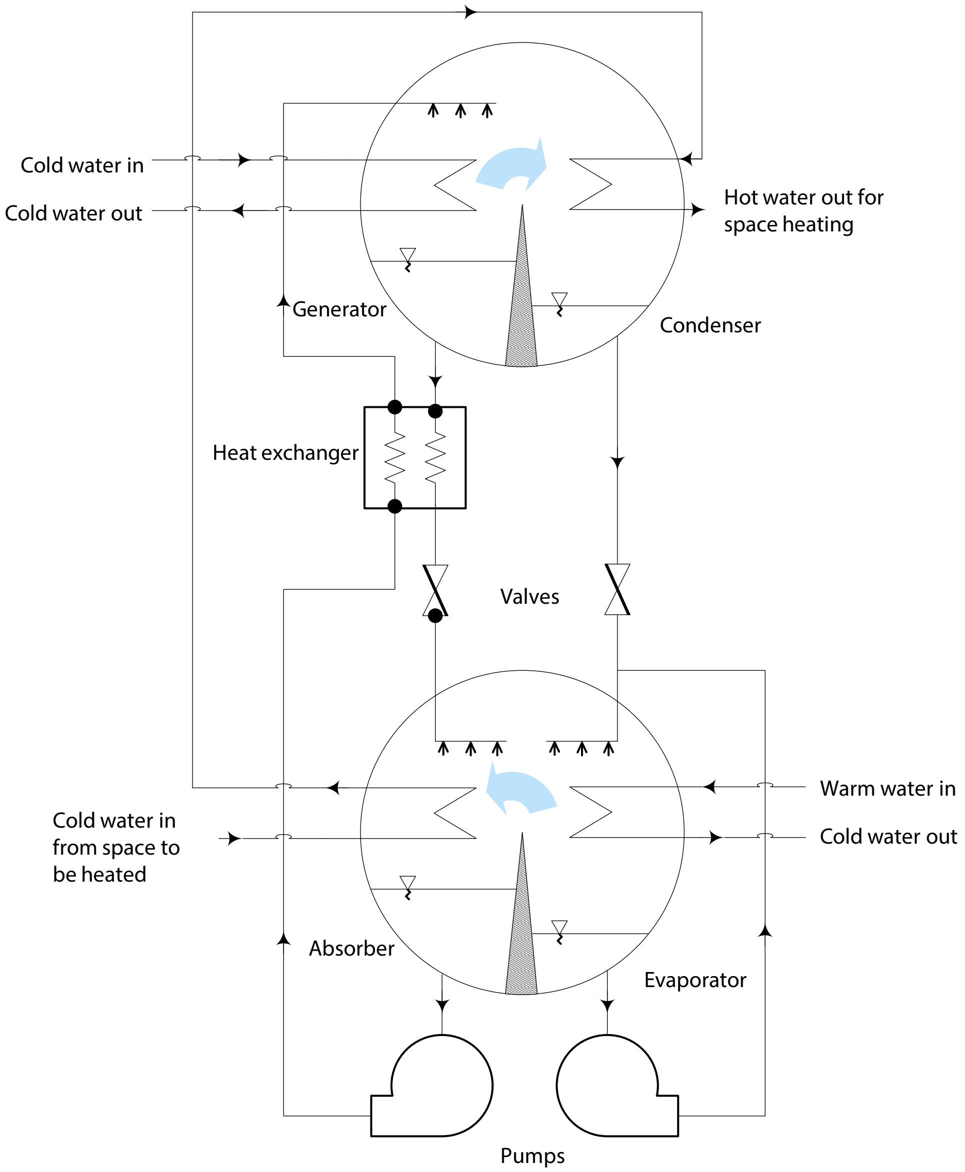


Figure 5.10: Single stage vapour absorption chiller/heat pump

Referring to figure 5.11, the temperature boosting which occurs is from an initial waste heat temperature T_i to a final temperature T_f. As can be seen, the variations from a single stage heat pump are indeed minimal. There are however three important differences:

- The positions of the evaporator/absorber shell and the condenser/generator shell are exchanged because in heat transformers it is the former shell which is at the highest pressure while the latter has the lowest pressure in the system.
- In the recuperative heat exchanger the heat is transferred to the strong H_2O solution, i.e. the solution pumped from the generator to the absorber.
- The aim of a heat transformer is to obtain a temperature boost $\Delta T = T_f - T_i$ while in a heat pump/refrigerator, the primary aim is to increase the overall COP.

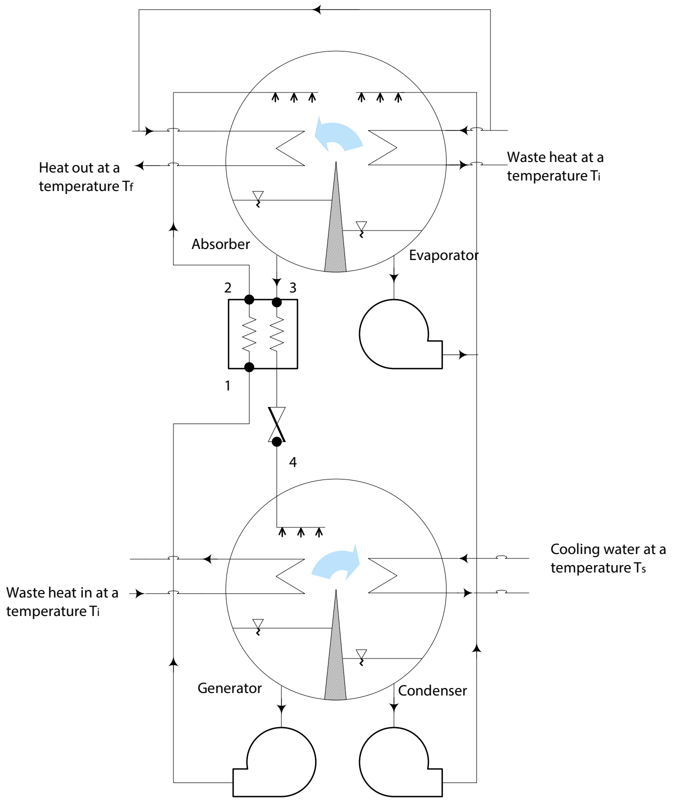


Figure 5.11: Single stage vapour absorption heat transformer

The COP of a heat transformer is defined in a slightly different manner than the COP of an absorption heat pump or chiller. The increase in heat energy at the output is considered and divided by the heat input from the source fluid (refer to Figure 5.11).

$$COP_{ht} = \frac{Q_{out,boost}}{Q_{in,source}} \tag{5.15}$$

The Dühring plot for the cycle is shown in figure 5.12. Again, apart from minor variation in the shape of the plot there is also a difference in the direction of the cycle. This is essentially due to the difference in pressure levels of the evaporator/absorber and condenser/generator pair.

The majority of heat transformer systems nowadays are single stage systems. However there may be the need of multistage systems (generally two) if larger temperature boosts are necessary.

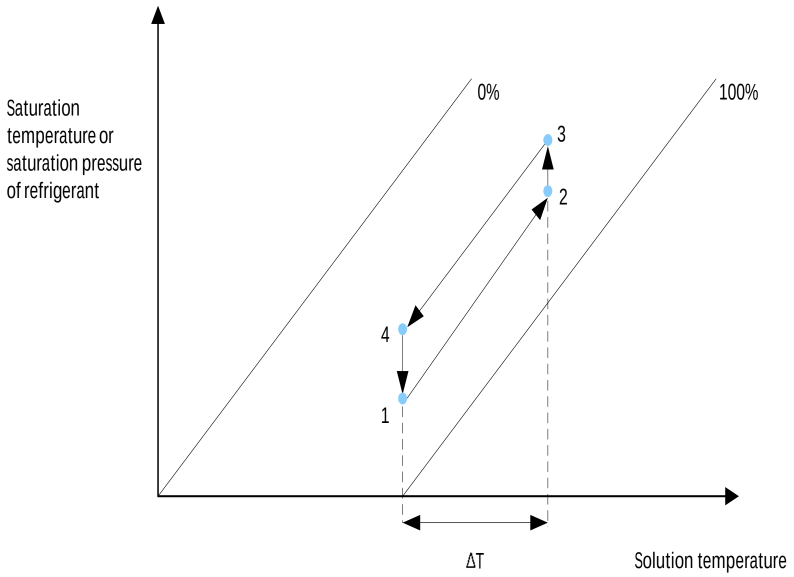


Figure 5.12: Dühring plot for a vapour absorption heat transformer

5.6 Multistage vapour absorption cycles

In general, when we talk about a multistage system we are referring to any absorption cycle where either the evaporator, absorber, generator or the condenser are present more than once in the system. For this reason, there may be various combinations and configurations of multistage systems giving different performances. Another term which needs due attention is multieffect. It signifies the number of times heat is utilized during an entire cycle. Multistaging can be obtained by one of the following ways:

- Sharing of the components between two or more systems. This enables an integration of two or more systems into one multistage system
- Having intrinsically separate systems and enabling an interaction of these systems via heat transfer

Multistage systems may also be thought of being composed of a number of elementary loops. The most fundamental loop would be the single effect absorption cycle. Thus, for each application of the absorption cycle to heat pumps, chillers or heat transformers various configurations may be chosen for a specific need such as higher COPs or say temperature boosting. In fact, multistage systems enable such increases in temperature and COPs when compared to the single effect systems. Of course, the more the elementary loops in the system the higher the cost. Indeed, economic analysis is hence essential.

A schematic diagram of a double effect absorption chiller is shown in Figure 5.13. In this case, heat energy is transferred directly to the high temperature generator by the burning of natural gas. However it is obvious that any other energy source maybe employed. The vapourised refrigerant from the high temperature generator is transferred to the low temperature generator where it condenses thus transferring heat to the solution in the generator. Some of the solution in the low temperature generator may actually flow to the high temperature generator. The rest of the cycle is the standard single effect loop.

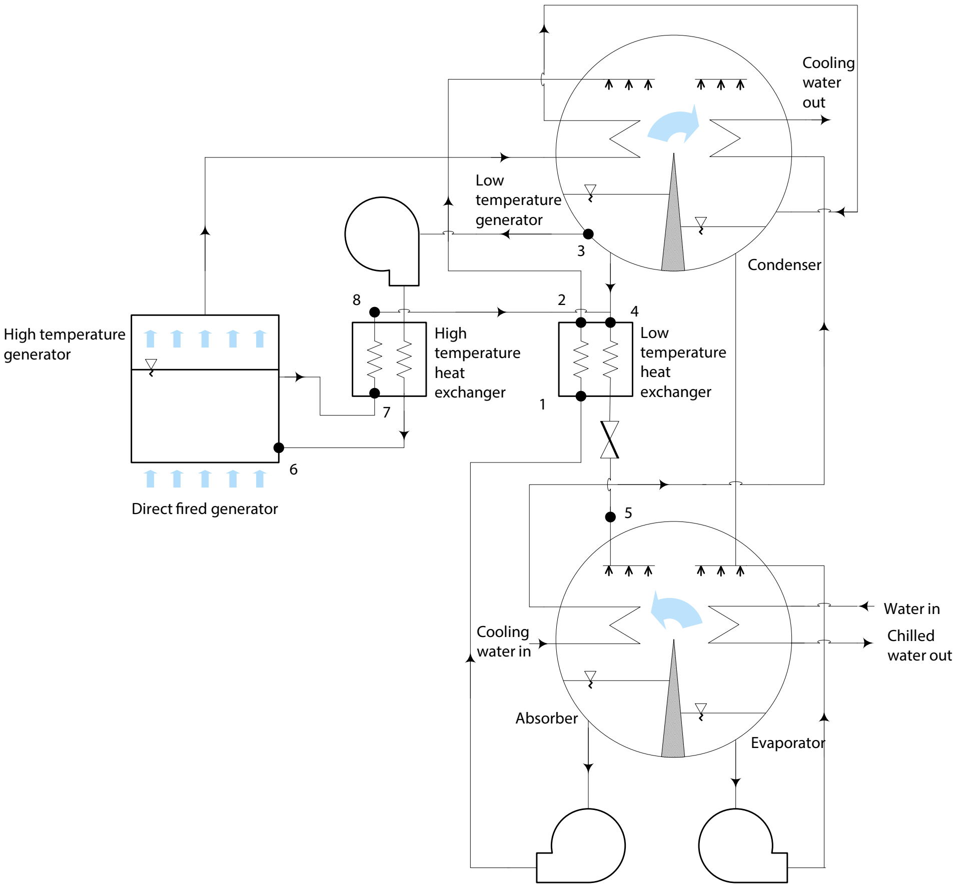


Figure 5.13: Double effect, direct fired LiBr-water absorption chiller as adapted from Wang (2000)

On Figure 5.14 the Dühring plot enables us to visualize the cycle. There are in fact three pressure levels being only a few mmHg absolute in the absorber/evaporator, an intermediate pressure in the low temperature generator/condenser and a high pressure in the high temperature absorber. One can also observe four concentration values in the entire cycle. At state 1 the LiBr concentration is a minimum and hence the solution is dilute. Heat is transferred to the solution via the low temperature heat exchanger, raising its temperature. In the low temperature generator some heat is again transferred and some refrigerant boils off increasing the temperature of the solution and the percentage of LiBr. As can be noticed some of the solution from the high temperature generator is transferred to state 4 and mixed with the solution from the low temperature generator. The solution therefore becomes weaker in refrigerant as the percentage of LiBr increases further. In the recuperator, heat is transferred from the solution to the strong refrigerant solution hence reducing its temperature to that at state 5. At state 6 on the other hand we have the same concentration as state 3, but now the stream has been heated by means of the high temperature heat exchanger. Heating of the solution at the high temperature generator causes more refrigerant to be boiled off than in the low temperature generator. Hence, at state 7 we have the highest LiBr concentration. This is then cooled in the same high temperature heat exchanger to state 8.

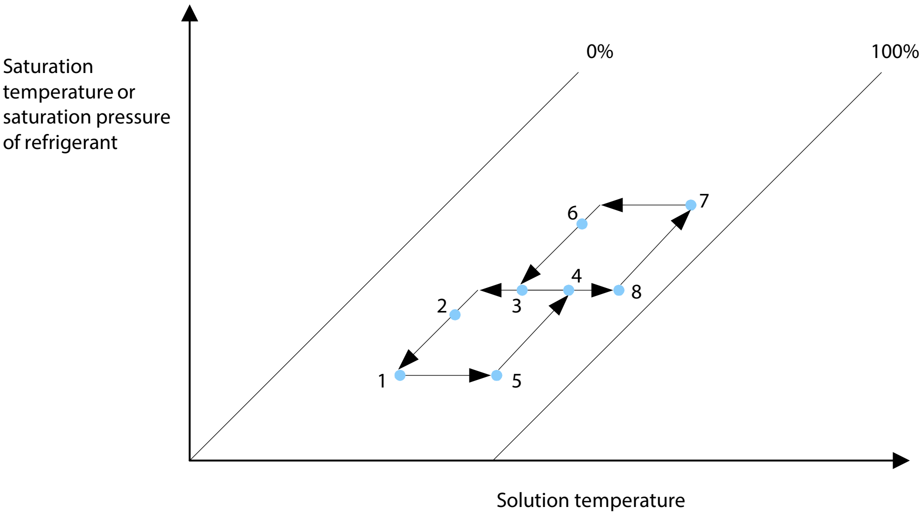
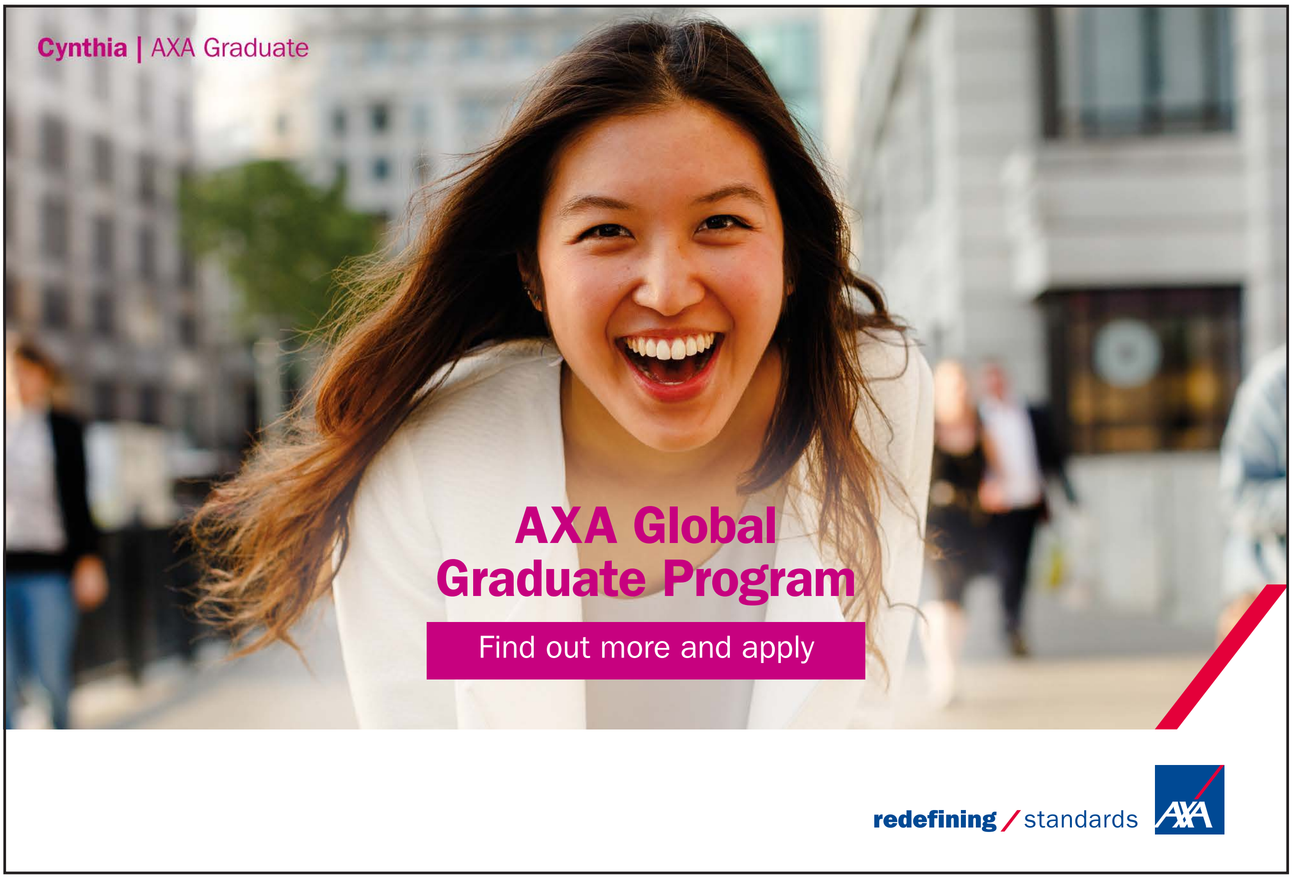


Figure 5.14: Dühring plot for a double effect vapour absorption system.



Due to the high pressures involved in $H_2O - NH_3$ systems, it would be impractical to think of multi-effect systems based on various pressure levels. The pressures would in fact be very high and much more robust equipment would be required which maybe both expensive as well as cumbersome. However, multi-effect $H_2O - NH_3$ systems do exist, but are based on concentration staging. The performance of water-ammonia cycles is improved by the use of a generator-absorber heat exchanger. Cycles employing such a concept are therefore more commonly known as GAX cycles. A single effect GAX cycle would give the same performance as a single effect $LiBr - H_2O$ cycle. GAX technology is not possible with $LiBr - H_2O$ cycles due to crystallization of the lithium bromide. These systems provide a very good COP in heating mode. The reader is referred to Kang et al. (2000), Jawahar and Saravanan (2010) and Mehr et al. (2013) for a detailed review on advanced vapour absorption cycles.

5.7 Gas liquefaction

Gases may be liquefied by cooling them (at atmospheric pressure) below a certain *critical temperature* which depends on the gas. Above the critical temperature, the fluid is in a gaseous state. Below the critical temperature a liquid starts forming. Refrigeration for the purpose of gas liquefaction is very important in the area of cryogenics which is a broad field encompassing low temperature systems (generally below 173K). Gases which are commonly liquefied are Air, Nitrogen, Oxygen and Argon.

The Linde-Hampson cycle is particularly common in gas liquefaction. The process schematic is shown in 5.15. Makeup gas is introduced at state 1 where it is mixed with the gas exiting the heat exchanger. The mixture is compressed to a high pressure at constant temperature at state 2. It is then cooled by means of the heat exchanger to state 3 where it is throttled irreversibly to state 4 where a mixture of gas and liquid is present. The liquid is removed at state 5 while the remaining gas is fed to the heat exchanger where it is heated to state 1 where it is mixed with more makeup gas.

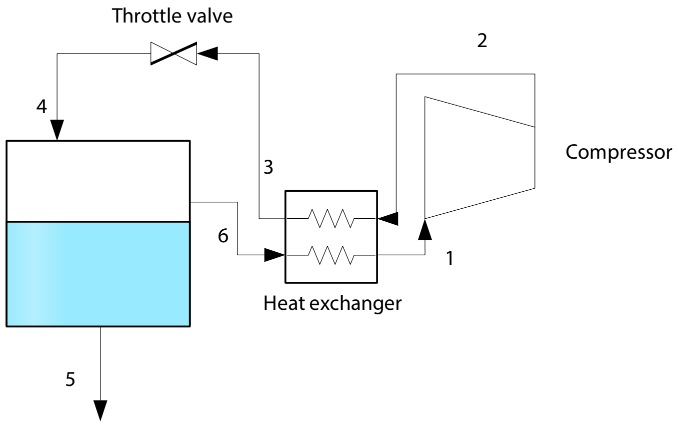


Figure 5.15: Linde-Hampson cycle components.

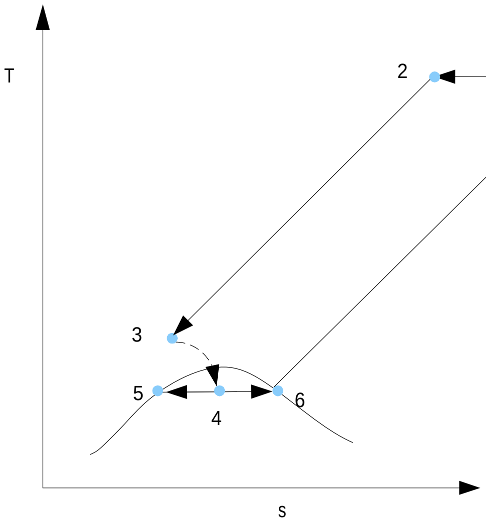


Figure 5.16: Linde-Hampson cycle T-s diagram.

5.8 Steam jet refrigeration

Using water as a refrigerant, steam jet refrigeration can provide the necessary cooling needs at temperatures above approximately *5°C*. Such systems find common use in air-conditioning since the temperatures usually required are not too low and also because water is perfectly safe if leakages occur.

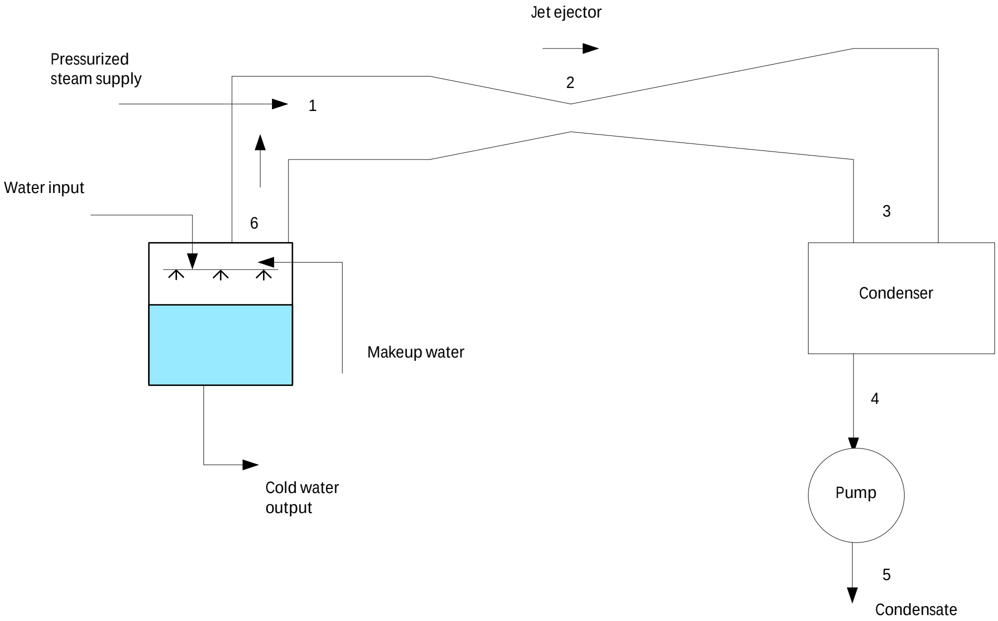


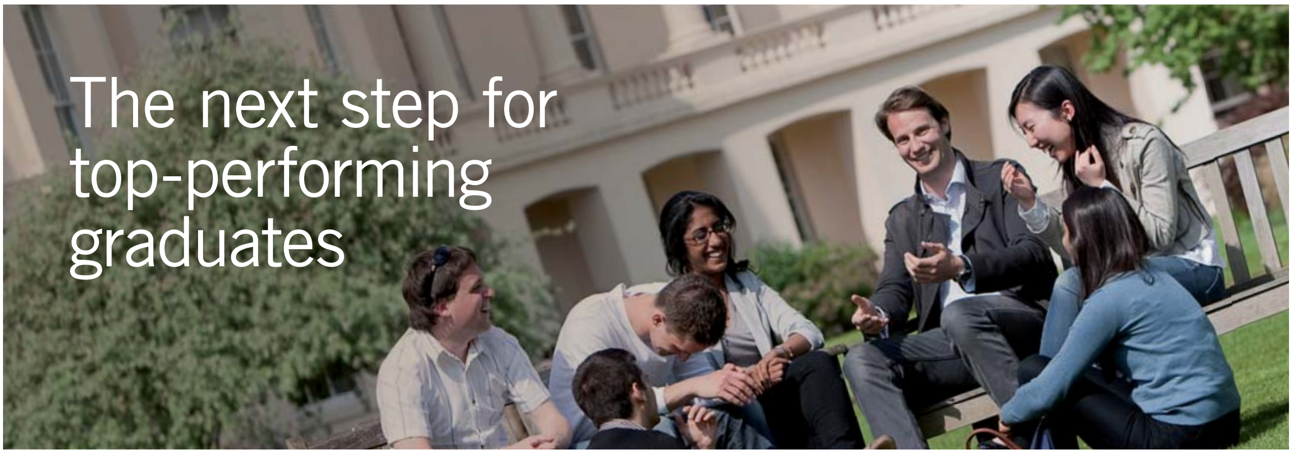
Figure 5.17: Steam jet refrigeration cycle.

With reference to Figure 5.17, high pressure steam is supplied at state 1 where it mixes with the steam from state 6. A jet ejecter is used where the mixture is accelerated at first with a consequential reduction in pressure but is then expanded in the diffuser section where it gradually attains a high pressure and is fed to the condenser. The condensate is pumped out of the system and into the boiler from where steam may be obtained and introduced back into the system at state 1. At state 6, vapour from the flash cooler is continuously drawn up to the jet ejector. The rest of the water is at the desired temperature and is transferred for cooling purposes. Make up water is continuously introduced into the system.

5.9 Air standard refrigeration cycle

Air can act as a suitable refrigerant. In such a case, a reversed *Brayton* cycle can be used. The cycle and the corresponding T-s diagram are shown in Figures 5.18 and 5.19 respectively.

The only difference from the standard vapour compression cycle is that the throttle valve is replaced by a turbine.



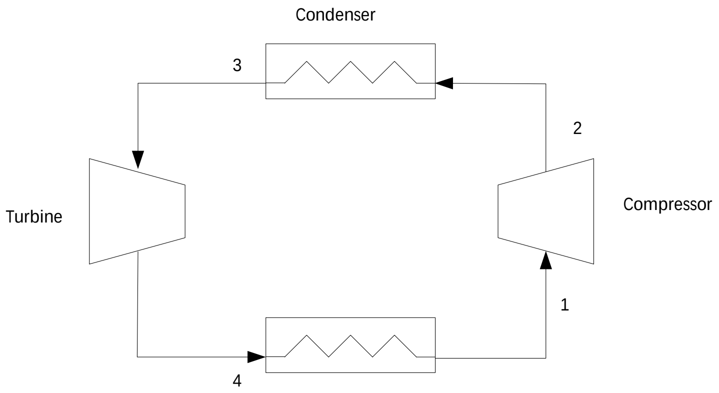


Figure 5.18: Air-standard refrigeration cycle.

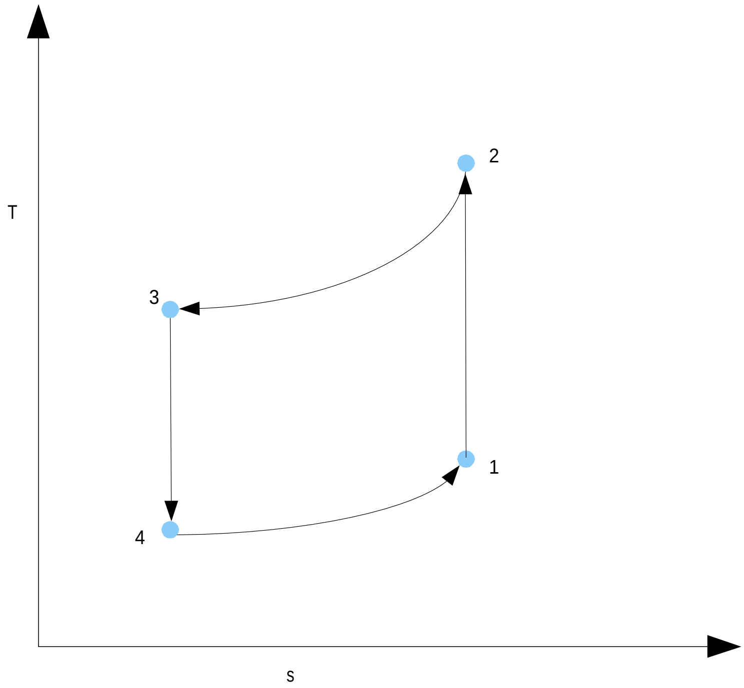


Figure 5.19: Air-standard refrigeration cycle T-s diagram.

The cycle shown in Figure 5.19 applies for a reversible (and hence isentropic) expansion and compression. The ideal gas law can be used to analyze the system since air is a perfect gas.

The work done by the compressor is given by:

$$W_{12} = h_2 - h_1 = c_p(T_2 - T_1) \tag{5.16}$$

$$W_{34} = -(h_4 - h_3) = -c_p(T_4 - T_3) \tag{5.17}$$

On the other hand, the refrigeration heat transfer is given by:

$$Q_1 = h_1 - h_4 = C_p(T_1 - T_4) \tag{5.18}$$

The relationship between temperature and pressure during the idealized adiabatic processes of compression and expansion is:

$$\frac{T_1}{T_2} = \frac{P_1^{\frac{\gamma-1}{\gamma}}}{P_2} \tag{5.19}$$

where γ is the adiabatic index. Multiple stages can be used in order to obtain the required low temperatures. The COP is given by:

$$COP_{ref} = \frac{-c_p(T_1 - T_4)}{c_p(T_2 - T_1) - c_p(T_4 - T_3)} \tag{5.20}$$

$$COP_{ref} = \frac{T_1 - T_4}{T_2 - T_3 - (T_1 + T_4)} \tag{5.21}$$

Such a system may easily be interfaced with another system such as air-conditioning units. This is yet another advantage of the air-standard cycle. Another application is in the aviation industry where compressed air is bled off in the early compression stages of the engine.

5.10 Conclusions

Student checklist

1. The work input required to transfer heat from a low to a high temperature reservoir can be avoided by replacing a compressor with various sub-components.
2. These sub-components are the absorber, generator (or desorber) and a pump.
3. Such a system is known as a vapour absorption cycle.
4. The vapour absorption cycle uses two fluids; a refrigerant and an absorbent. The absorber-generator part of the cycle uses a refrigerant-absorbent solution as the working fluid while the rest of the cycle operates by means of a pure refrigerant.
5. The most common absorbent-refrigerant pairs are $LiBr - H_2O$ and $H_2O - NH_3$.
6. A number of desirable properties for the absorbent-refrigerant solutions have been discussed.
7. Enthalpy-concentration charts are required in order to be able to solve for the energy flows in vapour absorption systems.
8. The use of Dühring plots is useful for representing the vapour absorption cycle.
9. Ammonia-water systems were introduced with particular advantages over $LiBr - H_2O$ systems. Due to the different characteristics of the $H_2O - NH_3$ pair, other system components need to be introduced which means increased system complexity. Still certain cooling requirements such as temperatures below 0^oC cannot be attained by means of the $LiBr - H_2O$ pair.
10. Heat transformers can be used to boost the temperature of a fluid. The COP for a heat transformer is defined as the ratio of heat energy of the fluid which has been boosted to a higher temperature to the heat energy from the source fluid.
11. An introduction to multistage vapour absorption systems has been given. Although multistage systems increase COP, they are more complex and require a larger capital investment.
12. Gas liquefaction was introduced an important field in cryogenics and the Linde-Hampson cycle was introduced.
13. The steam jet refrigeration cycle was described as an alternative to other refrigeration cycles when temperature above 5^oC are required.
14. Air can be used as a refrigerant in a reversed Brayton cycle refrigeration system. Sine air is an ideal gas, the ideal gas laws may be used.

6 Principles of numerical modelling and experimentation

6.1 General idea

Modelling thermodynamic systems can be performed on a sub-component level (such as valves, heat exchangers etc.). This type of modelling generally requires the use of thermo-fluid analysis. This can be done through the use of computational fluid dynamics (CFD). Such an approach is generally rather time-consuming and is in most cases adopted at a later stage in the design process. Another approach is to globally model the entire thermodynamic system. Each sub-component can be described by a set of equations based on mass and energy balance equations. Sub-components may either have a single input and output or even multiple inputs and outputs.

When combining all equations from every sub-component, it is not possible to find a solution to the problem. The operating conditions of the system must be input. This generally include pressure levels, ratures or for the refrigeration cases, cooling capacity. With the inputs provided by the , the resulting system of equations should be solved using one of various numerical methods which are available to the engineer (see Faires and Burden (2012) and Isaacson and Keller (1994)).

The resulting system of equations can either be *linear* or *non-linear* depending on the unknowns of the system. Linear systems of equations are rather easy to solve and do not require substantial computational time. When dealing with relatively complex non-linear systems, the problem becomes more time-consuming. Today's computational technology however generally still permits that such systems may be solved within acceptable time frames.

6.1.1 Equations for various components

The mathematical equations for the various thermodynamic components will be introduced in this section.

Connections

Connections link one sub-system to the next. Thermodynamic quantities of working fluids are assumed to remain the same at entry and exit of the connection. A representation is shown in Figure 6.1. The relevant mass balance, concentration balance, pressure and energy balance equations are given next.

Figure 6.1: Connection element.

Mass balance:

$$\dot{m}_i = \dot{m}_j \tag{6.1}$$

Concentration:

$$C_i = C_j \tag{6.2}$$

Pressure:

$$p_i = p_j \tag{6.3}$$

Energy balance:

$$h_i = h_j \tag{6.4}$$

Valve

The valve is used to create a pressure drop across it. A representation is shown in Figure 6.2. The relevant equations follow.

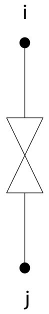


Figure 6.2: Throttle valve element.

Mass balance:

$$\dot{m}_i = \dot{m}_j \tag{6.5}$$

Concentration:

$$C_i = C_j \tag{6.6}$$

Pressure:

$$p_i \neq p_j \tag{6.7}$$

Energy balance:

$$h_i = h_j \tag{6.8}$$

Evaporator

The evaporator sub-system is shown in Figure 6.3 and the its mass balance, concentration, pressure and energy balance equations shown next.

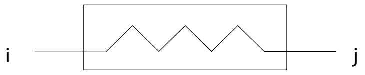


Figure 6.3: Evaporator element.

Mass balance:

$$\dot{m}_i = \dot{m}_j \tag{6.9}$$

Concentration:

$$C_i = C_j \tag{6.10}$$

Pressure:

$$p_i = p_j \tag{6.11}$$

Energy balance:

$$\dot{Q} = \dot{m}_i (h_j - h_i) \tag{6.12}$$

Condenser

The condenser sub-system can be described by the same equations as for the evaporator except for the energy balance equation where a heat rejection occurs rather than a heat input. The figure for the system is shown in Figure 6.4.

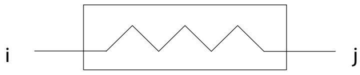


Figure 6.4: Condenser element.

Mass balance:

$$\dot{m}_i = \dot{m}_j \tag{6.13}$$

Concentration:

$$C_i = C_j \tag{6.14}$$

Pressure:

$$p_i = p_j \tag{6.15}$$

Energy balance:

$$\dot{Q} = \dot{m}_i (h_j - h_i) \tag{6.16}$$

Absorber

The absorber sub-system was introduced in Chapter 5. Figure 6.5 shows a diagram of an absorber unit.

Mass balance:

$$\dot{m}_i + \dot{m}_j = \dot{m}_k \tag{6.17}$$

Concentration:

$$\dot{m}_i C_i + \dot{m}_j C_j = \dot{m}_k C_k \tag{6.18}$$

Pressure:

$$p_i = p_j = p_k \tag{6.19}$$

Energy balance:

$$\dot{m}_i h_i + \dot{m}_j h_j - \dot{Q}_a = \dot{m}_k h_k \tag{6.20}$$

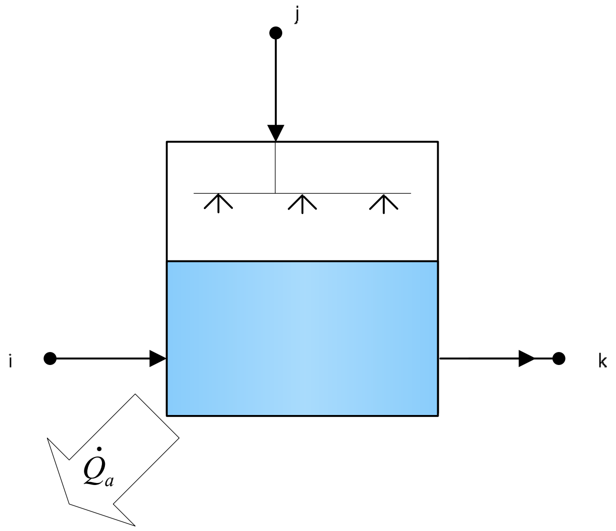


Figure 6.5: Absorber element.

Generator

The generator sub-system was also introduced in Chapter 5. The unit is shown in Figure 6.6. Its mass balance, concentration, pressure and energy balance are given next.

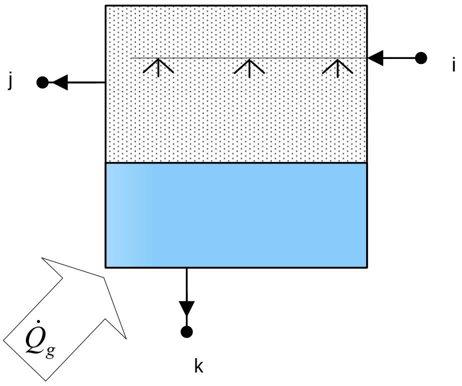


Figure 6.6: Generator element.

Mass balance:

$$\dot{m}_i = \dot{m}_{kj} + \dot{m}_{\mathbf{k}} \tag{6.21}$$

Concentration:

$$\dot{m}_i C_i = \dot{m}_j C_j + \dot{m}_{\mathbf{k}} C_k \tag{6.22}$$

Pressure:

$$p_i = p_j = p_k \tag{6.23}$$

Energy balance:

$$\dot{m}_i h_i + \dot{Q}_g = \dot{m}_j h_j + \dot{m}_k h_k \tag{6.24}$$

Heat exchanger

The heat exchanger component is shown in Figure 6.7. The mass balance, concentration, pressure and energy balance equations follow.

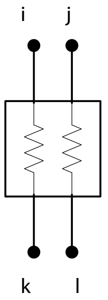


Figure 6.7: Heat exchanger element.

Mass balance:

$$\dot{m}_i = \dot{m}_k \tag{6.25}$$

$$\dot{m}_j = \dot{m}_l \tag{6.26}$$

Concentration:

$$C_i = C_k \tag{6.27}$$

$$C_j = C_l \tag{6.28}$$

Pressure:

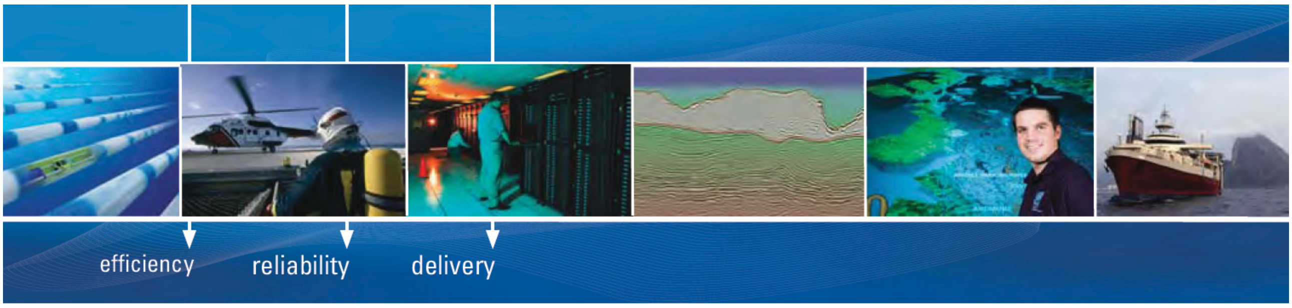
$$p_i = p_k \tag{6.29}$$

$$p_j = p_l \tag{6.30}$$

Energy balance:

$$\dot{Q} = \varepsilon(\dot{m}c)_{min}(T_{h,in} - T_{c,in}) = \dot{m}_i c_i (T_i - T_k) = \dot{m}_l c_l (T_j - T_l) \tag{6.31}$$

where ε is the heat exchanger effectiveness, c is the specific heat capacity of the working fluid, $T_{h,\,in}$ is the hot stream temperature at the inlet, $T_{c,in}$ is the cold stream temperature at the inlet.



Pump

The scope of the pump is to raise the pressure of the working fluid to a higher value. A figure is shown in Figure 6.8. The relevant equations are shown next.

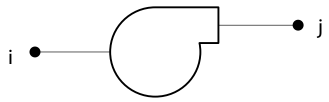


Figure 6.8: Pump element.

Mass balance:

$$\dot{m}_i = \dot{m}_j \tag{6.32}$$

Concentration:

$$C_i = C_j \tag{6.33}$$

Pressure:

$$p_i \neq p_j \tag{6.34}$$

Energy balance:

$$\dot{W} = \dot{m}\,(h_j - h_i) \tag{6.35}$$

6.1.2 Solving linear systems

A linear system of equations can be represented as follows (see Kreyszig (2010)):

$$Ax = b \tag{6.36}$$

where

$$A = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix}$$

$$\mathrm{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$\mathrm{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Such a system may have:

- A unique solution
- No solution
- Infinitely many solutions

The solution of linear systems is dependent on the number of equations *n* and the number of unknowns *m*. The following three definitions give the classification of such the system. In the modelling of thermodynamic systems we seek a determined solution where the number of equations is equal to the number of unknowns.

Definition

A system is said to be indetermined if $n < m$.

Definition

A system is said to be determined if $n = m$.

Definition

A system is said to be overdetermined if $n > m$.

A single stage vapour compression cycle, as will be shown in this chapter, can me modelled as a determined linear system of equations given the certain inputs from the user. A computer code can be used to solve such a system. Mathworks' Matlab(®) program will be introduced. Solving a linear system is rather straight forward and an example will be shown next for demonstration purposes.

Example: Solving a linear system of equations

Question: Find the mass flow rate of refrigerant flowing in an evaporator coil given that the cooling rate is 1kW and that the enthalpy of the refrigerant varies from 100kJ/kg to 200kJ/kg.

Solution: While the solution to this problem is trivial, it can be used to illustrate the how the solution is obtained. In this case we want to solve the two equations:

$$\dot{m}_1 + \dot{m}_2 = 0$$
$$h_1\dot{m}_1 + h_2\dot{m}_2 = Q_1$$

The Matlab implementation is shown below:

```
%% Inputs
h1 = 100;
h2 = 200;
Q1 = 1;

A = [1 1;h1 h2];
b = [0;Q1];

%% Solution
Ainverse = inv(A);

x = Ainverse*b;
```

Results: x = -0.0100 0.0100

6.1.3 Solving non-linear systems

In certain models of thermodynamic systems one or more of the system equations are non-linear. he entire system is said to be non-linear. Consider for instance an energy equation where re $\dot{m}$, $\dot{m}_2$, $\dot{m}_3$ as well as the specific enthalpy h_3:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3 \; Q_1 \tag{6.37}$$

In such a case, the product $\dot{m}_3 h_3$ makes eqn. 6.37 non-linear.

A numerical procedure which can be applied to solve non-linear systems of equations is the *Newton's method*. Suppose we have n functions $f_1(x), f_1(x) \dots, f_n(x)$. The set of functions must be re-arranged such that:

$$F(x) = 0 \tag{6.38}$$

where

$$F(x) \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{bmatrix}$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Starting from a particular guess x^g and having an elemental increase of dx we can expand $F(x^g + dx)$ by means of a Taylor series:

$$F(x^g + dx) = F(x^g) + \begin{bmatrix} \Sigma_{i=1}^{n} \dfrac{\partial f_1(x_i)}{\partial x_i} dx_i \\ \Sigma_{i=1}^{n} \dfrac{\partial f_2(x_i)}{\partial x_i} dx_i \\ \vdots \\ \Sigma_{i=1}^{n} \dfrac{\partial f_n(x_i)}{\partial x_i} dx_i \end{bmatrix}_{x=x^g} \tag{6.39}$$

In eqn. 6.39, all derivatives are evaluated at the guess value x^9. This equation maybe simplified further as follows:

$$F(x^g + dx) = F(x^g) + \begin{bmatrix} \dfrac{\partial f_1(x_1)}{\partial x_1} & \dfrac{\partial f_1(x_2)}{\partial x_2} \cdots & \dfrac{\partial f_1(x_n)}{\partial x_n} \\ \dfrac{\partial f_2(x_1)}{\partial x_1} & \dfrac{\partial f_2(x_2)}{\partial x_2} \cdots & \dfrac{\partial f_2(x_n)}{\partial x_n} \\ \vdots & \vdots \quad \vdots & \cdots \\ \dfrac{\partial f_2(x_1)}{\partial x_1} & \dfrac{\partial f_2(x_2)}{\partial x_2} \cdots & \dfrac{\partial f_2(x_n)}{\partial x_n} \end{bmatrix}_{x=x^g} \begin{bmatrix} dx_1 \\ dx_2 \\ \vdots \\ dx_n \end{bmatrix} \tag{6.40}$$

The matrix of partial derivatives is generally known as the *Jacobian* matrix with symbol *J*. Thus we have:

$$F(x^g + dx) = F(x^g) + J_{x=x^g} dx = 0 \tag{6.41}$$

Thus we have

$$-F(x^g) = J_{x=x^g} dx \tag{6.42}$$

The values of the vector *dx* can thus be found from:

$$-J^{-1}_{x=x^g} F(x^g) = dx \tag{6.43}$$

Thus a new iterate may be found from:

$$x^{g+1} = x^g + dx \tag{6.44}$$

The process should be repeated until the difference between the old and new iterate is below a certain tolerance Δ:

$$\left| x^{g+1} = x^g \right| < \Delta \tag{6.45}$$

In Matlab, a simple solution to solve non-linear systems exists by using *symbolic objects*. The details of what symbolic objects are will not be discussed here and the following discussion will only focus on how they can be implemented in order to solve a system of non-linear equations. An example is shown next.

Example: Solving a linear system of equations

Question: Find the mass flow rate of refrigerant flowing in a refrigerating system governed by the following equations:

$$\dot{m}(h_1 - h_3) = Q_1$$
$$\dot{m}(h_2 - h_1) = Q_2$$

Solution: Two symbolic objects must be created representing the mass flow rate and the specific enthalpy at state 1. This is done in line 8. The two equations are hence defined by $eq1$ and $eq2$. The system is solved using the solve() function. The symbolic objects m and $h1$ need to be converted to type double in lines 13 and 14.

The Matlab implementation is shown below:

```
%% inputs
Q1 = 1;
Q2 = -1;
h2 = 100;
h3 = 200;

%% Create symbolic objects representing variables
syms m h1
eq1 = Q1+m*(h1-h3);
eq2 = -Q2+m*(h2-h1);
sol = solve(eq1,eq2);

m = double(sol.m)
h1 = double(sol.h1)
```

Results: m = 0.0200 h1 = 150

6.2 Experimentation and uncertainty

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I odynamics and refrigeration, experimentation is carried out by means of various forms of cluding flow, temperature, volumetric, mass, electrical and power measurement. The measurement of these fundamental quantities can then be used to calculate derived quantities. There are various text books which explain the various measurement devices and techniques that are mentioned here. The reader is referred to Miller (1996), Holman (2001), Benedict (1984) and Granger (1994).

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Experimental test rigs include the system under investigation as well as all the necessary instrumentation. While it is possible to perform measurements by means of analogue devices, in modern experimentation the data is collected in digital form and logged by means of a multi-channel data-logger. The data can be effectively stored on a local drive and then analysed at a later stage. The reader is referred to Morris (2001) for more details of modern data gathering techniques. Before any measurement can take place, the measurement devices need to be calibrated. The output of a particular sensor is a voltage signal which must then be converted, by means of a calibration function to the value of the quantity in question. In principle, the calibration procedure should be carried out every time that there is a substantial time difference between one measurement and another. This is because environmental variables might cause the calibration procedure of a previous time to be invalid.

All experimental measurements contain a certain level of uncertainty. Dieck (2006) gives a detailed overview of the fundamentals related to uncertainty. In a way, uncertainty is considered to be a quantity representing how much the data disperses around the actual quantity that should be measured. It is a probabilistic quantity which depends on a number of factors which are not only associated with the measurement equipment but also with the environment (which here includes the operator) in which the test is carried out. The error of a particular reading from its true value can be divided into *systematic* and *random error.*

Definition

With systematic errors, the measured value has an offset compared to the actual measured value.

No matter how many times one repeats an experiment, the same offset will reappear in the following measurements. These kinds of errors are hence estimated by means of different types of measurements or calculations.

Definition

Random errors occur due to a lack of repeatability of a given measurement from the true value. This may occur due to uncontrolled factors related to the measurement equipment and the environment. This form of error is reduced by repeating measurements a number of times which enables a better average to be obtained.

The standard uncertainty can be estimated using the standard deviation of a set of readings. If the standard deviation of the mean is denoted by s, then the standard uncertainty for N samples in the set is given by:

$$u = \frac{S}{\sqrt{N}} \tag{6.46}$$

In order to obtain the combined uncertainty of n measurements on the derived variable ϕ

$$\phi = f(x_1, x_2, x_3, \ldots, x_n) \tag{6.47}$$

the uncertainty can be expressed as a sum of squares:

$$\Delta\phi^2 = \left(\frac{\partial\phi}{\partial x}\right)^2 \ \Delta x_1^2 \left(\frac{\partial\ \phi}{\partial x_2}\right)^2 \ \Delta x_2^2 + \left(\frac{\partial\phi}{\partial x_3}\right)\Delta x_3^2 + \ldots + \left(\frac{\partial\ \phi}{\partial x_\mathrm{n}}\right)^2 \ \Delta x_\mathrm{n}^2 \tag{6.48}$$

6.3 Limitations of numerical methods and experimental validation

Caution must be exercised in interpreting these results. The energy balances are based on the fundamental assumption that there are no energy losses from the system. In reality such energy losses do exist and certain processes are not really reversible as is assumed in the equations governing the system. Numerical results must always be validated through experiments. The models are in most cases tuned such that better agreement is obtained between what is calculated and what is actually measured.

6.4 Conclusions

Student checklist

1. Modelling of an entire thermodynamic system involves the solution of a system of linear or non-linear equations.
2. Detailed modelling of individual thermodynamic sub-systems can be carried out using a Computational Fluid Dynamics (CFD) approach.
3. The sub-system equations have been included for various components. The equations ultimately form the system of equations which needs to be solved.
4. Whether the system is linear or non-linear depends on the unknowns.
5. To solve the system of equations, the number of equations must be the same as the number of unknowns.
6. There are various methods of how to numerically solve linear and non-linear systems of equations.
7. Matlab provides various functions that give the user flexibility in solving such systems. Some basic examples of these tools have been presented to the reader.
8. Experimental measurements were discussed with particular reference to the kind of measurements which are usually carried out in thermodynamics and refrigeration.
9. The student should be familiar with methods of estimating the standard uncertainty as well as combined uncertainty.

7 Vapour compression system modelling

7.1 Single stage

7.1.1 Algorithm

Vapour compression system modelling will be considered in this chapter. The use of such models permits a number of interesting results to be produced. The single stage vapour compression cycle consists of four components. In this analysis we will consider the following inputs:

```
%% SYSTEM INPUTS

p_evap = 1.3 ; %pressure  in  bar
p_cond = 9.5 ;%pressure  in  bar
T1 = interp1 (psat_data , Tsat_data ,p_evap, 'cubic ' ) ;
     rature  at  exit  from evaporator ...in degC
   ercooling = 10; %undercooling  in degC
m = 1; %mass flow  rate  in kg/s
```

The state points and system components are shown in Figure 7.1 for the reader's reference. In this case, the exit temperature from the evaporator is taken as being the saturation temperature corresponding to the evaporator pressure. This can however be changed such that the refrigerant is superheated at exit from the evaporator. The refrigerant used in this analysis will be R134a.

The code listing for the calculation of the thermodynamic quantities at all state points is shown next. The algorithm first establishes the necessary thermodynamic properties at various state points. This is done by means of the refrigerant (R134a) table of thermodynamic properties. These are then used to calculate the energy inputs and outputs which are then used to find the COP.

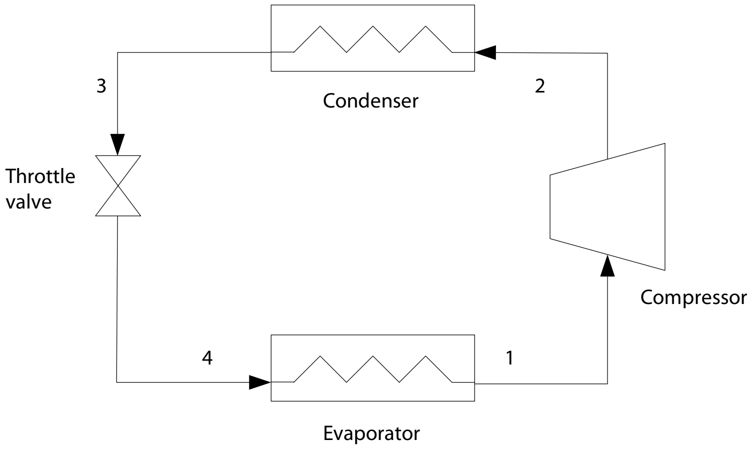


Figure 7.1: Vapour compression system being modelled.

```
% STATE 1

Tsat1 = interp1 (psat_data ,Tsat_data ,p_evap, ' cubic ') ;
Tsuper1 = T1-Tsat1 ;

if    Tsuper1 > 0
h1 = interp2 (Tsuper_superheated , Tsat_superheated , enthalpy_
superheateddata ...
,Tsuper1 , Tsat1 , ' cubic ' ) ;
s1 = interp2 (Tsuper_superheated , Tsat_superheated , entropy_
superheateddata ...
,Tsuper1 , Tsat1 , ' cubic ' ) ;
elseif    Tsuper1 == 0
h1 =     interp1 (Tsat_data , hg_data , Tsat1 , ' cubic ' ) ;
s1 =     interp1 (Tsat_data , sg_data , Tsat1 , ' cubic ' ) ;
 end

 %STATE 2

```

```
Tsat2 = interp1 (psat_data ,Tsat_data ,p_cond, ' cubic ') ;
s2 = s1;
position1 = find (Tsat_superheated (: ,1)<Tsat2);
position2 = find (Tsat_superheated (: ,1)≥Tsat2);
position1 = position1 (end) ;
position2 = position2 (1) ;


if Tsat_superheated (position2 ,1) == Tsat2
position1 = position2 ;
end

interpolation_factor = (s2   - ...
entropy_superheateddata (position1, 1))/(entropy_superheateddata
(position1, end)   - ...
entropy_superheateddata (position1 ,1)) ;

h_position1 = enthalpy_superheateddata (position1, 1) + ...
(enthalpy_superheateddata (position1, end)   - ...
enthalpy_superheateddata (position1, 1))*interpolation_factor ;
interpolation_factor = (s2   - ...
entropy_superheateddata (position2, 1))/(entropy_superheateddata
(position2, end)   -
 ... entropy_superheateddata (position2, 1) ) ;

h_position2 =    enthalpy_superheateddata (position2 ,1) + ...
(enthalpy_superheateddata (position2, end)   - ...
enthalpy_superheateddata (position2 ,1)) *interpolation_factor ;
interpolation_factor = ...
(Tsat2 - Tsat_superheated (position1, 1)) /(Tsat_superheated
(position2, 1)   - ...
Tsat_superheated (position1, 1)) ;
h2 = h_position1 + (h_position2 -h_position1)*interpolation_factor ;

interpolation_factor = (h2- h_position1 ) /( h_position2 - h_position1 ) ;
Tsuper2 = Tsuper_superheated(1, 1) + (Tsuper_superheated(1, end)   - ...
Tsuper_superheated (1, 1) )*interpolation_factor ;
T2 = Tsat2 + Tsuper2 ;

%STATE 3

Tsat3 = Tsat2 ;
```

```
T3 = Tsat3 - undercooling ;
h3 =    interp1 (Tsat_data , hf_data ,T3, ' cubic ') ;
s3 =    interp1 (Tsat_data , sf_data ,T3, ' cubic ' ) ;

%STATE 4

h4 = h3;
hf4 = interp1 (Tsat_data , hf_data , Tsat1 , ' cubic ' ) ;
hg4 = interp1 (Tsat_data ,hg_data ,Tsat1 , 'cubic ' ) ;
sf4 = interp1 (Tsat_data , sf_data , Tsat1 , ' cubic ' ) ;
sg4 = interp1 (Tsat_data , sg_data , Tsat1 , ' cubic ' ) ;
x = (h4- hf4)/(hg4- hf4) ;
s4 = sf4 + x*(sg4-sf4);

%% PRESSURES
p1 = p_evap;
p2 = p_cond;
p3 = p_cond;
     p_evap;
```

```

%% ENERGY/POWER CALCULATIONS

%All    quantities    calculated    in  kW
Q1  = m*(h1    -   h4);
W  = m*(h2-h1)  ;
Q2  = m*(h3-h2)  ;

%% COEFFICIENT OFPERFORMANCE
COPref  = Q1/W;
COPhp  = -Q2/W;
```

A summary of the algorithm is given here:

1. Calculate the degree of superheat at exit from the evaporator
2. Find the specific enthalpy and specific entropy for state point 1 from tables
3. For an ideal compression process (100 % isentropic efficiency) the entropy at state point 2 is equal to that at state point 1
4. The enthalpy at state point 2 is found using table interpolation
5. Determine the temperature at state point 3 and find the enthalpy from tables
6. The enthalpy at state point 4 is made equal to that at 3 due to the throttling process
7. All required energy quantities are established from the enthalpies at the state-points

7.1.2 Effect of pressure on refrigeration COP

The variation of refrigeration COP with evaporator and condenser pressures is shown in Figures 7.2 and 7.3. As can be noted, the COP increases with increasing evaporator pressure and decreasing condenser pressure. The variation of COP with evaporator pressure is rather linear. This is not the case for the variation of COP with condenser pressure.

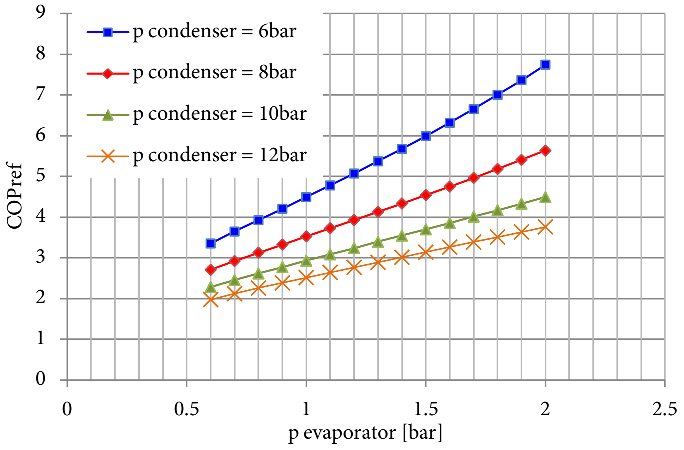


Figure 7.2: Refrigeration COP against evaporator pressure for various condenser pressures.

The variation of the rate of cooling with evaporator and condenser pressures is shown in Figures. 7.4 and 7.5. The corresponding compressor power input is shown in Figures 7.6 and 7.7. There is a high sensitivity of the power input to the evaporator pressure. For a range of less than 1.5bar in evaporator pressure, the compressor power input variation corresponds to round about a condenser pressure variation of 6bar.

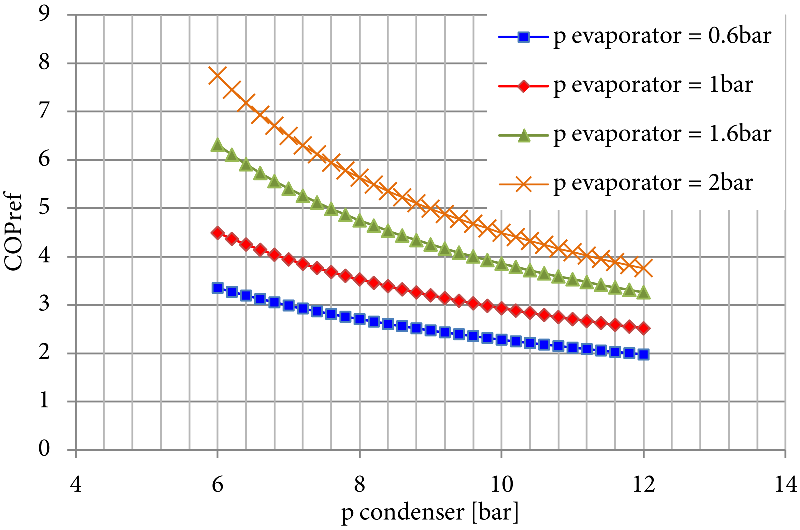


Figure 7.3: Refrigeration COP against condenser pressure for various evaporator pressures.

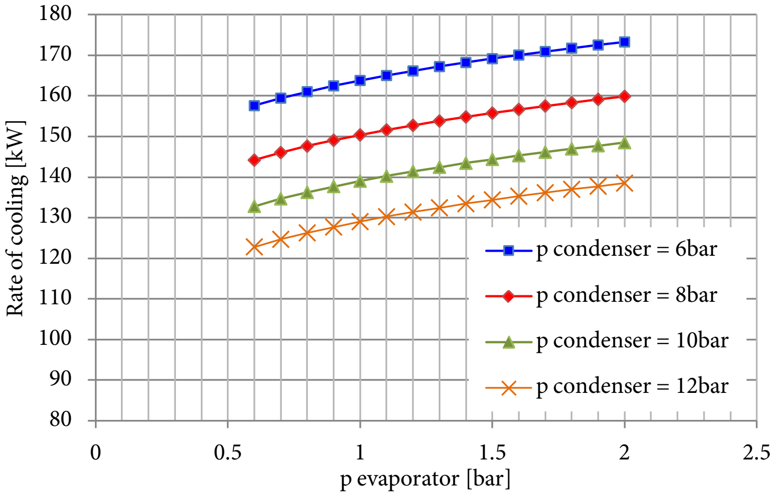


Figure 7.4: Rate of cooling against evaporator pressure for various condenser pressures.

7.1.3 Effect of undercooling and super heating on refrigeration COP

Figure 7.8 shows the variation of COP with condenser undercooling. The variation is relatively linear. The difference in COP between 0K and 14K of undercooling is 0.5. The superheat temperature used at the evaporator exit does not seem to affect this result substantially but a higher COP can be observed for a superheat temperature of 20K. The reason for this is the increased cooling effect in the evaporator (larger area under process 4-1).

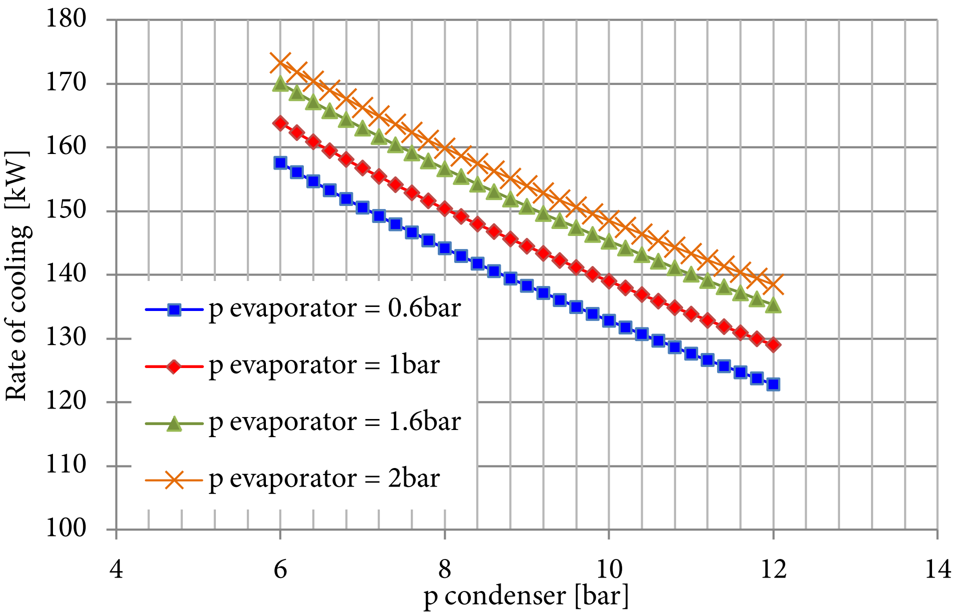


Figure 7.5: Rate of cooling against condenser pressure for various evaporator pressures.

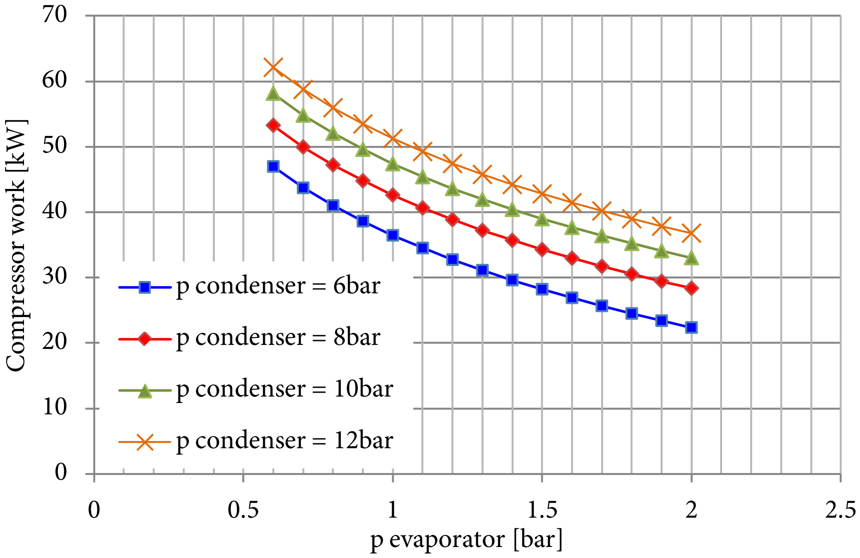


Figure 7.6: Compressor power against evaporator pressure for various condenser pressures.

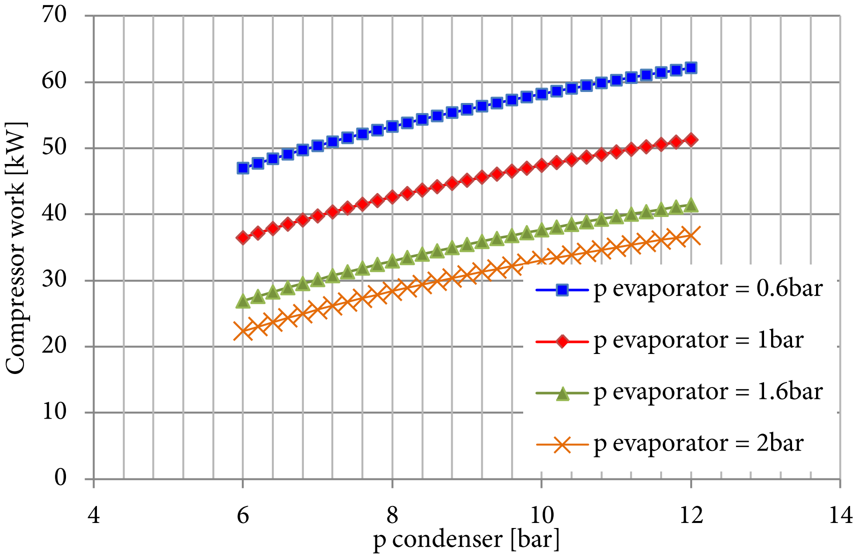


Figure 7.7: Compressor power against condenser pressure for various evaporator pressures.

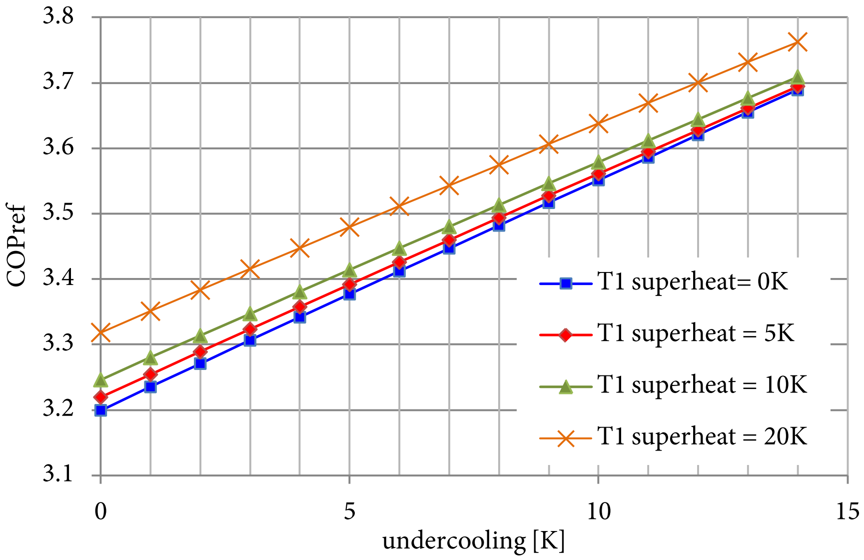


Figure 7.8: Refrigeration COP against undercooling temperature at exit from the condenser for various superheat temperatures at exit from the evaporator.

7.2 Cascaded system

7.2.1 Algorithm

The cascaded vapour compression system is certainly more complex than the single stage vapour compression cycle. The system modelling may therefore yield interesting results which can be useful for system design. Figure 7.10 shows the system and state points that are modelled in this section. The following are the system inputs used for the analysis. In certain cases, some of the inputs are varied such that certain plots can be obtained.

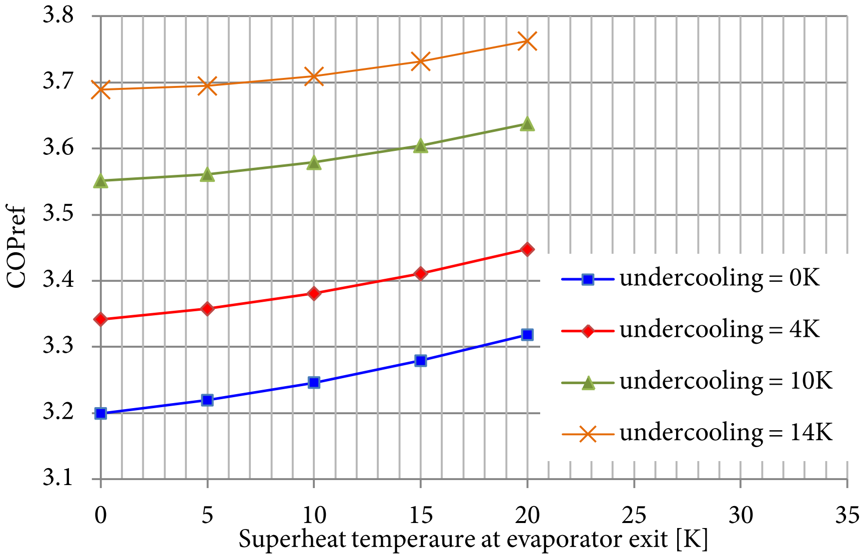


Figure 7.9: Refrigeration COP against superheat temperature at exit from the evaporator for various undercooling temperature at exit from the condenser.

```
%% SYSTEM INPUTS

p_evap = 1.3271; %pressure  in  bar
p_cond = 6;%pressure  in  bar
p_cond2 = 10;%pressure  in  bar
T1 = interp1 (psat_data , Tsat_data ,p_evap, 'cubic ' ) ;
%temperature  at  exit  from evaporator ... in degC
T5 = interp1 (psat_data , Tsat_data ,p_cond, 'cubic ' ) ;
%temperature  at  exit  from evaporator ... in degC
undercoolinsg = 0; %undercooling  in  degC
undercooling_secondstage = 0; %undercooling  in  degC

m_firststage = 1; %mass  flow  rate  in  kg/s
m_secondstage = 0.8; %mass  flow  rate  in  kg/s
```

The procedure adopted in this model is similar to that used for the single stage system. The same equations are used for the lower part of the system. Given the inputs, and hence not knowing the temperature at the output of the intermediate pressure evaporator, the problem becomes one which requires multiple numerical iterations since both state 5 and state 8 are unknowns. In the first iteration, state 8 is assumed to be in the liquid state (on the saturated liquid line). This enables the calculation of the enthalpy at state 5 since:

$$h_5 = -\frac{Q_2}{\dot{m}_H + h_8} \tag{7.1}$$

where Q_2 is the heat output from the intermediate pressure condenser (thus being negative). The rest of the calculation is the same as for the single stage vapour compression cycle. A new value of the enthalpy at state point 8 may hence be established and the enthalpy at state point 5 may be re-calculated in the next iteration. The process is repeated until an acceptable convergence is attained in the value of the enthalpy at state point 5.

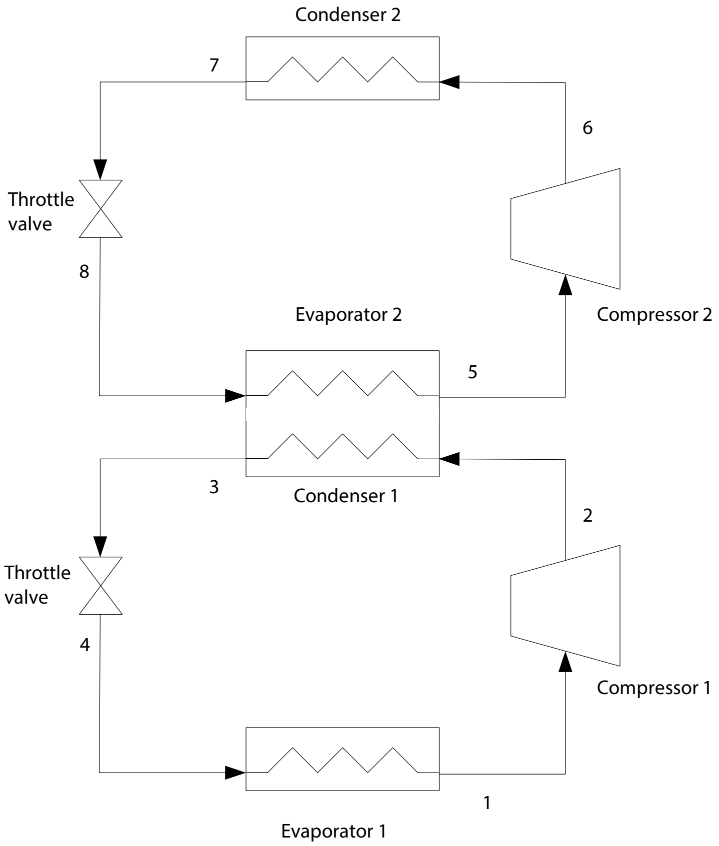


Figure 7.10: Model the cascaded system showing state points.

The algorithm is given here:

1. Solve the lower part of the system with the algorithm proposed in the previous section
2. Assume an initial enthalpy value at state point 8 (the first iterate can be taken as the enthalpy for a pure liquid at the intermediate pressure)
3. Find the enthalpy at state point 5 from eqn. 7.1.
4. Solve the rest of the system using the same algorithm proposed in the previous section
5. Calculate a new value for h_8 and calculate a new value for h_5
6. If the difference between the old and the new value of h_5 is below a certain tolerance, then convergence has been obtained. If not then repeat steps 3 to 5 using the new iterate of h_8
7. All required energy quantities are established from the enthalpies at the state-points

7.2.2 Mass flow rate ratio and refrigeration performance

It is useful to understand how the performance of the cascaded system is affected by the mass flow rate in the various system stages. The system pressures are set to an evaporator pressure of 1.33 bar, an intermediate pressure of 6 bar and a condenser pressure of 10 bar. The result of refrigerating COP against mass flow rate ratio (defined as $\dot{m}_L/\dot{m}_H$) is given in Figure 7.11. As can be observed, a monotonically increasing function is obtained. The COP is little affected when $\dot{m}_L/\dot{m}_H < 0.85$. When the mass flow rate on the lower stage is increased above the higher stage $\dot{m}_L/\dot{m}_H > 1$, the COP starts to increase relatively rapidly for these operating pressures. The behaviour may suggest that the COP will continue to increase indefinitely by increasing the mass flow rate ratio. This is not the case since the refrigerant condition at entry to the high pressure side compressor (must be dry saturated or superheated) dictates a certain limit on this mass flow rate ratio which can be used.

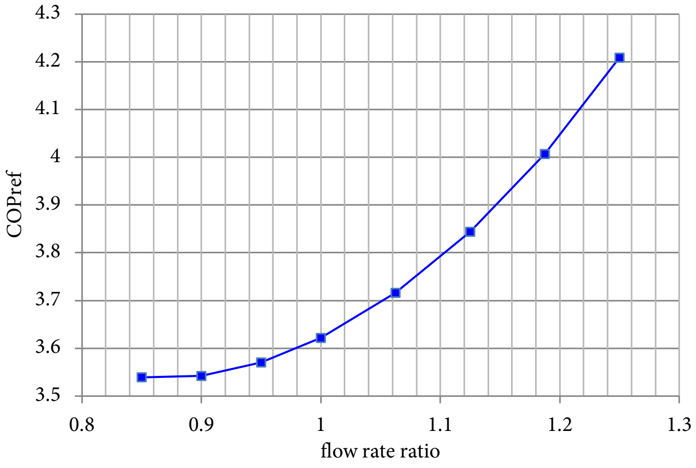


Figure 7.11: Refrigeration COP against mass flow rate ratio $\dot{m}_L/\dot{m}_H$. The evaporator pressure is set to 1.33 bar, the intermediate pressure at 6 bar and the high pressure condenser at 10 bar.

7.2.3 Effect of undercooling on performance

For the single stage vapour compression cycle, the undercooling of the refrigerant at the condenser exit provided an increase in refrigeration COP. For a cascaded system, the undercooling in the lower part of the system is expected to yield the same trend. Undercooling at the high pressure condenser exit will cause state point 8 to shift to the left of the p-h diagram. For a particular heat transfer from the intermediate pressure condenser, this shift will cause state point 5 to also shift to the left. This may cause the refrigerant to still be wet at the inlet to the compressor thus causing damage. The second negative effect is that this will cause the high pressure compressor to require more energy input. The effects of undercooling in the intermediate pressure condenser (undercooling1) and high pressure condenser (undercooling2) on the high pressure compressor power input is illustrated in Figures 7.12 and 7.13. As can be observed, the compressor energy requirements by undercooling the refrigerant at the intermediate pressure condenser exit are reduced. The opposite maybe observed with the high pressure condenser undercooling. In both cases the mass flowrates are set to 1kg/s and 0.8kg/s for the lower part and the upper part of the cycle respectively. The refrigeration COPs are given in Figures 7.14 and 7.15. It is clear that to maximize refrigeration COP no undercooling in the high pressure condenser should be present.

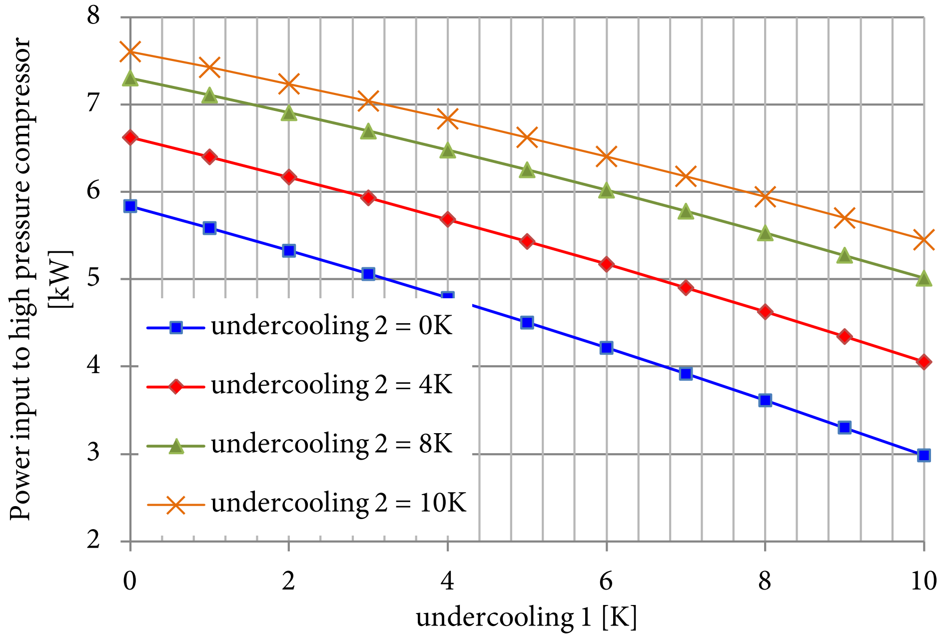


Figure 7.12: Power input to high pressure compressor against undercooling in the lower stage at various undercooling temperatures in the higher stage. The lower stage mass flow rate is set to 1kg/s while the higher stage compressor is set to 0.8kg/s.

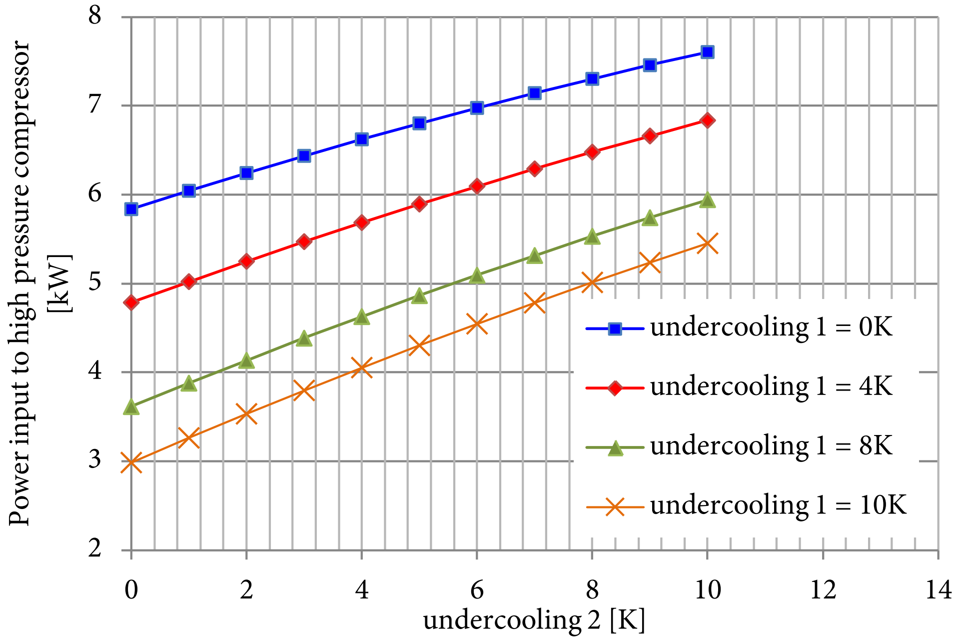


Figure 7.13: Power input to high pressure compressor against undercooling in the higher stage at various undercooling temperatures in the lower stage. The lower stage mass flow rate is set to 1kg/s while the higher stage compressor is set to 0.8kg/s.

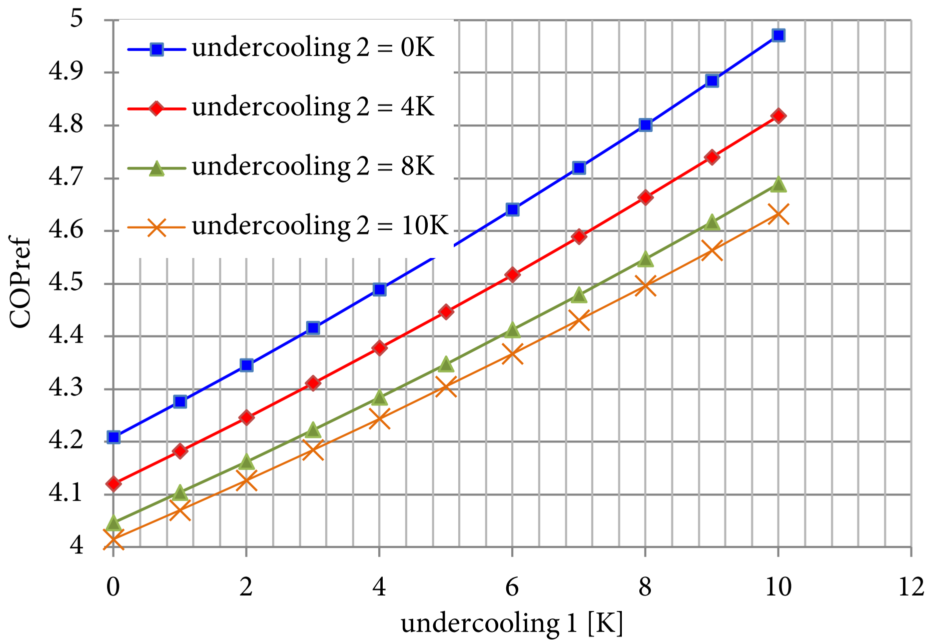


Figure 7.14: Refrigeration COP against undercooling in the lower stage at various undercooling temperatures in the higher stage. The lower stage mass flow rate is set to 1kg/s while the higher stage compressor is set to 0.8kg/s.

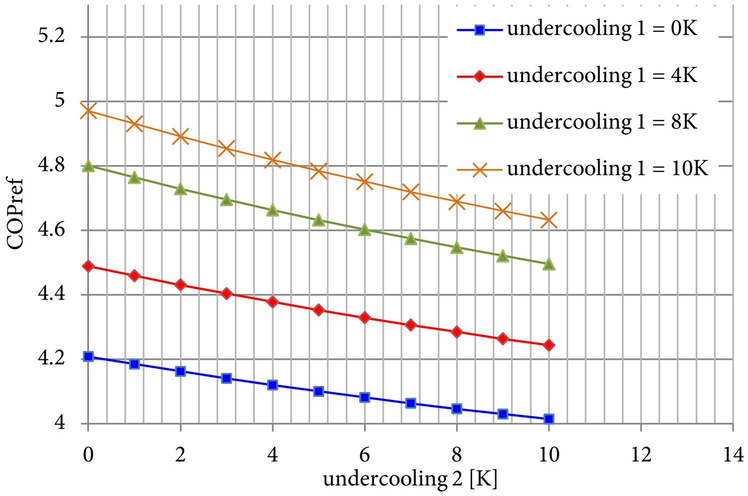


Figure 7.15: Refrigeration COP against undercooling in the higher stage at various undercooling temperatures in the lower stage. The lower stage mass flow rate is set to 1kg/s while the higher stage compressor is set to 0.8kg/s.

7.2.4 Effect of intermediate pressure on performance

The choice of intermediate pressure between the high pressure condenser and the low pressure evaporator will be investigated next. Figure 7.16 shows the variation of refrigeration COP with intermediate pressure. The variation is practically linear over the chosen range of pressures. Over a pressure change from 4bar to 8bar the COP difference is less than 0.1. This shows that the COP is not really sensitive to variations in the choice of intermediate pressure.

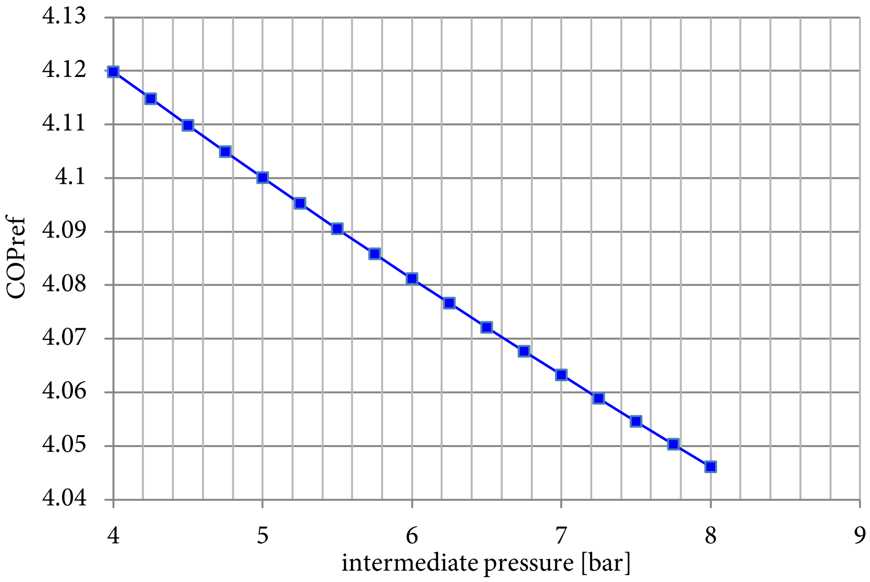


Figure 7.16: Refrigeration COP against intermediate pressure. The evaporator pressure is set to 1.33 bar and the high pressure condenser at 10 bar. The lower stage mass flow rate is set to 1kg/s while the higher stage compressor is set to 0.8kg/s.

7.3 Conclusions

Student checklist

1. The student should have an understanding of why numerical models are useful for the analysis of the behaviour of vapour compression cycles.
2. An algorithm for solving single stage vapour compression cycles has been presented. The student is encouraged try to program his/her own code with a different approach.
3. For the single stage system, the refrigerating COP increases with increasing evaporator pressure but decreases with increasing condenser pressure.
4. The refrigerating COP is rather sensitive to condenser undercooling but not to refrigerant superheating at the evaporator exit.
5. An algorithm has been presented for the analysis of cascaded systems. The approach used in such a case is an iterative one.
6. The results from the model show that the refrigerating COP of such a system is highly sensitive to the ratio of the mass flow rate of the low pressure and high pressure parts of the system.
7. The refrigerating COP increases with increased undercooling of the intermediate pressure condenser but reduces with the undercooling of the high pressure condenser.
8. The choice of intermediate pressure has little impact on refrigerating COP.

8 Vapour absorption system modelling

8.1 LiBr-water vapour absorption system

8.1.1 Algorithm

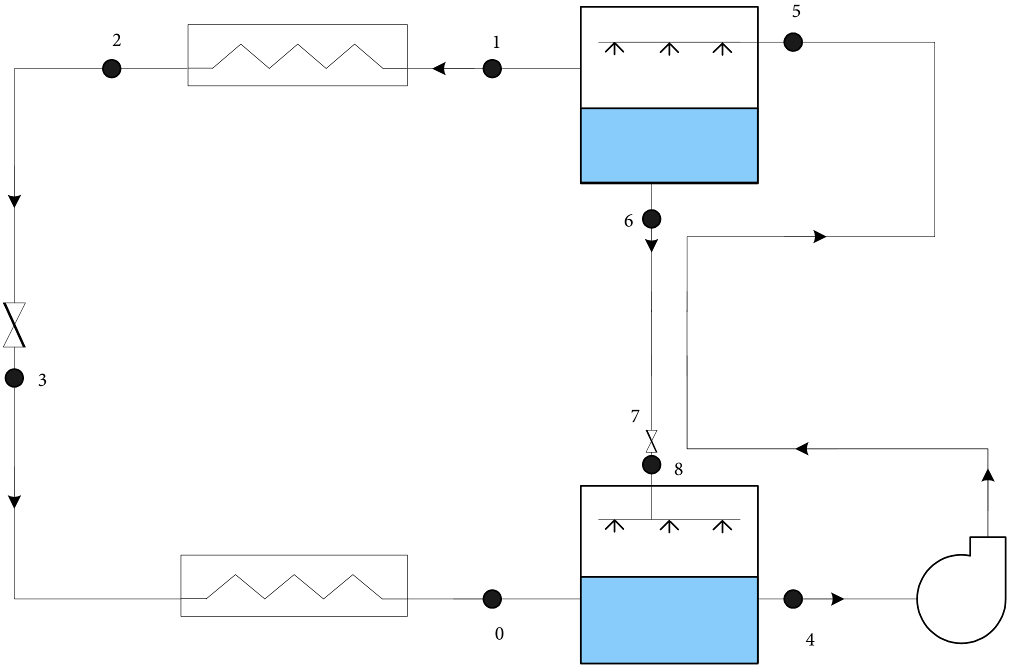
To demonstrate further the utility of numerical modelling applied to refrigeration, the vapour absorption system (using a LiBr-water combination) is modelled in this chapter. The modelled system is represented in Fig. 8.1.

The following fundamental inputs are used. Some of these variables are varied in order to analyse the effects of these on certain output quantities as was done in chapter 7. The cooling capacity Q_1 is used as a system input.

```
%% SYSTEM INPUTS

p_evap = 0.007575; %pressure  in  bar
p_cond = 0.0538;%pressure   in  bar
Ta = 25; %Absorber temperature  in degC
Tg = 80; %Generator  temperature  in degC
Cweak = 0.51; %weak  solution  concentration %lithium  bromide
Cstrong = 0.6; %strong   solution   concentration %lithium  bromide
undercooling = 0; %condenser  undercooling  in degC
Q1 = 100; %cooling  capacity
```

The thermodynamic properties at each state point needs to be established first. The quantities for water are established from available thermodynamic data by means of a separate function. The properties of the LiBr-water mixture are obtained from an interpolating polynomial as described in ASHRAE (2009). The polynomial is used to evaluate the enthalpy of the mixture at various state-points given the concentration of LiBr in water. This is given hereunder:



8.1: Vapour compression system being modelled.

$$h(C,T) = \sum_{i=1}^{5} a_i C^{i-1} + T\sum_{i=1}^{5} b_i C^{i-1} + T^2 \sum_{i=1}^{5} c_i C^{i-1} \tag{8.1}$$

where the constants a_i,b_i and c_i are given here:

```
a(1) = -2024.33 ;
a(2) = 163.309 ;
a(3) = -4.88161 ;
a(4) = 6.302948e -2;
a(5) = -2.913705e -4;

b(1) = 18.2829 ;
b(2) = -1.1691757 ;
b(3) = 3.248041e -2;
b(4) = -4.034184e -4;
b(5) = 1.8520569e -6;

c(1) = -3.7008214e -2;
c(2) = 2.8877666e -3;
c(3) = -8.1313015e -5;
c(4) = 9.9116628e -7;
c(5) = -4.4441207e -9;
```

The code is shown next. The function *steam* finds the thermodynamic properties for steam given the inputs provided for the function. On the other hand, the function *LiBr_water* finds the enthalpy of the LiBr-water mixture at the inputs of concentration and temperature provided by the user.

The system of equations that needs to be solved is linear and is made up of mass and energy conservation equations as described in chapter 6. An inverse matrix approach is used to solve the system such that no numerical iterations are necessary. This is performed by means of the *inv()* function in Matlab.

```
Tcond = steam( 'temperature ' ,p_cond) ;
h4 = LiBr_water(Cweak*100,Ta) ;
h5 = h4; %pump assumed to have negligible power input
h6 = LiBr_water(Cstrong*100,Tg) ;
h7=h6; %throttling process

h0 = XSteam( 'hV_p' ,p_evap) ;
h1 = XSteam( 'h_pT' ,p_cond,Tg) ;
h2 = XSteam( 'h_pT' ,p_cond, round(Tcond - undercooling));
h3 = h2; %throttling process

C0 = 0;
C1 = 0;
C2 = 0;
C3 = 0;
C4 = Cweak;
C5 = Cweak;
C6 = Cstrong ;
C7 = Cstrong ;

%% SOLVE LINEAR SYSTEMOFEQUATIONS

%m Q2 Qa Qg m4 m5 m6 m7

A = [1  0 0 0 0 0 0  0; % evaporator energy
-(h2-h1) 1 0 0 0 0 0  0; %condenser energy
-h0 0  -1 0 h4 0 0  -h7; %absorber energy
-h1 0 0 1 0 h5  -h6  0; %generator energy
-1 0 0 0 1 0 0   -1; %absorber mass
0 0 0 0 C4 0 0  -C7; %absorber LiBr mass
0 0 0 0 1  -1  0  0; %pump mass
0 0 0 0 0 0  -1   1]; %throttle valve mass

b = [Q1/(h0-h3) ;
```

```
0;
0;
0;
0;
0;
0;
0];

x = inv(A)*b;
m = x(1);
Q2 = x(2);
Qa = x(3);
Qg = x(4);
m4 = x(5);
m5 = x(6);
m6 = x(7);
m7 = x(8);

%% COP calculations

COPref = Q1/Qg;
COPhp = -Q2/Qg;
```

A summary of the algorithm is given here:

1. From the provided inputs, determine the properties of the refrigerant (in this case water)
2. From the inputs of concentration and temperature, determine the specific enthalpy of the LiBr-water mixture at various state-points
3. Determine the matrix of coefficients for the system of linear equations representing mass and energy conservation
4. Determine the right hand side of the mass and energy conservation linear equations
5. Find the inverse of the matrix of coefficients
6. Perform matrix multiplication of the inverse matrix of coefficients and the right hand side terms to determine the required quantities
7. Important parameters such as the COP may hence be found

8.1.2 Solution concentration

The concentration of LiBr in water at state points 4, 5, 6 and 7 will affect the performance of the vapour absorption system. This is due to the fact that such concentrations will determine the flow rates present in these state points and hence also the energy input in the generator and the energy rejected in the absorber.

At exit from the absorber, the solution will be weak since the percentage of LiBr is smaller than at exit from the generator. Due to the desorption of the water from the LiBr, the stream at exit from the generator is strongly concentrated with LiBr.

Fig. 8.2 show the refrigeration COP variation with the weak concentration at various strong concentrations. On the other hand Fig. 8.3 shows the COP variation against the strong solution concentration at various weak solution concentrations. It is clear that as the concentration of the weak solution is increased, there is a reduction in the system COP. Also, as the concentration of the strong solution is reduced the same problem happens. When the weak solution is at a concentration of 55% the system performance degrades dramatically when C_{strong} reduces to 56%. The reason for this behaviour is that in the limit as the weak solution concentration approaches that of the strong solution, it means that there is little desorption taking place in the generator and the energy required in the generator is practically equal to the exit enthalpy of the water at state 1.

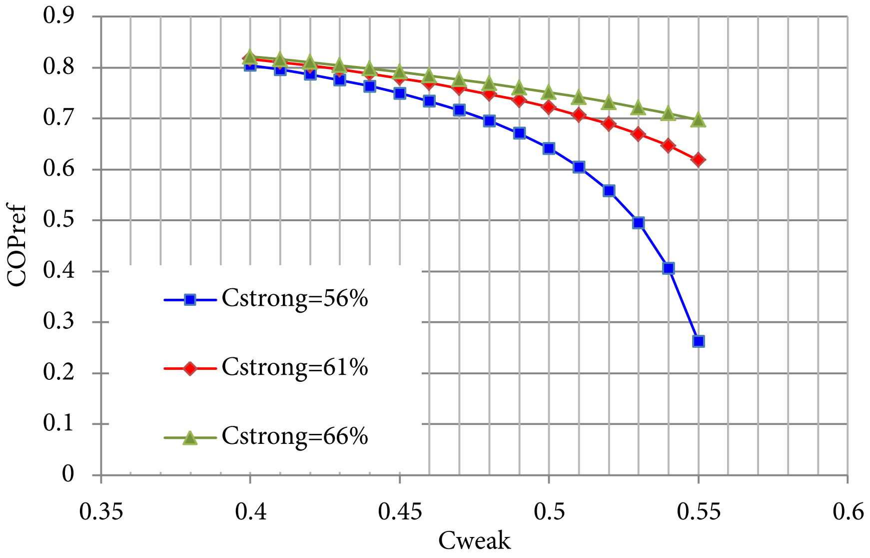


Figure 8.2: Effect of the weak solution concentration on the COP.

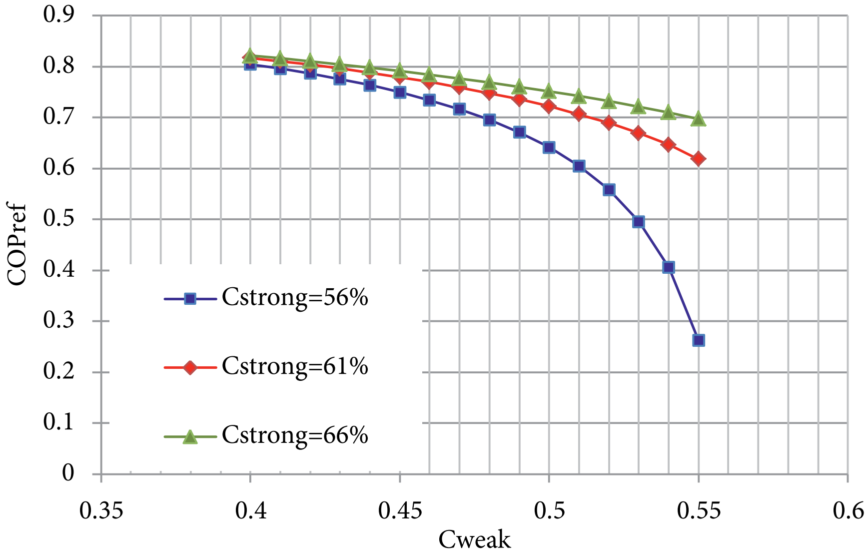


Figure 8.3: Effect of the strong solution concentration on the COP.

8.1.3 Absorber and generator temperature effects

Fig. 8.4 shows the generator heat input (on the primary axis) as well as the absorber heat rejected (on the secondary axis) as a function of absorber temperatures. The results are shown for various generator temperatures. The generator heat input reduces with increasing absorber temperature. The absorber cooling requirements also decrease. At a lower generator temperature, the heat input requirements to the generator is also reduced compared to a high temperature used in the generator. This means that for this system, if the difference between the generator and absorber temperatures is reduced, the generator heat input requirements reduce and hence the COP becomes higher.

The cooling requirements in the absorber also decrease with decreasing generator temperature. It is interesting to understand whether this effect persists when a solution heat exchanger is used. This will be answered in the next section of this chapter. The results for COP reflecting what has been said regarding the generator heat input are shown in Figs. 8.5 and 8.6.

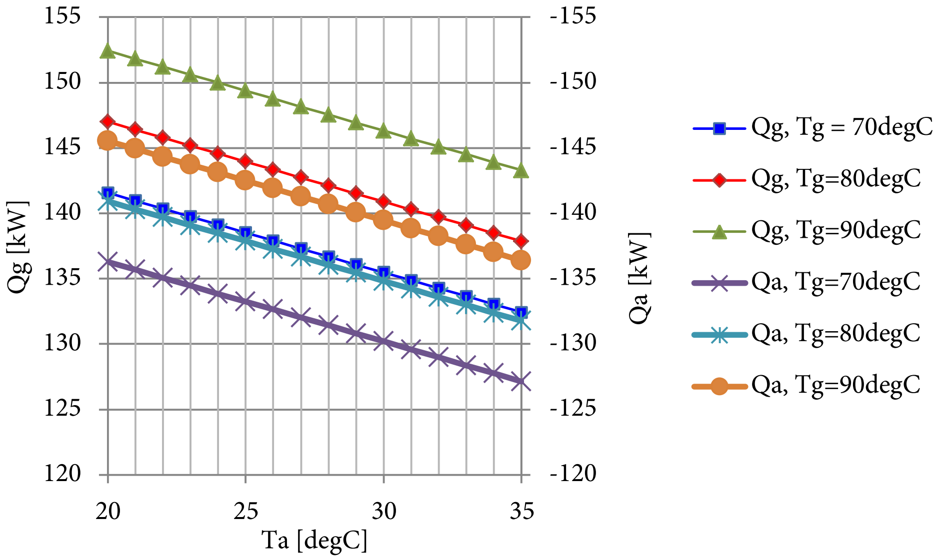


Figure 8.4: Heat transfers from the absorber and into the generator at different absorber and generator temperatures.

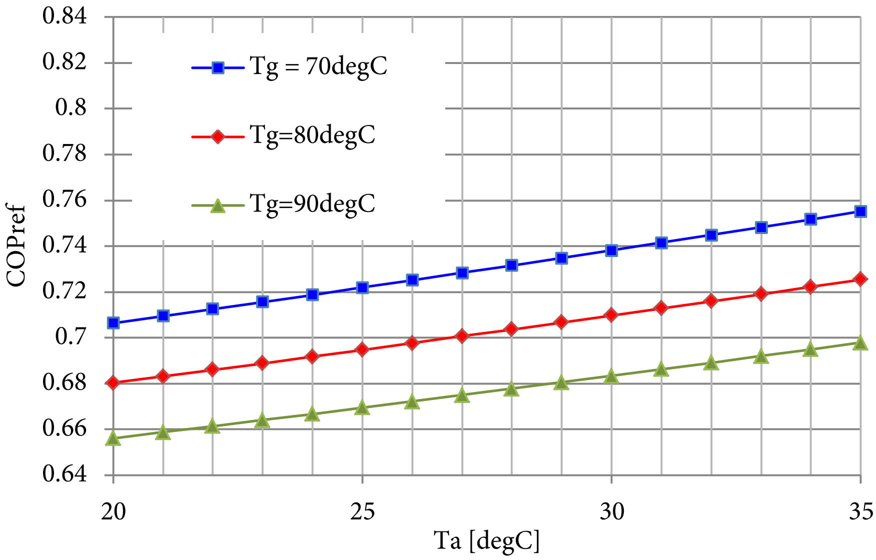


Figure 8.5: COP against absorber temperature.

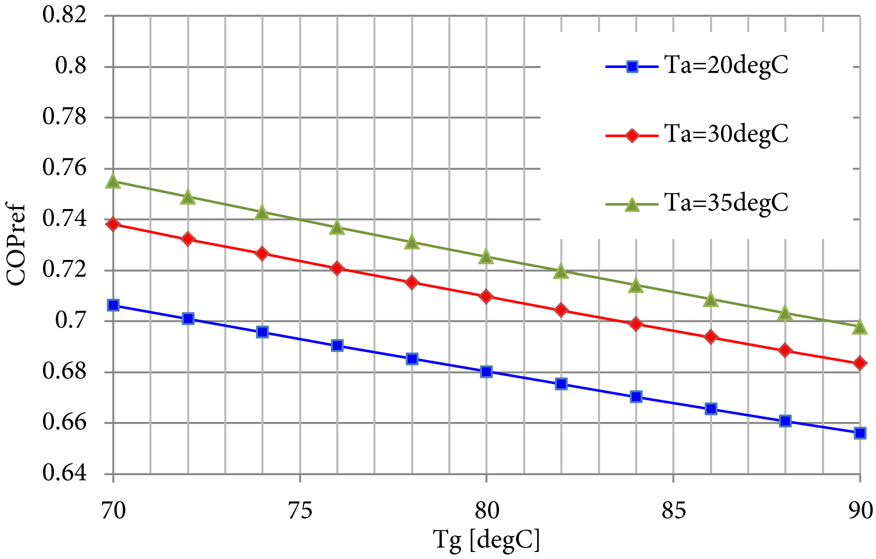


Figure 8.6: COP against generator temperature.

8.1.4 Effect of pressures on COP

The effect of evaporator and condenser pressures on the COP is shown in Fig. 8.7. It is clear that the sensitivity of the COP with evaporator and condenser pressures is relatively small. This goes to show that the performance of the unit is more dependent on the conditions found on the generator side.

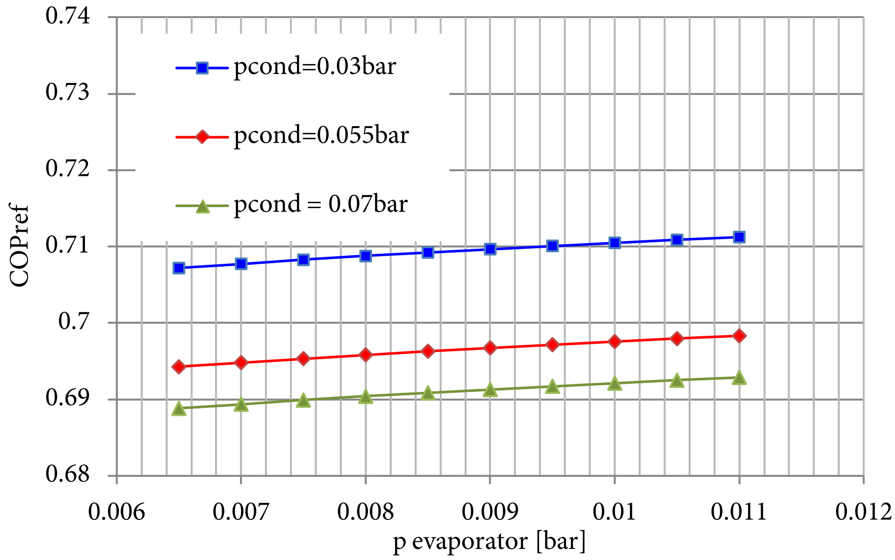


Figure 8.7: COP against evaporator pressure.

8.2 LiBr-water vapour absorption system using a solution heat exchanger

8.2.1 Algorithm

In this section, the solution heat exchanger is introduced. This sub-component has four inputs/outputs. The formulation of the system of equations results in 13 unknowns and 13 equations. The system is however non-linear. In this example, the in-built equation and system solver *solve()* in Matlab will be used. All inputs are maintained the same as for the no solution heat exchanger case in order to facilitate comparisons between the two systems. For the heat exchanger, an effectiveness of 0.775 is used. Also, in the heat exchanger equation used, the minimum heat capacity of the hot and cold stream is required. Kaita (2001) gives a comprehensive study of various thermodynamic properties of LiBr-water mixtures. At the temperatures and concentrations being considered, an average value of C_{min} = 1.9 is considered suitable. This is maintained constant throughout. Kaita (2001) showed that even for relatively high variations in temperatures and concentrations the value only varies by around 10%. Part of the program listing is shown here.

```
Tcond = steam( 'temperature ' ,p_cond) ;

h4 = LiBr_water(Cweak*100,Ta) ;
h5 = h4; %pump assumed to  have  negligible  power input
h7 = LiBr_water(Cstrong*100,Tg) ;

h0 = XSteam( 'hV_p' ,p_evap) ;
h1 = XSteam( 'h_pT' ,p_cond,Tg) ;
h2 = XSteam( 'h_pT' ,p_cond, round(Tcond  -  undercooling)) ;
h3 = h2; %throttling   process

C0 = 0;
C1 = 0;
C2 = 0;
C3 = 0;
C4 = Cweak;
C5 = Cweak;
C6 = Cweak;
C7 = Cstrong ;
C8 = Cstrong ;
C9 = Cstrong ;

%% SOLVE LINEAR SYSTEMOFEQUATIONS

syms Q2 Qa Qg mm4 m5 m6 m7 m8 m9 h6 h8 h9

eq1 = m-Q1/(h0-h3) ;
eq2 = -Q2+m*(h2-h1) ;
eq3 = -m*h0 - Qa +m4*h4-m9*h9;
eq4 = -m*h1 + Qg +m6*h6-m7*h7;
eq5 = m4-m5;
eq6 = hx_effectiveness*Cmin*(Tg-Ta) + (h8-h7) ;
eq7 = hx_effectiveness*Cmin*(Tg-Ta) - (h6-h5) ;
eq8 = m4*C4 - m9*C9;
eq9 = -m+ m4 - m9;

sol = solve (eq1 , eq2 , eq3 , eq4 , eq5 , eq6 , eq7 , eq8 , eq9 , eq10 , eq11 , eq12 , eq13) ;
```

```
eq10 = m5 - m6;
eq11 = m8 - m9;
eq12 = m7 - m8;
eq13 = h9 - h8;

```

The program algorithm is given here. Some of the steps are the same as for the case where no heat exchanger is used.

1. From the provided inputs, determine the properties of the refrigerant (in this case water)
2. From the inputs of concentration and temperature, determine the specific enthalpy of the LiBr-water mixture at various state-points
3. Define the variables that must be solved for using the command *symsQ2QaQgmm4m5m6m7m8m9h6h8h9*
4. Define each equation. The number of equations should be the same as the number of unknowns defined in the previous step.
5. Solve the system by means of the command *lve*(*eq*1, *eq*2, *eq*3, *eq*4, *eq*5, *eq*6, *eq*7, *eq*8, *eq*9, *eq*10, *eq*11, *eq*12, *eq*13)
6. Important parameters such as the COP may hence be found

8.2.2 Solution concentration with SHX

The effect of changing the solution concentration is somewhat similar to the case when no solution heat exchanger is used. Both the weak and strong solution concentrations will impact the system COP. The difference that can be noticed from the case where no heat exchanger is used is in the sensitivity of the COP to the solutions concentrations. Fig. 8.8 shows the variation of refrigeration COP with the weak solution concentration. At low concentration the COP remains rather constant and the strong solution concentration does not impact the COP. At higher weak solution concentrations, the COP reduces only by around 0.1 for high strong solution concentrations but reduces dramatically when the stron solution concentrations are low. When comparted to Fig. 8.2, it can be seen that the solution heat exchanger reduces the sensitivity of COP to solution concentrations. The heat transferred from the strong solution side compensates slightly for negative effect of having a relatively small strong solution concentrations or too high a weak solution concentration. Fig. 8.9 plots COP against the strong solution concentration in order to help the reader visualize better the effect of the strong solution concentration variation on the COP. The results from the simulation show the importance of having as low a weak solution concentration as possible.

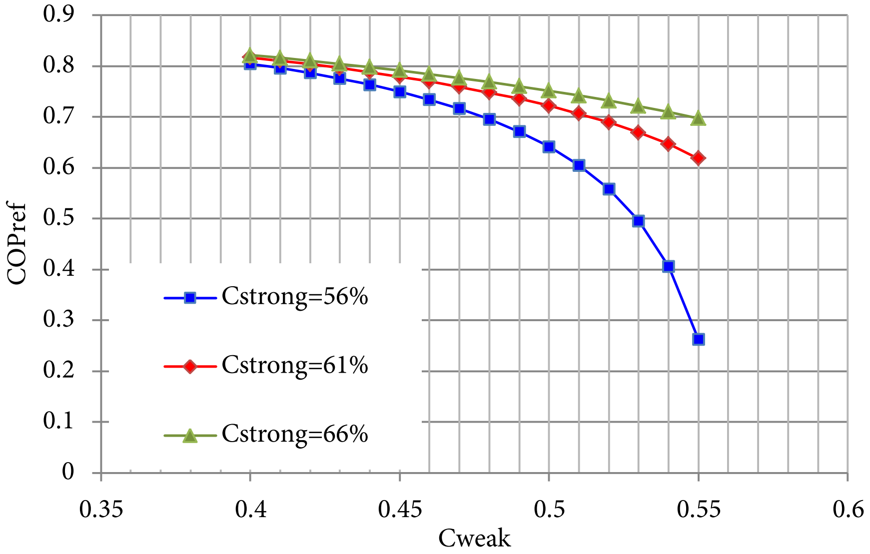


Figure 8.8: Effect of the weak solution concentration on the COP.

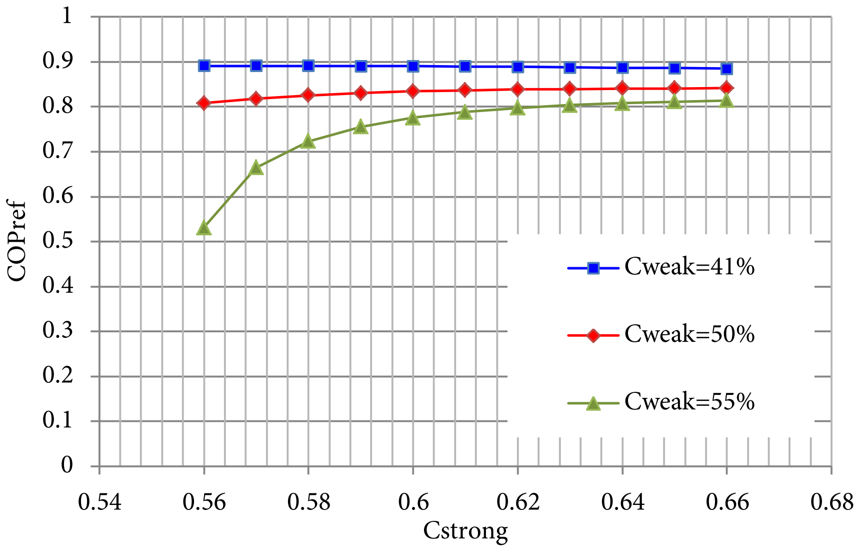


Figure 8.9: Effect of the strong solution concentration on the COP.

8.2.3 Absorber and generator temperature effects with SHX

Fig. 8.10 is similar to Fig. 8.4 and shows the generator heat input and the absorber heat rejected against absorber temperatures for various generator temperatures. The introduction of the solution heat exchanger enables a reduction in the generator heat input (hence the increase in COP) but also reduces the cooling requirements in the absorber. Figs. 8.11 and 8.12 show the variation of COP with absorber and generator temperatures respectively. When no solution heat exchanger is used, it was found that the COP increases by increasing the absorber temperature and reducing the generator temperature. The same behaviour can be observed when a solution heat exchanger is used. The COP is in all cases higher than for a system with no solution heat exchanger.

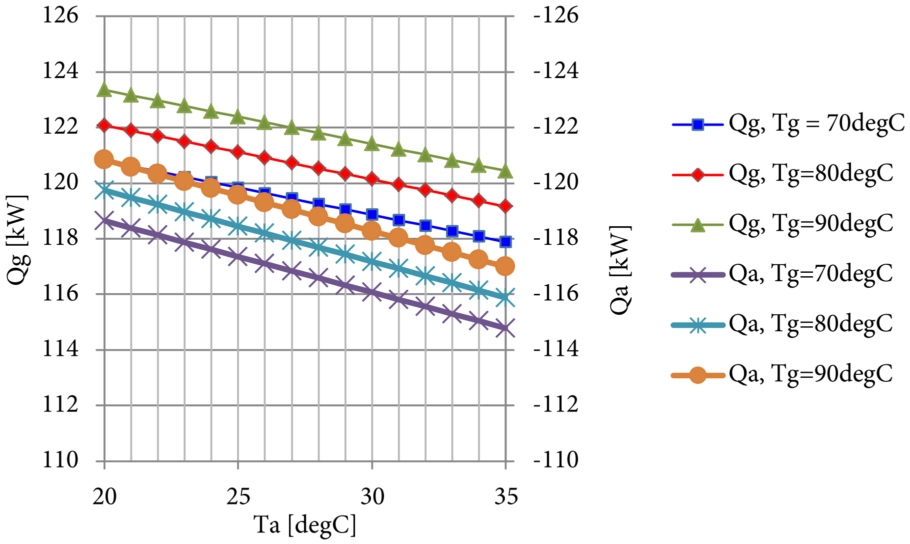


Figure 8.10: Heat transfers from the absorber and into the generator at different absorber and generator temperatures.

8.2.4 Effect of pressures on COP when using a SHX

The effect of evaporator and condenser pressures on the COP when a solution heat exchanger is used is shown in Fig. 8.13. As was the case for a system with no solution heat exchanger, the COP is not that much affected by the evaporator and condenser pressures. Again the COP for this case is higher than the original system with no solution heat exchanger.

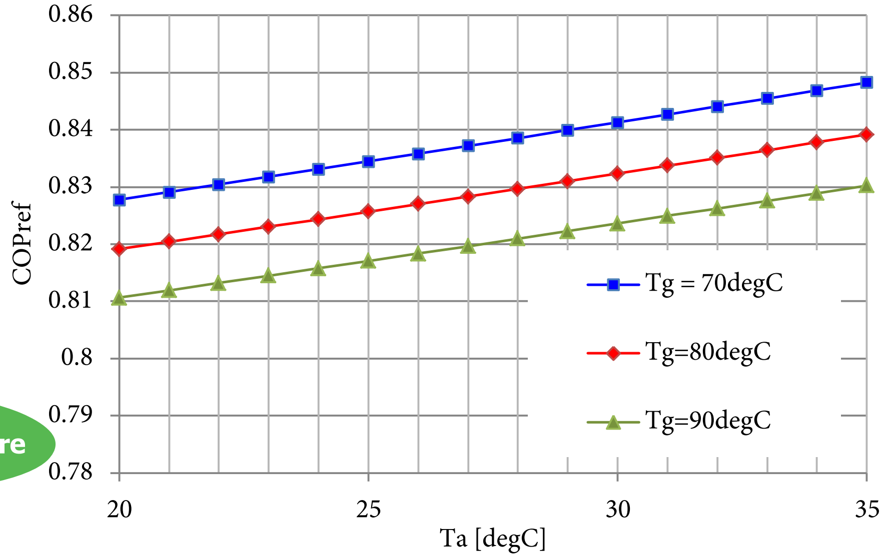


Figure 8.11: COP against absorber temperature.

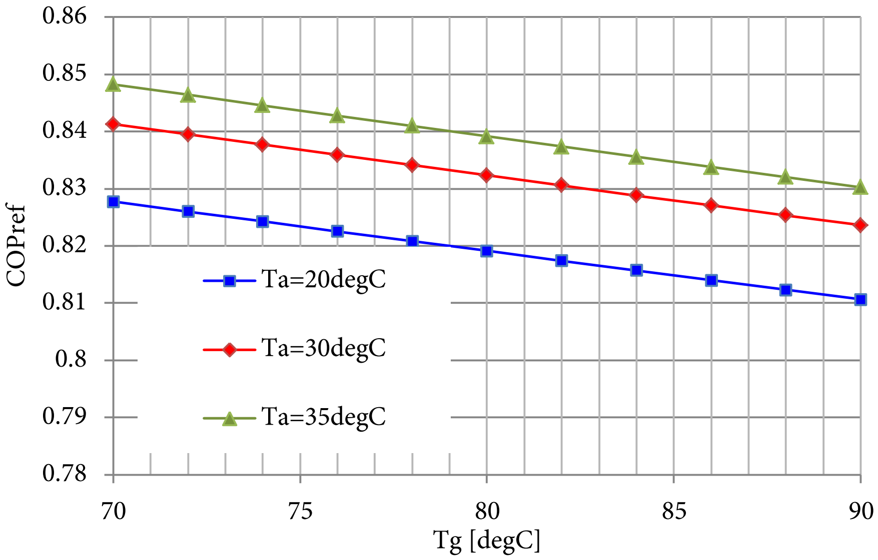


Figure 8.12: COP against generator temperature.

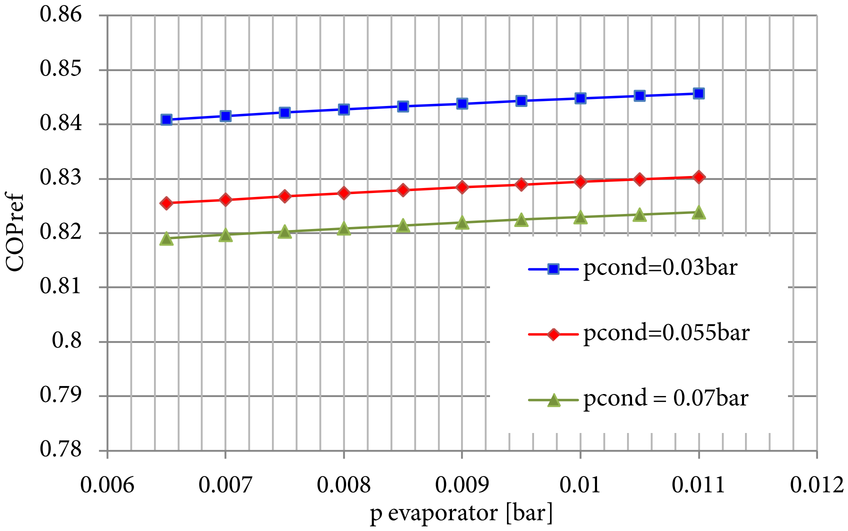


Figure 8.13: COP against evaporator pressure.

8.3 Conclusions

Student checklist

1. The student should appreciate the two different algorithms used to model vapour absorption systems.
2. The concentration of the weak solution should be as small as possible in order to increase COP.
3. If the weak solution has a low LiBr concentration, the COP is not very sensitive to the concentration of the strong solution.
4. The COP increases with increasing absorber temperature and with decreasing generator temperature.
5. The COP of a vapour absorption cycle is not very senstive to variations in evaporator pressure and is only slightly affected by the condenser pressure.
6. When a solution heat exchanger is used, the general trends of COP variation with the parameters discussed here remains in general the same. The COP is however always higher.

A Appendix

A.1 Properties of refrigerants

A.1.1 Properties of R134a

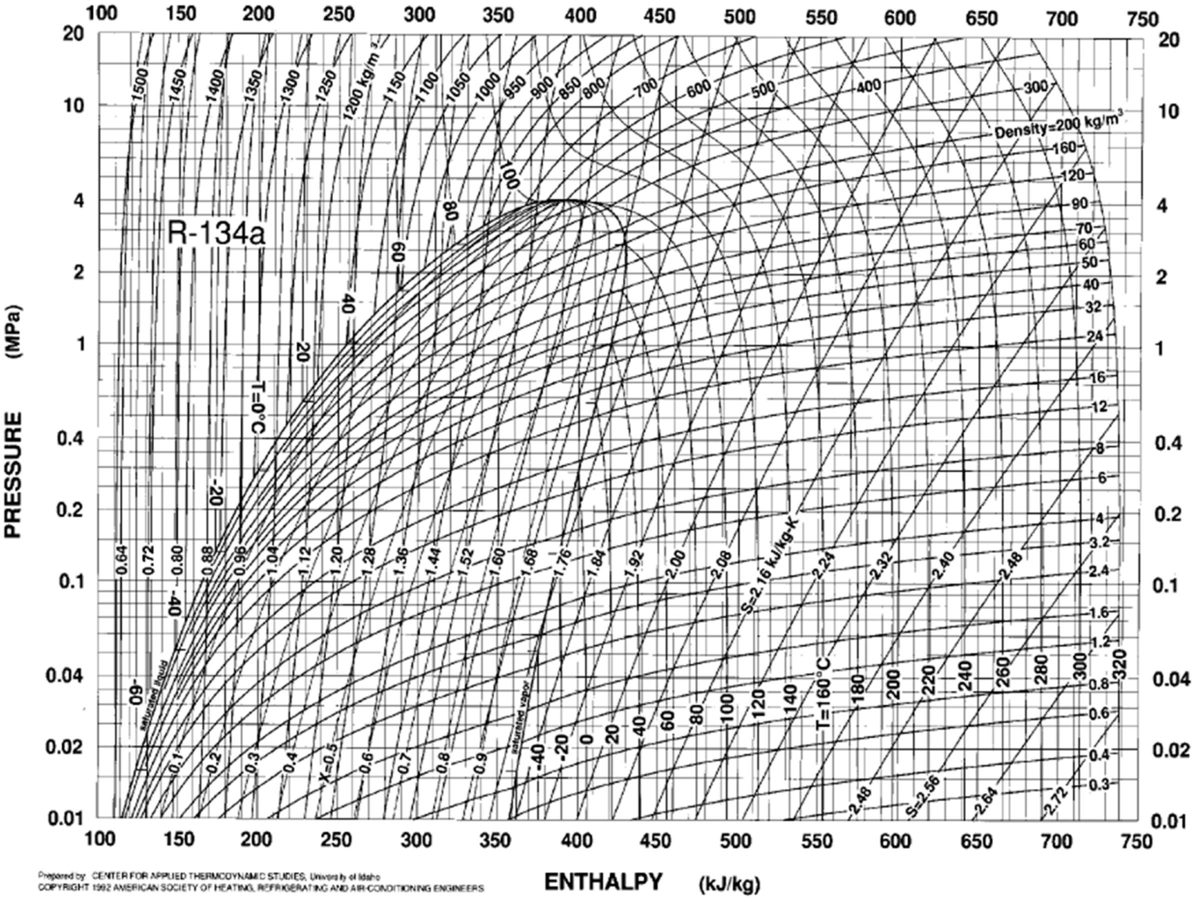
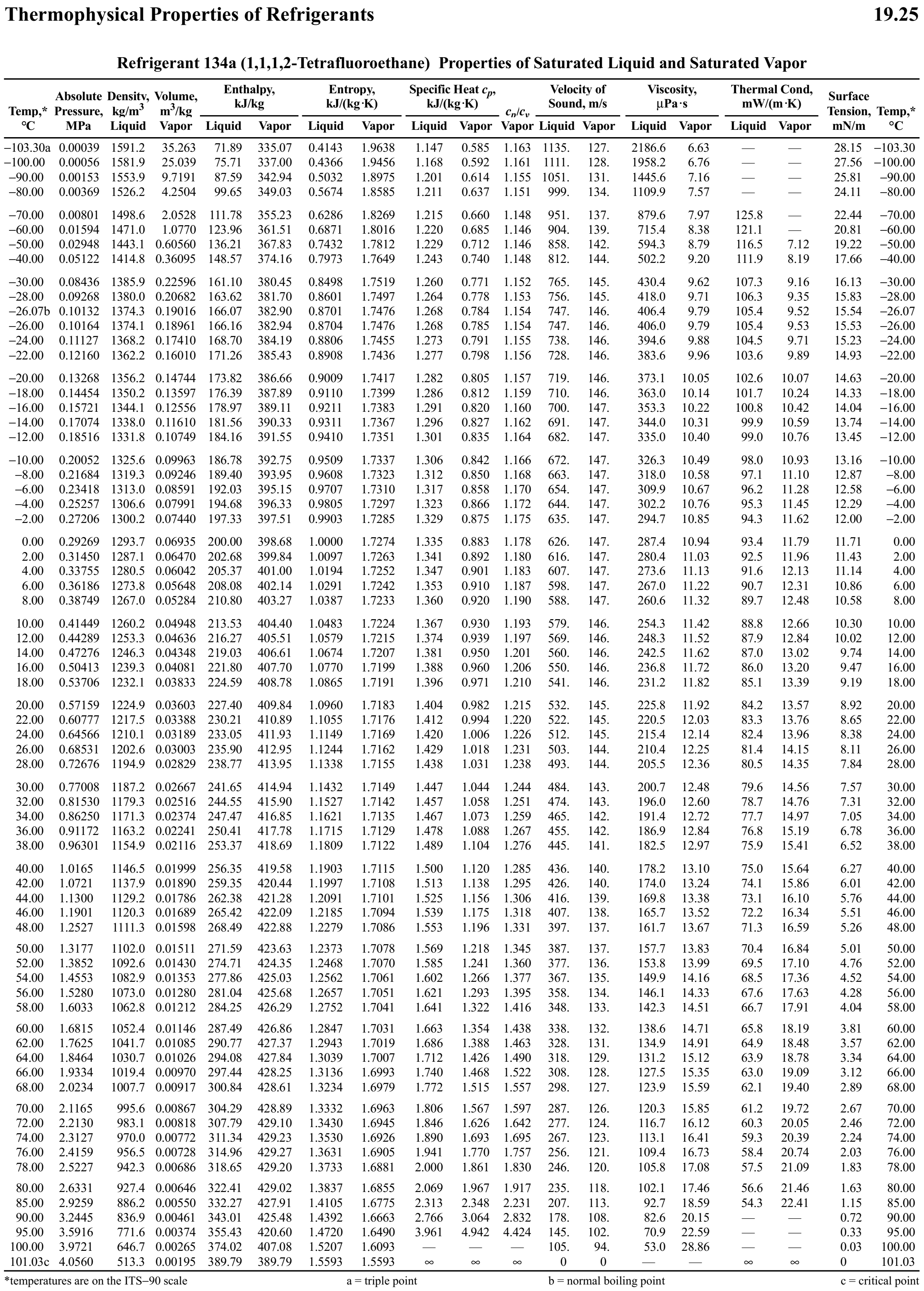


Figure A.1: *p* – *h* diagram for refrigerant R134a ASHRAE (2009). ©ASHRAE, www.ashrae.org. 2009 ASHRAE Handbook – Fundamentals.



Thermophysical Properties of Refrigerants 19.25

Refrigerant 134a (1,1,1,2-Tetrafluoroethane) Properties of Saturated Liquid and Saturated Vapor

Temp,* °C	Absolute Pressure, MPa	Density, kg/m^3	Volume, m^3/kg	Enthalpy, kJ/kg		Entropy, kJ/(kg·K)		Specific Heat c_p, kJ/(kg·K)		c_n/c_v	Velocity of Sound, m/s		Viscosity, μPa·s		Thermal Cond, mW/(m·K)		Surface Tension, mN/m	Temp,* °C
		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor		
–103.30a	0.00039	1591.2	35.263	71.89	335.07	0.4143	1.9638	1.147	0.585	1.163	1135.	127.	2186.6	6.63	—	—	28.15	–103.30
–100.00	0.00056	1581.9	25.039	75.71	337.00	0.4366	1.9456	1.168	0.592	1.161	1111.	128.	1958.2	6.76	—	—	27.56	–100.00
–90.00	0.00153	1553.9	9.7191	87.59	342.94	0.5032	1.8975	1.201	0.614	1.155	1051.	131.	1445.6	7.16	—	—	25.81	–90.00
–80.00	0.00369	1526.2	4.2504	99.65	349.03	0.5674	1.8585	1.211	0.637	1.151	999.	134.	1109.9	7.57	—	—	24.11	–80.00
–70.00	0.00801	1498.6	2.0528	111.78	355.23	0.6286	1.8269	1.215	0.660	1.148	951.	137.	879.6	7.97	125.8	—	22.44	–70.00
–60.00	0.01594	1471.0	1.0770	123.96	361.51	0.6871	1.8016	1.220	0.685	1.146	904.	139.	715.4	8.38	121.1	—	20.81	–60.00
–50.00	0.02948	1443.1	0.60560	136.21	367.83	0.7432	1.7812	1.229	0.712	1.146	858.	142.	594.3	8.79	116.5	7.12	19.22	–50.00
–40.00	0.05122	1414.8	0.36095	148.57	374.16	0.7973	1.7649	1.243	0.740	1.148	812.	144.	502.2	9.20	111.9	8.19	17.66	–40.00
–30.00	0.08436	1385.9	0.22596	161.10	380.45	0.8498	1.7519	1.260	0.771	1.152	765.	145.	430.4	9.62	107.3	9.16	16.13	–30.00
–28.00	0.09268	1380.0	0.20682	163.62	381.70	0.8601	1.7497	1.264	0.778	1.153	756.	145.	418.0	9.71	106.3	9.35	15.83	–28.00
–26.07b	0.10132	1374.3	0.19016	166.07	382.90	0.8701	1.7476	1.268	0.784	1.154	747.	146.	406.4	9.79	105.4	9.52	15.54	–26.07
–26.00	0.10164	1374.1	0.18961	166.16	382.94	0.8704	1.7476	1.268	0.785	1.154	747.	146.	406.0	9.79	105.4	9.53	15.53	–26.00
–24.00	0.11127	1368.2	0.17410	168.70	384.19	0.8806	1.7455	1.273	0.791	1.155	738.	146.	394.6	9.88	104.5	9.71	15.23	–24.00
–22.00	0.12160	1362.2	0.16010	171.26	385.43	0.8908	1.7436	1.277	0.798	1.156	728.	146.	383.6	9.96	103.6	9.89	14.93	–22.00
–20.00	0.13268	1356.2	0.14744	173.82	386.66	0.9009	1.7417	1.282	0.805	1.157	719.	146.	373.1	10.05	102.6	10.07	14.63	–20.00
–18.00	0.14454	1350.2	0.13597	176.39	387.89	0.9110	1.7399	1.286	0.812	1.159	710.	146.	363.0	10.14	101.7	10.24	14.33	–18.00
–16.00	0.15721	1344.1	0.12556	178.97	389.11	0.9211	1.7383	1.291	0.820	1.160	700.	147.	353.3	10.22	100.8	10.42	14.04	–16.00
–14.00	0.17074	1338.0	0.11610	181.56	390.33	0.9311	1.7367	1.296	0.827	1.162	691.	147.	344.0	10.31	99.9	10.59	13.74	–14.00
–12.00	0.18516	1331.8	0.10749	184.16	391.55	0.9410	1.7351	1.301	0.835	1.164	682.	147.	335.0	10.40	99.0	10.76	13.45	–12.00
–10.00	0.20052	1325.6	0.09963	186.78	392.75	0.9509	1.7337	1.306	0.842	1.166	672.	147.	326.3	10.49	98.0	10.93	13.16	–10.00
–8.00	0.21684	1319.3	0.09246	189.40	393.95	0.9608	1.7323	1.312	0.850	1.168	663.	147.	318.0	10.58	97.1	11.10	12.87	–8.00
–6.00	0.23418	1313.0	0.08591	192.03	395.15	0.9707	1.7310	1.317	0.858	1.170	654.	147.	309.9	10.67	96.2	11.28	12.58	–6.00
–4.00	0.25257	1306.6	0.07991	194.68	396.33	0.9805	1.7297	1.323	0.866	1.172	644.	147.	302.2	10.76	95.3	11.45	12.29	–4.00
–2.00	0.27206	1300.2	0.07440	197.33	397.51	0.9903	1.7285	1.329	0.875	1.175	635.	147.	294.7	10.85	94.3	11.62	12.00	–2.00
0.00	0.29269	1293.7	0.06935	200.00	398.68	1.0000	1.7274	1.335	0.883	1.178	626.	147.	287.4	10.94	93.4	11.79	11.71	0.00
2.00	0.31450	1287.1	0.06470	202.68	399.84	1.0097	1.7263	1.341	0.892	1.180	616.	147.	280.4	11.03	92.5	11.96	11.43	2.00
4.00	0.33755	1280.5	0.06042	205.37	401.00	1.0194	1.7252	1.347	0.901	1.183	607.	147.	273.6	11.13	91.6	12.13	11.14	4.00
6.00	0.36186	1273.8	0.05648	208.08	402.14	1.0291	1.7242	1.353	0.910	1.187	598.	147.	267.0	11.22	90.7	12.31	10.86	6.00
8.00	0.38749	1267.0	0.05284	210.80	403.27	1.0387	1.7233	1.360	0.920	1.190	588.	147.	260.6	11.32	89.7	12.48	10.58	8.00
10.00	0.41449	1260.2	0.04948	213.53	404.40	1.0483	1.7224	1.367	0.930	1.193	579.	146.	254.3	11.42	88.8	12.66	10.30	10.00
12.00	0.44289	1253.3	0.04636	216.27	405.51	1.0579	1.7215	1.374	0.939	1.197	569.	146.	248.3	11.52	87.9	12.84	10.02	12.00
14.00	0.47276	1246.3	0.04348	219.03	406.61	1.0674	1.7207	1.381	0.950	1.201	560.	146.	242.5	11.62	87.0	13.02	9.74	14.00
16.00	0.50413	1239.3	0.04081	221.80	407.70	1.0770	1.7199	1.388	0.960	1.206	550.	146.	236.8	11.72	86.0	13.20	9.47	16.00
18.00	0.53706	1232.1	0.03833	224.59	408.78	1.0865	1.7191	1.396	0.971	1.210	541.	146.	231.2	11.82	85.1	13.39	9.19	18.00
20.00	0.57159	1224.9	0.03603	227.40	409.84	1.0960	1.7183	1.404	0.982	1.215	532.	145.	225.8	11.92	84.2	13.57	8.92	20.00
22.00	0.60777	1217.5	0.03388	230.21	410.89	1.1055	1.7176	1.412	0.994	1.220	522.	145.	220.5	12.03	83.3	13.76	8.65	22.00
24.00	0.64566	1210.1	0.03189	233.05	411.93	1.1149	1.7169	1.420	1.006	1.226	512.	145.	215.4	12.14	82.4	13.96	8.38	24.00
26.00	0.68531	1202.6	0.03003	235.90	412.95	1.1244	1.7162	1.429	1.018	1.231	503.	144.	210.4	12.25	81.4	14.15	8.11	26.00
28.00	0.72676	1194.9	0.02829	238.77	413.95	1.1338	1.7155	1.438	1.031	1.238	493.	144.	205.5	12.36	80.5	14.35	7.84	28.00
30.00	0.77008	1187.2	0.02667	241.65	414.94	1.1432	1.7149	1.447	1.044	1.244	484.	143.	200.7	12.48	79.6	14.56	7.57	30.00
32.00	0.81530	1179.3	0.02516	244.55	415.90	1.1527	1.7142	1.457	1.058	1.251	474.	143.	196.0	12.60	78.7	14.76	7.31	32.00
34.00	0.86250	1171.3	0.02374	247.47	416.85	1.1621	1.7135	1.467	1.073	1.259	465.	142.	191.4	12.72	77.7	14.97	7.05	34.00
36.00	0.91172	1163.2	0.02241	250.41	417.78	1.1715	1.7129	1.478	1.088	1.267	455.	142.	186.9	12.84	76.8	15.19	6.78	36.00
38.00	0.96301	1154.9	0.02116	253.37	418.69	1.1809	1.7122	1.489	1.104	1.276	445.	141.	182.5	12.97	75.9	15.41	6.52	38.00
40.00	1.0165	1146.5	0.01999	256.35	419.58	1.1903	1.7115	1.500	1.120	1.285	436.	140.	178.2	13.10	75.0	15.64	6.27	40.00
42.00	1.0721	1137.9	0.01890	259.35	420.44	1.1997	1.7108	1.513	1.138	1.295	426.	140.	174.0	13.24	74.1	15.86	6.01	42.00
44.00	1.1300	1129.2	0.01786	262.38	421.28	1.2091	1.7101	1.525	1.156	1.306	416.	139.	169.8	13.38	73.1	16.10	5.76	44.00
46.00	1.1901	1120.3	0.01689	265.42	422.09	1.2185	1.7094	1.539	1.175	1.318	407.	138.	165.7	13.52	72.2	16.34	5.51	46.00
48.00	1.2527	1111.3	0.01598	268.49	422.88	1.2279	1.7086	1.553	1.196	1.331	397.	137.	161.7	13.67	71.3	16.59	5.26	48.00
50.00	1.3177	1102.0	0.01511	271.59	423.63	1.2373	1.7078	1.569	1.218	1.345	387.	137.	157.7	13.83	70.4	16.84	5.01	50.00
52.00	1.3852	1092.6	0.01430	274.71	424.35	1.2468	1.7070	1.585	1.241	1.360	377.	136.	153.8	13.99	69.5	17.10	4.76	52.00
54.00	1.4553	1082.9	0.01353	277.86	425.03	1.2562	1.7061	1.602	1.266	1.377	367.	135.	149.9	14.16	68.5	17.36	4.52	54.00
56.00	1.5280	1073.0	0.01280	281.04	425.68	1.2657	1.7051	1.621	1.293	1.395	358.	134.	146.1	14.33	67.6	17.63	4.28	56.00
58.00	1.6033	1062.8	0.01212	284.25	426.29	1.2752	1.7041	1.641	1.322	1.416	348.	133.	142.3	14.51	66.7	17.91	4.04	58.00
60.00	1.6815	1052.4	0.01146	287.49	426.86	1.2847	1.7031	1.663	1.354	1.438	338.	132.	138.6	14.71	65.8	18.19	3.81	60.00
62.00	1.7625	1041.7	0.01085	290.77	427.37	1.2943	1.7019	1.686	1.388	1.463	328.	131.	134.9	14.91	64.9	18.48	3.57	62.00
64.00	1.8464	1030.7	0.01026	294.08	427.84	1.3039	1.7007	1.712	1.426	1.490	318.	129.	131.2	15.12	63.9	18.78	3.34	64.00
66.00	1.9334	1019.4	0.00970	297.44	428.25	1.3136	1.6993	1.740	1.468	1.522	308.	128.	127.5	15.35	63.0	19.09	3.12	66.00
68.00	2.0234	1007.7	0.00917	300.84	428.61	1.3234	1.6979	1.772	1.515	1.557	298.	127.	123.9	15.59	62.1	19.40	2.89	68.00
70.00	2.1165	995.6	0.00867	304.29	428.89	1.3332	1.6963	1.806	1.567	1.597	287.	126.	120.3	15.85	61.2	19.72	2.67	70.00
72.00	2.2130	983.1	0.00818	307.79	429.10	1.3430	1.6945	1.846	1.626	1.642	277.	124.	116.7	16.12	60.3	20.05	2.46	72.00
74.00	2.3127	970.0	0.00772	311.34	429.23	1.3530	1.6926	1.890	1.693	1.695	267.	123.	113.1	16.41	59.3	20.39	2.24	74.00
76.00	2.4159	956.5	0.00728	314.96	429.27	1.3631	1.6905	1.941	1.770	1.757	256.	121.	109.4	16.73	58.4	20.74	2.03	76.00
78.00	2.5227	942.3	0.00686	318.65	429.20	1.3733	1.6881	2.000	1.861	1.830	246.	120.	105.8	17.08	57.5	21.09	1.83	78.00
80.00	2.6331	927.4	0.00646	322.41	429.02	1.3837	1.6855	2.069	1.967	1.917	235.	118.	102.1	17.46	56.6	21.46	1.63	80.00
85.00	2.9259	886.2	0.00550	332.27	427.91	1.4105	1.6775	2.313	2.348	2.231	207.	113.	92.7	18.59	54.3	22.41	1.15	85.00
90.00	3.2445	836.9	0.00461	343.01	425.48	1.4392	1.6663	2.766	3.064	2.832	178.	108.	82.6	20.15	—	—	0.72	90.00
95.00	3.5916	771.6	0.00374	355.43	420.60	1.4720	1.6490	3.961	4.942	4.424	145.	102.	70.9	22.59	—	—	0.33	95.00
100.00	3.9721	646.7	0.00265	374.02	407.08	1.5207	1.6093	—	—	—	105.	94.	53.0	28.86	—	—	0.03	100.00
101.03c	4.0560	513.3	0.00195	389.79	389.79	1.5593	1.5593	∞	∞	∞	0	0	—	—	∞	∞	0	101.03

*temperatures are on the ITS–90 scale a = triple point b = normal boiling point c = critical point

Figure A.2: R134a properties of saturated liquid and saturated vapour ASHRAE (2009).

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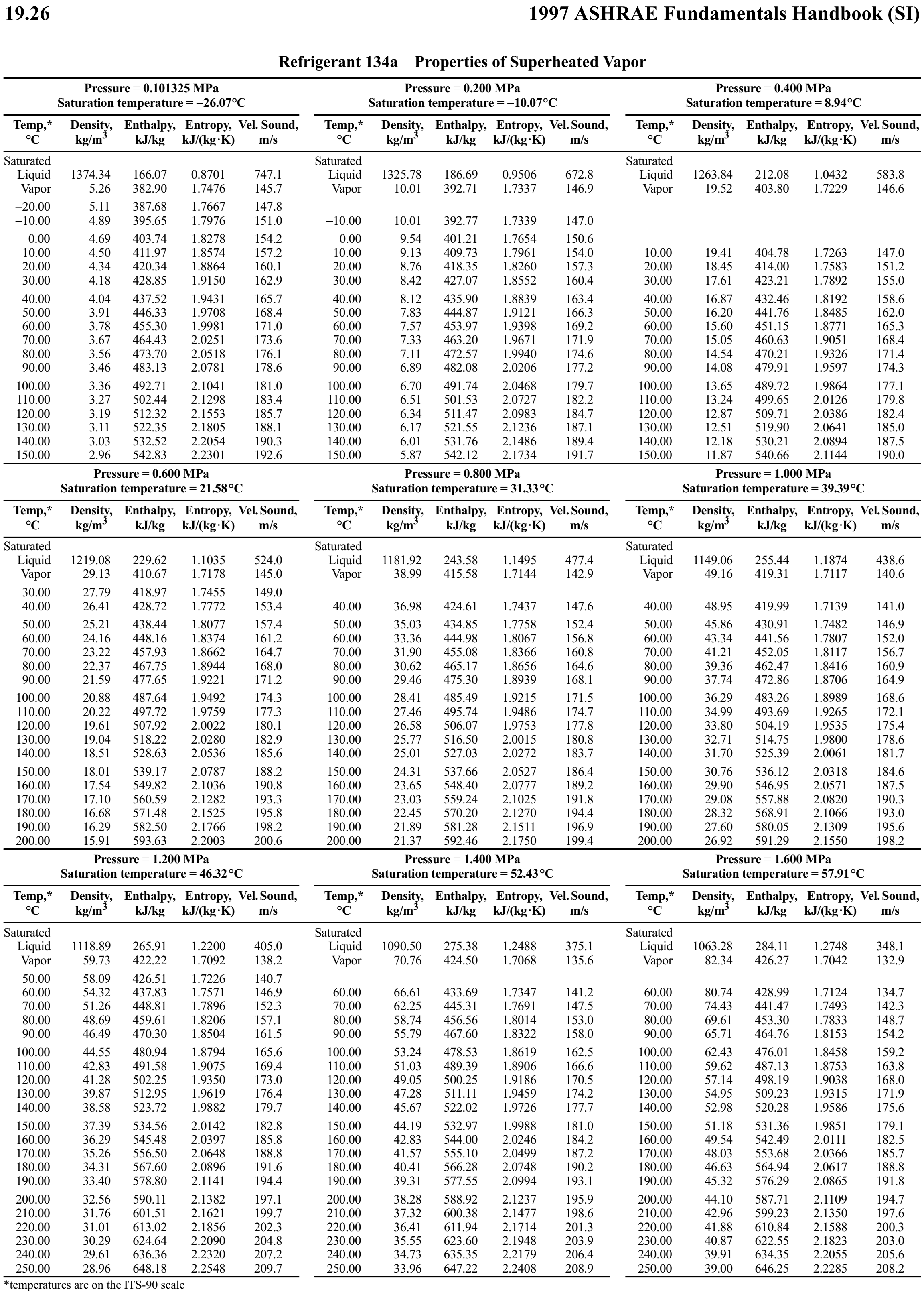

19.26 **1997 ASHRAE Fundamentals Handbook (SI)**

Refrigerant 134a Properties of Superheated Vapor

Pressure = 0.101325 MPa
Saturation temperature = –26.07°C

Temp.,* °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	1374.34	166.07	0.8701	747.1
Vapor	5.26	382.90	1.7476	145.7
–20.00	5.11	387.68	1.7667	147.8
–10.00	4.89	395.65	1.7976	151.0
0.00	4.69	403.74	1.8278	154.2
10.00	4.50	411.97	1.8574	157.2
20.00	4.34	420.34	1.8864	160.1
30.00	4.18	428.85	1.9150	162.9
40.00	4.04	437.52	1.9431	165.7
50.00	3.91	446.33	1.9708	168.4
60.00	3.78	455.30	1.9981	171.0
70.00	3.67	464.43	2.0251	173.6
80.00	3.56	473.70	2.0518	176.1
90.00	3.46	483.13	2.0781	178.6
100.00	3.36	492.71	2.1041	181.0
110.00	3.27	502.44	2.1298	183.4
120.00	3.19	512.32	2.1553	185.7
130.00	3.11	522.35	2.1805	188.1
140.00	3.03	532.52	2.2054	190.3
150.00	2.96	542.83	2.2301	192.6

Pressure = 0.200 MPa
Saturation temperature = –10.07°C

Temp.,* °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	1325.78	186.69	0.9506	672.8
Vapor	10.01	392.71	1.7337	146.9
–10.00	10.01	392.77	1.7339	147.0
0.00	9.54	401.21	1.7654	150.6
10.00	9.13	409.73	1.7961	154.0
20.00	8.76	418.35	1.8260	157.3
30.00	8.42	427.07	1.8552	160.4
40.00	8.12	435.90	1.8839	163.4
50.00	7.83	444.87	1.9121	166.3
60.00	7.57	453.97	1.9398	169.2
70.00	7.33	463.20	1.9671	171.9
80.00	7.11	472.57	1.9940	174.6
90.00	6.89	482.08	2.0206	177.2
100.00	6.70	491.74	2.0468	179.7
110.00	6.51	501.53	2.0727	182.2
120.00	6.34	511.47	2.0983	184.7
130.00	6.17	521.55	2.1236	187.1
140.00	6.01	531.76	2.1486	189.4
150.00	5.87	542.12	2.1734	191.7

Pressure = 0.400 MPa
Saturation temperature = 8.94°C

Temp.,* °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	1263.84	212.08	1.0432	583.8
Vapor	19.52	403.80	1.7229	146.6
10.00	19.41	404.78	1.7263	147.0
20.00	18.45	414.00	1.7583	151.2
30.00	17.61	423.21	1.7892	155.0
40.00	16.87	432.46	1.8192	158.6
50.00	16.20	441.76	1.8485	162.0
60.00	15.60	451.15	1.8771	165.3
70.00	15.05	460.63	1.9051	168.4
80.00	14.54	470.21	1.9326	171.4
90.00	14.08	479.91	1.9597	174.3
100.00	13.65	489.72	1.9864	177.1
110.00	13.24	499.65	2.0126	179.8
120.00	12.87	509.71	2.0386	182.4
130.00	12.51	519.90	2.0641	185.0
140.00	12.18	530.21	2.0894	187.5
150.00	11.87	540.66	2.1144	190.0

Pressure = 0.600 MPa
Saturation temperature = 21.58°C

Temp.,* °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	1219.08	229.62	1.1035	524.0
Vapor	29.13	410.67	1.7178	145.0
30.00	27.79	418.97	1.7455	149.0
40.00	26.41	428.72	1.7772	153.4
50.00	25.21	438.44	1.8077	157.4
60.00	24.16	448.16	1.8374	161.2
70.00	23.22	457.93	1.8662	164.7
80.00	22.37	467.75	1.8944	168.0
90.00	21.59	477.65	1.9221	171.2
100.00	20.88	487.64	1.9492	174.3
110.00	20.22	497.72	1.9759	177.3
120.00	19.61	507.92	2.0022	180.1
130.00	19.04	518.22	2.0280	182.9
140.00	18.51	528.63	2.0536	185.6
150.00	18.01	539.17	2.0787	188.2
160.00	17.54	549.82	2.1036	190.8
170.00	17.10	560.59	2.1282	193.3
180.00	16.68	571.48	2.1525	195.8
190.00	16.29	582.50	2.1766	198.2
200.00	15.91	593.63	2.2003	200.6

Pressure = 0.800 MPa
Saturation temperature = 31.33°C

Temp.,* °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	1181.92	243.58	1.1495	477.4
Vapor	38.99	415.58	1.7144	142.9
40.00	36.98	424.61	1.7437	147.6
50.00	35.03	434.85	1.7758	152.4
60.00	33.36	444.98	1.8067	156.8
70.00	31.90	455.08	1.8366	160.8
80.00	30.62	465.17	1.8656	164.6
90.00	29.46	475.30	1.8939	168.1
100.00	28.41	485.49	1.9215	171.5
110.00	27.46	495.74	1.9486	174.7
120.00	26.58	506.07	1.9753	177.8
130.00	25.77	516.50	2.0015	180.8
140.00	25.01	527.03	2.0272	183.7
150.00	24.31	537.66	2.0527	186.4
160.00	23.65	548.40	2.0777	189.2
170.00	23.03	559.24	2.1025	191.8
180.00	22.45	570.20	2.1270	194.4
190.00	21.89	581.28	2.1511	196.9
200.00	21.37	592.46	2.1750	199.4

Pressure = 1.000 MPa
Saturation temperature = 39.39°C

Temp.,* °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	1149.06	255.44	1.1874	438.6
Vapor	49.16	419.31	1.7117	140.6
40.00	48.95	419.99	1.7139	141.0
50.00	45.86	430.91	1.7482	146.9
60.00	43.34	441.56	1.7807	152.0
70.00	41.21	452.05	1.8117	156.7
80.00	39.36	462.47	1.8416	160.9
90.00	37.74	472.86	1.8706	164.9
100.00	36.29	483.26	1.8989	168.6
110.00	34.99	493.69	1.9265	172.1
120.00	33.80	504.19	1.9535	175.4
130.00	32.71	514.75	1.9800	178.6
140.00	31.70	525.39	2.0061	181.7
150.00	30.76	536.12	2.0318	184.6
160.00	29.90	546.95	2.0571	187.5
170.00	29.08	557.88	2.0820	190.3
180.00	28.32	568.91	2.1066	193.0
190.00	27.60	580.05	2.1309	195.6
200.00	26.92	591.29	2.1550	198.2

Pressure = 1.200 MPa
Saturation temperature = 46.32°C

Temp.,* °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	1118.89	265.91	1.2200	405.0
Vapor	59.73	422.22	1.7092	138.2
50.00	58.09	426.51	1.7226	140.7
60.00	54.32	437.83	1.7571	146.9
70.00	51.26	448.81	1.7896	152.3
80.00	48.69	459.61	1.8206	157.1
90.00	46.49	470.30	1.8504	161.5
100.00	44.55	480.94	1.8794	165.6
110.00	42.83	491.58	1.9075	169.4
120.00	41.28	502.25	1.9350	173.0
130.00	39.87	512.95	1.9619	176.4
140.00	38.58	523.72	1.9882	179.7
150.00	37.39	534.56	2.0142	182.8
160.00	36.29	545.48	2.0397	185.8
170.00	35.26	556.50	2.0648	188.8
180.00	34.31	567.60	2.0896	191.6
190.00	33.40	578.80	2.1141	194.4
200.00	32.56	590.11	2.1382	197.1
210.00	31.76	601.51	2.1621	199.7
220.00	31.01	613.02	2.1856	202.3
230.00	30.29	624.64	2.2090	204.8
240.00	29.61	636.36	2.2320	207.2
250.00	28.96	648.18	2.2548	209.7

Pressure = 1.400 MPa
Saturation temperature = 52.43°C

Temp.,* °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	1090.50	275.38	1.2488	375.1
Vapor	70.76	424.50	1.7068	135.6
60.00	66.61	433.69	1.7347	141.2
70.00	62.25	445.31	1.7691	147.5
80.00	58.74	456.56	1.8014	153.0
90.00	55.79	467.60	1.8322	158.0
100.00	53.24	478.53	1.8619	162.5
110.00	51.03	489.39	1.8906	166.6
120.00	49.05	500.25	1.9186	170.5
130.00	47.28	511.11	1.9459	174.2
140.00	45.67	522.02	1.9726	177.7
150.00	44.19	532.97	1.9988	181.0
160.00	42.83	544.00	2.0246	184.2
170.00	41.57	555.10	2.0499	187.2
180.00	40.41	566.28	2.0748	190.2
190.00	39.31	577.55	2.0994	193.1
200.00	38.28	588.92	2.1237	195.9
210.00	37.32	600.38	2.1477	198.6
220.00	36.41	611.94	2.1714	201.3
230.00	35.55	623.60	2.1948	203.9
240.00	34.73	635.35	2.2179	206.4
250.00	33.96	647.22	2.2408	208.9

Pressure = 1.600 MPa
Saturation temperature = 57.91°C

Temp.,* °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	1063.28	284.11	1.2748	348.1
Vapor	82.34	426.27	1.7042	132.9
60.00	80.74	428.99	1.7124	134.7
70.00	74.43	441.47	1.7493	142.3
80.00	69.61	453.30	1.7833	148.7
90.00	65.71	464.76	1.8153	154.2
100.00	62.43	476.01	1.8458	159.2
110.00	59.62	487.13	1.8753	163.8
120.00	57.14	498.19	1.9038	168.0
130.00	54.95	509.23	1.9315	171.9
140.00	52.98	520.28	1.9586	175.6
150.00	51.18	531.36	1.9851	179.1
160.00	49.54	542.49	2.0111	182.5
170.00	48.03	553.68	2.0366	185.7
180.00	46.63	564.94	2.0617	188.8
190.00	45.32	576.29	2.0865	191.8
200.00	44.10	587.71	2.1109	194.7
210.00	42.96	599.23	2.1350	197.6
220.00	41.88	610.84	2.1588	200.3
230.00	40.87	622.55	2.1823	203.0
240.00	39.91	634.35	2.2055	205.6
250.00	39.00	646.25	2.2285	208.2

*temperatures are on the ITS-90 scale

Figure A.3: R134a properties of superheated vapour ASHRAE (2009).
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Thermophysical Properties of Refrigerants **19.27**

Refrigerant 134a Properties of Superheated Vapor (*Concluded*)

Pressure = 1.800 MPa
Saturation temperature = 62.90°C

Temp,* °C	Density, kg/m³	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	1036.81	292.26	1.2987	323.2
Vapor	94.53	427.59	1.7014	130.1
70.00	88.23	437.17	1.7296	136.5
80.00	81.54	449.76	1.7657	144.0
90.00	76.38	461.74	1.7992	150.3
100.00	72.17	473.36	1.8308	155.9
110.00	68.64	484.78	1.8610	160.8
120.00	65.60	496.06	1.8900	165.4
130.00	62.91	507.29	1.9183	169.6
140.00	60.53	518.50	1.9457	173.5
150.00	58.37	529.71	1.9725	177.3
160.00	56.42	540.95	1.9988	180.8
170.00	54.62	552.24	2.0246	184.2
180.00	52.97	563.59	2.0499	187.4
190.00	51.44	575.01	2.0748	190.6
200.00	50.01	586.50	2.0993	193.6
210.00	48.68	598.08	2.1236	196.5
220.00	47.43	609.74	2.1475	199.4
230.00	46.25	621.50	2.1710	202.1
240.00	45.14	633.34	2.1944	204.9
250.00	44.09	645.28	2.2174	207.5

Pressure = 2.000 MPa
Saturation temperature = 67.49°C

Temp,* °C	Density, kg/m³	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	1010.74	299.96	1.3209	300.1
Vapor	107.46	428.52	1.6983	127.2
70.00	104.37	432.22	1.7091	129.9
80.00	94.85	445.86	1.7483	138.9
90.00	87.97	458.49	1.7835	146.2
100.00	82.58	470.57	1.8164	152.4
110.00	78.17	482.32	1.8474	157.8
120.00	74.44	493.86	1.8772	162.7
130.00	71.18	505.30	1.9059	167.2
140.00	68.33	516.68	1.9338	171.4
150.00	65.78	528.03	1.9609	175.4
160.00	63.47	539.39	1.9875	179.1
170.00	61.37	550.79	2.0135	182.6
180.00	59.45	562.23	2.0390	186.0
190.00	57.67	573.72	2.0641	189.3
200.00	56.02	585.28	2.0888	192.4
210.00	54.49	596.92	2.1131	195.5
220.00	53.05	608.64	2.1371	198.4
230.00	51.70	620.44	2.1608	201.3
240.00	50.43	632.33	2.1842	204.1
250.00	49.23	644.30	2.2073	206.8

Pressure = 2.200 MPa
Saturation temperature = 71.74°C

Temp,* °C	Density, kg/m³	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	984.76	307.32	1.3417	278.4
Vapor	121.25	429.08	1.6948	124.3
80.00	110.03	441.49	1.7303	133.3
90.00	100.70	454.98	1.7680	141.8
100.00	93.78	467.61	1.8023	148.7
110.00	88.25	479.75	1.8344	154.7
120.00	83.70	491.59	1.8649	160.0
130.00	79.79	503.25	1.8942	164.9
140.00	76.41	514.81	1.9226	169.3
150.00	73.40	526.32	1.9501	173.5
160.00	70.71	537.81	1.9769	177.4
170.00	68.28	549.31	2.0032	181.1
180.00	66.06	560.84	2.0289	184.6
190.00	64.02	572.42	2.0542	188.0
200.00	62.13	584.06	2.0790	191.3
210.00	60.38	595.76	2.1035	194.4
220.00	58.74	607.53	2.1276	197.5
230.00	57.21	619.38	2.1514	200.4
240.00	55.77	631.31	2.1749	203.3
250.00	54.42	643.33	2.1981	206.1

Pressure =2.400 MPa
Saturation temperature = 75.70°C

Temp,* °C	Density, kg/m³	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	958.58	314.40	1.3616	257.9
Vapor	136.07	429.27	1.6908	121.4
80.00	127.96	436.42	1.7112	126.9
90.00	114.90	451.12	1.7523	137.0
100.00	105.89	464.44	1.7885	144.8
110.00	99.00	477.04	1.8218	151.5
120.00	93.44	489.22	1.8532	157.2
130.00	88.79	501.14	1.8831	162.4
140.00	84.77	512.90	1.9119	167.2
150.00	81.27	524.57	1.9398	171.6
160.00	78.15	536.20	1.9670	175.7
170.00	75.35	547.82	1.9935	179.6
180.00	72.81	559.45	2.0195	183.3
190.00	70.48	571.11	2.0449	186.8
200.00	68.34	582.82	2.0699	190.2
210.00	66.36	594.58	2.0945	193.4
220.00	64.51	606.41	2.1188	196.6
230.00	62.79	618.31	2.1427	199.6
240.00	61.18	630.29	2.1662	202.5
250.00	59.66	642.35	2.1895	205.4

Pressure = 2.600 MPa
Saturation temperature = 79.41°C

Temp,* °C	Density, kg/m³	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	931.88	321.29	1.3806	238.2
Vapor	152.12	429.08	1.6863	118.3
80.00	150.48	430.22	1.6895	119.3
90.00	131.08	446.81	1.7359	131.7
100.00	119.15	461.03	1.7745	140.8
110.00	110.50	474.19	1.8093	148.1
120.00	103.72	486.75	1.8417	154.4
130.00	98.17	498.96	1.8724	160.0
140.00	93.46	510.94	1.9017	165.0
150.00	89.39	522.79	1.9301	169.7
160.00	85.80	534.57	1.9576	174.0
170.00	82.59	546.30	1.9844	178.1
180.00	79.70	558.04	2.0106	181.9
190.00	77.07	569.79	2.0362	185.5
200.00	74.65	581.57	2.0614	189.0
210.00	72.43	593.40	2.0861	192.4
220.00	70.36	605.29	2.1105	195.6
230.00	68.44	617.24	2.1345	198.8
240.00	66.64	629.27	2.1581	201.8
250.00	64.95	641.37	2.1815	204.8

Pressure = 2.800 MPa
Saturation temperature = 82.90°C

Temp,* °C	Density, kg/m³	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	904.29	328.05	1.3990	219.1
Vapor	169.71	428.50	1.6812	115.3
90.00	150.13	441.84	1.7183	125.9
100.00	133.85	457.32	1.7603	136.4
110.00	122.89	471.16	1.7970	144.6
120.00	114.63	484.17	1.8305	151.5
130.00	108.00	496.70	1.8620	157.5
140.00	102.49	508.93	1.8919	162.9
150.00	97.78	520.97	1.9207	167.8
160.00	93.66	532.90	1.9486	172.3
170.00	90.01	544.77	1.9757	176.5
180.00	86.74	556.61	2.0021	180.5
190.00	83.78	568.45	2.0279	184.3
200.00	81.08	580.31	2.0533	187.9
210.00	78.59	592.21	2.0782	191.4
220.00	76.29	604.16	2.1027	194.7
230.00	74.15	616.17	2.1268	198.0
240.00	72.16	628.25	2.1505	201.1
250.00	70.30	640.39	2.1740	204.1

Pressure = 3.000 MPa
Saturation temperatue = 86.20°C

Temp,* °C	Density, kg/m³	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	875.30	334.75	1.4171	200.4
Vapor	189.25	427.47	1.6752	112.2
90.00	173.82	435.84	1.6983	119.1
100.00	150.47	453.20	1.7455	131.8
110.00	136.36	467.93	1.7845	141.0
120.00	126.23	481.47	1.8194	148.5
130.00	118.34	494.36	1.8518	155.0
140.00	111.89	506.86	1.8824	160.7
150.00	106.45	519.11	1.9117	165.9
160.00	101.75	531.21	1.9399	170.6
170.00	97.62	543.21	1.9673	175.0
180.00	93.94	555.16	1.9940	179.2
190.00	90.62	567.10	2.0201	183.1
200.00	87.61	579.05	2.0456	186.8
210.00	84.84	591.02	2.0706	190.4
220.00	82.30	603.03	2.0952	193.8
230.00	79.94	615.10	2.1195	197.2
240.00	77.74	627.22	2.1433	200.4
250.00	75.69	639.41	2.1668	203.4
260.00	73.77	651.66	2.1900	206.5
270.00	71.96	664.00	2.2130	209.4
280.00	70.25	676.41	2.2356	212.2
290.00	68.63	688.89	2.2580	215.0
300.00	67.10	701.46	2.2801	217.8

Pressure = 4.000 MPa
Saturation temperature = 100.35°C

Temp,* °C	Density, kg/m³	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
Saturated				
Liquid	626.95	376.48	1.5272	101.3
Vapor	396.29	404.57	1.6024	93.4
110.00	233.68	446.28	1.7131	119.8
120.00	199.79	465.29	1.7621	132.5
130.00	179.83	481.11	1.8018	142.0
140.00	165.73	495.51	1.8371	149.7
150.00	154.89	509.13	1.8697	156.4
160.00	146.10	522.25	1.9004	162.4
170.00	138.74	535.07	1.9296	167.8
180.00	132.41	547.69	1.9578	172.7
190.00	126.88	560.17	1.9850	177.4
200.00	121.97	572.58	2.0115	181.7
210.00	117.55	584.95	2.0374	185.8
220.00	113.56	597.30	2.0627	189.7
230.00	109.90	609.66	2.0875	193.4
240.00	106.55	622.05	2.1119	197.0
250.00	103.44	634.47	2.1359	200.5
260.00	100.56	646.93	2.1595	203.8
270.00	97.87	659.45	2.1827	207.1
280.00	95.35	672.03	2.2057	210.2
290.00	92.98	684.67	2.2283	213.3
300.00	90.75	697.38	2.2507	216.2

Pressure = 6.00 MPa
Saturation temperature = n/a (supercritical)

Temp,* °C	Density, kg/m³	Enthalpy, kJ/kg	Entropy, kJ/(kg·K)	Vel. Sound, m/s
110.00	762.66	375.61	1.5174	173.6
120.00	591.77	405.75	1.5950	127.4
130.00	418.90	439.87	1.6807	120.4
140.00	333.91	465.19	1.7428	130.1
150.00	289.37	484.69	1.7894	139.9
160.00	260.70	501.52	1.8288	148.3
170.00	239.96	516.92	1.8639	155.7
180.00	223.87	531.45	1.8963	162.2
190.00	210.82	545.43	1.9269	168.1
200.00	199.88	559.04	1.9559	173.6
210.00	190.50	572.39	1.9839	178.6
220.00	182.31	585.57	2.0109	183.4
230.00	175.06	598.64	2.0371	187.8
240.00	168.56	611.63	2.0626	192.1
250.00	162.68	624.57	2.0876	196.2
260.00	157.33	637.50	2.1121	200.0
270.00	152.41	650.43	2.1361	203.8
280.00	147.88	663.38	2.1598	207.4
290.00	143.67	676.35	2.1830	210.9
300.00	139.75	689.36	2.2059	214.3

*temperatures are on the ITS-90 scale

Figure A.4: R134a properties of superheated vapour (continued) ASHRAE (2009).

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A.1.2 Properties of water/steam

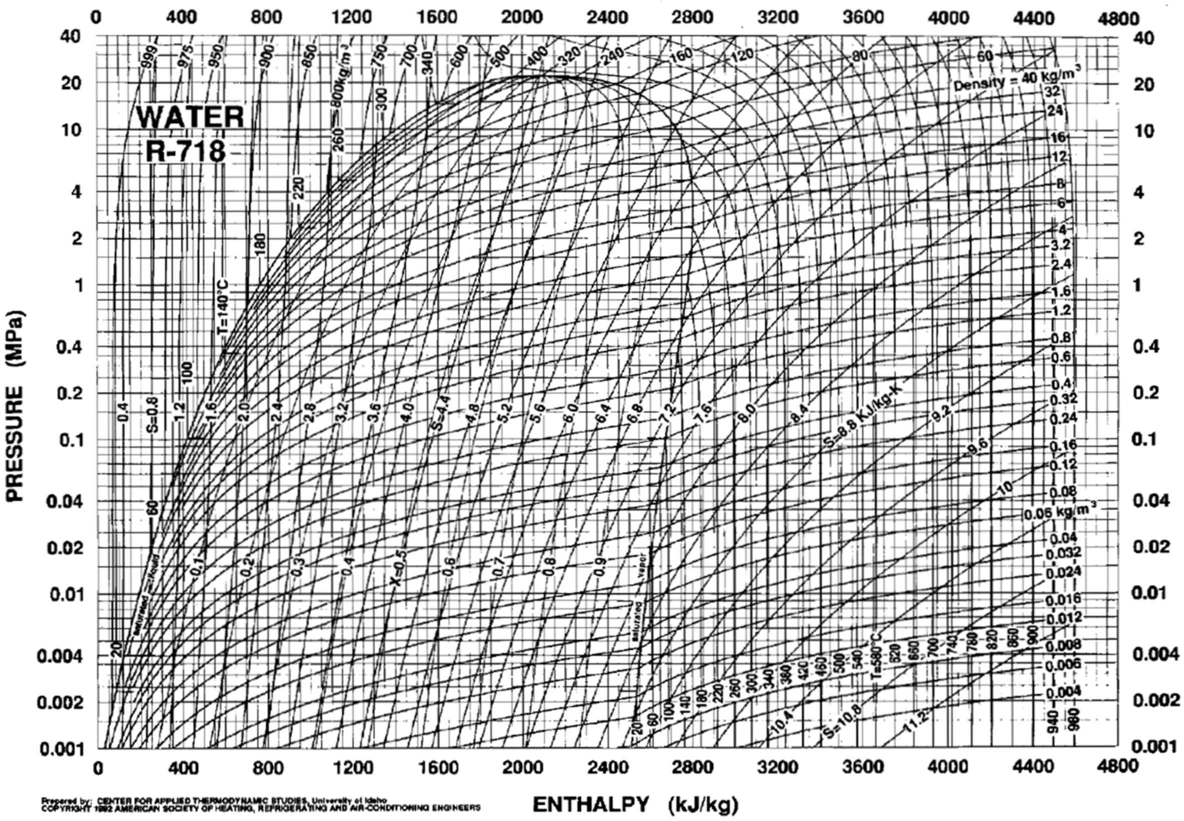

Figure A.5: $p - h$ diagram for refrigerant R718 water/steam ASHRAE (2009). ©ASHRAE, www.ashrae.org.
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Thermophysical Properties of Refrigerants **19.49**

Refrigerant 718 (Water/Steam) Properties of Saturated Liquid and Saturated Vapor

Temp,* °C	Absolute Pressure, MPa	Density, kg/m³ Liquid	Volume, m³/kg Vapor	Enthalpy, kJ/kg Liquid	Enthalpy, kJ/kg Vapor	Entropy, kJ/(kg·K) Liquid	Entropy, kJ/(kg·K) Vapor	Specific Heat c_p, kJ/(kg·K) Liquid	Specific Heat c_p, kJ/(kg·K) Vapor	c_p/c_v Vapor	Velocity of Sound, m/s Liquid	Velocity of Sound, m/s Vapor	Viscosity, µPa·s Liquid	Viscosity, µPa·s Vapor	Thermal Cond, mW/(m·K) Liquid	Thermal Cond, mW/(m·K) Vapor	Surface Tension, mN/m	Temp,* °C
0.01a	0.00061	999.8	205.98	0.0	2500.5	0.0000	9.1541	4.229	1.868	1.330	1401.	409.	1792.4	9.22	561.0	17.07	75.65	0.01
5.00	0.00087	999.9	147.02	21.0	2509.7	0.0763	9.0236	4.200	1.871	1.330	1426.	413.	1519.1	9.34	570.5	17.34	74.95	5.00
10.00	0.00123	999.7	106.32	42.0	2518.9	0.1510	8.8986	4.188	1.874	1.330	1448.	417.	1306.6	9.46	580.0	17.62	74.22	10.00
15.00	0.00171	999.1	77.900	62.9	2528.0	0.2242	8.7792	4.184	1.878	1.330	1467.	420.	1138.2	9.59	589.3	17.92	73.49	15.00
20.00	0.00234	998.2	57.777	83.8	2537.2	0.2962	8.6651	4.183	1.882	1.330	1483.	424.	1002.1	9.73	598.4	18.23	72.74	20.00
25.00	0.00317	997.0	43.356	104.8	2546.3	0.3670	8.5558	4.183	1.887	1.330	1497.	427.	890.5	9.87	607.1	18.55	71.98	25.00
30.00	0.00425	995.6	32.896	125.7	2555.3	0.4365	8.4513	4.183	1.892	1.330	1509.	431.	797.7	10.01	615.4	18.88	71.20	30.00
35.00	0.00563	994.0	25.221	146.6	2564.4	0.5050	8.3511	4.183	1.898	1.330	1520.	434.	719.6	10.16	623.2	19.23	70.41	35.00
40.00	0.00738	992.2	19.528	167.5	2573.4	0.5723	8.2550	4.182	1.905	1.330	1528.	437.	653.2	10.31	630.5	19.60	69.60	40.00
45.00	0.00959	990.2	15.263	188.4	2582.3	0.6385	8.1629	4.182	1.912	1.330	1535.	441.	596.3	10.46	637.3	19.97	68.78	45.00
50.00	0.01234	988.0	12.037	209.3	2591.2	0.7037	8.0745	4.182	1.919	1.330	1541.	444.	547.0	10.62	643.5	20.36	67.95	50.00
55.00	0.01575	985.6	9.5730	230.2	2600.0	0.7680	7.9896	4.182	1.928	1.331	1546.	447.	504.1	10.77	649.2	20.77	67.10	55.00
60.00	0.01993	983.2	7.6746	251.2	2608.8	0.8312	7.9080	4.183	1.937	1.331	1549.	450.	466.5	10.93	654.3	21.18	66.24	60.00
65.00	0.02502	980.5	6.1996	272.1	2617.5	0.8935	7.8295	4.184	1.947	1.331	1552.	453.	433.4	11.10	658.9	21.62	65.37	65.00
70.00	0.03118	977.8	5.0447	293.0	2626.1	0.9549	7.7540	4.187	1.958	1.332	1553.	456.	404.0	11.26	663.1	22.07	64.49	70.00
75.00	0.03856	974.8	4.1333	314.0	2634.6	1.0155	7.6813	4.190	1.970	1.333	1553.	459.	377.8	11.42	666.7	22.53	63.59	75.00
80.00	0.04737	971.8	3.4088	334.9	2643.1	1.0753	7.6112	4.194	1.983	1.334	1553.	462.	354.5	11.59	670.0	23.01	62.68	80.00
85.00	0.05781	968.6	2.8289	355.9	2651.4	1.1343	7.5436	4.199	1.996	1.335	1551.	465.	333.4	11.76	672.8	23.50	61.76	85.00
90.00	0.07012	965.3	2.3617	376.9	2659.6	1.1925	7.4784	4.204	2.011	1.336	1549.	467.	314.5	11.93	675.3	24.02	60.82	90.00
95.00	0.08453	961.9	1.9828	398.0	2667.7	1.2501	7.4154	4.210	2.027	1.338	1546.	470.	297.4	12.10	677.4	24.55	59.88	95.00
100.00b	0.10132	958.4	1.6736	419.1	2675.7	1.3069	7.3545	4.217	2.044	1.339	1542.	473.	281.8	12.27	679.1	25.09	58.92	100.00
105.00	0.12079	954.8	1.4200	440.2	2683.6	1.3630	7.2956	4.224	2.062	1.341	1537.	475.	267.7	12.44	680.6	25.66	57.95	105.00
110.00	0.14324	951.0	1.2106	461.3	2691.3	1.4186	7.2386	4.232	2.082	1.344	1532.	478.	254.8	12.61	681.7	26.24	56.97	110.00
115.00	0.16902	947.1	1.0370	482.5	2698.8	1.4735	7.1833	4.240	2.103	1.346	1525.	480.	243.0	12.78	682.6	26.84	55.98	115.00
120.00	0.19848	943.2	0.89222	503.8	2706.2	1.5278	7.1297	4.249	2.126	1.349	1518.	482.	232.1	12.96	683.2	27.46	54.97	120.00
125.00	0.23201	939.1	0.77089	525.1	2713.4	1.5815	7.0777	4.258	2.150	1.352	1511.	485.	222.2	13.13	683.6	28.10	53.96	125.00
130.00	0.27002	934.9	0.66872	546.4	2720.4	1.6346	7.0272	4.268	2.176	1.355	1503.	487.	213.0	13.30	683.7	28.76	52.94	130.00
135.00	0.31293	930.6	0.58234	567.8	2727.2	1.6873	6.9780	4.278	2.203	1.359	1494.	489.	204.5	13.47	683.6	29.44	51.91	135.00
140.00	0.36119	926.2	0.50898	589.2	2733.8	1.7394	6.9302	4.288	2.233	1.363	1484.	491.	196.6	13.65	683.3	30.13	50.86	140.00
145.00	0.41529	921.7	0.44643	610.8	2740.2	1.7910	6.8836	4.300	2.265	1.368	1475.	493.	189.3	13.82	682.8	30.85	49.81	145.00
150.00	0.47572	917.1	0.39287	632.3	2746.4	1.8421	6.8381	4.312	2.299	1.373	1464.	495.	182.5	13.99	682.1	31.59	48.75	150.00
155.00	0.54299	912.3	0.34681	654.0	2752.3	1.8927	6.7937	4.325	2.335	1.378	1453.	496.	176.2	14.16	681.1	32.35	47.68	155.00
160.00	0.61766	907.5	0.30709	675.6	2758.0	1.9429	6.7503	4.338	2.374	1.384	1441.	498.	170.3	14.34	680.0	33.12	46.60	160.00
165.00	0.70029	902.6	0.27270	697.4	2763.3	1.9927	6.7078	4.353	2.415	1.391	1429.	499.	164.8	14.51	678.6	33.92	45.51	165.00
170.00	0.79147	897.5	0.24283	719.3	2768.5	2.0421	6.6662	4.369	2.460	1.398	1416.	501.	159.6	14.68	677.1	34.74	44.41	170.00
175.00	0.89180	892.3	0.21679	741.2	2773.3	2.0910	6.6254	4.386	2.507	1.406	1403.	502.	154.7	14.85	675.3	35.58	43.31	175.00
180.00	1.0019	887.1	0.19403	763.2	2777.8	2.1397	6.5853	4.403	2.558	1.414	1389.	503.	150.2	15.02	673.4	36.44	42.20	180.00
185.00	1.1225	881.7	0.17406	785.4	2782.0	2.1879	6.5459	4.423	2.612	1.423	1375.	504.	145.9	15.20	671.2	37.32	41.08	185.00
190.00	1.2542	876.1	0.15650	807.6	2785.8	2.2358	6.5071	4.443	2.670	1.433	1360.	505.	141.8	15.37	668.8	38.23	39.95	190.00
195.00	1.3976	870.5	0.14102	829.9	2789.4	2.2834	6.4689	4.465	2.731	1.443	1345.	506.	138.0	15.54	666.2	39.15	38.82	195.00
200.00	1.5536	864.7	0.12732	852.4	2792.5	2.3308	6.4312	4.489	2.797	1.455	1329.	506.	134.4	15.71	663.4	40.10	37.68	200.00
205.00	1.7229	858.9	0.11517	875.0	2795.3	2.3778	6.3940	4.515	2.867	1.467	1313.	507.	130.9	15.89	660.3	41.08	36.54	205.00
210.00	1.9062	852.8	0.10438	897.7	2797.7	2.4246	6.3572	4.542	2.943	1.480	1296.	507.	127.6	16.06	657.1	42.07	35.39	210.00
215.00	2.1042	846.6	0.09475	920.5	2799.7	2.4712	6.3208	4.572	3.023	1.495	1279.	507.	124.5	16.23	653.5	43.10	34.24	215.00
220.00	2.3178	840.3	0.08615	943.5	2801.3	2.5175	6.2847	4.604	3.109	1.510	1262.	508.	121.6	16.41	649.8	44.15	33.08	220.00
225.00	2.5479	833.9	0.07846	966.7	2802.4	2.5637	6.2488	4.638	3.201	1.527	1243.	507.	118.7	16.59	645.7	45.24	31.91	225.00
230.00	2.7951	827.2	0.07155	990.0	2803.1	2.6097	6.2131	4.675	3.300	1.546	1225.	507.	116.0	16.76	641.4	46.35	30.75	230.00
235.00	3.0604	820.5	0.06534	1013.5	2803.3	2.6556	6.1777	4.715	3.405	1.565	1206.	507.	113.4	16.94	636.9	47.51	29.58	235.00
240.00	3.3447	813.5	0.05974	1037.2	2803.0	2.7013	6.1423	4.759	3.519	1.587	1186.	506.	110.9	17.12	632.0	48.70	28.40	240.00
245.00	3.6488	806.4	0.05469	1061.2	2802.1	2.7470	6.1070	4.806	3.641	1.610	1166.	505.	108.5	17.31	626.8	49.94	27.23	245.00
250.00	3.9736	799.1	0.05011	1085.3	2800.7	2.7926	6.0717	4.857	3.772	1.636	1145.	504.	106.2	17.49	621.4	51.22	26.05	250.00
255.00	4.3202	791.5	0.04596	1109.7	2798.8	2.8382	6.0363	4.912	3.914	1.664	1124.	503.	103.9	17.68	615.6	52.57	24.88	255.00
260.00	4.6894	783.8	0.04219	1134.4	2796.2	2.8838	6.0009	4.973	4.069	1.694	1102.	502.	101.7	17.88	609.4	53.98	23.70	260.00
265.00	5.0823	775.9	0.03876	1159.3	2793.0	2.9294	5.9652	5.039	4.236	1.728	1080.	500.	99.6	18.07	603.0	55.47	22.52	265.00
270.00	5.4999	767.7	0.03564	1184.6	2789.1	2.9751	5.9293	5.111	4.418	1.765	1057.	499.	97.5	18.28	596.1	57.04	21.35	270.00
275.00	5.9431	759.2	0.03278	1210.1	2784.5	3.0209	5.8931	5.191	4.617	1.806	1033.	497.	95.5	18.48	588.9	58.72	20.17	275.00
280.00	6.4132	750.5	0.03016	1236.1	2779.2	3.0669	5.8565	5.279	4.835	1.851	1009.	494.	93.6	18.70	581.4	60.52	19.00	280.00
285.00	6.9111	741.5	0.02777	1262.4	2773.0	3.1131	5.8195	5.377	5.077	1.902	984.	492.	91.6	18.92	573.5	62.47	17.84	285.00
290.00	7.4380	732.2	0.02556	1289.1	2765.9	3.1595	5.7818	5.485	5.345	1.959	959.	489.	89.7	19.15	565.2	64.59	16.68	290.00
295.00	7.9952	722.5	0.02354	1316.3	2757.8	3.2062	5.7434	5.607	5.644	2.024	933.	486.	87.8	19.39	556.6	66.91	15.52	295.00
300.00	8.5838	712.4	0.02167	1344.1	2748.7	3.2534	5.7042	5.746	5.981	2.097	906.	483.	85.9	19.65	547.7	69.49	14.37	300.00
310.00	9.8605	691.0	0.01834	1401.2	2727.0	3.3491	5.6226	6.084	6.799	2.279	849.	475.	82.2	20.21	529.0	75.61	12.10	310.00
320.00	11.279	667.4	0.01548	1461.3	2699.7	3.4476	5.5356	6.542	7.898	2.527	789.	466.	78.4	20.84	509.4	83.59	9.88	320.00
330.00	12.852	641.0	0.01298	1525.0	2665.3	3.5501	5.4407	7.201	9.458	2.886	725.	455.	74.6	21.60	489.2	94.48	7.71	330.00
340.00	14.594	610.8	0.01079	1593.8	2621.3	3.6587	5.3345	8.238	11.865	3.447	656.	443.	70.4	22.55	468.6	110.20	5.64	340.00
350.00	16.521	574.7	0.00881	1670.4	2563.5	3.7774	5.2105	10.126	16.110	4.448	581.	427.	65.9	23.81	447.6	134.65	3.68	350.00
360.00	18.655	528.1	0.00696	1761.0	2482.0	3.9153	5.0542	14.690	25.795	6.755	497.	406.	60.4	25.71	427.2	178.01	1.89	360.00
370.00	21.030	453.1	0.00499	1889.7	2340.2	4.1094	4.8098	41.955	78.751	19.373	396.	374.	52.3	29.57	428.0	299.38	0.39	370.00
373.99c	22.064	322.0	0.00311	2085.9	2085.9	4.4092	4.4092	∞	∞	∞	0	0	43.1	43.13	∞	∞	0.	373.99

*temperatures are on the IPTS–68 scale a = triple point b = normal boiling point c = critical point

Figure A.6: Water/steam properties of saturated liquid and saturated vapour ASHRAE (2009). ©ASHRAE, www.ashrae.org. 2009 ASHRAE Handbook – Fundamentals.

A.1.3 Properties of ammonia

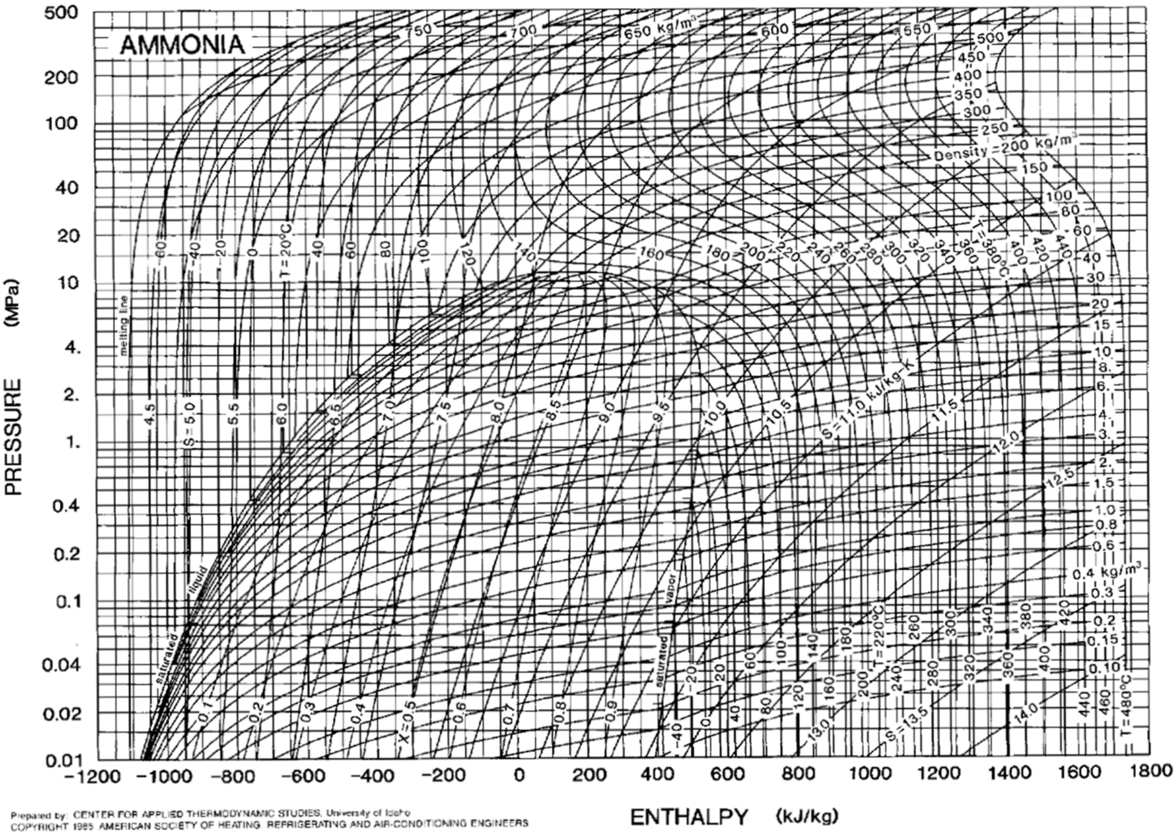

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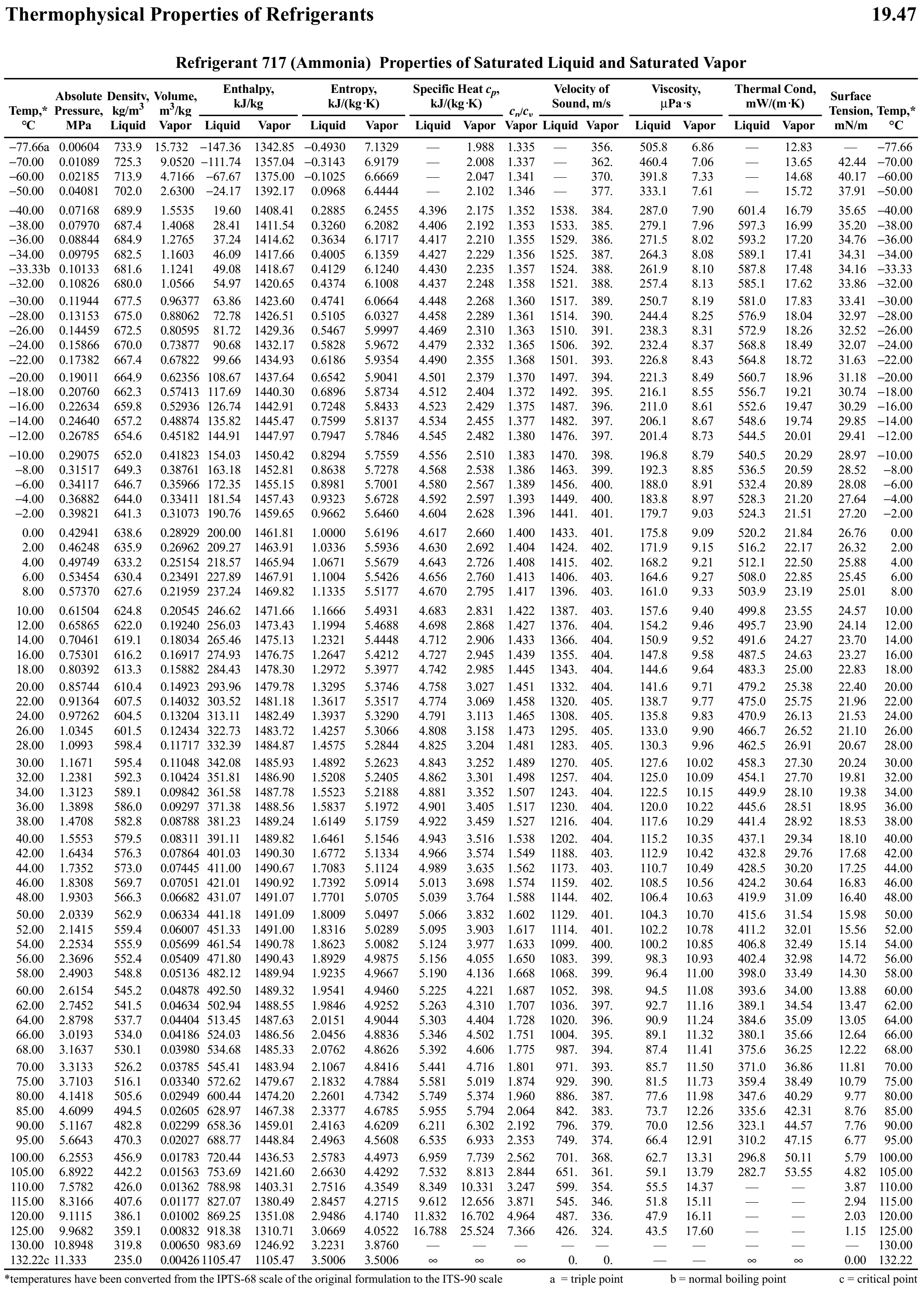
Figure A.7: *p* – *h* diagram for ammonia ASHRAE (2009). ©ASHRAE, www.ashrae.org. 2009 ASHRAE Handbook – Fundamentals.



Thermophysical Properties of Refrigerants 19.47

Refrigerant 717 (Ammonia) Properties of Saturated Liquid and Saturated Vapor

Temp,* °C	Absolute Pressure, MPa	Density, kg/m³ Liquid	Volume, m³/kg Vapor	Enthalpy, kJ/kg Liquid	Enthalpy, kJ/kg Vapor	Entropy, kJ/(kg·K) Liquid	Entropy, kJ/(kg·K) Vapor	Specific Heat c_p, kJ/(kg·K) Liquid	Specific Heat c_p, kJ/(kg·K) Vapor	c_p/c_v Vapor	Velocity of Sound, m/s Liquid	Velocity of Sound, m/s Vapor	Viscosity, µPa·s Liquid	Viscosity, µPa·s Vapor	Thermal Cond, mW/(m·K) Liquid	Thermal Cond, mW/(m·K) Vapor	Surface Tension, mN/m	Temp,* °C
−77.66a	0.00604	733.9	15.732	−147.36	1342.85	−0.4930	7.1329	—	1.988	1.335	—	356.	505.8	6.86	—	12.83	—	−77.66
−70.00	0.01089	725.3	9.0520	−111.74	1357.04	−0.3143	6.9179	—	2.008	1.337	—	362.	460.4	7.06	—	13.65	42.44	−70.00
−60.00	0.02185	713.9	4.7166	−67.67	1375.00	−0.1025	6.6669	—	2.047	1.341	—	370.	391.8	7.33	—	14.68	40.17	−60.00
−50.00	0.04081	702.0	2.6300	−24.17	1392.17	0.0968	6.4444	—	2.102	1.346	—	377.	333.1	7.61	—	15.72	37.91	−50.00
−40.00	0.07168	689.9	1.5535	19.60	1408.41	0.2885	6.2455	4.396	2.175	1.352	1538.	384.	287.0	7.90	601.4	16.79	35.65	−40.00
−38.00	0.07970	687.4	1.4068	28.41	1411.54	0.3260	6.2082	4.406	2.192	1.353	1533.	385.	279.1	7.96	597.3	16.99	35.20	−38.00
−36.00	0.08844	684.9	1.2765	37.24	1414.62	0.3634	6.1717	4.417	2.210	1.355	1529.	386.	271.5	8.02	593.2	17.20	34.76	−36.00
−34.00	0.09795	682.5	1.1603	46.09	1417.66	0.4005	6.1359	4.427	2.229	1.356	1525.	387.	264.3	8.08	589.1	17.41	34.31	−34.00
−33.33b	0.10133	681.6	1.1241	49.08	1418.67	0.4129	6.1240	4.430	2.235	1.357	1524.	388.	261.9	8.10	587.8	17.48	34.16	−33.33
−32.00	0.10826	680.0	1.0566	54.97	1420.65	0.4374	6.1008	4.437	2.248	1.358	1521.	388.	257.4	8.13	585.1	17.62	33.86	−32.00
−30.00	0.11944	677.5	0.96377	63.86	1423.60	0.4741	6.0664	4.448	2.268	1.360	1517.	389.	250.7	8.19	581.0	17.83	33.41	−30.00
−28.00	0.13153	675.0	0.88062	72.78	1426.51	0.5105	6.0327	4.458	2.289	1.361	1514.	390.	244.4	8.25	576.9	18.04	32.97	−28.00
−26.00	0.14459	672.5	0.80595	81.72	1429.36	0.5467	5.9997	4.469	2.310	1.363	1510.	391.	238.3	8.31	572.9	18.26	32.52	−26.00
−24.00	0.15866	670.0	0.73877	90.68	1432.17	0.5828	5.9672	4.479	2.332	1.365	1506.	392.	232.4	8.37	568.8	18.49	32.07	−24.00
−22.00	0.17382	667.4	0.67822	99.66	1434.93	0.6186	5.9354	4.490	2.355	1.368	1501.	393.	226.8	8.43	564.8	18.72	31.63	−22.00
−20.00	0.19011	664.9	0.62356	108.67	1437.64	0.6542	5.9041	4.501	2.379	1.370	1497.	394.	221.3	8.49	560.7	18.96	31.18	−20.00
−18.00	0.20760	662.3	0.57413	117.69	1440.30	0.6896	5.8734	4.512	2.404	1.372	1492.	395.	216.1	8.55	556.7	19.21	30.74	−18.00
−16.00	0.22634	659.8	0.52936	126.74	1442.91	0.7248	5.8433	4.523	2.429	1.375	1487.	396.	211.0	8.61	552.6	19.47	30.29	−16.00
−14.00	0.24640	657.2	0.48874	135.82	1445.47	0.7599	5.8137	4.534	2.455	1.377	1482.	397.	206.1	8.67	548.6	19.74	29.85	−14.00
−12.00	0.26785	654.6	0.45182	144.91	1447.97	0.7947	5.7846	4.545	2.482	1.380	1476.	397.	201.4	8.73	544.5	20.01	29.41	−12.00
−10.00	0.29075	652.0	0.41823	154.03	1450.42	0.8294	5.7559	4.556	2.510	1.383	1470.	398.	196.8	8.79	540.5	20.29	28.97	−10.00
−8.00	0.31517	649.3	0.38761	163.18	1452.81	0.8638	5.7278	4.568	2.538	1.386	1463.	399.	192.3	8.85	536.5	20.59	28.52	−8.00
−6.00	0.34117	646.7	0.35966	172.35	1455.15	0.8981	5.7001	4.580	2.567	1.389	1456.	400.	188.0	8.91	532.4	20.89	28.08	−6.00
−4.00	0.36882	644.0	0.33411	181.54	1457.43	0.9323	5.6728	4.592	2.597	1.393	1449.	400.	183.8	8.97	528.3	21.20	27.64	−4.00
−2.00	0.39821	641.3	0.31073	190.76	1459.65	0.9662	5.6460	4.604	2.628	1.396	1441.	401.	179.7	9.03	524.3	21.51	27.20	−2.00
0.00	0.42941	638.6	0.28929	200.00	1461.81	1.0000	5.6196	4.617	2.660	1.400	1433.	401.	175.8	9.09	520.2	21.84	26.76	0.00
2.00	0.46248	635.9	0.26962	209.27	1463.91	1.0336	5.5936	4.630	2.692	1.404	1424.	402.	171.9	9.15	516.2	22.17	26.32	2.00
4.00	0.49749	633.2	0.25154	218.57	1465.94	1.0671	5.5679	4.643	2.726	1.408	1415.	402.	168.2	9.21	512.1	22.50	25.88	4.00
6.00	0.53454	630.4	0.23491	227.89	1467.91	1.1004	5.5426	4.656	2.760	1.413	1406.	403.	164.6	9.27	508.0	22.85	25.45	6.00
8.00	0.57370	627.6	0.21959	237.24	1469.82	1.1335	5.5177	4.670	2.795	1.417	1396.	403.	161.0	9.33	503.9	23.19	25.01	8.00
10.00	0.61504	624.8	0.20545	246.62	1471.66	1.1666	5.4931	4.683	2.831	1.422	1387.	403.	157.6	9.40	499.8	23.55	24.57	10.00
12.00	0.65865	622.0	0.19240	256.03	1473.43	1.1994	5.4688	4.698	2.868	1.427	1376.	404.	154.2	9.46	495.7	23.90	24.14	12.00
14.00	0.70461	619.1	0.18034	265.46	1475.13	1.2321	5.4448	4.712	2.906	1.433	1366.	404.	150.9	9.52	491.6	24.27	23.70	14.00
16.00	0.75301	616.2	0.16917	274.93	1476.75	1.2647	5.4212	4.727	2.945	1.439	1355.	404.	147.8	9.58	487.5	24.63	23.27	16.00
18.00	0.80392	613.3	0.15882	284.43	1478.30	1.2972	5.3977	4.742	2.985	1.445	1343.	404.	144.6	9.64	483.3	25.00	22.83	18.00
20.00	0.85744	610.4	0.14923	293.96	1479.78	1.3295	5.3746	4.758	3.027	1.451	1332.	404.	141.6	9.71	479.2	25.38	22.40	20.00
22.00	0.91364	607.5	0.14032	303.52	1481.18	1.3617	5.3517	4.774	3.069	1.458	1320.	405.	138.7	9.77	475.0	25.75	21.96	22.00
24.00	0.97262	604.5	0.13204	313.11	1482.49	1.3937	5.3290	4.791	3.113	1.465	1308.	405.	135.8	9.83	470.9	26.13	21.53	24.00
26.00	1.0345	601.5	0.12434	322.73	1483.72	1.4257	5.3066	4.808	3.158	1.473	1295.	405.	133.0	9.90	466.7	26.52	21.10	26.00
28.00	1.0993	598.4	0.11717	332.39	1484.87	1.4575	5.2844	4.825	3.204	1.481	1283.	405.	130.3	9.96	462.5	26.91	20.67	28.00
30.00	1.1671	595.4	0.11048	342.08	1485.93	1.4892	5.2623	4.843	3.252	1.489	1270.	405.	127.6	10.02	458.3	27.30	20.24	30.00
32.00	1.2381	592.3	0.10424	351.81	1486.90	1.5208	5.2405	4.862	3.301	1.498	1257.	404.	125.0	10.09	454.1	27.70	19.81	32.00
34.00	1.3123	589.1	0.09842	361.58	1487.78	1.5523	5.2188	4.881	3.352	1.507	1243.	404.	122.5	10.15	449.9	28.10	19.38	34.00
36.00	1.3898	586.0	0.09297	371.38	1488.56	1.5837	5.1972	4.901	3.405	1.517	1230.	404.	120.0	10.22	445.6	28.51	18.95	36.00
38.00	1.4708	582.8	0.08788	381.23	1489.24	1.6149	5.1759	4.922	3.459	1.527	1216.	404.	117.6	10.29	441.4	28.92	18.53	38.00
40.00	1.5553	579.5	0.08311	391.11	1489.82	1.6461	5.1546	4.943	3.516	1.538	1202.	404.	115.2	10.35	437.1	29.34	18.10	40.00
42.00	1.6434	576.3	0.07864	401.03	1490.30	1.6772	5.1334	4.966	3.574	1.549	1188.	403.	112.9	10.42	432.8	29.76	17.68	42.00
44.00	1.7352	573.0	0.07445	411.00	1490.67	1.7083	5.1124	4.989	3.635	1.562	1173.	403.	110.7	10.49	428.5	30.20	17.25	44.00
46.00	1.8308	569.7	0.07051	421.01	1490.92	1.7392	5.0914	5.013	3.698	1.574	1159.	402.	108.5	10.56	424.2	30.64	16.83	46.00
48.00	1.9303	566.3	0.06682	431.07	1491.07	1.7701	5.0705	5.039	3.764	1.588	1144.	402.	106.4	10.63	419.9	31.09	16.40	48.00
50.00	2.0339	562.9	0.06334	441.18	1491.09	1.8009	5.0497	5.066	3.832	1.602	1129.	401.	104.3	10.70	415.6	31.54	15.98	50.00
52.00	2.1415	559.4	0.06007	451.33	1491.00	1.8316	5.0289	5.095	3.903	1.617	1114.	401.	102.2	10.78	411.2	32.01	15.56	52.00
54.00	2.2534	555.9	0.05699	461.54	1490.78	1.8623	5.0082	5.124	3.977	1.633	1099.	400.	100.2	10.85	406.8	32.49	15.14	54.00
56.00	2.3696	552.4	0.05409	471.80	1490.43	1.8929	4.9875	5.156	4.055	1.650	1083.	399.	98.3	10.93	402.4	32.98	14.72	56.00
58.00	2.4903	548.8	0.05136	482.12	1489.94	1.9235	4.9667	5.190	4.136	1.668	1068.	399.	96.4	11.00	398.0	33.49	14.30	58.00
60.00	2.6154	545.2	0.04878	492.50	1489.32	1.9541	4.9460	5.225	4.221	1.687	1052.	398.	94.5	11.08	393.6	34.00	13.88	60.00
62.00	2.7452	541.5	0.04634	502.94	1488.55	1.9846	4.9252	5.263	4.310	1.707	1036.	397.	92.7	11.16	389.1	34.54	13.47	62.00
64.00	2.8798	537.7	0.04404	513.45	1487.63	2.0151	4.9044	5.303	4.404	1.728	1020.	396.	90.9	11.24	384.6	35.09	13.05	64.00
66.00	3.0193	534.0	0.04186	524.03	1486.56	2.0456	4.8836	5.346	4.502	1.751	1004.	395.	89.1	11.32	380.1	35.66	12.64	66.00
68.00	3.1637	530.1	0.03980	534.68	1485.33	2.0762	4.8626	5.392	4.606	1.775	987.	394.	87.4	11.41	375.6	36.25	12.22	68.00
70.00	3.3133	526.2	0.03785	545.41	1483.94	2.1067	4.8416	5.441	4.716	1.801	971.	393.	85.7	11.50	371.0	36.86	11.81	70.00
75.00	3.7103	516.1	0.03340	572.62	1479.67	2.1832	4.7884	5.581	5.019	1.874	929.	390.	81.5	11.73	359.4	38.49	10.79	75.00
80.00	4.1418	505.6	0.02949	600.44	1474.20	2.2601	4.7342	5.749	5.374	1.960	886.	387.	77.6	11.98	347.6	40.29	9.77	80.00
85.00	4.6099	494.5	0.02605	628.97	1467.38	2.3377	4.6785	5.955	5.794	2.064	842.	383.	73.7	12.26	335.6	42.31	8.76	85.00
90.00	5.1167	482.8	0.02299	658.36	1459.01	2.4163	4.6209	6.211	6.302	2.192	796.	379.	70.0	12.56	323.1	44.57	7.76	90.00
95.00	5.6643	470.3	0.02027	688.77	1448.84	2.4963	4.5608	6.535	6.933	2.353	749.	374.	66.4	12.91	310.2	47.15	6.77	95.00
100.00	6.2553	456.9	0.01783	720.44	1436.53	2.5783	4.4973	6.959	7.739	2.562	701.	368.	62.7	13.31	296.8	50.11	5.79	100.00
105.00	6.8922	442.2	0.01563	753.69	1421.60	2.6630	4.4292	7.532	8.813	2.844	651.	361.	59.1	13.79	282.7	53.55	4.82	105.00
110.00	7.5782	426.0	0.01362	788.98	1403.31	2.7516	4.3549	8.349	10.331	3.247	599.	354.	55.5	14.37	—	—	3.87	110.00
115.00	8.3166	407.6	0.01177	827.07	1380.49	2.8457	4.2715	9.612	12.656	3.871	545.	346.	51.8	15.11	—	—	2.94	115.00
120.00	9.1115	386.1	0.01002	869.25	1351.08	2.9486	4.1740	11.832	16.702	4.964	487.	336.	47.9	16.11	—	—	2.03	120.00
125.00	9.9682	359.1	0.00832	918.38	1310.71	3.0669	4.0522	16.788	25.524	7.366	426.	324.	43.5	17.60	—	—	1.15	125.00
130.00	10.8948	319.8	0.00650	983.69	1246.92	3.2231	3.8760	—	—	—	—	—	—	—	—	—	—	130.00
132.22c	11.333	235.0	0.00426	1105.47	1105.47	3.5006	3.5006	∞	∞	∞	0.	0.	—	—	∞	∞	0.00	132.22

*temperatures have been converted from the IPTS-68 scale of the original formulation to the ITS-90 scale a = triple point b = normal boiling point c = critical point

Figure A.8: Ammonia properties of saturated liquid and saturated vapour ASHRAE (2009). ©ASHRAE, www.ashrae.org. 2009 ASHRAE Handbook – Fundamentals.

A.1.4 Properties of Lithium Bromide – water solutions

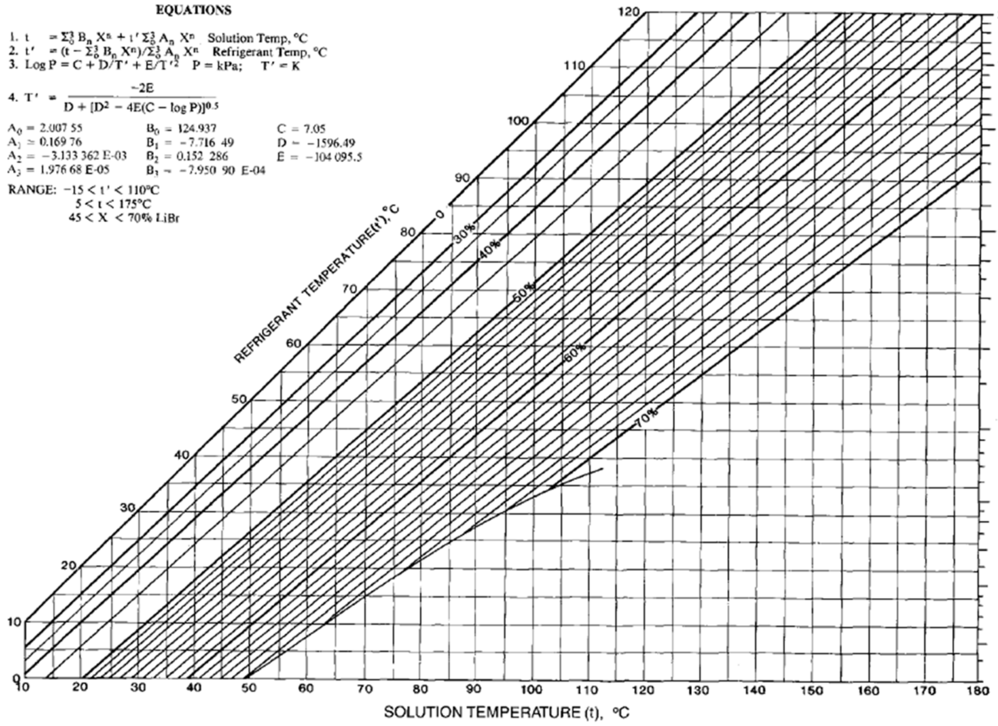
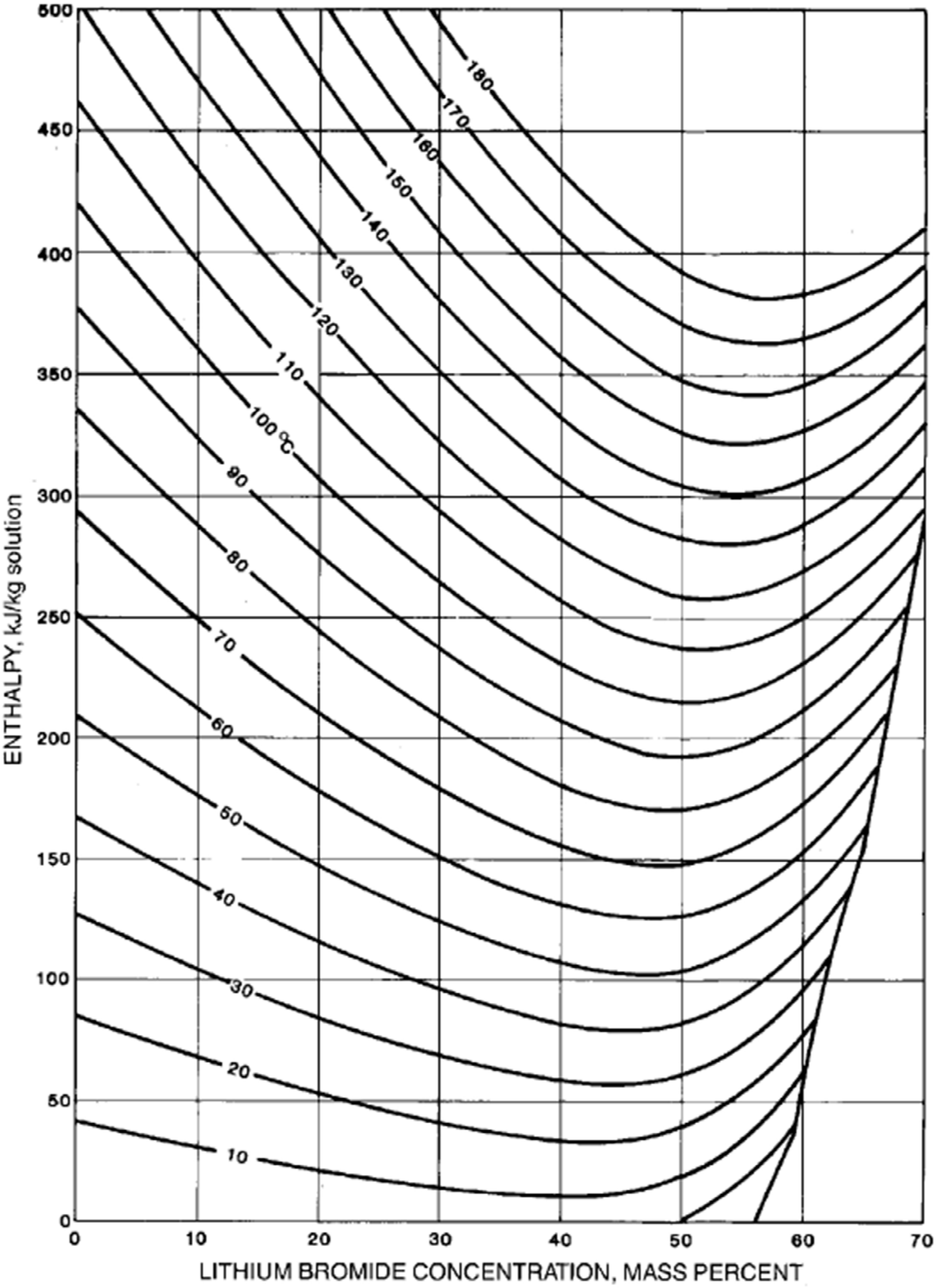


Figure A.9: Equilibrium chart for Aqueous LiBr solutions ASHRAE (2009). ©ASHRAE, www.ashrae.org. 2009 ASHRAE Handbook – Fundamentals.



EQUATIONS **CONCENTRATION RANGE** 40 < X < 70% LiBr **TEMPERATURE RANGE** 15 < t < 165°C

$h = \Sigma_0^4 A_n X^n + t\,\Sigma_0^4 B_n X^n + t^2\,\Sigma_0^4 C_n X^n$ in kJ/kg, where t = °C and X = %LiBr

$A_0 = -2024.33$
$A_1 = 163.309$
$A_2 = -4.88161$
$A_3 = 6.302948$ E-2
$A_4 = -2.913705$ E-4

$B_0 = 18.2829$
$B_1 = -1.1691757$
$B_2 = 3.248041$ E-2
$B_3 = -4.034184$ E-4
$B_4 = 1.8520569$ E-6

$C_0 = -3.7008214$ E-2
$C_1 = 2.8877666$ E-3
$C_2 = -8.1313015$ E-5
$C_3 = 9.9116628$ E-7
$C_4 = -4.4441207$ E-9

Figure A.10: Enthalpy concentration chart for water-LiBr solutions ASHRAE (2009). ©ASHRAE, www.ashrae.org. 2009 ASHRAE Handbook – Fundamentals.

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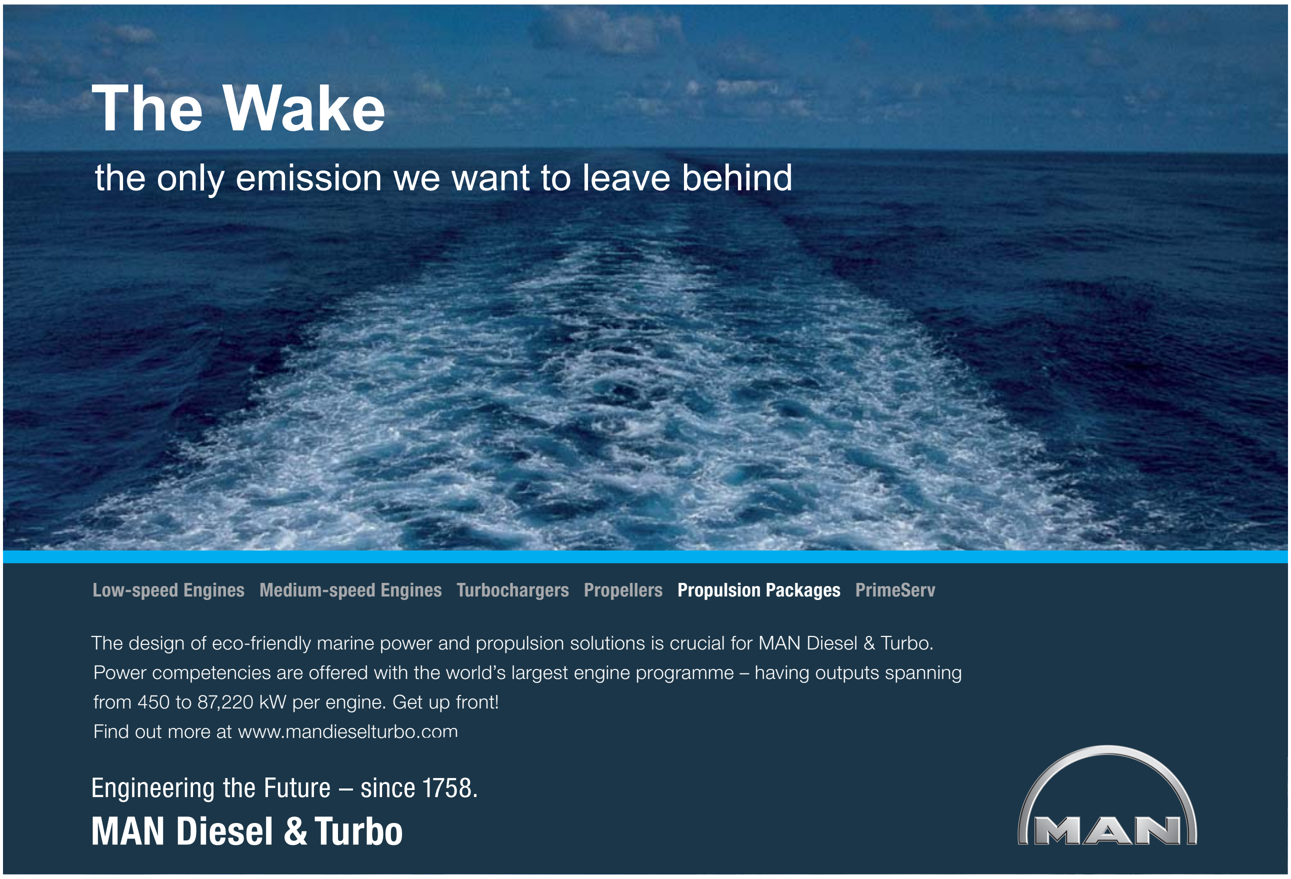
Nomenclature

Δ	Tolerance for the difference between the new and old iterates
F	Vector holding system functions $f_1, f_2, \dots f_n$
x	Vector of unknowns x_1, x_2, x_n
x^g	Vector of initial guesses in $x_1, x_2, \dots, x_n$
$\mathbf{b}$	Vector of right hand side of linear system of equations
$\dot{E}_k$	Rate of kinetic energy
$\dot{E}_p$	Rate of potential energy
$\dot{m}$	Mass flow rate
ε	Heat exchanger effectiveness
η	Heat engine efficiency
η_v	Compressor volumetric efficiency
η_{isen}	Compressor isentropic efficiency
γ	Adiabatic index
$\mathbf{F}$	Force vector
[illegible]	[illegible] direction vector
[illegible]	[illegible]splacement vector or vector of unknowns in linear system of equations
Φ	Function
ρ	Density



$\bar{m}$	Molar mass
$\bar{R}$	Universal gas constant
A	Area or matrix of coefficients for linear system of equations
A	Matrix of coefficients for linear system of equations
a_i	Constants for the enthalpy polynomial of the LiBr-water mixture
b_i	Constants for the enthalpy polynomial of the LiBr-water mixture
C	Concentration of absorbent in solution
c	Fluid speed *or* specific heat capacity of the working fluid *or* matrix terms
c_i	Constants for the enthalpy polynomial of the LiBr-water mixture
c_p	Specific heat capacity at constant pressure
c_v	Specific heat capacity at constant volume
COP	Coefficient of performance
COP_{hp}	Coefficient of performance for a heat pump
COP_{ref}	Coefficient of performance for a refrigerator
dx	Vector of elemental increments in x_1, x_2, ..., x_n
E	Primary energy supplied
g	Acceleration due to gravity
H	Enthalpy
h	Specific enthalpy
J	Jacobian matrix
K	Constant equal to pV^n in the polytropic process equation
m	Mass or number of equations
N	Number of inlets/outlets or number of sets
n	Number of moles, polytropic index, number of unknowns or number of variables
p	Pressure
p_a	Absorber pressure
pc	Condenser pressure
p_E	Evaporator pressure
p_g	Generator pressure
R	Specific gas constant
S	Entropy
s	Specific entropy or standard deviation
T	Temperature
t	Time
T_a	Absorber temperature
T_f	Final waste heat temperature
T_g	Generator temperature
T_i	Initial waste heat temperature
T_s	Cooling water temperature
U	Internal energy
u	Specific internal energy

V	Volume
v	Specific volume
V_c	Clearance volume
V_s	Overall compressor cylinder volume
W	Work done
x	Dryness fraction or general variable of a function
z	Height above some reference level