# Software Solutions to Problems on Heat Transfer

Radiation Heat Transfer – Part I

Dr. M. Thirumaleshwar



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# Software Solutions to Problems on Heat Transfer RADIATION HEAT TRANSFER – PART-I

(Laws of Radiation and Radiation properties, View factors & Radiation energy exchange between black bodies)

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# Preface to vol. 5

This is Vol. 5 of the book "Software Solutions to Problems on Heat Transfer".

In Vol. 1, problems on various aspects of CONDUCTION heat transfer were solved with Mathcad, EES, FEHT and EXCEL.

Vol. 2 contained solved problems on the topics of CONVECTION (i.e. Forced convection, Natural or Free convection).

In Vol. 3, we gave solved problems on the topics of BOILING and CONDENSATION. And,

In Vol. 4, we dealt with problems on the topic of HEAT EXCHANGERS.

### Present Vol. viz. RADIATION HEAT TRANSFER contains problems solved on following topics:

# Vol. 5. RADIATION HEAT TRANSFER Radiation Heat Transfer – Equation summary 5.A. Laws of Radiation & Radiation properties Problems solved with Mathcad (Prob. 5A.1 to 5A.10) Problems solved with EES (Prob. 5A.11 to 5A.12) Problems solved with EXCEL (Prob. 5A.13 to 5A.15) 5.B. View Factors & Radiation energy exchange between Black bodies: Problems solved with Mathcad (Prob. 5B.1 to 5B.7) Problems solved with EXCEL (Prob. 5B.8 to 5B.10) Problems solved with EXCEL (Prob. 5B.11 to 5B.18) 5.C. Electrical network method & Radiation energy exchange between gray surfaces 5.C.1 Radiation energy exchange in 2-zone enclosures 5.C.2 Radiation energy exchange in 3-zone enclosures 5.D. Radiation Shielding 5.E. Radiation error in temperature measurement

### References

Here also, problems are solved using the popular software, viz. "Mathcad", "Engineering Equation Solver (EES)", and MS EXCEL spreadsheet. Comments are included generously in the codes so that the logic behind the solutions is clear. An introductory chapter in Part-I gives a brief overview of the software used.

Many Functions are written on View Factors for various geometries, Radiation exchange in 2\_zone and 3-zone enclosures, Radiation shields etc. in Mathcad, EES and EXCEL-VBA. These Functions enable us to make the complicated calculations very easily.

As in Vol. I, II , III and IV, emphasis is given not only to solving a given problem but also to parametric analysis and graphical representation of results. Advantage of using Software to solve a variety of problems thus becomes evident. Also, problems with EXCEL spreadsheet are solved in greater details with relevant screen shots for immediate help to students, since EXCEL is available in practically every Personal Computer.

Acknowledgements: Firstly, I thank my students, since it is they who inspired me and motivated me.

Next, my thanks are due to the authorities at St. Joseph Engineering College, for their constant encouragement.

Also, my sincere thanks to **Bookboon.com** for publishing this book on the Internet. **Ms. Sophie** and her editorial team have been extremely helpful.

Finally, my heartfelt appreciation to **my wife, Kala,** but for whose unfailing and thoughtful support and encouragement, this work would not have been possible.

M. Thirumaleshwar *Author* November 2013

# About the Author

Dr. M. Thirumaleshwar graduated in Mechanical Engineering from Karnataka Regional Engineering College, Surathkal, Karnataka, India, in the year 1965. He obtained M.Sc (cryogenics) from University of Southampton, U.K. and Ph.D.(cryogenics) from Indian Institute of Science, Bangalore, India.

He is a Fellow of Institution of Engineers (India), Life Member, Indian Society for Technical Education, and a Foundation Fellow of Indian Cryogenics Council.

He has worked in India and abroad on large projects in the areas involving heat transfer, fluid flow, vacuum system design, cryo-pumping etc.

He worked as Head of Cryogenics Dept. in Bhabha Atomic Research Centre (BARC), Bombay and Centre for Advanced Technology (CAT), Indore, from 1966 to 1992.

He worked as Guest Collaborator with Superconducting Super Collider Laboratory of Universities Research Association, in Dallas, USA from 1990 to 1993.

He also worked at the Institute of Cryogenics, Southampton, U.K. as a Visiting Research Fellow from 1993 to 1994.

He was Head of the Dept. of Mechanical Engineering, Fr. Conceicao Rodrigues Institute of Technology, Vashi, Navi Mumbai, India for eight years.

He also worked as Head of Dept. of Mechanical Engineering and Civil Engineering, and then as Principal, Vivekananda College of Engineering and Technology, Puttur (D.K.), India.

## Presently, he is Professor and coordinator of Post-graduate program in the Dept. of Mechanical Engineering in St. Joseph Engineering College, Vamanjoor, Mangalore, India.

A book entitled **"Fundamentals of Heat and Mass Transfer"** authored by him and published by M/s Pearson Education, India (2006) **has been adopted as a Text book** for third year engineering students by the Visweswaraya Technological University (V.T.U.), Belgaum, India. He has also written and published three book-lets entitled as follows:

- 1. Towards Excellence... How to Study (A Guide book to Students)
- 2. Towards Excellence... How to teach (A guide book to Teachers)
- 3. Towards Excellence... Seminars, GD's and Personal Interviews (A guide book to Professional and Management students)

Dr. M. Thirumaleshwar has attended several National and International conferences and has more than 50 publications to his credit.

# About the Software used

Following three software are used while solving problems in this book:

- 1. Mathcad 2001 (Ref: www.ptc.com)
- 2. Engineering Equation Solver (EES) (Ref: <u>www.fchart.com</u>), and
- 3. Finite Element Heat Transfer (FEHT) (Ref: <u>www.fchart.com</u>)
- 4. MS EXCEL (2007) Spreadsheet (Ref: Microsoft)

Trial versions of the first three software and detailed Instruction Manuals may be downloaded from the websites indicated. EXCEL is a very popular spreadsheet which comes bundled with MS OFFICE software, and is generally available in every computer.

### See Part-I of this book for brief introduction to these four software.

While the information given there is enough to get going, for detailed instructions one should consult the respective Instruction manuals.



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# To the Student

### Dear Student:

I would like to remind you that Heat Transfer is an important subject useful in many branches of engineering. It is also a subject in which you can score high marks in the examinations, since the question paper generally consists of derivations and numerical problems, almost in the ratio 50:50. Therefore, it requires that:

- 1) you are thorough with the derivations, and
- 2) skillful in solving numerical problems.

To be thorough with derivations, you should refer to well known, standard Text books on the subject of Heat Transfer (See References at the end of this book). And, to develop your skill in solving problems... well, that is where I think that this book will help you.



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This book contains solutions to problems on heat transfer using four popular softwares, viz. Mathcad, Engineering Equation Solver (EES), Finite Element Heat Transfer (FEHT), and EXCEL spreadsheet. Trial versions of Mathcad, EES and FEHT can be downloaded from the websites indicated. EXCEL, which is a part of MS OFFICE, is generally preinstalled in most of the Personal Computers. Problems are chosen from the University question papers and standard heat transfer Text books.

Use of Software in solving problems has many advantages:

- 1. It helps in logical thinking
- 2. Problems are solved quickly and accurately
- 3. Parametric solutions (or 'what-if' solutions) are obtained easily
- 4. Solutions can be presented in tabular or graphical form, very easily and quickly
- 5. Once a particular type of problem is solved, solving a similar problem with different data input becomes very easy
- 6. Ease of getting solutions to problems in tabular or graphical form creates further interest and curiosity on the subject in the minds of students and encourages them to be creative and work further

### How to use this Book?

You need not worry if you don't know about these softwares. Since each problem is solved systematically step by step, and is well commented, just reading through the solution will make the logic of the solution clear to you. That is the most important thing in solving the problems. *Then, you must work out the problem yourself, by hand or using the software*. Of course, use of software has the above-mentioned advantages. *Simply reading the book won't do*. Have your favorite Text book nearby, in case you need to refer to it for any formulas or clarifications. There is no other 'easy method'.

As they say, 'there is no gain without pain!'

Lastly, I would like to tell you how greatly I enjoyed solving the problems presented in this book using the softwares mentioned.

I hope that you too will enjoy as much as I did in solving these problems and get benefitted.

Good Luck!

Author

# 5 Radiation heat transfer

### Learning objectives:

- 1. Radiation mode of heat transfer is radically different from conduction and convection modes in the sense that there is no need for a medium to be present for heat transfer to occur. But, just as conduction and convection heat transfers occur when there is a temperature gradient, net radiation heat transfer also occurs from a higher temperature level to a lower temperature level.
- 2. Radiation heat transfer is proportional to the fourth power of absolute temperature of the radiating surface. Therefore, radiation becomes the predominant mode of heat transfer when the temperature of the body is high.
- 3. Our interest in this chapter is on '**thermal radiation**', i.e. radiations in the wave length range of 0.1 to 100 microns.
- 4. Few important applications of radiation heat transfer:
  - i) industrial heating, such as in furnaces
  - ii) industrial air-conditioning, where the effect of solar radiation has to be considered in calculating the heat loads
  - iii) jet engine or gas turbine combustors
  - iv) industrial drying
  - v) energy conversion with fossil fuel combustion etc.
- 5. In this Chapter, first, we solve problems on: fundamental laws governing radiation heat transfer, radiation properties of surfaces, such as absorptivity (α), emissivity (ε) and transmissivity (τ). Calculation of 'view factor' for different geometries by analytical / graphical methods and by view factor algebra.
- 6. Next, we solve problems related to radiation heat transfer between surfaces in **two-surface and three-surface enclosures**, using the radiation network method. This method greatly simplifies the analysis and gives a 'physical feel' of the problem. Important practical examples of two-surface enclosure are: two infinite, parallel planes, long concentric cylinders and concentric spheres. Furnaces with re-radiating (insulated) surfaces are examples of three-surface enclosure.
- 7. **'Radiation shielding'** to reduce the radiation heat transfer between surfaces is studied next. Importance of radiation shielding in reducing the *radiation error in temperature measurement* is also studied.
- 8. Radiation has to be generally considered when the operating temperature level is high; as a rule, it will be prudent to check its relevance in problems involving natural convection and forced convection at high temperatures. Typical example is *heat transfer from walls and doors of furnaces*. In such problems, concept of 'radiation heat transfer coefficient' simplifies the numerical calculations.
- 9. Problems on above topics are worked out using Mathcad, EES and EXCEL software. Several Functions/Procedures are written to make the complicated calculations very easy.

### Formulas used: (Ref: 1)

### Absorptivity, Reflectivity and Transmissivity:

$\alpha + \rho + \tau = 1$	(13.1)
	(1011)

where,  $\alpha$  = absorptivity = fraction of incident radiation absorbed

 $\rho$  = reflectivity = fraction of incident radiation reflected

 $\tau$  = fraction of incident radiation transmitted

If  $\tau = 1$ , entire radiation passes through the body; such a body is 'transparent' or 'diathermaneous'.

If  $\alpha = 1$ , the body absorbs all the incident radiation and such a body is called a **'black body'**.

If  $\rho = 1$ , all the incident radiation is reflected, and it is a perfectly 'white body'.

**Emissive power (E):** The 'total (or hemispherical) emissive power' is the total thermal energy radiated by a surface per unit time and per unit area, over all the wave lengths and in all directions.



**Solid angle:** "Solid angle" is defined as a region of a sphere, which is enclosed by a conical surface with the vertex of the cone at the centre of the sphere. See Fig. below:



If a plane area A intercepts the line of propagation of radiation such that the normal to the surface makes an angle  $\theta$  with the line of propagation, then we project the incident area normal to the line of propagation, such that, the solid angle is now defined as:

$$\omega = \frac{A \cdot \cos(\theta)}{r^2} \text{ sr.}$$
(13.3)

Note that,  $A.cos(\theta) = A_n$  is the projected area of the incident surface, normal to the line of propagation.

**Intensity of radiation (I**<sub>b</sub>): Intensity of radiation for a black body,  $I_b$  is defined as the energy radiated per unit time per unit solid angle per unit area of the emitting surface projected normal to the line of view of the receiver from the radiating surface.

Mathematically, this is expressed as:

$$I_{b} = \frac{dQ_{b}}{(dA \cdot \cos(\theta)) \cdot d\omega} \quad W/(m2.sr)$$
(13.4)

i.e. 
$$I_{b} = \frac{dE_{b}}{\cos(\theta) \cdot d\omega}$$
 W/(m2.sr) (13.5)

Note that Emissive power  $E_b$  of a black body refers to unit *surface area* whereas Intensity  $I_b$  of a black surface refers to unit *projected area*.

 $I_b \lambda$  is the intensity of blackbody radiation for radiation of a given wave length  $\lambda$ . And,  $I_b$  is the summation over all the wave lengths, i.e.

$$I_{b} = \int_{0}^{\infty} I_{b\lambda} d\lambda \ W/(m2.sr)$$
(13.6)

**Lambert's cosine law:** "A diffuse surface radiates energy such that the rate of energy radiated in a direction  $\theta$  from the normal to the surface is proportional to the cosine of the angle  $\theta$ ". i.e.





### Planck's Law for spectral distribution:

Radiation energy emitted by a black surface depends on the wave length, temperature of the surface and the surface characteristics.

Planck's distribution Law relates to the **spectral black body emissive power**,  $E_b \lambda$  defined as 'the amount of radiation energy emitted by a black body at an absolute temperature T per unit time, per unit surface area, per unit wave length about the wave length  $\lambda$ '. Units of  $E_b \lambda$  are: W/(m<sup>2</sup>.µm). The first subscript 'b' indicates black body and the second subscript ' $\lambda$ ' stands for given wavelength, or monochromatic. Planck derived his equation for  $E_b \lambda$  in 1901 in conjunction with his 'quantum theory'.

Planck's distribution law is expressed as:

 $E_{b\lambda}(\lambda) = \frac{C_1 \cdot \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda \cdot T}\right) - 1} \qquad W/(m^2.\mu m).....(13.7)$ where,  $C_1 = 3.742 \cdot 10^8 \qquad W.\mu m^4/m^2$ and,  $C_2 = 1.4387 \cdot 10^4 \qquad \mu m.K$ 

### Plots of $E_b \lambda$ vs. $\lambda$ for a few different temperatures are shown in Fig. below:



### Wein's displacement law:

It is clear from Fig. 13.7 that the spectral distribution of emissive power of a black body at a given absolute temperature goes through a maximum. To find out the value of  $\lambda_{max}$ , the wave length at which this maximum occurs, differentiate Planck's equation w.r.t.  $\lambda$  and equate to zero. Finally, we get:

Wein's displacement law is stated as: "product of absolute temperature and wave length at which emissive power of a black body is a maximum, is constant".

**Value of maximum monochromatic emissive power of a black body at a given temperature** is obtained by substituting this value of  $\lambda_{max}T$  (= 2898 µm.K) in Planck's equation. And, we get:

$$E_{b\lambda_{max}} = 1.287 \cdot 10^{-5} \cdot T^5 \quad W/m^3$$
 (13,12)

(13.13)

### Stefan-Boltzmann Law:

$$E_b = \sigma \cdot T^* \quad W/m^2$$
  
where,  $\sigma = 5.67 \cdot 10^{-\circ} \quad W/(m^2.K^4)$ 

 $\sigma$  is known as 'Stefan-Boltzmann constant'.

### Net radiant energy exchange between two black bodies at temperatures $T_1$ and $T_2$ is given by:

 $Q_{net} = \sigma \cdot \left( T_1^4 - T_2^4 \right) W/m^2$  (13.14)

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Radiation heat transfer

### Radiation from a wave band:

Often, it is required to know the amount of radiation emitted in a given wave band, i.e. in a wave length interval between  $\lambda_1$  and  $\lambda_2$ . This is expressed as a fraction of the total emissive power and is written as  $F\lambda_1 \lambda_2$ . Then, we can write:

$$F_{\lambda 1\_\lambda 2} = \frac{\int_{\lambda 1}^{\lambda} 2}{\int_{0}^{\infty} E_{b\lambda} d\lambda} = \frac{1}{\sigma \cdot T^{4}} \cdot \int_{\lambda 1}^{\sigma \lambda 2} E_{b\lambda} d\lambda$$
  
i.e. 
$$F_{\lambda 1\_\lambda 2} = \frac{1}{\sigma \cdot T^{4}} \cdot \left( \int_{0}^{\sigma \lambda 2} E_{b\lambda} d\lambda - \int_{0}^{\sigma \lambda 1} E_{b\lambda} d\lambda \right)$$
  
i.e. 
$$F_{\lambda 1\_\lambda 2} = F_{0\_\lambda 2} - F_{0\_\lambda 1}$$
 (13.5)

Above formula is not very convenient to use, since  $E_b \lambda$  depends on absolute temperature T, and it is not practicable to tabulate  $F_{0-\lambda}$  for each T. This difficulty is overcome by expressing  $F_{0-\lambda}$  as follows:

$$F_{0} = \frac{\int_{0}^{\lambda} E_{b\lambda} d\lambda}{\sigma \cdot T^{4}} = \frac{\int_{0}^{\lambda} E_{b\lambda} d(\lambda \cdot T)}{\sigma \cdot T^{5}}$$
  
i.e. 
$$F_{0} = f(\lambda \cdot T)$$
(13.16)

i.e. Now,  $F_{0,\lambda}$  is expressed as a function of the product of wave length and absolute temperature ( =  $\lambda$ .T) only.

### Values of $F_{_{0\cdot\lambda}}$ vs. $\lambda.T$ are tabulated in Table 13.2 below:

	Radiation	Functions	
λT (μm.K)	F <sub>0-k</sub>	λT (µm.K)	F <sub>0-k</sub>
400	0	7000	0.8081
600	0	7200	0.8192
800	0.000016	7400	0.8295
1000	0.00032	7600	0.848
1200	0.00213	7800	0.848
1400	0.0078	8000	0.8563
1600	0.0197	8500	0.8746
1800	0.0393	9000	0.89
2000	0.0667	9500	0.9031
2200	0.1009	10000	0.9142
2400	0.1403	10500	0.9237
2600	0.1831	11000	0.9319
2800	0.2279	11500	0.9399
3000	0.2732	12000	0.9451
3200	0.3181	12500	0.9505
3400	0.3617	13000	0.9551
3600	0.4036	13500	0.9592
3800	0.4434	14000	0.9628
4000	0.4809	14500	0.9661
4200	0.516	15000	0.9689
4400	0.5488	16000	0.9738
4600	0.5793	17000	0.9776
4800	0.6075	18000	0.9808
5000	0.6337	19000	0.9834
5200	0.659	20000	0.9855
5400	0.6804	25000	0.9922
5600	0.701	30000	0.9953
5800	0.7201	35000	0.9969
6000	0.7378	40000	0.9979
6200	0.7541	45000	0.9985
6400	0.7962	50000	0.9989
6600	0.7832	75000	0.9997
6800	0.7961	100000	0.9999

Table 13.2.

### Note in the above Table that the units of product $\lambda$ .T is (micron.Kelvin).

Therefore,

$$F_{\lambda 1\_\lambda 2} = F_{\lambda 1.T\_\lambda 2.T} = \frac{1}{\sigma} \cdot \left[ \int_{0}^{\circ \lambda 2 \cdot T} \frac{E_{b\lambda}}{T^{5}} d(\lambda \cdot T) - \int_{0}^{\circ \lambda 1 \cdot T} \frac{E_{b\lambda}}{T^{5}} d(\lambda \cdot T) \right]$$
  
i.e. 
$$F_{\lambda 1\_\lambda 2} = F_{0\_\lambda 2.T} - F_{0\_\lambda 1.T}$$
(13.17)

Relation between Radiation intensity and Emissive power:

$$\mathbf{E}_{\mathbf{b}} = \pi \cdot \mathbf{I}_{\mathbf{b}} \tag{13.21}$$

### i.e. Total emissive power of a black (diffuse) surface is equal to $\pi$ times the intensity of radiation.

This is an important relation, which will be used while calculating the view factors required to determine net energy exchange between surfaces.

Total hemispherical emissivity ( $\epsilon$ ) of a surface is the average emissivity over all directions and all wave lengths and is expressed as:

$$\varepsilon(T) = \frac{E(T)}{E_{b}(T)} = \frac{E(T)}{\sigma \cdot T^{4}}$$
(13.22)

where E(T) is the emissive power of the real surface.



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Similarly, spectral emissivity is defined as:

$$\epsilon_{\lambda}(T) = \frac{E_{\lambda}(T)}{E_{b\lambda}(T)}$$
(13.23)

Emissivity values for a few surfaces at room temperature are given in handbooks.

A surface is said to be **gray** if its properties are independent of wave length, and a surface is **diffuse** if its properties are independent of direction.

### To calculate the average emissivity:

Let the variation of spectral emissivity with wave length be as follows:

$$\begin{split} & \varepsilon_1 = \text{constant, } 0 \leq \lambda \leq \lambda_1 \\ & \varepsilon_2 = \text{constant, } \lambda_1 \leq \lambda \leq \lambda_2 \\ & \varepsilon_3 = \text{constant, } \lambda_2 \leq \lambda \leq \infty \end{split}$$

Then, the average emissivity is calculated using eqn (13.24) as follows:

$$\epsilon(T) = \frac{\varepsilon_{1} \cdot \int_{0}^{\varepsilon_{\lambda}} 1}{\sigma \cdot T^{4}} + \frac{\varepsilon_{2} \cdot \int_{\lambda}^{\varepsilon_{\lambda}} 2}{\sigma \cdot T^{4}}$$
(13.25)

i.e. 
$$\varepsilon(T) = \varepsilon_1 \cdot F_0 \cdot \lambda_1(T) + \varepsilon_2 \cdot F \cdot \lambda_1 \cdot \lambda_2(T) + \varepsilon_3 \cdot F \cdot \lambda_2 \cdot \infty(T)$$
 (13.26)

Factors  $F_0\lambda_1(T)$  etc. can easily be determined using Tables in handbooks.

### 13.3.7 Kirchoff's Law:

$$\epsilon(T) = \alpha(T)....(13.27)$$

**Kirchoff's Law states** that the total hemispherical emissivity,  $\varepsilon$  of a gray surface at a temperature T is equal to its absorptivity,  $\alpha$  for black body radiation from a source at the same temperature T.

### The View factor:

View factor is also known by other names such as: 'configuration factor', 'shape factor', 'angle factor' etc.

**View factor is defined as** *the fraction of radiant energy leaving one surface which strikes a second surface directly.* 

Here, 'directly' means that reflection or re-radiated energy is not considered. View factor is denoted by  $F_{12}$ , where the first subscript, 1 stands for the emitting surface, and the second subscript, 2 stands for the receiving surface.

### View factor between Infinitesimal areas:

(Such a situation is encountered even when finite areas are involved, when the distance between these two areas 'r', is very large.)

$$F_{dA1_dA2} = \frac{\cos(\phi_1) \cdot \cos(\phi_2) \cdot dA_2}{\pi \cdot t^2} \qquad \dots (!3.31)$$

Note that the View factor involves geometrical quantities only.

### View factor between Infinitesimal and finite area:

$$F_{dA1-A2} = \int_{A2} \frac{\cos(\phi_1)\cos(\phi_2) dA_2}{\pi r^2} \dots \dots (13.32)$$

Practical situation of calculating view factors between infinitesimal to finite areas are encountered the case of a small thermocouple bead located inside a pipe or a small, spherical point source radiator located by the side of a wall etc.

### View factor from Finite to finite area:

$$F_{A1-A2} = \frac{1}{\pi A_1} \int_{A1} \int_{A2} \frac{\cos(\phi_1)\cos(\phi_2)}{r^2} dA_1 dA_2 \cdots \cdots (13.33)$$

In general, we write eqn. (13.33) compactly as:

$$F_{12} = \frac{1}{\pi A_1} \int_{A_1} \int_{A_2} \frac{\cos(\phi_1) \cos(\phi_2)}{r^2} dA_1 dA_2 \cdots \cdots (13.34)$$

**Reciprocity theorem:** 

$$A_1 \cdot F_{12} = A_2 \cdot F_{21}$$
 .....(13.36)

This is a very useful and important relation. It helps one to find out one of the view factors when the other one is known. In practice, one of the view factors which is easier to calculate is obtained first, and the other view factor is found out next, by using the reciprocity theorem.

Note: It is easier to remember the view factor relation given in eqn. (13.34) as:

$$A_{11}F_{12} = \int_{A1} \int_{A2} \frac{\cos(\phi_1)\cos(\phi_2)}{\pi r^2} dA_1 dA_2 \cdots \cdots (13.37)$$

### Radiation energy exchange between black bodies:

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot T_1^4 - A_2 \cdot F_{21} \cdot \sigma \cdot T_2^4$$

But, A 1 · F 12=A 2 · F 21 by reciprocity theorem

Therefore,

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) = A_2 \cdot F_{21} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)$$
 W....(13.40)

### Properties of View factor and view factor algebra:

### Properties of view factor:

- 1) The view factor depends only on the geometrics of bodies involved and not on their temperatures or surface properties.
- 2) Between two surfaces that exchange energy by radiation, the mutual shape factors are governed by the **'reciprocity relation'**, viz.  $A_1 \cdot F_{12} = A_2 \cdot F_{21}$ .



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- 3) When a convex surface 1 is completely enclosed by another surface 2, it is clear from Fig. 13.14(a) that all of the radiant energy emitted by surface 1 is intercepted by the enclosing surface 2. Therefore, view factor of surface 1 w.r.t. surface 2 is equal to unity. i.e. F<sub>12</sub> = 1. And, the view factor of surface 2 w.r.t. surface 1 is then easily calculated by applying the reciprocity relation, i.e. A<sub>1</sub>.1 = A<sub>2</sub>.F<sub>21</sub>, or, F<sub>21</sub> = A<sub>1</sub>/A<sub>2</sub>.
- 4) Radiation emitted from a flat surface never falls directly on that surface (see Fig. 13. 14 (b)),
  i.e. View factor of a flat surface w.r.t. itself is equal to zero, i.e. F<sub>11</sub> = 0. This is valid for a convex surface too, as shown in Fig. 13.14(c).
- 5) For a concave surface, it is clear from Fig. 13.14(d) that  $F_{11}$  is not equal to zero since some fraction of radiation emitted by a concave surface does fall on that surface directly.



(a) Surface 1 completely (b) Flat surface (c) Convex enclosed by surface 2

(b) Flat surface (c) Convex surface (d) Concave surface

Fig. 13.14 View factors for a few surfaces

6) If two, plane surfaces A1 and A2 are parallel to each other and separated by a short distance between them, practically all the radiation issuing from surface 1 falls directly on surface 2, and vice –versa. Therefore, F<sub>12</sub> = F<sub>21</sub> = 1



7) When the radiating surface 1 is divided into, say, two sub-areas A3 and A4, as shown in Fig. 13.15(a), we have:

$$A_{1} \cdot F_{12} = A_{3} \cdot F_{32} + A_{4} \cdot F_{42} \qquad \dots \qquad (13.41)$$

Obviously,  $F_{12} \neq F_{32} + F_{42}$ .

8) **Instead, if the receiving surface A2 is sub-divided into parts A3 and A4** as shown in Fig. 13.15 (b), we have:

$$\begin{array}{c} A_{1} \cdot F_{12} = A_{1} \cdot F_{13} + A_{1} \cdot F_{14} \\ \text{i.e.} & & \dots \\ F_{12} = F_{13} + F_{14} \end{array}$$
(13.42)

i.e. view factor from the emitting surface 1 to a sub-divided receiving surface is simply equal to the sum of the individual shape factors from the surface 1 to the respective parts of the receiving surface. This is known as 'Superposition rule'.



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- 9) Symmetry rule: If two (or more) surfaces are symmetrically located w.r.t. the radiating surface 1, then the view factors from surface 1 to these symmetrically located surfaces are identical. A close inspection of the geometry will reveal if there is any symmetry in a given problem.
- 10) **Summation rule:** Since radiation energy is emitted from a surface in all directions, invariably, we consider the emitting surface to be part of an enclosure. Even if there is an opening, we consider the opening as a surface with the radiative properties of that opening. Then, the conservation of energy principle requires that sum of all the view factors from the surface 1 to all other surfaces forming the enclosure, must be equal to 1. See Fig. 13.16, where the interior surface of a completely enclosed space is sub-divided into n parts, each of area A1, A2, A3...An.



Fig. 13.16 View factor – summation rule for radiation in an enclosure

Then,

$$F_{11} + F_{12} + \dots + F_{1n} = 1$$

$$F_{21} + F_{22} + \dots + F_{2n} = 1$$

$$\dots$$

$$F_{n1} + F_{n2} + \dots + F_{nn} = 1 \dots$$
(13.43)

i.e.

$$\sum_{j=1}^{n} F_{ij} = 1 \quad i = 1, 2, 3, \dots, n.....(13.44)$$

11) In an enclosure of 'n' black surfaces, maintained at temperatures  $T_1, T_2, ..., T_n$ , *net* radiation from any surface, say surface 1, is given by summing up the net radiation heat transfers from surface 1 to each of the other surfaces of the enclosure:

**Note:** Often, while solving radiation problems, determination of the view factor is the most difficult part. It will be useful to keep in mind the definition of view factor, summation rule, reciprocity relation, superposition rule and symmetry rule while attempting to find out the view factors.

TABLE 22-1				
View factor expressions for some common geometries of finite size (3D)				
Geometry	Relation			
Aligned parallel rectangles $L = \frac{j}{Y + \frac{j}{X}}$	$\begin{split} \overline{X} &= X/L \text{ and } \overline{Y} = Y/L \\ F_{i \to j} &= \frac{2}{\pi \overline{X} \overline{Y}} \left\{ \ln \left[ \frac{(1 + \overline{X}^2)(1 + \overline{Y}^2)}{1 + \overline{X}^2 + \overline{Y}^2} \right]^{1/2} \\ &+ \overline{X} (1 + \overline{Y}^2)^{1/2} \tan^{-1} \frac{\overline{X}}{(1 + \overline{Y}^2)^{1/2}} \\ &+ \overline{Y} (1 + \overline{X}^2)^{1/2} \tan^{-1} \frac{\overline{Y}}{(1 + \overline{X}^2)^{1/2}} \\ &- \overline{X} \tan^{-1} \overline{X} - \overline{Y} \tan^{-1} \overline{Y} \right\} \end{split}$			
Coaxial parallel disks	$\begin{split} R_i &= r_i / L \text{ and } R_j = r_j / L \\ S &= 1 + \frac{1 + R_j^2}{R_i^2} \\ F_{i \rightarrow j} &= \frac{1}{2} \left\{ S - \left[ S^2 - 4 \left( \frac{r_j}{r_i} \right)^2 \right]^{1/2} \right\} \end{split}$			

### View factor relations for a few geometries are given below: (Ref: Cengel)



\_\_\_\_\_

### TABLE 22-2

View factor expressions for some infinitely long (2D) geometries









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### View factor for two aligned parallel, rectangles of equal size:

### View factor between two coaxial parallel discs:





### View factor between two perpendicular rectangles with a common edge:

View factors for two concentric cylinders of finite length: (a) outer cylinder to inner cylinder (b) outer cylinder to itself:



### Radiation heat exchange between gray surfaces: (Ref: 1, 3)

### Radiation exchange between small, gray surfaces:

$$Q_{12} = \varepsilon_1 \cdot \varepsilon_2 \cdot A_1 \cdot F_{12} \cdot \sigma \cdot \left( T_1^4 - T_2^4 \right) = \varepsilon_1 \cdot \varepsilon_2 \cdot A_2 \cdot F_{21} \cdot \sigma \cdot \left( T_1^4 - T_2^4 \right) \qquad \dots (13.47)$$

### Remember that:

 $A_1 \cdot F_{12} = A_2 \cdot F_{21}$  ... by reciprocity

The product,  $(\epsilon_1, \epsilon_2)$  is known as 'equivalent emissivity  $(\epsilon_{eq})$ ' for a system of two 'small' gray bodies.

### The electrical network method:

### **Definitions:**

- 1. **Irradiation, (G):** is the total radiation incident upon a surface per unit time, per unit area (W/m<sup>2</sup>).
- 2. **Radiosity, (J):** is the total radiation leaving a surface, *with no regard for its origin* (i.e., reflected *plus* emitted from the surface) per unit time, per unit area (W/m<sup>2</sup>).



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Fig. 13.25 Irradiation and Radiosity

3. Surface resistance:

$$Q = \frac{E_{b} - J}{\frac{(1 - \varepsilon)}{A \cdot \varepsilon}} \qquad \qquad W.....(13.48)$$

By analogy with Ohm's law:

$$R = \frac{(1 - \varepsilon)}{A \cdot \varepsilon} \qquad \dots \text{ surface resistance}$$

$$E_{bi} \qquad \qquad \underbrace{O_i \\ K_i = \frac{O_i}{A_i \varepsilon_i}}_{R_i = \frac{1 - \varepsilon_i}{A_i \varepsilon_i}}$$

$$J_i = E_{bi} \qquad \dots \text{ for a black body} \dots (13.49)$$

$$J_i = E_{bi} = \sigma \cdot T_i^4$$
 .....for a re-radiating surface....(13.50)

4. Space resistance:



$$Q_{ij} = \frac{J_i - J_j}{\frac{1}{\langle A_i \cdot F_{ij} \rangle}} \quad W....(13.52)$$

By analogy with Ohm's law:

$$Q_{ij} = \frac{J_{i} - J_{j}}{R_{ij}} \quad W$$
  
where, 
$$R_{ij} = \frac{1}{A_{i} \cdot F_{ij}} \quad \dots (13.53)$$

 $R_{ij}$  is known as 'space resistance' and it represents the resistance to radiative heat flow between the radiosity potentials of the two surfaces, due to their relative orientation and spacing.

For a N surface enclosure, net heat transfer from surface i should be equal to the sum of net heat transfers from that surface to the remaining surfaces. i.e.

$$Q_{i} = \sum_{j=1}^{N} Q_{ij} = \sum_{j=1}^{N} A_{i} F_{ij} (J_{i} - J_{j}) = \sum_{j=1}^{N} \frac{J_{i} - J_{j}}{R_{ij}} \qquad \dots W....(13.54)$$
  
i.e. 
$$\frac{E_{bi} - J_{i}}{R_{i}} = \sum_{j=1}^{N} \frac{J_{i} - J_{j}}{R_{ij}} \qquad W...(13.55)$$

where  $R_{i}$  is the surface resistance and  $R_{ij}$  is the space resistance.

This situation is shown in Fig. (13.27).



Fig. 13.27 Radiation heat transfer from surface i to other surfaces in a N-surface enclosure

### Radiation heat exchange in two-zone enclosures: (Ref: 1, 3)





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#### A few special cases of two-surface enclosure:

Here, basic, governing eqn. is eqn.(13.56), which is modified depending upon the case considered.

#### Case (i): Radiant heat exchange between two black surfaces:

For a black body,  $\varepsilon = 1$ , and  $J = E_{b}$ , as explained earlier. i.e. surface resistance

 $[= (1 - \varepsilon)/(A.\varepsilon)]$  of a black body is zero.

Then, from eqn. (13.56), we get:

$$Q_{12} = \frac{\sigma \cdot \left(T_{1}^{4} - T_{2}^{4}\right)}{\frac{1}{A_{1} \cdot F_{12}}}$$
  
i.e.  $Q_{12} = A_{1} \cdot F_{12} \cdot \sigma \cdot \left(T_{1}^{4} - T_{2}^{4}\right)$  W....for two black surfaces forming an enclosure....(13.57)

Next, we shall consider four cases of practical interest where the view factor between the inner surface 1 and the outer surface 2 (i.e.  $F_{12}$ ) is equal to 1, and also the net radiation from a gray cavity:

#### Case (ii): Radiant heat exchange for a small object in a large cavity:

A practical example of a small object in a large cavity is the case of a steam pipe passing through a large plant room.

For this case, we have:

$$\frac{A_1}{A_2} = 0$$

And, eqn. (13.56) becomes:

 $Q_{12} = A_1 \cdot \sigma \cdot \epsilon_1 \cdot \left( T_1^4 - T_2^4 \right)$  ....for small object in a large cavity.....(13.58)

#### Case (iii): Radiant heat exchange between infinitely large parallel plates:

In this case,  $A_1 = A_2 = A$ , say, and  $F_{12} = 1$ 

Then, eqn. (13.56) becomes:

$$Q_{12} = \frac{A \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \qquad \dots \text{ for infinitely large parallel plates} \dots (13.59)$$

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#### Case (iv): Radiant heat exchange between infinitely long concentric cylinders:

In this case:

F<sub>12</sub>=1

Then, eqn. (13.56) becomes:

$$Q_{12} = \frac{A_1 \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)} \qquad \dots \text{ for infinitely long concentric cylinders.....(13.60)}$$

where

$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$

Remember that A<sub>1</sub> refers to the inner (or enclosed) surface.

#### Case (v): Radiant heat exchange between concentric spheres:

In this case:

Then, eqn. (13.56) becomes

$$Q_{12} = \frac{A_{1} \cdot \sigma \cdot \left(T_{1}^{4} - T_{2}^{4}\right)}{\frac{1}{\epsilon_{1}} + \left(\frac{A_{1}}{A_{2}}\right) \cdot \left(\frac{1}{\epsilon_{2}} - 1\right)} \qquad \dots \text{ for concentric spheres.....(13.61)}$$

where

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$$

Remember, again, that  $A_1$  refers to the inner (or enclosed) surface.

#### Case (vi): Energy radiated from a gray cavity:





(b) Radiation network

$$\begin{aligned} \mathbf{R}_{1} &= (1 - \boldsymbol{\epsilon}_{1}) / (\mathbf{A}_{1} \cdot \boldsymbol{\epsilon}_{1}) \\ \mathbf{R}_{12} &= 1 / (\mathbf{A}_{1} \cdot \mathbf{F}_{12}) \end{aligned}$$
$$\begin{aligned} \mathbf{Q}_{12} &= \mathbf{A}_{1} \cdot \boldsymbol{\epsilon}_{1} \cdot \boldsymbol{\sigma} \cdot \mathbf{T}_{1}^{4} \cdot \left[ \frac{1 - \mathbf{F}_{11}}{1 - (1 - \boldsymbol{\epsilon}_{1}) \cdot \mathbf{F}_{11}} \right] \end{aligned}$$
$$\begin{aligned} \text{W....net radiation from} \\ \text{gray cavity..}(13.62) \end{aligned}$$

Eqn. (13.62) is an *important result*, which gives net radiation from a gray cavity to surrounding space. If it is desired, for example, to calculate the net radiation from a blind hole drilled in a flange, then the relation to use is the eqn. (13.62).





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#### Radiation heat exchange in three zone enclosures: [Ref: 1]



#### Temperature of each surface is known; i.e. emissive power E<sub>b</sub> for each surface is known:

Then, the problem reduces to determining the radiosities  $J_1$ ,  $J_2$  and  $J_3$ .

This is done by applying Kirchoff's law of d.c. circuits to each node: i.e. sum of the currents (or, rate of heat transfers) entering *into* each node is zero:

Node 
$$J_1$$
:  $\frac{E_{b1} - J_1}{R_1} + \frac{J_2 - J_1}{R_{12}} + \frac{J_3 - J_1}{R_{13}} = 0$  .....(13.63,a)

Node 
$$J_2$$
:  $\frac{E_{b2} - J_2}{R_2} + \frac{J_1 - J_2}{R_{12}} + \frac{J_3 - J_2}{R_{23}} = 0$  .....(13.63,b)

Node 
$$J_3$$
:  $\frac{E_{b3} - J_3}{R_3} + \frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}} = 0$  .....(13.63,c)

Solving these three eqns. simultaneously, we get  $J_1$ ,  $J_2$  and  $J_3$ .

#### Once the radiosities are known, expressions for net heat flows between the surfaces are:

$$Q_{12} = \frac{J_{1} - J_{2}}{R_{12}} = \frac{J_{1} - J_{2}}{\frac{1}{A_{1} \cdot F_{12}}} \qquad \dots (13.64,a)$$

$$Q_{13} = \frac{J_{1} - J_{3}}{R_{13}} = \frac{J_{1} - J_{3}}{\frac{1}{A_{1} \cdot F_{13}}} \qquad \dots (13.64,b)$$

$$Q_{23} = \frac{J_2 - J_3}{R_{23}} = \frac{J_1 - J_2}{\frac{1}{A_2 \cdot F_{23}}}$$
 ....(13.64,c)



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And, net heat flow from each surface is:

$$Q_{1} = \frac{E_{b1} - J_{1}}{R_{1}} = \frac{E_{b1} - J_{1}}{\left(\frac{1 - \varepsilon_{1}}{A_{1} \cdot \varepsilon_{1}}\right)} \qquad \dots (13.65, a)$$

$$Q_{2} = \frac{E_{b2} - J_{2}}{R_{2}} = \frac{E_{b2} - J_{2}}{\left(\frac{1 - \varepsilon_{2}}{A_{2} \cdot \varepsilon_{2}}\right)} \qquad \dots (13.65,b)$$

$$Q_{3} = \frac{E_{b3} - J_{3}}{R_{3}} = \frac{E_{b3} - J_{3}}{\left(\frac{1 - \varepsilon_{3}}{A_{3} \cdot \varepsilon_{3}}\right)} \qquad \dots (13.65,c)$$

To the general eqns (13.64), apply the constraints such as:

If the surface is black or re-radiating:  $J_i = E_{bi} = \sigma . T_i^4$ .

And,  $Q_i = 0$  for a re-radiating surface.

If  $Q_i$  at any surface is specified instead of the temperature (i,e.  $E_{bi}$ ), then,  $(E_{bi} - J_i)/R_i$  is replaced by  $Q_i$ .

Few special cases of three-zone enclosures:

1. Two black surfaces connected to a third refractory surface:



Figure 13.33 Two black surfaces connected by a thrid re-radiating surface and its radiation network

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Fig. (13.33,a) shows the radiation network for this case. The radiation network is drawn very easily by *remembering the usual principles*: for a black surface, the surface resistance is zero, i.e.  $E_b = J$ . For a reradiating surface too,  $E_b = J$ , as already explained; further, for a re-radiating surface, Q = 0. Between two given surfaces, the radiosity potentials are connected by the respective space resistances, as shown. It may be observed that the system reduces to a series-parallel circuit of resistances as shown in Fig. (13.33,b).

We get:

$$Q_{12} = \sigma \cdot \left(T_{1}^{4} - T_{2}^{4}\right) \cdot \left[A_{1} \cdot F_{12} + \frac{1}{\left(\frac{1}{A_{1} \cdot F_{1R}} + \frac{1}{A_{2} \cdot F_{2R}}\right)}\right] \qquad \dots (13.66)$$

Here,  $Q_{12}$  is the net radiant heat transferred between surfaces 1 and 2. Similar expressions can be written for heat transfer between surfaces 2 and 3 (=  $Q_{23}$ ) and the heat transfer between surfaces 1 and 3 (=  $Q_{13}$ ).

\_\_\_\_\_

2. Two gray surfaces surrounded by a third re-radiating surface:



Figure 13.34 Two gray surfaces surrounded by a re-radiating surface

Here, we have:

i.e. 
$$\frac{E_{b3} - J_{3}}{\left(\frac{1 - \varepsilon_{3}}{A_{3} \cdot \varepsilon_{3}}\right)} = 0$$

i.e.  $E_{b3}=J_3$ 

i.e. once the radiosity of the re-radiating surface is known, its temperature can easily be calculated, since  $E_{b3} = \sigma T_3^4$ . Further, note that  $T_3$  is independent of the emissivity of surface 3.

Expression for heat flow rate is:

$$Q_1 = Q_2 = \frac{E_{b1} - E_{b2}}{R_{tot}}$$

where,  ${\rm R}_{\rm tot}$  is the total resistance, given by:

$$R_{tot} = R_1 + \left[\frac{1}{\frac{1}{R_{12}} + \frac{1}{(R_{13} + R_{23})}}\right] + R_2$$

i.e. 
$$R_{tot} = \left(\frac{1-\epsilon_1}{A_1\cdot\epsilon_1}\right) + \frac{1}{\left[A_1\cdot F_{12} + \frac{1}{\left(\frac{1}{A_1\cdot F_{13}} + \frac{1}{A_2\cdot F_{23}}\right)}\right]} + \left(\frac{1-\epsilon_2}{A_2\cdot\epsilon_2}\right) \qquad \dots (13.67)$$

#### Radiation heat exchange in *four zone enclosures*:

a) When all the four surfaces are black: Remembering the principles already explained, if the radiation network for an enclosure comprising of four black surfaces is drawn, it will look as shown in Fig. (13.35), below:



Fig 13.35 Radiation network for an enclosure of four black surfaces



Expression for net radiant heat flow rate from surface 1 is:

$$Q_{1} = \frac{J_{1} - J_{2}}{R_{12}} + \frac{J_{1} - J_{3}}{R_{13}} + \frac{J_{1} - J_{4}}{R_{14}}$$
  
i.e. 
$$Q_{1} = \frac{E_{b1} - E_{b2}}{R_{12}} + \frac{E_{b1} - E_{b3}}{R_{13}} + \frac{E_{b1} - E_{b4}}{R_{14}} \qquad \dots \text{since } E_{b1} = J_{1} \text{ ec.}$$
  
i.e. 
$$Q_{1} = A_{1} \cdot F_{12} \cdot (E_{b1} - E_{b2}) + A_{1} \cdot F_{13} \cdot (E_{b1} - E_{b3}) + A_{1} \cdot F_{14} \cdot (E_{b1} - E_{b4}) \qquad \dots (13.68)$$

Similar expressions can be written for the net heat flow from other three surfaces.

b) When all the four surfaces are gray: Now, for each surface, a surface resistance has also to be included, and the radiation network for this system will be as shown in Fig. (13.36):



Fig 13.36 Radiation network for an enclosure of four gray surfaces

Expression for net radiant heat flow rate from surface 1 is:

$$Q_{1} = \frac{E_{b1} - J_{1}}{R_{1}} = \frac{J_{1} - J_{2}}{R_{12}} + \frac{J_{1} - J_{3}}{R_{13}} + \frac{J_{1} - J_{4}}{R_{14}}$$
  
i.e. 
$$Q_{1} = \frac{E_{b1} - J_{1}}{\left(\frac{1 - \varepsilon_{1}}{A_{1} \cdot \varepsilon_{1}}\right)} = A_{1} \cdot F_{12} \cdot \left(J_{1} - J_{2}\right) + A_{1} \cdot F_{13} \cdot \left(J_{1} - J_{3}\right) + A_{1} \cdot F_{14} \cdot \left(J_{1} - J_{4}\right) \qquad \dots (13.69)$$

Similar expressions can be written for the net heat flow from other three surfaces.

#### **Radiation shielding:**

#### For two infinite, parallel plates 1 and 2, radiation shield 3, be placed between the plates:



Fig 13.37 Radiation shield between two parallel plates, and associated radiation network

When there is no shield, the radiation heat transfer between plates 1 and 2 is already shown to be:

$$Q_{12} = \frac{A \cdot \sigma \cdot \left(T_{1}^{4} - T_{2}^{4}\right)}{\frac{1}{\varepsilon_{1}} + \frac{1}{\varepsilon_{2}} - 1} \quad \dots \text{ for infinitely large parallel plates} \dots (13.59)$$

With one shield placed between plates 1 and 2, the radiation network will be as shown in Fig. (b) above. Note that now all the relevant resistances are in series. Net heat transfer between plates 1 and 2 is given as:

 $Q_{12-\text{one shield}} = (E_{b1}-E_{b2})/R_{tot}$  where  $R_{tot}$  is the total resistance.

i.e.

$$Q_{12\_one\_shield} = \frac{E_{b1} - E_{b2}}{\frac{1 - \varepsilon}{A_{1} \cdot \varepsilon} + \frac{1}{A_{1} \cdot F_{13}} + \frac{1 - \varepsilon}{A_{3} \cdot \varepsilon} \frac{3 - 1}{3 - 1} + \frac{1 - \varepsilon}{A_{3} \cdot \varepsilon} \frac{3 - 2}{3 - 2} + \frac{1}{A_{3} \cdot F_{32}} + \frac{1 - \varepsilon}{A_{2} \cdot \varepsilon} \frac{2}{2}}$$

....for two gray surfaces with one radiation shield placed in between.....(13.70)

Now, for two large parallel plates, we note:

$$F_{13}=F_{32}=1$$
 and,  $A_1=A_2=A_3=A$ 

Then, eqn. (13.70) simplifies to:

$$Q_{12\_one\_shield} = \frac{A \cdot \sigma \cdot \left(T_{1}^{4} - T_{2}^{4}\right)}{\left(\frac{1}{\varepsilon_{1}} + \frac{1}{\varepsilon_{2}} - 1\right) + \left(\frac{1}{\varepsilon_{3\_1}} + \frac{1}{\varepsilon_{3\_2}} - 1\right)} \qquad \dots (13.71)$$

Note that as compared to eqn. (13.59) for the case of no-shield, we have, with one shield, an additional term appearing in the denominator of eqn. (13.71). Therefore, if there are N radiation shields, we have, for net radiation heat transfer:

$$Q_{12_{N_{shields}}} = \frac{A \cdot \sigma \cdot \left(T_{1}^{4} - T_{2}^{4}\right)}{\left(\frac{1}{\varepsilon_{1}} + \frac{1}{\varepsilon_{2}} - 1\right) + \left(\frac{1}{\varepsilon_{3}} + \frac{1}{\varepsilon_{3}} - 1\right) + \dots + \left(\frac{1}{\varepsilon_{N_{s}}} + \frac{1}{\varepsilon_{N_{s}}} - 1\right)} \dots \dots (13.72)$$

If emissivities of all surfaces are equal, eqn. (13.72) becomes:

$$Q_{12_{N_{shields}}} = \frac{A \cdot \sigma \cdot \left(T_{1}^{4} - T_{2}^{4}\right)}{(N+1) \cdot \left(\frac{1}{\epsilon} + \frac{1}{\epsilon} - 1\right)} = \frac{1}{(N+1)} \cdot \left(Q_{12_{no_{shield}}}\right) \qquad ....(13.73)$$

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Note this important result, which implies that, when all emissivities are equal, presence of one radiation shield reduces the radiation heat transfer between the two surfaces to one-half, two radiation shields reduce the heat transfer to one-third, 9 radiation shields reduce the heat transfer to one-tenth etc.

For a more practical case of the two surfaces having emissivities of  $\varepsilon 1$  and  $\varepsilon 2$ , and all shields having the same emissivity of  $\varepsilon s$ , eqn. (13.72) becomes:

$$Q_{12_{N_{shields}}} = \frac{A \cdot \sigma \cdot \left(T_{1}^{4} - T_{2}^{4}\right)}{\left(\frac{1}{\varepsilon_{1}} + \frac{1}{\varepsilon_{2}} - 1\right) + N \cdot \left(\frac{2}{\varepsilon_{s}} - 1\right)} \qquad \dots (13.74)$$

To determine the equilibrium temperature of the radiation shield:

Once  $Q_{12}$  is determined from eqn. (13.71), the temperature of the shield is easily found out by applying the condition that in steady state:

We can use either of the conditions:  $Q_{12} = Q_{13}$  or  $Q_{12} = Q_{32}$ .

 $Q_{13}$  or  $Q_{32}$  is determined by applying eqn. (13.59); i.e. we get:

$$Q_{12} = Q_{13} = \frac{A \cdot \sigma \cdot \left(T_1^4 - T_3^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1}$$
 ...(13.76,a)

or,

$$Q_{12} = Q_{32} = \frac{A \cdot \sigma \cdot \left(T_{3}^{4} - T_{2}^{4}\right)}{\frac{1}{\epsilon_{3}} + \frac{1}{\epsilon_{2}} - 1} \qquad \dots (13.76,b)$$

In both the above eqns.  $T_{3}$  is the only unknown, which can easily be determined.

------

#### For a cylindrical radiation shield placed in between two, long concentric cylinders:

The radiation heat transfer between two long, concentric cylinders is:

$$Q_{12} = \frac{A_1 \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)} \qquad \dots \text{ for infinitely long concentric cylinders.....(13.60)}$$

where

$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$

Now, let a cylindrical radiation shield, 3, be placed in between the inner cylinder (1) and the outer cylinder (2), as shown in Fig.(13.38).



Fig 13.38 Radiation shield between two concentric cylinders, and associated radiation network

The radiation network for this system is shown in Fig. (13.38,b) and it is exactly the same as shown in Fig. (13.37,b).

And, the radiation heat transfer between cylinders 1 and 2, when the shield is present, is given by:

$$Q_{12\_one\_shield} = \frac{E_{b1} - E_{b2}}{\frac{1 - \varepsilon}{A_{1} \cdot \varepsilon} + \frac{1}{A_{1} \cdot F_{13}} + \frac{1 - \varepsilon}{A_{3} \cdot \varepsilon} \frac{3}{3} + \frac{1 - \varepsilon}{A_{3} \cdot \varepsilon} \frac{3}{2} + \frac{1}{A_{3} \cdot F_{32}} + \frac{1 - \varepsilon}{A_{2} \cdot \varepsilon} \frac{2}{2}}$$

....for two gray surfaces with one radiation shield placed in between.....(13.70)

Now, for the cylindrical system, we have:

F 
$$_{13}$$
=F  $_{32}$ =1  
A  $_{1}$ =2 $\cdot\pi$  ·r  $_{1}$ ·L  
A  $_{2}$ =2 $\cdot\pi$  ·r  $_{2}$ ·L  
and,  
A  $_{3}$ =2 $\cdot\pi$  ·r  $_{3}$ ·L

Then, eqn. (13.70) reduces to:

$$Q_{12_{one\_shield}} = \frac{A_1 \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right) + \left(\frac{A_1}{A_3}\right) \cdot \left(\frac{1}{\varepsilon_{3\_1}} + \frac{1}{\varepsilon_{3\_2}} - 1\right)}$$

... for concentric cylinders with one radiation shield...(13.77)

In eqn. (13.77), we have:  $(A_1/A_2) = (r_1/r_2)$ , and  $(A_1/A_3) = (r_1/r_3)$ .

**Note** that as compared to the relation for two concentric cylinders with no shield (i.e. eqn. 13.60), *an additional term appears in the denominator of eqn. (13.77)* (i.e. the third term) when one radiation shield is introduced; if there is a second radiation shield, say (4), then one more similar term will have to be added in the denominator to take care of the resistance of that shield.

In this case too, the **equilibrium temperature of the shield is determined** by applying the principle that, in steady state,

$$Q_{12} = Q_{13} = Q_{32}$$

#### For a spherical radiation shield placed in between two concentric spheres:

This case is also represented by Fig. (13.38,a), where inner sphere 1 is enclosed by an outer sphere 2, and a radiation shield 3, is placed in between. The radiation network for this system is shown in Fig. (13.38,b).

When there is no radiation shield, radiation heat transfer between surfaces 1 and 2 is given by eqn. (13.61), viz.

$$Q_{12} = \frac{A_1 \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)} \qquad \dots \text{ for concentric spheres.....(13.61)}$$

where

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$$





Again, when the radiation shield is present, the general relation for radiation heat transfer between surfaces 1 and 2 is eqn. (13.70). Remembering that for concentric spheres,

F 
$$_{13}$$
=F  $_{32}$ =1  
A  $_{1}$ =4 $\cdot\pi$  ·r  $_{1}^{2}$   
A  $_{2}$ =4 $\cdot\pi$  ·r  $_{2}^{2}$   
and,  
A  $_{3}$ =4 $\cdot\pi$  ·r  $_{3}^{2}$ 

Relation for radiant heat transfer between surfaces 1 and 2, is exactly as eqn. (13.77), i.e.

$$Q_{12_{one\_shield}} = \frac{A_1 \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right) + \left(\frac{A_1}{A_3}\right) \cdot \left(\frac{1}{\varepsilon_{3\_1}} + \frac{1}{\varepsilon_{3\_2}} - 1\right)}$$

...for concentric spheres with one radiation shield...(13.78)

In eqn. (13.78), we have:

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2 \qquad \text{and,} \qquad \frac{A_1}{A_3} = \left(\frac{r_1}{r_3}\right)^2$$

In this case also, **equilibrium temperature of the shield is determined** by applying the principle that, in steady state,

------

Q<sub>12</sub>=Q<sub>13</sub>=Q<sub>32</sub>

#### Radiation error in temperature measurement:



(a) Thermometer without radiation shield



(b) Thermometer with radiation shield

Let the reading *shown* by the thermometer be  $T_c$ . This reading, however, does not represent the true temperature of the fluid  $T_f$ , since the thermometer bulb will lose heat by radiation to the walls of the channel which are at a lower temperature  $T_w$  (which is usually the case).

The temperature  $T_c$  shown by the thermometer will be some value in between  $T_f$  and  $T_w$ .

We wish to find out the true temperature of the fluid T<sub>e</sub>, by knowing the thermometer reading T<sub>e</sub>.

Making an energy balance on the thermometer bulb, in steady state, we have:

#### Without radiation shield:

 $q_{conv}$  to the bulb =  $q_{rad}$  from the bulb

i.e. 
$$\mathbf{h} \cdot \mathbf{A}_{\mathbf{c}} \cdot \left(\mathbf{T}_{\mathbf{f}} - \mathbf{T}_{\mathbf{c}}\right) = \varepsilon_{\mathbf{c}} \cdot \mathbf{A}_{\mathbf{c}} \cdot \sigma \cdot \left(\mathbf{T}_{\mathbf{c}}^{4} - \mathbf{T}_{\mathbf{w}}^{4}\right)$$
  
i.e.  $\mathbf{T}_{\mathbf{f}} = \mathbf{T}_{\mathbf{c}} + \frac{\varepsilon_{\mathbf{c}} \cdot \sigma \cdot \left(\mathbf{T}_{\mathbf{c}}^{4} - \mathbf{T}_{\mathbf{w}}^{4}\right)}{h} \qquad \dots (13.79)$ 

Fig 13.39 Radiation shielding of thermometers

where  $A_c =$ surface area of thermometer bulb,

 $\varepsilon_{c}$  = emissivity of thermometer bulb surface

Eqn. (13.79) gives the true temperature of the fluid  $T_{f}$ .

Second term on the RHS of eqn. (13.79) represents the *error in temperature measurement* due to **radiation** effect.

#### Energy balance on the thermometer bulb:

Heat transferred to the bulb from the fluid by convection = Heat transferred from the bulb to the shield by radiation

i.e.

$$\mathbf{h} \cdot \mathbf{A}_{c} \cdot \left(\mathbf{T}_{f} - \mathbf{T}_{c}\right) = \frac{\sigma \cdot \left(\mathbf{T}_{c}^{4} - \mathbf{T}_{s}^{4}\right)}{\left(\frac{1 - \varepsilon}{\mathbf{A}_{c} \cdot \varepsilon}\right) + \frac{1}{\mathbf{A}_{c} \cdot \mathbf{F}_{cs}} + \left(\frac{1 - \varepsilon}{\mathbf{A}_{s} \cdot \varepsilon}\right)} \qquad \dots (13.80)$$

In eqn. (13.80),  $F_{cs}$  = view factor of thermometer bulb w.r.t the shield and is, generally equal to 1.

#### Making an energy balance on the shield:

heat transferred to shield from the fluid by convection + heat transferred to shield from bulb by radiation = heat transferred from shield to walls by radiation

i.e. 
$$2 \cdot \mathbf{A}_{s} \cdot \mathbf{h} \cdot \left(\mathbf{T}_{f} - \mathbf{T}_{s}\right) + \frac{\sigma \cdot \left(\mathbf{T}_{c}^{4} - \mathbf{T}_{s}^{4}\right)}{\left(\frac{1 - \varepsilon}{\mathbf{A}_{c} \cdot \varepsilon}\right) + \frac{1}{\mathbf{A}_{c} \cdot \mathbf{F}_{cs}} + \left(\frac{1 - \varepsilon}{\mathbf{A}_{s} \cdot \varepsilon}\right)} = \varepsilon_{s} \cdot \mathbf{A}_{s} \cdot \sigma \cdot \left(\mathbf{T}_{s}^{4} - \mathbf{T}_{w}^{4}\right) \qquad \dots (13.81)$$

where,  $\ \ \, A_{s}$  = area of shield on one side

 $\epsilon_{s}$  = emissivitty of shield surface

 $A_c$  = area of bulb surface

- $\epsilon_{c}$  = emissivity of bulb surface
- $F_{cs}$  = view factor of bulb w.r.t. shield

In the first term of the above eqn. factor 2 appears since convective heat transfer to the shield occurs on *both surfaces* of the shield. Also, in writing the RHS, the inherent assumption is that:

 $F_{sw}=1$  ...view factor between the shield and the walls

and,

 $\frac{A_s}{A_w}$  = 0 ...i.e. surface area of shield is negligible compared to the area of the channel walls

Solving eqns. (13.80) and (13.81) simultaneously, we obtain the shield temperature  $T_s$  and the thermometer reading  $T_c$ , (if  $T_f$  is known), or  $T_f$  (if  $T_c$  is known).



# 5A Laws of Radiation and Radiation properties:

Prob.5A.1: Draw a plot of Planck's Law for spectral distribution:

#### Mathcad Solution:

Planck's Law:

$$E_{b\lambda} = \frac{C_1 \cdot \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda \cdot T}\right) - 1} \qquad \dots W/(m^2.\mu m)....(13.7)$$

where:

We have, costants C1 and C2 in Planck's Law:

$$C_1 := 3.742 \cdot 10^8$$
 W.µm<sup>4</sup>/m<sup>2</sup>  
 $C_2 := 1.4387 \cdot 10^4$  µm.K

To plot the Planck's disribution for a black body, using Mathcad, first, define  $E_{b\lambda}$  as a function of T and  $\lambda$  as follows:

$$\begin{split} \mathtt{E}_{b\lambda}(\lambda,T) &:= \frac{C_1 \cdot \lambda^{-5}}{\text{exp}\!\left(\frac{C_2}{\lambda \cdot T}\right) - 1} \qquad \dots \mathtt{E}_{b\lambda} \text{ as a function of } \lambda \text{ and } T \end{split}$$

To draw the plot:

Define a range variable  $\lambda$  varying from 0.01  $\mu$ m o 1000  $\mu$ m:

 $\lambda := 0.01, 0.02...1000$  ....define the range variable

Now, select the x-y plot from the graph palette; on x-axis place holder, type  $\lambda$  and on the y-axis place holder, fill in  $E_{b\lambda}$  ( $\lambda$ ,100),  $E_{b\lambda}$  ( $\lambda$ ,500),  $E_{b\lambda}$  ( $\lambda$ ,1000), and  $E_{b\lambda}$  ( $\lambda$ ,5800). Click anywhere outside the graph region, and the curves appear immediately:

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See the ease with which this plot is drawn with Mathcad.

**Prob.5A.2.** Radiation strikes an object with transmissivity of 0.03 and reflectivity of 0.5. The absorbed flux is indirectly measured to be 94 W/m2. Compute the incident flux. (**M.U.**)

```
Mathcad Solution:

Data:

\tau := 0.03 transmissivity

\rho := 0.5 reflectivity

Q_a := 94 W/m<sup>2</sup>....absorbed flux

Let G = incident flux
```

Now, we have:  $\alpha + \tau + \rho = 1$  where,  $\alpha = Q_g/G$ 

Therefore,



**Prob.5A.3.** A hole of area dA = 2 cm2 is opened on the surface of a large spherical cavity whose inside is maintained at 1000 K. Calculate: (a) the radiation energy streaming through the hole in all directions into space, (b) the radiation energy streaming per unit solid angle in a direction making a 60 deg. angle with the normal to the surface of the opening. (M.U.)

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#### Mathcad Solution:



#### Data:

T := 1000 K....temperature

σ := 5.67·10<sup>-8</sup> W/(m<sup>2</sup>.K4)...Stefan-Boltzann constant

 $\theta = 60 \text{ deg.}$  ...angle made by the desired direction, with the normal to the surface

i.e.  $\theta := 60 \cdot \frac{\pi}{180}$  ...radians

#### (a) Radiation streaming out in all directions:

Since the spherical cavity can be considered as a black body, energy streaming out is given by Stefan-Boltzmann law:

i.e. 
$$Q := dA \cdot \sigma \cdot T^4$$

or, Q = 11.34 W...Radiaion energy streaming through the hole...Ans.

#### (b) Radiation streaming out through unit solid angle, in a direction making 60 deg. with normal:

Now, we have the relation:  $E_b = \pi$ .  $I_b$ , where  $E_b = Emissive$  power, and  $I_b =$  Intensity of radiation.

Therefore,  $I_b = \frac{E_b}{\pi}$ But,  $E_b = \sigma \cdot T^4$  W/m<sup>2</sup>...for a black body, by Stefan-Boltzmann Law i.e.  $I_b := \frac{\sigma \cdot T^4}{\pi}$  W/(m<sup>2</sup>.sr)....Intensity of radiation i.e.  $I_b = 1.805 \times 10^4$  W/(m<sup>2</sup>.sr)

#### And, radiation streaming out in that direction:

 $\label{eq:Q} \begin{aligned} & \mathsf{Q} \coloneqq \mathbf{I}_b \cdot \mathtt{d} \mathbf{A} \cdot \mathtt{cos}(\theta) \quad \mathsf{W} .... \mathsf{where} \ \theta \ \mathsf{is} \ \mathsf{in} \ \mathsf{radians} \ (\textbf{Note:} \ \mathsf{while} \ \mathsf{using} \ \mathsf{Mathcad}, \ \theta \ \mathsf{must} \ \mathsf{be} \\ & \mathsf{in} \ \mathsf{radians} \ \mathsf{while} \ \mathsf{calculating} \ \mathsf{cos}(\theta).) \end{aligned}$ 

i.e. Q = 1.805 W...radiaion through a solid angle of unity, in a direction of 60 deg. with normal to the surface...Ans.

**Prob.5A.4.** It is observed that intensity of radiation is maximum in case of solar radiation at a wavelength of 0.49 microns. Assuming the Sun as a black body, estimate its surface temperature and emissive power. Wein displacement constant =  $0.289 \times 10-2$  m.K. (M.U.)

\_\_\_\_\_

#### Mathcad Solution:

Data:

 $\lambda_{max} := 0.49$  microns...wave length of max. radiation

 $\sigma := 5.67 \cdot 10^{-8}$  W/(m<sup>2</sup>.K<sup>4</sup>)....Stefan-Boltzmann const.

#### By Wein's displacement law, we have:

$$\lambda_{max} \cdot T = 2890$$
 microns.K

Therefore,

$$T := \frac{2890}{\lambda_{max}}$$

i.e.  $T = 5.898 \times 10^3$  K...surface temp. of Sun...Ans.

#### Heat flux at the surface, E<sub>b</sub>:

Sun an be considered as a black body; then, from Stefan-Boltzmann Law:

$$E_b := \sigma \cdot T^4$$
  
i.e.  $E_b = 6.861 \times 10^7$  W/m<sup>2</sup>...heat flux at the surface of Sun...Ans.

**Prob.5A.5.** The temperature of a body of area 0.1m2 is 900 K. Calculate the total rate of energy emission, intensity of normal radiation in W/(m2.sr), maximum monochromatic emissive power, and wavelength at which it occurs. (**M.U.**)

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#### Mathcad Solution:

Data:

T := 900 K....temperature

A := 0.1 m<sup>2</sup>...area

σ := 5.67·10<sup>-8</sup> W/(m<sup>2</sup>.K<sup>4</sup>)....Stefan-Boltzmann const.

#### Total rate of energy emission:

$$E_b := \sigma \cdot T^4$$
 ... from Stefan-Boltzmann Law

i.e. 
$$E_b = 3.72 \times 10^4$$
 W/m<sup>2</sup>....Total emissive power

Therefore,

$$Q := E_b \cdot A$$

i.e.  $Q = 3.72 \times 10^3$  W...Total energy emission from surface...Ans.

#### Intensity of normal radiation:

(Note that for a black (diffuse) surface, intensity is the same in all directions.)

I := 
$$\frac{E_b}{\pi}$$
  
i.e. I = 1.184 × 10<sup>4</sup> W/(m<sup>2</sup>.sr).....Ans.

#### Wave length of max. monochromatic emissive power:

From Wein's displacement law, we have:

$$\lambda_m \cdot T = 2898 \quad \mu m.K$$

Therefore,

$$\lambda_m := \frac{2898}{T} \quad \mu m$$

i.e.  $\lambda_m = 3.22$  µm....wave length for max. monochromatic emissive power at 900 K...Ans



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#### Max. monochromatic emissive power:

We use Planck's law, with values of constants, C1 and C2:

C1 := 
$$3.742 \cdot 10^8$$
 W. $\mu$ m<sup>4</sup>/m<sup>2</sup>)

and, C2 := 1.4387.10<sup>4</sup> µm.K

$$E_{b\lambda max} := \frac{C1 \cdot \lambda_m^{-5}}{exp\left(\frac{C2}{\lambda_m \cdot T}\right) - 1} \dots Planck's Law$$

i.e.  $E_{b\lambda max} = 7.6 \times 10^3$ 

W/(m<sup>2</sup>.micron)...Max. monochr. emissive power....Ans.

#### Alternatively:

We can directly apply eqn.(13.12):

$$E_{b\lambda m} := 1.287 \cdot 10^{-5} \cdot T^5 \quad W/m^3....(13.12)$$
  
i.e.  $E_{b\lambda m} = 7.6 \times 10^9 \quad W/m^3 = 7.6 \times 10^{-3} W/(m^2.\mu m)$ ..same as obtained above...Ans.

**Prob.5A.6.** Write a Mathcad program to read the Radiation Functions (i.e. radiation emitted in a wave band as a fraction of Total emissive power) against  $\lambda T$  from the Table 13.2 and return its value at the desired  $\lambda T$  by interpolation:

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We have:

$$\begin{split} \mathbf{E}_{b\lambda}(\lambda,\mathbf{T}) &\coloneqq \frac{\mathbf{C}_{1} \cdot \lambda^{-5}}{\exp\!\left(\frac{\mathbf{C}_{2}}{\lambda \cdot \mathbf{T}}\right) - 1} \\ \mathbf{F}_{0} &\lambda = \frac{\displaystyle \int_{0}^{\lambda} \mathbf{E}_{b\lambda} \, d\lambda}{\sigma \cdot \mathbf{T}^{4}} = \frac{\displaystyle \int_{0}^{\lambda \cdot \mathbf{T}} \mathbf{E}_{b\lambda} \, d(\lambda \cdot \mathbf{T})}{\sigma \cdot \mathbf{T}^{5}} \end{split}$$

First, write the Table 13.2 for  $F_0_\lambda$  vs  $\lambda T$  in two columns only, and then be set it equal to Matrix M1. First column of the Matrix is  $\lambda T$  values and the second column is  $F_0_\lambda$  values.

Then:

$$\begin{split} F_{0\_\lambda}(M1,\lambda T) &:= & (\text{return } 0) \quad \text{if } \lambda T \leq 600 \\ & (\text{return } 1) \quad \text{if } \lambda T > 100000 \\ & \text{LambdaT\_vector} \leftarrow M1^{\langle 0 \rangle} \\ & F\_\text{vector} \leftarrow M1^{\langle 1 \rangle} \\ & F\_0\_\lambda \leftarrow \text{linterp}(\text{LambdaT\_vector},F\_\text{vector},\lambda T) \end{split}$$

Note that  $F_0_{\lambda} = 0$  for  $\lambda T < 600$ , and  $F_0_{\lambda} = 1$  for  $\lambda T > 100000$  (See the Table 13.2 at the beginning of this Chapter).

```
Ex: λT := 12000
```

$$F_{0,\lambda}(M1,\lambda T) = 0.9451$$

Also, if we need to get fraction between two given wave lengths  $\lambda 1$  and  $\lambda 2$ :

$$\begin{split} F_{\lambda 1\_\lambda 2}(M1,\lambda 1T,\lambda 2T) &:= F_{0\_\lambda}(M1,\lambda 2T) - F_{0\_\lambda}(M1,\lambda 1T) \\ \\ Ex: \quad \lambda 1T := 1600 \qquad \lambda 2T := 12000 \\ \\ F_{\lambda 1\_\lambda 2}(M1,\lambda 1T,\lambda 2T) = 0.925 \end{split}$$

#### Let us use this Function in the following problem:

**Prob.5A.7.** Window glass transmits radiant energy in the wave length range  $0.4\mu$ m to  $2.5\mu$ m. Determine the fraction of total radiant energy which is transmitted, when the source temperature is: (a) 5800 K (i.e. Sun's surface temp.), and (b) 300 K (i.e. room temp.)

#### Mathcad Solution:

#### Data:

T1 := 5800 K....source temperature in first case

T2 := 300 K....source temperature in second case

 $\lambda_1 := 0.4 \qquad \mu m...$ lower end of wave length band in which glass is transparent

 $\lambda_2 := 2.5$  µm...upper end of wave length band in which glass is transparent

 $\sigma := 5.67 \cdot 10^{-8}$  W/(m<sup>2</sup>.K<sup>4</sup>)....Stefan-Boltzmann const.

#### Case (a): Source temp. T1 = 5800 K

We use the Mathcad Functions written above, or Table 13.2 where radiation functions are tabulated against the product  $(\lambda, T)$ .

We have:

$$\lambda_1$$
·T1 = 2.32 × 10<sup>3</sup> µm.K  
 $\lambda_2$ ·T1 = 1.45 × 10<sup>4</sup> µm.K



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Corresponding to these  $\lambda T$  values, we get:

 $F_{0,\lambda_1} := F_{0,\lambda}(M1,\lambda_1 \cdot T1)$  ....using the Mathcad Function written above

i.e.  $F_{0_{\lambda_1}} = 0.1245$ 

and,  $F_{0,\lambda 2} := F_{0,\lambda}(M1,\lambda_2 \cdot T1)$  ....using the Mathcad Function written above

i.e. 
$$F_{0,\lambda,2} = 0.9661$$

Therefore, fraction transmmitted is equal to:

$$0.9661 - 0.12454 = 0.842$$

Alternatively:  $F_{\lambda 1 \ \lambda 2}(M1, \lambda_1 \cdot T1, \lambda_2 \cdot T1) = 0.842$ 

## i.e. 84.2% of the energy coming from the Sun (at 5800 K) is transmitted through the window glass.....Ans.

#### Case (b): Source temp. T2= 300 K

Again, we use the Mathcad Functions already written or Table 13.2 where radiation functions are tabulated against the product ( $\lambda$ .T).

We have:

$$\lambda_1 T_2 = 120$$
 µm.K

 $\lambda_2 \cdot T2 = 750$  µm.K

Corresponding to these  $\lambda T$  values, we get:

 $F_{0,\lambda 1} := F_{0,\lambda}(M1,\lambda_1,T2)$  ....using the Mathcad Function written above

i.e. 
$$F_{0,\lambda_1} = 0$$

and,  $F_{0_\lambda 2} := F_{0_\lambda}(M1, \lambda_2 \cdot T2)$  ....using the Mathcad Function written above

i.e. 
$$F_{0,\lambda 2} = 1.2 \times 10^{-5}$$
 = almost zero

## i.e. practically no energy will be transmitted through the window glass in this wave length range, if the source temp. = 300 K.

In other words, glass is 'opaque' to radiation at 300 K in the wave length range 0.4 µm to 2.5 µm.

This is the principle of 'Greenhouse effect', wherein radiation from a high temperature source (i.e. Sun) is allowed to pass through the glass into the enclosure of the green house, while radiation at a relatively low temperature from within the enclosure, is not allowed to escape out. This, in effect, causes an increase in the temperature of the space within the enclosure.

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**Prob.5A.8.** Spectral emissivity of a particular surface at 800 K is approximated by a step function, as follows:  $\varepsilon 1 = 0.1$  for  $\lambda = 0$  to 2 µm,  $\varepsilon 2 = 0.5$  for  $\lambda = 2$  to 15 µm, and  $\varepsilon 3 = 0.8$  for  $\lambda = 15$  to  $\infty$ . Calculate: (i) the total (hemispherical) emissive power, and (ii) total hemispherical emissivity  $\varepsilon$ , over all wave lengths.



#### Mathcad Solution:

#### Data:

Planck's Law gives spectral emissive power of a black body.

For a non-gray surface considered in this problem, we can write:

Total emissive power:  $E = \int_{-\infty}^{\infty} \epsilon_{\lambda}(\lambda) \cdot E_{b\lambda} d\lambda$ 

Variation of  $\varepsilon_{\lambda}$  wih  $\lambda$  is specified in the problem.

Therefore, splitting the above integral into parts:

$$\mathbf{E} = \int_{0}^{2} \mathbf{e}_{\lambda}(\lambda) \cdot \mathbf{E}_{b\lambda} \, d\lambda + \int_{2}^{15} \mathbf{e}_{\lambda}(\lambda) \cdot \mathbf{E}_{b\lambda} \, d\lambda + \int_{15}^{\infty} \mathbf{e}_{\lambda}(\lambda) \cdot \mathbf{E}_{b\lambda} \, d\lambda$$

i.e. 
$$\mathbf{E} = 0.1 \cdot \int_{0}^{2} \mathbf{E}_{b\lambda} d\lambda + 0.5 \cdot \int_{2}^{15} \mathbf{E}_{b\lambda} d\lambda + 0.8 \cdot \int_{15}^{\infty} \mathbf{E}_{b\lambda} d\lambda$$

Then, 
$$\epsilon = \frac{E}{E_b} = \epsilon 1 \cdot (F_{0}\lambda_1) + \epsilon 2 \cdot (F_{\lambda 1}\lambda_2) + \epsilon 3 \cdot (F_{\lambda 3} infinity)$$

i.e. 
$$\epsilon = \epsilon \mathbf{1} \cdot (\mathbf{F}_{0,\lambda 1}) + \epsilon \mathbf{2} \cdot (\mathbf{F}_{0,\lambda 2} - \mathbf{F}_{0,\lambda 1}) + \epsilon \mathbf{3} \cdot (\mathbf{F}_{0,infinity} - \mathbf{F}_{0,\lambda 2}) \qquad \dots (a)$$



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## Values of $F_{0-\lambda 1}$ etc. are obtained from the Mathcad Function written above, or from Table 13.2.

We have:

 $\lambda_1$ ·T = 1.6 × 10<sup>3</sup> ...correspondingly, we get:

$$F_{0_{\lambda_1}} := F_{0_{\lambda_1}}(M_{1,\lambda_1} \cdot T)$$
 i.e.  $F_{0_{\lambda_1}} = 0.0197$ 

And,

 $\lambda_2$ ·T = 1.2 × 10<sup>4</sup> ...correspondingly, we get:

$$F_{0_{\lambda_2}} := F_{0_{\lambda_2}}(M1, \lambda_2 \cdot T)$$
 i.e.  $F_{0_{\lambda_2}} = 0.9451$ 

And,

 $\lambda_3 \cdot T = \infty$  ...correspondingly, we get:

 $F_{0,\lambda3} := F_{0,\lambda}(M1, 100001)$  ....Note: For  $\lambda T$  = infinity, we have put  $\lambda T > 100000$ .

i.e.  $F_{0,\lambda,3} = 1$ 

OR:

 $F_{0,\lambda3} := 1$  ...from Table 13.2

Then, from eqn. (a):

 $\boldsymbol{\epsilon} := \boldsymbol{\epsilon} \mathbf{1} \cdot \boldsymbol{F}_{0\_\lambda 1} + \boldsymbol{\epsilon} \mathbf{2} \cdot \left( \boldsymbol{F}_{0\_\lambda 2} - \boldsymbol{F}_{0\_\lambda 1} \right) + \boldsymbol{\epsilon} \mathbf{3} \cdot \left( \boldsymbol{F}_{0\_\lambda 3} - \boldsymbol{F}_{0\_\lambda 2} \right)$ 

i.e. e = 0.509 ... Total hemisph. emissiviy over all wave lengths... Ans.

And, total emissive power of this surface is given by:

 $E := \epsilon \cdot \sigma \cdot T^4$  W/m<sup>2</sup>....total emissive power

i.e.  $E = 1.199 \times 10^4$  W/m<sup>2</sup>....total emissive power...Ans.

## Plot the variation of total emissivity $\epsilon$ and total emissive power E as Temp T varies from 500 K to 2000 K:

First, write the relevant quantities as functions of T:

$$\begin{split} F_{0\_\lambda 1}(T) &:= F_{0\_\lambda} \begin{pmatrix} M1, \lambda_1 \cdot T \end{pmatrix} & \text{check:} \quad F_{0\_\lambda 1}(T) = 0.0197 \\ F_{0\_\lambda 2}(T) &:= F_{0\_\lambda} \begin{pmatrix} M1, \lambda_2 \cdot T \end{pmatrix} & \text{check:} \quad F_{0\_\lambda 2}(T) = 0.9451 \\ F_{0\_\lambda 3} &:= F_{0\_\lambda} (M1, 100001) & \dots \text{Note:} \text{ For } \lambda T = \text{infinity, we have:} \quad F_{0\_\lambda 3} = 1 \\ \epsilon(T) &:= \epsilon 1 \cdot F_{0\_\lambda 1}(T) + \epsilon 2 \cdot \left(F_{0\_\lambda 2}(T) - F_{0\_\lambda 1}(T)\right) + \epsilon 3 \cdot \left(F_{0\_\lambda 3} - F_{0\_\lambda 2}(T)\right) & \dots \text{total hemisph. emissivity} \\ & \text{check:} \quad \epsilon(T) = 0.509 \\ E(T) &:= \epsilon(T) \cdot \sigma \cdot T^4 & W/m^2 \dots \text{total emissive power} \quad \text{check:} \quad E(T) = 1.181 \times 10^4 \end{split}$$

#### To plot the results:

T := 500,510..2000 ...define a range variable T

T =	= (T)	E(T) =
500	0.548	1.943·10 <sup>3</sup>
600	0.532	3.91·10 <sup>3</sup>
700	0.52	7.076·10 <sup>3</sup>
800	0.509	1.181·10 <sup>4</sup>
900	0.497	1.847·10 <sup>4</sup>
1000	0.483	2.737·10 <sup>4</sup>
1100	0.467	3.876.104
1200	0.45	5.287·10 <sup>4</sup>
1300	0.431	6.987·10 <sup>4</sup>
1400	0.413	8.991·10 <sup>4</sup>
1500	0.394	1.131·10 <sup>5</sup>
1600	0.376	1.395·10 <sup>5</sup>
1700	0.358	1.693·10 <sup>5</sup>
1800	0.341	2.027·10 <sup>5</sup>
1900	0.324	2.397·10 <sup>5</sup>
2000	0.309	2.804·10 <sup>5</sup>

#### Plot:

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**Prob.5A.9.** Spectral hemispherical emissivity (monochromatic emissivity) of fire brick at 750 K as a function of wave length is as follows:

 $\epsilon 1 = 0.1$  for  $\lambda = 0$  to 2 µm,  $\epsilon 2 = 0.6$  for  $\lambda = 2$  to 14 µm, and  $\epsilon 3 = 0.3$  for  $\lambda = 14$  to ∞.Calculate total hemispherical emissivity  $\epsilon$ , over all wave lengths. [VTU – July–Aug. 2004]

#### Mathcad Solution:

This problem is similar to the previous problem.

So, we will enter the data, and use the Mathcad Functions already written:

#### Data:

$$\begin{split} \text{T} &:= 750 \ \text{K....temperature} \\ \text{e1} &:= 0.1 \ \text{...emissiviy in wavelength range: 0 to 2 } \mu\text{m} \\ \text{e2} &:= 0.6 \ \text{...emissiviy in wavelength range: 2 to 14 } \mu\text{m} \\ \text{e3} &:= 0.3 \ \text{...emissiviy in wavelength range: 14 } \mu\text{m} \\ \text{to} &:= 5.67 \cdot 10^{-8} \ \text{W/(m}^2.\text{K}^4)....\text{Stefan-Boltzmann const.} \\ \lambda_1 &:= 2 \ \mu\text{m} \\ \lambda_2 &:= 14 \ \mu\text{m} \\ \lambda_3 &:= \infty \end{split}$$

#### **Results:**

$$\begin{split} F_{0\_\lambda 1}(T) &:= F_{0\_\lambda}(M1,\lambda_1 \cdot T) & \text{i.e.:} \quad F_{0\_\lambda 1}(T) = 0.01375 \\ F_{0\_\lambda 2}(T) &:= F_{0\_\lambda}(M1,\lambda_2 \cdot T) & \text{i.e.:} \quad F_{0\_\lambda 2}(T) = 0.9237 \\ F_{0\_\lambda 3} &:= F_{0\_\lambda}(M1,100001) & \dots \text{Note: For } \lambda T = \text{infinity, we have:} \quad F_{0\_\lambda 3} = 1 \\ \epsilon(T) &:= \epsilon 1 \cdot F_{0\_\lambda 1}(T) + \epsilon 2 \cdot \left(F_{0\_\lambda 2}(T) - F_{0\_\lambda 1}(T)\right) + \epsilon 3 \cdot \left(F_{0\_\lambda 3} - F_{0\_\lambda 2}(T)\right) \\ & \dots \text{total hemisph. emissivity} \end{split}$$

i.e.:  $\epsilon(T) = 0.57$  ...total hemisph. emissivity ... Ans.  $E(T) := \epsilon(T) \cdot \sigma \cdot T^4$  W/m<sup>2</sup>....total emissive power

i.e.: 
$$E(T) = 1.023 \times 10^4$$
 W/m<sup>2</sup>....total emissive power.... Ans.

Plot the variation of total emissivity  $\epsilon$  and total emissive power E as Temp T varies from 500 K to 2000 K:

T := 500, 510.. 2000 T := 500, 510.. 2000

T =	ε(T) =	E(T) =
500	0.542	1.922·10 <sup>3</sup>
600	0.56	4.117·10 <sup>3</sup>
700	0.569	7.747·10 <sup>3</sup>
800	0.571	1.325.104
900	0.566	2.105·10 <sup>4</sup>
1000	0.555	3.15·10 <sup>4</sup>
1100	0.541	4.489·10 <sup>4</sup>
1200	0.523	6.148·10 <sup>4</sup>
1300	0.503	8.143·10 <sup>4</sup>
1400	0.481	1.049·10 <sup>5</sup>
1500	0.459	1.319·10 <sup>5</sup>
1600	0.438	1.626·10 <sup>5</sup>
1700	0.416	1.972·10 <sup>5</sup>
1800	0.396	2.356·10 <sup>5</sup>
1900	0.376	2.78·10 <sup>5</sup>
2000	0.358	3.246·10 <sup>5</sup>





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#### Prob.5A.10. For a black body maintained at 390 K, determine:

- 1) total emissive power
- 2) wave length at which max. monochromatic emissive power occurs
- 3) Max. monochromatic emissive power [M.U. May 1999]

#### Mathcad Solution:

Data:

C1 :=  $3.742 \cdot 10^8$  W.µm<sup>4</sup>/m<sup>2</sup> C2 :=  $1.4387 \cdot 10^4$  µm.K T := 390 K  $\sigma$  :=  $5.67 \cdot 10^{-8}$  W/m<sup>2</sup>.K<sup>4</sup>... Stefan Boltzmann constant.

#### Calculations:

$$E_b := \sigma \cdot T^4$$
  
i.e.  $E_b = 1.312 \times 10^3$  W/m^2....Total emissive power.... Ans

We have:

$$\lambda_m := \frac{2897.8}{T}$$

i.e.  $\lambda_m = 7.43$  microns....wave length for max. mono. emissive power at 390 K.... Ans.

And:

$$E_{b\lambda} := \frac{C1 \cdot \lambda_m^{-5}}{\exp\left(\frac{C2}{\lambda_m \cdot T}\right) - 1}$$

i.e. E<sub>bb</sub> = 116.129 W/m<sup>2</sup>.micron...Max. monochr. emissive power...Ans.

"**Prob.5A.11.** For a body, of area 0.15 m<sup>2</sup>, at an effective temp of 900 K, calculate the following: (i) Total rate of energy emission (E) (ii) Intensity of normal radiation ( $I_{bn}$ ), and (iii) Wave length of max. monochromatic emissive power".

Also plot the variation of  $\lambda_m$  with T , varying T from 500 K to 2000 K.

#### "EES Solution:"

#### "Data:"

A = 0.15 [m^2] T = 900 [K] sigma = 5.67E-08 [W/m^2-K^4] "...Stefan Boltzmann constant"

#### "Calculations:"

E\_b = sigma \* T^4 "[W/m^2]...Total Emissive power"

 $E = E_b * A "[W]...Total rate of energy emission"$ 

 $I_bn = E_b / pi$  "[W/m^2]...Intensity of normal radiation"

lambda\_m \* T = 2898 "...by Wein's displacement Law.. finds lambda\_max, microns"

#### **Results:**

 Unit Settings: SI C Pa J mass deg

 A = 0.15 [m<sup>2</sup>]
 E = 5580 [W]
 Eb = 37201 [W/m<sup>2</sup>]

  $l_{bn}$  = 11841 [W/m<sup>2</sup>]
  $\lambda_m$  = 3.22 [microns]
  $\sigma$  = 5.670E-08 [W/m<sup>2</sup>K<sup>4</sup>]

 T = 900 [K]
  $\kappa$ 

Thus:

Total rate of energy emission = E = 5580 W

Intensity of normal radiation = 11841 W/m^2

Wave length of max. monochromatic emissive power = 3.22 microns ... Ans.

### Also plot the variation of $\lambda_m$ with T, varying T from 500 K to 2000 K:

#### First, compute the Parametric Table:

Es Parametri	😼 Parametric Table 📃 🗖 🔀								
Table 1									
116	1	2 Σ λ <sub>m</sub> [microns]	³ E [W]	4					
Run 1	500	5.796	531.6	1128					
Run 2	600	4.83	1102	2339					
Run 3	700	4.14	2042	4333					
Run 4	800	3.623	3484	7393					
Run 5	900	3.22	5580	11841					
Run 6	1000	2.898	8505	18048					
Run 7	1100	2.635	12452	26424					
Run 8	1200	2.415	17636	37425					
Run 9	1300	2.229	24291	51547					
Run 10	1400	2.07	32673	69334					
Run 11	1500	1.932	43057	91369					
Run 12	1600	1.811	55738	118280					
Run 13	1700	1.705	71035	150740					
Run 14	1800	1.61	89282	189462					
Run 15	1900	1.525	110838	235206					
Run 16	2000	1.449	136080	288771					

#### Now, plot the results:







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"**Prob.5A.12.** Considering an Industrial furnace at 2500 K as a black body, find out the following: (i) monochromatic emissive power at a wave length of 1.2 microns, (ii) wave length at which emission is maximum, (iii) max. emissive power, and (iv) total emissive power".

#### "EES Solution:"

#### "Data:"

T = 2500 [K] lambda = 1.2 [microns] sigma = 5.67E-08 [W/m^2-K^4]"...Stefan Boltzmann constant"

C1 = 3.742E08 [W-micron<sup>4</sup>/m<sup>2</sup>]"...const. in Planck's law eqn"

C2 = 1.4388E04 [m-K]"...const. in Planck's law eqn"

#### "Calculations:"

 $E_lambda_b = C1 * lambda^{(-5)} / (exp(C2 / (lambda * T)) -1) "[W/m^2]...monochr. emissive power at lambda = 2 microns"$ 

lambda\_m \* T = 2898 "...by Weins displacement Law.. finds lambda\_max, microns"

E\_lamda\_b\_max = 1.285E-05 \* T^5 "[W/m^2 per m length]...Max. mono. emissive power"

#### $E_b = sigma * T^4 "[W/m^2]...Total emissive power"$

#### **Results:**

#### Unit Settings: SI C kPa kJ mass deg

C1 = 3.742E+08 [W-micron <sup>4</sup> /m <sup>2</sup> ]						
E <sub>b</sub> = 2.215E+06 [W/m <sup>2</sup> ]						
E <sub>lamda,b,max</sub> = 1.255E+12 [W/m <sup>2</sup> ·per·m·length]						
$\lambda_{m} = 1.159 \text{ [microns]}$						
T = 2500 [K]						



#### Thus:

Monochr. emissive power at 1.2 µm = E\_lambda\_b = 1.253E06 W/m^2

Wave length at which emission is max =  $\lambda_m = 1.159 \ \mu m$ 

Max. mono. emissive power = E\_lambda\_b\_max = 1.255E12 W/m^2 per m length

Total emissive power =  $E_b = 2.215E06 \text{ W/m}^2 \dots \text{ Ans.}$ 

Prob.5A.13: Draw a plot of Planck's Law for spectral distribution with EXCEL:

#### **EXCEL Solution:**

We have already stated the Planck's Law, i.e.

$$E_{b\lambda}(\lambda, T) := \frac{C_1 \cdot \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda \cdot T}\right) - 1} \qquad \dots E_{b\lambda} \text{ as a function of } \lambda \text{ and } T$$

Where C1 and C2 are constants, given as:

C1 = 3.742E08 [W-micron^4/m^2]"...const. in Planck's law eqn" C2 = 1.4388E04 [m-K]"...const. in Planck's law eqn" Now, let us write a VBA Function in EXCEL to produce a Table of  $\lambda$  against T and plot the same for T = 5800 K, 1000 K, 500 K and 100 K:

#### Following are the steps in EXCEL:

1. In EXCEL go to: Developer - Visual Basic:

Home Insert Page Layout	Formulas Data	Review View Developer Ad	d-Ins CodeCogs
Visual Basic	Insert     Design     Insert     Insert </th <th>rties Map Properties In Code Source Refresh Data</th> <th>port Document Panel</th>	rties Map Properties In Code Source Refresh Data	port Document Panel
Code	Controls	XML	Modify



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2. Click on Visual Basic - Insert:



3. Click on Module and a blank window (Module 1) presents itself. Write the code in that window as shown below:

4. Now, this Function is available in EXCEL worksheet like any other built-in Function of EXCEL, under the 'User Defined' category. Let us use it to produce the Table and plot as shown below:

Only part of the Table is shown below. Note the formula used (i.e. the VBA Function written above) in the Formula bar:

C10 - 🕤			$f_{x}$	=Plancks	5_Law(B10,58	300)	
	А	В		С	D	E	F
4							
5		Planck's Law fo	or Sp	ectral d	istributio	n: (Ref: N	IT, p.645)
6							
7			T =	5800 K	T = 1000 K	T = 500 K	T = 100 K
8		Lambda (microns)	E_b_l	ambda (V	V/m^2/micro	on)	
9							
10		0.06	0.00	0534275			
11		0.07	0.09	0776083			
12		0.08	3.90	5862682			
13		0.09	67.9	4688745			
14		0.1	631.	4722659			
15		0.2	4803	3754.872			
16		0.3	3950	8670.79			
17		0.4	7421	6005.51	8.7562E-06		
18		0.5	8448	30620.57	0.00381825		
19		0.6	7832	25115.48	0.18564775		
20		0.7	6628	31071.45	2.64010463		
21		0.8	5383	35149.43	17.6770775		
22		0.9	4299	6046.38	72.3523627		
23		1	3418	31499.37	211.304778	0.0001193	
24		1.2	2178	8716.23	934.05258	0.0058015	
25		1.4	1425	3369.15	2395.97531	0.0825033	
26		1.6	9611	245.643	4440.54435	0.5524087	

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	А	В	С	D	E	F
145		740	0.000502235	8.5897E-05	4.253E-05	7.8579E-06
146		760	0.000451438	7.7226E-05	3.825E-05	7.0815E-06
147		780	0.000406904	6.9622E-05	3.449E-05	6.3986E-06
148		800	0.000367729	6.2931E-05	3.118E-05	5.7961E-06
149		820	0.000333157	5.7025E-05	2.826E-05	5.2629E-06
150		840	0.000302554	5.1795E-05	2.568E-05	4.7895E-06
151		860	0.000275386	4.7152E-05	2.338E-05	4.3682E-06
152		880	0.000251199	4.3018E-05	2.133E-05	3.9923E-06
153		900	0.000229611	3.9327E-05	1.951E-05	3.6559E-06
154		920	0.000210293	3.6023E-05	1.787E-05	3.3541E-06
155		940	0.000192964	3.3059E-05	1.64E-05	3.0829E-06
156		960	0.000177384	3.0394E-05	1.508E-05	2.8386E-06
157		980	0.000163346	2.7992E-05	1.389E-05	2.6179E-06
158		1000	0.000150669	2.5823E-05	1.282E-05	2.4183E-06

#### Now, plot the results:



Prob.5A.14: Write an EXCEL VBA program to generate the Radiation Functions (i.e. radiation emitted in a wave band as a fraction of Total emissive power) against  $\lambda T$ .

#### **EXCEL Solution:**

Recollect Planck's Law:

 $E_{b\lambda}(\lambda) = \frac{C_{1} \cdot \lambda^{-5}}{\exp\left(\frac{C_{2}}{\lambda \cdot T}\right) - 1}$  $C_{1} = 3.742 \cdot 10^{8} \qquad \text{W.} \mu \text{m} 4/\text{m} 2$  $W/(m_{2.\mu}m).....(13.7)$ where.  $C_2 = 1.438710^4$  µm.K

#### Radiation from a wave band:

and,

Often, it is required to know the amount of radiation emitted in a given wave band, i.e. in a wave length interval between  $\lambda_1$  and  $\lambda_2$ . This is expressed as a fraction of the total emissive power and is written as



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#### $F\lambda_1 \lambda_2$ . Then, we can write:

$$F_{\lambda 1} \lambda 2 = \frac{\int_{\lambda}^{0} \frac{2}{\lambda_{1}}}{\int_{0}^{\infty} E_{b\lambda} d\lambda} = \frac{1}{\sigma \cdot T^{4}} \cdot \int_{\lambda_{1}}^{0} E_{b\lambda} d\lambda$$
  
i.e. 
$$F_{\lambda 1} \lambda 2 = \frac{1}{\sigma \cdot T^{4}} \cdot \left( \int_{0}^{0} \frac{2}{\lambda_{1}} E_{b\lambda} d\lambda - \int_{0}^{0} \frac{1}{\lambda_{1}} E_{b\lambda} d\lambda \right)$$
  
i.e. 
$$F_{\lambda 1} \lambda 2 = F_{0} \lambda 2^{-1} F_{0} \lambda 1$$

Above formula is not very convenient to use, since  $E_b \lambda$  depends on absolute temperature T, and it is not practicable to tabulate  $F_{0} \lambda$  for each T. This difficulty is overcome by expressing  $F_{0} \lambda$  as follows:

$$F_{0} = \frac{\int_{0}^{\lambda} E_{b\lambda} d\lambda}{\sigma \cdot T^{4}} = \frac{\int_{0}^{\lambda} E_{b\lambda} d(\lambda \cdot T)}{\sigma \cdot T^{5}}$$
  
i.e. 
$$F_{0} = f(\lambda \cdot T) \qquad \dots (13.16)$$

So, we shall, first, write a VBA Function to calculate  $F_{0-\lambda}$  and then, use it to write another Function for  $F_{\lambda_1-\lambda_2}$ .

Following are the VBA Functions, where *integrals are calculated by numerical method*, using the Trapezoidal rule.

```
▼ F_zero_lambdaT1_lambdaT2_alternative
(General)
  Option Explicit
  Function F(x As Double) As Double
  'This function is used to calculate the fraction F 0 lambda, Ref: Incropera
  Dim C_1 As Double, C_2 As Double, sigma As Double
  C 1 = 374200000# 'W.micron^4/m2
  C_2 = 14388# 'micron.K
  sigma = 0.000000567 'W/m2.K^4
   F = C_1 / (sigma * x ^ 5 * (Exp(C_2 / x) - 1))
  End Function
  Function F_zero_lambdaT_Low_lambdaT_High_trapezoidal(LambdaT_Low As Double, _
  LambdaT_High As Double, n As Integer) As Double
  'Gives the fraction F zero Lambda by integration using Trapezoidal rule
  'Input: LambdaT Low, LambdaT High in micron.K, n is the no. of equal intervals
  Dim dx As Double, sum As Double, y As Double, i As Integer
  'If LambdaT Low is less than 21, F(x) goes to extremely low value and gives error!
  If LambdaT_Low < 21 Then
     LambdaT_Low = 21
  End If
  If LambdaT_High > 100000 Then
    LambdaT High = 100000
  End If
  dx = (LambdaT_High - LambdaT_Low) / n
  sum = (F(LambdaT Low) + F(LambdaT High)) / 2
  For i = 1 To (n - 1)
     y = F(LambdaT Low + (dx * i))
      sum = sum + y
  Next i
  sum = sum * dx
  F_zero_lambdaT_Low_lambdaT_High_trapezoidal = sum
  End Function
```

Now, we shall use these Functions to generate a Table of F0- $\lambda$ T against  $\lambda$ T as shown below. Note that we have taken lower value of  $\lambda$ T as 20 micron.K for reasons stated in the program. In addition, we have compared the thus calculated values to the Table values given in Incropera.

Also, note that 1000 steps have been taken while implementing the Trapezoidal rule. Errors as compared to Table values in Incropera are within 1.08%.

	E7	$\bullet$ $f_x$ =	F_zero_lambdaT_Low_lamb	odaT_High_trapezoidal(\$C\$	7,D7,1000)	
	В	С	D	E	F	G
1						
2						
3		<b>Band Emission: Fract</b>	ion of radiation emitt	ed in a wave length b	and: F_0_Lamb	oda
4		Ref: Incropera				
5						
6		LambdaT_Low (micron.K)	LambdaT_High (micron.K)	F_zero_Lambda1_Lambda2	Incropera_Value	Error (%)
7		20	200	4.77406E-27		
8			400	2.02204E-12		
9			600	9.62015E-08		
10			800	1.67442E-05		
11			1000	0.000324456	0.000321	-1.076699448
12			1200	0.002150487		
13			1400	0.00783215		
14			1600	0.019796645		
15			1800	0.039459178		
16			2000	0.066884133		
17			2200	0.10107475		
18			2400	0.140465168		
19			2600	0.183343203		
20			2800	0.228120007		
21			2898	0.250338338	0.250108	-0.092095456
22			3000	0.273461769		

	G37	$ f_x$	=(F37-E37)*100/F37			
	В	С	D	E	F	G
35			5600	0.701155044		
36			5800	0.720259702		
37			6000	0.737912358	0.737818	-0.012788728
38			6200	0.754228527		
39			6400	0.769316337		
40			6600	0.783276281		
41			6800	0.796201259		
42			7000	0.808176805		
43			7200	0.819281432		
44			7400	0.829587045		
45			7600	0.839159389		
46			7800	0.848058508		
47			8000	0.856339194		
48			8500	0.874652534		
49			9000	0.890069405		
50			9500	0.903121203		
51			10000	0.914231511	0.914199	-0.003556189

\_\_\_\_\_

\_\_\_\_\_

	E69	$\bullet$ ( $\circ$ $f_x$ =	F_zero_lambdaT_Low_lamb	odaT_High_trapezoidal(\$C\$	7,D69,1000)	
	В	С	D	E	F	G
62			19000	0.983468974		
63			20000	0.985616976	0.985602	-0.00151948
64			25000	0.992228641		
65			30000	0.995353822		
66			40000	0.997980671		
67			50000	0.998966543		
68			75000	0.999725719	0.999713	-0.001272301
69			100000	0.999917921	0.999905	-0.001292269



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#### Now, plot these results in EXCEL:



**Prob.5A.15.** Spectral hemispherical emissivity (monochromatic emissivity) of fire brick at 750 K as a function of wave length is as follows:

 $\epsilon 1 = 0.1$  for  $\lambda = 0$  to 2 µm,  $\epsilon 2 = 0.6$  for  $\lambda = 2$  to 14 µm, and  $\epsilon 3 = 0.3$  for  $\lambda = 14$  to  $\infty$ . Calculate total hemispherical emissivity  $\epsilon$ , over all wave lengths. [VTU – July–Aug. 2004]

Also, plot the variation of total emissivity  $\epsilon$  and total emissive power E as Temp T varies from 500 K to 2000 K:

Note: This is the same as Prob.5A.9 which was solved with Mathcad.

#### **EXCEL Solution:**

Let us use the VBA Function written above for F0- $\lambda$  while solving this problem.

1. Set up the EXCEL worksheet, enter data and name the cells, as shown below:

	lambda2 $\bullet$ (* $f_{x}$ 14								
	А	В	С	D	E				
96									
97									
98			Prob.5A.15.						
99									
100			Data:						
101			Т	750	К				
102			epsilon1	0.1	emissivity (0 - 2µ)				
103			epsilon2	0.6	emissivity (2 - 14µ)				
104			epsilon3	0.3	emissivity (14µ - infinity)				
105			sigma	5.67E-08	W/m^2.K^4				
106			lambda1	2	microns				
107			lambda2	14	microns				
108			lambda3	1.00E+10	microns				

2. Do the calculations:

F	F_0_lambdaT_1 🔹 🕤 🏂 =F_zero_lambdaT_Low_lambdaT_High_trapezoidal(0,lambdaT_1,1000)									
	А	В	С	D	E	F				
109										
110			Calculations:							
111										
112			lambdaT_1	1500	micron.K	alving lomb do and T				
113			lambdaT_2	10500	micron.K	Jiying lambua anu T				
114			lambdaT_3	7.5E+12	micron.K					
115					-					
116			Now, get F_0_lambdaT usi	ng the VBA Function:						
117										
118			F_0_lambdaT_1	0.012849488	Using VBA Fun	ctions for				
119			F_0_lambdaT_2	0.923721652	F_0_lambdaT					
120			F_0_lambdaT_3	0.999917917						
121										

Note: In the above screen shot, see the VBA Function invoked in cell D118, in the Formula bar.

#### 3. Continuing with the calculation:

	epsilo	n •	• (* fx =epsilon1*i	_0_lambdaT_1+epsilon2*(F_0_lambdaT_2-F_0_lambdaT_1)+epsilon3*(F_0_lambdaT_3-F_0_lambdaT_2)					
	А	В	С	D	E	F	G	Н	
121									
122			Therefore:	8	$\epsilon(T) := \epsilon 1 \cdot F_{0,\lambda 1}(T) + \epsilon 2 \cdot (F_{0,\lambda 1}(T))$	$\sum_{\lambda 2} (T) - F_{0\lambda 1}(T) + \epsilon 3 \cdot (1)$	$\mathbf{F}_{0,\lambda 3} - \mathbf{F}_{0,\lambda 2}(\mathbf{T})$		
123			Total hemisph. Emissivity:	1			total	hemisph. emissivity	
124			epsilon	0.570667127	Ans. E(T) :=	= ε(T)·σ·T <sup>4</sup> W/m <sup>2</sup>	total emissive p	ower	
125					7				
126			Total Emissive Power: E	10237.902	W/m^2 Ans.				

\_\_\_\_\_

#### Thus:

Total hemisph. Emissivity =  $e = 0.571 \dots Ans$ .

Total Emissive Power =  $E = 10237.9 \text{ W} / \text{m}^2 \dots \text{Ans.}$ 

## **XX RBS** Group

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### To plot the variation of total emissivity $\varepsilon$ and total emissive power E as Temp T varies from 500 K to 2000 K:

1. Set up a Table as shown, and enter the first row below the column titles, i.e. row 133, using the Formulas already written. However, take care to enter T with *relative reference* in the formulas, so that we can drag-copy the first row to fill up the entire Table:

G133		33	f_x =F_zero_lambdaT_Low_lambdaT_High_trapezoidal(0,D133,1000)								
	А	В	С	D	E	F	G	Н	1	J	K
132			T (K)	lambdaT_1(micron.K)	lambdaT_2(micron.K)	lambdaT_3(micron.K)	F_0_lambdaT_1	F_0_lambdaT_2	F_0_lambdaT_3	epsilon	E (W/m^2)
133			500	1000	7000	5E+12	0.000320733	0.80811908	0.999917917	0.54225	1921.601034
134			550					•			
135			600								
136			650								
137			700								
138			750								
139			800								
140			850								
141			900								
142			950								
143			1000								
144			1050								
145			1100								
146			1150								
147			1200								
148			1250								
149			1300								
150			1350								
151			1400								
152			1450								
153			1500								
154			1550								
155			1600								
156			1650								
157			1700								
158			1750								
159			1800								
160			1850								
161			1900								
162			1950								
163			2000								

**Note** how the VBA Function is entered in cell G133, in the Formula bar. Similarly, for all other columns, use relative references to appropriate quantities.

 Now, select row 133 i.e. from cell D133 to cell K133, and drag-copy to the end of the Table, i.e. up to cell K163. Immediately, all calculations are made and the Table is filled up:

	В	С	D	E	F	G	н	I.	J	К
132		Т (К)	lambdaT_1(micron.K)	lambdaT_2(micron.K)	lambdaT_3(micron.K)	F_0_lambdaT_1	F_0_lambdaT_2	F_0_lambdaT_3	epsilon	E (W/m^2)
133		500	1000	7000	5E+12	0.000320733	0.80811908	0.999917917	0.542	1921.601
134		550	1100	7700	5.5E+12	0.000911145	0.843645005	0.999917917	0.553	2867.181
135		600	1200	8400	6E+12	0.002134036	0.871206025	0.999917917	0.560	4117.044
136		650	1300	9100	6.5E+12	0.004316168	0.892826594	0.999917917	0.566	5725.270
137		700	1400	9800	7E+12	0.007789953	0.909975128	0.999917917	0.569	7747.171
138		750	1500	10500	7.5E+12	0.012849488	0.923721652	0.999917917	0.571	10237.902
139		800	1600	11200	8E+12	0.019718434	0.934852175	0.999917917	0.571	13251.142
140		850	1700	11900	8.5E+12	0.028533364	0.94394967	0.999917917	0.569	16837.973
141		900	1800	12600	9E+12	0.039341265	0.951450973	0.999917917	0.566	21046.021
142		950	1900	13300	9.5E+12	0.052107386	0.957686825	0.999917917	0.561	25918.902
143		1000	2000	14000	1E+13	0.066729199	0.962910186	0.999917917	0.555	31495.933
144		1050	2100	14700	1.05E+13	0.083052858	0.967316428	0.999917917	0.549	37812.099
145		1100	2200	15400	1.1E+13	0.10088958	0.971057862	0.999917917	0.541	44898.205
146		1150	2300	16100	1.15E+13	0.120030358	0.974254285	0.999917917	0.532	52781.176
147		1200	2400	16800	1.2E+13	0.140258237	0.977000715	0.999917917	0.523	61484.448
148		1250	2500	17500	1.25E+13	0.161357934	0.979373116	0.999917917	0.513	71028.425
149		1300	2600	18200	1.3E+13	0.18312292	0.981432674	0.999917917	0.503	81430.949
150		1350	2700	18900	1.35E+13	0.20536028	0.983229021	0.999917917	0.492	92707.775
151		1400	2800	19600	1.4E+13	0.227893746	0.984802687	0.999917917	0.481	104873.028
152		1450	2900	20300	1.45E+13	0.250565263	0.986186975	0.999917917	0.471	117939.633
153		1500	3000	21000	1.5E+13	0.273235479	0.987409416	0.999917917	0.460	131919.699
154		1550	3100	21700	1.55E+13	0.295783431	0.988492894	0.999917917	0.449	146824.876
155		1600	3200	22400	1.6E+13	0.318105697	0.989456534	0.999917917	0.438	162666.663
156		1650	3300	23100	1.65E+13	0.340115205	0.990316401	0.999917917	0.427	179456.677
157		1700	3400	23800	1.7E+13	0.361739836	0.99108605	0.999917917	0.416	197206.888
158		1750	3500	24500	1.75E+13	0.382920952	0.991776974	0.999917917	0.406	215929.819
159		1800	3600	25200	1.8E+13	0.40361191	0.992398958	0.999917917	0.396	235638.708
160		1850	3700	25900	1.85E+13	0.423776624	0.992960368	0.999917917	0.386	256347.652
161		1900	3800	26600	1.9E+13	0.443388211	0.993468383	0.999917917	0.376	278071.719
162		1950	3900	27300	1.95E+13	0.462427733	0.993929189	0.999917917	0.367	300827.043
163		2000	4000	28000	2E+13	0.480883053	0.994348131	0.999917917	0.358	324630.895

3. Now, plot the results in EXCEL:





Note that results obtained with EXCEL match very well with those obtained with Mathcad, as they should.

\_\_\_\_\_

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## 5B View factors (or, Shape factor, configuration factor) and Radiation energy exchange between Black bodies:

#### **Recollect:**

**Definition of view factor:** it is the fraction of energy emitted by surface 1, which is intercepted directly by surface 2.

**Note:** Often, while solving radiation problems, determination of the view factor is the most difficult part. It will be *useful to keep in mind* the definition of view factor, summation rule, reciprocity relation, superposition rule and symmetry rule while attempting to find out the view factors.



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#### Radiation energy exchange between black bodies:

Analysis of heat exchange between two black bodies is simpler since a black body absorbs all the radiation impinging on it and none is reflected.

Consider two black surfaces  $A_1$  and  $A_2$  at temperatures T1 and T2, exchanging radiation energy with each other.

Then:

 $Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) = A_2 \cdot F_{21} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)$  W....(13.40)

Prob.5B.1 Draw the plots for View factors of:

- 1) Aligned, parallel rectangles of equal size
- 2) Coaxial, parallel disks, and
- 3) Perpendicular rectangles with a common edge.

View factor relationships for a few important shapes, given at the beginning of this chapter, are reproduced below. (Ref: Cengel).

Now, we shall write Mathcad Functions for View factors, so that we need not refer to the graphs while doing calculations.

We will also use these Functions to draw the graphs.

-----

Geometry	Relation				
Aligned parallel rectangles	$\overline{X} = X/L$ and $\overline{Y} = Y/L$				
	$\begin{split} F_{i \to j} &= \frac{2}{\pi \overline{X} \overline{Y}} \left\{ \ln \left[ \frac{(1 + \overline{X}^2)(1 + \overline{Y}^2)}{1 + \overline{X}^2 + \overline{Y}^2} \right]^{1/2} \\ &+ \overline{X} (1 + \overline{Y}^2)^{1/2} \tan^{-1} \frac{\overline{X}}{(1 + \overline{Y}^2)^{1/2}} \\ &+ \overline{Y} (1 + \overline{X}^2)^{1/2} \tan^{-1} \frac{\overline{Y}}{(1 + \overline{X}^2)^{1/2}} \\ &- \overline{X} \tan^{-1} \overline{X} - \overline{Y} \tan^{-1} \overline{Y} \right\} \end{split}$				
Coaxial parallel disks					
	$\begin{aligned} R_i &= r_i / L \text{ and } R_j = r_j / L \\ S &= 1 + \frac{1 + R_j^2}{R_i^2} \\ F_{i \rightarrow j} &= \frac{1}{2} \left\{ S - \left[ S^2 - 4 \left( \frac{r_j}{r_i} \right)^2 \right]^{1/2} \right\} \end{aligned}$				





1. View factors for aligned, parallel rectangles: (See the fig. above for nomenclature)

We define:

$$XX = \frac{X}{L} \qquad YY = \frac{Y}{L}$$

$$A(XX, YY) := \frac{2}{\pi \cdot XX \cdot YY} \qquad B(XX, YY) := \ln \left[ \sqrt{\frac{(1 + XX^2) \cdot (1 + YY^2)}{1 + XX^2 + YY^2}} \right]$$

View factors (or, Shape factor, configuration factor) and Radiation energy exchange between Black bodies:

Software Solutions to Problems on Heat Transfer: Radiation Heat Transfer – Part I

$$C(XX, YY) := XX \cdot \left(1 + YY^{2}\right)^{\frac{1}{2}} \cdot \operatorname{atan}\left[\frac{XX}{\left(1 + YY^{2}\right)^{\frac{1}{2}}}\right]$$

$$D(XX, YY) := YY \cdot (1 + XX^2)^{\frac{1}{2}} \cdot \operatorname{atan}\left[\frac{YY}{\left(1 + XX^2\right)^{\frac{1}{2}}}\right]$$

 $E(XX) := XX \cdot atan(XX)$ 

 $F(YY) := YY \cdot atan(YY)$ 

 $F_{ij}(XX, YY) := A(XX, YY) \cdot (B(XX, YY) + C(XX, YY) + D(XX, YY) - E(XX) - F(YY))$ 

....View factor for aligned parallel rectangles



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\* Figures taken from London Business School's Masters in Management 2010 employment report



#### Now, to plot the graphs:

```
XX := 0.1,0.2..30 ....define a range variable:
```



We write the Mathcad Function for F12 for aligned, parallel rectangles as follows:

Here,

Inputs are: X, Y and L.

Output is: F12.

Software Solutions to Problems on Heat Transfer: Radiation Heat Transfer – Part I

$$\begin{split} \text{F12\_parallel\_rectangles}(X, Y, L) \coloneqq & XX \leftarrow \frac{X}{L} \\ & YY \leftarrow \frac{Y}{L} \\ & A \leftarrow \frac{2}{\pi \cdot XX \cdot YY} \\ & B \leftarrow \ln \Biggl[ \sqrt{\frac{\left(1 + XX^2\right) \cdot \left(1 + YY^2\right)}{1 + XX^2 + YY^2}} \Biggr] \\ & C \leftarrow XX \cdot \left(1 + YY^2\right)^{\frac{1}{2}} \cdot \operatorname{atan} \Biggl[ \frac{XX}{\frac{1}{\left(1 + YY^2\right)^2}} \Biggr] \\ & D \leftarrow YY \cdot \left(1 + XX^2\right)^{\frac{1}{2}} \cdot \operatorname{atan} \Biggl[ \frac{YY}{\frac{1}{\left(1 + XX^2\right)^{\frac{1}{2}}}} \Biggr] \\ & E \leftarrow XX \cdot \operatorname{atan}(XX) \\ & F \leftarrow YY \cdot \operatorname{atan}(YY) \\ & F_{12} \leftarrow A \cdot (B + C + D - E - F) \end{split}$$

Ex: F12\_parallel\_rectangles(0.5, 1, 1) = 0.117

#### 2. View factors for coaxial, parallel disks: (See the fig. above for nomenclature)

Define:

$$\begin{split} \mathtt{R}_{i} &= \frac{\mathtt{r}_{i}}{\mathtt{L}} \qquad \mathtt{R}_{j} = \frac{\mathtt{r}_{j}}{\mathtt{L}} \qquad \mathtt{S}(\mathtt{R}_{i}, \mathtt{R}_{j}) \coloneqq \mathtt{1} + \frac{\mathtt{1} + \mathtt{R}_{j}^{-2}}{\mathtt{R}_{i}^{-2}} \\ \mathtt{F}_{ij}(\mathtt{R}_{i}, \mathtt{R}_{j}) &\coloneqq \frac{\mathtt{1}}{2} \cdot \left[ \mathtt{S}(\mathtt{R}_{i}, \mathtt{R}_{j}) - \left[ \mathtt{S}(\mathtt{R}_{i}, \mathtt{R}_{j})^{2} - \mathtt{4} \cdot \left( \frac{\mathtt{R}_{j}}{\mathtt{R}_{i}} \right)^{2} \right]^{\frac{1}{2}} \right] \qquad \dots \texttt{View factor for coaxial parallel disks} \end{split}$$

#### Now, to plot the graphs:





We write the Mathcad Function for F12 for coaxial, parallel disks as follows:

Here,

Inputs are:  $r_i$ ,  $r_i$  and L.

Output is: F12.

Software Solutions to Problems on Heat Transfer: Radiation Heat Transfer – Part I View factors (or, Shape factor, configuration factor) and Radiation energy exchange between Black bodies:

$$\begin{split} \text{F12\_paralle1\_disks} \big( r_i, r_j, L \big) &\coloneqq & \mathsf{R}_i \leftarrow \frac{r_i}{L} \\ & \mathsf{R}_j \leftarrow \frac{r_j}{L} \\ & \mathsf{S} \leftarrow 1 + \frac{1 + \mathsf{R}_j^2}{\mathsf{R}_i^2} \\ & \mathsf{F12} \leftarrow \frac{1}{2} \cdot \left[ \mathsf{S} - \left[ \mathsf{S}^2 - 4 \cdot \left( \frac{r_j}{r_i} \right)^2 \right]^2 \right] \end{split}$$

Ex: F12\_parallel\_disks(0.5, 0.6, 1) = 0.232



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3. View factors for perpendicular rectangles with a common edge: (See the fig. above for nomenclature)

Define:

$$\begin{split} & \mathsf{H} = \frac{\mathsf{Z}}{\mathsf{X}} \quad \mathsf{W} = \frac{\mathsf{Y}}{\mathsf{X}} \qquad \mathsf{A}(\mathsf{W}) \coloneqq \frac{1}{\pi \cdot \mathsf{W}} \qquad \mathsf{B}(\mathsf{W}) \coloneqq \mathsf{W} \cdot \mathsf{atan} \bigg( \frac{1}{\mathsf{W}} \bigg) \\ & \mathsf{C}(\mathsf{H}) \coloneqq \mathsf{H} \cdot \mathsf{atan} \bigg( \frac{1}{\mathsf{H}} \bigg) \qquad \mathsf{D}(\mathsf{H}, \mathsf{W}) \coloneqq \left( \mathsf{H}^2 + \mathsf{W}^2 \right)^{\frac{1}{2}} \cdot \mathsf{atan} \bigg[ \frac{1}{\left( \mathsf{H}^2 + \mathsf{W}^2 \right)^2} \bigg] \\ & \mathsf{E}(\mathsf{H}, \mathsf{W}) \coloneqq \frac{\left( 1 + \mathsf{W}^2 \right) \cdot \left( 1 + \mathsf{H}^2 \right)}{\left( 1 + \mathsf{W}^2 + \mathsf{H}^2 \right)} \cdot \bigg[ \frac{\mathsf{W}^2 \cdot \left( 1 + \mathsf{W}^2 + \mathsf{H}^2 \right)}{\left( 1 + \mathsf{W}^2 + \mathsf{H}^2 \right)} \bigg]^{\mathsf{W}^2} \cdot \bigg[ \frac{\mathsf{H}^2 \cdot \left( 1 + \mathsf{H}^2 + \mathsf{W}^2 \right)}{\left( 1 + \mathsf{H}^2 + \mathsf{W}^2 \right)} \bigg]^{\mathsf{H}^2} \\ & \mathsf{F}_{ij}(\mathsf{H}, \mathsf{W}) \coloneqq \mathsf{A}(\mathsf{W}) \cdot \bigg( \mathsf{B}(\mathsf{W}) + \mathsf{C}(\mathsf{H}) - \mathsf{D}(\mathsf{H}, \mathsf{W}) + \frac{1}{4} \cdot \mathsf{ln}(\mathsf{E}(\mathsf{H}, \mathsf{W})) \bigg) \end{split}$$

...View factor for coaxial perpendicular rectangles with a comon edge

#### Now, to plot the graphs:





We write the Mathcad Function for F12 for perpendicular rectangles with a common edge as follows:

\_\_\_\_\_

Here,

Inputs are: X, Y and Z.

\_\_\_\_\_

Output is: F12.

Software Solutions to Problems on Heat Transfer: Radiation Heat Transfer – Part I

$$\begin{split} \text{F12\_perpendicular\_rectangles} \left( X,Y,Z \right) &\coloneqq \quad \left| H \leftarrow \frac{Z}{X} \\ W \leftarrow \frac{Y}{X} \\ A \leftarrow \frac{1}{\pi \cdot W} \\ B \leftarrow W \cdot \text{atan} \left( \frac{1}{W} \right) \\ C \leftarrow H \cdot \text{atan} \left( \frac{1}{H} \right) \\ D \leftarrow \left( H^2 + W^2 \right)^{\frac{1}{2}} \cdot \text{atan} \left[ \frac{1}{\left( H^2 + W^2 \right)^2} \right] \\ E \leftarrow \frac{\left( 1 + W^2 \right) \cdot \left( 1 + H^2 \right)}{\left( 1 + W^2 + H^2 \right)} \cdot \left[ \frac{W^2 \cdot \left( 1 + W^2 + H^2 \right)}{\left( 1 + W^2 \right) \cdot \left( W^2 + H^2 \right)} \right]^{W^2} \cdot \left[ \frac{H^2 \cdot \left( 1 + H^2 + W^2 \right)}{\left( 1 + H^2 \right) \cdot \left( H^2 + W^2 \right)} \right]^{H^2} \\ \text{F12} \leftarrow A \cdot \left( B + C - D + \frac{1}{4} \cdot \ln(E) \right) \end{split}$$

Ex: F12\_perpendicular\_rectangles(5,5,5) = 0.2

\_\_\_\_\_\_

Prob.5B.2. Write Mathcad Functions for view factors of following 2\_D geometries:

- 1) Parallel plates with mid-lines connected by perpendicular line
- 2) Inclined plates of equal width and with a common edge
- 3) Perpendicular plates with a common edge
- 4) Three sided enclosure
- 5) Infinite plane and row of cylinders

Geometries and view factor relations, given at the beginning of the chapter, are reproduced below: (Ref: Cengel)


## View factors for few two-dimensional geometries, infinitely long:





Software Solutions to Problems on Heat Transfer: Radiation Heat Transfer – Part I

## Mathcad Functions:

## (a) For parallel plates with mid-lines connected by perpendicular:

$$\begin{split} F_{ij\_parallel\_plates}(w_i, w_j, L) &\coloneqq & W_i \leftarrow \frac{w_i}{L} \\ & W_j \leftarrow \frac{w_j}{L} \\ & F_{ij} \leftarrow \frac{\left[ \left( W_i + W_j \right)^2 + 4 \right]^2 - \left[ \left( W_j - W_i \right)^2 + 4 \right]^2}{2 \cdot W_i} \end{split}$$

Example:

 $F_{ij\_parallel\_plates}(0.25, 0.5, 0.15) = 0.835$ 



\_\_\_\_\_



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## (b) For inclined parallel plates of equal width and with a common edge:

$$F_{ij\_inclined\_plates}(\alpha) := 1 - sin\left(\frac{\alpha \cdot \pi}{180}\right)$$

Note: a in degrees

Example: If the angle between plates is 30 deg.:

Fij\_inclined\_plates(30) = 0.5

(c) For perpendicular plates with a common edge:

$$F_{ij\_perpendicular\_plates}(w_i, w_j) := \frac{1}{2} \cdot \left[ 1 + \frac{w_j}{w_i} - \left[ 1 + \left( \frac{w_j}{w_i} \right)^2 \right]^{\frac{1}{2}} \right]$$

-

Example: when  $w_i = 0.5 \text{ m}$ ,  $w_i = 0.5 \text{ m}$ :

$$F_{ij}$$
 perpendicular plates(0.5, 0.5) = 0.293

(d) For three sided enclosure:

$$F_{ij\_three\_sided\_enclosure}(w_i, w_j, w_k) := \frac{w_i + w_j - w_k}{2 \cdot w_i}$$

Example:

$$F_{ij}$$
 three sided enclosure(0.7, 0.7, 0.5) = 0.643

(e) For infinite plane and row of cylinders:

$$F_{ij\_infinite\_plane\_to\_row\_of\_cyl}(D,s) \coloneqq 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^2 + \frac{D}{s} \cdot atan \left(\frac{s^2 - D^2}{D^2}\right)^2$$

Example: If D = 1 cm, s = 5 cm:

 $F_{ij\_infinite\_plane\_to\_row\_of\_cyl}(0.01, 0.05) = 0.268$ 

**Prob.5B.2.** Find out the net heat transferred between two circular disks 1 and 2, oriented one above the other, parallel to each other on the same centre line, as shown in Fig. Disk 1 has a radius of 0.5 m and is maintained at 1000 K, and disk 2 has a radius of 0.6 m and is maintained at 600 K. Assume both the disks to be black surfaces.



2

Fig. Prob.5B.2

## Mathcad Solution:

Data:

$$\label{eq:ri} \begin{array}{ll} r_i \coloneqq 0.5 & \mbox{m...radius of disk 1} \\ r_j \coloneqq 0.6 & \mbox{m...radius of disk 2} \\ \mbox{L} \coloneqq 1 & \mbox{m...distance between disk 1 and disk} \\ \mbox{T}_1 \coloneqq 1000 & \mbox{K...temp. of disk 1} \\ \mbox{T}_2 \coloneqq 600 & \mbox{K...temp. of disk 2} \end{array}$$

$$\begin{split} \sigma &:= 5.67 \cdot 10^{-8} \quad \text{W/m}^2.\text{K....Stefan-Boltzmann const.} \\ \text{A}_1 &:= \pi \cdot \mathbf{r_i}^2 \quad \text{i.e.} \quad \text{A}_1 = 0.785 \quad \text{m}^2....\text{area of disk 1} \\ \text{A}_2 &:= \pi \cdot \mathbf{r_j}^2 \quad \text{i.e.} \quad \text{A}_2 = 1.131 \quad \text{m}^2....\text{area of disk 2} \end{split}$$

This is the case of heat transfer between two black surfaces. So, we use eqn. (13.40):

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot (T_1^4 - T_2^4) = A_2 \cdot F_{21} \cdot \sigma \cdot (T_1^4 - T_2^4)$$
 W....(13.40)

So, the problem reduces to calculating the view factor  $F_{12}$  or  $F_{21}$ .

We can easily find out F12 using the graphs given earlier. However, we can determine F12 analytically *more accurately* with Mathcad Functions written above.

## We use the Mathcad Function already written above:

F12\_paratle1\_disks $(r_i, r_i, L) = 0.232$  ...View factor for coaxial parallel disks

## (Note: verify this result by referring to the graph.)

i.e. F<sub>12</sub> := 0.232

Therefore, net heat transfer between disks 1 and 2:

$$Q_{net} := A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)$$
 W...from eqn. (13.40)  
i.e.  $Q_{net} = 8.992 \times 10^3$  W....Ans.

**Prob.5B.3.** Find out the net heat transferred between two aligned parallel rectangles, as shown in Fig. Surface 1 is maintained at 500 K, and surface 2 is maintained at 1000 K. Assume both the surfaces to be black surfaces.



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Fig.Prob.5B.3

## Mathcad Solution:

Data:

X := 0.5 m....horizontal x-dimension of surfaces 1 and 2

Y := 1.0 m....horizontal y-dimension of surfaces 1 and 2

L := 1.0 ....vertical distane between surfaces 1 and 2

T1 := 500 K....temp. of surface 1

T<sub>2</sub> := 1000 K...temp. of surface 2

σ := 5.67·10<sup>-8</sup> W/m<sup>2</sup>.K....Stefan-Boltzmann const.

Calculations:

$A_1 := X \cdot Y$	i.e.	$A_1 = 0.5$	m <sup>2</sup> area of surface 1
$A_2 := X \cdot Y$	i.e.	A <sub>2</sub> = 0.5	m <sup>2</sup> area of surface 2

This is the case of heat transfer between two black surfaces. So, we use eqn. (13.40), viz.

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) = A_2 \cdot F_{21} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)$$
 W....(13.40)

So, the problem reduces to calculating the view factor  $F_{12}$  or  $F_{21}$ . We can find out  $F_{12}$  using the graphs. However, we can determine  $F_{12}$  analytically *more accurately* with Mathcad Function using the view factor relation given in Table for aligned parallel rectangles. We use the the Mathcad Function already written above for view factor as follows:

 $F12\_parallel\_rectangles(X, Y, L) = 0.117$ 

(Note: verify this result from the graph.)

i.e. F<sub>12</sub> := 0.117

i.e.

than surface 1.

Therefore, net heat transfer between surfaces 1 and 2:

$$Q_{net} := A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)$$
 W...from eqn. (13.40)  
 $Q_{net} = -3.11 \times 10^3$  W....Ans.

Note: -ve sign indicates that net heat transfer is to the surface 1, since surface 2 is at a higher temperature

**Prob 5B** 4 Find out the net heat transferred between the areas A1 and A2 shown in Fig. Area 1

**Prob.5B.4.** Find out the net heat transferred between the areas A1 and A2 shown in Fig. Area 1 is maintained at 900 K, and area 2 is maintained at 400 K. Assume both the surfaces to be black.



Fig.Prob.5B.4

## **Mathcad Solution:**

This is the case of heat transfer between two black surfaces. So, we use eqn. (13.40 ), viz.

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) = A_2 \cdot F_{21} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) \qquad W....(13.40)$$

So, the problem reduces to calculating the view factor  $F_{12}$  or  $F_{21}$ . We see that to calculate  $F_{12}$  for areas  $A_1$  and  $A_2$  as oriented in the Fig., we do not readily have an analytical relation or a graph.

Let us denote the combined areas  $(A_1 + A_3)$  by  $A_5$  and  $(A_2 + A_4)$  by  $A_8$ . Then, we see that  $A_5$  and  $A_8$  are *perpendicular rectangles which have a common edge*, and we have graphs or analytical relation for the view factor for such an orientation. Then, we resort to **view factor algebra**, as follows:

Remember that by definition, view factor  $F_{12}$  is the fraction of radiant energy emitted by surface 1 which falls directly on surface 2. Looking at the fig., we can say that fraction of energy leaving  $A_1$  and falling on  $A_2$  is equal to the fraction falling on  $A_6$  minus the fraction falling on  $A_4$ .

- i.e. F<sub>12</sub> = F<sub>16</sub> F<sub>14</sub> ... by definition of view factor

i.e. 
$$F_{12} = \frac{A_6}{A_1} \cdot (F_{65} - F_{63}) - \frac{A_4}{A_1} \cdot (F_{45} - F_{43})$$

eqn.(A)....using the definition of view factor, as done in first step above



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Now, observe that view factors F<sub>65</sub>, F<sub>63</sub>, F<sub>45</sub> and F<sub>43</sub> refer to perpendicular rectangles with a comon edge, and can be readily obtained from the graphs, or by analytical relation given in Table.

We have already written Mathcad Function for view factor relation for perpendicular rectangles with a common edge (See Prob.5B.1).

To find F<sub>85</sub>:

X := 5 Y := 5 Z := 5 ....See. Fig.

F12\_perpendicular\_rectangles(X,Y,Z) = 0.2 ....From the Mathcad Function

i.e. F<sub>65</sub> := 0.2 ...view factor from area A<sub>8</sub> to A<sub>5</sub>

Note: This value can be verified from graphs also.

To find F<sub>63</sub>:

 $X:=5 \quad Y:=5 \qquad Z:=3 \quad \dots \text{See. Fig.}$ 

F12\_perpendicular\_rectangles(X,Y,Z) = 0.161 ....From the Mathcad Function

i.e. F<sub>63</sub> := 0.161 ...view factor from area A<sub>8</sub> to A<sub>3</sub>

Note: This value also can be verified from graphs.

To find F45:

X := 5 Y := 3 Z := 5 ...See Fig.

F12\_perpendicular\_rectangles(X,Y,Z) = 0.269 ....From the Mathcad Function

i.e. F45 = 0.269 ... view factor from area A4 to A5

Note: This value also can be verified fromgraphs.

To find F<sub>43</sub>:

X := 5 Y := 3 Z := 3 ...See Fig.

F12\_perpendicular\_rectangles(X,Y,Z) = 0.231 ....From the Mathcad Function

i.e.  $F_{43} := 0.231$  ...view factor from area  $A_4$  to  $A_3$ 

Note: This value also can be verified fromgraphs.

\_\_\_\_\_

## Areas:

From Fig. we have:

 $A_1 := 10 \qquad A_2 := 10 \qquad A_3 := 15 \qquad A_4 := 15 \qquad m^2$ 

A<sub>5</sub> := 25 A<sub>6</sub> := 25 m<sup>2</sup>

Then, from eqn.(A):

$$\mathbf{F}_{12} := \left[ \frac{\mathbf{A}_{6}}{\mathbf{A}_{1}} \cdot \left( \mathbf{F}_{65} - \mathbf{F}_{63} \right) - \frac{\mathbf{A}_{4}}{\mathbf{A}_{1}} \cdot \left( \mathbf{F}_{45} - \mathbf{F}_{43} \right) \right]$$

i.e.  $F_{12} = 0.041$  ....view factor from A<sub>1</sub> to A<sub>2</sub>....Ans.

Note:  $F_{21}$  can be calculated, if required, by reciprocity relation, i.e.  $A_1$ . $F_{12} = A_2$ . $F_{21}$ 

Therefore, net heat transfer between surfaces 1 and 2:

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot (T_1^4 - T_2^4)$$
 W...from eqn. (13.40)

Here, we have:

$$T_1 := 900 \qquad \text{K....temp. of surface } A_1$$
  

$$T_2 := 400 \qquad \text{K...temp. of surface } A_2$$
  

$$\sigma := 5.67 \cdot 10^{-8} \qquad \text{W/m}^2.\text{K....Stefan-Boltzmann const.}$$

Therefore,

$$\mathsf{Q}_{net} \coloneqq \mathsf{A}_1 \cdot \mathsf{F}_{12} \cdot \sigma \cdot \left(\mathsf{T}_1^{4} - \mathsf{T}_2^{4}\right)$$

i.e. 
$$Q_{net} = 1.448 \times 10^4$$
 W....Ans.

\_\_\_\_\_

Prob.5B.5. Find out the relevant view factors for the geometries shown in Fig. below:

- a) a long tube with cross-section of an equilateral triangle
- b) a blackbody completely enclosed by another black body
- c) diagonal partition inside a long square duct
- d) sphere of diameter d inside a cubical box of sides, L = d
- e) hemispherical surface closed by a plane surface, and
- f) the end and surface of a circular cylinder whose length is equal to diameter.

Software Solutions to Problems on Heat Transfer: Radiation Heat Transfer – Part I View factors (or, Shape factor, configuration factor) and Radiation energy exchange between Black bodies:



(a) Equilateral triangle

(b) A blackbody completely enclosed by another black body

1



(c) Diagonal partition in a long square duct



(d) Sphere inside a cubical box



2 f Cylinder with length

= diameter

Fig.Prob.5B.5

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**General principle in solving these problems is** to invoke: Summation rule, reciprocity theorem, inspection of geometry for symmetry, and of course, remembering the definition of view factor:

(a) Long tube wih cross-section of equilateral triangle: See Fig. (a)

For surface 1:  $F_{11} + F_{12} + F_{13} = 1$  ....Summation rule But,  $F_{11} = 0$  ...since surface 1 is flat and can not see itself. Therefore,  $F_{12} + F_{13} = 1$ 

Now, *by inspection of geometry*, we find that surfaces 2 and 3 are located *symmetrically w.r.t. surface 1*, since it is an equilateral triangle. Therefore, radiation from surface 1 is divided equally between surfaces 2 and 3.

Similarly, for surface 2, we write:

F<sub>21</sub> + F<sub>23</sub> = 1  
i.e. F<sub>23</sub> = 1 - F<sub>21</sub>  
But, F<sub>21</sub> = 
$$\frac{A_1}{A_2}$$
·F<sub>12</sub> = F<sub>12</sub> ...since A<sub>1</sub> = A<sub>2</sub>

Therefore,  $F_{23} = 1 - F_{12} = 0.5$ 

Similarly for surface 3.

## (b) Black body enclosed inside a black enlosure: See Fig. (b)

For surface 1:  $F_{11} + F_{12} = 1$  ...by Summation rule

and,  $A_1 \cdot F_{12} = A_2 \cdot F_{21}$  ... by reciprocity

i.e. 
$$F_{12} = \frac{A_2}{A_1} \cdot F_{21}$$

Now,  $F_{11} = 1 - F_{12}$ 

i.e. 
$$F_{11} = 1 - \frac{A_2}{A_1} \cdot F_{21}$$

# But, F<sub>21</sub> = 1 since all the energy radiated by surface 2 is directly intercepted by surface 1.

Therefore,  $F_{11} = 1 - \frac{A_2}{A_1}$ 

## (c) Diagonal partition within a long square duct: See Fig. (c)

For surface 1:  $F_{11} + F_{12} + F_{13} = 1$  ... by Summation rule

But,  $F_{11} = 0$  ...since surface 1 is flat and can not 'see' itself.

Therefore,  $F_{12} + F_{13} = 1$ 

By symmetry:  $F_3 = F_{12} = 0.5$  ...since radiation emitted by surface 1 is divided equally between surfaces 2 and 3

By reciprocity:  $F_{21} = \frac{A_1}{A_2} \cdot F_{12}$ 

i.e. 
$$F_{21} = \frac{\sqrt{2} \cdot L}{L} \cdot 0.5 = 0.71$$

## (d) Sphere inside a cubical box: See Fig. (d)

For surface 1:  $F_{11} = 0$  ...since surface of the sphere is convex and can not 'see' itself.

Therefore, F12=1

By reciprocity:  $F_{21} = \frac{A_1}{A_2} \cdot F_{12}$ 

i.e. 
$$F_{21} = \frac{\pi \cdot d^2}{6 \cdot d^2} = \frac{\pi}{6}$$
 ...since L = d

## (e) Hemispherical surface closed by a flat surface: See Fig. (e)

For surface 1: F<sub>11</sub> + F<sub>12</sub> = 1 .....Summation rule

Also, F<sub>21</sub> = 1 ...since surface 2 is flat and can not 'see' itself, and all radiation emitted by surface 2 falls directly on the hemispherical surface 1.

i.e. 
$$F_{12} = \frac{\pi \cdot r^2}{2 \cdot \pi \cdot r^2} \cdot 1 = 0.5$$

 $\mathbf{F}_{12} = \frac{\mathbf{A}_2}{\mathbf{A}_1} \cdot \mathbf{F}_{21}$ 

Therefore,  $F_{11} = 1 - F_{12} = 0.5$ 

## (f) End and sides of a circular cylinder (L = d): See Fig. (f)

From the fig., note that the two end surfaces are denoted by 1 and 3 and the side surface is denoted by 2.



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<u>View factor F<sub>13</sub></u>: Surfaces 1 and 3 can be considered as two concentric parallel disks. Therefore, F<sub>13</sub> can be found out from graphs or by analytical relation given in Table. Let us use the analytical relation:

We have:

$$R_{i} = \frac{r_{i}}{L} \qquad R_{j} = \frac{r_{j}}{L} \qquad S(R_{i}, R_{j}) := 1 + \frac{1 + R_{j}^{2}}{R_{i}^{2}}$$
$$F_{ij}(R_{i}, R_{j}) := \frac{1}{2} \cdot \left[ S(R_{i}, R_{j}) - \left[ S(R_{i}, R_{j})^{2} - 4 \cdot \left(\frac{R_{j}}{R_{i}}\right)^{2} \right]^{\frac{1}{2}} \right]$$

...View factor for coaxial parallel disks

Now, for a cylinder with L = d:

$$R_i := 0.5$$
  $R_i := 0.5$ 

Therefore,  $F_{ij}(R_i, R_j) = 0.172$ 

i.e. F<sub>13</sub> := 0.172 ...view factor from surface 1 to surface 3

For surface 1:  $F_{11} + F_{12} + F_{13} = 1$  ... by Summation rule

But, F<sub>11</sub> = 0 ...since surface 1 is flat and can not 'see' itself.

Therefore,  $F_{12} = 1 - F_{13}$ 

By reciprocity:  $F_{21} = \frac{A_1}{A_2} \cdot F_{12}$ 

i.e. 
$$F_{21} = \frac{\frac{\pi \cdot d^2}{4}}{\pi \cdot d \cdot L} \cdot 0.828$$

i.e. 
$$F_{21} = \frac{\frac{\pi \cdot d^2}{4}}{\pi \cdot d^2} \cdot 0.828$$
 ...since L = d

i.e. 
$$F_{21} = \frac{0.828}{4} = 0.207$$

Software Solutions to Problems on Heat Transfer: Radiation Heat Transfer – Part I View factors (or, Shape factor, configuration factor) and Radiation energy exchange between Black bodies:

```
Also, by symmetry: F_{32} = F_{12} = 0.828
and, F_{23} = F_{21} = 0.207
```

**Prob.5B.6.** Find out the view factor  $(F_{11})$  of a cavity with respect to itself. Hence, find out the view factor  $F_{11}$  for the following:

\_\_\_\_\_\_

- a) a cylindrical cavity of diameter 'd' and depth 'h'
- b) a conical cavity of diameter 'd' and depth 'h'
- c) a hemispherical bowl of diameter 'd'



(a) Cavity (1) closed by a (b) Cylindrical cavity hypothetical flat surface (2)



Mathcad Solution:

View factor of a general cavity w.r.t. itself: See Fig.(a) above.

We desire to find  $F_{11}$ . It is obvious from the fig. that part of the radiation emitted by the cavity surface 1, falls on itself and therefore,  $F_{11}$  exists.

Close the opening (or mouth) of the cavity by a hypothetical flat surface 2. Then, surfaces 1 and 2 together form an enclosure. We can write:

For surface 1: F<sub>11</sub> + F<sub>12</sub> = 1 ...(a)....by Summation rule

For surface 2:  $F_{21} + F_{22} = 1$  ....by Summation rule

But,  $F_{22} = 0$  ...since surface 2 is flat and can not 'see' itself.

Therefore, F<sub>21</sub> = 1

Further,  $A_1 \cdot F_{12} = A_2 \cdot F_{21}$  ....by reciprocity

i.e. 
$$F_{12} = \frac{A_2}{A_1}$$

Now, F<sub>11</sub> = 1 - F<sub>12</sub> ... from eqn.(a)

i.e. 
$$F_{11} = 1 - \frac{A_2}{A_1}$$
 .....eqn.(b)





Eqn.(b) is an important result, since it gives the shape factor of any general cavity w.r.t. itself.

## Now, this result will be applied to following specific cavities:

(a) F11 for a cylindrical cavity of diameter 'd' and depth 'h': See Fig.(b)

We have: 
$$F_{11} = 1 - \frac{A_2}{A_1}$$

i.e. 
$$F_{11} = 1 - \frac{\frac{\pi \cdot d^2}{4}}{\frac{\pi \cdot d^2}{4} + \pi \cdot d \cdot h}$$
...Note that A<sub>1</sub> consists of the area of bottom

i.e. 
$$F_{11} = \frac{4 \cdot h}{4 \cdot h + d}$$
 ....Ans.

(b) F<sub>11</sub> for a conical cavity of diameter 'd' and depth 'h': See Fig. (c)

We have: 
$$F_{11} = 1 - \frac{A_2}{A_1}$$
  
i.e.  $F_{11} = 1 - \frac{\frac{\pi \cdot d^2}{4}}{\frac{\pi \cdot d \cdot L}{2}}$  ...where L is the slant height of the cone

i.e.  $F_{11} = 1 - \frac{d}{2 \cdot L}$ 

i.e.  $F_{11} = 1 - sin(\alpha)$  ...where  $\alpha$  is the half-vertex angle of the cone.

Alternatively: To get F11 in terms of depth 'h', we can write:

We have: 
$$F_{11} = 1 - \frac{d}{2 \cdot L}$$
  
i.e.  $F_{11} = 1 - \frac{d}{2 \cdot \sqrt{h^2 + \frac{d^2}{4}}}$   
i.e.  $F_{11} = 1 - \frac{d}{\sqrt{4 \cdot h^2 + d^2}}$ 

## (c) F11 for a hemispherical bowl of diameter 'd' : See Fig.(d)

We have: 
$$F_{11} = 1 - \frac{A_2}{A_1}$$

i.e. 
$$F_{11} = 1 - \frac{\frac{\pi \cdot d^2}{4}}{\frac{\pi \cdot d^2}{2}}$$

i.e. 
$$F_{11} = 1 - \frac{1}{2}$$

This result means that for any hemispherical cavity, half of the radiation emitted by the surface 1 falls on itself; it also means that the remaining half falls on the closing surface 2.

\_\_\_\_\_

**Prob.5B.7.** A 70 mm thick metal plate has a hole of 30 mm dia drilled through it. If the plate is held at 300 C and the surrounding is at 27 C, find the energy escaping to the surroundings from both ends of the hole. Assume the metal and surroundings as black bodies.

## Mathcad Solution:



## Data:

$r_i := 0.015$ m	rj ≔	• 0.015	m	L := (	0.07	m
A1 := $\pi \cdot 2 \cdot \mathbf{r_i} \cdot \mathbf{L}$	i.e.	A1 =	6.597 × 1	10 <sup>-3</sup>	m^	2
A2 := $\pi \cdot \mathbf{r}_i^2$	i.e.	A2 =	7.069 × 1	10 <sup>-4</sup>	m^	2
A3 := $\pi \cdot r_j^2$	i.e.	A3 =	7.069 × 1	10 <sup>-4</sup>	m^	2
T1 := 300 + 273 K	Ta	:= 27 + 2	273 K	C		

σ := 5.67·10<sup>-8</sup> W/m<sup>2</sup>.K<sup>4</sup>.....Stefan Boltzmann constant

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## Calculations:

 $F_{22} + F_{23} + F_{21} = 1$  ...eqn (a)....by summation rule

F<sub>22</sub> := 0 ...for a flat surface 2

F23 is the view factor from surface 2 to surface 3, i.e. between two coaxial disks Let us apply the Mathcad Function already written to calculate F23.

$$\label{eq:F23} \begin{array}{ll} \texttt{F}_{23} \coloneqq \texttt{F12\_paralle1\_disks} \big( r_i, r_j, \mathsf{L} \big) & \mbox{...using Mathcad Function} \\ \\ \texttt{i.e.} & \mbox{F}_{23} = 0.042 \end{array}$$

Then, from eqn. (a):  $F_{21} := 1 - (F_{22} + F_{23})$ 

i.e. 
$$F_{21} = 0.958$$

Now, applying reciprocal rule between surfaces 1 and 2:

 $A1 \cdot F_{12} = A2 \cdot F_{21}$  ... by reciprocity

Therefore:  $F_{12} := \frac{A2 \cdot F_{21}}{A1}$ 

Now, by symmetry:  $F_{13} := F_{12}$ 

Therefore: Heat lost by both ends:

$$Q_{tot} := 2 \cdot A1 \cdot F_{12} \cdot \sigma \cdot \left(T1^4 - T_a^4\right) \qquad W$$
  
i.e. 
$$Q_{tot} = 7.655 \qquad W....Ans.$$

\_\_\_\_\_

## **EES Functions to determine View Factors:**

Let us write EES Functions to find out View Factors for three practically important geometries:

- a) Aligned, parallel rectangles,
- b) Coaxial, parallel disks, and
- c) Perpendicular rectangles with a common edge

## a) Aligned, parallel rectangles:



\_\_\_\_\_

## \$UnitSystem SI Pa J Radians

FUNCTION F12\_parallel\_rectangles(X, Y, L)

{Gives View factor for coaxial, parallel rectangles separated by a distance L Note: Trigonometric functions are used with Radians; so, set the angles to Radians in Unit Systems}

XX := X / L

YY := Y / L

A := 2 / (pi \* XX \* YY)

 $B := \ln(sqrt((1 + XX^2) * (1 + YY^2) / (1 + XX^2 + YY^2)))$ 

 $C := XX * (1 + YY^{2})^{(1/2)} * \arctan(XX / (1 + YY^{2})^{(1/2)})$ 

 $D := YY * (1 + XX^2)^{(1/2)} * \arctan(YY / (1 + XX^2)^{(1/2)})$ 

E := XX \* arctan(XX)

F := YY \* arctan(YY)

F12\_parallel\_rectangles := A \* (B + C + D - E - F)

END

«\_\_\_\_\_»

Software Solutions to Problems on Heat Transfer: Radiation Heat Transfer – Part I

## b) Coaxial, parallel disks:



FUNCTION F12\_parallel\_disks(r\_1, r\_2, L)

{Gives View factor for coaxial, parallel disks separated by a distance L}

 $RR_1 := r_1 / L \\ RR_2 := r_2 / L$ 

 $S := 1 + (1 + RR_2^2) / RR_1^2$ 

F12\_parallel\_disks :=  $(1/2) * (S - (S^2 - 4 * (r_2 / r_1)^2)^{(1/2)})$ 

END

"\_\_\_\_\_"



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## c) Perpendicular rectangles with a common edge:



\$UnitSystem SI Pa J Radians

FUNCTION F12\_perpendicular\_rectangles(X, Y, Z)

{Gives View factor for coaxial, perpendicular rectangles with a common edge Note: Trigonometric functions are used with Radians; so, set the angles to Radians in Unit Systems}

 $\mathbf{H} := \mathbf{Z} \; / \; \mathbf{X}$ 

 $W:=Y \ / \ X$ 

A := 1 / (pi \* W)

 $B := W * \arctan(1 / W)$ 

 $C := H * \arctan(1 / H)$ 

 $D := (H^2 + W^2)^{(1/2)} * \arctan(1 / (H^2 + W^2)^{(1/2)})$ 

 $E := (1 + W^{2}) * (1 + H^{2}) / (1 + W^{2} + H^{2})$ 

 $F := (W^2 * (1 + W^2 + H^2) / ((1 + W^2) * (W^2 + H^2)))^{(W^2)}$ 

 $G := (H^2 * (1 + H^2 + W^2) / ((1 + H^2) * (H^2 + W^2)))^{(H^2)}$ 

F12\_perpendicular\_rectangles := A \* (B + C - D + (1/4)\*ln(E \* F \* G))

END

"\_\_\_\_\_"

## Now, let us use these EES Functions to solve some problems:

**Prob.5B.8.** Consider a truncated cone with top and bottom diameters of 10 and 20 cm and a height of 10 cm. Calculate:

- a) the shape factor from the top surface to the sides and also from the side surface to itself.
- b) If surface 1 is at 1000 K and surface 2 is at 500 K, find the net exchange of heat between 1 and 2 (Q\_12), if all the surfaces are black.
- c) Plot F\_12 and Q\_12 for L varying from 0.1 to 1 m



### Fig.Prob.5B.8

## **EES Solution:**

Let the top surface be designated as 1, bottom surface as 2, and the side surfaces as 3. See the fig. above.

## Then:

------

## "Data:"

 $r_1 = 0.05 [m]$ 

 $r_2 = 0.1 \ [m]$ 

L = 0.1 [m]

## "We have to find out F13 and F33:"

## "Calculations:"

A1 = pi \*  $r_1^2$  "[ $m^2$ ].... area of top surface"

A2 = pi \*  $r_2^2$  "[m^2]...area of bottom surface"

A3 = pi \*  $(r_1 + r_2)$  \* sqrt $((r_2 - r_1)^2 + L^2)$  "[m^2]...area of the side surface"

"Now:

F11 + F12 + F13 = 0 ..... by Summation rule

But, F11 = 0... since it is a flat surface.

We find F12 by the EES Function for parallel disks, written above.

So, we have:

F11 = 0

F12 = F12\_parallel\_disks(r\_1, r\_2, L) "using the EES Function for parallel disks"

F11 + F12 + F13 = 1 "....finds the view factor from top surface to side surface, F13"



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"Now, by reciprocal relation between surfaces 1 and 3:"

A1 \* F13 = A3 \* F31"...finds view factor F31"

"Now, by reciprocal relation between surfaces 1 and 2:"

A1 \* F12 = A2 \* F21 "...finds view factor from surface 2 to 1, F21"

"And, by reciprocal relation between surfaces 2 and 3:"

F22 = 0 "...since surface 2 is flat"

"For surface 2, by Summation rule:"

F22 + F21 + F23 = 1"...determines F23"

A2 \* F23 = A3 \* F32 "...finds view factor from surface 3 to 2, F32"

"Now, applying Summation rule to surface 3:"

F33 + F31 + F32 = 1"....determines F33"

## **Results:**

Main F12_parallel_disks			
Unit Settings: SI C P	a J mass rad		
A1 = 0.007854 [m <sup>2</sup> ]	A2 = 0.03142 [m <sup>2</sup> ]	A3 = 0.05269 [m <sup>2</sup> ]	F11 = 0
F12 = 0.4689	F13 = 0.5311	F21 = 0.1172	F22 = 0
F23 = 0.8828	F31 = 0.07918	F32 = 0.5264	F33 = 0.3944
L = 0.1 [m]	r <sub>1</sub> = 0.05 [m]	r <sub>2</sub> = 0.1 [m]	

Main F12\_parallel\_disks

Local variables in Function F12_parallel_disks (1 call, 0.00 sec)				
F12 <sub>parallel,disks</sub> =0.4689	L=0.1 [m]	RR <sub>1</sub> =0.5		
RR2=1	r <sub>1</sub> =0.05 [m]	r <sub>2</sub> =0.1 [m]		
S =9				

Thus:

Shape factor from the top surface to the sides = F13 = 0.5311 .... Ans.

Shape factor from the side surface to itself = F33 = 0.3944 .... Ans.

------

In addition: (b) If surface 1 is at 1000 K and surface 2 is at 500 K, find the net exchange of heat between 1 and 2, if all the surfaces are black.

Add the following code at the end of the above program and run it again:

\_\_\_\_\_

"If surface 1 is at 1000 K and surface 2 is at 500 K, find the net exchange of heat between 1 and 2, if all the surfaces are black."

T1 = 1000 [K]

T2 = 500 [K]

sigma = 5.67E-08 [W/m^2-K^4] "...Stefan Boltzmann const."

 $Q_{12} = A1 * F12 * sigma * (T1^4 - T2^4) "[W] \dots$  net radiant exchange between surfaces 1 and 2"

## We get:

## Unit Settings: SI C Pa J mass rad

A1 = 0.007854 [m <sup>2</sup> ]	A2 = 0.03142 [m <sup>2</sup> ]	A3 = 0.05269 [m <sup>2</sup> ]
F11 = 0	F12 = 0.4689	F13 = 0.5311
F21 = 0.1172	F22 = 0	F23 = 0.8828
F31 = 0.07918	F32 = 0.5264	F33 = 0.3944
L = 0.1 [m]	Q <sub>12</sub> = 195.7 [W]	r <sub>1</sub> = 0.05 [m]
r <sub>2</sub> = 0.1 [m]	σ = 5.670E-08 [W/m <sup>2</sup> -K <sup>4</sup> ]	T1 = 1000 [K]
T2 = 500 [K]		

i.e. Q\_12 = 195.7 W ... net heat exchange between surfaces 1 and 2 ... Ans.



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## Also, plot F\_12 and Q\_12 against L, with L varying from 0.1 m to 1 m:

Table 1 Table	92			
110	1 🔽 📕	<sup>2</sup> F12	<sup>3</sup> F33 ▼	₄ ⊈ Q <sub>12</sub> [W]
Run 1	0.1	0.4689	0.3944	195.7
Run 2	0.2	0.1922	0.6269	80.26
Run 3	0.3	0.09779	0.7367	40.83
Run 4	0.4	0.05802	0.7981	24.22
Run 5	0.5	0.03811	0.8367	15.91
Run 6	0.6	0.02685	0.8631	11.21
Run 7	0.7	0.0199	0.8822	8.309
Run 8	0.8	0.01533	0.8967	6.399
Run 9	0.9	0.01216	0.908	5.076
Run 10	1	0.009877	0.9171	4.123

## First, compute the Parametric Table. (Here, we have included F\_33 also)

## Now, plot the results:





**Prob.5B.9.** Consider two coaxial, parallel rectangles, each of size:  $1 \text{ m} \times 1 \text{m}$ , with a distance of 2 m from each other. If surface 1 is at 800 K and the surface 2 at 400 K, find the net heat interchange between the two surfaces. Consider the surfaces as black.

Software Solutions to Problems on Heat Transfer: Radiation Heat Transfer – Part I



Fig.Prob.5B.9

## **EES Solution:**

## "Data:"

X = 1 [m]

Y = 1 [m]

L =2 [m]

T1 = 800 [K]

T2 = 400 [K]

sigma = 5.67E-08[W/m^2-K^4] "...Stefan Boltzmann constant"

## "We have to find out F 12:"





## "Calculations:"

 $A1 = X * Y "[m^2]...$  area of top surface"

- A2 =A1 " $[m^2]$ .... area of bottom surface"
- F\_12 = F12\_parallel\_rectangles(X, Y, L)

"Net exchange of heat between A1 and A2:"

 $Q_{12} = A1 * F_{12} * sigma * (T1^4 - T2^4) "[W] \dots$  net exchange of heat between surfaces1 and 2"

## **Results:**

Unit Settings: SI C Pa J mass rad		
A1 =1 [m <sup>2</sup> ]	A2 =1 [m <sup>2</sup> ]	F <sub>12</sub> = 0.06859
L=2 [m]	Q <sub>12</sub> =1493 [W]	σ = 5.670E-08 [W/m <sup>2</sup> -K <sup>4</sup> ]
T1 =800 [K]	T2 = 400 [K]	×=1 [m]
Y =1 [m]		

## Thus:

Net exchange of heat between A1 and A2 =  $Q_{12} = 1493$  W ... Ans.

\_\_\_\_\_

## In addition:

Plot the variation of F\_12 and Q\_12 as the distance between the plates (L) varies from 0.5 m to 2.5 m:

## First, compute the Parametric Table:

Table 1			
▶ 121	1 L [m]	<sup>2</sup> F <sub>12</sub>	<sup>3</sup> Q <sub>12</sub> [W]
Run 1	0.5	0.4153	9041
Run 2	0.6	0.3545	7718
Run 3	0.7	0.3044	6628
Run 4	0.8	0.263	5726
Run 5	0.9	0.2286	4977
Run 6	1	0.1998	4351
Run 7	1.1	0.1757	3826
Run 8	1.2	0.1553	3382
Run 9	1.3	0.1381	3006
Run 10	1.4	0.1233	2686
Run 11	1.5	0.1107	2410
Run 12	1.6	0.09981	2173
Run 13	1.7	0.09036	1967
Run 14	1.8	0.08213	1788
Run 15	1.9	0.07492	1631
Run 16	2	0.06859	1493
Run 17	2.1	0.063	1372
Run 18	2.2	0.05804	1264
Run 19	2.3	0.05362	1167
Run 20	2.4	0.04968	1082
Run 21	2.5	0.04614	1005

## Now, plot the results:





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View Factors for two concentric cylinders of finite length:

Let us write EES Functions for the view factors for:

- a) From the outer cylinder to itself,
- b) From the outer cylinder to inner cylinder, and
- c) From the inner cylinder to itself
### We use the view factor equations provided by Modest. (Ref: 'Radiative Heat Transfer' by Modest)





Note: In the EES Functions we write below, height of the cylinders (i.e. h in the above figs.) is taken as L.

FUNCTION F22\_CoaxialCyl\_outer\_to\_itself (R1, R2, L)

"Finds the View factor F\_22 between two coaxial parallel cylinders – outer surface to itself" "Ref: Radiative Heat Transfer by Modest"

R := R2/R1 H := L / R1  $A = (sqrt(H \land 2 + 4 * R \land 2) - H) / (4 * R)$   $B = (2 * sqrt(R \land 2 - 1)) / H$  $C = sqrt(4 * R \land 2 + H \land 2) / H$   $D = H \land 2 + 4 * (R \land 2 - 1) - 2 * H \land 2 / R \land 2$   $E = H \land 2 + 4 * (R \land 2 - 1)$   $F = (R \land 2 - 2) / R \land 2$  $G = (2 / R) * \arctan(B)$ 

 $F22\_CoaxialCyl\_outer\_to\_itself = 1 - 1 / R - A + (1 / pi) * (G - (H / (2 * R)) * (C * arcsin(D / E) - arcsin(F)))$ 

END

«\_\_\_\_\_»

FUNCTION F21\_coaxialCyl\_outer\_to\_inner(R1, R2, L)

"Finds the View factor F\_21 between two coaxial parallel cylinders – outer surface to inner surface Ref: Radiative Heat Transfer by Modest"

$$\begin{split} R &:= R2/R1 \\ H &:= L / R1 \\ A &:= (H \land 2 + R \land 2 - 1) / (4 * H) \end{split}$$



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 $B := (H \land 2 - R \land 2 + 1) / (H \land 2 + R \land 2 - 1)$   $C := sqrt((H \land 2 + R \land 2 + 1) \land 2 - 4 * R \land 2) / (2 * H)$   $D := (H \land 2 - R \land 2 + 1) / (R * (H \land 2 + R \land 2 - 1))$  $E := (H \land 2 - R \land 2 + 1) / (2 * H)$ 

 $F21\_coaxialCyl\_outer\_to\_inner = (1 / R) * (1 - A - (1 / pi) * (arccos(B) - C * arccos(D) - E * arcsin(1 / R)))$ 

END

«\_\_\_\_\_\_»

FUNCTION F12\_Coaxial\_cyl\_inner\_to\_outer (R1, R2, L)

{Finds the View factor F12 between two coaxial parallel cylinders – inner surface to outer surface. Ref: F\_21 From Modest, see above.

Use: A2 \* F\_21 = A1 \* F\_12;

i.e. F\_12 = F\_21\*(A2/A1);

And, A2/A1 = R2/R1

F12\_Coaxial\_cyl\_inner\_to\_outer = (R2/R1) \* F21\_coaxialCyl\_outer\_to\_inner(R1, R2, L)

END

«\_\_\_\_\_»

### Now, let us use these Functions in solving a problem:

**Prob.5B.10.** Two concentric cylinders have diameters of 10 and 20 cm. Calculate the View factor between the two open ends.



Fig.Prob.5B.10

### **EES Solution:**

Let: Surface 1: outer surface of inner cylinder

Surface 2: inner surface of outer cylinder

Surfaces 3 and 4: surfaces at the two ends

Then, we have:

### "Data:"

R1 = 0.05 [m] R2 = 0.1 [m] L = 0.2 [m]

### "Calculations:"

A1 = 2 \* pi \* R1 \* L "[m^2] ... area of inner cylinder surface"

 $A2 = 2 * pi * R2 * L "[m^2] \dots$  area of outer cylinder surface"

A3 = pi \*  $(R2^2 - R1^2)$  "[m<sup>2</sup>] ... area of open, end surface"

A4 = A3 "[m^2] ... area of open, end surface"

F\_21 = F21\_CoaxialCyl\_outer\_to\_inner(R1, R2, L)"...finds View factor from outer to inner surface"

F\_22 = F22\_CoaxialCyl\_outer\_to\_itself (R1, R2, L)"...finds View factor from outer surface to itself"

"Now, by reciprocity:"

A1 \* F\_12 = A2 \* F\_21 "..finds F\_12"

"For surface 2, Summation rule gives:"

 $F_{21} + F_{22} + F_{23} + F_{24} = 1$ 

"But, by symmetry:"

F\_23 = F\_24 "...therefore, F\_23 and F\_24 are calculated."

"Again, applying reciprocity between surfaces 2 and 3:"

A2 \* F\_23 = A3 \* F\_32 "...finds F\_32"

"Now,"

F\_33 = 0 "...since surface is flat"

F\_44 = 0 "...since surface is flat"

F\_11 = 0 "...since surface is convex"





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### "Applying Summation rule for surface 3:"

 $F_{31} + F_{32} + F_{33} + F_{34} = 1$ "....eq. (A)"

"In the above, F\_31 and F\_34 are unknown."

"To find F\_31:"

F\_11 + F\_12 + F\_13 + F\_14 = 1 "...by Summation rule on surface 1"

"But,"

F\_13 = F\_14 "...by Symmetry"

"Therefore, F\_13 and F\_14 are determined, since F\_11 = 0 and F\_12 is already determined"

"Then, by reciprocity:"

A1 \* F\_13 = A3 \* F\_31 "...finds F\_31"

"And, from eqn.(A), F\_34 is immediately calculated."

### **Results:**

### Unit Settings: SI C Pa J mass rad

A1 = 0.06283 [m <sup>2</sup> ]	A2 = 0.1257 [m <sup>2</sup> ]	A3 = 0.02356 [m <sup>2</sup> ]	A4 = 0.02356 [m <sup>2</sup> ]
F <sub>11</sub> = 0	F <sub>12</sub> = 0.8253	F <sub>13</sub> = 0.08737	F <sub>14</sub> = 0.08737
F <sub>21</sub> = 0.4126	F <sub>22</sub> = 0.3286	F <sub>23</sub> = 0.1294	F <sub>24</sub> = 0.1294
F <sub>31</sub> = 0.233	F <sub>32</sub> = 0.6901	F <sub>33</sub> = 0	F <sub>34</sub> = 0.07694
F <sub>44</sub> = 0	L = 0.2 [m]	R1 = 0.05 [m]	R2 = 0.1 [m]

Thus:

View factor between the two open ends =  $F_{34} = 0.07694 \dots$  Ans.

Also, plot F\_34 for different values of L, i.e. 0.1 < L < 0.5 m:

### Parametric Table:

19	1 L [m]	<sup>2</sup> F <sub>34</sub> ▼
Run 1	0.1	0.2036
Run 2	0.15	0.1194
Run 3	0.2	0.07694
Run 4	0.25	0.05309
Run 5	0.3	0.03858
Run 6	0.35	0.02918
Run 7	0.4	0.02279
Run 8	0.45	0.01826
Run 9	0.5	0.01494

### Now, plot the results:



### Now, let us write VBA Functions for view factors for a few important geometries:

We shall write VBA Functions for:

- 1. Aligned, parallel rectangles,
- 2. Coaxial, parallel disks, and
- 3. Perpendicular rectangles with a common edge

Recollect the nomenclature of geometries:





- (a) Aligned parallel rectangles
- (b) Coaxial parallel disks



(c) Perpendicular rectangles with a common edge

Flg.13.18 Few three-dimensional geometries

And, the equations for View factors are given at the beginning of the chapter.

Now, we shall present the VBA programs for View factors:

### 1. Aligned, parallel rectangles:

```
Option Explicit
Function F_ij_Aligned_parallel_rectangles(XBYL As Double, YBYL As Double) As Double
'Finds the View factor F ij between two aligned parallel rectangles
'Note that VB does not have the ATAN function
'So, use the relation: ATAN(x) = ASIN(x/sqrt(x^2 + 1))
Dim XX As Double, YY As Double, A As Double, B As Double, C As Double
Dim D As Double, E As Double, F As Double, BB As Double, CC As Double
XX = XBYL
YY = YBYL
A = 2 / (Application.Pi() * XX * YY)
B = Application.Ln(Sqr((1 + XX ^ 2) * (1 + YY ^ 2) / (1 + XX ^ 2 + YY ^ 2)))
'C = XX * (1 + YY ^ 2) ^ (1 / 2) * Atan(XX / (1 + YY ^ 2) ^ (1 / 2))
D = YY * (1 + XX ^ 2) ^ (1 / 2) * Atan(YY / (1 + XX ^ 2) ^ (1 / 2))
C = XX * (1 + YY ^ 2) ^ (1 / 2) * Application.Asin(BB / Sqr(BB ^ 2 + 1))
D = YY * (1 + XX ^ 2) ^ (1 / 2) * Application.Asin(CC / Sqr(CC ^ 2 + 1))
'E = XX * Atan(XX)
'F = YY * Atan(YY)
E = XX * Application.Asin(XX / Sqr(XX ^ 2 + 1))
F = YY * Application.Asin(YY / Sqr(YY ^ 2 + 1))
F ij Aligned parallel rectangles = A * (B + C + D - E - F)
End Function
```

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Priyanka Sawant Manager



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### 2. Coaxial, parallel disks:

```
Function F_ij_Coaxial_parallel_discs(L_by_r_i As Double, r_j_by_L As Double) As Double
'Finds the View factor F_ij between two coaxial parallel discs
Dim RR_i As Double, RR_j As Double, S As Double
RR_i = 1 / L_by_r_i
RR_j = r_j_by_L
S = 1 + (1 + RR_j ^ 2) / RR_i ^ 2
F_ij_Coaxial_parallel_discs = (1 / 2) * (S - (S ^ 2 - 4 * (RR_j / RR_i) ^ 2) ^ 0.5)
End Function
```

### 3. Perpendicular rectangles with a common edge:

```
Function F ij Perpendicular rectangles(ZBYX As Double, YBYX As Double) As Double
'Finds the View factor F_ij between two perpendicular rectangles with a common edge
'common edge: X; other sides:Y, Z
Dim H As Double, W As Double, A As Double, B As Double, C As Double
Dim D As Double, E As Double, E_1 As Double, E_2 As Double, E_3 As Double
H = ZBYX
W = YBYX
A = 1 / (Application.Pi() * W)
B = W * Application.Asin((1 / W) / Sqr((1 / W) ^ 2 + 1))
C = H * Application.Asin((1 / H) / Sqr((1 / H) ^ 2 + 1))
D = (H ^ 2 + W ^ 2) ^ 0.5 * Application.Asin((1 / Sqr(H ^ 2 + W ^ 2)) / Sqr((1 / _
(H ^ 2 + W ^ 2)) + 1))
E 1 = (1 + W^{2}) * (1 + H^{2}) / (1 + W^{2} + H^{2})
E^{2} = (W^{2} + W^{2} + W^{2} + H^{2}) / ((1 + W^{2}) * (W^{2} + H^{2})))^{2} (W^{2})
E^{3} = (H^{2} + (1 + H^{2} + W^{2})) / ((1 + H^{2}) * (H^{2} + W^{2}))) ^{(H^{2})}
E = E 1 * E 2 * E 3
F_ij_Perpendicular_rectangles = A * (B + C - D + 0.25 * Application.Ln(E))
End Function
```

Prob.5B.11. Draw the View factor graphs for Aligned, parallel rectangles:

### **EXCEL Solution:**

We shall use the VBA Function written above.

### Following are the steps:

	D8	2	- (*	$f_{x}$	=F_ij_Al	igned_pa	arallel_r	ectangle	s(C82,\$D	\$81)		
	А	В	С		D	E	F	G	Н	- I	J	K
80												
81			XbyL\YbyL =		0.1	0.2	0.4	0.6	1	2	4	10
82			0.1	0.0	03162057	0.00626	0.01207	0.01715	0.02492	0.03514	0.04209	0.04671
83			0.2									
84			0.3									
85			0.4									
86			0.5									
87			0.6									
88			0.7									
89			0.8									
90			0.9									
91			1									
92			2									
93			3									
94			4									
95			5									
96			6									
97			7									
98			8									
99			9									
100			10									
101			20									
102			30									
103			40									
104			50									
105			60									
106			70									
107			80									
108			90									
109			100									
110												

### 1. Set up the EXCEL worksheet. Set up a Table as follows:

In the above screen shot, column C contains X/L values and row 81 contains Y/L values.

We use the VBA Function written above to calculate the view factor  $F_{ij}$  (i.e. view factor from surface i to surface j), for given values of X/L and Y/L. Formula entered in cellD82 can be seen in the Formula bar. Use relative references to X/L so that we can drag-copy to fill up the Table later. Similarly, write formulas for other Y/L values as shown above.

Recollect that to enter the desired VBA Function, click on 'Insert Function' mark:

Hor	ne Ins	ert Page La	ayout For	mulas [	Data R	eview	View	Deve
Paste 🖋	Calibri B I	- 11 U-	• A A () • A •	= = <mark>-</mark>	- ≫- 1= 1=		General \$ - %	•
Clipboard 🖻		Font	G	Alig	nment	G	Num	nber
D82	2	<b>-</b>	<i>f</i> x =F_ij	_Aligned_p	arallel_r	ectangle	s(C82,\$D	\$81)
A	В	С	Insert Funct	tion E	F	G	Н	1

Clicking on 'Insert Function', gives:

Insert Function		? 🔀
Search for a function:		
Type a brief descript Go	ion of what you want to do and then click	Go
Or select a category:	Most Recently Used 😽	
Select a function:	Most Recently Used	
MROUND CEILING TREND LINEST ASIN ABS ATAN MROUND(number, Returns a number rou	Financial Date & Time Math & Trig Statistical Lookup & Reference Database Text Logical Information User Defined	
Help on this function	ОК	Cancel

In the above window, click on 'User Defined' category and press OK:

Ir	sert Function		? 🛛
<u>S</u> e	earch for a function:		
	Type a brief descripti Go	on of what you want to do and then click	<u>G</u> o
	Or select a <u>c</u> ategory:		
Se	elect a functio <u>n</u> :		
	F_22_Coaxial_cyl_ou F_22_Coaxial_cyl_ou F_ii_Aligned_parallel F_ij_Coaxial_parallel F_ij_Perpendicular_re Formname G	ter_surface_to_inner_surface ter_to_itself rectangles discs ctangles	
	r_ij_Aligned_paral No help available.	IEI_rectangles(XBYL,YBYL)	
Н	elp on this function	ОК	Cancel

Choose 'F\_ij Aligned, parallel rectangles' and press OK. We get the following window; fill it up as shown. Note that cell C82 for X/L is in relative reference and cell D81 for Y/L is in absolute reference, so that when we drag-copy downwards, appropriate X/L values are automatically chosen:

Functio	on Arguments	? 🛛
F_ij_Alig	ned_parallel_rectangles	
XBYL	C82	= 0.1
YBYL	\$D\$81	= 0.1
		= 0.003162057
No help a	vailable.	
	YBYL	
Formula r	esult = 0.003162057	
<u>Help on t</u>	his function	OK Cancel

Click OK. And the value of F\_ij is calculated.

### Complete the row 82 likewise.

2. Now, select cells D82 to K82 and drag-copy to the end of the Table, i.e. up to the cell K109. The calculations are done immediately and the Table is filled up:

XbyL\YbyL =	0.1	0.2	0.4	0.6	1	2	4	10
0.1	0.0031621	0.00626	0.01207	0.01715	0.02492	0.03514	0.04209	0.04671
0.2	0.006263	0.0124	0.02392	0.03398	0.04941	0.06971	0.08353	0.09272
0.3	0.009247	0.01832	0.03532	0.0502	0.07305	0.10318	0.12372	0.13739
0.4	0.012073	0.02392	0.04614	0.0656	0.09554	0.13513	0.16219	0.18021
0.5	0.014711	0.02915	0.05625	0.08001	0.11665	0.16527	0.19859	0.22081
0.6	0.017148	0.03398	0.0656	0.09336	0.13627	0.19342	0.23271	0.25896
0.7	0.019379	0.0384	0.07417	0.10563	0.15435	0.21951	0.26447	0.29455
0.8	0.021411	0.04243	0.08199	0.11683	0.17092	0.24356	0.29387	0.32759
0.9	0.023255	0.04609	0.0891	0.12702	0.18605	0.26564	0.321	0.35816
1	0.024925	0.04941	0.09554	0.13627	0.19982	0.28588	0.34596	0.38638
2	0.035143	0.06971	0.13513	0.19342	0.28588	0.41525	0.50899	0.57338
3	0.039652	0.07868	0.15268	0.21888	0.32474	0.47558	0.58762	0.66599
4	0.042091	0.08353	0.16219	0.23271	0.34596	0.50899	0.63204	0.71933
5	0.043603	0.08654	0.16809	0.2413	0.35917	0.52993	0.6602	0.75363
6	0.044628	0.08858	0.17209	0.24712	0.36813	0.54421	0.67954	0.77741
7	0.045366	0.09005	0.17497	0.25132	0.37461	0.55454	0.69359	0.79482
8	0.045924	0.09116	0.17715	0.25449	0.3795	0.56236	0.70426	0.80811
9	0.046359	0.09202	0.17885	0.25697	0.38332	0.56847	0.71261	0.81856
10	0.046708	0.09272	0.18021	0.25896	0.38638	0.57338	0.71933	0.82699
20	0.048288	0.09586	0.18638	0.26795	0.40026	0.59563	0.7499	0.86563
30	0.048817	0.09691	0.18845	0.27096	0.40491	0.60309	0.76017	0.8787
40	0.049081	0.09744	0.18948	0.27246	0.40723	0.60682	0.76532	0.88526
50	0.049240	0.09775	0.1901	0.27337	0.40863	0.60906	0.76841	0.8892
60	0.049346	0.09797	0.19051	0.27397	0.40956	0.61056	0.77047	0.89183
70	0.049422	0.09812	0.19081	0.2744	0.41022	0.61162	0.77194	0.89371
80	0.049478	0.09823	0.19103	0.27472	0.41072	0.61243	0.77304	0.89512
90	0.049523	0.09832	0.1912	0.27497	0.41111	0.61305	0.7739	0.89622
100	0.049558	0.09839	0.19134	0.27517	0.41142	0.61355	0.77459	0.89709

3. Now draw the graphs in EXCEL:



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### Prob.5B.12. Draw the View factor graphs for coaxial, parallel disks:

### **EXCEL Solution:**

We shall use the VBA Function written above.

Following are the steps:

1. Set up the EXCEL worksheet. Set up a Table as follows:

D141	<b>-</b> (9	<i>f</i> <sub>x</sub> =F_ij_Co	axial_pa	rallel_di	iscs(C141	,\$D\$140	)	
A B	С	D	E	F	G	Н	- I	J
139								
140	L_byR_i\R_jby_L	0.3	0.6	1	1.5	2	4	8
141	0.1	0.000891081	0.00356	0.0099	0.02227	0.03959	0.15812	0.62344
142	0.2							
143	0.3							
144	0.4							
145	0.5							
146	0.6							
147	0.7							
148	0.8							
149	0.9							
150	1							
151	2							
152	3							
153	4							
154	5							
155	6							
156	7							
157	8							
158	9							
159	10							
160	20							
161	30							
162	40							
163	50							
164	60							
165	70							
166	80							
167	90							
168	100							

In the above screen shot, column C contains L/R\_i values and row 140 contains R\_j/L values.

We use the VBA Function written above to calculate the view factor  $F_{ij}$  (i.e. view factor from surface i to surface j), for given values of L/R\_i and R\_j/L. Formula entered in cell D141 can be seen in the Formula bar. Use relative references to L/R\_i so that we can drag-copy to fill up the Table later. Similarly, write formulas for other R\_j/L values as shown above.

2. Now, select cells D141 to J141 and drag-copy to the end of the Table, i.e. up to the cell J168. The calculations are done immediately and the Table is filled up:

L hup N.D. (hu L	0.2	0.6	4	1.0	2	4	0
L_DYK_I\K_DY_L	0.5	0.6	1	1.5	2	4	8
0.1	0.000891081	0.00356	0.0099	0.02227	0.03959	0.15812	0.62344
0.2	0.003461076	0.01384	0.0384	0.08623	0.15279	0.58388	0.97538
0.3	0.007426605	0.02965	0.08196	0.18242	0.31803	0.86472	0.98151
0.4	0.012392345	0.0493	0.13502	0.29352	0.48769	0.91153	0.98301
0.5	0.017934496	0.07092	0.19098	0.39754	0.60961	0.92482	0.98362
0.6	0.023671607	0.09278	0.24388	0.47901	0.67889	0.93063	0.98394
0.7	0.029306344	0.1136	0.28993	0.53614	0.71722	0.93375	0.98412
0.8	0.03463697	0.13258	0.32784	0.57481	0.73985	0.93564	0.98424
0.9	0.039547509	0.14937	0.35811	0.60123	0.75423	0.93688	0.98432
1	0.043988012	0.16393	0.38197	0.6198	0.76393	0.93774	0.98438
2	0.068027565	0.23196	0.46887	0.67544	0.79176	0.94035	0.98456
3	0.075457337	0.24941	0.48612	0.68493	0.7964	0.94081	0.98459
4	0.078424643	0.25595	0.49219	0.68818	0.79798	0.94097	0.9846
5	0.079871841	0.25906	0.495	0.68967	0.79871	0.94105	0.98461
6	0.080678654	0.26077	0.49653	0.69048	0.79911	0.94109	0.98461
7	0.081172376	0.26181	0.49745	0.69097	0.79935	0.94111	0.98461
8	0.081495782	0.26248	0.49805	0.69128	0.7995	0.94113	0.98461
9	0.081718871	0.26295	0.49846	0.6915	0.7996	0.94114	0.98461
10	0.081879129	0.26328	0.49875	0.69165	0.79968	0.94114	0.98461
20	0.082395398	0.26435	0.49969	0.69214	0.79992	0.94117	0.98461
30	0.082491655	0.26455	0.49986	0.69223	0.79996	0.94117	0.98462
40	0.082525393	0.26462	0.49992	0.69227	0.79998	0.94117	0.98462
50	0.082541017	0.26465	0.49995	0.69228	0.79999	0.94118	0.98462
60	0.082549507	0.26467	0.49997	0.69229	0.79999	0.94118	0.98462
70	0.082554627	0.26468	0.49997	0.69229	0.79999	0.94118	0.98462
80	0.08255795	0.26468	0.49998	0.6923	0.79999	0.94118	0.98462
90	0.082560228	0.26469	0.49998	0.6923	0.8	0.94118	0.98462
100	0.082561858	0.26469	0.49999	0.6923	0.8	0.94118	0.98462



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### 3. Now, draw the graphs in EXCEL:



**Prob.5B.13.** Draw the View factor graphs for Perpendicular rectangles with a common edge:

\_\_\_\_\_

### **EXCEL Solution:**

We shall use the VBA Function written above.

### Following are the steps:

	D20	07	<del>-</del> (•	<i>f</i> <sub>∞</sub> =F_ij_Pe	rpendic	ular_rect	angles(C	:207,\$D\$	206)			
	А	В	С	D	E	F	G	Н	1	J	K	L
205												
206			ZbyX\YbyX =	0.02	0.05	0.1	0.2	0.6	1	2	4	10
207			0.1	0.443749568	0.37202	0.28189	0.18108	0.07048	0.04325	0.02188	0.01097	0.00439
208			0.2	· · · · · · · · · · · · · · · · · · ·								
209			0.3									
210			0.4									
211			0.5									
212			0.6									
213			0.7									
214			0.8									
215			0.9									
216			1									
217			1.5									
218			2									
219			2.5									
220			3									
221			3.5									
222			4									
223			4.5									
224			5									
225			5.5									
226			6									
227			6.5									
228			7									
229			7.5									
230			8									
231			8.5									
232			9									
233			9.5									
234			10									

1. Set up the EXCEL worksheet. Set up a Table as follows:

In the above screen shot, column C contains Z/X values and row 206 contains Y/X values.

We use the VBA Function written above to calculate the view factor  $F_ij$  (i.e. view factor from surface i to surface j), for given values of Z/X and Y/X. Formula entered in cell D207 can be seen in the Formula bar. Use relative references to Z/X so that we can drag-copy to fill up the Table later. Similarly, write formulas for other Y/X values as shown above.

 Now, select cells D207 to L207 and drag-copy to the end of the Table, i.e. up to the cell L234. The calculations are done immediately and the Table is filled up:

ZbyX\YbyX =	0.02	0.05	0.1	0.2	0.6	1	2	4	10
0.1	0.443749568	0.37202	0.28189	0.18108	0.07048	0.04325	0.02188	0.01097	0.00439
0.2	0.466154321	0.42337	0.36216	0.27104	0.12295	0.07744	0.03971	0.02	0.00802
0.3	0.473183243	0.44044	0.39322	0.31714	0.16239	0.10512	0.05473	0.02768	0.01111
0.4	0.476459637	0.44852	0.40859	0.34295	0.19206	0.1277	0.06757	0.03434	0.0138
0.5	0.478288879	0.45305	0.41738	0.35864	0.21445	0.14619	0.07865	0.04018	0.01617
0.6	0.479423474	0.45587	0.4229	0.36884	0.23147	0.16138	0.08829	0.04536	0.01829
0.7	0.480178071	0.45775	0.4266	0.37581	0.24453	0.1739	0.09671	0.04998	0.02019
0.8	0.480705777	0.45906	0.4292	0.38078	0.25467	0.18425	0.10411	0.05413	0.02191
0.9	0.481089185	0.46002	0.43109	0.38443	0.26263	0.19286	0.11065	0.05789	0.02348
1	0.481376318	0.46073	0.43251	0.38719	0.26896	0.20004	0.11643	0.06131	0.02492
1.5	0.482110417	0.46257	0.43616	0.39435	0.28678	0.22232	0.13715	0.07458	0.03069
2	0.482390134	0.46327	0.43756	0.39711	0.29429	0.23285	0.1493	0.08365	0.03491
2.5	0.482524639	0.4636	0.43823	0.39844	0.29805	0.23847	0.15682	0.09015	0.03818
3	0.482599192	0.46379	0.4386	0.39919	0.30019	0.24176	0.16169	0.09495	0.04082
3.5	0.482644682	0.4639	0.43883	0.39964	0.30151	0.24383	0.16499	0.09857	0.043
4	0.48267443	0.46398	0.43897	0.39993	0.30238	0.24522	0.16731	0.10136	0.04484
4.5	0.48269493	0.46403	0.43908	0.40014	0.30298	0.24619	0.16899	0.10353	0.04641
5	0.482709646	0.46406	0.43915	0.40029	0.30342	0.2469	0.17024	0.10526	0.04777
5.5	0.482720563	0.46409	0.43921	0.4004	0.30374	0.24743	0.17119	0.10664	0.04895
6	0.482728883	0.46411	0.43925	0.40048	0.30399	0.24783	0.17193	0.10776	0.04998
6.5	0.482735367	0.46413	0.43928	0.40054	0.30418	0.24815	0.17252	0.10868	0.05089
7	0.482740519	0.46414	0.4393	0.40059	0.30433	0.2484	0.17299	0.10944	0.0517
7.5	0.482744679	0.46415	0.43933	0.40064	0.30446	0.2486	0.17338	0.11008	0.05242
8	0.482748086	0.46416	0.43934	0.40067	0.30456	0.24877	0.1737	0.11062	0.05306
8.5	0.482750912	0.46417	0.43936	0.4007	0.30464	0.24891	0.17397	0.11108	0.05363
9	0.482753281	0.46417	0.43937	0.40072	0.30471	0.24903	0.17419	0.11147	0.05414
9.5	0.482755287	0.46418	0.43938	0.40074	0.30477	0.24912	0.17438	0.11181	0.05461
10	0.482757001	0.46418	0.43939	0.40076	0.30482	0.24921	0.17455	0.1121	0.05502

1. Now, draw the graphs in EXCEL:



Prob.5B.14. Draw the View factor graphs for two concentric cylinders of finite length:

Recollect:

View Factors for two concentric cylinders of finite length:

Let us write EES Functions for the view factors for:

- a) From the outer cylinder to inner cylinder, F21,
- b) From the inner cylinder to outer cylinder, F12
- c) From outer cylinder to itself, F22

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### We use the view factor equations provided by Modest. (Ref: 'Radiative Heat Transfer' by Modest)





First, let us write VBA Functions for the View factors, using the above equations:

1. View Factor F\_21 from outer cylinder to inner cylinder for different L/R2:

### 2. View Factor F\_12 from inner cylinder to outer cylinder for different L/R2:

```
Function F_12_Coaxial_cyl_inner_surface_to_outer_surface(R1_by_R2 As Double, _
H_by_R2 As Double) As Double
'Ref: F_21 From Modest, see above
'Use: A2.F_21 = A1*F_12; i.e. F_12 = F_21*(A2/A1); And, A2/A1 = R2/R1
        F_12_Coaxial_cyl_inner_surface_to_outer_surface =
        F_21_Coaxial_cyl_outer_surface_to_inner_surface(R1_by_R2, H_by_R2) * (1# / R1_by_R2)
End Function
```

### 3. View Factor F\_22 from outer cylinder to itself for different L/R2:

```
Function F_22_Coaxial_cyl_outer_to_itself(R1_by_R2 As Double, H_by_R2 As Double) As Double
 'Finds the View factor F_22 between two coaxial parallel cylinders--outer surface to itself
 'Ref: Radiative Heat Transfer by Modest
Dim R As Double, H As Double, A As Double, B As Double, C As Double
Dim D As Double, E As Double, F As Double, G As Double
R = 1 / R1_by_R2H = H_by_R2 * R
C = Sqr(4 * R^{2} + H^{2}) / H
D = H \land 2 + 4 * (R \land 2 - 1) - 2 * H \land 2 / R \land 2
E = H \land 2 + 4 * (R \land 2 - 1)
F = (R^{2} - 2) / R^{2}
 'Note that VB does not have the ATAN function
 'So, use the relation: ATAN(x) = ASIN(x/sqrt(x^2 + 1))
 'G = (2 / R) * Application.Atan(B)
G = (2 / R) * Application.Asin(B / Sqr(B ^ 2 + 1))
F_22_Coaxial_cyl_outer_to_itself = 1 - 1 / R - A + (1 / Application.Pi()) * (G - (H / (2 * R)) *
 (C * Application.Asin(D / E) - Application.Asin(F)))
End Function
```

Now, using the above Functions, let us draw the graphs for different View factors for two concentric cylinders of finite length:

**Prob.5B.14.** Draw the View factor graph for F\_21 for two concentric cylinders of finite length:

### **EXCEL Solution:**

	E382	$\bullet$ $f_x$	=F_21_Coa	axial_cyl_o	outer_surfa	ce_to_inn	er_surface	(D382,\$E\$3{
	С	D	E	F	G	Н	I.	J
380								
381		(R1_by_R2)\L/R2	0.1	0.25	0.5	1	2	1.00E+06
382		0.001	6.35E-05	0.000156	0.000295	0.0005	0.000705	0.001
383		0.1						
384		0.15						
385		0.2						
386		0.25						
387		0.3						
388		0.35						
389		0.4						
390		0.45						
391		0.5						
392		0.55						
393		0.6						
394		0.65						
395		0.7						
396		0.75						
397		0.8						
398		0.85						
399		0.9						
400		0.95						
401		1						

1. Set up a Table in EXCEL worksheet as shown below:

In the above screen shot, column D contains R1/R2 values and row 381 contains L/R2 values.

We use the VBA Function written above to calculate the view factor  $F_{21}$  (i.e. view factor from surface 2 to surface 1), for given values of R1/R2 and L/R2. Formula entered in cell E382 can be seen in the Formula bar. Use relative references to R1/R2 so that we can drag-copy to fill up the Table later. Similarly, write formulas for other L/R2 values as shown above.

2. Now, select cells E382 to J382 and drag-copy to the end of the Table, i.e. up to the cell J401. The calculations are done immediately and the Table is filled up:

(R1_by_R2)\L/R2	0.1	0.25	0.5	1	2	1.00E+06
0.001	6.35E-05	0.000156	0.000295	0.0005	0.000705	0.001
0.1	0.006898	0.016903	0.031709	0.052666	0.072561	0.1
0.15	0.010836	0.026504	0.04945	0.081196	0.110487	0.15
0.2	0.015184	0.037054	0.0687	0.111382	0.149592	0.2
0.25	0.020024	0.048732	0.089691	0.143382	0.189938	0.25
0.3	0.02546	0.061759	0.112694	0.177367	0.231589	0.3
0.35	0.031626	0.076413	0.138032	0.213524	0.274613	0.35
0.4	0.038702	0.093054	0.166088	0.252053	0.319078	0.4
0.45	0.046929	0.112143	0.197321	0.29317	0.365058	0.45
0.5	0.056639	0.134292	0.232274	0.337106	0.412628	0.5
0.55	0.068305	0.160317	0.271598	0.384111	0.461868	0.55
0.6	0.082621	0.191324	0.316063	0.434452	0.512864	0.6
0.65	0.100646	0.228841	0.366573	0.488412	0.565709	0.65
0.7	0.124079	0.274996	0.424187	0.546297	0.620505	0.7
0.75	0.155797	0.332781	0.490118	0.608437	0.677368	0.75
0.8	0.201036	0.406394	0.565734	0.675197	0.736435	0.8
0.85	0.270142	0.501603	0.652549	0.746993	0.797878	0.85
0.9	0.385426	0.625941	0.752238	0.824349	0.86193	0.9
0.95	0.59705	0.788409	0.86678	0.908035	0.928971	0.95
1	1	1	1	1	1	1

3. Now, draw the graphs in EXCEL:

\_\_\_\_\_



\_\_\_\_\_

### **Prob.5B.15.** Draw the View factor graph for F\_12 for two concentric cylinders of finite length:

### **EXCEL Solution:**

1. Set up a Table in EXCEL worksheet as shown below:

	E433	•	• (• <i>f<sub>x</sub></i> =F	_22_Coaxial_cyl_o	uter_to_it	self(D433,	SE\$432)				
	А	В	С	D	E	F	G	Н	I.	J	К
431											
432				(R1_by_R2)\L/R2	0.1	0.25	0.5	1	2	4	1.00E+07
433				0.001	0.048719	0.117139	0.219068	0.381671	0.585286	0.763227	0.999
434				0.1							
435				0.15							
436				0.2							
437				0.25							
438				0.3							
439				0.35							
440				0.4							
441				0.45							
442				0.5							
443				0.55							
444				0.6							
445				0.65							
446				0.7							
447				0.75							
448				0.8							
449				0.85							
450				0.9							
451				0.95							
452				1							

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In the above screen shot, column D contains R1/R2 values and row 483 contains L/R2 values.

We use the VBA Function written above to calculate the view factor  $F_{12}$  (i.e. view factor from surface 1 to surface 2), for given values of R1/R2 and L/R2. Formula entered in cell E484 can be seen in the Formula bar. Use relative references to R1/R2 so that we can drag-copy to fill up the Table later. Similarly, write formulas for other L/R2 values as shown above.

2. Now, select cells E484 to K484 and drag-copy to the end of the Table, i.e. up to the cell K503. The calculations are done immediately and the Table is filled up:

(R1_by_R2)\L/R2	0.1	0.25	0.5	1	2	4	1.00E+07
0.001	0.063501	0.156076	0.295367	0.50025	0.705033	0.844159	1
0.1	0.068977	0.169028	0.317088	0.526657	0.725605	0.856116	1
0.15	0.072242	0.176691	0.329665	0.541305	0.736581	0.862386	1
0.2	0.075922	0.185272	0.343501	0.556909	0.74796	0.868814	1
0.25	0.080096	0.194929	0.358764	0.573527	0.759752	0.875401	1
0.3	0.084865	0.205863	0.375646	0.591225	0.771964	0.882152	1
0.35	0.09036	0.218324	0.394376	0.610069	0.784608	0.889069	1
0.4	0.096756	0.232634	0.415221	0.630133	0.797696	0.896156	1
0.45	0.104287	0.249207	0.438491	0.651488	0.81124	0.903421	1
0.5	0.113279	0.268584	0.464549	0.674212	0.825256	0.910868	1
0.55	0.124191	0.291485	0.493815	0.698384	0.83976	0.918508	1
0.6	0.137702	0.318874	0.526771	0.724086	0.854773	0.92635	1
0.65	0.154841	0.352063	0.563959	0.751403	0.870321	0.934408	1
0.7	0.177256	0.392851	0.605982	0.780424	0.886435	0.942699	1
0.75	0.20773	0.443708	0.653491	0.81125	0.903157	0.951246	1
0.8	0.251295	0.507992	0.707167	0.843996	0.920544	0.960079	1
0.85	0.317814	0.590121	0.767705	0.878816	0.93868	0.969245	1
0.9	0.428251	0.69549	0.83582	0.915943	0.9577	0.978815	1
0.95	0.628474	0.829904	0.9124	0.955826	0.977864	0.988926	1
1	1	1	1	1	1	1	1

### 3. Now, draw the graphs in EXCEL:



**Prob.5B.16.** Draw the View factor graph for F\_22 for two concentric cylinders of finite length:

### **EXCEL Solution:**

1. Set up a Table in EXCEL worksheet as shown below:

	E433	-	• (• <i>f<sub>x</sub></i> =F	_22_Coaxial_cyl_o	uter_to_it	self(D433,	\$E\$432)				
	А	В	С	D	E	F	G	Н	1	J	К
431											
432				(R1_by_R2)\L/R2	0.1	0.25	0.5	1	2	4	1.00E+07
433				0.001	0.048719	0.117139	0.219068	0.381671	0.585286	0.763227	0.999
434				0.1							
435				0.15							
436				0.2							
437				0.25							
438				0.3							
439				0.35							
440				0.4							
441				0.45							
442				0.5							
443				0.55							
444				0.6							
445				0.65							
446				0.7							
447				0.75							
448				0.8							
449				0.85							
450				0.9							
451				0.95							
452				1							

In the above screen shot, column D contains R1/R2 values and row 432 contains L/R2 values.

We use the VBA Function written above to calculate the view factor  $F_{22}$  (i.e. view factor from surface 2 to itself), for given values of R1/R2 and L/R2. Formula entered in cell E433 can be seen in the Formula bar. Use relative references to R1/R2 so that we can drag-copy to fill up the Table later. Similarly, write formulas for other L/R2 values as shown above.



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2. Now, select cells E433 to K433 and drag-copy to the end of the Table, i.e. up to the cell K452. The calculations are done immediately and the Table is filled up:

(R1_by_R2)\L/R2	0.1	0.25	0.5	1	2	4	1.00E+07
0.001	0.04872	0.11714	0.21907	0.38167	0.58529	0.76323	0.999
0.1	0.04557	0.10929	0.2036	0.35241	0.53573	0.69341	0.9
0.15	0.04396	0.1053	0.19574	0.33755	0.51061	0.65806	0.85
0.2	0.04235	0.10128	0.18783	0.32259	0.48536	0.62262	0.8
0.25	0.04071	0.09722	0.17983	0.30749	0.45994	0.58705	0.75
0.3	0.03906	0.0931	0.17173	0.29223	0.43431	0.55131	0.7
0.35	0.03738	0.08892	0.16351	0.27674	0.40842	0.51537	0.65
0.4	0.03566	0.08465	0.15512	0.261	0.38222	0.47919	0.6
0.45	0.03391	0.08028	0.14654	0.24493	0.35563	0.44271	0.55
0.5	0.0321	0.07579	0.13773	0.22848	0.3286	0.4059	0.5
0.55	0.03023	0.07114	0.12863	0.21157	0.30104	0.36871	0.45
0.6	0.02829	0.06632	0.11919	0.1941	0.27286	0.33107	0.4
0.65	0.02625	0.06126	0.10932	0.17596	0.24395	0.29291	0.35
0.7	0.0241	0.05591	0.09891	0.157	0.21418	0.25415	0.3
0.75	0.02179	0.05019	0.08782	0.137	0.18337	0.21471	0.25
0.8	0.01927	0.04397	0.07582	0.11567	0.1513	0.17444	0.2
0.85	0.01645	0.03703	0.06253	0.09259	0.11765	0.1332	0.15
0.9	0.01316	0.02895	0.0473	0.06702	0.08197	0.09074	0.1
0.95	0.00894	0.0187	0.02861	0.03758	0.04347	0.04669	0.05
1	0	0	0	5.6E-17	-2.8E-17	4.2E-17	2.9E-12

3. Now, draw the graphs in EXCEL:



**Prob.5B.17.** Consider a hemispherical oven of 1 m diameter. Find the View factor of curved surface to itself. (= F22). The curved surface is at 800 K and the base is at 300 K. If both the surfaces can be considered as black, determine the heat transfer rate per unit length. Assume the furnace to be very long.



Fig.Prob.5B.17

### **EXCEL Solution:**

Since the furnace is very long, there is no heat loss at the ends.

Excel worksheet is shown below. Formulas used are also shown in the worksheet.

С	D	E	F	G	Н	1
Data:						
	D	1	m			
	T1	300	К			
	T2	800	K			
	L	1	m			
	sigma	5.67E-08	W/m^2.K^4	Stefan	-Boltzmani	n const.
Calculations:						
	A1	1	m^2	→ A1=	D*L	
	A2	1.57080	m^2 🔍			••
				A2=	PI* (D/2)*	- L
For Surface 1:	F11 + F12 = 1	by Summation r	ule			
	But, F11 = 0since surface is flat					
Therefore:	F12 =	1				
Then, by reciprocity:	A1 * F12 = A2 * F21					
Therefore:	F21 = (A1 / A2) * F12					
i.e.	F21 =	0.63662				

For Surface 2:	F21 + F22 = 1	by Summation rule	e
Then:	F22 =	0.36338	Ans.
Heat Transferred:	Q12 = A2 * F21 * sigma * (T2^4 - T1^4)		
Then:	Q12 =	22765.05	WAns.

Thus:

F22 = View factor of curved surface to itself = 0.36338 ... Ans.

Q12 = Net heat transfer between surface 2 and 1 = 22765 W...Ans.



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\* Figures taken from London Business School's Masters in Management 2010 employment report



Prob.5B.18. Consider a thin, hollow cylinder of 10 cm dia and 10 cm length, as shown in Fig. below.

- a) What is the view factor of the curved surface to itself (i.e. F\_33)?
- b) If T1 = 800 K, T2 = 400 K, find the net heat transfer between surfaces 1 and 2 (Q\_12), considering all surfaces as black.
- c) Also, plot the variation of F\_33 and Q\_12 as L varies from 0.1 m to 0.5 m



Fig.Prob.5B.18

### **EXCEL Solution:**

Top, bottom and side surfaces are designated as 1, 2 and 3 respectively.

Let the radius be R and length, L.

Excel worksheet is shown below. Formulas used are also shown in the worksheet.

Following are the steps:

1. Set up the EXCEL worksheet, enter data:

	Α	В	С	D	E	F
565			Data:			
566				R	0.05	m
567				T1	800	К
568				T2	400	К
569				L	0.1	m
570				sigma	5.67E-08	W/m^2.K^4
571						

### 2. Perform the calculations:

	E584	•	f <sub>x</sub>	=F_ij_Co	xial_parallel_discs(\$E\$569/\$E\$566,\$E\$566/\$E\$569)				
	А	В	С		D	E	F		
571									
572			Calculations:						
573					A1 = PI() * R^2 =	0.007854	m^2		
574					A2 = A1	0.007854	m^2		
575					A3 = 2 * PI() * R * L =	0.031415927	m^2		
576									
577			For surface 1:		F_11 = 0, since surface 1 is flat				
578					F_11 + F_12 + F_13 = 1 By Summation	rule			
579			Therefore:						
580					F_12 + F_13 = 1				
581									
582					Calculate F_12 using the VBA Function:				
583							_		
584					F_12	0.171572875	_ <b> </b>		
585									
586			Then:		F_13 = (1 - F_12) =	0.828427125			

Note that F\_12 is calculated using the VBA Function written above for coaxial, parallel disks. See the Formula bar in the above screen shot.

Continuing, we have:

A	В	С	D	E	F
587					
588		Calculate F_31 using the	e reciprocity relation for surfaces 1 and 3	3:	
589					
590			i.e. A1 * F13 = A3 * F31		
591					
592		Therefore:	F_31 = (A1 * F_13) / A3 =	0.207106781	
593					
594		For surface 3:			
595			F_31 + F_32 + F_33 = 1 By Summation	rule	
596					
597		Also, for surface 3:	F_31 = F_32 by Symmetry		
598		Therefore:	F_32 =	0.207106781	
599					
600		Therefore:	F_33 = 1 - (F_31 + F_32)		
601					
602		So, we get:	F_33 =	0.5858	Ans.
603					
604		Net heat transfer Q_12:			
605			Q_12 = A1 * F_12 * sigma * (T1^4 - T2^4)		
606					
607			Q_12 =	29.340	W Ans.

Software Solutions to Problems on Heat Transfer: Radiation Heat Transfer – Part I

Thus:

View factor of the top surface to bottom surface (i.e.  $F_{12} = 0.1716 \dots Ans$ .

View factor of the curved surface to itself (i.e. F\_33) = 0.5858 ... Ans.

Net heat transfer between surfaces 1 and  $2 = Q_{12} = 29.34 \text{ W} \dots \text{ Ans.}$ 



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### To plot the variation of F\_33 and Q\_12 as L varies from 0.1 m to 0.5 m:

3. Set up a Table as shown below. Remember to enter L with relative reference in the equations for F\_12, F\_13 ... etc.

	D613	•	• (•	oaxial_parallel_discs(C613/\$E\$566,\$E\$56	56/C613)			
	А	В	С	D	E	F	G	Н
611								
612			L (m)	F_12	F_13	F_31	F_33	Q_12 (W)
613			0.1	0.1716	0.8284	0.2071	0.5858	29.340
614			0.12					
615			0.14					
616			0.16					
617			0.18					
618			0.2					
619			0.22					
620			0.24					
621			0.26					
622			0.28					
623			0.3					
624			0.32					
625			0.34					
626			0.36					
627			0.38					
628			0.4					
629			0.42					
630			0.44					
631			0.46					
632			0.48					
633			0.5					

Equation entered for F\_12 in cell D613 can be seen in the Formula bar in the above screen shot.

4. Now, select cells D613 to H613 and drag-copy to the end of the Table, i.e. up to cell H633. Then, immediately, the calculations are done and the Table gets filled up:

L (m)	F_12	F_13	F_31	F_33	Q_12 (W)
0.1	0.1716	0.8284	0.2071	0.5858	29.340
0.12	0.1311	0.8689	0.1810	0.6380	22.415
0.14	0.1027	0.8973	0.1602	0.6795	17.562
0.16	0.0823	0.9177	0.1434	0.7132	14.065
0.18	0.0671	0.9329	0.1296	0.7409	11.482
0.2	0.0557	0.9443	0.1180	0.7639	9.530
0.22	0.0469	0.9531	0.1083	0.7834	8.023
0.24	0.0400	0.9600	0.1000	0.8000	6.840
0.26	0.0345	0.9655	0.0928	0.8143	5.896
0.28	0.0300	0.9700	0.0866	0.8268	5.131
0.3	0.0263	0.9737	0.0811	0.8377	4.503
0.32	0.0233	0.9767	0.0763	0.8474	3.983
0.34	0.0207	0.9793	0.0720	0.8560	3.546
0.36	0.0186	0.9814	0.0682	0.8637	3.177
0.38	0.0167	0.9833	0.0647	0.8706	2.862
0.4	0.0152	0.9848	0.0616	0.8769	2.592
0.42	0.0138	0.9862	0.0587	0.8826	2.357
0.44	0.0126	0.9874	0.0561	0.8878	2.153
0.46	0.0115	0.9885	0.0537	0.8926	1.974
0.48	0.0106	0.9894	0.0515	0.8969	1.816
0.5	0.0098	0.9902	0.0495	0.9010	1.677

=====

### 5. Now, plot the results in EXCEL:





\_\_\_\_\_
To see the Part II download: Software Solutions to problems on heat transfer – Radiation Heat Transfer II