Software Solutions to Problems on Heat Transfer

Radiation Heat Transfer – Part II

Dr. M. Thirumaleshwar

bookboon.com
The eBook company

Dr. M. Thirumaleshwar

Software Solutions to Problems on Heat Transfer
RADIATION HEAT TRANSFER – PART-II

(Electrical network method & Radiation energy exchange in 2-zone and 3-zone enclosures, Radiation shielding, Radiation error in temperature measurement)

Software Solutions to Problems on Heat Transfer: Radiation Heat Transfer – Part II
(Electrical network method & Radiation energy exchange in 2-zone and 3-zone enclosures, Radiation shielding, Radiation error in temperature measurement)
1st edition

ISBN 978-87-403-0627-9

Contents

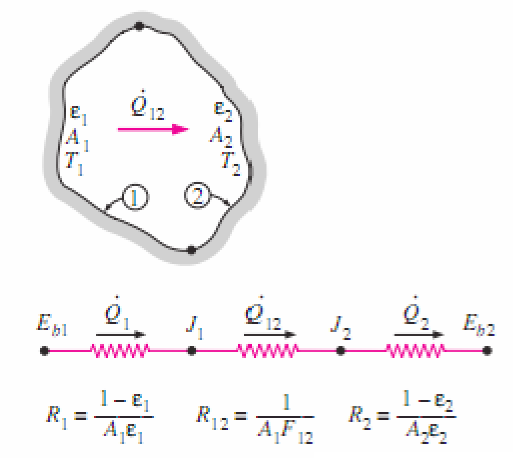
5C Electrical network method & Radiation energy exchange between gray surfaces:

5.C.1 Radiation energy exchange in 2-surface enclosures:

Prob. 5.C.1.1. Write Mathcad Functions for a general two-surface enclosures, and also for few special cases of two-surface enclosures:

Radiation heat exchange for a general two surface enclosure:

Following is the schematic diagram and the radiation network (Ref: Cengel):



Note: In the following Function, A1, A2 are areas of inner and outer surfaces(m), ε1,ε2 are emissivities of the surfaces;

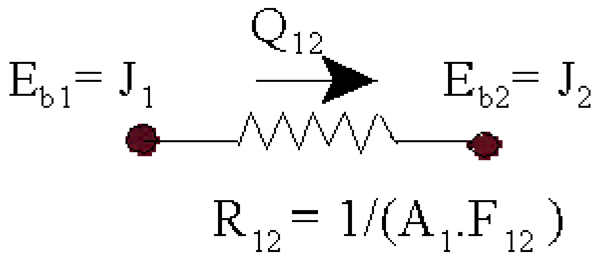
T1, T2 are temps. in Kelvin, F12 is the view factor from surface 1 to 2.

$$Q12\_two\_surface\_enclosure(A1, A2, F12, \varepsilon1, \varepsilon2, T1, T2) := \frac{5.67\cdot10^{-8}\cdot\left(T1^4 - T2^4\right)}{\left(\frac{1-\varepsilon1}{A1\cdot\varepsilon1}\right) + \frac{1}{A1\cdot F12} + \left(\frac{1-\varepsilon2}{A2\cdot\varepsilon2}\right)}$$

Example: $A1 := 6.283$ $A2 := 3.142$ $\varepsilon1 := 0.8$ $\varepsilon2 := 0.5$ $F12 := 0.5$ $T1 := 800$ $T2 := 600$

$$Q12\_two\_surface\_enclosure(A1, A2, F12, \varepsilon1, \varepsilon2, T1, T2) = 2.347\times10^4 \text{ W}$$

Radiation heat exchange between two black surfaces:



In the Function given below:

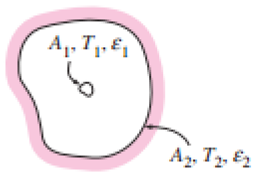
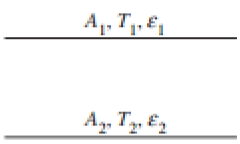
We have: A1 (m2), F12 is view factor from surface 1 to surface 2, T1, T2 are temps. in Kelvin

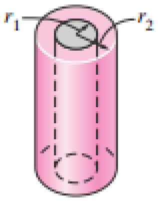
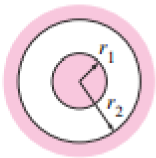
$$Q12\_black\_surfaces(A1,F12,T1,T2) := 5.67\cdot10^{-8}\cdot A1\cdot F12\cdot\left(T1^4 - T2^4\right) \quad W$$

Example: $A1 := 0.785$ $F12 := 0.232$ $T1 := 1000$ $T2 := 600$

$$Q12\_black\_surfaces(A1,F12,T1,T2) = 8.988\times10^{3} \quad W$$

Few special cases of two-surface enclosures: (Ref: Cengel)

Case		Conditions	Equation	
Small object in a large cavity		$\frac{A_1}{A_2} \approx 0$ $F_{12} = 1$	$\dot{Q}_{12} = A_1\sigma\varepsilon_1(T_1^4 - T_2^4)$	(22-37)
Infinitely large parallel plates		$A_1 = A_2 = A$ $F_{12} = 1$	$\dot{Q}_{12} = \dfrac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$	(22-38)

Infinitely long concentric cylinders 	$\frac{A_1}{A_2} = \frac{r_1}{r_2}$ $F_{12} = 1$	$\dot{Q}_{12} = \dfrac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$	(22-39)
Concentric spheres 	$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$ $F_{12} = 1$	$\dot{Q}_{12} = \dfrac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)^2}$	(22-40)

Mathcad Functions for the above cases are written below:

1. **Radiation heat exchange for a small object in a large enclosure: (ex: pipe in a large Plant room)**

   Note: In the Function below, A1 (m2), ε1 is emissivity of the small object, T1, T2 are temps. in Kelvin

   $$Q12\_small\_object(A1, \varepsilon 1, T1, T2) := 5.67 \cdot 10^{-8} \cdot A1 \cdot \varepsilon 1 \cdot \left(T1^4 - T2^4\right) \quad W$$

   Example: $A1 := 0.157 \quad \varepsilon 1 := 0.6 \quad T1 := 93 + 273 \quad T2 := 20 + 273$

   $$Q12\_small\_object(A1, \varepsilon 1, T1, T2) = 56.478 \quad W$$

---

2. **Radiation heat exchange between infinitely large parallel plates:**

   Note: In the Function below, A (m2), ε1, ε2 are emissivities of the surfaces, T1, T2 are temps. in Kelvin

   $$Q12\_parallel\_plates(A, \varepsilon 1, \varepsilon 2, T1, T2) := \frac{5.67 \cdot 10^{-8} \cdot A \cdot \left(T1^4 - T2^4\right)}{\frac{1}{\varepsilon 1} + \frac{1}{\varepsilon 2} - 1} \quad W$$

   Example: $A := 1 \quad T1 := 800 + 273 \quad T2 := 300 + 273 \quad \varepsilon 1 := 0.3 \quad \varepsilon 2 := 0.6$

   $$Q12\_parallel\_plates(A, \varepsilon 1, \varepsilon 2, T1, T2) = 1.726 \times 10^4 \quad W/m^2$$

---

3. **Radiation heat exchange between infinitely long concentric cylinders:**

   Note: In the Function below, L (m), r1, r2 are radii of inner and outer cylinders(m), ε1, ε2 are emissivities of the surfaces, T1, T2 are temps. in Kelvin

   $$Q12\_long\_cylinders(L, r1, r2, \varepsilon 1, \varepsilon 2, T1, T2) := \frac{5.67 \cdot 10^{-8} \cdot (2 \cdot \pi \cdot r1 \cdot L) \cdot \left(T1^4 - T2^4\right)}{\frac{1}{\varepsilon 1} + \frac{r1}{r2} \cdot \left(\frac{1}{\varepsilon 2} - 1\right)}$$

   Example: $\varepsilon 1 := 1 \quad \varepsilon 2 := 0.7 \quad T1 := 950 \quad T2 := 500 \quad r1 := 0.1 \quad r2 := 0.25 \quad L := 1$

   $$Q12\_long\_cylinders(L, r1, r2, \varepsilon 1, \varepsilon 2, T1, T2) = 2.287 \times 10^4 \quad W/m$$

---

4. **Radiation heat exchange between concentric spheres:**

Note: In the Function below, r1, r2 are radii of inner and outer spheres(m), ε1, ε2 are emissivities of the surfaces, T1, T2 are temps. in Kelvin

$$Q12\_concentric\_spheres(r1, r2, \varepsilon 1, \varepsilon 2, T1, T2) := \frac{5.67\cdot 10^{-8}\cdot\left(4\cdot\pi\cdot r1^2\right)\cdot\left(T1^4 - T2^4\right)}{\frac{1}{\varepsilon 1} + \left(\frac{r1}{r2}\right)^2\cdot\left(\frac{1}{\varepsilon 2} - 1\right)} \quad W$$

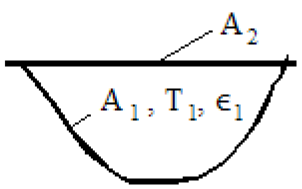
Example: $T1 := -183 + 273$

$T2 := 25 + 273 \quad \varepsilon 1 := 0.2 \quad \varepsilon 2 := 0.25 \quad r1 := 0.15 \quad r2 := 0.2$

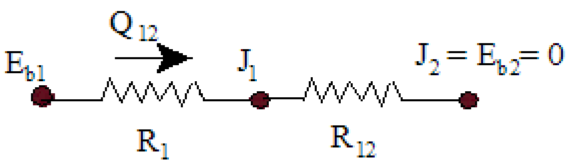
$Q12\_concentric\_spheres(r1, r2, \varepsilon 1, \varepsilon 2, T1, T2) = -18.748$ W... -ve sign indicates that heat flows from outer sphere to inner sphere, i.e. from surface 2 to surface 1.

---

5. **Energy radiated from a gray cavity:**



(a) Gray cavity



$R_1 = (1-\epsilon_1)/(A_1.\epsilon_1)$

$R_{12} = 1/(A_1.F_{12})$

(b) Radiation network

$$Q_{12} = A_1\cdot\varepsilon_1\cdot\sigma\cdot T_1^4\cdot\left[\frac{1-F_{11}}{1-\left(1-\varepsilon_1\right)\cdot F_{11}}\right]$$ W....net radiation from gray cavity..(13.62)

Note: In the Function below, A1 is area of cavity surface(m^2), ε1 is emissivity of the surface, T1 is temp. in Kelvin, F11 is the view factor from surface 1 to itself = 1-(A2/A1) where A2 is area of closing surface.

$$Q12\_from\_gray\_cavity(A1, F11, \varepsilon1, T1) := 5.67 \cdot 10^{-8} \cdot A1 \cdot \varepsilon1 \cdot T1^4 \cdot \left[\frac{1 - F11}{1 - (1 - \varepsilon1) \cdot F11} \right] \quad W$$

Example: $F11 := 0.857 \quad A1 := 2.199 \cdot 10^{-3} \quad \varepsilon1 := 0.6 \quad T1 := 350 + 273$

$$Q12\_from\_gray\_cavity(A1, F11, \varepsilon1, T1) = 2.452 \quad W$$

==

Prob. 5C.1.2. A convex grey body having a surface area of 4 m² has ε_1= 0.35 and T_1= 680 K. This is completely enclosed by a grey surface having an area of 36 m², ε_2=0.75 and T_2 = 310 K. Find the net rate of heat transfer Q_{12} between the two surfaces. [M.U. May, 1999]

In addition: If ε1 varies from 0.1 to 0.6, plot the variation of Q12, all other quantities remaining the same:

Mathcad Solution:

Data:

$T1 := 680$ K $\quad T2 := 310$ K $\quad \varepsilon_1 := 0.35 \quad \varepsilon_2 := 0.75$

$\sigma := 5.67 \cdot 10^{-8}$ W.m^2.K^4 $\quad A1 := 4$ m^2 $\quad A2 := 36$ m^2

$F_{12} := 1$ ..since all the heat radn. emitted by surface 1 is intercepted by surface 2.

Solution:

Starting from Fundamentals:

$E_{b1} := \sigma \cdot T1^4$ i.e. $E_{b1} = 1.212 \times 10^4$ W/m^2

$E_{b2} := \sigma \cdot T2^4$ i.e. $E_{b2} = 523.636$ W/m^2

$R12 := \dfrac{1}{A1 \cdot F_{12}}$ i.e. $R12 = 0.25$ 1/m^2

$R1 := \dfrac{1 - \varepsilon_1}{\varepsilon_1 \cdot A1}$ i.e. $R1 = 0.464$ 1/m^2

$R2 := \dfrac{1 - \varepsilon_2}{\varepsilon_2 \cdot A2}$ i.e. $R2 = 9.259 \times 10^{-3}$ 1/m^2

$R_{tot} := R1 + R12 + R2$ i.e. $R_{tot} = 0.724$ 1/m^2

Therefore:

$Q_{12} := \dfrac{E_{b1} - E_{b2}}{R_{tot}}$ i.e. $Q_{12} = 1.603 \times 10^4$ Watts......Ans.

Using the Mathcad Function for a general 2-surface enclosure:

We have the Function written earlier:

$$Q12\_two\_surface\_enclosure(A1, A2, F12, \varepsilon1, \varepsilon2, T1, T2) := \frac{5.67 \cdot 10^{-8} \cdot \left(T1^4 - T2^4\right)}{\left(\frac{1-\varepsilon1}{A1 \cdot \varepsilon1}\right) + \frac{1}{A1 \cdot F12} + \left(\frac{1-\varepsilon2}{A2 \cdot \varepsilon2}\right)}$$

Then, we write:

$$Q12 := Q12\_two\_surface\_enclosure\left(A1, A2, F_{12}, \varepsilon_1, \varepsilon_2, T1, T2\right)$$

i.e. $Q12 = 1.603 \times 10^4$ W.....Ans.. same as obtained above.

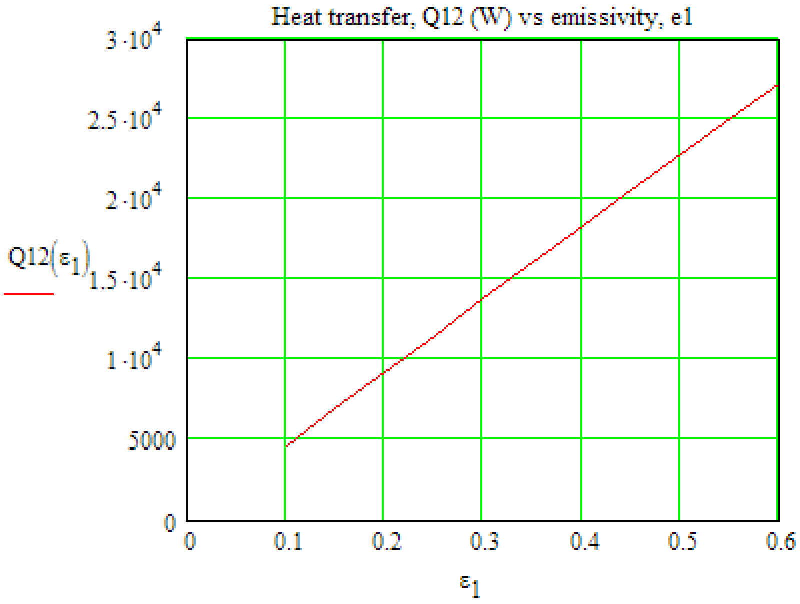
===

If ε1 varies from 0.1 to 0.6, plot the variation of Q12, all other quantities remaining the same:

Let: $Q12(\varepsilon_1) := Q12\_two\_surface\_enclosure\left(A1, A2, F_{12}, \varepsilon_1, \varepsilon_2, T1, T2\right)$ ...define Q12 as a function of ε1

$\varepsilon_1 := 0.1, 0.15 .. 0.6$ ...define a range variable

$\varepsilon_1 =$	$Q12(\varepsilon_1) =$
0.1	$4.623 \cdot 10^3$
0.15	$6.921 \cdot 10^3$
0.2	$9.211 \cdot 10^3$
0.25	$1.149 \cdot 10^4$
0.3	$1.377 \cdot 10^4$
0.35	$1.603 \cdot 10^4$
0.4	$1.829 \cdot 10^4$
0.45	$2.054 \cdot 10^4$
0.5	$2.278 \cdot 10^4$
0.55	$2.501 \cdot 10^4$
0.6	$2.723 \cdot 10^4$



==

Prob.5C.1.3. Calculate the net radiant heat interchange per sq. meter for two large parallel plates maintained at 800 C and 300 C. The emissivities of two plates are 0.3 and 0.6 respectively. [M.U. 1993]

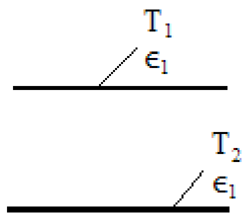


Fig. Two infinitely large parallel plates

Mathcad Solution:

Data:

$T1 := 800 + 273$ K $T2 := 300 + 273$ K $\varepsilon 1 := 0.3$ $\varepsilon 2 := 0.6$

$\sigma := 5.67 \cdot 10^{-8}$ W/m^2.K^4....Stefan Boltzmann const.

Solution:

We have:

$$q := \frac{\sigma \cdot \left(T1^4 - T2^4\right)}{\frac{1}{\varepsilon 1} + \frac{1}{\varepsilon 2} - 1}$$

i.e. $q = 1.726 \times 10^4$ W/m^2.....Ans.

===

Prob.5.C.1.4. Liquid O_2 at atmospheric pressure and temp –183 C is stored in a spherical vessel of outer diameter 0.3 m. The system is insulated by enclosing the container inside another concentric sphere of 0.5 m inner diameter, with space between them evacuated. Both the sphere surfaces are made of aluminum for which emissivity is 0.3. If the temp of outer surface is 40C. estimate the rate of heat flow due to radiation. What will be heat flow if polished Al with an emissivity of 0.05 is used for container walls. [M.U. 1992]

Mathcad Solution:

Data:

$T1 := -183 + 273$ K $\quad T2 := 40 + 273$ K $\quad \sigma := 5.67 \cdot 10^{-8}$ W/m^2.K^4

$D1 := 0.3$ m $\quad D2 := 0.5$ m

$\varepsilon 1 := 0.3 \quad \varepsilon 2 := 0.3$

Calculations:

$A1 := \pi \cdot D1^2$ i.e. $A1 = 0.283$ m^2

$A2 := \pi \cdot D2^2$ i.e. $A2 = 0.785$ m^2

Then:

$$Q12 := \frac{\sigma \cdot A1 \cdot \left(T1^4 - T2^4\right)}{\left(\frac{1}{\varepsilon 1}\right) + \frac{A1}{A2} \cdot \left(\frac{1}{\varepsilon 2} - 1\right)}$$

i.e. $Q12 = -36.618$ **W...Ans., -ve sign indicates heat flow *into* A1**

If the emissivity ε1 changes to 0.05:

$\varepsilon 1 := 0.05$

$$Q12' := \frac{\sigma \cdot A1 \cdot \left(T1^4 - T2^4\right)}{\left(\frac{1}{\varepsilon 1}\right) + \frac{A1}{A2} \cdot \left(\frac{1}{\varepsilon 2} - 1\right)}$$

i.e. $Q12' = -7.333$ **W...Ans., -ve sign indicates heat flow *into* A1**

Therefore, change in heat flow:

$\frac{Q12'}{Q12} = 0.2$ **...reduced to 20% of the earlier case.**

Note: We can also use the Mathcad Function written earlier:

We have:

$\varepsilon1 := 0.3 \quad \varepsilon2 := 0.3 \qquad r1 := 0.15 \ \text{m} \quad r2 := 0.25 \ \text{m}$

Then:

$Q12 := Q12\_concentric\_spheres(r1, r2, \varepsilon1, \varepsilon2, T1, T2)$ ..usind the Function

i.e. $Q12 = -36.618$ **W.... Ans. .... same as obtained above.**

In addition: plot the variation of Q12 as ε1 varies from 0.03 to 0.3, all other quantities remaining the same:

$\varepsilon1 := 0.03, 0.04 .. 0.3$ ...define a range variable

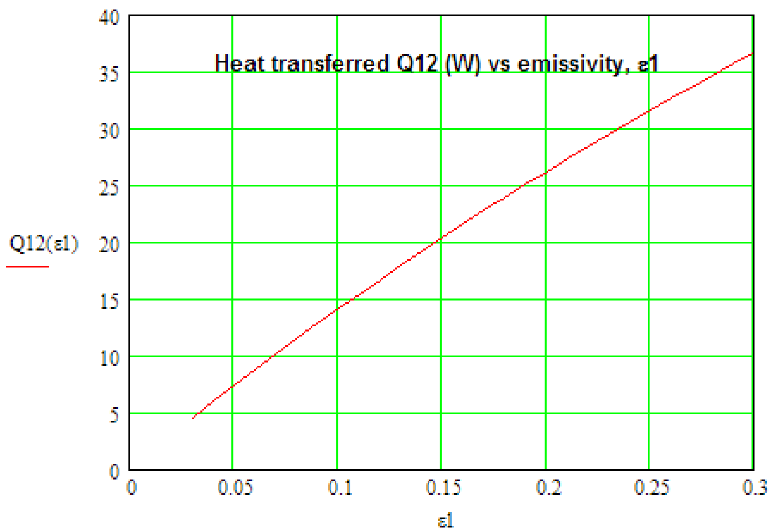
Let: $Q12(\varepsilon1) := |Q12\_concentric\_spheres(r1, r2, \varepsilon1, \varepsilon2, T1, T2)|$

Note that we taken the Absolute value of Q12 to get the heat loss as positive.

We get:

ε1 =	Q12(ε1) =
0.03	4.472
0.04	5.914
0.05	7.333
0.06	8.729
0.07	10.103
0.08	11.456
0.09	12.787
0.1	14.098
0.11	15.388
0.12	16.659
0.13	17.91
0.14	19.143
0.15	20.358
0.16	21.554

ε1 =	Q12(ε1) =
0.17	22.733
0.18	23.894
0.19	25.039
0.2	26.167
0.21	27.28
0.22	28.376
0.23	29.457
0.24	30.523
0.25	31.574
0.26	32.61
0.27	33.633
0.28	34.641
0.29	35.636
0.3	36.618



==

Prob.5.C.1.5. A blind cylindrical hole of diameter and length 3 cm is drilled into metal slab having emissivity 0.6. If the metal slab is maintained at temp 350 C. Find heat escaping out of the hole by radiation. [M.U. 1991]

Mathcad Solution:

This is a problem on determining energy escaping from gray cavity. We use eqn.(13.62), viz.

$$Q_{12} = A_1 \cdot \varepsilon_1 \cdot \sigma \cdot T_1^{\,4} \cdot \left[\frac{1 - F_{11}}{1 - (1 \cdot - \varepsilon_1) \cdot F_{11}} \right]$$ W....net radiation from gray cavity..(13.62)

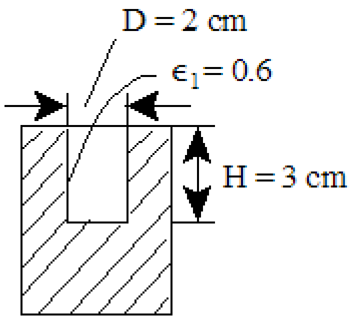


Fig.Prob.5.C.1.5

Data:

$D := 0.02$ m...dia. of cylindrical cavity

$H := 0.03$ m...height of the cylindrical cavity

$\varepsilon_1 := 0.6$ ...emissivity of cyl. surface

$T_1 := 350 + 273$ K....temp. of cavity

$\sigma := 5.67 \cdot 10^{-8}$ W/(m2.K)...Stefan-Boltzmann const.

Now, F_{11} for a cavity is already shown to be:

$$F_{11} = 1 - \frac{A_2}{A_1}$$ where A_2 = area of closing surface, A_1 = area of the cavity surface

i.e. $$F_{11} = 1 - \frac{\frac{\pi \cdot D^2}{4}}{\frac{\pi \cdot D^2}{4} + \pi \cdot D \cdot H}$$ ....for cylindrical cavity of this problem

i.e. $$F_{11} = 1 - \frac{D}{D + 4 \cdot H} = \frac{4 \cdot H}{4 \cdot H + D}$$

Then, from eqn. (13.62):

$$Q_{12} := A_1 \cdot \varepsilon_1 \cdot \sigma \cdot T_1^{\,4} \cdot \left[\frac{1 - F_{11}}{1 - (1 - \varepsilon_1) \cdot F_{11}} \right]$$

i.e. $Q_{12} = 2.45$ **W....energy escaping from the cavity....Ans.**

Therefore, $F_{11} := \frac{4 \cdot H}{4 \cdot H + D}$

i.e. $F_{11} = 0.857$ ...view factor of the cavity w.r.t. itself

And, $A_1 := \pi \cdot D \cdot H + \frac{\pi \cdot D^2}{4}$

i.e. $A_1 = 2.199 \times 10^{-3}$ m^2....area of surface of cylindrical cavity

---

Alternatively:

Use the Mathcad Function written above for the energy escaping from a cavity:

$$Q12 := Q12\_from\_gray\_cavity(A_1, F_{11}, \varepsilon_1, T_1)$$

i.e. $Q12 = 2.45$ W.....Ans.....same as obtained above.

===

Prob.5.C.1.6. A long pipe, 50 mm dia passes through a room and is exposed to air at 20 C. The pipe surface temp is 93 C. Assuming that the emissivity of pipe surface is 0.6, calculate the radiation heat loss per meter length of the pipe. [M.U. 1991]

Mathcad Solution:

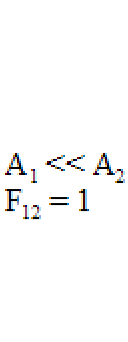


Fig.Prob.5.C.1.6.

The pipe is enclosed by the room; so, it **is two-surface enclosure problem**.

Further, area of the pipe is *very small*, compared to the area of the room. Therefore, this is a case of a *small object surrounded by a large area*, and we have:

$$\frac{A_1}{A_2} = 0$$

and, $F_{12} = 1$

$$Q_{12} = A_1 \cdot \sigma \cdot \varepsilon_1 \cdot \left(T_1^{\,4} - T_2^{\,4}\right)$$ ....for small object in a large cavity.....(13.58)

Data:

$d_1 := 0.05$ m....dia. of the pipe

$L := 1$ m....length of pipe

$\varepsilon_1 := 0.6$ ..emissivity of the surface of the pipe

$T_1 := 93 + 273$ K...temperature of the pipe

$T_2 := 20 + 273$ K...temperature of surroundings

$\sigma := 5.67 \cdot 10^{-8}$ W/(m².K)...Stefan-Boltzmann const.

Now, $A_1 := \pi \cdot d_1 \cdot L$

i.e. $A_1 = 0.157$ m²....surface area of the pipe per metre length

Then, applying eqn. (13.58), we get:

$$Q_{12} := A_1 \cdot \sigma \cdot \varepsilon_1 \cdot \left(T_1^{\ 4} - T_2^{\ 4}\right)$$

i.e. $Q_{12} = 56.507$ **W...net radiant heat loss from the pipe per metre length...Ans.**

Alternatively: Use the Function written earlier:

$Q12 := Q12\_small\_object(A_1, \varepsilon_1, T_1, T_2)$

i.e. $Q12 = 56.507$ **W.....Ans.....same as obtained above.**

==

Prob.5.C.1.7: A spherical steel ball, 50 mm in diameter, at a temperature of 600 deg. C, is taken out of a furnace and rests on the floor of a foundry room. Assuming that the surroundings are at a temperature of 30 deg.C, and the emissivity of the surface of the ball to be 0.8, calculate the net radiant heat loss from the ball.

(b) Also, plot the heat transferred Q12 as emissivity of ball surface varies from 0.1 to 0.9:

Mathcad Solution:

The steel ball is enclosed by the room; so, it is a *two-surface enclosure* problem.

Further, area of the ball is very small, compared to the area of the room.

Therefore, *this is a case of a small object surrounded by a large area,* and we use the Mathcad Function written earlier:

Data:

$r_1 := 0.025$ m....radius of the ball

$\varepsilon_1 := 0.8$ ..emissivity of the surface of the ball

$T_1 := 600 + 273$ K...temperature of the ball

$T_2 := 30 + 273$ K...temperature of surroundings

$\sigma := 5.67 \cdot 10^{-8}$ W/(m².K)...Stefan-Boltzmann const.

Now, $A_1 := 4 \cdot \pi \cdot r_1^{\,2}$

i.e. $A_1 = 7.854 \times 10^{-3}$ m²....surface area of the ball

Using the Mathcad Function written earlier:

$Q12 := Q12\_small\_object(A_1, \varepsilon_1, T_1, T_2)$

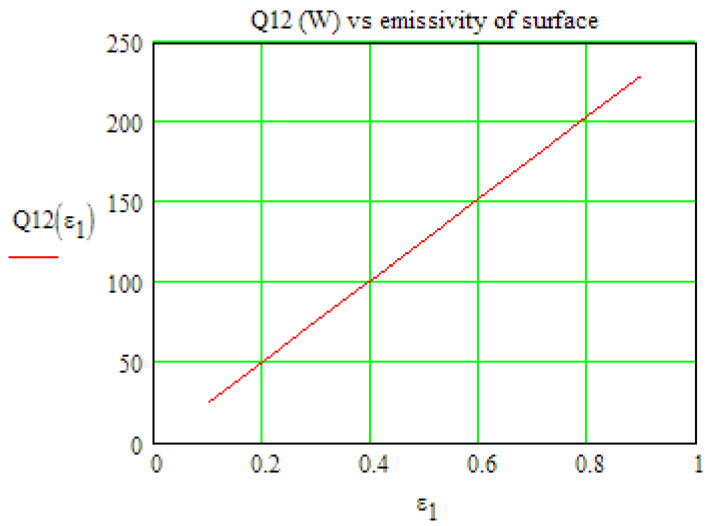
i.e. $Q12 = 203.925$ **W.....Ans.**

Also, plot the heat transferred Q12 as ε_1 varies from 0.1 to 0.9:

$Q12(\varepsilon_1) := Q12\_small\_object(A_1, \varepsilon_1, T_1, T_2)$ ...express Q12 as a Function of ε1

$\varepsilon_1 := 0.1, 0.2 .. 0.9$ ....define a range variable

$\varepsilon_1 =$	$Q12(\varepsilon_1) =$
0.1	25.491
0.2	50.981
0.3	76.472
0.4	101.963
0.5	127.453
0.6	152.944
0.7	178.435
0.8	203.925
0.9	229.416



==

Prob.5.C.1.8. A hemispherical furnace of radius 1.0 m has a roof temperature of T1 = 800 K and emissivity ε1 = 0.8. The flat circular floor of the furnace has a temperature of T2 = 600 K and emissivity ε2 = 0.5. Calculate the net radiant heat exchange between the roof and the floor. [M.U. 1998]

(b) Also, plot Q12 as the emissivity of base, ε_2 varies from 0.1 to 0.8.

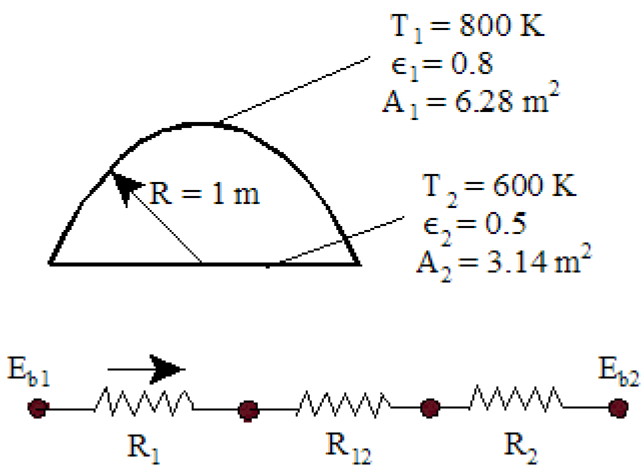


Fig.Prob.5.C.1.8

Mathcad Solution:

This is *a two-zone enclosure problem*.

Fig. above shows the radiation network for this problem.

We have:

$$Q_{12} = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2}$$

where R1 and R2 are the two *surface resistances* and,

R12 is the *space resistance* between the two radiosity potentials.

Data:

$T_1 := 800$ K...temp. of surface 1 (i.e. hemisphere)

$T_2 := 600$ K...temp. of surface 2 (i.e. base)

$\varepsilon_1 := 0.8$ ...emissivity of surface 1

$\varepsilon_2 := 0.5$ ...emissivity of surface 2

$R := 1$ m....radius of surface 1

$A_1 := \frac{4 \cdot \pi \cdot R^2}{2}$ m2....area of hemispherical surface 1

i.e. $A_1 = 6.283$ m2....area of surface 1

$A_2 := \pi \cdot R^2$ m2....area of surface 2

i.e. $A_2 = 3.142$ m2....area of surface 2

$\sigma := 5.67 \cdot 10^{-8}$ W/(m2.K)...Stefan-Boltzmann const.

View factors:

$F_{21} := 1$ **..since all the heat radiated by surface 2 is intercepted by hemispherical surface 1.**

Now, $A_1 \cdot F_{12} = A_2 \cdot F_{21}$ **...by reciprocity**

Therefore, $F_{12} := \frac{A_2 \cdot F_{21}}{A_1}$

i.e. $F_{12} = 0.5$ **...view factor from surface 1 to surface 2**

Resistances:

Now, $R_1 := \frac{1 - \varepsilon_1}{\varepsilon_1 \cdot A_1}$

i.e. $R_1 = 0.0398$ **m^{-2}....surface resistance of inner surface**

and, $R_2 := \frac{1 - \varepsilon_2}{\varepsilon_2 \cdot A_2}$

i.e. $R_2 = 0.318$ **m^{-2}....surface resistance of outer surface**

Also, $R_{12} := \frac{1}{A_1 \cdot F_{12}}$

i.e. $R_{12} = 0.318$ **m^{-2}....space resistance between inner and outer surface**

Therefore,

$$R_{tot} := R_1 + R_{12} + R_2$$

i.e. $R_{tot} = 0.676$ **m^{-2}....total resistance between inner and outer surface**

Also,

$E_{b1} := \sigma \cdot T_1^4$ i.e. $E_{b1} = 2.322 \times 10^4$ W/m2

$E_{b2} := \sigma \cdot T_2^4$ i.e. $E_{b2} = 7.348 \times 10^3$ W/m2

Then, net rate of heat transfer between surfaces 1 and 2 is given by:

$$Q_{12} := \frac{E_{b1} - E_{b2}}{R_{tot}}$$

i.e. $Q_{12} = 2.347 \times 10^{4}$ **Watts......Ans.**

Additionally:

If both the surfaces are black: Now, both the surface resistances become zero, since ε = 1 for both the black surfaces. Then,

$$Q_{12} := \frac{E_{b1} - E_{b2}}{R_{12}}$$

i.e. $Q_{12} = 4.988 \times 10^{4}$ **W....if both the surfaces are black.**

(b) Also, plot Q12 as the emissivity of base, ε_2 varies from 0.1 to 0.8, other quantities remaining the same:

Express relevant quantities as functions of ε_2:

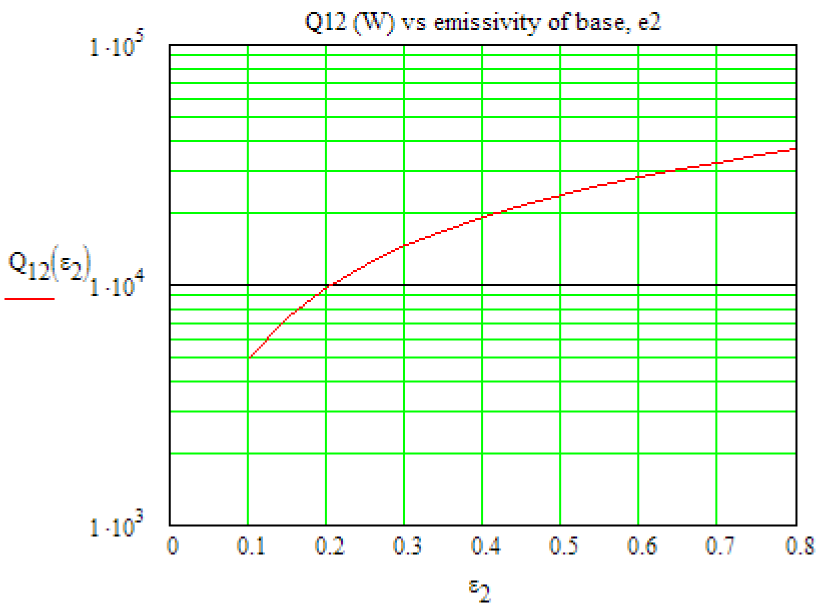
$$R_2(\varepsilon_2) := \frac{1 - \varepsilon_2}{\varepsilon_2 \cdot A_2}$$ ....R2 as a function of ε2

$$R_{tot}(\varepsilon_2) := R_1 + R_{12} + R_2(\varepsilon_2)$$ ....Rtot as a function of ε2

$$Q_{12}(\varepsilon_2) := \frac{E_{b1} - E_{b2}}{R_{tot}(\varepsilon_2)}$$ ....Q12 as a function of ε2

$\varepsilon_2 := 0.1, 0.15 .. 0.8$ ...define a range variable

$\varepsilon_2 =$	$Q_{12}(\varepsilon_2) =$
0.1	$4.926 \cdot 10^3$
0.15	$7.344 \cdot 10^3$
0.2	$9.732 \cdot 10^3$
0.25	$1.209 \cdot 10^4$
0.3	$1.442 \cdot 10^4$
0.35	$1.672 \cdot 10^4$
0.4	$1.9 \cdot 10^4$
0.45	$2.125 \cdot 10^4$
0.5	$2.347 \cdot 10^4$
0.55	$2.567 \cdot 10^4$
0.6	$2.784 \cdot 10^4$
0.65	$2.998 \cdot 10^4$
0.7	$3.21 \cdot 10^4$
0.75	$3.42 \cdot 10^4$
0.8	$3.627 \cdot 10^4$



==

Prob.5.C.1.9. A pipe carrying steam, having an outside dia 20 cm, runs in a large room and is exposed to air at a temp. of 30 C. The pipe surface temp. is 400 C.

i) Calculate the loss of heat to the surrounding per metre length of pipe due to thermal radiation. Emissivity of pipe surface is 0.8.
ii) What would be the rate of heat loss due to radiation if the pipe is enclosed in a 40 cm. dia brick conduit of emissivity 0.91? [M.U. 1997]

Mathcad Solution:

Case 1. Pipe in a large area:

This is a case of a small object surrounded by a large area, and we have:

$d := 0.2$ m $L := 1$ m $\varepsilon := 0.8$

$Ta := 30 + 273$ K $Ts := 400 + 273$ K

$\sigma := 5.67 \cdot 10^{-8}$ W/m^2.K^4... Stefan-Boltzmann const.

$A := \pi \cdot d \cdot L$ i.e. $A = 0.628$ m^2... pipe surface area

Therefore:

$Q := \sigma \cdot \varepsilon \cdot A \cdot \left(Ts^4 - Ta^4\right)$ .W..heat loss to srroundings

i.e. $Q = 5.607 \times 10^3$ **W......Ans.**

Case 2. Pipe enclosed in a 40 cm dia conduit:

This is a case of a concentric cylinders, and we have:

$d := 0.2$ m $\quad D := 0.4$ m

$\varepsilon 1 := 0.8 \quad \varepsilon 2 := 0.91$

$A1 := \pi \cdot d \cdot L$ i.e. $A1 = 0.628$ m^2... pipe surface area

$A2 := \pi \cdot D \cdot L$ i.e. $A2 = 1.257$ m^2... brick conduit surface area

Then, we have:

$$Q := \frac{\sigma \cdot A1 \cdot \left(Ts^4 - Ta^4\right)}{\left(\frac{1}{\varepsilon 1}\right) + \frac{A1}{A2} \cdot \left(\frac{1}{\varepsilon 2} - 1\right)}$$

i.e. $Q = 5.393 \times 10^3$ **W...Ans.**

===

Prob. 5.C.1.10. Write EES Functions for a general two-surface enclosures, and also for few special cases of two-surface enclosures:

Following are the EES Functions. Later, we shall use them in solving some problems.

```
$UnitSystem SI Pa J K

FUNCTION Q12_TwoSurfaceEnclosure(A_1, A_2, F_12, epsilon_1, epsilon_2, T_1, T_2)

{Returns Q12 from surface 1 to 2 in a general, two-surface enclosure; 1 is the internal surface.

Inputs: A1, A2 (m^2), T1, T2 (K)}

sigma := 5.67E-08 "[W/m^2-K^4]….Stefan Boltzmann constant"

R_12 := 1/(A_1 * F_12) "[1/m^2]…space resistance"

R_1 := (1 – epsilon_1) / (A_1 * epsilon_1) "[1/m^2]..surface resistance of surface 1"
```

```
R_2 := (1 – epsilon_2) / (A_2 * epsilon_2) "[1/m^2]..surface resistance of surface 2"

R_tot := R_1 + R_2 + R_12 "…total resistance"

E_b1 := sigma * T_1^4 "[W/m^2]"

E_b2 := sigma * T_2^4 "[W/m^2]"

Q12_TwoSurfaceEnclosure := (E_b1 – E_b2) / R_tot "[W]"

END

"------------------------------------------------------------------------------------------------------------"

FUNCTION Q12_SmallObject(A_1, epsilon_1, T_1, T_2)

{Returns Q12 from small object 1 to a large surrounding surface 2; 1 is the internal surface.

Inputs: A1 (m^2), T1, T2 (K)}

sigma := 5.67E-08 "[W/m^2-K^4]….Stefan Boltzmann constant"

Q12_SmallObject := sigma * A_1 * epsilon_1 * (T_1^4 – T_2^4) "[W]"

END

"------------------------------------------------------------------------------------------------------------"

FUNCTION Q12_parallel_plates(A, epsilon_1, epsilon_2,T_1, T_2)

{Returns net Q12 from plate 1 to a parallel plate 2;

Inputs: A (m^2), T1, T2 (K)}

sigma := 5.67E-08 "[W/m^2-K^4]….Stefan Boltzmann constant"

Q12_parallel_plates := sigma * A * (T_1^4 – T_2^4) / (1/epsilon_1 + 1/epsilon_2 – 1)"[W]"

END

"------------------------------------------------------------------------------------------------------------"
```

FUNCTION Q12_long_concentric_cylinders(L, R_1, R_2, epsilon_1, epsilon_2,T_1, T_2)

{Returns net Q12 from surface 1 to surface 2; 1 is the inner cylindrical surface.

Inputs: L, R_1, R_2 (m), T1, T2 (K)}

sigma := 5.67E-08 "[W/m^2-K^4]….Stefan Boltzmann constant"

Q12_long_concentric_cylinders := sigma * (2 * pi * R_1 * L) * (T_1^4 – T_2^4) / (1/epsilon_1 + (R_1 / R_2) * (1/epsilon_2 – 1))"[W]"

END

"---"

FUNCTION Q12_concentric_spheres(R_1, R_2, epsilon_1, epsilon_2,T_1, T_2)

{Returns net Q12 from surface 1 to surface 2; 1 is the inner spherical surface.

Inputs: R_1, R_2 (m), T1, T2 (K)}

sigma := 5.67E-08 "[W/m^2-K^4]….Stefan Boltzmann constant"

Q12_concentric_spheres := sigma * (4 * pi * R_1^2) * (T_1^4 – T_2^4) / (1/epsilon_1 + (R_1 / R_2)^2 * (1/epsilon_2 – 1))"[W]"

END

"---"

FUNCTION Q12_from_gray_cavity(A_1,F_11, epsilon_1, T_1)

{Returns net Q12 from surface 1 of cavity to a closing surface 2;

Inputs: A_1 (m^2), T1 (K)}

sigma := 5.67E-08 "[W/m^2-K^4]….Stefan Boltzmann constant"

Q12_from_gray_cavity := sigma * A_1 * epsilon_1 * T_1^4 * ((1 – F_11) / (1 – (1 – epsilon_1) * F_11))"[W]"

END

"---"

Prob. 5.C.1.11. Two large parallel plates are at 1000 K and 800 K. Determine the heat exchange per unit area, when:

i) The surfaces are black
ii) The hot surface has an emissivity of 0.9 and the cold surface has emissivity = 0.6 [VTU – Dec. 2006/Jan. 2007]

In addition, plot the variation of Q12 with e2, all other conditions remaining the same.

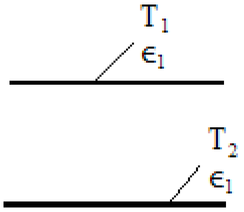


Fig. Two infinitely large parallel plates

EES Solution:

"Data:"

A = 1 [m^2]

epsilon_1 = 0.9

epsilon_2 = 0.6

T_1 = 1000 [K]

T_2 = 800 [K]

"Case 1: when both surfaces are black:"

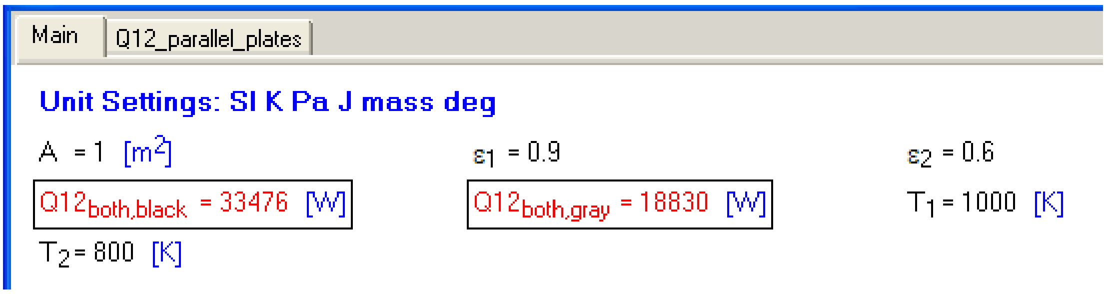
Q12_both_black = 5.67E-08[W/m^2-K^4] * A * (T_1^4 – T_2^4)

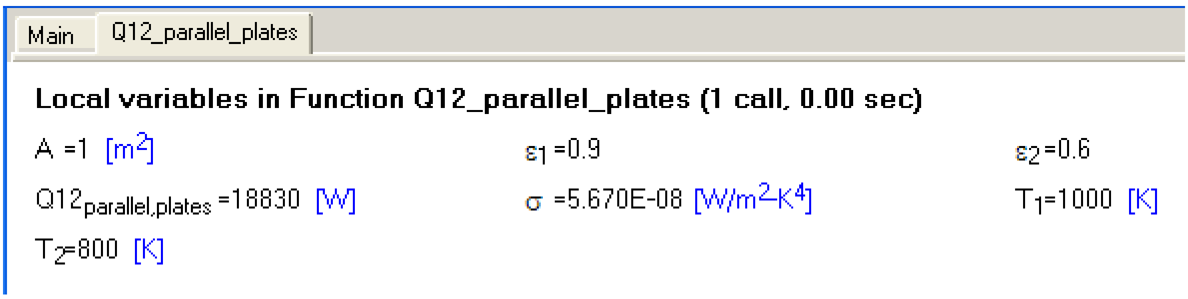
"Case 2: When epsilon_1 and epsilon_2 are 0.9 and 0.6 respectively:"

"Using the EES Function for parallel plates, written above:"

Q12_both_gray = Q12_parallel_plates(A, epsilon_1, epsilon_2,T_1, T_2)

Results:





Thus:

Q12 when both surfaces are black: 33476 W …. Ans.

Q12 when ε1 and ε2 are 0.9 and 0.6, respectively = 18830 W … Ans.

In addition, plot the variation of Q12 with e2, all other conditions remaining the same:

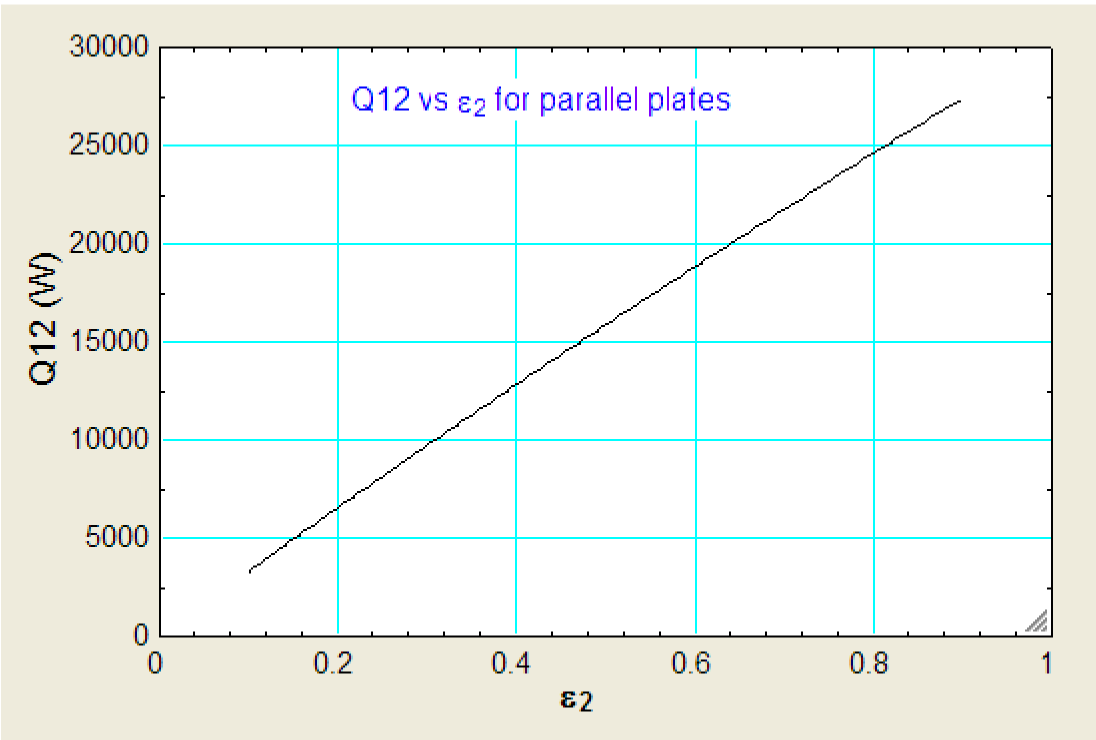
First, compute the parametric Table:

Parametric Table

Table 1

1..9	ε_2	$Q12_{both,gray}$ [W]
Run 1	0.1	3311
Run 2	0.2	6550
Run 3	0.3	9719
Run 4	0.4	12820
Run 5	0.5	15857
Run 6	0.6	18830
Run 7	0.7	21742
Run 8	0.8	24594
Run 9	0.9	27389

Now, plot the results:



==

Prob. 5.C.1.12. A solid copper sphere of 10 cm dia at 1000 C is suspended in a large enclosure. If the walls of the enclosure are at 30 C, determine: (i) the initial rate of cooling of the sphere (ii) time taken by the sphere to cool to 900 C.

Use the following properties for copper: density, ρ = 8954 kg/m^3, specific heat, cp = 381 J/kg.K, thermal conductivity, k = 386 W/m.K, emissivity, ε = 0.78. [VTU – Aug. 2001]

(b) Additionally, plot the variation of these quantities as emissivity of surface of sphere varies from 0.1 to 0.9.

EES Solution:

This is the case of a *small object in a large surrounding*.

So, to get Q12, use the EES Function written above.

"Data:"

R = 0.05 [m]"….radius of sphere"

epsilon_1 = 0.78"…emissivity of sphere surface"

T_1 = 1273 [K]"…sphere temp."

T_2 = 303 [K]"….surroundings temp."

rho = 8954[kg/m^3]"…density of copper"

cp = 381 [J/kg-K]"…sp. heat of copper"

k = 386 [W/m-K]"…thermal cond. of copper"

"Calculations:"

A_1 = 4 * pi * R^2 "[m^2]… surface area of sphere"

"To find Q12, use the EES Function already written for a small object in a large enclosure:"

Q12 = Q12_SmallObject(A_1, epsilon_1, T_1, T_2)"[W]…finds rate of initial heat loss from the sphere"

"But, Q12 is also equal to:

Q12 = mass * cp * dTbydtau, where dTbydtau is the initial rate of cooling"

"Therefore:"

mass = rho * (4/3) * pi * R^3 "…[kg]… mass of sphere"

Q12 = mass * cp * dTbydtau "…[C/s]….finds initial rate of cooling"

"Time taken to cool to 900 C:"

"Find the rate of heat loss when the sphere temp is 900 C. Then, take the average rate of heat loss from 1000 C to 900 C, and the find the approx. time taken to cool from 1000 C to 900 C, knowing the total amount of heat removed in the process:"

Q12_2 = Q12_SmallObject(A_1, epsilon_1, 1173, T_2)"[W]…finds rate of heat loss from the sphere when it is at 900 C"

"Therefore:"

Q12_avg = (Q12 + Q12_2) / 2

"And:"

Q_lost = mass * cp * (1000 – 900)"[J]"

"Therefore, approx. time taken to cool to 900 C:"

time = Q_lost / Q12_avg "[s]……time taken to cool to 900 C"

Results:

Main | Q12_SmallObject

Unit Settings: SI K Pa J mass deg

A_1 = 0.03142 [m^2]	cp = 381 [J/kg-K]	dTbydtau = 2.036 [C/s]	ε_1 = 0.78
k = 386 [W/m-K]	mass = 4.688 [kg]	Q12 = 3637 [W]	$Q12_2$ = 2619 [W]
$Q12_{avg}$ = 3128 [W]	Q_{lost} = 178624 [J]	R = 0.05 [m]	ρ = 8954 [kg/m^3]
time = 57.11 [s]	T_1 = 1273 [K]	T_2 = 303 [K]	

Main | Q12_SmallObject

Local variables in Function Q12_SmallObject (2 calls, 0.00 sec)

A_1=0.03142 [m^2]	ε_1 =0.78	$Q12_{SmallObject}$ =2619 [W]
σ =5.670E-08 [W/m^2-K^4]	T_1=1173 [K]	T_2=303 [K]

Thus:

Initial rate of heat loss, Q12 = 3637 W….Ans.

Initial rate of cooling, dT/dτ = 2.036 C/s … Ans.

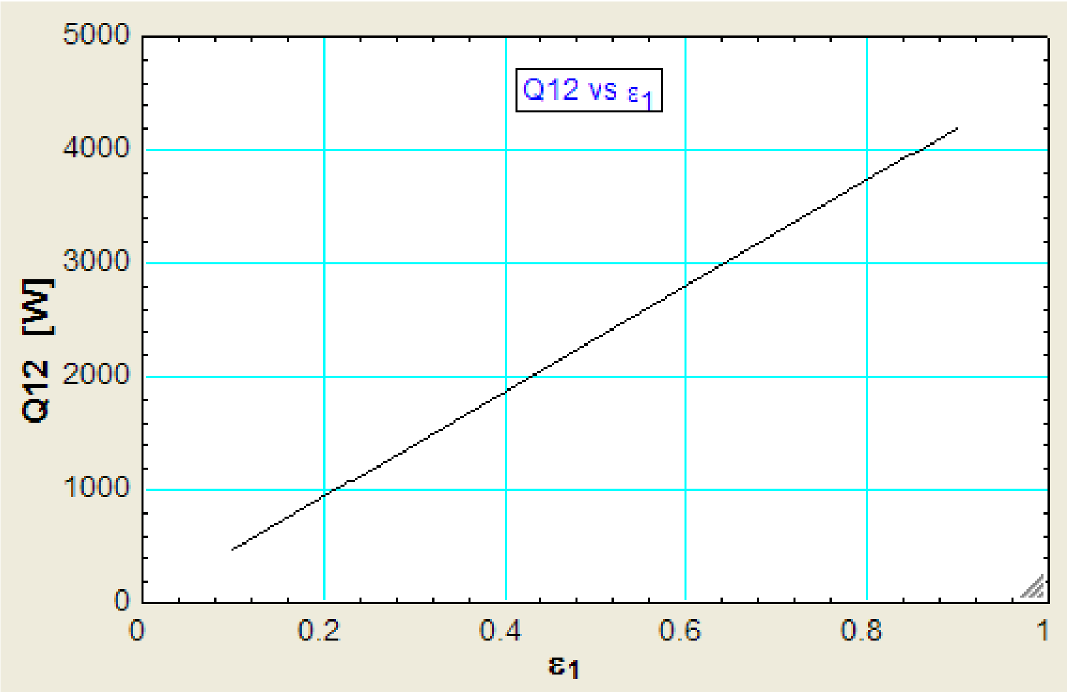
Approx. time taken to cool to 900 C = time = 57.11 s …. Ans.

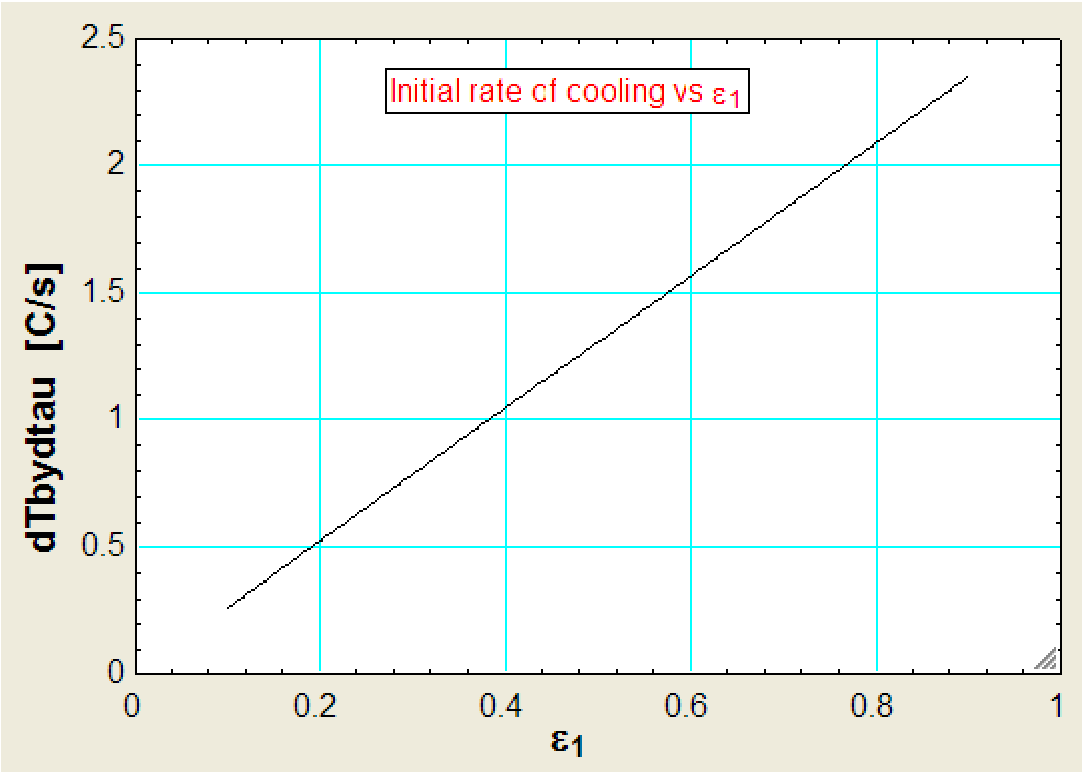
Additionally, plot the variation of these quantities as emissivity of surface of sphere varies from 0.1 to 0.9:

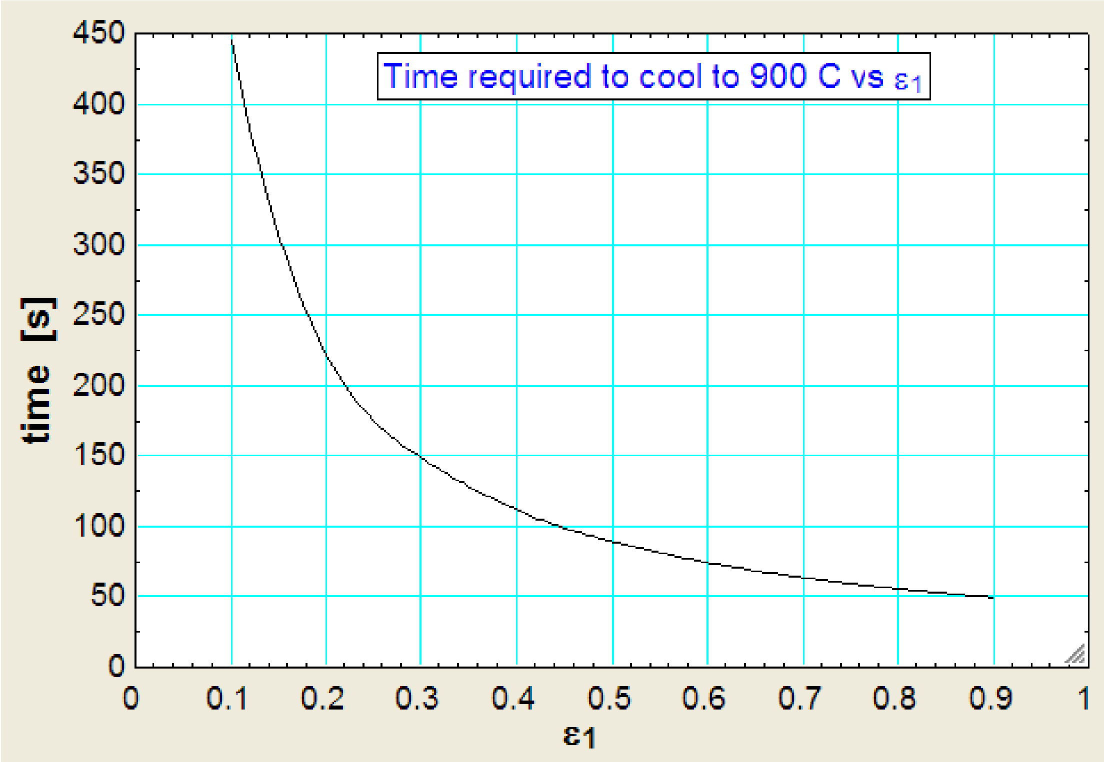
First, compute the Parametric Table:

1..9	ε_1	Q12 [W]	dTbydtau [C/s]	time [s]
Run 1	0.1	466.3	0.261	445.4
Run 2	0.2	932.6	0.5221	222.7
Run 3	0.3	1399	0.7831	148.5
Run 4	0.4	1865	1.044	111.4
Run 5	0.5	2331	1.305	89.09
Run 6	0.6	2798	1.566	74.24
Run 7	0.7	3264	1.827	63.63
Run 8	0.8	3730	2.088	55.68
Run 9	0.9	4197	2.349	49.49

Now, plot the results:




Initial rate cf cooling vs ε1
dTbydtau [C/s]
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
ε1



Prob. 5.C.1.13. Refer to Fig. below. Three thin walled, long, circular cylinders 1, 2 and 3, of diameters 15 cm, 25 cm and 35 cm respectively, are arranged concentrically as shown. Temperature of cylinder 1 is 80 K and that of cylinder 3 is 300 K. Emissivities of cylinders 1, 2 and 3 are 0.05, 0.1 and 0.2 respectively. Assuming that there is vacuum inside the annular spaces, determine the steady state temperature attained by cylinder 2.

(b) In addition, plot Q12 and T2 as ε_2 varies from 0.1 to 0.9.

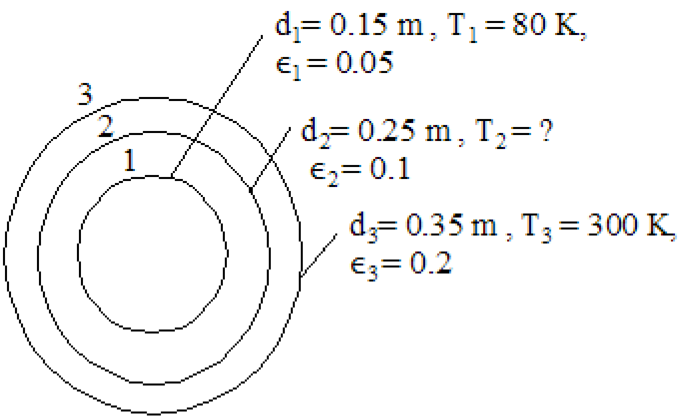


Fig.Prob.5C.1.13

EES Solution:

This is the case of long, concentric cylinders.

To find T2, use the fact that: in steady state, net radiant heat transfer between cylinders 1 and 2 must be equal to the net radiant heat transfer between cylinders 2 and 3.

To find Q12 between long, concentric cylinders, use the EES Function already written.

"Data:"

R1 = 0.075 [m]"….radius of inner cyl"

R2 = 0.125 [m]"….radius of middle cyl"

R3 = 0.175 [m]"….radius of outer cyl"

L = 1[m] "…assumed"

epsilon_1 = 0.05"…emissivity of surface 1"

epsilon_2 = 0.1"…emissivity of surface 2"

epsilon_3 = 0.2"…emissivity of surface 3"

T_1 = 80 [K]"…inner cylinder temp."

T_3 = 300 [K]"….outer cylinder temp."

"Calculations:"

"Net heat transfer between surfaces 1 and 2:

Using the EES Function already written:"

Q12 = Q12_long_concentric_cylinders(L, R1, R2, epsilon_1, epsilon_2,T_1, T_2)

"Similarly, Net heat transfer between surfaces 2 and 3:"

Q23 = Q12_long_concentric_cylinders(L, R2, R3, epsilon_2, epsilon_3,T_2, T_3)

"Equating Q12 and Q23, we get the unknown temp, T2:"

Q12 = Q23 "..equate Q12 to Q23 to get T2"

Results:

Main | Q12_long_concentric_cylinders

Unit Settings: SI K Pa J mass deg

ε_1 = 0.05	ε_2 = 0.1	ε_3 = 0.2
L = 1 [m]	Q12 = -6.503 [W]	Q23 = -6.503 [W]
R1 = 0.075 [m]	R2 = 0.125 [m]	R3 = 0.175 [m]
σ = 5.670E-08 [W/m^2-K^4]	T_1 = 80 [K]	T_2 = 280.9 [K]
T_3 = 300 [K]		

Main | Q12_long_concentric_cylinders

Local variables in Function Q12_long_concentric_cylinders (36 calls, 0.02 sec)

ε_1 =0.1	ε_2 =0.2
L=1 [m]	$Q12_{long,concentric,cylinders}$=-6.503 [W]
R_1 =0.125 [m]	R_2 =0.175 [m]
σ =5.670E-08 [W/m^2-K^4]	T_1=280.9 [K]
T_2=300 [K]	

Thus:

Temp of middle cylinder = T2 = 280.9 K … Ans.

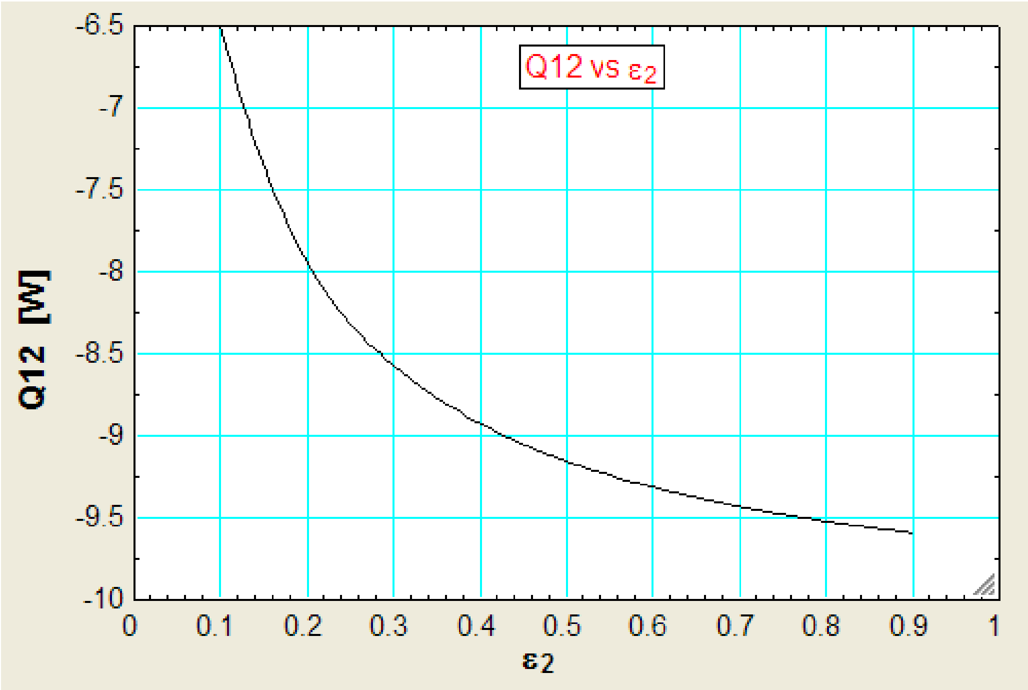
Net heat transfers Q12 = Q23 = -6.503 W.… Ans. (-ve sign indicates heat flow from outer cylinder to inner cylinder)

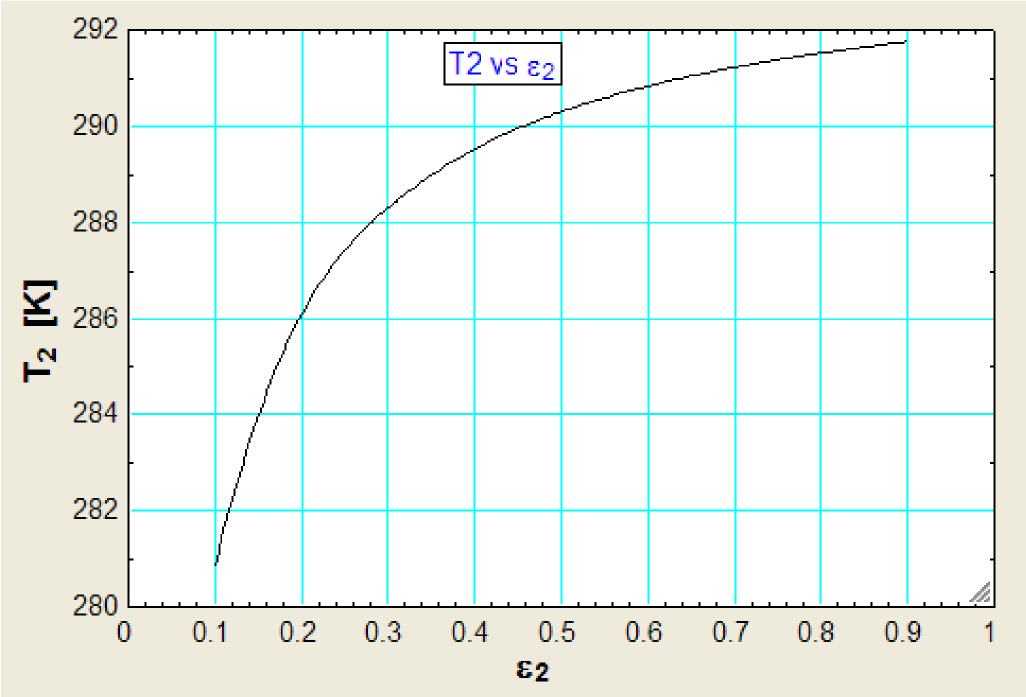
In addition, plot Q12 and T2 as ε_2 varies from 0.1 to 0.9:

First, compute the Parametric Table:

1..9	ε_2	Q12 [W]	T_2 [K]
Run 1	0.1	-6.503	280.9
Run 2	0.2	-7.942	286.1
Run 3	0.3	-8.574	288.3
Run 4	0.4	-8.93	289.5
Run 5	0.5	-9.157	290.3
Run 6	0.6	-9.316	290.8
Run 7	0.7	-9.433	291.2
Run 8	0.8	-9.522	291.5
Run 9	0.9	-9.593	291.8

Now, plot the results:





===

Prob.5.C.1.14. For a hemispherical furnace, the flat floor is at 700 K and has an emissivity of 0.5. The hemispherical roof is at 1000 K and has an emissivity of 0.25. Find the net radiant heat transfer from the roof to floor. [V.T.U – July–Aug. 2003]

(b) Also, plot Q12 as the emissivity of base, ε_2 varies from 0.1 to 0.9, all other conditions remaining the same.

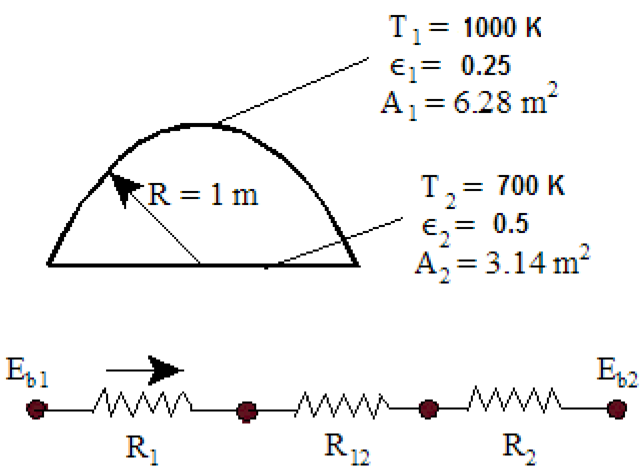


Fig.Prob.5.C.1.14

EES Solution:

Assume the radius of hemisphere as 1 m.

Note that *this problem is similar to Prob.5.C.1.8,* which was solved in Mathcad; but, now, temperatures and emissivity values are different.

This is a *two-surface enclosure problem.*

Use the EES Function written earlier, to find Q12 of a two-surface enclosure:

"Data:"

R1 = 1 [m]"….radius of hemisphere"

epsilon_1 = 0.25"…emissivity of hemisph. surface 1"

epsilon_2 = 0.5"…emissivity of base surface 2"

T_1 = 1000 [K]"…hemisph. surface temp."

T_2 = 700 [K]"….base temp."

"Calculations:"

A_1 = 4 * pi * R1^1 / 2 "[m^2]….area of hemisph. surface"

A_2 = pi * R1^2 "[m^2]..area of base surface"

"View factors:"

F_21 = 1 "…since all the energy leaving surface 2 is intercepted by surface 1"

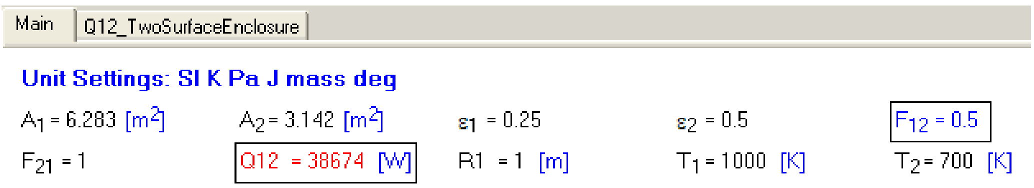
"To find F_12, View factor from surface 1 to surface 2:"

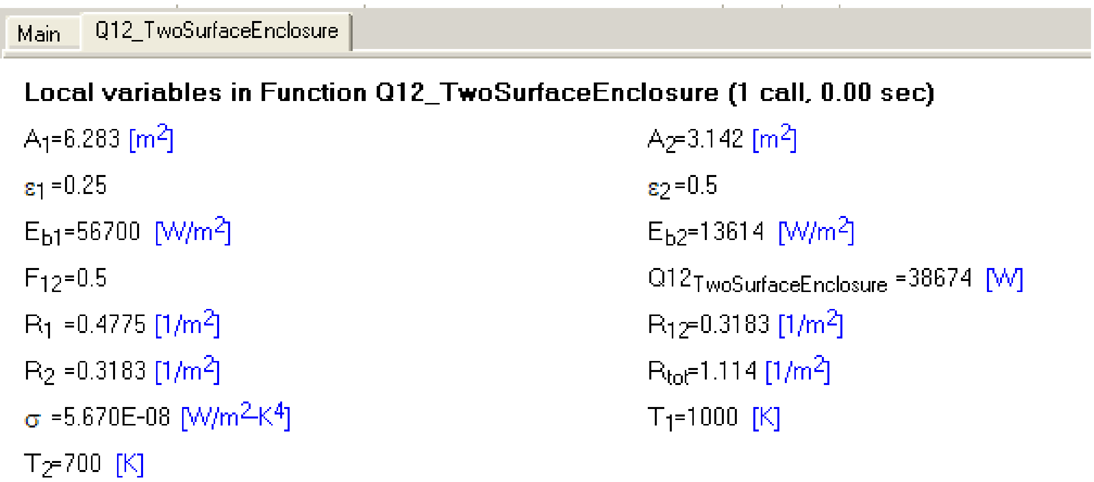
A_1 * F_12 = A_2 * F_21 "…by reciprocity… finds F_12"

"Now, to get Q12 use the EES Function for a general, two-surface enclosure, already written:"

Q12 = Q12_TwoSurfaceEnclosure(A_1, A_2, F_12, epsilon_1, epsilon_2, T_1, T_2)

Results:





Thus:

Net radiant heat transfer from roof to floor = Q12 = 38674 W …. Ans.

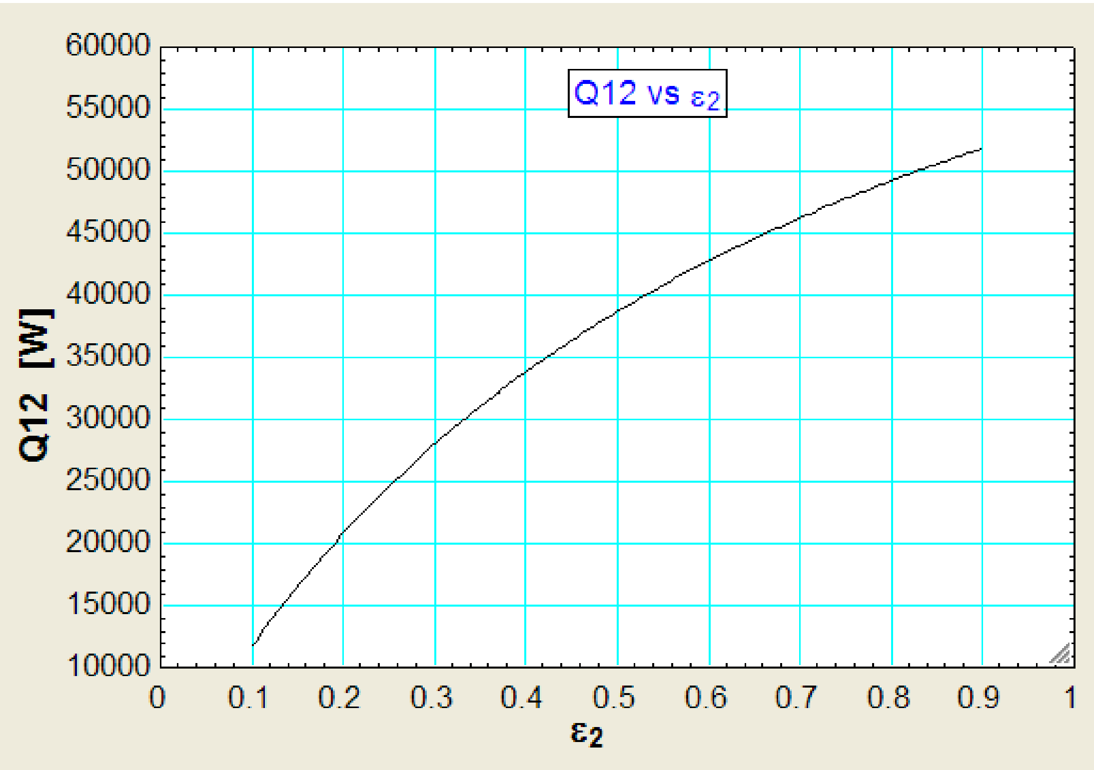
(b) Also, plot Q12 as the emissivity of base, ε_2 varies from 0.1 to 0.9:

First, compute the Parametric Table:

Table 1 | Table 2 | Table 3 | Table 4

1..9	ε_2	Q12 [W]
Run 1	0.1	11770
Run 2	0.2	20825
Run 3	0.3	28005
Run 4	0.4	33840
Run 5	0.5	38674
Run 6	0.6	42745
Run 7	0.7	46220
Run 8	0.8	49222
Run 9	0.9	51840

And, plot the results:



==

Prob.5.C.1.15. Write EXCEL – VBA Functions for heat transfer in a general, two-surface enclosure and also for a few special cases of the same.

1. **For a general, two-surface enclosure:**

```
Option Explicit
'General two-surface enclosure:
Function Q_12_Two_surface_enclosure(A_1 As Double, A_2 As Double, eps_1 As Double, _
eps_2 As Double, T_1 As Double, T_2 As Double, F_12 As Double) As Double

'Finds Q_12 = Q1 = -Q2 for a two-surface enclosure (W)

Dim sigma As Double, R1 As Double, R2 As Double, R_12 As Double

sigma = 0.0000000567  'W/m2.K^4
R1 = (1 - eps_1) / (A_1 * eps_1) 'surface resistance of surface 1
R2 = (1 - eps_2) / (A_2 * eps_2) 'surface resistance of surface 2
R_12 = 1 / (A_1 * F_12)    'space resistance between surface 1 and 2

Q_12_Two_surface_enclosure = (sigma * (T_1 ^ 4 - T_2 ^ 4)) / (R1 + R_12 + R2)

End Function
```

2. **Small object in a large cavity:**

```
'Special cases of Two-surface enclosures:

Function Q_12_small_object_in_large_cavity(A_1 As Double, eps_1 As Double, _
T_1 As Double, T_2 As Double) As Double

'Gives Q_12 (W) for a small object placed in a large enclosure, i.e. (A_1/A_2) = 0, F_12 = 1

Dim sigma As Double
sigma = 0.0000000567 'W/m2.K^4

Q_12_small_object_in_large_cavity = sigma * A_1 * eps_1 * (T_1 ^ 4 - T_2 ^ 4)

End Function
```

3. **Infinite, large, parallel plates:**

```
Function Q_12_Infinite_large_parallel_plates(Area As Double, eps_1 As Double, eps_2 As Double, _
T_1 As Double, T_2 As Double) As Double

'Gives Q_12 (W) between two infinite parallel plates of area A (m2) each,
'i.e. (A_1/A_2) = 1, F_12 = 1

Dim sigma As Double
sigma = 0.0000000567 'W/m2.K^4

Q_12_Infinite_large_parallel_plates = (sigma * Area * (T_1 ^ 4 - T_2 ^ 4)) / _
(1 / eps_1 + 1 / eps_2 - 1)

End Function
```

4. **Infinite, concentric cylinders:**

```
Function Q_12_Infinite_concentric_cylinders(A_1 As Double, R_1 As Double, R_2 As Double, _
eps_1 As Double, eps_2 As Double, T_1 As Double, T_2 As Double) As Double

'Gives Q_12 (W) between infinite concentric cylinders,
'i.e. (A_1/A_2) = R_1/R_2, F_12 = 1

Dim sigma As Double
sigma = 0.0000000567 'W/m2.K^4

Q_12_Infinite_concentric_cylinders = (sigma * A_1 * (T_1 ^ 4 - T_2 ^ 4)) / _
(1 / eps_1 + ((1 - eps_2) / eps_2) * (R_1 / R_2))

End Function
```

5. **Concentric spheres:**

```
Function Q_12_concentric_spheres(A_1 As Double, R_1 As Double, R_2 As Double, _
eps_1 As Double, eps_2 As Double, T_1 As Double, T_2 As Double) As Double

'Gives Q_12 (W) between concentric spheres,
'i.e. (A_1/A_2) = (R_1/R_2)^2, F_12 = 1

Dim sigma As Double
sigma = 0.0000000567 'W/m2.K^4

Q_12_concentric_spheres = (sigma * A_1 * (T_1 ^ 4 - T_2 ^ 4)) / _
(1 / eps_1 + ((1 - eps_2) / eps_2) * (R_1 / R_2) ^ 2)

End Function
```

Now, let us solve a few problems using these VBA Functions:

Prob.5.C.1.16. Consider the Prob.5.C.1.14 again. Solve it with EXCEL.

(b) Plot the variation of Q12 with ε1when ε1 varies from 0.1 to 0.9 and (c) also, plot the variation of Q12 with T1 when T1 varies from 800 K to 1300 K.

EXCEL Solution:

We have:

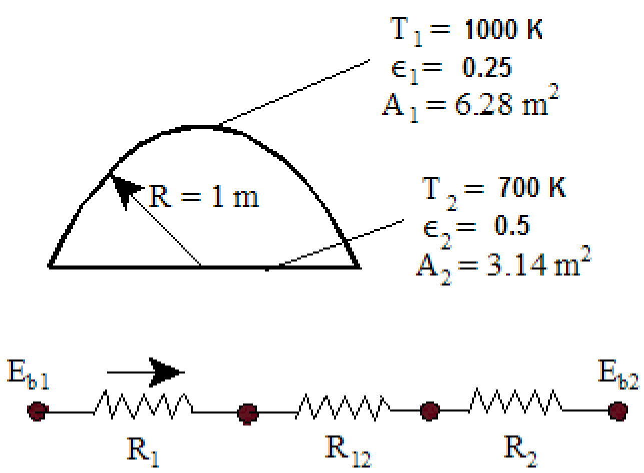


Fig.Prob.5.C.1.16

We have already shown that:

A1 = 6.283 m^2, A2 = 3.142 m^2,

ε1 = 0.25, ε2 = 0.5,

T1 = 1000 K, T2 = 700 K, and

View factor, F12 = 0.5

Set up the EXCEL worksheet as shown:

H8 =Q_12_Two_surface_enclosure(B8,C8,D8,E8,F8,G8,E6)

	A	B	C	D	E	F	G	H
1								
2		Two - surface enclosures:						
3							Using VBA Function	
4		1. General two-surface enclosure:						
5								
6		Stefan_const =	5.67E-08	ViewFactor_12 =	0.5			
7		Area_1 (m2)	Area_2 (m2)	epsilon_1	epsilon_2	Temp_1 (K)	Temp_2 (K)	Q_12(W)
8		6.283	3.142	0.25	0.5	1000	700	38674.82

The simple EXCEL worksheet shown above uses the VBA Function for Q12 for a general, two-surface enclosure, written earlier (available in 'User Defined' category of Functions).

See the Formula bar for the Function entered for Q12 in cell H8.

Note the advantage of this EXCEL spreadsheet: Above worksheet can be used as a *template* to solve any two-surface enclosure problem. Simply enter the relevant quantities from cell B8 to G8, and Q12 is immediately up-dated in cell H8.

(b) To plot Q12 against ε1, all other conditions remaining the same:

First, set up a Table as shown:

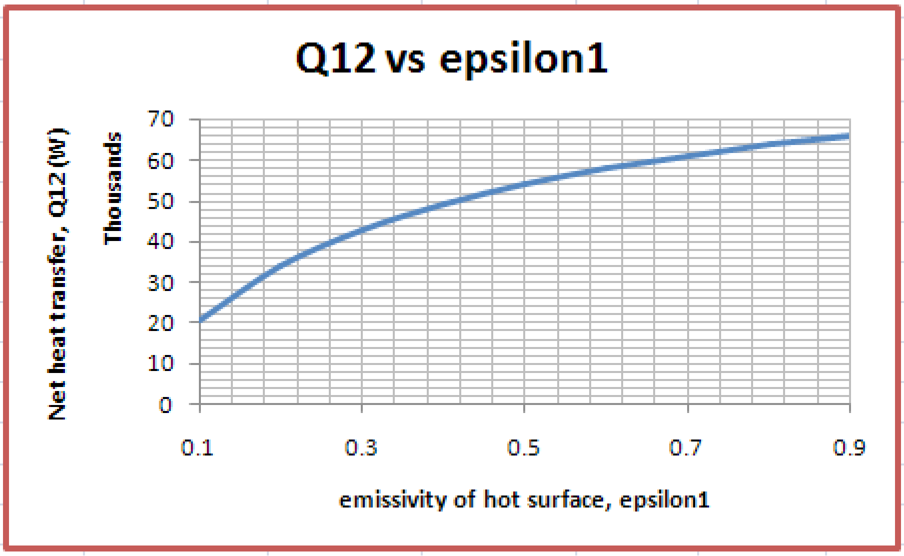
C12 | fx =Q_12_Two_surface_enclosure($B$8,$C$8,B12,$E$8,$F$8,$G$8,$E$6)

	A	B	C	D	E	F	G
10							
11		epsilon1	Q_12 (W/m^2)				
12		0.1	20824.465				
13		0.2					
14		0.3					
15		0.4					
16		0.5					
17		0.6					
18		0.7					
19		0.8					
20		0.9					

In the above, note that Q12 is calculated in cell C12 using the VBA Function. Take care to enter epsilon1 with *relative reference*, all other quantities are with *absolute reference*, so that when we drag-copy from cell C12 to cell C20, Q12 values are suitably calculated and the Table is filled up. See below:

C20 | fx =Q_12_Two_surface_enclosure($B$8,$C$8,B20,$E$8,$F$8,$G$8,$E$6)

	A	B	C	D	E	F	G
10							
11		epsilon1	Q_12 (W/m^2)				
12		0.1	20824.465				
13		0.2	33840.273				
14		0.3	42746.055				
15		0.4	49223.105				
16		0.5	54145.729				
17		0.6	58013.545				
18		0.7	61132.777				
19		0.8	63701.573				
20		0.9	65853.820				



(c) Now, plot Q12 vs T1, all other conditions remaining the same:

First, set up a Table as shown:

C25 =Q_12_Two_surface_enclosure($B$8,$C$8,$D$8,$E$8,B25,$G$8,$E$6)

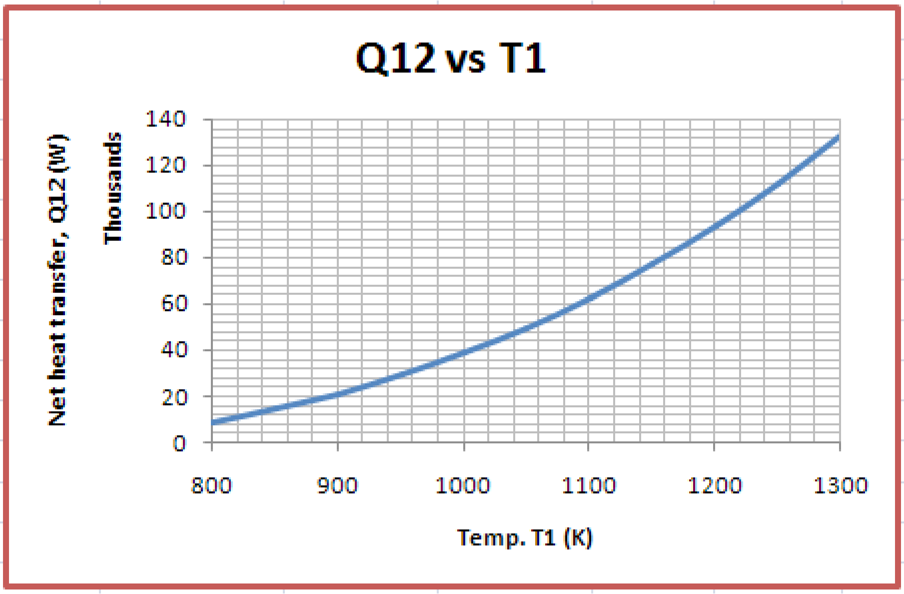
	A	B	C	D	E	F	G
23							
24		T_1 (K)	Q_12 (W/m^2)				
25		800	8626.637				
26		850					
27		900					
28		950					
29		1000					
30		1050					
31		1100					
32		1150					
33		1200					
34		1250					
35		1300					

In the above, note that Q12 is calculated in cell C25 using the VBA Function. Take care to enter T1 with *relative reference*, all other quantities are *with absolute reference*, so that when we drag-copy from cell C25 to cell C35, Q12 values are suitably calculated and the Table is filled up. See below:

C35   fx   =Q_12_Two_surface_enclosure($B$8,$C$8,$D$8,$E$8,B35,$G$8,$E$6)

	A	B	C	D	E	F	G
23							
24		T_1 (K)	Q_12 (W/m^2)				
25		800	8626.637				
26		850	14347.510				
27		900	21172.159				
28		950	29234.184				
29		1000	38674.817				
30		1050	49642.925				
31		1100	62295.008				
32		1150	76795.201				
33		1200	93315.275				
34		1250	112034.632				
35		1300	133140.310				

Now, plot the results:



==

Prob.5.C.1.17. Determine the rate of heat loss by radiation from a steel tube of outside dia 70 mm and 3 m long at a temp of 227 C if the tube is located within a square brick conduit of 0.3 m side and at 27 C. Take for steel ε1 = 0.79 and for brick, ε2 = 0.93. [P.U. 1998]

EXCEL Solution:

We have: 1 is inner surface and 2 is outer surface:

Length = 3 m

T1 = 227 + 273 = 500 K

T2 = 27 + 273 = 300 K

ε1 = 0.79

ε2 = 0.93

A1 = π * 0.07 * 3 = 0.659734 m^2

A2 = Perimeter * Length = (0.3 * 4) * 3 = 3.6 m^2

F12 = 1, since surface 1 is completely enclosed in 2.

Now, use the EXCEL Template prepared while solving the previous Problem.

Entering the above values of data, we get:

H8 =Q_12_Two_surface_enclosure(B8,C8,D8,E8,F8,G8,E6)

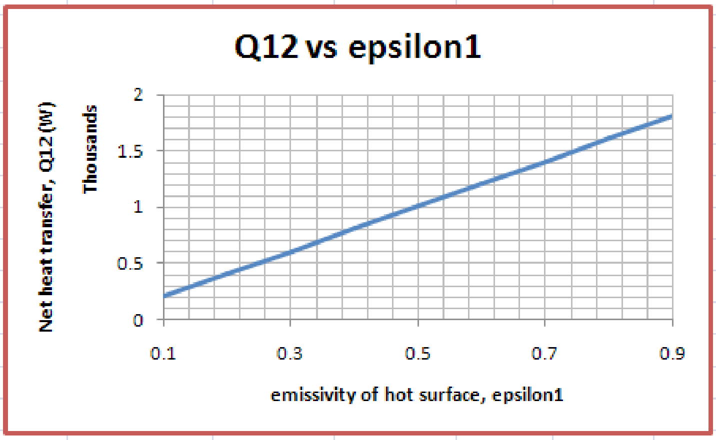
	A	B	C	D	E	F	G	H	I
3									
4		1. General two-surface enclosure:						Using VBA Function	
5									
6		Stefan_const =	5.67E-08	ViewFactor_12 =	1				
7		Area_1 (m2)	Area_2 (m2)	epsilon_1	epsilon_2	Temp_1 (K)	Temp_2 (K)	Q_12(W)	
8		0.659734	3.6	0.79	0.93	500	300	1590.272	

i.e. Q12 = 1590.27 W …. Ans.

Again, note the advantage of working with EXCEL spreadsheet:

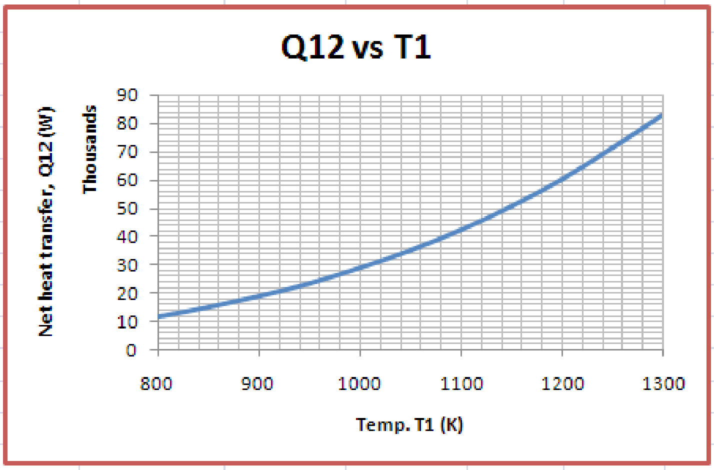
Variation of Q12 with epsilon1 and T1, in Tables and plots are immediately up-dated, as shown below:

epsilon1	Q_12 (W/m^2)
0.1	203.213
0.2	405.868
0.3	607.965
0.4	809.509
0.5	1010.500
0.6	1210.941
0.7	1410.834
0.8	1610.182
0.9	1808.987



And:

T_1 (K)	Q_12 (W/m^2)
800	11737.023
850	15022.987
900	18942.941
950	23573.620
1000	28996.146
1050	35296.026
1100	42563.150
1150	50891.796
1200	60380.623
1250	71132.678
1300	83255.393



==

Prob.5.C.1.18. Two very large plates are maintained at T1 = 600 K and T2 = 400 K. Emissivities are: epsilon1 = 0.5 and epsilon2 = 0.9. Determine the net radiation heat transfer between the plates.

(b) Also, plot Q12 as e1 varies from 0.1 to 0.9, all other conditions remaining the same.

(c) And, plot Q12 as T1 varies from 500 K to 1000 K, all other conditions remaining the same.[Ref:2]

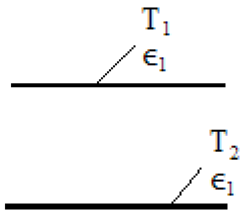
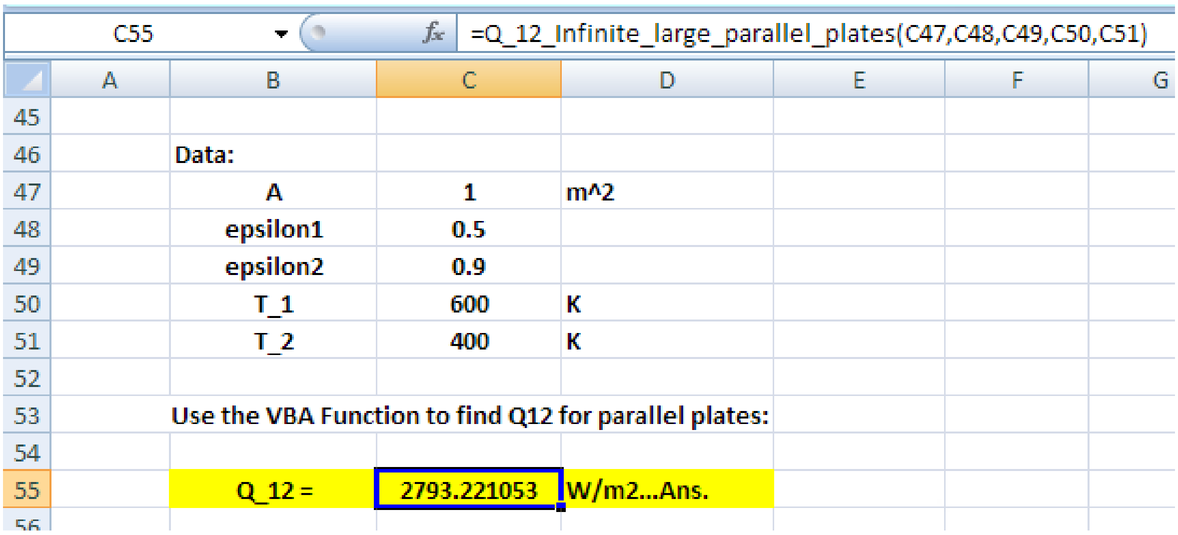


Fig. Two infinitely large parallel plates

EXCEL Solution:

We shall use the VBA Function written above for Q12 for, infinite, parallel plates (available in 'User Defined' category of Functions).

Set up the EXCEL worksheet as shown:



C55 =Q_12_Infinite_large_parallel_plates(C47,C48,C49,C50,C51)

	A	B	C	D	E	F	G
45							
46		Data:					
47		A	1	m^2			
48		epsilon1	0.5				
49		epsilon2	0.9				
50		T_1	600	K			
51		T_2	400	K			
52							
53		Use the VBA Function to find Q12 for parallel plates:					
54							
55		Q_12 =	2793.221053	W/m2...Ans.			

Note that Q12 is calculated in cell C55 and the VBA Function used can be seen in the Formula bar.

Thus: Q12 = 2793.22 W/m^2 …. Ans.

(b) Also, plot Q12 as ε1 varies from 0.1 to 0.9, all other conditions remaining the same:

First, set up a Table as shown:

C60 | =Q_12_Infinite_large_parallel_plates($C$47,B60,$C$49,$C$50,$C$51)

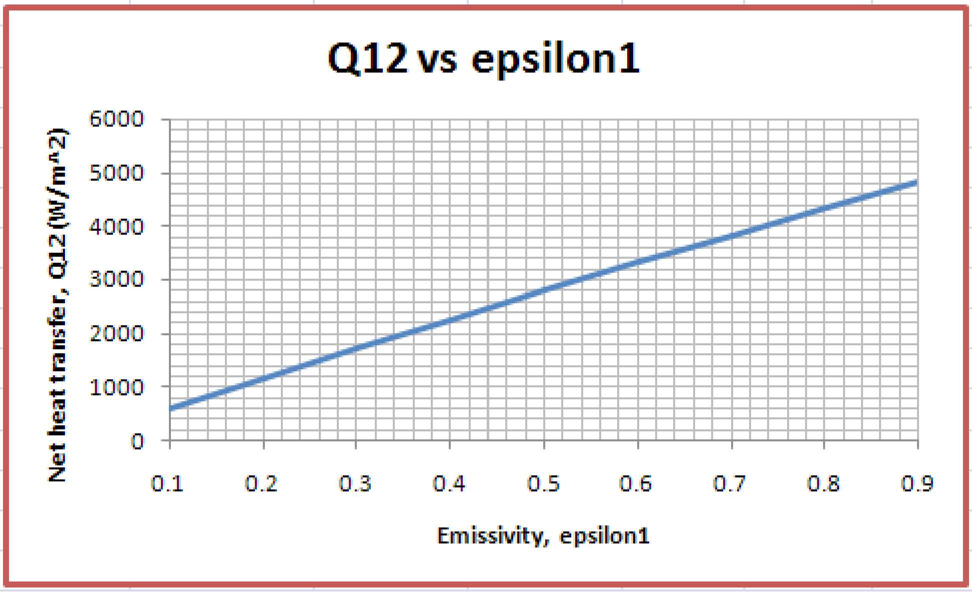
	A	B	C	D	E	F	G
58							
59		epsilon1	Q12 (W/m^2)				
60		0.1	583.2				
61		0.2					
62		0.3					
63		0.4					
64		0.5					
65		0.6					
66		0.7					
67		0.8					
68		0.9					

In the above, note that Q12 is calculated in cell C60 using the VBA Function. Take care to enter epsilon1 with *relative reference*, all other quantities are with *absolute reference*, so that when we drag-copy from cell C60 to cell C68, Q12 values are suitably calculated and the Table is filled up. See below:

C68 | =Q_12_Infinite_large_parallel_plates($C$47,B68,$C$49,$C$50,$C$51)

	A	B	C	D	E	F	G
58							
59		epsilon1	Q12 (W/m^2)				
60		0.1	583.200				
61		0.2	1153.722				
62		0.3	1711.974				
63		0.4	2258.349				
64		0.5	2793.221				
65		0.6	3316.950				
66		0.7	3829.880				
67		0.8	4332.343				
68		0.9	4824.655				

Now, plot the results:



(c) And, plot Q12 as T1 varies from 500 K to 1000 K, all other conditions remaining the same:

First, set up a Table as shown below:

C77 f_x =Q_12_Infinite_large_parallel_plates($C$47,$C$48,$C$49,B77,$C$51)

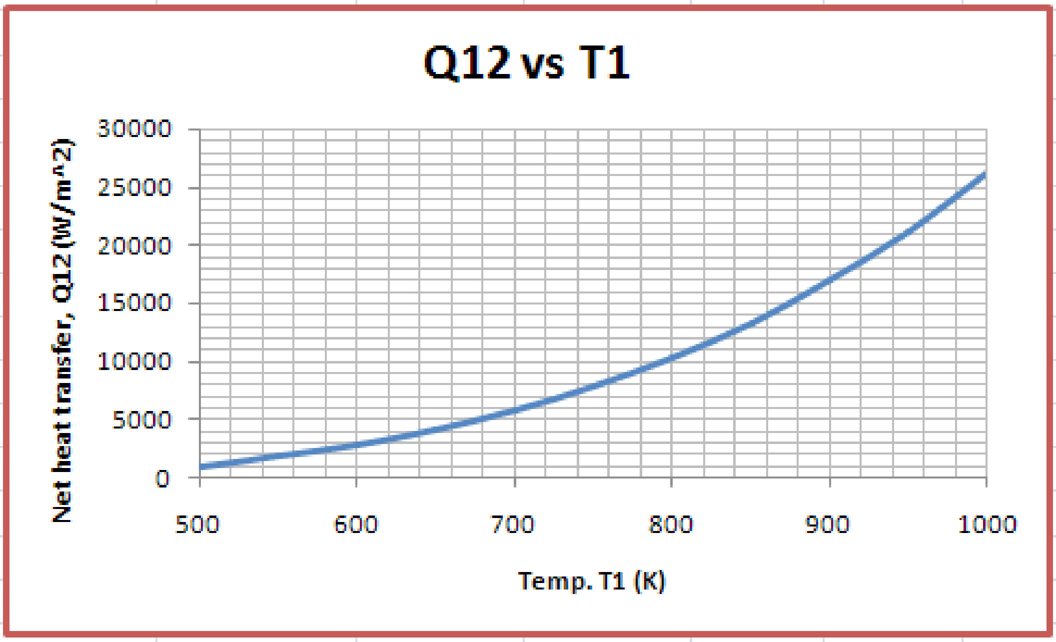
T1 (K)	Q12 (W/m^2)
500	991.0563158
550	
600	
650	
700	
750	
800	
850	
900	
950	
1000	

In the above, note that Q12 is calculated in cell C77 using the VBA Function. Take care to enter T1 with *relative reference*, all other quantities are with *absolute reference*, so that when we drag-copy from cell C77 to cell C87, Q12 values are suitably calculated and the Table is filled up. See below:

C87 f_x =Q_12_Infinite_large_parallel_plates($C$47,$C$48,$C$49,B87,$C$51)

T1 (K)	Q12 (W/m^2)
500	991.0563158
550	1770.103125
600	2793.221053
650	4106.739967
700	5761.018421
750	7810.443651
800	10313.43158
850	13332.42681
900	16933.90263
950	21188.36102
1000	26170.33263

Now, plot the results:



==

Prob.5.C.1.19. Consider a water tank of size: 2 m (L) × 1 m (W) × 1m (H), at a temperature of 30 C, which radiates from its sides and sides to surroundings at 5 C. Emissivity of tank surfaces is 0.9. Calculate the rate of heat radiation from the tank to the surroundings.

(b) Investigate the effect of emissivity of tank surface on the heat radiation rate. Let e vary from 0.1 to 0.9.

EXCEL Solution:

This is the case of a small object in large surroundings.

Use the VBA Function written earlier.

Following is the worksheet:

C103 | f_x =Q_12_small_object_in_large_cavity(C98,C101,C99,C100)

	A	B	C	D	E	F
94		Data:				
95		L	2	m		
96		W	1	m		
97		H	1	m		
98		A	8	m^2		
99		T_1	303	K		
100		T_2	278	K		
101		epsilon1	0.9			
102						
103		Q_12 =	1002.668	W...Ans.		

Q12 is calculated in cell C103 using the VBA Function for a small object in large surroundings, written earlier. See the Formula bar for the VBA Function entered in cell C103.

Thus: Q12 = 1002.67 W … Ans.

(b) Investigate the effect of emissivity of tank surface on the heat radiation rate. Let epsilon1 vary from 0.1 to 0.9:

First, set up a Table as shown:

C108 =Q_12_small_object_in_large_cavity($C$98,B108,$C$99,$C$100)

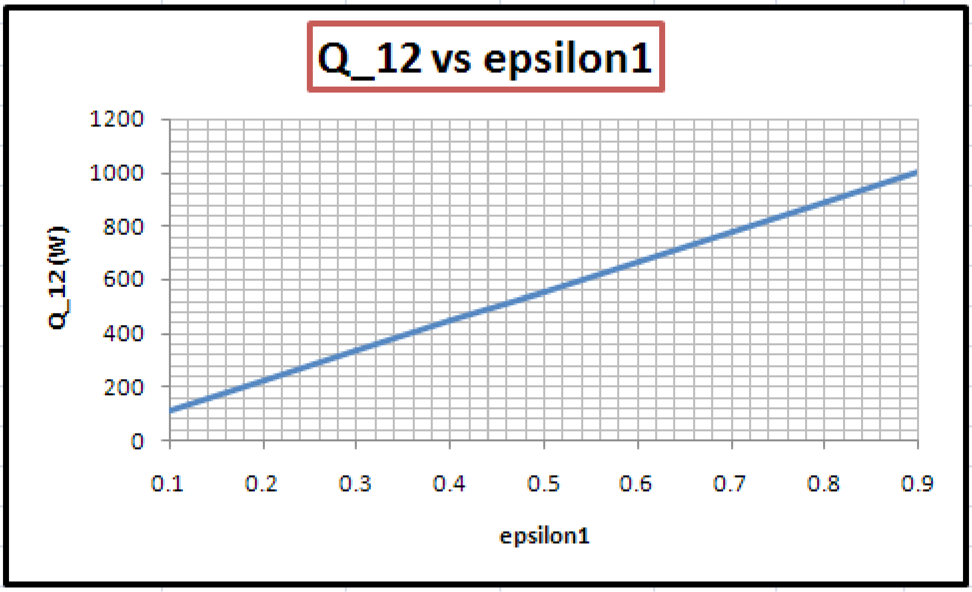
epsilon1	Q_12 (W)
0.1	111.408
0.2	
0.3	
0.4	
0.5	
0.6	
0.7	
0.8	
0.9	

In the above, note that Q12 is calculated in cell C108 using the VBA Function. Take care to enter epsilon1 with *relative reference*, all other quantities are with *absolute reference*, so that when we drag-copy from cell C108 to cell C116, Q12 values are suitably calculated and the Table is filled up. See below:

C116 =Q_12_small_object_in_large_cavity($C$98,B116,$C$99,$C$100)

epsilon1	Q_12 (W)
0.1	111.408
0.2	222.815
0.3	334.223
0.4	445.630
0.5	557.038
0.6	668.446
0.7	779.853
0.8	891.261
0.9	1002.668

Now, plot the results:



==

Prob.5.C.1.20. Three thin-walled, 3 m long, hollow cylinders of radii 5 cm, 10 cm and 15 cm are arranged concentrically. T1 = 1000 K and T3 = 300 K. Assuming epsilon1 = epsilon2 = epsilon3 = 0.05 and vacuum in the spaces between the cylinders, calculate the steady state temp of cylindrical surface 2, and the heat flow rate.

EXCEL Solution:

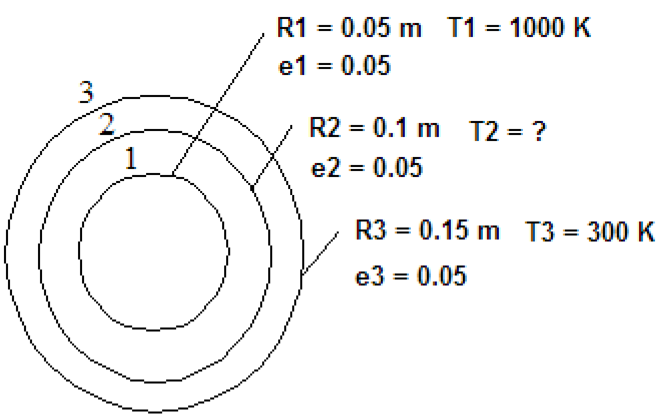


Fig.Prob.5.C.1.20

We shall use the condition that in steady state, Q12 = Q23 to calculate the temp. T2.

VBA Function for infinite, concentric cylinders is already written and we use it to calculate Q12 and Q23.

We also use Goal Seek in EXCEL to find T2 such that Q12 = Q23.

Following is the worksheet:

C140 | =Q_12_Infinite_concentric_cylinders(2*PI()*B136*C133,B136,C136,E136,F136,B138,C138)

	A	B	C	D	E	F	G	H	I
130									
131									
132		Data:							
133		L	3	m					
134									
135		R_1	R_2	R_3	e_1	e_2	e_3		
136		0.05	0.1	0.15	0.05	0.05	0.05		
137		T_1	T_2	T_3					
138		1000	400.000	300					
139									
140		Q_12=	1765.100531						
141		Q_23	57.25552611						
142		Diff^2	2916734.561						
143									
144									

Start with a trial value for T2. Use GoalSeek to make Diff^2 = 0 by changing T_2

In the above worksheet, **we have started with a trial value for T2 (= 400 K).**

Calculate Q12 and Q23 using the VBA Functions for Infinitely long, concentric cylinders, written earlier.

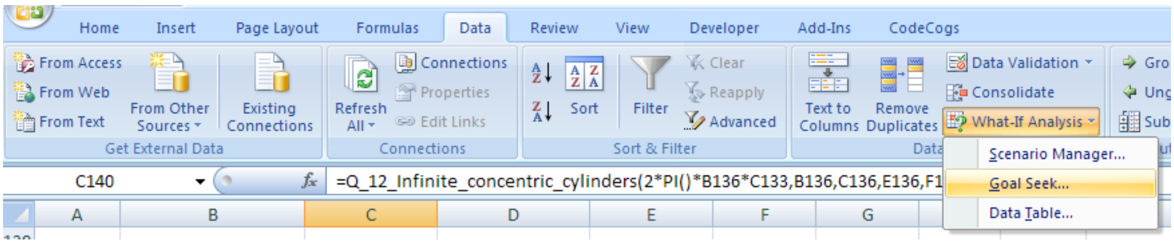
Formula bar shows the Function entered for Q12 in cell C140.

In steady state, Q12 should be equal to Q23. i.e. (Q12 – Q23)^2 = 0.i.e. cell C142 should be equal to zero. But, now it is not equal to zero since we have started with an arbitrary, trial value for T2.

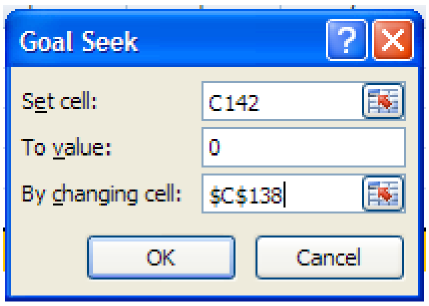
To find the correct value of T2:

Use Goal Seek to set (Q12 – Q23)^2 = 0, by changing T2, i.e. cell C138:

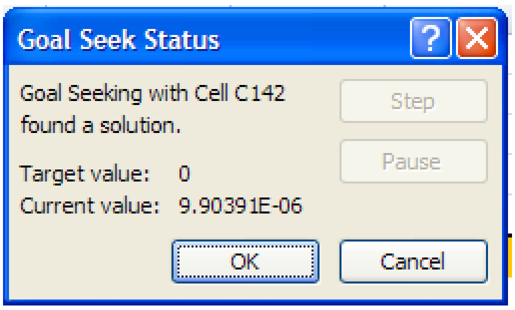
To apply Goal Seek: Go to: Data – What If analysis – Goal Seek:



Click on Goal Seek. We get following screen. Fill it up as shown:



Now, click OK and we get:



Goal Seek has found a solution.

Again click OK to see value of T2 in cell C138:

C142 fx =(C140-C141)^2

	A	B	C	D	E	F	G
130							
131							
132		Data:					
133		L	3	m			
134							
135		R_1	R_2	R_3	e_1	e_2	e_3
136		0.05	0.1	0.15	0.05	0.05	0.05
137		T_1	T_2	T_3			
138		1000	775.443	300			
139							
140		Q_12=	1156.487796				
141		Q_23	1156.484649				
142		Diff^2	9.90391E-06				
143							
144							

Start with a trial value for T2. Use GoalSeek to make Diff^2 = 0 by changing T_2

Therefore:

In steady state, T2 = 775.443 K …. Ans.

Q12 = Q23 = 1156.49 W …. Ans.

===

5.C.2 Radiation energy exchange in 3-surface enclosures:

Prob. 5.C.2.1. Write Mathcad Functions for a general three-surface enclosures.

Recollect the schematic and Radiation network for a general 3-surface enclosure (shown below).

Here, for the 3 surfaces, area (a1, A2, A3), Temperatures (T1, T2, T3) and emissivities (eps1, eps2, eps3) are given. View Factors F12, F3 and F23 are either given or calculated. We have to *find out the three Radiosities, and the rate of heat transfers.*

The procedure is:

i) *Apply Kirchoff's Law* to the three nodes to get three algebraic equations. Here, it is important to assume that the (heat) current flow is ***into*** the node. The signs adjust themselves while solving for Radiosities.

ii) Now, we write a Function to solve for the Radiosities J1, J2 and J3 with 'Solve Block' of Mathcad.

iii) Then, we use this Function in another Mathcad Function to get the values of heat transferred from each surface and between any two surfaces.

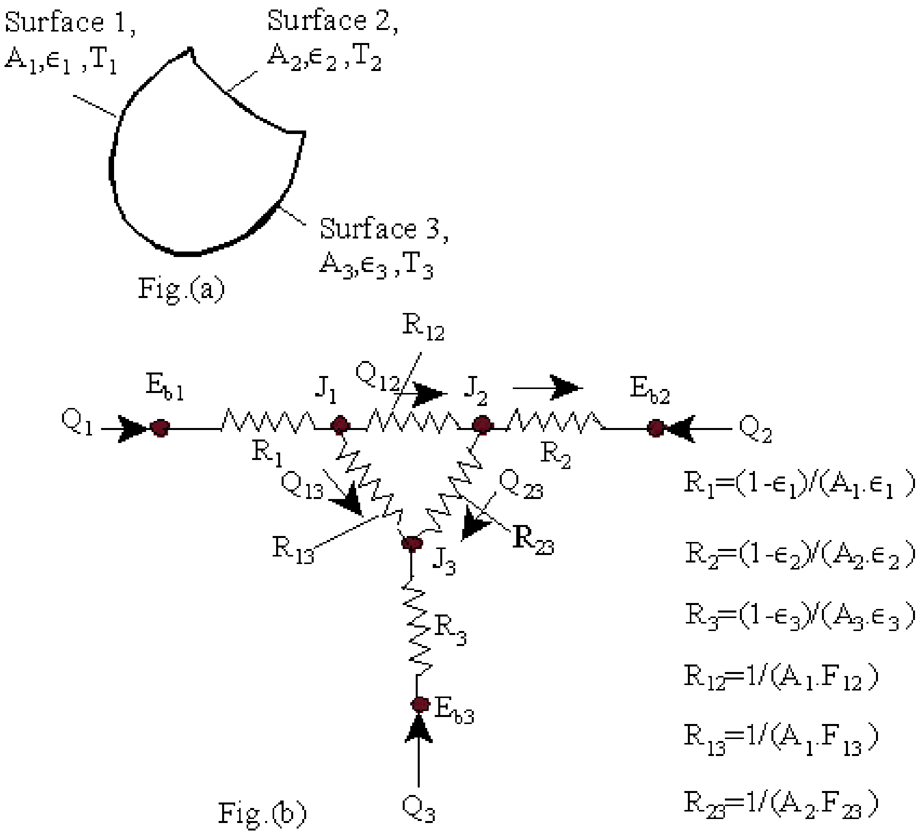


Fig. Prob.5.C.2.1

Writing Kirchoff's Law to Nodes J1, J2:

$$\frac{Eb1 - J1}{R1} + \frac{J2 - J1}{R12} + \frac{J3 - J1}{R13} = 0$$ ....for node J1

$$\frac{Eb2 - J2}{R2} + \frac{J1 - J2}{R12} + \frac{J3 - J2}{R23} = 0$$ ....for node J2

$$\frac{Eb3 - J3}{R3} + \frac{J1 - J3}{R13} + \frac{J2 - J3}{R23} = 0$$ ....for node J3

Now, write a Function to solve for J1, J2 and J3 using 'Solve Block' of Mathcad:

$J1 := 1000 \quad J2 := 1000 \quad J3 := 1000$ ...Guess values of Radiosities

Given

$$\frac{Eb1 - J1}{R1} + \frac{J2 - J1}{R12} + \frac{J3 - J1}{R13} = 0$$ ....for node J1

$$\frac{Eb2 - J2}{R2} + \frac{J1 - J2}{R12} + \frac{J3 - J2}{R23} = 0$$ ....for node J2

$$\frac{Eb3 - J3}{R3} + \frac{J1 - J3}{R13} + \frac{J2 - J3}{R23} = 0$$ ....for node J3

$Radiosities(Eb1, Eb2, Eb3, R1, R2, R3, R12, R13, R23) := Find(J1, J2, J3)$

Use the above Function in another Function to calculate the heat transfers:

$$HTransfer\_Three\_surface\_enclosure(A1, A2, A3, T1, T2, T3, eps1, eps2, eps3, F12, F13, F23) :=$$

$$R1 \leftarrow \frac{(1 - eps1)}{A1 \cdot eps1}$$
$$R2 \leftarrow \frac{(1 - eps2)}{A2 \cdot eps2}$$
$$R3 \leftarrow \frac{(1 - eps3)}{A3 \cdot eps3}$$
$$R12 \leftarrow \frac{1}{A1 \cdot F12}$$
$$R13 \leftarrow \frac{1}{A1 \cdot F13}$$
$$R23 \leftarrow \frac{1}{A2 \cdot F23}$$
$$Eb1 \leftarrow 5.67 \cdot 10^{-8} \cdot T1^4$$
$$Eb2 \leftarrow 5.67 \cdot 10^{-8} \cdot T2^4$$
$$Eb3 \leftarrow 5.67 \cdot 10^{-8} \cdot T3^4$$
$$M \leftarrow Radiosities(Eb1, Eb2, Eb3, R1, R2, R3, R12, R13, R23)$$
$$J1 \leftarrow M_0$$
$$J2 \leftarrow M_1$$
$$J3 \leftarrow M_2$$
$$Q12 \leftarrow \frac{J1 - J2}{R12}$$
$$Q13 \leftarrow \frac{J1 - J3}{R13}$$
$$Q23 \leftarrow \frac{J2 - J3}{R23}$$
$$Q1 \leftarrow \frac{Eb1 - J1}{R1}$$
$$Q2 \leftarrow \frac{Eb2 - J2}{R2}$$
$$Q3 \leftarrow \frac{Eb3 - J3}{R3}$$
$$HTransfer \leftarrow \begin{pmatrix} \text{"Q12(W)"} & \text{"Q13(W)"} & \text{"Q23(W)"} & \text{"Q1(W)"} & \text{"Q2(W)"} & \text{"Q3(W)"} \\ Q12 & Q13 & Q23 & Q1 & Q2 & Q3 \end{pmatrix}$$

In the above program:

Inputs: Areas, Temps, emissivities and View Factors, as shown above.

Ouputs: Q1, Q2, Q3 (i.e. heat transfer to/from each surface) and Q12, Q13, Q23 (i.e. net heat transfer between respective surfaces)

On the LHS we have the Function statement.

On RHS:

Lines 1,2,3: calculate the three 'surface resistances'

Lines 4,5,6: calculate the three 'space resistances'

Lines 7,8,9: calculate the Emissive powers of three surfaces

Line 10: use the Function, written above, to get three Radiosities J1, J2 and J3 in vector M

Lines 11,12,13: extract the values of J1,J2 and J3

Lines 14,15,16: calculate the various heat transfers

Line 17: return the calculated values in a Matrix

Let us work out a problem using the above Function:

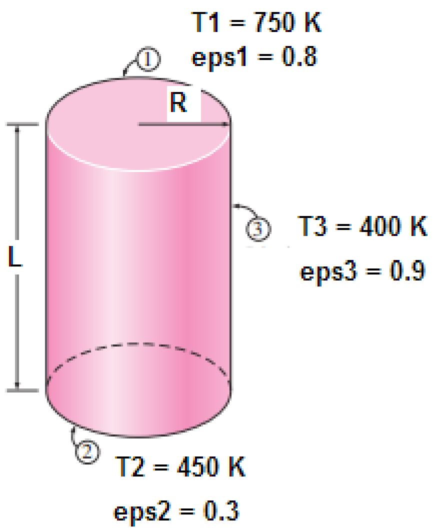
Prob. 5.C.2.2. Consider a cylindrical furnace with R = 1 m, L = 1.25 m as shown. Emissivities and temperatures of surfaces 1, 2 and 3 are: eps1 = 0.8, eps2 = 0.3, eps3 = 0.9, T1 = 750 K, T2 = 450 K and T3 = 400 K respectively. Determine the net rate of radiation to / from each surface.

(b) If surface 3 is black, what are the values of Q1, Q2 and Q3?

(c) For the case (a) plot the variation of Q13 as emissivity of surface 3 varies from 0.1 to 1.

Mathcad Solution:

This is a general, three-surface enclosure, for which the electrical network and the Mathcad Function are written above.



Data:

$R := 1$ m $\quad L := 1.25$ m $\quad eps1 := 0.8 \quad eps2 := 0.3 \quad eps3 := 0.9$

$T1 := 750$ K $\quad T2 := 450$ K $\quad T3 := 400$ K

Calculations:

$A1 := \pi \cdot R^2$ i.e. $A1 = 3.142$ m^2

$A2 := \pi \cdot R^2$ i.e. $A2 = 3.142$ m^2

$A3 := 2\pi \cdot R \cdot L$ i.e. $A3 = 7.854$ m^2

Now, we need View Factors: F12, F13 and F23:

Use the Mathcad Function for View factors for parallel disks, already written:

Recollect:

...View factor for coaxial parallel disks:

Define:

$R_i = \frac{r_i}{L}$ $\quad R_j = \frac{r_j}{L}$ $\quad S(R_i, R_j) := 1 + \frac{1 + R_j^2}{R_i^2}$

$$F_{ij}(R_i, R_j) := \frac{1}{2} \cdot \left[S(R_i, R_j) - \left[S(R_i, R_j)^2 - 4 \cdot \left(\frac{R_j}{R_i} \right)^2 \right]^{\frac{1}{2}} \right]$$ **...View factor for coaxial parallel disks**

Then, with the above notations:

$R_i := \frac{R}{L} \quad R_j := \frac{R}{L}$

$F12 := F_{ij}(R_i, R_j)$

i.e. $F12 = 0.307$ **....View factor from top surface 1 to bottom surface 2**

And,

$F11 + F12 + F13 = 1$ **.... by summation rule for surface 1**

But, $F11 := 0$ ...since surface 1 is Flat

Therefore: $F13 := 1 - F12$

i.e. $F13 = 0.693$ **....View factor from top surface 1 to side surfaces 3**

And, by symmetry: $F23 := F13$

Now, use the Mathcad Function for general, three-surface enclosure:

HTransfer_Three_surface_enclosure(A1, A2, A3, T1, T2, T3, eps1, eps2, eps3, F12, F13, F23) =

$$\begin{pmatrix} \text{"Q12(W)"} & \text{"Q13(W)"} & \text{"Q23(W)"} & \text{"Q1(W)"} & \text{"Q2(W)"} & \text{"Q3(W)"} \\ 9.7203 \times 10^3 & 2.8247 \times 10^4 & 6.3252 \times 10^3 & 3.7968 \times 10^4 & -3.3951 \times 10^3 & -3.4573 \times 10^4 \end{pmatrix}$$

i.e.

$$Q1 := \mathrm{HTransfer\_Three\_surface\_enclosure}(A1, A2, A3, T1, T2, T3, eps1, eps2, eps3, F12, F13, F23)_{1,3}$$

or: $Q1 = 3.797 \times 10^{4}$ **W.... heat *leaving* surface 1 .... Ans.**

And:

$$Q2 := \mathrm{HTransfer\_Three\_surface\_enclosure}(A1, A2, A3, T1, T2, T3, eps1, eps2, eps3, F12, F13, F23)_{1,4}$$

or: $Q2 = -3.395 \times 10^{3}$ **W.... heat *into* surface 2.... Ans.**

$$Q3 := \mathrm{HTransfer\_Three\_surface\_enclosure}(A1, A2, A3, T1, T2, T3, eps1, eps2, eps3, F12, F13, F23)_{1,5}$$

or: $Q3 = -3.457 \times 10^{4}$ **W.... heat *into* surface 3....Ans.**

Check: algebraic sum of Q1+Q2+Q3 should be equal to zero:

i.e. $Q1 + Q2 + Q3 = -1.455 \times 10^{-11}$ **..very nearly equal to zero...Checks.**

Note: In the results of the above Function, heat transfer between surfaces, i.e. Q12, Q13 and Q23 are also available.

(b) If the surface 3 is black, i.e. eps3 = 1:

We have, for surface 3, eps3 = 1. However, if we put eps3 = 1, looking at the Function, we see that R3 will be 0/0, i.e. indeterminate.

To overcome this problem, we write: eps3 = 0.9999, i.e. almost equal to 1, and we get sufficiently accurate results:

So, we write: $eps3 := 0.9999$

Now, use the Mathcad Function for general, three-surface enclosure:

$$\mathrm{HTransfer\_Three\_surface\_enclosure}(A1, A2, A3, T1, T2, T3, eps1, eps2, eps3, F12, F13, F23) = \begin{pmatrix} \text{"Q12(W)"} & \text{"Q13(W)"} & \text{"Q23(W)"} & \text{"Q1(W)"} & \text{"Q2(W)"} & \text{"Q3(W)"} \\ 9.886 \times 10^{3} & 2.913 \times 10^{4} & 6.834 \times 10^{3} & 3.902 \times 10^{4} & -3.052 \times 10^{3} & -3.596 \times 10^{4} \end{pmatrix}$$

i.e.

$$Q1 := \mathrm{HTransfer\_Three\_surface\_enclosure}(A1, A2, A3, T1, T2, T3, eps1, eps2, eps3, F12, F13, F23)_{1,3}$$

or: $Q1 = 3.902 \times 10^{4}$ **W.... heat *leaving* surface 1 .... Ans.**

And:

$$Q2 := \text{HTransfer\_Three\_surface\_enclosure}(A1, A2, A3, T1, T2, T3, eps1, eps2, eps3, F12, F13, F2$$

or: $Q2 = -3.052 \times 10^{3}$ **W.... heat *<u>into</u>* surface 2.... Ans.**

$$Q3 := \text{HTransfer\_Three\_surface\_enclosure}(A1, A2, A3, T1, T2, T3, eps1, eps2, eps3, F12, F13, F23$$

or: $Q3 = -3.596 \times 10^{4}$ **W.... heat *<u>into</u>* surface 3....Ans.**

Check: algebraic sum of Q1+Q2+Q3 should be equal to zero:

i.e. $Q1 + Q2 + Q3 = -7.385 \times 10^{-9}$ **..very nearly equal to zero...Checks.**

Note: In the results of the above Function, heat transfer between surfaces, i.e. Q12, Q13 and Q23 are also available.

(c) For the case (a), plot the variation of Q1 and Q13 as emissivity of surface 3 varies from 0.1 to 1:

Define the relevant quantities as functions of eps3:

Then:

$Q1(eps3) := HTransfer\_Three\_surface\_enclosure(A1, A2, A3, T1, T2, T3, eps1, eps2, eps3, F12, F13, F23)_{1,3}$

$Q12(eps3) := HTransfer\_Three\_surface\_enclosure(A1, A2, A3, T1, T2, T3, eps1, eps2, eps3, F12, F13, F23)_{1,0}$

$eps3 := (0.1\ \ 0.2\ \ 0.3\ \ 0.4\ \ 0.5\ \ 0.6\ \ 0.7\ \ 0.8\ \ 0.9\ \ 0.999)$ **...define a vector of eps3 values**

(**Note** that we have put eps3 = 0.999 instead of 1, to avoid dividing 0 by 0, as explained earlier).

Compute the Table:

$eps3^T =$

	0
0	0.1
1	0.2
2	0.3
3	0.4
4	0.5
5	0.6
6	0.7
7	0.8
8	0.9
9	0.999

$\left(\overrightarrow{Q1(eps3)}\right)^T =$

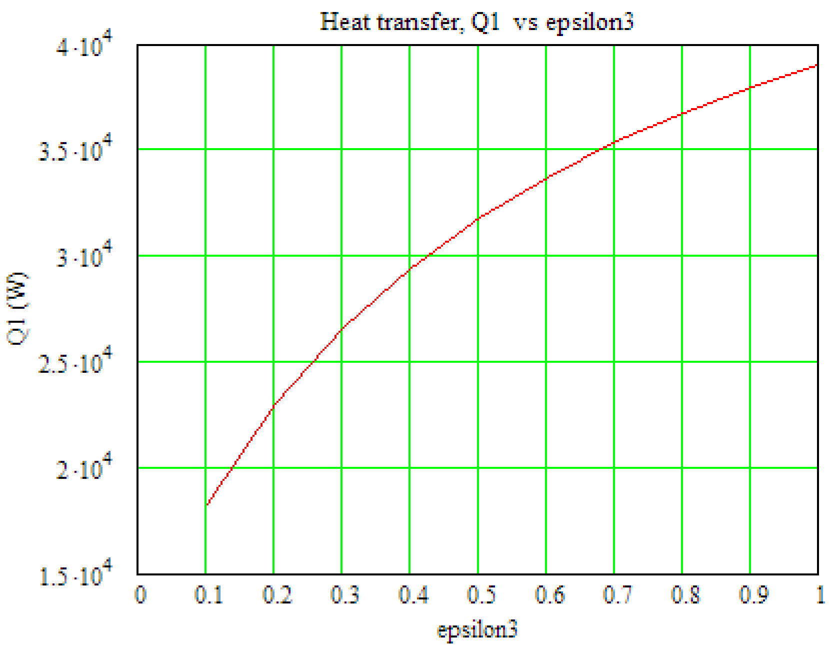
	0
0	$1.828\cdot10^4$
1	$2.299\cdot10^4$
2	$2.66\cdot10^4$
3	$2.948\cdot10^4$
4	$3.181\cdot10^4$
5	$3.374\cdot10^4$
6	$3.537\cdot10^4$
7	$3.677\cdot10^4$
8	$3.797\cdot10^4$
9	$3.901\cdot10^4$

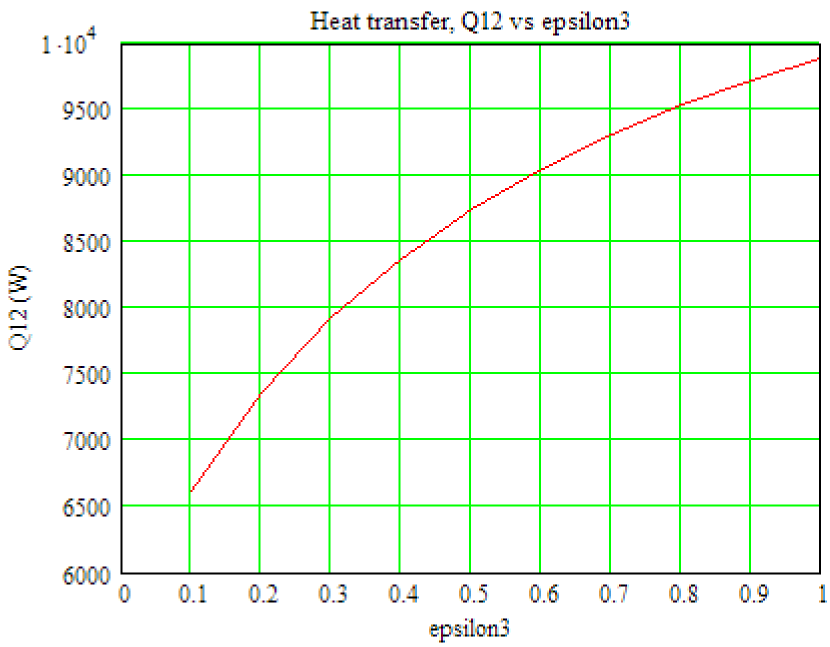
$\left(\overrightarrow{Q12(eps3)}\right)^T =$

	0
0	$6.61\cdot10^3$
1	$7.353\cdot10^3$
2	$7.925\cdot10^3$
3	$8.378\cdot10^3$
4	$8.747\cdot10^3$
5	$9.053\cdot10^3$
6	$9.311\cdot10^3$
7	$9.53\cdot10^3$
8	$9.72\cdot10^3$
9	$9.884\cdot10^3$

Plot the graphs:

$i := 0..9$


Heat transfer, Q1 vs epsilon3
4·10^4
3.5·10^4
3·10^4
2.5·10^4
2·10^4
1.5·10^4
Q1 (W)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
epsilon3


Heat transfer, Q12 vs epsilon3
1·10^4
9500
9000
8500
8000
7500
7000
6500
6000
Q12 (W)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
epsilon3

==

Prob. 5.C.2.3. Two parallel plates, 0.5 m × 1 m each, are spaced 0.5 m apart. The plates are at temperatures of 1000 C and 500 C and their emissivities are 0.2 and 0.5 respectively. The plates are located in a large room, the walls of which are at 27 C. The surfaces of the plates facing each other only exchange heat by radiation. Determine the rates of heat lost by each plate and heat gain of the walls by radiation. Use radiation network for solution. Assume shape factor between parallel plates: F12 = F21 = 0.285. [M.U. 1996]

(b) In addition, plot Q1 and Q12 against ε1, for 0.1 < ε1 < 0.9.

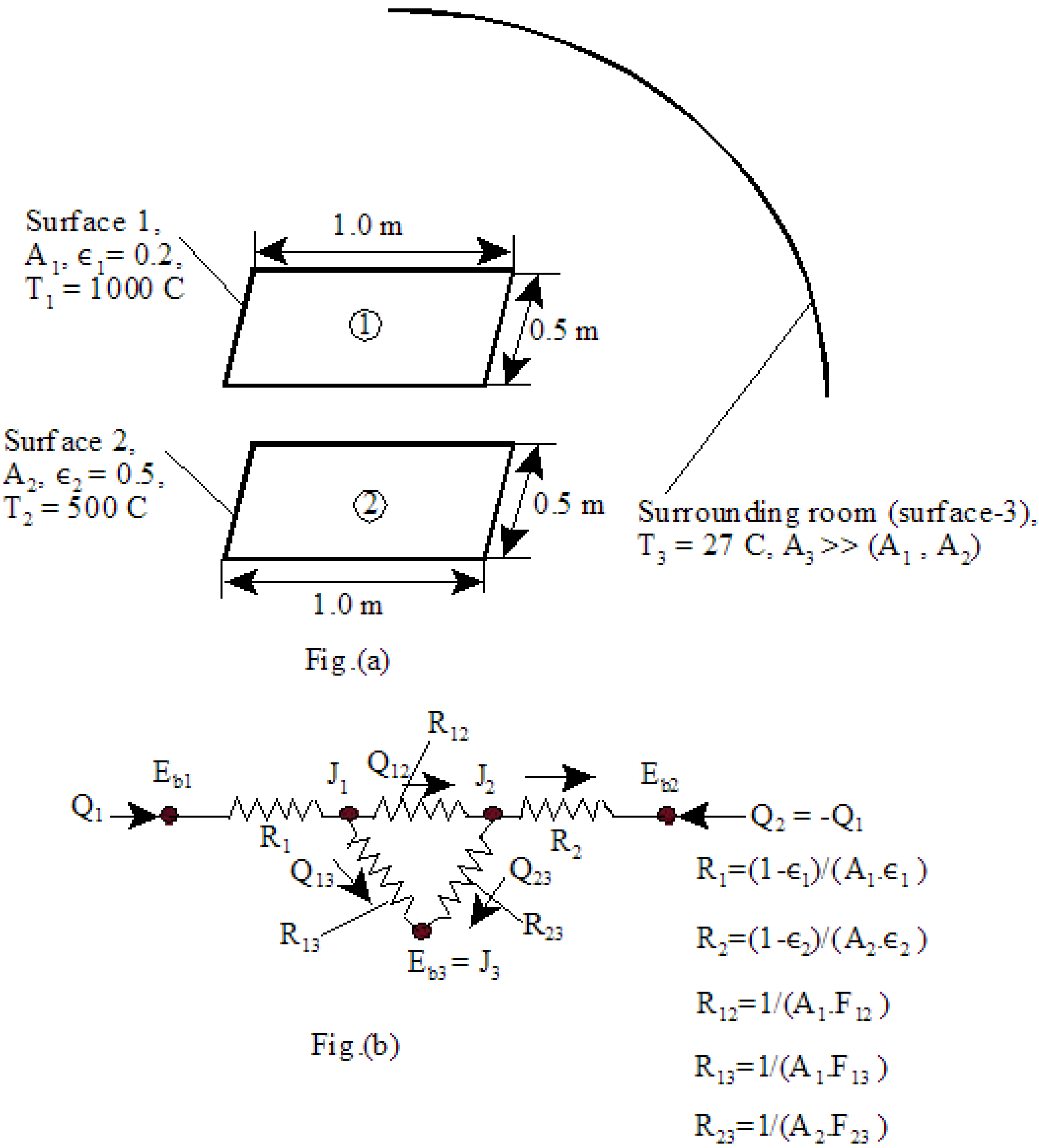


Fig. Prob.5.C.2.3

Mathcad Solution:

This type of problem is very popular with examiners in the University Question Papers.

Let the plate surfaces be designated as 1 and 2, and the surroundings as 3.

The schematic fig for this three surface enclosure and the Radiation network is shown in the Fig.(a) and (b) below.

We shall first solve this problem in the usual way, from fundamentals.

Then, we shall verify the results using the Mathcad Function for general, three-surface enclosure written earlier.

Since the area A_3 of the room is very large, we can take the surface resistance of A_3 as equal to zero.

i.e. $\frac{1-\varepsilon_3}{A_3 \cdot \varepsilon_3} = 0$

This means that Eb3 = J3, **i.e. a large room is equivalent to a black surface.**

Data:

$A_1 := 0.5$ m²...area of surface 1 $A_2 := 0.5$ m²...area of surface 2

$T_1 := 1000 + 273$ K...temp. of surface 1 $T_2 := 500 + 273$ K...temp. of surface 2

$T_3 := 27 + 273$ K...temp. of surface 3 (i.e. room)

$\varepsilon_1 := 0.2$ ..emissivity of surface 1 $\varepsilon_2 := 0.5$ ..emissivity of surface 2

$F_{12} := 0.285$ ...view factor of surface 1 w.r.t. surface 2

$F_{21} := 0.285$ ...view factor of surface 2 w.r.t. surface 1

$\sigma := 5.67 \cdot 10^{-8}$ W/(m².K)...Stefan-Boltzmann const.

Calculations:

Now, $F_{11} + F_{12} + F_{13} = 1$ ...by summation rule

But, $F_{11} = 0$ ...since surface 1 is flat and can not 'see' itself.

Therefore, $F_{12} + F_{13} = 1$

And, $F_{13} := 1 - F_{12}$

i.e. $F_{13} = 0.715$ ...view factor of surface 1 w.r.t. surface 3

Similarly, $F_{23} := 0.715$ ...view factor of surface 2 w.r.t. surface 3

Resistances:

$R_1 := \frac{1 - \varepsilon_1}{A_1 \cdot \varepsilon_1}$ i.e. $R_1 = 8$ m^{-2}....surface resistance of surface 1

$R_2 := \frac{1 - \varepsilon_2}{A_2 \cdot \varepsilon_2}$ i.e. $R_2 = 2$ m^{-2}....surface resistance of surface 2

$R_{12} := \frac{1}{A_1 \cdot F_{12}}$ i.e. $R_{12} = 7.018$ m^{-2}....space resistance between surfaces 1 and 2

$R_{13} := \frac{1}{A_1 \cdot F_{13}}$ i.e. $R_{13} = 2.797$ m^{-2}....space resistance between surfaces 1 and 3

$R_{23} := \frac{1}{A_2 \cdot F_{23}}$ i.e. $R_{23} = 2.797$ m^{-2}....space resistance between surfaes 2 and 3

Heat lost by each surface:

$Q_1 = \frac{E_{b1} - J_1}{R_1}$ = heat lost by surface 1

and, $Q_2 = \frac{E_{b2} - J_2}{R_2}$ = heat lost by surface 2

And, heat gain by surface 3:

$$Q_3 = Q_{13} + Q_{23}$$

i.e. $Q_3 = \frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}}$

Therefore, the problem reduces to calculating the radiosities, J_1, J_2 and J_3.

To calculate the radiosities J_1 and J_2, apply Kirchoff's law of electric circuits to nodes J_1 and J_2:

Node J1: $$\frac{E_{b1} - J_1}{R_1} + \frac{J_2 - J_1}{R_{12}} + \frac{E_{b3} - J_1}{R_{13}} = 0 \qquad \text{....eqn. (a)}$$

Node J2: $$\frac{J_1 - J_2}{R_{12}} + \frac{E_{b3} - J_2}{R_{23}} + \frac{E_{b2} - J_2}{R_2} = 0 \qquad \text{....eqn. (b)}$$

Emissive powers:

$E_{b1} := \sigma \cdot T_1^{\,4}$ i.e. $E_{b1} = 1.489 \times 10^5$ W/m^2....for surface 1

$E_{b2} := \sigma \cdot T_2^{\,4}$ i.e. $E_{b2} = 2.024 \times 10^4$ W/m^2....for surface 2

$E_{b3} := \sigma \cdot T_3^{\,4}$ i.e. $E_{b3} = 459.27$ W/m^2....for surface 3

Note that: $J_3 := E_{b3}$ ...for the large room

To get J_1 and J_2, solve eqns. (a) and (b) simultaneously. To do this, we shall use **'Solve block'** of Mathcad.

First, choose trial (or, guess) values for J_1 and J_2. Then, immediately after 'Given', write the constraints, viz. eqns. (a) and (b). Now, type 'Find (J_1,J_2) = ', and the result appears immediately:

$J_1 := 100 \quad J_2 := 100$ ....trial values

Given

$$\frac{E_{b1} - J_1}{R_1} + \frac{J_2 - J_1}{R_{12}} + \frac{E_{b3} - J_1}{R_{13}} = 0$$

$$\frac{J_1 - J_2}{R_{12}} + \frac{E_{b3} - J_2}{R_{23}} + \frac{E_{b2} - J_2}{R_2} = 0$$

$$\text{Find}(J_1, J_2) = \begin{pmatrix} 3.3476 \times 10^4 \\ 1.5057 \times 10^4 \end{pmatrix}$$

i.e. $J_1 := 3.3476 \cdot 10^4$ W/m2

and, $J_2 := 1.5057 \cdot 10^4$ W/m2

Therefore,

Heat lost by each surface:

$$Q_1 := \frac{E_{b1} - J_1}{R_1}$$

i.e. $Q_1 = 1.443 \times 10^4$ **W = heat lost by surface 1...Ans.**

and, $$Q_2 := \frac{E_{b2} - J_2}{R_2}$$

i.e. $Q_2 = 2.594 \times 10^3$ **W = heat lost by surface 2...Ans.**

Now, heat lost by both surfaces 1 and 2 is *gained* by the surroundings; so, heat gained by surroundings = $Q_3 = Q_1 + Q_2$

i.e. $Q_3 := Q_1 + Q_2$

i.e. $Q_3 = 1.702 \times 10^4$ **W = heat gained by surface 3...Ans.**

Verify:

We have: $Q_3 := \left(\frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}}\right)$

i.e. $Q_3 = 1.702 \times 10^4$ **W = heat gained by surface 3...verified.**

Now, let us solve the above problem using the Mathcad Function:

We have the data:

$A_1 := 0.5$ m²...area of surface 1 $A_2 := 0.5$ m²...area of surface 2

$T_1 := 1000 + 273$ K...temp. of surface 1 $T_2 := 500 + 273$ K...temp. of surface 2

$T_3 := 27 + 273$ K...temp. of surface 3 (i.e. room)

$\varepsilon_1 := 0.2$ ..emissivity of surface 1 $\varepsilon_2 := 0.5$ ..emissivity of surface 2

$F_{12} := 0.285$ ...view factor of surface 1 w.r.t. surface 2

$F_{21} := 0.285$ ...view factor of surface 2 w.r.t. surface 1

and:

$F_{13} := 0.715$ ...view factor of surface 1 w.r.t. surface 3

For a very large, (black) surrouding, we write:

$A_3 := 10^6$ m^2.... i.e. a very large value

$\varepsilon_3 := 0.9999$ ...for a Black body, emissivity = 1, but we take it as 0.9999 to avoid division by zero, as explained earlier.

Then, applying the Mathcad Function for a general, three-surface enclosure:

$$HTransfer\_Three\_surface\_enclosure(A_1, A_2, A_3, T_1, T_2, T_3, \varepsilon_1, \varepsilon_2, \varepsilon_3, F_{12}, F_{13}, F_{23}) =$$

$$\begin{pmatrix} \text{"Q12(W)"} & \text{"Q13(W)"} & \text{"Q23(W)"} & \text{"Q1(W)"} & \text{"Q2(W)"} & \text{"Q3(W)"} \\ 2.625 \times 10^3 & 1.18 \times 10^4 & 5.219 \times 10^3 & 1.443 \times 10^4 & 2.594 \times 10^3 & -1.702 \times 10^4 \end{pmatrix}$$

i.e. $Q1 = 1.443 \cdot 10^3$ **W....Ans.**

$Q2 = 2.594 \cdot 10^3$ **W....Ans.**

$Q3 = -1.702 \cdot 10^3$ **W...Ans....*negative sign* indicates that heat is gained by the surface 3, i.e. the surroundings.**

i.e. We get practically the same results, as obtained earlier.

Note how easy it is to solve this problem with the Mathcad Function.

Also, note that we have obtained the values for other heat transfer quantities, i.e. Q12 between surfaces 1 and 2, Q13 between surfaces 1 and 3, and Q23 between surfaces 2 and 3.

In addition:

Plot the variation of Q1, Q12 as e1 varies from 0.1 to 0.9:

$\varepsilon_1 := 0.1, 0.2..0.9$ ....define a range variable

Write Q1, Q12 as functions of ε1:

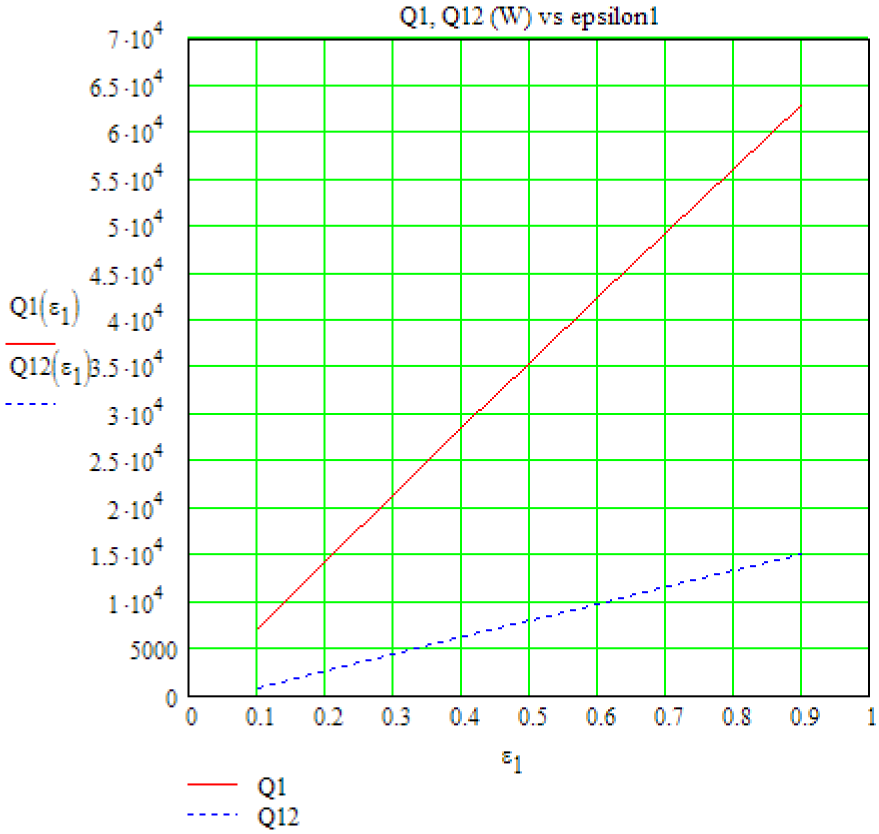
$$Q1(\varepsilon_1) := HTransfer\_Three\_surface\_enclosure(A_1, A_2, A_3, T_1, T_2, T_3, \varepsilon_1, \varepsilon_2, \varepsilon_3, F_{12}, F_{13}, F_{23})_{1,3}$$

$$Q12(\varepsilon_1) := HTransfer\_Three\_surface\_enclosure(A_1, A_2, A_3, T_1, T_2, T_3, \varepsilon_1, \varepsilon_2, \varepsilon_3, F_{12}, F_{13}, F_{23})_{1,0}$$

Now, compute the Table:

$\varepsilon_1 =$	$Q1(\varepsilon_1) =$	$Q12(\varepsilon_1) =$
0.1	$7.244 \cdot 10^3$	794.817
0.2	$1.443 \cdot 10^4$	$2.625 \cdot 10^3$
0.3	$2.155 \cdot 10^4$	$4.439 \cdot 10^3$
0.4	$2.862 \cdot 10^4$	$6.239 \cdot 10^3$
0.5	$3.562 \cdot 10^4$	$8.023 \cdot 10^3$
0.6	$4.257 \cdot 10^4$	$9.793 \cdot 10^3$
0.7	$4.946 \cdot 10^4$	$1.155 \cdot 10^4$
0.8	$5.629 \cdot 10^4$	$1.329 \cdot 10^4$
0.9	$6.307 \cdot 10^4$	$1.502 \cdot 10^4$

And, plot the results:



==

Prob. 5.C.2.4. Two parallel plates of size 1 m × 1 m are spaced 0.5 m apart and are located in a very large room, the walls of which are maintained at a temp of 27 C. One plate is maintained at a temp of 900 C and the other at 400 C. Their emissivities are 0.2 and 0.5 respectively. If the plates exchange heat between themselves and surroundings, find the net heat transfer to each plate and to the room. Consider only the plate surfaces facing each other. [VTU – M.Tech. – Dec. 2009–Jan. 2010](b) In addition, plot Q1 and Q12 against ε1, for 0.1 < ε1 < 0.9.

Mathcad Solution:

This problem is identical to the previous problem.

We shall quickly solve it using the Mathcad Function for general, three-surface enclosure:

We have the data:

$A_1 := 1$ m^2...area of surface 1 $A_2 := 1$ m^2...area of surface 2 $L := 0.5$ m

$T_1 := 900 + 273$ K...temp. of surface 1 $T_2 := 400 + 273$ K...temp. of surface 2

$T_3 := 27 + 273$ K...temp. of surface 3 (i.e. room)

$\varepsilon_1 := 0.2$ ..emissivity of surface 1 $\varepsilon_2 := 0.5$ ..emissivity of surface 2

For a very large, (black) surrouding, we write:

$A_3 := 10^6$ m^2.... i.e. a very large value

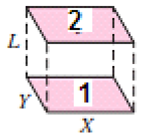
$\varepsilon_3 := 0.9999$ ...for a Black body, emissivity = 1, but we take it as 0.9999 to avoid division by zero, as explained earlier.

Now, we need the View factors F12, F13 and F23:

We can use the graphs, or use the Mathcad Function for ***View Factor between parallel, coaxial plates***, written earlier.

Let us recollect the Mathcad Function:

Geometry:



$$\text{F12\_parallel\_rectangles}(X,Y,L) := \left|\begin{array}{l} XX \leftarrow \dfrac{X}{L} \\ YY \leftarrow \dfrac{Y}{L} \\ A \leftarrow \dfrac{2}{\pi \cdot XX \cdot YY} \\ B \leftarrow \ln\left[\sqrt{\dfrac{(1+XX^2)\cdot(1+YY^2)}{1+XX^2+YY^2}}\right] \\ C \leftarrow XX\cdot(1+YY^2)^{\frac{1}{2}}\cdot\text{atan}\left[\dfrac{XX}{(1+YY^2)^{\frac{1}{2}}}\right] \\ D \leftarrow YY\cdot(1+XX^2)^{\frac{1}{2}}\cdot\text{atan}\left[\dfrac{YY}{(1+XX^2)^{\frac{1}{2}}}\right] \\ E \leftarrow XX\cdot\text{atan}(XX) \\ F \leftarrow YY\cdot\text{atan}(YY) \\ F_{12} \leftarrow A\cdot(B+C+D-E-F) \end{array}\right.$$

Then: $F12 := F12\_parallel\_rectangles(1,1,0.5)$

i.e. $F12 = 0.415$ **...View Factor from surface 1 to surface 2**

Also, from **Summation rule** for surface 1:

$F11 + F12 + F13 = 1$

But, $F11 := 0$ ..since it is a Flat surface

Therefore: $F13 := 1 - F12$

i.e. $F13 = 0.585$ **...View Factor from surface 1 to surface 3**

Then, by symmetery: $F23 := F13$ **...View Factor from surface 2 to surface 3**

Now, invoke the Mathcad Function to get various heat transfers:

$$HTransfer\_Three\_surface\_enclosure(A_1, A_2, A_3, T_1, T_2, T_3, \varepsilon_1, \varepsilon_2, \varepsilon_3, F12, F13, F23) =$$

$$\begin{pmatrix} "Q12(W)" & "Q13(W)" & "Q23(W)" & "Q1(W)" & "Q2(W)" & "Q3(W)" \\ 5.891 \times 10^3 & 1.4592 \times 10^4 & 6.2961 \times 10^3 & 2.0483 \times 10^4 & 405.1572 & -2.0888 \times 10^4 \end{pmatrix}$$

i.e. $Q1 = 20483$ **W....heat leaving surface 1....Ans.**

$Q2 = 405.1572$ **W....heat leaving surface 2....Ans.**

$Q3 = -20888$ **W....heat coming _into_ surface 3....Ans....**
.... negative ign indicates heat in to the surface.

(b) In addition, plot Q1 and Q12 against ε1, for 0.1 < ε1 < 0.9:

$\varepsilon_1 := 0.1, 0.2 .. 0.9$ ....define a range variable

Write Q1, Q12 as functions of ε1:

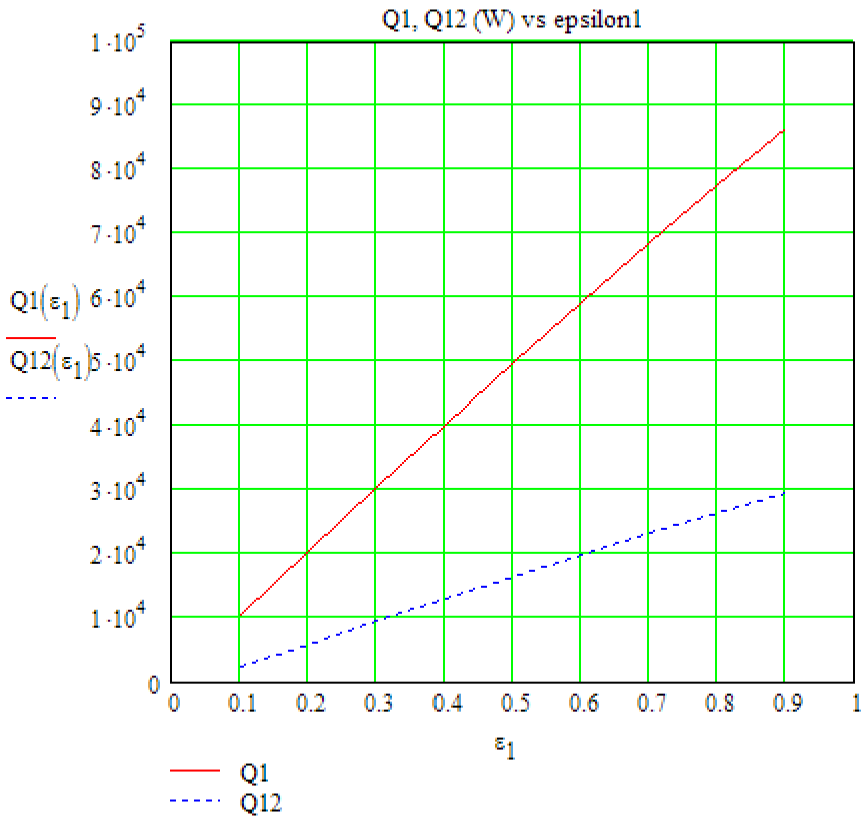
$$Q1(\varepsilon_1) := \text{HTransfer\_Three\_surface\_enclosure}(A_1, A_2, A_3, T_1, T_2, T_3, \varepsilon_1, \varepsilon_2, \varepsilon_3, F12, F13, F23)_{1,3}$$

$$Q12(\varepsilon_1) := \text{HTransfer\_Three\_surface\_enclosure}(A_1, A_2, A_3, T_1, T_2, T_3, \varepsilon_1, \varepsilon_2, \varepsilon_3, F12, F13, F23)_{1,0}$$

Now, compute the Table:

ε_1 =	$Q1(\varepsilon_1)$ =	$Q12(\varepsilon_1)$ =
0.1	$1.0337 \cdot 10^4$	$2.2377 \cdot 10^3$
0.2	$2.0483 \cdot 10^4$	$5.891 \cdot 10^3$
0.3	$3.0442 \cdot 10^4$	$9.4772 \cdot 10^3$
0.4	$4.022 \cdot 10^4$	$1.2998 \cdot 10^4$
0.5	$4.9822 \cdot 10^4$	$1.6456 \cdot 10^4$
0.6	$5.9253 \cdot 10^4$	$1.9851 \cdot 10^4$
0.7	$6.8517 \cdot 10^4$	$2.3187 \cdot 10^4$
0.8	$7.7618 \cdot 10^4$	$2.6464 \cdot 10^4$
0.9	$8.656 \cdot 10^4$	$2.9684 \cdot 10^4$

And, plot the results:



==

Prob. 5.C.2.5. Consider ***two special cases*** of 3-surface enclosures, viz.

i. Two black surfaces, connected by a third refractory surface, and
ii. Two gray surfaces, surrounded by a third re-radiating surface

Let us write Mathcad Functions for both the above cases:

Case 1: Two black surfaces connected by a reradiating surface: ex: top and bottom surfaces of a cylindrical furnce and its reradiating walls: (Q_R=0, E_{bR} = J_R)

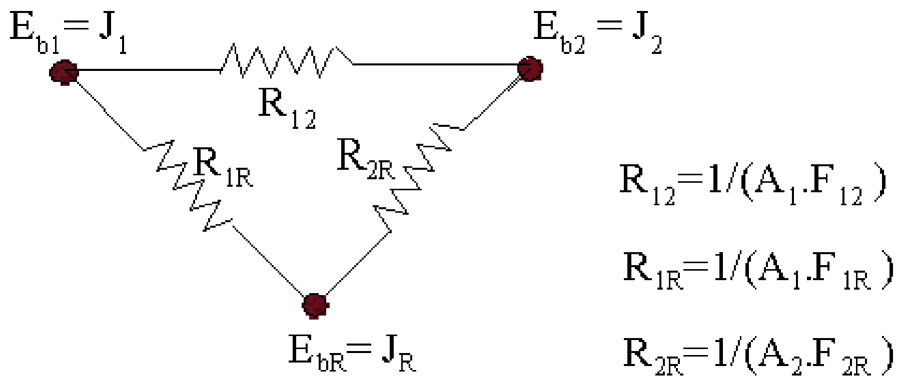


Fig.(a)

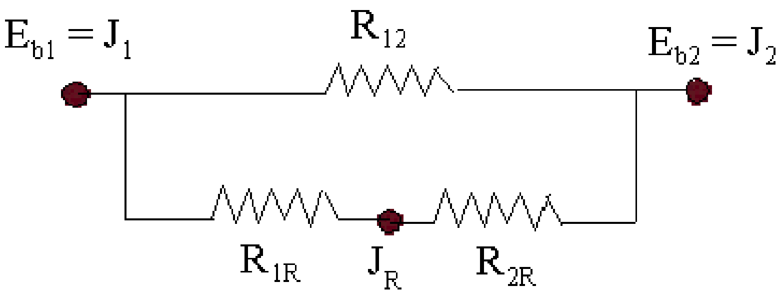


Fig.(b)

Fig. (a) shows the radiation network for this case. The radiation network is drawn very easily by *remembering the usual principles*: for a black surface, the surface resistance is zero, i.e. E_b = J. For a re-radiating surface too, E_b = J; further, for a re-radiating surface, Q = 0. Between two given surfaces, the radiosity potentials are connected by the respective space resistances, as shown. The system reduces to a series-parallel circuit of resistances as shown in Fig. (b).

Q_{12} is the net radiant heat transferred between surfaces 1 and 2.

TR is the temp of re-radiating surface.

$$Q_{12} = \sigma \cdot \left(T_1^{\,4} - T_2^{\,4}\right) \cdot \left[A_1 \cdot F_{12} + \frac{1}{\left(\frac{1}{A_1 \cdot F_{1R}} + \frac{1}{A_2 \cdot F_{2R}}\right)}\right] \qquad .....(13.66)$$

$$Q12\_TwoBlack\_OneRerad(A1,A2,F12,F1R,F2R,T1,T2) := \left|\begin{array}{l} R12 \leftarrow \dfrac{1}{A1\cdot F12} \\ R1R \leftarrow \dfrac{1}{A1\cdot F1R} \\ R2R \leftarrow \dfrac{1}{A2\cdot F2R} \\ Eb1 \leftarrow 5.67\cdot 10^{-8}\cdot T1^4 \\ Eb2 \leftarrow 5.67\cdot 10^{-8}\cdot T2^4 \\ Rtot \leftarrow \left(\dfrac{1}{R12} + \dfrac{1}{R1R + R2R}\right)^{-1} \\ Q12 \leftarrow \dfrac{Eb1 - Eb2}{Rtot} \\ JR \leftarrow \dfrac{\dfrac{Eb1}{R1R} + \dfrac{Eb2}{R2R}}{\dfrac{1}{R1R} + \dfrac{1}{R2R}} \\ Q1R \leftarrow (Eb1 - JR)\cdot\left(\dfrac{1}{R1R} + \dfrac{1}{R12 + R2R}\right) \\ Q2R \leftarrow (Eb2 - JR)\cdot\left(\dfrac{1}{R2R} + \dfrac{1}{R12 + R2R}\right) \\ TR \leftarrow \left(\dfrac{JR}{5.67\cdot 10^{-8}}\right)^{0.25} \\ \begin{pmatrix} \text{"Q12(W)"} & \text{"Q1R(W)"} & \text{"Q2R(W)"} & \text{"TR(K)"} \\ Q12 & Q1R & Q2R & TR \end{pmatrix} \end{array}\right.$$

Case 2. Two gray surfaces surrounded by a third reradiating surface: ex: top and bottom surfaces of a cylindrical furnce and its reradiating walls, or an equilateral duct etc.:

Note: For reradiating surface 3, $Q_3 = 0$, $E_{b3} = J_3$

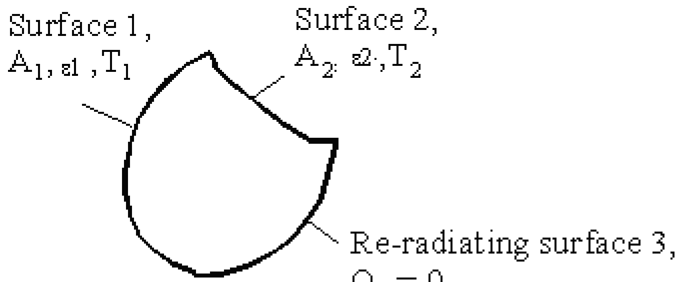


Fig.(a)

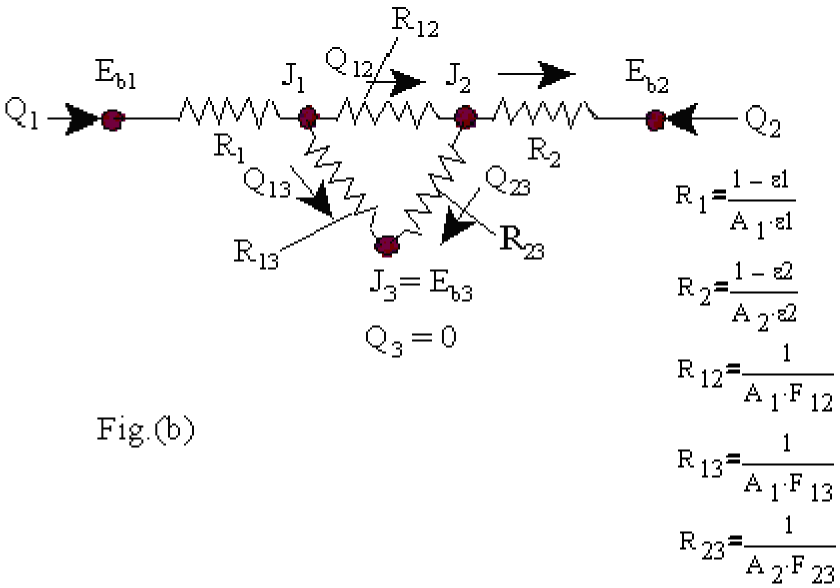


Fig.(b)

Here, we have:

$$Q_3 = 0$$

i.e. $$\frac{E_{b3} - J_3}{\left(\frac{1 - \varepsilon_3}{A_3 \cdot \varepsilon_3}\right)} = 0$$

i.e. $$E_{b3} = J_3$$

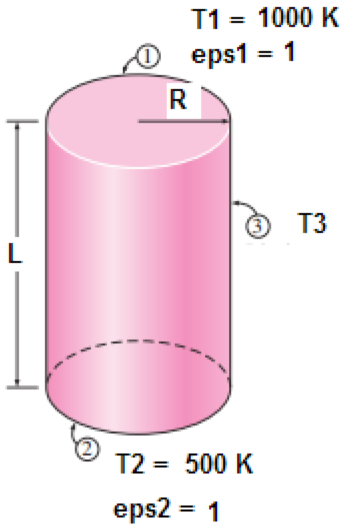
i.e. once the radiosity of the re-radiating surface is known, its temperature can easily be calculated, since $E_{b3} = \sigma.T_3^4$. Further, note that T_3 is independent of the emissivity of surface 3.

$$Q12\_TwoGray\_OneRerad(A1, A2, F12, F13, F23, eps1, eps2, T1, T2) := \left|\begin{array}{l} R12 \leftarrow \dfrac{1}{A1 \cdot F12} \\ R13 \leftarrow \dfrac{1}{A1 \cdot F13} \\ R23 \leftarrow \dfrac{1}{A2 \cdot F23} \\ R1 \leftarrow \dfrac{1 - eps1}{A1 \cdot eps1} \\ R2 \leftarrow \dfrac{1 - eps2}{A2 \cdot eps2} \\ Eb1 \leftarrow 5.67 \cdot 10^{-8} \cdot T1^4 \\ Eb2 \leftarrow 5.67 \cdot 10^{-8} \cdot T2^4 \\ Q3 \leftarrow 0 \\ Rtot \leftarrow R1 + \left(\dfrac{1}{\dfrac{1}{R12} + \dfrac{1}{R13 + R23}} \right) + R2 \\ Q1 \leftarrow \dfrac{Eb1 - Eb2}{Rtot} \\ Q2 \leftarrow -Q1 \\ J1 \leftarrow Eb1 - Q1 \cdot R1 \\ J2 \leftarrow Eb2 + Q1 \cdot R2 \\ J3 \leftarrow \dfrac{\dfrac{J1}{R13} + \dfrac{J2}{R23}}{\dfrac{1}{R13} + \dfrac{1}{R23}} \\ T3 \leftarrow \left(\dfrac{J3}{5.67 \cdot 10^{-8}} \right)^{0.25} \\ \begin{pmatrix} \text{"Q1(W)"} & \text{"Q2(W)"} & \text{"Q3(W)"} & \text{"T3(K)"} \\ Q1 & Q2 & Q3 & T3 \end{pmatrix} \end{array}\right.$$

Prob. 5.C.2.6. Two black discs, each of 500 mm dia, are placed directly opposite at a distance 1 m apart. The discs are maintained at 1000 K and 500 K respectively. Calculate the hat flow between the discs if they are connected by a cylindrical refractory surface. [P.U.]

(b) Also, plot Q12 against T1.

Mathcad Solution:



Data:

$R := 0.5$ m $L := 1$ m $eps1 := 1$ $eps2 := 1$

$T1 := 1000$ K $T2 := 500$ K

Calculations:

$A1 := \pi \cdot R^2$ i.e. $A1 = 0.7854$ m^2

$A2 := \pi \cdot R^2$ i.e. $A2 = 0.7854$ m^2

$A3 := 2\pi \cdot R \cdot L$ i.e. $A3 = 3.1416$ m^2

Now, we need View Factors: F12, F13 and F23:

Use the Mathcad Function for View factors for parallel disks, already written:

...View factor for coaxial parallel disks:

Define:

$R_i = \frac{r_i}{L}$ $R_j = \frac{r_j}{L}$ $S(R_i, R_j) := 1 + \frac{1 + R_j^2}{R_i^2}$

$$F_{ij}(R_i, R_j) := \frac{1}{2} \cdot \left[S(R_i, R_j) - \left[S(R_i, R_j)^2 - 4 \cdot \left(\frac{R_j}{R_i} \right)^2 \right]^{\frac{1}{2}} \right]$$ ...View factor for coaxial parallel disks

Then, with the above notations:

$$R_i := \frac{R}{L} \qquad R_j := \frac{R}{L}$$

$$F12 := F_{ij}(R_i, R_j)$$

i.e. $F12 = 0.1716$ **....View factor from top surface 1 to bottom surface 2**

And,

$F11 + F12 + F1R = 1$ **.... by summation rule for surface 1**

But, $F11 := 0$ ...since surface 1 is Flat

Therefore: $F1R := 1 - F12$

i.e. $F1R = 0.8284$ **....View factor from top surface 1 to side surfaces 3**

By symmetry: $F2R := F1R$

Now, use the Mathcad Function for two black surfaces, connected by a re-radiating surface:

$$Q12\_TwoBlack\_OneRerad(A1, A2, F12, F1R, F2R, T1, T2) = \begin{pmatrix} \text{"Q12(W)"} & \text{"Q1R(W)"} & \text{"Q2R(W)"} & \text{"TR(K)"} \\ 2.4456 \times 10^4 & 2.026 \times 10^4 & -2.026 \times 10^4 & 853.7382 \end{pmatrix}$$

i.e.

$Q12 = 24456$ **W....Net radn. between surfaces 1 and 2 .... Ans.**

$TR = 853.74$ **K....temp. of Rearadiating surface .... Ans.**

(b) Also, plot Q12, TR against T1 as T1 varies from 600 K to 1300 K:

$T1 := 600, 650 .. 1300$ ....define a range variable

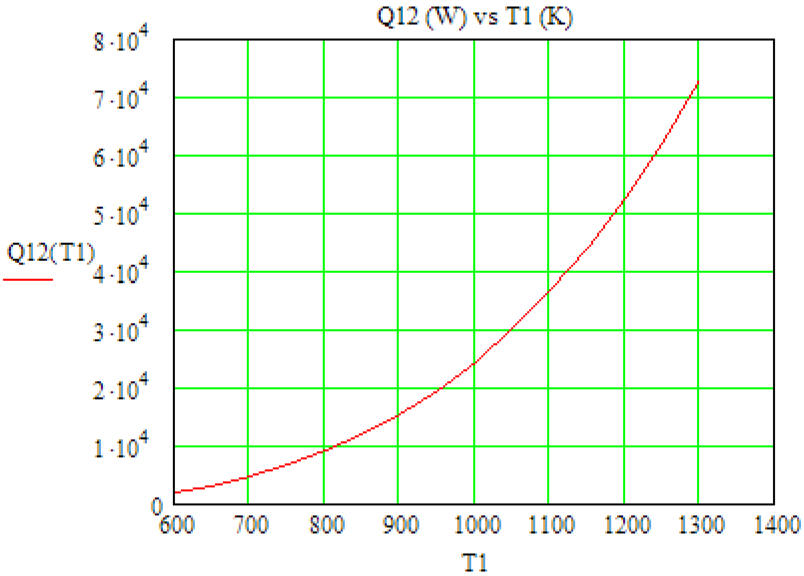
$Q12(T1) := Q12\_TwoBlack\_OneRerad(A1, A2, F12, F1R, F2R, T1, T2)_{1,0}$ ...define Q12 as a function of T1

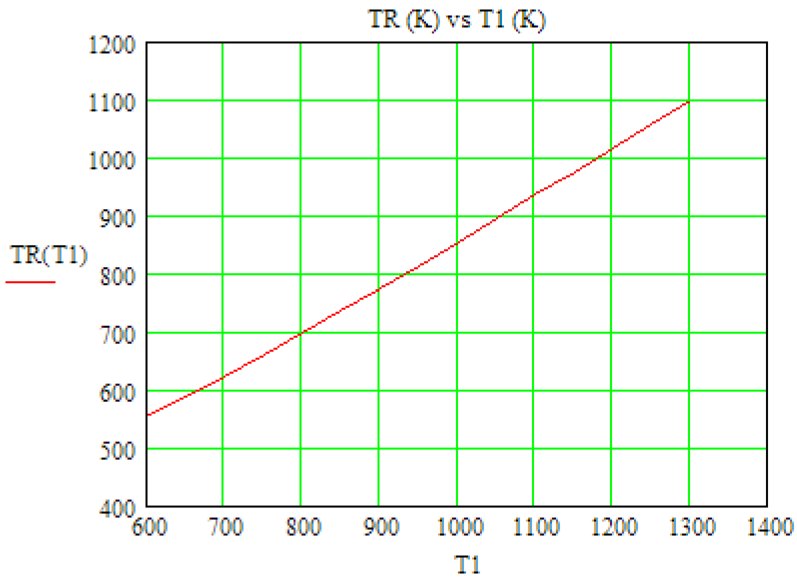
$TR(T1) := Q12\_TwoBlack\_OneRerad(A1, A2, F12, F1R, F2R, T1, T2)_{1,3}$ ...define TR as a function of T1

Compute the Table:

T1 =	Q12(T1) =	TR(T1) =
600	$1.7504 \cdot 10^{3}$	556.704
650	$3.0262 \cdot 10^{3}$	589.1821
700	$4.6329 \cdot 10^{3}$	623.677
750	$6.6235 \cdot 10^{3}$	659.744
800	$9.0545 \cdot 10^{3}$	697.0292
850	$1.1987 \cdot 10^{4}$	735.2581
900	$1.5485 \cdot 10^{4}$	774.2199
950	$1.9617 \cdot 10^{4}$	813.7541
1000	$2.4456 \cdot 10^{4}$	853.7382
1050	$3.0078 \cdot 10^{4}$	894.0787
1100	$3.6563 \cdot 10^{4}$	934.7034
1150	$4.3995 \cdot 10^{4}$	975.5566
1200	$5.2462 \cdot 10^{4}$	$1.0166 \cdot 10^{3}$
1250	$6.2057 \cdot 10^{4}$	$1.0578 \cdot 10^{3}$
1300	$7.2875 \cdot 10^{4}$	$1.0991 \cdot 10^{3}$

And, plot the results:





==

Prob. 5.C.2.7. Solve the above Problem if the emissivities of top and bottom disks are 0.8 and 0.4 respectively. Rest of the data are the same.

Mathcad Solution:

This is the case of two gray surfaces surrounded by a re-radiating surface.

So, we shall use the Mathcad Function written above.

Data:

$R := 0.5$ m $L := 1$ m $eps1 := 0.8$ $eps2 := 0.4$

$T1 := 1000$ K $T2 := 500$ K

Calculations:

$A1 := \pi \cdot R^2$ i.e. $A1 = 0.7854$ m^2

$A2 := \pi \cdot R^2$ i.e. $A2 = 0.7854$ m^2

$A3 := 2\pi \cdot R \cdot L$ i.e. $A3 = 3.1416$ m^2

And: $F12 := 0.1716$ $F1R := 0.8284$ $F2R := F1R$ ..calculated in previous problem

Now, we use the Mathcad Function for two gray surfaces surrounded by a re-radiating surface:

$$Q12\_TwoGray\_OneRerad(A1, A2, F12, F1R, F2R, eps1, eps2, T1, T2) = \begin{pmatrix} \text{"Q1(W)"} & \text{"Q2(W)"} & \text{"Q3(W)"} & \text{"T3(K)"} \\ 1.2076 \times 10^4 & -1.2076 \times 10^4 & 0 & 914.9328 \end{pmatrix}$$

i.e.

$Q1 = 12076$ **W....Net radn. at surface 1 .... Ans.**

$Q2 = -Q1 = -12076$ **W....Net radn. at surface 2 ..negative sign indicates heat flow *into* the surface.. Ans.**

$TR = 914.93$ **K....temp. of Rearadiating surface .... Ans.**

(b) Also, plot Q1 and T3 (i.e. temp of re-radiating surface) as T1 varies from 600 K to 1300 K, rest of the conditions being the same:

Define Q1 as a function of T1:

$$Q1(T1) := Q12\_TwoGray\_OneRerad(A1, A2, F12, F1R, F2R, eps1, eps2, T1, T2)_{1,0}$$

Define T3 as a function of T1:

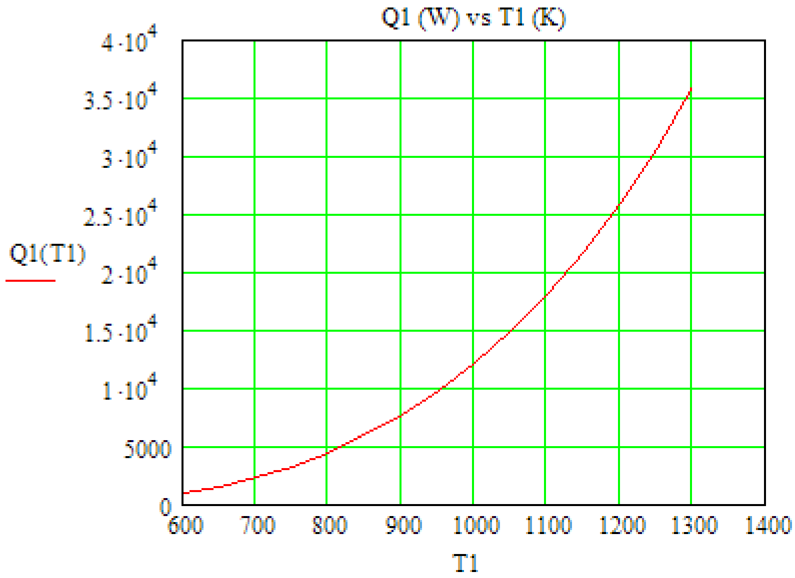
$$T3(T1) := Q12\_TwoGray\_OneRerad(A1, A2, F12, F1R, F2R, eps1, eps2, T1, T2)_{1,3}$$

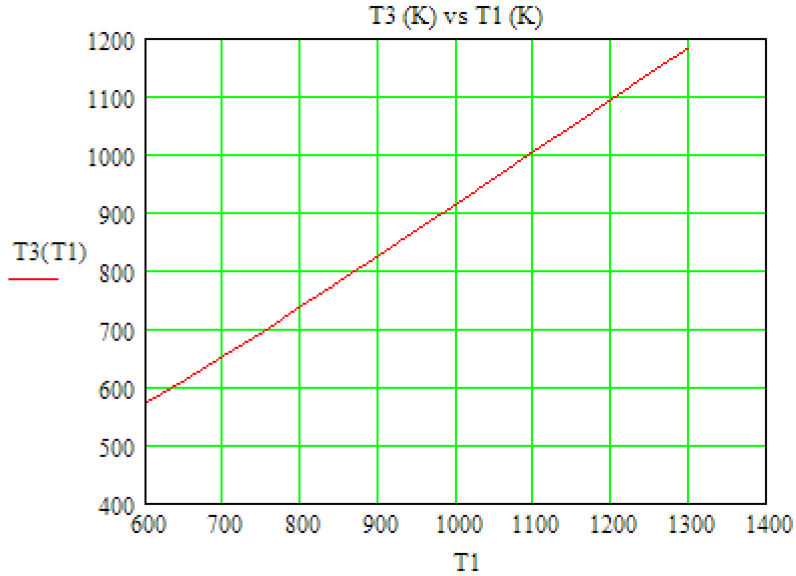
$T1 := 600, 650 .. 1300$ ....define a range variable

Compute the Table:

T1 =	Q1(T1) =	T3(T1) =
600	864.346	573.5056
650	$1.4943\cdot10^{3}$	613.2966
700	$2.2877\cdot10^{3}$	654.4171
750	$3.2707\cdot10^{3}$	696.5168
800	$4.4712\cdot10^{3}$	739.3428
850	$5.9191\cdot10^{3}$	782.7127
900	$7.6464\cdot10^{3}$	826.4946
950	$9.6869\cdot10^{3}$	870.5917
1000	$1.2076\cdot10^{4}$	914.9328
1050	$1.4852\cdot10^{4}$	959.4648
1100	$1.8055\cdot10^{4}$	$1.0041\cdot10^{3}$
1150	$2.1725\cdot10^{4}$	$1.049\cdot10^{3}$
1200	$2.5906\cdot10^{4}$	$1.0939\cdot10^{3}$
1250	$3.0644\cdot10^{4}$	$1.1388\cdot10^{3}$
1300	$3.5986\cdot10^{4}$	$1.1839\cdot10^{3}$

And, plot the results:




T3 (K) vs T1 (K)
T3(T1)
1200
1100
1000
900
800
700
600
500
400
600
700
800
900
1000
1100
1200
1300
1400
T1

==

Prob. 5.C.2.8. Two co-axial cylinders of 0.4 m and 1 m dia. are 1 m long. The annular top and bottom surfaces are well insulated and act as re-radiating surfaces. The inner surface is at 1000 K and has an emissivity of 0.6. The outer surface is maintained at 400 K and its emissivity is 0.4.

i) Determine the heat exchange between the surfaces
ii) If the annular base surfaces are open to the surroundings at 300 K, determine the radiant heat exchange.

If the outer cylinder is surface 2, take F21= 0.25 and F22= 0.27. [M.U. Dec. 1998]

Mathcad Solution:

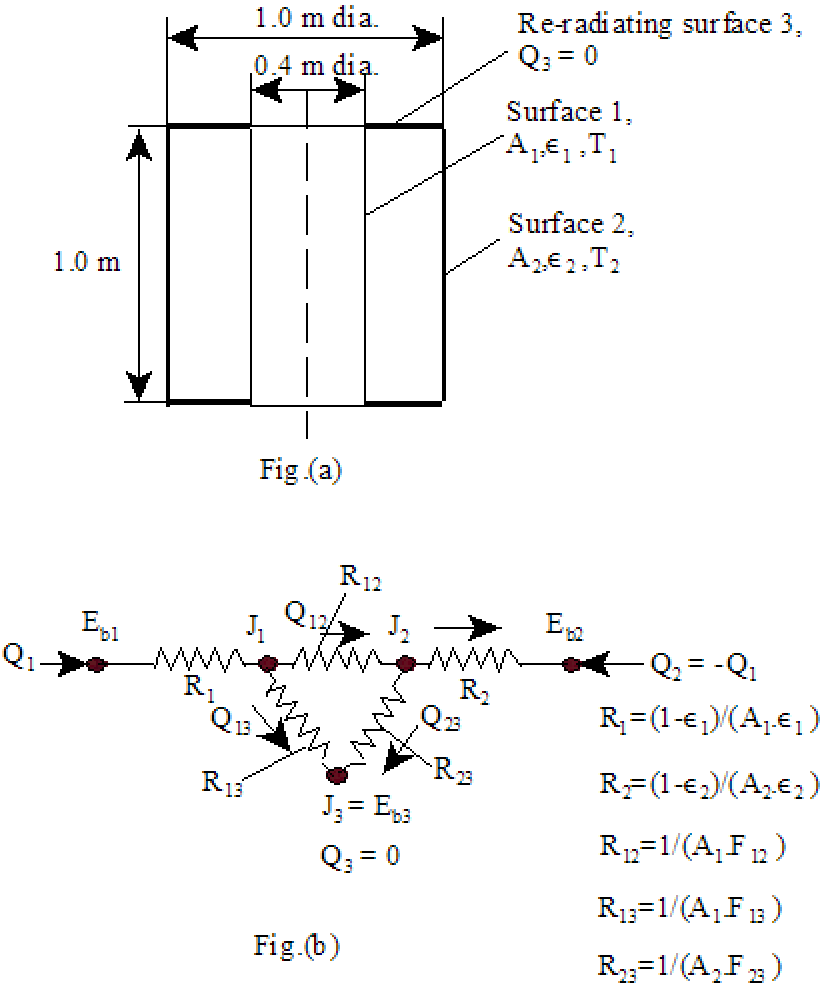


Fig.Prob.5.C.2.8

See Fig. above. Let the inner surface be denoted by 1, outer surface by 2, and the two annular surfaces by 3. Then, surfaces 1, 2 and 3 form an enclosure. And, the radiation network will look as shown in the fig.(b).

Data:

$D_1 := 0.4$ m. $D_2 := 1$ m. $L := 1$ m..

$T_1 := 1000$ K...temp. of inner surface (1) $T_2 := 400$ K...temp. of outer surface (2)

$\varepsilon_1 := 0.6$ ..emissivity of surface 1 $\varepsilon_2 := 0.4$ ..emissivity of surface 2

$F_{21} := 0.25$ ...view factor of surface 2 w.r.t. surface 1

$F_{22} := 0.27$ ...view factor of surface 2 w.r.t. itself

$\sigma := 5.67 \cdot 10^{-8}$ W/(m^2.K)...Stefan-Boltzmann const.

Calculations:

Areas:

$A_1 := \pi \cdot D_1 \cdot L$ i.e. $A_1 = 1.2566$ m^2....surface area of inner cylinder 1

$A_2 := \pi \cdot D_2 \cdot L$ i.e. $A_2 = 3.1416$ m^2....surface area of outer cylinder 2

To find F_{12}:

$F_{11} := 0$ ...since surface 1 is convex, and does not 'see' itself.

Then, $F_{12} := \dfrac{A_2}{A_1} \cdot F_{21}$ ...by reciprocity

i.e. $F_{12} = 0.625$ ..view factor from surface 1 to surface 2

Also, $F_{11} + F_{12} + F_3 = 1$ ....by summation rule

i.e. $F_{13} := 1 - (F_{11} + F_{12})$

i.e. $F_{13} = 0.375$ ..view factor from surface 1 to surface 3

Also, $F_{21} + F_{22} + F_{23} = 1$ ...by summaion rule

i.e. $F_{23} := 1 - (F_{21} + F_{22})$

i.e. $F_{23} = 0.48$ ..view factor from surface 2 to surface 3

Case(i): When both the annular surfaces act as re-radiating surfaces:

Now, we use the Mathcad Function for two gray surfaces surrounded by a re-radiating surface:

$$\mathrm{Q12\_TwoGray\_OneRerad}(A_1, A_2, F_{12}, F_{13}, F_{23}, \varepsilon_1, \varepsilon_2, T_1, T_2) = \left(\begin{array}{ccc|c} \text{"Q1(W)"} & \text{"Q2(W)"} & \text{"Q3(W)"} & \text{"T3(K)"} \\ 2.936\times10^4 & -2.936\times10^4 & 0 & 785.4289 \end{array}\right)$$

i.e. $Q1 = 2.936\cdot10^4$ **W....heat exchange between the surfaces...Ans.**

$T3 = 785.43$ **K....temp of re-radiating surface...Ans.**

Case(ii): When both the annular surfaces are open to surroundings at 300 K:

Now, $T_3 := 300$ K....temp. of surroundings

Also: $A_3 := 10^6$ m^2.... area of surroundings....take a very large value

$\varepsilon_3 := 0.9999$ ...emissivity of surroundings, equal to 1 for black surface, but take it as 0.9999 to avoid division by zero

Now, apply the Mathcad Function for general, three-surface enclosure:

$$HTransfer\_Three\_surface\_enclosure(A_1, A_2, A_3, T_1, T_2, T_3, \varepsilon_1, \varepsilon_2, \varepsilon_3, F_{12}, F_{13}, F_{23}) =$$

$$\begin{pmatrix} \text{"Q12(W)"} & \text{"Q13(W)"} & \text{"Q23(W)"} & \text{"Q1(W)"} & \text{"Q2(W)"} & \text{"Q3(W)"} \\ 2.2486 \times 10^4 & 1.6705 \times 10^4 & 1.0283 \times 10^4 & 3.9191 \times 10^4 & -1.2203 \times 10^4 & -2.6988 \times 10^4 \end{pmatrix}$$

i.e. We get:

$Q1 := 3.9191 \cdot 10^4$ **W....net heat transfer from surface 1 ... Ans.**

$Q2 := -1.2203 \cdot 10^4$ **W....net heat transfer from surface 2 ... Ans.**

$Q3 := -2.6988 \cdot 10^4$ **W....net heat transfer from surface 3 ... Ans.**

Verify: $Q2 + Q3 = -3.9191 \times 10^4$ **..W**

i.e. heat leaving surface 1 (= Q1) is equal to heat received by surface 2 and the surroundings...verified.

While using the above Function, note that we have obtained other heat transfers too, i.e. Q12 between surfaces 1 and 2, Q13 between surfaces 1 and 3, and, Q23 between surfaces 2 and 3.

==

Prob. 5.C.2.9. Two parallel plates, 0.5 m × 1 m each, are spaced 0.5 m apart. The plates are at temperatures of 1000 C and 500 C and their emissivities are 0.2 and 0.5 respectively. The plates are located in a large room, the walls of which are at 27 C. The surfaces of the plates facing each other only exchange heat by radiation. Determine the rates of heat lost by each plate and heat gain of the walls by radiation. Use radiation network for solution. Assume shape factor between parallel plates: $F_{12} = F_{21} = 0.285$. [M.U. 1996]

(b) Also, plot Q1 and Q3 as epsilon1 varies from 0.1 to 0.9

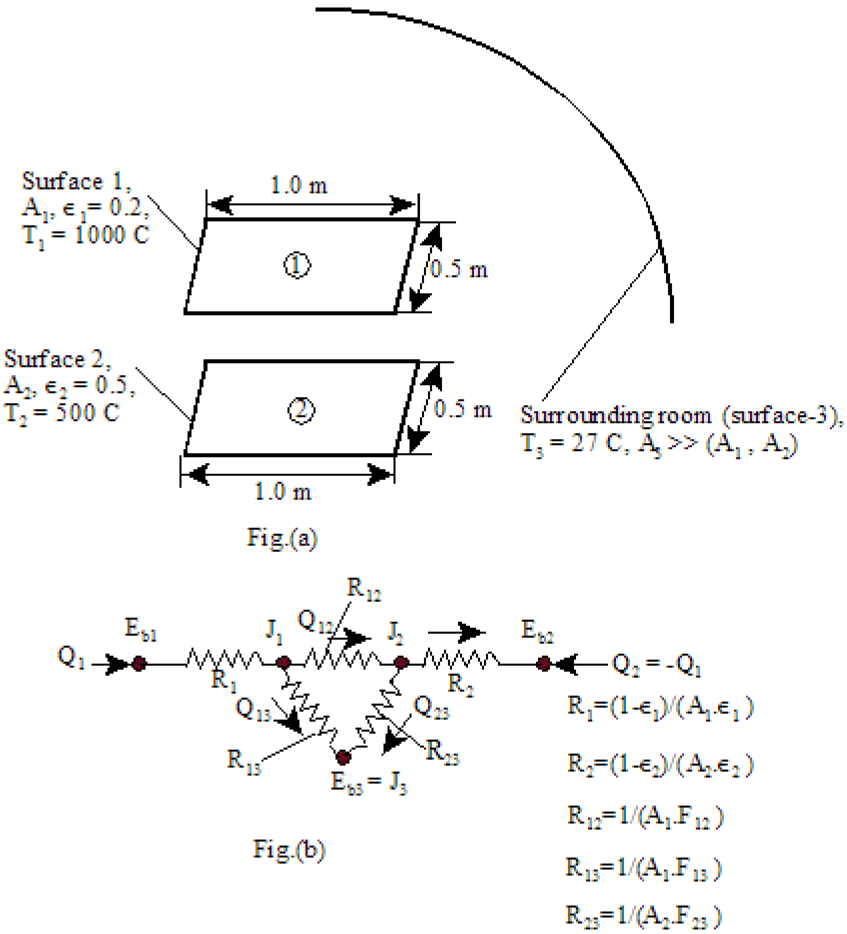


Fig.Prob.5.C.2.9

This problem is the same as Prob.5.C.2.3, which was solved with Mathcad.

However, now we shall solve it with EES:

EES Solution:

The schematic fig. and the radiation network are shown in Fig (a) and (b) above.

We shall, first write an EES PROCEDURE to calculate the Emissive powers and Resistances involved, to reduce labour; then, we shall use that Procedure in a Main EES program to find out the Radiosities by applying the Kirchoff's Law to the three nodes J1, J2 and J3 and solve the resulting three equations simultaneously to get J1, J2 and J3. Then, the heat transfers are easily calculated by applying the Ohm's Law between any two given radiosity potentials.

The EES Procedure is given below. Read the comments in the program:

```
$UnitSystem SI Pa J K

PROCEDURE
HTrans_Three_surface_enclosure(A_1,A_2,A_3,T_1,T_2,T_3,eps_1,eps_2,eps_3,F_12,F_13,F_23:Eb_
1,Eb_2,Eb_3,R_1,R_2,R_3,R_12,R_13,R_23)

{
Inputs: A1, A2 ,…etc

Outputs: Emissive powers: Eb_1,Eb_2,Eb_3 (W/m^2), and
         Surface and Space resistances: R_1, R_2, R_3, R_12, R_13, R_23 (1/m^2)
}
"Radiation Heat transfer in Three-Zone enclosure:"
"Note: For a insulated/re-radiating surface: Q_i = 0, and J_i = Eb_i (= sigma * T_i^4)"
"For a black surface: R_i = 0; so, J_i = Eb_i "

sigma := 5.67e-08 "[W/m^2-K^4]….Stefan Boltzmann const."
```

```
Eb_1 := sigma * T_1^4
Eb_2 := sigma * T_2^4
Eb_3 := sigma * T_3^4

R_1 := (1 – eps_1)/(A_1 * eps_1)          "surface resist of surface 1"
R_2 := (1 – eps_2)/(A_2 * eps_2)          "surface resist of surface 2"
R_3 := (1 – eps_3)/(A_3 * eps_3)          "surface resist of surface 2"

R_12 := 1/(A_1 * F_12)                    "space resist bet surfaces 1 and 2"
R_13 := 1/(A_1 * F_13)                    "space resist bet surfaces 1 and 3"
R_23 := 1/(A_2 * F_23)                    "space resist bet surfaces 2 and 3"

END

"================================================================"
```

Now, to solve the above problem:

"Data:"

```
A_1 = 0.5 [m^2]
A_2 = 0.5 [m^2]
A_3 = 1e10 [m^2] "…very large room"

T_1 = 1273 [K]
T_2 = 773 [K]
T_3 = 300 [K]

eps_1 = 0.2 "emissivity of plate 1"
eps_2 = 0.5 "emissivity of plate 2"
eps_3 = 0.9999 "…emissivity of surroundings = 1, taken as 0.9999 to avoid division by zero"

F_12 = 0.285 "…View Factor from surface 1 to 2… by data"
F_21 = 0.285 "…View Factor from surface 2 to 1… by data"
F_11 = 0 "…since surface 1 is flat"
"F_11 + F_12 + F_13 = 1 … by Summation rule for surface 1"

"Therefore:"

F_13 = 1- F_12 "…View Factor from surface 1 to 3"
```

"Similarly, for surface 2, F_22 = 0, and F_21 + F_22 + F_23 = 1 by Summation rule for surface 2, and we get:"

F_23 = 1- F_21 "…View Factor from surface 2 to 3"

"Now, calculate various Emissive powers and Resistances, using the EES PROCEDURE:"

CALL HTrans_Three_surface_enclosure(A_1,A_2,A_3,T_1,T_2,T_3,eps_1,eps_2,eps_3,F_12,F_13,F_23:Eb_1,Eb_2,Eb_3,R_1,R_2,R_3,R_12,R_13,R_23)

"Kirchoff's Law for Nodes J_1, J_2 and J_3:"

"For J_1:" (Eb_1 – J_1)/R_1 + (J_2 – J_1)/R_12 + (J_3 – J_1)/R_13 = 0
"For J_2:" (Eb_2 – J_2)/R_2 + (J_1 – J_2)/R_12 + (J_3 – J_2)/R_23 = 0
"For J_3:" (Eb_3 – J_3)/R_3 + (J_1 – J_3)/R_13 + (J_2 – J_3)/R_23 = 0

"Net heat transfer from each surface:"

Q_1 = (Eb_1 – J_1)/R_1
Q_2 = (Eb_2 – J_2)/R_2
Q_3 = (Eb_3 – J_3)/R_3

"Net heat transfer between surface 1 and 2:"

Q_12 = (J_1 – J_2)/R_12

"Net heat transfer between surface 1 and 3:"

Q_13 = (J_1 – J_3)/R_13

"Net heat transfer between surface 2 and 3:"

Q_23 = (J_2 – J_3)/R_23

"Check : Q_1 + Q_2 + Q_3 = 0 "

SumQ1Q2Q3 = Q_1 + Q_2 + Q_3

Results:

Main | HTrans_Three_surface_enclosure

Unit Settings: SI K Pa J mass deg

A_1 = 0.5 [m^2]	A_2 = 0.5 [m^2]	A_3 = 1.000E+10 [m^2]
Eb_1 = 148901 [W/m^2]	Eb_2 = 20244 [W/m^2]	Eb_3 = 459.3 [W/m^2]
eps_1 = 0.2	eps_2 = 0.5	eps_3 = 0.9999
F_{11} = 0 [-]	F_{12} = 0.285 [-]	F_{13} = 0.715 [-]
F_{21} = 0.285 [-]	F_{23} = 0.715 [-]	**J_1 = 33476** [W/m^2]
J_2 = 15057 [W/m^2]	**J_3 = 459.3** [W/m^2]	Q_1 = 14428 [W]
Q_{12} = 2625 [W]	Q_{13} = 11803 [W]	Q_2 = 2594 [W]
Q_{23} = 5219 [W]	Q_3 = -17022 [W]	R_1 = 8 [1/m^2]
R_{12} = 7.018 [1/m^2]	R_{13} = 2.797 [1/m^2]	R_2 = 2 [1/m^2]
R_{23} = 2.797 [1/m^2]	R_3 = 1.000E-14 [1/m^2]	SumQ1Q2Q3 = -0.0001729 [W]
T_1 = 1273 [K]	T_2 = 773 [K]	T_3 = 300 [K]

Main | HTrans_Three_surface_enclosure

Local variables in Procedure HTrans_Three_surface_enclosure (1 call, 0.00 sec)

A_1=0.5 [m^2]	A_2=0.5 [m^2]	A_3=1.000E+10 [m^2]
Eb_1 =148901 [W/m^2]	Eb_2 =20244 [W/m^2]	Eb_3 =459.3 [W/m^2]
eps_1=0.2	eps_2=0.5	eps_3=0.9999
F_{12}=0.285	F_{13}=0.715	F_{23}=0.715
R_1 =8 [1/m^2]	R_{12}=7.018 [1/m^2]	R_{13}=2.797 [1/m^2]
R_2 =2 [1/m^2]	R_{23}=2.797 [1/m^2]	R_3 =1.000E-14 [1/m^2]
σ =5.670E-08 [W/m^2-K^4]	T_1=1273 [K]	T_2=773 [K]
T_3=300 [K]		

Thus:

Heat leaving surface 1 = Q1 = 14428 W …. Ans.

Heat leaving surface 2 = Q2 = 2594 W … Ans.

Heat received by surface 3, i.e. ambient = Q3 = -17022 W … Ans. (negative sign indicates that heat is going *into* the surface).

Also, check: Q1 + Q2 + Q3 = 0 (which is satisfied… see SumQ1Q2Q3 = -0.0001729 = almost zero)

In addition, values of radiosities J1, J2 and J3, and also the various resistances, and heat transfers between surfaces, viz. Q12, Q13 and Q23 are also calculated.

Now, plot Q1 and Q3 as epsilon1 varies from 0.1 to 0.9:

First, produce the Parametric Table:

Table 1

1..9	eps_1	Q_1 [W]	Q_2 [W]	Q_3 [W]	SumQ1Q2Q3 [W]
Run 1	0.1	7244	3661	-10905	-0.001361
Run 2	0.2	14428	2594	-17022	0.0007215
Run 3	0.3	21552	1536	-23087	-0.0006374
Run 4	0.4	28616	486.5	-29102	-0.000599
Run 5	0.5	35622	-554.1	-35068	-0.001149
Run 6	0.6	42570	-1586	-40983	-0.0007688
Run 7	0.7	49460	-2610	-46851	0.0002317
Run 8	0.8	56295	-3625	-52670	-0.000083
Run 9	0.9	63073	-4632	-58442	0.0004825

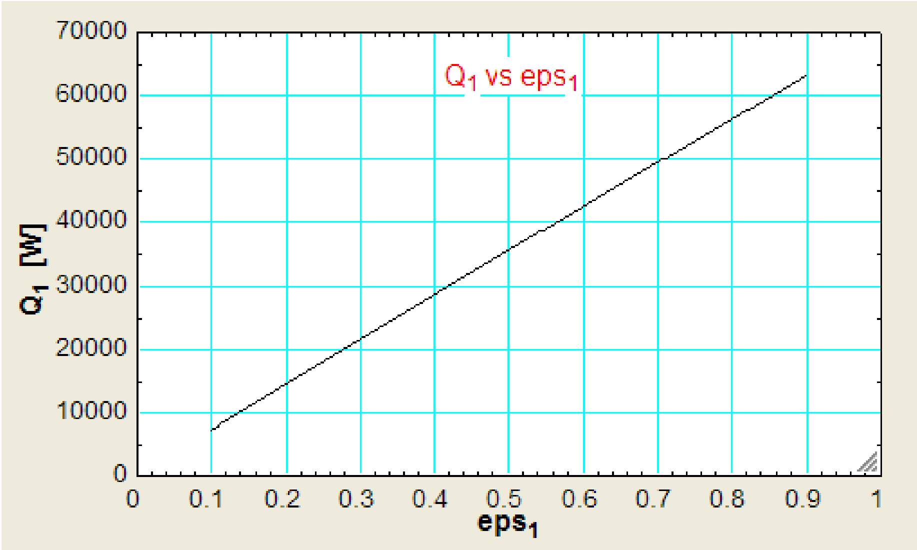
Note in the above Table that:

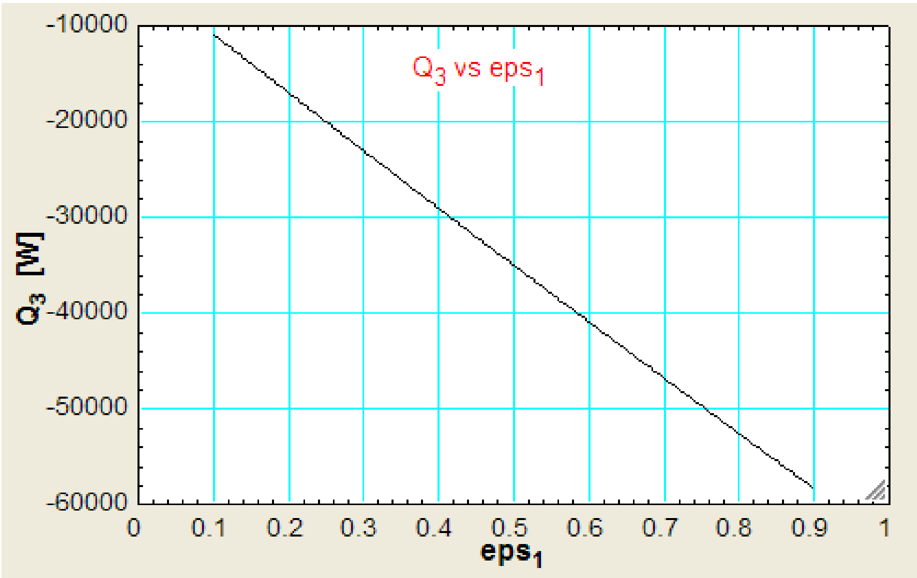
i) **For eps1 = > 0.5, Q2 is –ve, i.e. surface 2 receives heat**

ii) **Q3 is the heat received by the ambient (-ve)**

iii) **Sum of Q1, Q2 and Q3 is equal to almost zero, as it should be**

And, Plot the results:





==

Prob. 5.C.2.10. A square room, 3 m × 3 m, has a floor heated to 27 C and has a ceiling at 10 C. Walls are perfectly insulated. Height of room is 2.5 m. Emissivity of all surfaces is 0.8. Determine:

i) Net heat transfer from the ceiling
ii) Net heat transfer between ceiling and floor, and
iii) Temp of the walls [M.U.]

(b) Also, plot the variation of Q1 and T3 as eps1 varies from 0.1 to 0.9:

EES Solution:

This is a three-surface enclosure problem.

Important: Walls act as re-radiating surfaces, i.e. Eb_3 = J3, and Q_3 = 0.

Radiation network is shown below:

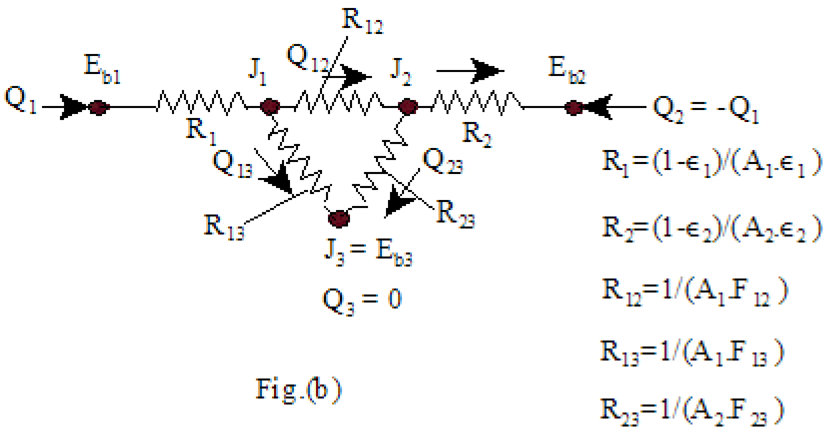


Fig.(b)

Let ceiling be designated as surface 1, floor as surface 2, and the surrounding walls as surface 3.

We shall use the EES PROCEDURE written above to calculate Emissive powers, various resistance etc.

We shall also use the EES Function written earlier *to determine the View Factor F_12 between the ceiling and the floor.*

"Data:"

A_1 = 9 [m^2]"….area of ceiling"
A_2 = 9 [m^2]"…area of floor"
A_3 = 30 [m^2] "…area of 4 walls"

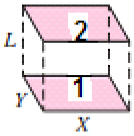
T_1 = 283 [K]"…temp of ceiling, by data"
T_2 = 300 [K]"…temp of floor, by data"
{T_3 = …to be found out}

sigma = 5.67E-08 [W/m^2-K^4]"…Stefan – Boltzmann constant"

eps_1 = 0.8 "emissivity of ceiling, i.e. surface 1"
eps_2 = 0.8 "emissivity of floor, i.e. surface 2"
eps_3 = 0.8 "…emissivity of surrounding walls, i.e. surface 3"

"Determine View Factors: F_12, F_13 etc."

"Use the EES Function written earlier for parallel rectangles to get F_12:"



"We have: for the ceiling and floors: X = 3 m, Y = 3 m, L = 2.5 m"

X = 3 [m]
Y = 3 [m]
L = 2.5 [m]

F_12 = F12_parallel_rectangles(X, Y, L) "…View Factor from surface 1 to 2…."

F_21 = F_12"…View Factor from surface 2 to 1… by symmetry"

F_11 = 0 "…since surface 1 is flat"

"F_11 + F_12 + F_13 = 1 … by Summation rule for surface 1"

"Therefore:"

F_13 = 1- F_12 "…View Factor from surface 1 to 3"

"Similarly, for surface 2, F_22 = 0 , and F_21 + F_22 + F_23 = 1 by Summation rule for surface 2, and we get:"

F_23 = 1- F_21 "…View Factor from surface 2 to 3"

"Now, calculate various Emissive powers and Resistances, using the EES PROCEDURE:"

CALL

HTrans_Three_surface_enclosure(A_1,A_2,A_3,T_1,T_2,T_3,eps_1,eps_2,eps_3,F_12,F_13,F_23:Eb_1,Eb_2,Eb_3,R_1,R_2,R_3,R_12,R_13,R_23)

"Kirchoff's Law for Nodes J_1, J_2 and J_3:"

"For J_1:" (Eb_1 – J_1)/R_1 + (J_2 – J_1)/R_12 + (J_3 – J_1)/R_13 = 0
"For J_2:" (Eb_2 – J_2)/R_2 + (J_1 – J_2)/R_12 + (J_3 – J_2)/R_23 = 0
"For J_3:" (J_1 – J_3)/R_13 + (J_2 – J_3)/R_23 = 0

Eb_3 = J_3 "….since surface 3 is re-radiating"

"Also:"

Eb_3 = sigma * T3 ^4 "[W/m^2]…by definition of Eb"

"Net heat transfer from each surface:"

Q_1 = (Eb_1 – J_1)/R_1"[W]…net heat transfer from surface 1"

Q_2 = (Eb_2 – J_2)/R_2"[W]…net heat transfer from surface 2"

Q_3 = (Eb_3 – J_3)/R_3"[W]…net heat transfer from surface 3"

"Net heat transfer between surface 1 and 2:"

Q_12 = (J_1 – J_2)/R_12 "[W]"

"Net heat transfer between surface 1 and 3:"

Q_13 = (J_1 – J_3)/R_13 "[W]"

"Net heat transfer between surface 2 and 3:"

Q_23 = (J_2 – J_3)/R_23 "[W]"

"Check : Q_1 + Q_2 + Q_3 = 0"

SumQ1Q2Q3 = Q_1 + Q_2 + Q_3

Results:

Main | HTrans_Three_surface_enclosure | F12_parallel_rectangles

Unit Settings: SI K Pa J mass rad

A_1 = 9 [m^2]	A_2 = 9 [m^2]	A_3 = 30 [m^2]
Eb_1 = 363.7 [W/m^2]	Eb_2 = 459.3 [W/m^2]	Eb_3 = 411.5 [W/m^2]
eps_1 = 0.8	eps_2 = 0.8	eps_3 = 0.8
F_{11} = 0	F_{12} = 0.2508	F_{13} = 0.7492
F_{21} = 0.2508	F_{23} = 0.7492	**J_1 = 375.1** [W/m^2]
J_2 = 447.9 [W/m^2]	**J_3 = 411.5** [W/m^2]	L = 2.5 [m]
Q_1 = -409.8 [W]	Q_{12} = -164.4 [W]	Q_{13} = -245.5 [W]
Q_2 = 409.8 [W]	Q_{23} = 245.5 [W]	Q_3 = 4.744E-15 [W]
R_1 = 0.02778 [1/m^2]	R_{12} = 0.443 [1/m^2]	R_{13} = 0.1483 [1/m^2]
R_2 = 0.02778 [1/m^2]	R_{23} = 0.1483 [1/m^2]	R_3 = 0.008333 [1/m^2]
σ = 5.670E-08 [W/m^2-K^4]	SumQ1Q2Q3 = 6.119E-13 [W]	T3 = 291.9
T_1 = 283 [K]	T_2 = 300 [K]	T_3 = 291.9 [K]
X = 3 [m]	Y = 3 [m]	

Thus:

i) **Net heat transfer from surface 1 = Q1 = -409.8 W …. Ans…. negative sign indicates heat coming *in to* the surface.**

ii) **Net heat transfer between ceiling and floor = Q12 = -164.4 W ….Ans.…again, negative sign indicates heat coming *in to* the surface 1.**

iii) **Temp of re-radiating walls = T3 = 291.9 K … Ans.**

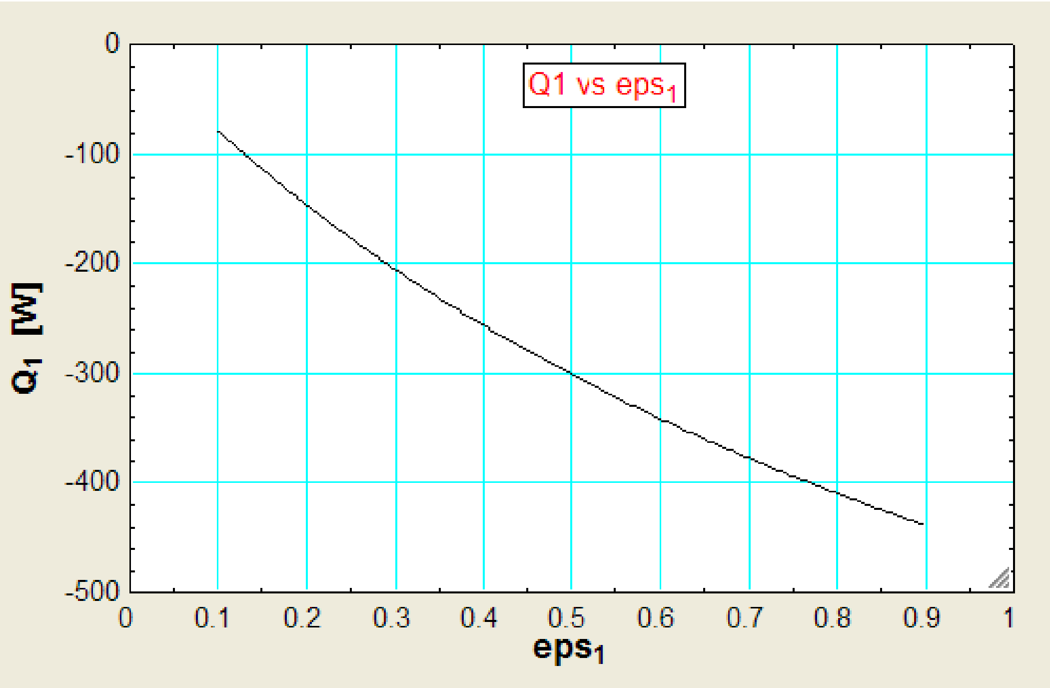
Note: Sum of Q1, Q2 and Q3 = 0, as it should be.

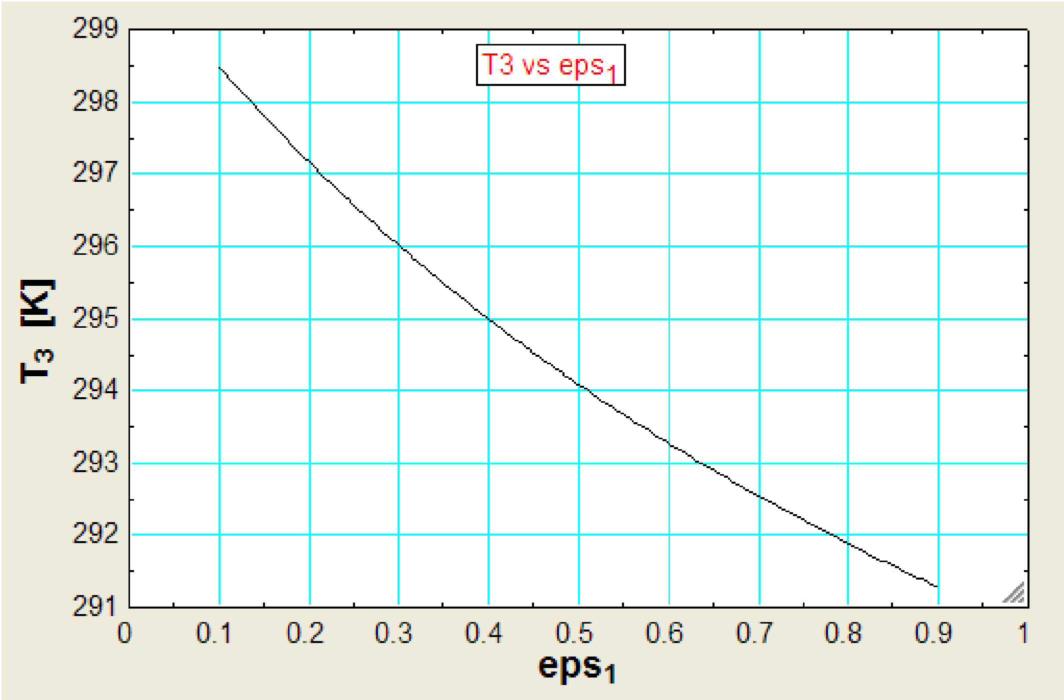
Plot Plot (b) Plot the variation of Q1 and T3 as eps1 varies from 0.1 to 0.9:

First, compute the Parametric Table:

1..9	eps_1	Q_1 [W]	T_3 [K]
Run 1	0.1	-79.29	298.5
Run 2	0.2	-147.1	297.2
Run 3	0.3	-205.7	296
Run 4	0.4	-256.9	295
Run 5	0.5	-301.9	294.1
Run 6	0.6	-342	293.3
Run 7	0.7	-377.7	292.5
Run 8	0.8	-409.8	291.9
Run 9	0.9	-438.9	291.3

Now, plot the results:




T3 vs eps1
T3 [K]
eps1
299
298
297
296
295
294
293
292
291
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

==

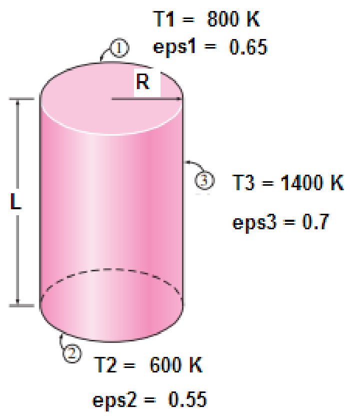
Prob. 5.C.2.11. In a cylindrical furnace with R = 0.75 m, L = 2 m as shown. Emissivities and temperatures of surfaces 1, 2 and 3 are: eps1 = 0.65, eps2 = 0.55, eps3 = 0.7, T1 = 800 K, T2 = 600 K and T3 = 1400 K respectively. Assume surface 3 to be re-radiating. Determine the net rate of radiation to / from each surface.

(b) If surface 3 is black, what are the values of Q1, Q2 and Q3?

(c) For the case (a) plot the variation of Q3 as emissivity of surface 3 varies from 0.1 to 1.

EES Solution:

This is a general, three-surface enclosure, for which the electrical network is shown below:



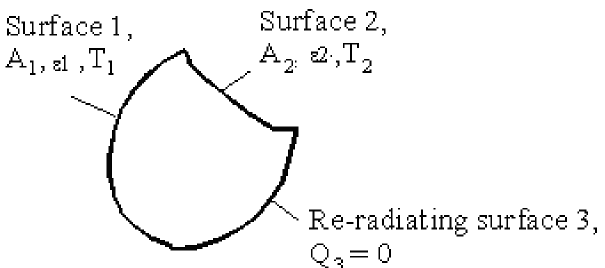


Fig.(a)

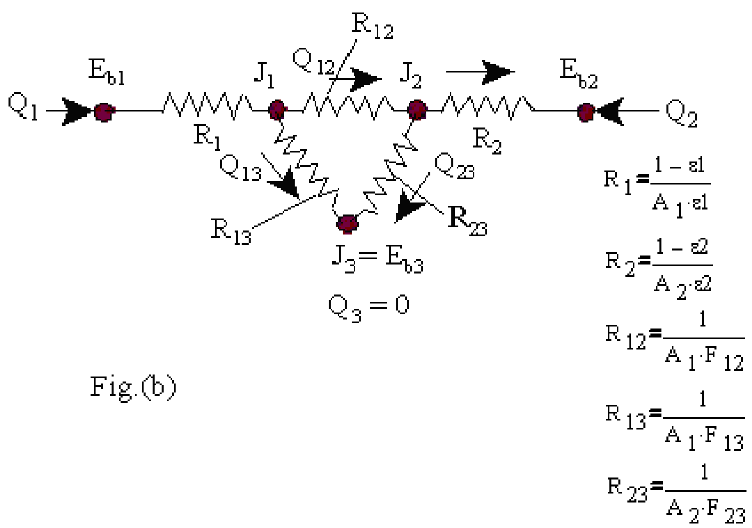


Fig.(b)

Writing Kirchoff's Law to Nodes J1, J2 and J3:

$$\frac{Eb1 - J1}{R1} + \frac{J2 - J1}{R12} + \frac{J3 - J1}{R13} = 0 \quad \text{....for node J1}$$

$$\frac{Eb2 - J2}{R2} + \frac{J1 - J2}{R12} + \frac{J3 - J2}{R23} = 0 \quad \text{....for node J2}$$

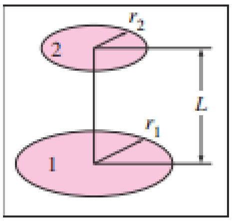
$$\frac{Eb3 - J3}{R3} + \frac{J1 - J3}{R13} + \frac{J2 - J3}{R23} = 0 \quad \text{....for node J3}$$

While writing the above equations, remember to consider all heat currents as flowing *in to* the respective node.

First, get the Emissive powers and various resistances using the EES Function written earlier.

Then, solve simultaneously the three equations obtained by applying the Kirchoff's Law to the three Nodes, written above.

To get the View Factors F12, F13 and F23 use the EES Function for two parallel disks, written earlier.



Following is the EES program:

"Data:"

R = 0.75 [m]
L = 2 [m]

A_1 = pi * R^2 "[m^2]…area of top disk"
A_2 = A_1 "[m^2]…area ofbottom disk"
A_3 = 2 * pi * R * L "[m^2] …area of walls"

T_1 = 800 [K]"…temp of top disk"
T_2 = 600 [K]"…temp of bottom disk"
T_3 =1400 [K]"….temp of walls"

sigma = 5.67E-08 [W/m^2-K^4]"…Stefan – Boltzmann constant"

eps_1 = 0.65 "emissivity of surface 1"
eps_2 = 0.55 "emissivity of surface 2"
eps_3 = 0.7 "…emissivity of surface 3"

"Determine View Factors: F_12, F_13 etc."

"Use the EES Function written earlier for parallel disks to get F_12:

i.e. F_12 = F12_parallel_disks(r_1, r_2, L)"

"We have: for the top and bottom disks, in the above Function: r_1 = R, r_2 = R = 0.75 m, L = 2 m"

F_12 = F12_parallel_disks(R, R, L) "…View factor from top disk to bottom disk"

F_21 = F_12 "…by symmetry"

"F_11 + F_12 + F_13 = 1 … by Summation rule for surface 1"

"Therefore:"

F_13 = 1- F_12 "…View Factor from surface 1 to 3"

"Similarly, for surface 2, F_22 = 0 , and F_21 + F_22 + F_23 = 1 by Summation rule for surface 2, and we get:"

F_23 = 1- F_21 "…View Factor from surface 2 to 3"

"Now, calculate various Emissive powers and Resistances, using the EES PROCEDURE:"

CALL HTrans_Three_surface_enclosure(A_1,A_2,A_3,T_1,T_2,T_3,eps_1,eps_2,eps_3,F_12,F_13,F_23:Eb_1,Eb_2,Eb_3,R_1,R_2,R_3,R_12,R_13,R_23)

"Kirchoff's Law for Nodes J_1, J_2 and J_3:"

"For J_1:" (Eb_1 – J_1)/R_1 + (J_2 – J_1)/R_12 + (J_3 – J_1)/R_13 = 0

"For J_2:" (Eb_2 – J_2)/R_2 + (J_1 – J_2)/R_12 + (J_3 – J_2)/R_23 = 0

"For J_3:" (Eb_3 – J_3)/R_3 + (J_1 – J_3)/R_13 + (J_2 – J_3)/R_23 = 0

"Net heat transfer from each surface:"

Q_1 = (Eb_1 – J_1)/R_1 "[W]"

Q_2 = (Eb_2 – J_2)/R_2 "[W]"

Q_3 = (Eb_3 – J_3)/R_3 "[W]"

"Net heat transfer between surface 1 and 2:"

Q_12 = (J_1 – J_2)/R_12

"Net heat transfer between surface 1 and 3:"

Q_13 = (J_1 – J_3)/R_13

"Net heat transfer between surface 2 and 3:"

Q_23 = (J_2 – J_3)/R_23

"Check: Q_1 + Q_2 + Q_3 = 0 "

SumQ1Q2Q3 = Q_1 + Q_2 + Q_3

Results:

Main | HTrans_Three_surface_enclosure | F12_parallel_disks

Unit Settings: SI K Pa J mass rad

A_1 = 1.767 [m^2]	A_2 = 1.767 [m^2]	A_3 = 9.425 [m^2]
Eb_1 = 23224 [W/m^2]	Eb_2 = 7348 [W/m^2]	Eb_3 = 217819 [W/m^2]
eps_1 = 0.65	eps_2 = 0.55	eps_3 = 0.7
F_{12} = 0.1111	F_{13} = 0.8889	F_{21} = 0.1111
F_{23} = 0.8889	**J_1 = 81136** [W/m^2]	**J_2 = 88578** [W/m^2]
J_3 = 201198 [W/m^2]	L = 2 [m]	Q_1 = -190056 [W]
Q_{12} = -1461 [W]	Q_{13} = -188594 [W]	Q_2 = -175443 [W]
Q_{23} = -176904 [W]	Q_3 = 365499 [W]	R = 0.75 [m]
R_1 = 0.3047 [$1/m^2$]	R_{12} = 5.093 [$1/m^2$]	R_{13} = 0.6366 [$1/m^2$]
R_2 = 0.463 [$1/m^2$]	R_{23} = 0.6366 [$1/m^2$]	R_3 = 0.04547 [$1/m^2$]
σ = 5.670E-08 [W/m^2-K^4]	SumQ1Q2Q3 = 0.0122 [W]	T_1 = 800 [K]
T_2 = 600 [K]	T_3 = 1400 [K]	

Thus:

Net heat transfer from surface 1 = Q1 = -190056 W … negative sign indicates heat flowing *in to* the surface 1… Ans.

Net heat transfer from surface 2 = Q2 = -175443 W … negative sign indicates heat flowing *in to* the surface 2… Ans.

Net heat transfer from surface 3 = Q3 = 365499 W … heat flowing *from* the surface 3… Ans.

Also, SumQ1Q2Q3 = 0.0122, i.e. almost equal to zero, as it should be.

(b) If surface 3 is black, what are the values of Q1, Q2 and Q3?

Now, eps3 = 1.

In EES data, enter: eps3 = 0.9999 to avoid division by zero.

Press F2 to calculate, and we get:

Unit Settings: SI K Pa J mass rad

A_1 = 1.767 [m^2]	A_2 = 1.767 [m^2]	A_3 = 9.425 [m^2]
Eb_1 = 23224 [W/m^2]	Eb_2 = 7348 [W/m^2]	Eb_3 = 217819 [W/m^2]
eps_1 = 0.65	eps_2 = 0.55	eps_3 = 0.9999
F_{12} = 0.1111	F_{13} = 0.8889	F_{21} = 0.1111
F_{23} = 0.8889	**J_1 = 86574** [W/m^2]	**J_2 = 95496** [W/m^2]
J_3 = 217814 [W/m^2]	L = 2 [m]	Q_1 = -207904 [W]
Q_{12} = -1752 [W]	Q_{13} = -206152 [W]	Q_2 = -190385 [W]
Q_{23} = -192137 [W]	Q_3 = 398289 [W]	R = 0.75 [m]
R_1 = 0.3047 [1/m^2]	R_{12} = 5.093 [1/m^2]	R_{13} = 0.6366 [1/m^2]
R_2 = 0.463 [1/m^2]	R_{23} = 0.6366 [1/m^2]	R_3 = 0.00001061 [1/m^2]
σ = 5.670E-08 [W/m^2-K^4]	SumQ1Q2Q3 = 0.0002028 [W]	T_1 = 800 [K]
T_2 = 600 [K]	T_3 = 1400 [K]	

i.e.

Net heat transfer from surface 1 = Q1 = -207904 W … negative sign indicates heat flowing *in to* the surface 1… Ans.

Net heat transfer from surface 2 = Q2 = -190385 W … negative sign indicates heat flowing *in to* the surface 2… Ans.

Net heat transfer from surface 3 = Q3 = 398289 W … heat flowing *from* the surface 3… Ans.

Also, SumQ1Q2Q3 = 0.0002028, i.e. almost equal to zero, as it should be.

(c) For the case (a), plot the variation of Q3 as emissivity of surface 3 varies from 0.1 to 1:

First, prepare the Parametric Table:

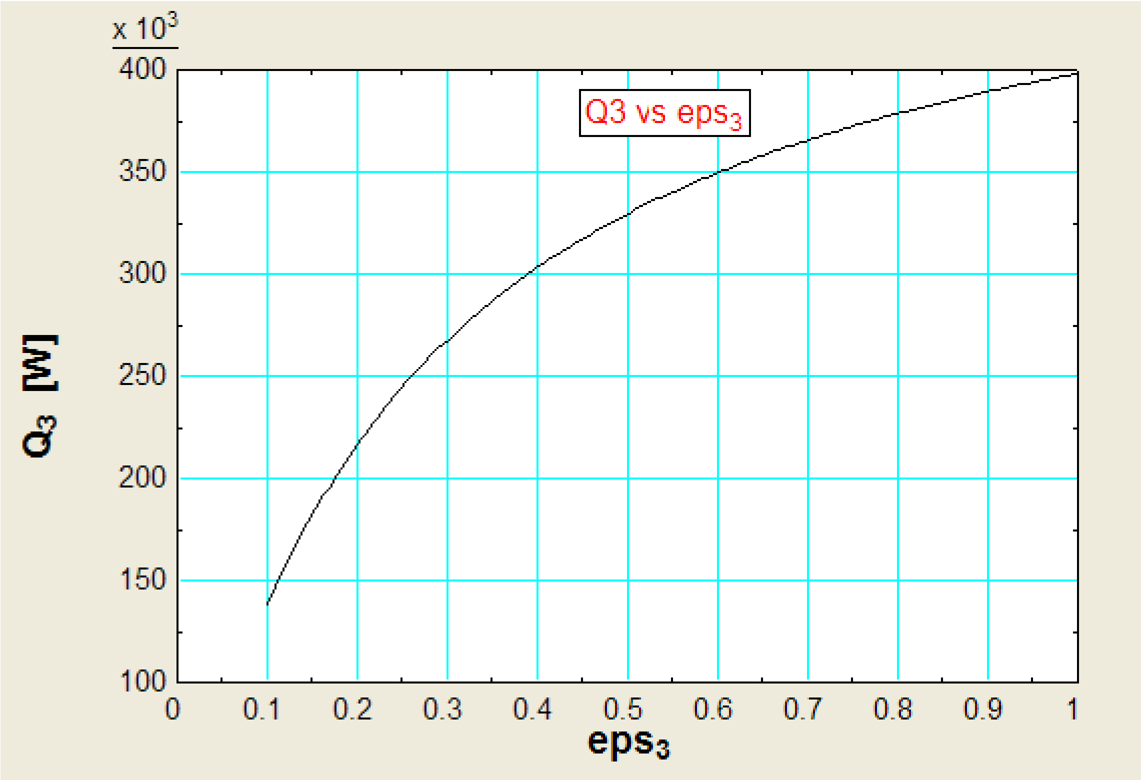
Table 1 | Table 2 | Table 3

1..10	eps_3	Q_1 [W]	Q_2 [W]	Q_3 [W]	SumQ1Q2Q3 [W]	Q_{13} [W]
Run 1	0.1	-66271	-71811	138082	-0.00002081	-66825
Run 2	0.2	-109093	-107661	216754	-2.050E-09	-108950
Run 3	0.3	-136752	-130818	267570	-6.125E-10	-136159
Run 4	0.4	-156091	-147008	303099	1.051E-10	-155183
Run 5	0.5	-170373	-158965	329338	2.337E-10	-169232
Run 6	0.6	-181352	-168157	349509	-6.355E-11	-180033
Run 7	0.7	-190056	-175443	365499	-1.197E-10	-188594
Run 8	0.8	-197124	-181361	378485	3.075E-11	-195548
Run 9	0.9	-202979	-186263	389242	1.459E-10	-201308
Run 10	0.9999	-207904	-190385	398289	-4.741E-09	-206152

In the above Table, in addition to Q3, we have also obtained Q1, Q2, Q13 and SumQ1Q2Q3. Note that SumQ1Q2Q3 is almost equal to zero, in all cases.

As said earlier, negative sign indicates that heat is flowing ***in to*** the surface.

Plot the result:



==

Prob. 5.C.2.12. A long duct of equilateral triangular section, of side w = 0.75 m, shown in Fig. Ex. 13.28, has its surface 1 at 700 K, surface 2 at 1000 K, and surface 3 is insulated. Further, surface 1 has an emissivity of 0.8 and surface 2 is black. Determine the rate at which energy must be supplied to surface 2 to maintain these operating conditions.

(b) Plot the variation of Q1 and T3 as epsilon1 varies from 0.1 to 0.9.

EES Solution:

Since the duct is very long, end effects can be neglected, and this is a ***three-surface enclosure*** problem.

Surface 1 is gray, surface 2 is black and surface 3 is insulated.

Radiation network is shown in Fig.(b) below:

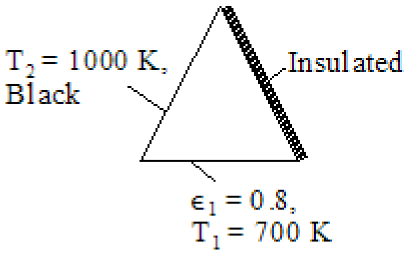


Fig.(a)

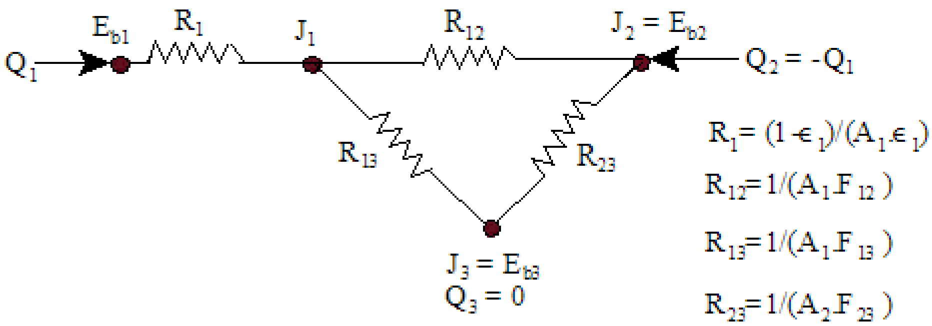


Fig.(b)

We use the EES Function written earlier to calculate Emissive powers and resistances.

T3 is unknown, but is calculated from the relations: Eb3 = J3 = σ.T3^4, where σ is the Stefan-Boltzmann constant.

EES code is given below:

```
"Data:"

L = 1 [m]"…assumed length of duct"
W = 0.75 [m]"…side of equilateral triangle"
A_1 = W * L "[m^2]….area of side 1"
A_2 = A_1"[m^2]…area ofside 2"
A_3 = A_1 "[m^2] …area of side 3"

T_1 = 700 [K]"…temp of ceiling, by data"
T_2 = 1000 [K]"…temp of floor, by data"

{T_3 = …to be found out}

sigma = 5.67E-08 [W/m^2-K^4]"…Stefan – Boltzmann constant"

eps_1 = 0.8 "emissivity of surface 1"
eps_2 = 0.9999 "emissivity of surface 2 …. black"
eps_3 = 0.9999 "…emissivity of surface 3 …. "

"View Factors: F_12, F_13 etc."
"F_11 + F_12 + F_13 = 1 … by Summation rule for surface 1"
"But, "

F_11 = 0 "…since surface 1 is flat;

Also, F_12 = F_13, by symmetry."

"Therefore:"

F_12 = 0.5 "…View Factor from surface 1 to 2"

F_13 = 0.5 "…View Factor from surface 1 to 3"
```

"Similarly, for surface 2, F_22 = 0 , and F_21 + F_22 + F_23 = 1 by Summation rule for surface 2, and we get:"

F_23 = 0.5 "…View Factor from surface 2 to 3"

"Now, calculate various Emissive powers and Resistances, using the EES PROCEDURE:"

CALL
HTrans_Three_surface_enclosure(A_1,A_2,A_3,T_1,T_2,T_3,eps_1,eps_2,eps_3,F_12,F_13,F_23:Eb_1,Eb_2,Eb_3,R_1,R_2,R_3,R_12,R_13,R_23)

"Kirchoff's Law for Nodes J_1:"

"For J_1:" (Eb_1 – J_1)/R_1 + (J_2 – J_1)/R_12 + (J_3 – J_1)/R_13 = 0

Eb_2 = J_2 "….since surface 2 is black"

Eb_3 = J_3 "….since surface 3 is insulated"

{Eb_3 = sigma * T_3 ^4 "[W/m^2]…by definition of Eb"}

"Net heat transfer from each surface:"

Q_1 = (Eb_1 – J_1)/R_1"[W]…net heat transfer from surface 1"

Q_2 = -Q_1"[W]…net heat transfer from surface 2"

Q_3 = 0 "[W]…net heat transfer from surface 3"

"Net heat transfer between surface 1 and 2:"

Q_12 = (J_1 – J_2)/R_12 "[W]"

"Net heat transfer between surface 1 and 3:"

Q_13 = (J_1 – J_3)/R_13 "[W]"

"Net heat transfer between surface 2 and 3:"

Q_23 = (J_2 – J_3)/R_23 "[W]"

"Check: Q_1 + Q_2 + Q_3 = 0"

SumQ1Q2Q3 = Q_1 + Q_2 + Q_3

"=="

Results:

Unit Settings: SI K Pa J mass rad

A_1 = 0.75 [m^2]	A_2 = 0.75 [m^2]	A_3 = 0.75 [m^2]
Eb_1 = 13614 [W/m^2]	Eb_2 = 56700 [W/m^2]	Eb_3 = 38558 [W/m^2]
eps_1 = 0.8	eps_2 = 0.9999	eps_3 = 0.9999
F_{11} = 0	F_{12} = 0.5	F_{13} = 0.5
F_{23} = 0.5	**J_1 = 20417** [W/m^2]	**J_2 = 56700** [W/m^2]
J_3 = 38558 [W/m^2]	L = 1 [m]	Q_1 = -20409 [W]
Q_{12} = -13606 [W]	Q_{13} = -6803 [W]	Q_2 = 20409 [W]
Q_{23} = 6803 [W]	Q_3 = 0 [W]	R_1 = 0.3333 [1/m^2]
R_{12} = 2.667 [1/m^2]	R_{13} = 2.667 [1/m^2]	R_2 = 0.0001333 [1/m^2]
R_{23} = 2.667 [1/m^2]	R_3 = 0.0001333 [1/m^2]	σ = 5.670E-08 [W/m^2-K^4]
SumQ1Q2Q3 = 7.727E-07 [W]	T_1 = 700 [K]	T_2 = 1000 [K]
T_3 = 908.1 [K]	W = 0.75 [m]	

Thus:

Net heat transfer from surface 1 = Q1 = -20409 W … negative sign indicates heat coming *into* the surface … Ans.

Net heat transfer from surface 2 = Q2 = 20409 W … positive sign indicates heat *leaving* the surface … Ans.

Temp of insulated surface 3 = T3 = 908.1 K … ans.

Note that as a check: SumQ1Q2Q3 = 0…is satisfied.

(b) Plot the variation of Q1 and T3 as epsilon1 varies from 0.1 to 0.9:

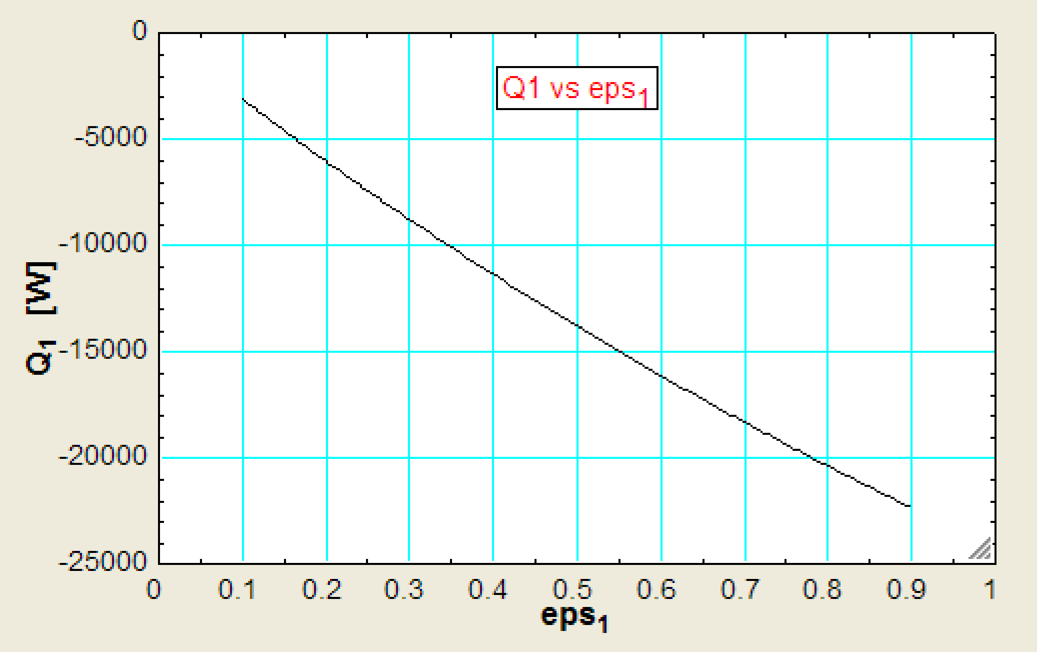
First, compute the Parametric Table:

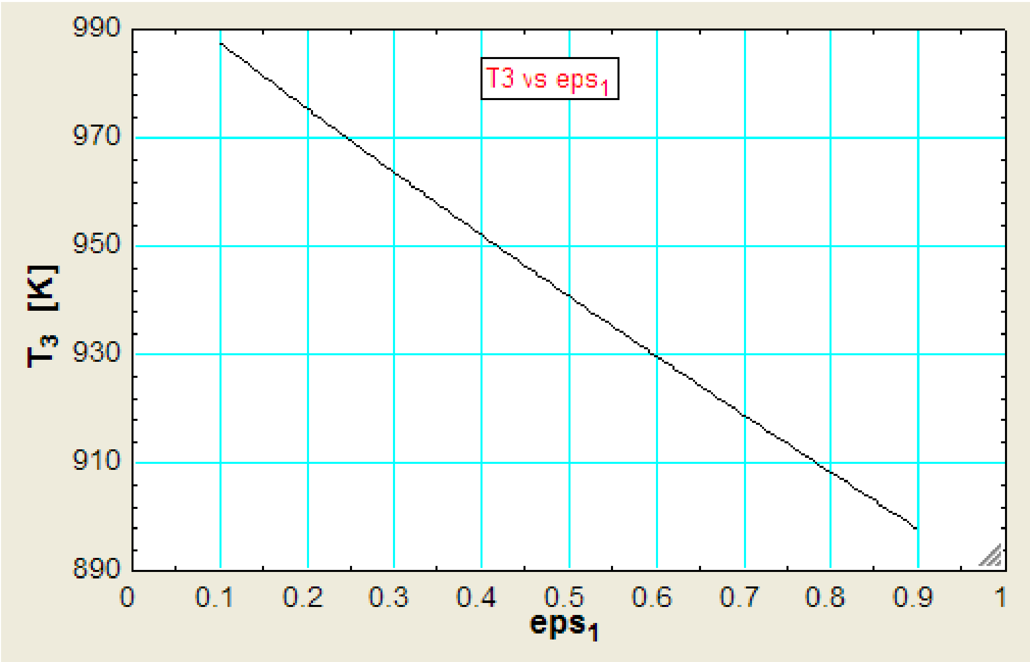
Table 1 | Table 2 | Table 3 | Table 4

1..9	eps_1	Q_1 [W]	T_3 [K]
Run 1	0.1	-3127	987.5
Run 2	0.2	-6059	975.4
Run 3	0.3	-8813	963.5
Run 4	0.4	-11405	951.9
Run 5	0.5	-13849	940.6
Run 6	0.6	-16157	929.6
Run 7	0.7	-18341	918.7
Run 8	0.8	-20409	908.1
Run 9	0.9	-22372	897.7

Note: Negative sign for Q1 denotes heat flowing *into* the surface 1.

Now, plot the results:





Note: The four problems solved above with EES cover the important variations of the three-surface enclosure problem.

==

Prob. 5.C.2.13. A furnace is in the shape of a frustum of a cone with base (i.e. surface 1) diameter of 2 m, top (i.e. surface 2) diameter of 1 m, and height 1.5 m. The curved surface (i.e. surface 3) has an emissivity of eps3 = 0.65 and is maintained at T3 = 1600 K. Emissivities and temperatures of surfaces 1, 2 are: eps1 = 0.6, eps2 = 0.7, T1 = 700 K, T2 = 1200 K respectively. Determine the net rate of radiation to / from each surface.

(b) If surface 3 is black, what are the values of Q1, Q2 and Q3?

(c) For the case (a) plot the variation of Q13 as emissivity of surface 3 varies from 0.1 to 1.

EXCEL Solution:

This is a general, three-surface enclosure, for which the schematic fig and the Radiation network are shown below:

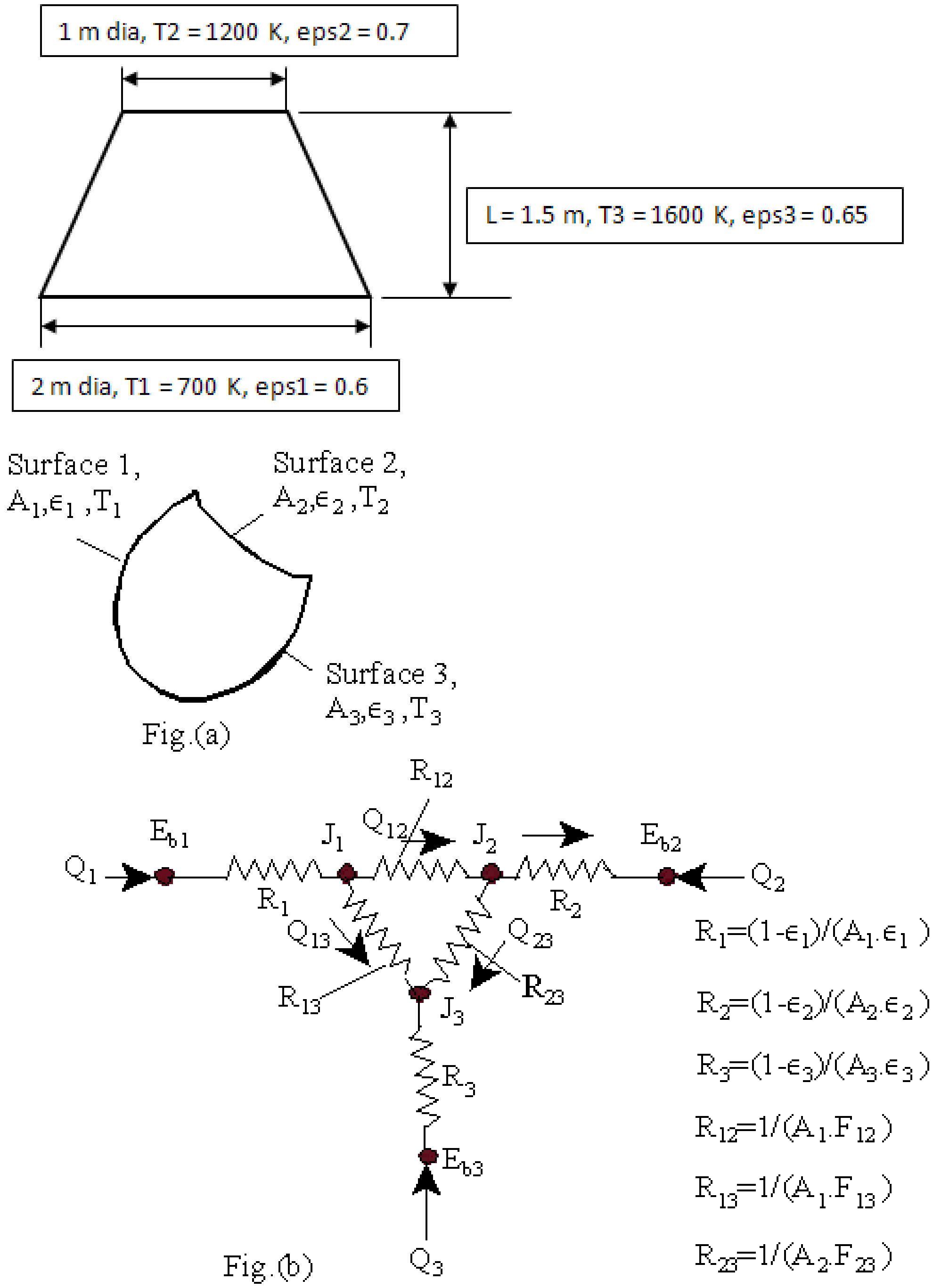


Fig.5.C.2.13

In the above fig., base and top are circles whose area is calculated as (π * R^2).

Area of frustum of a cone: $A3 = \pi \cdot (R\_1 + R\_2) \cdot \sqrt{(R\_2 - R\_1)^2 + L^2}$

Now, let us prepare an EXCEL template to solve general, three surface enclosure problem.

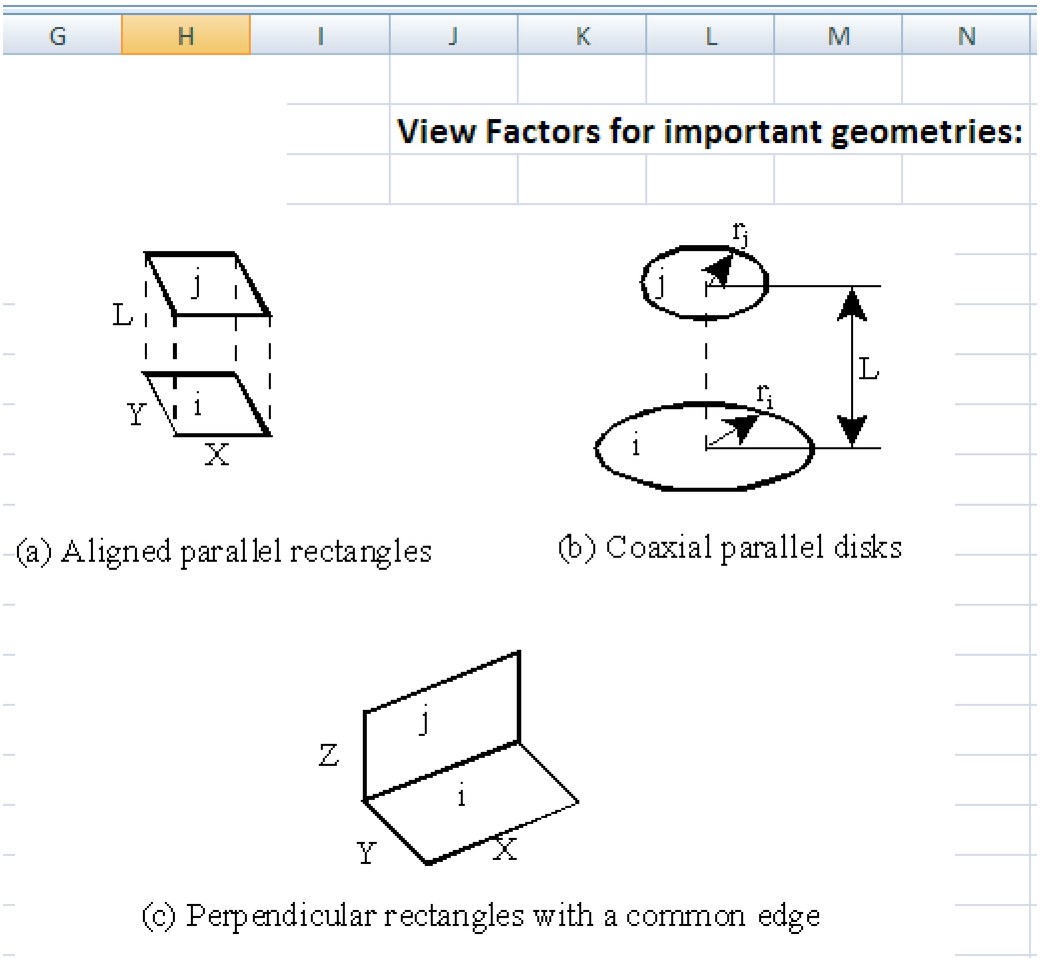
Here, in the worksheet, we will first have a portion where we calculate View Factors, say for the three important geometries, viz. parallel rectangles, coaxial parallel discs and perpendicular rectangles with a common edge.

Recollect that we have already written EXCEL – VBA Functions for View factors for these geometries. See Problems 5B.11, 5B.12 and 5B.13.

Then, in the template, we will calculate the Emissive powers and various 'surface resistances' and 'space resistances'. Then, we will solve the three equations obtained by writing the Kirchoff's Law for the three nodes, to obtain the radiosities J1, J2 and J3. We shall use the EXCEL Solver to do this. Knowing J1, J2 and J3, the heat flows from a given surface or between any two surfaces are easily calculated.

Following is the worksheet:

First, a template for View Factors:



=F_ij_Coaxial_parallel_discs(M78/M76,M77/M78)

F	G	H	I	J	K	L	M	N
		(a) Aligned, parallel rectangles:				(b) Coaxial, parallel disks:		
		X =	1			r_i =	0.75	
		Y =	1			r_j =	0.75	
		L =	1			L =	2	
		F_ij =	0.1998249			F_ij =	0.111111	
		(c) Perpendicular rectangles with a common edge:						
		X =	1					
		Y =	1					
		Z =	1					
		F_ij =	0.2000438					

Note in the above that in a corner of the worksheet, there is a template to calculate the View Factors for the aforesaid three geometries using the VBA Functions already written. For example, View factor for Coaxial, parallel disks is calculated in cell M79, using the VBA Function. The Function entered can be seen in the Formula bar. If we change the values of r_i, r_j or L in cells M76, M77 and M78 respectively, immediately the View factor in cell M79 will change. Similarly, we can find out the View factors for other two geometries by entering data in respective cells.

Let us remind ourselves the radiation network for the three surface enclosure (given above) and the equations obtained by applying the Kirchoff's Law to the three Nodes:

Node J_1: $$\frac{E_{b1}-J_1}{R_1}+\frac{J_2-J_1}{R_{12}}+\frac{J_3-J_1}{R_{13}}=0 \quad .....(13.63,a)$$

Node J_2: $$\frac{E_{b2}-J_2}{R_2}+\frac{J_1-J_2}{R_{12}}+\frac{J_3-J_2}{R_{23}}=0 \quad .....(13.63,b)$$

Node J_3: $$\frac{E_{b3}-J_3}{R_3}+\frac{J_1-J_3}{R_{13}}+\frac{J_2-J_3}{R_{23}}=0 \quad .....(13.63,c)$$

Solving these three eqns. simultaneously, we get J_1, J_2 and J_3.

And, when the three Radiosities, viz. J1, J2 and J3 are obtained by solving the above three equations, the various heat flows are calculated as follows:

Once the magnitudes of the radiosities are known, expressions for net heat flows between the surfaces are:

$$Q_{12} = \frac{J_1 - J_2}{R_{12}} = \frac{J_1 - J_2}{\frac{1}{A_1 \cdot F_{12}}} \qquad ....(13.64,a)$$

$$Q_{13} = \frac{J_1 - J_3}{R_{13}} = \frac{J_1 - J_3}{\frac{1}{A_1 \cdot F_{13}}} \qquad ....(13.64,b)$$

$$Q_{23} = \frac{J_2 - J_3}{R_{23}} = \frac{J_1 - J_2}{\frac{1}{A_2 \cdot F_{23}}} \qquad ....(13.64,c)$$

And, net heat flow from each surface is:

$$Q_1 = \frac{E_{b1} - J_1}{R_1} = \frac{E_{b1} - J_1}{\left(\frac{1 - \varepsilon_1}{A_1 \cdot \varepsilon_1}\right)} \quad \text{.....(13.65,a)}$$

$$Q_2 = \frac{E_{b2} - J_2}{R_2} = \frac{E_{b2} - J_2}{\left(\frac{1 - \varepsilon_2}{A_2 \cdot \varepsilon_2}\right)} \quad \text{.....(13.65,b)}$$

$$Q_3 = \frac{E_{b3} - J_3}{R_3} = \frac{E_{b3} - J_3}{\left(\frac{1 - \varepsilon_3}{A_3 \cdot \varepsilon_3}\right)} \quad \text{.....(13.65,c)}$$

Next, In the EXCEL worksheet, enter data for the three surface enclosure problem, and name the cells:

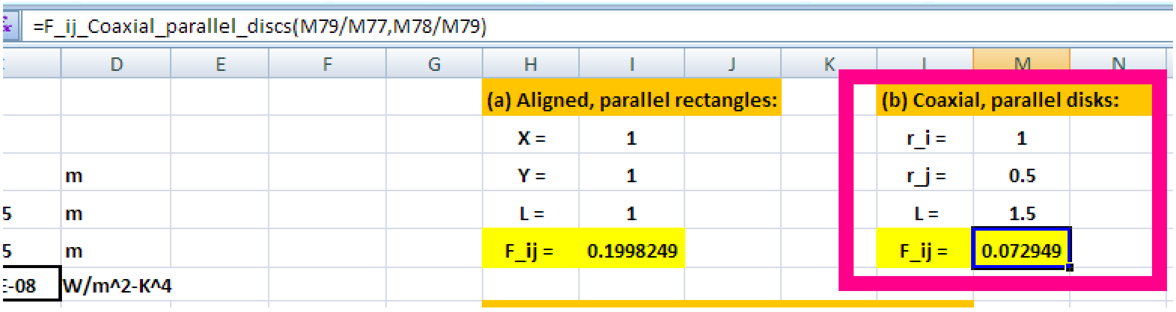
A_3 | fx =PI()*(C78+C79)*SQRT((C79-C78)^2+C80^2)

	A	B	C	D	E	F
76		**Data:**				
77						
78		**R_1 =**	1	m		
79		**R_2 =**	0.5	m		
80		**L =**	1.5	m		
81		**sigma (W/m2.K^4):**	5.67E-08	W/m^2-K^4		
82						
83		**Areas (m2):**	A_1	A_2	A_3	
84			3.141592654	0.7853982	7.450941	
85						
86		**Emissivities:**	eps_1	eps_2	eps_3	
87			0.6	0.7	0.65	
88						
89		**View Factors:**	F_11	F_12	F_13	
90			0	0.072949	0.927051	
91			F_21	F_22	F_23	
92			0.291796	0	0.927051	
93			F_31	F_32	F_33	
94			0.390879022	0.0977198	0.511401	

In the above, A_3 is the area of frustum and the equation entered in cell E84 can be seen in the Formula bar.

View Factors:

View factor F12 is determined with the template explained earlier, using the VBA Function for View Factor of parallel disks. See the Formula bar below:


=F_ij_Coaxial_parallel_discs(M79/M77,M78/M79)

	D	E	F	G	H	I	J	K	L	M	N
					(a) Aligned, parallel rectangles:				(b) Coaxial, parallel disks:		
					X =	1			r_i =	1	
	m				Y =	1			r_j =	0.5	
5	m				L =	1			L =	1.5	
5	m				F_ij =	0.1998249			F_ij =	0.072949	
:-08	W/m^2-K^4										

This value of F_ij is transferred to cell D90 above. Other View factors are calculated using the View factor algebra:

$F\_11 + F\_12 + F\_13 = 1$ ...by Summation rule for surface 1

But, $F\_11 = 0$ ..for Flat surface 1

Therefore: $F\_13 = 1 - F\_12$ ...View factor from surface 1 to 3

$A\_1 \cdot F\_12 = A\_2 \cdot F\_21$ ....by reciprocity

Then: $F\_21 = \frac{A\_1}{A\_2} \cdot F\_12$ ...View factor from surface 2 to 1

But, $F\_22 = 0$ ..for Flat surface 2

Therefore: $F\_23 = 1 - F\_21$ ...View factor from surface 2 to 3

And:

$A\_1 \cdot F\_13 = A\_3 \cdot F\_31$ ....by reciprocity

Then: $F\_31 = \frac{A\_1}{A\_3} \cdot F\_13$ ...View factor from surface 3 to 1

And, by reciprocity: $A\_2 \cdot F\_23 = A\_3 \cdot F\_32$

Therefore: $F\_32 = \frac{A\_2}{A\_3} \cdot F\_23$ ...View factor from surface 3 to 2

And: $F\_33 = 1 - F\_31 - F\_32$ ...using Summation rule for surface 3

Using the above formulas, all View Factors are entered in the above part of worksheet.

Next, enter the Temp values, and continue the calculations:

E_b1 | f_x =sigma*T_1^4

	A	B	C	D	E
97					
98		Temps. (K)	T_1	T_2	T_3
99			700	1200	1600
100					
101		E_b (W/m2)	E_b1	E_b2	E_b3
102			1.36E+04	1.18E+05	3.72E+05
103					
104		Resistances(m^-2):	R_1	R_2	R_3
105			0.212206591	0.5456741	7.23E-02
106					
107			R_12	R_13	R_23
108			4.363457843	0.3433575	1.37343

In the above, Emissive powers E_b1 etc are calculated from:

$$E\_b = sigma \cdot T^4$$

See the formula bar above for the formula entered for E_b1 in cell C102.

Similarly, surface resistances R_1, R_2 and R_3 and space resistances R_12, R_13 and R_23 are calculated with the formulas given above in Fig.5.C.2.13.

Next, important step to calculate Radiosities J1, J2 and J3:

C114 | f_x =(E_b1-J_1)/R_1+(J_2-J_1)/R_12+(J_3-J_1)/R_13

	A	B	C	D	E	F
109						
110		Radiosities (W/m2):	J_1	J_2	J_3	
111			1000	1000	1000	.....Ans.
112						
113		Apply Kirchoff's Law at each Node:				
114		Node J1:	59440.51951			
115		Node J2:	213631.4001			
116		Node J3:	5128012.95			
117		Sum_diff^2=	2.63457E+13			
118						
119		Apply Solver to make Sumdiff^2 a minimum, by varying J_1, J_2, and J_3:				

In the above, first, put guess values of 1000 each for J1, J2 and J3.

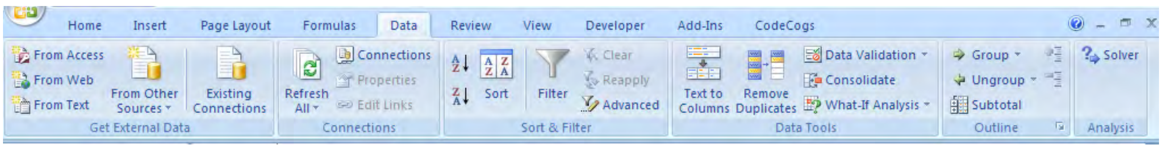
Then, for Nodes 1, 2 and 3 enter the corresponding equations, obtained by applying Kirchoff's Law, given earlier.

See the Formula bar in the above screen shot for formula entered for Node 1, in cell C114. Similarly, formulas are entered for Nodes 2 and 3 in cells C115 and C116.

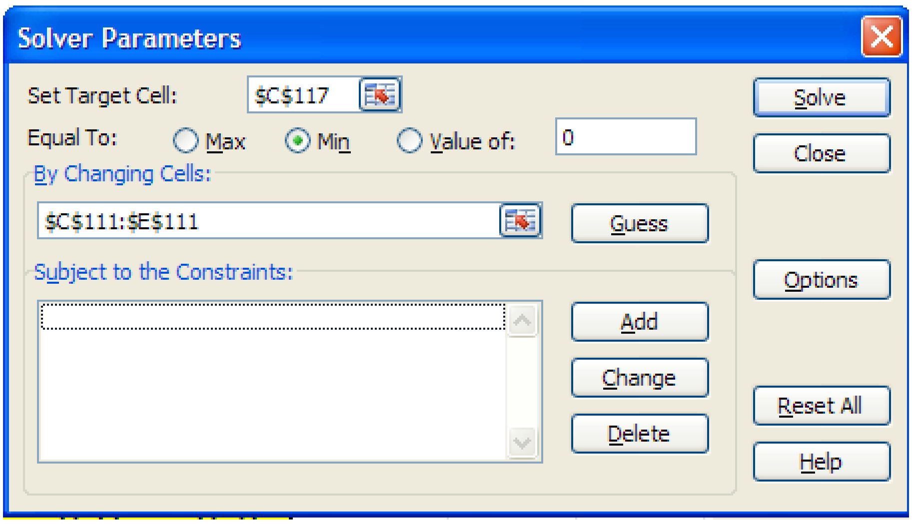
Sum of the squares of C114 to C116 should be zero, but, in this case since J1, J2 and J3 are assumed values, the sum of squares is not zero.

Apply Solver to make C117 zero by changing J1, J2 and J3. (i.e. cells C11, D111 and E111.)

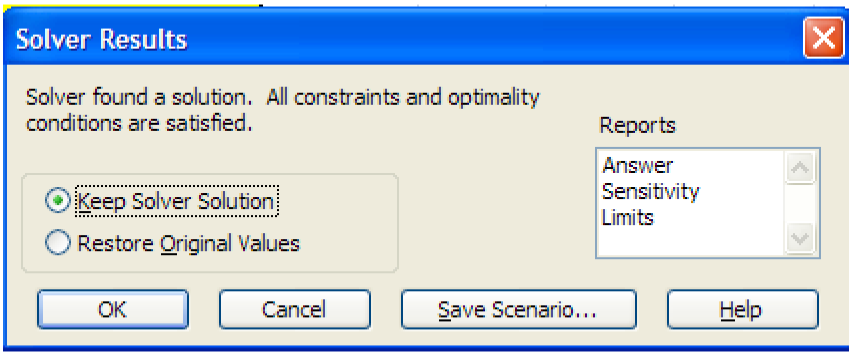
To do this: Go to Data – Solver:



Click on Solver. We get the following window. Fill it up as shown:



Click Solve. We get following message:



Solver has found a solution. Click OK to keep the solution. We get:

C114 | =(E_b1-J_1)/R_1+(J_2-J_1)/R_12+(J_3-J_1)/R_13

	A	B	C	D	E	F
109						
110		Radiosities (W/m2):	J_1	J_2	J_3	
111			133193.6206	172619	323575.7	.....Ans.
112						
113		Apply Kirchoff's Law at each Node:				
114		Node J1:	2.93716E-06			
115		Node J2:	2.48438E-06			
116		Node J3:	-5.9306E-05			
117		Sum_diff^2=	3.53202E-09			
118						
119		Apply Solver to make Sumdiff^2 a minimum, by varying J_1, J_2, and J_3:				

Observe that values of J1, J2 and J3 are shown in the respective cells.

Sum_diff^2 is almost equal to zero.

And, results of equations at Nodes 1, 2 and 3 are almost equal to zero.

Next, continue the calculations for heat transfers:

C125 | =(E_b1-J_1)/R_1

	A	B	C	D	E	F
112						
113		Apply Kirchoff's Law at each Node:				
114		Node J1:	2.93716E-06			
115		Node J2:	2.48438E-06			
116		Node J3:	-5.9306E-05			
117		Sum_diff^2=	3.53202E-09			
118						
119		Apply Solver to make Sumdiff^2 a minimum, by varying J_1, J_2, and J_3:				
120						
121		Q_betwn_surfaces (W):	Q_12	Q_13	Q_23	
122			-9035.35191	-554471.89	-109912	.....Ans.
123						
124		Qnet_from_surfaces (W):	Q_1	Q_2	Q_3	
125			-563507.241	-100876.84	664384.1	.....Ans.
126						
127		Check: Q1 + Q2 + Q3 = 0	SumQ1Q2Q3 =		0.0	...Verified.

In the above heat transfers from/to each surface (Q_1, Q_2 and Q_3), and also between surfaces (Q_12, Q_13, Q_23) are calculated using formulas given earlier. For ex. see in the Formula bar, the formula entered to calculate Q_1 in cell C125.

Negative sign for a heat transfer indicates heat coming ***in to*** the surface.

As a check: Sum of Q1, Q2 and Q3 should be zero.

Thus:

Q1 = -563507.24 W, Q2 = -100876.84 W and Q3 = 664384.1 W … Ans.

(b) If surface 3 is black, what are the values of Q1, Q2 and Q3?

Now, simply change eps3 = 0.9999. Remember not to put eps3 = 1, to avoid division by zero, as explained earlier.

Then, all other related quantities will change in the worksheet.

	A	B	C	D	E
85					
86		Emissivities:	eps_1	eps_2	eps_3
87			0.6	0.7	0.9999
88					
89		View Factors:	F_11	F_12	F_13
90			0	0.072949	0.927051
91			F_21	F_22	F_23
92			0.291796	0	0.927051
93			F_31	F_32	F_33
94			0.390879022	0.0977198	0.511401
95					
96					
97					
98		Temps. (K)	T_1	T_2	T_3
99			700	1200	1600
100					
101		E_b (W/m2)	E_b1	E_b2	E_b3
102			1.36E+04	1.18E+05	3.72E+05
103					
104		Resistances(m^-2):	R_1	R_2	R_3
105			0.212206591	0.5456741	1.34E-05
106					
107			R_12	R_13	R_23
108			4.363457843	0.3433575	1.37343

However, J1, J2 and J3, do not change o their own:

110		Radiosities (W/m2):	J_1	J_2	J_3	
111			133193.6206	172619	323575.7	.....Ans.
112						
113		Apply Kirchoff's Law at each Node:				
114		Node J1:	0			
115		Node J2:	0			
116		Node J3:	3576430614			
117		Sum_diff^2=	1.27909E+19			
118						
119		Apply Solver to make Sumdiff^2 a minimum, by varying J_1, J_2, and J_3:				

Observe that Sum_diff^2 is not zero now.

Apply Solver to get correct values of J1, J2 and J3.

(Remember that for a black body Eb_3 should be equal to J3. Since we have taken eps3 = 0.9999, Eb_3 is not *exactly* equal to, but *almost* equal to J3).

109						
110	Radiosities (W/m2):	J_1	J_2	J_3		
111		151403.3259	186642.71	371578.7	.....Ans.	
112						
113	Apply Kirchoff's Law at each Node:					
114	Node J1:	-5.7975E-07				
115	Node J2:	-3.4051E-09				
116	Node J3:	0.000318623				
117	Sum_diff^2=	1.01521E-07				
118						
119	Apply Solver to make Sumdiff^2 a minimum, by varying J_1, J_2, and J_3:					
120						
121	Q_betwn_surfaces (W):	Q_12	Q_13	Q_23		
122		-8076.02333	-641242.43	-134653	.....Ans.	
123						
124	Qnet_from_surfaces (W):	Q_1	Q_2	Q_3		
125		-649318.456	-126576.64	775895.1	.....Ans.	
126						
127	Check: Q1 + Q2 + Q3 = 0	SumQ1Q2Q3 =		0.0	...Verified.	

We get:

Q1 = -649318.46 W, Q2 = -126576.64 W, Q3 = 775895.1 W …. Ans.

(c) For the case (a) plot the variation of Q3, Q13 as emissivity of surface 3 varies from 0.1 to 0.9:

Now, change eps3 = 0.65 to go back to case (a).

First, prepare a Table as shown below:

G152

	A	B	C	D	E
128					
129		**Plot Q3, Q_13 against eps3:**			
130					
131		eps_3	Q_3 (W)	Q_13 (W)	SumQ1Q2Q3
132		0.1			
133		0.2			
134		0.3			
135		0.4			
136		0.5			
137		0.6			
138		0.7			
139		0.8			
140		0.9			
141		0.99			
142		0.9999			
143					

Now, write a VBA program to do the following:

Copy the first value of eps3 from this Table to cell E87 in the Data section.

Then, all other values except J1, J2, J3 and the heat transfers, will change.

Apply the Solver to get J1, J2 and J3.

Immediately all heat transfers will get updated.

Then, copy Q_3, Q_13 and also SumQ1Q2Q3 (to check that it is zero) to the respective places in the Table.

Repeat these steps for the next value of eps3, etc.

Following is the VBA program, operated by a control button:

```
Sub Macro1()
'
' Macro1 Macro
' Finds J1, J2 and J3 making Sumdiff^2 minimum
'
' Keyboard Shortcut: Ctrl+Shift+J

Dim i As Integer

For i = 0 To 10 'start of For ... Next loop

    Range("E87") = Cells(132 + i, 2) 'copy the first value of eps3 from the Table to cell E87

    'Following part of the code applies Solver to minimise cell C117 by changing cells C111:E111

    '------------------------------------------------

    SolverOk SetCell:="$C$117", MaxMinVal:=2, ValueOf:="0", ByChange:= _
        "$C$111:$E$111"
    SolverSolve UserFinish:=True
    SolverFinish KeepFinal:=1

    '-------------------------------------------------

    Cells(132 + i, 3) = Range("E125") 'copies up-dated value of Q_3 to its place in Table

    Cells(132 + i, 4) = Range("D122") 'copies up-dated value of Q_13 to its place in Table

    Cells(132 + i, 5) = Range("E127") 'copies up-dated value of SUMQ1Q2Q3 to its place in Table

 Next i

End Sub
```

Read the comments in the above program.

Now, press the command button and the Solver is applied for each value of eps3:

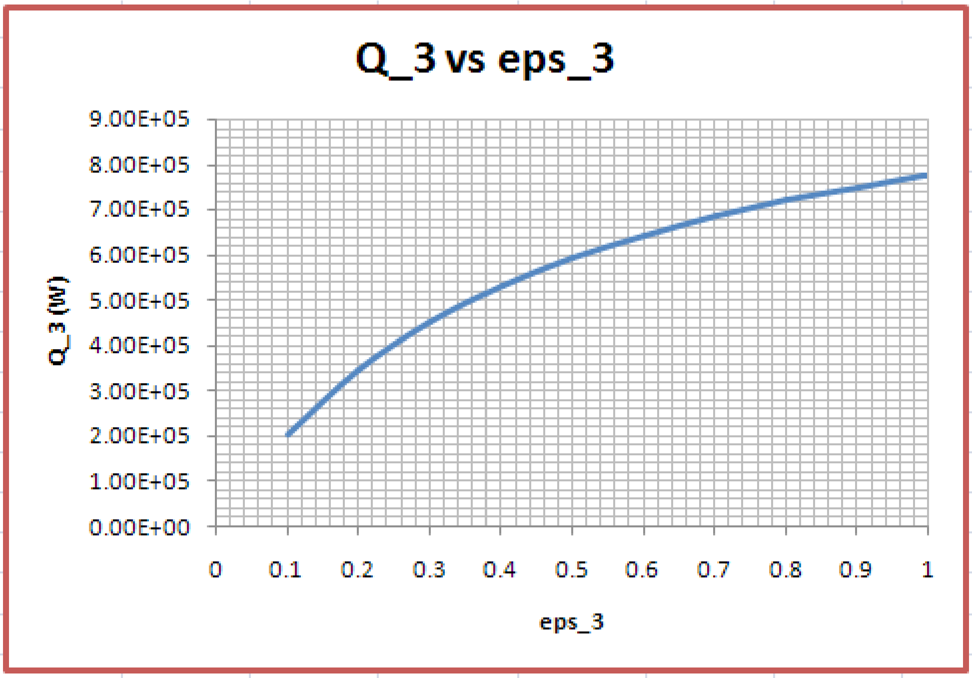
Apply Kirchoff's Law at each Node:				
Node J1:	1.31084E-07			
Node J2:	8.31787E-08			
Node J3:	1.74704E-06			
Sum_diff^2=	3.07627E-12			
Apply Solver to make Sumdiff^2 a minimum, by varying J_1, J_2, and J_3:				
Q_betwn_surfaces (W):	Q_12	Q_13	Q_23	
	-8076.023329	-641242.433	-134652.664	.....Ans.
Qnet_from_surfaces (W):	Q_1	Q_2	Q_3	
	-649318.4563	-126576.641	775895.097	.....Ans.
Check: Q1 + Q2 + Q3 = 0	SumQ1Q2Q3 =		0.0	...Verified.

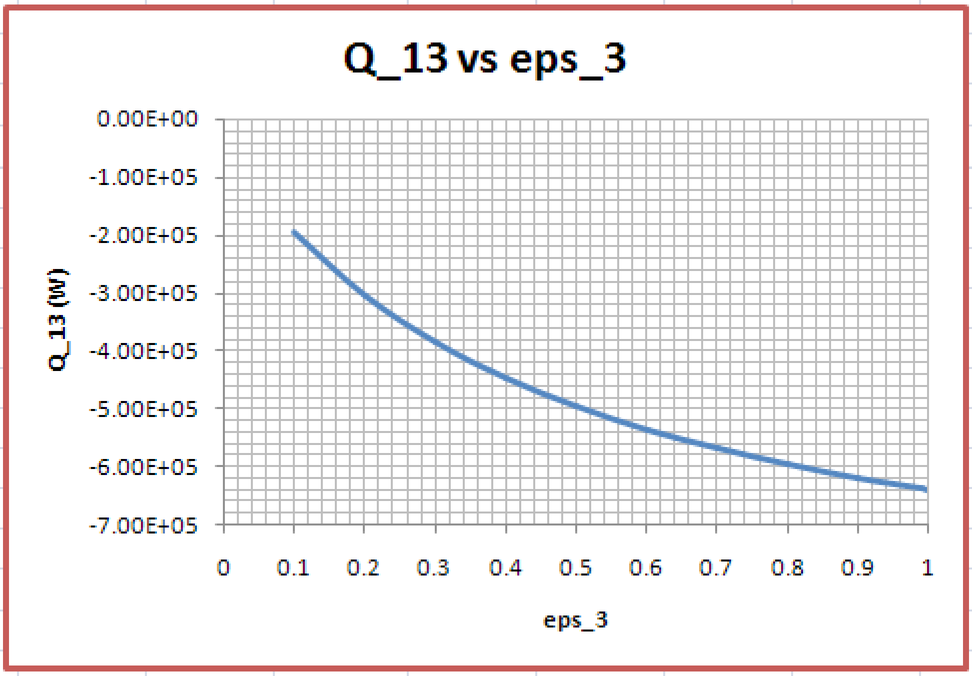
Click to
Apply Solver to find:
J1, J2 and J3

And, the Table gets filled up:

eps_3	Q_3 (W)	Q_13 (W)	SumQ1Q2Q3
0.1	203869.841	-196129.975	0
0.2	345299.485	-306181.229	1.04774E-09
0.3	449165.229	-387002.724	0
0.4	528678.165	-448874.465	0
0.5	591504.376	-497761.693	0
0.6	642397.878	-537363.671	1.39698E-09
0.7	684463.429	-570096.318	0
0.8	719814.651	-597604.316	1.28057E-09
0.9	749940.300	-621046.116	-3.25963E-09
0.99	773483.421	-639365.825	3.48082E-08
0.9999	775895.097	-641242.433	1.96125E-06

Now, plot the results in EXCEL:





Note: negative sign for Q_13 indicates heat flowing *in to* the surface 1 from surface 3.

===

Prob. 5.C.2.14. Two parallel plates of size 1 m × 1 m are spaced 0.5 m apart and are located in a very large room, the walls of which are maintained at a temp of 27 C. One plate is maintained at a temp of 900 C and the other at 400 C. Their emissivities are 0.2 and 0.5 respectively. If the plates exchange heat between themselves and surroundings, find the net heat transfer to each plate and to the room. Consider only the plate surfaces facing each other. [VTU – M.Tech. – Dec. 2009–Jan. 2010]

(b) In addition, plot Q1 and Q12 against ε1, for 0.1 < ε1 < 0.9.

This is the same as Prob. 5.C.2.4. which was solved with Mathcad earlier.

Now, let us solve it with EXCEL:

Use the Template prepared in the previous Problem.

Use A3 = Area of the room as very large, say 1E06 m^2, and the emissivity as 0.99 (black body).

Next, In the EXCEL worksheet, enter data for the three surface enclosure problem, and name the cells:

	A	B	C	D	E
76		**Data:**			
77					
78		X =	1	m	
79		Y =	1	m	
80		L =	0.5	m	
81		sigma (W/m2.K^4):	5.67E-08	W/m^2-K^4	
82					
83		Areas (m2):	A_1	A_2	A_3
84			1	1	1.00E+06
85					
86		Emissivities:	eps_1	eps_2	eps_3
87			0.2	0.5	0.99
88					
89		View Factors:	F_11	F_12	F_13
90			0	0.4152533	0.5847467
91			F_21	F_22	F_23
92			0.4152533	0	0.5847467
93			F_31	F_32	F_33
94			5.84747E-07	5.84747E-07	0.999998831

In the above, A_3 is the area of the room, taken as 1E06 m^2 (i.e. very large).

View Factors:

View factor F12 is determined with the template explained earlier, using the VBA Function for View Factor of parallel plates. See the Formula bar below:

=F_ij_Aligned_parallel_rectangles(I77/I79,I78/I79)

E	F	G	H	I	J	K
			(a) Aligned, parallel rectangles:			
			X =	1		
			Y =	1		
			L =	0.5		
			F_ij =	0.4152533		

This value of F_ij is transferred to cell D90 above. Other View factors are calculated using the View factor algebra:

Next, enter the Temp values, and continue the calculations:

R_1 | fx =(1-eps_1)/(A_1*eps_1)

	A	B	C	D	E
96					
97					
98		Temps. (K)	T_1	T_2	T_3
99			1173	673	300
100					
101		E_b (W/m2)	E_b1	E_b2	E_b3
102			1.07E+05	1.16E+04	4.59E+02
103					
104		Resistances(m^-2):	R_1	R_2	R_3
105			4	1	1.01E-08
106					
107			R_12	R_13	R_23
108			2.408168701	1.710142186	1.710142186

In the above, Emissive powers E_b1 etc and Resistances are calculated:

Next, important step to calculate Radiosities J1, J2 and J3:

C114 | fx =(E_b1-J_1)/R_1+(J_2-J_1)/R_12+(J_3-J_1)/R_13

	A	B	C	D	E	F
109						
110		Radiosities (W/m2):	J_1	J_2	J_3	
111			1000	1000	459	.....Ans.
112						
113		Apply Kirchoff's Law at each Node:				
114		Node J1:	26269.48723			
115		Node J2:	10315.35534			
116		Node J3:	26730632.7			
117		Sum_diff^2=	7.14528E+14			
118						
119		Apply Solver to make Sumdiff^2 a minimum, by varying J_1, J_2, and J_3:				

In the above, first, put guess values of 1000 each for J1, J2 and 459 for J3 (since Eb_3 = J3 for black body).

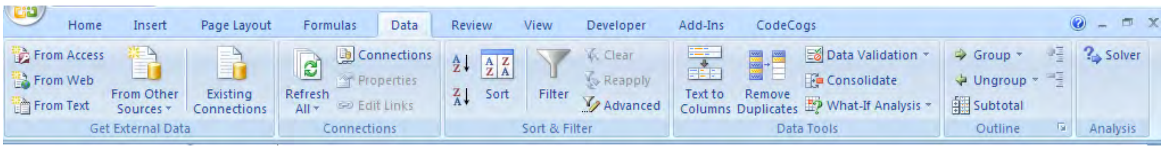
Then, for Nodes 1, 2 and 3 enter the corresponding equations, obtained by applying Kirchoff's Law, given earlier.

See the Formula bar in the above screen shot for formula entered for Node 1, in cell C114. Similarly, formulas are entered for Nodes 2 and 3 in cells C115 and C116.

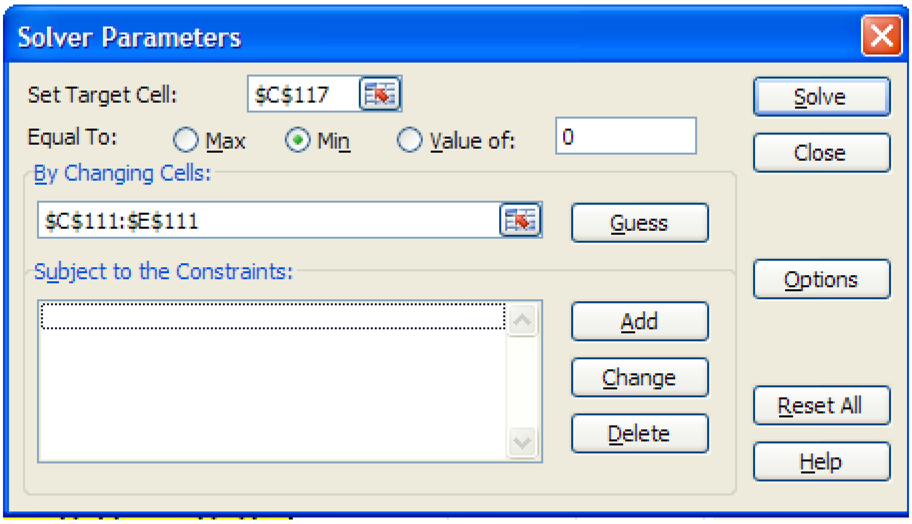
Sum of the squares of C114 to C116 should be zero, but, in this case since J1, J2 and J3 are assumed values, the sum of squares is not zero.

Apply Solver to make C117 zero by changing J1, J2 and J3. (i.e. cells C11, D111 and E111.)

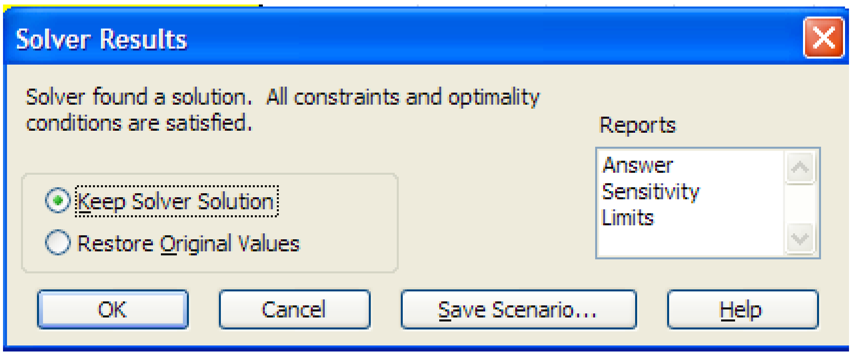
To do this: Go to Data – Solver:



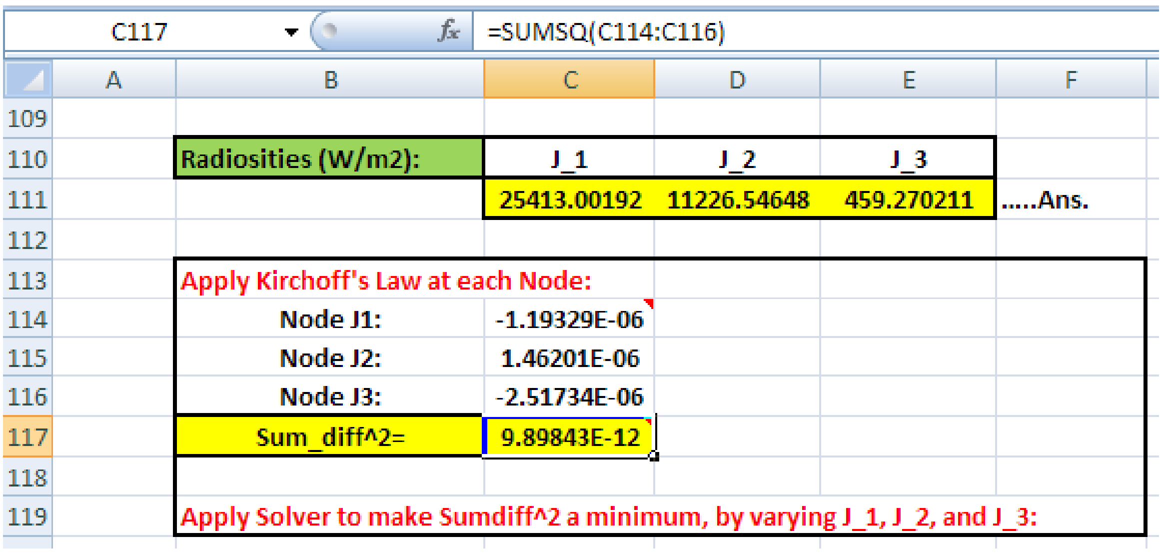
Click on Solver. We get the following window. Fill it up as shown:



Click Solve. We get following message:



Solver has found a solution. Click OK to keep the solution. We get:



	A	B	C	D	E	F
109						
110		Radiosities (W/m2):	J_1	J_2	J_3	
111			25413.00192	11226.54648	459.270211	.....Ans.
112						
113		Apply Kirchoff's Law at each Node:				
114		Node J1:	-1.19329E-06			
115		Node J2:	1.46201E-06			
116		Node J3:	-2.51734E-06			
117		Sum_diff^2=	9.89843E-12			
118						
119		Apply Solver to make Sumdiff^2 a minimum, by varying J_1, J_2, and J_3:				

Observe that values of J1, J2 and J3 are shown in the respective cells.

Sum_diff^2 is almost equal to zero.

And, results of equations at Nodes 1, 2 and 3 are almost equal to zero.

Next, continue the calculations for heat transfers:

C125 | f_x =(E_b1-J_1)/R_1

	A	B	C	D	E	F
112						
113		Apply Kirchoff's Law at each Node:				
114		Node J1:	-1.19329E-06			
115		Node J2:	1.46201E-06			
116		Node J3:	-2.51734E-06			
117		Sum_diff^2=	9.89843E-12			
118						
119		Apply Solver to make Sumdiff^2 a minimum, by varying J_1, J_2, and J_3:				
120						
121		Q_betwn_surfaces (W):	Q_12	Q_13	Q_23	
122			5890.97244	14591.61227	6296.129264	.....Ans.
123						
124		Qnet_from_surfaces (W):	Q_1	Q_2	Q_3	
125			20482.58471	405.1568258	-20887.74154	.....Ans.
126						
127		Check: Q1 + Q2 + Q3 = 0	SumQ1Q2Q3 =		0.0	...Verified.

In the above heat transfers from/to each surface (Q_1, Q_2 and Q_3), and also between surfaces (Q_12, Q_13, Q_23) are calculated using formulas given earlier. For ex. see in the Formula bar, the formula entered to calculate Q_1 in cell C125.

Negative sign for a heat transfer indicates heat coming ***in to*** the surface.

As a check: Sum of Q1, Q2 and Q3 should be zero.

Thus:

Q1 = 20482.58 W, Q2 = 405.16 W and Q3 = -20887.74 W … Ans.

Note: Results match with those obtained with Mathcad earlier.

(b) In addition, plot Q1 and Q12 against ε1, for 0.1 < ε1 < 0.9:

First, set up a Table as follows:

D143

	A	B	C	D	E
130					
131		eps_1	Q_1 (W)	Q_12 (W)	SumQ1Q2Q3
132		0.1			
133		0.2			
134		0.3			
135		0.4			
136		0.5			
137		0.6			
138		0.7			
139		0.8			
140		0.9			

Now, write a VBA program to do the following:

Copy the first value of eps1 from this Table to cell C87 in the Data section.

Then, all other values except J1, J2, J3 and the heat transfers, will change.

Apply the Solver to get J1, J2 and J3.

Immediately all heat transfers will get up-dated.

Then, copy Q_1, Q_12 and also SumQ1Q2Q3 (to check that it is zero) to the respective places in the Table.

Repeat these steps for the next value of eps1, etc.

Following is the VBA program, operated by a control button:

```
Sub Macro1()
'
' Macro1 Macro
' Finds J1, J2 and J3 making Sumdiff^2 minimum
'
' Keyboard Shortcut: Ctrl+Shift+J

Dim i As Integer

For i = 0 To 8 'start of For ... Next loop

    Range("C87") = Cells(132 + i, 2) 'copy the first value of eps1 from the Table to cell C87

    'Following part of the code applies Solver to minimise cell C117 by changing cells C111:E111

    '------------------------------------------------

    SolverOk SetCell:="$C$117", MaxMinVal:=2, ValueOf:="0", ByChange:= _
        "$C$111:$E$111"
    SolverSolve UserFinish:=True
    SolverFinish KeepFinal:=1

    '-------------------------------------------------

    Cells(132 + i, 3) = Range("C125") 'copies up-dated value of Q_1 to its place in Table

    Cells(132 + i, 4) = Range("C122") 'copies up-dated value of Q_12 to its place in Table

    Cells(132 + i, 5) = Range("E127") 'copies up-dated value of SUMQ1Q2Q3 to its place in Table

 Next i

End Sub
```

Read the comments in the above program.

Now, press the command button and the Solver is applied for each value of eps1:

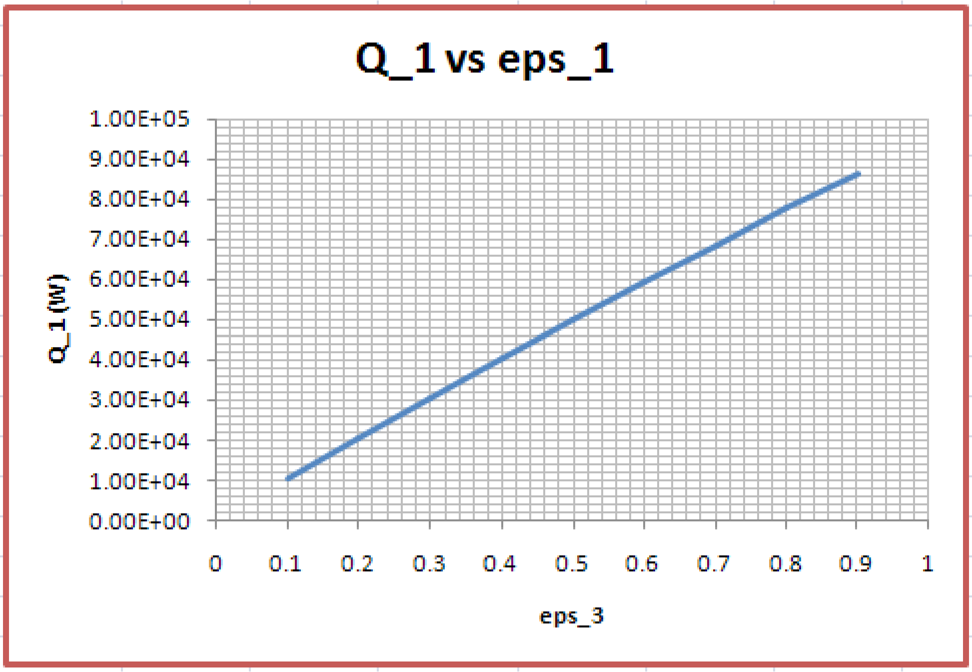
Apply Kirchoff's Law at each Node:				
Node J1:	-1.71713E-09			
Node J2:	-2.13731E-09			
Node J3:	-1.09903E-08			
Sum_diff^2=	1.28304E-16			
Apply Solver to make Sumdiff^2 a minimum, by varying J_1, J_2, and J_3:				
Q_betwn_surfaces (W):	Q_12	Q_13	Q_23	
	29684.3372	56875.95647	15075.40191	.....Ans.
Qnet_from_surfaces (W):	Q_1	Q_2	Q_3	
	86560.29367	-14608.9353	-71951.35838	.....Ans.
Check: Q1 + Q2 + Q3 = 0	SumQ1Q2Q3 =		0.0	...Verified.

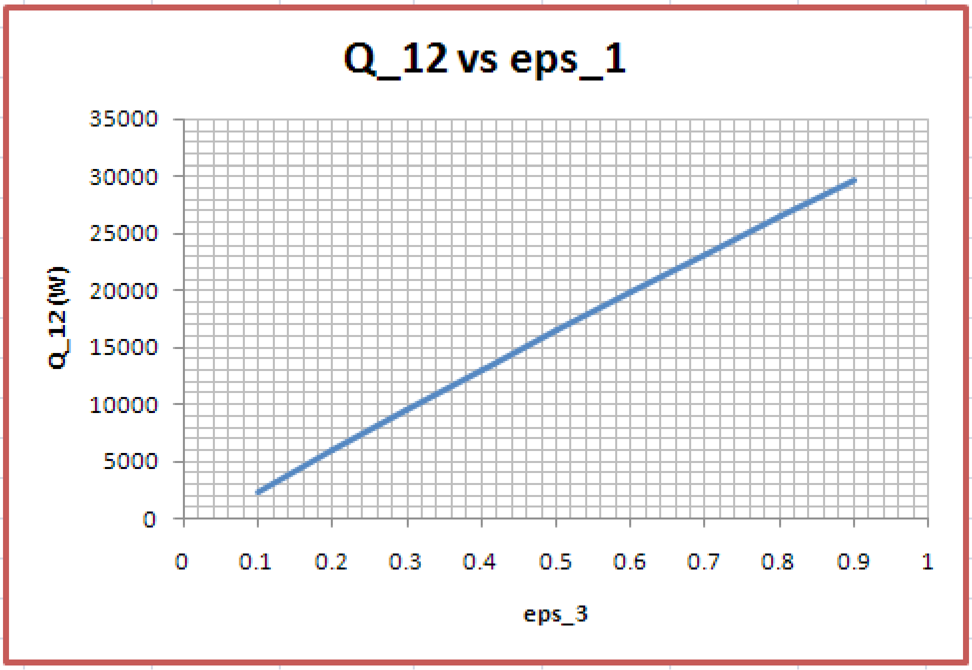
And, the Table gets filled up:

eps_1	Q_1 (W)	Q_12 (W)	SumQ1Q2Q3
0.1	10337.014	2237.752367	6.2355E-09
0.2	20482.572	5890.97896	1.437E-08
0.3	30441.943	9477.163321	1.1154E-08
0.4	40220.207	12998.13416	-1.01427E-08
0.5	49822.259	16455.65428	6.80302E-09
0.6	59252.819	19851.42351	-2.82307E-09
0.7	68516.441	23187.08151	6.0827E-09
0.8	77617.519	26464.21038	-1.47847E-08
0.9	86560.294	29684.3372	-1.4843E-08

Note that for each case, SumQ1Q2Q3 = 0 is satisfied.

Now, plot the results:





===

Prob. 5.C.2.15. A cylindrical shaped furnace is 1 m dia and 1.2 m high. The top surface, having an emissivity of 0.7 emits a uniform heat flux of 7 kW/m^2. The bottom surface with an emissivity of 0.4 is maintained at 350 K. The sides are insulated and function as reradiating surfaces. Determine the heat transfer to bottom surface and also the temperature of top and sides.

(b) Also, plot Q1, T1 and T3 against J1, as J1 varies from 300 to 1100 W/m^2, other conditions remaining the same.

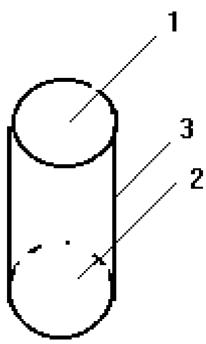


Fig.Prob.5.C.2.15 (a)

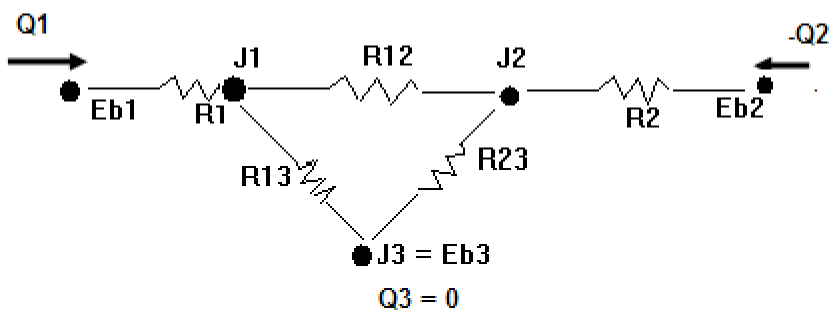


Fig.Prob.5.C.2.15 (b)

EXCEL Solution:

Note that in this case heat flux at the top is given and not its emissive power. So, the flux is to be taken as radiosity of the surface. i.e. J1 = 7000 W/m^2. The equivalent radiation circuit is shown in Fig. (b) above.

Recollect that the Kirchoff's eqns for a general three surface enclosure are:

Node J_1: $$\frac{E_{b1}-J_1}{R_1}+\frac{J_2-J_1}{R_{12}}+\frac{J_3-J_1}{R_{13}}=0 \qquad .....(13.63,a)$$

Node J_2: $$\frac{E_{b2}-J_2}{R_2}+\frac{J_1-J_2}{R_{12}}+\frac{J_3-J_2}{R_{23}}=0 \qquad .....(13.63,b)$$

Node J_3: $$\frac{E_{b3}-J_3}{R_3}+\frac{J_1-J_3}{R_{13}}+\frac{J_2-J_3}{R_{23}}=0 \qquad .....(13.63,c)$$

Solving these three eqns. simultaneously, we get J_1, J_2 and J_3.

However, in the above case, J1 is known, and Eb1 is to be found out.

Also, for Node J3, since Eb3 = J3, the first term in eqn for Node J3 will be zero. Therefore, eps3 does not enter in to calculations. Anyway, we enter eps3 = 0.99.

So, apply the Solver to solve for Eb1, J2 and J3 from the above three eqns; have a trial values for Eb1, J2 and J3 to start with, say equal to 1000 each.

Once Eb1 is obtained, get T1 from Eb1 = sigma * T1^4.

And, when J3 is known, get T3 from Eb3 = J3 = sigma * T3^4.

Use the Template prepared for the three surface enclosure.

Next, In the EXCEL worksheet, enter data for the three surface enclosure problem, and name the cells:

F_13 | =1-F_12

	A	B	C	D	E
76		**Data:**			
77					
78		R_1 =	0.5	m	
79		R_2 =	0.5	m	
80		L =	1.2	m	
81		sigma (W/m2.K^4):	5.67E-08	W/m^2-K^4	
82					
83		Areas (m2):	A_1	A_2	A_3
84			0.785398163	0.785398163	3.769911184
85					
86		Emissivities:	eps_1	eps_2	eps_3
87			0.7	0.4	0.99
88					
89		View Factors:	F_11	F_12	F_13
90			0	0.13108	0.86892
91			F_21	F_22	F_23
92			0.13108	0	0.86892
93			F_31	F_32	F_33
94			0.181025	0.181025	0.63795

In the above, A_3 is the area of the cylindrical surface = $2.\pi.R.L$

View Factors:

View factor F12 is determined with the template explained earlier, using the VBA Function for View Factor of parallel discs. See the Formula bar below:

fx =F_ij_Coaxial_parallel_discs(M79/M77,M78/M79)

L	M	N	O	P
(b) Coaxial, parallel disks:				
r_i =	0.5			
r_j =	0.5			
L =	1.2			
F_ij =	0.13108			

This value of F_ij is transferred to cell D90 above. Other View factors are calculated using the View factor algebra:

Next, enter the T2 = 350 K, For T1 and T3 enter the formulas and put Eb3 = J3.

For Eb1, J2, J3 enter trial values of say, 1000 each, and continue the calculations:

T_1 | fx | =(E_b1/sigma)^0.25

	A	B	C	D	E
97					
98		Temps. (K)	T_1	T_2	T_3
99			364.4217046	350	364.4217046
100					
101		E_b (W/m2)	E_b1	E_b2	E_b3
102			1.00E+03	8.51E+02	1.00E+03
103					
104		Resistances(m^-2):	R_1	R_2	R_3
105			0.545674091	1.909859317	2.68E-03
106					
107			R_12	R_13	R_23
108			9.713453957	1.465312738	1.465312738
109					
110		Radiosities (W/m2):	J_1	J_2	J_3
111			7000	1000	1000 ....

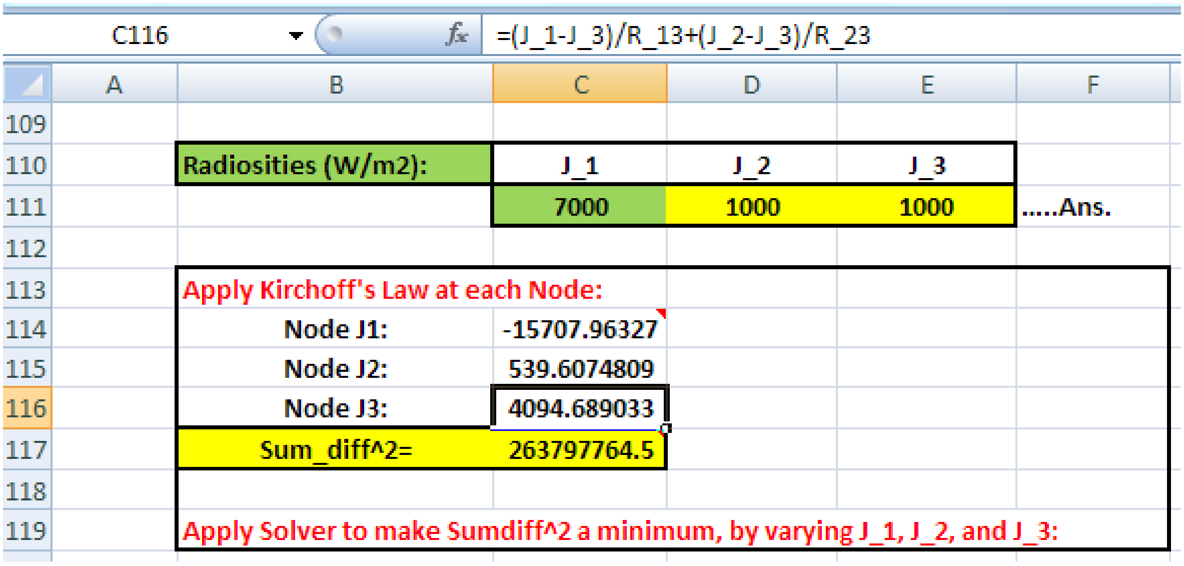
In the above, trial values for E_b1, J2, J3 are entered. T1, T3 in cells C99 and E99 are calculated as:

$$T1 := \left(\frac{Eb1}{\sigma}\right)^{0.25}$$

$$T3 := \left(\frac{J3}{\sigma}\right)^{0.25}$$

Next, important step to calculate Eb1, J2 and J3:

Enter the three Nodal equations as shown:


C116 f_x =(J_1-J_3)/R_13+(J_2-J_3)/R_23

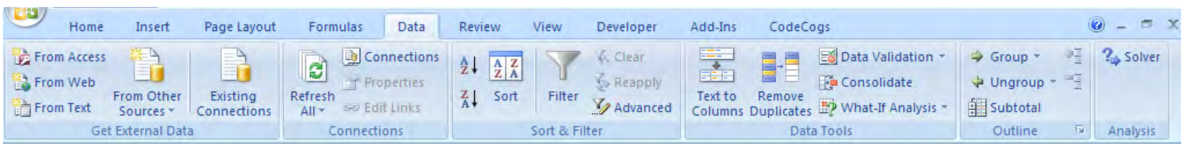
	A	B	C	D	E	F
109						
110		Radiosities (W/m2):	J_1	J_2	J_3	
111			7000	1000	1000	.....Ans.
112						
113		Apply Kirchoff's Law at each Node:				
114		Node J1:	-15707.96327			
115		Node J2:	539.6074809			
116		Node J3:	4094.689033			
117		Sum_diff^2=	263797764.5			
118						
119		Apply Solver to make Sumdiff^2 a minimum, by varying J_1, J_2, and J_3:				

For Nodes 1, 2 and 3 enter the corresponding equations, obtained by applying Kirchoff's Law, are entered. Eqn for Node J3 can be seen in the Formula bar above.

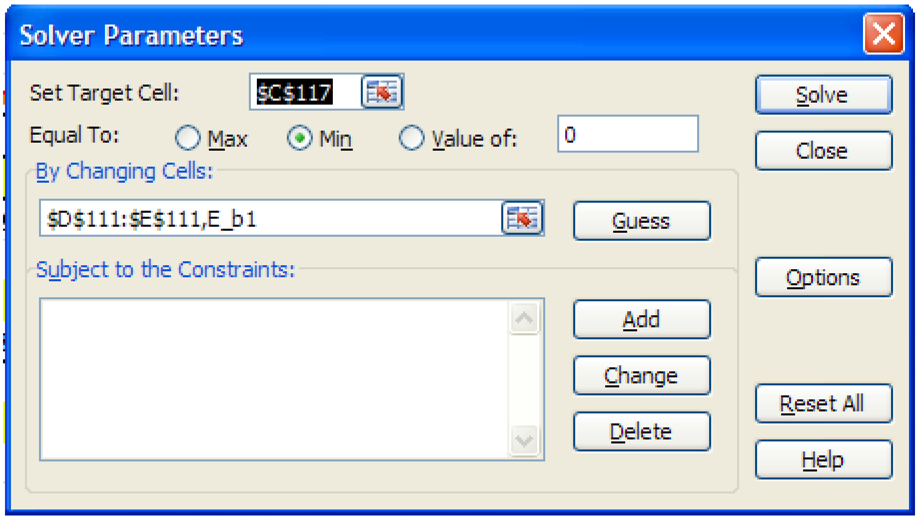
Sum of the squares of C114 to C116 should be zero, but, in this case since Eb1, J2 and J3 are assumed values, the sum of squares is not zero.

Apply Solver to make C117 zero by changing Eb1, J2 and J3. (i.e. cells C102, D111 and E111.)

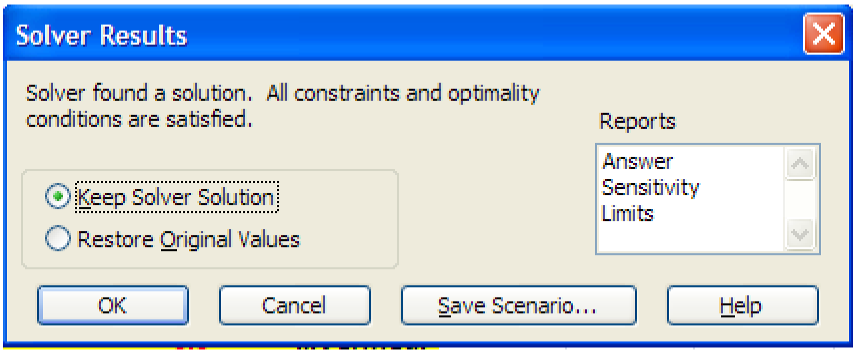
To do this: Go to Data – Solver:



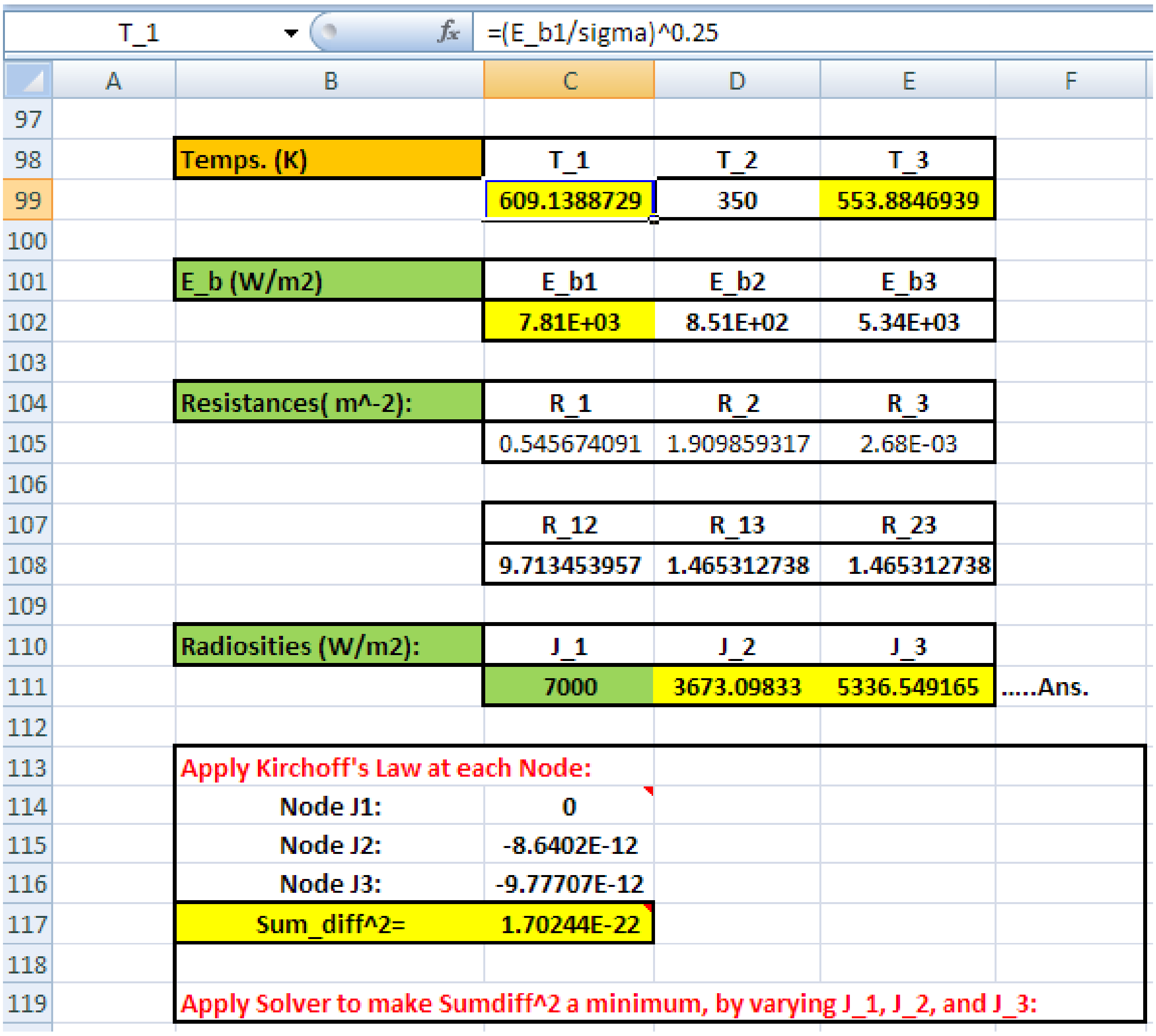
Click on Solver. We get the following window. Fill it up as shown:



Click Solve. We get following message:



Solver has found a solution. Click OK to keep the solution. We get:



T_1 =(E_b1/sigma)^0.25

	A	B	C	D	E	F
97						
98		Temps. (K)	T_1	T_2	T_3	
99			609.1388729	350	553.8846939	
100						
101		E_b (W/m2)	E_b1	E_b2	E_b3	
102			7.81E+03	8.51E+02	5.34E+03	
103						
104		Resistances(m^-2):	R_1	R_2	R_3	
105			0.545674091	1.909859317	2.68E-03	
106						
107			R_12	R_13	R_23	
108			9.713453957	1.465312738	1.465312738	
109						
110		Radiosities (W/m2):	J_1	J_2	J_3	
111			7000	3673.09833	5336.549165	.....Ans.
112						
113		Apply Kirchoff's Law at each Node:				
114		Node J1:	0			
115		Node J2:	-8.6402E-12			
116		Node J3:	-9.77707E-12			
117		Sum_diff^2=	1.70244E-22			
118						
119		Apply Solver to make Sumdiff^2 a minimum, by varying J_1, J_2, and J_3:				

Observe that values of Eb1, J2 and J3 are shown in the respective cells.

Also, note that temps T1 and T3 are calculated.

Sum_diff^2 is almost equal to zero.

And, results of equations at Nodes 1, 2 and 3 are almost equal to zero.

Next, continue the calculations for heat transfers:

C125 | f_x =(E_b1-J_1)/R_1

	A	B	C	D	E	F
112						
113		Apply Kirchoff's Law at each Node:				
114		Node J1:	0			
115		Node J2:	-8.6402E-12			
116		Node J3:	-9.77707E-12			
117		Sum_diff^2=	1.70244E-22			
118						
119		Apply Solver to make Sumdiff^2 a minimum, by varying J_1, J_2, and J_3:				
120						
121		Q_betwn_surfaces (W):	Q_12	Q_13	Q_23	
122			342.5044978	1135.218982	-1135.218982	.....Ans.
123						
124		Qnet_from_surfaces (W):	Q_1	Q_2	Q_3	
125			1477.723479	-1477.72348	0	.....Ans.
126						
127		Check: Q1 + Q2 + Q3 = 0	SumQ1Q2Q3 =		0.0	...Verified.

In the above, heat transfers from/to each surface (Q_1, Q_2 and Q_3), and also between surfaces (Q_12, Q_13, Q_23) are calculated using formulas given earlier. For ex. see in the Formula bar, the formula entered to calculate Q_1 in cell C125.

Negative sign for a heat transfer indicates heat coming ***in to*** the surface.

As a check: Sum of Q1, Q2 and Q3 should be zero.

Thus:

Q1 = 1477.72 W, Q2 = -1477.72 W , Eb1 = 7810 W/m^2, T1 = 609.14 K, T3 = 553.88 K … Ans.

(b) Plot Q1, T1 and T3 against J1, as J1 varies from 3000 to 11000 W/m^2:

First, set up a Table as follows:

	A	B	C	D	E	F
130						
131		J1 (W/m^2)	Q_1 (W)	T1 (K)	T3 (K)	SumQ1Q2Q3
132		3000				
133		4000				
134		5000				
135		6000				
136		7000				
137		8000				
138		9000				
139		10000				
140		11000				

Now, write a VBA program to do the following:

Copy the first value of J1 from this Table to cell C111 in the Data section.

Then, all related values will change.

Apply the Solver to get Eb1, J2 and J3.

Immediately T1, T3 and all heat transfers will get up-dated.

Then, copy Q_1, T_1, T_3 and also SumQ1Q2Q3 (to check that it is zero) to the respective places in the Table.

Repeat these steps for the next value of J1, etc.

Following is the VBA program, operated by a control button:

```
' Macro1 Macro
' Finds J1, J2 and J3 making Sumdiff^2 minimum
'
' Keyboard Shortcut: Ctrl+Shift+J

Dim i As Integer

For i = 0 To 8 'start of For ... Next loop

    Range("C111") = Cells(132 + i, 2) 'copy the first value of J1 from the Table to cell C111

    'Following part of the code applies Solver to minimise cell C117 by changing cells D111:E111
    'and cell C102

    '-----------------------------------------------

    SolverOk SetCell:="$C$117", MaxMinVal:=2, ValueOf:="0", ByChange:= _
        "$D$111:$E$111, $C$102"
    SolverSolve UserFinish:=True
    SolverFinish KeepFinal:=1

    '------------------------------------------------

    Cells(132 + i, 3) = Range("C125") 'copies up-dated value of Q_1 to its place in Table

    Cells(132 + i, 4) = Range("C99") 'copies up-dated value of T1 to its place in Table

    Cells(132 + i, 5) = Range("E99") 'copies up-dated value of T3 to its place in Table

    Cells(132 + i, 6) = Range("E127") 'copies up-dated value of SUMQ1Q2Q3 to its place in Table

 Next i

End Sub
```

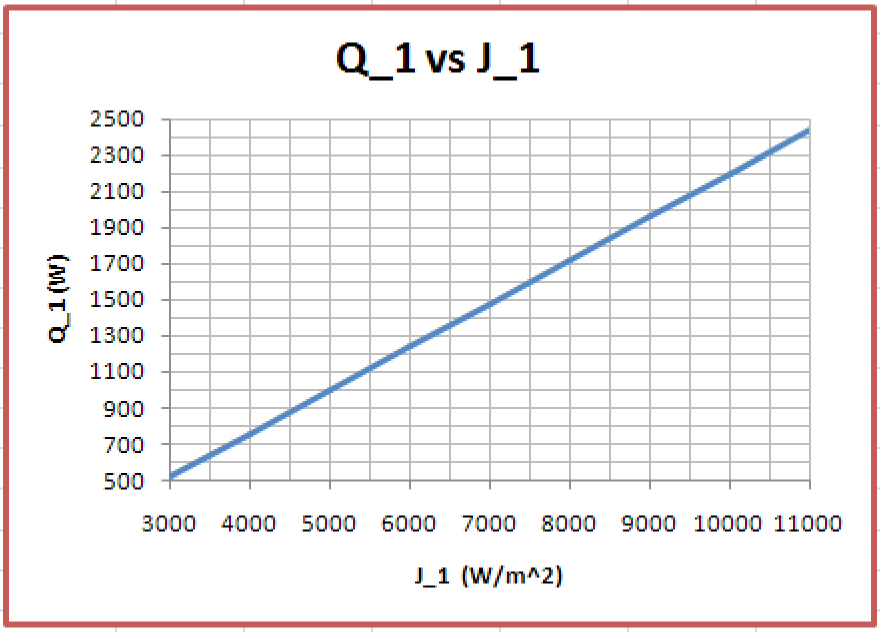
Read the comments in the above program.

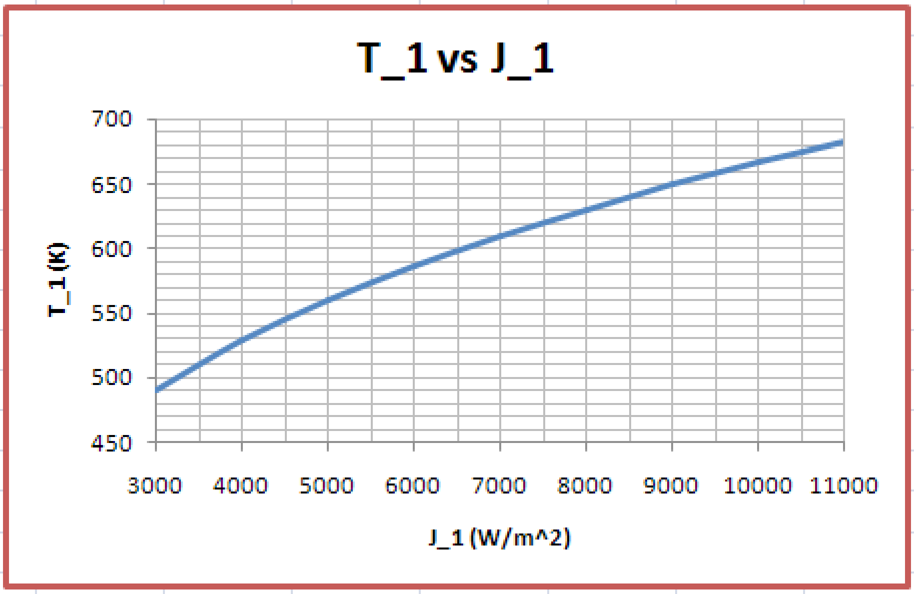
Now, press the command button and the Solver is applied for each value of J1 and the Table gets filled up:

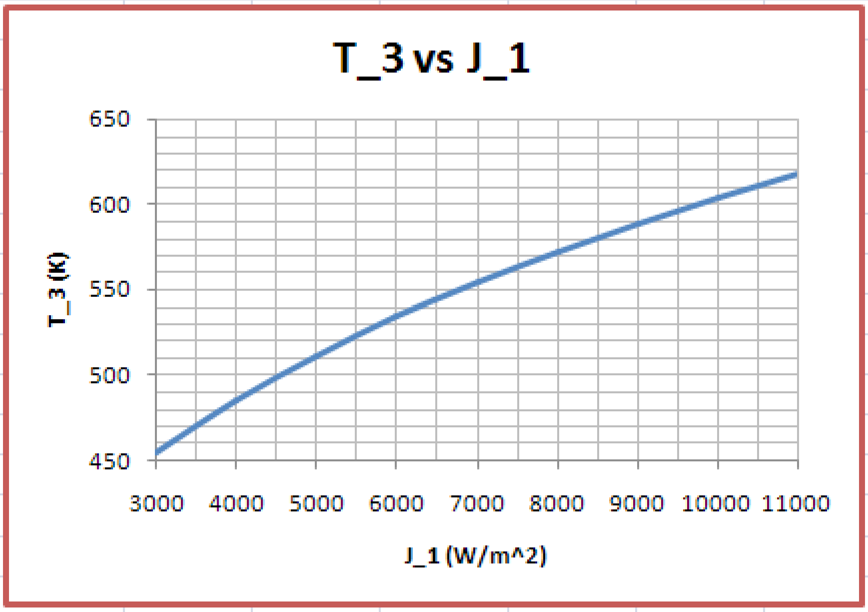
	A	B	C	D	E	F
130						
131		J1 (W/m^2)	Q_1 (W)	T1 (K)	T3 (K)	SumQ1Q2Q3
132		3000	516.469	490.493	454.460	1.14824E-11
133		4000	756.783	528.186	485.419	-6.82121E-13
134		5000	997.096	559.193	511.381	-7.95808E-13
135		6000	1237.410	585.761	533.901	2.04636E-12
136		7000	1477.723	609.139	553.885	-2.50111E-12
137		8000	1718.037	630.098	571.914	2.27374E-13
138		9000	1958.351	649.153	588.383	6.82121E-13
139		10000	2198.664	666.663	603.574	3.18323E-12
140		11000	2438.978	682.893	617.699	-4.54747E-13

Note that in each case, SumQ1Q2Q3 = 0 is satisfied.

Now, plot the results:







==

Prob. 5.C.2.16. Consider a cubical furnace, 1 m × 1 m × 1 m size. One of the vertical surfaces is at T1 = 1000 K, eps1 = 0.8, and the base is at 400 K, eps2 = 0.4. All other sides and top surface are insulated and can be considered as re-radiating. Calculate the net heat transfer from the surface 1, and the equilibrium temp of the re-radiating surfaces.

(b) Also, plot Q1 and T3 as T1 varies from 600 K to 1400 K.

EXCEL Solution:

This problem is similar to Prob.5.C.2.15.

The schematic and the radiation circuit are shown below:

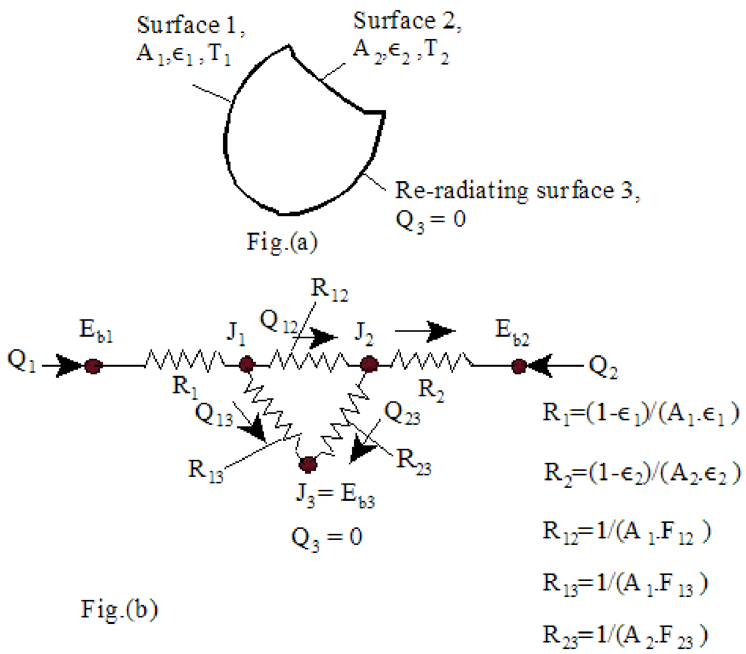


Fig.Prob.5.C.2.16

Now, instead of solving it with the template prepared earlier, **let us write VBA Functions for Q1 and temp of re-radiating surface, TR.**

Recollect that the formulas for this case are:

To get Q1:

$$Q_1 = -Q_2 = \frac{E_{b1} - E_{b2}}{R_{tot}}$$

where, R_{tot} is the total resistance, given by:

$$R_{tot} = R_1 + \left[\frac{1}{\frac{1}{R_{12}} + \frac{1}{(R_{13} + R_{23})}}\right] + R_2$$

i.e. $$R_{tot} = \left(\frac{1-\varepsilon_1}{A_1 \cdot \varepsilon_1}\right) + \frac{1}{\left[A_1 \cdot F_{12} + \frac{1}{\left(\frac{1}{A_1 \cdot F_{13}} + \frac{1}{A_2 \cdot F_{23}}\right)}\right]} + \left(\frac{1-\varepsilon_2}{A_2 \cdot \varepsilon_2}\right) \quad \text{....(13.67)}$$

To get TR, first get JR by applying Kirchoff's Law to Node J3:

Applying Kirchoff's Law at J_3:

$$\frac{J1 - JR}{\frac{1}{A1 \cdot F1R}} = -\frac{J2 - JR}{\frac{1}{A2 \cdot F2R}}$$

i.e. $$JR \cdot (A1 \cdot F1R + A2 \cdot F2R) = J1 \cdot A1 \cdot F1R + J2 \cdot A2 \cdot F2R$$

i.e. $$JR = \frac{J1 \cdot A1 \cdot F1R + J2 \cdot A2 \cdot F2R}{(A1 \cdot F1R + A2 \cdot F2R)}$$

Then: $$TR = \left(\frac{JR}{\sigma}\right)^{0.25}$$

where σ = Stefan Boltzmann constant.

Following is the VBA Functions to calculate Q1 and TR, using the above equations:

To determine heat transfer from surface 1, Q1:

```
Function Q_1_ThreeZoneEnclosure_with_ReradiatingSurface(A_1 As Double, A_2 As Double, _
F_12 As Double, F_1R As Double, F_2R As Double, eps_1 As Double, eps_2 As Double, _
T_1 As Double, T_2 As Double) As Double

'Gives Q_1 = -Q_2 (W) for the two grey surfaces,Q_R = 0 for re-radiating surface

Dim sigma As Double, R_1 As Double, R_2 As Double, XX As Double, YY As Double
Dim Eb_1 As Double, Eb_2 As Double

sigma = 0.0000000567 'W/m2.K^4
Eb_1 = sigma * T_1 ^ 4 'W/m^2, Emissive power of surface 1
Eb_2 = sigma * T_2 ^ 4 'W/m^2, Emissive power of surface 2

R_1 = (1 - eps_1) / (eps_1 * A_1) 'surface resistance of surface 1
R_2 = (1 - eps_2) / (eps_2 * A_2) 'surface resistance of surface 2

XX = ((1 / (A_1 * F_1R)) + (1 / (A_2 * F_2R))) ^ (-1)
YY = 1 / (A_1 * F_12 + XX)

 Q_1_ThreeZoneEnclosure_with_ReradiatingSurface = (Eb_1 - Eb_2) / (R_1 + YY + R_2)
End Function
```

To determine temp of re-radiating surface, TR:

```
Function T_R_ThreeZoneEnclosure_with_ReradiatingSurface(A_1 As Double, A_2 As Double, _
F_12 As Double, F_1R As Double, F_2R As Double, eps_1 As Double, eps_2 As Double, _
T_1 As Double, T_2 As Double) As Double
'Gives T_R (K) ..temp of re-radiating surface

Dim sigma As Double, R_1 As Double, R_2 As Double, XX As Double, YY As Double
Dim Eb_1 As Double, Eb_2 As Double, Q_1 As Double, Q_2 As Double
Dim J_1 As Double, J_2 As Double, J_R As Double

sigma = 0.0000000567 'W/m2.K^4
Eb_1 = sigma * T_1 ^ 4 'W/m^2, Emissive power of surface 1
Eb_2 = sigma * T_2 ^ 4 'W/m^2, Emissive power of surface 2
R_1 = (1 - eps_1) / (eps_1 * A_1) 'surface resistance of surface 1
R_2 = (1 - eps_2) / (eps_2 * A_2) 'surface resistance of surface 2
Q_1 = Q_1_ThreeZoneEnclosure_with_ReradiatingSurface(A_1, A_2, F_12, F_1R, F_2R, eps_1, _
eps_2, T_1, T_2) 'W/m2
Q_2 = -Q_1 '...by heat balance
J_1 = Eb_1 - Q_1 * R_1 'W/m2, Radiosity of surface 1
J_2 = Eb_2 - Q_2 * R_2 'W/m2, Radiosity of surface 2

'Apply Kirchoff's Law at J_3:

'(J_1-J_R)/((1/(A_1*F_1R))) = -(J_2-J_R)/((1/(A_2*F_2R)))
'J_R*(A_1*F_1R+A_2*F_2R) = J_1*A_1*F_1R+J_2*A_2*F_2R

'Then, Radiosity of re-radiating surface 3 i.e. JR is given by:

J_R = (J_1 * A_1 * F_1R + J_2 * A_2 * F_2R) / (A_1 * F_1R + A_2 * F_2R)

T_R_ThreeZoneEnclosure_with_ReradiatingSurface = (J_R / sigma) ^ 0.25

End Function
```

Now, let us solve the above problem using these VBA Functions:

First, use the template to determine the View Factors for perpendicular rectangles:

fx =F_ij_Perpendicular_rectangles(I85/I83,I84/I83)

H	I	J	K	L	M
(c) Perpendicular rectangles with a common edge:					
X =	1				
Y =	1				
Z =	1				
F_ij =	0.2000438				

Thus, F12 = 0.2 and the other View Factors are determined using View Factor algebra, as explained earlier.

Now, set up the EXCEL worksheet, enter data and name the cells.

F_21 | fx | =F_12*A_1/A_2

	A	B	C	D	E
76		**Data:**			
77					
78		X =	1	m	
79		Y =	1	m	
80		Z =	1	m	
81		sigma (W/m2.K^4):	5.67E-08	W/m^2-K^4	
82					
83		Areas (m2):	A_1	A_2	A_3
84			1	1	4.00E+00
85					
86		Emissivities:	eps_1	eps_2	eps_3
87			0.8	0.4	0.999
88					
89		View Factors:	F_11	F_12	F_13
90			0	0.2	0.8
91			F_21	F_22	F_23
92			0.2	0	0.8
93			F_31	F_32	F_33
94			0.2	0.2	0.6

Note that eps3 does not enter into any calculations; so, we just fill up eps_3 = 0.999

Next, fill up the temperatures T1 and T2, given in data. T3 (i.e. temp of reradiating surface is to be found out later, after finding out J3).

	A	B	C	D
97				
98		Temps. (K)	T_1	T_2
99			1000	400

Now, to get Q1 and T3:

Use the VBA Functions written above:

H93 =Q_1_ThreeZoneEnclosure_with_ReradiatingSurface(A_1,A_2,F_12,F_13,F_23,eps_1,eps_2,T_

	A	B	C	D	E	F	G	H	I
85								Z =	1
86		Emissivities:	eps_1	eps_2	eps_3			F_ij =	0.2000438
87			0.8	0.4	0.999				
88									
89		View Factors:	F_11	F_12	F_13				
90			0	0.2	0.8				
91			F_21	F_22	F_23		Results:		
92			0.2	0	0.8				
93			F_31	F_32	F_33		Q1	16170.29	W .... Ans.
94			0.2	0.2	0.6		TR	911.7513	K...Ans.

Note that the Function for Q1 can be seen in the Formula bar above.

Similarly, Function for T3 can be seen in the Formula bar in the screen shot given below:

H94 =T_R_ThreeZoneEnclosure_with_ReradiatingSurface(A_1,A_2,F_12,F_13,F_23,eps_1,eps_2,T_1,T_2)

	A	B	C	D	E	F	G	H	I	J
91			F_21	F_22	F_23		Results:			
92			0.2	0	0.8					
93			F_31	F_32	F_33		Q1	16170.29	W .... Ans.	
94			0.2	0.2	0.6		TR	911.7513	K...Ans.	

Thus:

Q1 = net heat transfer from surface 1 = 16170.29 W …. Ans.

TR = temp of re-radiating surfaces = 911.75 K … Ans.

(b) Also, plot Q1 and T3 as T1 varies from 600 K to 1400 K.

First, prepare a Table as shown below:

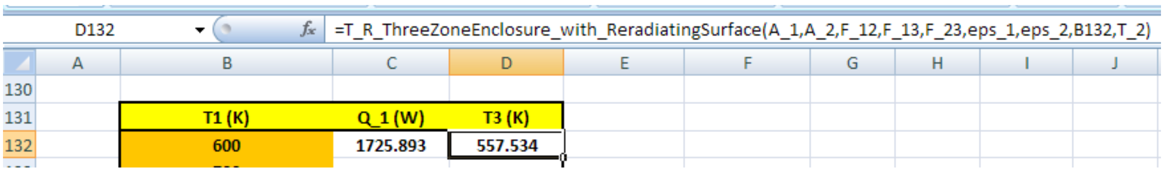
C132 =Q_1_ThreeZoneEnclosure_with_ReradiatingSurface(A_1,A_2,F_12,F_13,F_23,eps_1,eps_2,B132,T_2)

	A	B	C	D
130				
131		T1 (K)	Q_1 (W)	T3 (K)
132		600	1725.893	557.534
133		700		
134		800		
135		900		
136		1000		
137		1100		
138		1200		
139		1300		
140		1400		

Formula bar shows the Function entered for Q1 in cell C132.

Note that reference to T1 is by 'relative reference' so that we can 'drag-copy' till the end of Table.

Similarly, for T3 also, refer to T1 by relative reference, as shown below:

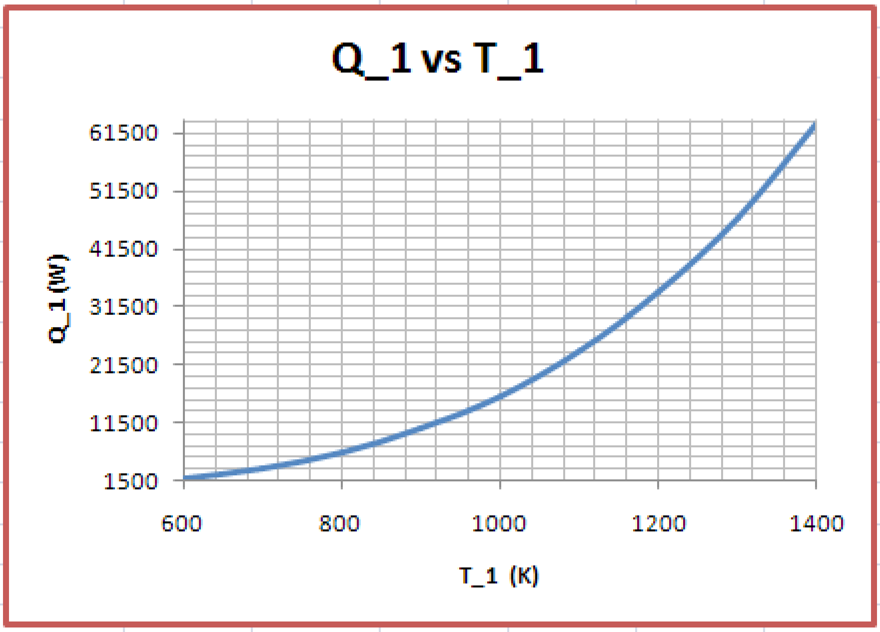


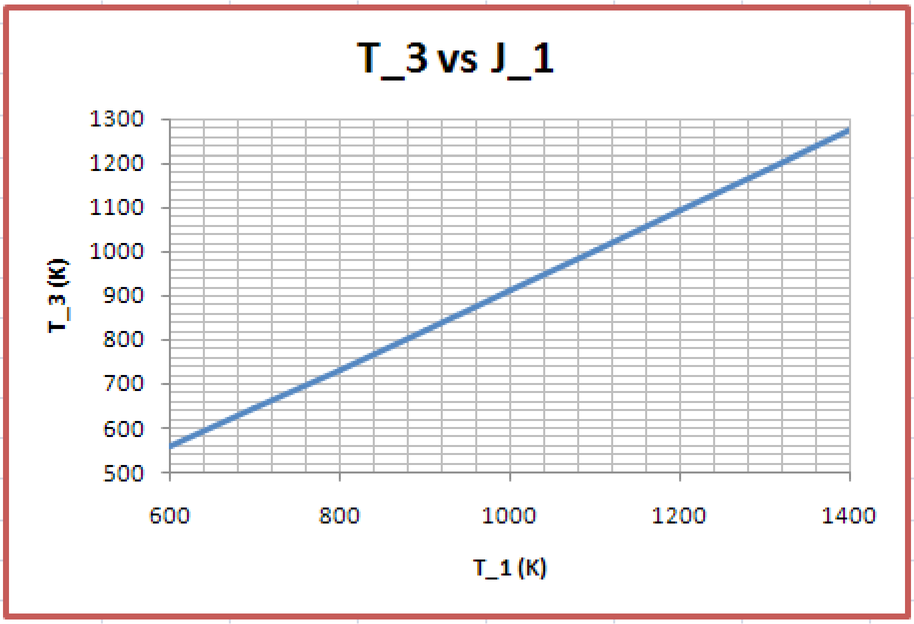
Now, select cells C132 to D132 and drag-copy till the end of the Table, i.e. up to cell D140.

Immediately, all calculations are made and the Table is filled up:

	A	B	C	D
130				
131		T1 (K)	Q_1 (W)	T3 (K)
132		600	1725.893	557.534
133		700	3559.654	644.077
134		800	6372.527	732.469
135		900	10463.224	821.836
136		1000	16170.287	911.751
137		1100	23872.083	1001.992
138		1200	33986.810	1092.434
139		1300	46972.493	1183.008
140		1400	63326.985	1273.670

Now, plot the results in EXCEL:





Next, consider what happens if surface 1 is black:

i.e. put eps1 = 0.999.

Immediately the results up date themselves in the worksheet:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
76		Data:						(a) Aligned, parallel rectangles:				(b) Coaxial, parallel disks:		
77								X =	1			r_i =	0.5	
78		X =	1	m				Y =	1			r_j =	0.5	
79		Y =	1	m				L =	0.5			L =	1.2	
80		Z =	1	m				F_ij =	0.415253			F_ij =	0.13108	
81		sigma (W/m2.K^4):	5.67E-08	W/m^2-K^4										
82								(c) Perpendicular rectangles with a common edge:						
83		Areas (m2):	A_1	A_2	A_3			X =	1					
84			1	1	4.00E+00			Y =	1					
85								Z =	1					
86		Emissivities:	eps_1	eps_2	eps_3			F_ij =	0.200044					
87			0.999	0.4	0.999									
88														
89		View Factors:	F_11	F_12	F_13									
90			0	0.2	0.8									
91			F_21	F_22	F_23		Results:							
92			0.2	0	0.8									
93			F_31	F_32	F_33		Q1	17441.4	W .... Ans.					
94			0.2	0.2	0.6		TR	928.536	K...Ans.					

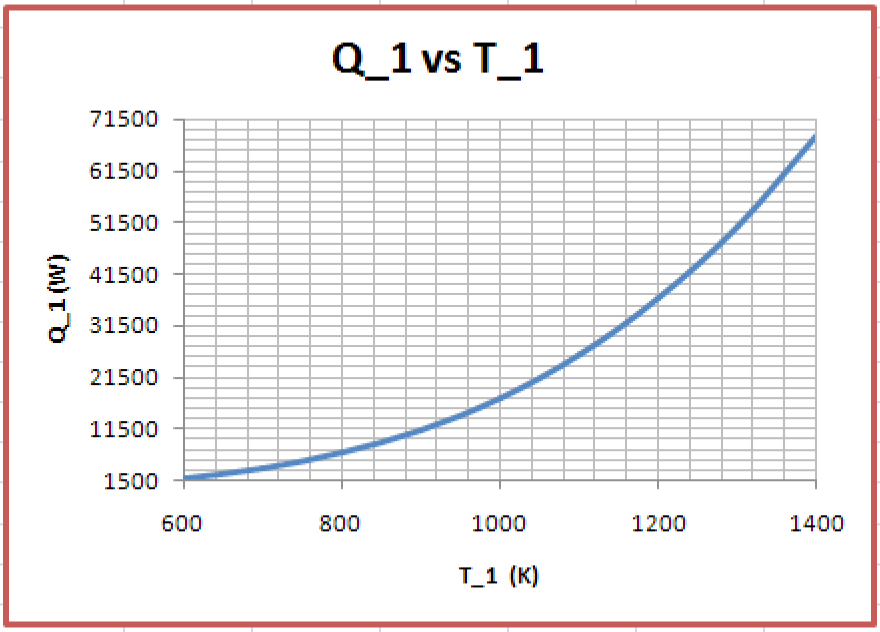
Now, Q1 = 17441.4 W, TR = 928.54 K … Ans.

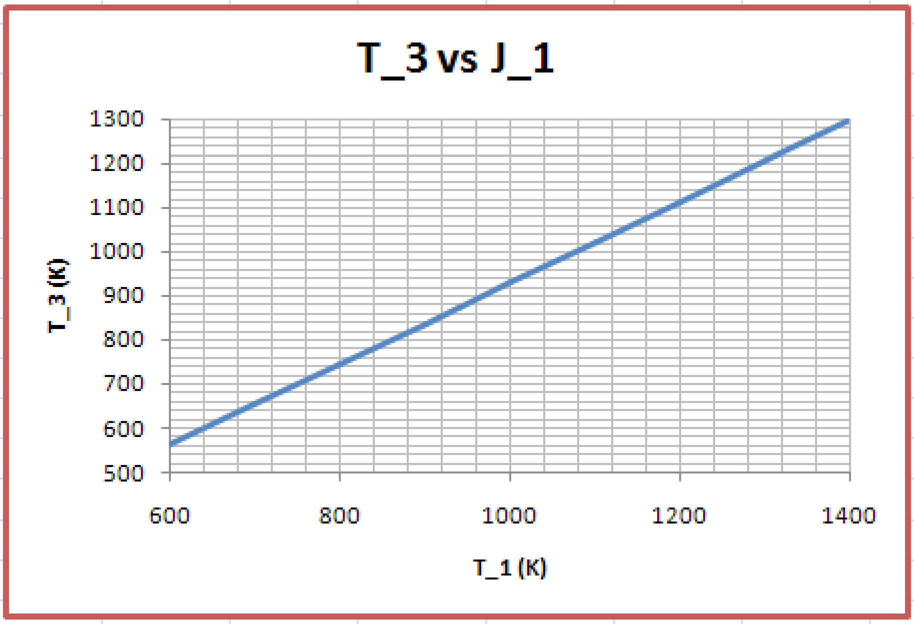
Compare these values with Q1 = 16170.29 W and TR = 911.75 K obtained with eps1 = 0.8 earlier.

Also, observe that the Table and plots have also updated themselves:

T1 (K)	Q_1 (W)	T3 (K)
600	1861.559	565.419
700	3839.465	654.591
800	6873.448	745.245
900	11285.701	836.674
1000	17441.375	928.536
1100	25748.582	1020.654
1200	36658.391	1112.932
1300	50664.832	1205.314
1400	68304.893	1297.764

And:




T_3 vs J_1
1300
1200
1100
1000
900
800
700
600
500
T_3 (K)
600
800
1000
1200
1400
T_1 (K)

===

5D Radiation shields

Recollect: [Ref: 1]

For two infinite, parallel plates 1 and 2, radiation shield 3, be placed between the plates:

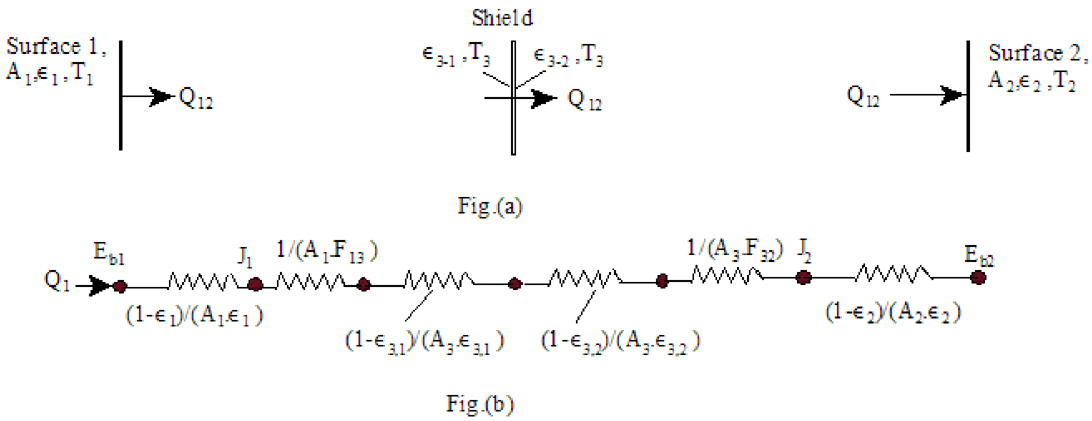


Fig.13.37 Radiation shield between two parallel plates, and associated radiation network

When there is no shield, the radiation heat transfer between plates 1 and 2 is already shown to be:

$$Q_{12} = \frac{A \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$ ....for infinitely large parallel plates.....(13.59)

$$Q_{12\_one\_shield} = \frac{A \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3\_1}} + \frac{1}{\varepsilon_{3\_2}} - 1\right)}$$ ....(13.71)

If there are N radiation shields, we have, for net radiation heat transfer:

$$Q_{12_{N\_shields}} = \frac{A \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon 3\_1} + \frac{1}{\varepsilon_{3\_2}} - 1\right) + ... + \left(\frac{1}{\varepsilon_{N\_1}} + \frac{1}{\varepsilon_{N\_2}} - 1\right)}$$ ...(13.72)

If emissivities of all surfaces are equal, eqn. (13.72) becomes:

$$Q_{12_{N\_shields}} = \frac{A \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{(N+1) \cdot \left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1\right)} = \frac{1}{(N+1)} \cdot \left(Q_{12_{no\_shield}}\right)$$ ....(13.73)

==

Prob.5.D.1. Write Mathcad Functions for: radiation heat transfer and temp of radiation shield for:

i) Heat transfer between two infinite, parallel plates with no radiation shield,
ii) Heat transfer between two infinite, parallel plates with ***one radiation shield*** in between,
iii) Heat transfer between two infinite, parallel plates with ***N radiation shields*** in between, and
iv) Temp of radiation shield for the case of one radiation shield between two parallel plates.

Mathcad Solution:

Inputs:

A … area of plates, m^2

eps1, eps2 …emissivities of two plates

eps_s1, eps_s2 .. emissivities of radiation shield, on the surfaces looking towards plates 1 and 2 respectively

T1, T2 … temps of two plates, Kelvin

Output:

Q12 … W

T_shield … Kelvin

1. **Infinite parallel plates with no shield:**

$$Q12\_plate\_no\_shield(A, eps1, eps2, T1, T2) := \frac{5.67 \cdot 10^{-8} \cdot A \cdot \left(T1^4 - T2^4\right)}{\left(\frac{1}{eps1} + \frac{1}{eps2} - 1\right)}$$

---

2. **Infinite parallel plates with one shield in between:**

$$Q12\_plate\_one\_shield(A, eps1, eps2, eps\_s1, eps\_s2, T1, T2) := \frac{5.67 \cdot 10^{-8} \cdot A \cdot \left(T1^4 - T2^4\right)}{\left(\frac{1}{eps1} + \frac{1}{eps2} - 1\right) + \left(\frac{1}{eps\_s1} + \frac{1}{eps\_s2} - 1\right)}$$

---

3. **To determine the equilibrium temperature of the radiation shield:**

We can use either of the conditions: Q12 = Q13 or Q12 = Q32.

$$Q_{12} = Q_{13} = \frac{A \cdot \sigma \cdot \left(T_1^4 - T_3^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} \qquad \ldots(13.76,a)$$

or,

$$Q_{12} = Q_{32} = \frac{A \cdot \sigma \cdot \left(T_3^4 - T_2^4\right)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1} \qquad \ldots.(13.76,b)$$

Function for temp of Shield:

$T3 := 500$ .K...trial value

Given

$$Q12\_plate\_one\_shield(A, eps1, eps2, eps\_s1, eps\_s2, T1, T2) = \frac{A \cdot 5.67 \cdot 10^{-8} \cdot \left(T1^4 - T3^4\right)}{\frac{1}{eps1} + \frac{1}{eps\_s1} - 1}$$

$$T\_shield(A, eps1, eps2, eps\_s1, eps\_s2, T1, T2) := Find(T3)$$

When there are N shields, all of emissivity eps_s1 and eps_s2 on either side, plates 1 and 2 having emissivities eps1 and eps2:

$$Q12\_plate\_N\_shields(N, A, eps1, eps2, eps\_s1, eps\_s2, T1, T2) := \frac{A \cdot 5.67 \cdot 10^{-8} \cdot \left(T1^4 - T2^4\right)}{\left(\frac{1}{eps1} + \frac{1}{eps2} - 1\right) + N \cdot \left(\frac{1}{eps\_s1} + \frac{1}{eps\_s2} - 1\right)}$$

Note: In the above Function, if we put N = 1, we get Q12 with one shield.

===

Prob.5.D.2. Two large plates at 800 K and 600 K have emissivities of 0.5 and 0.8 respectively. A radiation shield having emissivity of 0.1 on the surface facing 800 K plate and 0.05 on the surface facing 600 K plate is placed between the two plates in parallel direction with respect to the plates. Calculate:

1) The radiation heat transfer between the two plates in presence of the radiation shield
2) The equilibrium temp of the shield
3) The rate of heat transfer between the two plates without the presence of radiation shield [VTU – July–Aug. 2004]

(b) In addition, if the emissivity on the surface facing 800 K plate (i.e. eps_s1) varies from 0.05 to 0.35, plot the heat transfer between the two plates (Q12) and the temp of radiation shield.

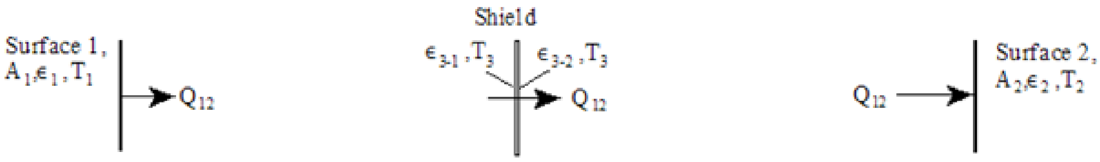


Fig.(a)

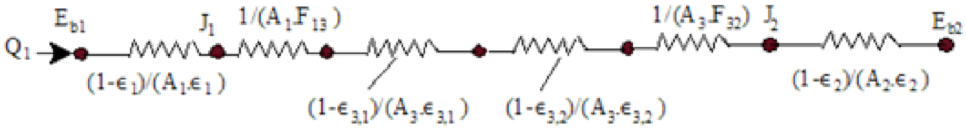


Fig.(b)

Mathcad Solution:

Data:

$A := 1$ m^2

$eps1 := 0.5$ $eps2 := 0.8$ $eps\_s1 := 0.1$ $eps\_s2 := 0.05$

$T1 := 800$ K $T2 := 600$ K

Calculations:

Use the Mathcad Functions written above:

Heat transfer with one shield:

$Q12\_plate\_one\_shield(A, eps1, eps2, eps\_s1, eps\_s2, T1, T2) = 508.032$ **W.....Ans.**

Temp of the radiation shield:

$T\_shield(A, eps1, eps2, eps\_s1, eps\_s2, T1, T2) = 746.8$ **K.....Ans.**

Heat transfer with no shield:

$Q12\_plate\_no\_shield(A, eps1, eps2, T1, T2) = 7.056 \times 10^{3}$ **W.....Ans.**

Note that it is very convenient to make these calculations quickly with Mathcad Functions.

(b) In addition, plot the heat transfer between the two plates (Q12) and the temp of radiation shield if the emissivity on the surface facing 800 K plate (i.e. eps_s1) varies from 0.05 to 0.35:

Write the relevant quantities as functions of eps_s1:

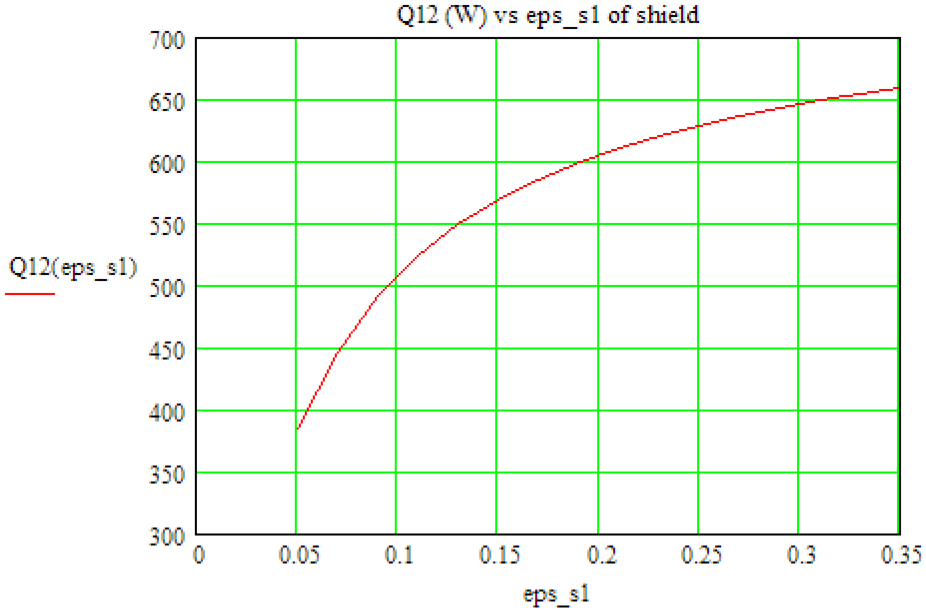
Q12(eps_s1) := Q12_plate_one_shield(A, eps1, eps2, eps_s1, eps_s2, T1, T2)

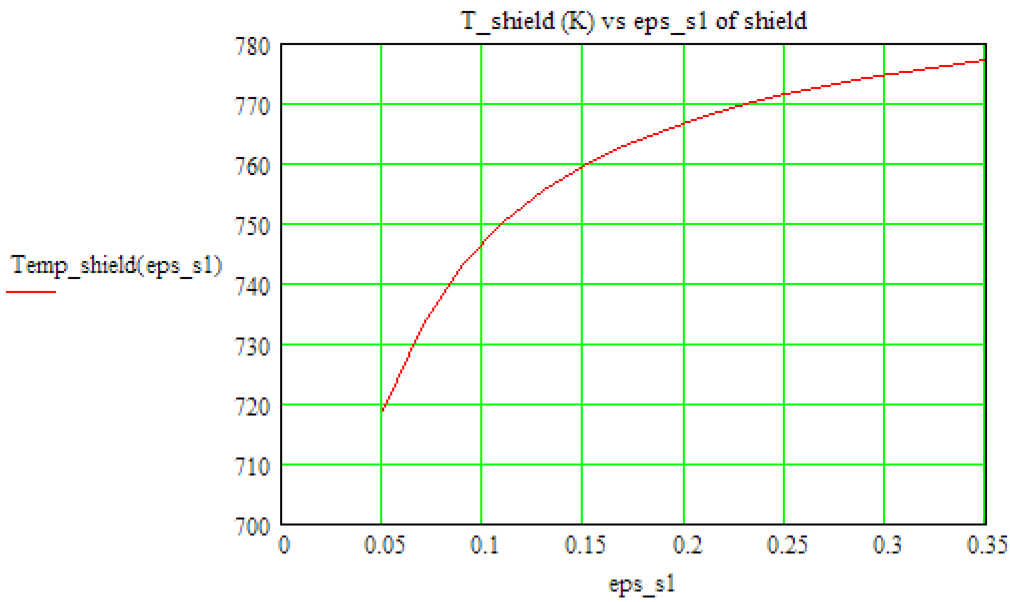
Temp_shield(eps_s1) := T_shield(A, eps1, eps2, eps_s1, eps_s2, T1, T2)

eps_s1 := 0.05, 0.07 .. 0.35 ....define a range variable

eps_s1 =	Q12(eps_s1) =	Temp_shield(eps_s1)
0.05	384.873	718.869
0.07	446.762	733.303
0.09	490.589	743.032
0.11	523.254	750.041
0.13	548.54	755.336
0.15	568.693	759.477
0.17	585.132	762.806
0.19	598.797	765.54
0.21	610.336	767.826
0.23	620.209	769.766
0.25	628.752	771.433
0.27	636.218	772.881
0.29	642.798	774.15
0.31	648.641	775.272
0.33	653.863	776.271
0.35	658.56	777.166

Now, plot the results:





==

Prob.5.D.3. Consider two large parallel plates, one at 1000 K with emissivity 0.8 and the other at 300 K having emissivity of 0.6. A radiation shield is placed between them. The shield has emissivity of 0.1 on the side facing hot plate and 0.3 on the surface facing cold plate. Calculate the percentage reduction in radiation heat transfer as a result of radiation shield. [VTU – May 2007]

Mathcad Solution:

Data:

$A := 1$ m^2

$eps1 := 0.8$ $eps2 := 0.6$ $eps\_s1 := 0.1$ $eps\_s2 := 0.3$

$T1 := 1000$ K $T2 := 300$ K

Calculations:

Use the Mathcad Functions written above:

Heat transfer with no shield:

$Q12\_no\_shield := Q12\_plate\_no\_shield(A, eps1, eps2, T1, T2)$

i.e. $Q12\_no\_shield = 2.934 \times 10^4$ **W....Ans.**

Heat transfer with one shield:

Q12_with_shield := Q12_plate_one_shield(A, eps1, eps2, eps_s1, eps_s2, T1, T2)

i.e. Q12_with_shield = 3.947×10^3 **W....Ans.**

Therefore:

$$\text{Percent\_reduction} := \frac{\text{Q12\_no\_shield} - \text{Q12\_with\_shield}}{\text{Q12\_no\_shield}} \cdot 100$$

i.e. Percent_reduction = 86.55 **%....percent reduction due to radiation shield ... Ans.**

==

Prob.5.D.4. Two large plates at 800 C and 300 C have emissivities of 0.3 and 0.5 respectively. A radiation shield having emissivity of 0.05 on both sides is placed between the two plates. Calculate:

1) The heat transfer between the two plates without radiation shield
2) The equilibrium temp of the shield
3) The rate of heat transfer between the two plates with the presence of radiation shield [VTU – June 2012]

Mathcad Solution:

Data:

A := 1 m^2

eps1 := 0.3 eps2 := 0.5 eps_s1 := 0.05 eps_s2 := 0.05

T1 := 1073 K T2 := 573 K

Calculations:

Use the Mathcad Functions written above:

Heat transfer with no shield:

Q12_plate_no_shield(A, eps1, eps2, T1, T2) = 1.593×10^4 **W.....Ans.**

Temp of the radiation shield:

T_shield(A, eps1, eps2, eps_s1, eps_s2, T1, T2) = 914.019 K.....Ans.

Heat transfer with one shield:

Q12_plate_one_shield(A, eps1, eps2, eps_s1, eps_s2, T1, T2) = 1.593×10^{3} W.....Ans.

(b) In addition, plot the heat transfer between the two plates (Q12) and the temp of radiation shield if the emissivities on either side of shield (i.e. eps_s1 and eps_s2) vary from 0.05 to 0.35:

Let eps_s = eps_s1 = eps_s2

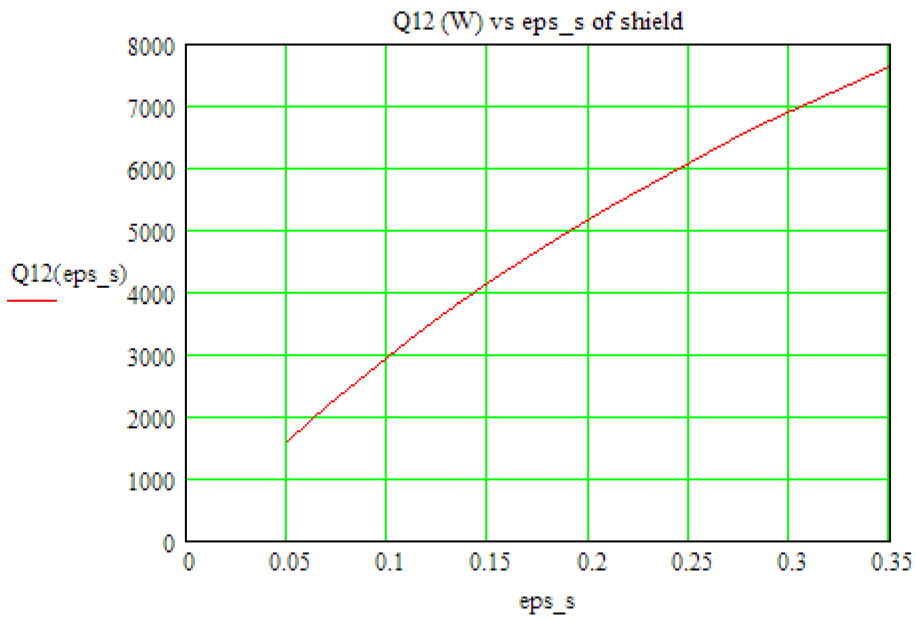
Write the relevant quantities as functions of eps_s:

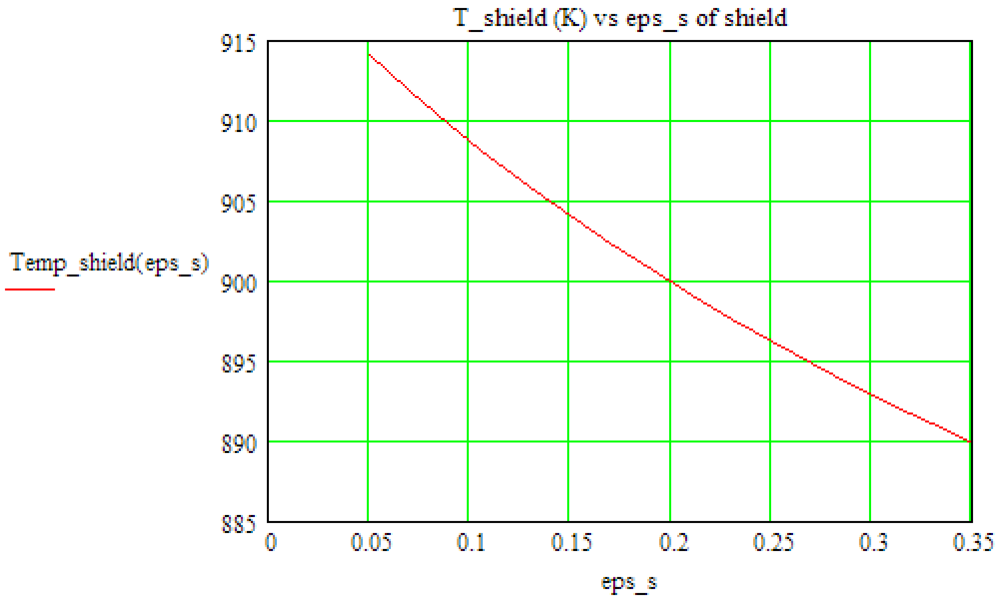
Q12(eps_s) := Q12_plate_one_shield(A, eps1, eps2, eps_s, eps_s, T1, T2)

Temp_shield(eps_s) := T_shield(A, eps1, eps2, eps_s, eps_s, T1, T2) eps_s := 0.05, 0.07 .. 0.35 ....define a range variable

eps_s =	Q12(eps_s) =	Temp_shield(eps_s)
0.05	$1.593\cdot10^3$	914.019
0.07	$2.164\cdot10^3$	911.814
0.09	$2.702\cdot10^3$	909.722
0.11	$3.209\cdot10^3$	907.734
0.13	$3.689\cdot10^3$	905.844
0.15	$4.143\cdot10^3$	904.043
0.17	$4.573\cdot10^3$	902.326
0.19	$4.982\cdot10^3$	900.686
0.21	$5.37\cdot10^3$	899.119
0.23	$5.74\cdot10^3$	897.621
0.25	$6.092\cdot10^3$	896.185
0.27	$6.429\cdot10^3$	894.809
0.29	$6.75\cdot10^3$	893.489
0.31	$7.056\cdot10^3$	892.222
0.33	$7.35\cdot10^3$	891.004
0.35	$7.632\cdot10^3$	889.832

Now, plot the results:





==

Prob.5.D.5. Two large plates have emissivities of 0.8 each. A radiation shield is place in between them. What should be its emissivity on either surface to reduce the radiation heat transfer between the surfaces by a factor of 10?

Mathcad Solution:

Data:

eps1 := 0.8 eps2 := 0.8

Let eps_s = eps_s1 = eps_s2 ...emissivity of shield

Calculations:

We have:

$$Q12\_plate\_no\_shield = \frac{5.67 \cdot 10^{-8} \cdot A \cdot \left(T1^4 - T2^4\right)}{\left(\frac{1}{eps1} + \frac{1}{eps2} - 1\right)}$$

$$Q12\_plate\_one\_shield = \frac{5.67 \cdot 10^{-8} \cdot A \cdot \left(T1^4 - T2^4\right)}{\left(\frac{1}{eps1} + \frac{1}{eps2} - 1\right) + \left(\frac{1}{eps\_s} + \frac{1}{eps\_s} - 1\right)}$$

Taking their ratio:

$$\frac{Q12\_plate\_one\_shield}{Q12\_plate\_no\_shield} = \frac{\left(\frac{1}{eps1} + \frac{1}{eps2} - 1\right)}{\left[\left(\frac{1}{eps1} + \frac{1}{eps2} - 1\right) + \left(\frac{1}{eps\_s} + \frac{1}{eps\_s} - 1\right)\right]}$$

Then, since their ratio is 1/10 by data, we have:

$$\frac{\left(\frac{1}{eps1} + \frac{1}{eps2} - 1\right)}{\left[\left(\frac{1}{eps1} + \frac{1}{eps2} - 1\right) + \left(\frac{1}{eps\_s} + \frac{1}{eps\_s} - 1\right)\right]} = \frac{1}{10}$$

Solving:

$$\left[\left(\frac{1}{eps1}+\frac{1}{eps2}-1\right)+\left(\frac{1}{eps\_s}+\frac{1}{eps\_s}-1\right)\right]=10\cdot\left(\frac{1}{eps1}+\frac{1}{eps2}-1\right)$$

i.e. $$\frac{2}{eps\_s}-1=10\cdot\left(\frac{1}{eps1}+\frac{1}{eps2}-1\right)-\left(\frac{1}{eps1}+\frac{1}{eps2}-1\right)$$

i.e. $$eps\_s:=\frac{2}{1+\left[10\cdot\left(\frac{1}{eps1}+\frac{1}{eps2}-1\right)-\left(\frac{1}{eps1}+\frac{1}{eps2}-1\right)\right]}$$

i.e. $eps\_s=0.138$ **..required emissivity of shield to reduce heat transfer to one-tenth..Ans.**

==

Prob.5.D.6. The net radiation from the surface of two parallel plates maintained at temperatures T1 and T2 is to be reduced by 79 times. Calculate the no. of screens to be placed between two surfaces to achieve this reduction in heat exchange, assuming the emissivity of screens as 0.05 and that of surfaces as 0.8. [M.U. 1997]

Mathcad Solution:

Data:

$eps1:=0.8$ $eps2:=0.8$ $eps\_s:=0.05$

Calculations:

Let N be the no. of screens required.

$N := 5$ ... trial value

Given

$$\frac{\frac{1}{eps1}+\frac{1}{eps2}-1}{\left(\frac{1}{eps1}+\frac{1}{eps2}-1\right)+N\cdot\left(\frac{1}{eps\_s}+\frac{1}{eps\_s}-1\right)} = \frac{1}{79}$$

$Find(N) = 3$ **....No. of screens reqd.... Ans.**

==

Prob.5.D.7. Two very large parallel plates with emissivities 0.3 and 0.7, exchange heat. Find the percentage reduction in radiation in heat transfer when *two* polished Aluminium radiation shields ($\varepsilon\_s = 0.4$) are placed between them. [M.U. Dec. 2000]

Mathcad Solution:

Data:

$eps1 := 0.3$ $eps2 := 0.7$ $eps\_s := 0.4$

Calculatiobs:

We have:

$$\frac{\frac{1}{eps1}+\frac{1}{eps2}-1}{\left(\frac{1}{eps1}+\frac{1}{eps2}-1\right)+2\cdot\left(\frac{1}{eps\_s}+\frac{1}{eps\_s}-1\right)} = 0.32$$

i.e. by introducing 2 radiation shields, the heat transfer is reduced to 32% of that without the shields.….Ans.

==

Prob.5.D.8. Write Mathcad Functions for radiation heat transfer and temp of radiation shield for:

1) Heat transfer between two infinite, concentric cylinders with no radiation shield,
2) Heat transfer between two infinite, concentric cylinders with ***one radiation shield*** in between,
3) Heat transfer between two infinite, concentric cylinders with ***two radiation shields*** in between, and,
4) Temp of radiation shield for the case of ***one*** radiation shield between two concentric cylinders.

Recollect:

For concentric cylinders:

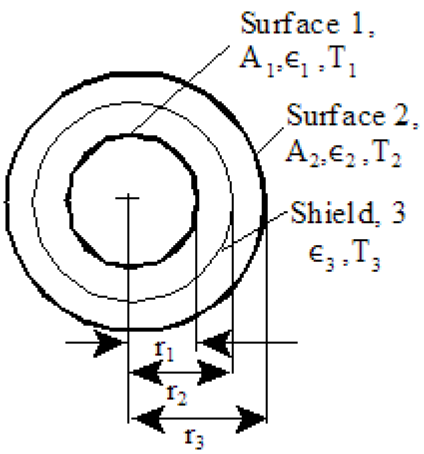


Fig.(a)

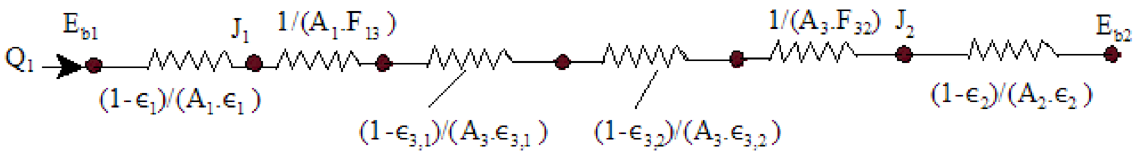


Fig.(b)

For concentric cylinders with no radiation shield:

$$Q_{12} = \frac{A_1 \cdot \sigma \cdot \left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)}$$ ....for infinitely long concentric cylinders.....(13.60)

where

$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$

For concentric cylinders with one radiation shield in between:

$$Q_{12_{one\_shield}} = \frac{A_1 \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right) + \left(\frac{A_1}{A_3}\right) \cdot \left(\frac{1}{\varepsilon_{3\_1}} + \frac{1}{\varepsilon_{3\_2}} - 1\right)}$$

...for concentric cylinders with one radiation shield...(13.77)

In eqn. (13.77), we have: $(A_1/A_2) = (r_1/r_2)$, and $(A_1/A_3) = (r_1/r_3)$.

Mathcad Solution:

Inputs:

R1, R2, R_shield …radii of inner cylinder, outer cylinder and the radiation shield placed between them

A1, A2 … surface areas of inner and outer cylinders, m^2

eps1, eps2 …emissivities of inner and outer cylinders facing each other

eps_s1, eps_s2 .. emissivities of radiation shield, on the surfaces looking towards inner cylinder 1 and outer cylinder 2 respectively

T1, T2 … temps of two cylinders, Kelvin

Output:

Q12 … W,

T_shield … Kelvin

1. Q12 for Long, concentric cylinders, with no radiation shield:

$$Q12\_Concentric\_cyl(R1, R2, L, eps1, eps2, T1, T2) := \frac{5.67 \cdot 10^{-8} \cdot (2 \cdot \pi \cdot R1 \cdot L) \cdot \left(T1^4 - T2^4\right)}{\frac{1}{eps1} + \left(\frac{R1}{R2}\right) \cdot \left(\frac{1}{eps2} - 1\right)}$$

---

2. Q12 for Long, concentric cylinders, with one radiation shield:

Q12_Concentric_cyl_One_shield(R1, R2, R_shield, L, eps1, eps2, eps_s1, eps_s2, T1, T2) :=

$$\frac{5.67\cdot 10^{-8}\cdot(2\cdot\pi\cdot R1\cdot L)\cdot\left(T1^{4}-T2^{4}\right)}{\frac{1}{eps1}+\left(\frac{R1}{R2}\right)\cdot\left(\frac{1}{eps2}-1\right)+\left(\frac{R1}{R\_shield}\right)\cdot\left(\frac{1}{eps\_s1}+\frac{1}{eps\_s2}-1\right)}$$

---

3. Q12 for Long, concentric cylinders, with two radiation shields:

Q12_Concentric_cyl_Two_shields(R1, R2, R_s1, R_s2, L, eps1, eps2, eps_s1, eps_s2, T1, T2) :=

$$\frac{5.67\cdot 10^{-8}\cdot(2\cdot\pi\cdot R1\cdot L)\cdot\left(T1^{4}-T2^{4}\right)}{\frac{1}{eps1}+\left(\frac{R1}{R2}\right)\cdot\left(\frac{1}{eps2}-1\right)+\left(\frac{R1}{R\_s1}\right)\cdot\left(\frac{1}{eps\_s1}+\frac{1}{eps\_s2}-1\right)+\left(\frac{R1}{R\_s2}\right)\cdot\left(\frac{1}{eps\_s1}+\frac{1}{eps\_s2}-1\right)}$$

---

4. Temp. of radiation shield (T3), kept between concentric cylinders:

Equilibrium temp. of shield:

Let the equilibrium temp. of shield be T_3.

In steady state, we have:

$$Q_{12\_one\_shield} = Q_{13} = Q_{32}$$

Q_{12} with one shield is already calculated. Q_{13} or Q_{32} is calculated using eqn.for concentric cylinders with no shield.

Let us take: $Q_{12\_one\_shield} = Q_{13}$

i.e.

$$Q_{12\_one\_shield} = \frac{A1 \cdot \sigma \cdot \left(T1^4 - T3^4\right)}{\frac{1}{\varepsilon_1} + \left(\frac{A1}{A3}\right) \cdot \left(\frac{1}{\varepsilon_{31}} - 1\right)}$$

Applying the above equation, we write, using the 'Solve block' of Mathcad:

$T3 := 300$ K....trial value

Given

$$\frac{\left(T1^4 - T2^4\right)}{\frac{1}{eps1} + \left(\frac{R1}{R2}\right) \cdot \left(\frac{1}{eps2} - 1\right) + \left(\frac{R1}{R\_shield}\right) \cdot \left(\frac{1}{eps\_s1} + \frac{1}{eps\_s2} - 1\right)} = \frac{\left(T1^4 - T3^4\right)}{\frac{1}{eps1} + \left(\frac{R1}{R\_shield}\right) \cdot \left(\frac{1}{eps\_s1} - 1\right)}$$

$T_{shield}(R1, R2, R\_shield, eps1, eps2, eps\_s1, eps\_s2, T1, T2) := Find(T3)$

Note: In the above, T_shield is written as a function of R1, R2,….T1, T2.

This will make it very convenient to calculate the temp of shield, and also to make parametric analysis and draw graphs.

==

Prob.5.D.9. Write Mathcad Functions for radiation heat transfer and temp of radiation shield for:

1) Heat transfer between two concentric spheres with no radiation shield,
2) Heat transfer between two concentric spheres with ***one radiation shield*** in between,
3) Heat transfer between two concentric spheres with ***two radiation shields*** in between, and,
4) Temp of radiation shield for the case of ***one*** radiation shield between two concentric spheres

Recollect:

For concentric spheres:

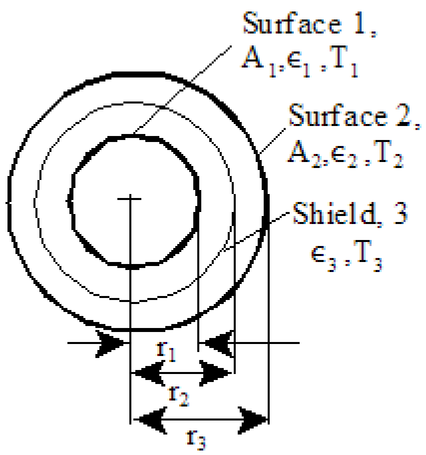


Fig.(a)

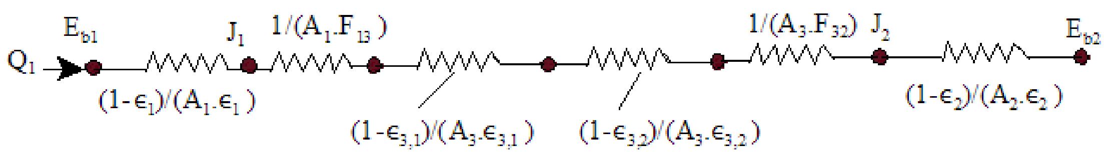


Fig.(b)

For concentric spheres with no radiation shield:

$$Q_{12} = \frac{A_1 \cdot \sigma \cdot \left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)} \quad \text{.... for concentric spheres}$$

For concentric spheres with one radiation shield in between:

$$Q_{12_{one\_shield}} = \frac{A_1 \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right)\cdot\left(\frac{1}{\varepsilon_2} - 1\right) + \left(\frac{A_1}{A_3}\right)\cdot\left(\frac{1}{\varepsilon_{3\_1}} + \frac{1}{\varepsilon_{3\_2}} - 1\right)}$$

.... for concentric spheres with one radiation shield

In the above eqns. we have:

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2 \quad \text{and,} \quad \frac{A_1}{A_3} = \left(\frac{r_1}{r_3}\right)^2$$

Mathcad Solution:

Inputs:

R1, R2, R_shield …radii of inner cylinder, outer cylinder and the radiation shield placed between them

A1, A2 … surface areas of inner and outer cylinders, m^2

eps1, eps2 …emissivities of inner and outer cylinders facing each other

eps_s1, eps_s2 .. emissivities of radiation shield, on the surfaces looking towards inner cylinder 1 and outer cylinder 2 respectively

T1, T2 … temps of two cylinders, Kelvin

Output:

Q12 … W

Temp_shield … Kelvin

1. Q12 for concentric spheres, with no radiation shield:

$$Q12\_Concentric\_spheres(R1,R2,eps1,eps2,T1,T2) := \frac{5.67\cdot 10^{-8}\cdot\left(4\cdot\pi\cdot R1^2\right)\cdot\left(T1^4 - T2^4\right)}{\frac{1}{eps1} + \left(\frac{R1}{R2}\right)^2\cdot\left(\frac{1}{eps2} - 1\right)}$$

---

2. Q12 for concentric spheres, with *one* radiation shield:

$$Q12\_Concentric\_sph\_One\_shield(R1,R2,R\_shield,eps1,eps2,eps\_s1,eps\_s2,T1,T2) :=$$

$$\frac{5.67\cdot 10^{-8}\cdot\left(4\cdot\pi\cdot R1^2\right)\cdot\left(T1^4 - T2^4\right)}{\frac{1}{eps1} + \left(\frac{R1}{R2}\right)^2\cdot\left(\frac{1}{eps2} - 1\right) + \left(\frac{R1}{R\_shield}\right)^2\cdot\left(\frac{1}{eps\_s1} + \frac{1}{eps\_s2} - 1\right)}$$

---

3. Q12 for concentric spheres, with *two* radiation shields:

Q12_Concentric_sph_Two_shields(R1, R2, R_s1, R_s2, eps1, eps2, eps_s1, eps_s2, T1, T2) :=

$$\frac{5.67\cdot 10^{-8}\cdot\left(4\cdot\pi\cdot R1^{2}\right)\cdot\left(T1^{4}-T2^{4}\right)}{\frac{1}{eps1}+\left(\frac{R1}{R2}\right)^{2}\cdot\left(\frac{1}{eps2}-1\right)+\left(\frac{R1}{R\_s1}\right)^{2}\cdot\left(\frac{1}{eps\_s1}+\frac{1}{eps\_s2}-1\right)+\left(\frac{R1}{R\_s2}\right)^{2}\cdot\left(\frac{1}{eps\_s1}+\frac{1}{eps\_s2}\right)}$$

---

4. Temp of radiation shield (T3), kept between concentric spheres:

Equilibrium temp. of shield:

Let the equilibrium temp. of shield be T_3.

In steady state, we have:

$$Q_{12\_one\_shield} = Q_{13} = Q_{32}$$

Q_{12} with one shield is already calculated. Q_{13} or Q_{32} is calculated using eqn.for concentric spheres with no shield.

Let us take: $Q_{12\_one\_shield} = Q_{13}$

i.e. $$Q_{12\_one\_shield} = \frac{A1\cdot\sigma\cdot\left(T1^{4}-T3^{4}\right)}{\frac{1}{\varepsilon_1}+\left(\frac{A1}{A3}\right)\cdot\left(\frac{1}{\varepsilon_{31}}-1\right)}$$

Remember that now, surface area ratio, (A1/A3) = (R1/R_shield)^2 for a sphere.

Applying the above equation, we write, using the 'Solve block' of Mathcad:

T3 := 300 K....trial value

Given

$$\frac{\left(T1^{4}-T2^{4}\right)}{\frac{1}{eps1}+\left(\frac{R1}{R2}\right)^{2}\cdot\left(\frac{1}{eps2}-1\right)+\left(\frac{R1}{R\_shield}\right)^{2}\cdot\left(\frac{1}{eps\_s1}+\frac{1}{eps\_s2}-1\right)} = \frac{\left(T1^{4}-T3^{4}\right)}{\frac{1}{eps1}+\left(\frac{R1}{R\_shield}\right)^{2}\cdot\left(\frac{1}{eps\_s1}-1\right)}$$

$Temp_{shield}$(R1, R2, R_shield, eps1, eps2, eps_s1, eps_s2, T1, T2) := Find(T3)

Note: In the above, Temp_shield is written as a function of R1, R2,….T1, T2.

This will make it very convenient to calculate the temp of shield, and also to make parametric analysis and draw graphs.

===

Prob.5.D.10. Two long, concentric cylinders have diameters of 40 and 80 mm. The inside cylinder is at 800 C and the outer cylinder is at 100 C. Inside and outside emissivities are 0.8 and 0.4. Calculate the percent reduction in heat transfer if a cylindrical shield of 60 mm dia and emissivity of 0.3 is placed between the cylinders. Also, calculate the equilibrium temp of the shield.

(b) Plot the percent reduction in heat transfer and the Shield temp as emissivity (on either side) of shield varies from 0.05 to 0.35.

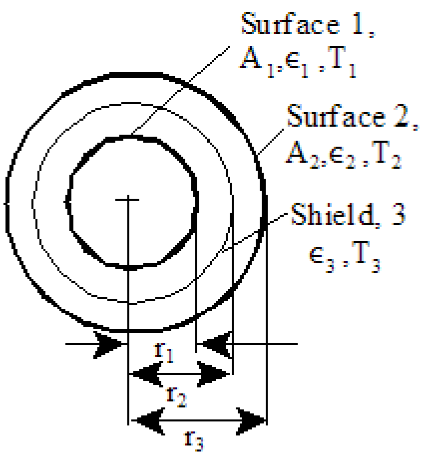


Fig.(a)

Mathcad Solution:

Let us use the Mathcad Functions written above in Prob.5.D.8.

Data:

R1 := 0.02 m   R2 := 0.04 m   R_shield := 0.03 m   L := 1 m .... assumed

eps1 := 0.8   eps2 := 0.4   eps_s1 := 0.3   eps_s2 := 0.3

T1 := 1073 K   T2 := 373 K

Calculations:

Heat transfer with no shield:

Q12_no_shield := Q12_Concentric_cyl(R1, R2, L, eps1, eps2, T1, T2)

i.e.   Q12_no_shield = 4.653×10^{3} **W .... Ans.**

Heat transfer with one radiation shield:

Q12_with_shield := Q12_Concentric_cyl_One_shield(R1, R2, R_shield, L, eps1, eps2, eps_s1, eps_s2, T1, T2)

i.e.   Q12_with_shield = 1.611×10^{3} **W .... Ans.**

Therefore:

$$Percent\_reduction := \frac{Q12\_no\_shield - Q12\_with\_shield}{Q12\_no\_shield} \cdot 100$$

i.e. $Percent\_reduction = 65.385$ **% .... reduction because of the shield....Ans.**

Equilibrium temp of the Shield:

$$T\_shield := T_{shield}(R1, R2, R\_shield, eps1, eps2, eps\_s1, eps\_s2, T1, T2)$$

i.e. $T\_shield = 911.835$ **K = 638.835 C .... Ans.**

Plot the percent reduction in heat transfer and the Shield temp as emissivity (on either side) of shield varies from 0.05 to 0.35:

Write the relevant quantities as functions of eps_s:

Let: $eps\_s1 = eps\_s2 = eps\_s$ ....since emissivity on either side of shield is same

Then, we have:

$Q12\_no\_shield := Q12\_Concentric\_cyl(R1, R2, L, eps1, eps2, T1, T2)$ ..does not depend on eps_s

$Q12\_with\_shield(eps\_s) := Q12\_Concentric\_cyl\_One\_shield(R1, R2, R\_shield, L, eps1, eps2, eps\_s, eps\_s, T1, T2)$

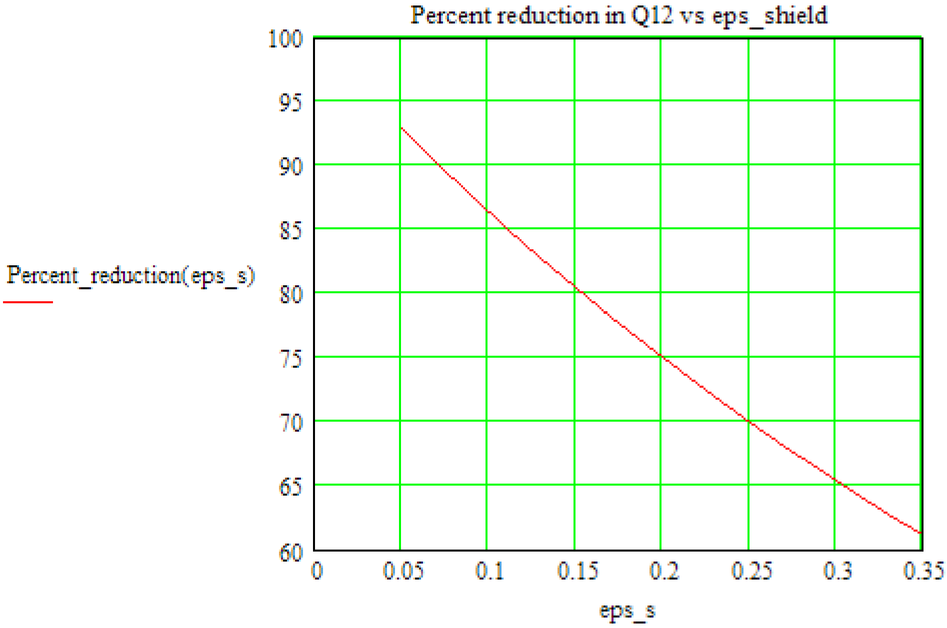
$$Percent\_reduction(eps\_s) := \frac{Q12\_no\_shield - Q12\_with\_shield(eps\_s)}{Q12\_no\_shield} \cdot 100$$

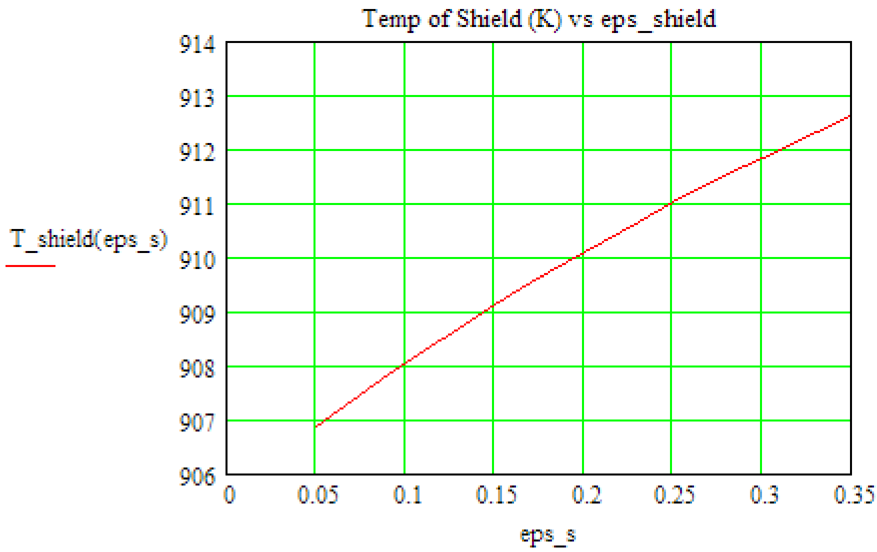
$$T\_shield(eps\_s) := T_{shield}(R1, R2, R\_shield, eps1, eps2, eps\_s, eps\_s, T1, T2)$$

Now, compute the Parametric Table and plot the graphs:

eps_s := 0.05, 0.07 .. 0.35 .....define a range variable

eps_s =	Percent_reduction(eps_s)	T_shield(eps_s)
0.05	92.857	906.864
0.07	90.187	907.351
0.09	87.615	907.819
0.11	85.135	908.269
0.13	82.743	908.703
0.15	80.435	909.122
0.17	78.205	909.525
0.19	76.05	909.915
0.21	73.967	910.291
0.23	71.951	910.654
0.25	70	911.005
0.27	68.11	911.345
0.29	66.279	911.674
0.31	64.504	911.993
0.33	62.782	912.302
0.35	61.111	912.601





===

Prob.5.D.11. A spherical tank with diameter D1 = 40 cm, filled with a cryogenic fluid at T1 = 100 K, is placed inside a spherical container of diameter D2 = 60 cm, maintained at T2 = 300 K. Emissivities of inner and outer tanks are **ε**1 = 0.10 and **ε**2 = 0.20 respectively.

1) Find the rate of heat loss into the inner vessel by radiation
2) If a spherical radiation shield of diameter D3 = 50 cm, with an emissivity **ε**3 = 0.05 on both surfaces is placed between the spheres, what is the new rate of heat loss?

[M.U. Jan. 2002]

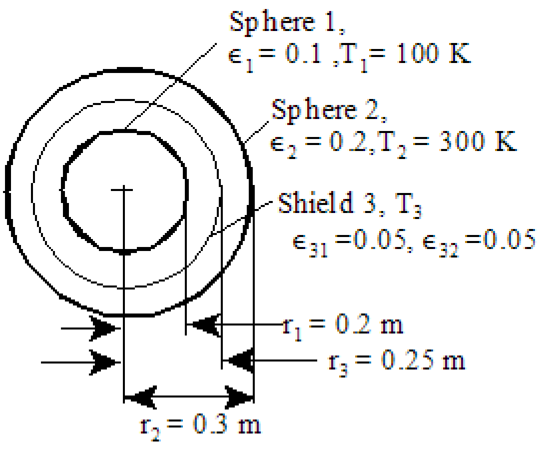


Fig.(a)

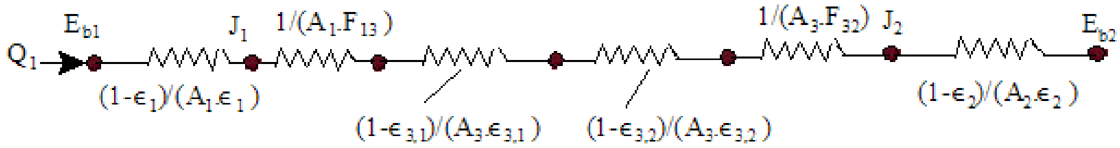


Fig.(b)

Fig. Prob.5.D.11

Mathcad Solution:

Let us use the Mathcad Functions written above in Prob.5.D.9.

Data:

R1 := 0.2 m R2 := 0.3 m R_shield := 0.25 m

eps1 := 0.1 eps2 := 0.2 eps_s1 := 0.05 eps_s2 := 0.05

T1 := 100 K T2 := 300 K

Calculations:

Heat transfer with no shield:

$Q12\_no\_shield := Q12\_Concentric\_spheres(R1, R2, eps1, eps2, T1, T2)$

i.e. $Q12\_no\_shield = -19.359$ **W ....negative sign indicates heat flow into the inner cylinder... Ans.**

Heat transfer with one radiation shield:

$Q12\_with\_shield := Q12\_Concentric\_sph\_One\_shield(R1, R2, R\_shield, eps1, eps2, eps\_s1, eps\_s2, T1, T2)$

i.e. $Q12\_with\_shield = -6.206$ **W ..negative sign indicates heat flow into the inner cylinder.... Ans.**

Therefore:

$$Percent\_reduction := \frac{Q12\_no\_shield - Q12\_with\_shield}{Q12\_no\_shield} \cdot 100$$

i.e. $Percent\_reduction = 67.941$ **% .... reduction because of the shield....Ans.**

Equilibrium temp of the Shield:

$T\_shield := Temp_{shield}(R1, R2, R\_shield, eps1, eps2, eps\_s1, eps\_s2, T1, T2)$

i.e. $T\_shield = 264.919$ **K = - 8.08 C .... Ans.**

Plot the percent reduction in heat transfer and the Shield temp as emissivity (on either side) of shield varies from 0.05 to 0.35:

Write the relevant quantities as functions of eps_s:

Let: $eps\_s1 = eps\_s2 = eps\_s$ ....since emissivity on either side of shield is same

Then, we have:

$$\text{Q12\_no\_shield} := \text{Q12\_Concentric\_spheres}(\text{R1}, \text{R2}, \text{eps1}, \text{eps2}, \text{T1}, \text{T2})$$ ..does not depend on eps_s

$$\text{Q12\_with\_shield}(\text{eps\_s}) := \text{Q12\_Concentric\_sph\_One\_shield}(\text{R1}, \text{R2}, \text{R\_shield}, \text{eps1}, \text{eps2}, \text{eps\_s}, \text{eps\_s}, \text{T1}, \text{T2})$$

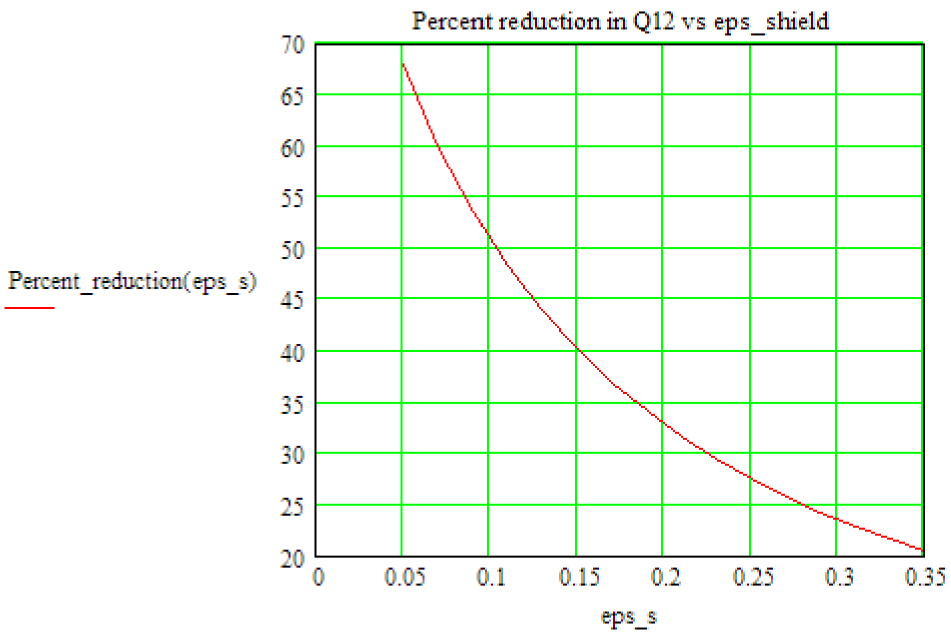
$$\text{Percent\_reduction}(\text{eps\_s}) := \frac{\text{Q12\_no\_shield} - \text{Q12\_with\_shield}(\text{eps\_s})}{\text{Q12\_no\_shield}} \cdot 100$$

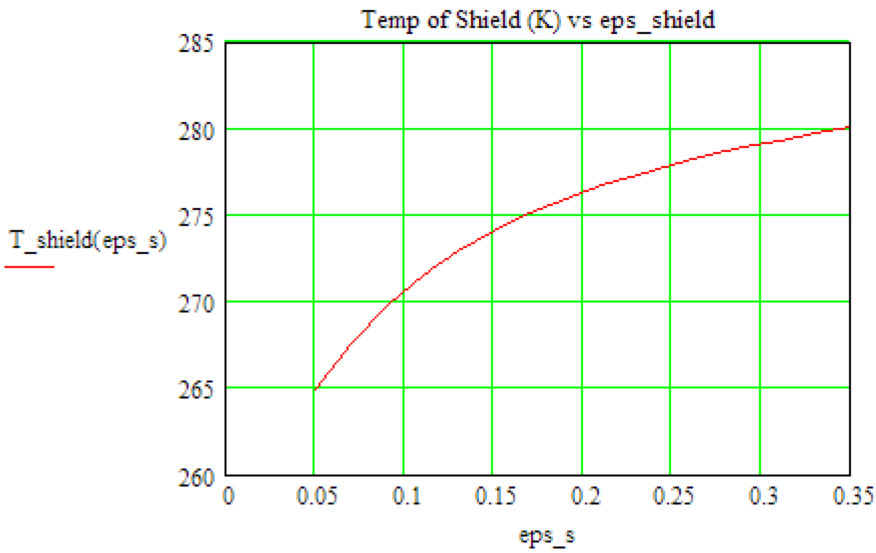
$$\text{T\_shield}(\text{eps\_s}) := \text{Temp}_{\text{shield}}(\text{R1}, \text{R2}, \text{R\_shield}, \text{eps1}, \text{eps2}, \text{eps\_s}, \text{eps\_s}, \text{T1}, \text{T2})$$

Now, compute the Parametric Table and plot the graphs:

eps_s := 0.05, 0.07 .. 0.35 .....define a range variable

eps_s =	Percent_reduction(eps_s)	T_shield(eps_s)
0.05	67.941	264.919
0.07	59.972	267.637
0.09	53.558	269.765
0.11	48.284	271.478
0.13	43.872	272.886
0.15	40.127	274.066
0.17	36.907	275.067
0.19	34.109	275.928
0.21	31.656	276.677
0.23	29.487	277.334
0.25	27.556	277.915
0.27	25.826	278.432
0.29	24.266	278.896
0.31	22.854	279.315
0.33	21.568	279.694
0.35	20.393	280.039





===

Prob.5.D.12. Write EES Functions for radiation heat transfer without and with radiation shield for the cases of: (i) infinite, parallel plates (ii) infinite, concentric cylinders, and (iii) concentric spheres. Also write Functions to determine the equilibrium temp of the radiation shield.

EES Solutions:

Equations for these cases are already given earlier.

Now, the EES Functions:

In the following Functions:

Inputs:

In case of cylinders and spheres:

1…. denotes the inner cylinder / sphere, and]

2… denotes outer cylinder / sphere

R1 …. Radius of inner cylinder / sphere, (m)

R2 …. Radius of outer cylinder / sphere, (m)

R_shield …. Radius of radation shield placed between 1 and 2, (m)

A … area of plates, (m^2)

eps1, eps2 …emissivities of two plates or cylinders or spheres

eps_s1, eps_s2 .. emissivities of radiation shield, on the faces looking towards surfaces 1 and 2 respectively

T1, T2 … temps of two plates or cylinders or spheres, (K)

Output:

Q12 … net heat transfer between surfaces 1 and 2, (W)

T_shield … equilibrium temp of shield, (K)

```
$UnitSystem SI Pa J C

FUNCTION Q12_parallel_plates(A,eps_1,eps_2,T1,T2)

sigma:=5.67E-08 "[W/m^2-K^4]"
```

```
Q12_parallel_plates:=(A * sigma * (T1^4 – T2^4)) / (1/eps_1 + 1/eps_2 -1)

END

"-----------------------------------------------------------------------------------------------------"

FUNCTION Q12_parallel_plates_one_shield(A, eps_1, eps_2, eps_s1, eps_s2, T1,T2)

sigma:=5.67E-08 "[W/m^2-K^4]"

Q12_parallel_plates_one_shield:=(A * sigma * (T1^4 – T2^4)) / ((1/eps_1 + 1/eps_2 -1) + (1 / eps_s1 + 1/eps_s2 -1))

END

"-----------------------------------------------------------------------------------------------------"

FUNCTION Temp_Shield_parallel_plates(A, eps_1, eps_2, eps_s1, eps_s2, T1,T2)

sigma:=5.67E-08 "[W/m^2-K^4]"

Temp_Shield_parallel_plates:=(T1^4 – Q12_parallel_plates_one_shield(A, eps_1, eps_2, eps_s1, eps_s2, T1,T2) * (1 / eps_1 + 1 / eps_s1 – 1) / (A * sigma))^0.25

END

"-----------------------------------------------------------------------------------------------------"

FUNCTION Q12_parallel_plates_N_shields(A, N, eps_1, eps_2, eps_s1, eps_s2, T1,T2)

sigma:=5.67E-08 "[W/m^2-K^4]"

Q12_parallel_plates_N_shields:=(A * sigma * (T1^4 – T2^4)) / ((1/eps_1 + 1/eps_2 -1) + N * (1 / eps_s1 + 1/eps_s2 -1))

END

"-----------------------------------------------------------------------------------------------------"
```

```
FUNCTION Q12_concentric_cyl(R1, R2, L , eps_1, eps_2,T1,T2)

sigma:=5.67E-08 "[W/m^2-K^4]"

Q12_concentric_cyl:=(2 * pi * R1 * L * sigma * (T1^4 – T2^4)) / (1/eps_1 + (R1 / R2) * (1/eps_2 -1))

END

"----------------------------------------------------------------------------------------------------"

FUNCTION Q12_concentric_cyl_one_shield(R1, R2,R_shield, L , eps_1, eps_2, eps_s1, eps_s2, T1,T2)

sigma:=5.67E-08 "[W/m^2-K^4]"

Q12_concentric_cyl_one_shield:=(2 * pi * R1 * L * sigma * (T1^4 – T2^4)) / (1/eps_1 + (R1 / R2) * (1/
eps_2 -1)+ (R1 / R_shield) * (1/eps_s1 + 1/eps_s2 -1))

END

"----------------------------------------------------------------------------------------------------"

FUNCTION Temp_Shield_concentric_cyl(R1, R2,R_shield, L , eps_1, eps_2, eps_s1, eps_s2, T1,T2)

sigma:=5.67E-08 "[W/m^2-K^4]"

Temp_Shield_concentric_cyl:=(T1^4 – Q12_concentric_cyl_one_shield(R1, R2,R_shield, L, eps_1,
eps_2, eps_s1, eps_s2, T1,T2) *(1/eps_1 + (R1 / R_shield) * (1/eps_s1 -1)) / (2 * pi * R1 * L * sigma))^0.25

END

"----------------------------------------------------------------------------------------------------"

FUNCTION Q12_concentric_spheres(R1, R2, eps_1, eps_2,T1,T2)

sigma:=5.67E-08 "[W/m^2-K^4]"

Q12_concentric_spheres :=(4 * pi * R1^2 * sigma * (T1^4 – T2^4)) / (1/eps_1 + (R1 / R2)^2 * (1/eps_2 -1))

END

"----------------------------------------------------------------------------------------------------"
```

```
FUNCTION Q12_concentric_sph_one_shield(R1, R2,R_shield, eps_1, eps_2, eps_s1, eps_s2, T1,T2)

sigma:=5.67E-08 "[W/m^2-K^4]"

Q12_concentric_sph_one_shield:=(4 * pi * R1^2 * sigma * (T1^4 – T2^4)) / (1/eps_1 + (R1 / R2)^2 * (1/eps_2 -1)+ (R1 / R_shield)^2 * (1/eps_s1 + 1/eps_s2 -1))

END

"-------------------------------------------------------------------------------------------------------"

FUNCTION Temp_Shield_concentric_spheres(R1, R2,R_shield, eps_1, eps_2, eps_s1, eps_s2, T1,T2)

sigma:=5.67E-08 "[W/m^2-K^4]"

Temp_Shield_concentric_spheres:=(T1^4 – Q12_concentric_sph_one_shield(R1, R2,R_shield, eps_1, eps_2, eps_s1, eps_s2, T1,T2) *(1/eps_1 + (R1 / R_shield)^2 * (1/eps_s1 -1)) / (4 * pi * R1^2 * sigma))^0.25

END

=
-------------------------------------------------------------------------------------------------------
```

Now, let us use the above EES Functions to solve some problems:

Prob.5.D.13. Two very large parallel planes, with emissivities 0.3 and 0.8 exchange heat. Find the percentage reduction in heat transfer when a polished Aluminium shield (eps = 0.04) is placed between them. [VTU – May/June 2010]

EES Solution:

We have:

$$Q12_{parallel,plates} := \frac{A \cdot \sigma \cdot (T1^4 - T2^4)}{\frac{1}{eps_1} + \frac{1}{eps_2} - 1}$$

$$Q12_{parallel,plates,one,shield} := \frac{A \cdot \sigma \cdot (T1^4 - T2^4)}{\frac{1}{eps_1} + \frac{1}{eps_2} - 1 + \frac{1}{eps_{s1}} + \frac{1}{eps_{s2}} - 1}$$

Therefore, taking the ratio of Q12 with shield to that without shield:

$$Ratio = \frac{\frac{1}{eps_1} + \frac{1}{eps_2} - 1}{\frac{1}{eps_1} + \frac{1}{eps_2} - 1 + \frac{1}{eps_{s1}} + \frac{1}{eps_{s2}} - 1}$$

In EES data and this eqn for Ratio are entered:

"Data:"

eps_1 = 0.3
eps_2 = 0.8
eps_s1 = 0.04
eps_s2 = 0.04

Ratio = (1/eps_1 + 1/eps_2 – 1) / ((1/eps_1 + 1/eps_2 – 1) + (1/eps_s1 + 1/eps_s2 – 1))

And, Results:

Unit Settings: SI C Pa J mass deg

$eps_1 = 0.3$ $eps_2 = 0.8$ $eps_{s1} = 0.04$ $eps_{s2} = 0.04$ $Ratio = 0.06815$

i.e. Q12 with shield is 6.815% of that without shield…. Ans.

==

Prob.5.D.14. Two large parallel planes with emissivity 0.6 are at 900 K and 300 K. A radiation shield with one side polished and having emissivity of 0.05 and the other side unpolished with emissivity 0.4 is proposed to be used between them. Which side of the shield should face the hotter plane if the temp of the shield is to be kept minimum? Justify your answer. [VTU – June 2012]

(b) Also, for the case 1, plot Q12_with_shield and T_shield as eps_s1 (i.e. emissivity of shield surface facing the 900 K surface) varies from 0.05 to 0.4:

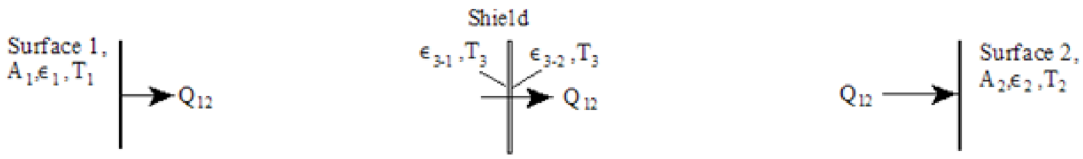


Fig.(a)

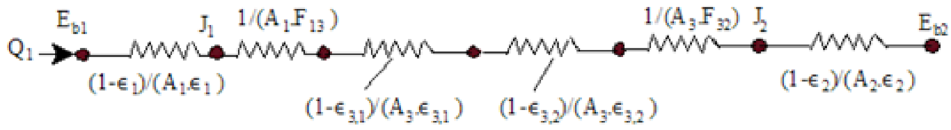


Fig.(b)

EES Solution:

We shall use the EES Functions written above:

"Data:"

A = 1"[m^2]"

eps_1 = 0.6
eps_2 = 0.6
eps_s1 = 0.05 "..facing surface 1"
eps_s2 = 0.4 ".... facing surface 2"

T1 = 900"[K]"

T2 = 300 "[K]"

"Let:

Case 1: polished surface of shield is facing the hot surface 1

Case 2: unpolished surface of shield is facing the hot surface 1"

Q12 = Q12_parallel_plates(A,eps_1,eps_2,T1,T2)"….when there is no shield"

Q12_shield_case1 = Q12_parallel_plates_one_shield(A, eps_1, eps_2, eps_s1, eps_s2, T1,T2) "..polished surface facing hotter plate"

Q12_shield_case2 = Q12_parallel_plates_one_shield(A, eps_1, eps_2, eps_s2, eps_s1, T1,T2) "..unpolished surface facing hotter plate"

T_shield_case1 = Temp_Shield_parallel_plates(A, eps_1, eps_2, eps_s1, eps_s2, T1,T2) "…for case 1"

T_shield_case2 = Temp_Shield_parallel_plates(A, eps_1, eps_2, eps_s2, eps_s1, T1,T2) "…for case 2"

Results:

Unit Settings: SI C Pa J mass deg

(Table 1, Run 8)

$A = 1$ [m^2]	$eps_1 = 0.6$	$eps_2 = 0.6$	$eps_{s1} = 0.4$
$eps_{s2} = 0.4$	$Q12 = 15746$ [W]	$Q12_{shield,case1} = 5801$ [W]	$Q12_{shield,case2} = 5801$ [W]
$T1 = 900$ [K]	$T2 = 300$ [K]	$T_{shield,case1} = 759.1$ [K]	$T_{shield,case2} = 759.1$ [K]

Thus:

For case 1: Temp of shield = 554 K … Ans.

For case 2: Temp of shield = 868.9 K …. Ans.

Therefore, use option 1 for minimum temp of shield.

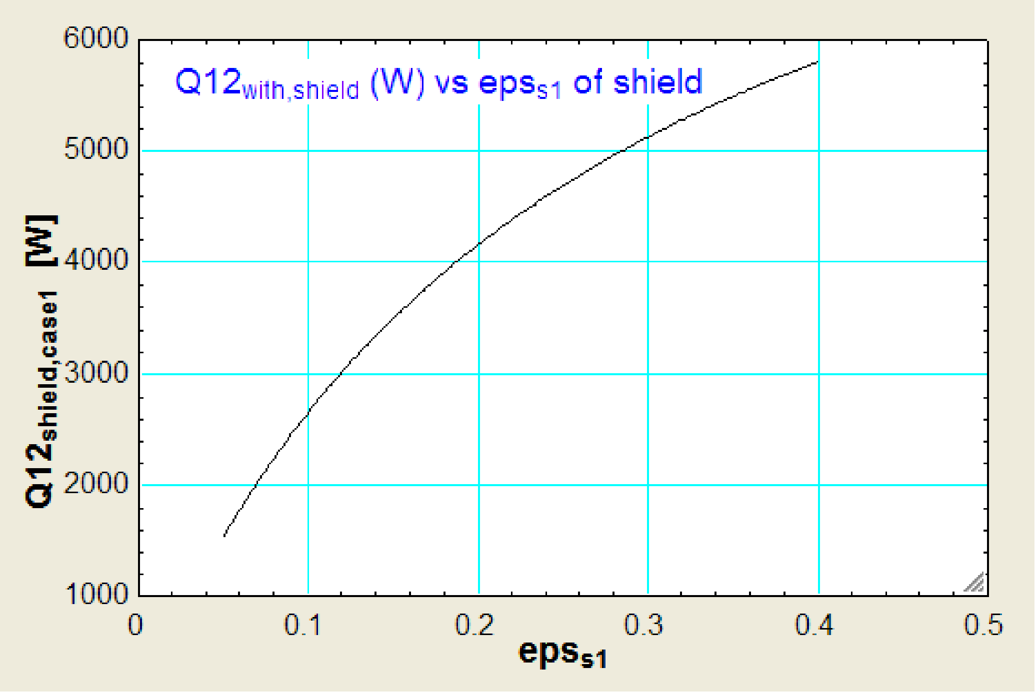
(b) Also, for case 1, plot Q12_with_shield and T_shield as eps_s1 (i.e. emissivity of shield surface facing the 900 K surface) varies from 0.05 to 0.4:

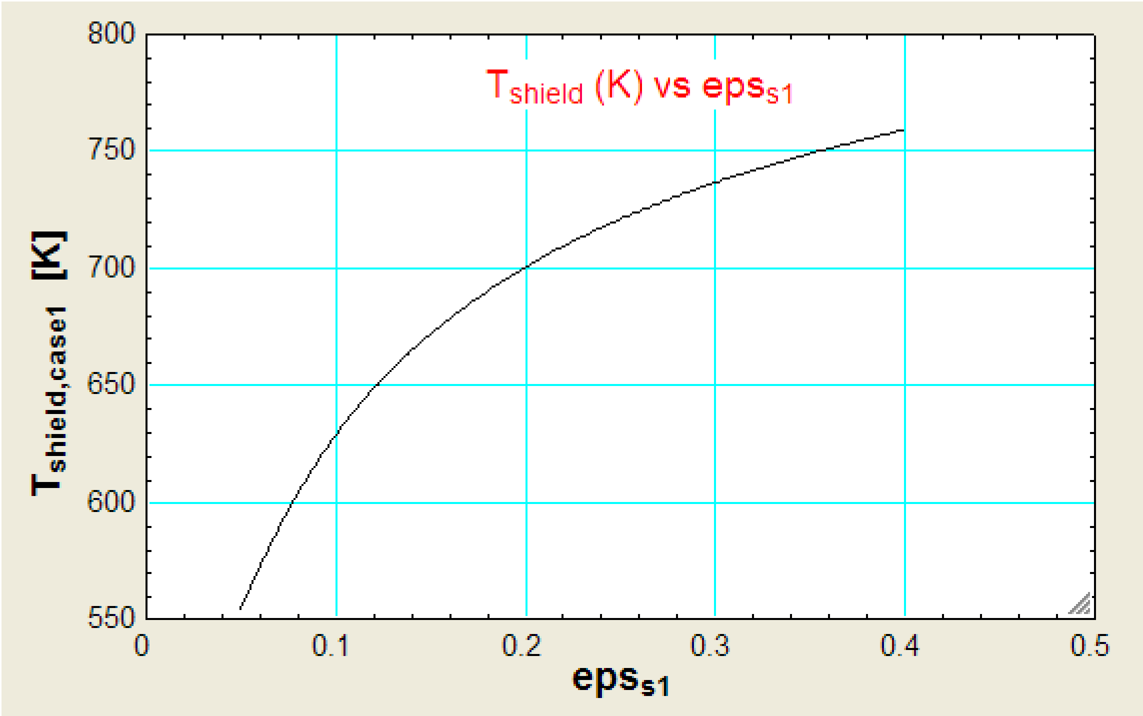
First, compute the Parametric Table:

Table 1

1..8	eps_{s1}	$Q12_{shield,case1}$ [W]	$T_{shield,case1}$ [K]
Run 1	0.05	1542	554
Run 2	0.1	2656	628.9
Run 3	0.15	3499	671.7
Run 4	0.2	4159	700.2
Run 5	0.25	4690	720.9
Run 6	0.3	5127	736.6
Run 7	0.35	5492	749
Run 8	0.4	5801	759.1

Next, plot the results:





==

Prob.5.D.15. Two parallel plates at T1 = 900 K and T2 = 500 K have emissivities eps1 = 0.6 and eps2 = 0.9 respectively. A radiation shield having an emissivity eps_s1 = 0.15 on one side and epas_s2 = 0.06 on the other side is placed between the plates. Calculate the heat transfer rate per sq. m. with and without radiation shield.

(b) Also. Find out the equilibrium temp of the shield (c) Plot Temp of shield as the emissivity eps_s1 of shield varies from 0.05 to 0.4.

EES Solution:

We shall use the EES Functions written above.

"Data:"

```
^ =Z=N≤xã { Oz≤

Éé ë| N=Z=MKS
Éé ë| O=Z=MKV
Éé ë| ëN=Z=MKNR=±KÑ Åå Ö=ëì êÑ ÅÉ=N≤
Éé ë| ëO=Z=MKMS=±KÑ Åå Ö=ëì êÑ ÅÉ=O≤

q N=Z=VMM±xh z≤

q O=Z=RMM=±xh z≤
```

"Calculations:"

Q12 = Q12_parallel_plates(A,eps_1,eps_2,T1,T2)"….when there is no shield"

Q12_shield_case1 = Q12_parallel_plates_one_shield(A, eps_1, eps_2, eps_s1, eps_s2, T1,T2) "..polished surface facing hotter plate"

T_shield = Temp_Shield_parallel_plates(A, eps_1, eps_2, eps_s1, eps_s2, T1,T2) "…equilibrium temp of shield"

Results:

Unit Settings: SI C Pa J mass deg

A = 1 [m^2]	eps_1 = 0.6	eps_2 = 0.9	eps_{s1} = 0.15
eps_{s2} = 0.06	Q12 = 18932 [W]	$Q12_{shield,case1}$ = 1396 [W]	T1 = 900 [K]
T2 = 500 [K]	T_{shield} = 830.4 [K]		

Thus:

Q12 without shield = 18932 W/m^2 …. Ans.

Q12 with shield = 1396 W/m^2 … Ans.

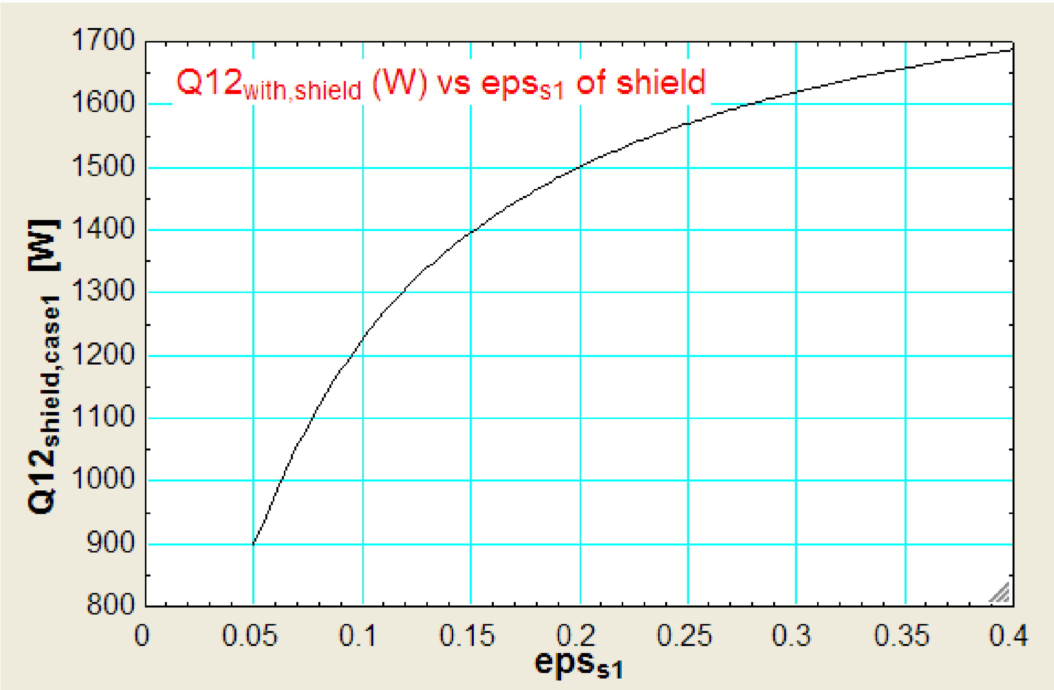
Temp of shield = 830.4 K … Ans.

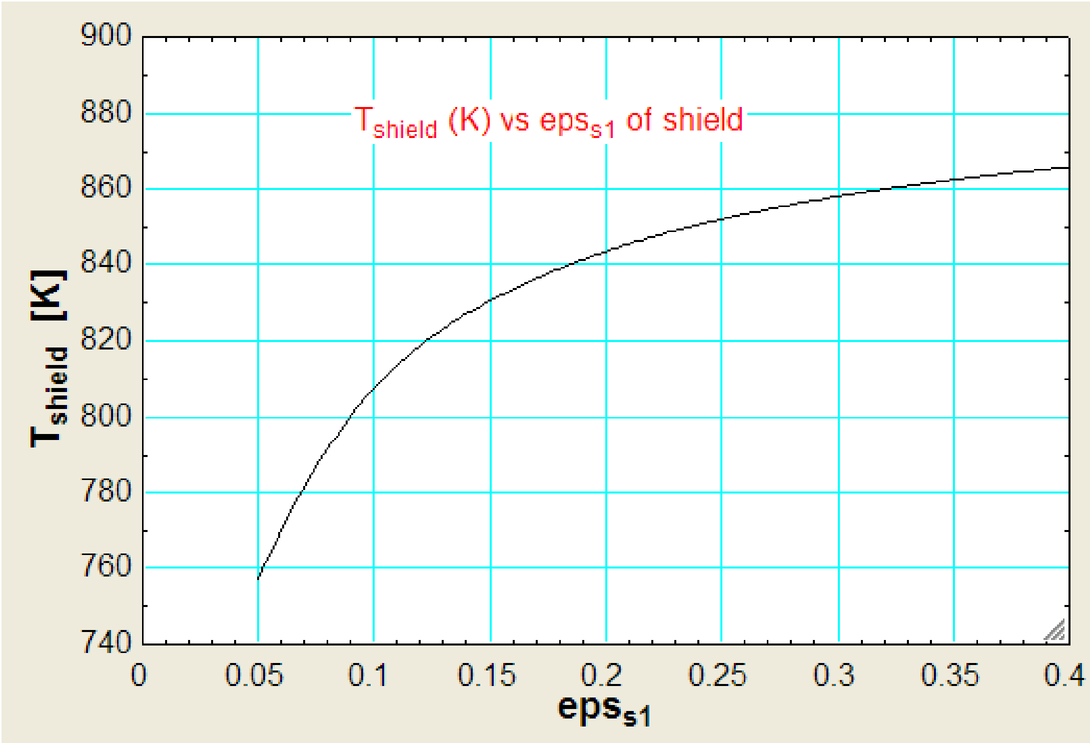
(b) Also, plot Q12_with_shield and T_shield as eps_s1 (i.e. emissivity of shield surface facing the 900 K surface) varies from 0.05 to 0.4:

First, compute the Parametric Table:

1..8	eps_{s1}	$Q12_{shield,case1}$ [W]	T_{shield} [K]
Run 1	0.05	898.9	757.1
Run 2	0.1	1226	807.6
Run 3	0.15	1396	830.4
Run 4	0.2	1500	843.5
Run 5	0.25	1570	852
Run 6	0.3	1620	858
Run 7	0.35	1658	862.4
Run 8	0.4	1688	865.8

And, plot the results:





==

"**Prob.5.D.16**. A cryogenic fluid flows in a pipe of 10 mm OD at a temp of 100 K. It is surrounded by another coaxial pipe of 13 mm OD. Space between the pipes is evacuated. The outer pipe is at 280 K. Emissivities of both surfaces is 0.3. (b) If a shield of emissivity of 0.05 on either face and dia of 11.5 mm is placed between the pipes, determine the percentage reduction in heat flow. Determine the radiant heat flow for a 3 m length. Also, calculate the equilibrium temp of the shield.

(c) Plot the Percentage reduction and the equilibrium temp of the shield as emissivity of shield varies from 0.05 to 0.3"

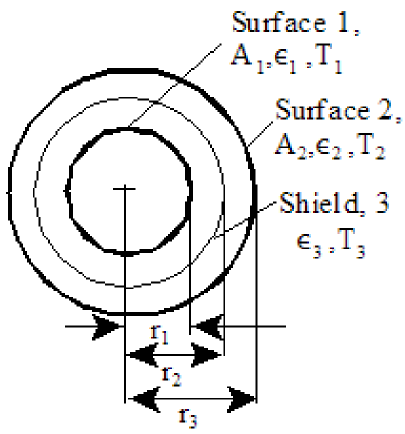


Fig.(a)

EES Solution:

"Data:"

R1 = 0.005 "[m]"
R2 = 0.0065"[m]"
R_shield = 0.00575 "[m]"

L = 3 "[m]"

eps_1 = 0.3
eps_2 = 0.3
eps_s1 = 0.05
eps_s2 = 0.05
T1 = 100 "[K]"

T2 = 280 "[K]"

"Calculations:"

Q12 = Q12_concentric_cyl(R1, R2, L, eps_1, eps_2,T1,T2)

Q12_shield = Q12_concentric_cyl_one_shield(R1, R2,R_shield, L, eps_1, eps_2, eps_s1, eps_s2, T1,T2)

T_shield = Temp_Shield_concentric_cyl(R1, R2,R_shield, L, eps_1, eps_2, eps_s1, eps_s2, T1,T2)

PercentReduction = (Q12 – Q12_shield) * 100 / Q12

Results:

Unit Settings: SI C Pa J mass deg

$eps_1 = 0.3$	$eps_2 = 0.3$	$eps_{s1} = 0.05$
$eps_{s2} = 0.05$	L = 3 [m]	PercentReduction = 86.86 [%]
Q12 = -6.301 [W]	$Q12_{shield}$ = -0.8276 [W]	R1 = 0.005 [m]
R2 = 0.0065 [m]	R_{shield} = 0.00575 [m]	T1 = 100 [K]
T2 = 280 [K]	T_{shield} = 237.4 [K]	

Thus:

Q12 without shield = -6.301 W…. negative sign indicates flow towards the inner pipe which is at a lower temp …. Ans.

Q12 with radiation shield = – 0.8276 W … negative sign indicates flow towards the inner pipe which is at a lower temp …. Ans.

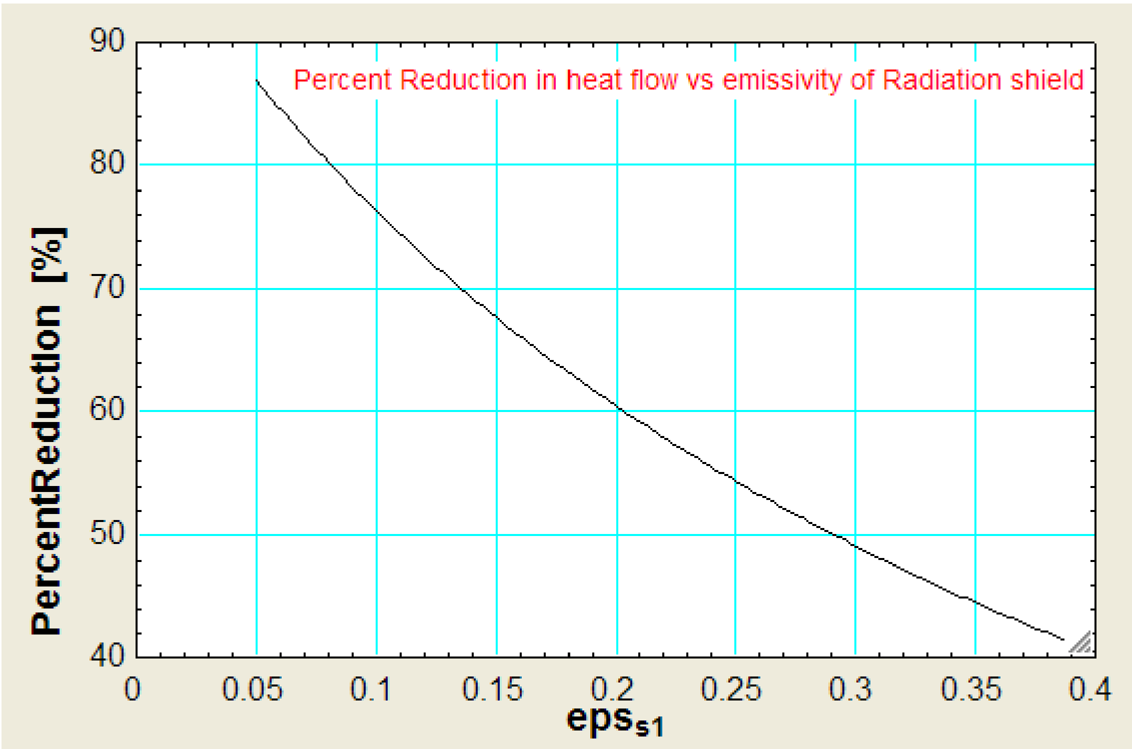
Equilibrium temp of shield = T_shield = 237.4 K … Ans.

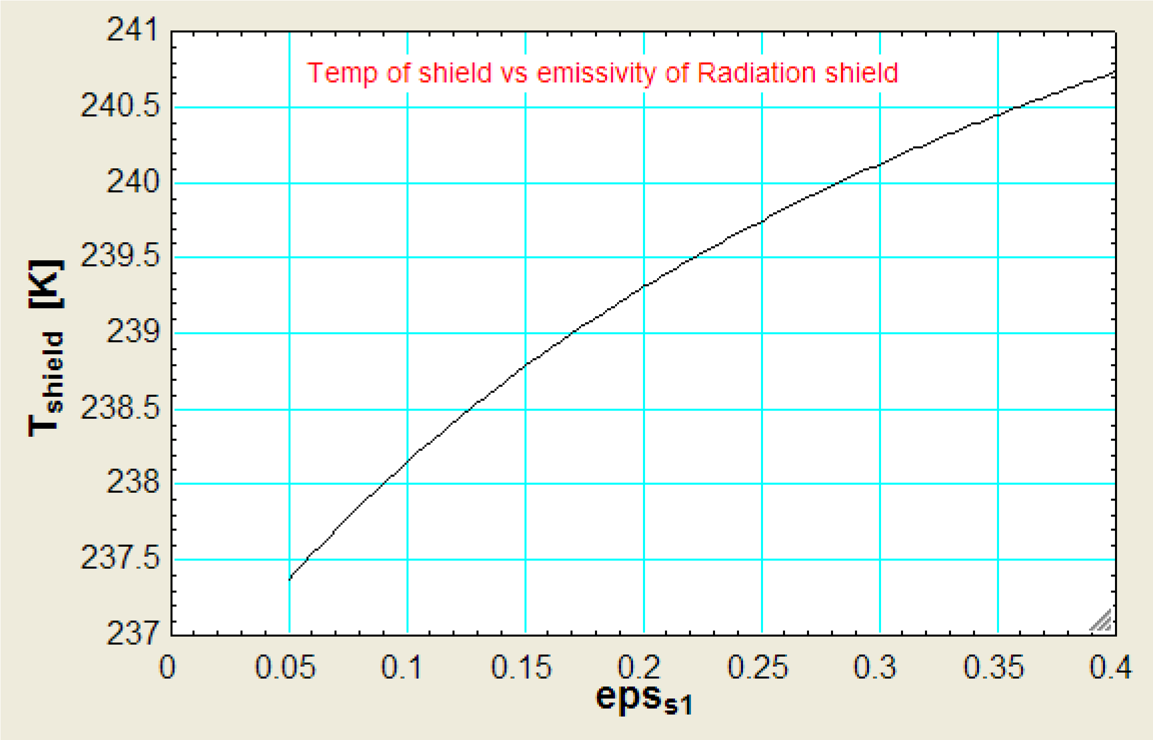
To plot the Percentage reduction and the equilibrium temp of the shield as emissivity of shield varies from 0.05 to 0.3:

First, compute the Parametric Table, keeping both eps_s1 = eps_s2 = 0.05, 0.1, 0.15 ….. 0.4.

1..8	eps_{s1}	eps_{s2}	PercentReduction [%]	T_{shield} [K]
Run 1	0.05	0.05	86.86	237.4
Run 2	0.1	0.1	76.31	238.2
Run 3	0.15	0.15	67.65	238.8
Run 4	0.2	0.2	60.41	239.3
Run 5	0.25	0.25	54.27	239.7
Run 6	0.3	0.3	49	240.1
Run 7	0.35	0.35	44.43	240.4
Run 8	0.4	0.4	40.41	240.7

Now, plot the results:




Temp of shield vs emissivity of Radiation shield
T_{shield} [K]
eps_{s1}
241
240.5
240
239.5
239
238.5
238
237.5
237
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4

==

"**Prob.5.D.17.** Two coaxial cylinders of diameters D1 = 0.1 m and D2 = 0.3 m and emissivities eps1 = 0.7 and eps2 = 0.4 are maintained at temps T1 = 800 K and T2 = 500 K respectively. A coaxial shield of diameter D3 = 0.2 m and emissivity eps_s on both its surfaces is placed between the cylinders. Determine eps_s if the radiation heat transfer between the cylinders is to be reduced to 15% of that without the radiation shield. What is the equilibrium temp of the shield at that time?"

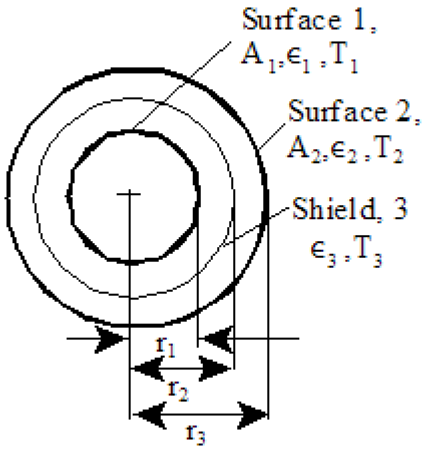


Fig.(a)

EES Solution:

"Data:"

R1 = 0.05 "[m]"
R2 = 0.15"[m]"
R_shield = 0.1 "[m]"

L = 1 "[m]"

eps_1 = 0.7
eps_2 = 0.4

"Let: eps_s1 = eps_s2 = eps_s … emissivity of shield"

T1 = 800 "[K]"

T2 = 500 "[K]"

"Calculations:"

Q12 = Q12_concentric_cyl(R1, R2, L , eps_1, eps_2,T1,T2) "….. heat transfer without radiation shield"

Q12_shield = Q12_concentric_cyl_one_shield(R1, R2,R_shield, L , eps_1, eps_2, eps_s, eps_s, T1,T2) "….heat transfer with shield"

Q12_shield = 0.15 * Q12 "….given condition that heat transfer with shield is 15% of that without shield"

T_shield = Temp_Shield_concentric_cyl(R1, R2,R_shield, L , eps_1, eps_2, eps_s, eps_s, T1,T2)

Results:

Unit Settings: SI C Pa J mass deg

$eps_1 = 0.7$	$eps_2 = 0.4$	$eps_s = 0.0875$	L = 1 [m]
Q12 = 3206 [W]	$Q12_{shield}$ = 480.9 [W]	R1 = 0.05 [m]	R2 = 0.15 [m]
R_{shield} = 0.1 [m]	T1 = 800 [K]	T2 = 500 [K]	T_{shield} = 692.7 [K]

Thus:

Q12 without shield = 3206 W …. Ans.

Q12 with shield = 480.9 W …. Ans.

Their ratio is = (480.9 * 100) / 3206 = 15% …. Verified

Emissivity of shield to achieve this = eps_s = 0.0875 …. Ans.

Temp of shield (eps_s = 0.0875) is: 692.7 K … Ans.

==

"**Prob.5.D.18.** Liquid nitrogen (LN2) is stored in a dewar made of two concentric spheres, with the space between them evacuated. The inner sphere has an outer dia D1 = 1 m and for outer sphere the inner dia D2 = 1.2 m and emissivities eps1 = 0.2 and eps2 = 0.2. Temperatures are maintained at temps T1 = 78 K and T2 = 300 K respectively. A coaxial shield of diameter D3 = 1.1 m and emissivity eps_s = 0.05 on both its surfaces is placed between the two spheres. If the latent heat of vaporization of LN2 is 2 × 10^5 J/kg, determine:

a) the boil-off rate of liquid nitrogen when there is no shield,
b) the boil-off rate of liquid nitrogen when the shield is present. What is the equilibrium temp of the shield at that time?
c) Plot the variation of boil-off rate and the shield temp as emissivity of shield (on its either side) varies from 0.05 to 0.4."

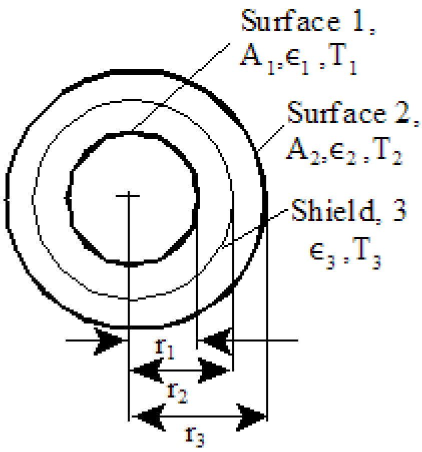


Fig.(a)

EES Solution:

"Data:"

R1 = 0.5 "[m]"

R2 = 0.6"[m]"

R_shield = 0.55 "[m]"

eps_1 = 0.2

eps_2 = 0.2

eps_s1 = 0.05

eps_s2 = 0.05

T1 = 78 "[K]"

T2 = 300 "[K]"

h_fg = 2E05 [J/kg]

"Calculations:"

Q12 = Q12_concentric_spheres(R1, R2, eps_1, eps_2,T1,T2)"[W]…Q12 with no shield"

Q12_shield = Q12_concentric_sph_one_shield(R1, R2,R_shield, eps_1, eps_2, eps_s1, eps_s2, T1,T2) "[W]….Q12 with shield"

T_shield=Temp_Shield_concentric_spheres(R1,R2,R_shield,eps_1,eps_2,eps_s1,eps_s2,T1,T2)"[K]…. Temp of shield"

BoilOff_with_shield = Abs(Q12_shield) / h_fg * 3600"[kg/h]….boil off rate of liq. nitrogen, when there is shield"

BoilOff_no_shield = Abs(Q12) / h_fg * 3600"[kg/h]….boil off rate of liq. nitrogen, when there is no shield"

Results:

Unit Settings: SI C Pa J mass deg

$BoilOff_{no,shield}$ = 3.324 [kg/h]	$BoilOff_{with,shield}$ = 0.6462 [kg/h]	eps_1 = 0.2
eps_2 = 0.2	eps_{s1} = 0.05	eps_{s2} = 0.05
h_{fg} = 200000 [J/kg]	Q12 = -184.7 [W]	$Q12_{shield}$ = -35.9 [W]
R1 = 0.5 [m]	R2 = 0.6 [m]	R_{shield} = 0.55 [m]
T1 = 78 [K]	T2 = 300 [K]	T_{shield} = 254.7 [K]

Thus:

LN2 Boil off rate without shield = 3.324 kg/h …. Ans.

LN2 Boil off with shield = 0.6462 kg/h …. Ans.

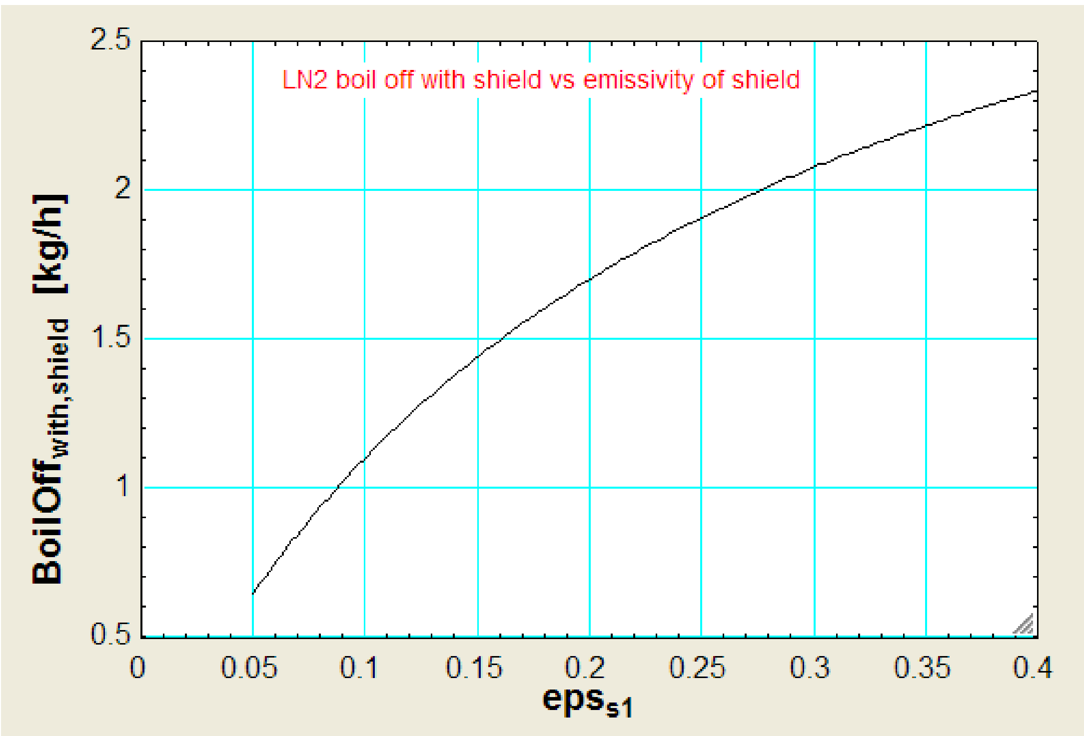
Equilibrium temp of shield = 254.7 K … Ans.

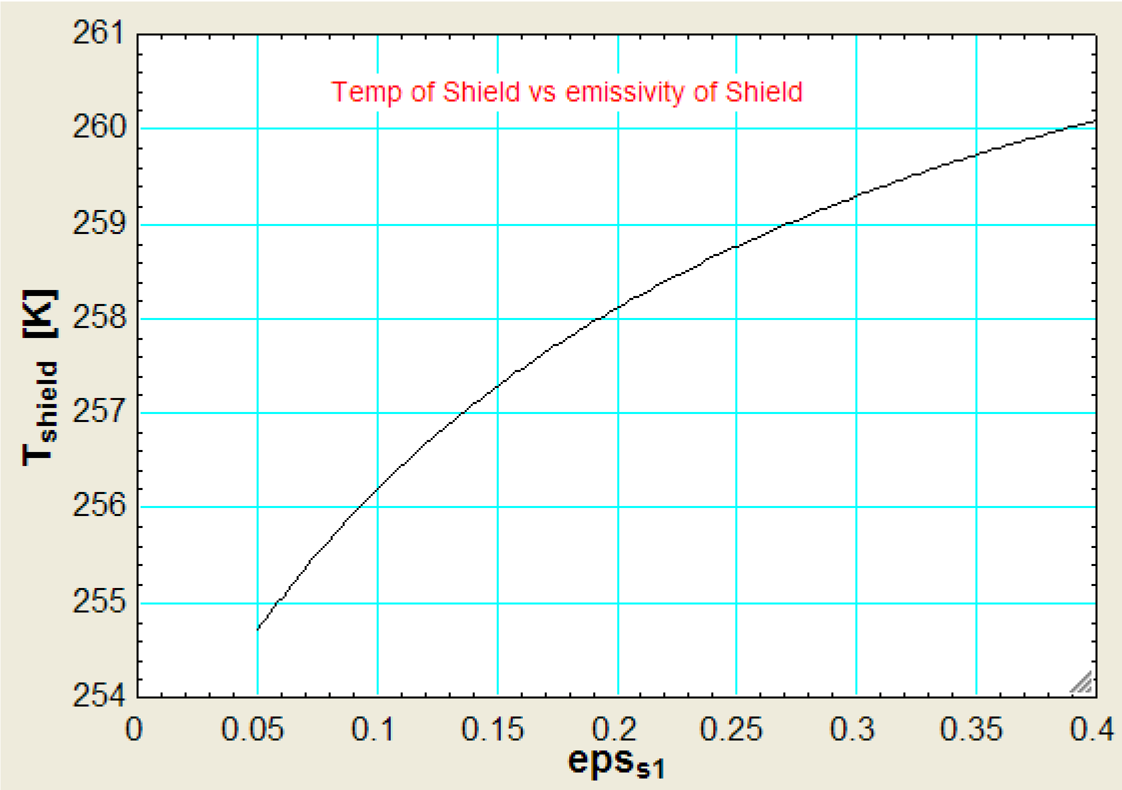
To plot the variation of boil-off rate and the shield temp as emissivity of shield (on its either side) varies from 0.05 to 0.4:

First, compute the Parametric Table, keeping both eps_s1 = eps_s2 = 0.05, 0.1, 0.15 ….. 0.4.

1..8	eps_{s1}	eps_{s2}	$BoilOff_{no,shield}$ [kg/h]	$BoilOff_{with,shield}$ [kg/h]	T_{shield} [K]
Run 1	0.05	0.05	3.324	0.6462	254.7
Run 2	0.1	0.1	3.324	1.101	256.2
Run 3	0.15	0.15	3.324	1.439	257.3
Run 4	0.2	0.2	3.324	1.699	258.1
Run 5	0.25	0.25	3.324	1.906	258.8
Run 6	0.3	0.3	3.324	2.075	259.3
Run 7	0.35	0.35	3.324	2.215	259.7
Run 8	0.4	0.4	3.324	2.333	260.1

Now, plot the results:





===

Prob.5.D.19. Write VBA Functions for heat transfer when radiation shield is present for the cases of:

1) Parallel plates,
2) concentric cylinders, and
3) concentric spheres

Also write Functions for the equilibrium temp of the shield in each case.

EXCEL Solution:

Before we write VBA Functions for heat transfer when the shield is present, let us recollect the Functions we wrote earlier for heat transfer without the shield for the above three cases, since in most of the problems, we are asked to compare the heat transfer with and without the shields:

Heat transfer Q12 (W), when there is *no* radiation shield:

```
Function Q_12_Infinite_large_parallel_plates(Area As Double, eps_1 As Double, eps_2 As Double, _
T_1 As Double, T_2 As Double) As Double

'Gives Q_12 (W) between two infinite parallel plates of area A (m2) each,
'i.e. (A_1/A_2) = 1, F_12 = 1

Dim sigma As Double
sigma = 0.0000000567 'W/m2.K^4

Q_12_Infinite_large_parallel_plates = (sigma * Area * (T_1 ^ 4 - T_2 ^ 4)) / _
(1 / eps_1 + 1 / eps_2 - 1)

End Function
```

```
Function Q_12_Infinite_concentric_cylinders(A_1 As Double, R_1 As Double, R_2 As Double, _
eps_1 As Double, eps_2 As Double, T_1 As Double, T_2 As Double) As Double

'Gives Q_12 (W) between infinite concentric cylinders,
'i.e. (A_1/A_2) = R_1/R_2, F_12 = 1

Dim sigma As Double
sigma = 0.0000000567 'W/m2.K^4

Q_12_Infinite_concentric_cylinders = (sigma * A_1 * (T_1 ^ 4 - T_2 ^ 4)) / _
(1 / eps_1 + ((1 - eps_2) / eps_2) * (R_1 / R_2))

End Function
```

```
Function Q_12_concentric_spheres(A_1 As Double, R_1 As Double, R_2 As Double, _
eps_1 As Double, eps_2 As Double, T_1 As Double, T_2 As Double) As Double

'Gives Q_12 (W) between concentric spheres,
'i.e. (A_1/A_2) = (R_1/R_2)^2, F_12 = 1

Dim sigma As Double
sigma = 0.0000000567 'W/m2.K^4

Q_12_concentric_spheres = (sigma * 4 * Application.Pi() * R_1 ^ 2 * (T_1 ^ 4 - T_2 ^ 4)) / _
(1 / eps_1 + ((1 - eps_2) / eps_2) * (R_1 / R_2) ^ 2)

End Function
```

Heat transfer Q12 (W), when a *radiation shield is present* between the two surfaces:

```
Function Q_12_ParallelPlates_with_NRadiationShields(A As Double, N As Integer, _
eps_1 As Double, eps_2 As Double, eps_s1 As Double, eps_s2 As Double, _
T_1 As Double, T_2 As Double) As Double

'Gives Q_12 (W) ..heat tr with N radiation shieds

Dim sigma As Double, XX As Double, YY As Double, ZZ As Double
sigma = 0.0000000567 'W/m2.K^4

XX = A * sigma * (T_1 ^ 4 - T_2 ^ 4)
YY = 1 / eps_1 + 1 / eps_2 - 1
ZZ = 1 / eps_s1 + 1 / eps_s2 - 1
Q_12_ParallelPlates_with_NRadiationShields = XX / (YY + N * ZZ)
End Function
```

***Note** that the above Function can be used when there are N shields in between the two parallel plates. If there is only one shield, simply put N = 1.*

```
Function Q_12_ConcentricCylinders_with_RadiationShield(L As Double, R_1 As Double, _
R_2 As Double, R_3 As Double, eps_1 As Double, eps_2 As Double, eps_31 As Double, _
eps_32 As Double, T_1 As Double, T_2 As Double) As Double

'Gives Q_12 (W) ..heat tr with a cyl. radiation shied

Dim sigma As Double, A_1 As Double, XX As Double, YY As Double, ZZ As Double
sigma = 0.0000000567 'W/m2.K^4
A_1 = 2 * Application.Pi() * R_1 * L
XX = A_1 * sigma * (T_1 ^ 4 - T_2 ^ 4)
YY = (R_1 / R_2) * (1 / eps_2 - 1)
ZZ = (R_1 / R_3) * (1 / eps_31 + 1 / eps_32 - 1)

Q_12_ConcentricCylinders_with_RadiationShield = XX / (1 / eps_1 + YY + ZZ)

End Function
```

```
Function Q_12_ConcentricSpheres_with_RadiationShield(R_1 As Double, _
R_2 As Double, R_3 As Double, eps_1 As Double, eps_2 As Double, eps_31 As Double, _
eps_32 As Double, T_1 As Double, T_2 As Double) As Double

'Gives Q_12 (W) ..heat tr with a spherical radiation shied

Dim sigma As Double, A_1 As Double
sigma = 0.0000000567 'W/m2.K^4

A_1 = 4 * Application.Pi() * R_1 ^ 2

 Q_12_ConcentricSpheres_with_RadiationShield = (A_1 * sigma * (T_1 ^ 4 - T_2 ^ 4)) / _
 (1 / eps_1 + (R_1 / R_2) ^ 2 * (1 / eps_2 - 1) + (R_1 / R_3) ^ 2 * (1 / eps_31 + 1 / eps_32 - 1))


End Function
```

Equilibrium temp of shield, T_shield (K), when a *radiation shield is present* between the two surfaces:

```
Function T_shield_ParallelPlates(eps_1 As Double, eps_2 As Double, eps_31 As Double, _
eps_32 As Double, T_1 As Double, T_2 As Double) As Double
'Gives Q_12 (W) ..heat tr with N radiation shieds

Dim sigma As Double, XX As Double, YY As Double, ZZ As Double
Dim Q_12 As Double, AA As Double, A As Integer

A = 1 'm^2 .... area of plate
sigma = 0.0000000567 'W/m2.K^4
XX = A * sigma * (T_1 ^ 4 - T_2 ^ 4)
YY = 1 / eps_1 + 1 / eps_2 - 1
ZZ = 1 / eps_31 + 1 / eps_32 - 1

Q_12 = XX / (YY + ZZ) '..heat transfer when one shield is present

AA = 1 / eps_1 + 1 / eps_31 - 1

T_shield_ParallelPlates = (T_1 ^ 4 - Q_12 * AA / (A * sigma)) ^ 0.25

End Function
```

```
Function T_shield_ConcentricCylinders(L As Double, R_1 As Double, _
R_2 As Double, R_3 As Double, eps_1 As Double, eps_2 As Double, eps_31 As Double, _
eps_32 As Double, T_1 As Double, T_2 As Double) As Double

'Gives T_shield ..equilibrium temp of radiation shied, (K)


Dim sigma As Double, A_1 As Double, XX As Double, YY As Double, ZZ As Double
Dim Q_12 As Double, AA As Double

sigma = 0.0000000567 'W/m2.K^4

A_1 = 2 * Application.Pi() * R_1 * L
XX = A_1 * sigma * (T_1 ^ 4 - T_2 ^ 4)
YY = (R_1 / R_2) * (1 / eps_2 - 1)
ZZ = (R_1 / R_3) * (1 / eps_31 + 1 / eps_32 - 1)

Q_12 = XX / (1 / eps_1 + YY + ZZ) '..heat transfer when one shield is present

AA = 1 / eps_1 + (R_1 / R_3) * (1 / eps_31 - 1)

T_shield_ConcentricCylinders = (T_1 ^ 4 - Q_12 * AA / (A_1 * sigma)) ^ 0.25

End Function
```

```
Function T_shield_ConcentricSpheres(R_1 As Double, R_2 As Double, R_3 As Double, _
eps_1 As Double, eps_2 As Double, eps_31 As Double, _
eps_32 As Double, T_1 As Double, T_2 As Double) As Double

'Gives T_shield (K)) ..equilibrium temp of a spherical radiation shied

Dim sigma As Double, A_1 As Double, Q_12 As Double, AA As Double

sigma = 0.0000000567 'W/m2.K^4

A_1 = 4 * Application.Pi() * R_1 ^ 2

'Heat transfer when one shield is present is given by:

Q_12 = (A_1 * sigma * (T_1 ^ 4 - T_2 ^ 4)) / _
 (1 / eps_1 + (R_1 / R_2) ^ 2 * (1 / eps_2 - 1) + (R_1 / R_3) ^ 2 * (1 / eps_31 + 1 / eps_32 - 1))

AA = 1 / eps_1 + (R_1 / R_3) ^ 2 * (1 / eps_31 - 1)

T_shield_ConcentricSpheres = (T_1 ^ 4 - Q_12 * AA / (A_1 * sigma)) ^ 0.25

End Function
```

===

Now, let us solve some problems in EXCEL using the above Functions.

Prob.5.D.20. Consider two large parallel plates, one at 1000 K with emissivity 0.8 and the other at 300 K having emissivity 0.6. A radiation shield is placed between them. The shield has emissivity of 0.1 on the side facing the hot plate and 0.3 on the side facing the cold plate. Calculate the percentage reduction in radiation heat transfer as a result of radiation shield. [P.U. 2000]

(b) Plot the Percentage reduction in heat transfer and the temp of Shield as emissivity of shield on both sides (eps_s1 = eps_s2 = eps_s) varies from 0.05 to 0.4:

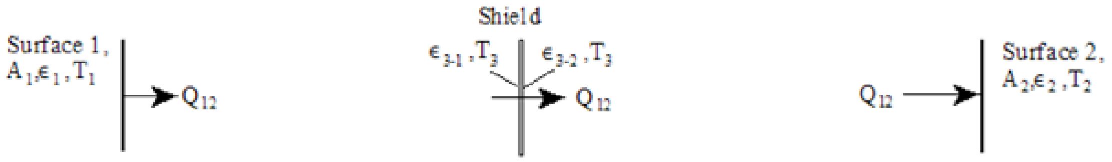


Fig.(a)

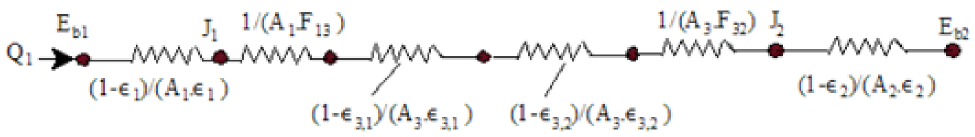


Fig.(b)

EXCEL Solution:

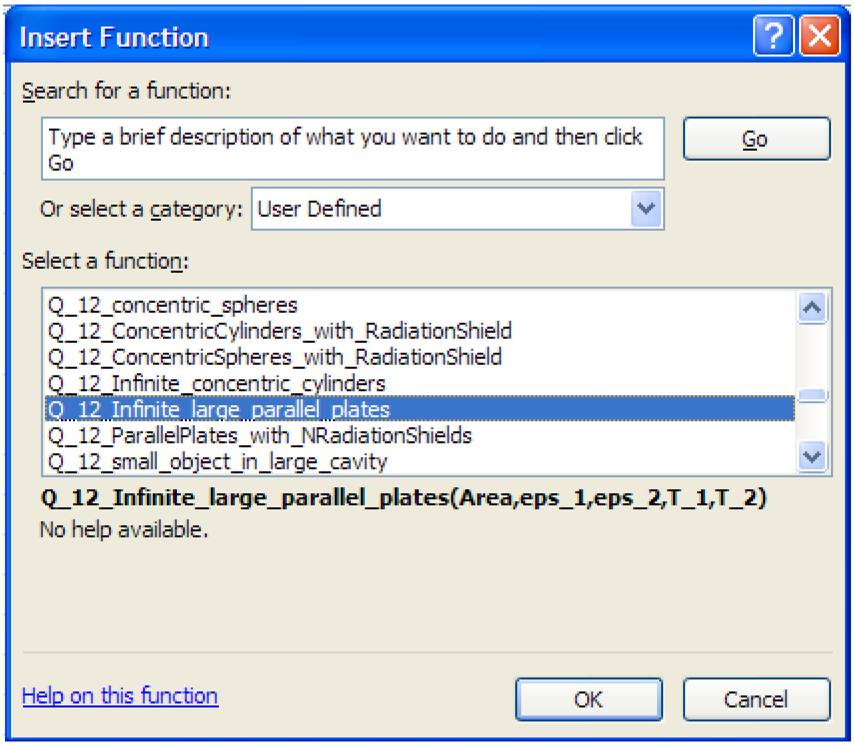
Following are the steps:

1. Set up the EXCEL worksheet, enter data:

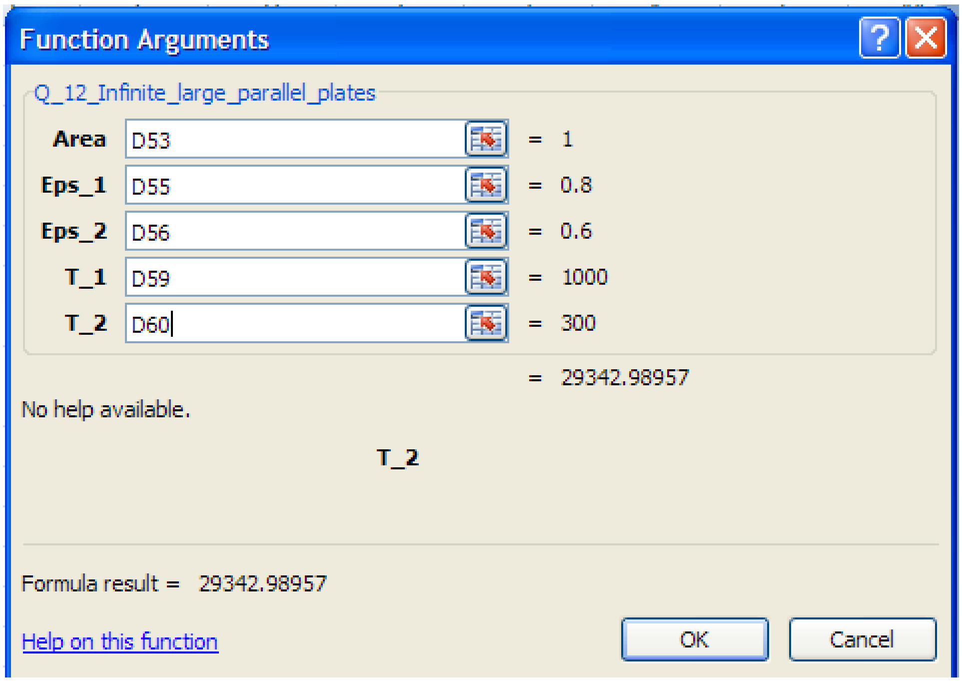
	A	B	C	D	E
51		Data:			
52					
53		Area of plate	A	1	m^2
54		No. of shields	N	1	
55		emissivity of plate 1	eps1	0.8	
56		emissivity of plate 2	eps2	0.6	
57		emissivity of shield surface facing plate 1	epsilon_31	0.1	
58		emissivity of shield surface facing plate 2	epsilon_32	0.3	
59		Temp, plate 1	T_1	1000	K
60		Temp, plate 2	T_2	300	K

2. Do calculations using the VBA Functions written above. Functions are available under 'User Defined' category:

For the case of parallel plates with no radiation shield, choose the appropriate Function:



Press OK. We get the following window; fill it up as shown:



Press OK. We get the result in cell D62:

D62 | =Q_12_Infinite_large_parallel_plates(D53,D55,D56,D59,D60)

	A	B	C	D	E	F	G
60		Temp, plate 2	T_2	300	K		
61							
62		Heat transfer with no shield	Q_12_noShield	29342.99	W		

In the above, see the Formula entered in cell D62 for Q_12 _noshield.

Similarly, enter Function for Q12 with one Shield (i.e. N = 1):

D64 | =Q_12_ParallelPlates_with_NRadiationShields(D53,D54,D55,D56,D57,D58,D59,D60)

	A	B	C	D	E	F
61						
62		Heat transfer with no shield	Q_12_noShield	29342.98957	W	
63						
64		Heat transfer with shield	Q_12_shield	3946.717895	W	

And, see the Formula entered in cell D64 for Q_12 _shield.

And enter Function for the Temp of the Shield:

D66 | =T_shield_ParallelPlates(D55,D56,D57,D58,D59,D60)

	A	B	C	D	E	F
63						
64		Heat transfer with shield	Q_12_shield	3946.718	W	
65						
66		Equilibrium temp of shield	T_shield	731.6305	K	

And, see the formula entered in cell D66 for the temp of Shield, in the Formula bar above

And, percentage reduction in heat transfer because of the shield is:

D68 | =(D62-D64)*100/D62

	A	B	C	D	E
63					
64		Heat transfer with shield	Q_12_shield	3946.718	W
65					
66		Equilibrium temp of shield	T_shield	731.6305	K
67					
68		Percentage reduction in heat transfer, due to shield	Percent_reduction	86.54971	%.....Ans.

Thus:

Q12 with no shield = 29342.99 W …. Ans.

Q12 with shield = 3946.718 W … Ans.

Percentage reduction due to shield = 86.55% …. Ans.

Equilibrium temp of shield = 731.635 K … Ans.

Plot the Percentage reduction in heat transfer and the temp of Shield as emissivity of shield on both sides (eps_s1 = eps_s2 = eps_s) varies from 0.05 to 0.4:

First set up a Table as shown:

C75 =Q_12_ParallelPlates_with_NRadiationShields(1,1,$D$55,$D$56,B75,B75,$D$59,$D$60)

	A	B	C	D	E	F	G
73							
74		eps_s	Q_12_shield(W)	Percent_reduction	T_shield (K)		
75		0.05	1374.519	95.316	844.697		
76		0.1					
77		0.15					
78		0.2					
79		0.25					
80		0.3					
81		0.35					
82		0.4					

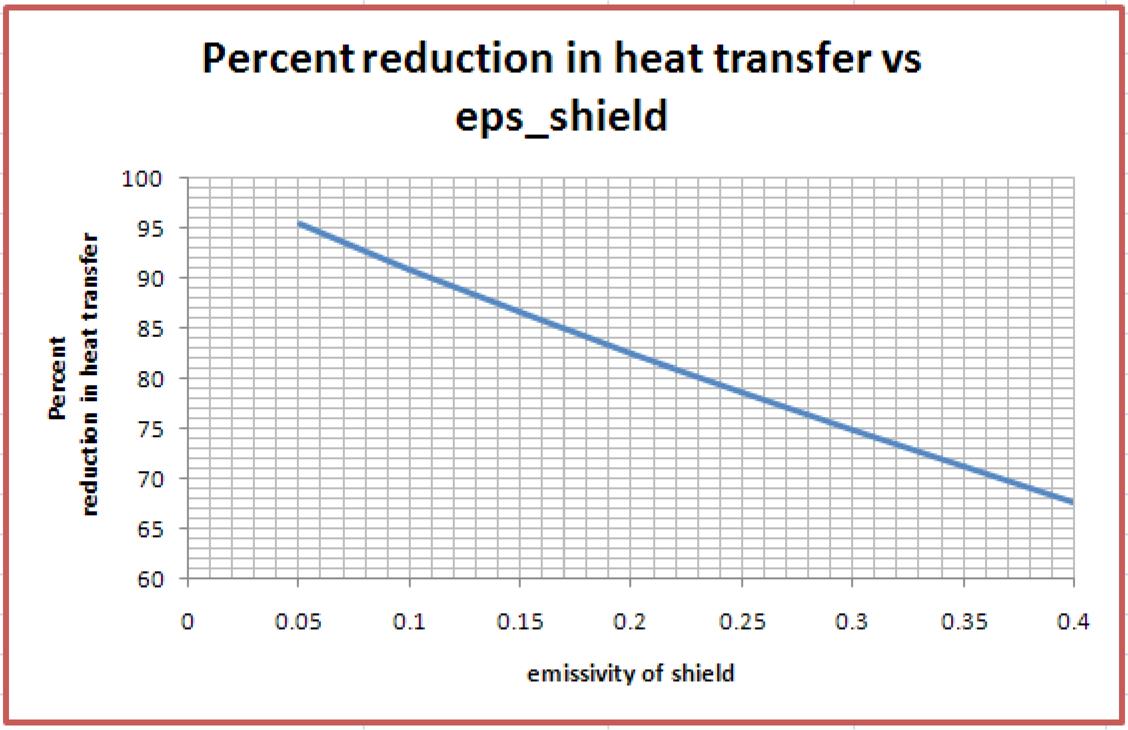
In the above Table, in row 75 enter formulas for Q_12 with shield, % reduction in heat transfer and T_shield as shown. Formula for Q12 with shield, entered in cell C75, can be seen in the Formula bar. Remember to enter references to eps_s1 ans eps_s2 with relative reference so that we can drag-copy to get values at other values of eps_s. Similarly, for % reduction and T_shield.

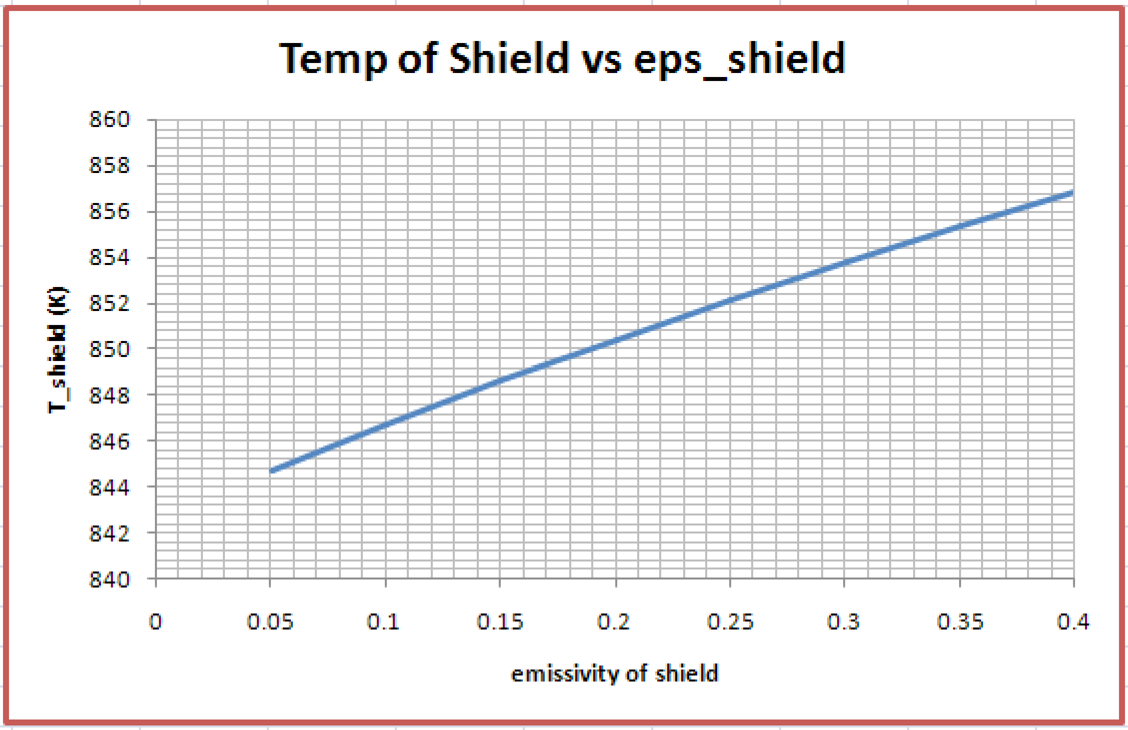
Now, select cells C75 to E75 and drag-copy to the end of Table, i.e. up to cell E82. Immediately, all calculations are made and the Table is filled up:

E82 =T_shield_ParallelPlates($D$55,$D$56,B82,B82,$D$59,$D$60)

	A	B	C	D	E
73					
74		eps_s	Q_12_shield(W)	Percent_reduction	T_shield (K)
75		0.05	1374.519	95.316	844.697
76		0.1	2688.800	90.837	846.693
77		0.15	3946.718	86.550	848.590
78		0.2	5151.823	82.443	850.396
79		0.25	6307.372	78.505	852.117
80		0.3	7416.360	74.725	853.758
81		0.35	8481.546	71.095	855.326
82		0.4	9505.475	67.606	856.825

Now, plot the results in EXCEL:





==

Prob.5.D.21. Two parallel plates are at T1 = 700 K and T2 = 350 K. Their emissivities are 0.6 and 0.9 respectively. Determine the emissivity of a radiation shield if heat transfer between the plates is to be reduced to 10% of that without the radiation shield.

EXCEL Solution:

Following are the steps:

1. Set up the EXCEL worksheet, enter data:

	A	B	C	D	E
107		Data:			
108					
109		Area of plate	A	1	m^2
110		No. of shields	N	1	
111		emissivity of plate 1	eps1	0.6	
112		emissivity of plate 2	eps2	0.9	
113		emissivity of shield	epsilon_3	0.1	....assumed
114					
115		Temp, plate 1	T_1	700	K
116		Temp, plate 2	T_2	350	K

In the above, emissivity of shield (on its either side) is assumed as 0.1; its correct value will be found out later by Goal Seek.

2. Now, do calculations. Find out Q12_withnoshield and Q12_with shield and put the condition that Q12 with shield should be equal to 10% of Q12 with no shield:

For Q12_noshield:

D118 f_x =Q_12_Infinite_large_parallel_plates(1,D111,D112,D115,D116)

	A	B	C	D	E
113		emissivity of shield	epsilon_3	0.1	....assumed
114					
115		Temp, plate 1	T_1	700	K
116		Temp, plate 2	T_2	350	K
117					
118		Heat transfer with no shield	Q_12_noShield	7179.083789	W

For Q12_with shield:

D120 f_x =Q_12_ParallelPlates_with_NRadiationShields(1,1,D111,D112,D113,D113,D115,D116)

	A	B	C	D	E	F
116		Temp, plate 2	T_2	350	K	
117						
118		Heat transfer with no shield	Q_12_noShield	7179.083789	W	
119						
120		Heat transfer with shield	Q_12_shield	614.2531584	W	

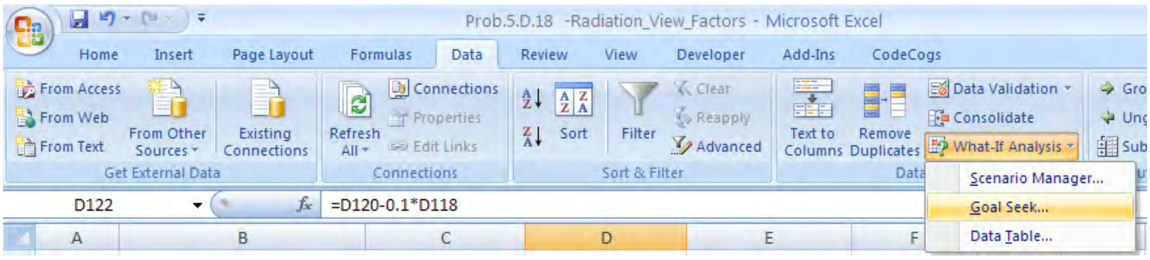
And enter the constraint:

D122 f_x =D120-0.1*D118

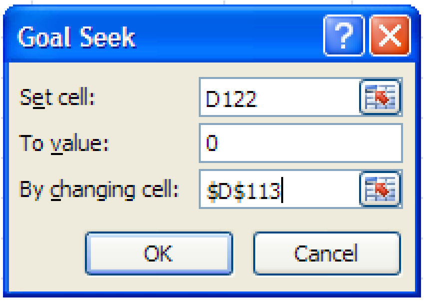
	A	B	C	D	
116		Temp, plate 2	T_2	350	K
117					
118		Heat transfer with no shield	Q_12_noShield	7179.083789	W
119					
120		Heat transfer with shield	Q_12_shield	614.2531584	W
121					
122		Contraint:	Q12_shield - 0.1*Q12_noshield	-103.6552205	W

Now, apply Goal Seek to make value in D122 (see the eqn in Formula bar above) zero by changing the value of eps_shield in cell C113:

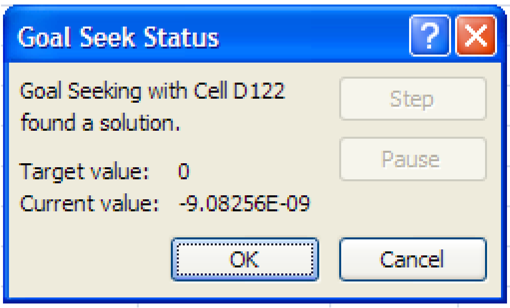
Go to Data – What If Analysis – Goal eek:



Click on Goal Seek. We get the following window. Fill it up as shown:



Press OK. We get:



Goal Seek has found a solution. Click OK. See the value of eps_shield in cell D113:

C113 | f_x | epsilon_3

	A	B	C	D	E
107		Data:			
108					
109		Area of plate	A	1	m^2
110		No. of shields	N	1	
111		emissivity of plate 1	eps1	0.6	
112		emissivity of plate 2	eps2	0.9	
113		emissivity of shield	epsilon_3	0.117647059	....assumed
114					
115		Temp, plate 1	T_1	700	K
116		Temp, plate 2	T_2	350	K
117					
118		Heat transfer with no shield	Q_12_noShield	7179.083789	W
119					
120		Heat transfer with shield	Q_12_shield	717.9083789	W
121					
122		Contraint:	Q12_shield - 0.1*Q12_noshield	-9.08256E-09	W
123					

Therefore the emissivity of shield required is: 0.118 …. Ans.

===

Prob.5.D.22. Two coaxial cylinders of diameters D1 = 0.1 m and D2 = 0.3 m, are at T1 = 1000 K and T2 = 500 K. Their emissivities are 0.7 and 0.4 respectively. Now, a coaxial radiation shield of dia D3 = 0.2 m and emissivity = 0.2 (on either face) is placed between the two cylinders. Determine (a) the net rate of heat transfer between the cylinders per unit length of cylinders, with and without the shield.

(b) equilibrium temp of the shield, and

(c) plot Q12_with shield and temp of shield as emissivity of shield is varied from 0.05 to 0.4.

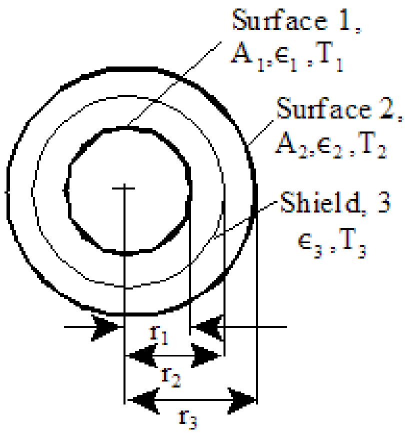


Fig.(a)

EXCEL Solution:

Following are the steps:

1) Set up the EXCEL worksheet, enter data:

D134 =2*PI()*D131*D130

	A	B	C	D	
128		Data:			
129					
130		Length of cyl.	L	1	m
131		Radius, inner cyl	R_1	0.05	m
132		Radius, outer cyl.	R_2	0.15	m
133		Radius, shield	R_3	0.1	m
134		Surface area of inner cyl	A_1	0.31416	m^2
135					
136		emissivity of inner cyl. 1	eps1	0.7	
137		emissivity of outer cyl.2	eps2	0.4	
138		emissivity of shield	epsilon_3	0.2	
139					
140		Temp, inner cyl. 1	T_1	1000	K
141		Temp, outer cyl. 2	T_2	500	K

2. Do the calculations for Q12 with and without shield and for the temp of shield using the VBA Functions written earlier:

Heat transfer without the Shield:

D143 =Q_12_Infinite_concentric_cylinders(D134,D131,D132,D136,D137,D140,D141)

	A	B	C	D	E	F
140		Temp, inner cyl. 1	T_1	1000	K	
141		Temp, outer cyl. 2	T_2	500	K	
142						
143		Heat transfer with no shield	Q_12_noShield	8659.014751	W	

In the above, eqn entered in cell D143 can be seen in the Formula bar.

Heat transfer with the Shield being present:

D145 =Q_12_ConcentricCylinders_with_RadiationShield(D130,D131,D132,D133,D136,D137,D138,D138,D140,D141)

	A	B	C	D	E	F	G	H	I
140		Temp, inner cyl. 1	T_1	1000	K				
141		Temp, outer cyl. 2	T_2	500	K				
142									
143		Heat transfer with no shield	Q_12_noShield	8659.014751	W				
144									
145		Heat transfer with shield	Q_12_shield	2597.7044	W				

In the above, eqn entered in cell D145 can be seen in the Formula bar.

Equilibrium temp of the Shield:

D147 =T_shield_ConcentricCylinders(D130,D131,D132,D133,D136,D137,D138,D138,D140,D141)

	A	B	C	D	E	F	G
140		Temp, inner cyl. 1	T_1	1000	K		
141		Temp, outer cyl. 2	T_2	500	K		
142							
143		Heat transfer with no shield	Q_12_noShield	8659.014751	W		
144							
145		Heat transfer with shield	Q_12_shield	2597.7044	W		
146							
147		Equilibrium temp of Shield	T_shield	840.896	K		

In the above, eqn entered in cell D147 can be seen in the Formula bar.

Thus:

Q12 with no shield = 8659.01 W …. Ans.

Q12 with shield = 2597.7 W … Ans.

Equilibrium temp of shield = 840.9 K … Ans.

(c) To plot Q12_with shield and temp of shield as emissivity of shield is varied from 0.05 to 0.4:

First, set up a Table as follows:

	A	B	C	D
151				
152		eps_s	Q_12_shield(W)	T_shield (K)
153		0.05	779.31	849.95
154		0.1		
155		0.15		
156		0.2		
157		0.25		
158		0.3		
159		0.35		
160		0.4		

In cells C153 and D153 above, we have used the respective VBA Functions. In the Functions, take care to refer to eps_shield by 'relative reference' so that we can drag-copy to calculate for other values of eps_shield.

For Q12_with shield, in cell C153, we have the Function:

=Q_12_ConcentricCylinders_with_RadiationShield($D$130,$D$131,$D$132,$D$133,$D$136,$D$137,B153,B153,$D$140,$D$141)

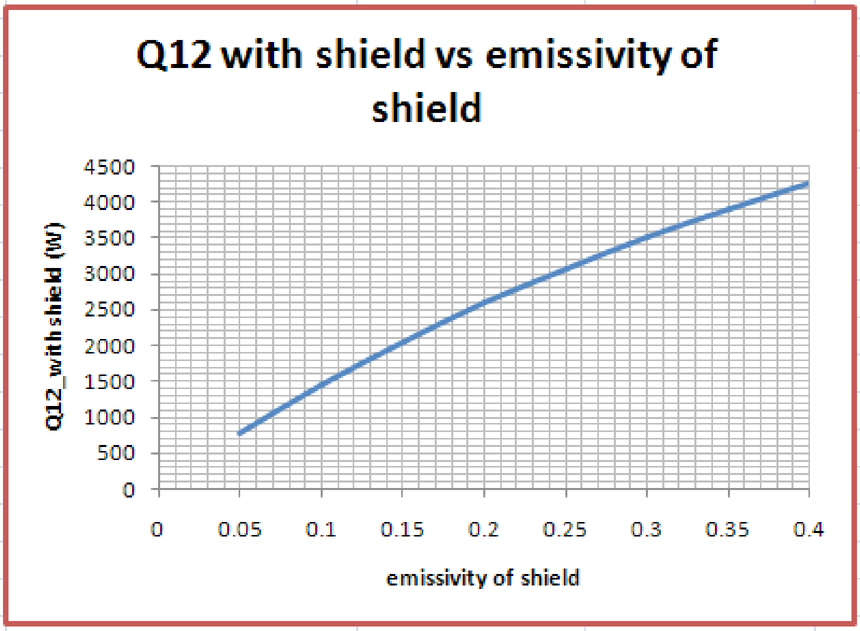
For T_ shield, in cell D153, we have the Function:

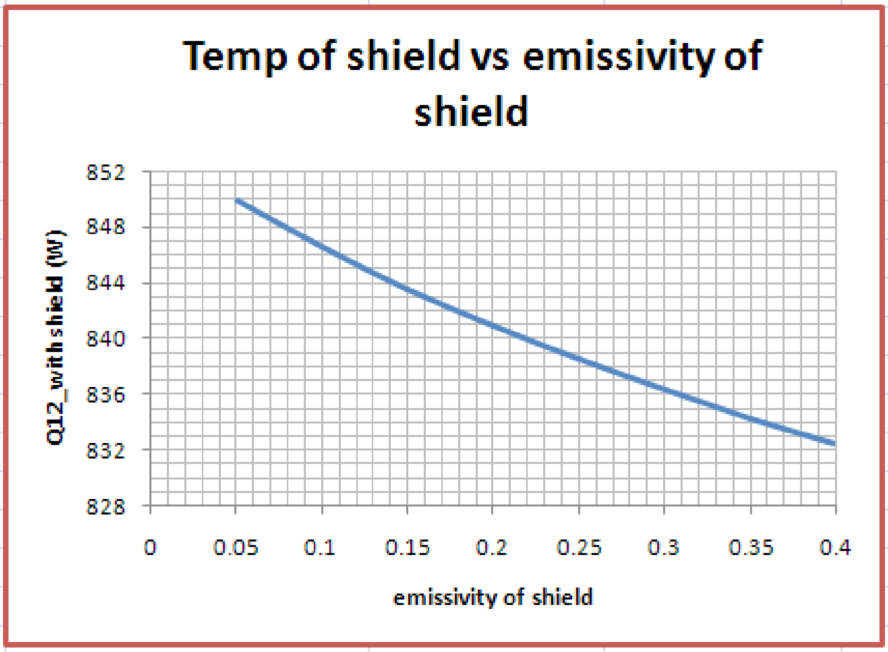
=T_shield_ConcentricCylinders($D$130,$D$131,$D$132,$D$133,$D$136,$D$137,B153,B153,$D$140,$D$141)

Now, select cells C153 to D153, and drag-copy to the end of Table, i.e. up to cell D160. Immediately, all calculations are done and the Table gets filled up:

	A	B	C	D
151				
152		eps_s	Q_12_shield(W)	T_shield (K)
153		0.05	779.31	849.95
154		0.1	1461.21	846.59
155		0.15	2062.88	843.59
156		0.2	2597.70	840.90
157		0.25	3076.23	838.47
158		0.3	3506.90	836.26
159		0.35	3896.56	834.25
160		0.4	4250.79	832.41

Now, plot the results in EXCEL:





==

Prob.5.D.23. Liquid oxygen (LOX) is stored in a dewar made of two concentric spheres, with the space between them evacuated. The inner sphere has an outer dia D1 = 0.3 m and for outer sphere the inner dia D2 = 0.4 m and emissivities eps1 = 0.2 and eps2 = 0.25. Temperatures are maintained at temps T1 = 90K and T2 = 300 K respectively. A coaxial shield of diameter D3 = 0.35 m and emissivity eps_s = 0.05 on both its surfaces is placed between the two spheres. If the latent heat of vaporization of LOX is 2.13 × 10^5 J/kg, determine:

a) the boil-off rate of liquid oxygen when there is no shield,
b) the boil-off rate of liquid oxygen when the shield is present. What is the equilibrium temp of the shield at that time?
c) Plot the variation of boil-off rate and the shield temp as emissivity of shield (on its either side) varies from 0.05 to 0.4.

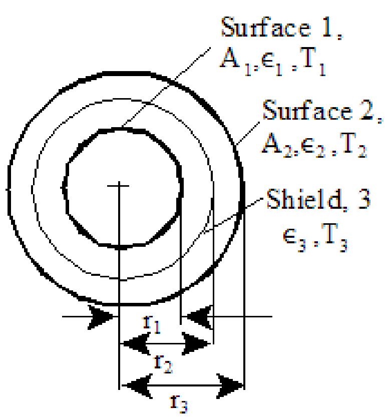


Fig.(a)

EXCEL Solution:

We shall use the VBA Functions for concentric spheres.

Following are the steps:

1. Set up the EXCEL worksheet, enter data:

	A	B	C	D	E
202					
203		Data:			
204					
205					
206		Radius, inner sphere	R_1	0.15	m
207		Radius, outer sphere	R_2	0.2	m
208		Radius, shield	R_3	0.175	m
209		Surface area of inner sphere	A_1	0.28274	m^2
210					
211		emissivity of inner sphere. 1	eps1	0.2	
212		emissivity of outer sphere.2	eps2	0.25	
213		emissivity of shield	epsilon_3	0.05	
214					
215		Temp, inner cyl. 1	T_1	90	K
216		Temp, outer cyl. 2	T_2	300	K
217		Latent heat of evap. Of LOX	h_fg	2.13E+05	J/kg

2. Do the calculations for Q12 with and without shield and for the temp of shield using the VBA Functions written earlier:

Heat transfer without the Shield:

D219 =Q_12_concentric_spheres(D209,D206,D207,D211,D212,D215,D216)

	A	B	C	D	E
217		Latent heat of evap. Of LOX	h_fg	2.13E+05	J/kg
218					
219		Heat transfer with no shield	Q_12_noShield	-19.2604	W

In the above, eqn entered in cell D219 can be seen in the Formula bar.

Negative sign indicates heat flow ***in to*** *the inner cylinder.*

Heat transfer with the Shield being present:

D221 | f_x | =Q_12_ConcentricSpheres_with_RadiationShield(D206,D207,D208,D211,D212,D213,D213,D215,D216)

	A	B	C	D	E	F	G	H
217		Latent heat of evap. Of LOX	h_fg	2.13E+05	J/kg			
218								
219		Heat transfer with no shield	Q_12_noShield	-19.2604	W			
220								
221		Heat transfer with shield	Q_12_shield	-3.6446	W			

In the above, eqn entered in cell D221 can be seen in the Formula bar.

Negative sign indicates heat flow ***in to*** *the inner cylinder.*

Percent reduction in heat transfer due to Shield:

D223 =(D219-D221)*100/D219

	A	B	C	D	E
217		Latent heat of evap. Of LOX	h_fg	2.13E+05	J/kg
218					
219		Heat transfer with no shield	Q_12_noShield	-19.2604	W
220					
221		Heat transfer with shield	Q_12_shield	-3.6446	W
222					
223		% reduction in heat transfer due to radiation shield	Percent_Reduction	81.077	%

In the above, eqn entered in cell D223 can be seen in the Formula bar.

Equilibrium temp of the Shield:

D225 =T_shield_ConcentricSpheres(D206,D207,D208,D211,D212,D213,D213,D215,D216)

	A	B	C	D	E	F
223		% reduction in heat transfer due to radiation shield	Percent_Reduction	81.077	%	
224						
225		Equilibrium temp of Shield	T_shield	257.196	K	

In the above, eqn entered in cell D225 can be seen in the Formula bar.

Evaporation Rate of LOX:

D227 =ABS(D221/D217)

	A	B	C	D	E
220					
221		Heat transfer with shield	Q_12_shield	-3.6446	W
222					
223		% reduction in heat transfer due to radiation shield	Percent_Reduction	81.077	%
224					
225		Equilibrium temp of Shield	T_shield	257.196	K
226					
227		Evapn. rate of LOX, with shield	m_evapn	1.7111E-05	kg/s
228		i.e.	m_evapn=	0.06160	kg/h

In the above, eqn entered in cell D227 can be seen in the Formula bar.

Note that evaporation rate is taken as positive quantity by using EXCEL Function ABS().

Thus:

Q12 with no shield = 19.2604 W …. Ans.

Q12 with shield = 3.6446 W … Ans.

% reduction in heat flow = 81.077% …. Ans.

Equilibrium temp of shield = 267.196 K … Ans.

Evaporation rate of LOX = 0.0616 kg/h … Ans.

(c) Plot the variation of boil-off rate and the shield temp as emissivity of shield (on its either side) varies from 0.05 to 0.4.

First, set up a Table as follows:

	A	B	C	D	E
229					
230					
231		eps_s	Q_12_shield (W)	m_evapn (kg/h)	T_shield (K)
232		0.05	3.64	0.0616	257.1964841
233		0.1			
234		0.15			
235		0.2			
236		0.25			
237		0.3			
238		0.35			
239		0.4			

In cells C232 and D232 above, we have used the respective VBA Functions. In the Functions, take care to refer to eps_shield by 'relative reference' so that we can drag-copy to calculate for other values of eps_shield.

For Q12_with shield, in cell C232, we have the Function:

=ABS(Q_12_ConcentricSpheres_with_RadiationShield($D$206,$D$207,$D$208,$D$211,$D$212, B232,B232,$D$215, $D$216))

For evaporation rate of LOX, m_evapn in cell D232, we have the equation:

=C232*3600/$D$217

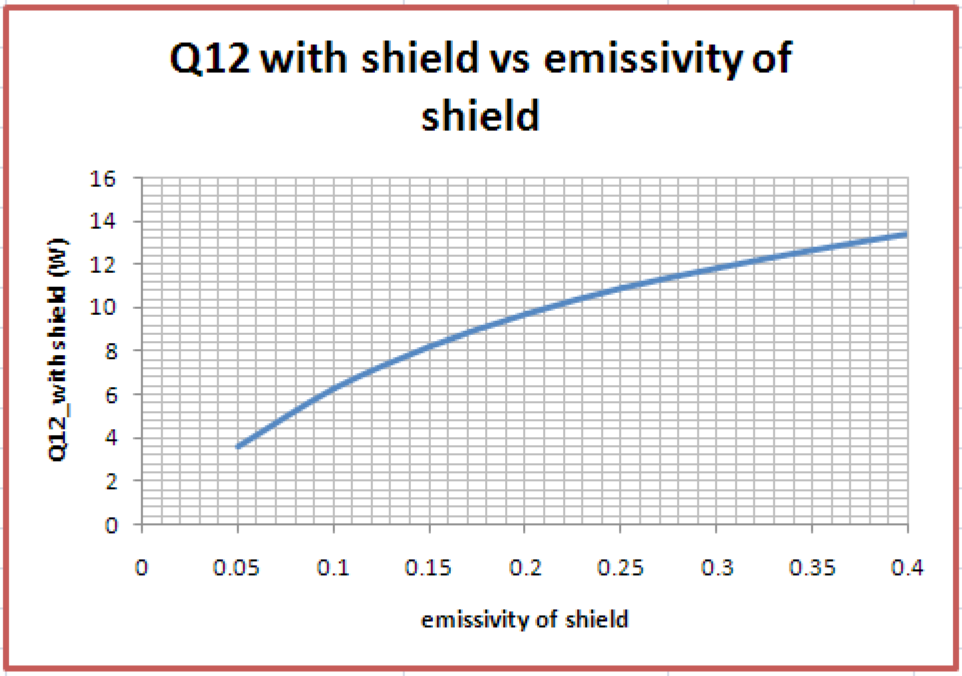
For T_ shield, in cell E232, we have the Function:

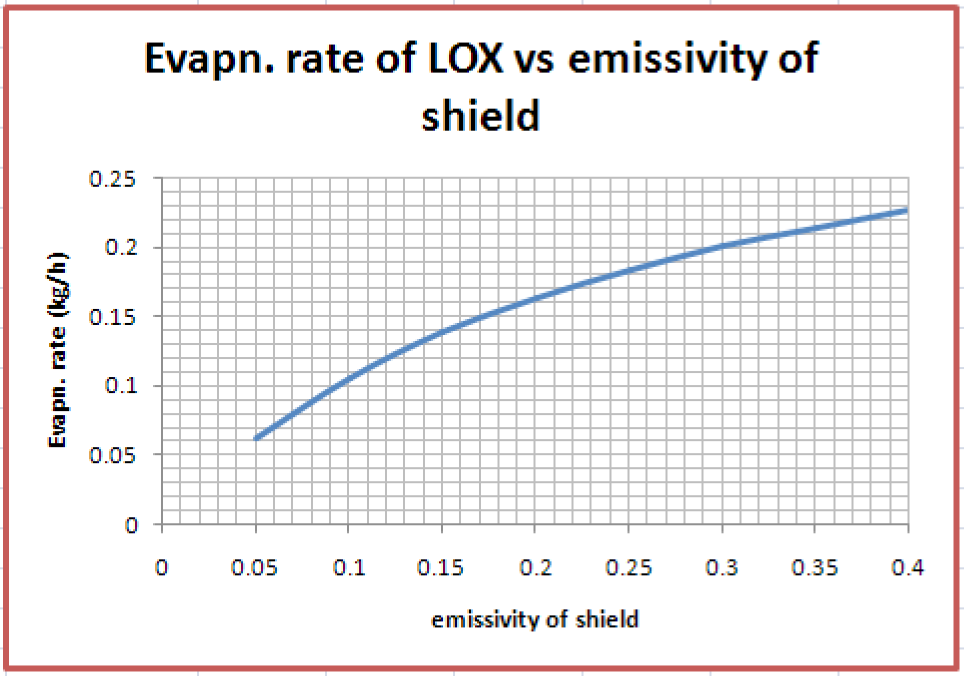
=T_shield_ConcentricSpheres($D$206,$D$207,$D$208,$D$211,$D$212,B232,B232,$D$215, $D$216)

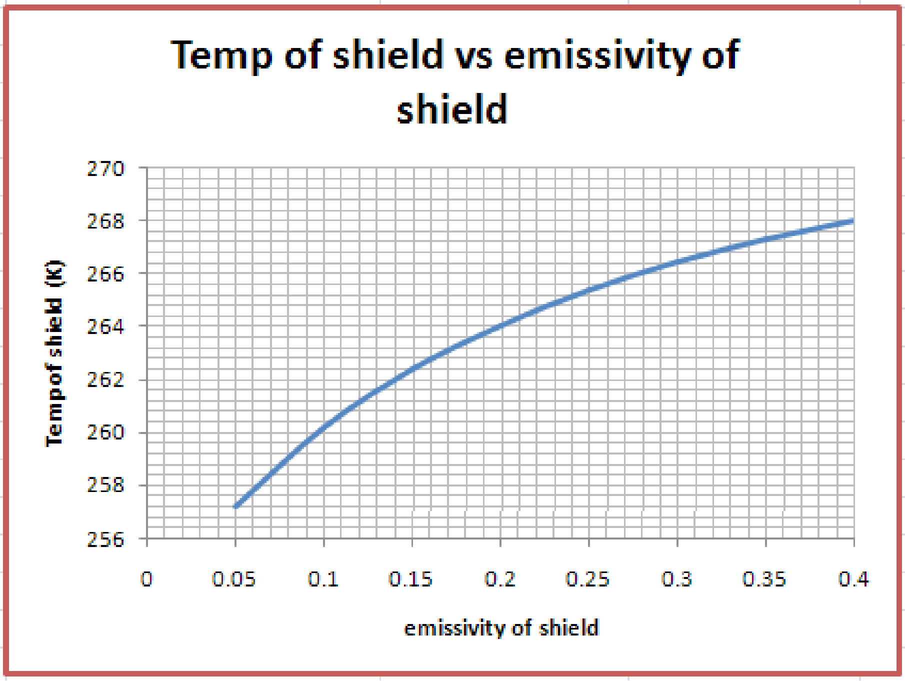
Now, select cells C232 to E232, and drag-copy to the end of Table, i.e. up to cell E239. Immediately, all calculations are done and the Table gets filled up:

	A	B	C	D	E
229					
230					
231		eps_s	Q_12_shield (W)	m_evapn (kg/h)	T_shield (K)
232		0.05	3.64	0.0616	257.196
233		0.1	6.24	0.1054	260.207
234		0.15	8.18	0.1382	262.393
235		0.2	9.68	0.1637	264.053
236		0.25	10.89	0.1840	265.357
237		0.3	11.87	0.2006	266.408
238		0.35	12.69	0.2145	267.273
239		0.4	13.38	0.2261	267.998

Now, plot the results in EXCEL:






Temp of shield vs emissivity of shield
Temp of shield (K)
270
268
266
264
262
260
258
256
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
emissivity of shield

==

5E 'Radiation error' in temperature measurement:

Recollect that:

T_f = actual temp of fluid
T_c = temp reading by the Thermocouple (TC)
T_w = temp of channel walls
T_s = temp of radiation shield
A_c = surface area of TC bead
A_s = surface area of shield (one side)
h = convection coeff. between the fluid and TC bead / shield
ε_c = emissivity of TC bead
ε_s = emissivity of shield

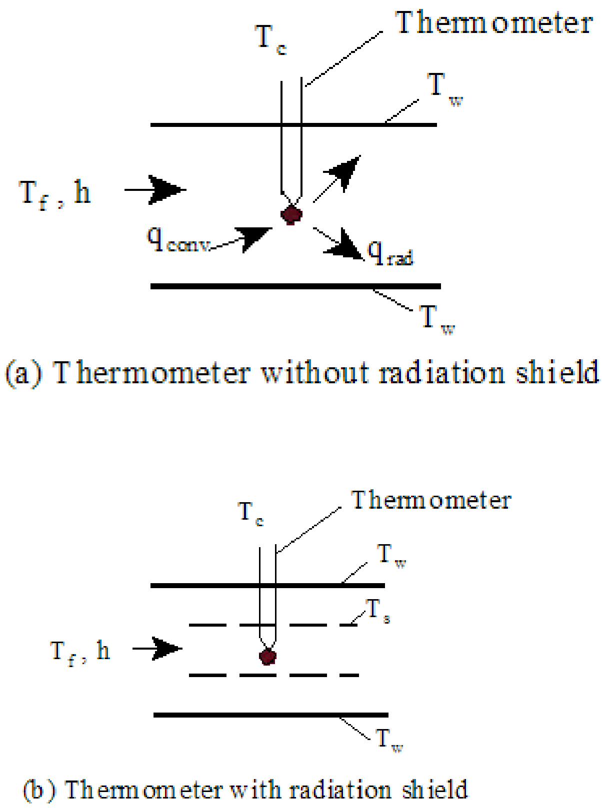


Fig.13.39 Radiation shielding of thermometers

Making an **energy balance on the thermometer bulb,** in steady state, we have:

Without radiation shield:

q_{conv} to the bulb = q_{rad} from the bulb

i.e. $$h \cdot A_c \cdot (T_f - T_c) = \varepsilon_c \cdot A_c \cdot \sigma \cdot (T_c^4 - T_w^4)$$

i.e. $$T_f = T_c + \frac{\varepsilon_c \cdot \sigma \cdot (T_c^4 - T_w^4)}{h} \quad .....(13.79)$$

With radiation shield being present:

Energy balance on the *thermometer bulb*:

$$h \cdot A_c \cdot (T_f - T_c) = \frac{\sigma \cdot (T_c^4 - T_s^4)}{\left(\frac{1-\varepsilon_c}{A_c \cdot \varepsilon_c}\right) + \frac{1}{A_c \cdot F_{cs}} + \left(\frac{1-\varepsilon_s}{A_s \cdot \varepsilon_s}\right)} \quad ....(13.80)$$

In eqn. (13.80), F_{cs} = view factor of thermometer bulb w.r.t the shield and is, generally equal to 1.

Making an energy balance on the *shield*:

$$2 \cdot A_s \cdot h \cdot (T_f - T_s) + \frac{\sigma \cdot (T_c^4 - T_s^4)}{\left(\frac{1-\varepsilon_c}{A_c \cdot \varepsilon_c}\right) + \frac{1}{A_c \cdot F_{cs}} + \left(\frac{1-\varepsilon_s}{A_s \cdot \varepsilon_s}\right)} = \varepsilon_s \cdot A_s \cdot \sigma \cdot (T_s^4 - T_w^4) \qquad \text{....(13.81)}$$

Here the assumption is,

$F_{sw} = 1$ ...view factor between the shield and the walls

and,

$\frac{A_s}{A_w} = 0$ ...i.e. surface area of shield is negligible compared to the area of the channel walls

Solving eqns. (13.80) and (13.81) simultaneously, we obtain the shield temperature T_s and the thermometer reading T_c, (if T_f is known), or T_f (if T_c is known).

Prob.5E.1. Write Mathcad Functions for Radiation error in temp. measurement:

Mathcad Functions for 'Radiation error' in temp. measurement:

1. Thermocouple with *<u>no</u>* radiation shield:

In steady state, making a heat balance on the tthermocouple bead, we have:

$$q_{conv} = q_{rad}$$

i.e. $h \cdot A_c \cdot (T_f - T_c) = \sigma \cdot \varepsilon_c \cdot A_c \cdot (T_c^4 - T_w^4)$

i.e. $$T_f = T_c + \frac{\varepsilon_c \cdot \sigma \cdot (T_c^4 - T_w^4)}{h} \qquad \text{....(13.79)}$$

We write the Function to find T_f when other quantities are known:

$$\text{RadnError\_TC\_no\_shield\_Tf}(T_c, T_w, \varepsilon_c, h) := T_c + \frac{\varepsilon_c \cdot 5.67 \cdot 10^{-8} \cdot (T_c^4 - T_w^4)}{h}$$

2. Thermocouple *with* radiation shield being present:

Making a heat balance on the thermocouple bead:

heat received by convection from the gas = heat lost by radiation to shield

$$q_{conv} = q_{rad}$$

i.e. $$h \cdot A_c \cdot (T_f - T_c) = \frac{\sigma \cdot \left(T_c^4 - T_s^4\right)}{\left(\frac{1 - \varepsilon_c}{A_c \cdot \varepsilon_c}\right) + \frac{1}{A_c \cdot F_{cs}} + \left(\frac{1 - \varepsilon_s}{A_s \cdot \varepsilon_s}\right)} \quad \text{....(13.80)}$$

where, $F_{cs} := 1$ ....view factor for thermocouple bead w.r.t. shield

Then, eqn. (13.80) becomes:

$$h \cdot A_c \cdot (T_f - T_c) = \frac{\sigma \cdot A_c \cdot \left(T_c^4 - T_s^4\right)}{\frac{1}{\varepsilon_c} + \left(\frac{A_c}{A_s}\right) \cdot \left(\frac{1}{\varepsilon_s} - 1\right)}$$

But, for a Thermocouple, A_c / A_s << 1.

Therefore:

$$h \cdot A_c \cdot (T_f - T_c) = \sigma \cdot \varepsilon_c \cdot A_c \cdot \left(T_c^4 - T_s^4\right)$$

i.e. $$h \cdot (T_f - T_c) = \sigma \cdot \varepsilon_c \cdot \left(T_c^4 - T_s^4\right) \quad \text{.....eqn. (A)}$$

Next, making a heat balance on the shield:

heat received by convection from the gas by *both* surfaces of shield + heat received by radiation from the Thermocouple bead = heat lost by radiation from the shield to the channel walls.

So, we write:

$$2 \cdot A_s \cdot h \cdot (T_f - T_s) + \frac{\sigma \cdot \left(T_c^4 - T_s^4\right)}{\left(\frac{1 - \varepsilon_c}{A_c \cdot \varepsilon_c}\right) + \frac{1}{A_c \cdot F_{cs}} + \left(\frac{1 - \varepsilon_s}{A_s \cdot \varepsilon_s}\right)} = \varepsilon_s \cdot A_s \cdot \sigma \cdot \left(T_s^4 - T_w^4\right) \qquad ....(13.81)$$

i.e.

$$2 \cdot A_s \cdot h \cdot (T_f - T_s) + \sigma \cdot \varepsilon_c \cdot A_c \cdot \left(T_c^4 - T_s^4\right) = \varepsilon_s \cdot A_s \cdot \sigma \cdot \left(T_s^4 - T_w^4\right)$$

Dividing by A_s, and since $A_c / A_s << 1$, we get:

$$2 \cdot h \cdot (T_f - T_s) = \varepsilon_s \cdot \sigma \cdot \left(T_s^4 - T_w^4\right) \qquad .....eqn. (B)$$

Solving eqns (A) and (B) simultaneously using 'Solve Block' of Mathcad, we get T_c and T_s (if T_f is known) or T_f and T_s (if T_c is known).

Now, when T_f is known, to find T_c:

Now, when T_f is known, to find T_c:

$T_w := 500$ K $\quad \varepsilon_s := 0.1 \quad \varepsilon_c := 0.7 \quad h := 70$ W/m^2.K

$T_f := 1110.5$ K

$T_c := 100$ K $\quad T_s := 100$ K ....trial values

Given

$$h \cdot (T_f - T_c) = 5.67 \cdot 10^{-8} \cdot \varepsilon_c \cdot \left(T_c^{\,4} - T_s^{\,4}\right)$$ .....eqn. (A) ...by heat balance on the TC junction...eqn. (A)

$$2 \cdot h \cdot (T_f - T_s) = \varepsilon_s \cdot 5.67 \cdot 10^{-8} \cdot \left(T_s^{\,4} - T_w^{\,4}\right)$$ .....eqn. (B) ...by heat balance on the shield...eqn. (B)

$$\text{RadnError\_TC\_with\_shield\_TcTs}(T_w, T_f, \varepsilon_c, \varepsilon_s, h) := \text{Find}(T_c, T_s)$$

Note that the in the above Solve block, T_c and T_s are written as Functions of T_w, T_f, ε_c, ε_s and h. This helps us to vary any of those variables and find T_c and T_s and to draw graphs.

Result of above Function for the above example is:

$$\text{RadnError\_TC\_with\_shield\_TcTs}(T_w, T_f, \varepsilon_c, \varepsilon_s, h) = \begin{pmatrix} 1.075 \times 10^3 \\ 1.062 \times 10^3 \end{pmatrix}$$

T_c is the first item of the result vector, and T_s is the second item.

We extract these from the output vector as:

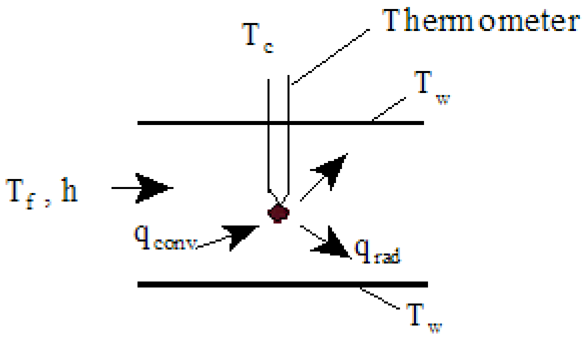
$$\text{RadnError\_TC\_with\_shield\_TcTs}(T_w, T_f, \varepsilon_c, \varepsilon_s, h)_0 = 1.075 \times 10^3$$ K....temp. Tc

$$\text{RadnError\_TC\_with\_shield\_TcTs}(T_w, T_f, \varepsilon_c, \varepsilon_s, h)_1 = 1.062 \times 10^3$$ K....temp. Ts

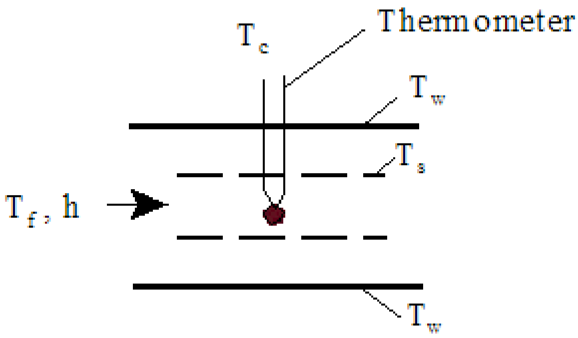
==

Prob.5E.2. Hot gas at 1350 K is flowing in a duct whose walls are maintained at a temperature Tw = 530 K. A thermocouple (TC) placed in the stream to measure temp. If the emissivity of the thermocouple junction is εc = 0.5 and the convective heat transfer coefficient between the flowing air and the thermocouple is h = 115 W/(m2.C), find out the temperature shown by the TC.

(b) Now, if a radiation shield ($\varepsilon_S = 0.1$) is placed between the thermocouple and the walls, what will be new value of T_C read by the thermocouple? And how much is the radiation error?



(a) Thermometer without radiation shield



(b) Thermometer with radiation shield

Mathcad Solution:

Data:

$T_f := 1350$ K $\quad T_w := 530$ K $\quad \varepsilon_s := 0.1 \quad \varepsilon_c := 0.5 \quad h := 115$ W/m^2.K

Here T_f is given. We have to find out T_c. We do this easily as follows:

We use the Mathcad Function written above for T_f in a 'Solve block'.

Start with a guess value for T_c and apply the Solve Block to fulfill the condition that T_f = 1350 K

$T_c := 500$ K ...trial value

Given

$RadnError\_TC\_no\_shield\_Tf(T_c, T_w, \varepsilon_c, h) = T_f$

$Find(T_c) = 1.059 \times 10^3$ **K ... TC temp with no shield... Ans.**

i.e. $T_c := 1.059 \cdot 10^3$ K

$Radiation\_error := T_f - T_c$

i.e. $Radiation\_error = 291$ **K....error when no shield is present ... Ans.**

With the radiation shield being present:

Use the Mathcad Function written above:

Let:

$AA := RadnError\_TC\_with\_shield\_TcTs(T_w, T_f, \varepsilon_c, \varepsilon_s, h)$

$Temp\_TC := AA_0$

i.e. $Temp\_TC = 1.306 \times 10^3$ **....K..temp reading of TC when shield is present...Ans.**

$Temp\_shield := AA_1$

i.e. $Temp\_shield = 1.285 \times 10^3$ **....K..temp of shield...Ans.**

Therefore:

$Radiation\_error := T_f - Temp\_TC$

i.e. $Radiation\_error = 44.468$ **K....error when no shield is present ... Ans.**

Note the great reduction in temp error, due to the presence of radiation shield.

In addition:

Plot the variation of 'Radiation error', with the shield being present, and also the Shield temp when the emissivity of shield varies from 0.05 to 0.5:

Express relevant quantities as functions of ε_s:

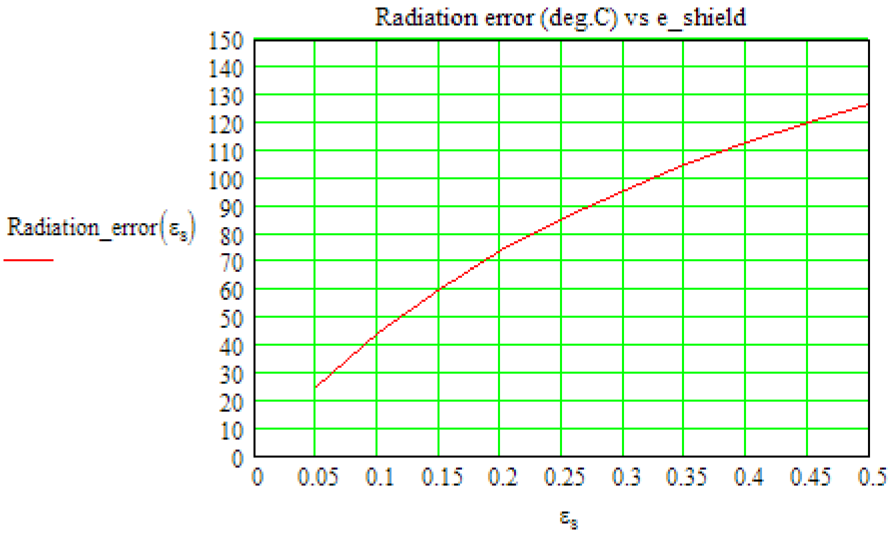
$$Radiation\_error(\varepsilon_s) := T_f - RadnError\_TC\_with\_shield\_TcTs(T_w, T_f, \varepsilon_c, \varepsilon_s, h)_0$$

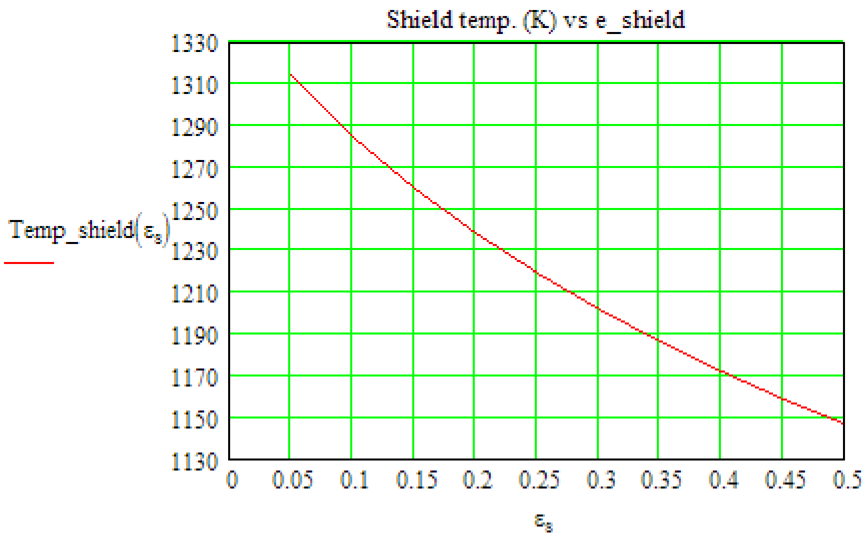
$$Temp\_shield(\varepsilon_s) := RadnError\_TC\_with\_shield\_TcTs(T_w, T_f, \varepsilon_c, \varepsilon_s, h)_1$$

$\varepsilon_s := 0.05, 0.1 .. 0.5$ ...define a range variable

ε_s =	Radiation_error(ε_s)	Temp_shield(ε_s)
0.05	24.837	$1.314 \cdot 10^3$
0.1	44.468	$1.285 \cdot 10^3$
0.15	60.564	$1.26 \cdot 10^3$
0.2	74.111	$1.238 \cdot 10^3$
0.25	85.737	$1.219 \cdot 10^3$
0.3	95.869	$1.202 \cdot 10^3$
0.35	104.807	$1.186 \cdot 10^3$
0.4	112.773	$1.172 \cdot 10^3$
0.45	119.933	$1.159 \cdot 10^3$
0.5	126.415	$1.147 \cdot 10^3$

Now, plot the results:





==

Prob. 5.E.3. Hot air is flowing in a duct whose walls are maintained at a temperature Tw = 450 K. A thermocouple (TC) placed in the stream shows a reading of 650 K. If the emissivity of the thermocouple junction is εc = 0.8 and the convective heat transfer coefficient between the flowing air and the thermocouple is h = 85 W/(m2.C), find out the true temperature of the flowing stream.

(b) Now, if a radiation shield (εs = 0.3) is placed between the thermocouple and the walls, what will be new value of Tc read by the thermocouple? And how much is the temperature error?

(c) Plot the radiation error and shield temp as ε_s varies from 0.05 to 0.5.

EES Solution:

"Data:"

T_w = 450 [K] "…temp of walls"
T_c = 650 [K] "..TC reading"
epsilon_c = 0.8 "…emissivity of TC"
h = 85 [W/m^2-K] "…conv. heat transfer coeff."
epsilon_s = 0.3 "…emissivity of shield"
sigma = 5.67E-08 [W/m^2-K^4] "..Stefan-Boltzmann const."

"Calculations:"

"When there is no radiation shield:"

T_f = T_c + epsilon_c * sigma * (T_c^4 – T_w^4) / h "…finds true temp of fluid"

"When radiation shield is present:"

"By heat balance on the Thermocouple bead:"

h * (T_f – T_c_with_shield) = sigma * epsilon_c * (T_c_with_shield^4 – T_s^4) "…. eqn, (A)."

"By heat balance on the Shield:"

2 * h * (T_f – T_s) = epsilon_s * sigma * (T_s^4 – T_w^4) "…eqn. (B)."

"Solving eqns. (A) and (B) simultaneously, we get thermocouple reading (T_c) and the Shield temp. (T_s)"

"Radiation error:"

Error_no_shield = T_f – T_c "[deg.C]…. error when there is no shield"

Error_with_shield = T_f – T_c_with_shield "[deg.C]…. error when the shield is present"

Results:

Unit Settings: SI C kPa kJ mass deg

ε_c = 0.8
$Error_{with,shield}$ = 8.785 [C]
T_c = 650 [K]
T_s = 703 [K]
ε_s = 0.3
h = 85 [W/m^2-K]
$T_{c,with,shield}$ = 714.6 [K]
T_w = 450 [K]
$Error_{no,shield}$ = 73.38 [C]
σ = 5.670E-08 [W/m^2-K^4]
T_f = 723.4 [K]

Thus:

True temp of fluid = T_f = 723.4 K …. Ans.

TC reading when the shield is present = 714.6 deg.C … Ans.

Shield temp = T_s = 703 K … Ans.

Radiation error when there is no shield = 73.38 deg.C … Ans.

Radiation error when shield is present = 8.785 deg.C … Ans.

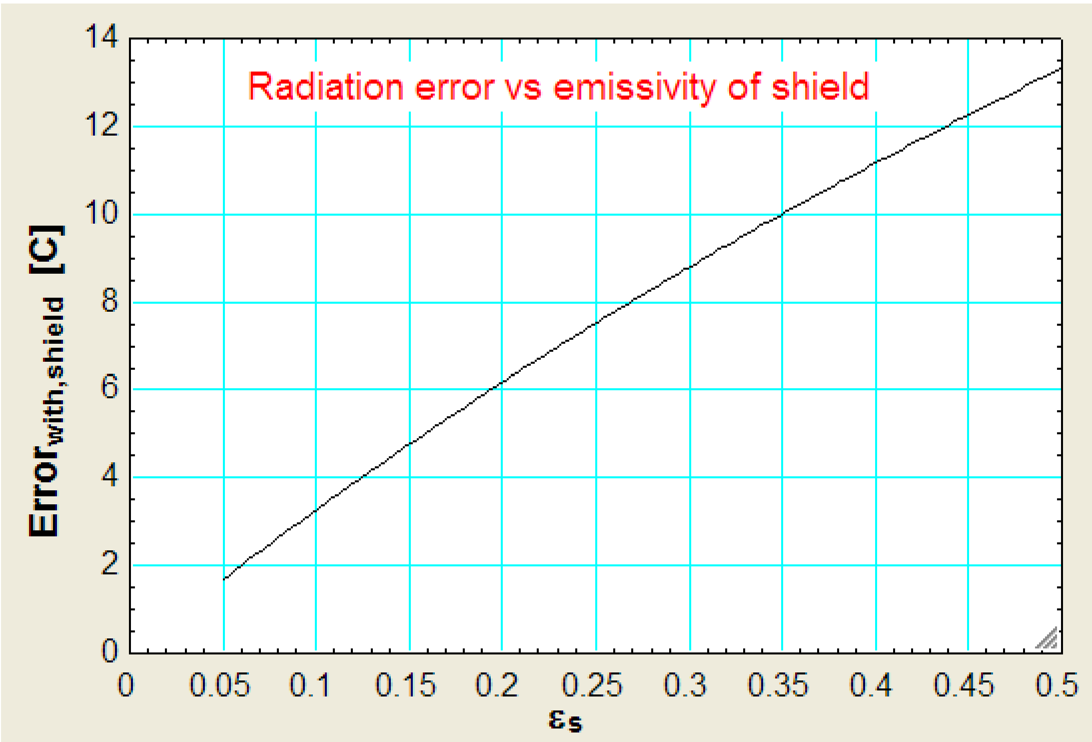
(c) Plot the 'Radiation error' and shield temp as ε_s varies from 0.05 to 0.5:

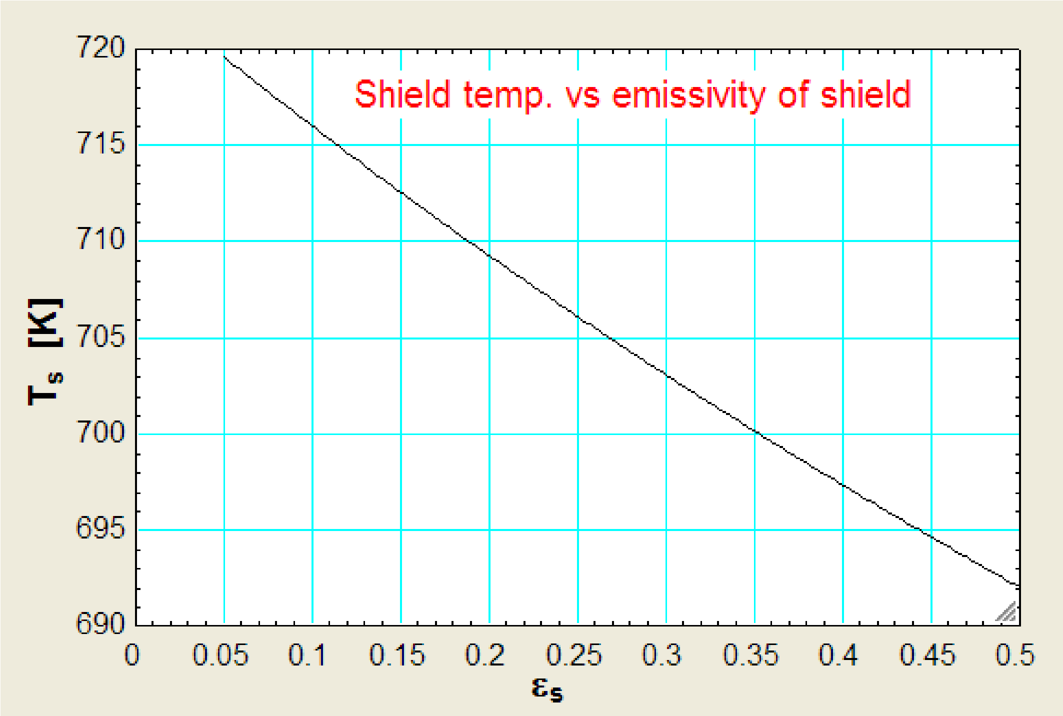
First, compute the Parametric Table:

Table 1

1..10	ε_s	$Error_{with,shield}$ [C]	T_s [K]
Run 1	0.05	1.682	719.6
Run 2	0.1	3.265	716
Run 3	0.15	4.759	712.5
Run 4	0.2	6.173	709.2
Run 5	0.25	7.512	706.1
Run 6	0.3	8.785	703
Run 7	0.35	9.996	700.1
Run 8	0.4	11.15	697.3
Run 9	0.45	12.25	694.6
Run 10	0.5	13.31	692

Now, plot the results:




Shield temp. vs emissivity of shield
720
715
710
705
700
695
690
T_s [K]
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
ε_s

==

"**Prob. 5.E.4.** Temp of hot exhaust gases flowing in to a room whose walls are maintained at a temperature T_w = 20 C is measured with a thermocouple(TC) whose emissivity is 0.6. The thermocouple shows a reading of 500 C. If the convective heat transfer coefficient between the flowing gases and the thermocouple is h = 200 W/(m2.C), find out the true temperature of the flowing stream, and the radiation error.

(b) Now, if a radiation shield (ε_s = 0.3) is placed between the thermocouple and the walls, what will be new value of Tc read by the thermocouple? And how much is the temperature error?

(c) Plot the radiation error and shield temp as emissivity of shield varies from 0.05 to 0.5."

EES Solution:

"Data:"

T_w = 293 [K] "…temp of walls"
T_c = 773 [K] "..TC reading"
epsilon_c = 0.6 "…emissivity of TC"
h = 200 [W/m^2-K] "…conv. heat transfer coeff."
epsilon_s = 0.3 "…emissivity of shield"

sigma = 5.67E-08 [W/m^2-K^4] "..Stefan-Boltzmann const."

"Calculations:"

"When there is no radiation shield:"

T_f = T_c + epsilon_c * sigma * (T_c^4 – T_w^4) / h "…finds true temp of fluid"

"When radiation shield is present:"

"By heat balance on the Thermocouple bead:"

h * (T_f – T_c_with_shield) = sigma * epsilon_c * (T_c_with_shield^4 – T_s^4) "…. eqn, (A)."

"By heat balance on the Shield:"

2 * h * (T_f – T_s) = epsilon_s * sigma * (T_s^4 – T_w^4) "…eqn. (B)."

"Solving eqns. (A) and (B) simultaneously, we get thermocouple reading (T_c) and the Shield temp. (T_s)"

"Radiation error:"

Error_no_shield = T_f – T_c "[deg.C]…. error when there is no shield"

Error_with_shield = T_f – T_c_with_shield "[deg.C]…. error when the shield is present"

Results:

Unit Settings: SI C kPa kJ mass deg

ε_c = 0.6
$Error_{with,shield}$ = 5.021 [C]
T_c = 773 [K]
T_s = 814.1 [K]

ε_s = 0.3
h = 200 [W/m²-K]
$T_{c,with,shield}$ = 827.5 [K]
T_w = 293 [K]

$Error_{no,shield}$ = 59.48 [C]
σ = 5.670E-08 [W/m²-K⁴]
T_f = 832.5 [K]

Thus:

True temp of fluid = T_f = 832.5 K …. Ans.

TC reading when the shield is present = 827.5 deg.C … Ans.

Shield temp = T_s = 814.1 K … Ans.

Radiation error when there is no shield = 59.48 deg.C … Ans.

Radiation error when shield is present = 5.021 deg.C … Ans.

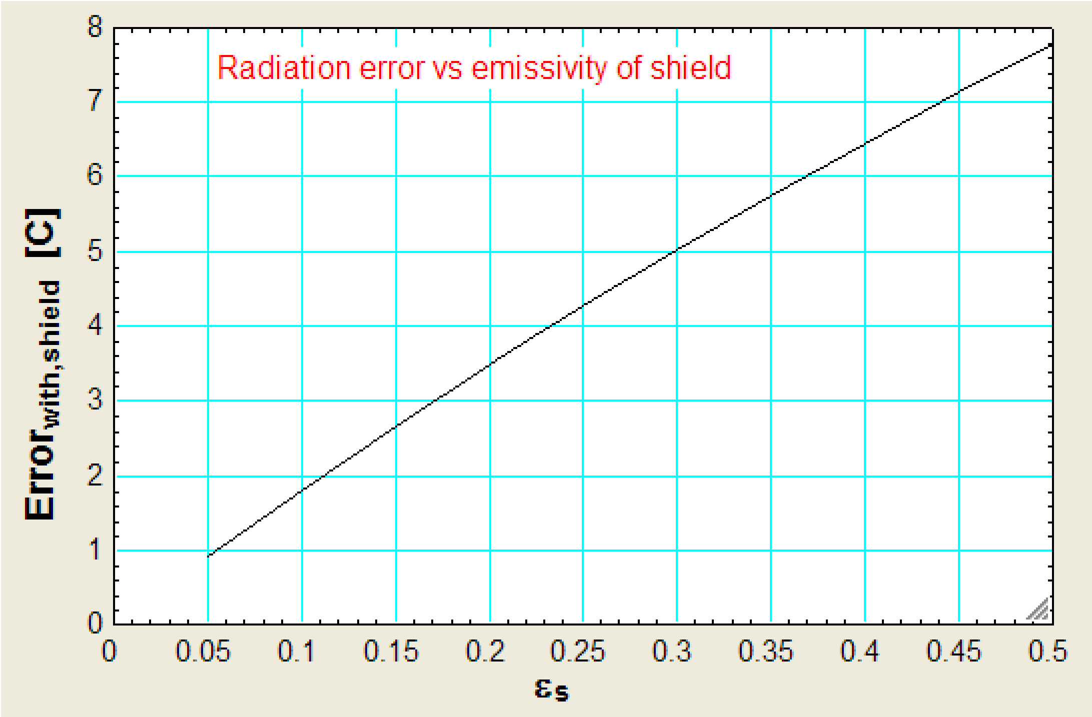
(c) Plot the 'Radiation error' and shield temp as emissivity of shield varies from 0.05 to 0.5:

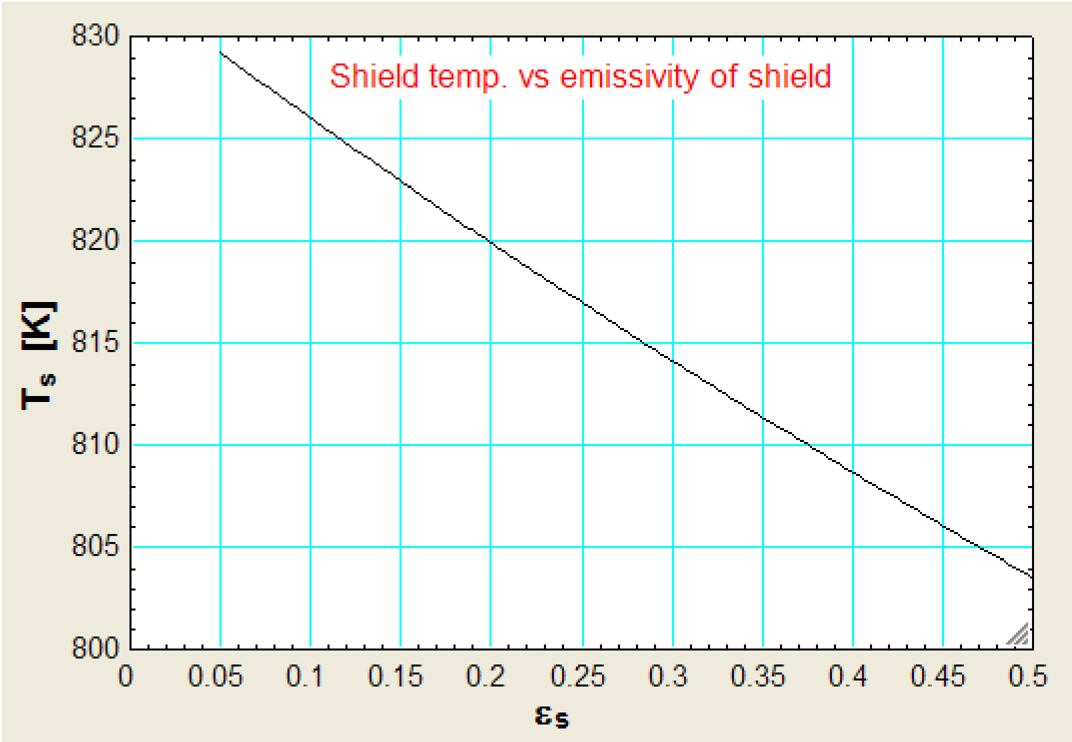
First, compute the Parametric Table:

Table 1 | Table 2

1..10	ε_s	$Error_{with,shield}$ [C]	T_s [K]
Run 1	0.05	0.9246	829.2
Run 2	0.1	1.811	826
Run 3	0.15	2.661	822.9
Run 4	0.2	3.478	819.9
Run 5	0.25	4.264	817
Run 6	0.3	5.021	814.1
Run 7	0.35	5.751	811.3
Run 8	0.4	6.455	808.7
Run 9	0.45	7.135	806
Run 10	0.5	7.792	803.5

Now, plot the results:





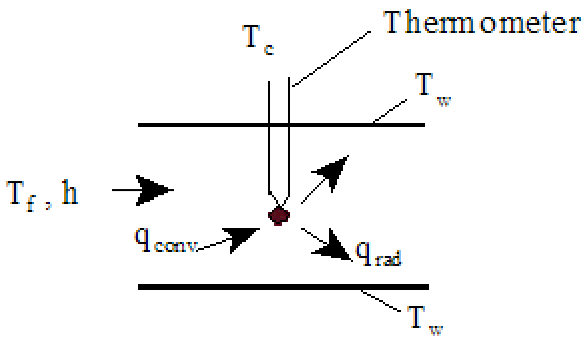
==

Prob. 5.E.5. A thermocouple (TC) whose emissivity is 0.4 is used to measure the temp of a gas flowing in a duct, and it records a temp of 280 C. If the film coeff of heat transfer is h = 150 W/(m^2.C), find out the true temperature of the flowing stream, and the radiation error. Temp of duct wall is 140 C.

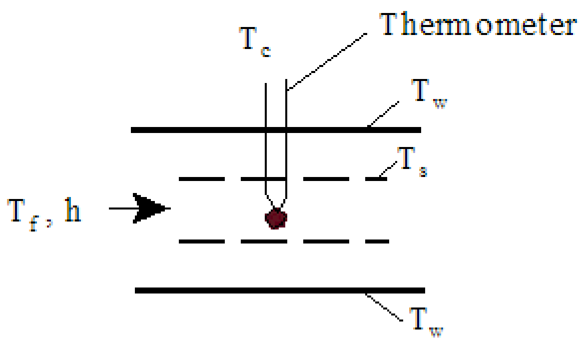
(b) Now, what should be the emissivity of junction to reduce the temp error by 30%?

(c) Now, if a shield of emissivity 0.3 is kept surrounding the TC bead, what is the new temp error? And what is the shield temp?

(d) Plot the radiation error and shield temp as emissivity of shield varies from 0.1 to 0.55



(a) Thermometer without radiation shield



(b) Thermometer with radiation shield

EXCEL Solution:

Following are the steps in EXCEL Solution:

1. Set up the worksheet, name the cells:

h | f_x 200

	A	B	C	D	E
4					
5		Data:			
6					
7		Stefan Boltzmann constant	sigma	5.67E-08	W/m^2.K^4
8		Temp of walls	T_w	413	K
9		Thermocouple reading	T_c	553	K
10		emissivity of TC bead	eps_c	0.4	
11		conv. coeff.	h	200	W/m^2.K

2. Continue with the calculations:

T_f | =T_c + (eps_c * sigma * (T_c^4 - T_w^4))/h

	A	B	C	D	E
10		emissivity of TC bead	eps_c	0.4	
11		conv. coeff.	h	200	W/m^2.K
12					
13		Calculations:			
14					
15		True temp of gas	T_f	560.306	K...Ans.
16		Therefore:			
17		Radn. error	Error_no_shield	7.306	deg.C

Note the eqn for true temp of gas entered in cell D15, in the Formula bar.

Then, Error (with no radn. Shield) = (T_f – T_c) = 7.306 deg. C …. Ans.

(b) Next, what should be the emissivity of TC to reduce the error by 30%?

i.e. the New_error = 7.306 × 0.7 = 5.114 deg.C.

3. Next pert of calculations is shown below:

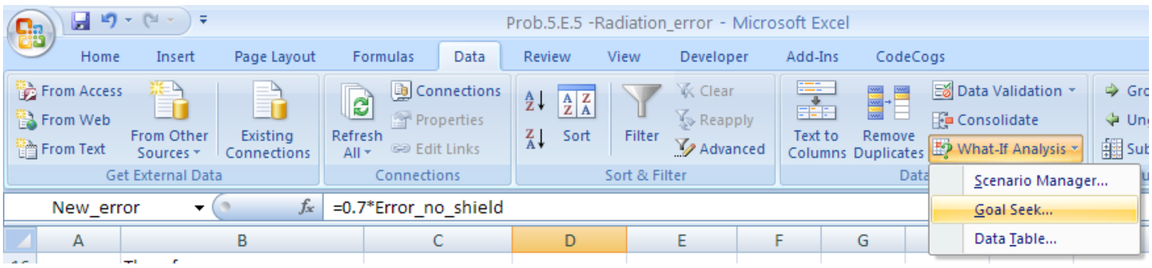
	A	B	C	D	E	F
16		Therefore:				
17		**Radn. error**	**Error_no_shield**	**7.306**	**deg.C**	
18						
19		**If radn_error is reduced by 30 % :**				
20		**Then, new error = 5.114 C.**				
21						
22		**Find eps_c such that:**				
23		Error_no_shield = New_error				
24		**Apply Goal Seek to make value in cell D17 equal to 5.114 by changing cell D10 (i.e. eps_c):**				

Now, the condition to be satisfied is: **Error_no_shield = New_error**

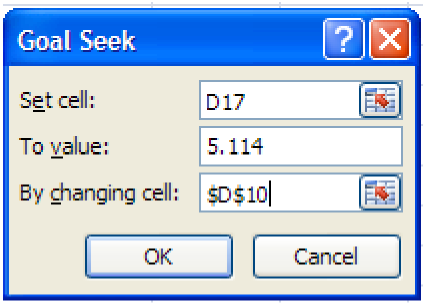
We have to find eps_c such that this condition is satisfied.

So, apply Goal seek to make value in cell D17 equal to 5.114 by changing cell D10 (i.e. eps_c):

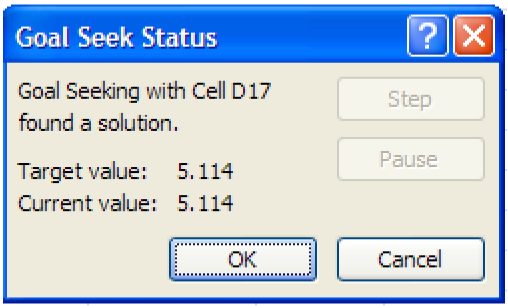
Go to Data-WhatIf Analysis – Goal Seek:



Click on Goal Seek; we get the following window, fill it up as shown:



Press OK. We get:

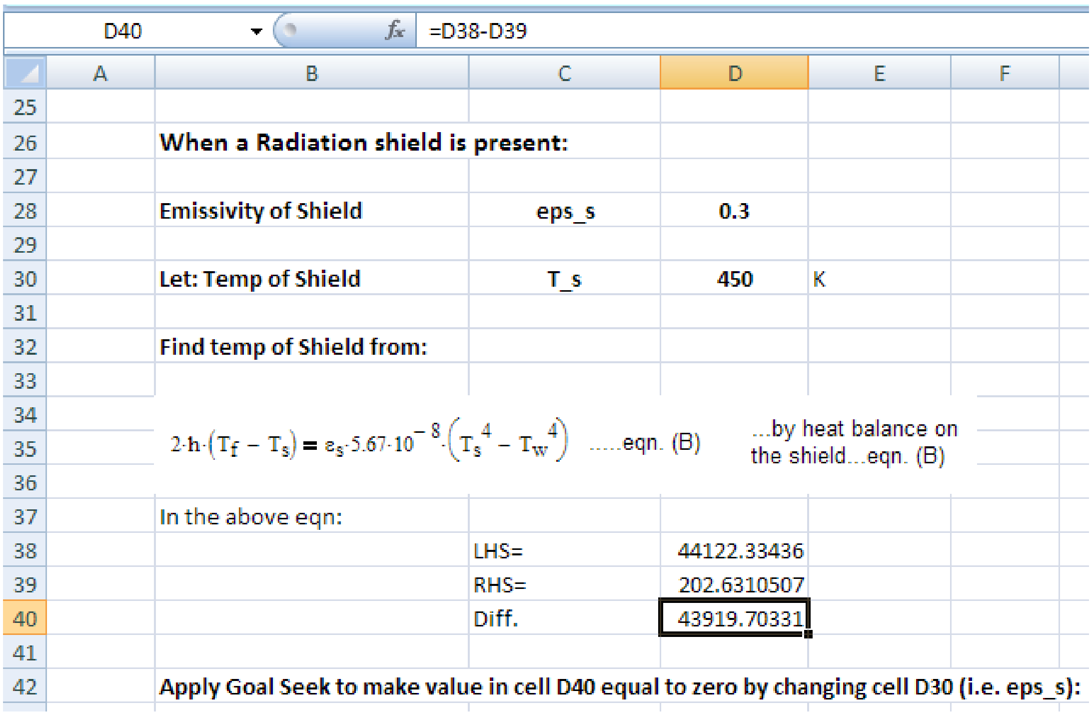


Goal Seek has found a solution. Again, press OK, and see the value of new eps_c in cell D10:

	A	B	C	D	E
7		Stefan Boltzmann constant	sigma	5.67E-08	W/m^2.K^4
8		Temp of walls	T_w	413	K
9		Thermocouple reading	T_c	553	K
10		emissivity of TC bead	eps_c	0.279995339	
11		conv. coeff.	h	200	W/m^2.K
12					
13		Calculations:			
14					
15		True temp of gas	T_f	558.114	K...Ans.
16		Therefore:			
17		Radn. error	Error_no_shield	5.114	deg.C

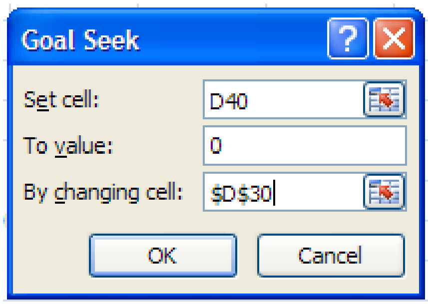
Ths, eps_c required to reduce the radiation error by 30% is: 0.28 …. Ans.

(c) Now, if a shield of emissivity 0.3 is kept surrounding the TC bead, what is the new temp error? And what is the shield temp?

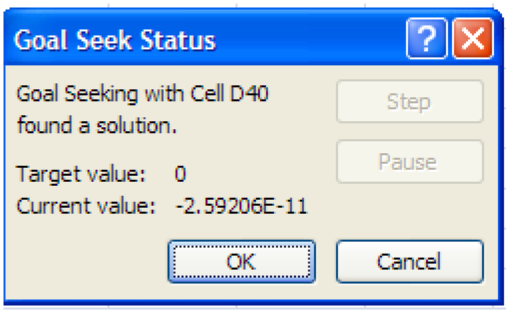


Note in the above screen shot that we have started with a guess value of 450 for T_s in cell D30.

Applying Goal Seek to make cell D40 equal to zero by changing cell D30 (i.e. T_s), we get the value of temp of Shield T_s:



Press OK. We get:

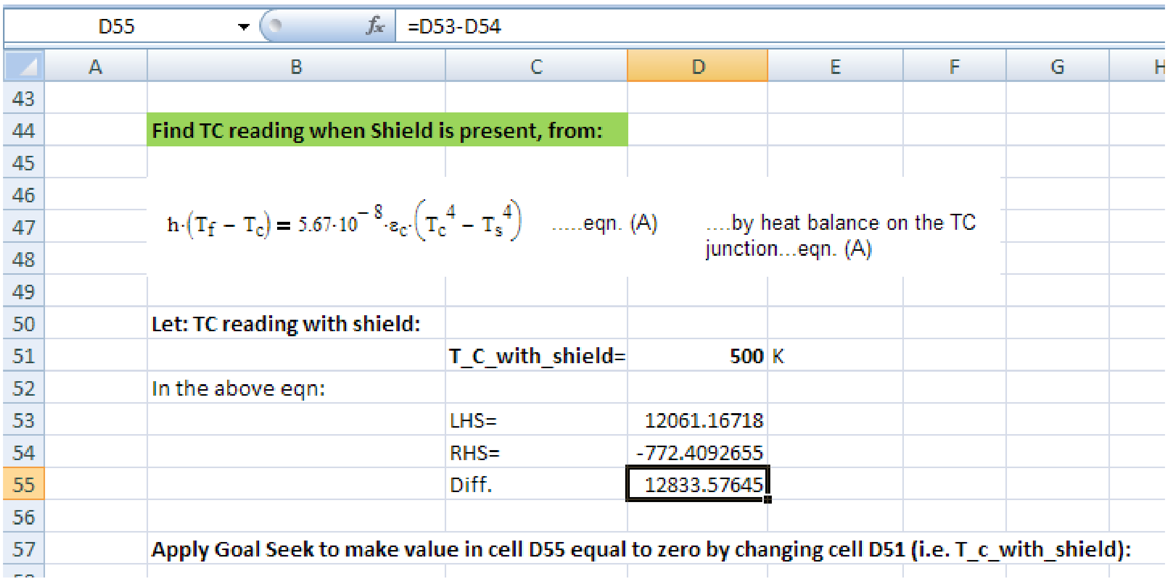


Again, pres OK, and see the value of T_s in cell D30:

	A	B	C	D	E	F
28		**Emissivity of Shield**	**eps_s**	**0.3**		
29						
30		**Let: Temp of Shield**	**T_s**	**557.4369692**	K	
31						
32		**Find temp of Shield from:**				
33						
34						
35		$2\cdot h\cdot\left(T_f - T_s\right) = \varepsilon_s\cdot 5.67\cdot 10^{-8}\cdot\left(T_s^{\,4} - T_w^{\,4}\right)$	.....eqn. (B)		...by heat balance on the shield...eqn. (B)	
36						
37		In the above eqn:				
38			LHS=	1147.546687		
39			RHS=	1147.546687		
40			Diff.	-2.59206E-11		
41						
42		**Apply Goal Seek to make value in cell D40 equal to zero by changing cell D30 (i.e.T_s):**				

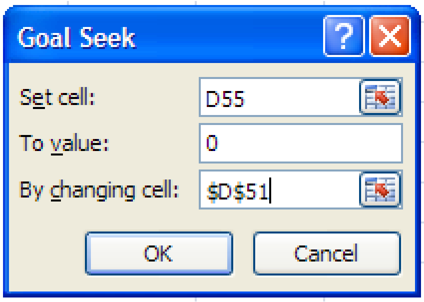
i.e. Shield temp = T_s = 557.437 K … Ans.

Now, continue to calculate the new reading of TC:

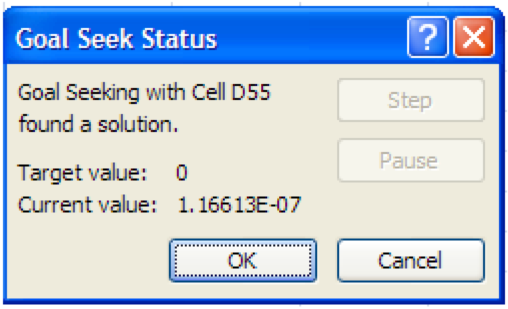


Note in the above screen shot that we have started with a guess value of 500 for T_c_with_shield in cell D51.

Applying Goal Seek to make cell D55 equal to zero by changing cell D51 (i.e. T_c_with_shield), we get the value of temp of Shield T_c_with_shield:



Press OK We get:



Again, press OK and see the value of T_c_with_shield in cell D51:

D59 =T_f - T_C_with_shield

	A	B	C	D	E	F	G
43							
44		Find TC reading when Shield is present, from:					
45							
46							
47		$h\cdot(T_f - T_c) = 5.67\cdot10^{-8}\cdot\varepsilon_c\cdot(T_c^4 - T_s^4)$	.....eqn. (A)		....by heat balance on the TC junction...eqn. (A)		
48							
49							
50		Let: TC reading with shield:					
51			T_C_with_shield=	560.0955	K		
52		In the above eqn:					
53			LHS=	42.07563757			
54			RHS=	42.07563746			
55			Diff.	1.16613E-07			
56							
57		Apply Goal Seek to make value in cell D55 equal to zero by changing cell D51 (i.e. T_c_with_shield):					
58							
59		And, new temp error is:	Error_with_shield	0.21038	C.....Ans.		

Thus:

TC reading when the shield is present = 560.096 K …. Ans.

Temp error when shield is present = 0.2104 C … Ans.

(c) Plot the 'Radiation error' and shield temp as emissivity of shield varies from 0.1 to 0.55:

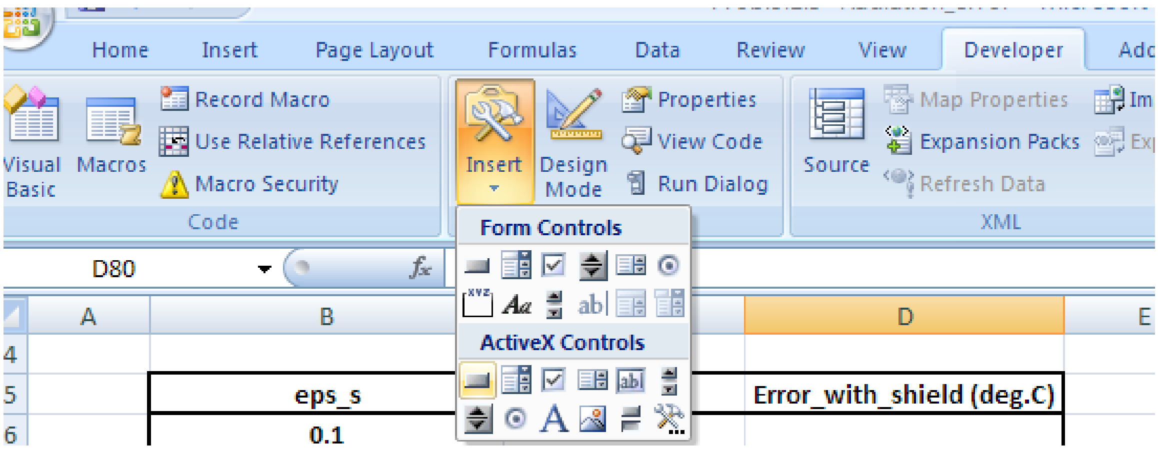
Following are the steps:

1. First, prepare a Table as shown below:

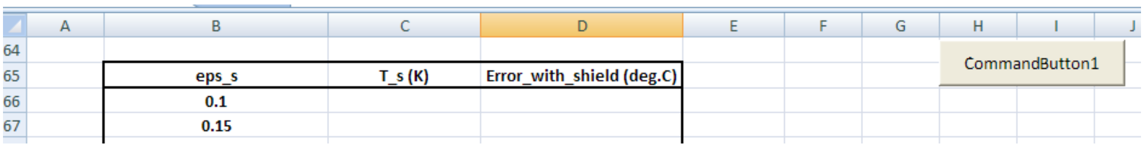
	A	B	C	D
64				
65		eps_s	T_s (K)	Error_with_shield (deg.C)
66		0.1		
67		0.15		
68		0.2		
69		0.25		
70		0.3		
71		0.35		
72		0.4		
73		0.45		
74		0.5		
75		0.55		

2. Now, we need a VBA program to read eps_s values one by one and apply Goal Seek **twice** to get T_s and T_c_with_shield, and thereby get Error_with_shield, and then copy the calculated values in the main worksheet to their respective places in the Table. Also, this program should be run from a command button.

To do this, first go to: Developer- Insert-ActiveX Controls:



Click on the command button i.e. first top left button under ActiveX Controls. Then click at the required place in the worksheet and draw the button to a suitable size:



Then, click on Developer-View Code, and complete the code as shown below:

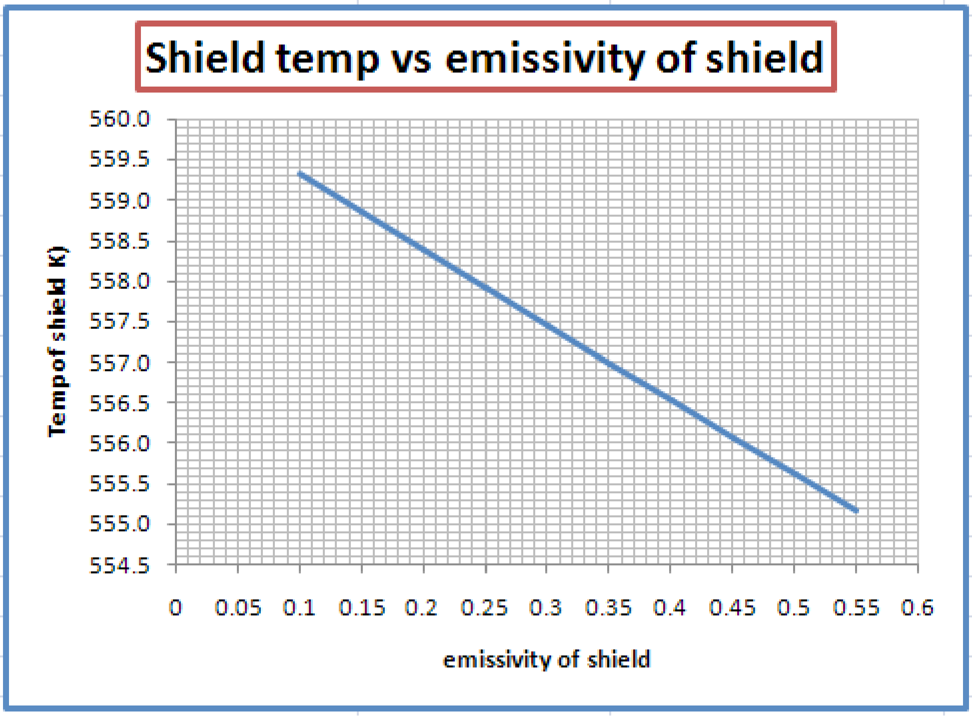
```
Private Sub CommandButton1_Click()

Dim i As Integer

For i = 0 To 9 ' start of For....Next loop

Range("D28") = Cells(66 + i, 2) 'copy the first value of eps_s from the Table to cell D28

' Goal Seek to find Temp of Shield, T_s, in cell D30:
'
'
    Range("D40").Select
    Range("D40").GoalSeek Goal:=0, ChangingCell:=Range("D30")
    ActiveWindow.SmallScroll Down:=18

 Cells(66 + i, 3) = Range("D30") 'copy the value of T_s from cell D30 to its place in Table

' Goal Seek to find temp of Thermocouple when shield is present, T_C_with_shield, in cell D51:
'
'
    Range("D55").Select
    Range("D55").GoalSeek Goal:=0, ChangingCell:=Range("D51")
    ActiveWindow.SmallScroll Down:=18

Cells(66 + i, 4) = Range("D59") 'copy the value of Error_with_shield from cell D59
                                'to its place in Table

Next i  'Go to next value of i

End Sub
```

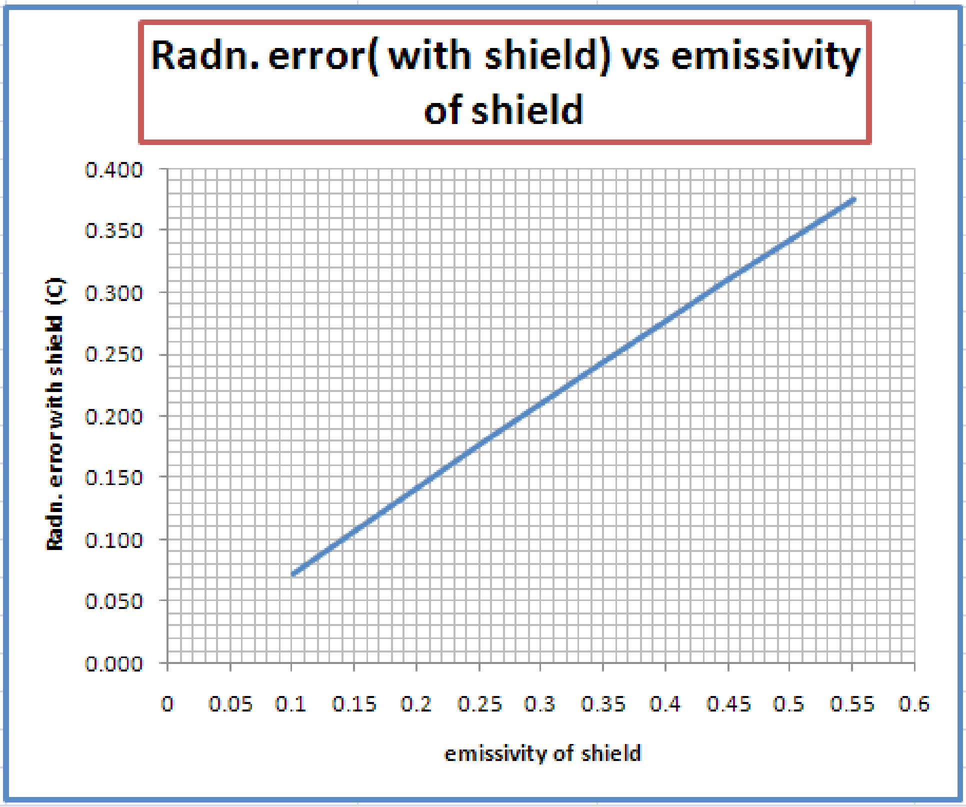
Read the comments in the above program to know what each step does.

3. Now, click on the Command Button. Immediately, calculations are completed and the Table is filled up:

eps_s	T_s (K)	Error_with_shield (deg.C)
0.1	559.331	0.072
0.15	558.850	0.107
0.2	558.375	0.142
0.25	557.904	0.176
0.3	557.437	0.210
0.35	556.975	0.244
0.4	556.517	0.277
0.45	556.063	0.310
0.5	555.613	0.342
0.55	555.168	0.374

4. Now, plot the results in EXCEL:





==

References

1. M. Thirumaleshwar, *Fundamentals of Heat & Mass Transfer*, Pearson Education, India, 2006
2. Yunus A Cengel, *Heat and Mass Transfer*, 3rd Ed., McGraw Hill Co.
3. F.P. Incropera and D.P.DeWitt, *Fundamentals of Heat and Mass Transfer*, 5th Ed., John Wiley & Sons
4. Domkundwar et al, *A Course in Heat & Mass Transfer*, Dhanpat Rai & Co, 5th Ed, 1999
5. Frank Kreith and Mark S Bohn, *Principles of Heat Transfer*, PWS Publ. Co. (Intl. Thomson Publ.), 5th Ed., 1997