Software Solutions to Problems on Heat Transfer

Conduction: Part III

Dr. M. Thirumaleshwar

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Dr. M. Thirumaleshwar

Software Solutions to Problems on Heat Transfer

Conduction – Part III

Numerical methods in heat conduction

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Conduction – Part III
Numerical methods in heat conduction
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Part 1:

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1I Numerical Methods in Heat conduction

Learning objectives:

1. Here, the differential equation is substituted by a set of algebraic equations (obtained by applying heat balance at the nodes) and simultaneous solution of these algebraic equations gives the temperatures at selected, 'discrete points' in the system. So, the important difference to be noted is that while in an analytical solution, temperature is obtained at any point in the body, in a numerical solution temperatures are obtained only at selected, discrete points or 'nodes'. *By selecting these nodes close enough, sufficiently accurate results are obtained.*
2. Advantages of numerical methods are:
   1) easy to apply, with the availability of high speed computers
   2) desired accuracy can be obtained by controlling the no. of nodes or 'mesh size'.
   3) variation in area, thermal conductivity or heat transfer coefficients, and complicated boundary conditions can be easily taken in to account.
   4) mathematical model for a numerical solution is more likely to be a better representative of the actual system
   5) parametric study to observe the effect of variation of different parameters on the solution, or 'what-if' analysis, is easier with numerical methods in conjunction with high speed computers.
3. We shall study problems to formulate set of algebraic equations from the differential equations in Cartesian, cylindrical and spherical coordinates and solve them for one-dimensional, steady state conduction. Then, we shall also study the finite difference representation and solution of two-dimensional, steady state conduction problems.

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Formulas:[Ref. 1]

1D Steady State conduction, with heat generation, in Cartesian coordinates:

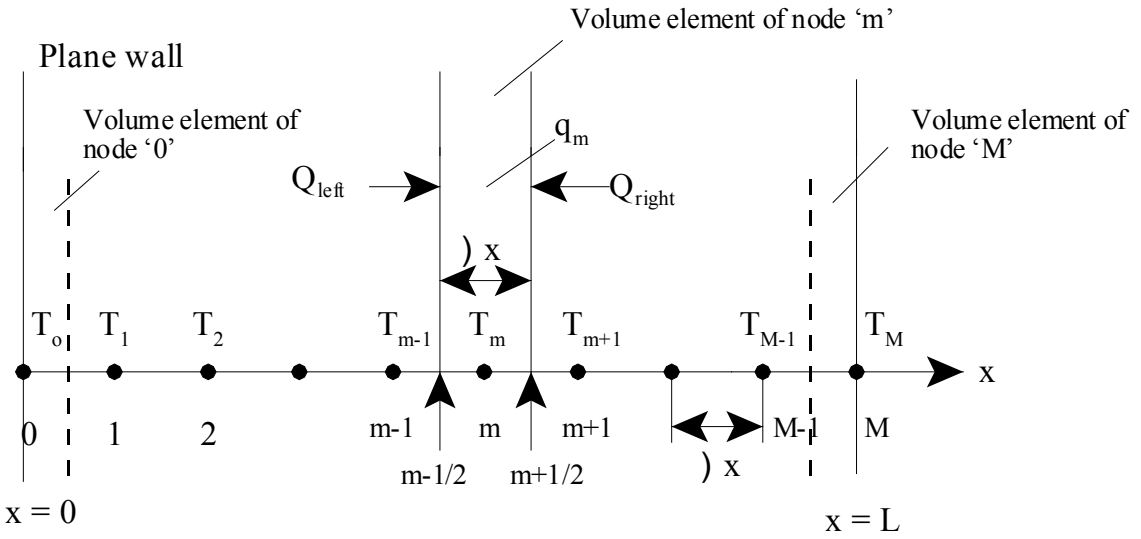

Fig. 8.2 Finite difference formulaion in a plane wall by energy balance

For Internal Nodes:

$$k \cdot A \cdot \frac{T_{m-1} - T_m}{\Delta x} + k \cdot A \cdot \frac{T_{m+1} - T_m}{\Delta x} + q_m \cdot A \cdot \Delta x = 0 \tag{8.7}$$

i.e.

$$\left(T_{m-1} - 2 \cdot T_m + T_{m+1}\right) + \frac{q_m \cdot (\Delta x)^2}{k} = 0 \tag{8.8}$$

For Boundary Nodes:

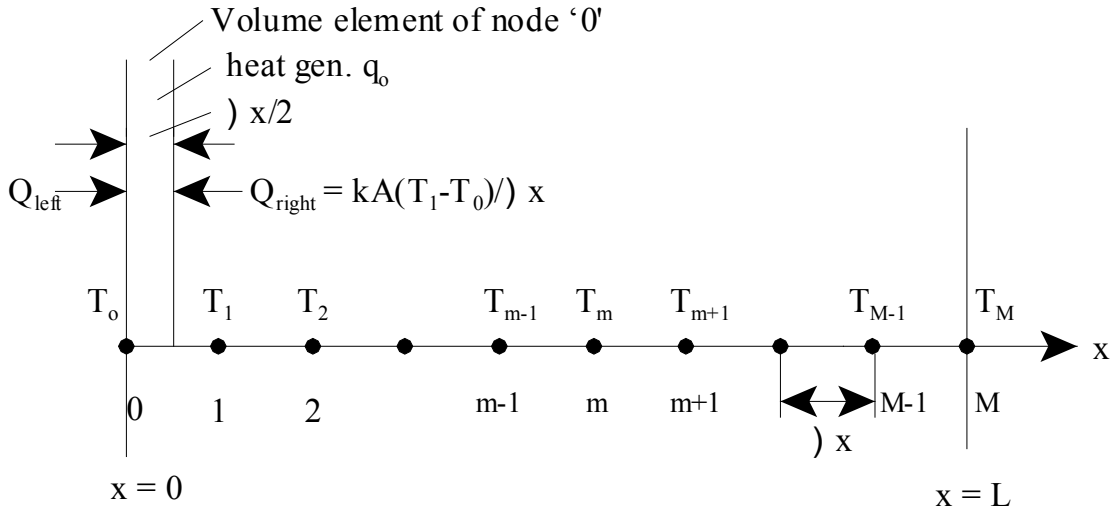

Fig. 8.3 Finite difference formulation for left boundary of a plane wall

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1. **Specified temps at the boundaries: Here, the temps at boundaries are given.**
2. **With Prescribed heat flux at the boundaries:**

By energy balance:

$$Q_{left} + k \cdot A \cdot \frac{(T_1 - T_0)}{\Delta x} + q_o \cdot \left(\frac{A \cdot \Delta x}{2}\right) = 0 \tag{8.10}$$

For node '0':

$$q_{left} \cdot A + k \cdot A \cdot \frac{(T_1 - T_0)}{\Delta x} + q_o \cdot \left(\frac{A \cdot \Delta x}{2}\right) = 0 \tag{8.11}$$

i.e.

$$2 \cdot T_1 - 2 \cdot T_0 + \frac{(\Delta x)^2 \cdot q_o}{k} + \frac{2 \cdot \Delta x \cdot q_{left}}{k} = 0 \tag{8.12}$$

For node 'M': Replace the subscript '0' by 'M' and subscript '1' by 'M-1':

$$q_{right} \cdot A + k \cdot A \cdot \frac{(T_{M-1} - T_M)}{\Delta x} + q_M \cdot \left(\frac{A \cdot \Delta x}{2}\right) = 0 \tag{8.13}$$

i.e.

$$2 \cdot T_{M-1} - 2 \cdot T_M + \frac{(\Delta x)^2 \cdot q_M}{k} + \frac{2 \cdot \Delta x \cdot q_{right}}{k} = 0 \tag{8.14}$$

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For insulated boundary condition and for a plane of thermal symmetry: It is a special case of case with prescribed heat flux at boundaries: i.e. q_left = q_right = 0

$$2\cdot T_1 - 2\cdot T_0 + \frac{(\Delta x)^2\cdot q_o}{k} = 0 \tag{8.15}$$

$$2\cdot T_{M-1} - 2\cdot T_M + \frac{(\Delta x)^2\cdot q_M}{k} = 0 \tag{8.16}$$

We can also get the above eqns by applying the 'mirror image concept' at the insulated (or symmetry) boundary.

3. **Convection boundary condition:**

For node '0':

$$h\cdot A\cdot\left(T_a - T_0\right) + k\cdot A\cdot\frac{\left(T_1 - T_0\right)}{\Delta x} + q_o\cdot\left(\frac{A\cdot\Delta x}{2}\right) = 0 \tag{8.19}$$

i.e.

$$2\cdot T_1 - 2\cdot T_0\cdot\left(1 + \frac{h\cdot\Delta x}{k}\right) + \frac{(\Delta x)^2\cdot q_o}{k} + \frac{2\cdot h\cdot\Delta x}{k}\cdot T_a = 0 \tag{8.20}$$

For node 'M': Replace the subscript '0' by 'M' and subscript '1' by 'M-1':

We get:

$$h\cdot A\cdot\left(T_a - T_M\right) + k\cdot A\cdot\frac{\left(T_{M-1} - T_M\right)}{\Delta x} + q_M\cdot\left(\frac{A\cdot\Delta x}{2}\right) = 0 \tag{8.21}$$

i.e.

$$2\cdot T_{M-1} - 2\cdot T_M\cdot\left(1 + \frac{h\cdot\Delta x}{k}\right) + \frac{(\Delta x)^2\cdot q_M}{k} + \frac{2\cdot h\cdot\Delta x}{k}\cdot T_a = 0 \tag{8.22}$$

4. **Radiation boundary condition:**

For node '0':

$$\varepsilon\cdot\sigma\cdot A\cdot\left(T_a^4 - T_0^4\right) + k\cdot A\cdot\frac{\left(T_1 - T_0\right)}{\Delta x} + q_o\cdot\left(\frac{A\cdot\Delta x}{2}\right) = 0 \tag{8.23}$$

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For node 'M': Replace the subscript '0' by 'M' and subscript '1' by 'M-1':

We get:

$$\varepsilon \cdot \sigma \cdot A \cdot \left(T_a{}^4 - T_M{}^4 \right) + k \cdot A \cdot \frac{\left(T_{M-1} - T_M \right)}{\Delta x} + q_M \cdot \left(\frac{A \cdot \Delta x}{2} \right) = 0 \tag{8.24}$$

5. **Combined convection and radiation boundary condition:**

Let the combined heat transfer coefficient be: h_{comb}

For node '0':

$$h_{comb} \cdot A \cdot \left(T_a - T_0 \right) + k \cdot A \cdot \frac{\left(T_1 - T_0 \right)}{\Delta x} + q_o \cdot \left(\frac{A \cdot \Delta x}{2} \right) = 0 \tag{8.25}$$

i.e.

$$2 \cdot T_1 - 2 \cdot T_0 \cdot \left(1 + \frac{h_{comb} \cdot \Delta x}{k} \right) + \frac{(\Delta x)^2 \cdot q_o}{k} + \frac{2 \cdot h_{comb} \cdot \Delta x}{k} \cdot T_a = 0 \tag{8.26}$$

For node 'M': Replace the subscript '0' by 'M' and subscript '1' by 'M-1':

We get:

$$h_{comb} \cdot A \cdot \left(T_a - T_M \right) + k \cdot A \cdot \frac{\left(T_{M-1} - T_M \right)}{\Delta x} + q_M \cdot \left(\frac{A \cdot \Delta x}{2} \right) = 0 \tag{8.27}$$

i.e.

$$2 \cdot T_{M-1} - 2 \cdot T_M \cdot \left(1 + \frac{h_{comb} \cdot \Delta x}{k} \right) + \frac{(\Delta x)^2 \cdot q_M}{k} + \frac{2 \cdot h_{comb} \cdot \Delta x}{k} \cdot T_a = 0 \tag{8.28}$$

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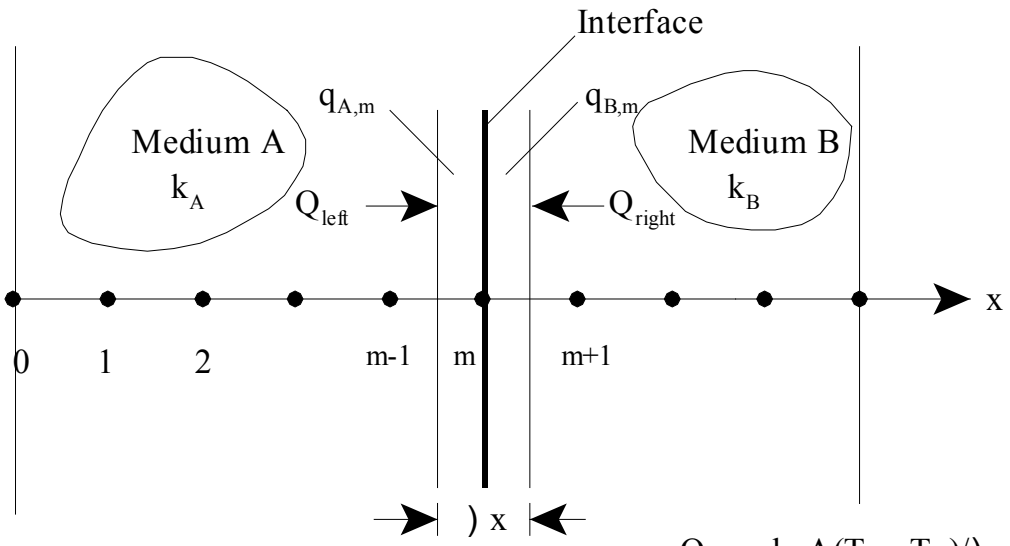
6. **Interface Boundary condition:**



$$Q_{left} = k_A A(T_{m-1}-T_m)/) \ x$$

$$Q_{right} = k_B A(T_{m+1}-T_m)/) \ x$$

Fig. 8.4 Finite difference formulation for interface boundary condition

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With no contact resistance:

The finite difference formulation for this boundary condition is given by:

$$k_A \cdot A \cdot \frac{T_{m-1} - T_m}{\Delta x} + k_B \cdot A \cdot \frac{T_{m+1} - T_m}{\Delta x} + q_{A,m} \cdot \left(\frac{A \cdot \Delta x}{2}\right) + q_{B,m} \cdot \left(\frac{A \cdot \Delta x}{2}\right) = 0 \tag{8.29}$$

With contact resistance:

If there is a contact resistance Rc at the interface, we use the resistance concept to write the difference equation. (See eqn. (8.9)). Now, at the interface, there is a temperature drop. Let the temperature at the interface drop from Tc_1 to Tc_2.

Then, we can write:

$$k_A \cdot A \cdot \frac{T_{m-1} - Tc_1}{\Delta x} + k_B \cdot A \cdot \frac{T_{m+1} - Tc_2}{\Delta x} + q_{A,m} \cdot \left(\frac{A \cdot \Delta x}{2}\right) + q_{B,m} \cdot \left(\frac{A \cdot \Delta x}{2}\right) = 0 \tag{8.30}$$

And, temperature drop at the interface is calculated as:

$$\Delta T_c = \left(Tc_1 - Tc_2\right) = Q \cdot \frac{Rc}{A} \tag{8.31}$$

where Q is the heat flow rate through the interface (i.e. between nodes (m-1) and (m+1)) and (Rc/A) is the interface thermal resistance.

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One-dimensional, steady state conduction in cylindrical systems:

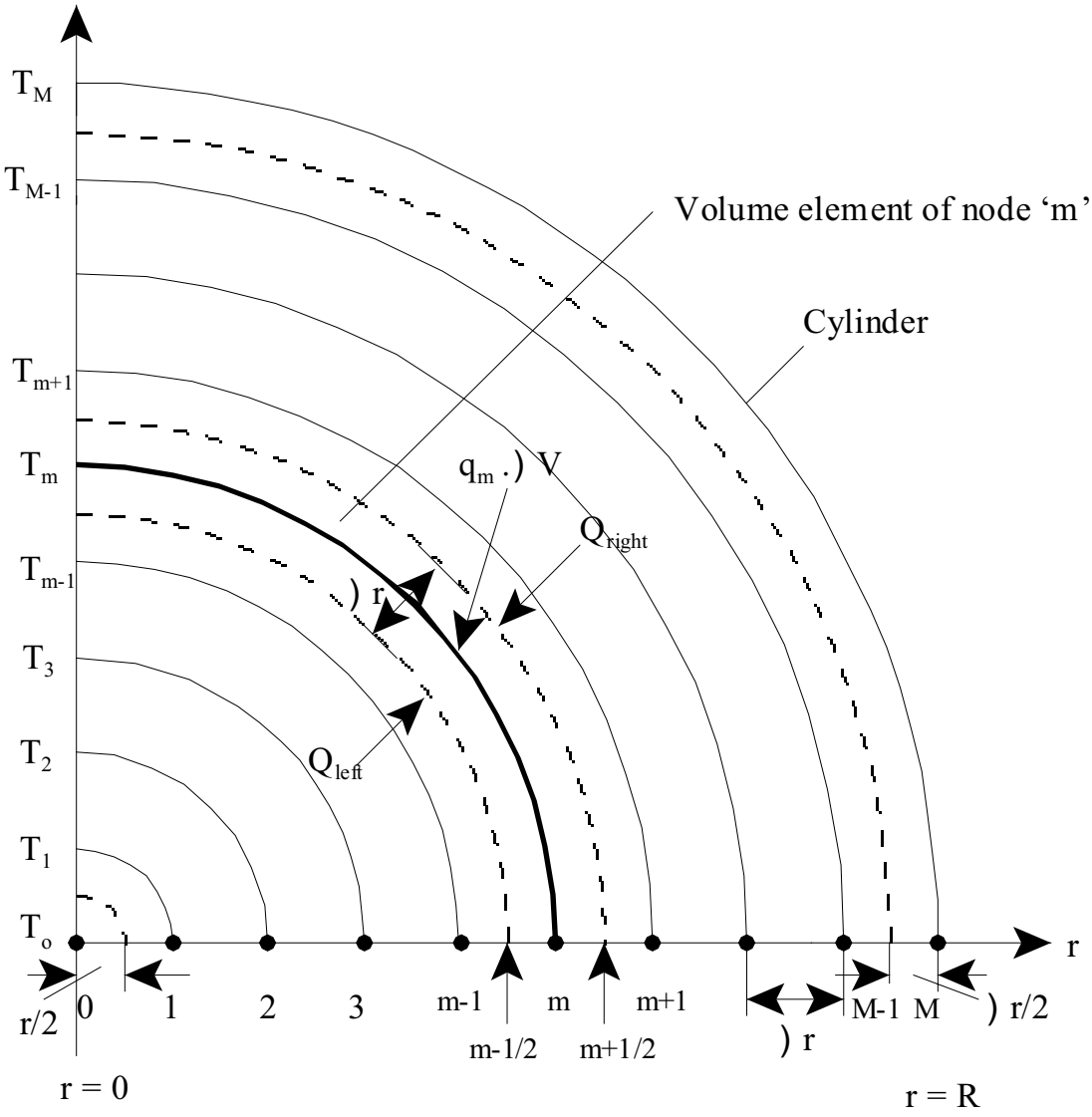

Fig. 8.5 Finite difference formulaion in a cylindrical/spherical system

For Internal node:

Writing an energy balance for the volume element around node 'm', remembering that all heat flows are ***into*** the volume, we get:

$$\frac{T_{m-1} - T_m}{\Delta r} + \frac{T_{m+1} - T_m}{\Delta r} + (2\cdot\pi\cdot m\cdot\Delta r\cdot\Delta r)\cdot L\cdot q_{m=0}$$

$$\frac{}{2\cdot\pi\cdot\left(m\cdot\Delta r - \frac{\Delta r}{2}\right)\cdot L\cdot k} \qquad \frac{}{2\cdot\pi\cdot\left(m\cdot\Delta r + \frac{\Delta r}{2}\right)\cdot L\cdot k}$$

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Simplifying the above equation, we get:

$$\left(1-\frac{1}{2\cdot m}\right)\cdot T_{m-1}-2\cdot T_m+\left(1+\frac{1}{2\cdot m}\right)\cdot T_{m+1}+\frac{(\Delta r)^2\cdot q_m}{k}=0 \qquad (8.32)$$

Eqn. (8.32) is the finite difference eqn. for internal nodes i.e. for nodes 1, 2, ….(M-1), with constant thermal conductivity and internal heat generation.

At the centre: i.e. at r = 0:

Writing the energy balance for the half-volume (of thickness Δr/2) around node '0', we get:

$$\frac{\frac{T_1-T_0}{\Delta r}}{2\cdot\pi\cdot\frac{\Delta r}{2}\cdot L\cdot k}+\pi\cdot\left(\frac{\Delta r}{2}\right)^2\cdot L\cdot q_0=0$$

Simplifying the above equation, we get:

$$4\cdot(T_1-T_0)+\frac{(\Delta r)^2\cdot q_0}{k}=0 \qquad (8.33)$$

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At the periphery: i.e. at node 'M':

For convection boundary conditions, where heat transfer from the periphery is with an ambient at temperature T_a with a heat transfer coeff. of h, energy balance around node 'M', gives:

$$\frac{\frac{T_{M-1}-T_M}{\Delta r}}{2\cdot\pi\cdot\left(M\cdot\Delta r-\frac{\Delta r}{2}\right)\cdot L\cdot k}+(2\cdot\pi\ M\cdot\Delta r\cdot L)\cdot h\cdot\left(T_a-T_M\right)+2\cdot\pi\cdot M\cdot\Delta r\cdot\frac{\Delta r}{2}\cdot L\cdot q_M=0$$

Simplifying the above equation, we get:

$$\left(1-\frac{1}{2\cdot M}\right)\cdot T_{M-1}-\left[\left(1-\frac{1}{2\cdot M}\right)+\frac{\Delta r\cdot h}{k}\right]\cdot T_M+\frac{\Delta r\cdot h}{k}\cdot T_a+\frac{(\Delta r)^2\cdot q_M}{2\cdot k}=0 \tag{8.34}$$

One-dimensional, steady state conduction in spherical systems:

See the fig. for cyl. system, i.e. fig. 8.5 above.

For Internal node:

Writing an energy balance for the volume element around node 'm', remembering that all heat flows are ***into*** the volume, we get:

$$\frac{\frac{T_{m-1}-T_m}{\Delta r}}{4\cdot\pi\cdot\left(m\cdot\Delta r-\frac{\Delta r}{2}\right)^2\cdot k}+\frac{\frac{T_{m+1}-T_m}{\Delta r}}{4\cdot\pi\cdot\left(m\cdot\Delta r+\frac{\Delta r}{2}\right)^2\cdot k}+\left[4\cdot\pi\cdot(m\cdot\Delta r)^2\cdot\Delta r\right]\cdot q_m=0 \tag{8.35}$$

Simplifying the above equation, we get:

$$\left(1-\frac{1}{2\cdot m}\right)^2\cdot\left(T_{m-1}-T_m\right)+\left(1+\frac{1}{2\cdot m}\right)^2\cdot\left(T_{m+1}-T_m\right)+\frac{(\Delta r)^2\cdot q_m}{k}=0 \tag{8.36}$$

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At the centre, r = 0:

Applying the energy balance to the half-volume around node '0',

$$\frac{\frac{T_1 - T_0}{\Delta r}}{4\cdot\pi\cdot\left(\frac{\Delta r}{2}\right)^2\cdot k} + \frac{4}{3}\cdot\pi\cdot\left(\frac{\Delta r}{2}\right)^3\cdot q_0 = 0$$

Simplifying,

$$6\cdot\left(T_1 - T_0\right) + \frac{(\Delta r)^2\cdot q_0}{k} = 0 \tag{8.37}$$

For the boundary node 'M':

Let there be heat transfer at the boundary with a fluid flowing at a temperature of T_a with a heat transfer coeff. of 'h'. Then, writing an energy balance for the half-volume around node 'M', we get:

$$\frac{\frac{T_{M-1} - T_M}{\Delta r}}{4\cdot\pi\cdot\left(M\cdot\Delta r - \frac{\Delta r}{2}\right)^2\cdot k} + 4\cdot\pi\cdot(M\cdot\Delta r)^2\cdot h\cdot\left(T_a - T_M\right) + 4\cdot\pi\cdot(M\cdot\Delta r)^2\cdot\frac{\Delta r}{2}\cdot q_M = 0 \tag{8.38}$$

Simplifying the above equation, we get:

$$\left(1 - \frac{1}{2\cdot M}\right)^2\cdot T_{M-1} - \left[\left(1 - \frac{1}{2\cdot M}\right)^2 + \frac{\Delta r\cdot h}{k}\right]\cdot T_M + \frac{\Delta r\cdot h}{k}\cdot T_a + \frac{(\Delta r)^2\cdot q_M}{2\cdot k} = 0 \tag{8.39}$$

==========

Table 8.1

Summary of steady state, finite difference equations for different boundary conditions:

(q = heat flux, h = conv. heat tr. coeff., k = thermal cond., no int. heat gen., and $\Delta x = \Delta y$)

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Situation **Finite difference eqn. (with $\Delta x = \Delta y$, no heat generation)**

1) **Node at an internal corner with convection (Fig. (8.7,a):**

$$T_{m,n-1} + 2\cdot T_{m-1,n} + 2\cdot T_{m,n+1} + T_{m+1,n} - \left(6 + \frac{2\cdot h\cdot \Delta x}{k}\right)\cdot T_{m,n} + \frac{2\cdot h\cdot \Delta x}{k}\cdot T_a = 0 \qquad (8.44)$$

2) **Node at a plane surface with convection (Fig. (8.7,b):**

$$\left(2\cdot T_{m-1,n} + T_{m,n+1} + T_{m,n-1}\right) + \frac{2\cdot h\cdot \Delta x}{k}\cdot T_a - 2\cdot\left(\frac{h\cdot \Delta x}{k} + 2\right)\cdot T_{m,n} = 0 \qquad (8.45)$$

3) **Node at an external corner with convection (Fig. (8.7,c):**

$$\left(T_{m,n-1} + T_{m-1,n}\right) + \frac{2\cdot h\cdot \Delta x}{k}\cdot T_a - 2\cdot\left(\frac{h\cdot \Delta x}{k} + 1\right)\cdot T_{m,n} = 0 \qquad (8.46)$$

4) **Node at a plane surface with uniform heat flux (Fig. (8.7,d):**

$$\left(2\cdot T_{m-1,n} + T_{m,n+1} + T_{m,n-1}\right) + \frac{2\cdot q\cdot \Delta x}{k} - 4\cdot T_{m,n} = 0 \qquad (8.47)$$

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Note: In eqns.(8.42) and (8.44), put h = 0 or q = 0, to get difference equations for an insulated surface or a surface with thermal symmetry.

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One-dimensional Transient heat conduction in a plane wall:

Applying the general energy balance:

$$k\cdot A\cdot\frac{T_{m-1}-T_m}{\Delta x}+k\cdot A\cdot\frac{T_{m+1}-T_m}{\Delta x}+q_m\cdot(A\cdot\Delta x)=\rho\cdot A\cdot\Delta x\cdot C_p\cdot\frac{T_m^{i+1}-T_m^{i}}{\Delta\tau} \tag{8.51}$$

Simplifying,

$$T_{m-1}-2\cdot T_m+T_{m+1}+\frac{q_m\cdot(\Delta x)^2}{k}=\frac{(\Delta x)^2}{\alpha\cdot\Delta\tau}\cdot\left(T_m^{i+1}-T_m^{i}\right) \tag{8.52}$$

where, $\alpha=\dfrac{k}{\rho\cdot C_p}$ = thermal diffisivity of the material.

Now, the term $\dfrac{\alpha\cdot\Delta\tau}{(\Delta x)^2}$ is the finite difference form of the Fourier number, Fo

So, eqn. (8.52) reduces to:

$$T_{m-1}-2\cdot T_m+T_{m+1}+\frac{q_m\cdot(\Delta x)^2}{k}=\frac{\left(T_m\right)^{i+1}-\left(T_m\right)^{i}}{Fo} \tag{8.53}$$

Explicit method:

$$\left(T_{m-1}\right)^{i}-2\cdot\left(T_m\right)^{i}+\left(T_{m+1}\right)^{i}+\frac{q_m^{i}\cdot(\Delta x)^2}{k}=\frac{\left(T_m\right)^{i+1}-\left(T_m\right)^{i}}{Fo} \tag{8.54}$$

So, we write for T_m^{i+1}:

$$\left(T_m\right)^{i+1}=Fo\cdot\left[\left(T_{m-1}\right)^{i}+\left(T_{m+1}\right)^{i}\right]+(1-2\cdot Fo)\cdot\left(T_m\right)^{i}+Fo\cdot\frac{\left(q_m\right)^{i}\cdot(\Delta x)^2}{k} \tag{8.55}$$

When there is no heat generation, eqn. (8.55) reduces to:

$$\left(T_m\right)^{i+1}=Fo\cdot\left[\left(T_{m-1}\right)^{i}+\left(T_{m+1}\right)^{i}\right]+(1-2\cdot Fo)\cdot\left(T_m\right)^{i} \tag{8.56}$$

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Implicit method:

If in the LHS of eqn. (8.53), we use the values at time step (i + 1), we get the implicit relation for the node temperatures:

$$\text{i.e } \left(T_{m-1}\right)^{i+1} - 2\cdot\left(T_m\right)^{i+1} + \left(T_{m+1}\right)^{i+1} + \frac{\left(q_m\right)^{i+1}\cdot(\Delta x)^2}{k} = \frac{\left(T_m\right)^{i+1} - \left(T_m\right)^{i}}{Fo} \tag{8.57}$$

Eqn. (8.57) is simplified to:

$$(1 + 2\cdot Fo)\cdot\left(T_m\right)^{i+1} - Fo\cdot\left[\left(T_{m-1}\right)^{i+1} + \left(T_{m+1}\right)^{i+1} + \frac{\left(q_m\right)^{i+1}\cdot(\Delta x)^2}{k}\right] - \left(T_m\right)^{i} = 0 \tag{8.58}$$

When there is no heat generation, eqn. (8.58) reduces to:

$$(1 + 2\cdot Fo)\cdot\left(T_m\right)^{i+1} - Fo\cdot\left[\left(T_{m-1}\right)^{i+1} + \left(T_{m+1}\right)^{i+1}\right] - \left(T_m\right)^{i} = 0 \tag{8.59}$$

For Boundary Nodes:

Exact nature of the difference eqns. depends on the specific boundary condition. For example:

For node '0' with convection boundary condition:

Explicit formulation:

$$h\cdot A\cdot\left[T_a - \left(T_0\right)^{i}\right] + k\cdot A\cdot\frac{\left(T_1\right)^{i} - \left(T_0\right)^{i}}{\Delta x} + \left(q_0\right)^{i}\cdot A\cdot\frac{\Delta x}{2} = \rho\cdot A\cdot\frac{\Delta x}{2}\cdot C_p\cdot\frac{\left(T_0\right)^{i+1} - \left(T_0\right)^{i}}{\Delta\tau} \tag{8.60}$$

Simplifying:

$$\left(T_0\right)^{i+1} = (1 - 2\cdot Fo - 2\cdot Fo\cdot Bi)\cdot\left(T_0\right)^{i} + Fo\cdot\left[2\cdot\left(T_1\right)^{i} + 2\cdot Bi\cdot T_a + \frac{\left(q_0\right)^{i}\cdot(\Delta x)^2}{k}\right] \tag{8.61}$$

where $Bi - \frac{h\cdot\Delta x}{k}$ = Biot number

When there is no heat generation, eqn. (8.61) for explicit formulation becomes:

$$\left(T_0\right)^{i+1} = (1 - 2\cdot Fo - 2\cdot Fo\cdot Bi)\cdot\left(T_0\right)^{i} + Fo\cdot\left[2\cdot\left(T_1\right)^{i} + 2\cdot Bi\cdot T_a\right] \tag{8.62}$$

Stability criterion foe Explicit method:

"Coefficients of all $T_m^{\,i}$ in the $T_m^{\,i+1}$ expressions (called 'primary coefficients') must be greater than or equal to zero for all nodes 'm'".

Generally, boundary nodes with convection conditions are more restrictive and in such cases, coeff. of $T_m^{\,i}$ from the most restrictive eqn. must be considered for the stability criterion and the time step $\Delta\tau$ must be determined with respect to that coefficient.

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Table 8.2

Summary of transient, finite difference equations for different boundary conditions:

(q = heat flux, h = conv. heat tr. coeff., k = thermal cond., no int. heat gen., and $\Delta x = \Delta y$)

Situation	Finite difference eqn. (with $\Delta x = \Delta y$, no heat generation)

1. Node at an interior corner with convection (Fig. (8.12,a):

Explicit method:

$$\left(T_{m,n}\right)^{i+1} = \frac{2}{3}\cdot Fo\cdot\left[2\cdot\left(T_{m,n+1}\right)^{i} + 2\cdot\left(T_{m+1,n}\right)^{i} + \left(T_{m-1,n}\right)^{i} + \left(T_{m,n-1}\right)^{i} + 2\cdot Bi\cdot T_{a}\right] \ldots + \left(1 - 4\cdot Fo - \frac{4}{3}\cdot Fo\cdot Bi\right)\cdot\left(T_{m,n}\right)^{i} \quad (8.71)$$

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Stability criterion for above:

$$\mathrm{Fo}\cdot(3+\mathrm{Bi})\leq\frac{3}{4} \tag{8.72}$$

Implicit method:

$$\left[1+4\cdot\mathrm{Fo}\cdot\left(1+\frac{\mathrm{Bi}}{3}\right)\right]\cdot\left(\mathrm{T}_{m,n}\right)^{i+1}-\frac{2\cdot\mathrm{Fo}}{3}\cdot\left[\begin{matrix}\left(\mathrm{T}_{m-1,n}\right)^{i+1}+\left(\mathrm{T}_{m,n-1}\right)^{i+1}+2\cdot\left(\mathrm{T}_{m,n+1}\right)^{i+1}\ldots\\+2\cdot\left(\mathrm{T}_{m+1,n}\right)^{i+1}+2\cdot\mathrm{Bi}\cdot\mathrm{T}_{a}\end{matrix}\right]-\left(\mathrm{T}_{m,n}\right)^{i}=0 \tag{8.73}$$

2. Node at a plane surface with convection (Fig. (8.12,b):

Explicit method:

$$\begin{aligned}\left(\mathrm{T}_{m,n}\right)^{i+1}=&\mathrm{Fo}\cdot\left[2\cdot\left(\mathrm{T}_{m-1,n}\right)^{i}+\left(\mathrm{T}_{m,n+1}\right)^{i}+\left(\mathrm{T}_{m,n-1}\right)^{i}+2\cdot\mathrm{Bi}\cdot\mathrm{T}_{a}\right]\ldots\\&+(1-4\cdot\mathrm{Fo}-2\cdot\mathrm{Fo}\cdot\mathrm{Bi})\cdot\left(\mathrm{T}_{m,n}\right)^{i}\end{aligned} \tag{8.74}$$

Stability criterion for above:

$$\mathrm{Fo}\cdot(2+\mathrm{Bi})\leq\frac{1}{2} \tag{8.75}$$

Implicit method:

$$\left[1+2\cdot\mathrm{Fo}(2+\mathrm{Bi})\cdot\left(\mathrm{T}_{m,n}\right)^{i+1}-\mathrm{Fo}\cdot\left[2\cdot\left(\mathrm{T}_{m-1,n}\right)^{i+1}+\left(\mathrm{T}_{m,n+1}\right)^{i+1}+\left(\mathrm{T}_{m,n-1}\right)^{i+1}\right]=\left(\mathrm{T}_{m,n}\right)^{i}+2\cdot\mathrm{Bi}\cdot\mathrm{Fo}\cdot\mathrm{T}_{a}\right. \tag{8.76}$$

3. Node at a plane surface, insulated:

To obtain finite difference eqn. or stability criterion for an insulated surface (or a surface of thermal symmetry), set Bi = 0 (i.e. h = 0) in eqns. (8.74), (8.75) or (8.76).

4. Node at exterior corner, with convection (Fig. 8.12, c):

Explicit method:

$$\begin{aligned}\left(\mathrm{T}_{m,n}\right)^{i+1}=&2\cdot\mathrm{Fo}\cdot\left[\left(\mathrm{T}_{m-1,n}\right)^{i}+\left(\mathrm{T}_{m,n-1}\right)^{i}+2\cdot\mathrm{Bi}\cdot\mathrm{T}_{a}\right]\ldots\\&+(1-4\cdot\mathrm{Fo}-4\cdot\mathrm{Fo}\cdot\mathrm{Bi})\cdot\left(\mathrm{T}_{m,n}\right)^{i}\end{aligned} \tag{8.77}$$

Stability criterion for above:

$$\mathrm{Fo}\cdot(1+\mathrm{Bi})\leq\frac{1}{4} \tag{8.78}$$

Implicit method:

$$(1+4\cdot\mathrm{Fo}\cdot(1+\mathrm{Bi}))\cdot\left(\mathrm{T}_{m,n}\right)^{i+1}-2\cdot\mathrm{Fo}\cdot\left[\left(\mathrm{T}_{m-1,n}\right)^{i+1}+\left(\mathrm{T}_{m,n-1}\right)^{i+1}\right]=\left(\mathrm{T}_{m,n}\right)^{i}+4\cdot\mathrm{Bi}\cdot\mathrm{Fo}\cdot\mathrm{T}_{a} \tag{8.79}$$

==========

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1IA. One-dimensional, steady state conduction:

Prob. 1I.A.1. Heat is generated uniformly in a stainless steel plate having k = 20 W/m.K. The thickness of the plate is 1 cm and heat generation rate is 500 MW/m³. If the two sides of the plate are maintained at 200 and 100 C respectively, calculate the temperature at the centre of the plate. Also find the distance of the plate at which maximum temperature occurs from the 200 C surface.

Mathcad Solution:

Data:

$L := 0.01$ m...thickness of slab

$M := 10$ number of sub-regions to which the slab is divided

$k := 20$ W/(m.C).....thermal cond. of slab material

$T_0 := 200$ C...temp. of left surface, node '0'.

$T_{10} := 100$ C...temp. of right surface, node '10'

$q_g := 500 \cdot 10^6$ W/m³...uniform heat gen. rate in slab

$\Delta x = \frac{L}{M} = \frac{0.01}{10}$ i.e. $\Delta x := 0.001$ m...distance between adjacent nodes

Note that there are 11 nodes, numbered as: 0,1,2,3,4,…..10. Out of these, nodes '0' and '10' are boundary nodes and the nodes 1,2,3…9 are internal nodes. Temperature of node '0' is given, i.e. T_0 = 200 C, and T_{10} = 100 C.

Fig. 1I.A.1 shows the schematic of finite difference nodes for this problem.

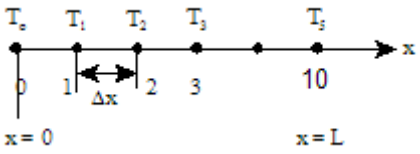

Fig.1I.A.1. Finite difference nodes

Apply eqn. (8.8) for interior nodes, 1,2,3…9:

$$\left(T_{m-1} - 2 \cdot T_m + T_{m+1}\right) + \frac{q_g \cdot (\Delta x)^2}{k} = 0 \qquad (8.78)$$

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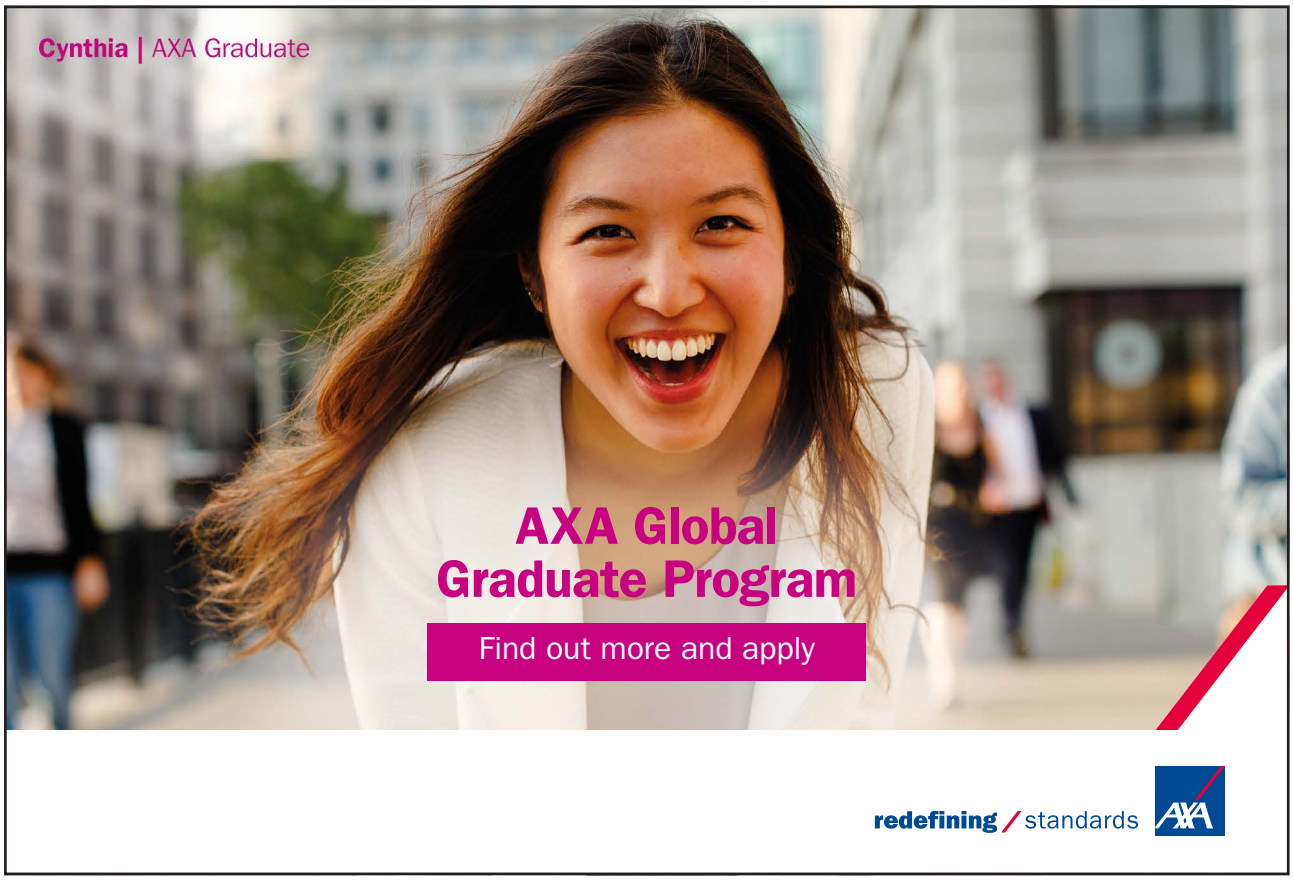
We have: $\frac{q_g \cdot (\Delta x)^2}{k} = 25$

Node 0: $T_0 = 200$ C....by data.....(a)

Node 1: $T_0 - 2 \cdot T_1 + T_2 + 25 = 0$ .....(b)

Node 2: $T_1 - 2 \cdot T_2 + T_3 + 25 = 0$ ....(c)

Node 3: $T_2 - 2 \cdot T_3 + T_4 + 25 = 0$ ....(d)



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Node 4: $T_3 - 2 \cdot T_4 + T_5 + 25 = 0$ .....(e)

Node 5: $T_4 - 2 \cdot T_5 + T_6 + 25 = 0$ .....(f)

Node 6: $T_5 - 2 \cdot T_6 + T_7 + 25 = 0$ ....(g)

Node 7: $T_6 - 2 \cdot T_7 + T_8 + 25 = 0$ ....(h)

Node 8: $T_7 - 2 \cdot T_8 + T_9 + 25 = 0$ .....(i)

Node 9: $T_8 - 2 \cdot T_9 + T_{10} + 25 = 0$ .....(j)

For Node 10: For this boundary condition, temp is given:

Node 10: $T_{10} = 100$ .....(k)

Equations (a) to (k) have to be solved simultaneously to get 10 nodal temperatures. Of course, in this case temp. at node '0' and node '10' is already known.

We will use 'Solve block' of Mathcad to solve these 10 equations simultaneously.

We start with assumed or trial values for all the variables i.e. for the temperatures at nodes 1 to 9. Then, in the solve block, immediately below 'Given' write all the constraint equations. Then, the command 'Find(T0,T1,T2...T10)' immediately gives a vector of temperature values:

$T_1 := 50$ $T_2 := 50$ $T_3 := 50$ $T_4 := 50$

....trial values of temperatures

$T_5 := 50$ $T_6 := 50$ $T_7 := 50$ $T_8 := 50$

$T_9 := 50$ $T_0 := 200$ $T_{10} := 100$

Given

Node 0: $T_0 = 200$ C....by data.....(a)

Node 1: $T_0 - 2 \cdot T_1 + T_2 + 25 = 0$ .....(b)

Node 2: $T_1 - 2 \cdot T_2 + T_3 + 25 = 0$ ....(c)

Node 3: $T_2 - 2 \cdot T_3 + T_4 + 25 = 0$ ....(d)

Node 4: $T_3 - 2 \cdot T_4 + T_5 + 25 = 0$ .....(e)

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Node 5: $T_4 - 2 \cdot T_5 + T_6 + 25 = 0$ .....(f)

Node 6: $T_5 - 2 \cdot T_6 + T_7 + 25 = 0$ ....(g)

Node 7: $T_6 - 2 \cdot T_7 + T_8 + 25 = 0$ ....(h)

Node 8: $T_7 - 2 \cdot T_8 + T_9 + 25 = 0$ .....(i)

Node 9: $T_8 - 2 \cdot T_9 + T_{10} + 25 = 0$ .....(j)

Node 10: $T_{10} = 100$ .....(k)

$$Temp := Find(T_0, T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9, T_{10})$$

…'Temp' is the vector containing values of temperatures T0, T1…T10 .. shown below:

Therefore,

Temp =

	0
0	200
1	302.5
2	380
3	432.5
4	460
5	462.5
6	440
7	392.5
8	320
9	222.5
10	100

i.e. Temperatures at different nodes are: T0 = 200 C, T1 = 302.5 C, T3 = 380 C…etc.

To draw the temp. distribution:

In the above, temperatures at various nodes are contained in vector 'Temp'.

$i := 0, 1 .. 10$ ...define the range variable i, varying from 0 to 10 with an increment of 1

'i' is the node no. and (i. Δx) gives the distance from left edge of slab.

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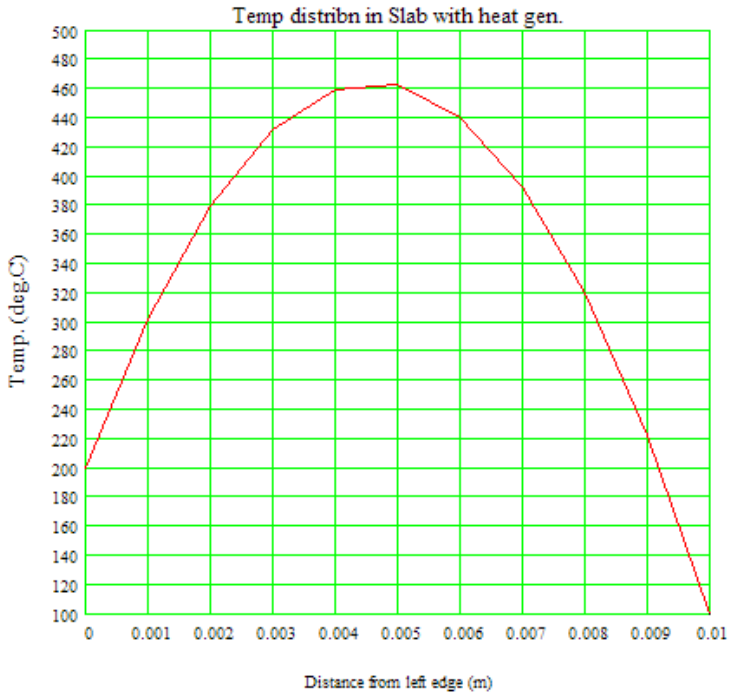
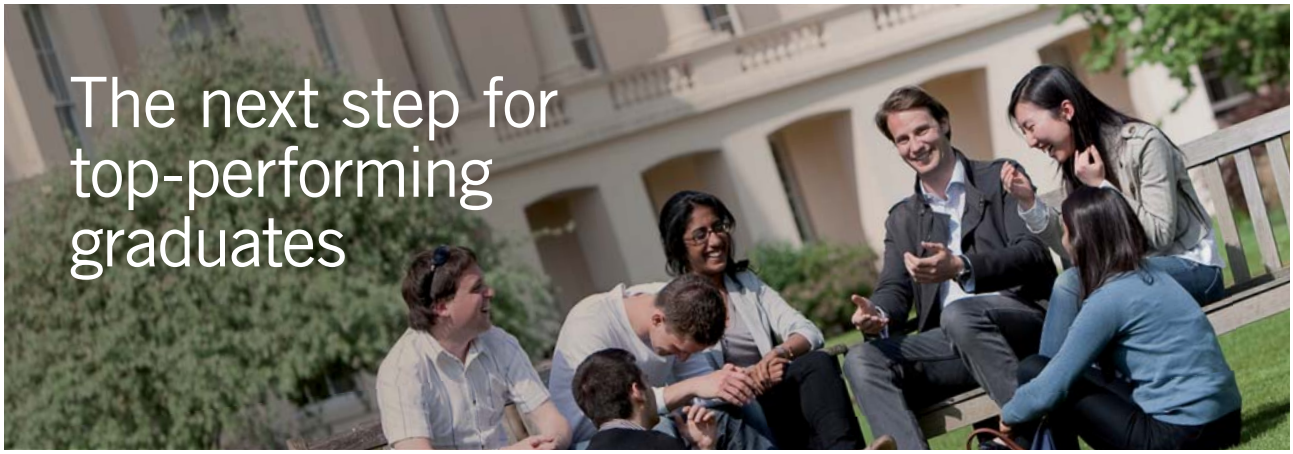


The graph is not very smooth, since only 10 nodes are chosen. It may be noted that if more no. of nodes are chosen, obviously the graph will be smoother.



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Also note that mid-point temp. T5 = 462.5 C.' Tmax occurs at slighly before x = 0.005 m.

This problem was solved analytically in Prob. 1F.2.

Let us compare the results from analytical relation and numerical methods:

Analytical relation for temp distribution is:

$T1 := 200\ C \qquad T2 := 100\ C$

$$T(x) := T1 + \left[(L - x) \cdot \frac{q_g}{2 \cdot k} + \frac{(T2 - T1)}{L} \right] \cdot x \qquad T(0.01) = 100$$

....checks.

$x := 0, 0.001 .. 0.01$ ....define the range variable x

$i := 0, 1 .. 10$ ....define the range variable i

Analytical method **Numerical method**

x	T(x)	$i \cdot \Delta x$	$Temp_i$
0	200	0	200
0.001	302.5	0.001	302.5
0.002	380	0.002	380
0.003	432.5	0.003	432.5
0.004	460	0.004	460
0.005	462.5	0.005	462.5
0.006	440	0.006	440
0.007	392.5	0.007	392.5
0.008	320	0.008	320
0.009	222.5	0.009	222.5
0.01	100	0.01	100

As we can see from the above Table, there is an excellent agreement between the analytical results and results by finite difference method.

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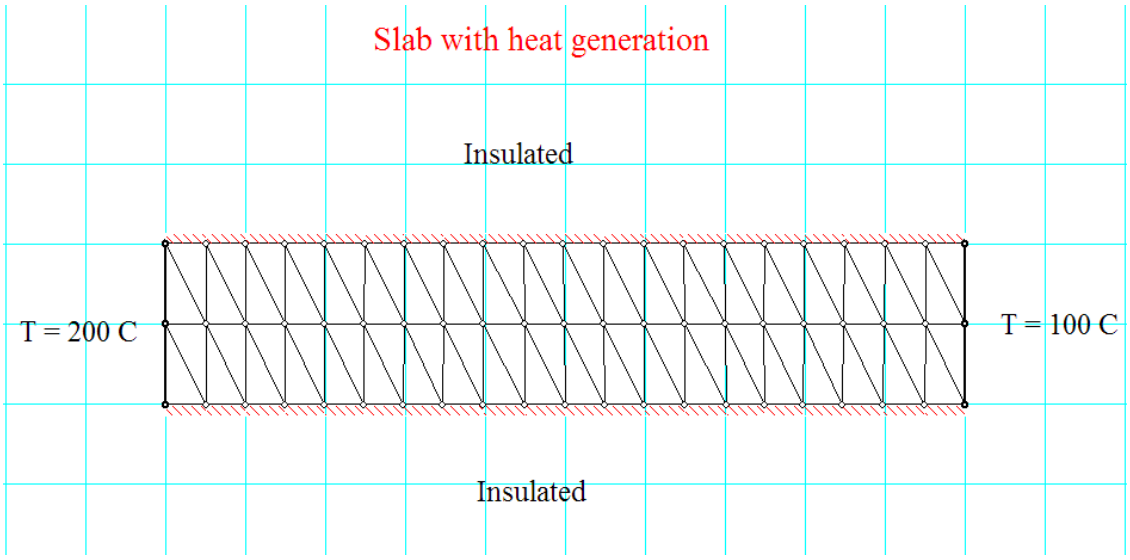
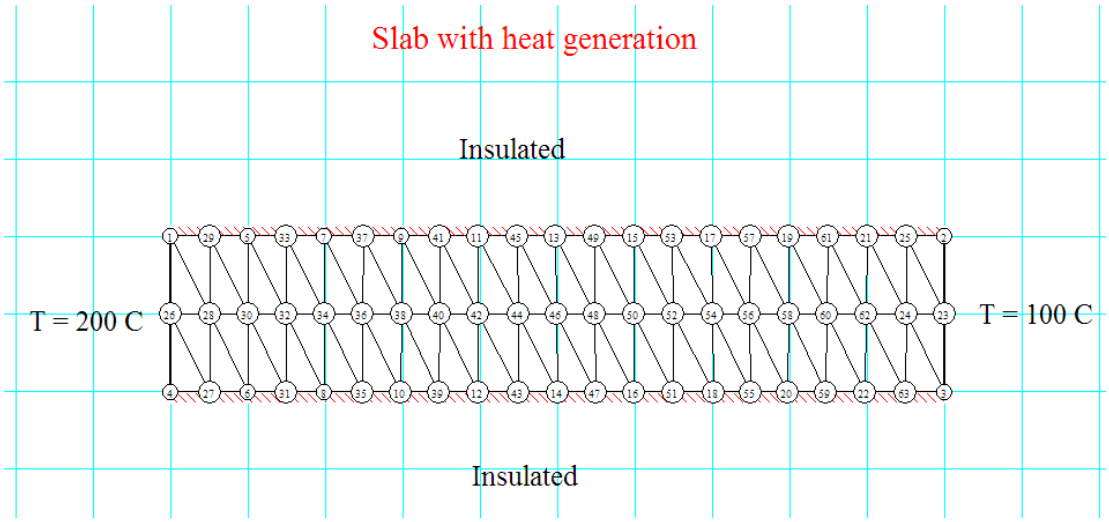
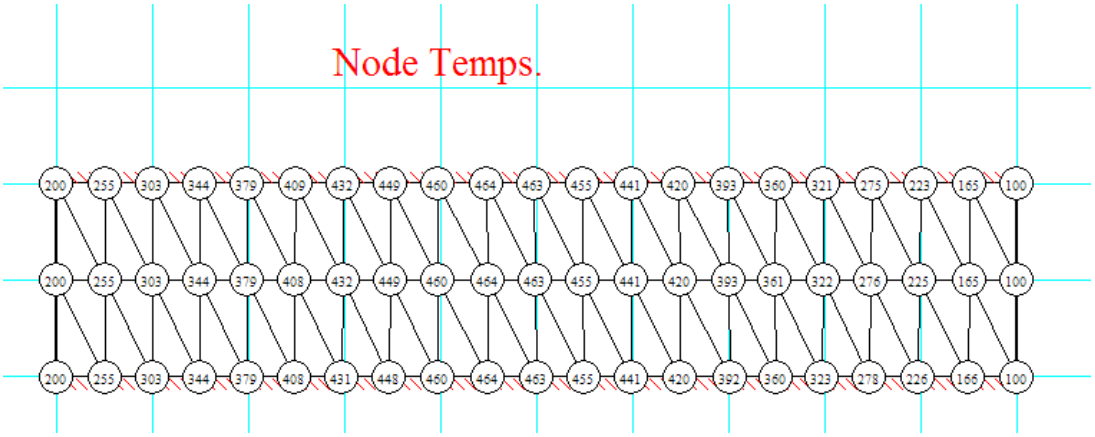
Solution by Finite Element Heat Transfer (FEHHT) Software:



Node Nos.:



Node Temperatures:



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Plot of x vs Temp:

LHs = 200C, RHS = 100C

Dist (mm)	T(deg.C)
0	200
1	303
2	379
3	432
4	460
5	463
6	441
7	393
8	322
9	225
10	100

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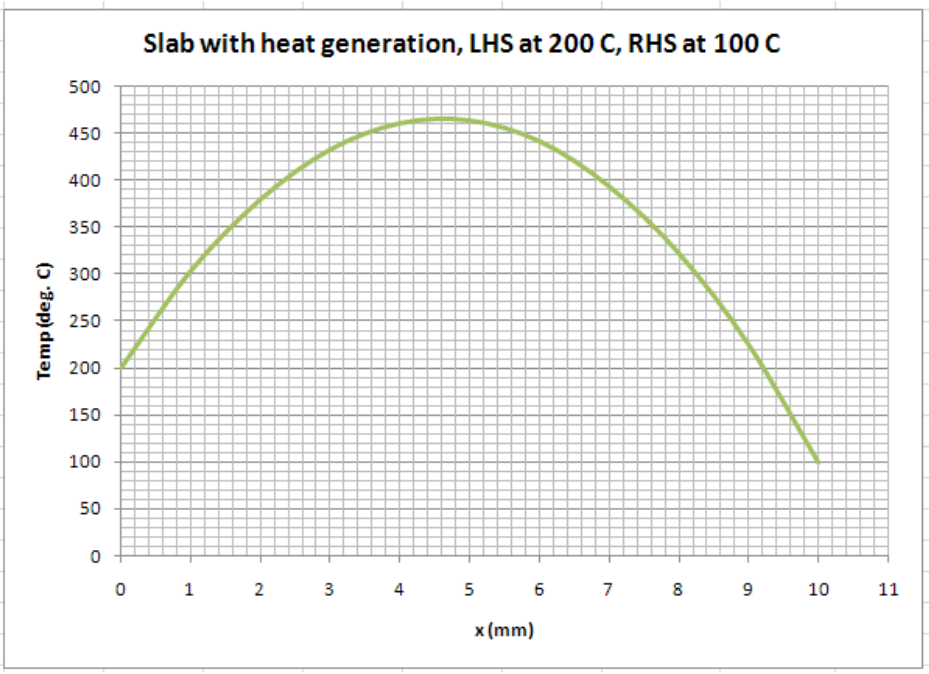

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(b) When left surface is insulated, right face maintained at 100 C:

Now, the node '0' is on an insulated boundary. Difference equation for node '0' is obtained now treating it as an internal node if the insulated surface is imagined to be a mirror i.e. node '1' extends to the left of node '0' and eqn. (8.8) is applicable.

$$\text{i.e.} \qquad \left(T_{m-1} - 2\cdot T_m + T_{m+1}\right) + \frac{q_m\cdot(\Delta x)^2}{k} = 0 \qquad \text{.....(8.8)}$$

$$\text{For m = 0:} \qquad T_{-1} - 2\cdot T_o + T_1 + \frac{q_g\cdot(\Delta x)^2}{k} = 0$$

From mirror image concept: $T_{-1} = T_1$

Therefore, for node '0', we get:

$$2\cdot T_1 - 2\cdot T_0 + 25 = 0 \qquad \text{.....(a)}$$

Equations for other nodes remain unchanged.

Therefore, solving eqn (a')….(k) simultaneously will give the temperatures at nodes 0 to 10.

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Use 'solve block' to solve the set of algebraic equations (a') to (k) simultaneously, in Mathcad.

Start with assumed or trial values of temperatures:

$T_0 := 50$ $T_1 := 50$ $T_2 := 50$ $T_3 := 50$ $T_4 := 50$ $T_5 := 50$ ....trial values of temperatures

$T_6 := 50$ $T_7 := 50$ $T_8 := 50$ $T_9 := 50$ $T_{10} := 100$

Given

Node 0: $2 \cdot T_1 - 2 \cdot T_0 + 25 = 0$ .....(a')

Node 1: $T_0 - 2 \cdot T_1 + T_2 + 25 = 0$ .....(b)

Node 2: $T_1 - 2 \cdot T_2 + T_3 + 25 = 0$ ....(c)

Node 3: $T_2 - 2 \cdot T_3 + T_4 + 25 = 0$ ....(d)

Node 4: $T_3 - 2 \cdot T_4 + T_5 + 25 = 0$ .....(e)

Node 5: $T_4 - 2 \cdot T_5 + T_6 + 25 = 0$ .....(f)

Node 6: $T_5 - 2 \cdot T_6 + T_7 + 25 = 0$ ....(g)

Node 7: $T_6 - 2 \cdot T_7 + T_8 + 25 = 0$ ....(h)

Node 8: $T_7 - 2 \cdot T_8 + T_9 + 25 = 0$ .....(i)

Node 9: $T_8 - 2 \cdot T_9 + T_{10} + 25 = 0$ .....(j)

Node 10: $T_{10} = 100$ .....(k)

$Temp := Find(T_0, T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9, T_{10})$

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…'Temp' is the vector containing values of temperatures T0, T1…T10 .. shown below:

Temp =

	0
0	1350
1	1337.5
2	1300
3	1237.5
4	1150
5	1037.5
6	900
7	737.5
8	550
9	337.5
10	100

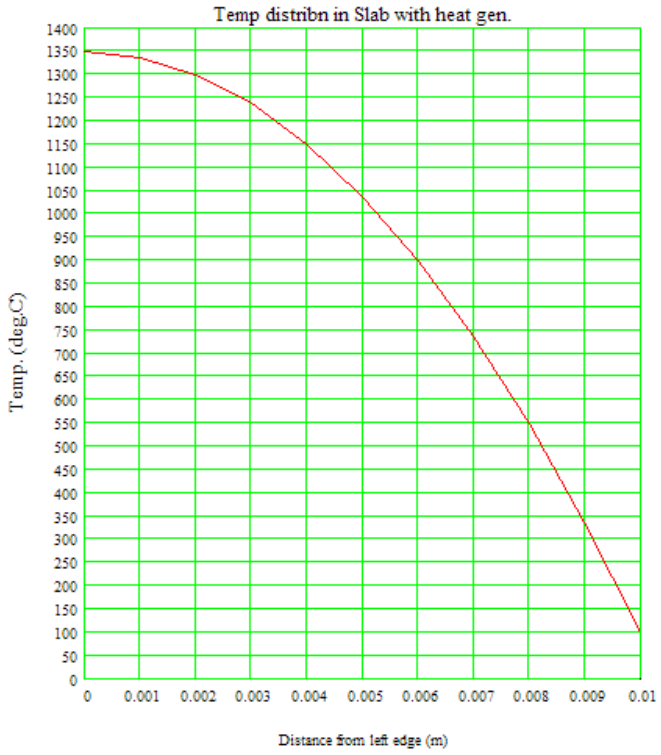
i.e. T0 = 1350 C, T1 = 1337.5 C, T3 = 1300 C … T10 = 100 C. etc.

To draw the temp. distribution:

In the above, temperatures at various nodes are contained in vector 'Temp'.

$i := 0, 1 .. 10$ …define the range variable i, varying from 0 to 10 with an increment of 1

'i' is the node no. and (i. Δx) gives the distance from left edge of slab.



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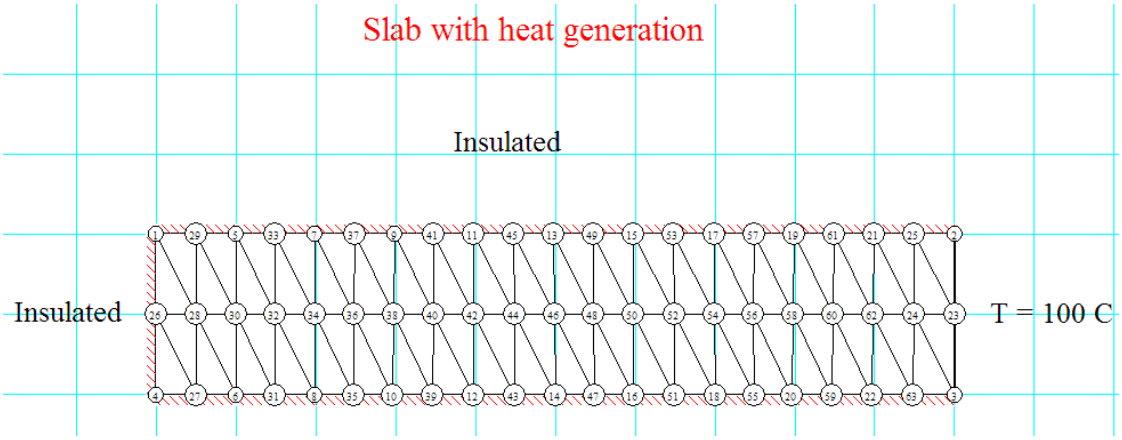
Obviously, max. temp occurs at the insulated left face, and Tmax = 1350 C. and at mid-point i.e. x = 0.005 m, the temp is 1037.5 C.

Solution with FEHT:

Node Nos.:



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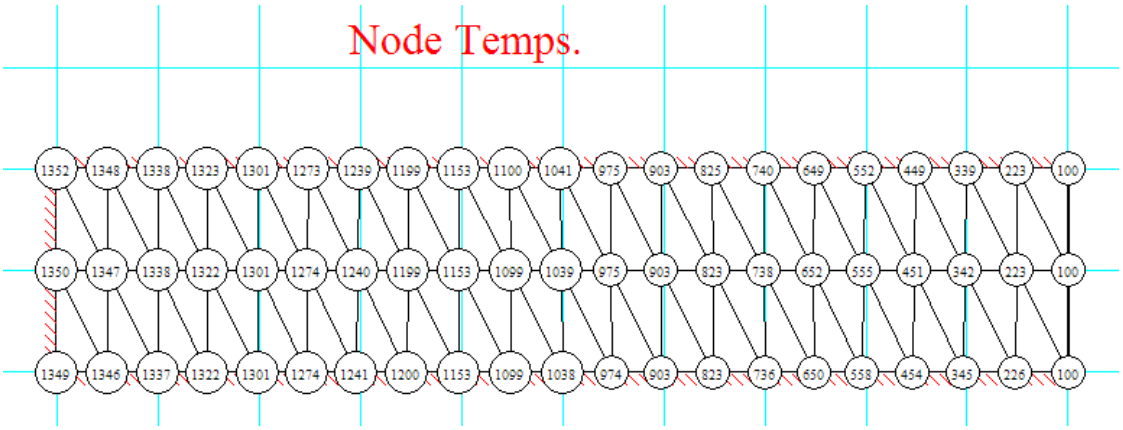
Node Temps.:



Plot of x vs Temp:

LHs = Ins., RHS = 100C

Dist (mm)	T(deg.C)
0	1350
1	1338
2	1301
3	1240
4	1153
5	1039
6	903
7	738
8	555
9	342
10	100

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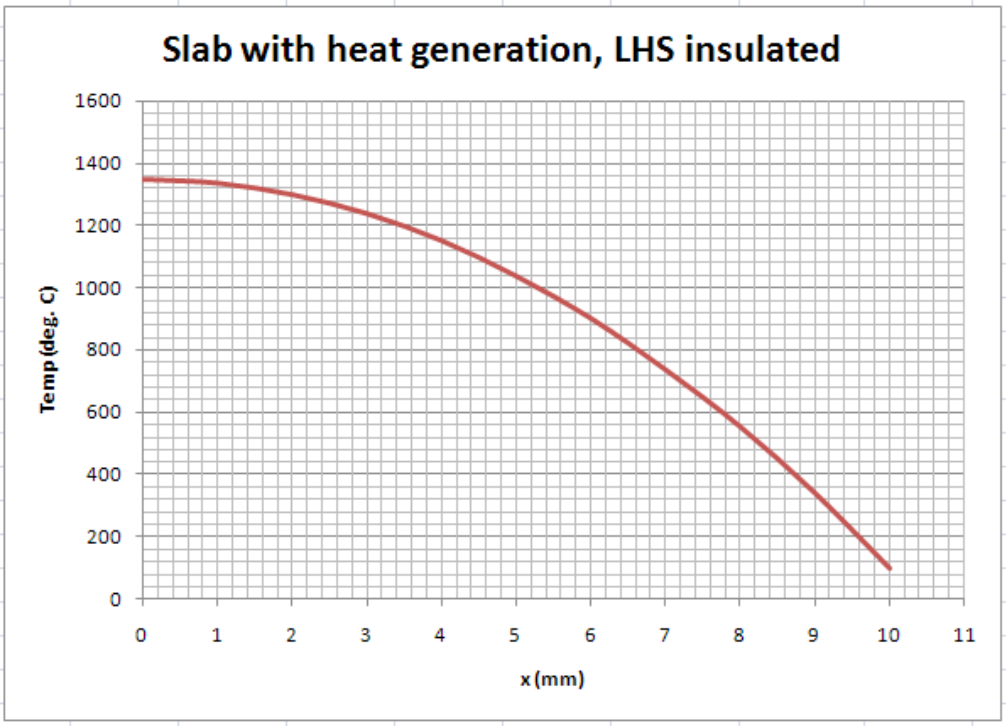

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(c) When left surface is maintained at 200 C, right face subjected to convection by a fluid at 100 C with a heat transfer coeff. h = 4000 W/m^2.C:

Now: $T_a := 100$ C... temp of fluid $h := 4000$ W/m^2.C.....convection coeff

Now, all other eqns. except for node 0 and 10 remain the same as for previous problem.
For node 10, write the difference eqn.:

For Node 10: here, we have convection boundary condition. So, apply eqn.(8.22):

$$2 \cdot T_{M-1} - 2 \cdot T_M \cdot \left(1 + \frac{h \cdot \Delta x}{k}\right) + \frac{(\Delta x)^2 \cdot q_M}{k} + \frac{2 \cdot h \cdot \Delta x}{k} \cdot T_a = 0 \qquad (8.22)$$

OR: we can directly write the energy balance for node 10, remembering that all heat flows to be considered as flowing 'in to' the node: We get:

$$k \cdot \left(\frac{T_9 - T_{10}}{\Delta x}\right) + h \cdot \left(T_a - T_{10}\right) + q_g \cdot \left(\frac{\Delta x}{2}\right) = 0 \qquad \text{....eqn. (k)}$$

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For node '0', we have T0 = 200 C.

Equations (a) to (k) have to be solved simultaneously to get 10 nodal temperatures.

Use 'solve block' to solve the set of algebraic equations (a') to (k) simultaneously, in Mathcad.
Start with assumed or trial values of temperatures:

$T_0 := 200$ $T_1 := 50$ $T_2 := 50$ $T_3 := 50$ $T_4 := 50$ $T_5 := 50$ ....trial values of temperatures

$T_6 := 50$ $T_7 := 50$ $T_8 := 50$ $T_9 := 50$ $T_{10} := 100$

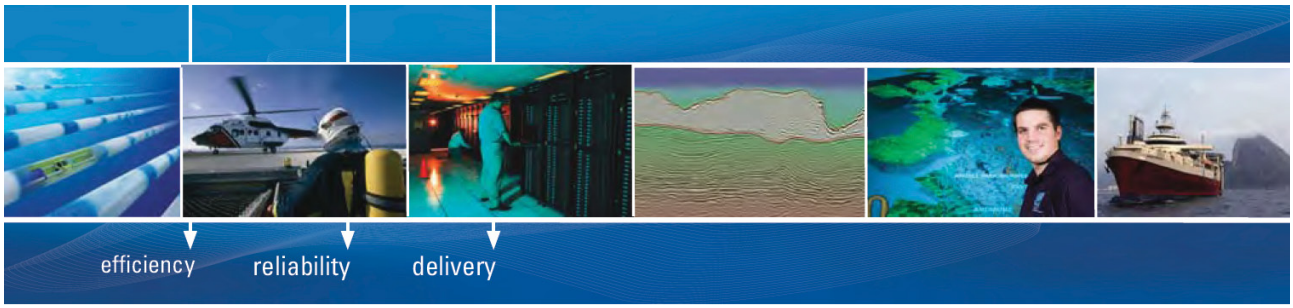
Given

Node 0: $T_0 = 200$ .....(a')

Node 1: $T_0 - 2 \cdot T_1 + T_2 + 25 = 0$ .....(b)

Node 2: $T_1 - 2 \cdot T_2 + T_3 + 25 = 0$ ....(c)

Node 3: $T_2 - 2 \cdot T_3 + T_4 + 25 = 0$ ....(d)



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Node 4: $T_3 - 2 \cdot T_4 + T_5 + 25 = 0$ .....(e)

Node 5: $T_4 - 2 \cdot T_5 + T_6 + 25 = 0$ .....(f)

Node 6: $T_5 - 2 \cdot T_6 + T_7 + 25 = 0$ ....(g)

Node 7: $T_6 - 2 \cdot T_7 + T_8 + 25 = 0$ ....(h)

Node 8: $T_7 - 2 \cdot T_8 + T_9 + 25 = 0$ .....(i)

Node 9: $T_8 - 2 \cdot T_9 + T_{10} + 25 = 0$ .....(j)

Node 10: $k \cdot \left(\frac{T_9 - T_{10}}{\Delta x} \right) + h \cdot \left(T_a - T_{10} \right) + q_g \cdot \left(\frac{\Delta x}{2} \right) = 0$ .....(k)

$Temp := Find\left(T_0, T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9, T_{10}\right)$

…'Temp' is the vector containing values of temperatures T0, T1…T10 .. shown below:

$Temp =$

	0
0	200
1	347.5
2	470
3	567.5
4	640
5	687.5
6	710
7	707.5
8	680
9	627.5
10	550

To draw the temp. distribution:

In the above, temperatures at various nodes are contained in vector 'Temp'.

$i := 0, 1 .. 10$ ...define the range variable i, varying from 0 to 10 with an increment of 1

'i' is the node no. and (i. Δx) gives the distance from left edge of slab.

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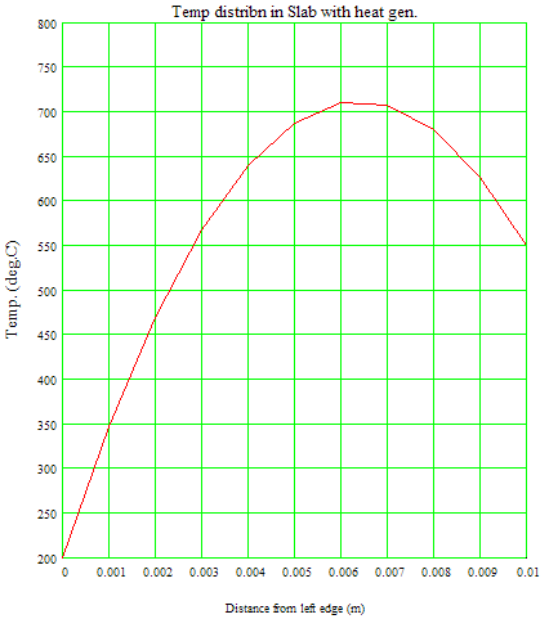
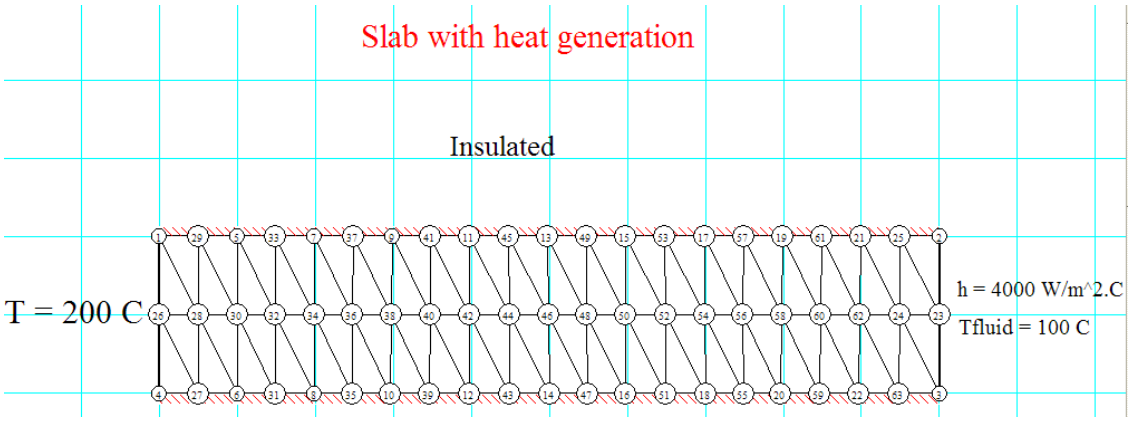


Now, temp at the mid-point, i.e. x = 0.005 m is 687.5 C and temp at the right face, i.e. at x = 0.01 m is = 550 C. and at mid-point i.e. x = 0.005 m, the temp is 687.5 C. T_{max} occurs at around x = 0.065 m.

Solution with FEHT:

Node Nos.:



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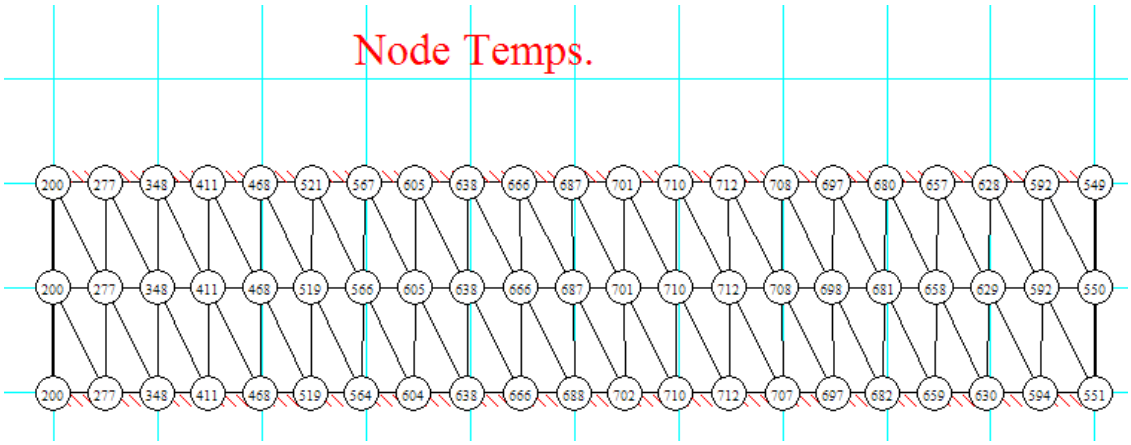
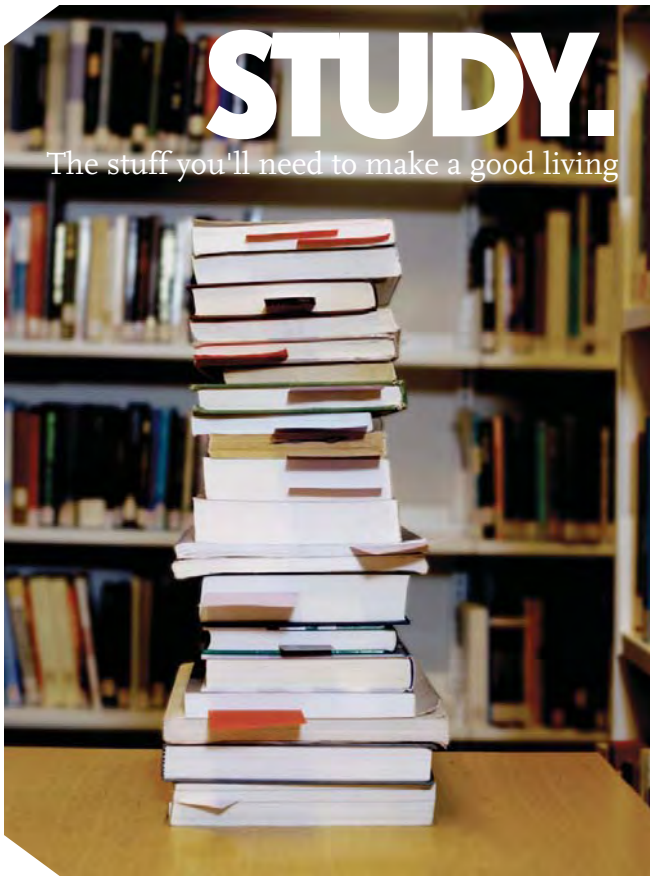
Node Temps:





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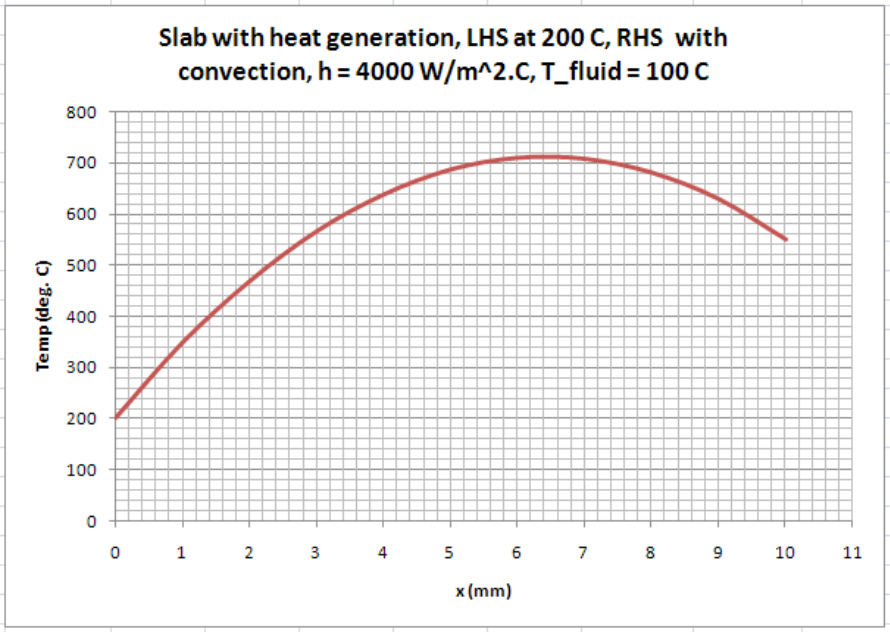
Plot x vs Temp:

LHs = 200 C., RHS = Convn, h = 4000 W/m^2.C, T_inf = 100 C

Dist (mm)	T(deg.C)
0	200
1	348
2	468
3	566
4	638
5	687
6	710
7	708
8	681
9	629
10	550



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"**Prob. 1I.A.2.** Consider a large plane wall of thickness L = 0.3 m, k = 2.5 W/m.C, and surface area A = 24 m^2. The left side of the wall is subjected to a heat flux of q_0 = 350 W/m^2 while the temp at that surface is measured to be T_0 = 60 C. Assuming steady, 1D heat transfer and taking a nodal spacing of 6 cm, (a) obtain the finite difference formulation for the six nodes and (b) determine the temp of the other surface of the wall by solving those eqns. [Ref. 2]"

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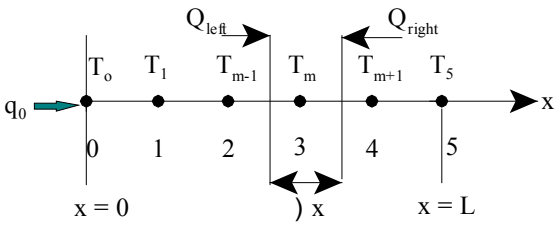

Fig.Prob.1I.A.2

EES Solution:

"Data:"

k = 2.5 [W/m-C]

L = 0.3[m]

A = 24 [m^2]

"There are 5 equal divisions (M = 5) so that DELTAx = 0.06 m.
Of these, 0 and 5 are boundary nodes and 1 to 4 are internal nodes:
On the LHS, there is constant heat flux, and the temp is also given; on the RHS no condition is given."

DELTAx = 0.06[m]

q_0 = 350[W/m^2]

T[0]=60[C] "..for node '0' … by data"

"Find T_1 by energy balance at node 0:"

q_0 * A + k * A * (T[1] – T[0]) / DELTAx = 0 "…finds T[1] … energy balance at node 0"

"Similarly, eqn for node 1 gives temp T_2, ….. and eqn for node 4 gives temp T_5:"

"Now for internal nodes, for the mth node: the difference eqn. is:

T_(m-1) – 2 * T_m +T_(m+1) = 0"

"i.e. for nodes 1 to 4, we get:"

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Duplicate m = 1,4

T[m-1] – 2*T[m] + T[m+1] = 0

end

"To draw the plot of x vs Temp.:"

duplicate i = 0,5

X[i] = i * DELTAx

End

Results:

Unit Settings: SI C kPa kJ mass deg

A = 24 [m^2] Δx = 0.06 [m] k = 2.5 [W/m-C] L = 0.3 [m]

q_0 = 350 [W/m^2]

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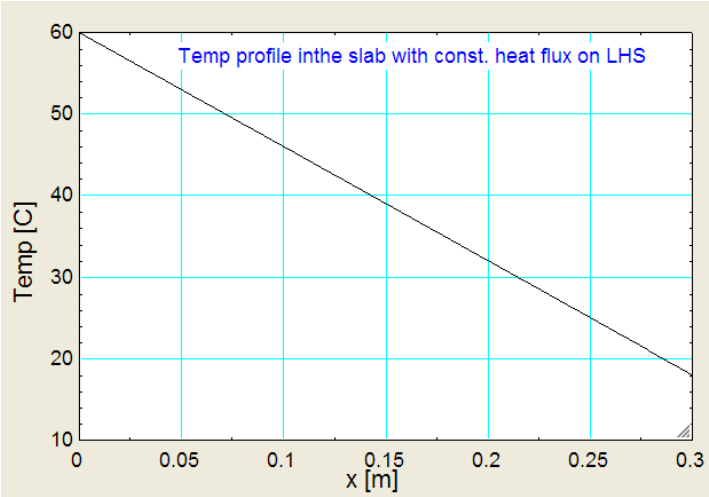
Temp at various distances from LHS:

Sort	1 T_i [C]	2 X_i [m]
[0]	60	0
[1]	51.6	0.06
[2]	43.2	0.12
[3]	34.8	0.18
[4]	26.4	0.24
[5]	18	0.3

Plot the temp distribution:



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"**Prob. 1I.A.3.** Consider a plane wall of thickness L = 0.02 m, k = 25 W/m.C. Internal heat generation rate is 0.25 MW/m^3. On both sides, heat is transferred only by radiation to the surroundings at 30 C. Determine centre and surface temperatures. Take 10 equal divisions."

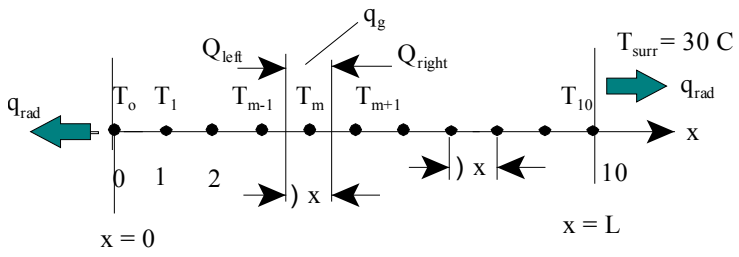

Fig.Prob.1I.A.3

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EES Solution:

"Data:"

k = 25 [W/m-C]

L = 0.02[m]

A = 1 [m^2]"…assumed"

sigma = 5.67e-08 [W/m^2-K^4] "..Stefan – Boltzmann constant"

epsilon = 1 "…emissivity of surfaces = 1, assumed"

"There are 10 equal divisions (M = 10) so that DELTAx = 0.002 m.
Of these, 0 and 10 are boundary nodes and 1 to 9 are internal nodes:"

DELTAx = 0.002[m]

q_g = 0.25e06[W/m^2]

T_surr = 30[C]

"**To find temp at internal nodes:**

By energy balance at node m, remembering to write all energy terms as flowing ***in to*** the node:

k * A * (T[m-1] – T[m]) / DELTAx + k * A * (T[m+1] – T[m]) / DELTAx + q_g * A * DELTAx = 0

This is the eqn used for for temperatures T[1] to T[9].

Enter the eqn in EES as follows, to get the eqns for temperatures T[1] to T[9]:"

Duplicate m = 1,9

k * A * (T[m-1] – T[m]) / DELTAx + k * A * (T[m+1] – T[m]) / DELTAx + q_g * A * DELTAx = 0

end

"**Now for boundary nodes '0' and '10**':"

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"For node '0':"

k * A * (T[1] – T[0]) / DELTAx + sigma * A * epsilon * ((T_surr + 273)^4 – (T[0] + 273)^4) + q_g * A * DELTAx/2 = 0 " T[0] in deg.C"

"For node 10, we get:"

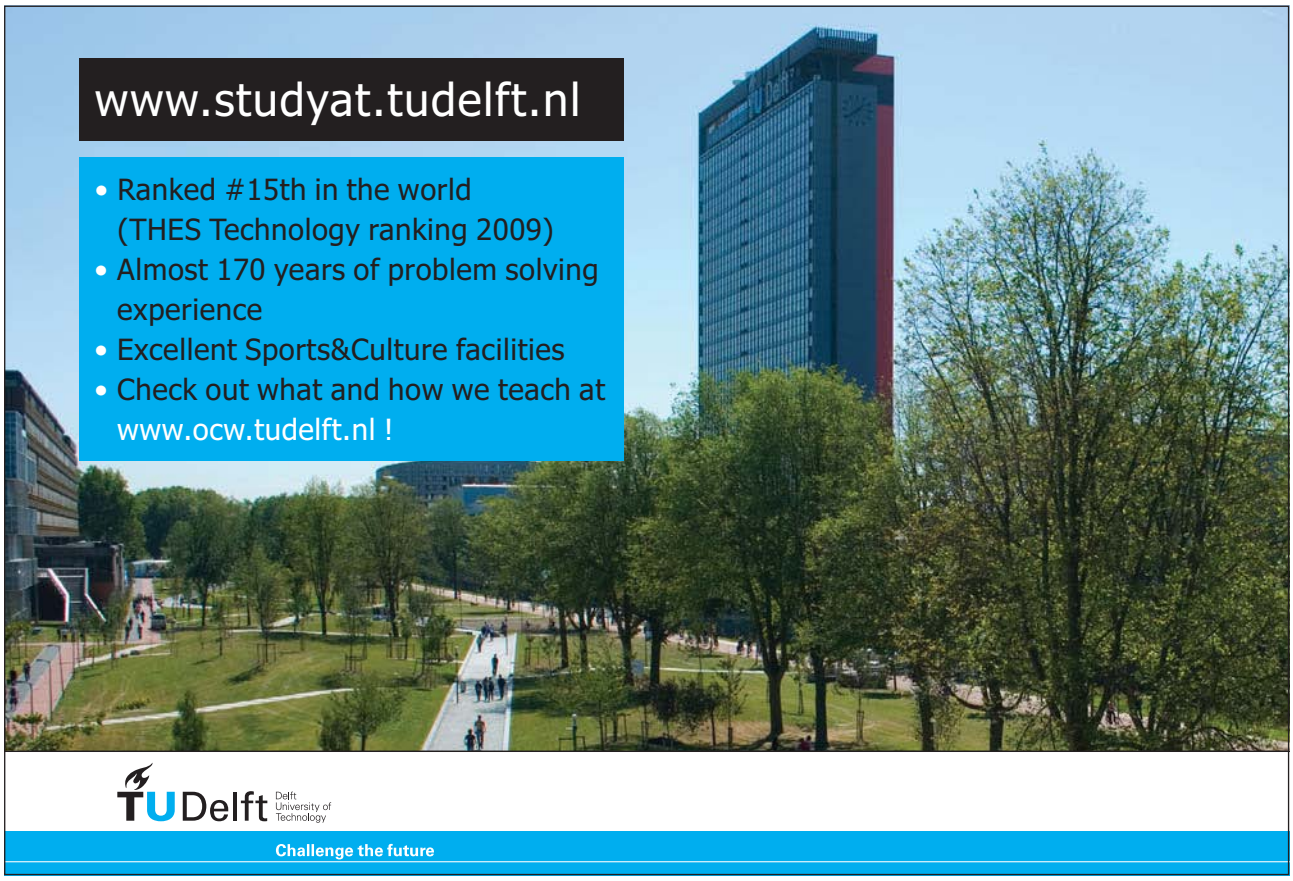
k * A * (T[9] – T[10]) / DELTAx + sigma * A * epsilon * ((T_surr + 273)^4 – (T[10] + 273)^4) + q_g * A * DELTAx/2 = 0 " T[10] in deg.C"

"**To draw the plot of x vs Temp.**:"

duplicate i = 0,10

X[i] = i * DELTAx "…creates an array of x's"

end



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Results:

Unit Settings: SI C kPa kJ mass deg

$A = 1$ [m^2]	$\Delta x = 0.002$ [m]
$\varepsilon = 1$	$k = 25$ [W/m-C]
$L = 0.02$ [m]	$q_g = 250000$ [W/m^2]
$\sigma = 5.670E-08$ [W/m^2K^4]	$T_{surr} = 30$ [C]

Temp. at various locations:

Sort	1 T_i [C]	2 X_i [m]
[0]	205.7	0
[1]	205.9	0.002
[2]	206	0.004
[3]	206.1	0.006
[4]	206.2	0.008
[5]	206.2	0.01
[6]	206.2	0.012
[7]	206.1	0.014
[8]	206	0.016
[9]	205.9	0.018
[10]	205.7	0.02

Therefore, centre temp = 206.2 C at x = 0.01 m, and

Surface temp = 205.7 C at x = 0.02 m…..(Ans)

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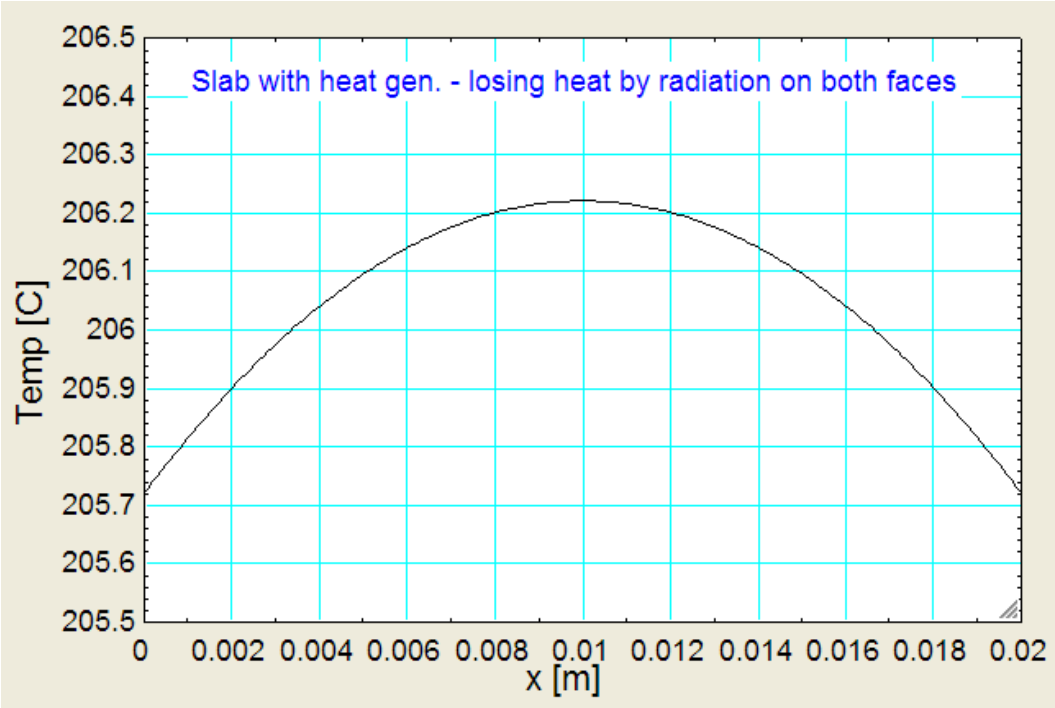
Plot the temp profile:



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"**Prob. 1I.A.4.** In a slab (k = 15 W/m.K), 14 cm thick, and insulated at the left face (i.e. at x = 0), heat generation rate varies with position as: q_g = q_0 * exp(-50 * x), W/m^3, where q_0 = 10^6 and x is in metres. Right surface is subjected to convection with h = 4000 W/m^2.C with a fluid at 80 C. Determine the temperatures on both the faces and also the heat transfer from the surface. Take DELTAx = 1 cm."

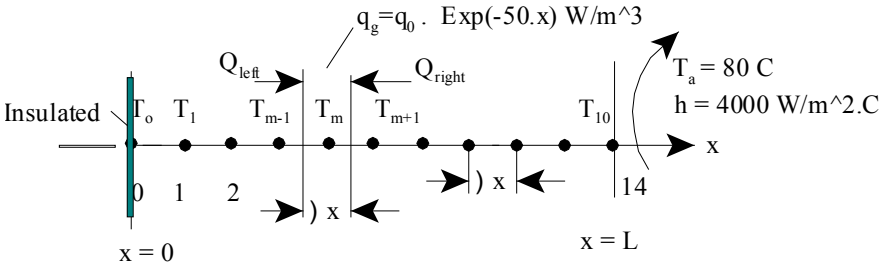

Fig.Prob.1I.A.4

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EES Solution:

"Data:"

"First define a function for heat generation:"

function q_g(x)

q_g = 1e06 * exp(– 50 * x) "[W/m^3]….defines q-g"

end

"----------"

k = 15 [W/m-C]

L = 0.14 [m]

A = 1 [m^2]"…assumed"

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h = 4000 [W/m^2-C]

T_a = 80 [C]

"There are 14 equal divisions (M = 14) so that DELTAx = 0.01 m. Nodes: 0 to 14

Of these, 0 and 14 are boundary nodes and 1 to 13 are internal nodes:"

DELTAx = 0.01[m]

"q_g = q_0 * exp (-50 * x) … [W/m^3] …. variable internal heat gen. rate"

"To find temp at internal nodes:

By energy balance at node m, remembering to write all energy terms as flowing in to the node:

k * A * (T[m-1] – T[m]) / DELTAx + k * A * (T[m+1] – T[m]) / DELTAx + q_g(m*DELTAx) * A * DELTAx = 0

This is the eqn for temperatures T[1] to T[13]"

Duplicate m = 1,13

k * A * (T[m-1] – T[m]) / DELTAx + k * A * (T[m+1] – T[m]) / DELTAx + q_g(m*DELTAx) * A * DELTAx = 0

end

"Now for boundary nodes '0' and '14':"

"For node '0':"

k * A * (T[1] – T[0]) / DELTAx + q_g(0) * A * DELTAx/2 = 0 "for T[0]"

"For nodes 14, we get:"

k * A * (T[13] – T[14]) / DELTAx + q_g (L)* A * DELTAx/2 + h * A * (T_a – T[14]) = 0 "for T[14]"

"To draw the plot of x vs Temp.:"

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duplicate i = 0,14

X[i] = i * DELTAx

end

"Heat transfer from the surface:"

Q = h * A * (T[14] – T_a) "[W]…..per m^2 of surface"

Results:

Unit Settings: SI C kPa kJ mass deg

A = 1 [m^2]	Δx = 0.01 [m]	h = 4000 [W/m^2-C]	k = 15 [W/m-C]
L = 0.14 [m]	Q = 20396 [W]	T_a = 80 [C]	

Sort	1 T_i [C]	2 X_i [m]
[0]	249.5	0
[1]	246.2	0.01
[2]	238.8	0.02
[3]	229	0.03
[4]	217.7	0.04
[5]	205.5	0.05
[6]	192.7	0.06
[7]	179.6	0.07
[8]	166.3	0.08
[9]	152.9	0.09
[10]	139.4	0.1
[11]	125.8	0.11
[12]	112.3	0.12
[13]	98.69	0.13
[14]	85.1	0.14

i.e. Temp at left (insulated) surface = 249.5 C …. Ans.

Temp. at right surface = 85.1 C .. Ans.

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Obviously, temp at the insulated surface is maximum.

Heat transferred from right surface = heat transferred by convection = 20396 W.

Q should also be equal to total heat generated in the slab. i.e.

Total heat generated in the slab:

$$Q := 10^6 \cdot \int_0^{0.14} \exp(-50 \cdot x)\, dx$$

$$Q = 1.998 \cdot 10^4 \quad W/m^2$$

This is 19980 W, whereas the heat transfer by convection is 20396 W. This small difference

(2.08%) is due to errors of finite difference numerical method.

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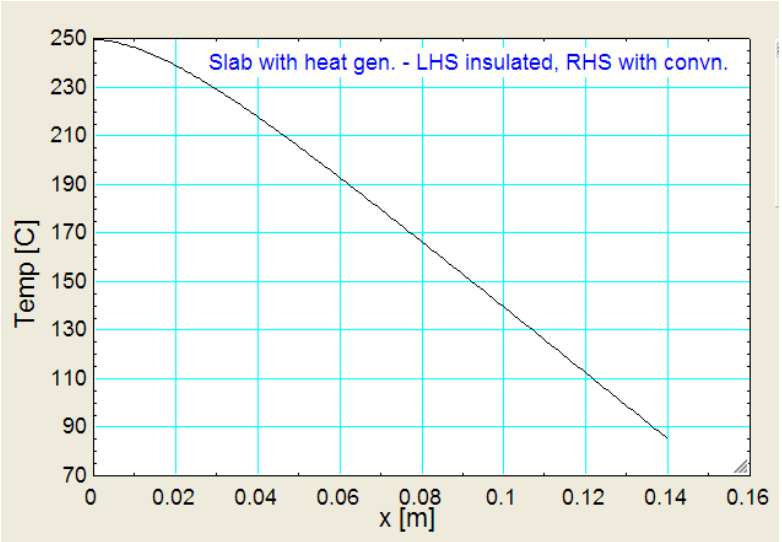
Plot T vs x for the slab:



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"**Prob. 1I.A.5.** A nuclear fuel element is in the form of hollow cylinder insulated at the inner surface. Its inner and outer radii are 50 mm and 100 mm respectively. Outer surface gives heat to a fluid at 50 C where the unit surface conductance is 100 W/m^2.C. The thermal cond. of material is 50 W/m.C. Find the rate of heat generation so that max. temp. in the system will not exceed 200 C. [P.U.]"

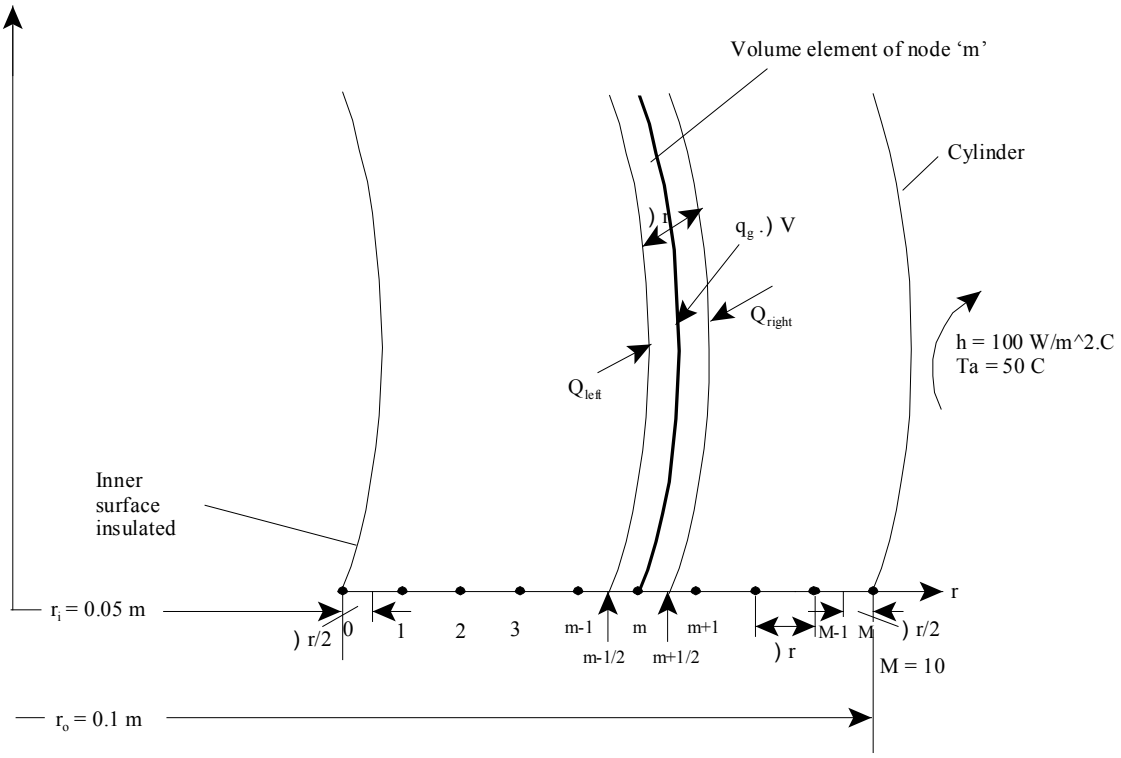

Fig.Prob.1I.A.5

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EES Solution:

"Data:"

k = 50 [W/m-C]

r_i = 0.05 [m]

r_0 = 0.1 [m]

L = 1 [m]"….assumed"

h = 100 [W/m^2-C]

T_a = 50 [C]

"q_g = heat gen. W/m^3, … to be found out:"

"Calculations:"

"Let there be 10 equal divisions (M = 10) of the radial distance (r_0 – r_i), so that DELTAr = 0.005 m.

Nodes: 0 to 10.

Of these, 0 and 10 are boundary nodes and 1 to 9 are internal nodes:"

DELTAr = 0.005[m]

"To find temp at internal nodes:

By energy balance at node m, remembering to write all energy terms as flowing in to the node:

(T[m-1] – T[m]) / (DELTAr / (k * 2 * pi * (r[m] – DELTAr/2) * L)) + (T[m+1] – T[m]) / (DELTAr / (k * 2 * pi * (r[m] + DELTAr/2) * L)) + q_g * (L * 2 * pi * r[m] * DELTAr) = 0

In the above eqn.: first term is heat flowing in from LHS, 2nd term is heat flowing in from RHS, and the 3rd tem is heat gen.

Also, note that effect of curvature of cylinder is neglected and thermal resistance is calculated treating the cyl. shell as a slab of thickness DELTAr.

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This is the eqn for temperatures T[1] to T[9]"

Duplicate m = 1,9

(T[m-1] – T[m]) / (DELTAr / (k * 2 * pi * (r[m] – DELTAr/2) * L)) + (T[m+1] – T[m]) / (DELTAr / (k * 2 * pi * (r[m] + DELTAr/2) * L)) + q_g * (L * 2 * pi * r[m] * DELTAr) = 0

end

"Now for boundary nodes '0' and '10':"

"For node '0':
Max. temp of 200 C occurs on this surface since it is insulated:"

T[0] = 200 [C] "for T[0] … by data"

"For node 10, we get:"

(k * 2 * pi* (r_0 – DELTAr/2) * L) * (T[9] – T[10]) / DELTAr + q_g * (L * 2 * pi * r_0 * DELTAr / 2) + h * (2 * pi * r_0 * L) * (T_a – T[10]) = 0

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"Also, heat generated = heat convected from outer surface:"

pi * (r_0^2 – r_i^2) * L * q_g = h * (2 * pi * r_0 * L) * (T[10] – T_a)

"To draw the plot of r vs Temp.:"

duplicate m = 0,10

r[m] = r_i + m * DELTAr
end

Results:

Unit Settings: SI C kPa kJ mass deg

$\Delta r = 0.005$ [m]	$h = 100$ [W/m^2-C]	$k = 50$ [W/m-C]	$L = 1$ [m]
$q_g = 379616$ [W/m^3]	$r_0 = 0.1$ [m]	$r_i = 0.05$ [m]	$T_a = 50$ [C]

Thus:

q_g = 379616 W/m^3 …. Heat gen. rate …. Ans.

And radial temp. distribution is given by:

Sort	1 r_i [m]	2 T_i [C]
[0]	0.05	200
[1]	0.055	199.9
[2]	0.06	199.6
[3]	0.065	199.2
[4]	0.07	198.6
[5]	0.075	197.9
[6]	0.08	197.1
[7]	0.085	196.1
[8]	0.09	195
[9]	0.095	193.7
[10]	0.1	192.4

i.e. temp. at the outer surface (i.e. at r = 0.1 m): T[10] = 192.4 C … Ans.

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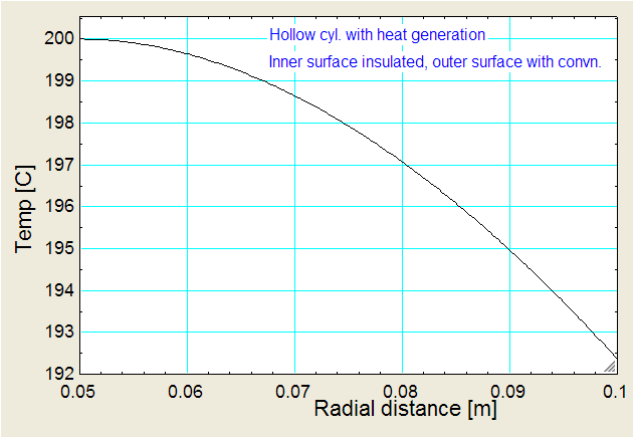
Plot the temp distribution:



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"Verify the results with Analytical relations:

We have the analytical relation for Tmax:"

{Tmax = T_w + (q_g * r_i^2 / (4 * k)) * ((r_0 / r_i)^2 – 2 * ln (r_0 / r_i) -1)}

200 – T_w = (q_g * r_i^2 / (4 * k)) * ((r_0 / r_i)^2 – 2 * ln (r_0 / r_i) -1) "..finds T_w, the outer wall temp."

"We note that T_w is 192.3 C as per this formula, whereas the result by numerical method was 192.4 C.

So, the variation is:"

Variation = (192.4 – 192.3) * 100 / 192.4 "[%] …. variation in the value of T_w"

"i.e. Variation is only about 0.052%"

==========

Solution to the above problem from Finite Element Heat Transfer (FEHT) software:

Choose 'cylindrical' (instead of Cartesian) from Setup, since we are dealing with cylindrical system.

So, horizontally, it is variation of R and vertically, Z.

After drawing the geometry, show top and bottom surfaces as 'insulated' for 1D effect.

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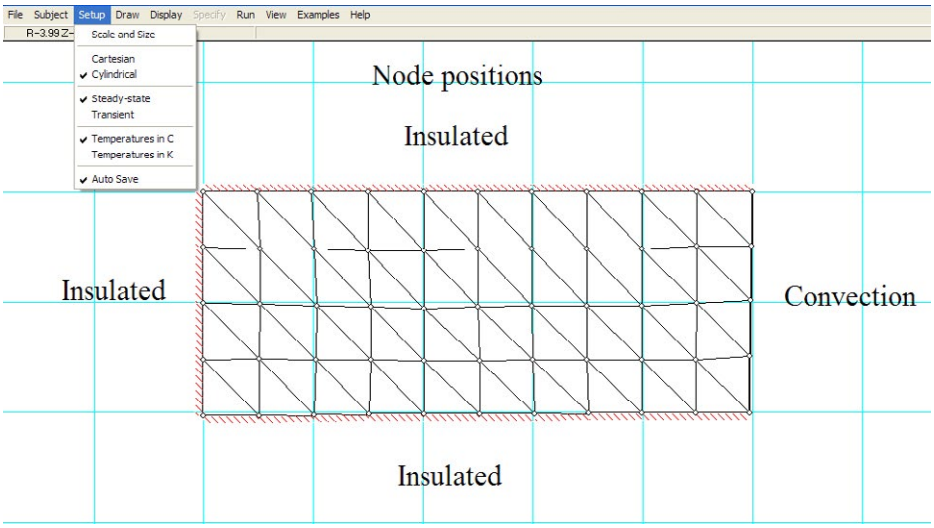
By data, RHS has convection and LHS is insulated.

Heat gen. rate is to be found out so that LHS (max temp) is 200 C.

To start with assume qg = 1E5 W/m^3 and calculate to find the temp on LHS; By trial and error, go on changing qg so that we get T = 200 C on the node corresponding to LHS.

Node positions:



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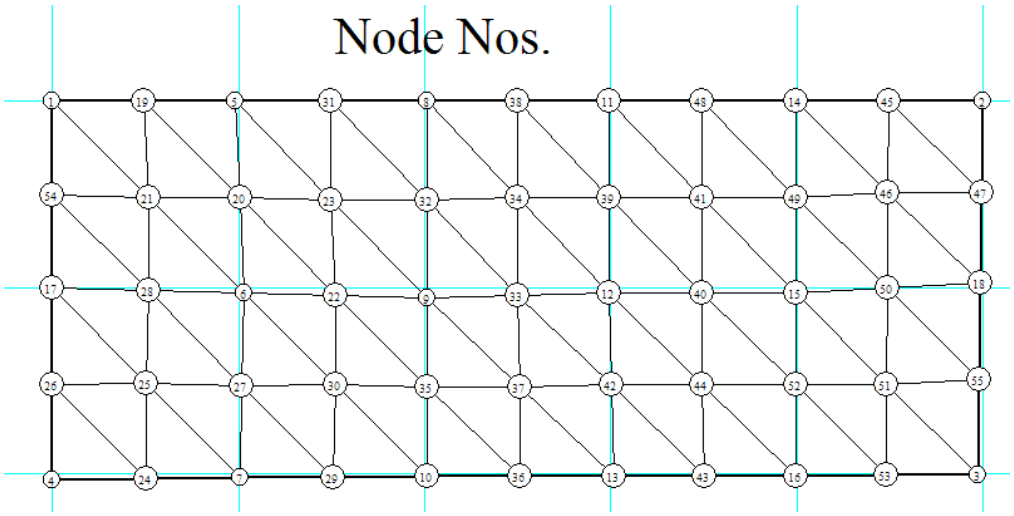
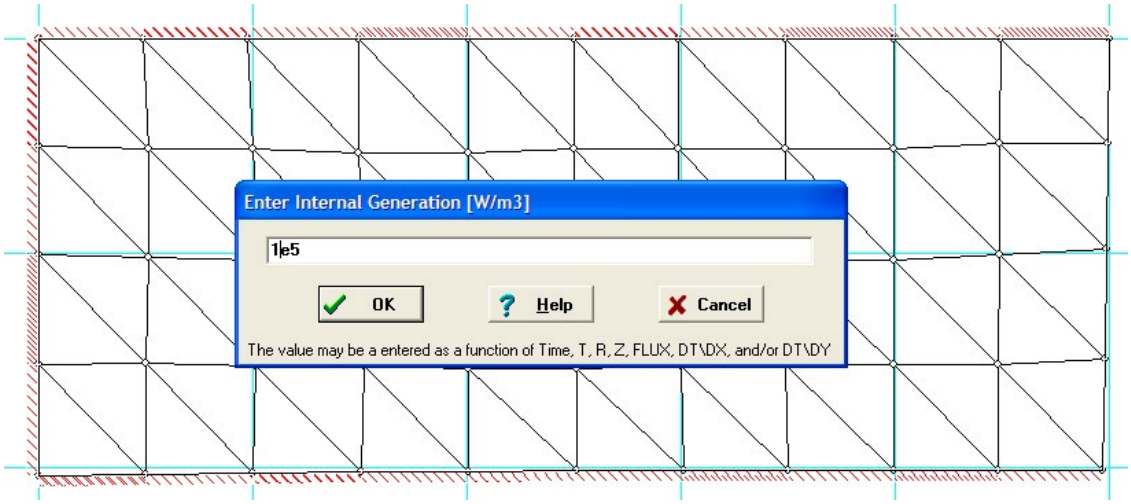
Node Nos.



Note that node no. on LHS is 17.

We try to change qg such that temp on this node becomes 200 C:

First trial with qg = 1E5 W/m^3:



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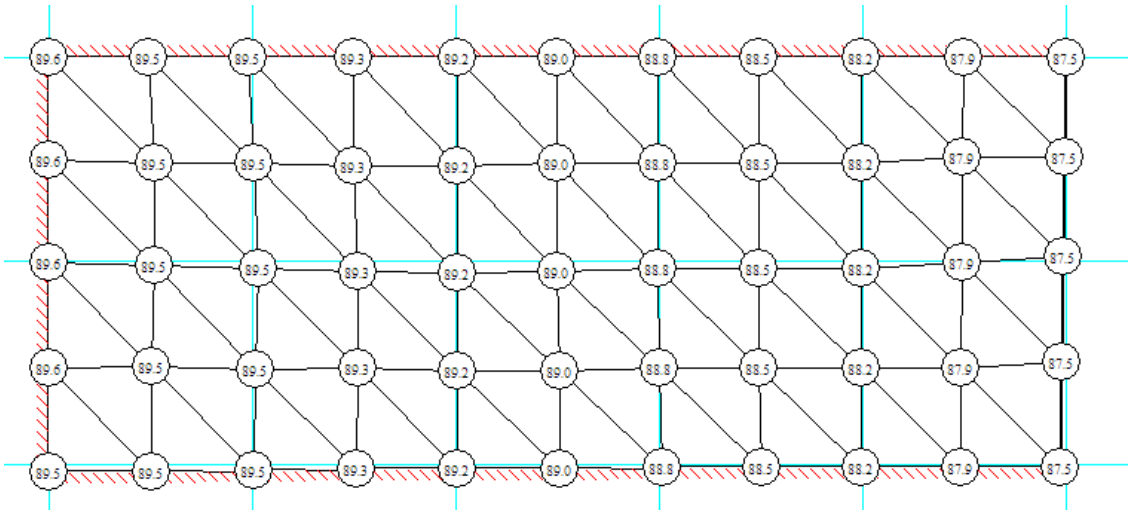
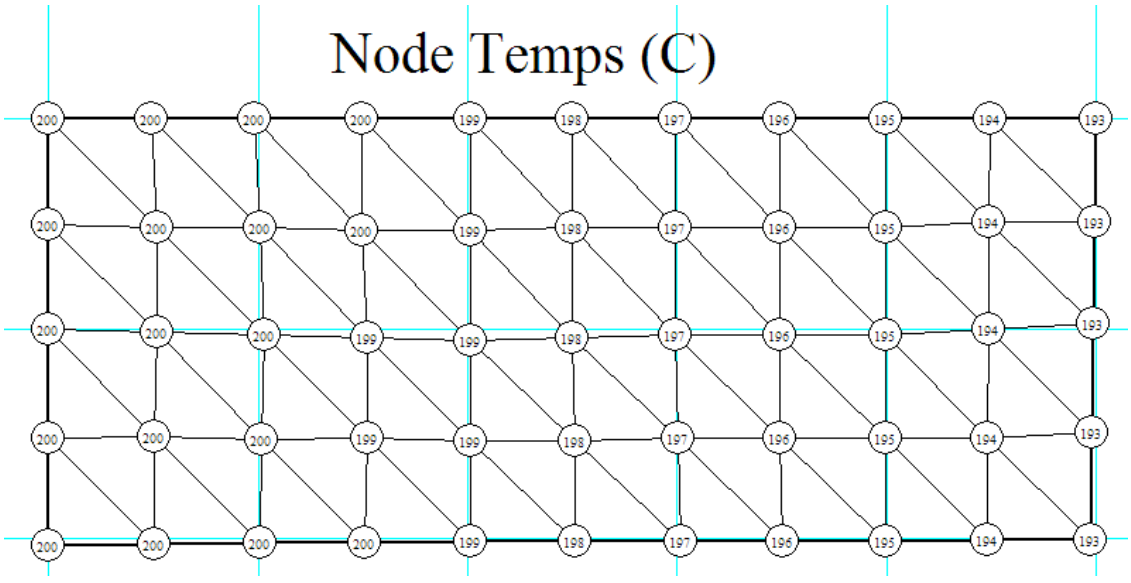
For this trial, Node temps are:



i.e. at Node 17, T = 89.6 C.

So, try qg = 3.8E5 W/m^3:

Now, we get Node temps as:



i.e. At Node 17, temp is 200 C.

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We can get the result in Tabular form and copy it into Excel to edit and draw a graph:

Node No.	R (m)	Z (m)	T (deg. C)	Node Bal. (W)
17	**0.05001**	-0.06006	**200.3**	-6.98E-16
6	**0.06033**	-0.06033	**199.9**	-1.40E-15
9	**0.07011**	-0.06059	**198.9**	0
12	**0.0799**	-0.06033	**197.4**	-3.49E-16
15	**0.08996**	-0.06033	**195.2**	1.05E-15
18	**0.09988**	-0.0598	**192.6**	-45.01

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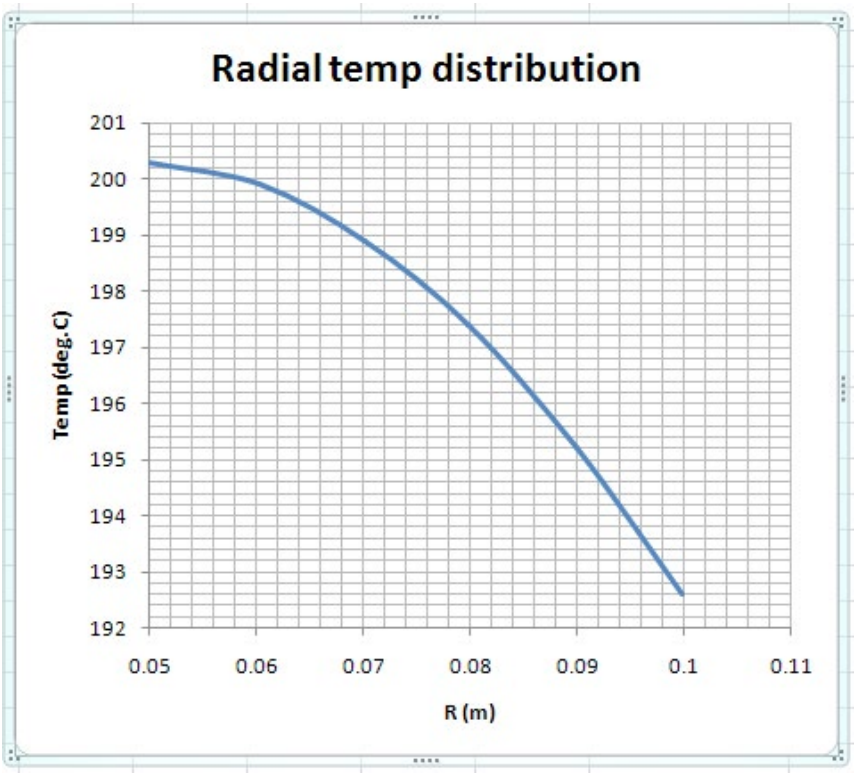

Note that at Node 17, in the Table, temp is shown accurately as: 200.3 C.

With more patience, one can continue to change the value of qg and continue the trial and error calculation to make it exactly 200 C.

Also note that temp on the outer surface subjected to convection is: 192.6 C. Compare this with the value of 192.4 C obtained with EES.

==========

"**Prob. 1I.A.6.** A solid sphere of radius 10 mm and k = 14 W/m.C has a uniform heat generation rate of 2 × 10^6 W/m^3. Heat is conducted from its outer surface to a fluid at 25 C by convection with h = 2200 W/m^2.C. Dividing the radius to 10 equal divisions and using numerical method, determine: (i) the steady state temp at the centre and at the outer surface (ii) draw the temp profile along the radius."

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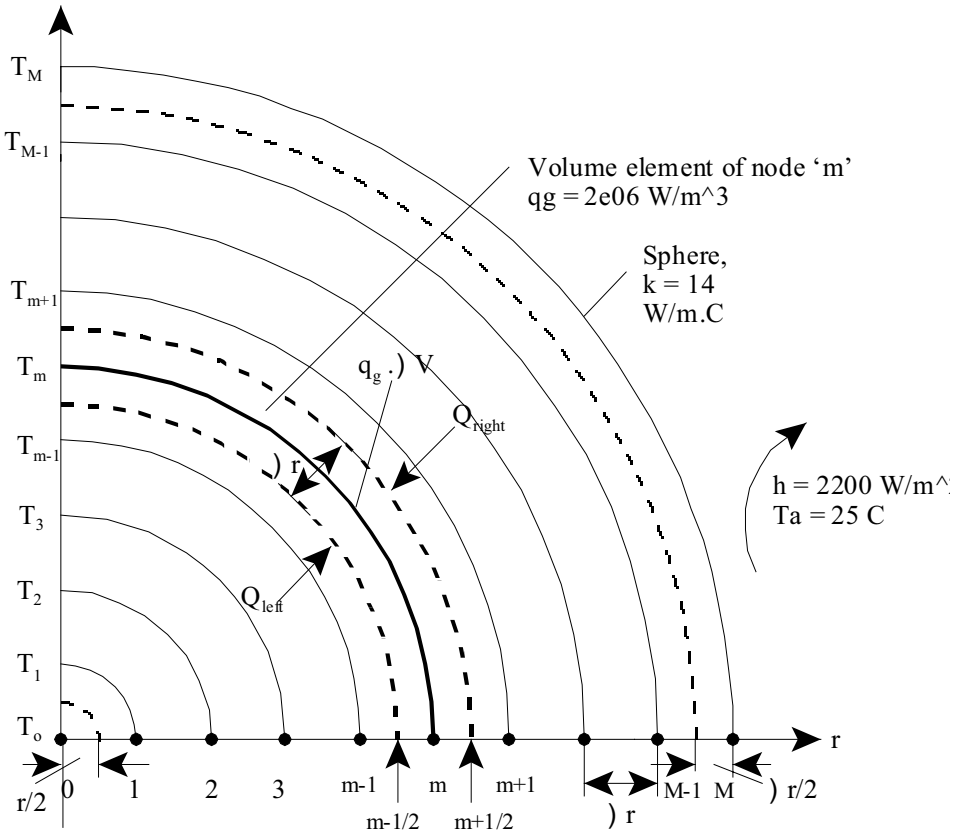

Fig.Prob.1I.A.6

EES Solution:

"Data:"

k = 14 [W/m-C]

r_0 = 0.01 [m]

h = 2200 [W/m^2-C]

T_a = 25 [C]

q_g = 2e06 [W/m^3] "…heat gen. W/m^3 "

"Calculations:"

"Let there be 10 equal divisions (MM = 10) of the radial distance r_0, so that DELTAr = 0.001 m.

Nodes: 0 to 10.

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Of these, 0 and 10 are boundary nodes and 1 to 9 are internal nodes:"

MM = 10 "no. of equal divisions"

DELTAr = 0.001[m]

"To find temp at internal nodes:

By energy balance at node m, remembering to write all energy terms as flowing in to the node:

$$\frac{\frac{T_{m-1} - T_m}{\Delta r}}{4 \cdot \pi \cdot \left[m \cdot \Delta r - \frac{\Delta r}{2} \right]^2 \cdot k} + \frac{\frac{T_{m+1} - T_m}{\Delta r}}{4 \cdot \pi \cdot \left[m \cdot \Delta r + \frac{\Delta r}{2} \right]^2 \cdot k} + q_g \cdot 4 \cdot \pi \cdot (m \cdot \Delta r)^2 \cdot \Delta r = 0$$

In the above eqn.: first term is heat flowing in from LHS, 2nd term is heat flowing in from RHS, and the 3rd tem is heat gen.

Also, note that effect of curvature of sphere is neglected and thermal resistance is calculated treating the spherical shell as a slab of thickness DELTAr.

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The above eqn is entered in EES as follows:

This is the eqn for temperatures T[1] to T[9]"

Duplicate m = 1,9

(T[m-1] – T[m]) / (DELTAr / (4 * pi * (m * DELTAr – DELTAr/2)^2 * k)) + (T[m+1] – T[m]) / (DELTAr / (4 * pi * (m * DELTAr + DELTAr/2)^2 * k)) + q_g * (4 * pi * (m * DELTAr)^2 * DELTAr) = 0

end

"Now for boundary nodes '0' and '10':"

"For node '0':

This is the centre node. Considering the half volume around node '0' ans writing the heat balance:"

$$\frac{\frac{T_1 - T_0}{\Delta r}}{k \cdot 4 \cdot \pi \cdot \left[\frac{\Delta r}{2}\right]^2} + 4 / 3 \cdot \pi \cdot \left[\frac{\Delta r}{2}\right]^3 \cdot q_g = 0$$

This eqn. is entered in EES as:

(T[1] – T[0]) / (DELTAr / (k * 4 * pi * (DELTAr/2)^2)) + (4 / 3) * pi * (DELTAr / 2)^3 * q_g = 0 "for T[0] "

"Similarly, for node 10, we get:"

$$\frac{\frac{T_9 - T_{10}}{\Delta r}}{4 \cdot \pi \cdot \left[r_0 - \frac{\Delta r}{2}\right]^2 \cdot k} + q_g \cdot 4 \cdot \pi \cdot r_0^2 \cdot \frac{\Delta r}{2} + h \cdot 4 \cdot \pi \cdot r_0^2 \cdot (T_a - T_{10}) = 0$$

And this eqn is entered in EES as:

(T[9] – T[10]) / (DELTAr / (4 * pi * (r_0 – DELTAr / 2)^2 * k)) + q_g * 4 * pi * (r_0)^2 * DELTAr / 2 + h * (4 * pi * (r_0)^2) * (T_a – T[10]) = 0

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"To draw the plot of r vs Temp.:"

duplicate m = 0,10

r[m] = m * DELTAr

end

"=="

"Compare the results with Analytical relations: We have the analytical relation for T_{max}:"

The analytical eqn. for temp distribution is:

$$T_{analyt,i} = T_a + q_g \cdot \frac{r_0}{3 \cdot h} + \frac{q_g}{6 \cdot k} \cdot (r_0^2 - (i \cdot \Delta r)^2)$$

duplicate i = 0, 10

T_analyt[i] =T_a + q_g * r_0 / (3 * h) + (q_g / (6 * k)) * (r_0^2 – (i * DELTAr)^2)

end

Results:

$\Delta r = 0.001$ [m]
$h = 2200$ [W/m^2-C]
$k = 14$ [W/m-C]
MM = 10
$q_g = 2.000E+06$ [W/m^3]
$r_0 = 0.01$ [m]
$T_a = 25$ [C]

Sort	1 r_i	2 T_i [C]	3 $T_{analyt,i}$ [C]
[0]	0	30.4	30.41
[1]	0.001	30.38	30.39
[2]	0.002	30.31	30.32
[3]	0.003	30.2	30.2
[4]	0.004	30.04	30.03
[5]	0.005	29.82	29.82
[6]	0.006	29.56	29.55
[7]	0.007	29.26	29.24
[8]	0.008	28.9	28.89
[9]	0.009	28.5	28.48
[10]	0.01	28.05	28.03

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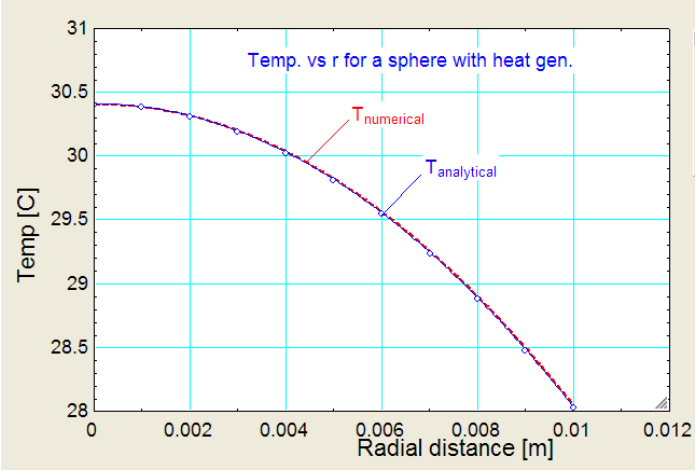
So, by numerical method:

temp at the centre (i.e. at r = 0): 30.4 C, and

temp at the surface (i.e. at r = 0.01 m): 28.05 C …. Ans.

Above Table also gives the temperatures at different radii calculated analytically. It may be noted that results by numerical methods match well with those calculated from analytical formulas.

Plot of temp vs radial distances:



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"**Prob. 1I.A.7.** A hollow sphere (k = 30 W/m.C) of inner radius 6 cm and outside radius 8 cm has a heat gen. rate of 4 × 10^6 W/m^3. The inside surface is insulated and heat is removed from the outside surface by a fluid at 100 C with h = 300 W/m^2.C. Calculate the temperatures on the inside and outside surfaces. Divide the radial distance into 20 equal divisions."

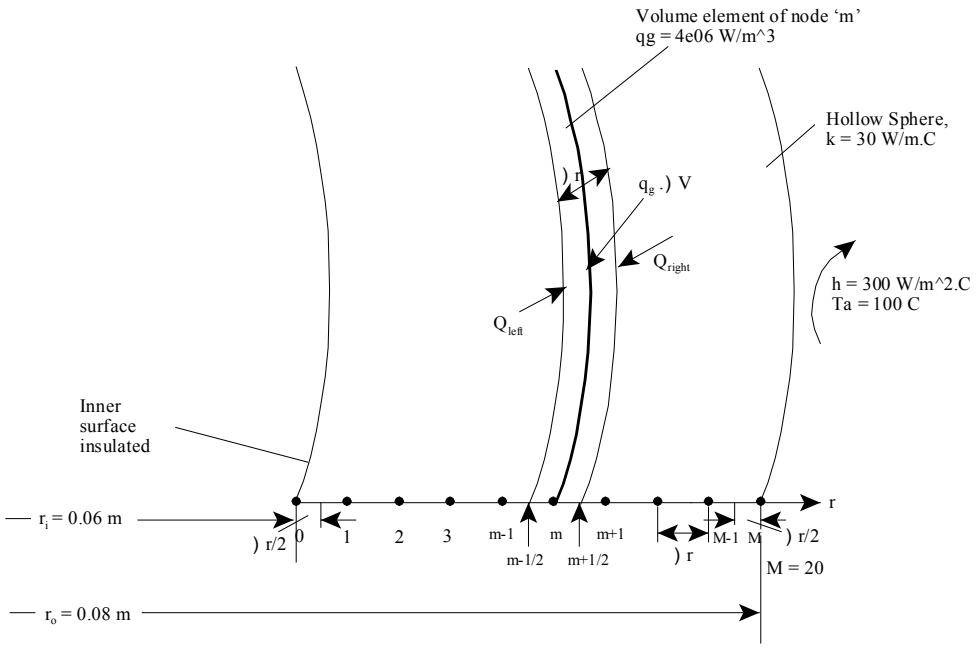

Fig.Prob.1I.A.7

EES Solution:

"Data:"

k = 30 [W/m-C]

r_i = 0.06 [m]

r_0 = 0.08 [m]

h = 300 [W/m^2-C]

T_a = 100 [C]

q_g = 4e06 [W/m^3] "…heat gen. W/m^3"

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"Calculations:"

"Let there be 20 equal divisions (MM = 20) of the radial distance (r_0 – r_i), so that DELTAr = 0.001 m.

Nodes: 0 to 10.

Of these, 0 and 20 are boundary nodes and 1 to 19 are internal nodes:"

MM = 20 "no. of equal divisions"

DELTAr = 0.001[m]

"To find temp at internal nodes:

$$\frac{\frac{T_{m-1} - T_m}{\Delta r}}{4 \cdot \pi \cdot \left[r_i + m \cdot \Delta r - \frac{\Delta r}{2}\right]^2 \cdot k} + \frac{\frac{T_{m+1} - T_m}{\Delta r}}{4 \cdot \pi \cdot \left[r_i + m \cdot \Delta r + \frac{\Delta r}{2}\right]^2 \cdot k} + q_g \cdot 4 \cdot \pi \cdot (r_i + m \cdot \Delta r)^2 \cdot \Delta r = 0$$

"In the above eqn.: first term is heat flowing in from LHS, 2nd term is heat flowing in from RHS, and the 3rd tem is heat gen.

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Also, note that effect of curvature of sphere is neglected and thermal resistance is calculated treating the sphericalshell as a slab of thickness DELTAr."

This is the eqn for temperatures T[1] to T[19]

It is entered in EES as:"

Duplicate m = 1,19

(T[m-1] – T[m]) / (DELTAr / (4 * pi * ((r_i + m * DELTAr) – DELTAr/2)^2 * k)) + (T[m+1] – T[m]) / (DELTAr / (4 * pi * ((r_i + m * DELTAr) + DELTAr/2)^2 * k)) + q_g * (4 * pi * (r_i + m * DELTAr)^2 * DELTAr) = 0

end

"Now for boundary nodes '0' and '20':"

"For node '0':

The inside surface is insulated. So, no heat transfer from LHS: Considering the half volume around node '0' and writing the heat balance:"

$$\frac{T_1 - T_0}{\frac{\Delta r}{k \cdot 4 \cdot \pi \cdot \left[r_i + \frac{\Delta r}{2}\right]^2}} + 4 / 3 \cdot \pi \cdot \left[\left(r_i + \frac{\Delta r}{2}\right)^3 - r_i^3\right] \cdot q_g = 0$$

It is entered in EES as:

(T[1] – T[0]) / (DELTAr / (k * 4 * pi * (r_i + DELTAr/2)^2)) + (4 / 3) * pi * ((r_i + DELTAr / 2)^3 – r_i^3) * q_g = 0 "for T[0]"

"For node 20:

We get T[20] by a heat balance at the outer surface. i.e. all the heat generated within the spherical shell must be dissipated by convection from the outer surface: i.e."

$$q_g \cdot 4 / 3 \cdot \pi \cdot (r_0^3 - r_i^3) = h \cdot 4 \cdot \pi \cdot r_0^2 \cdot (T_{20} - T_a)$$

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In EES, it is entered as:

q_g * (4/3) * pi * (r_0 ^3 – r_i^3) = h * (4 * pi * r_0^2) * (T[20] – T_a) "..gives temp at node 20, i.e. at the outer surface"

"To draw the plot of r vs Temp.:"

duplicate m = 0,20

r[m] = r_i + m * DELTAr

end

"=="

T_w = T[20] "outside surface temp."

"Compare the results with Analytical relations: Analytical relation for Temp distribution is:"

$$T_{analyt,i} = T_w + \frac{q_g}{6 \cdot k} \cdot (r_0^2 - (r_i + i \cdot \Delta r)^2) - \frac{q_g}{3 \cdot k} \cdot r_i^3 \cdot \left[\frac{1}{r_i + i \cdot \Delta r} - \frac{1}{r_0}\right] \quad \text{for } i = 0 \text{ to } 20$$

In EES, this eqn. is:

duplicate i = 0, 20

T_analyt[i] =T_w + (q_g / (6 * k)) * (r_0^2 – (r_i + i * DELTAr)^2) – (q_g / (3 * k)) * r_i^3 * (1 / (r_i + i * DELTAr) – 1/ r_0)

End

Results:

Unit Settings: SI C kPa kJ mass deg

Δr = 0.001 [m]	h = 300 [W/m^2-C]	k = 30 [W/m-C]
MM = 20	q_g = 4.000E+06 [W/m^3]	r_0 = 0.08 [m]
r_i = 0.06 [m]	T_a = 100 [C]	T_w = 305.6 [C]

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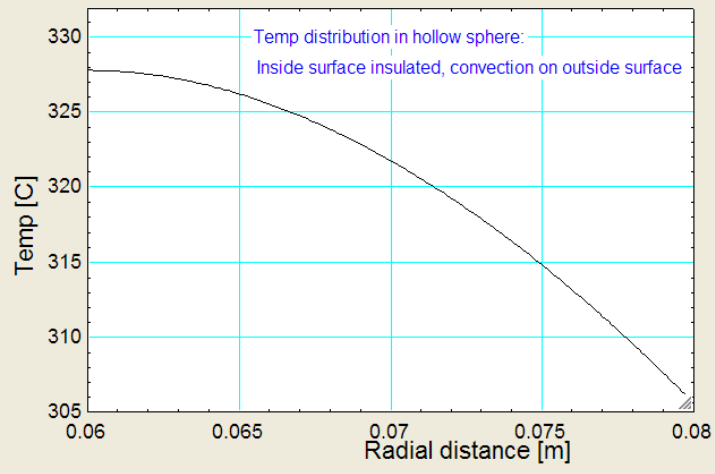
Temp. distribution by numerical method and analytical method:

Sort	1 r_i [m]	2 T_i [C]	3 $T_{analyt,i}$ [C]
[0]	0.06	327.8	327.8
[1]	0.061	327.7	327.7
[2]	0.062	327.5	327.5
[3]	0.063	327.2	327.2
[4]	0.064	326.8	326.8
[5]	0.065	326.2	326.2
[6]	0.066	325.5	325.5
[7]	0.067	324.7	324.7
[8]	0.068	323.8	323.8
[9]	0.069	322.8	322.8
[10]	0.07	321.7	321.7
[11]	0.071	320.5	320.5
[12]	0.072	319.2	319.2
[13]	0.073	317.8	317.8
[14]	0.074	316.4	316.4
[15]	0.075	314.8	314.8
[16]	0.076	313.1	313.1
[17]	0.077	311.3	311.3
[18]	0.078	309.5	309.5
[19]	0.079	307.6	307.6
[20]	0.08	305.6	305.6

Note that the results by numerical methods and analytical methods match extremely well.

Plot of Temp vs radial distance:



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"**Prob. 1I.A.8.** A turbine blade (k = 29 W/m.C) is 60 mm long, 500 mm^2 cross-sectional area and 120 mm perimeter. Temp of root of blade is 480 C and it is exposed to products of combustion passing through the turbine at 820 C. If the film coeff between the blade and the combustion gases is 320 W/m^2.C, determine: (i) the temp at the middle of the blade, (ii) rate of heat flow from the blade. Use 20 equal divisions, i.e. DELTAx = 3 mm while adopting numerical method."

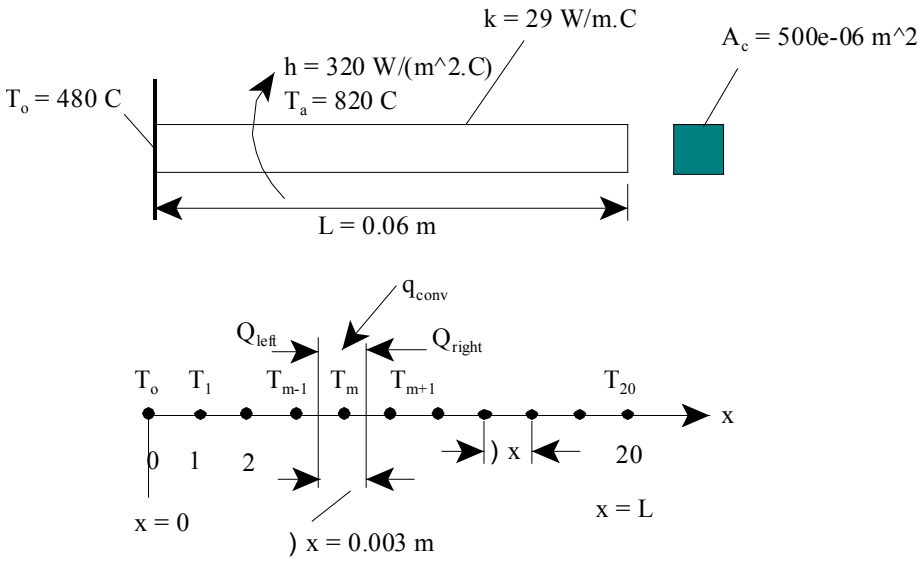

Fig.Prob.1I.A.8

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EES Solution:

"Data:"

"The turbine blade is treated as a fin losing heat from its tip."

k = 29 [W/m-C]

L = 0.06 [m] "… length of fin"

A_c = 500e-06 [m^2] "… area of cross-section of fin"

P = 0.12 [m] "… perimeter of fin"

h = 320 [W/m^2-C]

T_a = 820 [C]

"Calculations:"

"Let there be 20 equal divisions (MM = 20) of the fin length so that DELTAx = 0.003 m. Nodes: 0 to 20.

Of these, 0 and 20 are boundary nodes and 1 to 19 are internal nodes:"

MM = 20 "no. of equal divisions"

DELTAx = 0.003[m]

"To find temp at internal nodes:

$$\frac{T_{m-1} - T_m}{\frac{\Delta x}{k \cdot A_c}} + \frac{T_{m+1} - T_m}{\frac{\Delta x}{k \cdot A_c}} + h \cdot P \cdot \Delta x \cdot (T_a - T_m) = 0 \qquad \text{for } m = 1 \text{ to } 19$$

This is the eqn for temperatures T[1] to T[19]"

"Above eqn. is entered in EES:"

Duplicate m = 1,19

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(T[m-1] – T[m]) / (DELTAx / (k * A_c)) + (T[m+1] – T[m]) / (DELTAx / (k * A_c)) + h * P * DELTAx * (T_a – T[m]) = 0

end

"In the above eqn.: first term is heat flowing in to the node by conduction from LHS, 2nd term is heat flowing in by conduction in to the node from RHS, and the 3rd term is heat flow into the node by convection from the gases."

"Now for boundary nodes '0' and '20':"

"For node '0':

Temp at the root of fin i.e. at node '0' is given as 480 C:"

T[0] = 480 [C] "by data … for T[0] "

"**For node 20,** write the heat balance, including the convection from the tip, remembering to show all heat flows as flowing into the node: i.e."

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$$\frac{T_{19} - T_{20}}{\frac{\Delta x}{k \cdot A_c}} + h \cdot \left[P \cdot \frac{\Delta x}{2} + A_c \right] \cdot (T_a - T_{20}) = 0$$

"Above eqn. is entered in EES as:"

(T[19] – T[20]) / (DELTAx / (k * A_c)) + h * (P * DELTAx / 2 + A_c) * (T_a – T[20]) = 0 "..for node 20"

"For heat transfer from the fin: Write a heat balance at node '0':"

$$Q_{fin} + \frac{T_1 - T_0}{\frac{\Delta x}{k \cdot A_c}} + h \cdot P \cdot \frac{\Delta x}{2} \cdot (T_a - T_0) = 0$$

"In EES this eqn. is entered as:"

Q_fin + (T[1] – T[0]) / (DELTAx / (k * A_c)) + h * (P * DELTAx/2) * (T_a – T[0]) = 0

"To draw the plot of x vs Temp.:"

duplicate i = 0,20

x[i] = i * DELTAx

end

"=="

"Compare the results with Analytical relations: We have the analytical relation for Temp distribution:"

"Fin parameter m_p:"

$$m_p = \sqrt{\frac{h \cdot P}{k \cdot A_c}}$$

"In EES, above eqn. becomes:"

m_p = sqrt((h * P) / (k * A_c)) "[1/m] … fin parameter"

{

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"Temp distribution is given by:

(T_analyt – T_a) / (T[0] – T_a) = (A + B) / (C + D) where:

A = cosh(m_p * (L – i * DELTAx))

B = (h / (k * m_p)) * sinh (m_p * (L – i * DELTAx))

C = cosh (m_p * L)

D = (h / (k * m_p)) *sinh (m_p * L) "

$$T_{analyt,j} = T_a + (T_0 - T_a) \cdot \left[\frac{\cosh(m_p \cdot (L - j \cdot \Delta x)) + \frac{h}{k \cdot m_p} \cdot \sinh(m_p \cdot (L - j \cdot \Delta x))}{C + D} \right] \quad \text{for } j = 0 \text{ to } 20$$

}

"Enter these eqns in EES as follows:"

C = cosh (m_p * L)

D = (h / (k * m_p)) *sinh (m_p * L)

duplicate j = 0, 20

T_analyt[j] = T_a + (T[0] – T_a) * ((cosh(m_p * (L – j * DELTAx)) + (h / (k * m_p)) * sinh (m_p * (L – j * DELTAx))) / (C + D))

end

"Analytical relation for heat flow from fin:"

$$Q_{fin,analyt} = k \cdot A_c \cdot m_p \cdot (T_0 - T_a) \cdot \left[\frac{\tanh(m_p \cdot L) + \frac{h}{k \cdot m_p}}{1 + \frac{h}{k \cdot m_p} \cdot \tanh(m_p \cdot L)} \right]$$

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"Enter it in EES:"

Q_fin_analyt = k * A_c * m_p * (T[0] – T_a) * ((tanh (m_p * L) + (h / (k * m_p))) / (1 + (h / (k * m_p)) * tanh (m_p * L)))

Results:

Unit Settings: SI C kPa kJ mass deg

A_c = 0.0005 [m^2]	C = 10.99	D = 2.346
Δx = 0.003 [m]	h = 320 [W/m^2-C]	k = 29 [W/m-C]
L = 0.06 [m]	MM = 20	m_p = 51.46 [1/m]
P = 0.12 [m]	Q_{fin} = -253.8 [W]	$Q_{fin,analyt}$ = -253 [W]
T_a = 820 [C]		

Thus:

Q_fin = -253.8 W … heat transfer from fin … by numerical method … Ans.

Q_fin_analyt = -253 W …. heat transfer from fin … by analytical method … Ans.

Note: -ve sign indicates that heat is transferred into the fin from the combustion gases.

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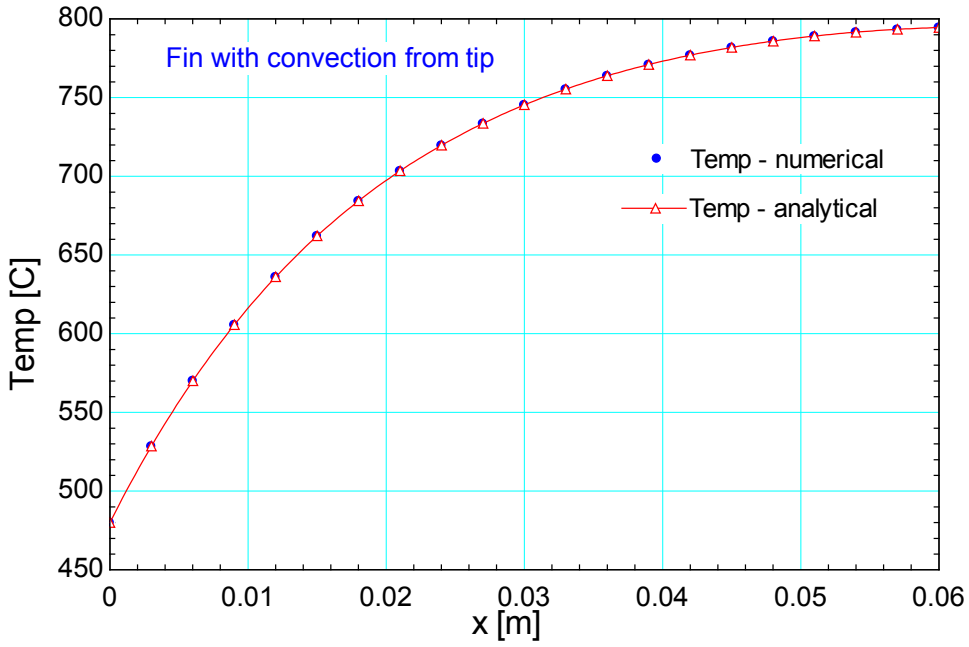
Temperatures along the fin:

Temperatures at various x values as per numerical method and analytical method are given below:

Sort	1 T_i [C]	2 x_i [m]	3 $T_{analyt,i}$ [C]
[0]	480	0	480
[1]	528.5	0.003	528.5
[2]	570	0.006	570
[3]	605.5	0.009	605.6
[4]	635.9	0.012	636
[5]	662	0.015	662.1
[6]	684.3	0.018	684.4
[7]	703.3	0.021	703.4
[8]	719.6	0.024	719.7
[9]	733.4	0.027	733.5
[10]	745.2	0.03	745.3
[11]	755.2	0.033	755.4
[12]	763.7	0.036	763.8
[13]	770.9	0.039	771
[14]	776.8	0.042	776.9
[15]	781.8	0.045	781.9
[16]	785.8	0.048	785.9
[17]	789	0.051	789.1
[18]	791.5	0.054	791.6
[19]	793.3	0.057	793.3
[20]	794.4	0.06	794.5

Plot of temp vs x:



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In the above Table and plot, temperatures calculated by both the methods are compared.
It is observed that the values match very closely.

==========

Prob. 1I.A.9. A conical cylinder of length L and radii R1 and R2, (R1 < R2) is fully insulated on the outer surface. The two ends are maintained at T1 and T2, (T1 > T2). Considering one-dimensional steady state heat flow, derive expressions for heat flow and temperature distribution.

As a numerical example, taking: R1 = 1.25 cm, R2 = 2.5 cm, L = 20 cm, T1 = 227 C, T2 = 27 C, k = 40 W/(m.C), find:

1) steady state heat transfer rate, Q
2) temperature at mid-plane
3) temperature at a plane 14 cm from the small end
4) draw the temperature profile in the solid

Use numerical method, with 10 equal divisions, i.e. $\Delta x = 0.02$ m

Mathcad Solution:

Data:

R1 := 0.0125 m.....Radius at small end of truncated cone

R2 := 0.025 m.....Radius at larger end of truncated cone

T1 := 227 C.....temp. at small end of truncated cone

T2 := 27 C.....temp. at larger end of truncated cone

L := 0.2 m....length of section

k := 40 W/(m.C).....thermal cond. of the solid

MM := 10 ...no. of equal divisions

Δx := 0.02 m....

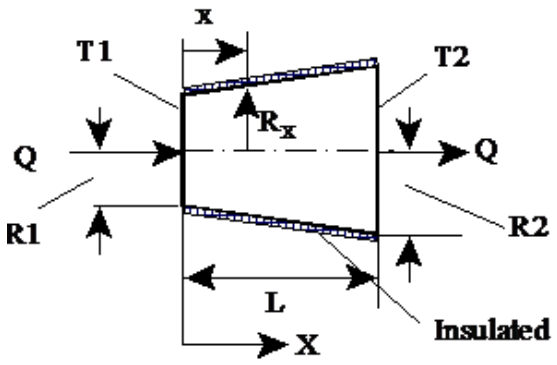

Fig.Prob.1I.A.9

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Note that here, area of cross-section varies with the length x.

Write a function for variation of area with x:

$$R(x) := R1 + \frac{(R2 - R1)}{L} \cdot x$$ m.....radius at any x

$$A(x) := \pi \cdot R(x)^2$$ m^2.....area at any x

Difference equations:

For Internal nodes:

Consider an internal node 'm':

Write the heat balance, remembering to have all heat flows ***into*** the node:

$$\frac{T_{m-1} - T_m}{\left(\frac{m \cdot \Delta x}{k \cdot A\left(m \cdot \Delta x - \frac{\Delta x}{2}\right)}\right)} + \frac{T_{m+1} - T_m}{\left(\frac{m \cdot \Delta x}{k \cdot A\left(m \cdot \Delta x + \frac{\Delta x}{2}\right)}\right)} = 0$$ ..for m = 1 to 9.....eqn.(A)

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In eqn. (A) above, first term is the heat flowing from LHS and the second term is the heat flowing from RHS.

For Boundary nodes '0' and '10':

Temperatures are given:

T0=227 C.... at node '0' T10=27 C.... at node '10'

Now, use Solve Block of Mathcad to solve the 11 eqns simultaneously.

Start with guess values for unknown temperatures:

T0 := 227 T10 := 27 T1 := 100 T2 := 100 T3 := 100 T4 := 100 .....guess values

T5 := 100 T6 := 100 T7 := 100 T8 := 100 T9 := 100 .....guess values

Given

T0=227 ...(1) ... for node '0'.. by data

$$\frac{T0 - T1}{\left(\frac{1\cdot\Delta x}{k\cdot A\left(1\cdot\Delta x - \frac{\Delta x}{2}\right)}\right)} + \frac{T2 - T1}{\left(\frac{1\cdot\Delta x}{k\cdot A\left(1\cdot\Delta x + \frac{\Delta x}{2}\right)}\right)} = 0 \qquad \text{...(2)....for node 1}$$

$$\frac{T1 - T2}{\left(\frac{2\cdot\Delta x}{k\cdot A\left(2\cdot\Delta x - \frac{\Delta x}{2}\right)}\right)} + \frac{T3 - T2}{\left(\frac{2\cdot\Delta x}{k\cdot A\left(2\cdot\Delta x + \frac{\Delta x}{2}\right)}\right)} = 0 \qquad \text{...(3)....for node 2}$$

$$\frac{T2 - T3}{\left(\frac{3\cdot\Delta x}{k\cdot A\left(3\cdot\Delta x - \frac{\Delta x}{2}\right)}\right)} + \frac{T4 - T3}{\left(\frac{3\cdot\Delta x}{k\cdot A\left(3\cdot\Delta x + \frac{\Delta x}{2}\right)}\right)} = 0 \qquad \text{...(4)....for node 3}$$

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$$\frac{T3 - T4}{\left(\frac{4\cdot\Delta x}{k\cdot A\left(4\cdot\Delta x - \frac{\Delta x}{2}\right)}\right)} + \frac{T5 - T4}{\left(\frac{4\cdot\Delta x}{k\cdot A\left(4\cdot\Delta x + \frac{\Delta x}{2}\right)}\right)} = 0 \qquad \text{...(5)....for node 4}$$

$$\frac{T4 - T5}{\left(\frac{5\cdot\Delta x}{k\cdot A\left(5\cdot\Delta x - \frac{\Delta x}{2}\right)}\right)} + \frac{T6 - T5}{\left(\frac{5\cdot\Delta x}{k\cdot A\left(5\cdot\Delta x + \frac{\Delta x}{2}\right)}\right)} = 0 \qquad \text{...(6)....for node 5}$$

$$\frac{T5 - T6}{\left(\frac{6\cdot\Delta x}{k\cdot A\left(6\cdot\Delta x - \frac{\Delta x}{2}\right)}\right)} + \frac{T7 - T6}{\left(\frac{6\cdot\Delta x}{k\cdot A\left(6\cdot\Delta x + \frac{\Delta x}{2}\right)}\right)} = 0 \qquad \text{...(7)....for node 6}$$

$$\frac{T6 - T7}{\left(\frac{7\cdot\Delta x}{k\cdot A\left(7\cdot\Delta x - \frac{\Delta x}{2}\right)}\right)} + \frac{T8 - T7}{\left(\frac{7\cdot\Delta x}{k\cdot A\left(7\cdot\Delta x + \frac{\Delta x}{2}\right)}\right)} = 0 \qquad \text{...(8)....for node 7}$$

$$\frac{T7 - T8}{\left(\frac{8\cdot\Delta x}{k\cdot A\left(8\cdot\Delta x - \frac{\Delta x}{2}\right)}\right)} + \frac{T9 - T8}{\left(\frac{8\cdot\Delta x}{k\cdot A\left(8\cdot\Delta x + \frac{\Delta x}{2}\right)}\right)} = 0 \qquad \text{...(9)....for node 8}$$

$$\frac{T8 - T9}{\left(\frac{9\cdot\Delta x}{k\cdot A\left(9\cdot\Delta x - \frac{\Delta x}{2}\right)}\right)} + \frac{T10 - T9}{\left(\frac{9\cdot\Delta x}{k\cdot A\left(9\cdot\Delta x + \frac{\Delta x}{2}\right)}\right)} = 0 \qquad \text{...(10)....for node 9}$$

$T10 = 27$ ....(11) .... for node 10...by data

$$Temp := Find(T0, T1, T2, T3, T4, T5, T6, T7, T8, T9, T10)$$

Temp above is the vector that contains the temperatures T0 … T10.

i.e.

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Temp =

	0
0	227
1	190.666
2	160.376
3	134.739
4	112.759
5	93.707
6	77.033
7	62.319
8	49.239
9	37.535
10	27

Therefore, temp at the mid-plane (i.e. x = 0.1 m) is: **T[5] = 93.707 C …. Ans.**

And, temp at x = 0.14 m is: **T[7] = 62.319 C …. Ans.**

To draw the temp profile:

i := 0, 1.. 10 ….define a range variable i

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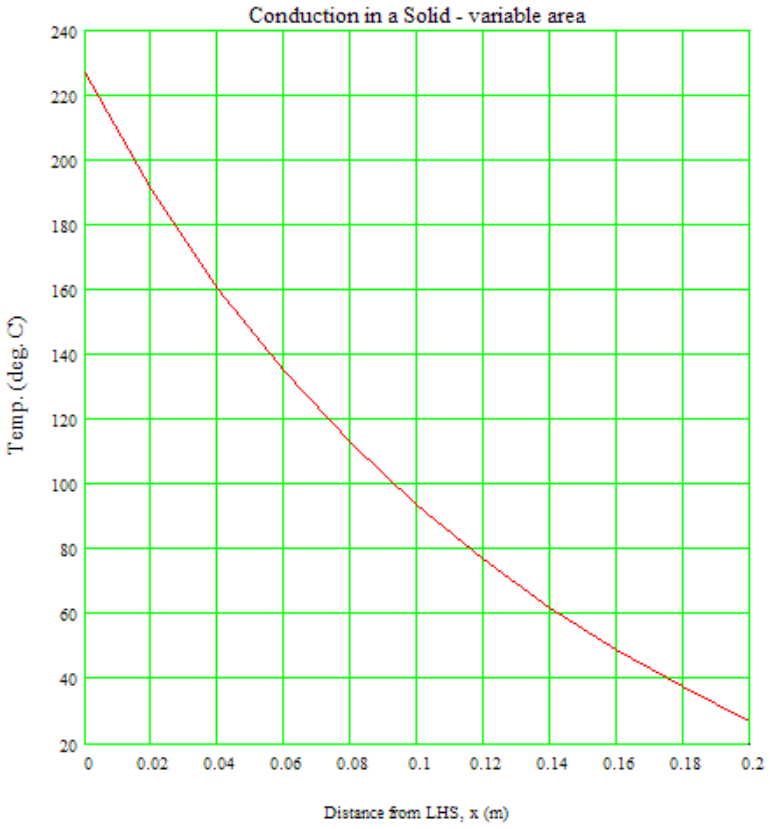

Heat transfer Q by numerical method:

$$Q\_numerical := \frac{Temp_0 - Temp_1}{\frac{\Delta x}{k \cdot A\left(\frac{\Delta x}{2}\right)}}$$

W ... heat transfer from node 0 to node 1.
This is the same Q flowing through the rod in steady state.

i.e. Q_numerical = 39.327 **W .... Ans.**

Now, compare these results with those obtained by Analytical method:

From Ref. [1], we have:

Heat transfer rate, Q:

$$Q := \frac{k \cdot \pi \cdot (T0 - T10) \cdot R1 \cdot R2}{L}$$ ...define Q

Substituting values:

Q = 39.27 W....**heat transfer rate through the section....Ans.**

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Note: By numerical method, we obtained Q = 39.327 W …and, results match very well.

Temperature at mid-plane i.e. at x = 0.1 m:

$x := 0.1$ m.....at midplane of the section

We have, at x = 0.1 m, Rx = R1 + (R2 - R1).(x/L)

Therefore: $Rx(x) := R1 + (R2 - R1)\cdot\left(\frac{x}{L}\right)$ ...define Rx as a function of x

i.e. $Rx(0.1) = 0.019$ m....radius at x = 0.1 m

Now, temp. distribution is given by equation: [Ref. 1]:

From Ref. [1]: $T(x) := T0 - (T0 - T10)\cdot\frac{\left(1 - \frac{R1}{Rx(x)}\right)}{\left(1 - \frac{R1}{R2}\right)}$ ....define T as a function of x

Therefore, $T(0.1) = 93.667$ C...**temp. at midplane....Ans.**

Note: By numerical method, we obtained temp at mid-plane, T[5] = 93.707 C …and, results match very well.

Temperature at x = 14 cm from LHS:

Simply substitute x = 0.14 in T(x):

$T(0.14) = 62.294$ C....**temp. at a plane 14 cm from LHS....Ans.**

Note: Compare this value with the value of T[7] = 62.319 C, obtained by Numerical method.

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Finally, temperatures obtained for various values of x, by both the Numerical and Analytical methods, are tabulated below:

x := 0, 0.02.. 0.2 m ... along the length

---

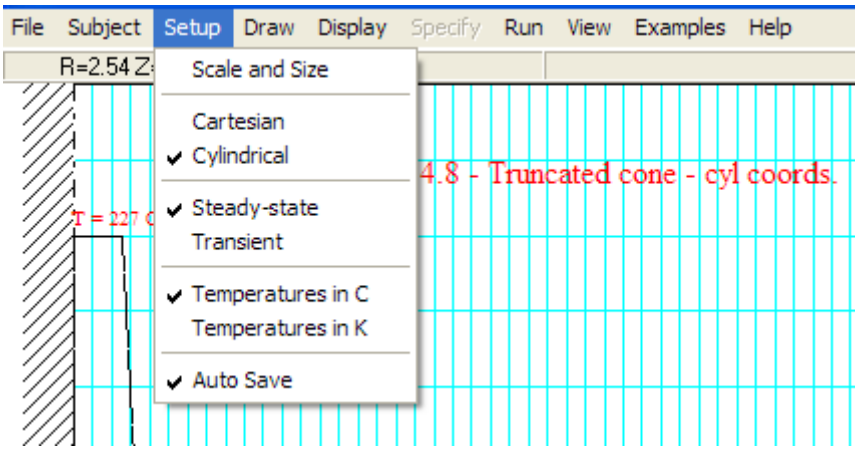
x (m) T_analytical (C) T_numerical (C)

---

x	T(x)	$Temp_i$
0	227	227
0.02	190.636	190.666
0.04	160.333	160.376
0.06	134.692	134.739
0.08	112.714	112.759
0.1	93.667	93.707
0.12	77	77.033
0.14	62.294	62.319
0.16	49.222	49.239
0.18	37.526	37.535
0.2	27	27

Solve the above problem by Finite Element Heat Transfer (FEHT) Software:

First choose cylindrical coordinates:



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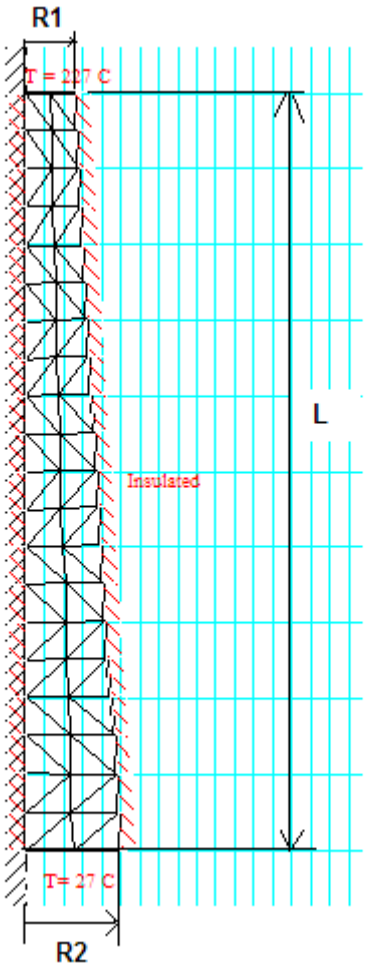
Next draw the outline:



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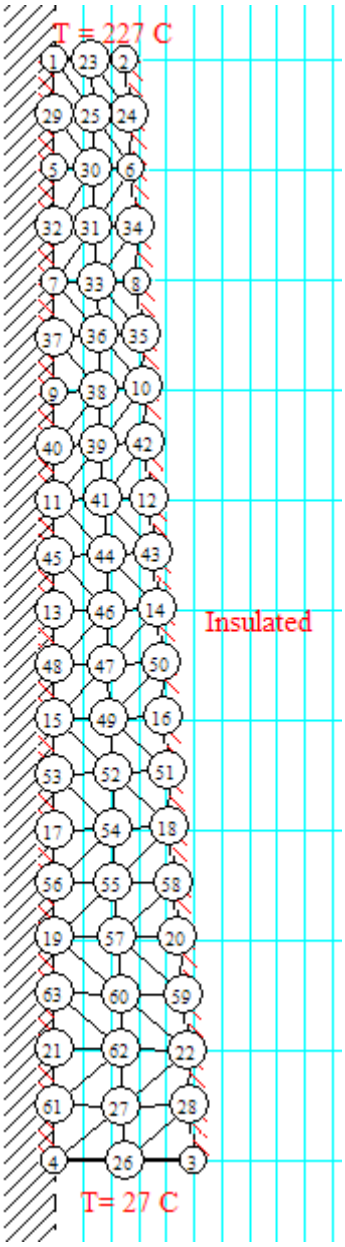
Then, draw the elements, show node nos.:



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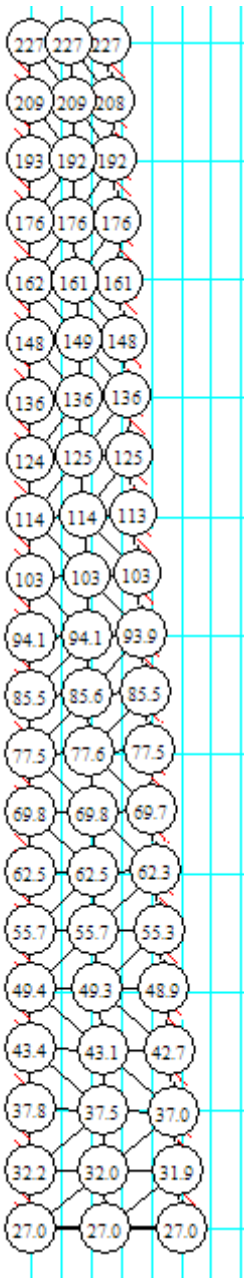
Run and show results:

Temps. at corresponding Nodes:



Note that at mid-plane, i.e. at 10 cms from top, the temp is: 94.1 C.

Compare this with the value of 93.667 C obtained with Mathcad.

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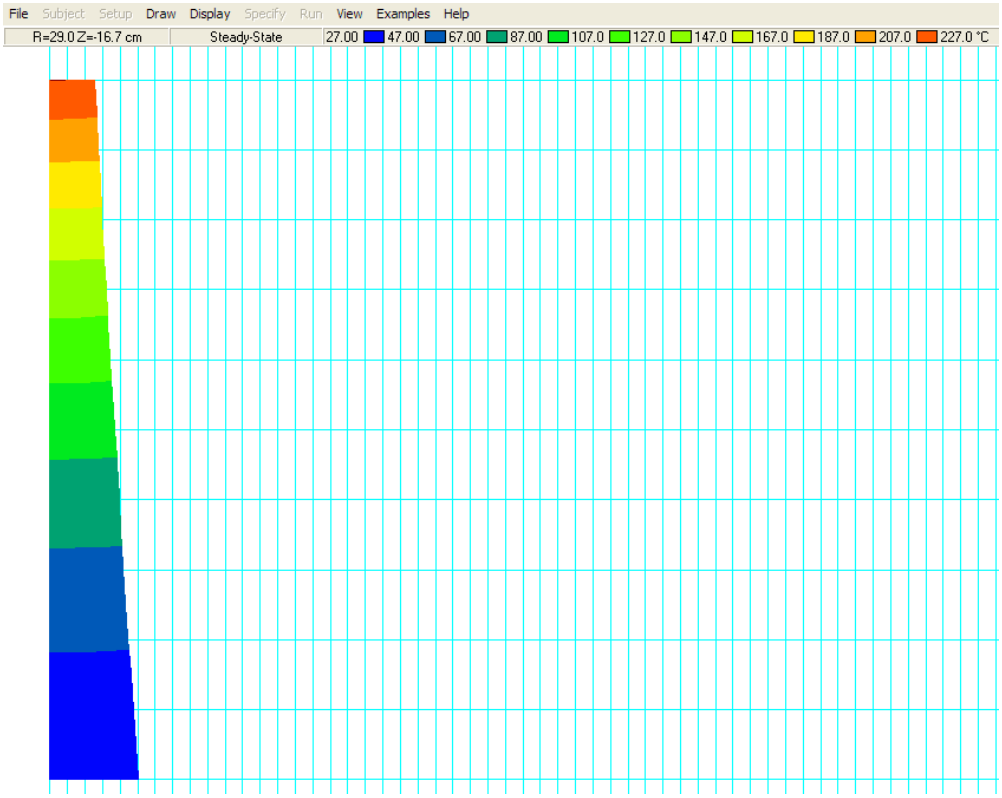
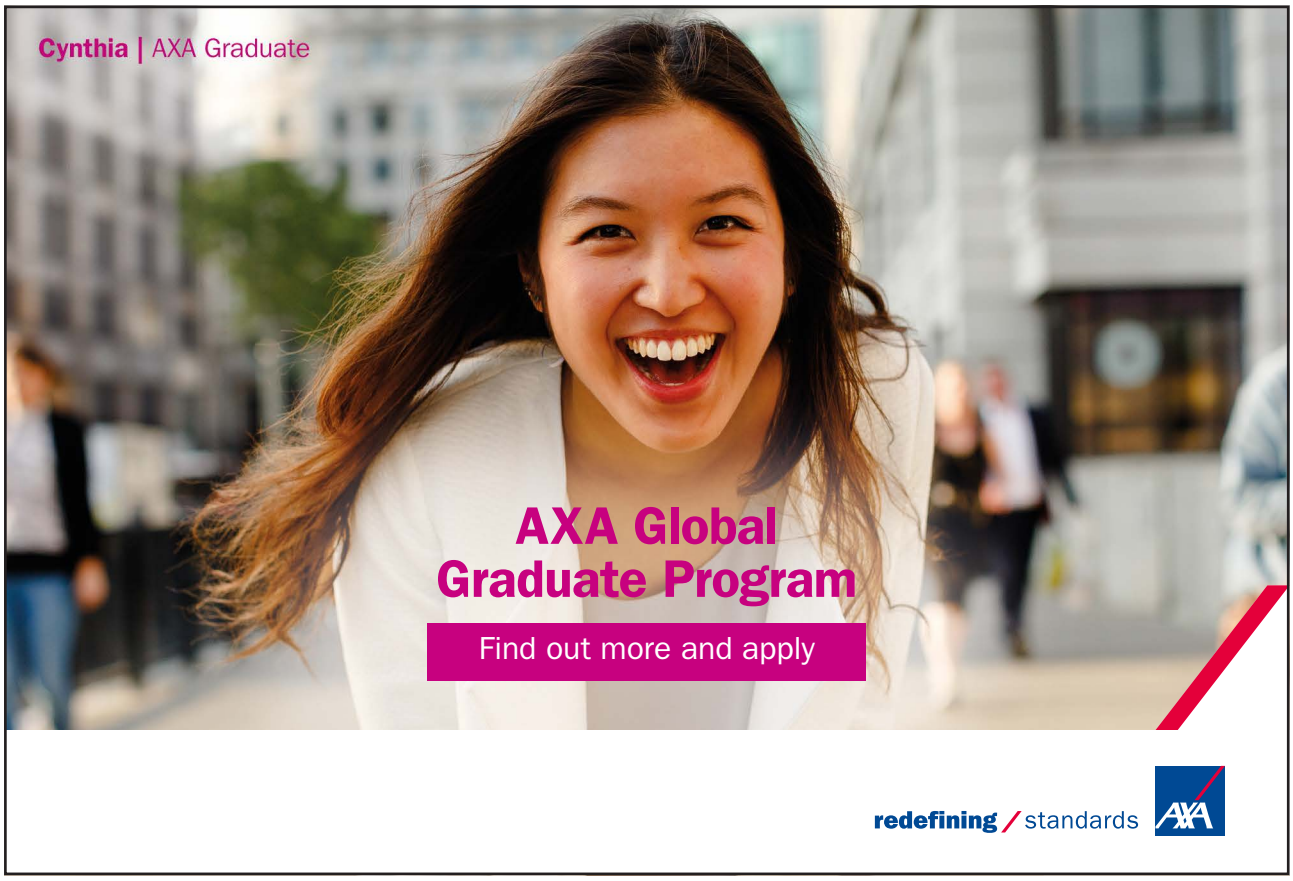
Temp. contours:





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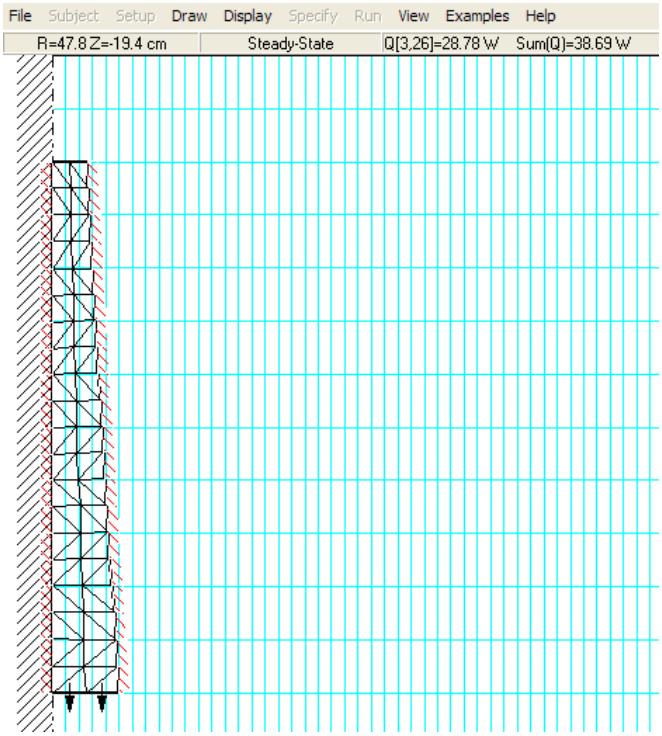
Heat transferred:



We note that heat transferred is 38.69 W.

Compare this with Q = 39.27 W obtained with Mathcad.

Tabular results: (Edited in Excel)

Node N0.	Dist(cm)	T (Deg.C)
23	0	227
25	1	208.7
30	2	192.3
31	3	176.3
33	4	161.4
36	5	148.6
38	6	136.1
39	7	124.7
41	8	113.6
44	9	103.4
46	10	94.1
47	11	85.62
49	12	77.6
52	13	69.82
54	14	62.47
55	15	55.73
57	16	49.26
60	17	43.09
62	18	37.47
27	19	32
26	20	27

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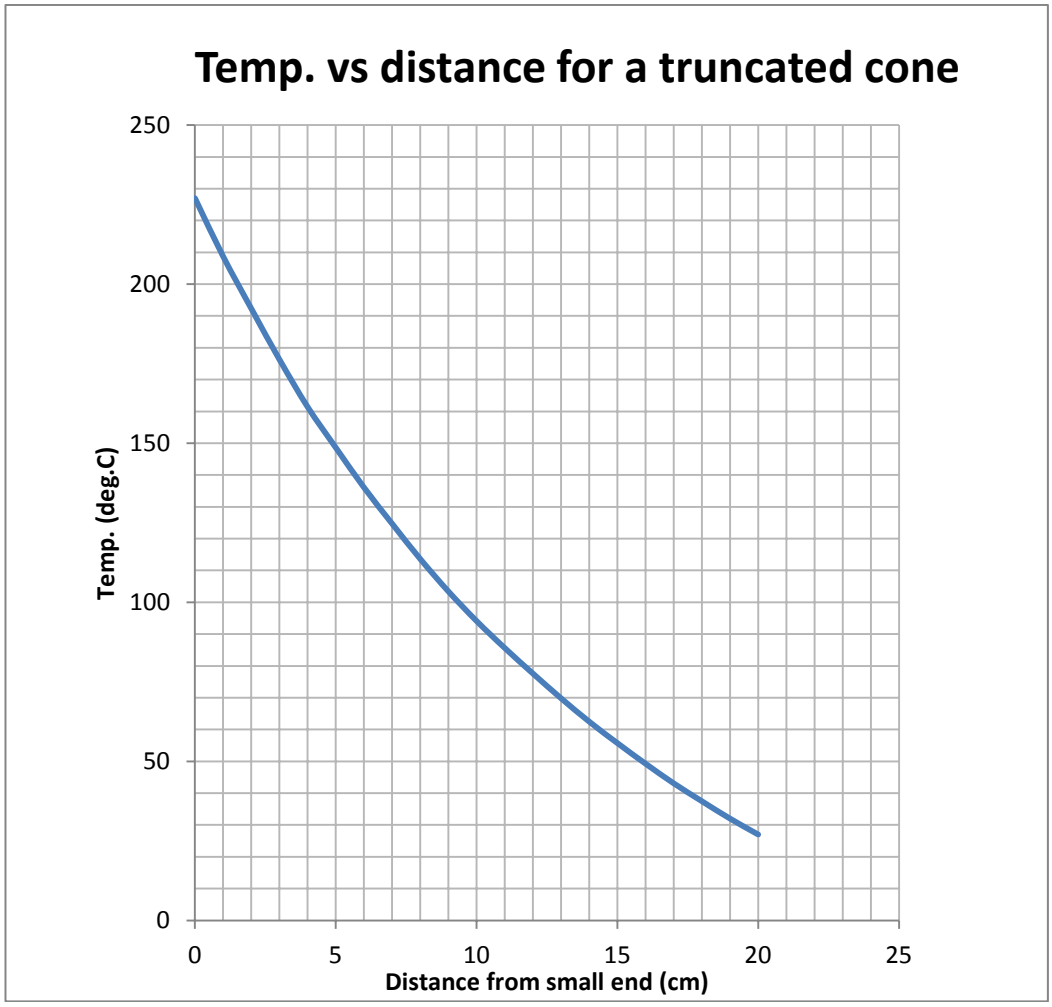
Note that at 16 cm from small end, T = 49.26 C, whereas by Mathcad, this value was 49.22 C. So, the results match very well.

Plot Temp vs distance from top:



==========

"**Prob. 1I.A.10.** A structural support has the shape of a truncated cone (see fig. 1IA.10) of length 0.2 m and its area varies with x as A = (p/4).x3. Circumference is perfectly insulated. Thermal conductivity of the material varies with temperature and is given by:

k(T) = 14.695 (1 + 0010208 T), where T is in deg. C and k is in W/(m.C). What is the steady state heat transfer rate through this strut if the two ends are maintained at 400 C and 150 C, as shown? Also find the temperature at the mid-plane. Draw the temperature profile in the solid. [Ref:1, p. 70]"

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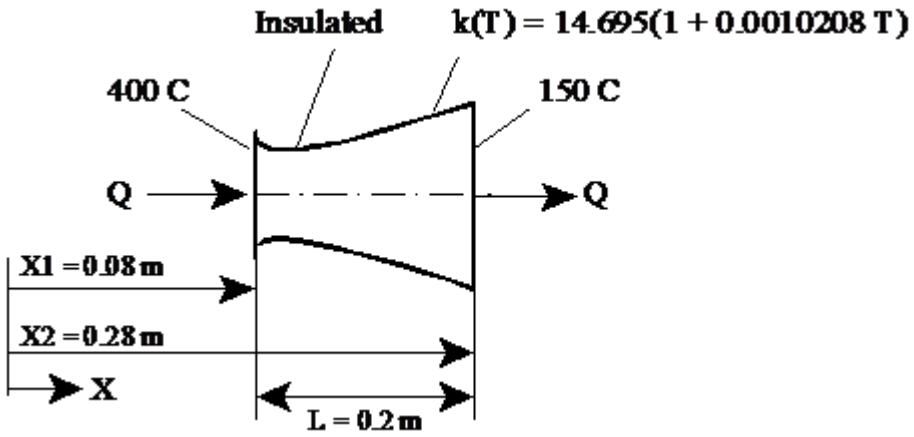

Fig.Prob.1l.A.10. Conduction with variable Area an d variable thermal conductivity

EES Solution:

"First define functions for Area and thermal conductivity:"

function A(x)

"Input: x in metres; Output: A in m^2"

A = (pi / 4) * x^3 "…m^2"

end
"----------"

function k(T)

"Input: T in C , Output: k in W/m.C"

k = 14.695 * (1 + 0.0010208 * T) "[W/m-C]…"

end
"----------"

"Data:"

L = 0.2 [m] "… length of fin"

k_0 = 14.695

beta = 0.0010208

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"Calculations:"

"Let there be 20 equal divisions (MM = 20) of the length so that DELTAx = 0.01 m. Nodes: 0 to 20.

Of these, 0 and 20 are boundary nodes and 1 to 19 are internal nodes:"

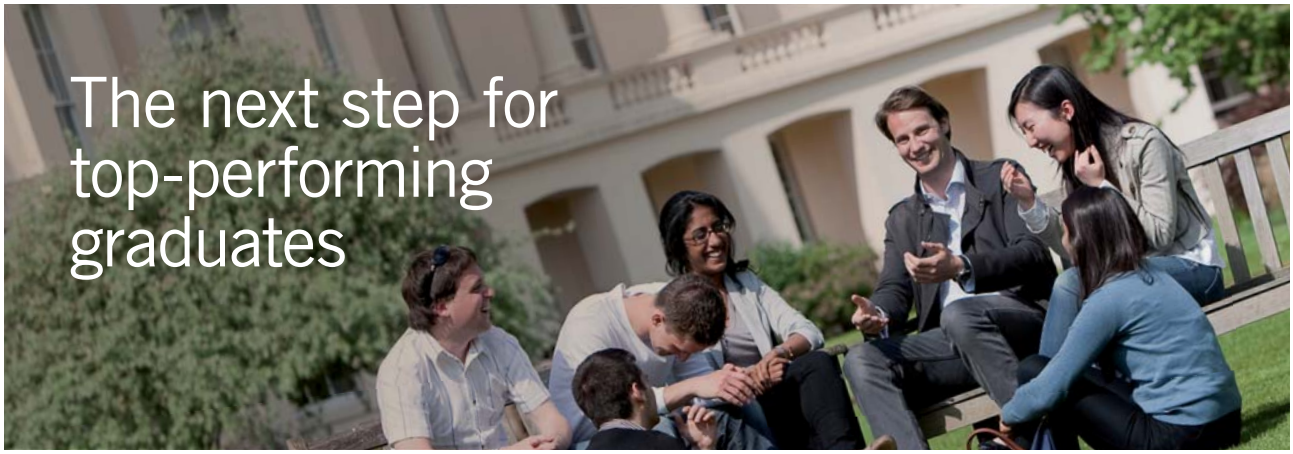
MM = 20 "no. of equal divisions"

DELTAx = 0.01[m]

"To find temp at internal nodes:

By energy balance at node m, remembering to write all energy terms as flowing in to the node:

$$\frac{\frac{T_{m-1} - T_m}{\Delta x}}{k\left[\frac{T_{m-1} + T_m}{2}\right] \cdot A\left[m \cdot \Delta x - \frac{\Delta x}{2} + 0.08\right]} + \frac{\frac{T_{m+1} - T_m}{\Delta x}}{k\left[\frac{T_m + T_{m+1}}{2}\right] \cdot A\left[m \cdot \Delta x + \frac{\Delta x}{2} + 0.08\right]} = 0 \quad \text{for } m = 1 \text{ to } 19$$



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* Figures taken from London Business School's Masters in Management 2010 employment report

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In EES, it is entered as:

Duplicate m = 1,19

(T[m-1] – T[m]) / (DELTAx / (k ((T[m-1] + T[m]) / 2) * A(m * DELTAx – DELTAx / 2 + 0.08))) + (T[m+1] – T[m]) / (DELTAx / (k ((T[m] + T[m + 1]) / 2) * A(m * DELTAx + DELTAx / 2 + 0.08))) = 0

end

"Note: In the above eqn.: first term is heat flowing in to the node by conduction from LHS, 2nd term is heat flowing in by conduction in to the node from RHS."

"Now for boundary nodes '0' and '10':"

"For node '0':

Temp at the node '0' is given as 400 C:"

T[0] = 400 [C] "for node 0, by data "

"For node 10: temp is given, i.e."

T[20] = 150[C] "..for node 20, by data"

"**For heat transfer:**

Write expression for heat transfer between node '0' and node '1'. In steady state, this must be the rate of heat transfer in the rod.:"

$$Q = \frac{T_0 - T_1}{\frac{\Delta x}{k\,(Tm)\cdot A\left[\frac{\Delta x}{2} + 0.08\right]}}$$

$$Tm = \frac{T_0 + T_1}{2}$$

In EES this is entered:

Q = (T[0] – T[1]) / (DELTAx / (k(Tm) * A(DELTAx/2+ 0.08)))

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Tm = (T[0] + T[1]) / 2

"To draw the plot of x vs Temp.:"

duplicate i = 0,20

x[i] = i * DELTAx + 0.08

end

"=="

"Compare with the results from Analytical relations for Q and Tx:"

$$Q_{analyt} = \frac{k_0 \cdot \frac{\pi}{2} \cdot \left[T_{20} - T_0 + \frac{\beta}{2} \cdot \left(T_{20}^{\,2} - T_0^{\,2} \right) \right]}{\frac{1}{0.28^{\,2}} - \frac{1}{0.08^{\,2}}}$$

This is entered in EES:

Q_analyt =((k_0 * pi /2) * ((T[20] – T[0]) + beta / 2 * (T[20]^2 – T[0]^2))) / (1 / 0.28^2 – 1/ 0.08^2)

"And, for temp at various x's:"

$$\beta \cdot \frac{T_{analyt,i}^{\,2}}{2} + T_{analyt,i} - \left[T_0 + \beta \cdot \frac{T_0^{\,2}}{2} \right] - \frac{Q_{analyt}}{k_0 \cdot \frac{\pi}{4} \cdot 2 \cdot x_i^{\,2}} + \frac{Q_{analyt}}{k_0 \cdot \frac{\pi}{4} \cdot 2 \cdot 0.08^{\,2}} = 0 \qquad \text{for } i = 0 \text{ to } 20$$

And, in EES it is entered as:

duplicate i = 0, 20

beta * T_analyt[i]^2 / 2 + T_analyt[i] – (T[0] + beta * T[0]^2/2) – Q_analyt / (k_0 * (pi /4) *2 * x[i]^2) + Q_analyt / (k_0 * (pi / 4) * 2 * 0.08^2) = 0

end

Results:

Unit Settings: SI C kPa kJ mass deg

$\beta = 0.001021$	$\Delta x = 0.01$ [m]	$k_0 = 14.7$	L = 0.2 [m]
MM = 20 [-]	Q = 51.72 [W]	$Q_{analyt} = 51.5$ [W]	Tm = 373.6 [C]

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Thus:

By Numerical method: Q = 51.72 W … Ans.

By Analytical method, Q_analyt = 51.5 W … Ans.

Temp at mid-plane (i.e. at node 10, or x = 0.18 m) = 184.6 C …. Ans.

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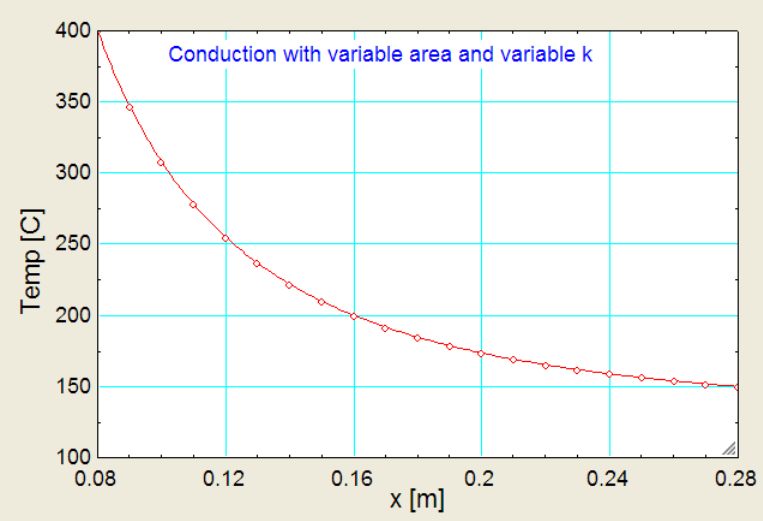
Compare the temp at various x's by both the methods:

Sort	1 T_i [C]	2 x_i [m]	3 $T_{analyt,i}$
[0]	400	0.08	400
[1]	347.2	0.09	347
[2]	308	0.1	307.8
[3]	278.2	0.11	278
[4]	255	0.12	254.8
[5]	236.7	0.13	236.5
[6]	221.9	0.14	221.8
[7]	209.9	0.15	209.7
[8]	199.9	0.16	199.8
[9]	191.6	0.17	191.5
[10]	184.6	0.18	184.5
[11]	178.6	0.19	178.6
[12]	173.5	0.2	173.5
[13]	169.1	0.21	169
[14]	165.2	0.22	165.2
[15]	161.9	0.23	161.8
[16]	158.9	0.24	158.9
[17]	156.3	0.25	156.3
[18]	154	0.26	153.9
[19]	151.9	0.27	151.9
[20]	150	0.28	150

Note that the temperatures calculated by numerical and analytical methods match very well.

Plot of temp profile:



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"**Prob. 1I.A.11.**Two ends of a copper rod (k = 380 W/m.K), 15 mm dia and 300 mm long are connected to two walls. The two ends are maintained at 300 C and 260 C. Air is blown across the rod with a heat transfer coeff. of 20 W/(m^2.K). Air temp. is 40 C. Determine:

1) location and value of min. temp. in the rod
2) mid point temp. of the rod
3) draw the temp. profile
4) net heat transfer to air
5) heat transferred from the first 0.12 m length of the rod from LHS.
6) heat transferred from the left end (i.e. at x = 0)

b) If in this example, if there is an uniform heat generation q_g = 1.5 × 10^5 W/m3 in the rod, determine:

1) location and value of min. temp. in the rod
2) mid-point temp. of the rod
3) draw the temp. profile [Ref:[1], p. 243]"

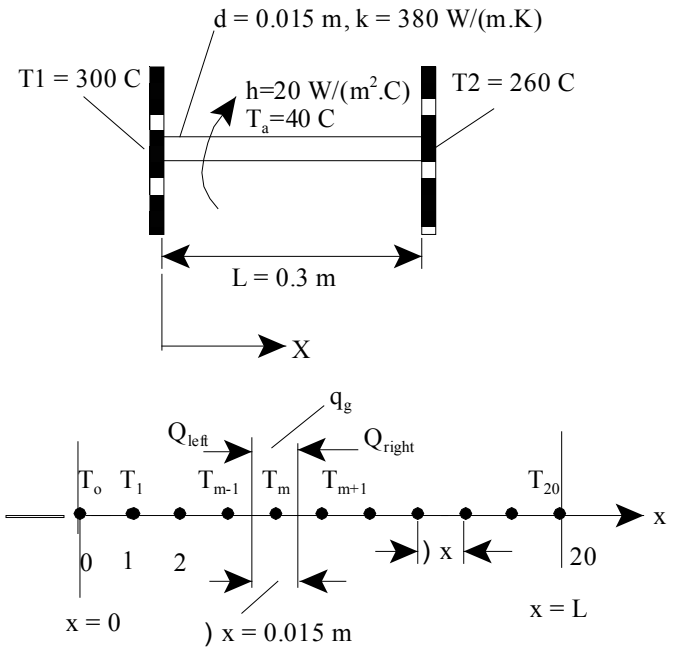

Fig.Prob.1I.A.11

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EES Solution:

"Data:"

L = 0.3 [m]"…length of rod"
k = 380 [W/m-C]
d = 0.015[m]
T1 = 300[C]
T2 = 260 [C]
T_a = 40 [C]
h = 20 [W/m^2-C]

"Calculations:"

"Let there be 20 equal divisions (MM = 20) of the length so that DELTAx = 0.015 m. Nodes: 0 to 20.

Of these, 0 and 20 are boundary nodes and 1 to 19 are internal nodes:"

MM = 20 "no. of equal divisions"

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DELTAx = 0.015[m]

A_c = pi * d^2 /4 "[m^2] … area of cross-section"

P = pi * d "[m] … perimeter"

"To find temp at internal nodes:

By energy balance at node m, remembering to write all energy terms as flowing in to the node:

$$\frac{T_{m-1} - T_m}{\frac{\Delta x}{k \cdot A_c}} + \frac{T_{m+1} - T_m}{\frac{\Delta x}{k \cdot A_c}} + h \cdot P \cdot \Delta x \cdot (T_a - T_m) = 0 \qquad \text{for } m = 1 \text{ to } 19$$

In EES, above eqn is entered as:

Duplicate m = 1,19

(T[m-1] – T[m]) / (DELTAx / (k * A_c)) + (T[m+1] – T[m]) / (DELTAx / (k * A_c)) + h * P * DELTAx * (T_a – T[m]) = 0

end

"In the above eqn.: first term is heat flowing in to the node by conduction from LHS, 2nd term is heat flowing in by conduction in to the node from RHS, and the 3rd term is the heat flow by convection."

"Now for boundary nodes '0' and '10':"

"For node '0':

Temp at the node '0' is given as 300 C:"

T[0] = 300 [C] "by data … for T[0] "

"For node 20: temp is given, i.e."

T[20] = 260[C] "..for node 20"

"**For heat transfer:** Write energy balance at node '0' and node '1', to get Q_left and Q_right. Add them up to get total heat transfer from the rod to the ambient:"

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Node 0:

$$Q_{left} + \frac{T_1 - T_0}{\frac{\Delta x}{k \cdot A_c}} + h \cdot P \cdot \frac{\Delta x}{2} \cdot (T_a - T_0) = 0$$

In EES:

Q_left + (T[1] – T[0]) / (DELTAx / (k * A_c)) + h * P * (DELTAx/2) * (T_a – T[0]) = 0 "…for node 0 … gives Q_left"

Node 0 :

$$Q_{right} + \frac{T_{19} - T_{20}}{\frac{\Delta x}{k \cdot A_c}} + h \cdot P \cdot \frac{\Delta x}{2} \cdot (T_a - T_0) = 0$$

And, in EES it is entered:

Q_right + (T[19] – T[20]) / (DELTAx / (k * A_c)) + h * P * (DELTAx/2) * (T_a – T[0]) = 0 "…for node 20 … gives Q_right"

Q_total = Abs(Q_left) + Abs(Q_right) "[W] …total heat transfer from rod"

"Also, verify by calculating the heat lost by the rod along its length by convection:"

duplicate i = 1,19

Q_conv[i] = h * P * DELTAx * (T[i] – T_a)

end

Q_conv[0] = h * P * DELTAx/2 * (T[0] – T_a)

Q_conv[20] = h * P * DELTAx/2 * (T[20] – T_a)

"To draw the plot of x vs Temp.:"

duplicate i = 0,20

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x[i] = i * DELTAx

end

"Min. temp in the rod:"

Tmin = min(T[0..20]) "…finds the min. temp of the 20 nodal temperatures"

"=="

Results:

Unit Settings: SI C kPa kJ mass deg

A_c = 0.0001767 [m^2]	d = 0.015 [m]	Δx = 0.015 [m]
h = 20 [W/m^2-C]	k = 380 [W/m-C]	L = 0.3 [m]
MM = 20 [-]	P = 0.04712 [m]	Q_{left} = 40.65 [W]
Q_{right} = 21.17 [W]	Q_{total} = 61.82 [W]	T1 = 300 [C]
T2 = 260 [C]	Tmin = 243.8 [C]	T_a = 40 [C]

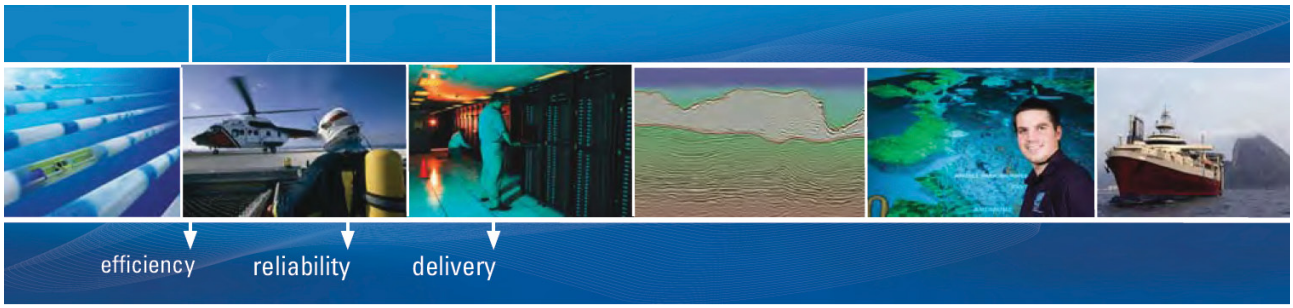

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Table of x vs Temp:

Sort	1 x_i [m]	2 T_i [C]	3 $Q_{conv,i}$ [C]
[0]	0	300	1.838
[1]	0.015	291.3	3.553
[2]	0.03	283.5	3.442
[3]	0.045	276.3	3.341
[4]	0.06	270	3.251
[5]	0.075	264.4	3.172
[6]	0.09	259.4	3.102
[7]	0.105	255.2	3.042
[8]	0.12	251.6	2.992
[9]	0.135	248.8	2.951
[10]	0.15	246.5	2.92
[11]	0.165	245	2.898
[12]	0.18	244	2.884
[13]	0.195	243.8	2.88
[14]	0.21	244.1	2.886
[15]	0.225	245.1	2.9
[16]	0.24	246.8	2.923
[17]	0.255	249.1	2.956
[18]	0.27	252	2.998
[19]	0.285	255.7	3.049
[20]	0.3	260	1.555

Thus:

Min. temp in the rod: Tmin = 243.8 C, occurring at x_{min} = 0.195 m … Ans.

Mid-point temp of rod: at x = 0.15 m (i.e. node 10): 246.5 C … Ans.

Net heat transfer to air: Q_total = 61.82 W … Ans.

Verify: Total convection heat transfer from surface of rod:

i.e. Sum of Q_conv at all the nodes (i.e. 3rd column in the Table above): 59.695 W … Ans.

Compare this value of Q_conv with Qtotal = 61.82 W.

Heat transferred from left end: Q_left = 40.65 W … Ans.

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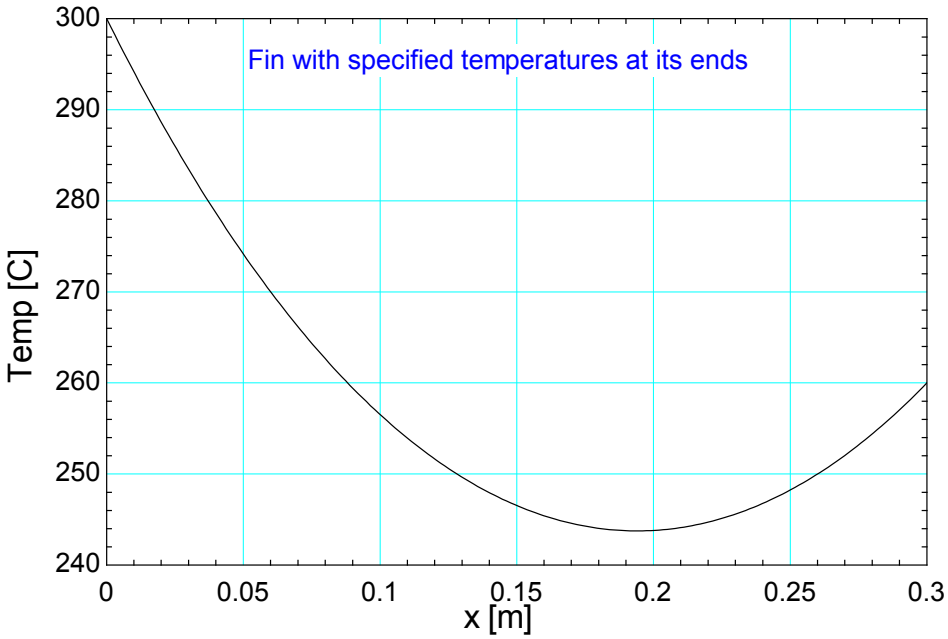
Heat transferred from first 0.12 m from LHS:

Sum the corresponding values (from Node 0 to Node 8) in column 3 of above Table.

We get: Q_conv from first 0.12 m of rod = 25.895 W … Ans.

Temp. profile in the rod:



d) **If there is uniform heat generation in the rod:**

Add the following code to the earlier EES code:

"(b) When there is heat generation in the rod:"

q_g = 1.5e05 [W/m^3]"…uniform heat generation"

"Now, the nodal equations for heat balance get modified since heat gen. is also to be included:"

"Let us write the temperatures in another vector called 'Temp':"

"For Node 0:"

Temp[0] = 300[C] "..by data"

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"For Node 20:"

Temp[20] = 260[C] "…by data"

"For Internal nodes 1 to 19:"

"Write the heat balance, including the heat generation in the present case, remembering to write all heat flows as flowing ***into*** the node:"

$$\frac{Temp_{m-1} - Temp_m}{\frac{\Delta x}{k \cdot A_c}} + \frac{Temp_{m+1} - Temp_m}{\frac{\Delta x}{k \cdot A_c}} + h \cdot P \cdot \Delta x \cdot (T_a - Temp_m) + q_g \cdot \Delta x \cdot A_c = 0 \qquad \text{for } m = 1 \text{ to } 19$$

Duplicate m = 1,19

(Temp[m-1] – Temp[m]) / (DELTAx / (k * A_c)) + (Temp[m+1] – Temp[m]) / (DELTAx / (k * A_c)) + h * P * DELTAx * (T_a – Temp[m]) + q_g * DELTAx * A_c = 0

End

"In the above eqn.: first term is heat flowing in to the node by conduction from LHS, 2nd term is heat flowing in by conduction in to the node from RHS, 3rd term is the heat flow by convection, and the 4th term is the heat generation term."

Results:

Sort	1 x_i [m]	2 $Temp_i$ [C]
[0]	0	300
[1]	0.015	292.1
[2]	0.03	284.9
[3]	0.045	278.4
[4]	0.06	272.5
[5]	0.075	267.3
[6]	0.09	262.7
[7]	0.105	258.8
[8]	0.12	255.4
[9]	0.135	252.6
[10]	0.15	250.5
[11]	0.165	248.8
[12]	0.18	247.8
[13]	0.195	247.3
[14]	0.21	247.4
[15]	0.225	248.1
[16]	0.24	249.3
[17]	0.255	251.1
[18]	0.27	253.5
[19]	0.285	256.4
[20]	0.3	260

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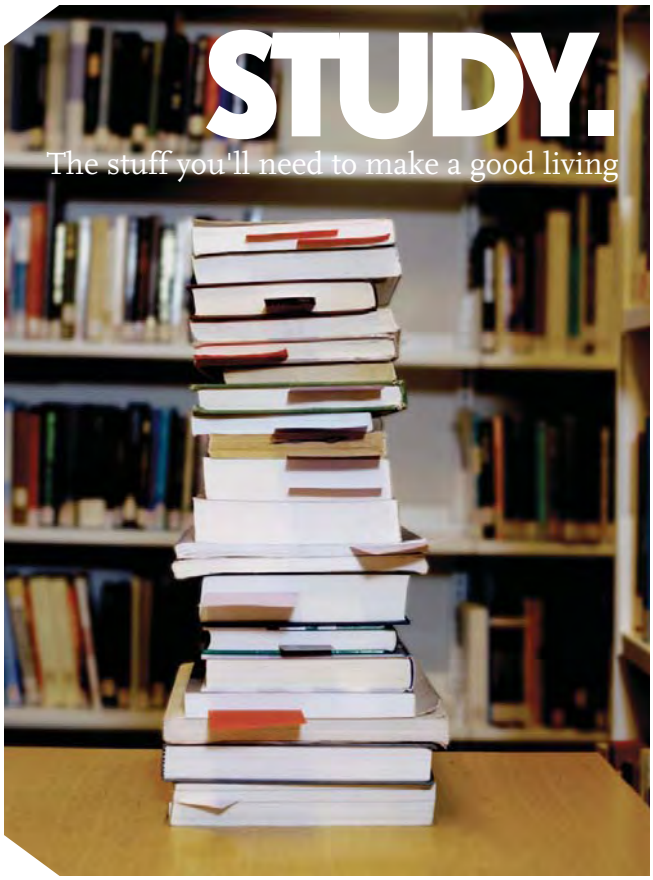
Thus:

From the Table above,

Min. temp in the rod = 247.3 C, at x = 0.195 m .. Ans.

Mid-point temp of rod (i.e. at x = 0.15 m) = 250.5 C … Ans.



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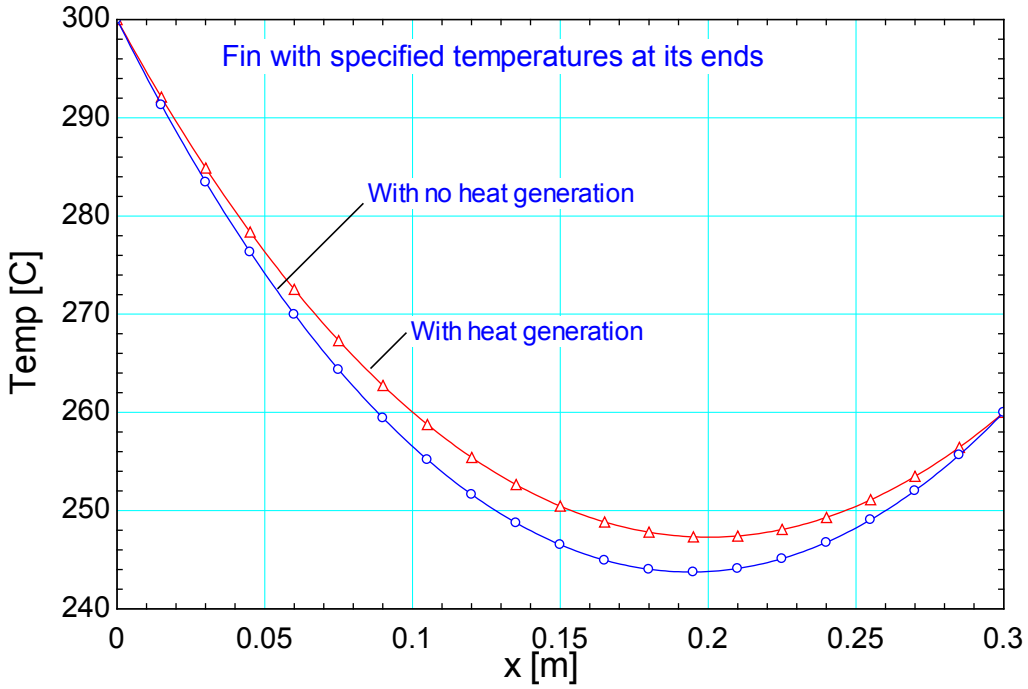
Temp. profile:



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Prob. 1I.A.12. Consider an Aluminium alloy fin (k = 180 W/m.C) of triangular cross-section with L = 5 cm, base thickness b = 1 cm and width w in the direction perpendicular to paper is very large. Base of the fin is at T0 = 180 C. Fin is losing heat by convection to ambient air at T_inf = 25 C with h = 25 W/m^2.C, and by radiation to the surroundings at an average temp T_surr = 290 K. Using finite difference method with six equally spaced nodes along the fin in the x-direction, determine: (a) temperatures at the nodes (b) rate of heat transfer from the fin for w = 1 m. Take emissivity of fin surface as 0.9 and assume steady, one – dimensional heat transfer

(c) Also, investigate the effect of fin base temp on the fin tip temp and the rate of heat transfer from the fin. Let the fin base temp vary from 100 C to 200 C. Plot the results. [Ref. 2]

Use numerical method, with 5 equal divisions, i.e. $\Delta x = 0.01$ m

i.e. there are six equally spaced nodes. Nodes 0 and 5 are boundary nodes and nodes 1 to 4 are internal nodes.

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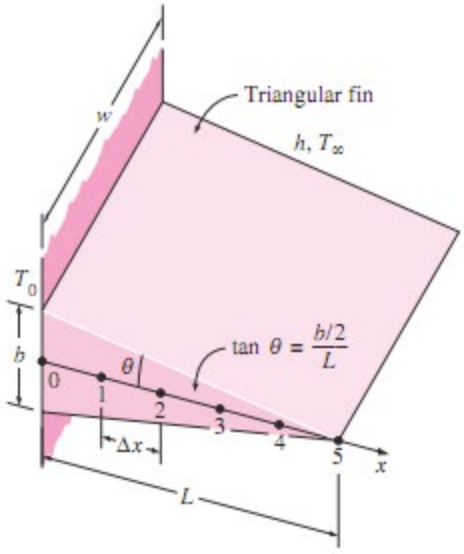
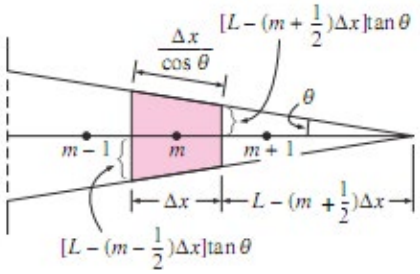
Mathcad Solution:





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Data:

$L := 0.05$ m.....Length of fin

$b := 0.01$ m.....base thickness

$w := 1$ m....width of fin

$T0 := 180$ C.....temp. at base of fin

$T_{inf} := 25$ C ... fluid temp

$T_{surr} := 290$ K .... surrounding temp for radiation heat transfer

$k := 180$ W/(m.C).....thermal cond. of Al alloy

$h := 25$ W/m^2.C ... heat transfer coeff

$MM := 5$ ...no. of equal divisions

$\Delta x := 0.01$ m.

$\sigma := 5.67 \cdot 10^{-8}$ W/m^2.K^4 ... Stefan-Boltzmann constant

$\varepsilon := 0.9$ ...emissivity

Note that here, area of cross-section varies with the length x.
Write a function for variation of area with x:

tan(θ) = (b/2) / L

$\theta := \operatorname{atan}\left(\frac{\frac{b}{2}}{L}\right)$ i.e. $\theta = 0.1$ radians = $\frac{180}{\pi} \cdot \theta = 5.711$ degrees.

At any x from LHS, the thickness is:

$b(x) := 2 \cdot (L - x) \cdot \tan(\theta)$ ex: at x = 0: $b(0) = 0.01$ m...verified.

And, area of cross-section at any x is:

$A(x) := b(x) \cdot w$ m^2 ... area of cross-section at any x

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Difference equations:

For Internal nodes:

Consider an internal node 'm':

Write the heat balance, remembering to have all heat flows ***into*** the node:

$$\frac{T_{m-1}-T_m}{\left(\frac{\Delta x}{k\cdot A\left(m\cdot\Delta x-\frac{\Delta x}{2}\right)}\right)}+\frac{T_{m+1}-T_m}{\left(\frac{\Delta x}{k\cdot A\left(m\cdot\Delta x+\frac{\Delta x}{2}\right)}\right)}+\frac{h\cdot 2\cdot\Delta x\cdot w}{\cos(\theta)}\cdot\left(T_{inf}-T_m\right)+\frac{\sigma\cdot\varepsilon\cdot 2\cdot\Delta x\cdot w}{\cos(\theta)}\cdot\left[\left(T_{surr}\right)^4-\left(T_m+273\right)^4\right]=0$$

..for m = 1 to 4.....eqn.(A)

In eqn. (A) above, first term is the heat flowing from LHS and the second term is the heat flowing from RHS. The third term is the heat flowing from the surface by convection; and, the 4th term is heat transfer by radiation.

For Boundary nodes '0' and '5':

T0=180 C.... at node '0'

At node 5, by heat balance:

$$\frac{T4-T5}{\frac{\Delta x}{k\cdot A\left(5\cdot\Delta x-\frac{\Delta x}{2}\right)}}+\frac{h\cdot 2\cdot\frac{\Delta x}{2}\cdot w}{\cos(\theta)}\cdot\left(T_{inf}-T5\right)+\frac{\sigma\cdot\varepsilon\cdot 2\cdot\frac{\Delta x}{2}\cdot w}{\cos(\theta)}\cdot\left[\left(T_{surr}\right)^4-(T5+273)^4\right]=0$$

Now, use Solve Block of Mathcad to solve the 6 eqns simultaneously.

Start with guess values for unknown temperatures:

$T_{base}:=180$ T1 := 100 T2 := 100 T3 := 100 T4 := 100 T5 := 100 .....guess values

Given

$T0=T_{base}$ ...(1) ... for node '0'.. by data

For node 1:

$$\frac{T0-T1}{\left(\frac{\Delta x}{k\cdot A\left(1\cdot\Delta x-\frac{\Delta x}{2}\right)}\right)}+\frac{T2-T1}{\left(\frac{\Delta x}{k\cdot A\left(1\cdot\Delta x+\frac{\Delta x}{2}\right)}\right)}+\frac{h\cdot 2\cdot\Delta x\cdot w}{\cos(\theta)}\cdot\left(T_{inf}-T1\right)+\frac{\sigma\cdot\varepsilon\cdot 2\cdot\Delta x\cdot w}{\cos(\theta)}\cdot\left[\left(T_{surr}\right)^4-(T1+273)^4\right]=0$$

....(2) for node 1

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For node 2:

$$\frac{T1 - T2}{\left(\frac{\Delta x}{k \cdot A\left(2 \cdot \Delta x - \frac{\Delta x}{2}\right)}\right)} + \frac{T3 - T2}{\left(\frac{\Delta x}{k \cdot A\left(2 \cdot \Delta x + \frac{\Delta x}{2}\right)}\right)} + \frac{h \cdot 2 \cdot \Delta x \cdot w}{\cos(\theta)} \cdot \left(T_{inf} - T2\right) + \frac{\sigma \cdot \varepsilon \cdot 2 \cdot \Delta x \cdot w}{\cos(\theta)} \cdot \left[\left(T_{surr}\right)^4 - (T2 + 273)^4\right] = 0$$

...(3)....for node 2

For node 3:

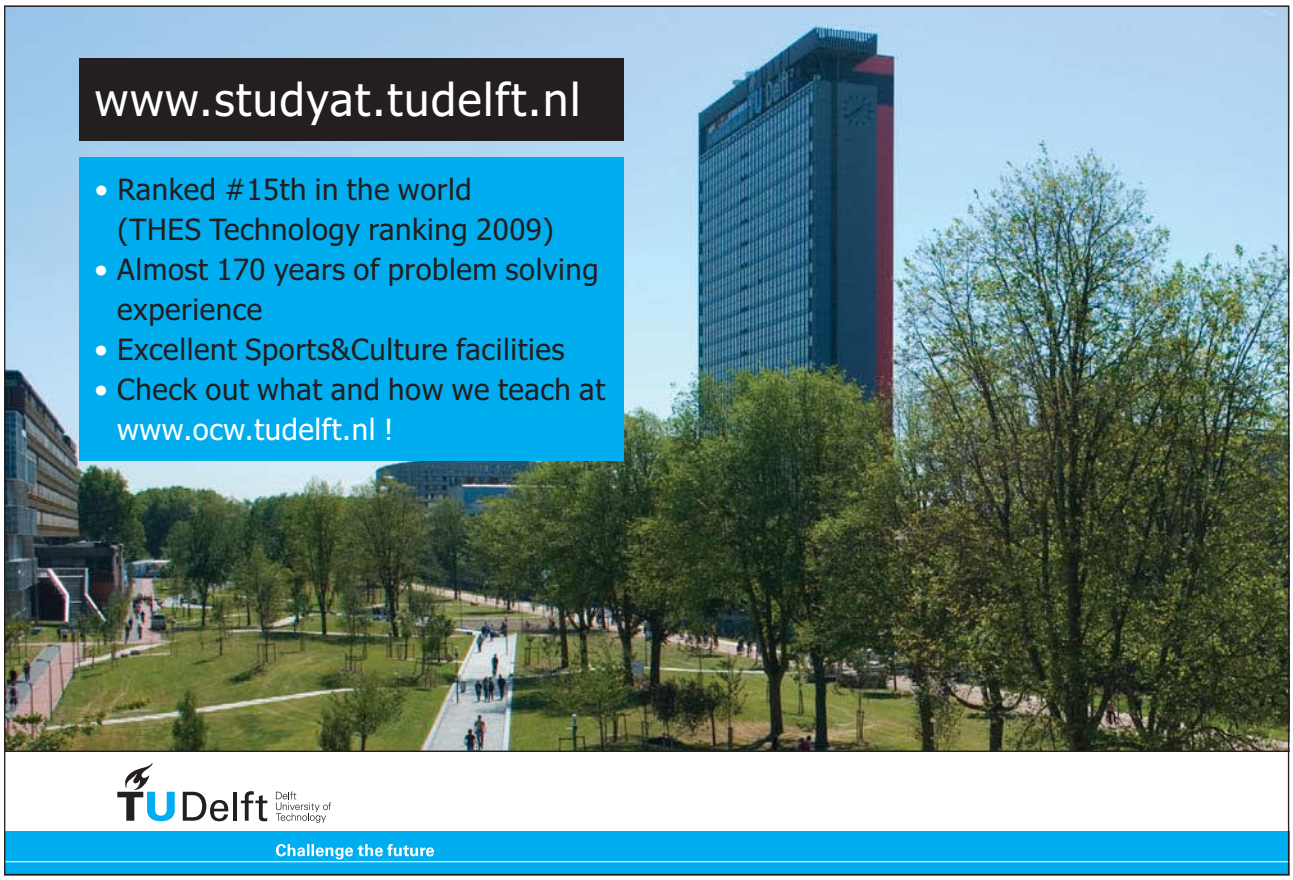
$$\frac{T2 - T3}{\left(\frac{\Delta x}{k \cdot A\left(3 \cdot \Delta x - \frac{\Delta x}{2}\right)}\right)} + \frac{T4 - T3}{\left(\frac{\Delta x}{k \cdot A\left(3 \cdot \Delta x + \frac{\Delta x}{2}\right)}\right)} + \frac{h \cdot 2 \cdot \Delta x \cdot w}{\cos(\theta)} \cdot \left(T_{inf} - T3\right) + \frac{\sigma \cdot \varepsilon \cdot 2 \cdot \Delta x \cdot w}{\cos(\theta)} \cdot \left[\left(T_{surr}\right)^4 - (T3 + 273)^4\right] = 0$$

...(4)....for node 3

For node 4:

$$\frac{T3 - T4}{\left(\frac{\Delta x}{k \cdot A\left(3 \cdot \Delta x - \frac{\Delta x}{2}\right)}\right)} + \frac{T5 - T4}{\left(\frac{\Delta x}{k \cdot A\left(3 \cdot \Delta x + \frac{\Delta x}{2}\right)}\right)} + \frac{h \cdot 2 \cdot \Delta x \cdot w}{\cos(\theta)} \cdot \left(T_{inf} - T4\right) + \frac{\sigma \cdot \varepsilon \cdot 2 \cdot \Delta x \cdot w}{\cos(\theta)} \cdot \left[\left(T_{surr}\right)^4 - (T4 + 273)^4\right] = 0$$

...(5)....for node 4



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For node 5:

$$\frac{\frac{T4 - T5}{\Delta x}}{k \cdot A\left(5 \cdot \Delta x - \frac{\Delta x}{2}\right)} + \frac{h \cdot 2 \cdot \frac{\Delta x}{2} \cdot w}{\cos(\theta)} \cdot \left(T_{inf} - T5\right) + \frac{\sigma \cdot \varepsilon \cdot 2 \cdot \frac{\Delta x}{2} \cdot w}{\cos(\theta)} \cdot \left[\left(T_{surr}\right)^4 - (T5 + 273)^4\right] = 0$$

...(6)....for node 5

$$Temp\left(T_{base}\right) := Find(T0, T1, T2, T3, T4, T5)$$

Now, note that the vector 'Temp' contains all the node temperatures.

Also note that Temp is written as a function of $_{Tbase}$. By doing so, the same Solve Block is used again and again to calculate the node temperatures for different values of T_{base}.

Thus, we get, for T_{base} = 180 C:

$$Temp\left(T_{base}\right) = \begin{bmatrix} 180 \\ 177.04 \\ 174.117 \\ 171.233 \\ 168.391 \\ 165.576 \end{bmatrix}$$

$Temp\left(T_{base}\right)_5 = 165.576$ **C ... tip temp....Ans.**

And, $T1 := 177.04$ $T2 := 174.117$ $T3 := 171.233$ $T4 := 168.391$ $T5 := 165.576$ **C....Ans.**

As an example, at another value of T_{base}, say T_{base} = 200 C we get various node temps as:

$$Temp(200) = \begin{bmatrix} 200 \\ 196.579 \\ 193.203 \\ 189.874 \\ 186.597 \\ 183.353 \end{bmatrix}$$

To draw the graph of Tip temp (i.e. T5) against base temp T0:

$T_{base} := 100, 105 .. 200$ ..define a range variable T_{base}

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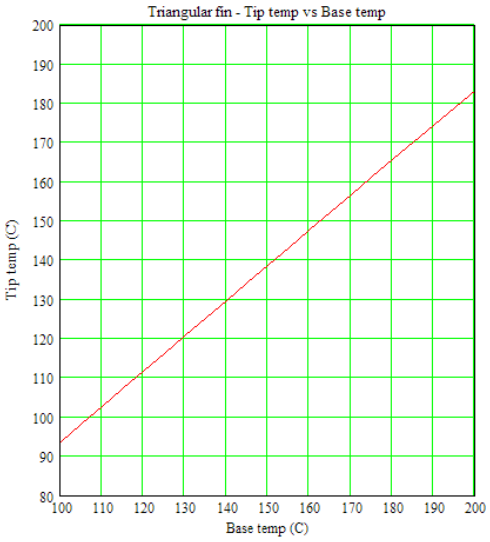

Rate of heat transfer from the fin:

Add up the convection and radiation heat transfers from all the Nodes:

$$T_{base} := 180 \quad C \qquad T_{inf} = 25 \quad C \qquad T_{surr} = 290 \quad C$$

$$\begin{aligned} Q\_fin(T_{base}) := {} & h \cdot \frac{2 \cdot \frac{\Delta x}{2} \cdot w}{\cos(\theta)} \cdot (T_{base} - T_{inf}) + h \cdot \frac{2 \cdot \frac{\Delta x}{2} \cdot w}{\cos(\theta)} \cdot (Temp(T_{base})_5 - T_{inf}) \dots \\ & + \left[h \cdot \frac{2 \cdot \Delta x \cdot w}{\cos(\theta)} \cdot \left[(Temp(T_{base})_1 + Temp(T_{base})_2 + Temp(T_{base})_3 + Temp(T_{base})_4) - 4 \cdot T_{inf} \right] \right] \dots \\ & + \left[\varepsilon \cdot \sigma \cdot \frac{2 \cdot \frac{\Delta x}{2} \cdot w}{\cos(\theta)} \cdot \left[(T_{base} + 273)^4 - T_{surr}^4 \right] \right] + \varepsilon \cdot \sigma \cdot \left[\frac{2 \cdot \frac{\Delta x}{2} \cdot w}{\cos(\theta)} \cdot \left[(Temp(T_{base})_5 + 273)^4 - T_{surr}^4 \right] \right] \dots \\ & + \varepsilon \cdot \sigma \cdot \frac{2 \cdot \Delta x \cdot w}{\cos(\theta)} \cdot \left[(Temp(T_{base})_1 + 273)^4 + (Temp(T_{base})_2 + 273)^4 + (Temp(T_{base})_3 + 273)^4 + (Temp(T_{base})_4 + 273)^4 - 4 \cdot T_{surr}^4 \right] \end{aligned}$$

In the above eqn, the different terms are:

1st term: convection from node '0'. Note that the area for convection (and also for radiation) is = 2. $[(\Delta x/2)/\cos(\theta)]$. w. See the fig. above.

2nd term: Convection from node '5'. Same comments as for the previous case.

3rd term: Convection from the interior nodes, 1 to 4. Now the area for convection (and also for radiation) is = 2. $[(\Delta x)/\cos(\theta)]$. w. See the fig. above.

4th term: Radiation from node '0'. Note that the temperatures must be in Kelvin.

5th term: Radiation from node '5'. Note that the temperatures must be in Kelvin.

6th term: Radiation from interior nodes, i.e. nodes 1 to 4.

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We get:

Q_fin(180) = 537.369 **W....heat transfer from fin when T0 = 180 C .... Ans.**

To draw the graph of Q_fin against base temp T0:

T0 := 100, 105.. 200 ..define a range variable T_{base}

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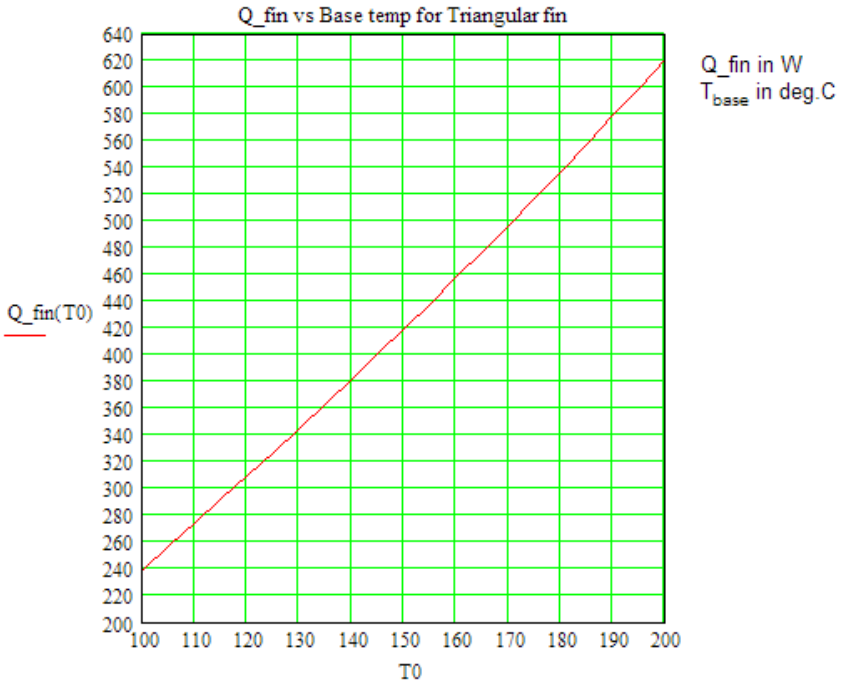
T0	Q_fin(T0)
100	239.814
105	256.84
110	274.054
115	291.46
120	309.063
125	326.867
130	344.877
135	363.099
140	381.536
145	400.193
150	419.075
155	438.188
160	457.535
165	477.122
170	496.953
175	517.034
180	537.369
185	557.964
190	578.823
195	599.951
200	621.355



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Prob. 1I.A.13. A turbine blade is 5 cm long with cross-sectional area A = 4.5 cm^2 and perimeter P = 12 cm. It is made of high alloy steel (k = 25 W/m.K). The temp of the blade attachment point is 500 C and the blade is exposed to combustion gases at 900 C. The heat transfer coeff between the blade surface and the gases is 500 W/m^2.K. Using 5 equally spaced nodes determine: (a) temp distribution in the blade, rate of heat transfer to the blade, and the fin efficiency, and (b) compare the fin efficiency calculated numerically with that calculated by the exact method. [Ref. 5]

Use numerical method, with 5 equal divisions, i.e. Δx = 0.01 m

i.e. there are six equally spaced nodes. Nodes 0 and 5 are boundary nodes and nodes 1 to 4 are internal nodes.

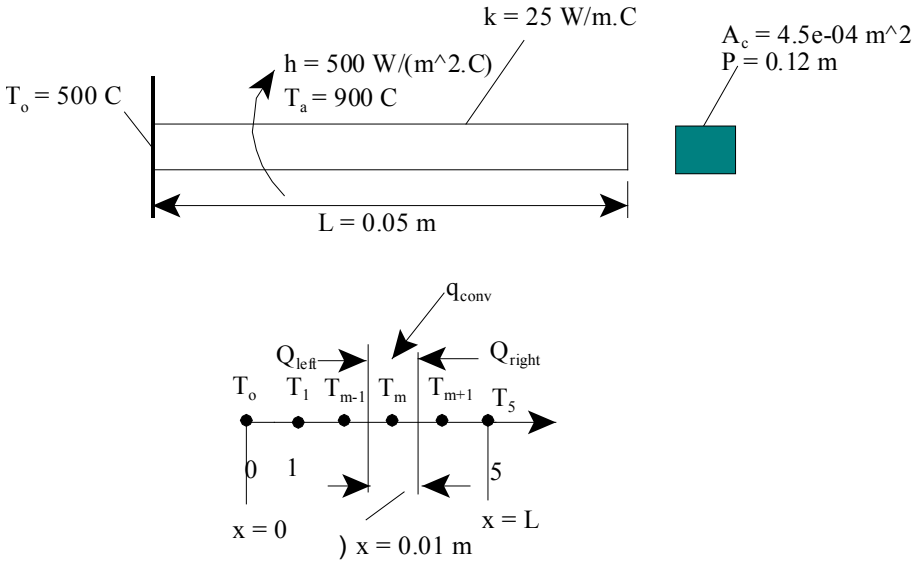

Fig.Prob.1I.A.13

Mathcad Solution:

Data:

L := 0.05 m.....Length of fin

A := $4.5 \cdot 10^{-4}$ m^2.....area of cross-section of fin

P := 0.12 m....Perimeter of fin

T0 := 500 C.....temp. at base of fin

T_a := 900 C ... temp of combustion gases

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k := 25 W/(m.C).....thermal cond. of fin material

h := 500 W/m^2.C ... heat transfer coeff

MM := 5 ...no. of equal divisions

Δx := 0.01 m.

Difference equations:

For Internal nodes:

Consider an internal node 'm':

Write the heat balance, remembering to have all heat flows ***into*** the node:

$$\frac{T_{m-1} - T_m}{\left(\frac{\Delta x}{k \cdot A}\right)} + \frac{T_{m+1} - T_m}{\left(\frac{\Delta x}{k \cdot A}\right)} + h \cdot P \cdot \Delta x \cdot \left(T_a - T_m\right) = 0 \qquad \text{..for m = 1 to 4.....eqn.(A)}$$

In eqn. (A) above, first term is the heat flowing from LHS and the second term is the heat flowing from RHS, and the third term is the heat transfer from the fin surface by convection.

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For Boundary nodes '0' and '5':

T0=500 C.... at node '0'

At node 5, by heat balance:

$$\frac{T4 - T5}{\frac{\Delta x}{k \cdot A}} + h \cdot P \cdot \frac{\Delta x}{2} \cdot \left(T_a - T5\right) = 0$$

Now, use Solve Block of Mathcad to solve the 6 eqns simultaneously.

Start with guess values for unknown temperatures:

$T_{base} := 500$ $T1 := 100$ $T2 := 100$ $T3 := 100$ $T4 := 100$ $T5 := 100$ .....guess values

Given

$T0 = T_{base}$ ...(1) ... for node '0'.. by data

For node 1:

$$\frac{T0 - T1}{\left(\frac{\Delta x}{k \cdot A}\right)} + \frac{T2 - T1}{\left(\frac{\Delta x}{k \cdot A}\right)} + h \cdot P \cdot \Delta x \cdot \left(T_a - T1\right) = 0$$ ....(2) for node 1

For node 2:

$$\frac{T1 - T2}{\left(\frac{\Delta x}{k \cdot A}\right)} + \frac{T3 - T2}{\left(\frac{\Delta x}{k \cdot A}\right)} + h \cdot P \cdot \Delta x \cdot \left(T_a - T2\right) = 0$$ ....(3) for node 2

For node 3:

$$\frac{T2 - T3}{\left(\frac{\Delta x}{k \cdot A}\right)} + \frac{T4 - T3}{\left(\frac{\Delta x}{k \cdot A}\right)} + h \cdot P \cdot \Delta x \cdot \left(T_a - T3\right) = 0$$ ....(4) for node 3

For node 4:

$$\frac{T3 - T4}{\left(\frac{\Delta x}{k \cdot A}\right)} + \frac{T5 - T4}{\left(\frac{\Delta x}{k \cdot A}\right)} + h \cdot P \cdot \Delta x \cdot \left(T_a - T4\right) = 0$$ ....(5) for node 4

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For node 5:

$$\frac{T4 - T5}{\frac{\Delta x}{k \cdot A}} + h \cdot \left(P \cdot \frac{\Delta x}{2} + A\right) \cdot \left(T_a - T5\right) = 0 \qquad \text{....(6) for node 5}$$

$$Temp(k) := Find(T0, T1, T2, T3, T4, T5)$$

Note that Temp is written as a function of k, so that graph can be drawn for different k values. Therefore, for k = 25 W/m.K, temp distribution is:

$$Temp(25) = \begin{bmatrix} 500 \\ 704.029 \\ 803.54 \\ 851.606 \\ 873.863 \\ 882.179 \end{bmatrix}$$

i.e. $T1 := 704.029$ $T2 := 803.54$ $T3 := 851.606$ $T4 := 873.863$ $T5 := 882.179$ **C....Ans.**

And, $Temp(25)_5 = 882.179$ **C ... tip temp....Ans.**

Again, when k = 65 W/m.K, temp distribution is easily found out by writing Temp(65) = , as shown below:.

$$Temp(65) = \begin{bmatrix} 500 \\ 641.77 \\ 730.569 \\ 784.613 \\ 814.988 \\ 827.925 \end{bmatrix}$$

i.e. $T1 := 641.77$ $T2 := 730.569$ $T3 := 784.613$ $T4 := 814.988$ $T5 := 827.925$ **C....Ans.**

And, $Temp(65)_5 = 827.925$ **C ... tip temp....Ans.**

To draw the graph of Temp distribution along the fin for k = 25 and 65 W/m.K:

$i := 0, 1 .. 5$ ..define a range variable i

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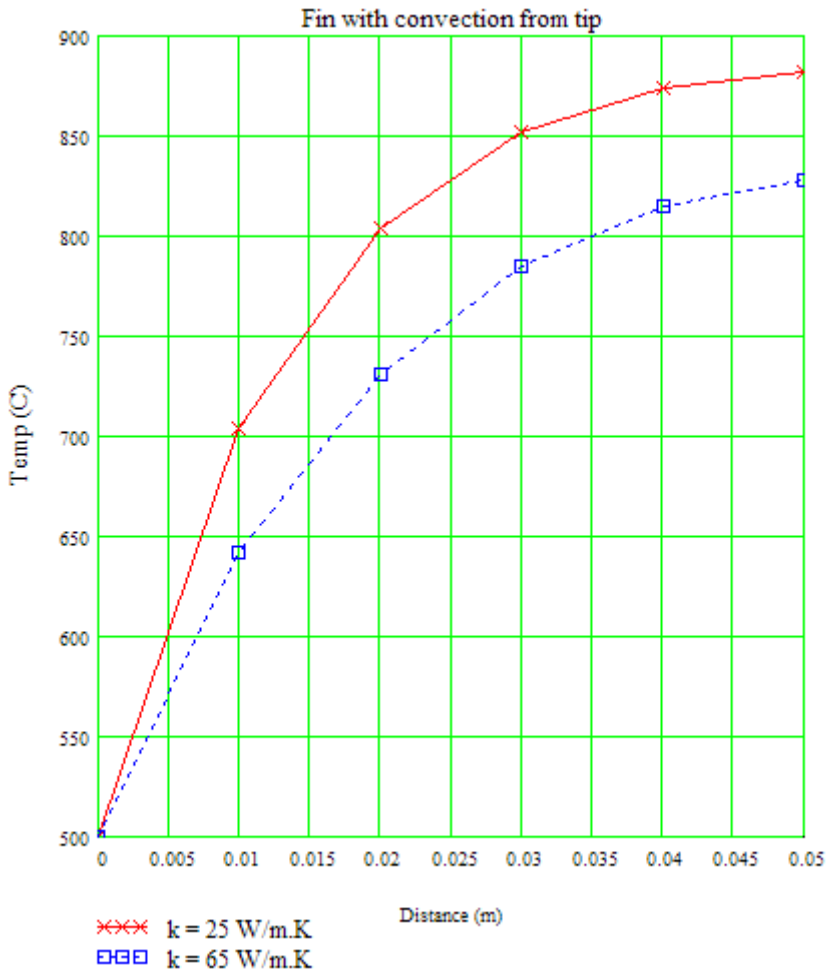

Rate of heat transfer from the fin:

Write the energy balance at the base of the fin, i.e. at node '0', taking care to see that all heat flows are written as flowing ***into*** the node:

When k = 25 W/m.K, we have the temp distribution:

T1 := 704.029 T2 := 803.54 T3 := 851.606 T4 := 873.863 T5 := 882.179 **C..as calculated earlier.**

$$Q_{fin} + \frac{(T1 - T0)}{\frac{\Delta x}{k \cdot A}} + h \cdot P \cdot \frac{\Delta x}{2} \cdot \left(T_a - T0\right) = 0$$

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i.e. $$Q_{fin} := -\left[\frac{(T1 - T0)}{\frac{\Delta x}{k \cdot A}} + h \cdot P \cdot \frac{\Delta x}{2} \cdot \left(T_a - T0\right)\right]$$

i.e. $Q_{fin} = -349.533$ **W .... -ve sign indicates that heat is flowing from the gases to the fin**

Fin efficiency:

$$\eta_f = \frac{Q_{fin}}{Q_{max}}$$ where Qmax is the heat transferred from the fin if the entire fin surface were at a temp of T0.

Now: $$Q_{max} := h \cdot (P \cdot L + A) \cdot \left(T_a - T0\right)$$

Note that area of fin tip is also included for calculation of convective heat transfer.

i.e. $Q_{max} = 1.29 \cdot 10^3$ W .... Max heat transfer from the fin

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Therefore: $\eta_f := \frac{|Q_{fin}|}{Q_{max}}$

i.e. $\eta_f = 0.271$ **....fin efficiency .... Ans.**

Comparison with Analytical calculations:

For a fin with convection from tip, we have:

$m := \sqrt{\frac{h \cdot P}{k \cdot A}}$ i.e. $m = 73.03$ 1/m....fin parameter

Then,

$$Q_{analyt} := k \cdot A \cdot m \cdot (T0 - T_a) \cdot \frac{\tanh(m \cdot L) + \frac{h}{m \cdot k}}{1 + \frac{h}{m \cdot k} \cdot \tanh(m \cdot L)}$$

i.e. $Q_{analyt} = -328.381$ **W.....-ve sign indicating that heat is flowing into the fin from the combustion gases.**

Compare this value of Q = 328.381 W with that obtained by numerical method, i.e. Qfin = 349.533 W. The difference is about 6.1%. Accuracy will certainly be better if we choose to have more no. of nodes.

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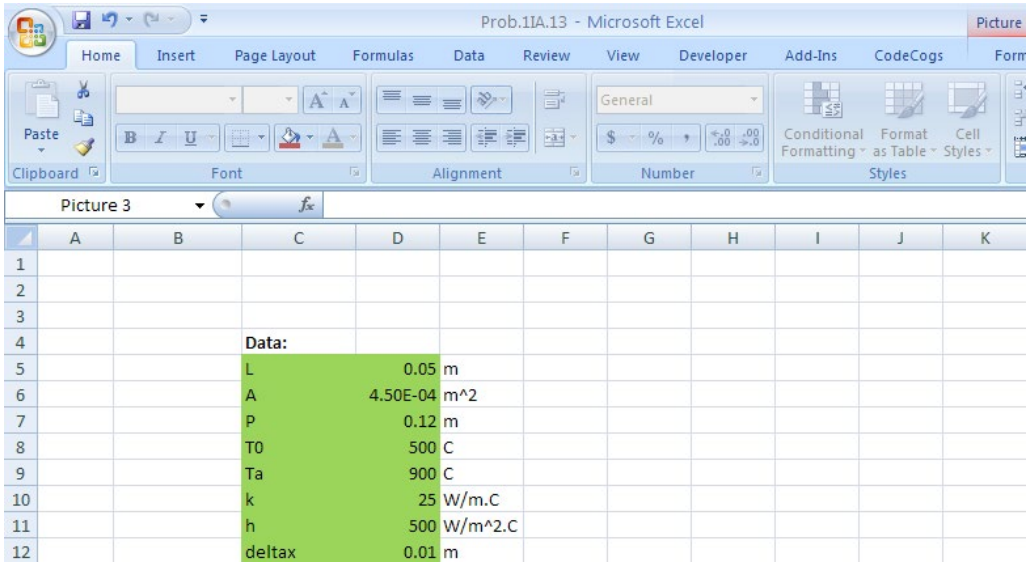
Now, solve the above problem with EXCEL:

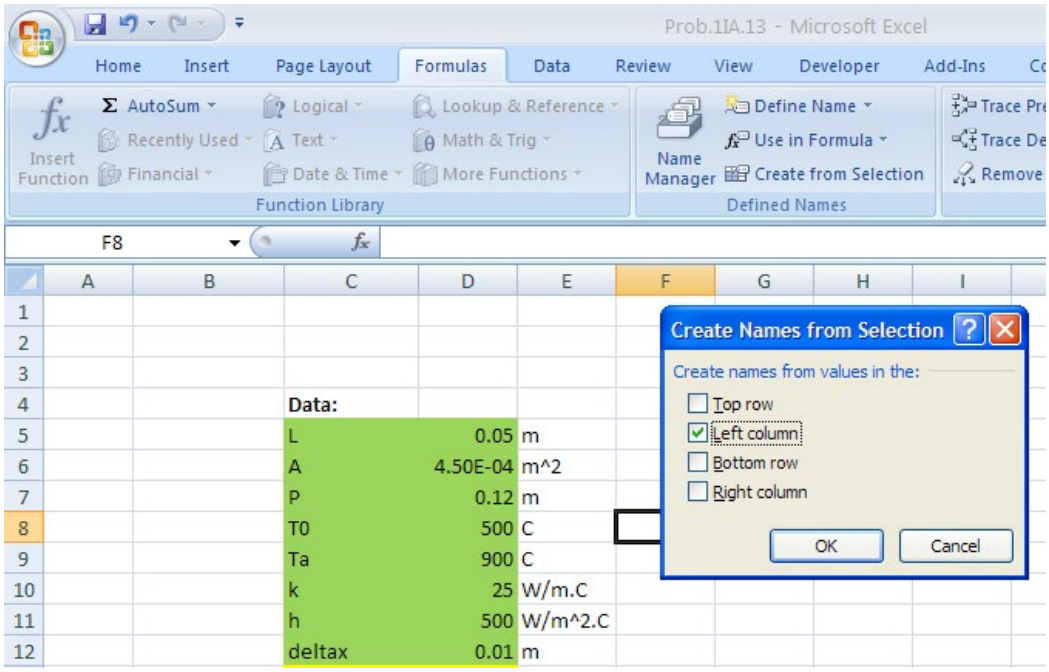
EXCEL Solution:

Following are the steps involved:

1. Start EXCEL and enter the data. Write the notation of the quantity on the left, its corresponding value on the adjacent cell on the right, and the units to its right, as shown below:



2. Immediately, name the cells, which will be useful in entering equations. Select the two columns containing the quantity notation and the values, and Click on Formulas – Create from selection and we get:



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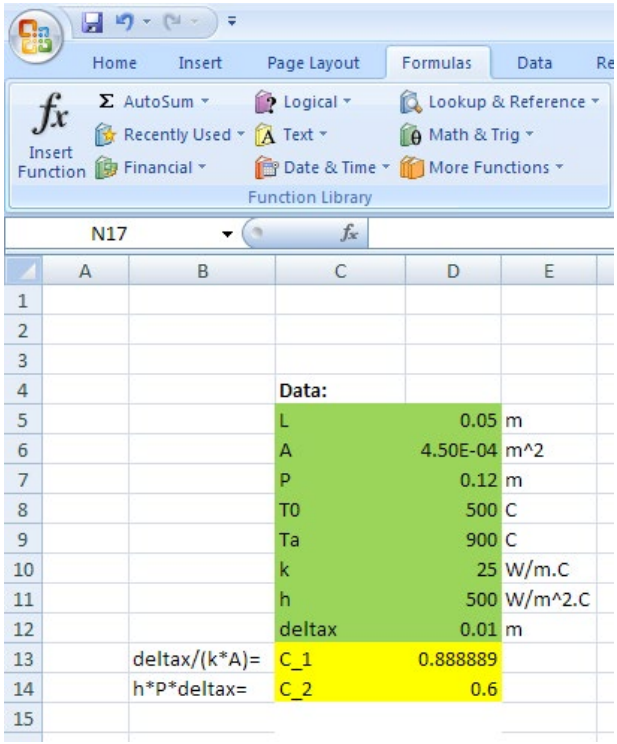
Select Left column and press OK. This means that the selected values in column D are named with corresponding names in column C, e.g. 0.05 as L, 0.12 as P, 0.01 as deltax etc. Now, we can use L, P, etc while entering equations, instead of referring to cells containing them. You can use some colour code also to identify data, calculations etc.

3. Calculate two quantities, viz. deltax/(k*A) and h * P *deltax and name them as C_1 and C_2. Take care to see that you don't name them as C1 and C2, since they refer to cells C1 and C2.



	A	B	C	D	E
1					
2					
3					
4			**Data:**		
5			L	0.05	m
6			A	4.50E-04	m^2
7			P	0.12	m
8			T0	500	C
9			Ta	900	C
10			k	25	W/m.C
11			h	500	W/m^2.C
12			deltax	0.01	m
13		deltax/(k*A)=	C_1	0.888889	
14		h*P*deltax=	C_2	0.6	
15					

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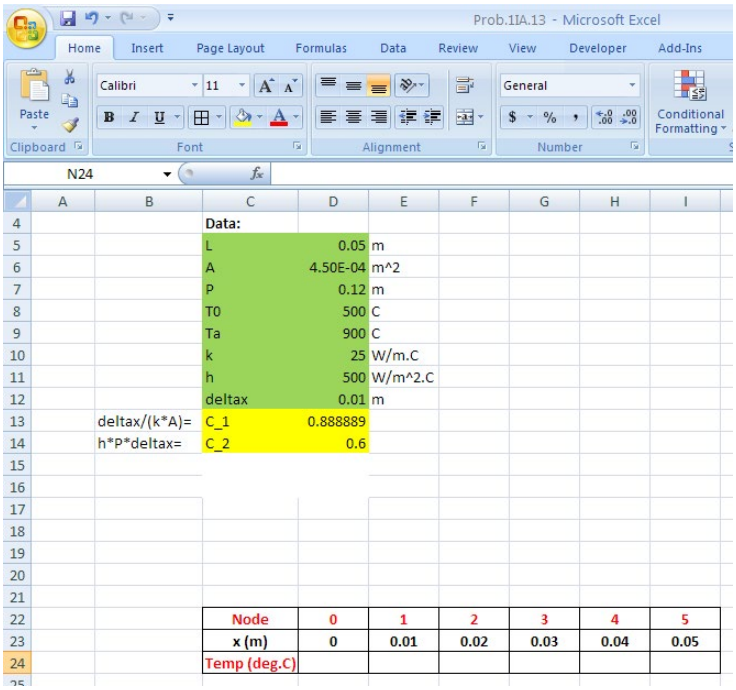
4. Next step is to do the calculations for different nodes, i.e. for internal nodes and the boundary nodes on the left and right. Take advantage of the cell structure of EXCEL and plan to arrange the Nodes, corresponding distances (to draw graphs later) and the corresponding temperatures. One arrangement is shown below:



	A	B	C	D	E	F	G	H	I
4			Data:						
5			L	0.05	m				
6			A	4.50E-04	m^2				
7			P	0.12	m				
8			T0	500	C				
9			Ta	900	C				
10			k	25	W/m.C				
11			h	500	W/m^2.C				
12			deltax	0.01	m				
13		deltax/(k*A)=	C_1	0.888889					
14		h*P*deltax=	C_2	0.6					
15									
16									
17									
18									
19									
20									
21									
22			Node	0	1	2	3	4	5
23			x (m)	0	0.01	0.02	0.03	0.04	0.05
24			Temp (deg.C)						

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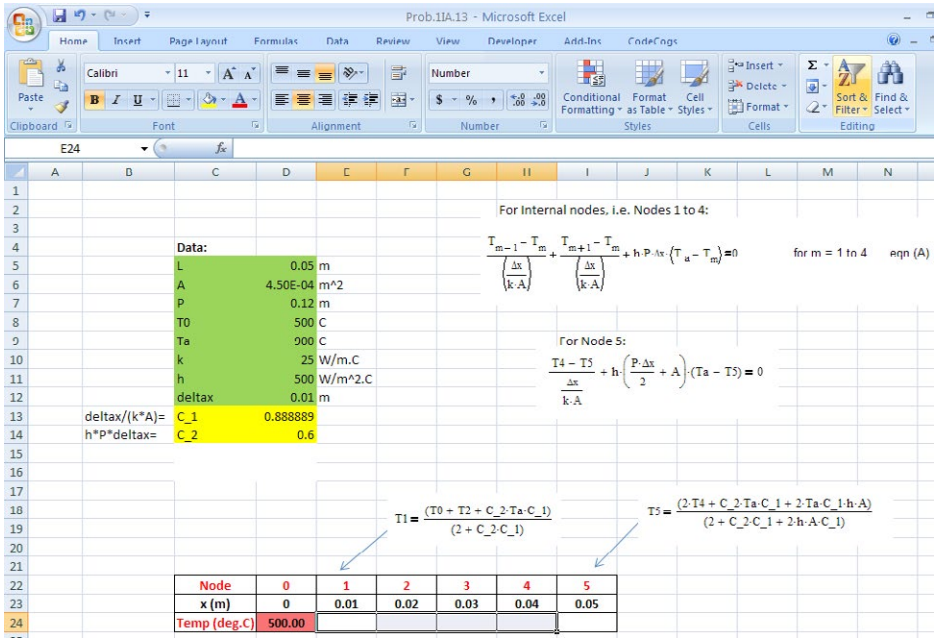
5. Now, by data, T0 = 500 C; enter it. Nodes 1 to 4 are Internal Nodes. Node 5 is boundary node on RHS. The heat balance equations and the equations for temperatures in each of the nodes are already presented in the Mathcad solution above. For clarity, show them in EXCEL worksheet. **Great advantage** of EXCEL is that, to enter eqns for temperatures of Internal nodes, you have to enter the equation for just one node, viz. Node 1 and then copy the formula in the rest of the Internal nodes, i.e. upto Node 4.



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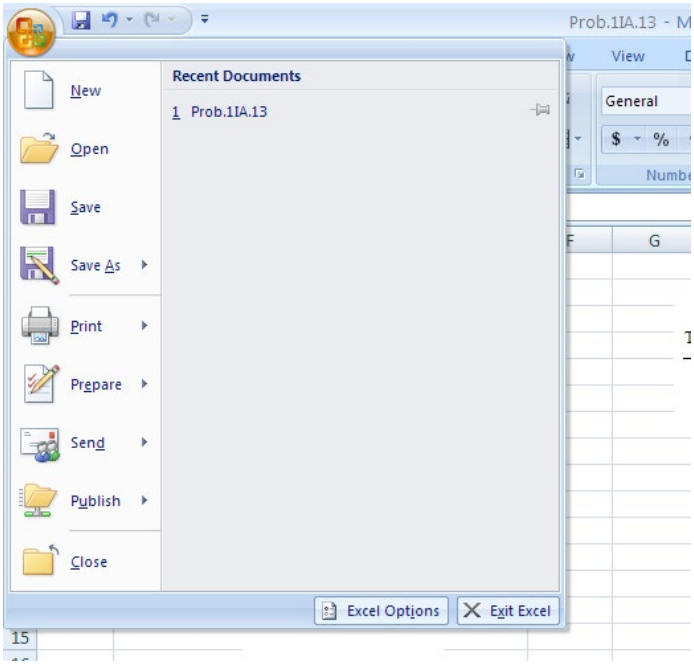
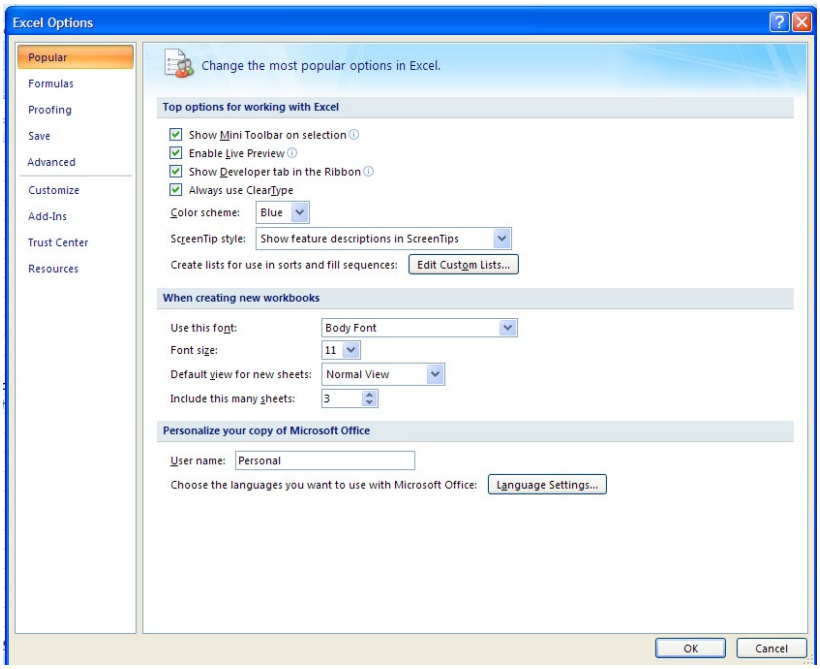
6. **Next step is important:** Observe the eqn for T1. It contains references to T0 and T2, i.e. to cells to the left and right of cell E24. Rest of the quantities in the eqn are 'named' constants. When you copy the eqn for T1, i.e. the eqn entered in cell E24 to cells upto H24, the references to adjacent cells automatically adjust themselves. However, the cells refer to each other and we have to solve the equations simultaneously by 'iteration'. For this purpose, we have to enable Iteration in EXCEL. To do this: click on EXCEL Office button (on the left, top corner)



And, then click on Excel Options; we get:



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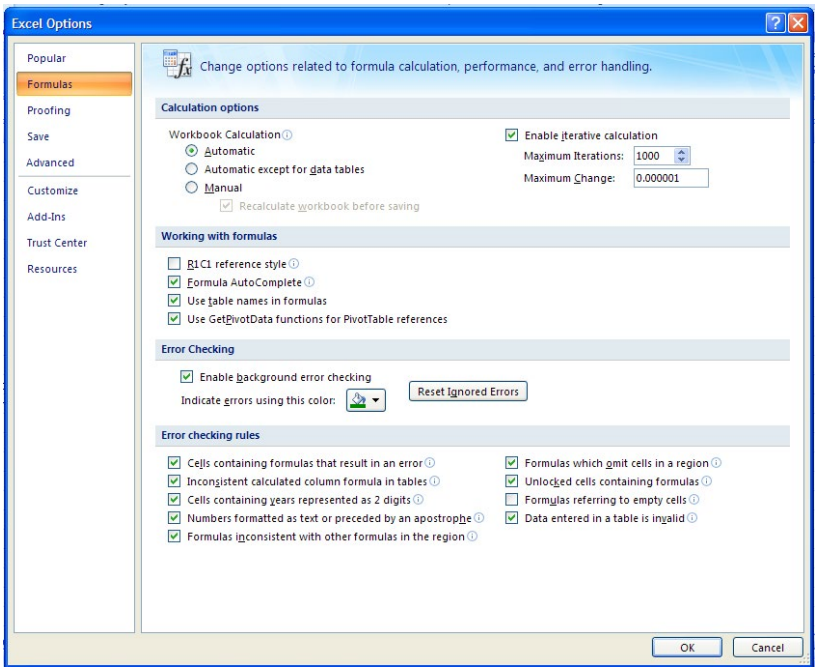
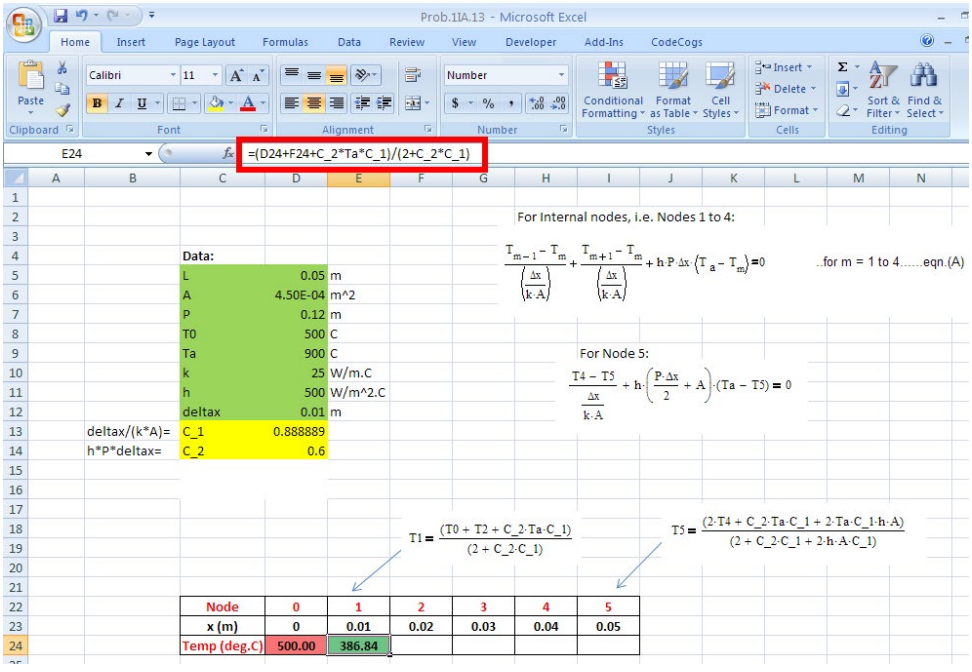
Click on Formulas (on the left, second item from top), and then check mark on Enable Iterations, and enter Max. Iterations as 1000 and Max. change as 0.000001:



Then, click OK.

7. Now, we are ready to enter eqns for Nodes 1 to 5:

Enter the eqn for Temp T1 in Node 1. See the Formula bar where it appears as you enter the eqn.



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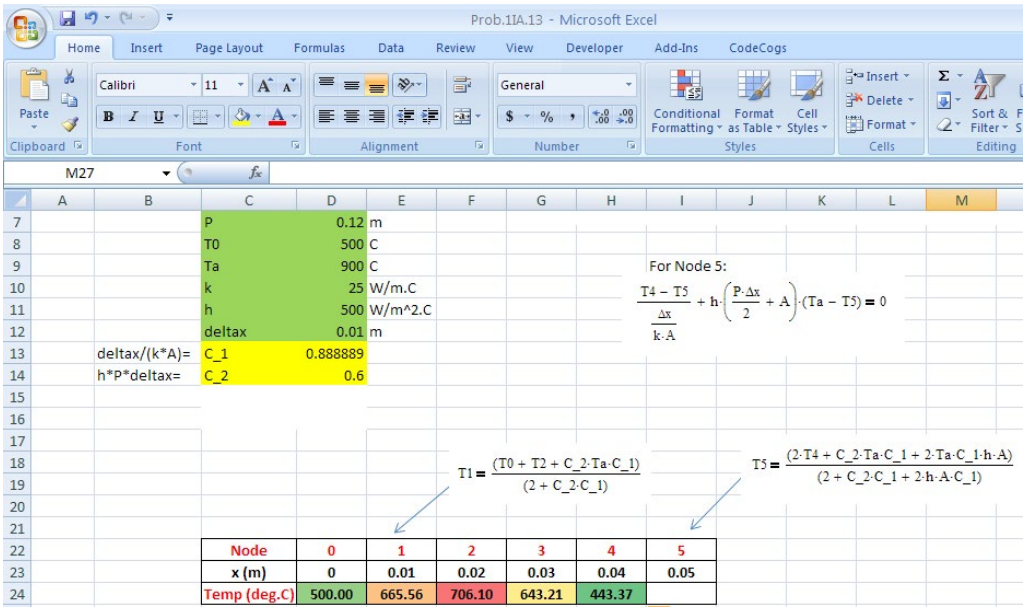
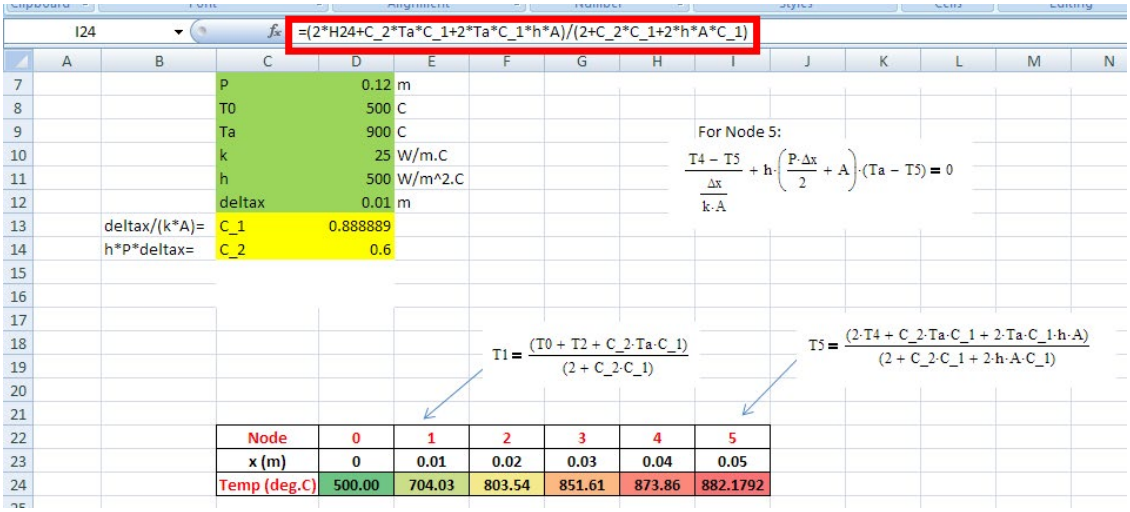
8. Now, copy the eqn in cell E24 upto Node 4, i.e. upto cell H24, by dragging the bottom right, black corner of cell E24 upto H24. Check that cell references to the left and right of each cell is automatically adjusted (by seeing the Formula bar). See below:



7			P	0.12	m								
8			T0	500	C								
9			Ta	900	C								
10			k	25	W/m.C								
11			h	500	W/m^2.C								
12			deltax	0.01	m								
13		deltax/(k*A)=	C_1	0.888889									
14		h*P*deltax=	C_2	0.6									
15													
16													
17													
18													
19													
20													
21													
22			Node	0	1	2	3	4	5				
23			x (m)	0	0.01	0.02	0.03	0.04	0.05				
24			Temp (deg.C)	500.00	665.56	706.10	643.21	443.37					

Don't worry about temperatures appearing for T1 to T4, ***since we have not yet entered the eqn for T5.*** When we enter eqn for T5, by Iteration, all values will adjust themselves.

9. Now, enter eqn for T5 in cell I24: See the eqn in Formula bar. Note that immediately the temperatures in all Nodes adjust themselves. The screen print is shown below:



7			P	0.12	m								
8			T0	500	C								
9			Ta	900	C								
10			k	25	W/m.C								
11			h	500	W/m^2.C								
12			deltax	0.01	m								
13		deltax/(k*A)=	C_1	0.888889									
14		h*P*deltax=	C_2	0.6									
15													
16													
17													
18													
19													
20													
21													
22			Node	0	1	2	3	4	5				
23			x (m)	0	0.01	0.02	0.03	0.04	0.05				
24			Temp (deg.C)	500.00	704.03	803.54	851.61	873.86	882.1792				

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10. So, the final temperatures in all the Nodes are:

Node	0	1	2	3	4	5
x (m)	0	0.01	0.02	0.03	0.04	0.05
Temp (deg.C)	500.00	704.03	803.54	851.61	873.86	882.1792

Thus: T0 = 500 C, T1 = 704.03 C, T2 = 803.54 C, T3 = 851.61 C, T4 = 873.86 C and T5 = 882.179 C.

Compare these values with those obtained with Mathcad, i.e.

T0 = 500 C, T1 = 704.029 C, T2 = 803.54 C, T3 = 851.606 C, T4 = 873.863 C and T5 = 882.179 C.

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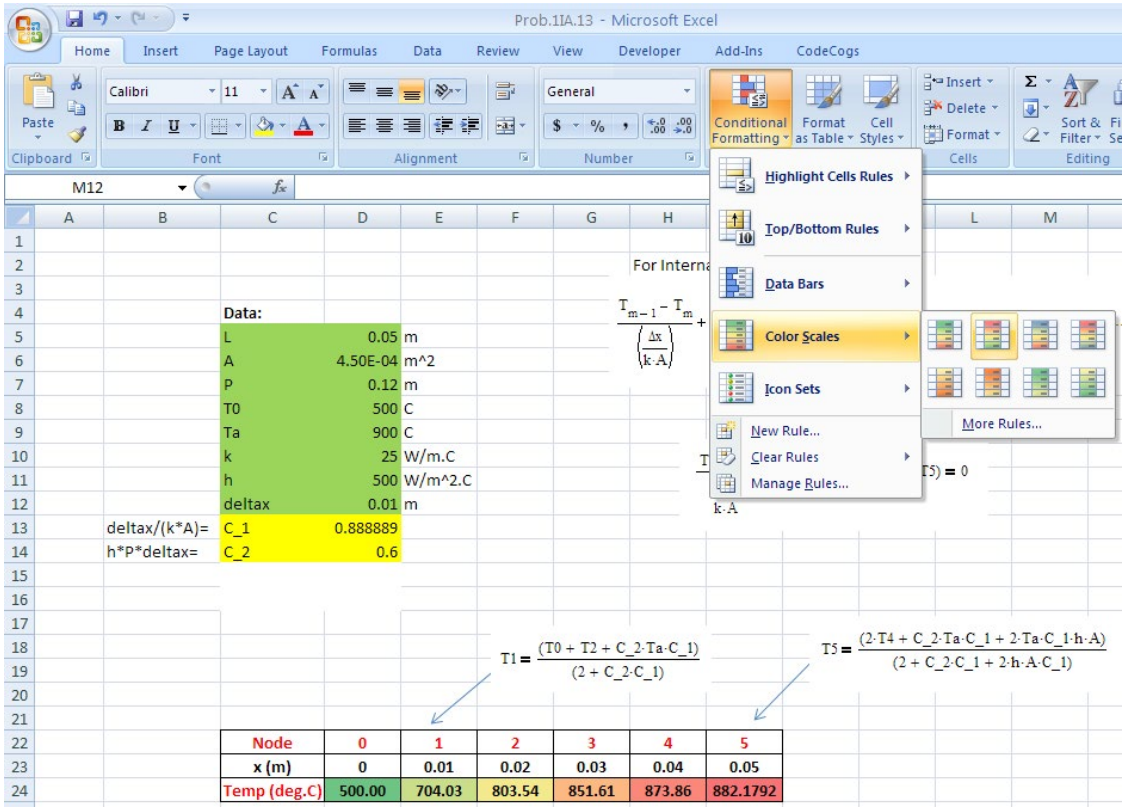
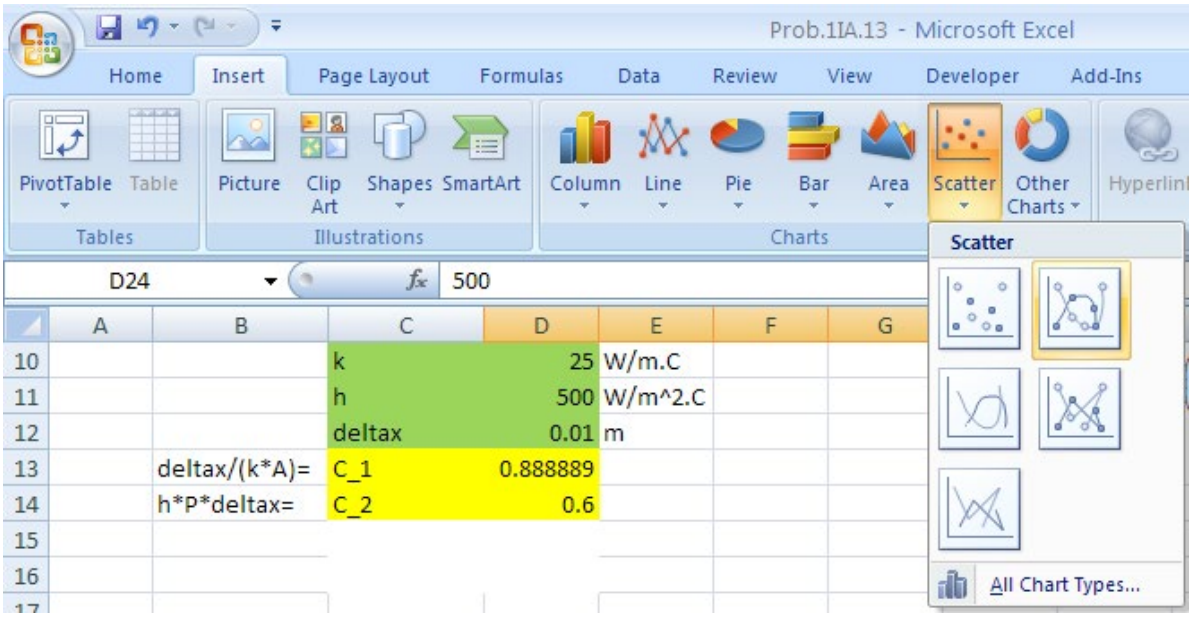
Note that in EXCEL, colour coding of the temp values is done by going to Conditional Formatting – color Scales and clicking on upper, second item, as shown below:



11. **Now, draw the graph T vs x:** This is very easy in EXCEL. Just select x and T values, (i.e. from cell D23 to I23 and D24 to I24), click on Insert – Scatter – 2nd item in top row, i.e. Scatter with smooth lines and Markers.



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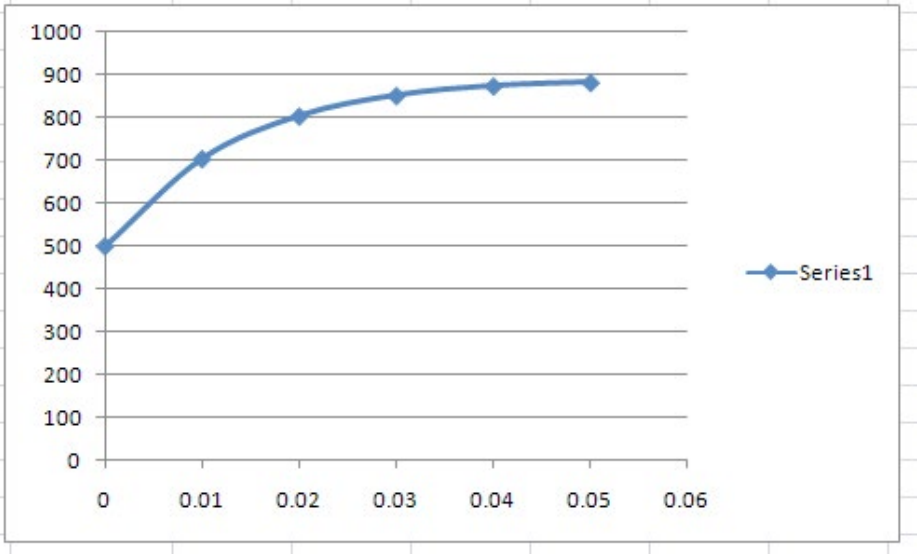
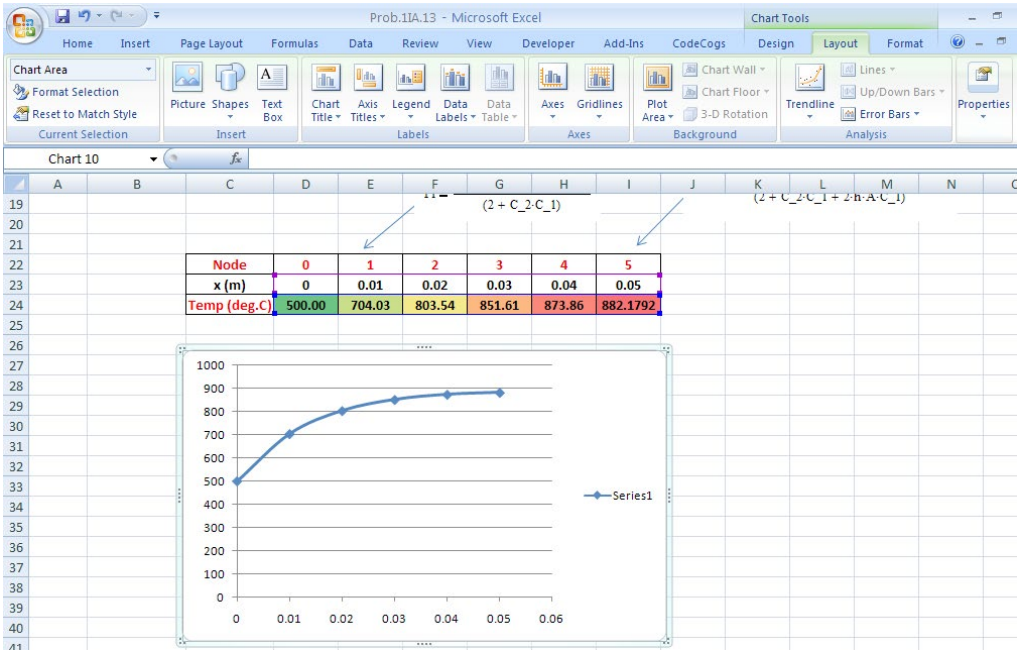
We get:

Node	0	1	2	3	4	5
x (m)	0	0.01	0.02	0.03	0.04	0.05
Temp (deg.C)	500.00	704.03	803.54	851.61	873.86	882.1792



12. Now, Format this plot. Select the plot, then click on Chart Tools – Layout:

We see the following screen:



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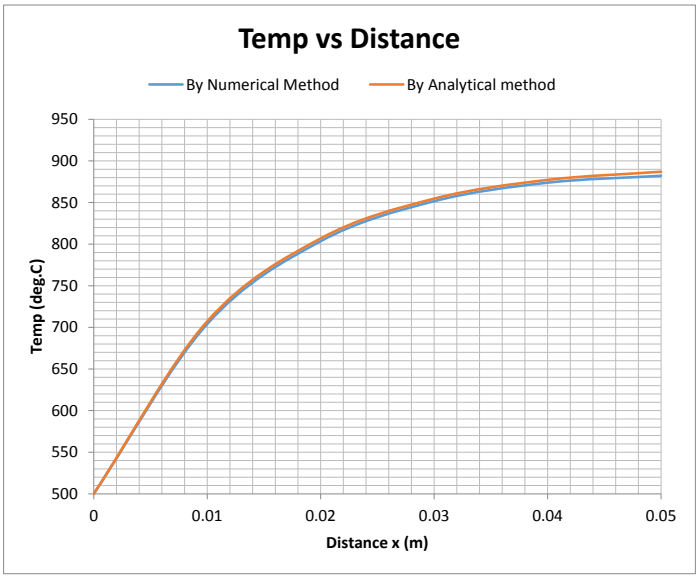
Now, you can format grid lines, legend, Axis labels, Chart label etc.

13. Finally, after formatting, we have the plot:

Node	0	1	2	3	4	5
x (m)	0	0.01	0.02	0.03	0.04	0.05
Temp (deg.C)	500.00	704.03	803.54	851.61	873.86	882.1792



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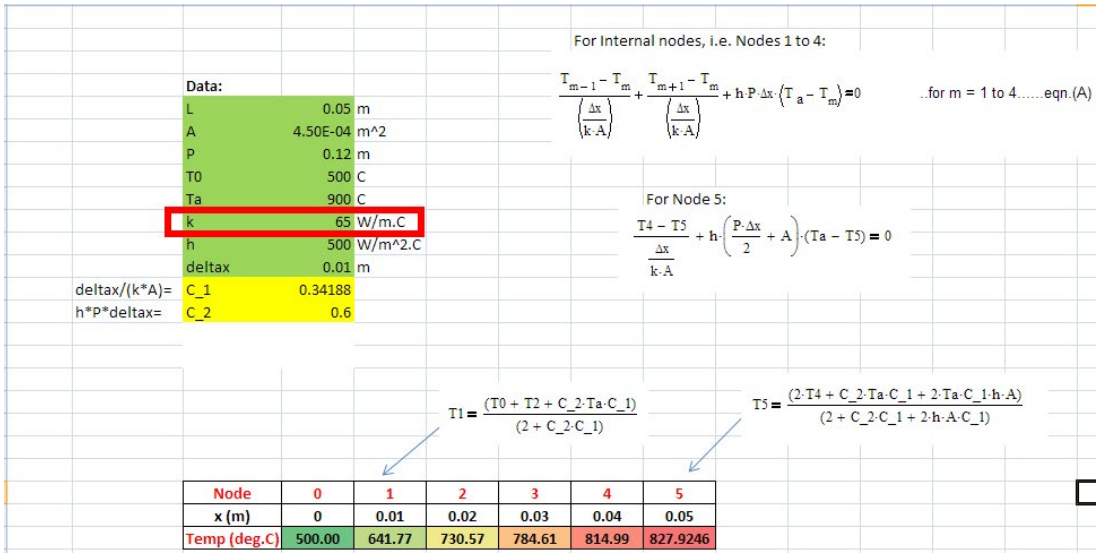
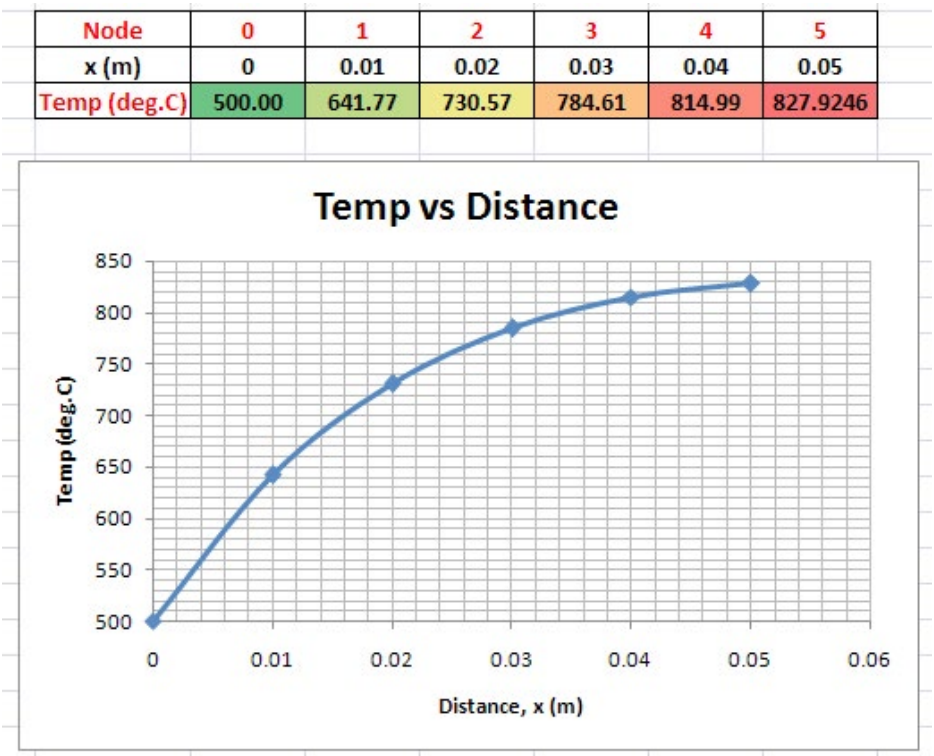
14. Now, if we have to change any of the Inputs, it is very easy and the resulting temp distribution presents itself immediately. For example, if we change the thermal conductivity value k = 65 W/m.C, we get:



Compare these values with those obtained with Mathcad, i.e.

T0 = 500 C, T1 = 641.77 C, T2 = 730.569 C, T3 = 784.613 C, T4 = 814.988 C and T5 = 827.925 C.

And, the plot also changes immediately to:



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To find Q_{fin} and η_{fin} when k = 25 W/m.C:

EXCEL calculation is reproduced below:

Node	0	1	2	3	4	5
x (m)	0	0.01	0.02	0.03	0.04	0.05
Temp (deg.C)	500.00	704.03	803.54	851.61	873.86	882.1792

Qfin=	-349.533	W
Qmax=	1290	W
η_fin=	0.270956	

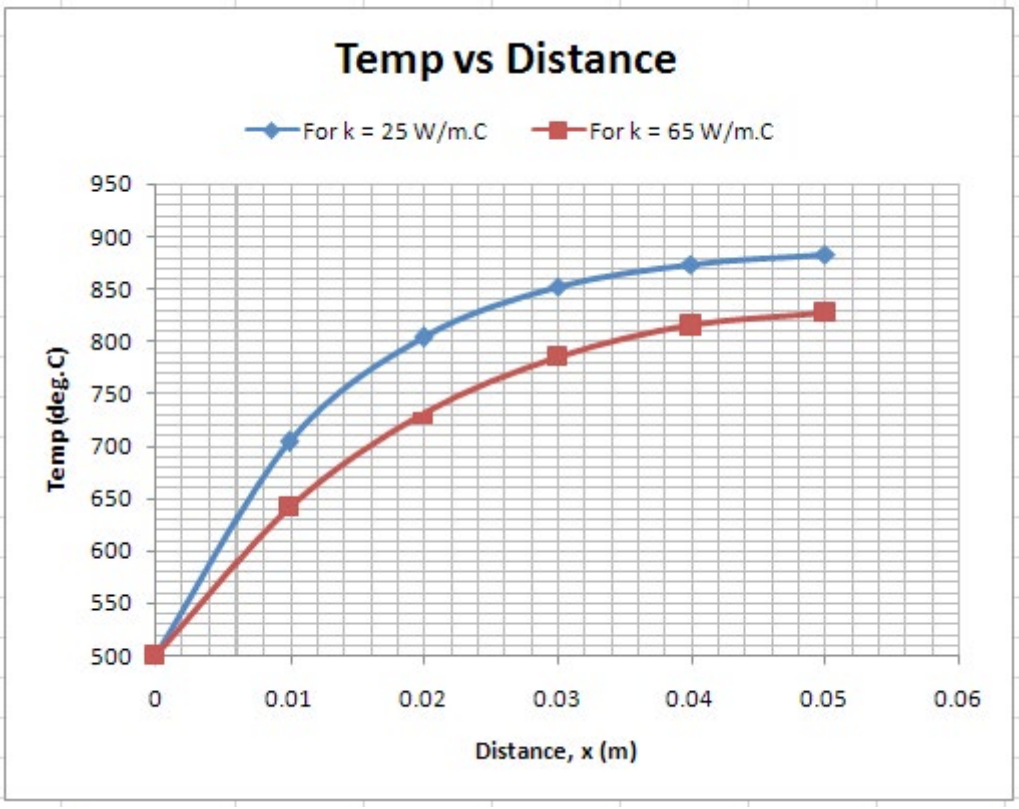
$$Q_{fin} = -\left[\frac{T1 - T0}{\frac{\Delta x}{k \cdot A}} + \frac{h \cdot P \cdot \Delta x}{2} \cdot (Ta - T0)\right]$$

$$Q_{max} = h \cdot (P \cdot L + A) \cdot (Ta - T0)$$

$$\eta_{fin} = \frac{Q_{fin}}{Q_{max}}$$

Finally, Temp distribution for k = 25 and k = 65 W/m.C are drawn in the same plot:

x (m)	0	0.01	0.02	0.03	0.04	0.05	k (W/m.C)
T(deg.C)	500	704.03	803.54	851.61	873.86	882.18	25
T(deg.C)	500	641.7696	730.5688	784.613	814.9879	827.9246	65



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15. It is of interest to compare the values of temps obtained by numerical method with those obtained by exact, analytical solution. For this case, analytical solution exists and temp at any x is given by:

$$\frac{\theta(x)}{\theta_o} = \frac{\cosh(m\cdot(L-x)) + \frac{h}{m\cdot k}\cdot\sinh(m\cdot(L-x))}{\cosh(m\cdot L) + \frac{h}{m\cdot k}\cdot\sinh(m\cdot L)}$$

where

$$\theta(x) = T(x) - T_a$$

$$\theta_o = T_o - T_a$$

So, in EXCEL, we set it up as:

Fin parameter	m	73.02967
h/(m.k)=	C_2	0.273861
theta_0=	theta_0	-400

E78 =(theta_0*(COSH(m*(L-E76))+C_2*SINH(m*(L-E76)))/(COSH(m*L)+C_2*SINH(m*L)))+Ta

	A	B	C	D	E	F	G	H	I	J	K	L
73												
74												
75												
76			x (m)	0	0.01	0.02	0.03	0.04	0.05			
77			Temp (deg.C)	500	704.03	803.54	851.61	873.86	882.18	By Numerical method		
78			Temp (deg.C)	500	707.19	806.89	854.68	877.21	887.03	By Exact analytical formulas		

Eqn for node 1 i.e. x = 0.01 m, i.e. cell 78 is shown in Formula bar.

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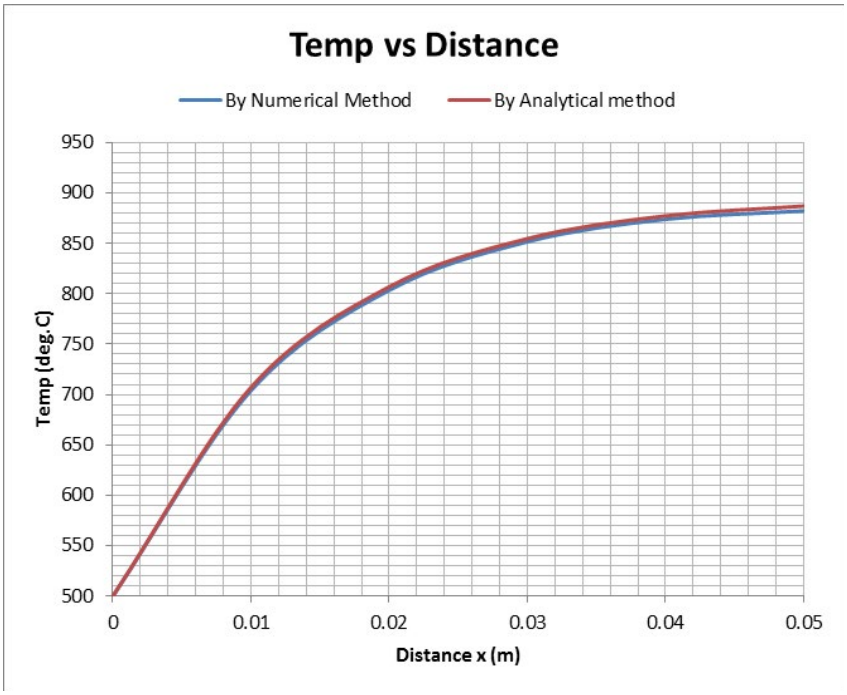
Now, draw the plot to compare the temp values:



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Prob.1IA.14. Consider a slab of thickness, L = 1 cm. Thermal conductivity of the slab material varies linearly with temperature as: k(T) = 26.679.(1 + 8.621 × 10^{-4} T), W/(m.C), where T is in deg. C. Surface at x = 0 is insulated and the other surface at x = L is subjected to a convection heat transfer with a fluid at 100 C with a heat transfer coeff. of 4000 W/(m^2.C).There is uniform internal heat generation in the slab at a rate of 8 × 10^7 W/m^3. Dividing the slab into 5 equally spaced sub-regions, find the temperatures at the different nodes. Assume one-dimensional, steady state conduction.

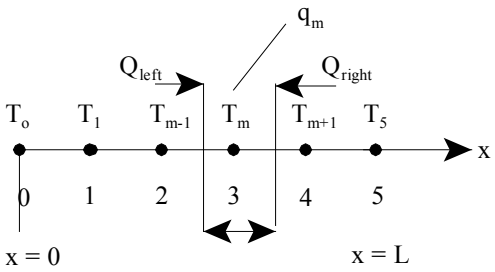

Fig.Prob.1IA.14

EXCEL Solution:

First, derive the difference equations for the Internal nodes and boundary nodes:

For Internal nodes:

Consider any internal node 'm' and apply the energy balance for the differential volume around node 'm'. Remember to consider that *all energy flows are into the control volume*. Using the thermal resistance concept, we get:

$$\frac{T_{m-1}-T_m}{\frac{\Delta x}{k_0\cdot\left[1+\beta\cdot\left(\frac{T_{m-1}+T_m}{2}\right)\right]\cdot A}}+\frac{T_{m+1}-T_m}{\frac{\Delta x}{k_0\cdot\left[1+\beta\cdot\left(\frac{T_{m+1}+T_m}{2}\right)\right]\cdot A}}+q_m\cdot(A\cdot\Delta x)=0$$

i.e.

$$\left(T_{m-1}-T_m\right)\cdot k_0\cdot\left[1+\beta\cdot\left(\frac{T_{m-1}+T_m}{2}\right)\right]+\left(T_{m+1}-T_m\right)\cdot k_0\cdot\left[1+\beta\cdot\left(\frac{T_{m+1}+T_m}{2}\right)\right]+q_m\cdot(\Delta x)^2=0$$

i.e.

$$\left(T_{m-1}-2\cdot T_m+T_{m+1}\right)+\frac{\beta}{2}\cdot\left[\left(T_{m-1}\right)^2-2\cdot\left(T_m\right)^2+\left(T_{m+1}\right)^2\right]+\frac{q_m\cdot(\Delta x)^2}{k_0}=0$$

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$$\text{i.e.} \quad \left(T_{m-1} - 2\cdot T_m + T_{m+1}\right) + \frac{\beta}{2}\cdot\left[\left(T_{m-1}\right)^2 - 2\cdot\left(T_m\right)^2 + \left(T_{m+1}\right)^2\right] + C\_1 = 0 \qquad \text{.....(A)}$$

$$\text{where} \quad q_m = q_g \quad \text{and,} \quad \frac{q_g\cdot(\Delta x)^2}{k_0} = C\_1 = 11.994$$

Eqn. (A) gives the difference eqn. for the interior nodes 1,2,3,and 4. It is seen that this eqn. is non-linear and solving the set of non-linear equations by conventional methods is difficult. But, as we shall presently see, in EXCEL, it is very easy to get solution using the **'Solver'.**

In eqn.(A), let us put m =1, 2, 3 and 4 to get the difference eqns. for the respective nodes:

Node 1: $$\left(T_0 - 2\cdot T_1 + T_2\right) + \frac{\beta}{2}\cdot\left[\left(T_0\right)^2 - 2\cdot\left(T_1\right)^2 + \left(T_2\right)^2\right] + 11.994 = 0 \qquad \text{.....(b)}$$

Node 2: $$\left(T_1 - 2\cdot T_2 + T_3\right) + \frac{\beta}{2}\cdot\left[\left(T_1\right)^2 - 2\cdot\left(T_2\right)^2 + \left(T_3\right)^2\right] + 11.994 = 0 \qquad \text{.....(c)}$$

Node 3: $$\left(T_2 - 2\cdot T_3 + T_4\right) + \frac{\beta}{2}\cdot\left[\left(T_2\right)^2 - 2\cdot\left(T_3\right)^2 + \left(T_4\right)^2\right] + 11.994 = 0 \qquad \text{.....(d)}$$

Node 4: $$\left(T_3 - 2\cdot T_4 + T_5\right) + \frac{\beta}{2}\cdot\left[\left(T_3\right)^2 - 2\cdot\left(T_4\right)^2 + \left(T_5\right)^2\right] + 11.994 = 0 \qquad \text{.....(e)}$$

In EXCEL, we have to enter the eqn for only Node 1. Then, we can drag-copy it to Nodes 2, 3 and 4.

Difference eqns. for boundary nodes:

For node '0': Apply the energy balance to the half-volume around the node '0'; *all heat lines flowing into the volume.*

There is no heat flowing from the left side of the control volume into node '0' since the surface is insulated. Writing other terms, we get:

$$\frac{T_1 - T_0}{\Delta x} + q_g\cdot\left(A\cdot\frac{\Delta x}{2}\right) = 0$$
$$\overline{\left[k_0\cdot\left[1+\beta\cdot\left(\frac{T_0+T_1}{2}\right)\right]\cdot A\right]}$$

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i.e. $$\left(T_1 - T_0\right)\cdot k_0\cdot\left[1+\beta\cdot\left(\frac{T_0+T_1}{2}\right)\right]+\frac{q_g\cdot(\Delta x)^2}{2}=0$$

i.e. $$\left(T_1 - T_0\right)+\frac{\beta}{2}\cdot\left(T_1^{\,2}-T_0^{\,2}\right)+\frac{q_g\cdot(\Delta x)^2}{2\cdot k_0}=0$$

i.e. $$\left(T_1 - T_0\right)+\frac{\beta}{2}\cdot\left(T_1^{\,2}-T_0^{\,2}\right)+\frac{C\_1}{2}=0 \quad \text{....(a)}$$

Eqn.(a) is the difference eqn. for node '0'. This eqn. is also a non-linear eqn.

For node 5: Apply the energy balance to the half-volume around the node 5; all heat lines flowing into the volume. There is convection condition on the right surface. Writing the energy balance, we get:

$$\frac{T_4 - T_5}{\frac{\Delta x}{\left[k_0\cdot\left[1+\beta\cdot\left(\frac{T_4+T_5}{2}\right)\right]\cdot A\right]}}+h\cdot A\cdot\left(T_a - T_5\right)+q_g\cdot\left(A\cdot\frac{\Delta x}{2}\right)=0$$

i.e. $$\left(T_4 - T_5\right)+\frac{\beta}{2}\cdot\left(T_4^{\,2}-T_5^{\,2}\right)+\frac{h\cdot\left(T_a - T_5\right)\cdot\Delta x}{k_0}+\frac{q_g\cdot(\Delta x)^2}{2\cdot k_0}=0$$

i.e. $$\left(T_4 - T_5\right)+\frac{\beta}{2}\cdot\left(T_4^{\,2}-T_5^{\,2}\right)+C\_2\cdot\left(Ta - T_5\right)+\frac{C\_1}{2}=0 \quad \text{.....(f)}$$

where $$\frac{h\cdot\Delta x}{k_0}=C\_2=0.3$$

Eqn.(f) is the difference eqn. for node 5. This eqn. is also non-linear.

Now, we have got 6 equations viz. eqn. (a), (b)…(f) and there are 6 unknown node temperatures. So, solving these 6 coupled equations simultaneously, we get the temperatures T0, T1 … T6.

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The steps involved to solve such non-linear equations in EXCEL are explained below:

1. Enter the data, and name the cells:

	A	B	C	D	E	F
1						
2						
3						
4			**Data:**			
5			L	0.01	m	
6			A	1.00E+00	m^2.. Assumed	
7			Ta	100	C	
8			k_0	26.679	W/m.C	
9			beta	8.62E-04	1/C	
10		Th. conductivity:	k = k0 * (1 + beta*T)			
11			h	4000	W/m^2.C	
12			qg	8.00E+07	W/m^3	
13			deltax	0.002	m	
14		qg*deltax^2/k_0=	C_1	11.99445		
15		h*deltax/k_0=	C_2	0.299861		
16						

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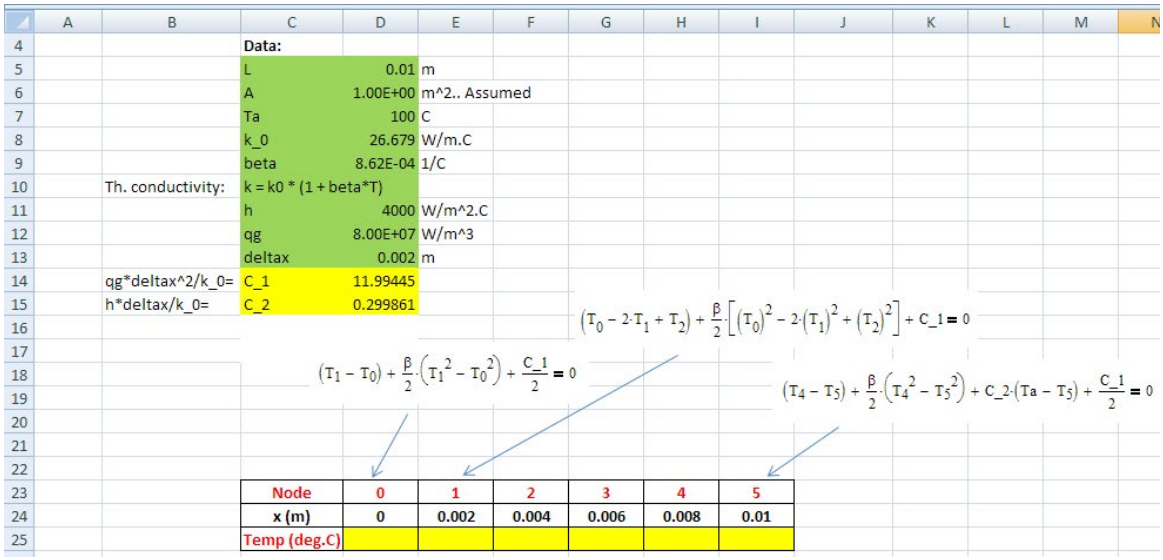
Note that we have also calculated [qg * deltax^2 / k_0] and [h * deltax / k_0] separately and named them as C_1 and C_2 respectively. They are required for use in the difference eqns.

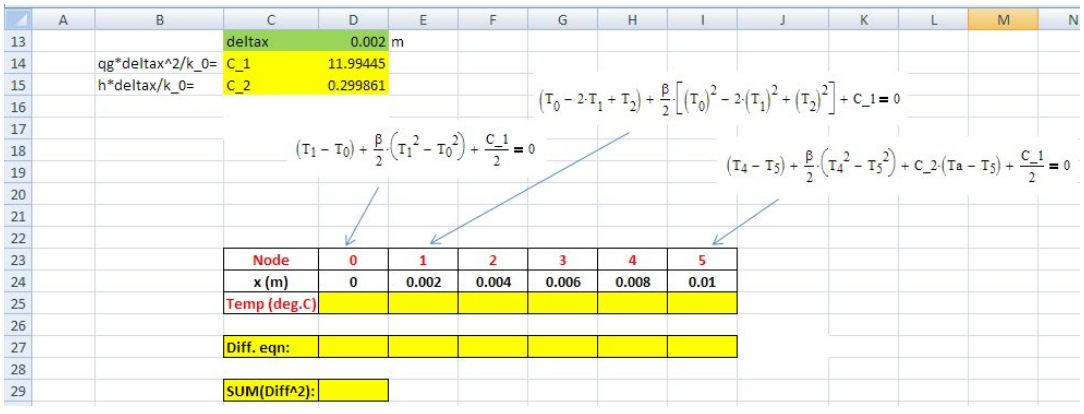
2. Set up the scheme for calculations as shown below. Note that the difference eqns to be entered are also shown in the worksheet:



3. Now, since we don't have explicit eqns for T0, T1…..T4, and T5, we use the SOLVER in EXCEL.

First, *enable solver*, as explained earlier in Introduction on EXCEL.

4. Now, create a parallel row below Temp row for 'Difference eqns' and SUM(Diff^2) as shown:



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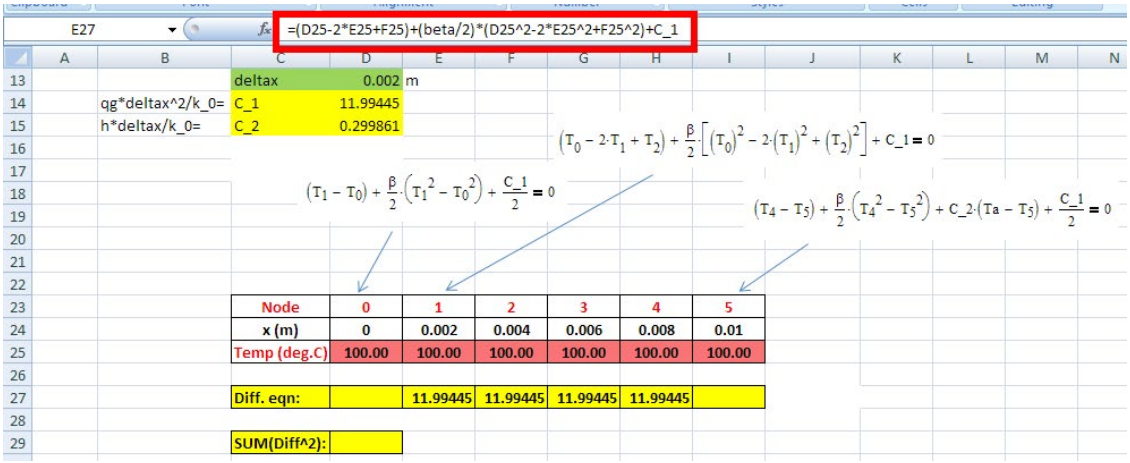
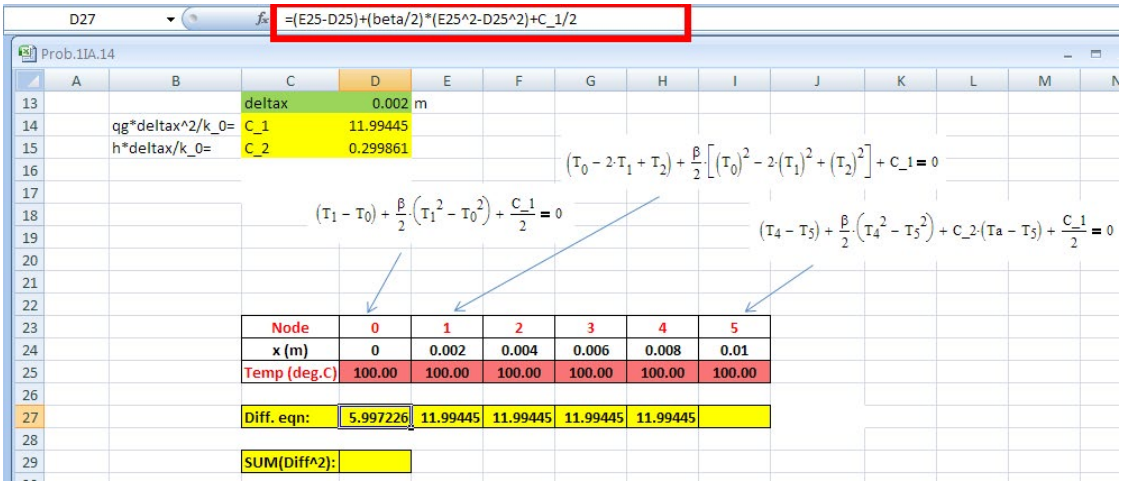
5. In the Temp row (i.e. row 25), fill in trial values for all temps, say 100 C; This is necessary while using Solver. In the Diff. eqn row (i.e. row 27), enter the Difference eqn for Node 1 under the Node 1 i.e. in cell E27. Then, drag copy it upto Node 4, i.e. upto cell H27.



Note that Difference eqn for Node 1 is entered in cell E27, and it is shown in the Formula bar.

6. Likewise, enter the Difference eqn for Node 0 in cell D27:



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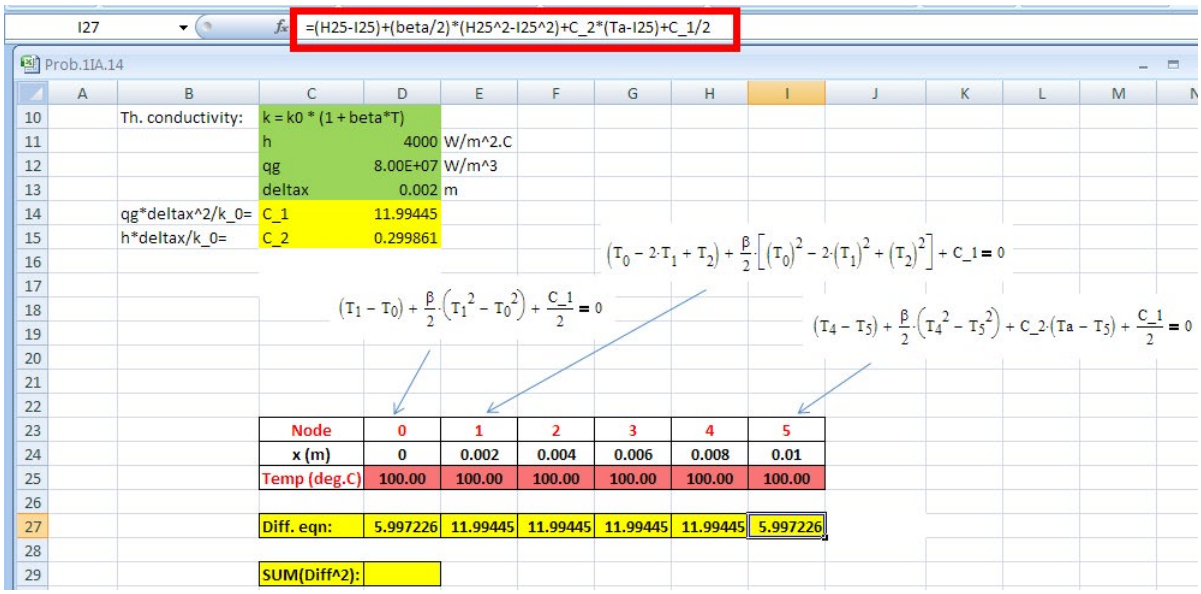
7. And, Difference eqn for Node 5 in cell I27:



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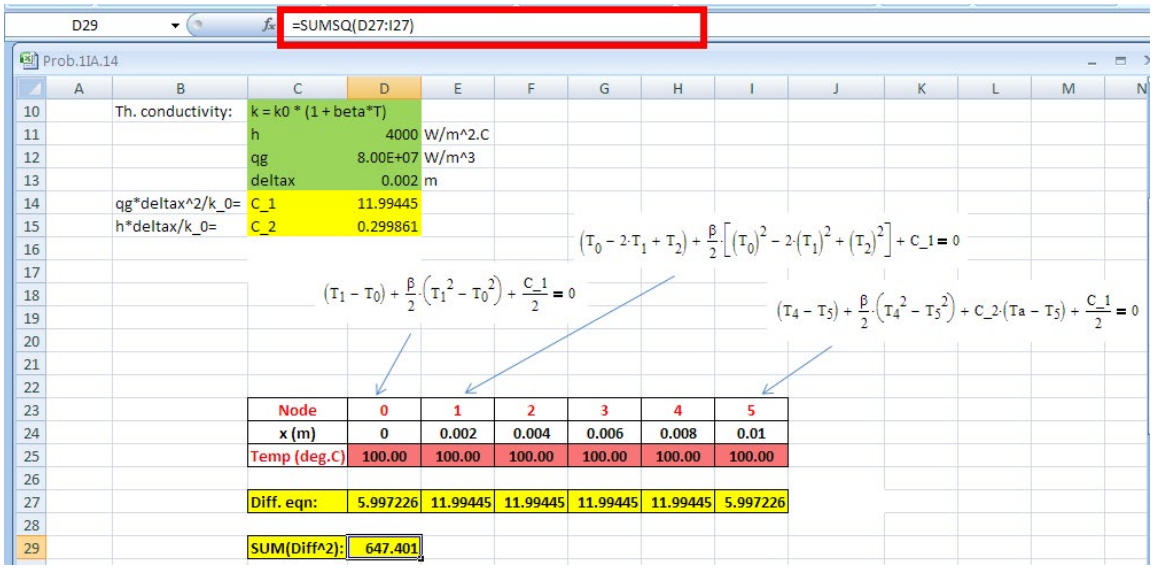
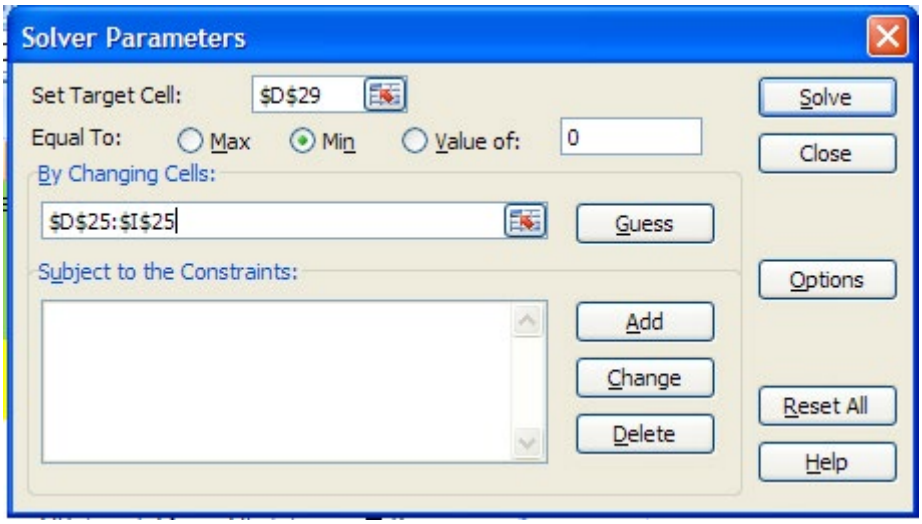
8. Next, important step: As we know, difference eqns should be individually equal to zero for a final solution. We have used trial (guess) values for t0, T1…..T5. Now, the technique to get the correct values of T0, T1,…T5 is to minimize the Sum of squares of cells D27 to I27, by changing the temp values in cells D25 to I25. So, in cell D29, enter the Excel Function SUMSQ (D27:I27) to get the sum of the square:



9. Now, use the Solver. To do this, go to Data tab, and click Solver. We get following screen.

Fill in D29 for **Set Target cell** and choose Min in the space for **Equal to:** and in the place for **By changing cells: fill in D25:I25.** This means that we wish to minimize the value in Target cell (i.e. D29) by changing the values of temps in the range D25:I25. Note that we do not put the Target value to zero since getting an exact value of zero by the numerical method may not be possible. However, minimizing that target value will give acceptable results.



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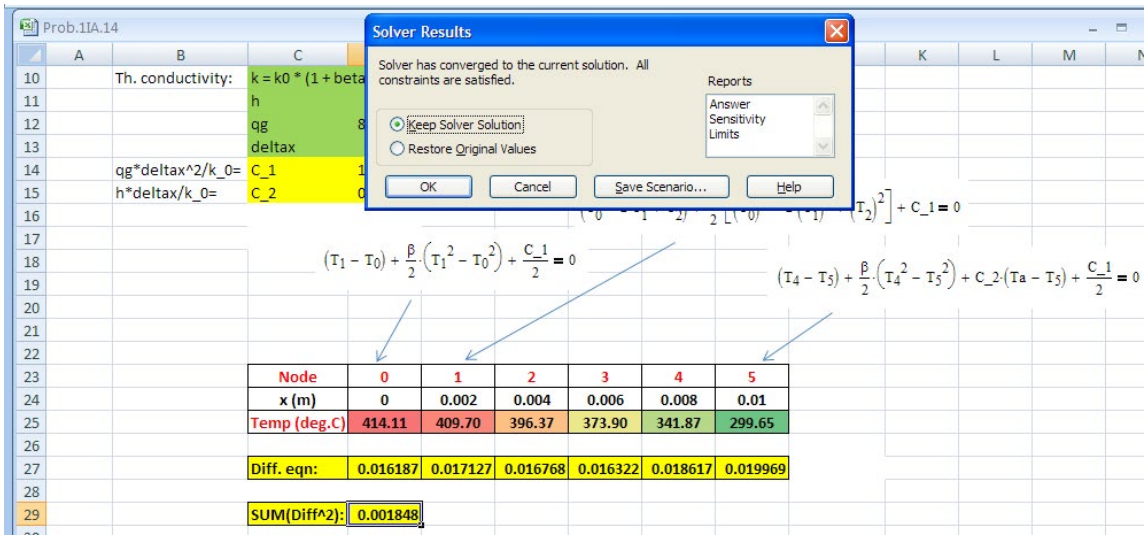
Now, press Solve. We get:



Note that Temp values T0 to T5 have appeared. Also, note that Target cell D29 is minimized to 0.001848 (not exactly equal to zero).

Press Keep Solver Solution.

Once again use the Solver by repeating Step 9. i.e. now, the Temp values obtained above become trial values and we see the final result:

Node	0	1	2	3	4	5
x (m)	0	0.002	0.004	0.006	0.008	0.01
Temp (deg.C)	414.10	409.69	396.36	373.89	341.87	299.66
Diff. eqn:	0.018124	0.018802	0.01899	0.017904	0.016057	0.013075
SUM(Diff^2):	0.001792					

Note that the temps have changed very slightly. And, the Target cell is minimized to 0.001792.

We accept these values. **So, the temps are:**

T0 = 414.10 C, T1 = 409.69 C, T2 = 396.36 C, T3 = 373.89 C, T4 = 341.87 C and T5 = 299.66 C.

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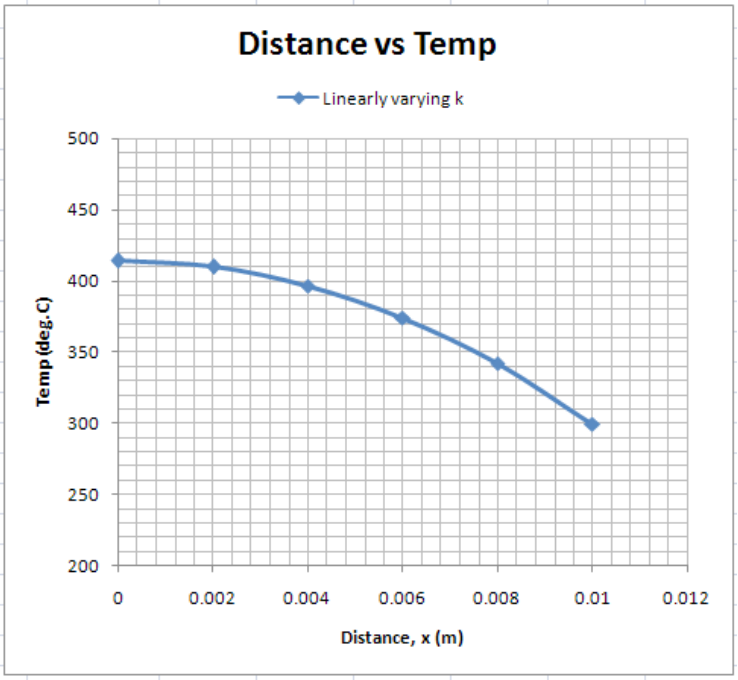
10. **Now, draw the plot of x vs T:**

The plot, suitably formatted, as explained earlier, is shown below:



11. **Now, what happens if the k value is constant, instead of linearly varying, say k = 26.679 W/m.C?**

i.e. beta = 0. Change the value of beta to zero:

	Data:		
	L	0.01	m
	A	1.00E+00	m^2.. Assumed
	Ta	100	C
	k_0	26.679	W/m.C
	beta	0.00E+00	1/C
Th. conductivity:	k = k0 * (1 + beta*T)		
	h	4000	W/m^2.C
	qg	8.00E+07	W/m^3
	deltax	0.002	m
qg*deltax^2/k_0=	C_1	11.99445	
h*deltax/k_0=	C_2	0.299861	

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And run the Solver again. We get:

Node	0	1	2	3	4	5
x (m)	0	0.002	0.004	0.006	0.008	0.01
Temp (deg.C)	448.97	443.01	425.08	395.18	353.32	299.48
Diff. eqn:	0.030982	0.030495	0.029036	0.02613	0.0218	0.01691
SUM(Diff^2):	0.004177					

12. **Now, plot the temp values for both the cases:**

x (m)	0	0.002	0.004	0.006	0.008	0.01	k (W/m.C)
T(deg.C)	414.10	409.69	396.36	373.89	341.87	299.66	linearly varying
T(deg.C)	448.97	443.01	425.08	395.18	353.32	299.48	26.679

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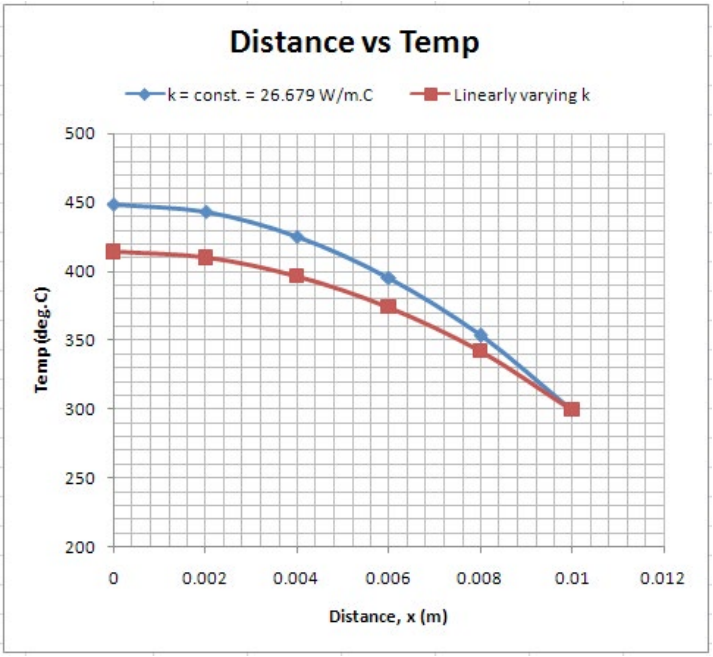
And, draw the graph:



13. **Now, with the temp linearly varying, if the heat generation rate is halved, what happens to temp distribution?**

i.e. now, qg = 4E07 W/m^3. Change qg in the original Worksheet, and run Solver again. We get:

	Data:		
	L	0.01	m
	A	1.00E+00	m^2.. Assumed
	Ta	100	C
	k_0	26.679	W/m.C
	beta	8.62E-04	1/C
Th. conductivity:	k = k0 * (1 + beta*T)		
	h	4000	W/m^2.C
	qg	4.00E+07	W/m^3
	deltax	0.002	m
qg*deltax^2/k_0=	C_1	5.997226	
h*deltax/k_0=	C_2	0.299861	

Node	0	1	2	3	4	5
x (m)	0	0.002	0.004	0.006	0.008	0.01
Temp (deg.C)	262.48	260.03	252.67	240.30	222.81	199.99
Diff. eqn:	0.00102	0.000886	0.000759	0.000705	0.000626	0.000465
SUM(Diff^2):	3.51E-06					

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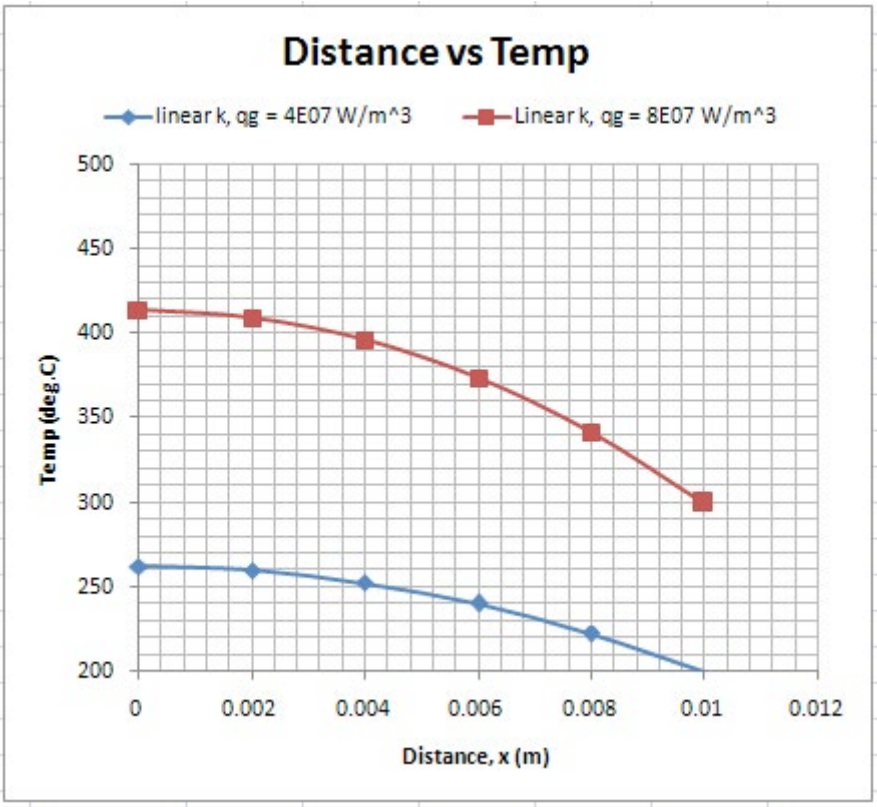
And, produce the plot for comparison:

x (m)	0	0.002	0.004	0.006	0.008	0.01	k (W/m.C)
T(deg.C)	414.10	409.69	396.36	373.89	341.87	299.66	linearly varying
T(deg.C)	262.48	260.03	252.67	240.30	222.81	199.99	linear k, qg halved



Prob.1IA.15. Consider a brick wall (k = 0.72 W/m.K), 0.2 m thick, exposed to a heat flux of 2000 W/m^2 on the RHS. On the LHS, there is convection with room air at 20 C, with a heat transfer coeff of: h = 10 W/m^2.C. Dividing the wall into 4 equally spaced sub-regions, find the temp distribution.

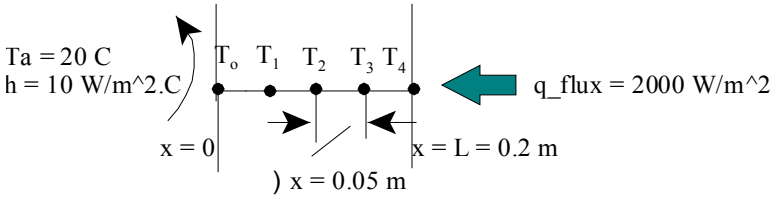


Fig.Prob.1IA.15

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EXCEL Solution:

First, derive the difference equations for the Internal nodes and boundary nodes:

For Internal nodes:

Consider node '1, for example, and apply the energy balance for the differential volume around node 1. Remember to consider that *all energy flows are into the control volume*. We get:

$$k \cdot A \cdot \frac{T0 - T1}{\Delta x} + k \cdot A \cdot \frac{(T2 - T1)}{\Delta x} = 0$$

i.e. $$T0 + T2 - 2 \cdot T1 = 0$$

We get similar eqns for Nodes 2 and 3.

For Boundary Node '0':

Again, apply the energy balance for the differential volume around node 0. Remember to consider that *all energy flows are into the control volume*. We get:

$$\frac{k \cdot A}{\Delta x} \cdot (T\_1 - T\_0) + h \cdot A \cdot (Ta - T\_0) = 0$$

i.e. $$C\_2 \cdot (T\_1 - T\_0) + h \cdot A \cdot (Ta - T\_0) = 0 \quad \text{where} \quad C\_2 = \frac{k \cdot A}{\Delta x}$$

i.e. $$T\_0 = \frac{(C\_2 \cdot T\_1 + h \cdot A \cdot Ta)}{(C\_2 + h \cdot A)}$$

For Boundary Node '4':

Again, apply the energy balance for the differential volume around node 0. Remember to consider that *all energy flows are into the control volume*. We get:

$$\frac{k \cdot A}{\Delta x} \cdot (T\_3 - T\_4) + q\_flux \cdot A = 0$$

i.e. $$T\_4 = \frac{(C\_2 \cdot T\_3 + q\_flux \cdot A)}{C\_2} \quad \text{where} \quad C\_2 = \frac{k \cdot A}{\Delta x}$$

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The steps involved to solve this problem in EXCEL are explained below:

1. Enter the data, and name the cells:

deltax | fx | 0.05

	A	B	C	D	E	F
4			Data:			
5			L	0.2	m	
6			A	1.00E+00	m^2.. Assumed	
7		Environment temp.	Ta	20	C	
8		Radiation Source temp.	q_flux	2000	W/m^2	
9		conv. heat tr coeff	h	10	W/m^2.C	
10		Th. conductivity	k	7.20E-01	W/m.K	
11			T_0	6.50E+01	C	
12			deltax	0.05	m	
13		k*A/deltax=	C_2	14.4		

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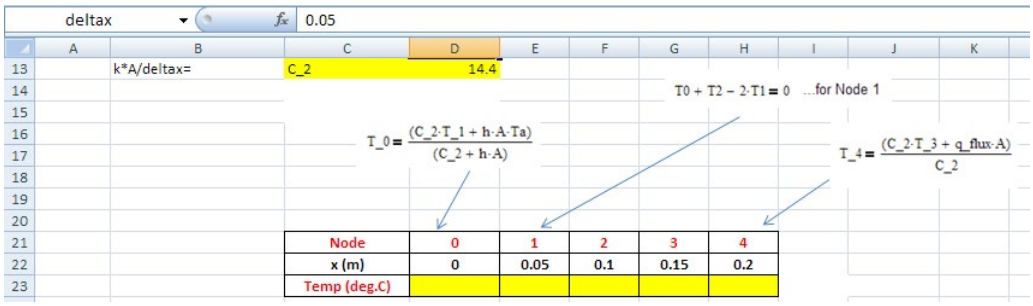
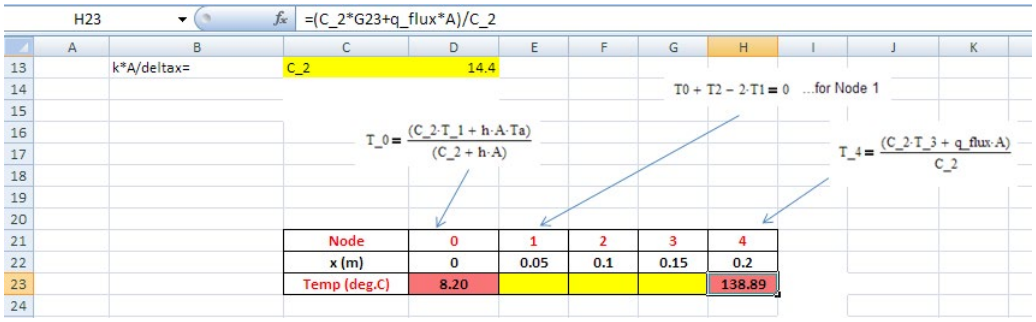
2. Set up the scheme for calculations as shown below. Note that the eqns to be entered are also shown in the worksheet:

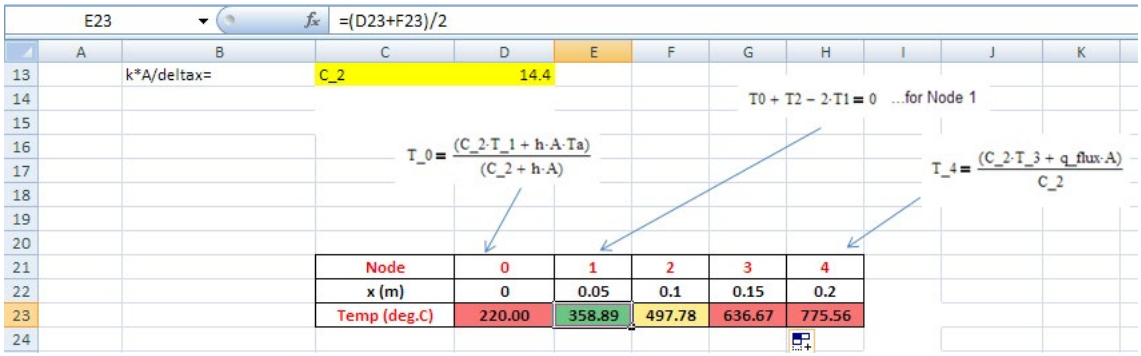


3. In the Temp row (i.e. Row 23) fill in the eqn for T_0 in cell D23, and the eqn for T_4 in cell H23. Eqn in cell H23 can be seen in the formula bar in the screenshot shown below:



Some values are seen in Temp row in cells D23 and H23, but do not worry about them, since they will change automatically when we enter formulas for other cells.

4. Now, enter the eqn for T_1 in cell E23. Since eqns for T_2 and T_3 are also similar o thie eqn for T_1, drag-copy the eqn in cell 23 to E24 and E25:



Eqn. for T_1 (i.e. cell E23) is seen in the formula bar.

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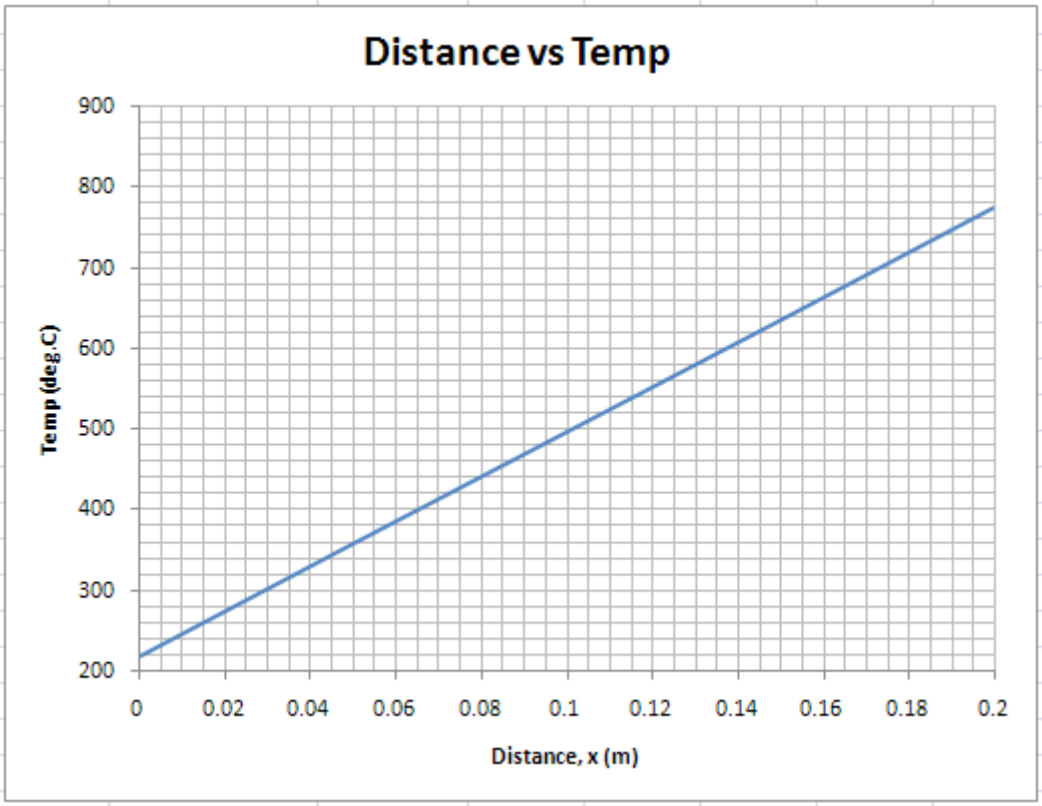
Observe that immediately all the temperatures have updated themselves.

5. Now, draw the graph of x vs T:



Prob.1IA.16. Consider a plane wall of thickness L = 0.4 m, area, A = 20 m^2, thermal cond. k = 2.3 W/m.C. The LHs is maintained at 80 C and the RHS loses heat by convection to surroundings at 15 C with heat transfer coeff. h = 24 W/m^2.C. Considering 4 equally spaced nodes, determine the nodal temperatures for one dimensional steady state conduction. Also, determine the heat transfer through the wall. [Ref. 2]

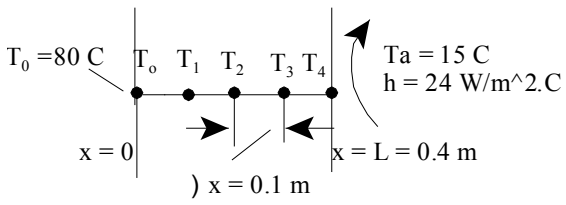


Fig.Prob.1IA.16

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EXCEL Solution:

Here T0 = 80 C, by data.

T1, T2 and T3 are Internal nodes.

On node 4, there is convection.

Get the difference eqns for node on RHS and Internal nodes, by writing the energy balance at the node concerned, taking care to see that all heat lines are considered as flowing into the node:

For Interior nodes:

$$\left(T_{m-1} - 2 \cdot T_m + T_{m+1}\right) + \frac{q_m \cdot (\Delta x)^2}{k} = 0 \qquad \text{.....(8.8)}$$

Here, internal energy generation, $q_m = 0$

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For Node '0':

T0 = 80 C, by data

$$2 \cdot T_{M-1} - 2 \cdot T_M \cdot \left(1 + \frac{h \cdot \Delta x}{k}\right) + \frac{(\Delta x)^2 \cdot q_M}{k} + \frac{2 \cdot h \cdot \Delta x}{k} \cdot T_a = 0 \quad \ldots(8.22)$$

Here, M = 4, $q_m = 0$

Following are the steps in EXCEL calculation: Here, M = 4, qm = qg = 0

1. First, ensure that iteration in EXCEL is enabled. Now, enter the data in EXCEL worksheet and name the cells, and set up the scheme:

	A	B	C	D	E	F	G	H	I	J
17			Data:	L=	0.4	m				
18				k=	2.3	W/m.C				
19				qg	0	W/m^3				
20				Ta=	15	deg.C				
21				h=	24	W/m2.C				
22				deltax=	0.1	m				
23				C_1	1.043478					
24				A=	20	m^2				
25										
26				Node	0	1	2	3	4	
27				x	0	0.1	0.2	0.3	0.4	
28				Temp (deg.C)	80.00					
29										
30				Qconv=						
31										

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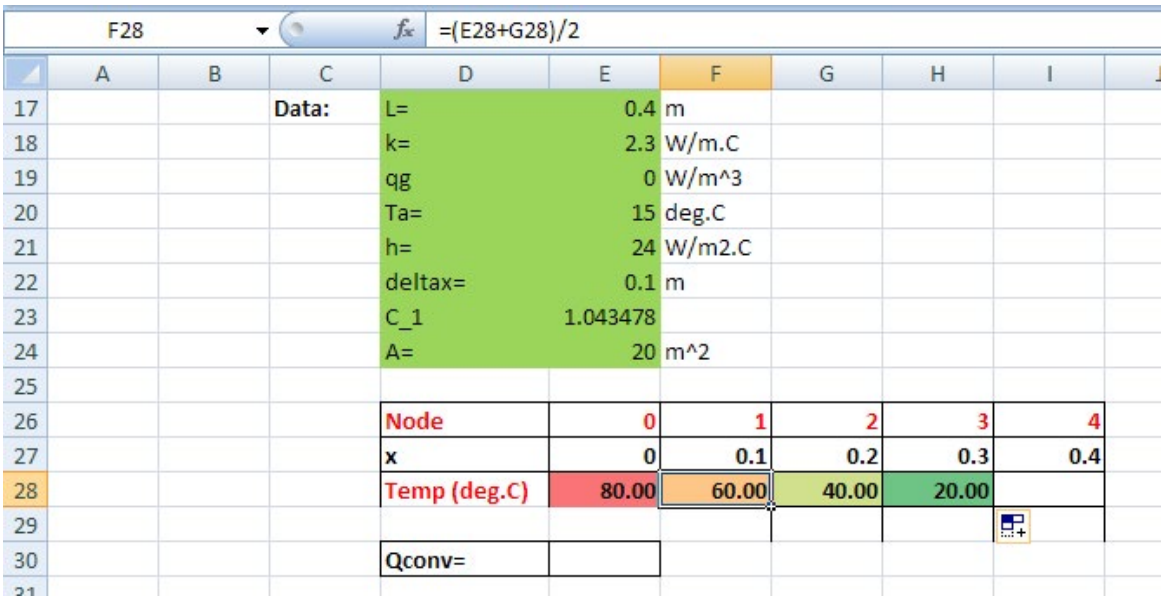
2. Enter the eqn for T1, then drag-copy it for nodes 2 and 3:

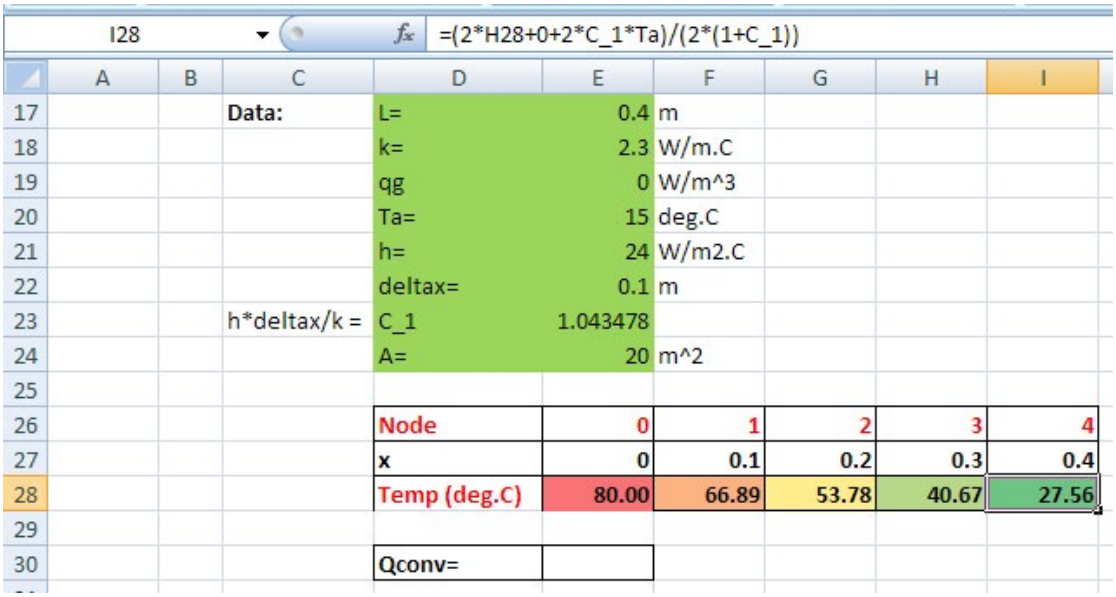


F28 =(E28+G28)/2

	A	B	C	D	E	F	G	H	I
17			Data:	L=	0.4	m			
18				k=	2.3	W/m.C			
19				qg	0	W/m^3			
20				Ta=	15	deg.C			
21				h=	24	W/m2.C			
22				deltax=	0.1	m			
23				C_1	1.043478				
24				A=	20	m^2			
25									
26				Node	0	1	2	3	4
27				x	0	0.1	0.2	0.3	0.4
28				Temp (deg.C)	80.00	60.00	40.00	20.00	
29									
30				Qconv=					

Note the eqn for T1 in the Formula bar.

3. Now, enter the eqn fot T4:



I28 =(2*H28+0+2*C_1*Ta)/(2*(1+C_1))

	A	B	C	D	E	F	G	H	I
17			Data:	L=	0.4	m			
18				k=	2.3	W/m.C			
19				qg	0	W/m^3			
20				Ta=	15	deg.C			
21				h=	24	W/m2.C			
22				deltax=	0.1	m			
23			h*deltax/k =	C_1	1.043478				
24				A=	20	m^2			
25									
26				Node	0	1	2	3	4
27				x	0	0.1	0.2	0.3	0.4
28				Temp (deg.C)	80.00	66.89	53.78	40.67	27.56
29									
30				Qconv=					

Note the eqn for T4 in the Formula bar.

Note that temperatures at different nodes are automatically calculated by iteration, since iteration is enabled.

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4. Also calculate heat transfer by convection from RHS:

$Q_{conv} = h * A * (T4-Ta)$:

E30 =h*A*(I28-Ta)

	A	B	C	D	E	F	G	H	I
26				Node	0	1	2	3	4
27				x	0	0.1	0.2	0.3	0.4
28				Temp (deg.C)	80.00	66.89	53.78	40.67	27.56
29									
30				Qconv=	6030.25	W			

We note that Q_{conv} = 6030.25 W.

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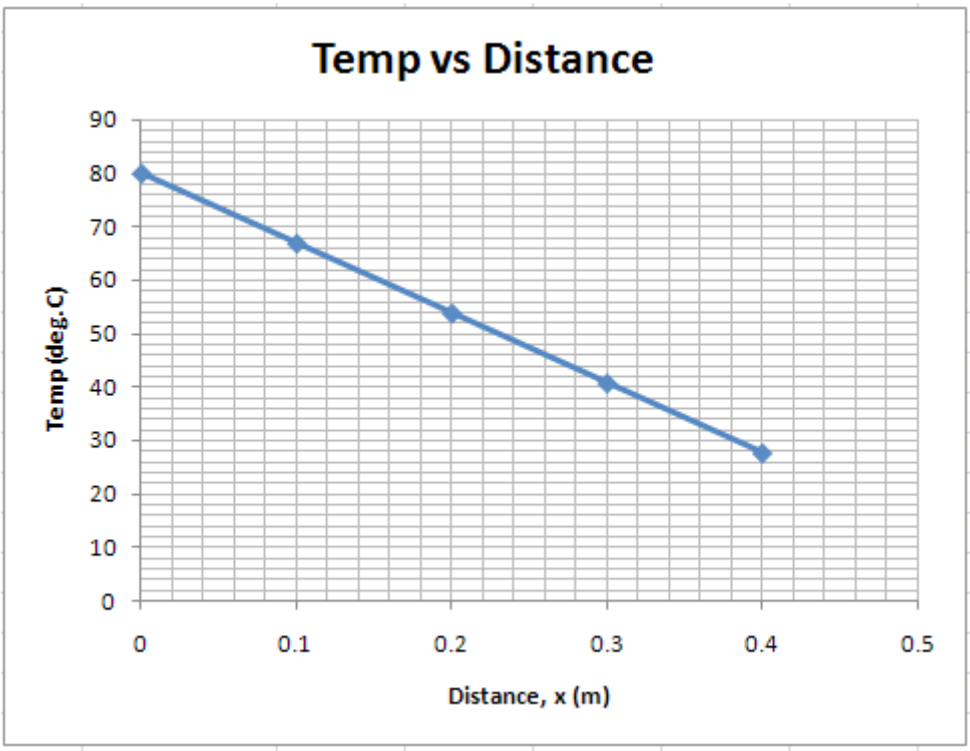
5. **Now, draw the plot of T vs x** and format it as required:



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1IB. Two-dimensional, steady state conduction:

Prob. 1I.B.1. Determine the steady state temperatures at the 6 nodal points in the two dimensional bar shown in the fig. Use numerical method, with the grids as shown, $\Delta x = \Delta y$.

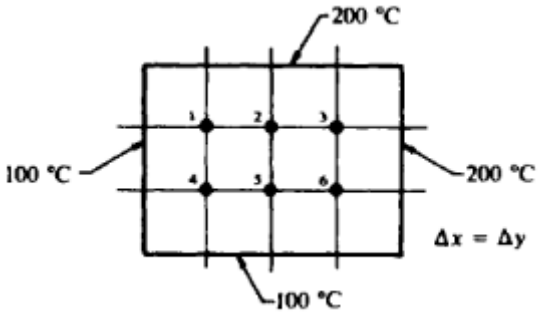

Fig.Prob.1I.B.1

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EES Solution:

"Data:"

T_west = 100 [C]

T_north = 200 [C]

T_east = 200 [C]

T_south = 100 [C]

"Calculations:"

"Note that all the six nodes viz. nodes 1 … 6 are interior nodes.

For interior nodes, in two dimensional steady state conduction, for DELTAx = DELTAy, with internal heat generation rate of q_g (W/m^3), we have:

For any general node (m,n):

T[m-1,n] + T[m,n+1] + T[m+1,n] + T[m,n-1] – 4*T[m,n] + (DELTAx^2/k) * q_g = 0"

"In the present case, q_g = 0:

So, we get the six nodal equations as follows:"

"For Node 1:" T_west + T_north + T2 + T4 – 4 * T1 = 0

"For Node 2:" T1 + T_north + T3 + T5 – 4 * T2= 0

"For Node 3:" T2 + T_north + T_east + T6 – 4 * T3= 0

"For Node 4:" T_west + T1 + T5 + T_south – 4 * T4 = 0

"For Node 5:" T4 + T2 + T6 + T_south – 4 * T5= 0

"For Node 6:" T5 + T3 + T_east + T_south – 4 * T6 = 0

"Solve the above six equations simultaneously, to get the six nodal temperatures, T1….T6:"

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Results:

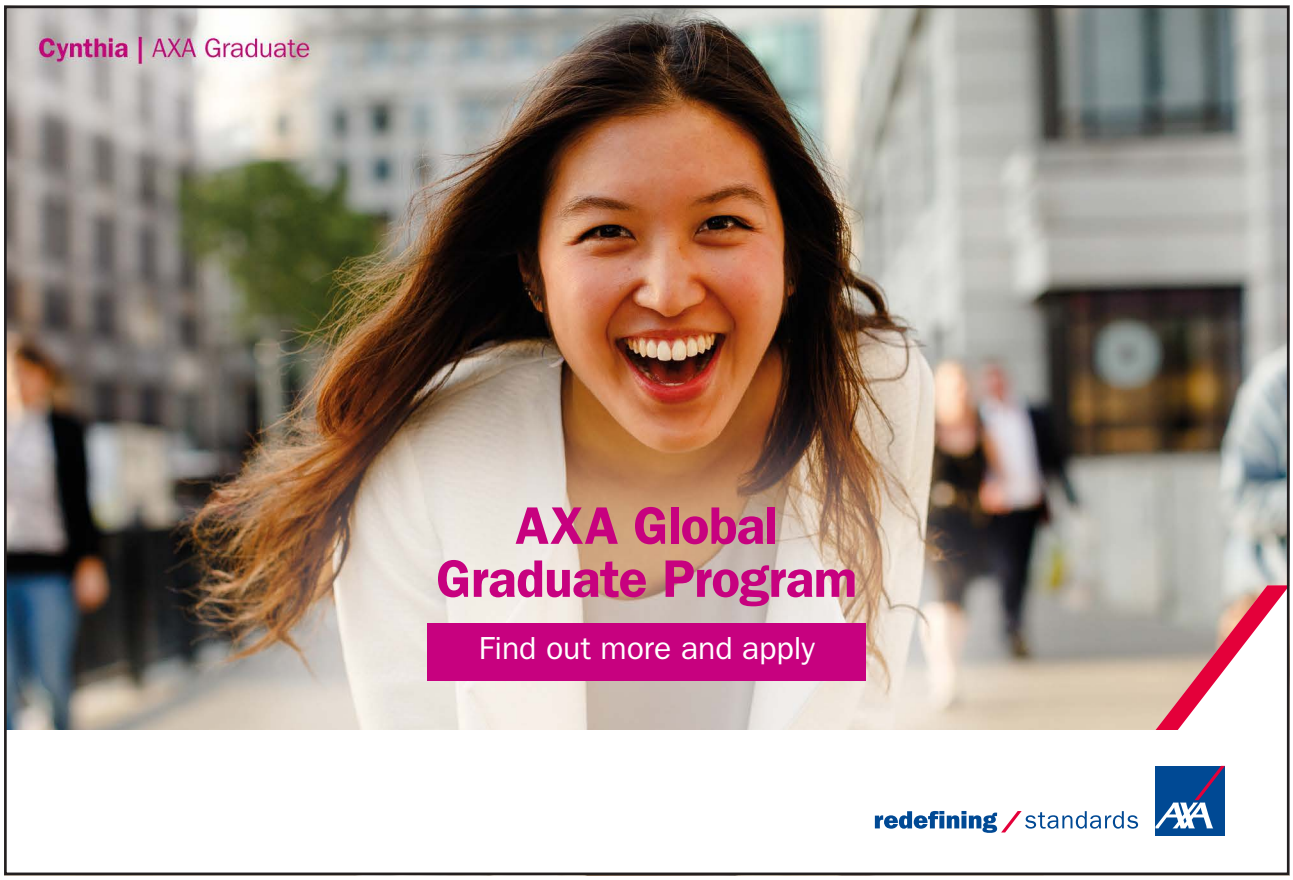
Unit Settings: SI C kPa kJ mass deg

T1 = 146.4 [C]	T2 = 165.2 [C]	T3 = 179.7 [C]	T4 = 120.3 [C]
T5 = 134.8 [C]	T6 = 153.6 [C]	T_{east} = 200 [C]	T_{north} = 200 [C]
T_{south} = 100 [C]	T_{west} = 100 [C]		

All the six nodal temperatures are given above … Ans.

==========

Prob. 1I.B.2. Considering the fig. given below, calculate the temperatures at the four discrete points and the heat flow from the 500 C and 100 C faces. Take k = 1 W/m.K. Use numerical method, with the grids as shown, $\Delta x = \Delta y$.



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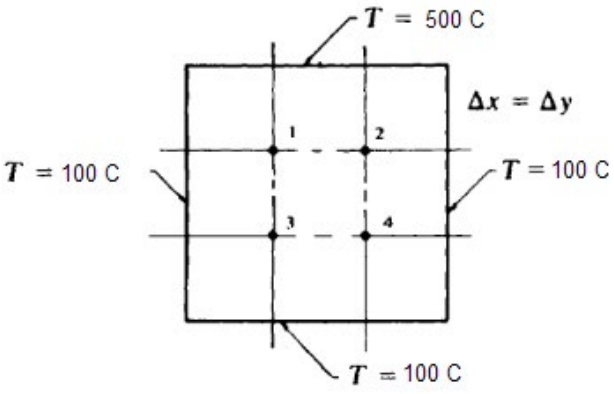

Fig.Prob.1I.B.2

EES Solution:

"Data:"

T_west = 100 [C]

T_north = 500 [C]

T_east = 100 [C]

T_south = 100 [C]

k= 1 [W/m-C]

"Calculations:"

"Note that all the 4 nodes viz. nodes 1 … 4 are interior nodes.

For interior nodes, in two dimensional steady state conduction, for DELTAx = DELTAy, with internal heat generation rate of q_g W/m^3,

we have:

For any general interior node (m,n):

T[m-1,n] + T[m,n+1] + T[m+1,n] + T[m,n-1] – 4 * T[m,n] + (DELTAx^2/k) * q_g = 0 "

"In the present case, q_g = 0:

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So, we get the 4 nodal equations as follows:"

"For Node 1:" T_west + T_north + T2 + T3 – 4 * T1 = 0

"For Node 2:" T1 + T_north + T_east + T4 – 4 * T2= 0

"For Node 3:" T_west + T1 + T4 + T_south – 4 * T3= 0

"For Node 4:" T3 + T2 + T_east + T_south – 4 * T4 = 0

"Solve the above 4 equations simultaneously, to get the six nodal temperatures, T1….T4:"

"Heat transfer from 500 C and 100 C surfaces:"

"Q_north,= heat transfer from the north surface at 500 C:"

"Put DELTAx = DELTAy = say, 1:"

DELTAx = 1

DELTAx = DELTAy

Q_north = k * (DELTAx/2) * (T_west – T_north) / DELTAy + k * (DELTAx) * (T1 – T_north) / DELTAy + k * (DELTAx) * (T2 – T_north) / DELTAy + k * (DELTAx/2) * (T_east – T_north) / DELTAy

"Q_west,= heat transfer from the west surface at 100 C:"

Q_west = k * (DELTAy/2) * (T_north – T_west) / DELTAx + k * (DELTAy) * (T1 – T_west) / DELTAx + k * (DELTAy) * (T3 – T_west) / DELTAx + k * (DELTAy/2) * (T_south – T_west) / DELTAx

"Q_east,= heat transfer from the east surface at 100 C:"

Q_east = k * (DELTAy/2) * (T_north – T_east) / DELTAx + k * (DELTAy) * (T2 – T_east) / DELTAx + k * (DELTAy) * (T4 – T_east) / DELTAx + k * (DELTAy/2) * (T_south – T_east) / DELTAx

"Q_south,= heat transfer from the south surface at 100 C:"

Q_south = k * (DELTAx/2) * (T_west – T_south) / DELTAy + k * (DELTAx) * (T3 – T_south) / DELTAy + k * (DELTAx) * (T4 – T_north) / DELTAy + k * (DELTAx/2) * (T_east – T_south) / DELTAy

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Results:

Unit Settings: SI C kPa kJ mass deg

$\Delta x = 1$	$\Delta y = 1$	$k = 1$ [W/m-C]
$Q_{east} = 400$ [W]	$Q_{north} = -900$ [W]	$Q_{south} = -300$ [W]
$Q_{west} = 400$ [W]	T1 = 250 [C]	T2 = 250 [C]
T3 = 150 [C]	T4 = 150 [C]	$T_{east} = 100$ [C]
$T_{north} = 500$ [C]	$T_{south} = 100$ [C]	$T_{west} = 100$ [C]

Thus, we get:

T1 = T2 = 250 C, T3 = T4 = 150 C ….. Ans.

Q_east = Q_west = 400 W , from inside the plate towards the respective faces.

Q_north = – 900 W, -ve sign indicating that flow of heat is from the surface towards the inside, and,

Q_south = – 300 W, -ve sign indicating that flow of heat is from the surface towards the inside.

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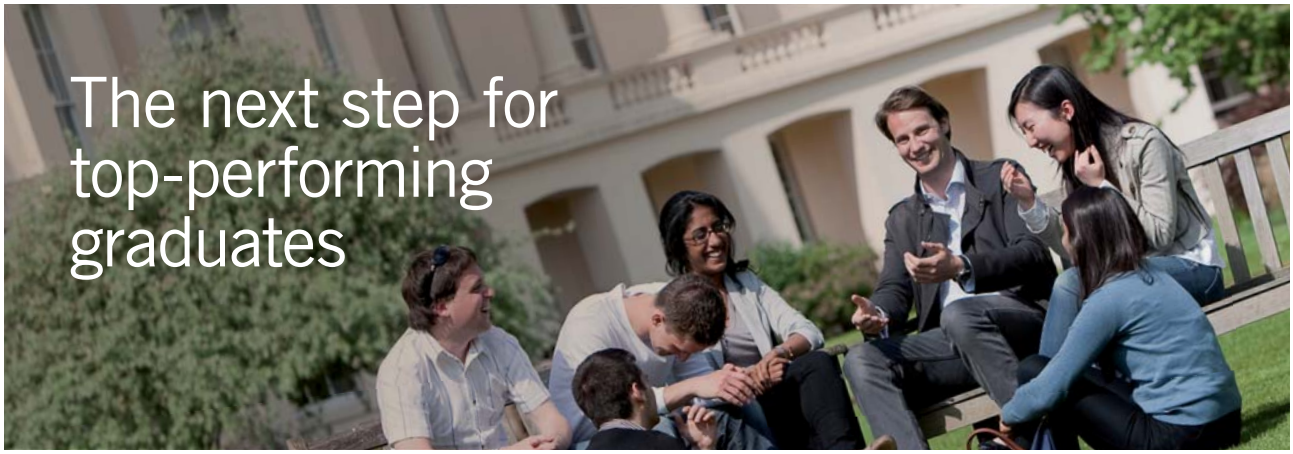

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"**Prob. 1I.B.3.** A long conducting rod of rectangular cross-section (20 mm x 30 mm) and k = 20 W/m.K experiences uniform heat generation of q = 5 × 10^7 W/m^3, while its surfaces are maintained at 300 K. (a) Using a finite difference method with a grid spacing of 5 mm, determine the temp distribution in the rod. (b) With the boundary conditions unchanged, what heat generation rate would cause the mid-point temp to reach 600 K? [Ref. 3]"

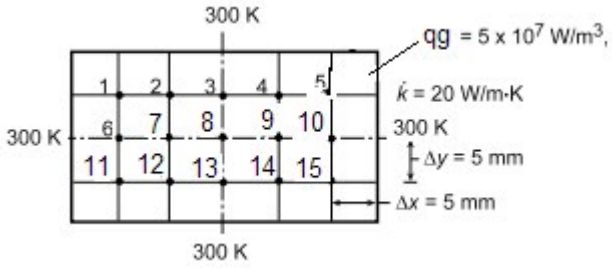

Fig.Prob.1I.B.3

EES Solution:

"Data:"

T_west = 300 [K]

T_north = 300 [K]

T_east = 300 [K]

T_south = 300 [K]

L = 0.03 [m]

B = 0.02 [m]

k= 20 [W/m-C]

DELTAx = 0.005 [m]

DELTAy =0.005 [m]

q_g = 5e07 [W/m^3]

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"Calculations:"

"Note that all the 15 nodes viz. nodes 1 … 15 are interior nodes.

For interior nodes, in two dimensional steady state conduction, for DELTAx = DELTAy, with internal heat generation rate of q_g (W/m^3), we have:

For any general interior node (m,n):

T[m-1,n] + T[m,n+1] + T[m+1,n] + T[m,n-1] – 4 * T[m,n] + (DELTAx^2/k) * q_g = 0"

"Now, by symmetry, we can write:"

T1 = T5

T2 = T4

T6 = T10

T7 = T9

T11 = T15

T12 = T14

T3 = T13

T4 = T14

T5 = T15

"So, we need to find out only T1, T2, T3, T6, T7 and T8.

These are all internal nodes.

So, we get the 6 nodal equations as follows:"

"For Node 1:" T_west + T_north + T2 + T6 – 4 * T1 + q_g * (DELTAx ^2 /k)= 0

"For Node 2:" T1 + T_north + T3 + T7 – 4 * T2 +q_g * (DELTAx ^2 /k) = 0

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"For Node 3:" T2 + T_north + T4 + T8 – 4 * T3 + q_g * (DELTAx ^2 /k)= 0

"For Node 6:" T_west + T1 + T7 + T11 – 4 * T6 +q_g * (DELTAx ^2 /k) = 0

"For Node 7:" T6 + T2 + T8 + T12 – 4 * T7 +q_g * (DELTAx ^2 /k) = 0

"For Node 8:" T7+ T3 + T9 + T13 – 4 * T8 +q_g * (DELTAx ^2 /k) = 0

"Solve the above 6 equations simultaneously, to get the six nodal temperatures, T1, T2, T3, T6, T7 and T8:"

Results:

B = 0.02 [m]	Δx = 0.005 [m]	Δy = 0.005 [m]
k = 20 [W/m-C]	L = 0.03 [m]	q_g = 5.000E+07 [W/m^3]
T1 = 348.5 [K]	T10 = 362.4 [K]	T11 = 348.5 [K]
T12 = 368.9 [K]	T13 = 374.6 [K]	T14 = 368.9 [K]
T15 = 348.5 [K]	T2 = 368.9 [K]	T3 = 374.6 [K]
T4 = 368.9 [K]	T5 = 348.5 [K]	T6 = 362.4 [K]
T7 = 390.2 [K]	T8 = 398 [K]	T9 = 390.2 [K]
T_{east} = 300 [K]	T_{north} = 300 [K]	T_{south} = 300 [K]
T_{west} = 300 [K]		

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Note the values of all temperatures obtained above…. Ans.

(b) What is the value of q_g which will cause the centre temp (i.e. T8) to become 600 K?

We get:

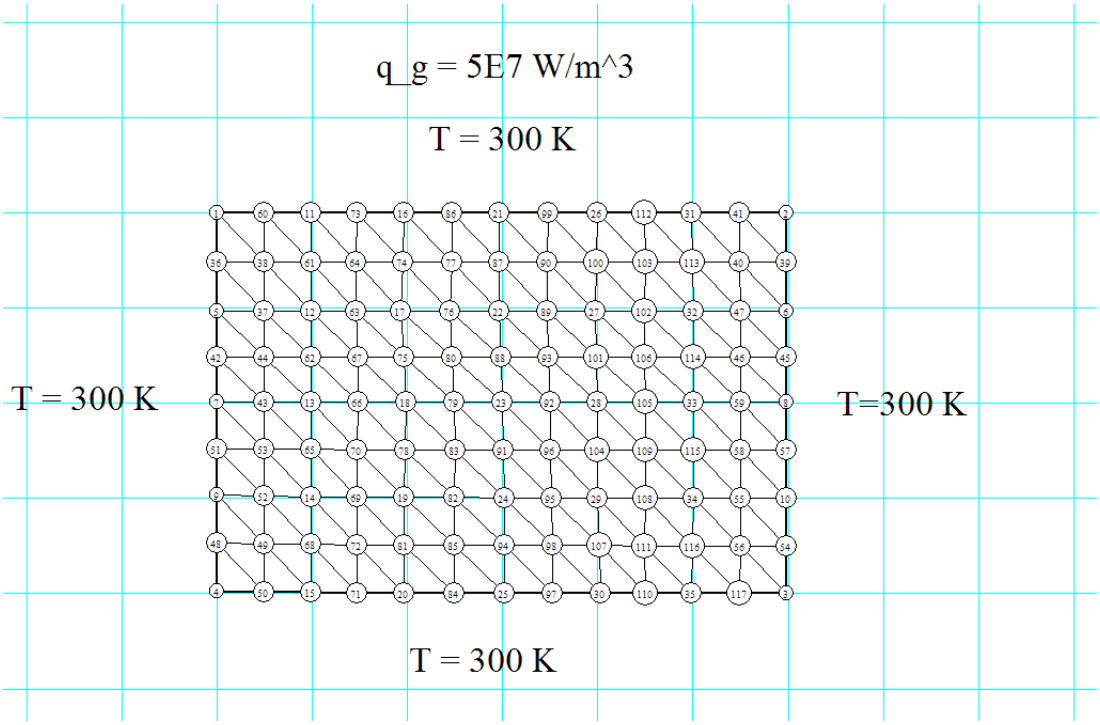
Table 1

	1 q_g [W/m³]	2 T8 [K]
1..1		
Run 1	1.530E+08	600

i.e. q_g = 1.53 × 10^8 W/m^3 will cause the centre temp to become T8 = 600 K …. Ans.

Solution to the above problem by Finite Element Heat Transfer (FEHT) Software:

Node positions:



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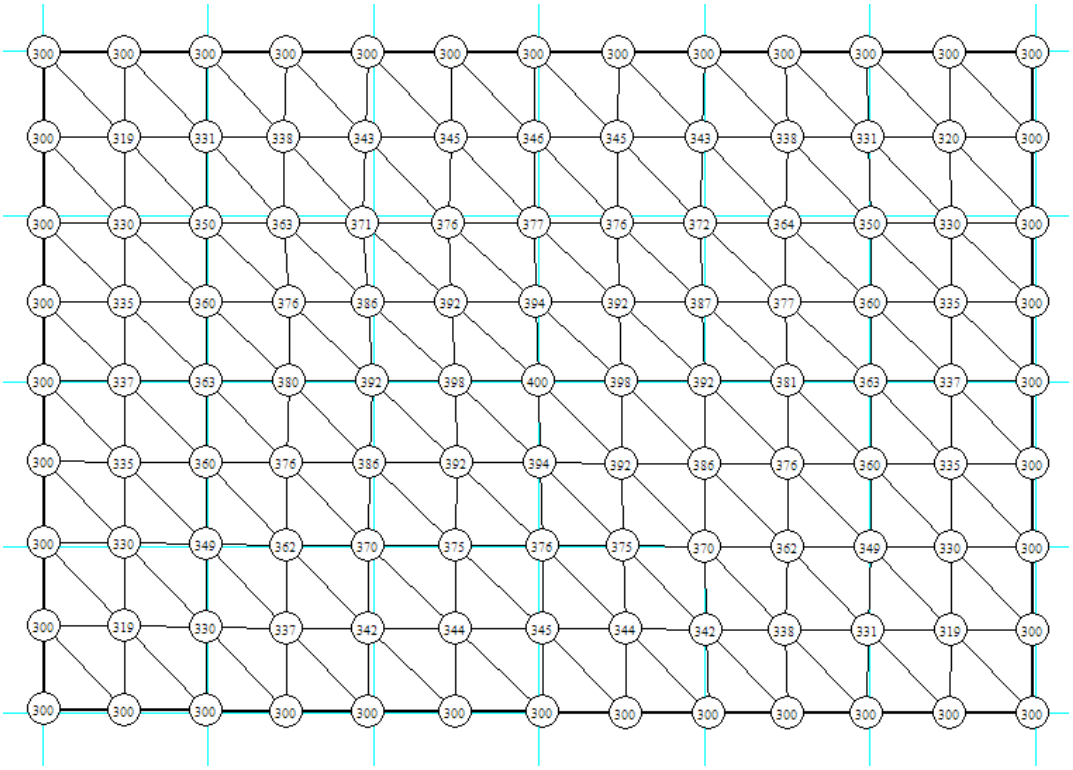
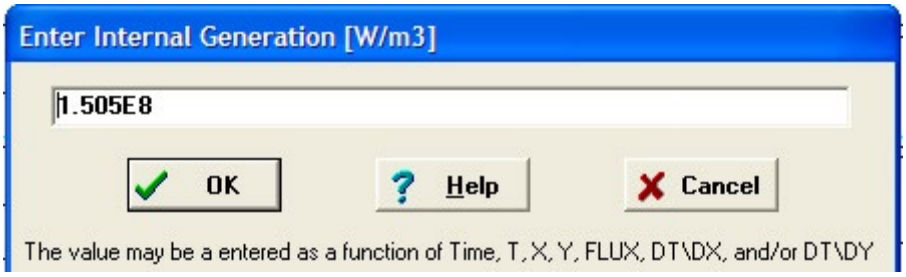
Node Temps (K):



Note that, here, centre temp = 400 K. Compare this with 398 K obtained earlier with EES.

What should be the value of qg to make centre temp = 600 K?

Let us change the value of internal heat generation by *trial and error* and check the centre temp for each value of qg. We get:



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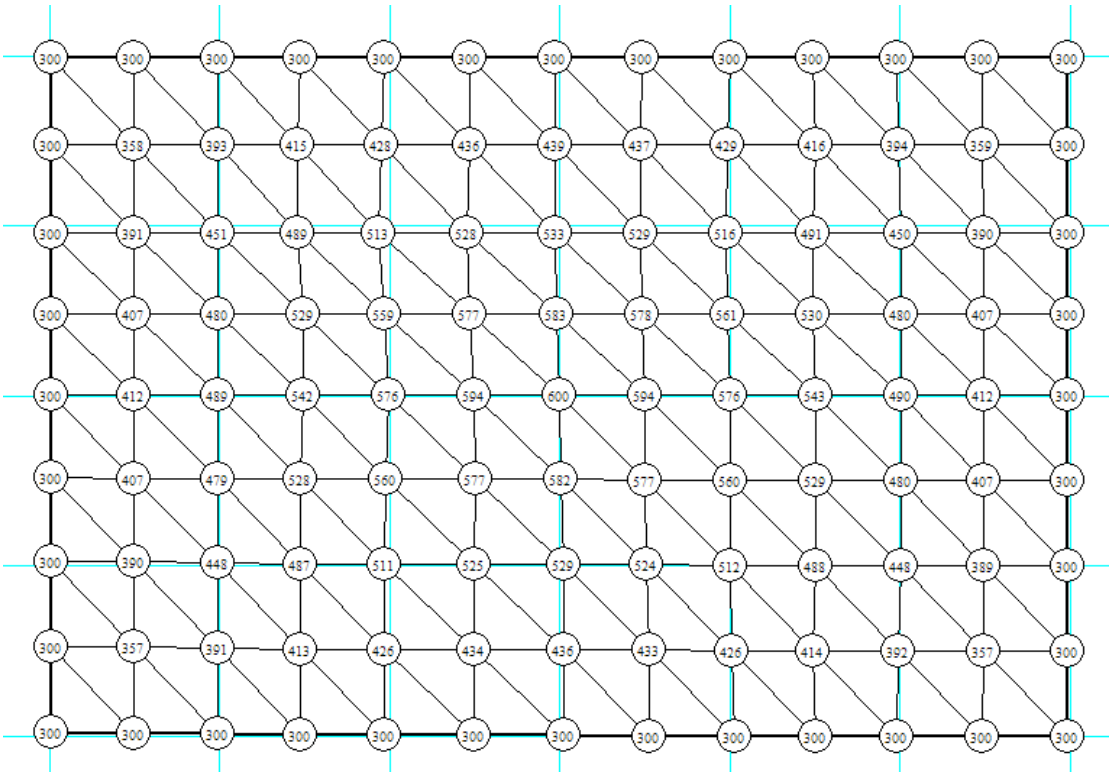
And the temp distribution for qg = 1.505E8 W/m^3 is:



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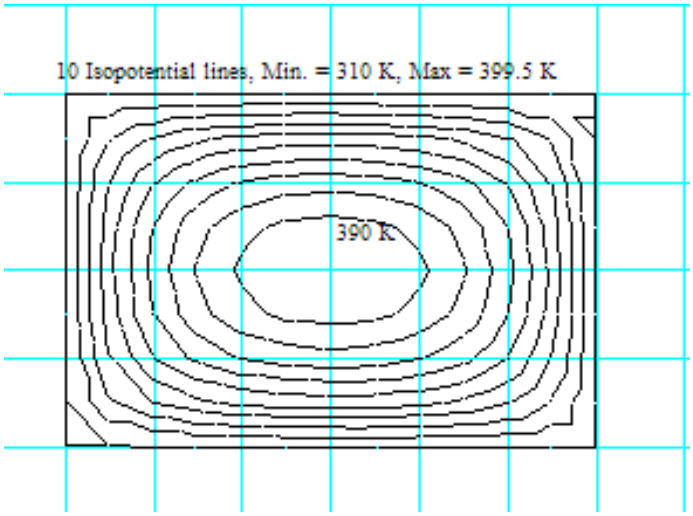
Note that, now, the centre temp is 600 K.

Compare this value of qg with that obtained earlier with EES, i.e. 1.53E08 W/m^3.

And, 10 isotherms are drawn in FEHT as follows:



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Prob. 1I.B.4. Consider steady two dimensional heat transfer in a long solid body of cross-section shown in fig. The temperatures at the selected nodes and the thermal conditions on the boundaries are shown. For the body, k = 180 W/m.C and heat gen. rate is qg = 10^7 W/m^3. Using finite difference method with a mesh of Δx = Δy = 10 cm, determine: (a) temperatures at the nodes 1, 2, 3 and 4, and (b) rate of heat loss from the top surface through a 1 meter long section of the body (c) Plot the effect of variation of k and qg on T1 and T3. [Ref. 2]

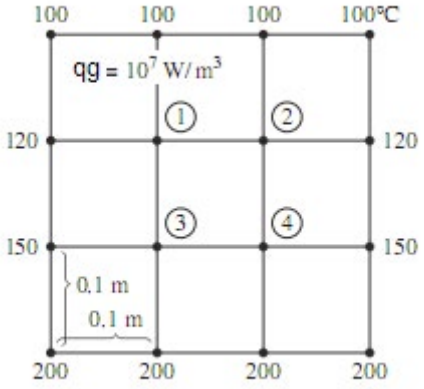

Fig.Prob.1I.B.4

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Mathcad Solution:

Data:

$k := 180$ W/(m.C).....thermal cond. of material

$q_g := 10^7$ W/m^3

$\Delta x := 0.1$ m

$\Delta y := 0.1$ m

Difference equations:

For Internal nodes: For any internal node, the temp is given by:

$$T_{west} + T_{north} + T_{east} + T_{south} - 4 \cdot T_{node} + q_g \cdot \frac{\Delta x^2}{k} = 0$$

where Tnode is the temp of the node in question.

So, write the eqns for nodes 1, 2, 3 and 4, and use the Solve block of Mathcad to solve these eqns simultaneously.:

Start with guess values for unknown temperatures:

$T1 := 100$ $T2 := 100$ $T3 := 100$ $T4 := 100$ .....guess values

Given

$$120 + 100 + T2 + T3 - 4 \cdot T1 + \frac{q_g \cdot \Delta x^2}{k} = 0 \quad \text{..for node 1}$$

$$T1 + 100 + 120 + T4 - 4 \cdot T2 + \frac{q_g \cdot \Delta x^2}{k} = 0 \quad \text{..for node 2}$$

$$150 + T1 + T4 + 200 - 4 \cdot T3 + \frac{q_g \cdot \Delta x^2}{k} = 0 \quad \text{..for node 3}$$

$$T3 + T2 + 150 + 200 - 4 \cdot T4 + \frac{q_g \cdot \Delta x^2}{k} = 0 \quad \text{..for node 4}$$

$$Temp\left(k, q_g\right) := Find(T1, T2, T3, T4)$$

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Note that Temp is written as a function of k and qg, so that graph can be drawn for different k and qg values.

Therefore, for k = 180 W/m.C, and qg = 10^7 W/m^3, the temp distribution is:

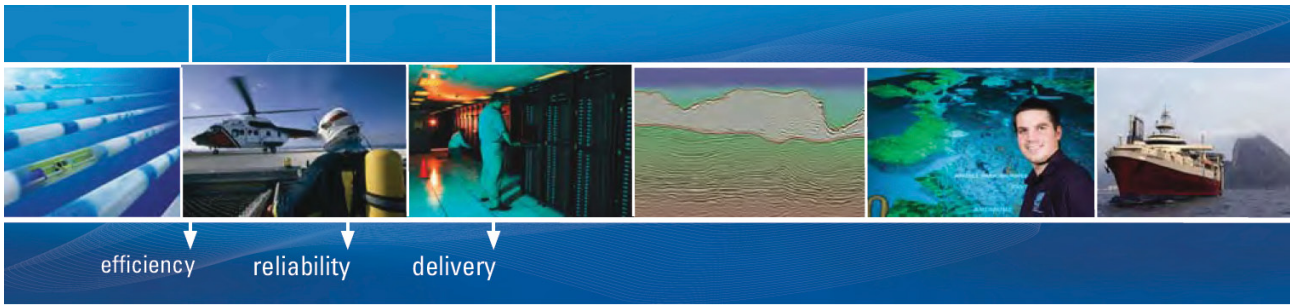
$$\text{Temp}(k, q_g) = \begin{bmatrix} 404.028 \\ 404.028 \\ 436.528 \\ 436.528 \end{bmatrix}$$

i.e. $T1 := \text{Temp}(k, q_g)_0$ $\quad T1 = 404.028$

$T2 := \text{Temp}(k, q_g)_1$ $\quad T2 = 404.028$

$T3 := \text{Temp}(k, q_g)_2$ $\quad T3 = 436.528$

$T4 := \text{Temp}(k, q_g)_3$ $\quad T4 = 436.528$ $\quad$ **C....Ans.**



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Again, for example, when k = 10 W/m.K, qg = 10^7 W/m^3, temp distribution is easily found out by writing Temp(k,qg) = , as shown below:

$k := 10$ W/m.C $\quad q_g := 10^7$ W/m^3

$$Temp(k, q_g) = \begin{bmatrix} 5.126 \cdot 10^3 \\ 5.126 \cdot 10^3 \\ 5.159 \cdot 10^3 \\ 5.159 \cdot 10^3 \end{bmatrix}$$

To calculate the heat transferred from the top surface:

Write the heat balance on the top surface, of thickness Δy/2:

$$Q_{top} + 2 \cdot k \cdot \frac{\Delta x}{2} \cdot \frac{(120 - 100)}{\Delta y} + 2 \cdot k \cdot \Delta x \cdot \frac{Temp(k, q_g)_0 - 100}{\Delta y} + q_g \cdot \left(\frac{\Delta y}{2} \cdot 3 \cdot \Delta x\right) = 0$$

i.e $$Q_{top}(k, q_g) := -\left[2 \cdot k \cdot \frac{\Delta x}{2} \cdot \frac{(120 - 100)}{\Delta y} + 2 \cdot k \cdot \Delta x \cdot \frac{Temp(k, q_g)_0 - 100}{\Delta y} + q_g \cdot \left(\frac{\Delta y}{2} \cdot 3 \cdot \Delta x\right)\right]$$

i.e. $Q_{top}(180, q_g) = -2.631 \cdot 10^5$ **W .... -ve sign indicating that heat flow is from the top surface outwards.**

To draw the graph of variation of T1 and T3 with k, keeping qg = 10^7 W/m^3:

Let k vary from 10 to 400 W/m.C

$q_g := 10^7$ W/m^3

$k := 10, 15 .. 400$ ..define a range variable k

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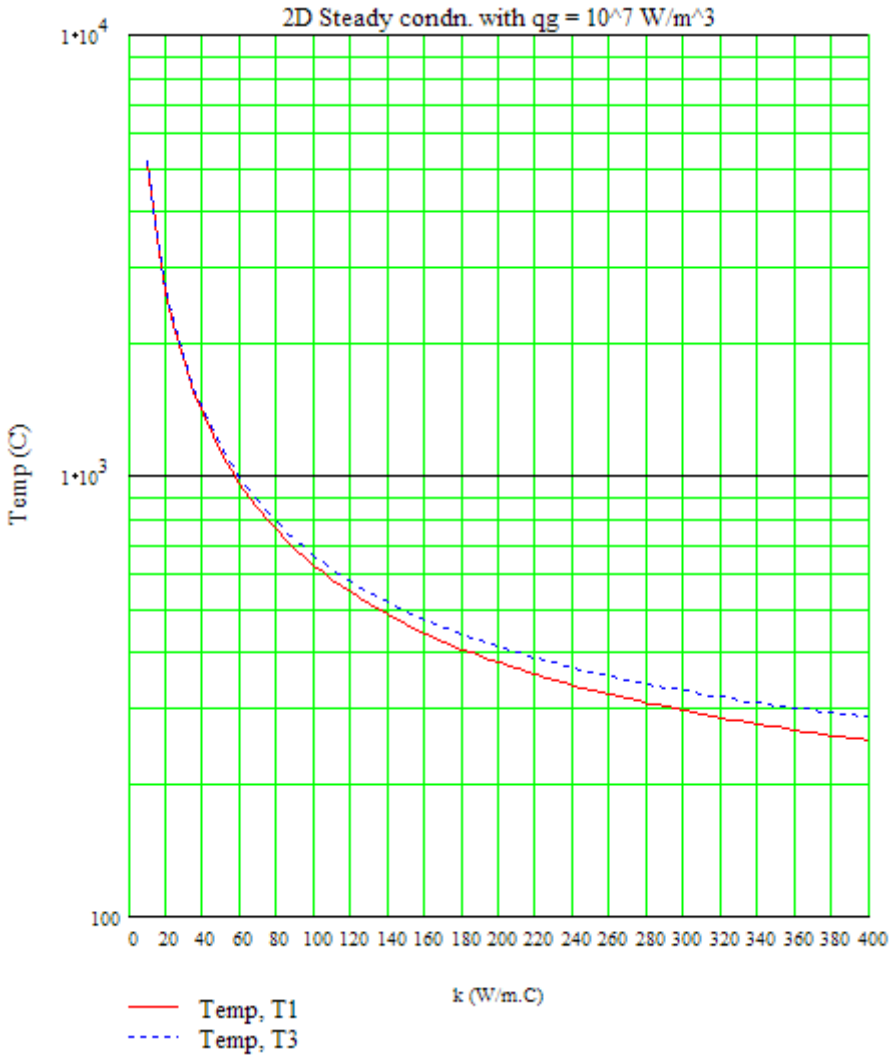

To draw the graph of variation of Q_{top} with k, keeping qg = 10^7 W/m^3:

Let k vary from 10 to 400 W/m.C

$q_g := 10^7$ W/m^3

ex: $-Q_{top}\left\langle 400, 10^7 \right\rangle = 2.79 \cdot 10^5$ W

k := 10, 15.. 400 ..define a range variable k

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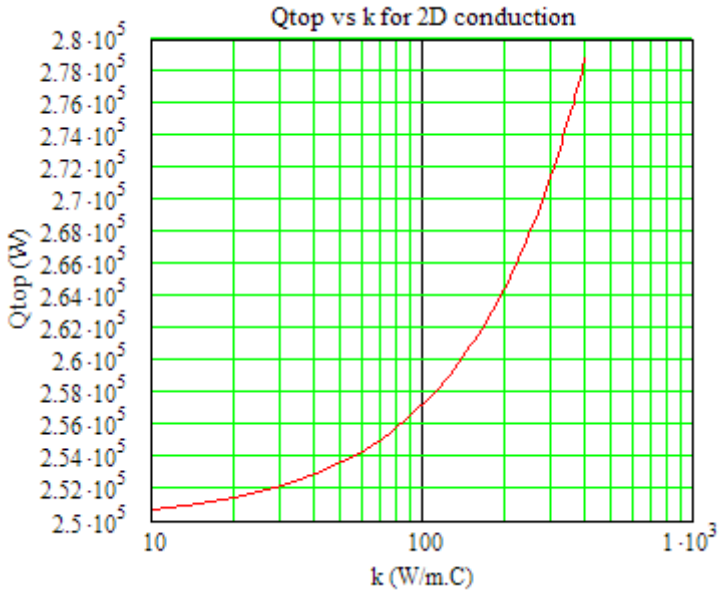

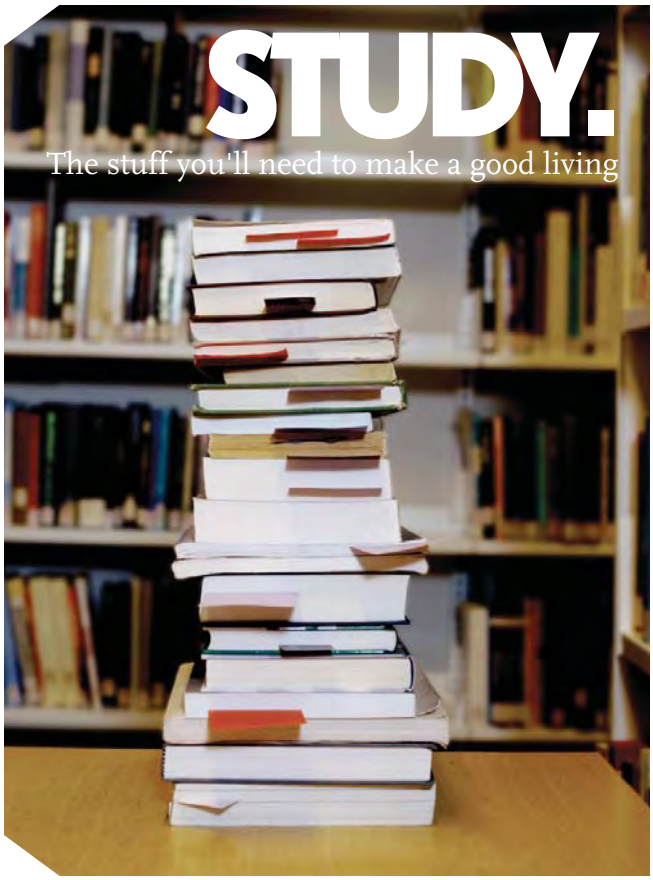

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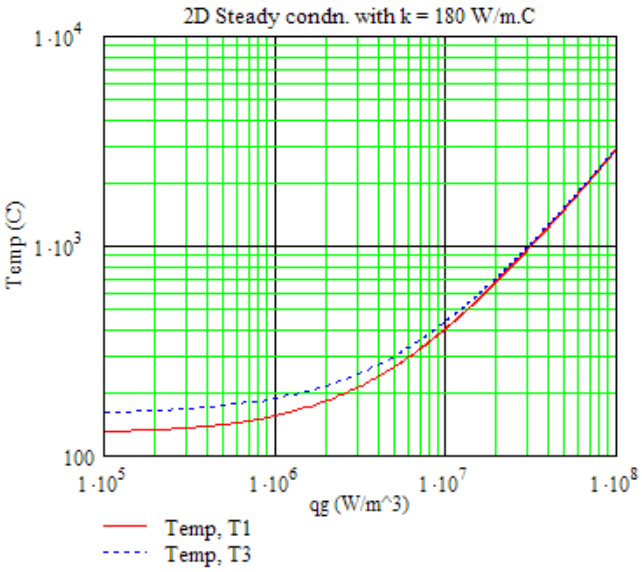
To draw the graph of variation of T1 and T3 with qg, keeping k = 180 W/m.C:

Let qg vary from 10^5 to 10^8 W/m^3

$k := 180$ W/m.C

$q_g := 10^5, 2 \cdot 10^5 .. 10^8$ ..define a range variable qg



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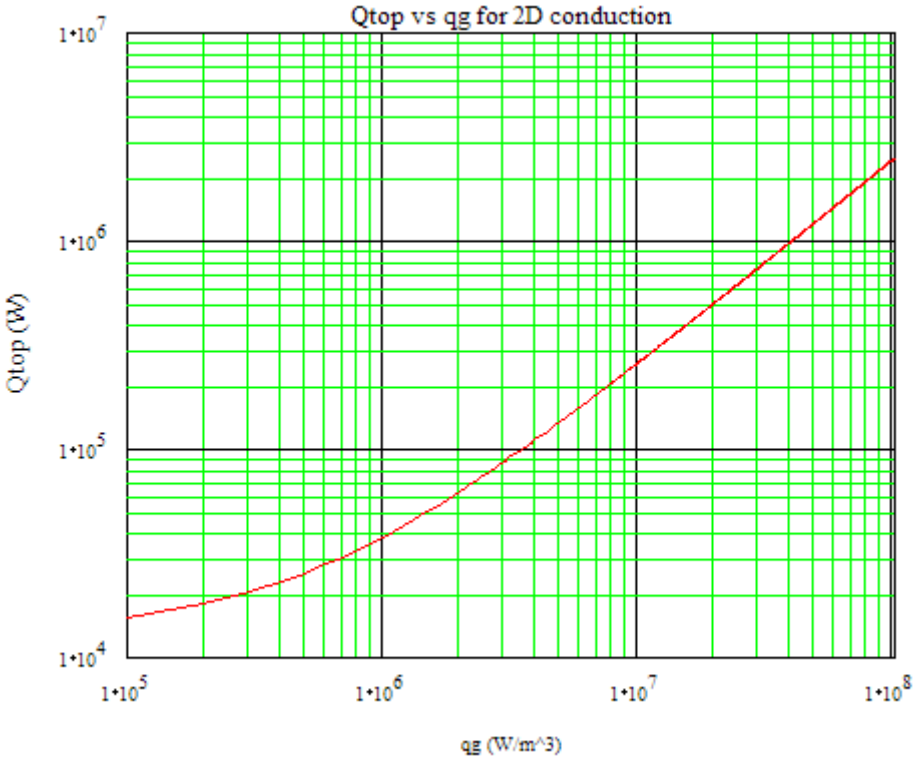
To draw the graph of variation of Qtop with qg, keeping k = 180 W/m.C:

Let qg vary from 10^5 to 10^8 W/m^3

$q_g := 10^5, 2 \cdot 10^5 .. 10^8$ ..define a range variable qg

k := 180 W/m.C



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"**Prob. 1I.B.5.** A long bar of rectangular cross-section, 0.4 m × 0.6 m on a side and having k = 1.5 W/m.K is subjected to the boundary conditions shown below. (See the Diagram Window). Two of the sides are maintained at a uniform temp of 200 C. One of the sides is adiabatic. And the remaining side is subjected to a convection process with Ta = 30 C and h = 50 W/m^2.K. Using appropriate numerical technique and a grid spacing of 0.1 m, determine the temp distribution in the bar and the heat transfer rate between the bar and the fluid per unit length of the bar. [Ref.3]"

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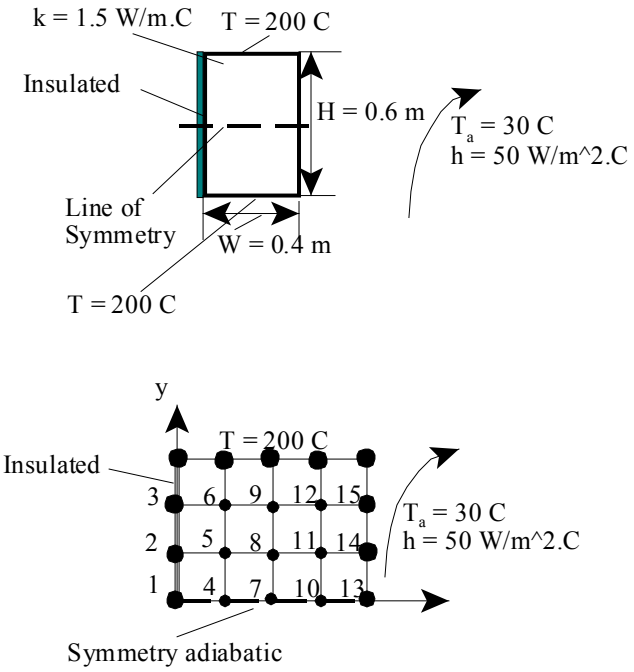

Fig.Prob.1I.B.5

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EES Solution:

"Note that there is symmetry about the line drawn at mid-height of the section. So, we consider the upper half section, and it suffices if we find out the temp distribution only in that half.

There are 15 nodes; Nodes 1, 2, 3 are on the left, insulated face. Nodes 13, 14, 15 are on the RHS, with convection. And, Nodes 1, 4, 7, 10, 13 are on the symmetry adiabatic line. Rest of the nodes, i.e. Nodes 5, 6, 8, 9, 11 and 12 are internal nodes."

"Data:"

T_north = 200 [C]

T_south = 200 [C]

T_a = 30 [C]

h = 50 [W/m^2-C]

Height = 0.6 [m]

Width = 0.4 [m]

k= 1.5 [W/m-C]

DELTAx = 0.1 [m]

DELTAy =0.1 [m]

"Calculations:"

"Note that of the 15 nodes, following are the interior nodes: i.e. nodes 5, 6, 8, 9, 11 and 12.

For interior nodes, in two dimensional steady state conduction, for DELTAx = DELTAy, with internal heat generation rate of q_g W/m^3, we have:

For any general interior node (m,n):

T[m-1,n] + T[m,n+1] + T[m+1,n] + T[m,n-1] – 4 * T[m,n] + (DELTAx^2/k) * q_g = 0

In the present case, internal heat generation does not exist, i.e. q_g = 0"

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"So, we write the nodal equations for internal nodes as follows:"

"For Node 5:" T2 + T6 + T8 + T4 – 4 * T5 = 0

"For Node 6:" T3 + T_north + T9 + T5 – 4 * T6 = 0

"For Node 8:" T5 + T9 + T11 + T7 – 4 * T8 = 0

"For Node 9:" T6 + T_north + T12 + T8 – 4 * T9 = 0

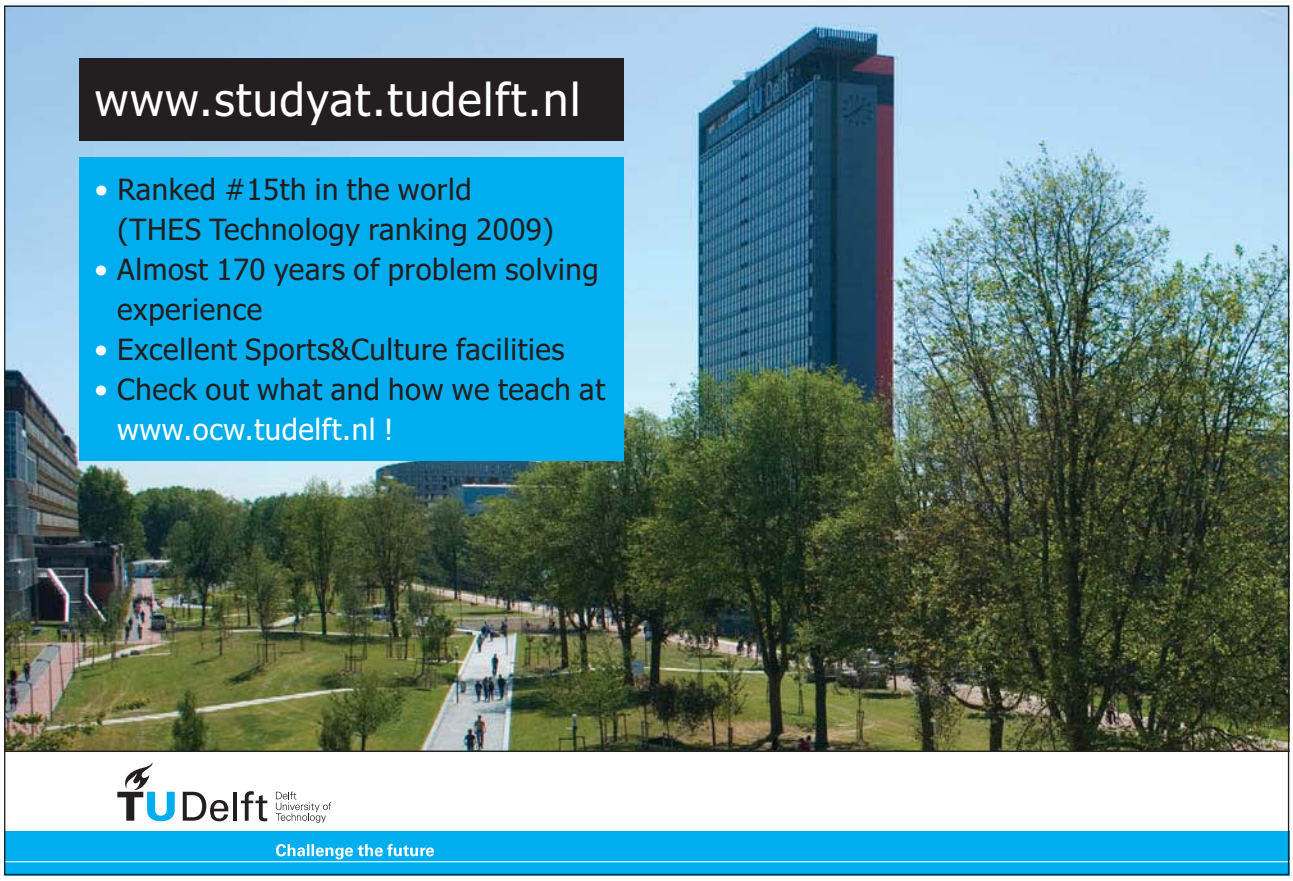
"For Node 11:" T8 + T12 + T14 + T10 – 4 * T11 = 0

"For Node 12:" T9+ T_north + T15 + T11 – 4 * T12 = 0

"Nodes on the left, insulated face, i.e. Nodes 1, 2 and 3:

Write energy balances around these nodes, taking care to see that all heat flows are into the respective nodes:"

"For Node 1:" 0 + k * DELTAx/2 * (T2 – T1)/DELTAy + k * DELTAy/2 * (T4 – T1)/DELTAx + 0 = 0



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"For Node 2:" 0 + k * DELTAx/2 * (T3 – T2)/DELTAy + k * DELTAy * (T5 – T2)/DELTAx + k * DELTAx/2 * (T1 – T2)/DELTAy = 0

"For Node 3:" 0 + k * DELTAx/2 * (T_north – T3)/DELTAy + k * DELTAy * (T6 – T3)/DELTAx + k * DELTAx/2 * (T2 – T3)/DELTAy = 0

"Nodes on the symmetry, adiabatic face, i.e. Nodes 4, 7 and 10:

Considering the symmetry, adiabatic face as mirror, we get:"

"For Node 4:" T1 + 2 * T5 + T7 – 4 * T4 = 0

"For Node 7:" T4 + 2 * T8 + T10 – 4 * T7 = 0

"For Node 10:" T7 + 2 * T11 + T13 – 4 * T10 = 0

"Nodes on RHS, with convection i.e. Nodes 13, 14 and 15:

Write energy balances around these nodes, taking care to see that all heat flows are into the respective nodes:"

"For Node 13:" k * DELTAy/2 * (T10 – T13) / DELTAx + k * DELTAx / 2 * (T14 – T13) / DELTAy + h * DELTAy/2 * (T_a – T13) + 0 = 0

"For Node 14:" k * DELTAy * (T11 – T14) / DELTAx + k * DELTAx / 2 * (T15 – T14) / DELTAy + h * DELTAy * (T_a – T14) + k * DELTAx / 2 * (T13 – T14) / DELTAy = 0

"For Node 15:" k * DELTAy * (T12 – T15) / DELTAx + k * DELTAx / 2 * (T_north – T15) / DELTAy + h * DELTAy * (T_a – T15) + k * DELTAx / 2 * (T14 – T15) / DELTAy = 0

"Solve the above 15 equations simultaneously, to get the six nodal temperatures, T1…. T15"

"Heat transfer from the face exposed to convection (i.e. RHS):"

Q_conv =2 * (h * (DELTAy/2) * (T13 – T_a) + h * (DELTAy) * (T14 – T_a) +h * (DELTAy) * (T15 – T_a)+ h * (DELTAy/2) * (T_north – T_a))

"Note that in the above eqn. the first factor 2 appears since the heat transfer is twice the value for the uper half.

For the two nodes at the edges, area for convection is (1. DELTAy/2)"

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Results:

Unit Settings: SI C kPa kJ mass deg

Δx = 0.1 [m]	Δy = 0.1 [m]	h = 50 [W/m^2-C]	Height = 0.6 [m]
k = 1.5 [W/m-C]	Q_{conv} = 1486 [W]	T1 = 153.9 [C]	T10 = 95.56 [C]
T11 = 103.5 [C]	T12 = 132.8 [C]	T13 = 45.81 [C]	T14 = 48.73 [C]
T15 = 66.97 [C]	T2 = 159.8 [C]	T3 = 176.4 [C]	T4 = 148 [C]
T5 = 154.4 [C]	T6 = 172.9 [C]	T7 = 129.4 [C]	T8 = 137 [C]
T9 = 160.7 [V]	T_a = 30 [C]	T_{north} = 200 [C]	T_{south} = 200 [C]
Width = 0.4 [m]			

Thus:

Temperatures at various Nodes, T1 … T15 are shown above. …. Ans.

Note that temperatures T13, T14, and T15 on the face with convection are less as compared to temperatures at other nodes, as expected.

Also, Convection heat transfer from the exposed RHS = Q_conv = 1486 W (for both the symmetrical halves considered) … Ans.

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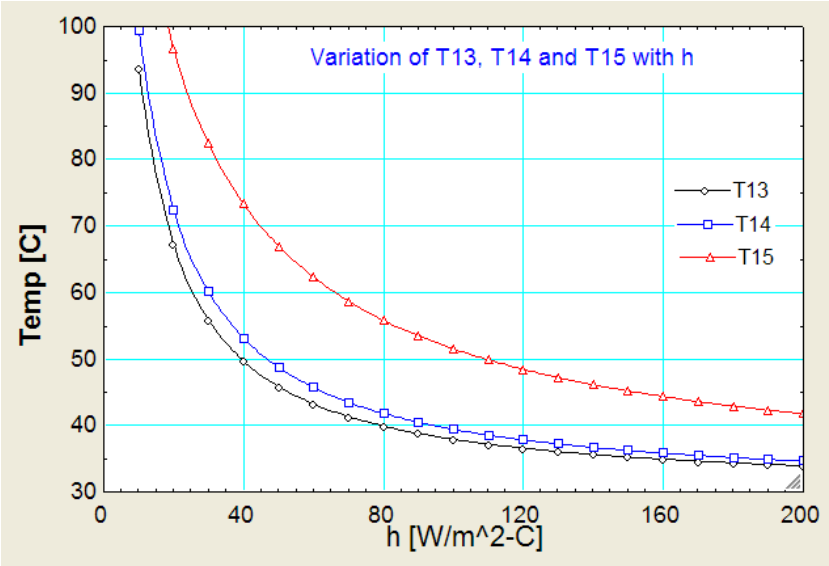
Variation of temperatures on the face exposed to convection, viz. T13, T14, T15 and Q_conv with heat transfer coeff. h:

1..20	1 h [W/m2-C]	2 T13 [C]	3 T14 [C]	4 T15 [C]	5 Q_{conv} [W]
Run 1	10	93.64	99.48	123.2	559
Run 2	20	67.24	72.4	96.64	850.6
Run 3	30	55.86	60.09	82.38	1082
Run 4	40	49.66	53.15	73.3	1290
Run 5	50	45.81	48.73	66.97	1486
Run 6	60	43.19	45.69	62.28	1675
Run 7	70	41.31	43.47	58.66	1859
Run 8	80	39.89	41.79	55.78	2040
Run 9	90	38.78	40.47	53.43	2219
Run 10	100	37.9	39.41	51.48	2397
Run 11	110	37.18	38.54	49.83	2573
Run 12	120	36.57	37.82	48.41	2748
Run 13	130	36.06	37.2	47.19	2923
Run 14	140	35.63	36.68	46.12	3097
Run 15	150	35.25	36.22	45.17	3271
Run 16	160	34.92	35.82	44.33	3444
Run 17	170	34.63	35.47	43.58	3617
Run 18	180	34.37	35.16	42.9	3789
Run 19	190	34.14	34.88	42.29	3961
Run 20	200	33.93	34.63	41.74	4133

Plot of T13, T14 and T15 against h:



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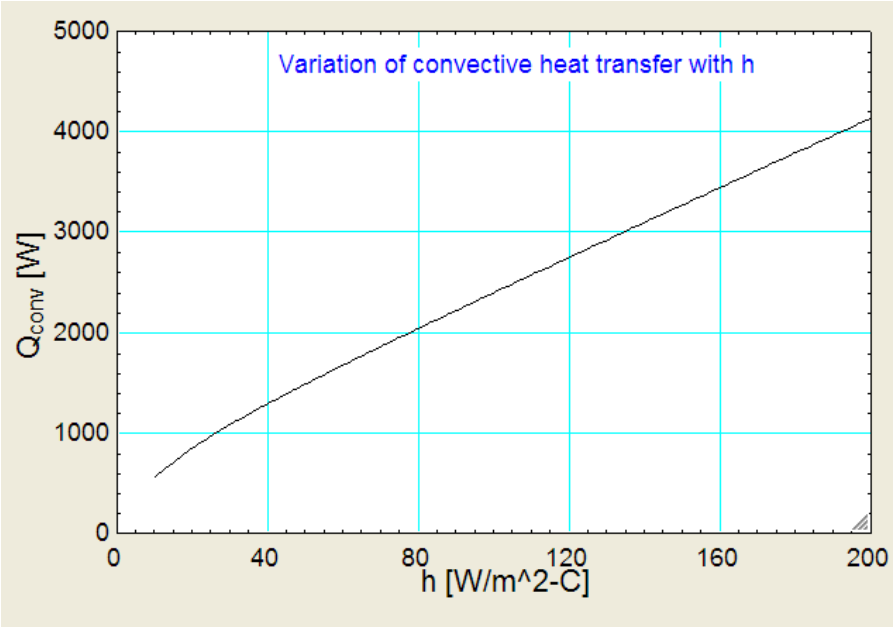
Plot of Q_conv against h:



==========

"**Prob. 1I.B.6.** Consider an Aluminium heat sink (k = 240 W/m.K) as shown. The inner and outer widths of the square channel are w = 20 mm and W = 40 mm. An outer surface temp of Ts = 50 C is maintained by the array of electronic chips. At the inner surface, the coolant temp T_a = 20 C and h = 5000 W/m^2.K. (a) Determine the unknown temperatures T1 … T7 and the rate of heat transfer to the coolant per unit length of the channel. For this purpose, consider a symmetrical section of the channel and a two dimensional grid with DELTAx = DELTAy = 5 mm. and the rate of heat transfer
(b) Assess the effect of variation in h on the unknown temperatures and the heat rate. [Ref.3]"

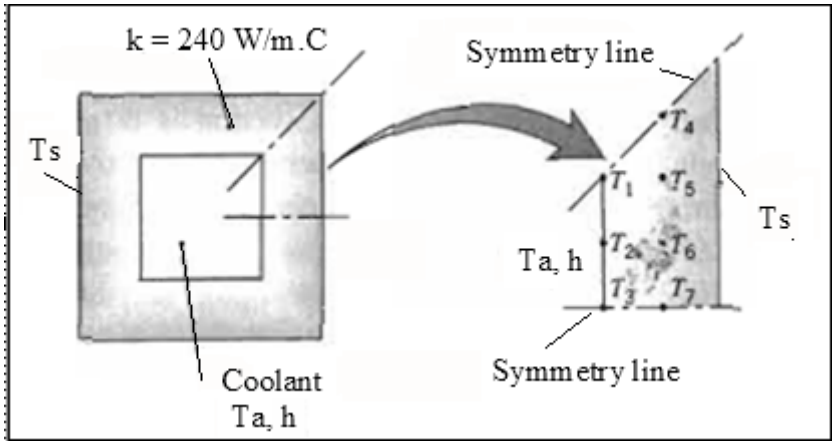


Fig.Prob.1I.B.6

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"EES Solution:"

"Note that the symmetry lines shown in the fig form one-eigth of the section.

So, we consider this section, and it suffices if we find out the temp distribution only in that section. There are 7 nodes; Nodes 1, 2, 3 are on the left face, exposed to convection. Nodes 4, 5, 6, 7 are internal nodes and the RHS is maintained at a constant temp of Ts = 50 C."

"Data:"

T_s = 50 [C]

T_a = 20 [C]

h = 5000 [W/m^2-C]

k= 240 [W/m-C]

DELTAx = 0.005 [m]

DELTAy =0.005 [m]

"Calculations:"

"Note that of the 7 nodes, following are the interior nodes: i.e. nodes 4, 5, 6, and 7.

For interior nodes, in two dimensional steady state conduction, for DELTAx = DELTAy, with internal heat generation rate of q_g W/m^3, we have:

For any general interior node (m,n):

T[m-1,n] + T[m,n+1] + T[m+1,n] + T[m,n-1] – 4 * T[m,n] + (DELTAx^2/k) * q_g = 0

In the present case, internal heat generation does not exist, i.e. q_g = 0"

"So, we write the nodal equations for internal nodes as follows:"

"For Node 5:" T1 + T4 + T_s + T6 – 4 * T5 = 0

"For Node 6:" T2 + T5 + T_s + T7 – 4 * T6 = 0

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"For Node 7. This node is on the symmetry adiabatic line. So, using mirror concept:"

T3 + 2 * T6 + T_s – 4 * T7 = 0

"For Node 4.This node is also on the symmetry adiabatic line. So, using mirror concept:"

T5 + 2 * T_s + T5 – 4 * T4 = 0

"For Node 1: Writing the energy balance:"

k * DELTAy * (T5 – T1) / DELTAx + k * DELTAx/2 * (T2 – T1) / DELTAy + h * DELTAy / 2 * (T_a – T1) = 0

"For Node 2: Writing the energy balance:"

k * DELTAx/2 * (T1 – T2) / DELTAy+ k * DELTAy * (T6 – T2) / DELTAx + k * DELTAx/2 * (T3 – T2) / DELTAy + h * DELTAy * (T_a – T2) = 0

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"For Node 3: Writing the energy balance:"

$$k * DELTAx/2 * (T2 - T3) / DELTAy + k * DELTAy/2 * (T7 - T3) / DELTAx + h * DELTAy / 2 * (T_a - T3) = 0$$

"Solve the above 7 equations simultaneously, to get the six nodal temperatures, T1…. T7"

"Heat transfer from the face exposed to convection (i.e. RHS):"

$$Q\_conv = 8 * (h * (DELTAy/2) * (T1 - T_a) + h * (DELTAy) * (T2 - T_a) + h * (DELTAy / 2) * (T3 - T_a))$$

"Note that in the above eqn. the first factor 8 appears since the heat transfer is 8 times the value for the symmetrical section shown."

Results:

Unit Settings: SI C kPa kJ mass deg

$\Delta x = 0.005$ [m]	$\Delta y = 0.005$ [m]	$h = 5000$ [W/m^2-C]	$k = 240$ [W/m-C]
$Q_{conv} = 10339$ [W]	T1 = 46.61 [C]	T2 = 45.67 [C]	T3 = 45.44 [C]
T4 = 49.23 [C]	T5 = 48.46 [C]	T6 = 48 [C]	T7 = 47.86 [C]
$T_a = 20$ [C]	$T_s = 50$ [C]		

Temperatures T1 … T7 are shown above … Ans.

Convective heat transfer to the inner surface = Q_conv = 10339 W …. Ans.

(b) Variation of T1 … T7 and Q_conv with h:

Let h vary from 200 to 5000 W/m^2.K :

1..10	1 h [W/m2-C]	2 T1 [C]	3 T2 [C]	4 T3 [C]	5 T4 [C]	6 T5 [C]	7 T6 [C]	8 T7 [C]	9 Q_{conv} [W]
Run 1	200	49.84	49.8	49.79	49.96	49.93	49.91	49.9	476.9
Run 2	400	49.69	49.6	49.58	49.93	49.86	49.82	49.8	947.8
Run 3	600	49.54	49.41	49.37	49.9	49.79	49.73	49.71	1413
Run 4	800	49.39	49.21	49.17	49.86	49.72	49.64	49.61	1872
Run 5	1000	49.24	49.02	48.97	49.83	49.65	49.55	49.52	2325
Run 6	1500	48.88	48.56	48.48	49.74	49.49	49.33	49.29	3434
Run 7	2000	48.53	48.11	48	49.66	49.33	49.13	49.06	4510
Run 8	3000	47.85	47.25	47.1	49.51	49.02	48.73	48.64	6567
Run 9	4000	47.21	46.44	46.24	49.37	48.73	48.35	48.24	8507
Run 10	5000	46.61	45.67	45.44	49.23	48.46	48	47.86	10339

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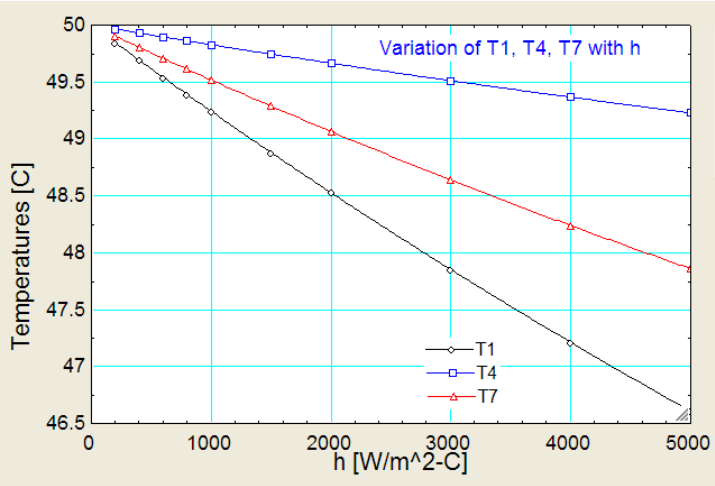
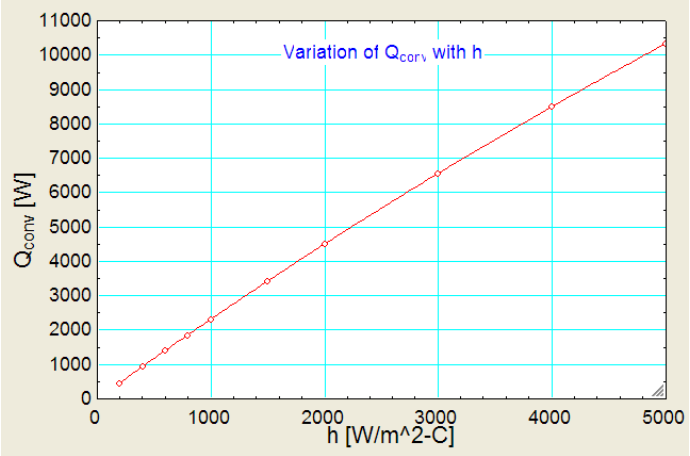
Plot the variation of typical temperatures T1, T4 and T7 with h:



Plot the variation of Q_conv with h:



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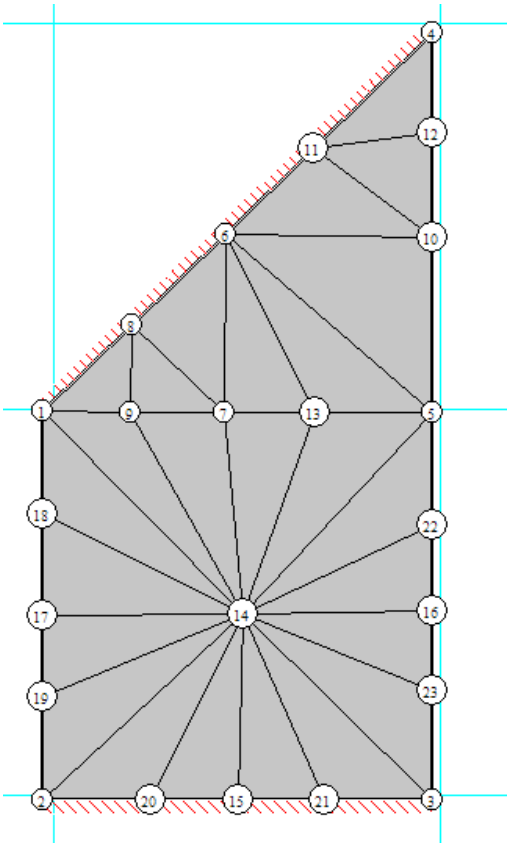
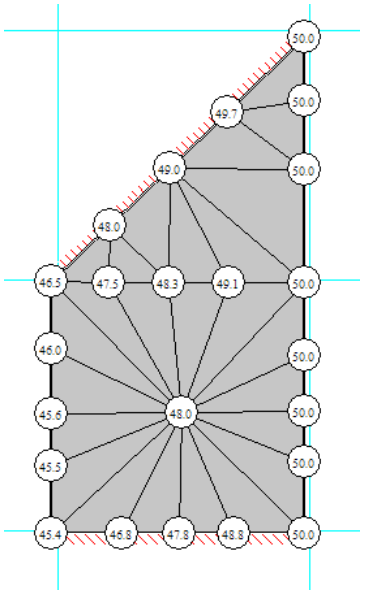
Solution to the above problem by Finite Element Heat Transfer (FEHT) Software:

1. **Node positions:**



2. **Node temperatures:**



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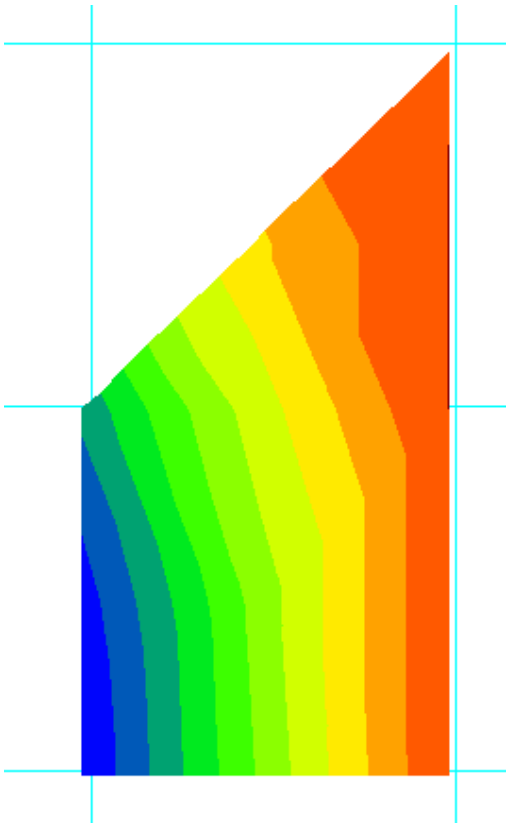
3. **Temperature contours:**



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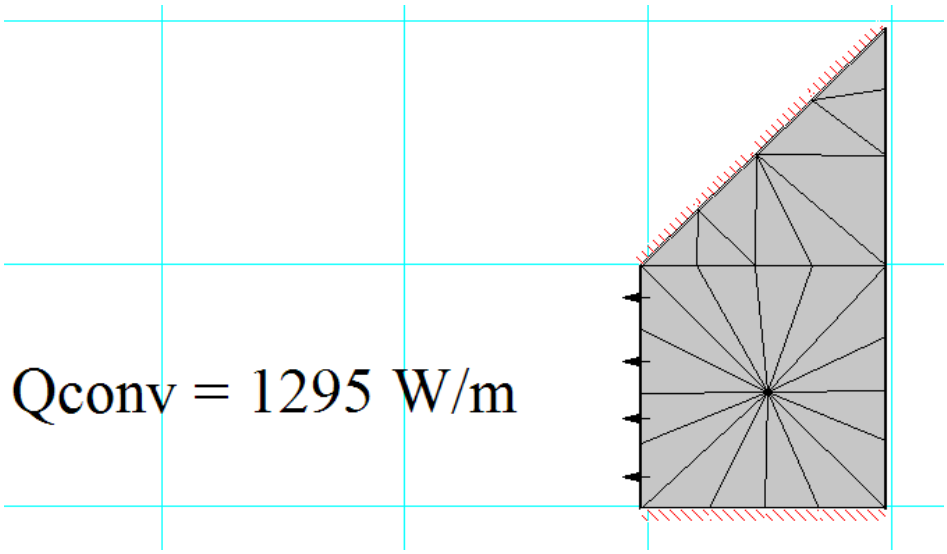
4. **Heat flows:**



Remembering that Q_conv calculated here is for one-eighth of the section, total heat transfer to inside surface by convection is:

Qtotal = 1295 × 8 = 10360 W.…Ans.

This matches very well with the value obtained earlier with Finite difference method, viz. Qtotal = 10339 W.

5. **Tabular Nodal results:**

Node	X [m]	Y [m]	T [°C]	Node Balance [W/m]
1	0.09975	-0.06006	46.46	-174.8
2	0.09975	-0.07011	45.41	-170.2
3	0.1098	-0.07011	**50**	153.2
4	0.1098	-0.05027	**50**	-5.601
5	0.1098	-0.06009	**50**	255.2
6	0.1045	-0.05549	48.99	-4.4409E-16
7	0.1044	-0.06008	48.34	4.4409E-15
8	0.1021	-0.05781	48.04	1.7764E-15
9	0.102	-0.06007	47.46	-7.1054E-15
10	0.1098	-0.05557	**50**	132.9
11	0.1067	-0.05325	49.66	1.1102E-16
12	0.1098	-0.05282	**50**	70.26
13	0.1068	-0.06009	49.1	1.0214E-14

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14	0.1049	-0.06529	48.04	1.3322E-15
15	0.1048	-0.07011	47.84	5.3291E-15
16	0.1098	-0.06521	**50**	206.8
17	0.09975	-0.06537	45.62	-302.8
18	0.09975	-0.06272	45.97	-344.8
19	0.09975	-0.06744	45.48	-302.5
20	0.1025	-0.07011	46.78	-6.6613E-15
21	0.107	-0.07011	48.83	-5.3291E-15
22	0.1098	-0.06299	**50**	246.4
23	0.1098	-0.06728	**50**	235.9

From the above Table, Nodal temperatures can be noted.

Also, at any given surface, the total heat transfer from that surface can be computed by adding up the nodal heat balances corresponding to the nodes on that surface. For example, considering the inner surface subjected to convection, nodes on that surface are (from top to down): 1, 18, 17, 19 and 2. And, the corresponding nodal balances are: -174.8, -344.8, -302.8, -302.5 and -170.2 W/m. (-ve sign indicates heat leaving the surface). Adding up, the Q_{conv} for this one-eighth section is: 1295.1W/m as obtained directly.

==========

Prob. 1I.B.7. A very long bar of square cross-section has its four sides held at constant temperatures as shown in Fig. Determine the temperatures at the internal nodes. Compare the results with analytical solution.

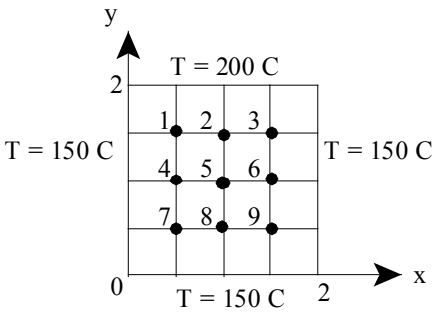

Fig.Prob.1IB.7

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Analytical solution for this problem is a little complicated and is given in terms of an infinite series, as follows:

$$\theta = \theta_c \cdot \frac{2}{\pi} \cdot \sum_{n=1}^{\infty} \frac{(-1)^{n+1}+1}{n} \cdot \frac{\sinh\left(n \cdot \pi \cdot \frac{y}{L}\right)}{\sinh\left(n \cdot \pi \cdot \frac{W}{L}\right)} \cdot \sin\left(n \cdot \pi \cdot \frac{x}{L}\right)$$

Nomenclature for the above eqn. for the present problem is as follows:

$\theta = T - 150$ ….T = temp. at the desired point; 150 C is the const. temp. on three sides

$\theta_c := 200 - 150$ ..temp. difference between the temp. of fourth side and the const. temp. of three sides.

n…no. of terms considered in the infinite series
x, y ….coordinates of the point where temp. is desired
L – 2 m…length along x-axis
W – 2 m ….length along y-axis

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However, this problem can be solved in EXCEL very easily, as shown below:

EXCEL Solution:

All the nine nodes are Internal nodes.

We have the following eqns for temperatures of Internal nodes in 2D conduction: [Ref. 1]

$$T_{node} = \frac{(T_{left} + T_{top} + T_{right} + T_{bottom})}{4}$$ ..when there is no heat gen.

And,

$$T_{node} = \frac{(T_{left} + T_{top} + T_{right} + T_{bottom}) + \frac{q_g \cdot \Delta x^2}{k}}{4}$$ ..when there is heat gen.

In EXCEL, we will solve this problem very easily by iteration.

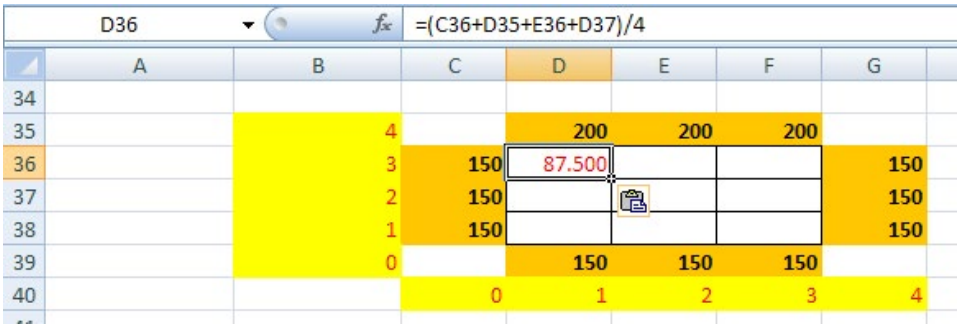
Following are the steps:

1. First, enable Iteration in EXCEL, as already explained.
2. Set up the 2D scheme as follows:

	A	B	C	D	E	F	G	H
34								
35		4		200	200	200		
36		3	150				150	
37		2	150				150	
38		1	150				150	
39		0		150	150	150		
40			0	1	2	3	4	
41								

Note how the Boundary temps are shown around the 9 Internal nodes. The 9 Nodes from 1, 2, …. 9 are: D36, E36, F36, D37, E37, F37, D38, E38 and F38 respectively.

3. Now, fill in the formula for Node 1, i.e. in cell D36.



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Note the eqn for T1 in the Formula bar.

4. Next, to fill in the eqns for other nodes is very easy: simply drag-copy the eqn from cell D36 to other cells:

F38 f_x =(E38+F37+G38+F39)/4

	A	B	C	D	E	F	G
34							
35		4		200	200	200	
36		3	150	171.429	176.339	171.429	150
37		2	150	159.375	162.500	159.375	150
38		1	150	153.571	154.911	153.571	150
39		0		150	150	150	
40			0	1	2	3	4

In the above fig. the formula for cell F38 can be seen in the Formula bar, after copying from D36.

The temperatures are immediately calculated by EXCEL by iteration.

Temperatures T1, T2, ….T9 can be read off from the above fig.

==========

Prob.1IB.8. Solve Prob.1IB.3 using EXCEL:

Let us repeat the problem statement below:

A long conducting rod of rectangular cross-section (20 mm × 30 mm) and k = 20 W/m.K experiences uniform heat generation of q_g = 5 × 10^7 W/m^3, while its surfaces are maintained at 300 K. (a) Using a finite difference method with a grid spacing of 5 mm, determine the temp distribution in the rod. (b) With the boundary conditions unchanged, what heat generation rate would cause the mid-point temp to reach 600 K? [Ref. 3]

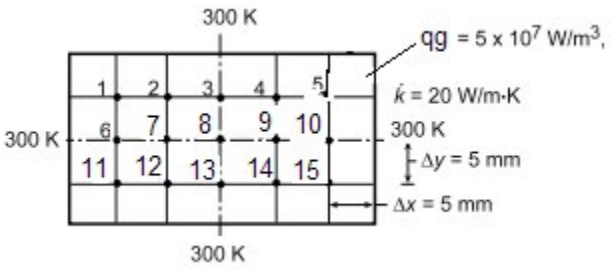

Fig.Prob.1I.B.3

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Note that all the 15 Nodes are Internal nodes.

When there is heat generation, the 2D difference equation for an Internal node is given by:

$$T_{node} = \frac{\left(T_{left} + T_{top} + T_{right} + T_{bottom}\right) + \frac{q_g \cdot \Delta x^2}{k}}{4} \quad \text{..when there is heat gen.}$$

So, for Node 1, we get:

T1 = (300 + 300 + T2 + T6 + qg *Δx^2 /k) / 4

For other nodes, eqns can be filled up in EXCEL, simply by drag-copy.

Following are the steps in EXCEL Solution:

1. Since solution is by iteration, first, enable iteration in EXCEL, as already explained.
2. Next, set up the scheme of Nodes and Boundary conditions. Lso, enter the data and name the cells. Calculate C_1 as shown:

	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Data:	k =	20	W/m.K	deltax=	0.005	m	qg*deltax^2/k =	C_1 =	62.5					
2		qg =	5.00E+07	W/m^3	deltay =	0.005	m								
3										Difference equations:					
4										For Internal nodes: For any internal node, the temp is given					
5		4		300	300	300	300	300							
6		3	300	1	2	3	4	5	300	$T_{west} + T_{north} + T_{east} + T_{south} - 4 \cdot T_{node} + q_g \frac{\Delta x^2}{k} = 0$					
7		2	300	6	7	8	9	10	300						
8		1	300	11	12	13	14	15	300	where Tnode is the temp of the node in question.					
9		0		300	300	300	300	300							
10			0	1	2	3	4	5	6						
11															

3. In the above Fig. Node nos. 1, 2, …15 are shown in the respective cells. Now, enter the eqn for Node 1 in cell E6. The eqn entered is shown in the Formula bar:

E6 f_x =((D6+E5+F6+E7)+C_1)/4

	B	C	D	E	F	G	H	I	J
4									
5		4		300	300	300	300	300	
6		3	300	167.63	2	3	4	5	300
7		2	300	6	7	8	9	10	300
8		1	300	11	12	13	14	15	300
9		0		300	300	300	300	300	
10			0	1	2	3	4	5	6

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4. Now, drag-copy horizontally up to Node 5, i.e. up to cell I6:

E6 =((D6+E5+F6+E7)+ C_1)/4

	B	C	D	E	F	G	H	I	J
4									
5		4		300	300	300	300	300	
6		3	300	215.54	193.68	189.66	194.48	216.74	300
7		2	300	6	7	8	9	10	300
8		1	300	11	12	13	14	15	300
9		0		300	300	300	300	300	
10			0	1	2	3	4	5	6

5. And, drag-copy the entire line vertically up to Node 15, i.e. upto cell I8:

	B	C	D	E	F	G	H	I	J
4									
5		4		300	300	300	300	300	
6		3	300	348.46	368.94	374.60	368.94	348.46	300
7		2	300	362.41	390.21	398.03	390.21	362.41	300
8		1	300	348.46	368.94	374.60	368.94	348.46	300
9		0		300	300	300	300	300	
10			0	1	2	3	4	5	6

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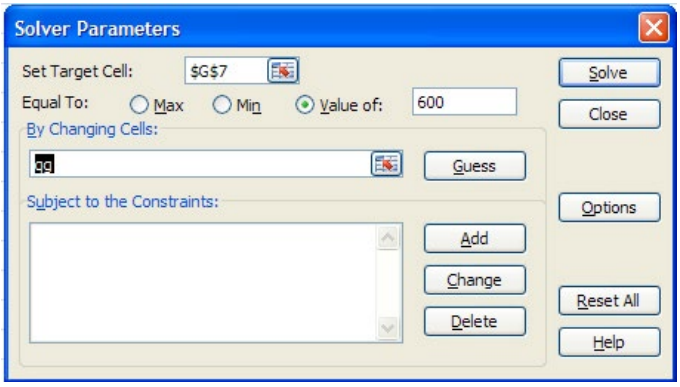
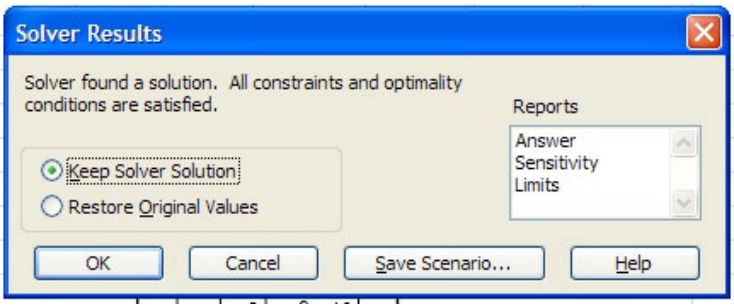
Note in the above Fig that the temps in all the 15 Nodes are calculated immediately. Compare these temps with those obtained using EES earlier.

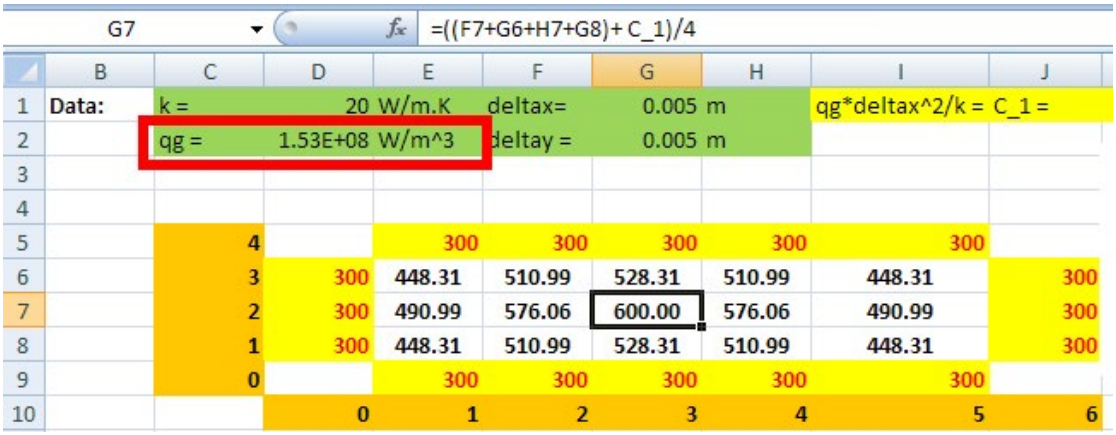
6. Next, we have to find out the qg required to get the centre temp (i.e. Node 8, or cell G7) as 600K. We shall use the Solver, where **Target cell is** Cell G7, **Equal to:** 600, **By changing cell:** D2 (named as qg). We get:



Press Solve. We get:



Press OK and, observe that the value of qg has changed to 1.53 E08 W/m^3:



1	Data:	k =	20	W/m.K	deltax=	0.005	m	qg*deltax^2/k =	C_1 =
2		qg =	1.53E+08	W/m^3	deltay =	0.005	m		
3									
4									
5		4		300	300	300	300	300	
6		3	300	448.31	510.99	528.31	510.99	448.31	300
7		2	300	490.99	576.06	600.00	576.06	490.99	300
8		1	300	448.31	510.99	528.31	510.99	448.31	300
9		0		300	300	300	300	300	
10			0	1	2	3	4	5	6

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Also, observe that all the Node temps, with this value of qg, are also automatically calculated.

We can note that in cases such as this, where all the boundary temps are known, calculating the internal node temps is extremely easy while using EXCEL.

==========

Prob.1IB.9. LHS and RHS of a 1 cm × 2 cm ceramic strip are maintained at constant temp of 300 C and the bottom side is insulated. Top surface is exposed to convection with a fluid at Ta = 50 C with heat transfer coeff. h = 200 W/m^2.C. Determine the steady state temps on nodes 1 to 9.

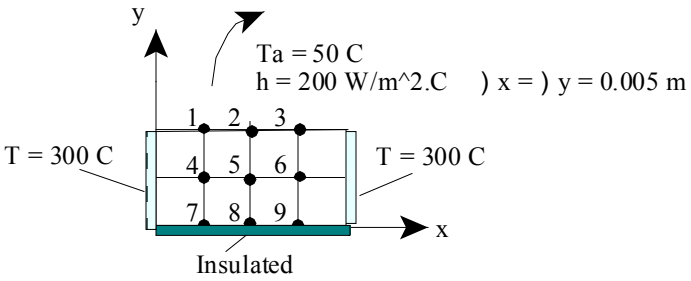

Fig.Prob.1IB.9

EXCEL Solution:

Difference eqns for various Nodes are obtained by making an energy balance at each node, remembering to write all the heat flows as going ***into*** the node.

Difference eqns for the 9 Nodes are:

$$\text{Node 1:}\quad k\cdot\frac{\Delta y}{2}\cdot\frac{(T_L - T1)}{\Delta x} + \left[k\cdot\frac{\Delta y}{2}\cdot\frac{(T2 - T1)}{\Delta x}\right] + \left[k\cdot\Delta x\cdot\frac{(T4 - T1)}{\Delta y}\right] + h\cdot\Delta x\cdot(Ta - T1) = 0$$

$$\text{Node 2:}\quad k\cdot\frac{\Delta y}{2}\cdot\frac{(T1 - T2)}{\Delta x} + \left[k\cdot\frac{\Delta y}{2}\cdot\frac{(T3 - T2)}{\Delta x}\right] + \left[k\cdot\Delta x\cdot\frac{(T5 - T2)}{\Delta y}\right] + h\cdot\Delta x\cdot(Ta - T2) = 0$$

$$\text{Node 3:}\quad k\cdot\frac{\Delta y}{2}\cdot\frac{(T2 - T3)}{\Delta x} + \left[k\cdot\frac{\Delta y}{2}\cdot\frac{(T_R - T3)}{\Delta x}\right] + \left[k\cdot\Delta x\cdot\frac{(T6 - T3)}{\Delta y}\right] + h\cdot\Delta x\cdot(Ta - T3) = 0$$

$$\text{Node 4:}\quad k\cdot\Delta y\cdot\frac{(T_L - T4)}{\Delta x} + \left[k\cdot\Delta y\cdot\frac{(T5 - T4)}{\Delta x}\right] + \left[k\cdot\Delta x\cdot\frac{(T7 - T4)}{\Delta y}\right] + k\cdot\Delta x\cdot\frac{(T1 - T4)}{\Delta y} = 0$$

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Node 5: $$k\cdot\Delta y\cdot\frac{(T4-T5)}{\Delta x}+\left[k\cdot\Delta y\cdot\frac{(T6-T5)}{\Delta x}\right]+\left[k\cdot\Delta x\cdot\frac{(T8-T5)}{\Delta y}\right]+k\cdot\Delta x\cdot\frac{(T2-T5)}{\Delta y}=0$$

Node 6: $$k\cdot\Delta y\cdot\frac{(T5-T6)}{\Delta x}+\left[k\cdot\Delta y\cdot\frac{(T_R-T6)}{\Delta x}\right]+\left[k\cdot\Delta x\cdot\frac{(T9-T6)}{\Delta y}\right]+k\cdot\Delta x\cdot\frac{(T3-T6)}{\Delta y}=0$$

Node 7: $$k\cdot\frac{\Delta y}{2}\cdot\frac{(T_L-T7)}{\Delta x}+\left[k\cdot\frac{\Delta y}{2}\cdot\frac{(T8-T7)}{\Delta x}\right]+\left[k\cdot\Delta x\cdot\frac{(T4-T7)}{\Delta y}\right]+0=0$$

Node 8: $$k\cdot\frac{\Delta y}{2}\cdot\frac{(T7-T8)}{\Delta x}+\left[k\cdot\frac{\Delta y}{2}\cdot\frac{(T9-T8)}{\Delta x}\right]+\left[k\cdot\Delta x\cdot\frac{(T5-T8)}{\Delta y}\right]+0=0$$

Node 9: $$k\cdot\frac{\Delta y}{2}\cdot\frac{(T8-T9)}{\Delta x}+\left[k\cdot\frac{\Delta y}{2}\cdot\frac{(T_R-T9)}{\Delta x}\right]+\left[k\cdot\Delta x\cdot\frac{(T6-T9)}{\Delta y}\right]+0=0$$

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1. We will use Solver in EXCEL; so, enable Solver, as already explained.
2. Set up the scheme in EXCEL as shown. Enter the data, and name the cells.

	A	B	C	D	E	F	G	H	I
1			Data:						
2				k=	3	W/m.C			
3				deltay	0.005	m	h	200	W/m^2.C
4				deltax=	0.005	m	Ta	50	C
5									
6					h=200 W/m^2.C, Ta = 50 C				
7				300	1	2	3	300	
8				300	4	5	6	300	
9				300	7	8	9	300	
10					Insulated	Insulated	Insulated		
11									
12									
13				Diff. eqns:					
14									
15									
16									
17									
18				SumSq=	0				

In the above fig, 1, 2…9 are the Nodes. i.e. through the cells E7:G9. The conditions at the boundaries are also shown.

Below the Temps in the Nodes, we have a similar Table where Difference eqns are entered for each node (i.e. in cells E14:G16), and the Sum of the squares of these difference eqns is also shown in cell E18.

3. Now, enter the difference eqns in the respective nodes.

E14 =k*(deltay/2)*(D7-E7)/deltax+k*(deltay/2)*(F7-E7)/deltax+h*deltax*(Ta-E7)+k*deltax*(E8-E7)/deltay

	A	B	C	D	E	F	G	H	I	J	K	L	M
13				Diff. eqns:									
14					508	57	500						
15					891	0	879						
16					432	-9	426						
17													
18				SumSq=	2446016								

In the above Fig. difference eqn for Node 1 can be seen in the Formula bar.

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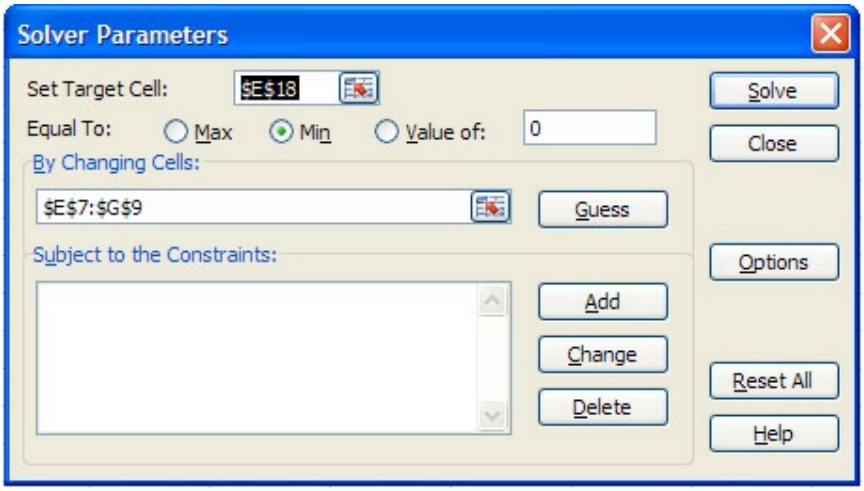
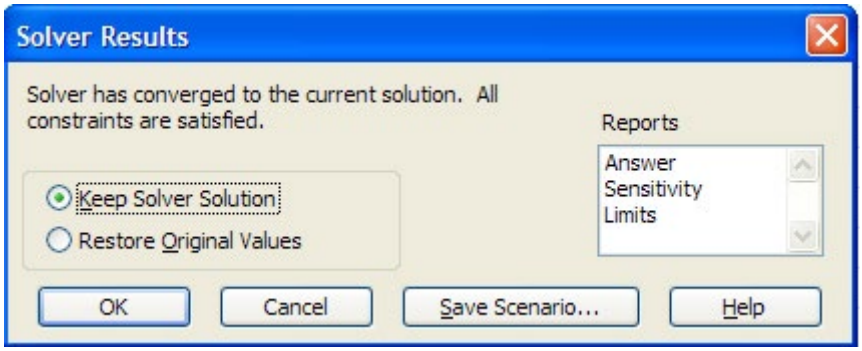
4. Now, our aim is to make Sum of squares of difference eqns (i.e. cell E18) to a minimum, by changing the cells corresponding to T1to Ts, i.e. cells E7 to G9. So, apply the Solver from the Data tab:



Press Solve, and we get:



Press Keep the Solver Solution, and we get the Node temps as follows:

	h=200 W/m^2.C, Ta = 50 C			
300	229.504	212.448	229.504	300
300	262.619	249.543	262.619	300
300	271.431	260.487	271.431	300
	Insulated	Insulated	Insulated	

Diff. eqns:			
0.003585	0.003548	0.003469	
0.003549	0.003533	0.003416	
0.001542	7.37E-05	0.001371	

SumSq=	7.85E-05

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Note that Sum of squares is almost zero (as it should be). Read off the temps T1 to T9.

==========

1IC. One-dimensional, transient conduction:

Prob. 1I.C.1. A large Uranium plate of thickness L = 8 cm, (k = 28 W/(m.C), $\alpha = 12.5 \times 10^{-6}$ m^2/s) is initially at an uniform temperature of 100 C. Heat gen. rate in the plate is 10^5 W/m^3. At time $\tau = 0$, the left side is insulated, and right side of the plate is subjected to convection with a fluid at temperature of 20 C and a heat transfer coeff. of 35 W/(m^2.C). Using a uniform nodal spacing of 2 cm, develop the explicit finite difference formulations for all nodes, and determine the temperature distribution in the plate after 5 min. Also, find out how long it will take for steady conditions to be reached in the plate. [Ref.2]

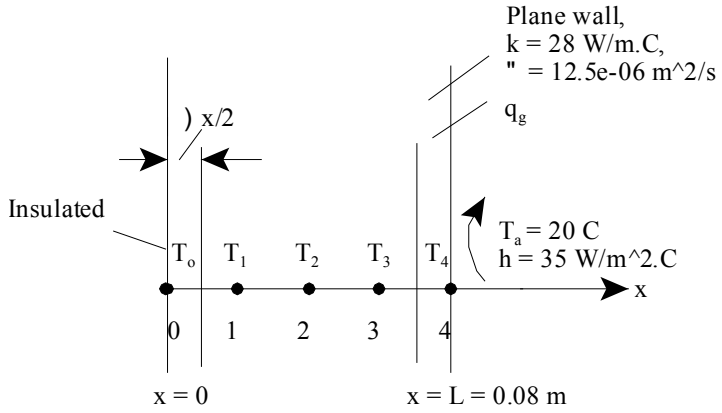

Fig.Prob.1I.C.1

Mathcad Solution:

Data:

$L := 0.08$ m....thickness of plate

$k := 28$ W/(m.C)..thermal cond. of plate

$\alpha := 12.5 \cdot 10^{-6}$ m^2/s....thermal diffusivity of plate

$q_g := 10^6$ W/m^3...heat gen. rate in the plate

$T := 100$ C...initial temp. of plate

$T_a := 20$ C....temp. of ambient fluid

$h := 35$ $W/(m^2.C)$....heat tr. coeff. between the ambient fluid and the plate surface.

$\Delta x := 0.02$ m....nodal spacing

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$\tau := 300$ s...time after which temp. distribution in plate is desired

$A := 1$ m^2.... area of cross-section, assumed.

Now, we have to fix the upper limit of $\Delta\tau$ from stability criterion. To do that, we have to ensure that the smaller coeff. of T_m^i in the eqns for temperatures is greater than (or equal to) zero, i.e. (1 - 2. Fo -2.Fo.Bi) must be greater than or equal to zero. Putting this condition, we get:

$$1 - 2\cdot Fo - 2\cdot Fo\cdot\frac{h\cdot\Delta x}{k} \geq 0$$

i.e. $$Fo \leq \frac{1}{2\cdot\left(1+\frac{h\cdot\Delta x}{k}\right)}$$

i.e. $$\Delta\tau \leq \frac{(\Delta x)^2}{2\cdot\alpha\cdot\left(1+\frac{h\cdot\Delta x}{k}\right)}$$

i.e. $\Delta\tau \leq 15.61$ s

This means that a time step less than 15.61 s has to be employed from stability criterion.

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Let us choose:

$$\Delta\tau := 15 \quad s$$

i.e. $Fo := \frac{\alpha \cdot \Delta\tau}{(\Delta x)^2}$ $Bi := \frac{h \cdot \Delta x}{k}$ $Fo = 0.469$ $Bi = 0.025$

Also, $\alpha = \frac{k}{\rho \cdot cp}$ Therefore: $\rho \cdot cp = \frac{k}{\alpha}$

Difference eqns. for interior nodes:

Nodes 1, 2, 3 are interior nodes. Finite difference equations for these nodes by explicit method are obtained from:

$$T_{left} - 2 \cdot T_m + T_{right} + \frac{q_m \cdot \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^{i}}{Fo} \quad \text{..for m = 1,2,3.....eqn.(A)}$$

where 'left' and 'right' refer to nodes immediately to the left and right of node 'm'.
And, the Fourier no. given by:

$$Fo = \frac{\alpha \cdot \Delta\tau}{\Delta x^2} \quad \text{...finite difference form of Fourier no.}$$

However, we will not use eqn.(A), but will work from fundamentals and write the energy balance for an internal node as follows (Remember: all heat flow lines **into** the Node):

$$k \cdot A \cdot \frac{\left(T_{m-1}\right)^i - \left(T_m\right)^i}{\Delta x} + k \cdot A \cdot \frac{\left(T_{m+1}\right)^i - \left(T_m\right)^i}{\Delta x} + q_g \cdot (A \cdot \Delta x) = \rho \cdot (A \cdot \Delta x) \cdot cp \cdot \frac{T_m^{i+1} - T_m^{i}}{\Delta\tau} \quad \text{...eqn.(B)}$$

Further, in explicit method, at a given node, the temperature on the LHS of above eqn., are chosen at the previous step 'i'.

Eqn.(B) is written for all internal nodes.

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Difference eqns. for boundary nodes:

For node '0':

Node '0' is on the left surface, insulated. Applying the energy balance to the half-volume surrounding node '0'::

Node 0 ... on insulated surface:

$$k \cdot A \cdot \frac{\left(T_1\right)^i - \left(T_0\right)^i}{\Delta x} + q_g \cdot \left(A \cdot \frac{\Delta x}{2}\right) = \rho \cdot \left(A \cdot \frac{\Delta x}{2}\right) \cdot cp \cdot \frac{T_0^{\,i+1} - T_0^{\,i}}{\Delta \tau} \qquad \text{.....eqn.(C)}$$

For node 4… on surface with convection:

This is a node with convection boundary condition. So, applying the energy balance to the half-volume around node 4, with all the heat lines flowing into the element, we get:

$$h \cdot A \cdot \left(T_a - T_4^{\,i}\right) + k \cdot A \cdot \left\{\frac{T_3^{\,i} - T_4^{\,i}}{\Delta x}\right\} + q_g \cdot A \cdot \frac{\Delta x}{2} = \rho \cdot A \cdot \frac{\Delta x}{2} \cdot C_p \cdot \frac{T_4^{\,i+1} - T_4^{\,i}}{\Delta \tau} \qquad \text{...eqn.(D)}$$

In the explicit method, in all the above eqns. (B)…(D), the unknown temp at the 'next step (i+1)' at each node can be explicitly calculated since it is the only unknown in each eqn. However, while using Mathcad (or EES), it is preferable to write the energy balance and solve, since it is more instructive.

So, let us write all the eqns for all the nodes 0…4 and then use the Solve Block of Mathcad to get the temperatures:

This calculation is easily done in Mathcad.

We slightly change the notation for convenience in calculation: we write the superscripts as subscripts to work with matrix notation, as shown below:

$T0_0 := 100$ $T1_0 := 100$ $T2_0 := 100$ $T3_0 := 100$ $T4_0 := 100$ ....Initial values

$T0_1 := 100$ $T1_1 := 100$ $T2_1 := 100$ $T3_1 := 100$ $T4_1 := 100$ ...guess values for temperatures at next time step

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Given

$$k \cdot A \cdot \frac{T1_0 - T0_0}{\Delta x} + q_g \cdot \left(A \cdot \frac{\Delta x}{2}\right) = \frac{k}{\alpha} \cdot \left(A \cdot \frac{\Delta x}{2}\right) \cdot \frac{T0_1 - T0_0}{\Delta \tau}$$ for Node 0: from.....eqn.(C)

$$k \cdot A \cdot \frac{T0_0 - T1_0}{\Delta x} + k \cdot A \cdot \frac{T2_0 - T1_0}{\Delta x} + q_g \cdot (A \cdot \Delta x) = \frac{k}{\alpha} \cdot (A \cdot \Delta x) \cdot \frac{T1_1 - T1_0}{\Delta \tau}$$ ..for Node 1... from eqn.(A)

$$k \cdot A \cdot \frac{T1_0 - T2_0}{\Delta x} + k \cdot A \cdot \frac{T3_0 - T2_0}{\Delta x} + q_g \cdot (A \cdot \Delta x) = \frac{k}{\alpha} \cdot (A \cdot \Delta x) \cdot \frac{T2_1 - T2_0}{\Delta \tau}$$ ..for Node 2...

$$k \cdot A \cdot \frac{T2_0 - T3_0}{\Delta x} + k \cdot A \cdot \frac{T4_0 - T3_0}{\Delta x} + q_g \cdot (A \cdot \Delta x) = \frac{k}{\alpha} \cdot (A \cdot \Delta x) \cdot \frac{T3_1 - T3_0}{\Delta \tau}$$ ..for Node 3...

$$h \cdot A \cdot \left(T_a - T4_0\right) + k \cdot A \cdot \left(\frac{T3_0 - T4_0}{\Delta x}\right) + q_g \cdot A \cdot \frac{\Delta x}{2} = \frac{k}{\alpha} \cdot A \cdot \frac{\Delta x}{2} \cdot \frac{T4_1 - T4_0}{\Delta \tau}$$ for Node 4... from...eqn.(D)

$$Temp\left(T0_0, T1_0, T2_0, T3_0, T4_0\right) := Find\left(T0_1, T1_1, T2_1, T3_1, T4_1\right)$$

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Note: 'Temp' vector is defined as a function of initial temperatures $T0_0$….$T4_0$.

This is useful to repeatedly use this function to get temperatures at the next step, using the temperatures obtained in the previous step.

So, after the first time step, we get the new temperatures as:

$$\text{Temp}\left(T0_0, T1_0, T2_0, T3_0, T4_0\right) = \begin{bmatrix} 106.696 \\ 106.696 \\ 106.696 \\ 106.696 \\ 104.821 \end{bmatrix}$$

i.e. New Temps at Nodes 0 to 4 are: 106.696, 106.696, 106.696, 106.696 and 104.821 C.

Now, set these temperatures as starting temperatures in the Temp function and get the new temperatures at the second time step…and so on.

In the small Mathcad program given below, LHS defines a function Temp(n) where n is the no. of time steps, which we can specify. Output is a vector containing step no., total time elapsed, and node temperatures T0, T1,…T4.

On the RHS, first 5 lines define the initial temperatures at the nodes, all equal to 100 C.

Then, a 'while loop' uses the Temp function defined above with these initial temperatures and determines the new temperatures at each node. Then, again this is repeated by setting the new temperatures as initial temperatures and calling the Temp function. Here, the no. of time steps, 'n' can be changed since it is included in function definition on the LHS.

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```
Explcit_Temp(n) := | T0← 100
                   | T1← 100
                   | T2← 100
                   | T3← 100
                   | T4← 100
                   | i← 0
                   | while i<n
                   |     TTemp← Temp(T0, T1, T2, T3, T4)
                   |     T0← TTemp_0
                   |     T1← TTemp_1
                   |     T2← TTemp_2
                   |     T3← TTemp_3
                   |     T4← TTemp_4
                   |     i← i + 1
                   | (i  15·i  T0  T1  T2  T3  T4)
```

Check:

Explcit_Temp(0) = [0 0 100 100 100 100 100] ..Temps at Step 0....i.e. initial conditions

i = step no.; Δτ = one time step = 15 s; τ = time duration from beginning = i. Δτ, s

i	τ (s)	T0 (C)	T1 (C)	T2 (C)	T3 (C)	T4 (C)
Explcit_Temp(1) =	1	15	106.696	106.696	106.696	106.696	104.821
Explcit_Temp(2) =	2	30	113.393	113.393	113.393	112.514	111.288

Temps after 5 min. i.e. after 20 Time steps:

Explcit_Temp(20) =	20	300	228.928	228.396	226.83	224.038	219.894

i.e. After 5 min (=300 s), the temperatures are:

T0 = 228.928, T1 = 228.396, T2 = 226.83, T3 = 224.038, T4 = 219.894 deg.C….Ans.

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At what time Steady State is reached?

When steady state is reached, temperatures at successive time steps should not change much.

Explcit_Temp(2000) = [2000 30000 2412.083 2404.964 2383.607 2348.012 2298.177]

Explcit_Temp(2020) = [2020 30300 2412.52 2405.4 2384.039 2348.437 2298.593]

We see from the above that at 2000th time step, i.e. after 30000 s (= 500 min.), the temperatures have stabilized. There is not much difference in temperatures when we compare the values for 2000th and 2020th time steps.

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Program to get the temps at all Nodes with Time:

```
Explicit_NodeTemps(n) :=
    T0 ← 100
    T1 ← 100
    T2 ← 100
    T3 ← 100
    T4 ← 100
    i ← 0
    while i<n
        TTemp ← Temp(T0, T1, T2, T3, T4)
        T0 ← TTemp_0
        T1 ← TTemp_1
        T2 ← TTemp_2
        T3 ← TTemp_3
        T4 ← TTemp_4
        Step_i ← i
        Time_i ← i·Δτ
        A0_0 ← 100
        A1_0 ← 100
        A2_0 ← 100
        A3_0 ← 100
        A4_0 ← 100
        A0_i ← T0
        A1_i ← T1
        A2_i ← T2
        A3_i ← T3
        A4_i ← T4
        C1 ← augment(A3, A4)
        C2 ← augment(A2, C1)
        C3 ← augment(A1, C2)
        C4 ← augment(A0, C3)
        TProfile ← augment(Time, C4)
        TProfile ← augment(Step, TProfile)
        i ← i + 1
    return TProfile
```


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Program shown above returns a Matrix as follows:

0th column: gives Time step

1st column: gives Time (s)

2nd to 6th column: Temps. T0, T1, T2, T3 and T4 at different times

Note: In Mathcad Matrix, Rows and Columns are numbered from 0 onwards (though it can be set to start from 1).

As an example, after 20 steps, we get for Nodal temperatures:

Explicit_NodeTemps(20) =

	0	1	2	3	4	5	6
0	0	0	100	100	100	100	100
1	1	15	113.393	113.393	113.393	112.514	111.288
2	2	30	120.089	120.089	119.677	119.048	116.994
3	3	45	126.786	126.593	126.272	125.077	123.342
4	4	60	133.301	133.229	132.558	131.52	129.243
5	5	75	139.93	139.645	139.083	137.636	135.514
6	6	90	146.359	146.211	145.364	144.016	141.492
7	7	105	152.917	152.58	151.826	150.161	147.707
8	8	120	159.298	159.081	158.096	156.487	153.711
9	9	135	165.791	165.417	164.5	162.636	159.877
10	10	150	172.137	171.859	170.753	168.913	165.882
11	11	165	178.573	178.167	177.105	175.051	172.001
12	12	180	184.889	184.556	183.336	181.28	177.994
13	13	195	191.273	190.837	189.641	187.4	184.068
14	14	210	197.56	197.177	195.848	193.585	190.043
15	15	225	203.897	203.43	202.107	199.682	196.075
16	16	240	210.156	209.725	208.287	205.824	202.026
17	17	255	216.449	215.949	214.503	211.894	208.017
18	18	270	222.677	222.202	220.655	217.996	213.942
19	19	285	228.928	228.396	226.83	224.038	219.894

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To plot temps T0 and T4 with Time:

Let us consider time steps upto 2700.

Define: TTemp := Explicit_NodeTemps(2701)

TTemp =

	0	1	2	3	4	5	6
2634	$2.634 \cdot 10^3$	$3.951 \cdot 10^4$	$2.419 \cdot 10^3$	$2.412 \cdot 10^3$	$2.39 \cdot 10^3$	$2.354 \cdot 10^3$	$2.304 \cdot 10^3$
2635	$2.635 \cdot 10^3$	$3.953 \cdot 10^4$	$2.419 \cdot 10^3$	$2.412 \cdot 10^3$	$2.39 \cdot 10^3$	$2.354 \cdot 10^3$	$2.304 \cdot 10^3$
2636	$2.636 \cdot 10^3$	$3.954 \cdot 10^4$	$2.419 \cdot 10^3$	$2.412 \cdot 10^3$	$2.39 \cdot 10^3$	$2.354 \cdot 10^3$	$2.304 \cdot 10^3$
2637	$2.637 \cdot 10^3$	$3.956 \cdot 10^4$	$2.419 \cdot 10^3$	$2.412 \cdot 10^3$	$2.39 \cdot 10^3$	$2.354 \cdot 10^3$	$2.304 \cdot 10^3$
2638	$2.638 \cdot 10^3$	$3.957 \cdot 10^4$	$2.419 \cdot 10^3$	$2.412 \cdot 10^3$	$2.39 \cdot 10^3$	$2.354 \cdot 10^3$	$2.304 \cdot 10^3$
2639	$2.639 \cdot 10^3$	$3.959 \cdot 10^4$	$2.419 \cdot 10^3$	$2.412 \cdot 10^3$	$2.39 \cdot 10^3$	$2.354 \cdot 10^3$	$2.304 \cdot 10^3$
2640	$2.64 \cdot 10^3$	$3.96 \cdot 10^4$	$2.419 \cdot 10^3$	$2.412 \cdot 10^3$	$2.39 \cdot 10^3$	$2.354 \cdot 10^3$	$2.304 \cdot 10^3$
2641	$2.641 \cdot 10^3$	$3.962 \cdot 10^4$	$2.419 \cdot 10^3$	$2.412 \cdot 10^3$	$2.39 \cdot 10^3$	$2.354 \cdot 10^3$	$2.304 \cdot 10^3$
2642	$2.642 \cdot 10^3$	$3.963 \cdot 10^4$	$2.419 \cdot 10^3$	$2.412 \cdot 10^3$	$2.39 \cdot 10^3$	$2.354 \cdot 10^3$	$2.305 \cdot 10^3$
2643	$2.643 \cdot 10^3$	$3.965 \cdot 10^4$	$2.419 \cdot 10^3$	$2.412 \cdot 10^3$	$2.39 \cdot 10^3$	$2.354 \cdot 10^3$	$2.305 \cdot 10^3$

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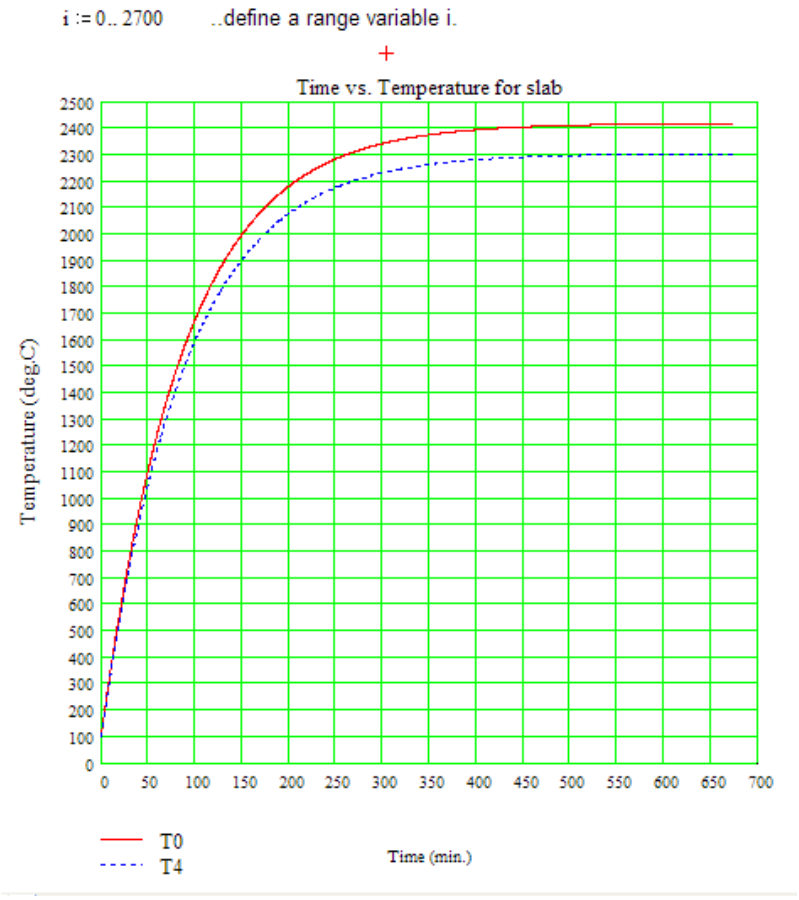

Note: At Time = 660 min (= 11 hrs), i.e. Step no. 2640: T0 = 2419 C, T4 = 2304 C

==========

Prob. 1I.C.2. Solve the above problem by Implicit method. Use the same data.

Mathcad Solution:

In Implicit method, the heat balance equations are the same as earlier; however, the temperatures are considered at the 'future time' i.e. at time step (I + 1). So, at each step all the nodal equations have to be solved simultaneously. While using Mathcad (or EES), this is not a problem.

So, instead of using readily available formula for interior and boundary nodes for various situations, let us work from fundamentals, by writing energy balance eqns at each node and solving them in Mathcad (Solve Block) or EES.

Also, there is no restriction on the choice of time step $\Delta\tau$ as in the case of Explicit Method. So, we can choose a higher time step.

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In the present case, however, let us choose $\Delta\tau$ = 15 s, for comparison with Explicit Method.

$\Delta\tau := 15$ $A := 1$

$Fo := \frac{\alpha \cdot \Delta\tau}{(\Delta x)^2}$ $Bi := \frac{h \cdot \Delta x}{k}$ $Fo = 0.469$ $Bi = 0.025$

Difference eqns. for interior nodes:

Nodes 1, 2, 3 are interior nodes. Applying energy balance, with all heat flow lines going into the node:

$$k \cdot A \cdot \frac{\left(T_{m-1}\right)^{i+1} - \left(T_m\right)^{i+1}}{\Delta x} + k \cdot A \cdot \frac{\left(T_{m+1}\right)^{i+1} - \left(T_m\right)^{i+1}}{\Delta x} + q_g \cdot (A \cdot \Delta x) = \rho \cdot (A \cdot \Delta x) \cdot cp \cdot \frac{T_m^{\,i+1} - T_m^{\,i}}{\Delta\tau}$$

...eqn.(B)... for nodes 1,2,3

Here, note that all temperatures are considered at 'future time', (i + 1). Since they are all unknown yet, all the nodal the eqns have to be solved simultaneously.

Difference eqns. for boundary nodes:

For node '0':

Node '0' is on the left surface, insulated. Applying the energy balance for the half-volume around node '0':

$$k \cdot A \cdot \frac{\left(T_1\right)^{i+1} - \left(T_0\right)^{i+1}}{\Delta x} + q_g \cdot \left(A \cdot \frac{\Delta x}{2}\right) = \rho \cdot \left(A \cdot \frac{\Delta x}{2}\right) \cdot cp \cdot \frac{T_0^{\,i+1} - T_0^{\,i}}{\Delta\tau} \qquad \text{.....eqn.(C)}$$

For node 4:

This is a node with convection boundary condition. So, applying the energy balance to the half-volume around node 4, with all the heat lines flowing into the element, we get:

$$h \cdot A \cdot \left(T_a - T_4^{\,i+1}\right) + k \cdot A \cdot \left(\frac{T_3^{\,i+1} - T_4^{\,i+1}}{\Delta x}\right) + q_g \cdot A \cdot \frac{\Delta x}{2} = \rho \cdot A \cdot \frac{\Delta x}{2} \cdot C_p \cdot \frac{T_4^{\,i+1} - T_4^{\,i}}{\Delta\tau} \qquad \text{...eqn.(D)}$$

So, let us write all the eqns for all the nodes 0 … 4 and then use the Solve Block of Mathcad to get the temperatures.

This calculation is easily done in Mathcad. We slightly change the notation for convenience in calculation: we write the superscripts as subscripts to work with matrix notation, as shown below.

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$T0_0 := 100$ $T1_0 := 100$ $T2_0 := 100$ $T3_0 := 100$ $T4_0 := 100$

$T0_1 := 50$ $T1_1 := 50$ $T2_1 := 50$ $T3_1 := 50$ $T4_1 := 50$ ...guess values

Given

$$k \cdot A \cdot \frac{T1_1 - T0_1}{\Delta x} + q_g \cdot \left(A \cdot \frac{\Delta x}{2}\right) = \frac{k}{\alpha} \cdot \left(A \cdot \frac{\Delta x}{2}\right) \cdot \frac{T0_1 - T0_0}{\Delta \tau}$$ for Node 0: from.....eqn.(C)

$$k \cdot A \cdot \frac{T0_1 - T1_1}{\Delta x} + k \cdot A \cdot \frac{T2_1 - T1_1}{\Delta x} + q_g \cdot (A \cdot \Delta x) = \frac{k}{\alpha} \cdot (A \cdot \Delta x) \cdot \frac{T1_1 - T1_0}{\Delta \tau}$$ ..for Node 1...

$$k \cdot A \cdot \frac{T1_1 - T2_1}{\Delta x} + k \cdot A \cdot \frac{T3_1 - T2_1}{\Delta x} + q_g \cdot (A \cdot \Delta x) = \frac{k}{\alpha} \cdot (A \cdot \Delta x) \cdot \frac{T2_1 - T2_0}{\Delta \tau}$$ ..for Node 2...

$$k \cdot A \cdot \frac{T2_1 - T3_1}{\Delta x} + k \cdot A \cdot \frac{T4_1 - T3_1}{\Delta x} + q_g \cdot (A \cdot \Delta x) = \frac{k}{\alpha} \cdot (A \cdot \Delta x) \cdot \frac{T3_1 - T3_0}{\Delta \tau}$$ ..for Node 3...

$$h \cdot A \cdot \left(T_a - T4_1\right) + k \cdot A \cdot \left(\frac{T3_1 - T4_1}{\Delta x}\right) + q_g \cdot A \cdot \frac{\Delta x}{2} = \frac{k}{\alpha} \cdot A \cdot \frac{\Delta x}{2} \cdot \frac{T4_1 - T4_0}{\Delta \tau}$$ for Node 4... from...eqn.(D)

$$Temp\left(T0_0, T1_0, T2_0, T3_0, T4_0\right) := Find\left(T0_1, T1_1, T2_1, T3_1, T4_1\right)$$

Note that the Vector Temp is written as a function of starting temperatures.

Then, the new temperatures, viz. $T0_1 \ldots T4_1$ are given as:

$$Temp\left(T0_0, T1_0, T2_0, T3_0, T4_0\right) = \begin{bmatrix} 106.686 \\ 106.675 \\ 106.617 \\ 106.391 \\ 105.514 \end{bmatrix}$$ ....after one time step of 15 s

i.e. New Temps at Nodes 0 to 4 are: 106.686, 106.675, 106.617, 106.391 and 105.514 C.

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Now, set these temperatures as starting temperatures in the Temp function and get the new temperatures at the second time step…and so on.

In the small Mathcad program given below, LHS defines a function Temp(n) where n is the no. of time steps, which we can specify. Output is a vector containing step no., total time elapsed, and node temperatures T0, T1,…T4.

On the RHS, first 5 lines define the initial temperatures at the nodes, all equal to 100 C.

Then, a 'while loop' uses the Temp function defined above with these initial temperatures and determines the new temperatures at each node. Then, again this is repeated by setting the new temperatures as initial temperatures and calling the Temp function. Here, the no. of time steps, 'n' can be changed since it is included in function definition on the LHS.

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```
Implcit_Temp(n) := | T0 ← 100
                   | T1 ← 100
                   | T2 ← 100
                   | T3 ← 100
                   | T4 ← 100
                   | i ← 0
                   | while i<n
                   |     TTemp ← Temp(T0, T1, T2, T3, T4)
                   |     T0 ← TTemp_0
                   |     T1 ← TTemp_1
                   |     T2 ← TTemp_2
                   |     T3 ← TTemp_3
                   |     T4 ← TTemp_4
                   |     i ← i + 1
                   | (i  15·i  T0  T1  T2  T3  T4)
```

Check: At time = zero, i.e. at the start, the initial temps are:

$$\text{Implcit\_Temp}(0) = [\ 0 \quad 0 \quad 100 \quad 100 \quad 100 \quad 100 \quad 100\ ]$$

Example: After 5 time steps, i.e. after 75 s, the temps are:

$$\text{Implcit\_Temp}(5) = [\ 5 \quad 75 \quad 133.151 \quad 133.012 \quad 132.517 \quad 131.43 \quad 129.377\ ]$$

---

i = step no.; $\Delta\tau$ = one time step = 15 s; τ = time duration from beginning = i. $\Delta\tau$, s

i	τ	T0	T1	T2	T3	T4

$$\text{Implcit\_Temp}(1) = [\ 1 \quad 15 \quad 106.686 \quad 106.675 \quad 106.617 \quad 106.391 \quad 105.514\ ]$$

$$\text{Implcit\_Temp}(8) = [\ 8 \quad 120 \quad 152.658 \quad 152.42 \quad 151.638 \quad 150.123 \quad 147.598\ ]$$

$$\text{Implcit\_Temp}(20) = [\ 20 \quad 300 \quad 228.56 \quad 228.049 \quad 226.473 \quad 223.709 \quad 219.555\ ]$$

$$\text{Implcit\_Temp}(240) = [\ 240 \quad 3600 \quad 1244.686 \quad 1241.099 \quad 1230.315 \quad 1212.269 \quad 1186.856\ ]$$

$$\text{Implcit\_Temp}(2700) = [\ 2700 \quad 40500 \quad 2418.892 \quad 2411.752 \quad 2390.334 \quad 2354.636 \quad 2304.659\ ]$$

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i.e. After 5 min (=300 s), the temperatures are:

T0 = 228.56, T1 = 228.049, T2 = 226.473, T3 = 223.709, T4 = 219.555 deg.C….Ans.

==========

At what time Steady State is reached?

When steady state is reached, temperatures at successive time steps should not change much.

$$\text{Implcit\_Temp}(2000) = [\ 2000 \quad 30000 \quad 2411.954 \quad 2404.836 \quad 2383.48 \quad 2347.887 \quad 2298.055\ ]$$

$$\text{Implcit\_Temp}(2020) = [\ 2020 \quad 30300 \quad 2412.397 \quad 2405.278 \quad 2383.918 \quad 2348.318 \quad 2298.477\ ]$$

We see from the above that at 2000th time step, i.e. after 30000 s (= 500 min.), the temperatures have stabilized. There is not much difference in temperatures when we compare the values for 2000th and 2020th time steps.

Program to get the temps at all Nodes with Time:

```
Implcit_NodeTemps(n) := | T0← 100
                        | T1← 100
                        | T2← 100
                        | T3← 100
                        | T4← 100
                        | i← 0
                        | while i<n
                        |     TTemp← Temp(T0, T1, T2, T3, T4)
                        |     T0← TTemp_0
                        |     T1← TTemp_1
                        |     T2← TTemp_2
                        |     T3← TTemp_3
                        |     T4← TTemp_4
                        |     Step_i← i
                        |     A0_0← 100
                        |     A1_0← 100
                        |     A2_0← 100
                        |     A3_0← 100
                        |     A4_0← 100
                        |     A0_i← T0
                        |     Time_i← i·Δτ
```


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```
    A1_i ← T1
    A2_i ← T2
    A3_i ← T3
    A4_i ← T4
    C1 ← augment(A3, A4)
    C2 ← augment(A2, C1)
    C3 ← augment(A1, C2)
    C4 ← augment(A0, C3)
    TProfile ← augment(Time, C4)
    TProfile ← augment(Step, TProfile)
    i ← i + 1
return TProfile
```

Program shown above returns a Matrix as follows:

0th column: gives Time step

1st column: gives Time (s)

2nd to 6th column: Temps. T0, T1, T2, T3 and T4 at different times

Note: In Mathcad Matrix, Rows and Columns are numbered from 0 onwards (though it can be set to start from 1).

As an example, after 20 steps, we get for Nodal temperatures:

Implcit_NodeTemps(20) =

	0	1	2	3	4	5	6
0	0	0	100	100	100	100	100
1	1	15	113.35	113.315	113.161	112.681	111.333
2	2	30	119.984	119.917	119.65	118.939	117.293
3	3	45	126.584	126.482	126.099	125.187	123.319
4	4	60	133.151	133.012	132.517	131.43	129.377
5	5	75	139.684	139.51	138.912	137.669	135.449
6	6	90	146.186	145.978	145.284	143.9	141.524
7	7	105	152.658	152.42	151.638	150.123	147.598
8	8	120	159.104	158.836	157.972	156.335	153.665
9	9	135	165.524	165.23	164.288	162.535	159.723
10	10	150	171.921	171.6	170.586	168.722	165.77
11	11	165	178.294	177.95	176.866	174.894	171.805
12	12	180	184.646	184.279	183.129	181.052	177.826
13	13	195	190.978	190.589	189.373	187.194	183.834
14	14	210	197.289	196.878	195.6	193.321	189.826
15	15	225	203.581	203.149	201.81	199.432	195.804
16	16	240	209.854	209.402	208.002	205.526	201.765
17	17	255	216.107	215.635	214.176	211.604	207.711
18	18	270	222.343	221.851	220.333	217.665	213.641
19	19	285	228.56	228.049	226.473	223.709	219.555

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To plot temps T0 and T4 with Time:

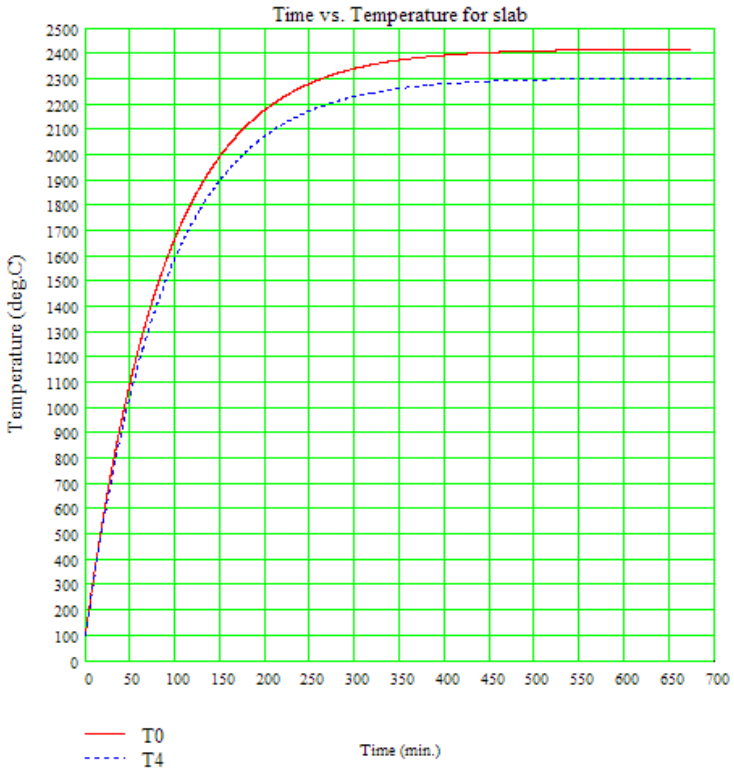
Consider Time steps upto 2700.

Define: TTTemp := Implcit_NodeTemps(2701)

TTTemp =

	0	1	2	3	4	5	6
2637	$2.637\cdot10^3$	$3.956\cdot10^4$	$2.419\cdot10^3$	$2.412\cdot10^3$	$2.39\cdot10^3$	$2.354\cdot10^3$	$2.304\cdot10^3$
2638	$2.638\cdot10^3$	$3.957\cdot10^4$	$2.419\cdot10^3$	$2.412\cdot10^3$	$2.39\cdot10^3$	$2.354\cdot10^3$	$2.304\cdot10^3$
2639	$2.639\cdot10^3$	$3.959\cdot10^4$	$2.419\cdot10^3$	$2.412\cdot10^3$	$2.39\cdot10^3$	$2.354\cdot10^3$	$2.304\cdot10^3$
2640	$2.64\cdot10^3$	$3.96\cdot10^4$	$2.419\cdot10^3$	$2.412\cdot10^3$	$2.39\cdot10^3$	$2.354\cdot10^3$	$2.304\cdot10^3$
2641	$2.641\cdot10^3$	$3.962\cdot10^4$	$2.419\cdot10^3$	$2.412\cdot10^3$	$2.39\cdot10^3$	$2.354\cdot10^3$	$2.304\cdot10^3$
2642	$2.642\cdot10^3$	$3.963\cdot10^4$	$2.419\cdot10^3$	$2.412\cdot10^3$	$2.39\cdot10^3$	$2.354\cdot10^3$	$2.304\cdot10^3$
2643	$2.643\cdot10^3$	$3.965\cdot10^4$	$2.419\cdot10^3$	$2.412\cdot10^3$	$2.39\cdot10^3$	$2.354\cdot10^3$	$2.304\cdot10^3$
2644	$2.644\cdot10^3$	$3.966\cdot10^4$	$2.419\cdot10^3$	$2.412\cdot10^3$	$2.39\cdot10^3$	$2.354\cdot10^3$	$2.304\cdot10^3$
2645	$2.645\cdot10^3$	$3.968\cdot10^4$	$2.419\cdot10^3$	$2.412\cdot10^3$	$2.39\cdot10^3$	$2.354\cdot10^3$	$2.304\cdot10^3$
2646	$2.646\cdot10^3$	$3.969\cdot10^4$	$2.419\cdot10^3$	$2.412\cdot10^3$	$2.39\cdot10^3$	$2.354\cdot10^3$	$2.304\cdot10^3$

i := 0.. 2700 ....define a range variable i.



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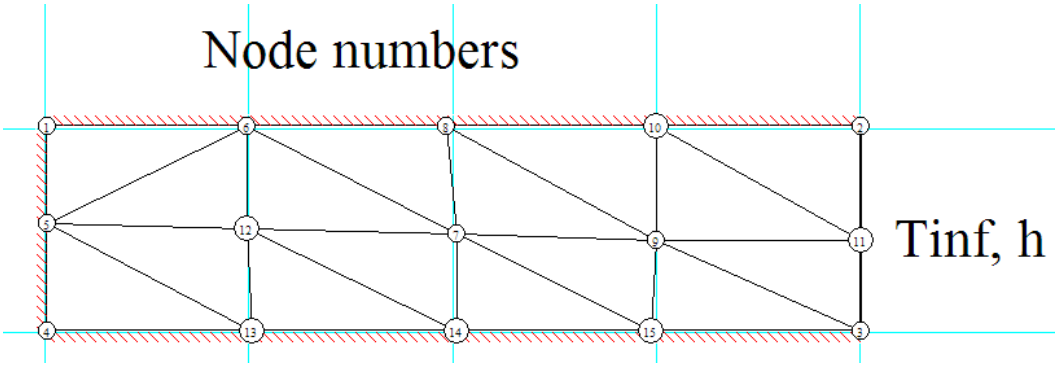
Note: At Time = 660 min (= 11 hrs), i.e. Step no. 2640: T0 = 2419 C, T4 = 2304 C.

These values are the same as obtained for Explicit method.

==========

Solution to the above problem by Finite Element Heat Transfer (FEHT) Software:

1. **Node positions:**



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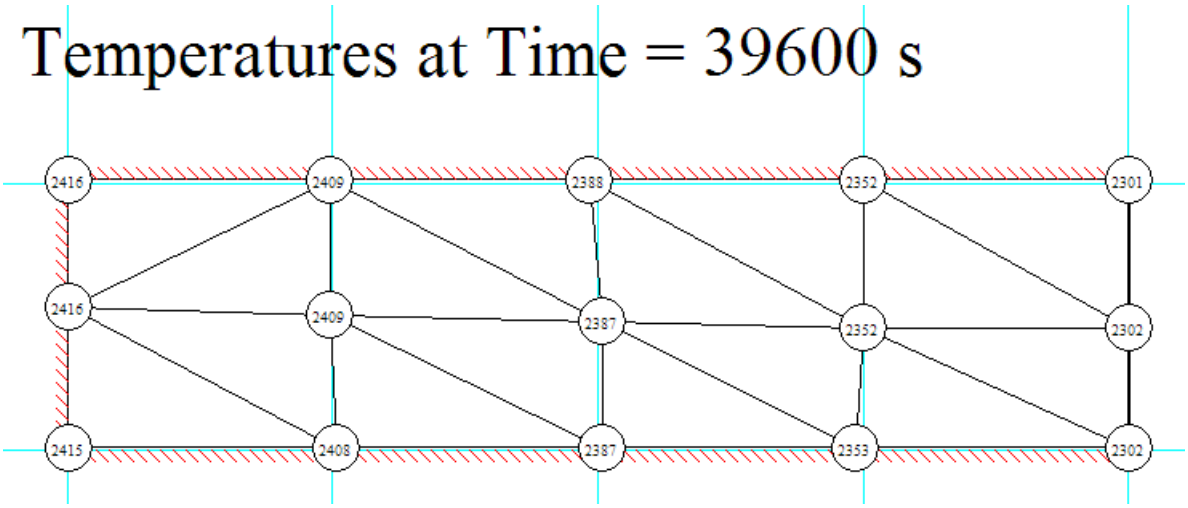
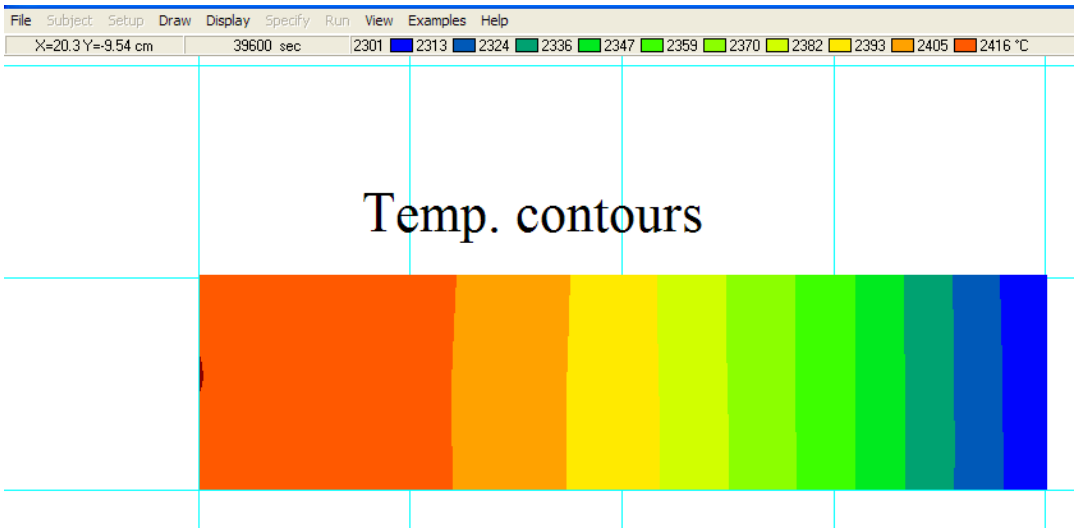
2. **Node temperatures:**



Note: At Time = 660 min (= 39600 s): T0 = 2416 C, T4 = 2302 C.

These values are almost the same as those obtained for Explicit and Implicit methods.

3. **Temperature contours:**



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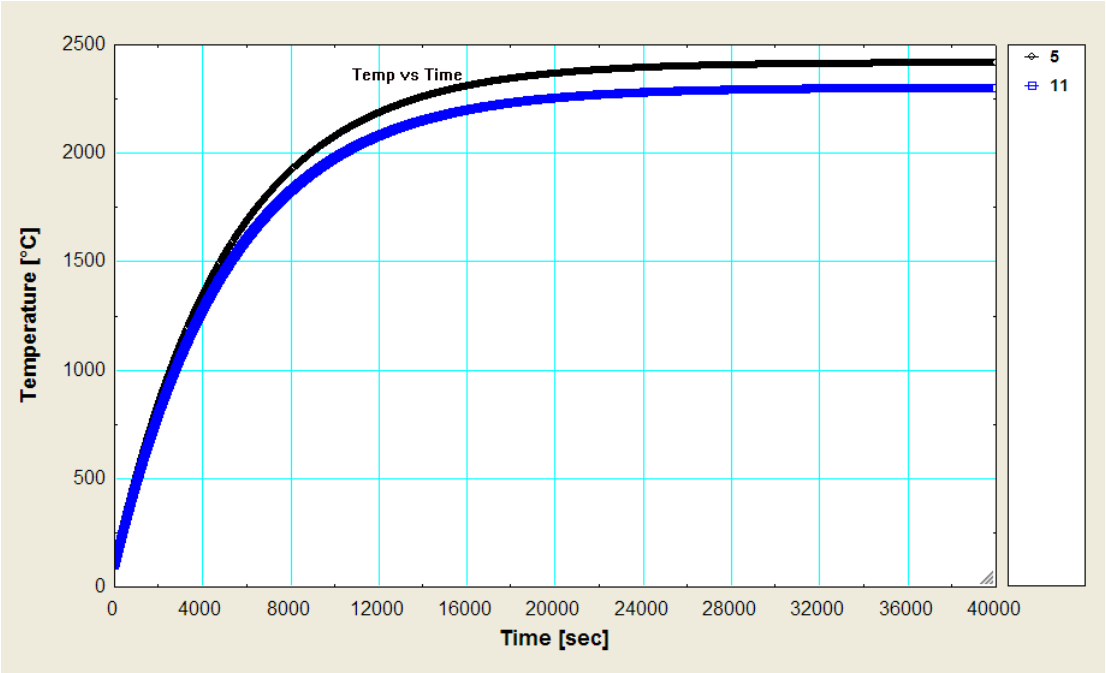
4. **Plot of Temp vs Time for Nodes '0' and '4':**



==========

Prob. 1I.C.3. In areas where ambient temperature drops to subzero temperatures and remains so for prolonged periods, freezing of water in underground pipelines is a major concern. It is of interest to know at what depth the water pipes should be buried so that the water does not freeze.

At a particular location, the soil is initially at an uniform temperature of 15 C and the soil is subjected to a subzero temperature of -20 C continuously for 50 days.

1. What is the minimum burial depth required to ensure that the water in the pipes does not freeze? i.e. pipe surface temperature should not fall below 0 C.
2. Plot the temp. distributions in the soil for different times i.e. after 1 day, 2 days etc. Properties of soil may be taken as: $\alpha = 0.138 \times 10^{-6}$ m^2/s, $\rho = 2050$ kg/m^3, k = 0.52 W/(m.K), Cp = 1840 J/kg.K. [Ref. 1]

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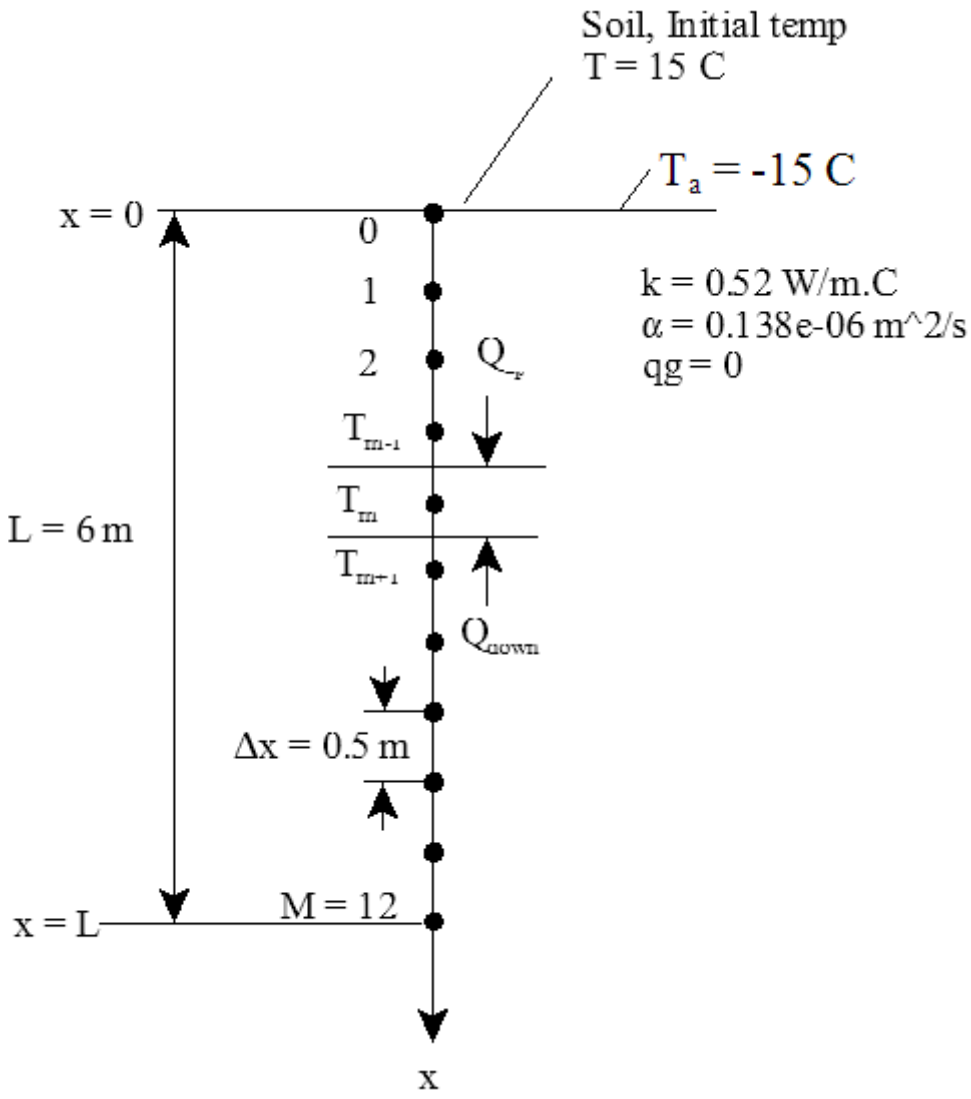

Fig.Prob.1I.C.3

Mathcad Solution:

While adopting numerical procedure, in this case, let us make a **reasonable assumption that temp at a depth of 6 m does not change from the initial condition of 15 C, even after 50 days.**

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Data:

$L := 6$ m....thickness of soil considered

$k := 0.52$ W/(m.C)..thermal cond. of soil

$\alpha := 0.138 \cdot 10^{-6}$ m^2/s....thermal diffusivity of soil

$q_g := 0$ W/m^3...heat gen. rate

$T := 15$ C...initial temp. of soil

$T_a := -20$ C....temp. of ambient

$\Delta x := 0.5$ m....nodal spacing

$M := 12$ no. of equal spacings, i.e. nodes 0,1,2....12

$\tau := 50 \cdot 24 \cdot 3600$ s (= 50 days)...time after which temp. distribution in soil is desired

$A := 1$ m2

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Solve this problem by Implicit formulation:

Difference eqns. for interior nodes:

Nodes 1, 2, 3…11 are interior nodes. Finite difference equations for these nodes by implicit method are obtained from eqn. (A), by setting m = 1, 2, 3, 4.. etc. i.e.

$$(1+2\cdot Fo)\cdot\left(T_m\right)^{i+1} - Fo\cdot\left[\left(T_{m-1}\right)^{i+1} + \left(T_{m+1}\right)^{i+1} + \frac{\left(q_m\right)^{i+1}\cdot(\Delta x)^2}{k}\right] - \left(T_m\right)^{i} = 0 \quad \text{....(A)}$$

Difference eqns. for boundary nodes:

Nodes 0 and 12 are boundary nodes; both the nodes have constant temperatures:

For node '0':

T0 = - 20 C.... by data.

For node '12' …. at a depth of 6 m from surface:

T12 = 15 C ... temp at a depth of 6 m ... remains constant... by assumption

Now, we can choose any $\Delta\tau$, since there is no problem of stability in implicit formulation.
Let us choose $\Delta\tau$ = 1 hr:

$$\Delta\tau := 3600 \ \text{s}$$

Therefore, $Fo := \frac{\alpha\cdot\Delta\tau}{(\Delta x)^2}$ i.e. $Fo = 1.9872\cdot 10^{-3}$

Now, to start with, i.e. at $\tau = 0$, all the node temperatures T0, T1,….T12 are known (= 15 C). Then, at the next time step, solve all the eqns. simultaneously to get the node temperatures at that time step. Using these results, solve the eqns. at the next time step, etc. till you reach the given time limit of 50 days.

This calculation is easily done in Mathcad. We slightly change the notation for convenience in calculation: we write the superscripts as subscripts to work with matrix notation, as shown below.

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Initial values:

$T0_0 := 15$ $T1_0 := 15$ $T2_0 := 15$ $T3_0 := 15$ $T4_0 := 15$

$T5_0 := 15$ $T6_0 := 15$ $T7_0 := 15$ $T8_0 := 15$ $T9_0 := 15$

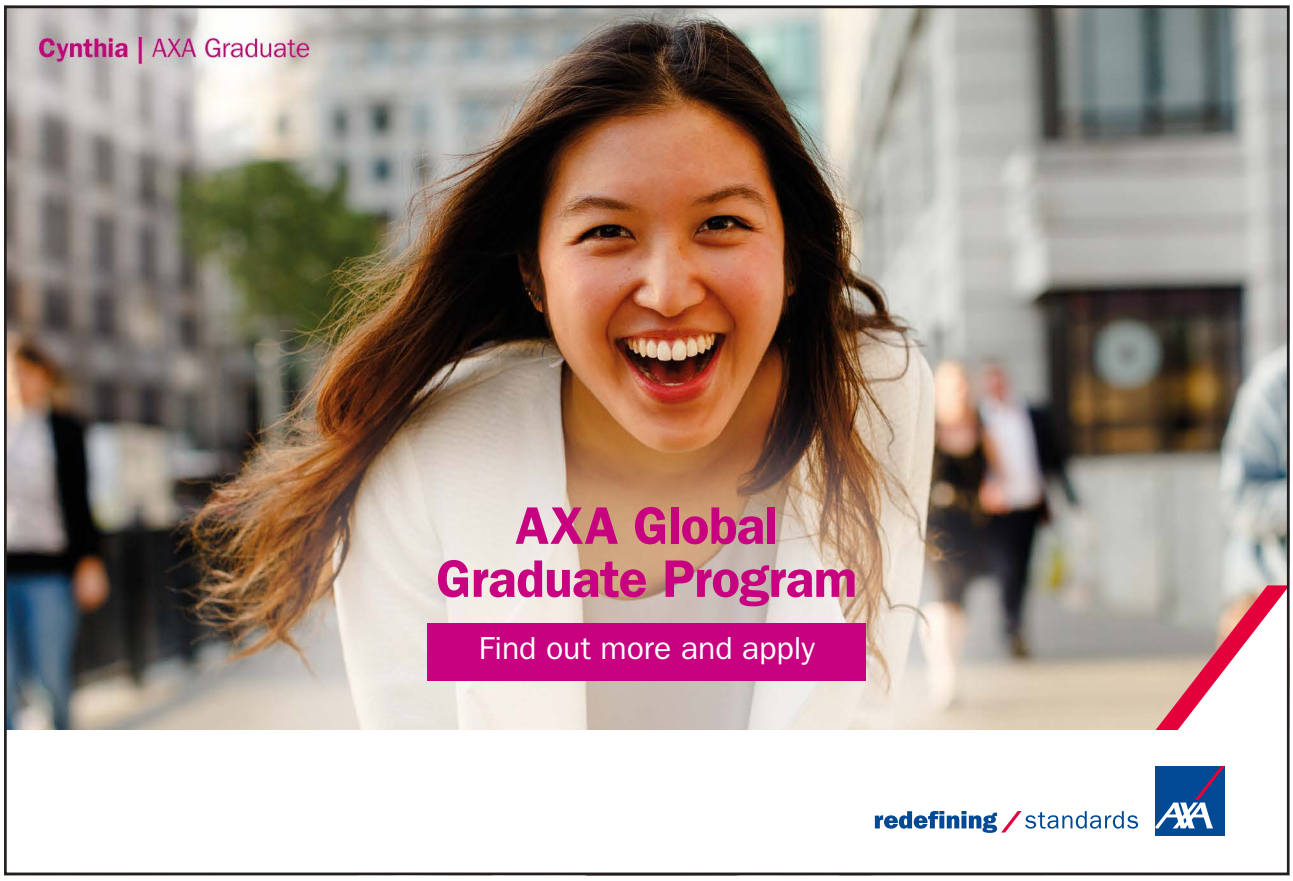
$T10_0 := 15$ $T11_0 := 15$ $T12_0 := 15$ deg.C

...guess values:

$T0_1 := -20$ $T1_1 := 50$ $T2_1 := 50$ $T3_1 := 50$ $T4_1 := 50$

$T5_1 := 50$ $T6_1 := 50$ $T7_1 := 50$ $T8_1 := 50$ $T9_1 := 50$

$T10_1 := 50$ $T11_1 := 50$ $T12_1 := 15$



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Use Mathcad Solve Block to get updated temps:

Given

Node 0 : $T0_1 = -20$ .....(a)

Node 1 : $(1+2\cdot Fo)\cdot T1_1 - Fo\cdot(T0_1 + T2_1) - T1_0 = 0$ ....(b)

Node 2 : $(1+2\cdot Fo)\cdot T2_1 - Fo\cdot(T1_1 + T3_1) - T2_0 = 0$ ....(c)

Node 3 : $(1+2\cdot Fo)\cdot T3_1 - Fo\cdot(T2_1 + T4_1) - T3_0 = 0$ ....(d)

Node 4 : $(1+2\cdot Fo)\cdot T4_1 - Fo\cdot(T3_1 + T5_1) - T4_0 = 0$ ....(e)

Node 5 : $(1+2\cdot Fo)\cdot T5_1 - Fo\cdot(T4_1 + T6_1) - T5_0 = 0$ ....(f)

Node 6 : $(1+2\cdot Fo)\cdot T6_1 - Fo\cdot(T5_1 + T7_1) - T6_0 = 0$ ....(g)

Node 7 : $(1+2\cdot Fo)\cdot T7_1 - Fo\cdot(T6_1 + T8_1) - T7_0 = 0$ ....(h)

Node 8 : $(1+2\cdot Fo)\cdot T8_1 - Fo\cdot(T7_1 + T9_1) - T8_0 = 0$ ....(i)

Node 9 : $(1+2\cdot Fo)\cdot T9_1 - Fo\cdot(T8_1 + T10_1) - T9_0 = 0$ ....(j)

Node 10: $(1+2\cdot Fo)\cdot T10_1 - Fo\cdot(T9_1 + T11_1) - T10_0 = 0$ ....(k)

Node 11: $(1+2\cdot Fo)\cdot T11_1 - Fo\cdot(T10_1 + T12_1) - T11_0 = 0$ ....(l)

Node 12: $T12_1 = 15$ ....(m)

$TempNew(T0_0, T1_0, T2_0, T3_0, T4_0, T5_0, T6_0, T7_0, T8_0, T9_0, T10_0, T11_0, T12_0) := Find(T0_1, T1_1, T2_1, T3_1, T4_1, T5_1, T6_1, T7_1, T8_1, T9_1, T10_1, T11_1, T12_1)$

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In the above Solve block, TempNew is the function that calculates and returns the new Node temperatures, viz. $T0_1$, $T1_1$, … $T12_1$, after the next time step. Note that it is written as a function of initial temps $T0_0$, $T1_0$….etc.

Node Temps after 1 Timestep:

$\text{TempNew}\left(T0_0, T1_0, T2_0, T3_0, T4_0, T5_0, T6_0, T7_0, T8_0, T9_0, T10_0, T11_0, T12_0\right) =$

	0
0	-20
1	14.931
2	15
3	15
4	15
5	15
6	15
7	15
8	15
9	15
10	15
11	15
12	15

We see that, after 1 time step, the temps are: starting from top $T0_1$ = -20 C, $T1_1$ = 14.931 C, $T2_1$ = 15 C … $T12_1$ = 15 C.

Now, write a program / Function to get temps after N time steps:

Function Name: Implicit_Temp_1(……)

Inputs: Initial temps and No. of Time steps

Outputs: Time steps, Total time, and temps at all nodes at that time step.

The Mathcad program is shown below:

In this program:

LHS … gives the name of the program with Initial temps and the No. of Time steps as Inputs.

RHS….calculates the new temps at the nodes using the TempNew function explained above. Then, the new temperatures are set as initial temperatures and the calculations are repeated by the 'while loop' till the time steps (Nsteps) are completed. New Node temperatures obtained at the last step are stored in a vector Y and returned.

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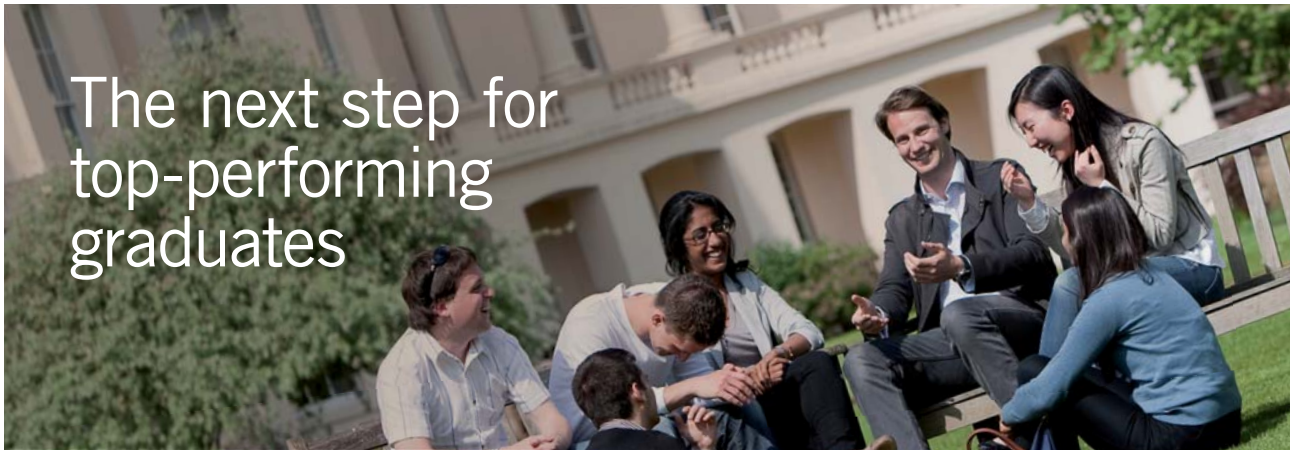
Note that Y is a Mathcad vector which gives Step no., Total time, and Node temps T0….T12 in columns 0, 1, 2….14 respectively.

```
Implcit_Temp_1(T0,T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,T11,T12,Nsteps) := | i← 0
                                                                    | while i<Nsteps
                                                                    |   TTemp← TempNew(T0,T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,T11,T1
                                                                    |   T0← TTemp_0
                                                                    |   T1← TTemp_1
                                                                    |   T2← TTemp_2
                                                                    |   T3← TTemp_3
                                                                    |   T4← TTemp_4
                                                                    |   T5← TTemp_5
                                                                    |   T6← TTemp_6
                                                                    |   T7← TTemp_7
                                                                    |   T8← TTemp_8
                                                                    |   T9← TTemp_9
                                                                    |   T10← TTemp_10
                                                                    |   T11← TTemp_11
                                                                    |   T12← TTemp_12
                                                                    |   i← i+ 1
                                                                    | Y← (i  i·Δτ  T0  T1  T2  T3  T4  T5  T6  T7  T8  T9  T10  T11  T12)
```



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* Figures taken from London Business School's Masters in Management 2010 employment report

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To get the temp distribution after 50 days:

Initial values:

T0 := 15 T1 := 15 T2 := 15 T3 := 15 T4 := 15

T5 := 15 T6 := 15 T7 := 15 T8 := 15 T9 := 15

T10 := 15 T11 := 15 T12 := 15

Nsteps := 1200 ...i.e. 1200 hrs = 50 days

Store the above Function in another vector called TProfile:

TProfile := Implcit_Temp_1(T0, T1, T2, T3, T4, T5, T6, T7, T8, T9, T10, T11, T12, Nsteps)

Then:

TProfile =

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	$1.2 \cdot 10^3$	$4.32 \cdot 10^6$	-20	-7.56	2.424	8.938	12.457	14.064	14.694	14.911	14.976	14.994	14.999	15	15

i.e. 0th column: Timestep = 1200, 1st column: Total time = 4.32E06 s = 50 days.

Columns 2 to 14 .. give the Node temps T0 …. T12 respectively.

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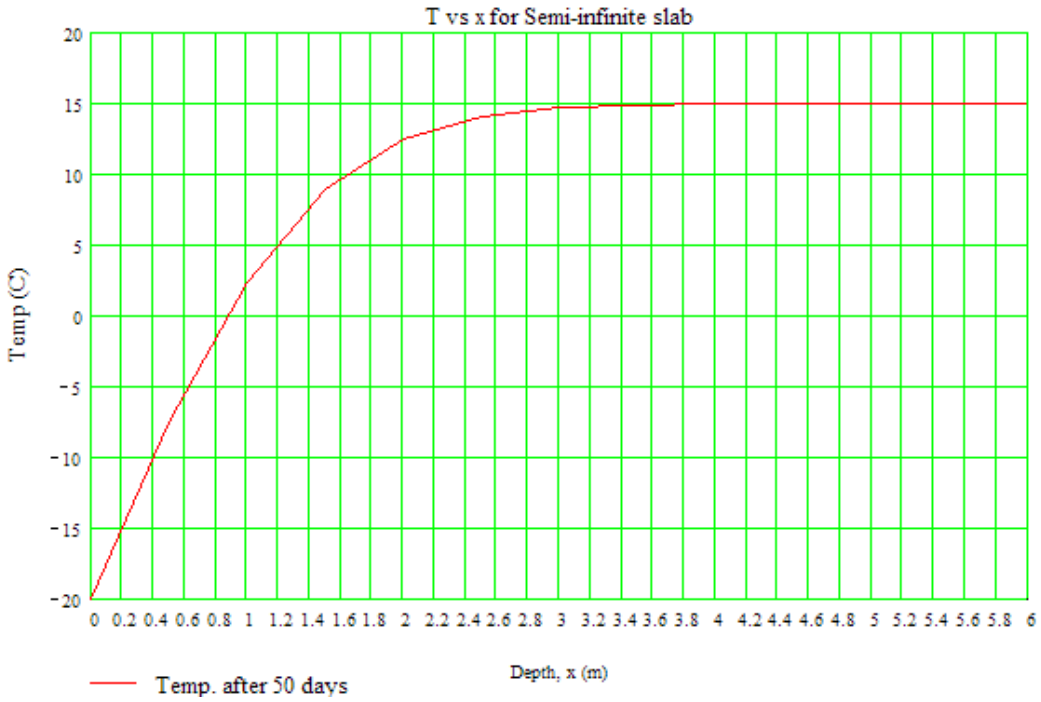
Now, plot these temps against distance (i.e. depth in the soil):

i := 2, 3.. 14 ...define range variable i



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At what depth the temp will reach 0 C in the soil after 50 days?

In the above graph, we see that temp is zero at a depth of around 0.9 m.

If we need to know it more accurately:

Using the Trace graph facility in Mathcad (Format – Graph – Trace menu), we get:

y = 0 C at x = 0.87078 m;

i.e. at a depth of 0.871 m, the temp will reach 0 C after 50 days duration …. Ans.

To draw Temp profiles in soil after different time periods:

Use the Implicit_Temp_1 Function at different, desired time steps and store them in different vectors and then plot them together on the same graph:

After 1 day: i.e. 24 hrs, i.e. NSteps = 24.

Store the results in vector TProfile1, as shown below:

TProfile1 := Implcit_Temp_1(T0, T1, T2, T3, T4, T5, T6, T7, T8, T9, T10, T11, T12, 24)

TProfile1 =

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	24	$8.64 \cdot 10^4$	-20	13.41	14.961	14.999	15	15	15	15	15	15	15	15	15

Temps T0….T12 after 1 day (Timestep = 24) are shown in columns 2 to 14 in the vector TProfile1.

Similarly, results for Node temps after 7 days, 15 days, 30 days and 50 days are stored in vectors TProfile7, TProfile15, TProfile30, TProfile50 respectively, as shown below:

After 7 days: i.e. 188 hrs, i.e. NSteps = 188

TProfile7 := Implcit_Temp_1(T0, T1, T2, T3, T4, T5, T6, T7, T8, T9, T10, T11, T12, 188)

After 15 days: i.e. 360 hrs, i.e. NSteps = 360

TProfile15 := Implcit_Temp_1(T0, T1, T2, T3, T4, T5, T6, T7, T8, T9, T10, T11, T12, 360)

After 30 day: i.e. 720 hrs, i.e. NSteps = 720

TProfile30 := Implcit_Temp_1(T0, T1, T2, T3, T4, T5, T6, T7, T8, T9, T10, T11, T12, 720)

After 50 day: i.e. 1200 hrs, i.e. NSteps = 1200

TProfile50 := Implcit_Temp_1(T0, T1, T2, T3, T4, T5, T6, T7, T8, T9, T10, T11, T12, 1200)

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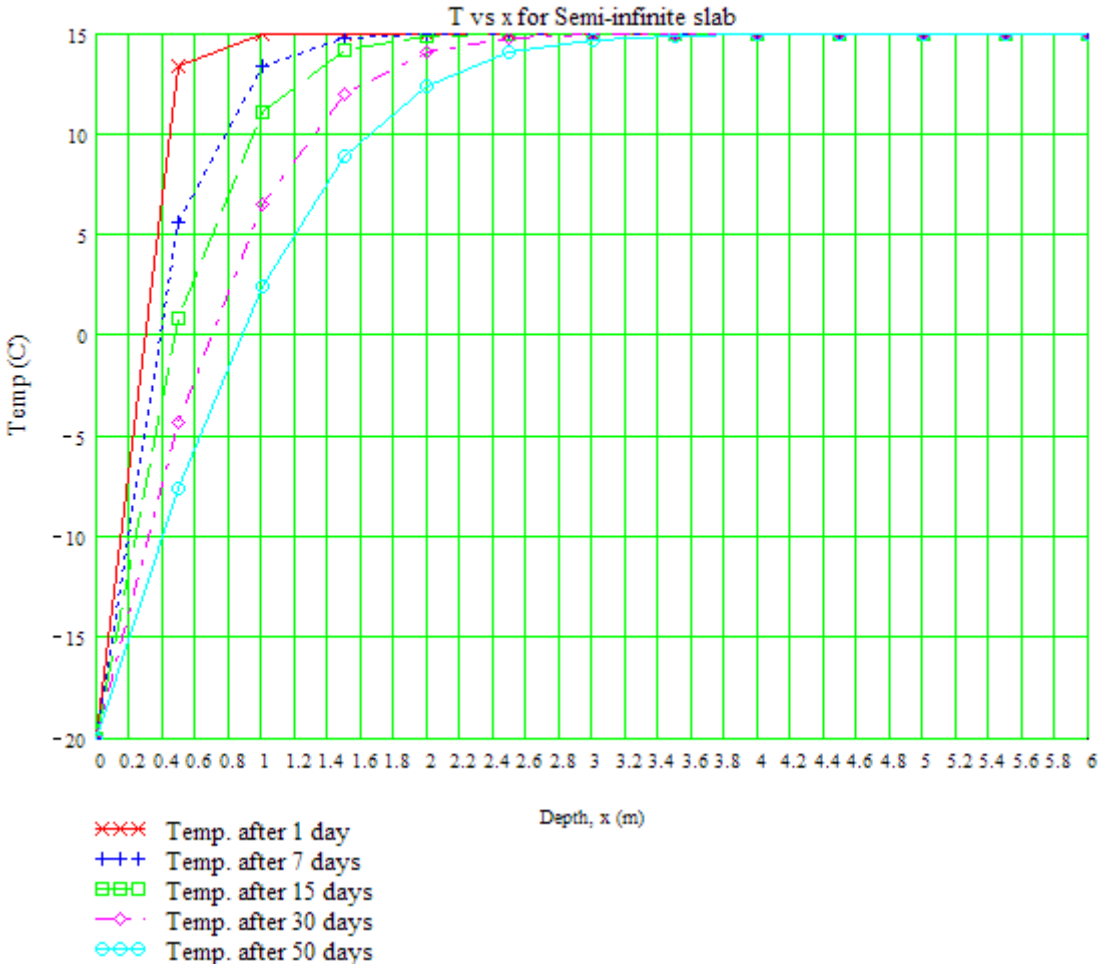
To plot the temps against x at different times:

i := 2, 3 .. 14 ...define range variable i



It may be observed from the above plot that: even after 50 days of duration, depths beyond 1 m have not yet reached the freezing temp of zero deg. C.

==========

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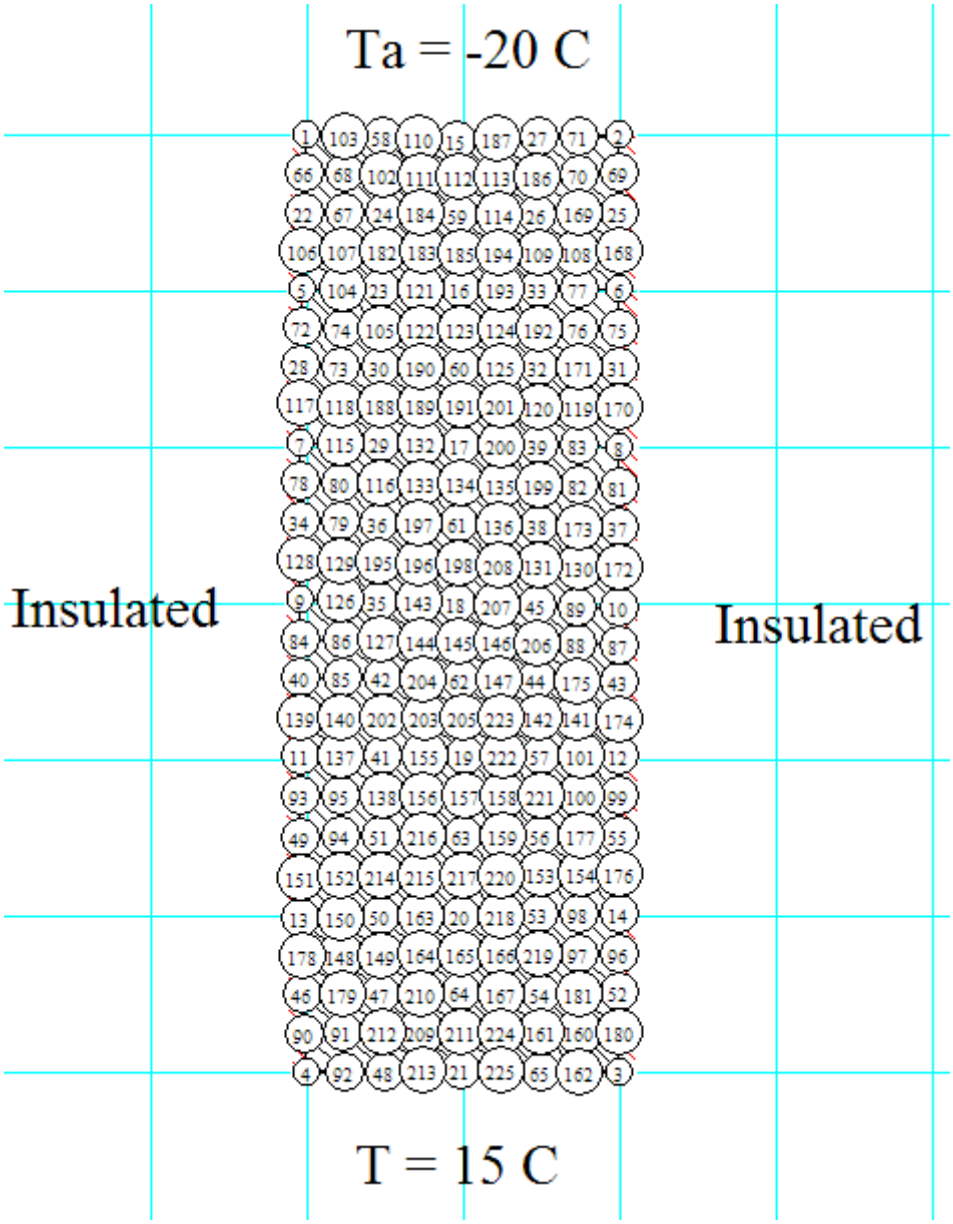
Solution to the above problem by Finite Element Heat Transfer (FEHT) Software:

1. **Node positions:**



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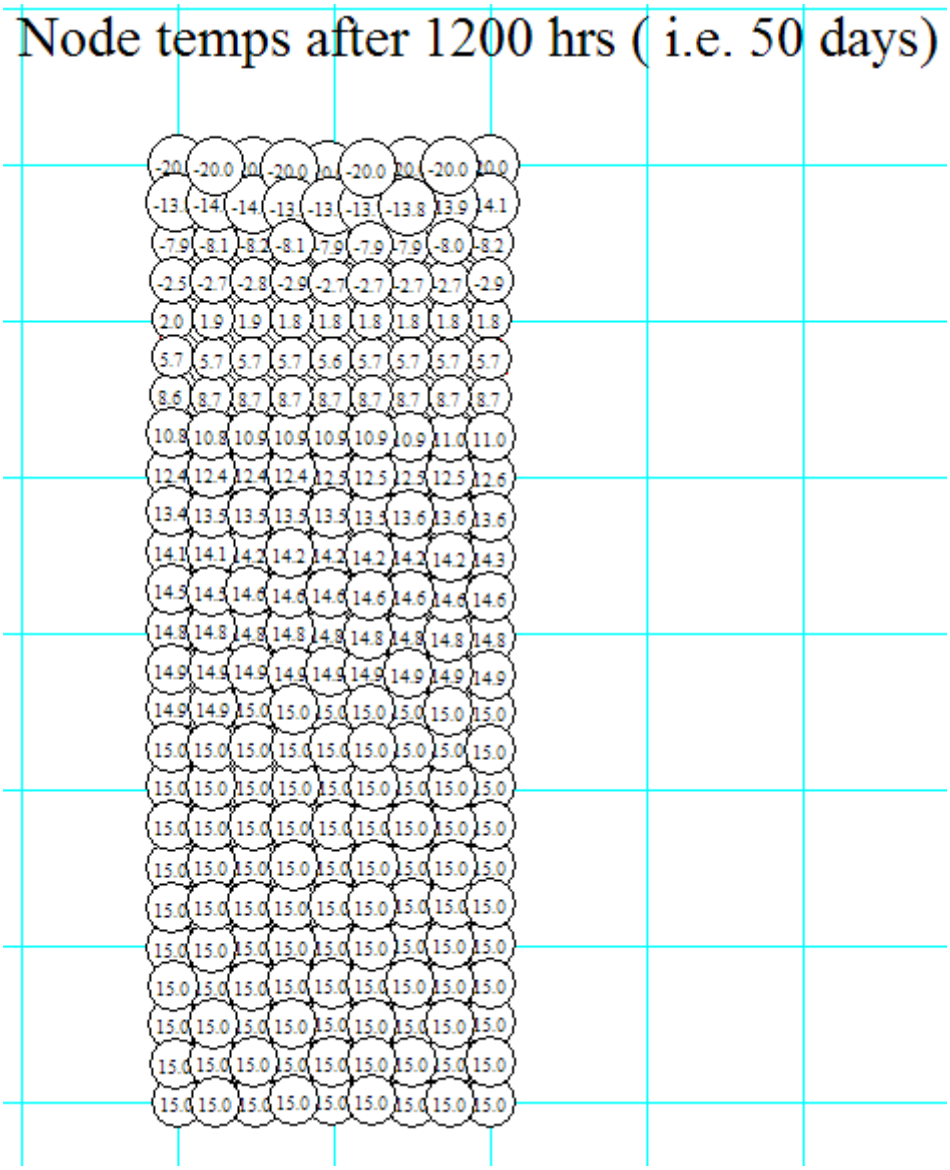
2. **Node Temperatures after 50 days:**



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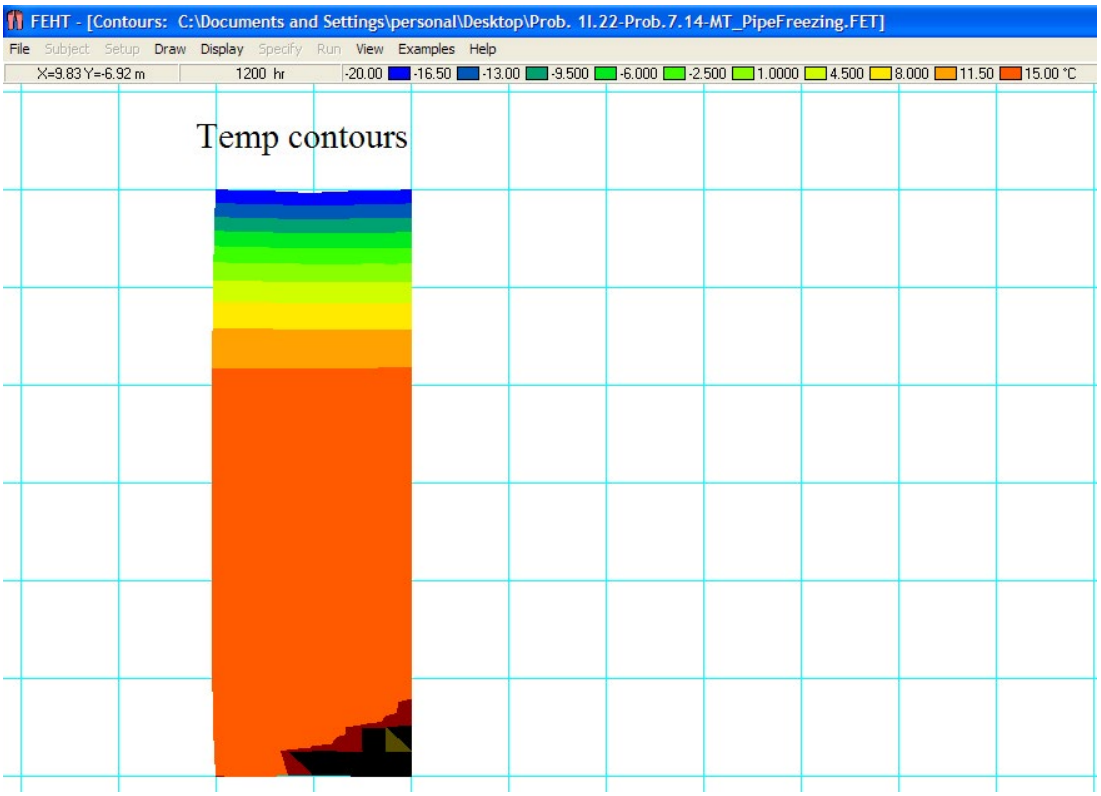
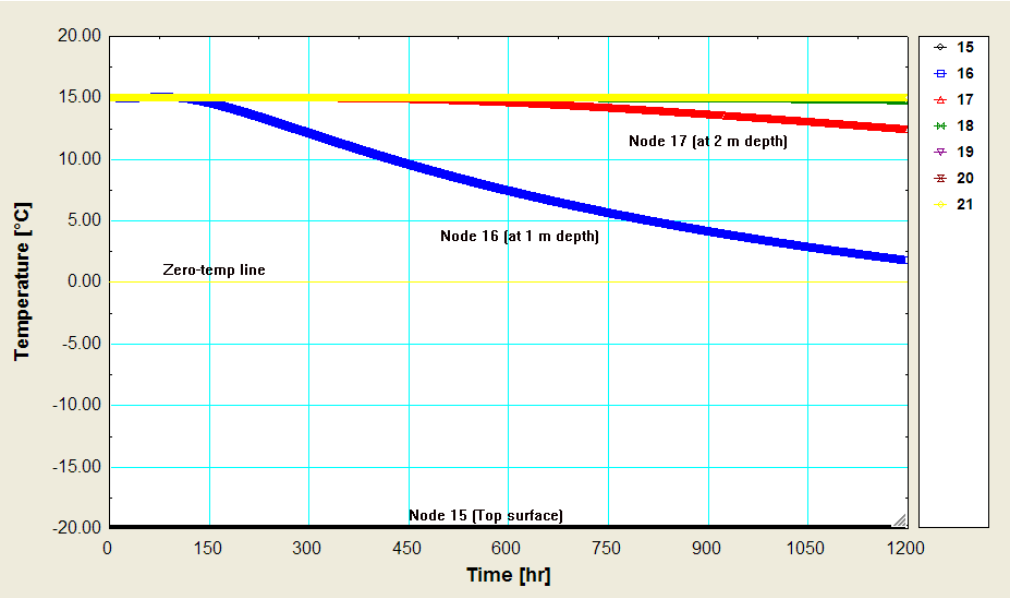
3. **Temp contours (shaded bands) – at 1200 hrs:**
   Temp range of colours shown in tool bar at the top of the fig.:



4. **Temp vs Time for different Nodes:**



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To draw plot of Temp vs depth from surface, at various times:

First, click on View-Tabular output, and you get the following Table (only part of Table is shown):

Tabular Nodal Results

Close | Print | Select All | Copy | Save as

Node	X [m]	Y [m]	T (#0) [°C]	T (#1) [°C]
10	8.996	-5.027	15	15
11	6.96	-5.98	15	15
12	8.996	-5.98	15	15
13	6.959	-7.011	15	15
14	8.996	-6.985	15	15
15	7.964	-2.037	**-20**	**-20**
16	7.99	-2.99	15	15
17	7.99	-3.982	15	15
18	7.964	-5.001	15	15
19	8.017	-5.98	15	15
20	7.99	-6.998	15	15
21	7.99	-7.99	**15**	**15**
22	6.986	-2.495	15	15

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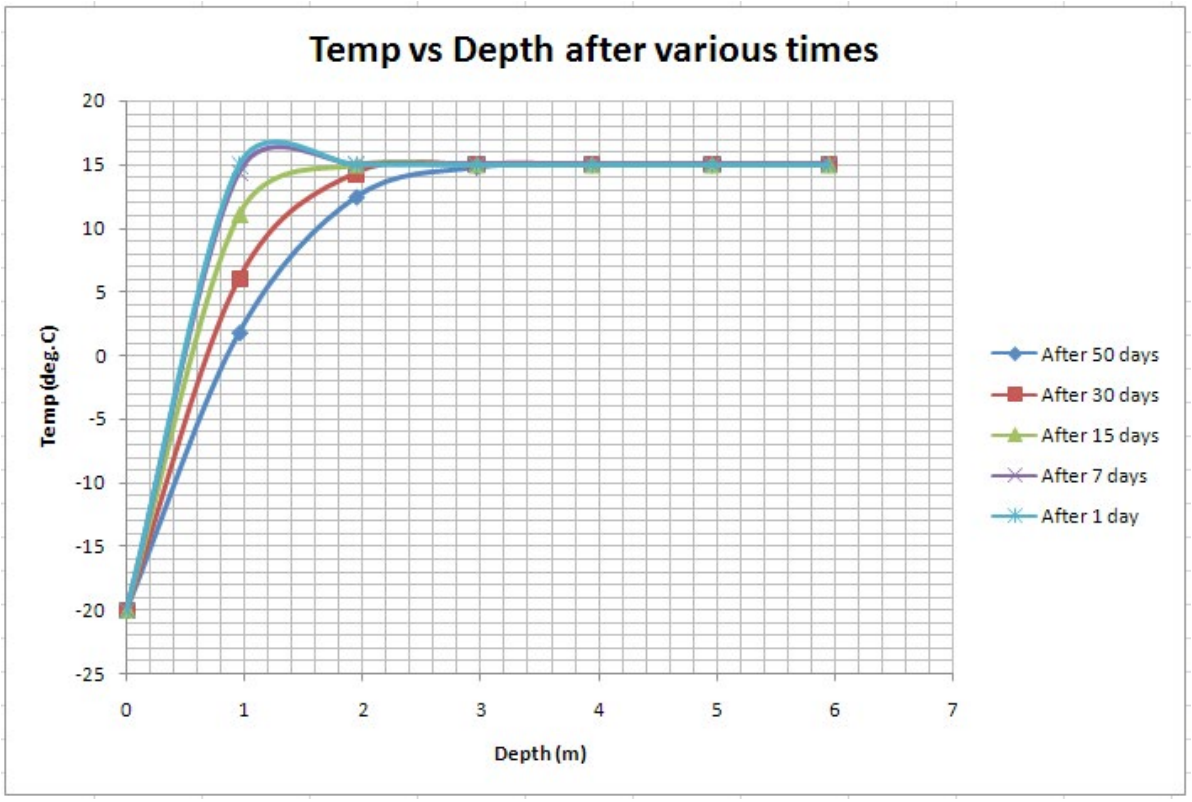
Now copy this to Excel, and edit the Table:

We get:

Node No.	X (m)	50 days T(1200 h)	30 days T(720 h)	15 days T(360 h)	7 days T(188 h)	1day T(24 h)
15	0	-20	-20	-20	-20	-20
16	0.953	1.789	5.996	11.08	14.37	14.97
17	1.945	12.45	14.3	14.98	15	15
18	2.964	14.79	14.99	15	15	15
19	3.943	14.99	15	15	15	15
20	4.961	15	15	15	15	15
21	5.953	15	15	15	15	15

Now, draw the graph for various times:



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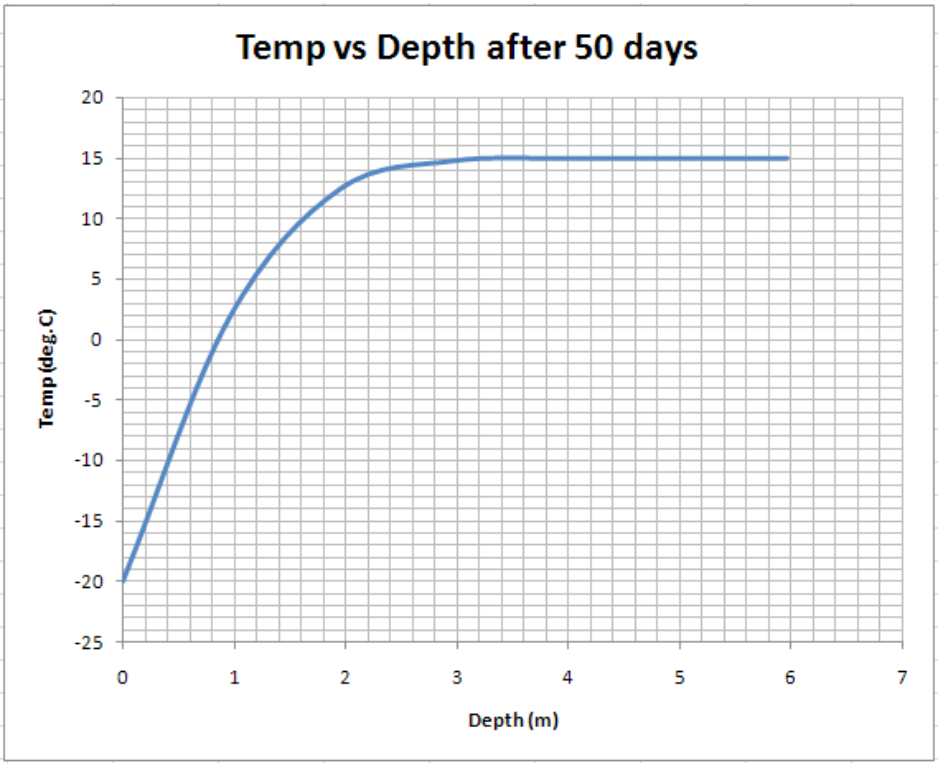
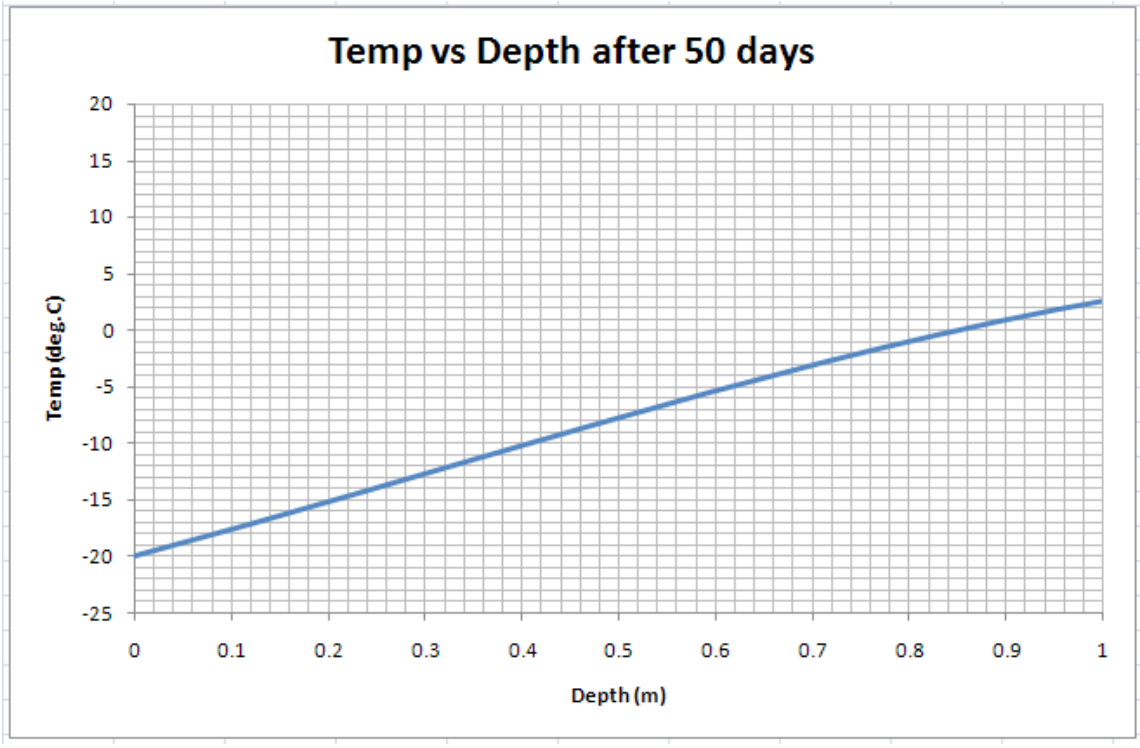
To find at what depth the temp reaches 0 C after 50 days, draw the Temp vs Depth graph at time = 50 days, again:



Draw the same graph enlarged, i.e. with x-axis from 0 to 1 m:



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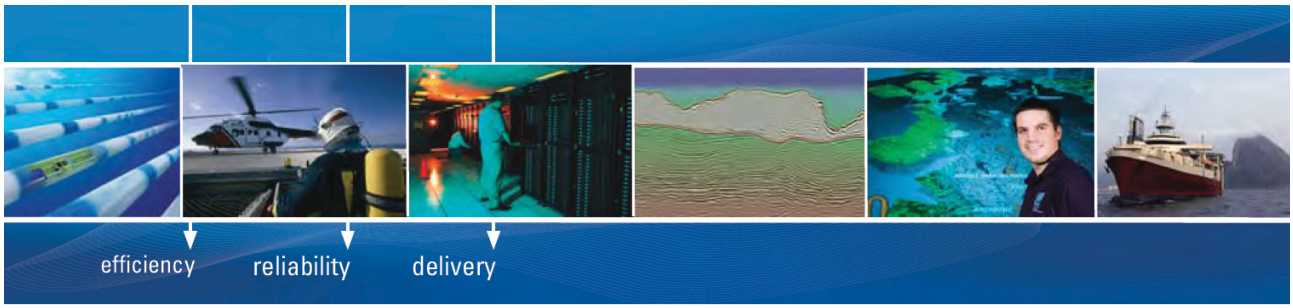
We see that temp reaches zero at a depth, x = 0.84 m, after 50 days.

Compare this with the value of 0.87 m obtained earlier with Mathcad.

==========

Prob. 1I.C.4. Consider a plane wall with k = 1.5 W/m.C, cp = 1000 J/kg.C, ρ = 200 kg/m^3, α = 7.5E-06 m^2/s. Its thickness L = 50 mm. Initial uniform temp = 25 C. Suddenly, the boundary at x = L is subjected to heating by a fluid at a temp of 50 C and h = 75 W/m^2.C. And, the boundary at x = 0 is subjected to a heat flux = 2000 W/m^2. Take Δx = 5 mm and Δt = 20 s. Plot the temp distribution in the wall for the initial condition, at t = 160 s, at t = 300 s and at t = 1300 s.

Adopt the Implicit, finite difference method.



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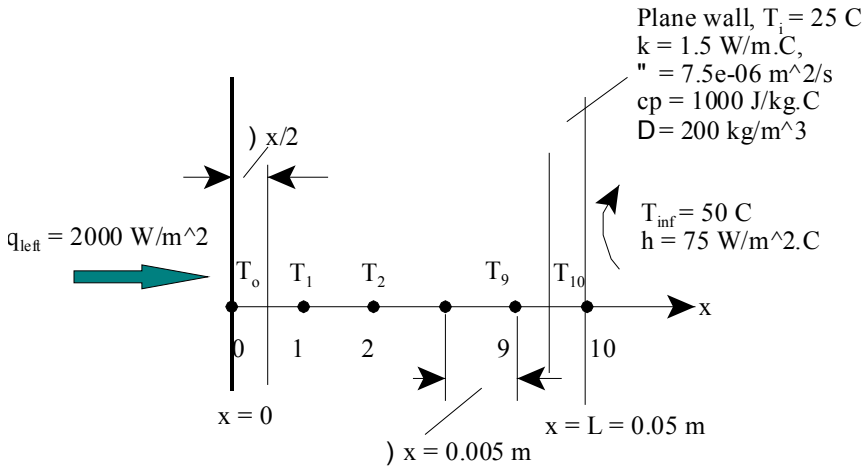

Fig.Prob.1I.C.4

EES Solution:

"Data:"

L=0.05 "[m]"
k=1.5 "[W/m-C]"
alpha=7.5E-6 "[m^2/s]"
"T_i=25" "[C]"
T_infinity=50 "[C]"
h=75 "[W/m^2-C]"
q_left = 2000 [W/m^2]
DELTAx=0.005 "[m]"
{Time=300 [s] "parameter to be varied"}

"Calculations:"

M=L/DELTAx+1 "Number of nodes"
DELTAt=20 "[s]"
Fo=(alpha*DELTAt)/DELTAx^2

"**The technique is to:** Set up the parametric Table. Store the temperatures in the parametric table at a given time as old temperatures using the variable ROW. The first row contains the initial values; So, Solve Table must begin at row 2.

Use the DUPLICATE statement to reproduce the eqns for the internal nodes. Column 1 contains the time, column 2 the value of T[0], column 3, the value of T[1], etc., and column 12 the Row."

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Row = 1+Time/DELTAt

Duplicate i=0,10

T_old[i]=TableValue('Table 1',Row-1,#T[i])

end

"Using the implicit finite difference approach, the equations for the eleven unknown temperatures are:"

"Node 0: constant heat flux, q_left"

$$q_{left} + k \cdot \left[\frac{T_1 - T_0}{\Delta x}\right] = \frac{k}{\alpha} \cdot \frac{\Delta x}{2} \cdot \left[\frac{T_0 - T_{old,0}}{\Delta t}\right]$$

In EES this is entered:

q_left+ k*(T[1]-T[0])/DELTAx = (k/alpha)*(DELTAx/2)*(T[0]-T_old[0])/DELTAt

"Node 1 to 9: Applying the heat balance:"

$$k \cdot \left[\frac{T_{i-1} - T_i}{\Delta x}\right] + k \cdot \left[\frac{T_{i+1} - T_i}{\Delta x}\right] = \frac{k}{\alpha} \cdot \Delta x \cdot \left[\frac{T_i - T_{old,i}}{\Delta t}\right] \qquad \text{for } i = 1 \text{ to } 9$$

And in EES this eqn is:

Duplicate i = 1,9

k*(T[i – 1]-T[i])/DELTAx + k*(T[i + 1]-T[i])/DELTAx = (k/alpha)*(DELTAx)*(T[i]-T_old[i])/DELTAt

end

"Node 10 – convection: Applying the heat balance:"

$$k \cdot \left[\frac{T_9 - T_{10}}{\Delta x}\right] + h \cdot (T_\infty - T_{10}) = \frac{k}{\alpha} \cdot \frac{\Delta x}{2} \cdot \left[\frac{T_{10} - T_{old,10}}{\Delta t}\right]$$

In EES this eqn is entered as:

k*(T[9]-T[10])/DELTAx + h*(T_infinity-T[10])= (k/alpha)*(DELTAx/2)*(T[10]-T_old[10])/DELTAt

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Results:

Time	**T[0]**	**T[1]**	**T[2]**	**T[3]**	**T[4]**	**T[5]**
	T[6]	**T[7]**	**T[8]**	**T[9]**	**T[10]**	**Row**
[s]	**[C]**	**[C]**	**[C]**	**[C]**	**[C]**	**[C]**
	[C]	**[C]**	**[C]**	**[C]**	**[C]**	
0	25	25	25	25	25	25
	25	25	25	25	1	
20	41.33	36.02	32.55	30.34	29.02	28.37
	28.28	28.74	29.83	31.72	34.73	2
40	50.41	44.5	40.01	36.75	34.57	33.31
	32.87	33.19	34.26	36.06	38.59	3
60	57.69	51.63	46.75	43	40.3	38.54
	37.67	37.59	38.24	39.56	41.45	4
80	64.11	57.98	52.9	48.85	45.78	43.62
	42.31	41.77	41.92	42.69	43.99	5
100	69.95	63.77	58.55	54.28	50.91	48.4
	46.68	45.69	45.36	45.6	46.32	6
120	75.32	69.1	63.77	59.31	55.68	52.85
	50.77	49.36	48.57	48.31	48.5	7
140	80.28	74.02	68.59	63.96	60.11	56.99
	54.57	52.78	51.56	50.83	50.53	8
160	**84.87**	**78.58**	**73.06**	**68.28**	**64.22**	**60.84**
	58.1	**55.96**	**54.34**	**53.18**	**52.42**	**9**
180	89.12	82.8	77.2	72.28	68.02	64.41
	61.38	58.91	56.92	55.37	54.18	10
200	93.06	86.72	81.03	75.98	71.56	67.72
	64.43	61.64	59.32	57.39	55.81	11
220	96.71	90.35	84.59	79.42	74.83	70.78
	67.25	64.19	61.55	59.28	57.32	12
240	100.1	93.71	87.88	82.61	77.87	73.63
	69.87	66.54	63.61	61.02	58.72	13
260	103.2	96.83	90.94	85.57	80.68	76.27
	72.3	68.73	65.53	62.64	60.03	14
280	106.1	99.72	93.78	88.31	83.3	78.72
	74.55	70.76	67.3	64.14	61.23	15
300	**108.8**	**102.4**	**96.41**	**90.85**	**85.72**	**80.99**
	76.64	**72.64**	**68.95**	**65.54**	**62.35**	**16**
320	111.3	104.9	98.85	93.21	87.97	83.1
	78.58	74.38	70.48	66.83	63.39	17

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1160	142	135.3	128.7	122.1	115.5	108.9
	102.3	95.73	89.18	82.64	76.11	59
180	142.1	135.4	128.8	122.2	115.5	108.9
	102.4	95.79	89.24	82.69	76.15	60
1200	142.2	135.5	128.9	122.2	115.6	109
	102.4	95.86	89.29	82.73	76.18	61
1220	142.3	135.6	129	122.3	115.7	109.1
	102.5	95.92	89.34	82.78	76.22	62
1240	142.3	135.7	129	122.4	115.8	109.2
	102.6	95.97	89.39	82.82	76.25	63
1260	142.4	135.7	129.1	122.5	115.8	109.2
	102.6	96.02	89.43	82.86	76.28	64
1280	142.5	135.8	129.2	122.5	115.9	109.3
	102.7	96.07	89.48	82.89	76.31	65
1300	**142.5**	**135.9**	**129.2**	**122.6**	**116**	**109.3**
	102.7	**96.11**	**89.51**	**82.92**	**76.34**	**66**

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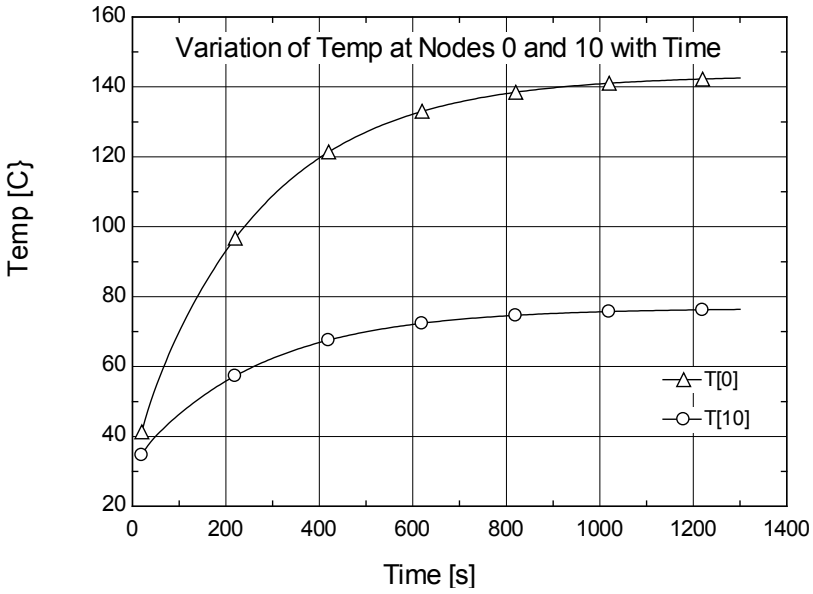

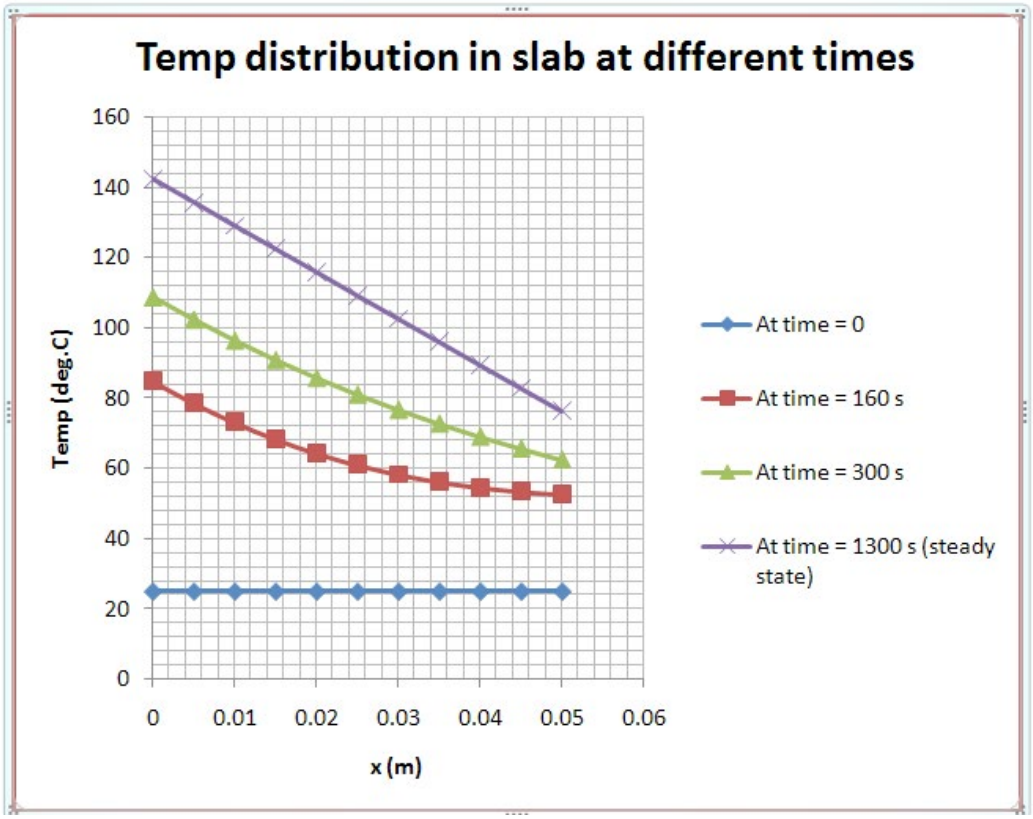

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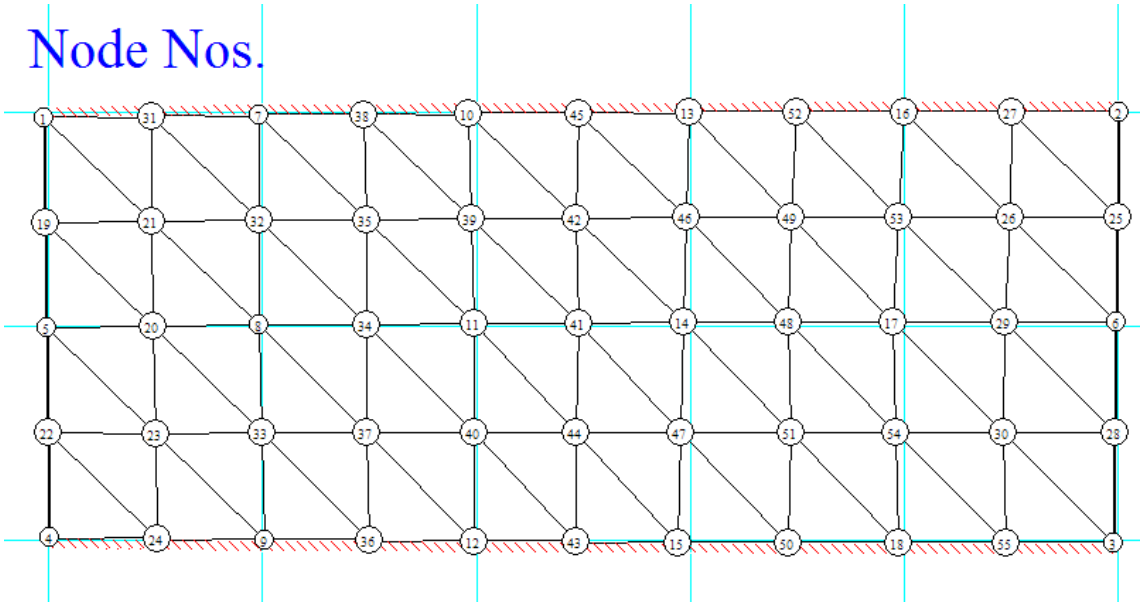
Solution to the above problem by Finite Element Heat Transfer (FEHT) Software:

1. **Node positions:**



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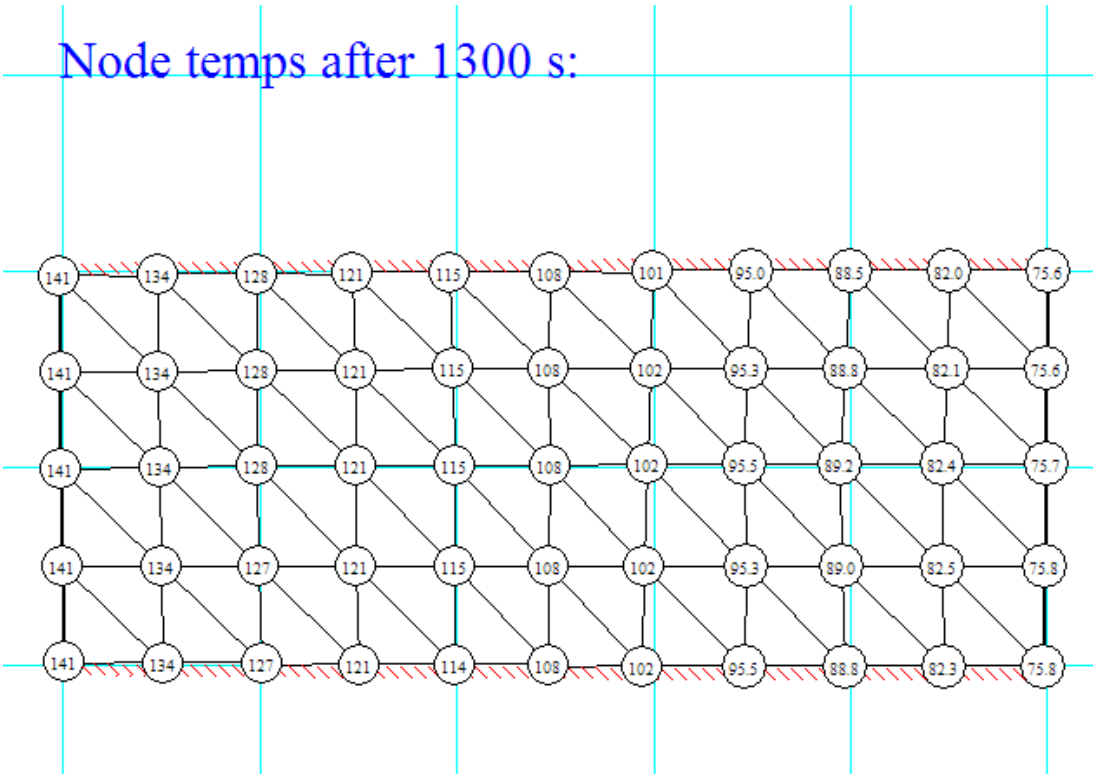
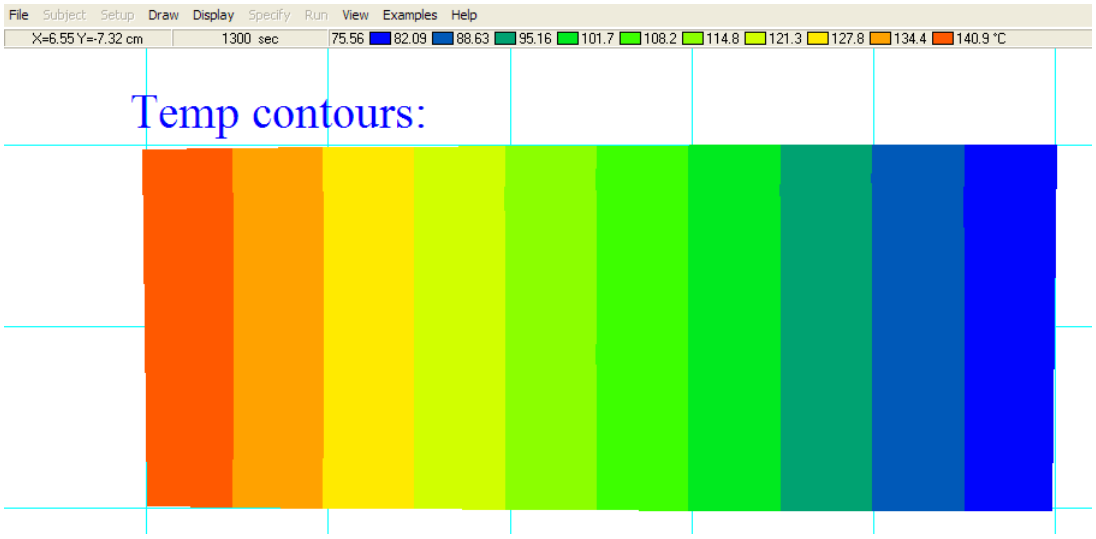
2. **Node temps:**



3. **Temp contours (shaded bands) –**
   Temp range of colours shown in tool bar at the top of the fig.:



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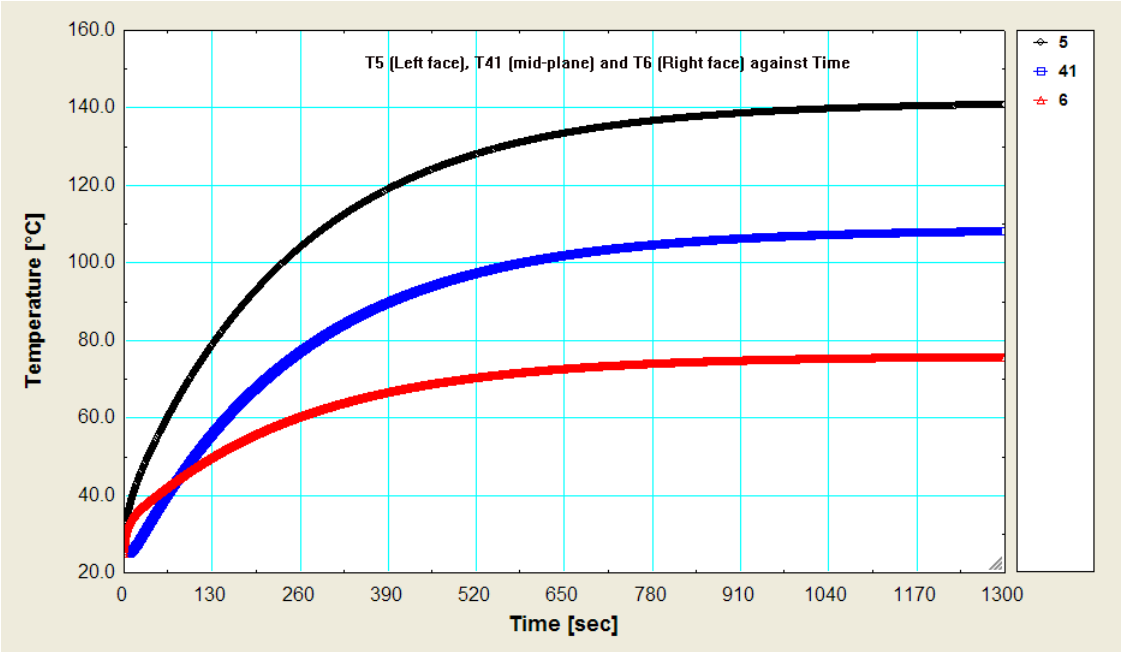
4. **Temp vs Time at different Nodes:**



==========

Prob. 1I.C.5: Consider a fuel element, which is a plane wall of L = 20 mm, cooled on both the faces with a fluid of temp = T_infinity = 250 C, with h = 1100 W/m^2.C. For the material, k = 30 W/m.C and α = 5 E-06 m^2/s. Initially the wall is at a uniform temp of 250 C, with no heat generation. Suddenly, the element is inserted into the reactor causing a uniform heat generation of q_g = 1E08 W/m^3. Using the Explicit method of finite difference s, with Δx = 2 mm, determine the temp distribution 1.5 s after the element is inserted into the core.[Ref. 3]

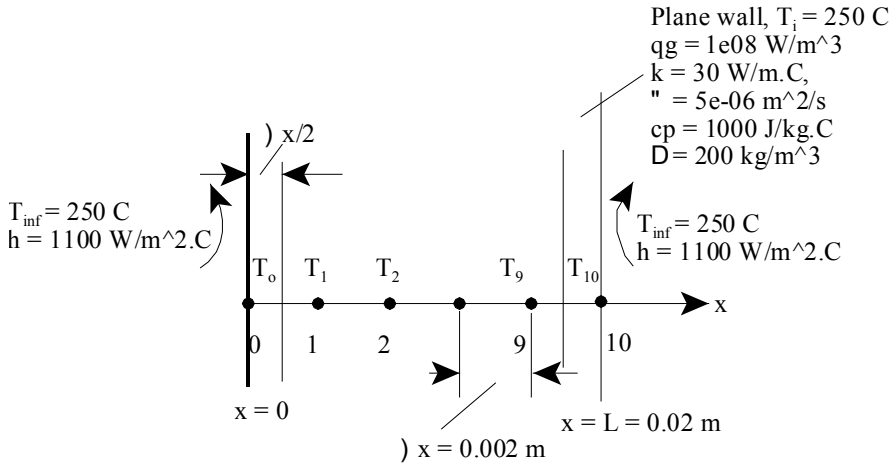


Fig.Prob.1I.C.5

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EES Solution:

"Data:"

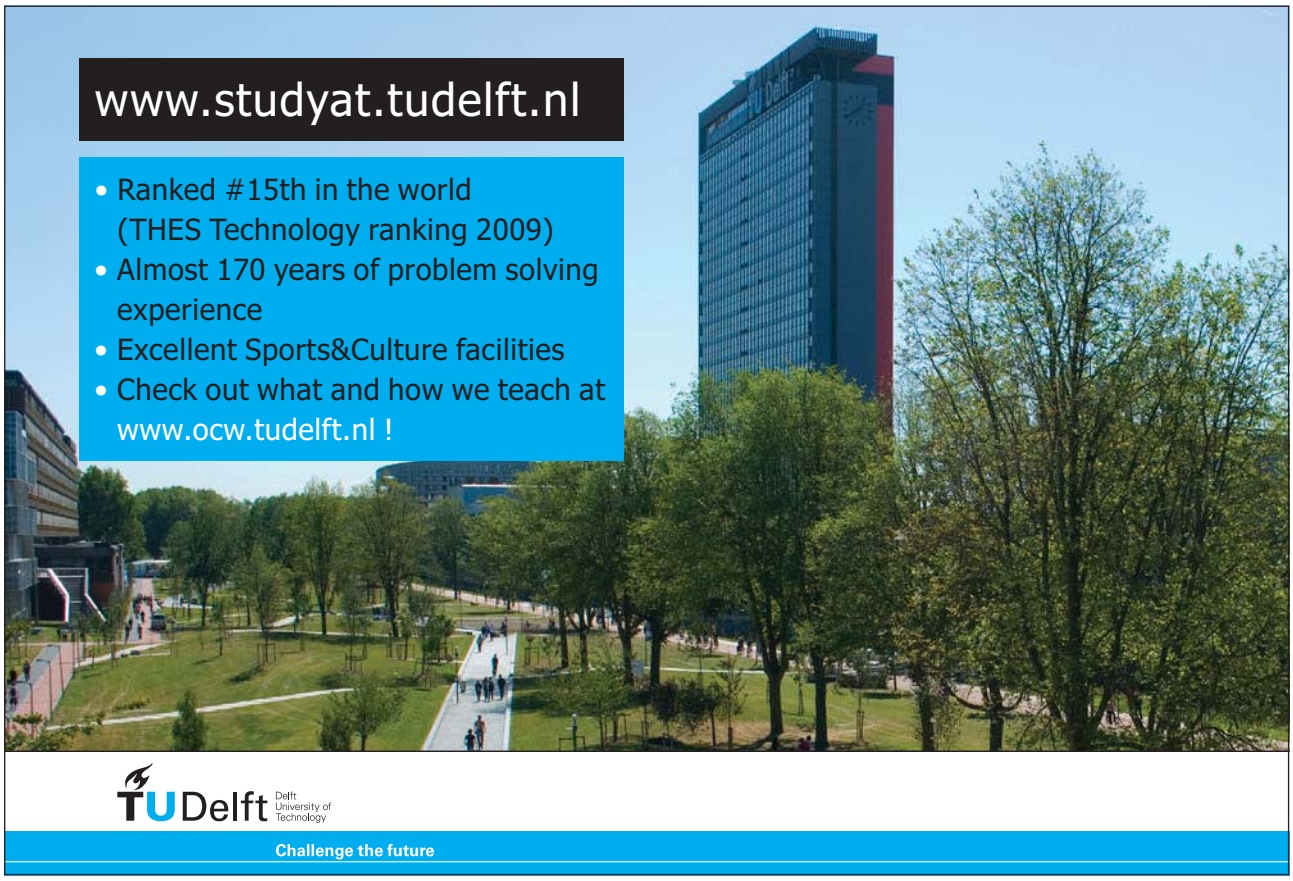
L=0.02 "[m]"
k=30 "[W/m-C]"
alpha=5E-6 "[m^2/s]"
T_i=250 "[C] … Initial temp."
q_g=1E8 "[W/m^3]"
T_infinity=250 "[C]…. fluid temp"
h=1100 "[W/m^2-C]"
DELTAx=0.002 "[m]"

{time=300 [s] "parameter to be varied"}

"Calculations:"

M=L/DELTAx+1 "Number of nodes"

Fo=(alpha*DELTAt)/DELTAx^2 "…mesh Fourier No."



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"Stability criteria:"

Biot = h * DELTAx/k

{Fo * (1 + Biot) = 0.5 "…determines Fo from Stability criterion, and, we get: DELTAt = 0.3727 s from stability criteria.

Let us use DELTAt = 0.3 s"}

DELTAt=0.3 "[s]"

"The temperatures in the parametric table as 'old temps'; and , then, in each step, they are recovered using the variable ROW. The first row contains the initial values. So, Solve Table must begin at row 2. Note the use the DUPLICATE statement to reduce the number of equations to be typed. Column 1 contains the time, column 2 the value of T[1], column 3, the value of T[2], etc., and column 7 the Row."

Row = 1 + Time/DELTAt

Duplicate i=0,10
    T_old[i]=TableValue('Table 1',Row -1,#T[i])
end

"Using the explicit finite difference approach, writing the energy balance eqns.,to get the eleven equations for the eleven unknown temperatures:"

"Node 0, convection:"

$$h \cdot (T_\infty - T_{old,0}) + k \cdot \left[\frac{T_{old,1} - T_{old,0}}{\Delta x}\right] + q_g \cdot \frac{\Delta x}{2} = \frac{k}{\alpha} \cdot \frac{\Delta x}{2} \cdot \left[\frac{T_0 - T_{old,0}}{\Delta t}\right]$$

In EES, it is entered as:

h * (T_infinity – T_old[0]) + k * (T_old[1] – T_old[0]) / DELTAx + q_g * DELTAx/2 = (k / alpha) * DELTAx/2 * (T[0] – T_old[0])/DELTAt

"Internal nodes: 1 to 9:"

Duplicate m = 1,9

$$k \cdot \left[\frac{T_{old,m-1} - T_{old,m}}{\Delta x}\right] + k \cdot \left[\frac{T_{old,m+1} - T_{old,m}}{\Delta x}\right] + q_g \cdot \Delta x = \frac{k}{\alpha} \cdot \Delta x \cdot \left[\frac{T_m - T_{old,m}}{\Delta t}\right] \quad \text{for } m = 1 \text{ to } 9$$

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In EES it is typed as:

k * (T_old[m-1] – T_old[m]) / DELTAx + k * (T_old[m+1] – T_old[m]) / DELTAx + q_g * DELTAx = (k / alpha) * DELTAx * (T[m] – T_old[m])/DELTAt

end

"Node 10: with convection:"

$$h \cdot (T_\infty - T_{10}) + k \cdot \left[\frac{T_9 - T_{10}}{\Delta x}\right] + q_g \cdot \frac{\Delta x}{2} = \frac{k}{\alpha} \cdot \frac{\Delta x}{2} \cdot \left[\frac{T_{10} - T_{old,10}}{\Delta t}\right]$$

And, it is entered as:

h * (T_infinity – T[10]) + k * (T[9] – T[10]) / DELTAx + q_g * DELTAx/2 = (k / alpha) * DELTAx/2 * (T[10] – T_old[10])/DELTAt

Results:

Time	**T[0]**	**T[1]**	**T[2]**	**T[3]**	**T[4]**	**T[5]**
	T[6]	**T[7]**	**T[8]**	**T[9]**	**T[10]**	**Row**
[s]	**[C]**	**[C]**	**[C]**	**[C]**	**[C]**	**[C]**
	[C]	**[C]**	**[C]**	**[C]**	**[C]**	
0	250	250	250	250	250	250
	250	250	250	250	250	1
0.3	255	255	255	255	255	255
	255	255	255	255	254.8	2
0.6	259.7	260	260	260	260	260
	260	260	260	259.9	259.6	3
0.9	264.4	264.9	265	265	265	265
	265	265	265	264.8	264.2	4
1.2	269	269.7	270	270	270	270
	270	270	269.9	269.7	268.8	5
1.5	**273.5**	**274.5**	**274.9**	**275**	**275**	**275**
	275	**275**	**274.9**	**274.5**	**273.4**	**6**
1.8	278	279.3	279.8	280	280	280
	280	279.9	279.7	279.2	277.8	7

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14.1	439.2	451.1	459.6	465.4	468.7	469.8
	468.7	465.4	459.6	451	439.1	48
14.4	442.7	454.8	463.6	469.5	472.9	474
	472.9	469.5	463.5	454.7	442.6	49
14.7	446.2	458.6	467.5	473.5	477	478.2
	477	473.5	467.5	458.5	446.1	50

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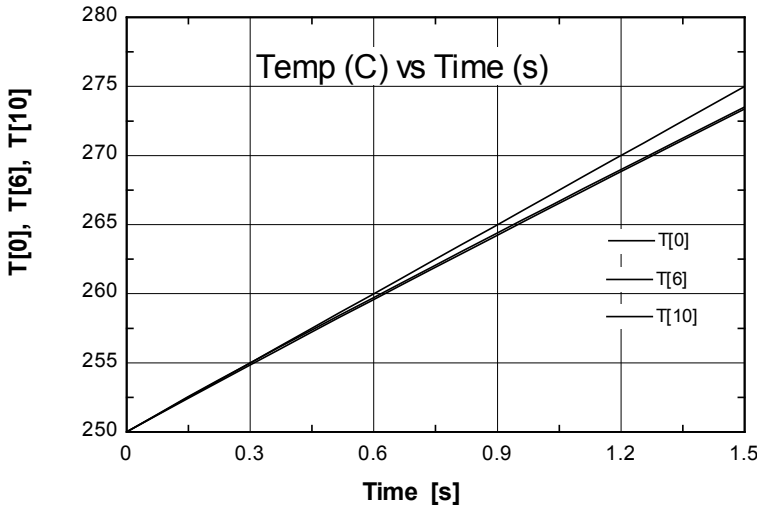
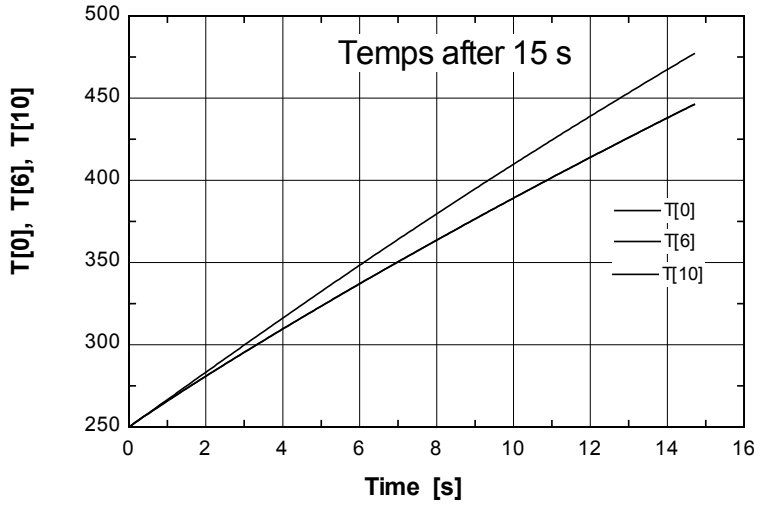
Plot the variation of T0, T6 and T10 for the first 1.5 s:



Plot the variation of T0, T6 and T10 upto15 s:



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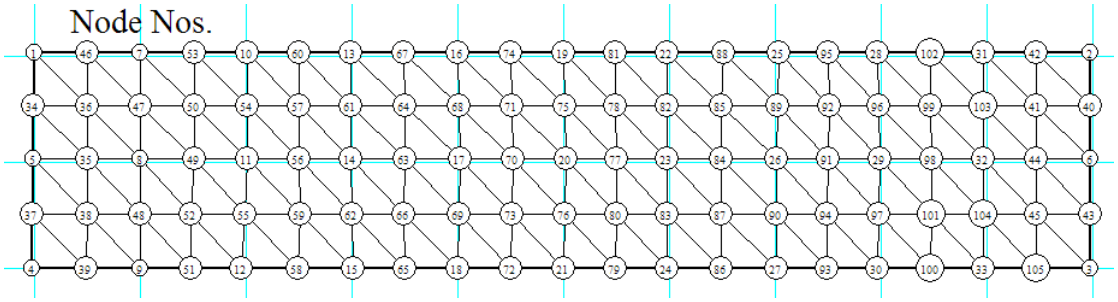
Solution to the above problem by Finite Element Heat Transfer (FEHT) Software:

1. **Node positions:**



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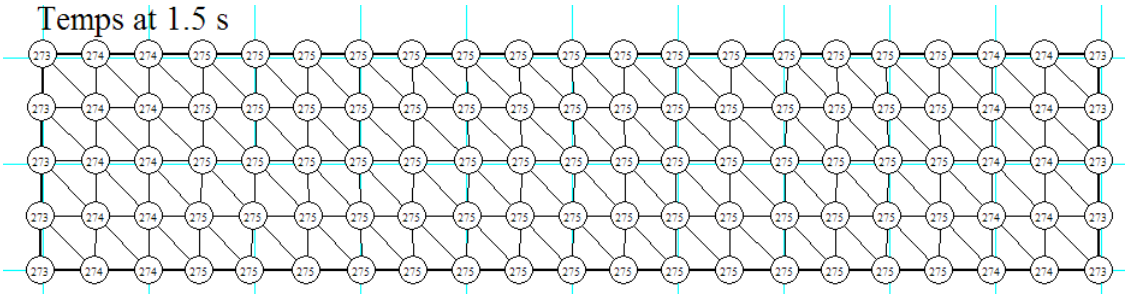
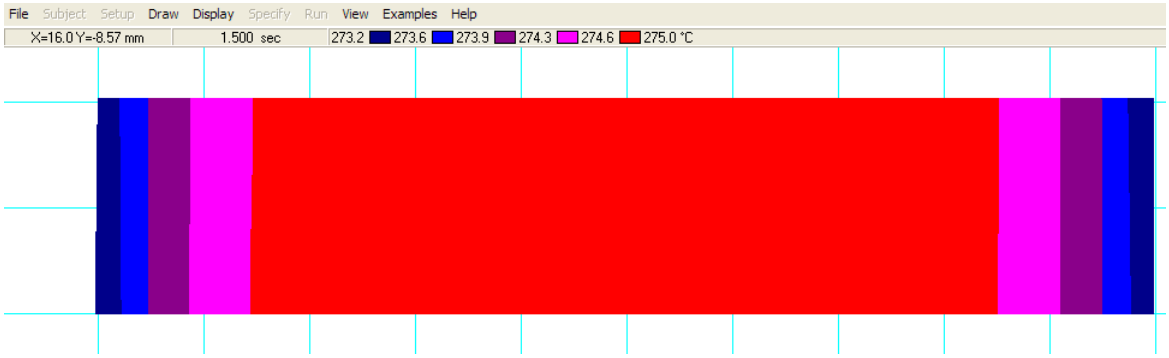
2. **Node Temps:**

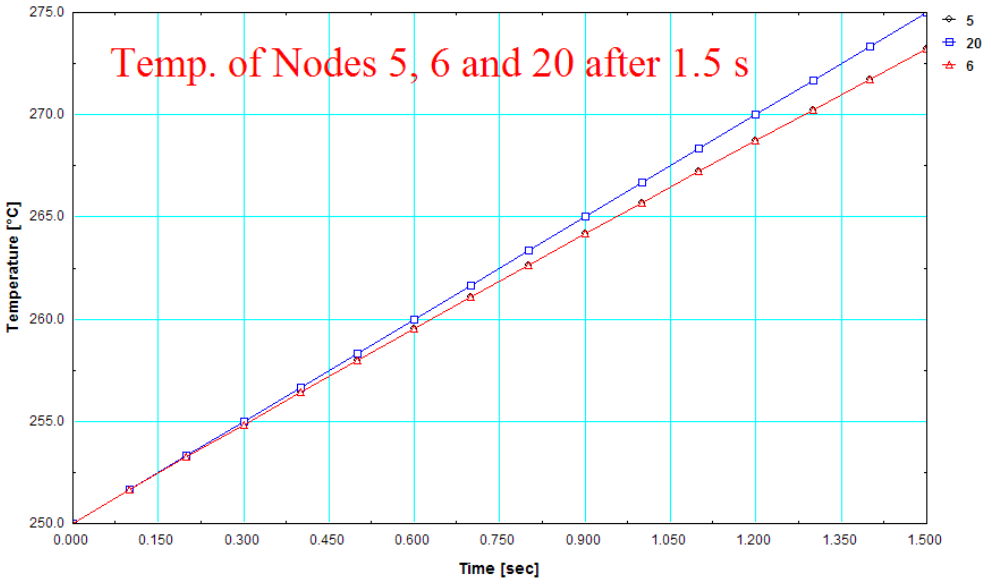


3. **Temp contours (shaded bands) –**

Temp range of colours shown in tool bar at the top of the fig.



4. **Temps of Nodes 5, 6 and 20 after 1.5 s:**



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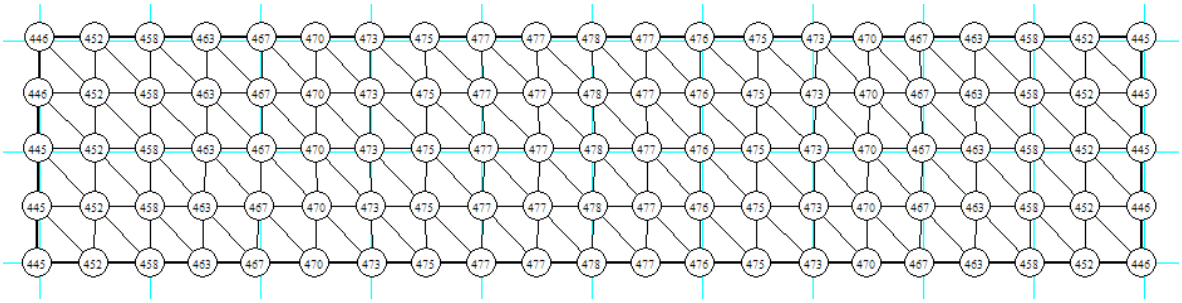
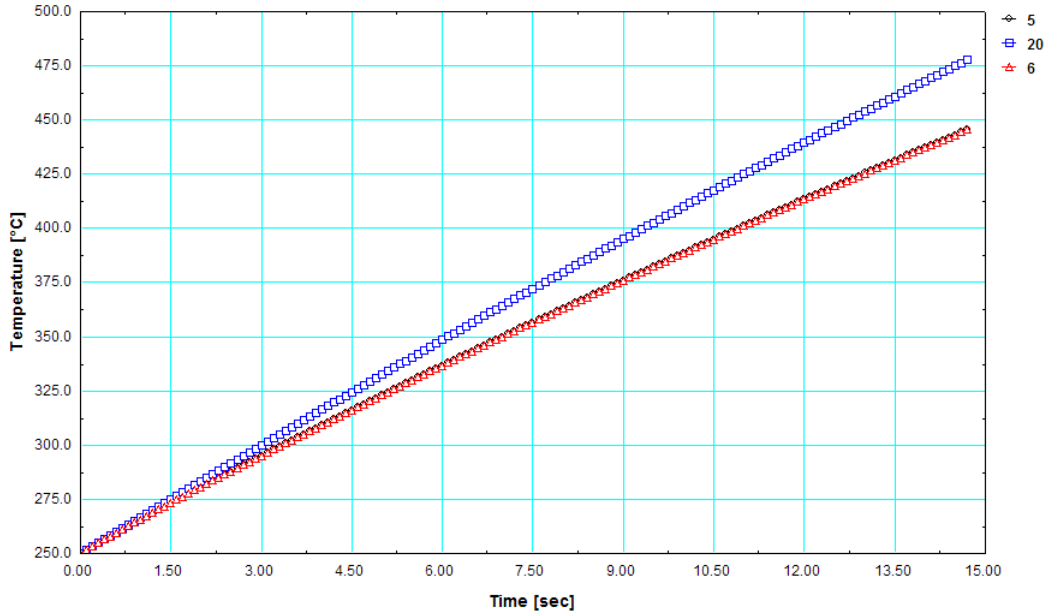
After 14.7 s:

Node Temps after 14.7 s:



Temp vs Time for Nodes 5, 6 and 20:



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"**Prob. 1I.C.6.**The ceramic wall shown is initially at a uniform temp of 25 C. Its thickness = 3 cm. It is suddenly exposed to a radiation source at 1200 C on the RHS. The LHS is exposed to room air at 25 C with a radiation surrounding temp of 25 C, with h = 1.92 × DELTAT^(5/4) W/m^2.K. Convection on the RHS is negligible. For ceramic: k = 3.0 W/m.K, rho = 1600 kg/m^3, cp = 0.8 kJ/kg.K. Determine: temp distribution in the plate after 15, 30, 45, 60, 90, 120 and 150 s. Also, determine steady state temp distribution. Calculate the total heat gained by the plate for these times."

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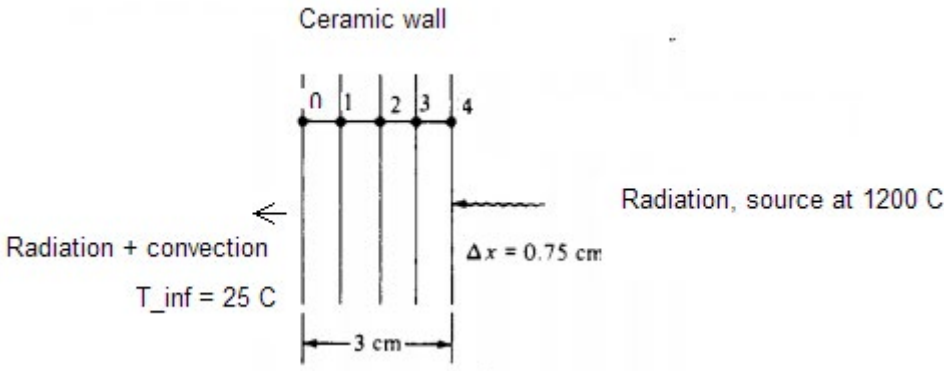

Fig.Prob.11.C.6

"Data:"

L=0.03 "[m]"
k=3 "[W/m-C]"
rho = 1600 "[kg/m^3]"
cp = 800 "[J/kg.K]"
T_ini=25 "[C]"
T_infinity=25.0 "[C]"
T_rad = 1200 "[C]"

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{h=1.92 x DELTAT^(5/4) "[W/m^2-C]"}
DELTAx=0.0075 "[m]"
sigma = 5.67E-08 "[W/m^2.K^4]"
epsilon = 0.8

"time=150 [s]" "parameter to be varied"

"Calculations:"

M=L/DELTAx+1 "Number of nodes"
A=1"[m^2]"

"Node 0: Convection + Radiation: By energy balance:"

sigma * epsilon * ((273 + T_infinity)^4 – (T[0] + 273)^4) – 1.92 * (T[0] – T_infinity)^(5/4) + k*A*(T[1]-T[0])/DELTAx =rho * cp *A*(DELTAx/2)*dTdt[0]

"Node 1:"

k*A*(T[0]-T[1])/DELTAx + k*A*(T[2]-T[1])/DELTAx = rho * cp*A*(DELTAx)*dTdt[1]

"Node 2:"

k*A*(T[1]-T[2])/DELTAx + k*A*(T[3]-T[2])/DELTAx = rho * cp*A*(DELTAx)*dTdt[2]

"Node 3:"

k*A*(T[2]-T[3])/DELTAx + k*A*(T[4]-T[3])/DELTAx = rho * cp*A*(DELTAx)*dTdt[3]

"Node 4:"

sigma * epsilon * ((T_rad + 273)^4 – (T[4] + 273)^4)+ k*A*(T[3]-T[4])/DELTAx= rho * cp*A*(DELTAx/2)*dTdt[4]

"Integrate using the INTEGRAL command:"

```
duplicate i=0,4
        T[i]=T_ini+INTEGRAL(dTdt[i],time)
end
```

"To get the temps in Kelvin:"

```
duplicate i=0,4
        TKelvin[i]=T[i] + 273
End
```


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Results:

time [s]	TKelvin[0] [K]	TKelvin[1]	TKelvin[2] [K]	TKelvin[3]	TKelvin[4] [K]
0	**298**		**298**		**298**
	298		**298**		
15	**299.4**		**301.6**		**315.3**
	384.2		**729.2**		
30	**307.2**		**317.6**		**368**
	531.7		**891.3**		
45	**327.5**		**350.5**		**438.9**
	641		**996.8**		
60	**361.5**		**395.4**		**507.8**
	726.9		**1067**		
75	405		446.3		571.8
	795.6		1118		
90	**453.1**		**499.3**		**630.7**
	852.2		**1156**		
105	502.5		551.9		684.7
	900.2		1187		
120	**550.7**		**602.8**		**734.4**
	942		**1212**		
135	596.3		650.9		780
	979.1		1234		
50	**638.5**		**695.7**		**821.9**
	1012		**1253**		
165	676.9		737		860.3
	1042		1269		
840	954		1063		1172
	1282		1391		
855	954.1		1063		1172
	1282		1391		
870	954.1		1063		1172
	1282		1391		
885	**954.2**		**1063**		**1172**
	1282		**1391**		

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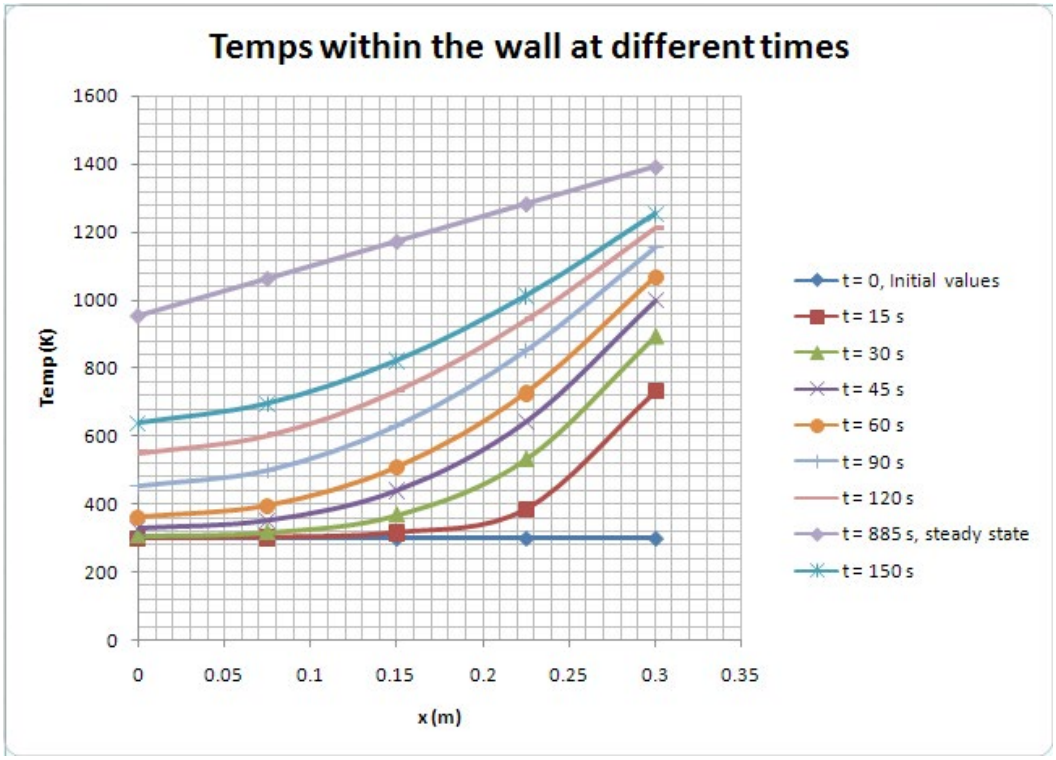
Plot the temp variation in the wall with time:



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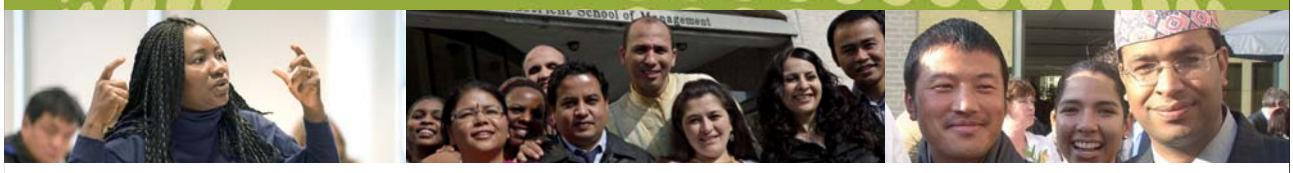

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Prob.1I.C.7. Solve Prob. 1IC.4 with EXCEL:

Let us state the problem again:

Prob. 1I.C.4. Consider a plane wall with k = 1.5 W/m.C, cp = 1000 J/kg.C, ρ = 200 kg/m^3, α = 7.5E-06 m^2/s. Its thickness L = 50 mm. Initial uniform temp = 25 C. Suddenly, the boundary at x = L is subjected to heating by a fluid at a temp of 50 C and h = 75 W/m^2.C. And, the boundary at x = 0 is subjected to a heat flux = 2000 W/m^2. Take Δx = 5 mm and Δt = 20 s. Plot the temp distribution in the wall for the initial condition, at t = 160 s, at t = 300 s and at t = 1300 s.

Adopt the Implicit, finite difference method.

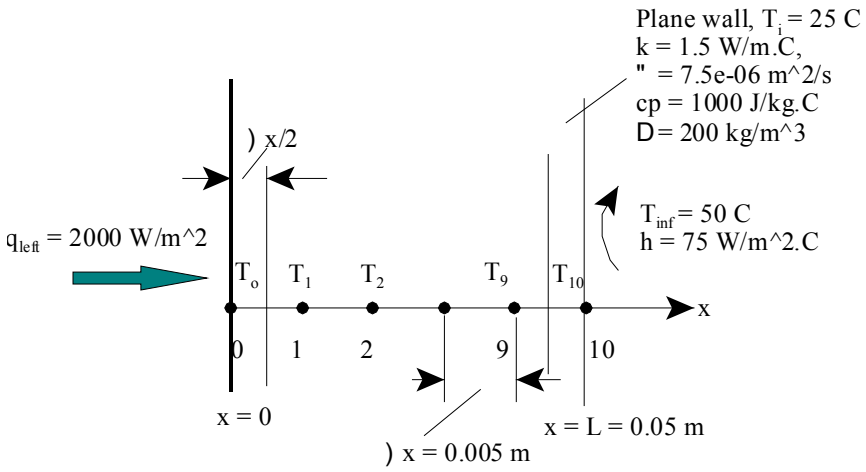

Fig.Prob.1I.C.4

EXCEL Solution:

We derive the difference eqns for Boundary Node '0', Internal Nodes 1 to 9, and Boundary Node 10, by writing an energy balance at the respective nodes:

For Implicit method:

For Boundary Node '0':

$$q_{left} + k \cdot \left[\frac{T_1 - T_0}{\Delta x}\right] = \frac{k}{\alpha} \cdot \frac{\Delta x}{2} \cdot \left[\frac{T_0 - T_{old,0}}{\Delta t}\right]$$

i.e. $$q_{left} \cdot \Delta x + k \cdot (T_1 - T_0) = \frac{k}{Fo} \cdot \frac{1}{2} \cdot (T_0 - T_{old\_0})$$

i.e. $$T_0 = \frac{(2 \cdot q_{left} \cdot \Delta x \cdot Fo + 2 \cdot k \cdot T_1 \cdot Fo + k \cdot T_{old\_0})}{k \cdot (2 \cdot Fo + 1)}$$ ....For Node 0 ..

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Here, T_old_0 refers to the temp T_0 at the 'previous' time step.

For Internal Nodes '1 to 9':

$$k \cdot \left[\frac{T_{i-1} - T_i}{\Delta x}\right] + k \cdot \left[\frac{T_{i+1} - T_i}{\Delta x}\right] = \frac{k}{\alpha} \cdot \Delta x \cdot \left[\frac{T_i - T_{old,i}}{\Delta t}\right] \qquad \text{for } i = 1 \text{ to } 9$$

Then, for Node 1:

$$k \cdot (T_0 - T_1) + k \cdot (T_2 - T_1) = \frac{k}{Fo} \cdot (T_1 - T_{1\_old})$$

i.e. $$(T_0 - T_1) + (T_2 - T_1) = \frac{1}{Fo} \cdot (T_1 - T_{1\_old})$$

i.e. $$T_1 = \frac{(T_0 \cdot Fo + T_2 \cdot Fo + T_{1\_old})}{(2 \cdot Fo + 1)}$$ ....For internal node 1 ..

Here, again, T1_old refers to the temp T1 at the 'previous' time step.

In EXCEL, now, it is very easy to write the eqns for Nodes 2 to 9: simply drag and copy from Node 1.

For Boundary Node '10':

$$k \cdot \left[\frac{T_9 - T_{10}}{\Delta x}\right] + h \cdot (T_\infty - T_{10}) = \frac{k}{\alpha} \cdot \frac{\Delta x}{2} \cdot \left[\frac{T_{10} - T_{old,10}}{\Delta t}\right]$$

i.e. $$k \cdot (T_9 - T_{10}) + h \cdot (T_{inf} - T_{10}) \cdot \Delta x = \frac{k}{Fo} \cdot \frac{1}{2} \cdot (T_{10} - T_{10\_old})$$

i.e. $$T_{10} = \frac{(2 \cdot k \cdot T_9 \cdot Fo + 2 \cdot h \cdot T_{inf} \cdot \Delta x \cdot Fo + k \cdot T_{10\_old})}{(2 \cdot k \cdot Fo + 2 \cdot h \cdot \Delta x \cdot Fo + k)}$$ ....For Node 10 .

Here also, T_10_old refers to the temp T_10 at the 'previous' time step.

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Note: In the above eqns, **Fo is the mesh Fourier No.** defined as:

$$Fo = \frac{\alpha \cdot \Delta T}{\Delta x^2}$$

where α is the thermal diffusivity, given by:

$$\alpha = \frac{k}{\rho \cdot cp}$$

ρ is the density, cp is the sp.heat.

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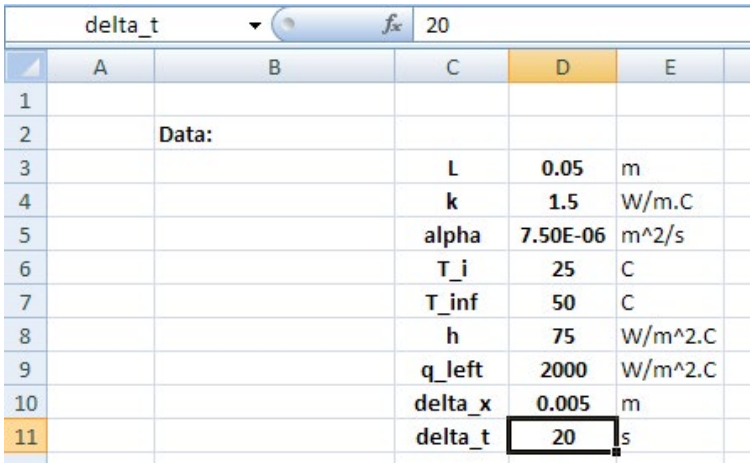
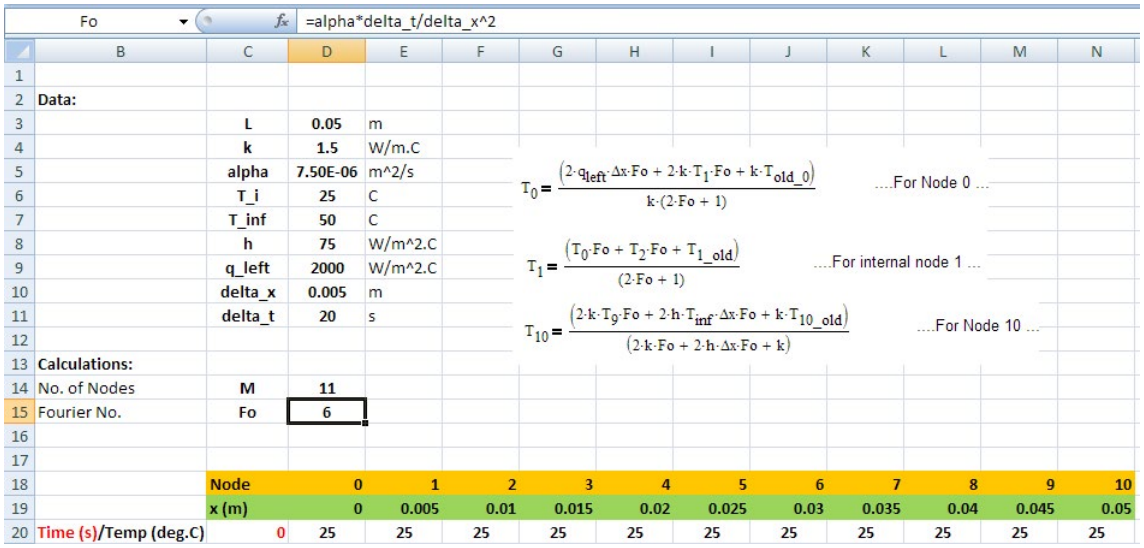
Following are the steps in EXCEL Solution:

1. Set up the EXCEL worksheet, enter data and name the cells:



	A	B	C	D	E
1					
2		Data:			
3			L	0.05	m
4			k	1.5	W/m.C
5			alpha	7.50E-06	m^2/s
6			T_i	25	C
7			T_inf	50	C
8			h	75	W/m^2.C
9			q_left	2000	W/m^2.C
10			delta_x	0.005	m
11			delta_t	20	s

2. Now, enter calculations. No. of Nodes M and mesh Fourier No. Fo are calculated. Scheme for calculations at different times is shown in the Table. Formulas used for different nodes are also shown alongside for ready reference. Initial temperatures, i.e. at time = 0, are also entered.



	B	C	D	E	F	G	H	I	J	K	L	M	N
2	Data:												
3		L	0.05	m									
4		k	1.5	W/m.C									
5		alpha	7.50E-06	m^2/s									
6		T_i	25	C									
7		T_inf	50	C									
8		h	75	W/m^2.C									
9		q_left	2000	W/m^2.C									
10		delta_x	0.005	m									
11		delta_t	20	s									
13	Calculations:												
14	No. of Nodes	M	11										
15	Fourier No.	Fo	6										
18		Node	0	1	2	3	4	5	6	7	8	9	10
19		x (m)	0	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04	0.045	0.05
20	Time (s)/Temp (deg.C)	0	25	25	25	25	25	25	25	25	25	25	25

$$T_0 = \frac{(2 \cdot q_{left} \cdot \Delta x \cdot Fo + 2 \cdot k \cdot T_1 \cdot Fo + k \cdot T_{old\_0})}{k \cdot (2 \cdot Fo + 1)} \quad \text{....For Node 0 ...}$$

$$T_1 = \frac{(T_0 \cdot Fo + T_2 \cdot Fo + T_{1\_old})}{(2 \cdot Fo + 1)} \quad \text{....For internal node 1 ...}$$

$$T_{10} = \frac{(2 \cdot k \cdot T_9 \cdot Fo + 2 \cdot h \cdot T_{inf} \cdot \Delta x \cdot Fo + k \cdot T_{10\_old})}{(2 \cdot k \cdot Fo + 2 \cdot h \cdot \Delta x \cdot Fo + k)} \quad \text{....For Node 10 ...}$$

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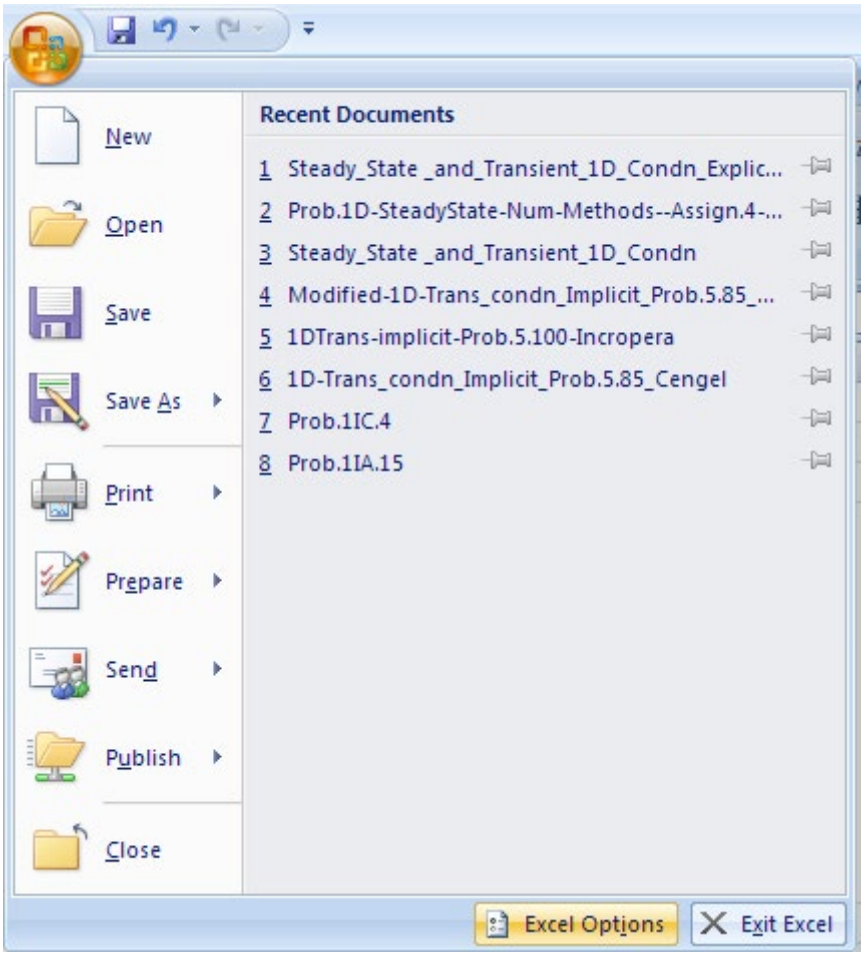
3. Now, enter the time steps in column C, up to time = 1300 s. Part of that Table is shown below:

	B	C	D	E	F	G	H	I	J	K	L	M	N
13	**Calculations:**												
14	No. of Nodes	**M**	**11**										
15	Fourier No.	**Fo**	**6**										
16													
17													
18		**Node**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
19		**x (m)**	**0**	**0.005**	**0.01**	**0.015**	**0.02**	**0.025**	**0.03**	**0.035**	**0.04**	**0.045**	**0.05**
20	**Time (s)/Temp (deg.C)**	**0**	25	25	25	25	25	25	25	25	25	25	25
21		**20**											
22		**40**											
23		**60**											
24		**80**											
25		**100**											
26		**120**											
27		**140**											
28		**160**											
29		**180**											
30		**200**											

4. Now, since all the nodal eqns for implicit method will be solved by iteration, make sure that iteration is enabled in EXCEL. i.e. Press Office button, go to EXCEL options:



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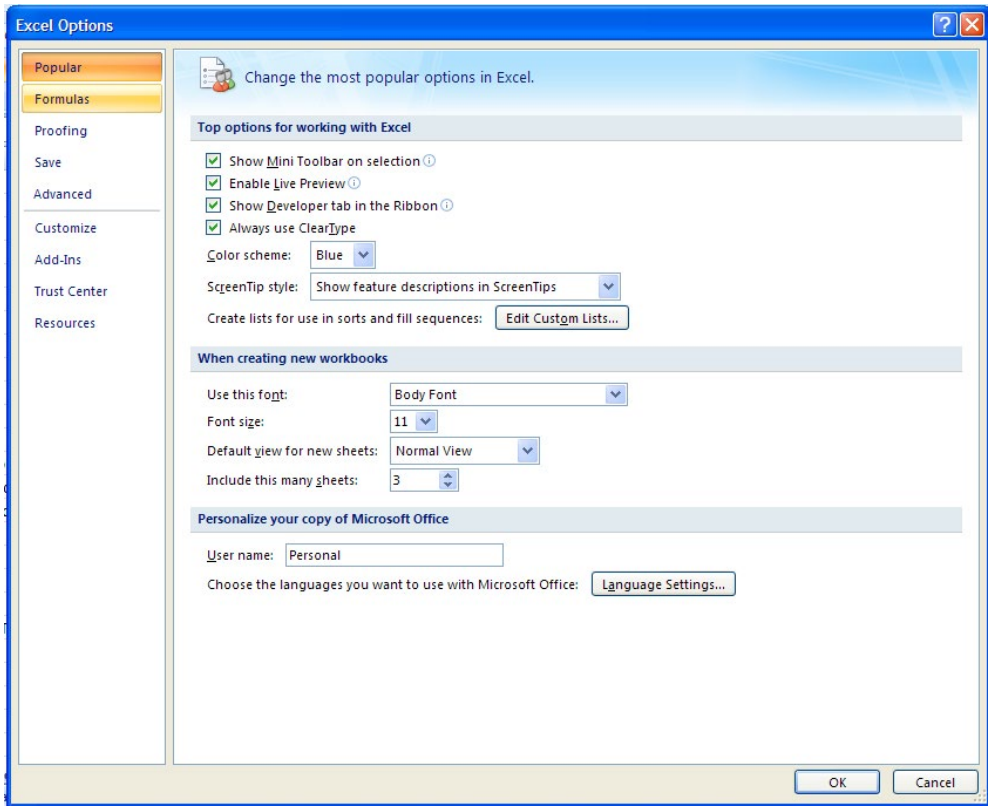
Press EXCEL options:



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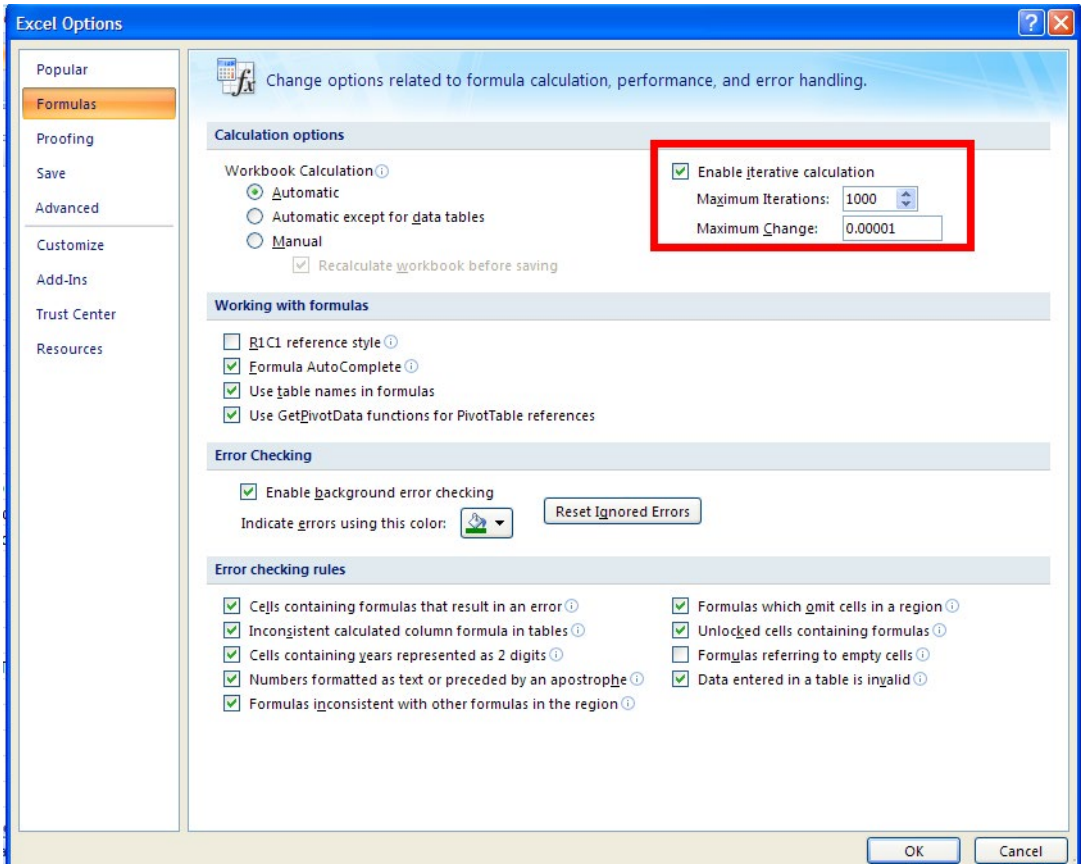
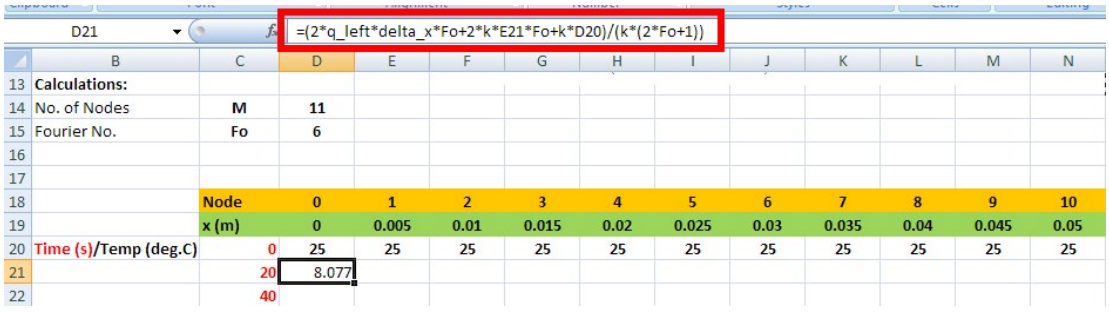


Note that we have put a check mark on Enable Iterative Calculation, and changed the Max. Iterations to 1000, and Max. change to 0.00001. Press OK.

5. Now, enter the eqn for Node '0'. Eqn can be seen in the Formula bar:



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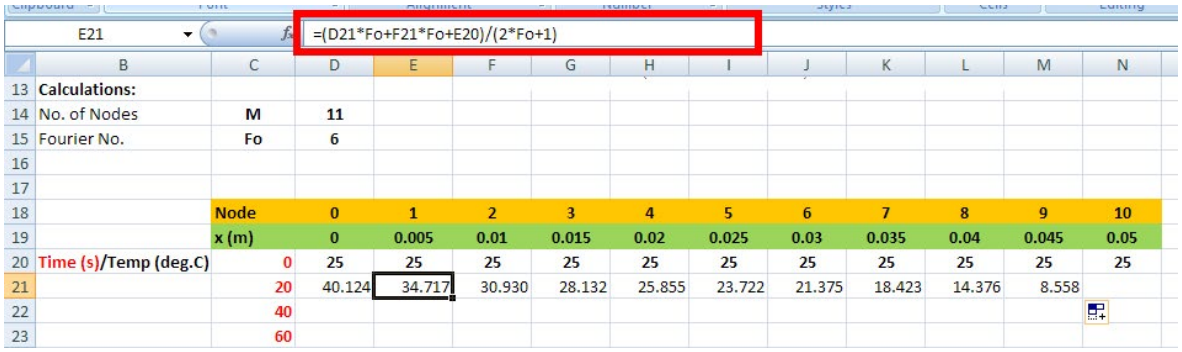
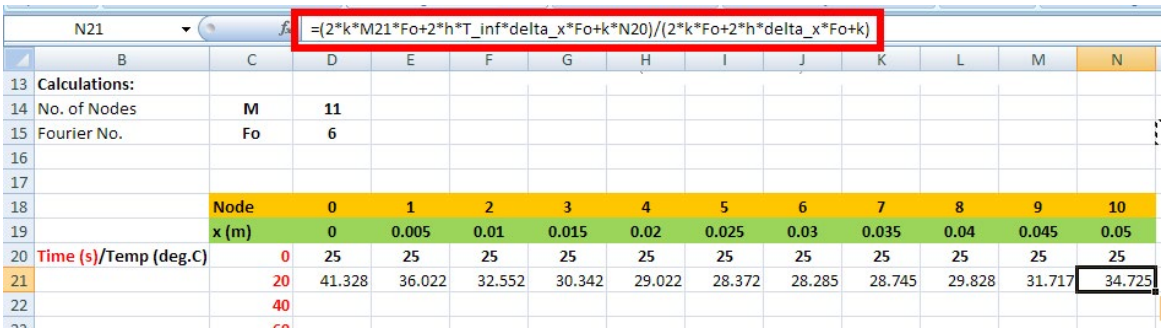
6. Similarly enter the eqn. for Node 1, and then drag-copy it up to Node 9:



E21 =(D21*Fo+F21*Fo+E20)/(2*Fo+1)

	B	C	D	E	F	G	H	I	J	K	L	M	N
13	Calculations:												
14	No. of Nodes	M	11										
15	Fourier No.	Fo	6										
16													
17													
18		Node	0	1	2	3	4	5	6	7	8	9	10
19		x (m)	0	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04	0.045	0.05
20	Time (s)/Temp (deg.C)	0	25	25	25	25	25	25	25	25	25	25	25
21		20	40.124	34.717	30.930	28.132	25.855	23.722	21.375	18.423	14.376	8.558	
22		40											
23		60											

7. And, enter eqn for Node 10. Now, we see that all temperatures have adjusted themselves in row 21:



N21 =(2*k*M21*Fo+2*h*T_inf*delta_x*Fo+k*N20)/(2*k*Fo+2*h*delta_x*Fo+k)

	B	C	D	E	F	G	H	I	J	K	L	M	N
13	Calculations:												
14	No. of Nodes	M	11										
15	Fourier No.	Fo	6										
16													
17													
18		Node	0	1	2	3	4	5	6	7	8	9	10
19		x (m)	0	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04	0.045	0.05
20	Time (s)/Temp (deg.C)	0	25	25	25	25	25	25	25	25	25	25	25
21		20	41.328	36.022	32.552	30.342	29.022	28.372	28.285	28.745	29.828	31.717	34.725
22		40											

8. Now, it is very easy to complete the Table: Select cells D21 to N21 in row 21 and simply drag-copy up to the time desired in column C: And, we get following Table: (only partly shown.)

	B	C	D	E	F	G	H	I	J	K	L	M	N
18		Node	0	1	2	3	4	5	6	7	8	9	10
19		x (m)	0	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04	0.045	0.05
20	Time (s)/Temp (deg.C)	0	25	25	25	25	25	25	25	25	25	25	25
21		20	41.328	36.022	32.552	30.342	29.022	28.372	28.285	28.745	29.828	31.717	34.725
22		40	50.412	44.503	40.007	36.753	34.567	33.306	32.868	33.193	34.259	36.064	38.593
23		60	57.686	51.626	46.753	43.004	40.297	38.545	37.666	37.586	38.239	39.556	41.454
24		80	64.107	57.975	52.902	48.854	45.780	43.620	42.306	41.766	41.922	42.692	43.985
25		100	69.947	63.767	58.553	54.280	50.911	48.398	46.681	45.693	45.360	45.600	46.324
26		120	75.317	69.098	63.768	59.306	55.682	52.853	50.767	49.362	48.568	48.310	48.502
27		140	80.277	74.023	68.591	63.962	60.109	56.995	54.570	52.779	51.558	50.835	50.532
28		160	84.866	78.581	73.057	68.277	64.216	60.839	58.103	55.956	54.338	53.184	52.421
29		180	89.116	82.804	77.195	72.276	68.024	64.406	61.383	58.906	56.921	55.367	54.177
30		200	93.056	86.717	81.031	75.984	71.555	67.715	64.426	61.645	59.320	57.394	55.807
31		220	96.707	90.345	84.587	79.422	74.830	70.784	67.250	64.186	61.545	59.276	57.320
32		240	100.093	93.709	87.885	82.611	77.868	73.631	69.869	66.543	63.610	61.021	58.723
33		260	103.233	96.828	90.943	85.567	80.684	76.271	72.298	68.729	65.525	62.641	60.026
34		280	106.144	99.720	93.778	88.309	83.297	78.720	74.551	70.757	67.302	64.143	61.233
35		300	108.844	102.403	96.408	90.852	85.719	80.990	76.640	72.638	68.950	65.536	62.354
36		320	111.348	104.890	98.847	93.210	87.966	83.096	78.578	74.382	70.478	66.828	63.393
37		340	113.670	107.197	101.109	95.397	90.050	85.050	80.375	76.000	71.895	68.026	64.356
38		360	115.824	109.337	103.206	97.425	91.982	86.861	82.042	77.501	73.209	69.137	65.250
39		380	117.821	111.321	105.151	99.306	93.774	88.541	83.588	78.892	74.428	70.168	66.079
40		400	119.673	113.161	106.955	101.050	95.436	90.099	85.021	80.182	75.559	71.123	66.848
41		420	121.391	114.867	108.628	102.668	96.977	91.543	86.350	81.379	76.607	72.010	67.560

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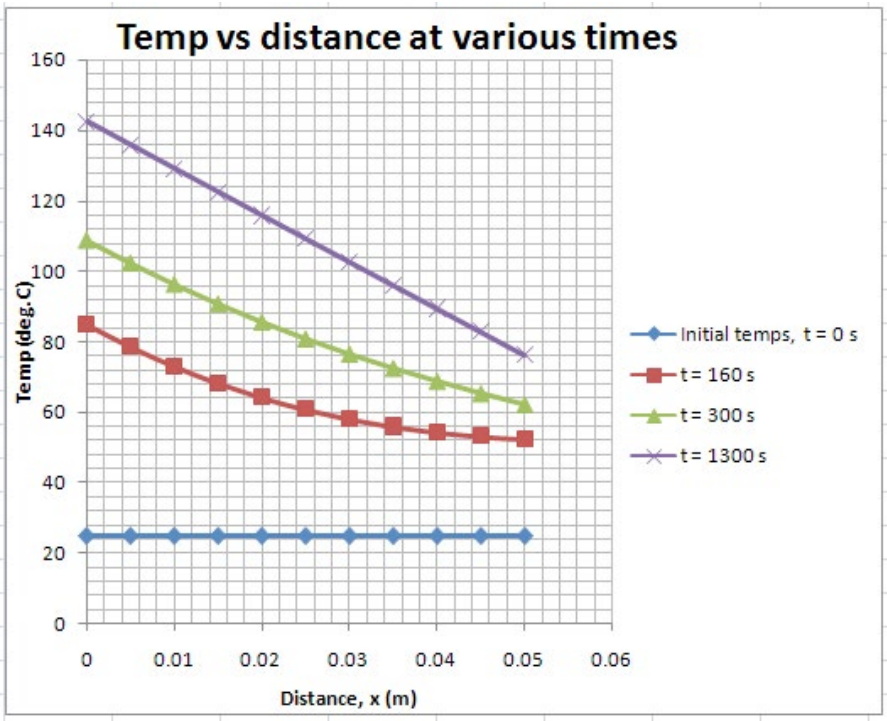
75	1100	141.641	134.986	128.352	121.740	115.148	108.577	102.024	95.488	88.967	82.460	75.964
76	1120	141.764	135.108	128.472	121.855	115.259	108.680	102.119	95.573	89.042	82.524	76.015
77	1140	141.878	135.221	128.583	121.963	115.361	108.776	102.207	95.653	89.112	82.582	76.063
78	1160	141.984	135.326	128.685	122.062	115.456	108.865	102.289	95.726	89.176	82.637	76.107
79	1180	142.082	135.423	128.781	122.155	115.544	108.947	102.365	95.795	89.236	82.687	76.147
80	1200	142.173	135.513	128.869	122.240	115.625	109.024	102.435	95.858	89.291	82.734	76.185
81	1220	142.257	135.597	128.951	122.319	115.701	109.094	102.500	95.917	89.343	82.778	76.220
82	1240	142.335	135.675	129.028	122.393	115.771	109.160	102.561	95.971	89.391	82.818	76.252
83	1260	142.407	135.747	129.098	122.461	115.836	109.221	102.617	96.022	89.435	82.856	76.282
84	1280	142.475	135.814	129.164	122.525	115.896	109.278	102.669	96.068	89.476	82.890	76.310
85	1300	142.537	135.875	129.224	122.583	115.952	109.330	102.717	96.112	89.514	82.922	76.336

Note that the temperature values obtained match very well with those obtained with EES.

9. Now, it is easy to draw the graphs:

a) **Temp distribution in the slab at various times:**



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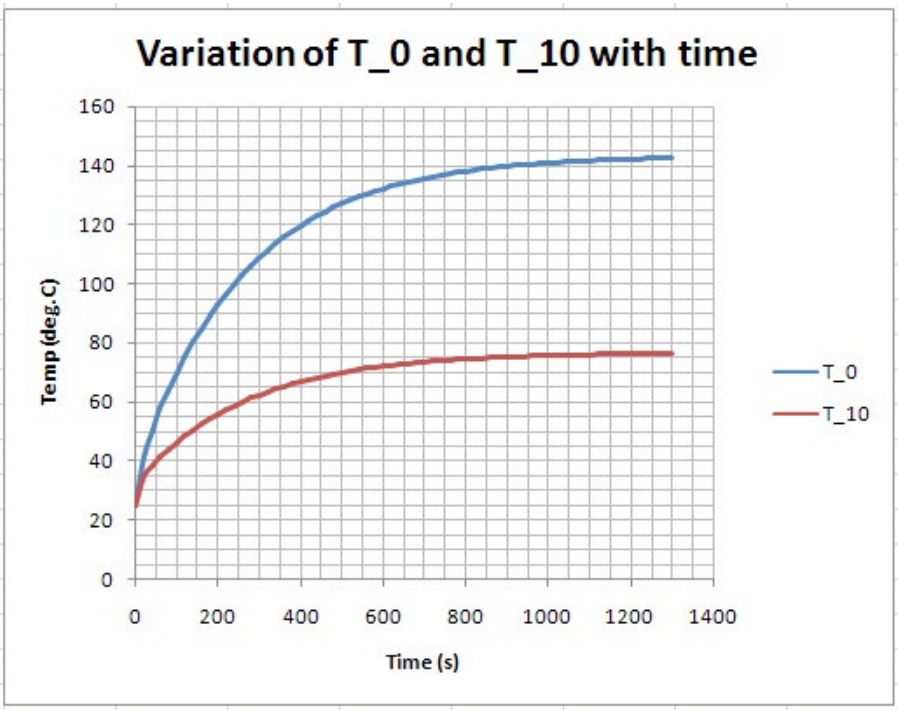
b) **Temp variation of LHS and RHS of slab with time:**



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Prob.1I.C.8. A steel plate (α = 1.2 × 10^{-5} m^2/s, k = 43 W/(m.C)), of thickness 2L = 10 cm, initially at an uniform temperature of 250 C is suddenly immersed in an oil bath at T_a = 45 C. Convection heat transfer coeff. between the fluid and the surfaces is 700 W/(m^2.C).

a) How long will it take for the centre plane to cool to 100 C?
b) Draw the temp. profile in the slab at different times.

EXCEL Solution:

Let us divide the thickness in to 10 equally spaced divisions and solve by **explicit method**.

We derive the difference eqns for Boundary Node '0', Internal Nodes 1 to 9, and Boundary Node 10, by writing an energy balance at the respective nodes. (See Ref. [1])

For Explicit method:

For Internal node 'm', with heat generation:

$$\left(T_m\right)^{i+1}=Fo\cdot\left[\left(T_{m-1}\right)^{i}+\left(T_{m+1}\right)^{i}\right]+(1-2\cdot Fo)\cdot\left(T_m\right)^{i}+Fo\cdot\frac{\left(q_m\right)^{i}\cdot(\Delta x)^{2}}{k} \tag{8.56}$$

where Fo is the mesh Fourier no. = α * Δt /Δ x^2, and q_m = heat gen. rate (W/m^3).

Eqn.(8.55) is the explicit difference eqn. valid for all interior nodes, 1, 2….(M-1), when there is internal heat generation. Now, the new temperature T_m^{i+1} can be explicitly solved since the other terms involved at the previous time step 'i', are already known. Here, superscript 'i' refers to 'previous' time step.

With no heat generation:

$$\left(T_m\right)^{i+1}=Fo\cdot\left[\left(T_{m-1}\right)^{i}+\left(T_{m+1}\right)^{i}\right]+(1-2\cdot Fo)\cdot\left(T_m\right)^{i} \tag{8.56}$$

For node '0' with convection boundary condition:

Explicit formulation:

$$h\cdot A\cdot\left[T_a-\left(T_0\right)^{i}\right]+k\cdot A\cdot\frac{\left(T_1\right)^{i}-\left(T_0\right)^{i}}{\Delta x}+\left(q_0\right)^{i}\cdot A\cdot\frac{\Delta x}{2}=\rho\cdot A\cdot\frac{\Delta x}{2}\cdot C_p\cdot\frac{\left(T_0\right)^{i+1}-\left(T_0\right)^{i}}{\Delta\tau} \tag{8.60}$$

Simplifying:

$$\left(T_0\right)^{i+1}=(1-2\cdot Fo-2\cdot Fo\cdot Bi)\cdot\left(T_0\right)^{i}+Fo\cdot\left[2\cdot\left(T_1\right)^{i}+2\cdot Bi\cdot T_a+\frac{\left(q_0\right)^{i}\cdot(\Delta x)^{2}}{k}\right] \tag{8.61}$$

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where $Bi = \frac{h \cdot \Delta x}{k}$ = Biot number

In this particular case, heat generation, $q_0 = 0$

For node '10' with convection boundary condition:

Since conditions on the RHS are identical to those on LHS, use the eqn for Node '0', i.e. eqn. (8.61), but changing T_0 to T_10 and T_1 to T_9. Remember that $q_0 = 0$.

Stability criterion:

The limit on Δτ is determined from mathematical and thermodynamic as follows:

"Coefficients of all $T_m^{\ i}$ in the $T_m^{\ i+1}$ expressions (called 'primary coefficients') must be greater than or equal to zero for all nodes 'm'".

Generally, boundary nodes with convection conditions are more restrictive and in such cases, coeff. of $T_m^{\ i}$ from the most restrictive eqn. must be considered for the stability criterion and the time step Δτ must be determined with respect to that coefficient.

Following are the steps in EXCEL Solution:

1. Set up the worksheet, enter data and name the cells:

deltax | 0.01

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2		Explicit method:				For internal nodes:							
3						With no heat generation:							
4		Data:				$(T_m)^{i+1}=Fo\cdot\left[(T_{m-1})^i+(T_{m+1})^i\right]+(1-2\cdot Fo)\cdot(T_m)^i$ .....(8.56)							
5		Ta	45										
6		L	0.1	m									
7		h	700	W/m2.C		For Node '0':							
8		k	42.3	W/m.C		$(T_0)^{i+1}=(1-2\cdot Fo-2\cdot Fo\cdot Bi)\cdot(T_0)^i+Fo\cdot\left[2\cdot(T_1)^i+2\cdot Bi\cdot T_a+\frac{(q_0)^i\cdot(\Delta x)^2}{k}\right]$ .....(8.61)							
9		rho	7858	kg/m3									
10		cp	442	J/kg.C									
11		alpha	1.20E-05	m2/s		where $Bi=\frac{h\cdot\Delta x}{k}$ = Biot number							
12		deltax	0.01	m									
13		deltax^2/(2*alpha*(1+Bi))	3.575051	s...stability critera		For Node '0':		Use eqn. (8.61), but change T_0 to T_10 and T_1 to T_9.					
14		deltat	2	s									
15	Fourier No.	Fo	0.24										
16	Biot No.	Bi	0.165485										

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Note that *stability criterion* is also calculated, and it is equal to: 3.575; so, Δt is chosen as 2 s which is less than that required for stability criterion. Also, we have provided a cell comment regarding stability criterion as shown below:

12	deltax	0.01	m	
13	deltax^2/(2*alpha*(1+Bi))	3.575051	s...stability c	
14	deltat	2	s	
15	Fo	0.24		
16	Bi	0.165485		

Stability criteria from eqn. (8.61). Deltat must be less than this value.

2. Set up the worksheet, suitable for calculations of transient conduction, as shown below:

17													
18		Node	0	1	2	3	4	5	6	7	8	9	10
19		x (m)	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
20	t (sec.)	0	250	250	250	250	250	250	250	250	250	250	250 Initial
21		2											
22		4											
23		6											
24		8											
25		10											
26		12											
27		14											
28		16											

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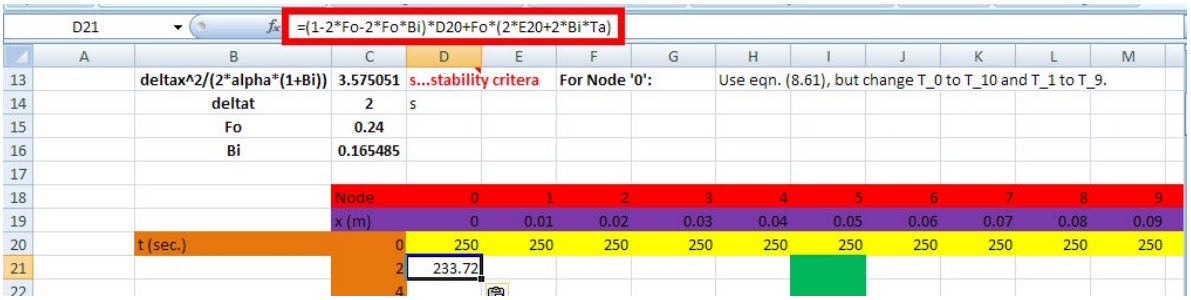
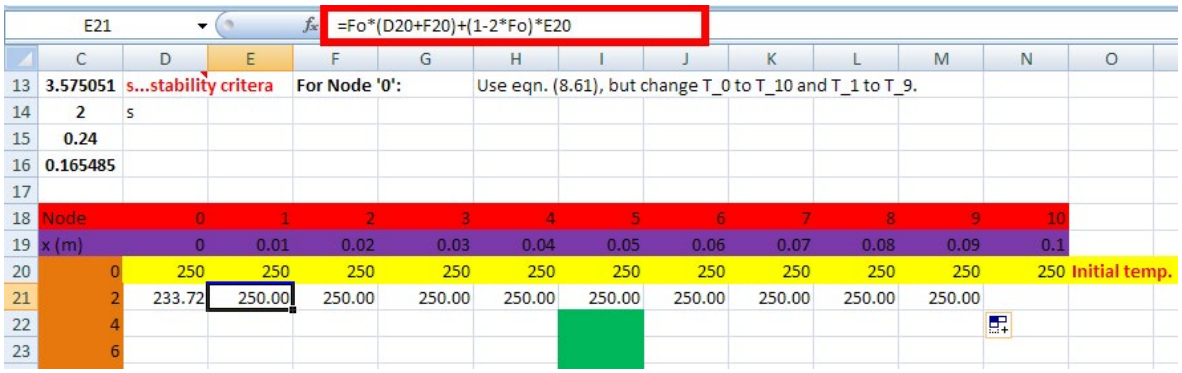
3. Now, after making sure that Iteration calculations are enabled in EXCEL, enter the eqn for Node '0' in cell D21: See the formula bar in fig. below:

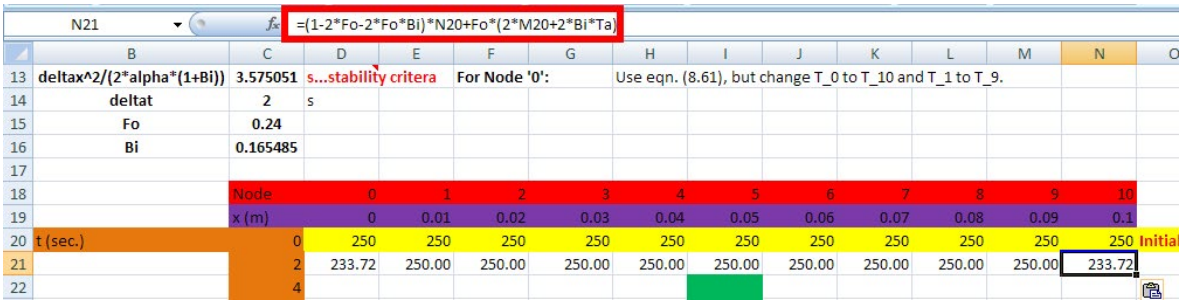


Note that while entering eqn for T_0 in cell D21, T_0_old refers to D20, i.e. T_0 at the previous step. Take care to enter eqns in this way for cells E21 and N21.

4. Now, enter eqn. for Node 1 in cell E21 and then drag-copy it up to Node 9, i.e. up to cellM21:



5. Now, enter eqn for T_10 in cell N21:



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6. Now, select the cells from D21 up to N21, and drag-copy downwards, up to the desired time, say up to 510 s. Part of the Table is shown below. Observe how the temp in the centre line of plate (i.e. Node '5') changes. We have to continue till it reaches 100 C:

18		Node	0	1	2	3	4	5	6	7	8	9	10	
19		x (m)	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	
20	t (sec.)	0	250	250	250	250	250	250	250	250	250	250	250	Initial
21		2	233.72	250.00	250.00	250.00	250.00	250.00	250.00	250.00	250.00	250.00	233.72	
22		4	226.54	246.09	250.00	250.00	250.00	250.00	250.00	250.00	250.00	246.09	226.54	
23		6	221.51	242.34	249.06	250.00	250.00	250.00	250.00	250.00	249.06	242.34	221.51	
24		8	217.48	238.95	247.67	249.77	250.00	250.00	250.00	249.77	247.67	238.95	217.48	
25		10	214.09	235.89	246.08	249.32	249.95	250.00	249.95	249.32	246.08	235.89	214.09	
26		12	211.12	233.11	244.42	248.70	249.81	249.97	249.81	248.70	244.42	233.11	211.12	
27		14	208.48	230.54	242.73	247.94	249.58	249.90	249.58	247.94	242.73	230.54	208.48	
28		16	206.09	228.17	241.05	247.08	249.26	249.74	249.26	247.08	241.05	228.17	206.09	
29		18	203.89	225.96	239.41	246.16	248.85	249.51	248.85	246.16	239.41	225.96	203.89	
30		20	201.87	223.89	237.80	245.19	248.37	249.20	248.37	245.19	237.80	223.89	201.87	
31		22	199.98	221.94	236.24	244.18	247.80	248.80	247.80	244.18	236.24	221.94	199.98	
32		24	198.21	220.10	234.71	243.14	247.17	248.32	247.17	243.14	234.71	220.10	198.21	
33		26	196.55	218.36	233.23	242.09	246.48	247.77	246.48	242.09	233.23	218.36	196.55	
34		28	194.98	216.69	231.78	241.01	245.73	247.15	245.73	241.01	231.78	216.69	194.98	
35		30	193.49	215.10	230.38	239.93	244.94	246.47	244.94	239.93	230.38	215.10	193.49	

230		420	87.91	94.46	99.74	103.61	105.97	106.76	105.97	103.61	99.74	94.46	87.91	
231		422	87.65	94.16	99.40	103.25	105.59	106.38	105.59	103.25	99.40	94.16	87.65	
232		424	87.38	93.85	99.06	102.89	105.22	106.00	105.22	102.89	99.06	93.85	87.38	
233		426	87.12	93.55	98.73	102.53	104.85	105.63	104.85	102.53	98.73	93.55	87.12	
234		428	86.86	93.25	98.40	102.17	104.48	105.25	104.48	102.17	98.40	93.25	86.86	
235		430	86.60	92.95	98.07	101.82	104.11	104.88	104.11	101.82	98.07	92.95	86.60	
236		432	86.35	92.66	97.74	101.47	103.75	104.51	103.75	101.47	97.74	92.66	86.35	
237		434	86.09	92.36	97.42	101.12	103.38	104.14	103.38	101.12	97.42	92.36	86.09	
238		436	85.84	92.07	97.09	100.78	103.02	103.78	103.02	100.78	97.09	92.07	85.84	
239		438	85.59	91.78	96.77	100.43	102.67	103.42	102.67	100.43	96.77	91.78	85.59	
240		440	85.34	91.49	96.45	100.09	102.31	103.06	102.31	100.09	96.45	91.49	85.34	
241		442	85.09	91.20	96.13	99.75	101.96	102.70	101.96	99.75	96.13	91.20	85.09	
242		444	84.84	90.92	95.82	99.41	101.60	102.34	101.60	99.41	95.82	90.92	84.84	
243		446	84.59	90.64	95.50	99.08	101.25	101.99	101.25	99.08	95.50	90.64	84.59	
244		448	84.35	90.35	95.19	98.74	100.91	101.64	100.91	98.74	95.19	90.35	84.35	
245		450	84.11	90.07	94.88	98.41	100.56	101.29	100.56	98.41	94.88	90.07	84.11	
246		452	83.86	89.80	94.58	98.08	100.22	100.94	100.22	98.08	94.58	89.80	83.86	
247		454	83.62	89.52	94.27	97.75	99.88	100.59	99.88	97.75	94.27	89.52	83.62	
248		456	83.39	89.24	93.97	97.43	99.54	100.25	99.54	97.43	93.97	89.24	83.39	
249		458	83.15	88.97	93.66	97.10	99.20	99.91	99.20	97.10	93.66	88.97	83.15	

310		580	71.15	75.14	78.36	80.71	82.15	82.64	82.15	80.71	78.36	75.14	71.15	
311		582	70.99	74.95	78.15	80.49	81.92	82.41	81.92	80.49	78.15	74.95	70.99	
312		584	70.83	74.77	77.95	80.27	81.70	82.17	81.70	80.27	77.95	74.77	70.83	
313		586	70.67	74.59	77.74	80.06	81.47	81.94	81.47	80.06	77.74	74.59	70.67	
314		588	70.51	74.40	77.54	79.84	81.24	81.72	81.24	79.84	77.54	74.40	70.51	
315		590	70.35	74.22	77.34	79.63	81.02	81.49	81.02	79.63	77.34	74.22	70.35	
316		592	70.20	74.04	77.14	79.41	80.80	81.26	80.80	79.41	77.14	74.04	70.20	
317		594	70.04	73.86	76.94	79.20	80.58	81.04	80.58	79.20	76.94	73.86	70.04	
318		596	69.89	73.68	76.74	78.99	80.36	80.82	80.36	78.99	76.74	73.68	69.89	
319		598	69.73	73.51	76.55	78.78	80.14	80.60	80.14	78.78	76.55	73.51	69.73	
320		600	69.58	73.33	76.35	78.57	79.92	80.38	79.92	78.57	76.35	73.33	69.58	

We see that the mid-plane temp reaches 100 C after about 457 s…..Ans.

And, at that time, surface temp is about 83.3 deg.C

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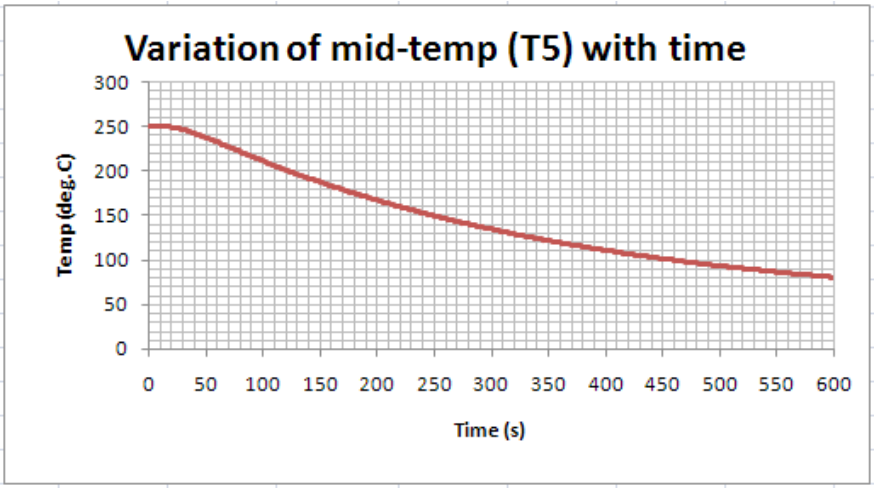
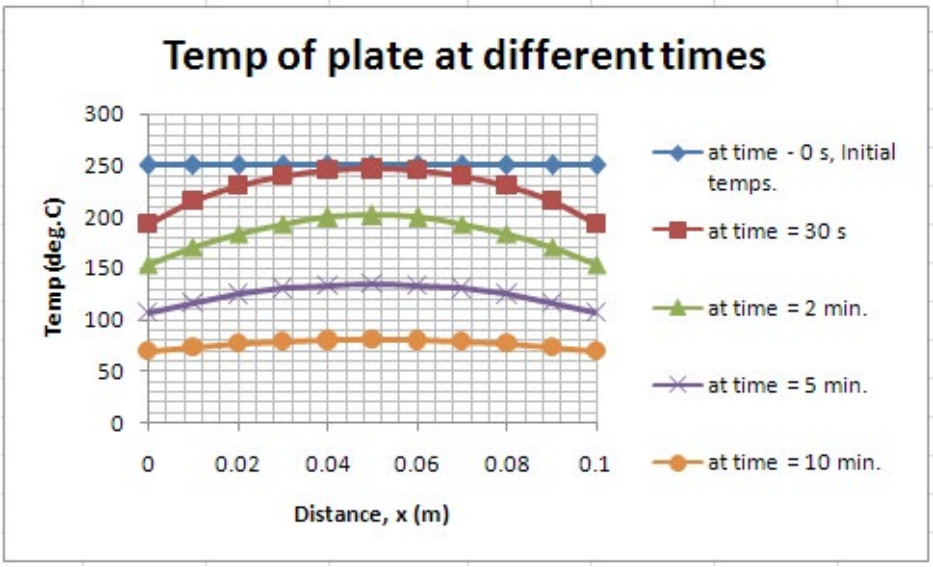
7. Next, draw plots in EXCEL:





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1ID. Two-dimensional, transient conduction:

Prob. 1I.D.1. Consider 2D transient heat transfer in a L-shaped solid bar, initially at a uniform temp of 140 C (see Fig.). k = 15 W/m.K and α = 3.2 × 10^-6 m^2/s. Heat gen. rate qg = 2 × 10^7 W/m^3. The right surface is insulated and the bottom surface is maintained at a uniform temp of 140 C at all times. At time = 0, entire top surface is subjected to convection with ambient air at Ta = 25 C and h = 80 W/m^2.K and left surface is subjected to uniform heat flux of 8000 W/m^2. Nodal spacing is Δx = Δy = 1.5 cm. Using Implicit method, determine the temp at the top corner (Node 3) after 2, 5 and 30 min. (Ref. 3]

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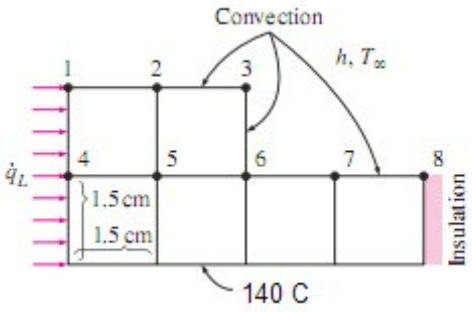

Fig.Prob.1I.D.1

Mathcad Solution:

Data:

$\Delta x := 0.015$ m

$\Delta y := 0.015$ m

$T_a := 25$ C.....ambient temp.

$h := 80$ W/(m^2.C).....heat transfer coeff.

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$k := 15$ W/(m.C)....thermal cond.

$\alpha := 3.2 \cdot 10^{-6}$ m^2/s ... thermal diffusivity

$q_g := 2 \cdot 10^7$ W/m^3 .... heat gen. rate

$q_{left} := 8000$ W/m^2....heat flux on left surface

$T_{bottom} := 140$ C...const temp at bottom surface

Initial values of temps in all nodes: 140 C. i.e.

$T1_0 := 140$ $T2_0 := 140$ $T3_0 := 140$ $T4_0 := 140$ $T5_0 := 140$ $T6_0 := 140$ $T7_0 := 140$

$T8_0 := 140$ C

Nodes are represented by numbers 1, 2,.....8.

Let us adopt 'Implicit' method of finite difference solution, so that there is no restriction on the value of time step, Δt

Let: $\Delta t := 5$ s

We shall develop finite difference eqn. for each node by writing the energy balance for the corresponding elemental volume around that node, and solve the resulting equations with solve block of Mathcad. T1, T2….T8 are the temps at the 'next' time step.

Start with guess values for T1, T2, …. T8:

$T1 := 150$ $T2 := 150$ $T3 := 150$ $T4 := 150$ $T5 := 150$ $T6 := 150$ $T7 := 150$

$T8 := 150$ ...guess values of temperatures

Given

Node 1:

$$q_{left} \cdot \frac{\Delta y}{2} + h \cdot \frac{\Delta x}{2} \cdot (T_a - T1) + k \cdot \frac{\Delta y}{2} \cdot \frac{(T2 - T1)}{\Delta x} + k \cdot \frac{\Delta x}{2} \cdot \frac{(T4 - T1)}{\Delta y} + q_g \cdot \frac{\Delta x}{2} \cdot \frac{\Delta y}{2} = \frac{k}{\alpha} \cdot \left(\frac{\Delta x}{2} \cdot \frac{\Delta y}{2}\right) \cdot \frac{(T1 - T1_0)}{\Delta t}$$

Node 2:

$$k \cdot \frac{\Delta y}{2} \cdot \frac{(T1 - T2)}{\Delta x} + h \cdot \Delta x \cdot (T_a - T2) + k \cdot \frac{\Delta y}{2} \cdot \frac{(T3 - T2)}{\Delta x} + k \cdot \Delta x \cdot \frac{(T5 - T2)}{\Delta y} + q_g \cdot \Delta x \cdot \frac{\Delta y}{2} = \frac{k}{\alpha} \cdot \left(\Delta x \cdot \frac{\Delta y}{2}\right) \cdot \frac{(T2 - T2_0)}{\Delta t}$$

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Node 3:

$$k\cdot\frac{\Delta y}{2}\cdot\frac{(T2-T3)}{\Delta x}+\left[h\cdot\left(\frac{\Delta x}{2}+\frac{\Delta y}{2}\right)\right]\cdot\left(T_a-T3\right)+k\cdot\frac{\Delta x}{2}\cdot\frac{(T6-T3)}{\Delta y}+q_g\cdot\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}=\frac{k}{\alpha}\cdot\left(\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}\right)\cdot\frac{\left\langle T3-T3_0\right\rangle}{\Delta t}$$

Node 4:

$$q_{left}\cdot\Delta y+k\cdot\frac{\Delta x}{2}\cdot\frac{(T1-T4)}{\Delta y}+k\cdot\Delta y\cdot\frac{(T5-T4)}{\Delta x}+k\cdot\frac{\Delta x}{2}\cdot\left\langle\frac{T_{bottom}-T4}{\Delta y}\right\rangle+q_g\cdot\frac{\Delta x}{2}\cdot\Delta y=\frac{k}{\alpha}\cdot\left(\frac{\Delta x}{2}\cdot\Delta y\right)\cdot\frac{\left\langle T4-T4_0\right\rangle}{\Delta t}$$

Node 5:

$$k\cdot\Delta y\cdot\frac{(T4-T5)}{\Delta x}+k\cdot\Delta x\cdot\frac{(T2-T5)}{\Delta y}+k\cdot\Delta y\cdot\frac{(T6-T5)}{\Delta x}+k\cdot\Delta x\cdot\left\langle\frac{T_{bottom}-T5}{\Delta y}\right\rangle+q_g\cdot\Delta x\cdot\Delta y=\frac{k}{\alpha}\cdot(\Delta x\cdot\Delta y)\cdot\frac{\left\langle T5-T5_0\right\rangle}{\Delta t}$$

Node 6:

$$k\cdot\Delta y\cdot\frac{(T5-T6)}{\Delta x}+\left[h\cdot\left(\frac{\Delta x}{2}+\frac{\Delta y}{2}\right)\right]\cdot\left(T_a-T6\right)+k\cdot\frac{\Delta x}{2}\cdot\frac{(T3-T6)}{\Delta y}+k\cdot\frac{\Delta y}{2}\cdot\frac{(T7-T6)}{\Delta x}\ldots=\frac{k}{\alpha}\cdot\left(\Delta x\cdot\Delta y\cdot\frac{3}{4}\right)\cdot\frac{\left\langle T6-T6_0\right\rangle}{\Delta t}$$
$$+k\cdot\Delta x\cdot\left\langle\frac{T_{bottom}-T6}{\Delta y}\right\rangle+q_g\cdot\Delta x\cdot\Delta y\cdot\frac{3}{4}$$

Node 7:

$$k\cdot\frac{\Delta y}{2}\cdot\frac{(T6-T7)}{\Delta x}+h\cdot\Delta x\cdot\left(T_a-T7\right)+k\cdot\frac{\Delta y}{2}\cdot\frac{(T8-T7)}{\Delta x}+k\cdot\Delta x\cdot\frac{\left\langle T_{bottom}-T7\right\rangle}{\Delta y}+q_g\cdot\frac{\Delta x\cdot\Delta y}{2}=\frac{k}{\alpha}\cdot\frac{\Delta x\cdot\Delta y}{2}\cdot\frac{\left\langle T7-T7_0\right\rangle}{\Delta t}$$

Node 8:

$$k\cdot\frac{\Delta y}{2}\cdot\frac{(T7-T8)}{\Delta x}+h\cdot\frac{\Delta x}{2}\cdot\left(T_a-T8\right)+k\cdot\frac{\Delta x}{2}\cdot\frac{\left\langle T_{bottom}-T8\right\rangle}{\Delta y}+q_g\cdot\frac{\Delta x\cdot\Delta y}{4}=\frac{k}{\alpha}\cdot\frac{\Delta x\cdot\Delta y}{4}\cdot\frac{\left\langle T8-T8_0\right\rangle}{\Delta t}$$

$$Temp\left\langle T1_0,T2_0,T3_0,T4_0,T5_0,T6_0,T7_0,T8_0\right\rangle:=Find(T1,T2,T3,T4,T5,T6,T7,T8)$$

Note: Node temps. after one time step are stored in vector 'Temp', written as afunction of starting temps..

We get, after one time step:

$$Temp\left\langle T1_0,T2_0,T3_0,T4_0,T5_0,T6_0,T7_0,T8_0\right\rangle=\begin{bmatrix}160.792\\159.792\\158.497\\160.848\\159.89\\158.614\\157.428\\157.366\end{bmatrix}$$

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O the RHS, we have temperatures T1, T2,… T8 from top downwards.

Now, to go to the next time step, use the values of T1, T2, …. T8 obtained above as the 'old' temp and apply the Solve Block again.

We write a small Mathcad program to do this:

```
Implcit_Temp(n) := | T1 ← 140
                   | T2 ← 140
                   | T3 ← 140
                   | T4 ← 140
                   | T5 ← 140
                   | T6 ← 140
                   | T7 ← 140
                   | T8 ← 140
                   | i ← 0
                   | while i < n
                   |     TTemp ← Temp(T1, T2, T3, T4, T5, T6, T7, T8)
                   |     T1 ← TTemp_0
                   |     T2 ← TTemp_1
                   |     T3 ← TTemp_2
                   |     T4 ← TTemp_3
                   |     T5 ← TTemp_4
                   |     T6 ← TTemp_5
                   |     T7 ← TTemp_6
                   |     T8 ← TTemp_7
                   |     i ← i + 1
                   | (i  Δt·i  T1  T2  T3  T4  T5  T6  T7  T8)
```

In the above program, LHS defines the program as a function of n, the no. of time steps. (Remember each time step = 5 s).

In the RHS, first 8 lines define the initial temps to the 8 Nodes.

Then, a while loop is used wherein the earlier defined Temp (T1, T2,…T8) Solve Block is called and is assigned to a vector TTemp. Its elements contain the newly calculated Node temps.

In the next 8 lines, old T1….T8 are updated with these new values; and, the Solve Block is called successively till the desired no. of time steps are covered.

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Results for n = 0, 1, 6, 24, 60 and 360 time steps are given below:

i = step no.; Δτ = one time step = 5 s; τ = time duration from beginning = i. Δτ, s

i	τ	T1	T2	T3	T4	T5	T6	T7	T8

Implcit_Temp(0) =

	0	1	2	3	4	5	6	7	8	9
0	0	0	140	140	140	140	140	140	140	140

....Initial temps.

Implcit_Temp(1) =

	0	1	2	3	4	5	6	7	8	9
0	1	5	160.792	159.792	158.497	160.848	159.89	158.614	157.428	157.366

Implcit_Temp(6) =

	0	1	2	3	4	5	6	7	8	9
0	6	30	256.169	251.458	243.361	246.268	241.722	230.769	216.98	215.473

Implcit_Temp(24) =

	0	1	2	3	4	5	6	7	8	9
0	24	120	467.46	454.157	424.039	409.675	395.825	350.303	285.087	273.689

Implcit_Temp(60) =

	0	1	2	3	4	5	6	7	8	9
0	60	300	579.069	559.515	514.699	491.936	471.476	403.85	305.905	287.109

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Implcit_Temp(360) =

	0	1	2	3	4	5	6	7	8	9
0	360	$1.8 \cdot 10^3$	596.312	575.718	528.544	504.611	483.066	411.936	308.847	288.869

Thus, we see from the above that temp of top corner of the body, i.e. T3, after 2, 5 and 30 min (i.e. time step n = 24, 60 and 360) are, respectively, as follows:

T3 = 424.039, 514.699 and 528.544 C…after 2, 5 and 30 min..Ans.

To plot T3 against Time:

Modify the above program slightly, to return T3:

```
Implicit_TempT3(n) := | T1 ← 140
                      | T2 ← 140
                      | T3 ← 140
                      | T4 ← 140
                      | T5 ← 140
                      | T6 ← 140
                      | T7 ← 140
                      | T8 ← 140
                      | ATempT3_0 ← 140
                      | i ← 0
                      | while i < n
                      |     TTemp ← Temp(T1, T2, T3, T4, T5, T6, T7, T8)
                      |     T1 ← TTemp_0
                      |     T2 ← TTemp_1
                      |     T3 ← TTemp_2
                      |     T4 ← TTemp_3
                      |     T5 ← TTemp_4
                      |     T6 ← TTemp_5
                      |     T7 ← TTemp_6
                      |     T8 ← TTemp_7
                      |     ATempT3_(i+1) ← T3
                      |     ATempT3_0 ← 140
                      |     i ← i + 1
                      | return ATempT3
```


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Above program returns a vector called ATempT3 wherein only the values of T3 are stored for each time step.

TTempT3 := Implicit_TempT3(360) ...Store the T3 values for 360 time steps (i.e. 1800 s) in the vector TTempT3

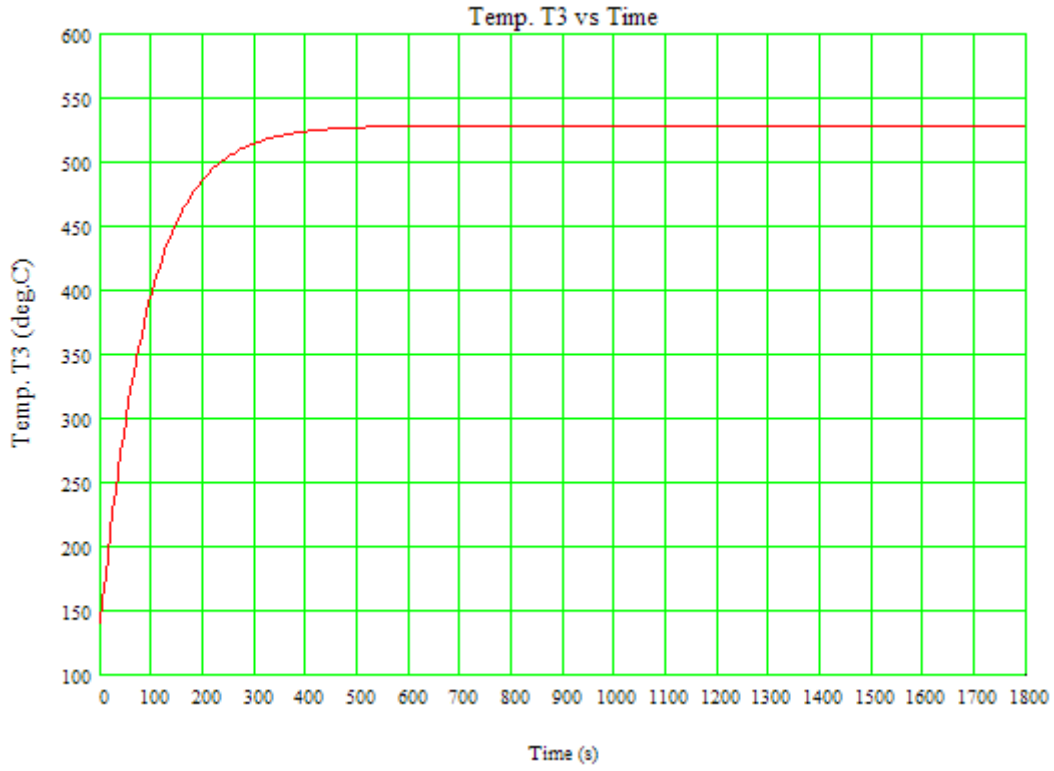
$TTempT3_0 = 140$ ..check: temp at time step 0

$TTempT3_{24} = 424.039$ ..check: temp at time step 24

$TTempT3_{360} = 528.544$ ..check: temp at time step 360

Now, draw the Plot:

i := 0, 1.. 360 ...define range variable i, i.e. time step



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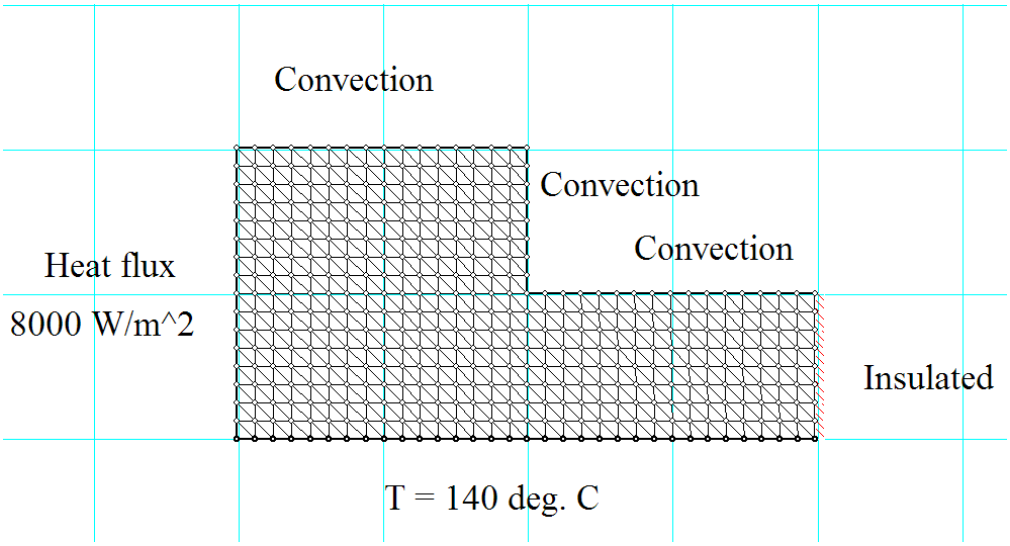
FEHT Solution for the above problem:

Node positions:



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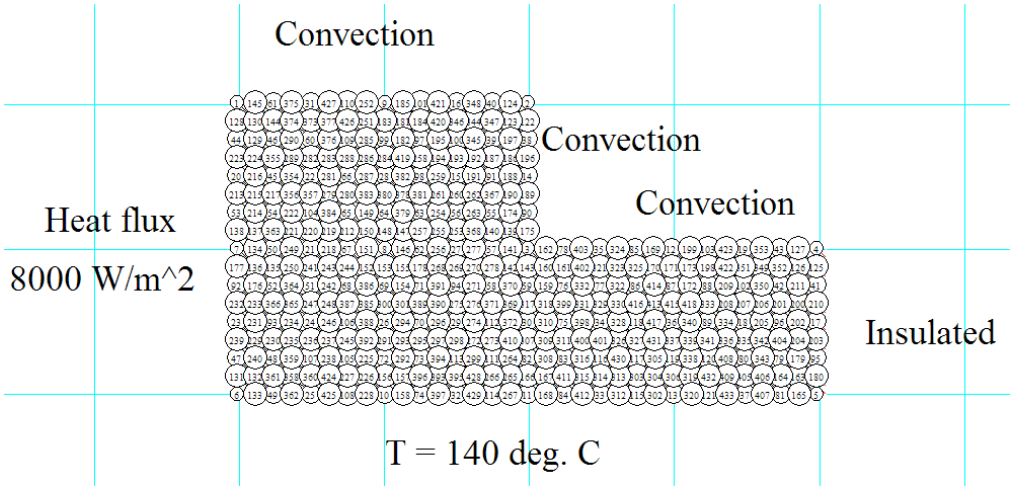
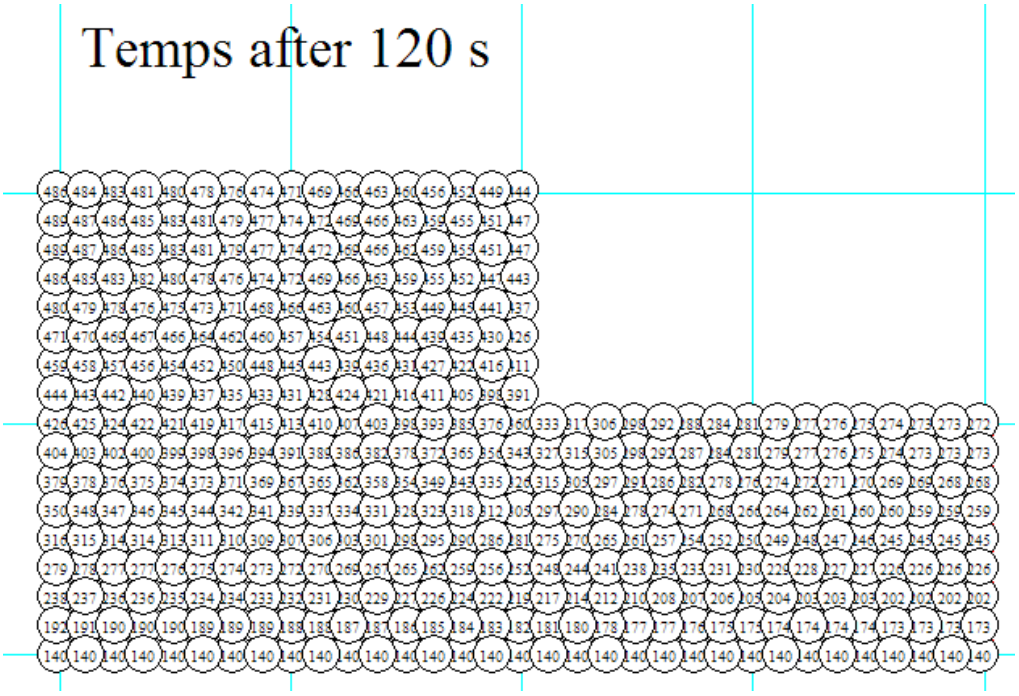
Node Numbers:



Note that top edge is Node No. 2. in the above set up.

Node Temps after 120 s:



Note that temp of Node 2 after 120 s is: 444 deg.C. Compare this value with 424.04 C obtained with Mathcad.

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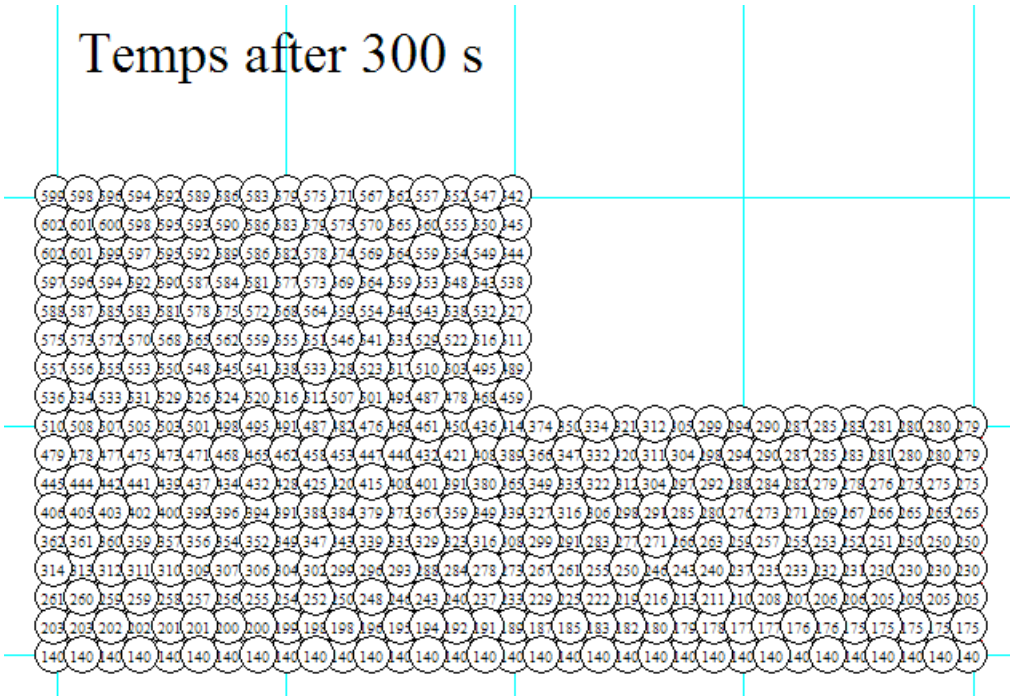
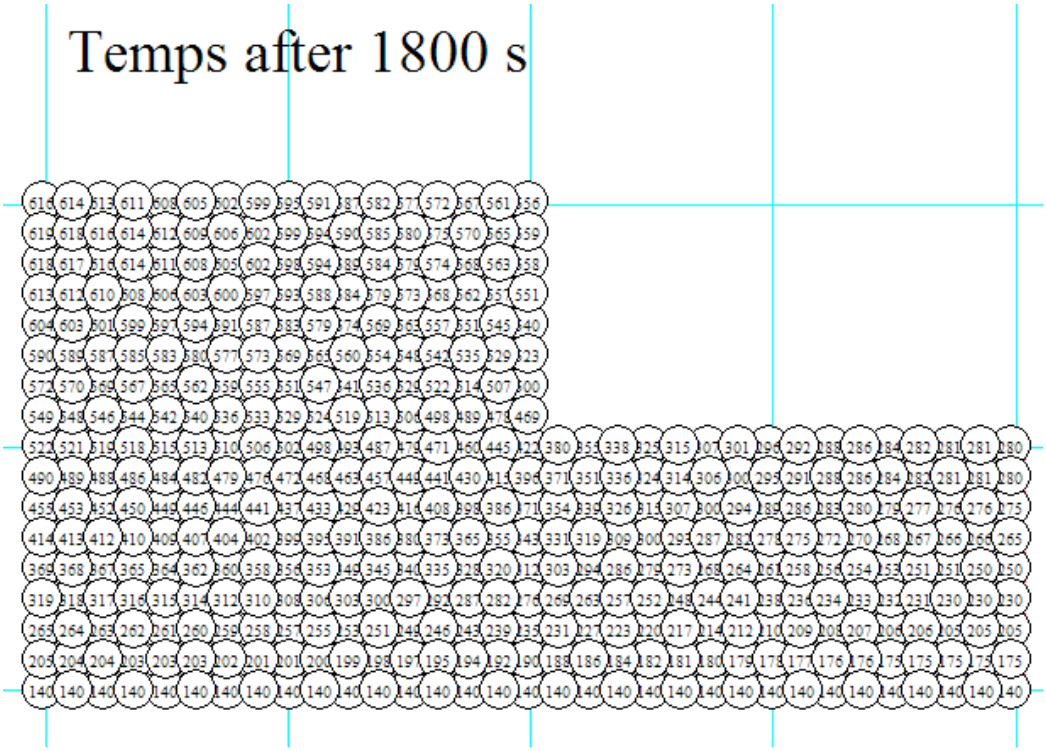
Temps after 300 s:



Note that temp of Node 2 after 300 s is: 542 deg.C. Compare this value with 514.7 C obtained with Mathcad.

Temps after 1800 s:



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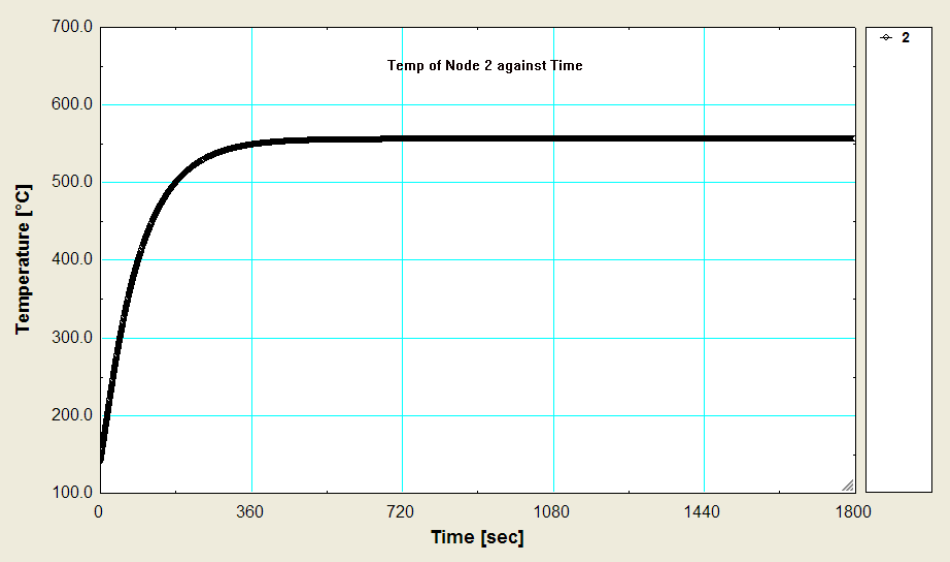
Note that temp of Node 2 after 1800 s is: 556 deg.C. Compare this value with 528.5 C obtained with Mathcad. (Difference = 5.6%)

Temp of Top corner Node (i.e. Node no. 2) against time:



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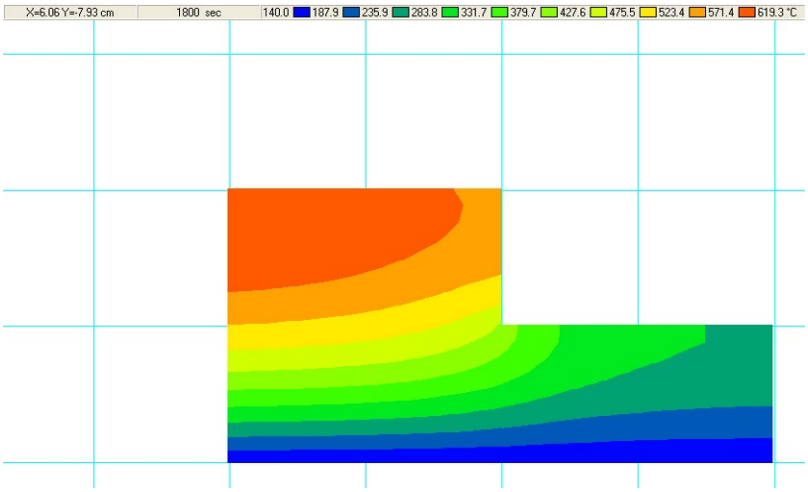
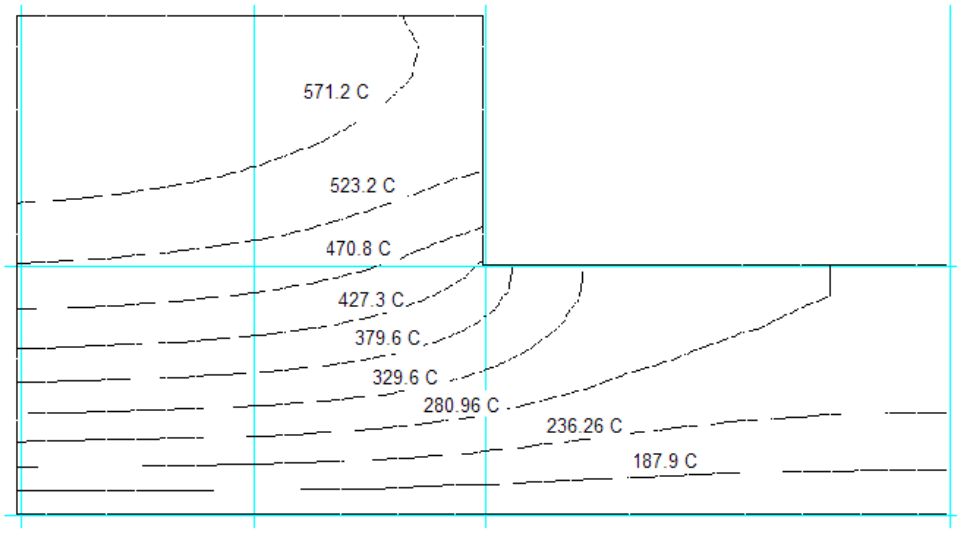
Temp contours in the L-bar as colour bands after 1800 s:



Temp contours in the L-bar as Iso-potential lines after 1800 s:

Min = 187.9 C, Max = 619.3 C



==========

Prob.1I.D.2. : Consider a long solid bar (k = 28 W/m.K and α = 12 × 10^-6 m^2/s) of square cross-section (size: 20 cm × 20 cm) that is initially at a uniform temp of 20 C. Heat is generated in the bar t a rate of qg = 8 × 10^5 W/m^3. 4 sides of the bar are subjected to convection to ambient air at Ta = 30 C and h = 45 W/m^2.K. Using Explicit finite difference method and a mesh size of Δx = Δy = 10 cm, determine the centre temp of the bar after 20 min. Also, find out when steady state temp is reached. [(Ref. 3)]

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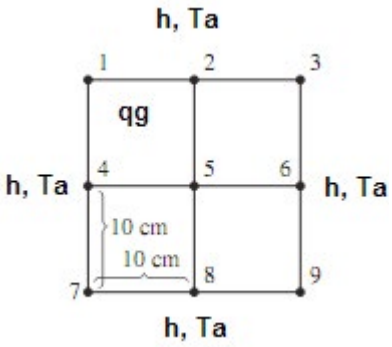

Fig.Prob.1I.D.2

Mathcad Solution:

Data:

$\Delta x := 0.1$ m

$\Delta y := 0.1$ m

$T_a := 30$ C.....ambient temp.

$h := 45$ W/(m^2.C).....heat transfer coeff.

$k := 28$ W/(m.C)....thermal cond.

$\alpha := 12 \cdot 10^{-6}$ m^2/s ... thermal diffusivity

$q_g := 8 \cdot 10^5$ W/m^3 .... heat gen. rate

Initial values of temps in all nodes: 20 C. i.e.

$T1_0 := 20$ $T2_0 := 20$ $T3_0 := 20$ $T4_0 := 20$ $T5_0 := 20$ $T6_0 := 20$ $T7_0 := 20$

$T8_0 := 20$ $T9_0 := 20$ C

Nodes are represented by numbers 1, 2,…..9.
Temps at these nodes have to be found out.
We observe that **there is symmetry** along the vertical and horizontal centre lines.
i.e. T1 = T3 = T7 = T9, and T2 = T4 = T6 = T8.
Therefore, we need to find out only three temps, viz. T1, T2 and T5.
So, consider only one -quarter of the section, i.e. the square area 1-2-5-4. For this section, faces 2–5 and 4–5 are symmetry lines, i.e. these faces can be considered as insulated.

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Let us adopt 'Explicit' method of finite difference solution.

So, now, there is restriction on the value of time step, Δt, which should be properly chosen to prevent numerical oscillations.

We should see that Δt should be such that the coeff. of T_m in the most restrictive node (generally, adjacent to a node with convection) is more than zero:

In the present case, Node 3 is subjected to convection from both sides.

The stability criterion for a corner node subjected to convection on both sides is:

$$Fo\cdot(1+Bi)<\frac{1}{4}$$ where Fo = mesh Fourier No., and Bi = Biot No.

$$Fo=\frac{\alpha\cdot\Delta t}{\Delta x^2} \qquad Bi=\frac{h\cdot\Delta x}{k}=0.161$$

Therefore: $$\Delta t<\frac{\Delta x^2}{\alpha}\cdot\frac{1}{4\cdot(1+Bi)}$$

i.e. $\Delta t<179.443$ s

i.e. use any timestep less than 179 s from stability criterion.

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Let us use: $\Delta t := 60\ s$

Then: $Fo := \frac{\alpha \cdot \Delta t}{\Delta x^2}$ i.e. $Fo = 0.072$ ....Fourier No.

We shall develop finite difference eqn. for each of the three nodes, viz. 1, 2 and 5 by writing the energy balance for the corresponding elemental volume around that node, and solve the resulting three equations with solve block of Mathcad. T1, T2, T5 are the temps at the 'next' time step.

Start with guess values for T1, T2, T5:

$T1 := 100$ $T2 := 100$ $T5 := 100$ C...guess values

Given

Node 1: $h \cdot \frac{\Delta x + \Delta y}{2} \cdot \left(T_a - T1_0\right) + k \cdot \frac{\Delta y}{2} \cdot \frac{\left(T2_0 - T1_0\right)}{\Delta x} + k \cdot \frac{\Delta x}{2} \cdot \frac{\left(T2_0 - T1_0\right)}{\Delta x} + \frac{\Delta x}{2} \cdot \frac{\Delta y}{2} \cdot q_g = \frac{k}{\alpha} \cdot \left(\frac{\Delta x}{2} \cdot \frac{\Delta y}{2}\right) \cdot \frac{\left(T1 - T1_0\right)}{\Delta t}$

since T4 = T2.

Node 2: $h \cdot \frac{\Delta x}{2} \cdot \left(T_a - T2_0\right) + k \cdot \frac{\Delta y}{2} \cdot \frac{\left(T1_0 - T2_0\right)}{\Delta x} + \frac{k \cdot \Delta x}{2} \cdot \frac{\left(T5_0 - T2_0\right)}{\Delta y} + \frac{\Delta x}{2} \cdot \frac{\Delta y}{2} \cdot q_g = \frac{k}{\alpha} \cdot \left(\frac{\Delta x}{2} \cdot \frac{\Delta y}{2}\right) \cdot \frac{\left(T2 - T2_0\right)}{\Delta t}$

Node 5: $\frac{k \cdot \Delta x}{2} \cdot \frac{\left(T2_0 - T5_0\right)}{\Delta y} + \frac{k \cdot \Delta y}{2} \cdot \frac{\left(T2_0 - T5_0\right)}{\Delta x} + \frac{\Delta x}{2} \cdot \frac{\Delta y}{2} \cdot q_g = \frac{k}{\alpha} \cdot \left(\frac{\Delta x}{2} \cdot \frac{\Delta y}{2}\right) \cdot \frac{\left(T5 - T5_0\right)}{\Delta t}$

$Temp\left(T1_0, T2_0, T5_0\right) := Find(T1, T2, T5)$

Note that Temp is written as a function of initial temps T10, T20, T50. It helps in finding out 'new temps' at successive time steps as shown later.

We get:

$$Temp\left(T1_0, T2_0, T5_0\right) = \begin{bmatrix} 41.034 \\ 40.803 \\ 40.571 \end{bmatrix}$$

....On the RHS, we have: Temps T1,T2, T5 after 1 time step of 60 s

$$Temp(41.034, 40.803, 40.571) = \begin{bmatrix} 61.028 \\ 61.124 \\ 61.209 \end{bmatrix}$$

....Here, on the RHS, we have: Temps T1,T2, T5 after 2 time steps, i.e. after 120 s

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Now, to go to any desired time step, use the values of T1, T2, T5 obtained above as the 'old' temp and apply the Solve Block again.

We write a small Mathcad program to do this:

```
Explcit_Temp(n) := | T1 ← 20
                   | T2 ← 20
                   | T5 ← 20
                   | TempT5_0 ← 20        Store the temps T5 in a separate
                   |                      vector called 'TempT5'
                   | i ← 0
                   | while i<n
                   |     TTemp ← Temp(T1, T2, T5)
                   |     T1 ← TTemp_0
                   |     T2 ← TTemp_1
                   |     T5 ← TTemp_2
                   |     TempT5_(i+1) ← T5     ..Add the value of T5 at each
                   |                           time step to the vector TempT5
                   |     i ← i + 1
                   | return T5                 ...To get only the value of T5 at
                   |                           the given time step
```

T5 at start, i.e. time step = 0: Explcit_Temp(0) = 20 C

T5 at time step = 1, i.e. after 1 min: Explcit_Temp(1) = 40.571 C

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Similarly:

```
-------------------------------------------------
          Time      T5
          (min.)
-------------------------------------------------
Explcit_Temp(10) = 217.227    C

 Explcit_Temp(15) = 302.769    C

 Explcit_Temp(20) = 379.314    C

  Explcit_Temp(25) = 447.74     C

  Explcit_Temp(30) = 508.899    C

  Explcit_Temp(40) = 612.414    C
  Explcit_Temp(50) = 695.101    C

   Explcit_Temp(60) = 761.151    C

   Explcit_Temp(360) = 1.023·10^3     C     ....after 6 hrs.
```

To plot T5 against Time for say, 6 hrs (= 360 time steps):

Let us slightly change the above program, to return stored values of T5:

```
ExplcitTempT5(n) := | T1 ← 20
                    | T2 ← 20
                    | T5 ← 20
                    | TempT5_0 ← 20          Store the temps T5 in a separate
                    |                        vector called 'TempT5'
                    | i ← 0
                    | while i < n
                    |     TTemp ← Temp(T1, T2, T5)
                    |     T1 ← TTemp_0
                    |     T2 ← TTemp_1
                    |     T5 ← TTemp_2
                    |     TempT5_(i+1) ← T5   ..Add the value of T5 at each
                    |                         time step to the vector TempT5
                    |     i ← i + 1
                    | return TempT5           ...To get all the stored values of
                                              T5 till the end of given time step
```


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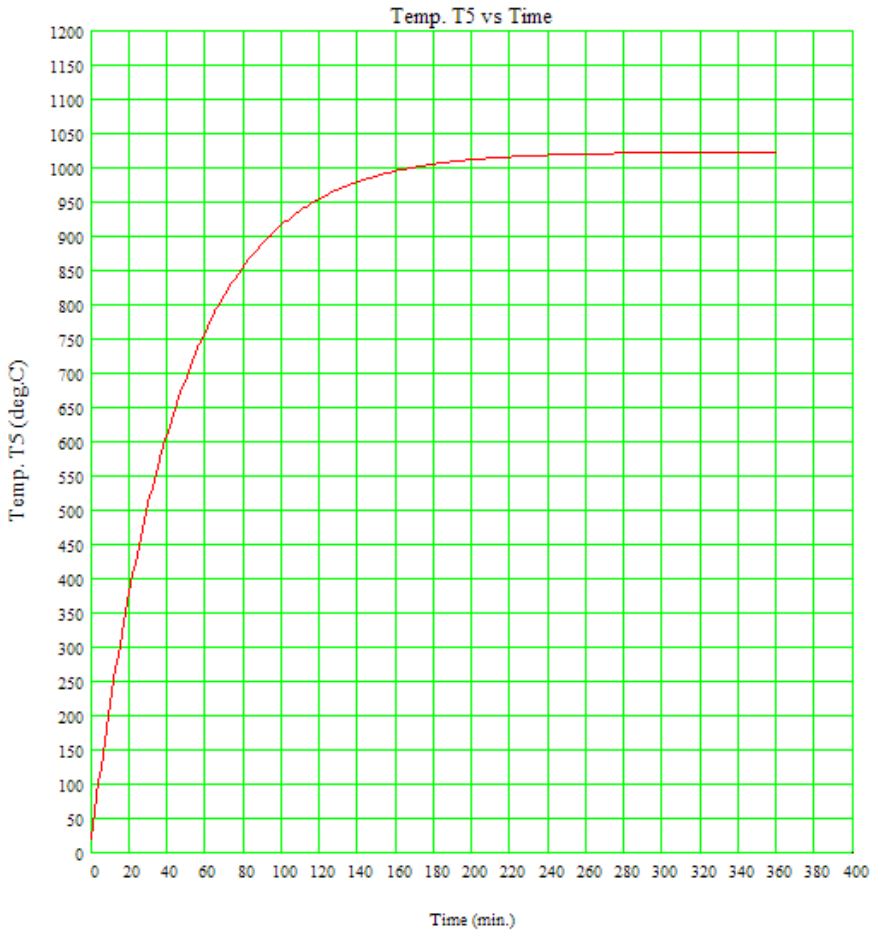
Now, the vector TTempT5 shown below contains all values of T5 for the 360 Timesteps:

TTempT5 := Explcit_TempT5(360)

Now, plot T5 against Time:

i := 0, 1.. 360 ....define the range variable i



We observe from the graph that the steady state is reached after about 240 min. (= 5 hrs), and at that time, T5 = 1023 deg. C.

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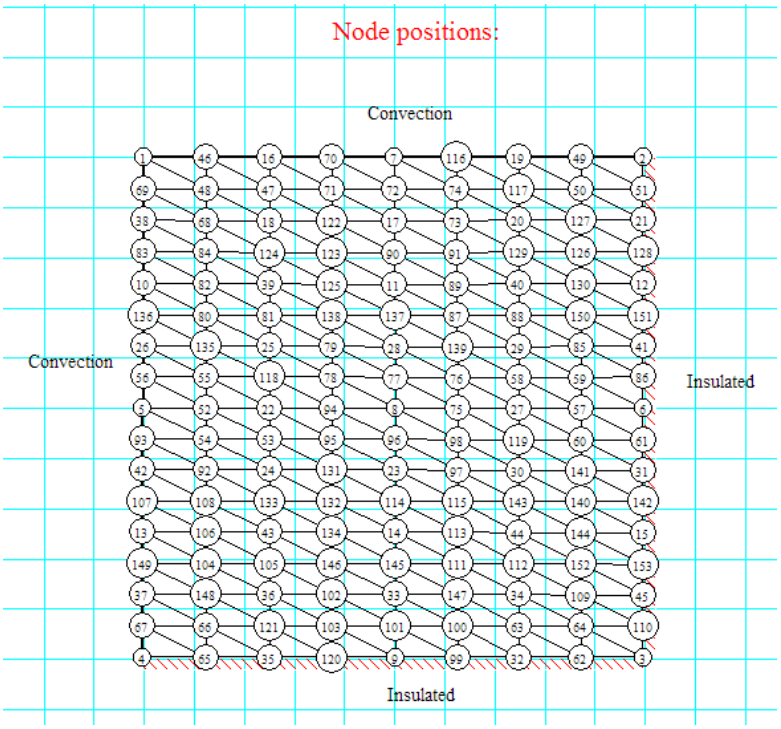
Solution to the above problem by Finite Element Heat Transfer (FEHT) software:

Node positions:



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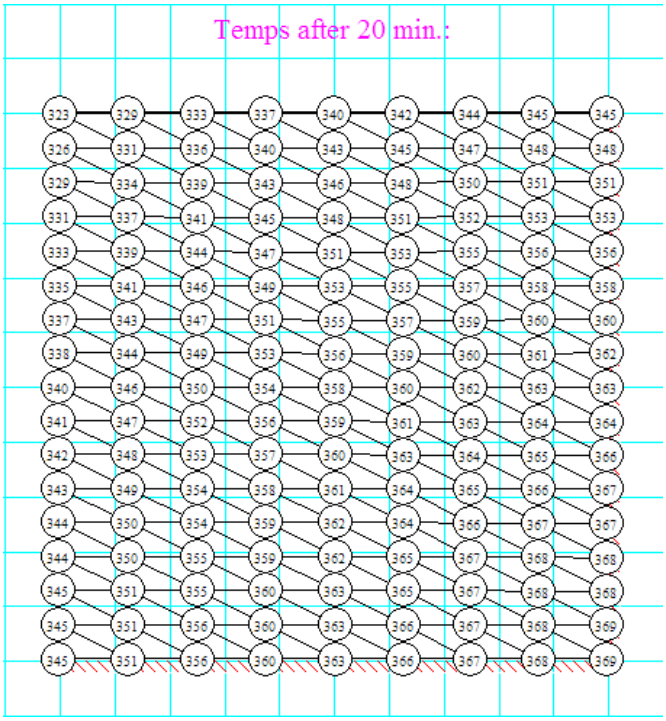
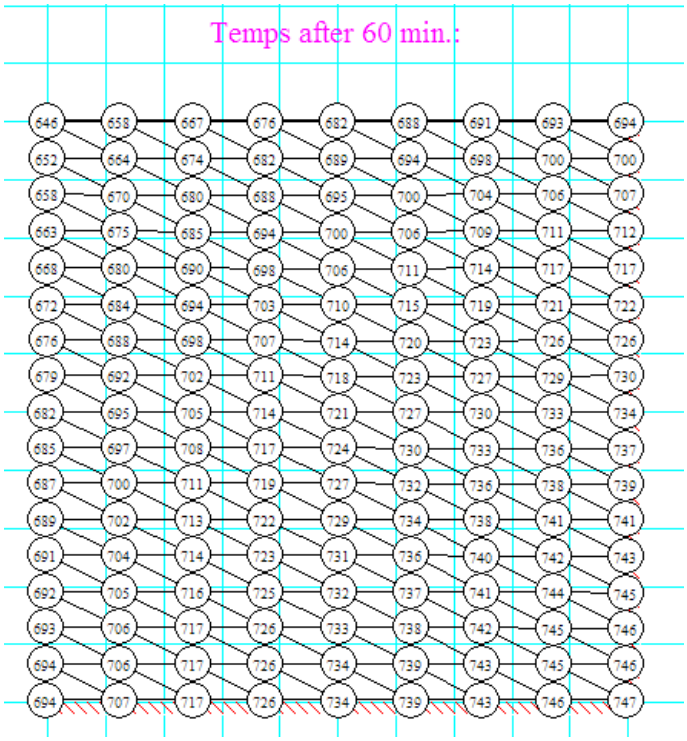
Temp at Node 3 after 20 min:



Note that temp T5 (at Node 3) is 369 C. Compare this with T5 = 379. 3 obtained earlier with Mathcad calculations.

Temp after 60 min.:



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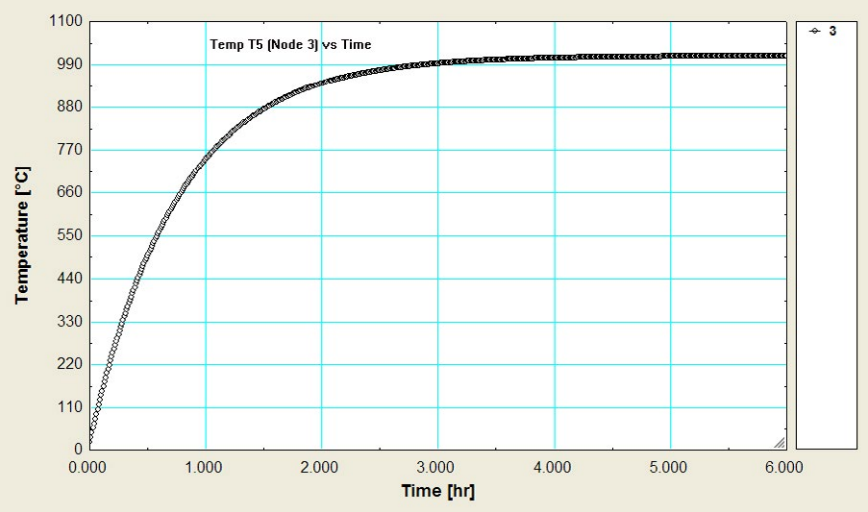
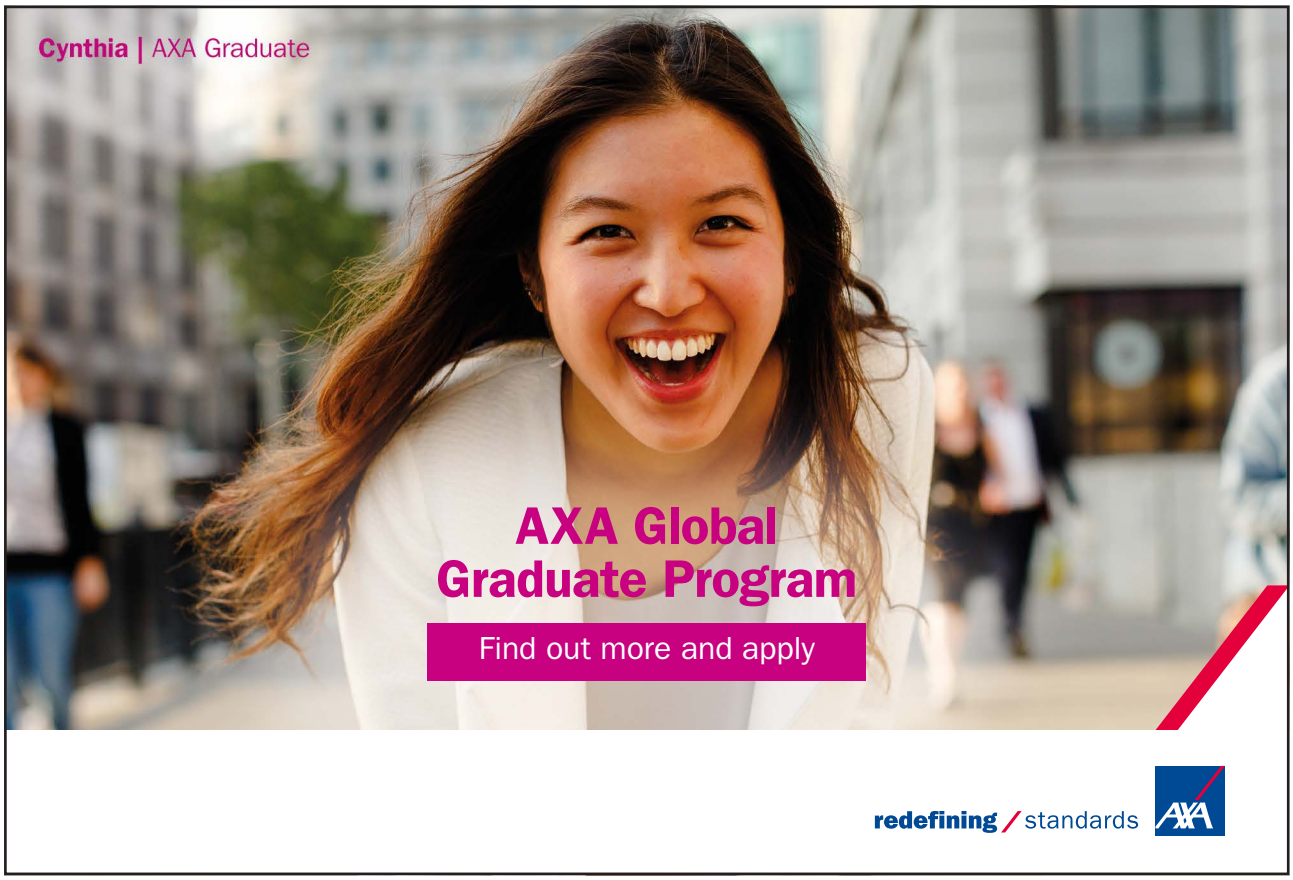
Note that temp T5 (at Node 3) is 747 C. Compare this with T5 = 761.2 obtained earlier with Mathcad calculations.

Plot Temp T5 vs Time:



Note that steady state temp is reached after about 4 hrs, and its value is: about 1015 C. Compare this with T5 = 1023 C obtained with Mathcad.

==========



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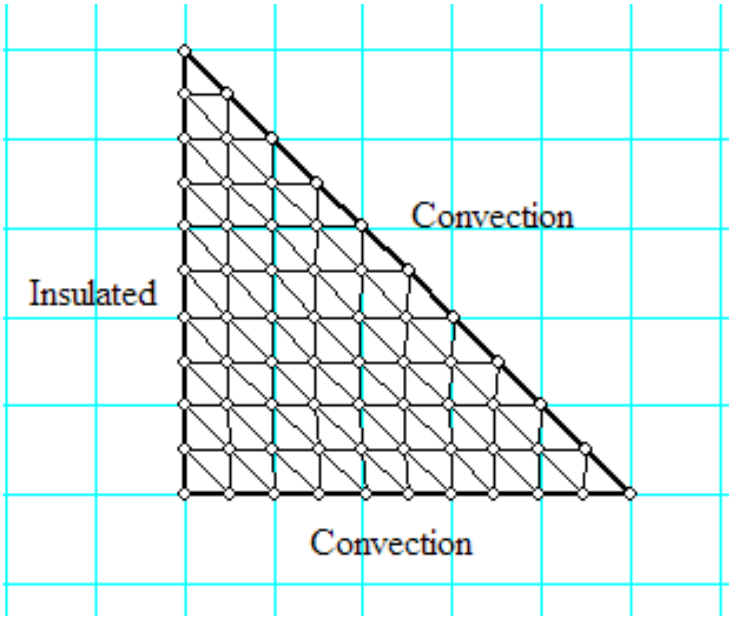
Prob.1I.D.3. A long concrete beam of triangular cross-section is shown, with the two short faces measuring 20 cm each. It is initially at a uniform temp of 20 C. Suddenly, one of the short faces and the long face are exposed to hot gases at 400 C with h = 10 W/m^2.K; the other short face is insulated.

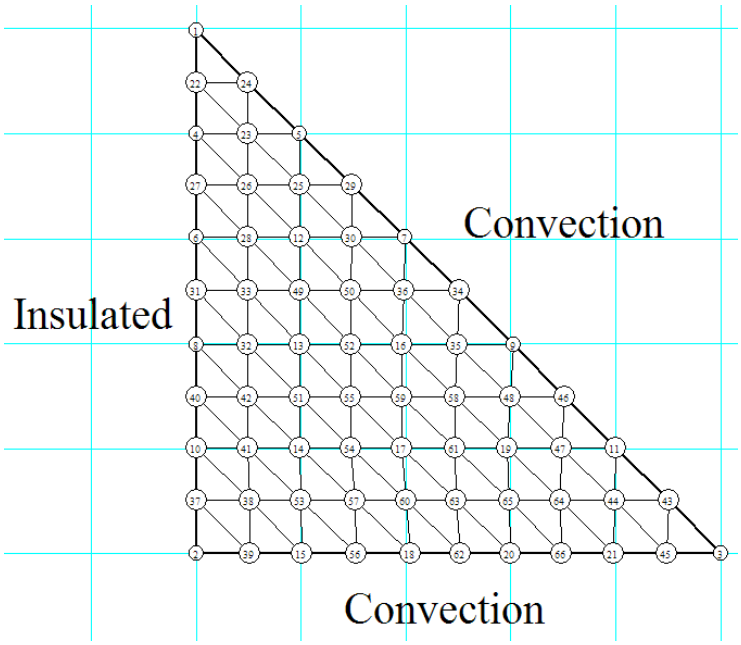
After one hour of exposure, where do the highest and lowest temperatures occur? Plot the variation of these temperatures from time = 0 to 3600 s.

FEHT Solution:

Node positions:



Node Nos.:



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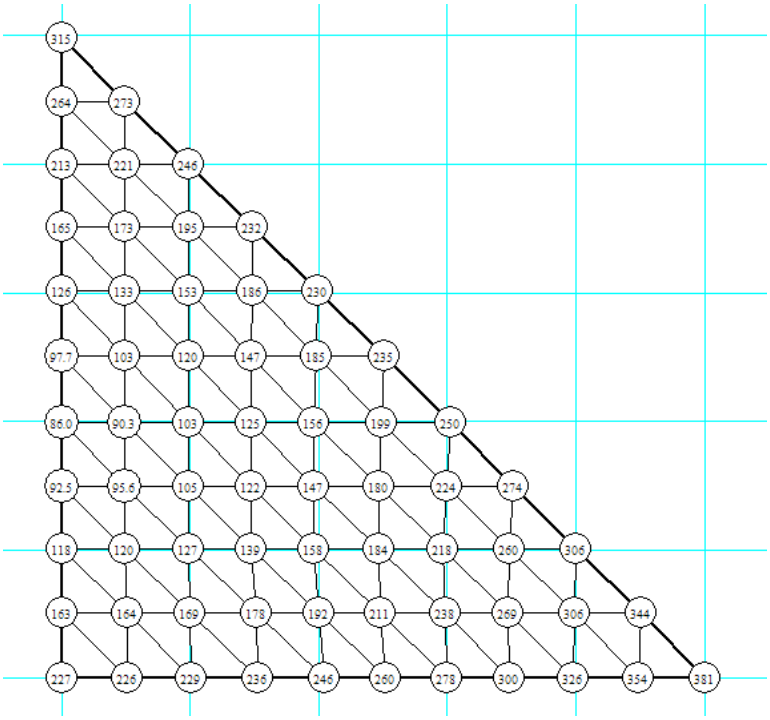
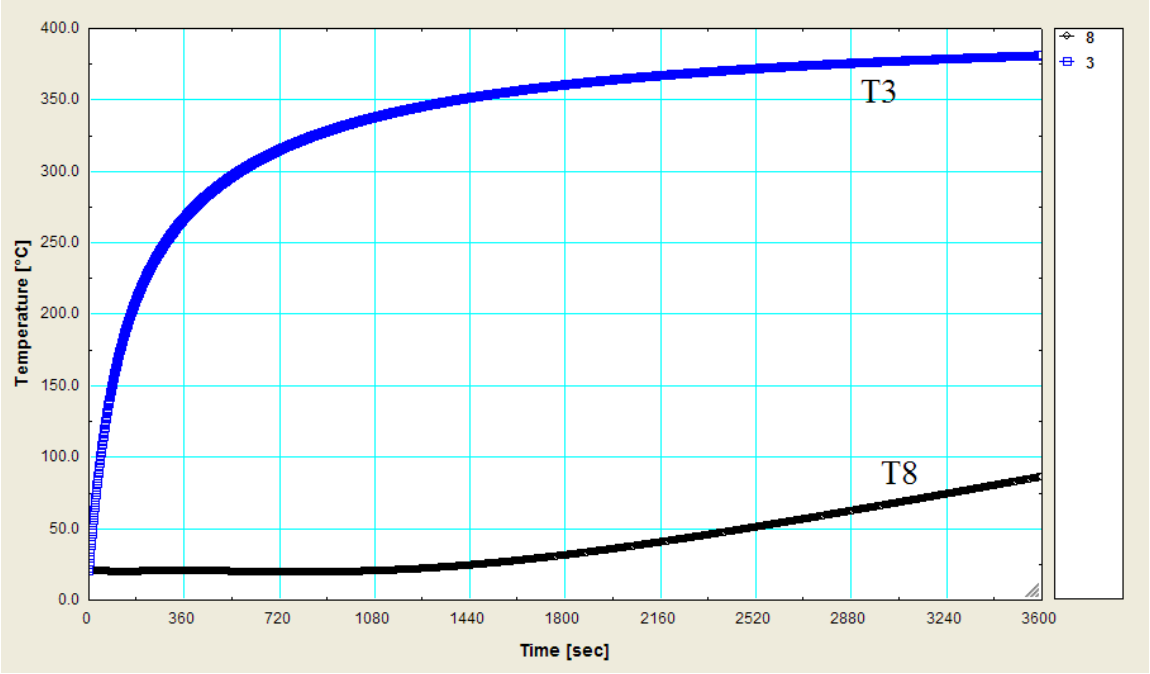
Temps after 3600 s:



We observe that after 1 h, highest temp is 381 C occurring at Node 3 and lowest temp is 86 C occurring at Node 8.

Plot of variation of T3 and T8 with Time:



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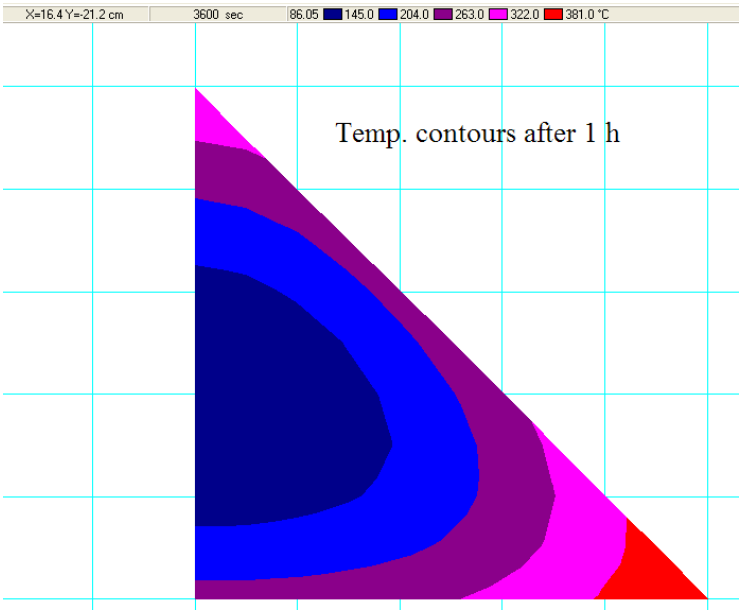
Temp. contours in the beam after 1h:



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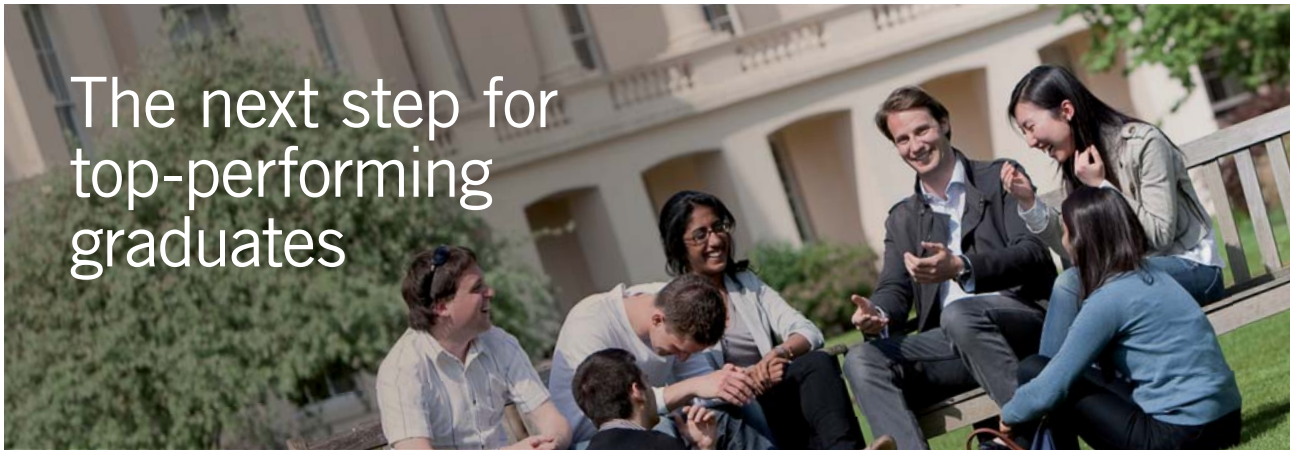

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Prob.1I.D.4. Consider a long solid bar (k = 28 W/m.K and α = 12 × 10^-6 m^2/s) of square cross-section (size: 20 cm × 20 cm) that is initially at a uniform temp of 20 C. Heat is generated in the bar t a rate of qg = 8 x 10^5 W/m^3. 4 sides of the bar are subjected to convection to ambient air at Ta = 30 C and h = 45 W/m^2.K. Using Explicit finite difference method and a mesh size of Δx = Δy = 10 cm, determine the centre temp of the bar after 20 min. Also, find out when steady state temp is reached.
[(Ref. 3)]

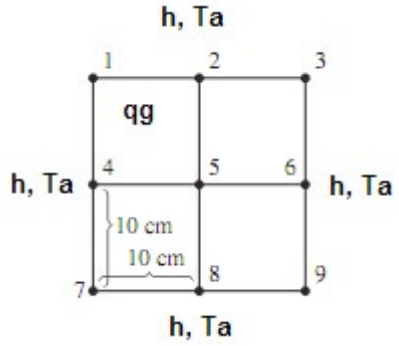

Fig.Prob.1I.D.4

This is the same Problem as 1ID.2.

But, let us work it out in EXCEL:

Nodes are represented by numbers 1, 2,…..9.
Temps at these nodes have to be found out.
Because of **symmetry** along the vertical and horizontal centre lines, we have:
T1 = T3 = T7 = T9, and T2 = T4 = T6 = T8.
Therefore, we need to find out only three temps, viz. T1, T2 and T5.
Let us adopt **'Explicit' method of finite difference solution**.

Now, finite difference eqns:

'Explicit' finite difference eqns are derived by applying heat balance at each node, remembering to consider all heat flows as flowing ***in to*** the node:

For Node 1: This is subjected to convection on both sides. We get:

$$h\cdot\frac{\Delta x+\Delta y}{2}\cdot(T_a-T1_0)+k\cdot\frac{\Delta y}{2}\cdot\frac{(T2_0-T1_0)}{\Delta x}+k\cdot\frac{\Delta x}{2}\cdot\frac{(T4_0-T1_0)}{\Delta x}+\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}\cdot q_g=\rho\cdot\left(\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}\right)\cdot cp\cdot\frac{(T1-T1_0)}{\Delta t}$$

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Here, subscript '0' refers to 'previous' time step.

$$\text{i.e.} \quad h\cdot\Delta x\cdot(Ta - T1_0) + \frac{k}{2}\cdot(T2_0 - T1_0 + T2_0 - T1_0) + \frac{\Delta x^2}{4}\cdot q_g = \frac{k}{\alpha}\cdot\frac{\Delta x^2}{4\cdot\Delta t}\cdot(T1 - T1_0)$$

$$\text{since } \Delta x = \Delta y, \text{and, } T4 = T2, \text{ and } \alpha = \frac{k}{\rho\cdot cp}$$

$$\text{i.e.} \quad Bi\cdot(Ta - T1_0) + (T2_0 - T1_0) + \frac{\Delta x^2\cdot q_g}{4\cdot k} = \frac{1}{4\cdot Fo}\cdot(T1 - T1_0) \qquad \text{where} \quad Bi := \frac{h\cdot\Delta x}{k}$$

$$\text{i.e.} \quad T1 = 4\cdot Fo\cdot\left[Bi\cdot(Ta - T1_0) + (T2_0 - T1_0) + C1\right] + T1_0 \qquad \text{where} \quad C1 := \frac{\Delta x^2\cdot q_g}{4\cdot k}$$

For Node 2: This is a surface node, subjected to convection. We get:

$$h\cdot\Delta x\cdot(T_a - T2_0) + k\cdot\frac{\Delta y}{2}\cdot\frac{(T1_0 - T2_0)}{\Delta x} + \frac{k\cdot\Delta y}{2}\cdot\frac{(T3_0 - T2_0)}{\Delta x} + \left[k\cdot\Delta x\cdot\frac{(T5_0 - T2_0)}{\Delta y}\right] + \Delta x\cdot\frac{\Delta y}{2}\cdot q_g = \frac{k}{\alpha}\cdot\left(\Delta x\cdot\frac{\Delta y}{2}\right)\cdot\frac{(T2 - T}{\Delta t}$$

$$\text{i.e.} \quad Bi\cdot(Ta - T2_0) + (T1_0 - T2_0) + (T5_0 - T2_0) + 2\cdot C1 = \frac{1}{2\cdot Fo}\cdot(T2 - T2_0)$$

$$\text{i.e.} \quad T2 = T2_0 + 2\cdot Fo\cdot\left[Bi\cdot(Ta - T2_0) + (T1_0 - T2_0) + (T5_0 - T2_0) + 2\cdot C1\right]$$

For Node 5: This is an internal node. We get:

$$4\cdot k\cdot(T2_0 - T5_0) + \Delta x^2\cdot q_g = \frac{k}{\alpha}\cdot\Delta x^2\cdot\frac{(T5 - T5_0)}{\Delta t}$$

$$\text{i.e.} \quad 4\cdot(T2_0 - T5_0) + 4\cdot C1 = \frac{T5 - T5_0}{Fo}$$

$$\text{i.e.} \quad T5 = T5_0 + 4\cdot Fo\cdot(T2_0 - T5_0 + C1)$$

Stability criterion:

While adopting 'Explicit' form of solution, there is restriction on the value of time step, Δt, which should be properly chosen to prevent numerical oscillations.

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We should see that Δt should be such that the coeff. of T_m in the most restrictive node (generally, adjacent to a node with convection) is more than zero:

In the present case, Node 1 is subjected to convection from both sides.
Eqn for T1 can be re-written as:

$$T1 = [-4 \cdot Fo \cdot (Bi + 1) + 1] \cdot T1_0 + 4 \cdot Fo \cdot (Bi \cdot Ta + T2_0 + C1)$$

Therefore: equating the coeff of T10 to zero, we get:

$$Fo \cdot (1 + Bi) < \frac{1}{4}$$ where Fo = mesh Fourier No., and Bi = Biot No.

$$Fo = \frac{\alpha \cdot \Delta t}{\Delta x^2} \qquad Bi = \frac{h \cdot \Delta x}{k} = 0.161$$

Therefore: $$\Delta t < \frac{\Delta x^2}{\alpha} \cdot \frac{1}{4 \cdot (1 + Bi)}$$

i.e. $\Delta t < 179.443$ s

i.e. use any timestep less than 179 s from stability criterion.

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Let us use: $\Delta t := 60\ s$

Then: $Fo := \frac{\alpha \cdot \Delta t}{\Delta x^2}$ i.e. $Fo = 0.072$ ....Fourier No.

Following are the steps in EXCEL calculations:

1. Set up the EXCEL worksheet, and name the cells:

Biot | =h*deltax/k

	A	B	C	D	E
1					
2		Data:			
3					
4		k	28	W/m.C	
5		Ta	30	C	
6		alpha	0.000012	m2/s	
7		h	45	W/m2.C	
8		qg	8.00E+05	W/m3	
9		deltax	0.1	m2/s	
10		deltat	60	s	
11		Fo	0.072	Fourier No.	
12		Biot	0.160714	Biot No.	
13		C_1	71.42857		
14		T_initial	20	C	

2. Prepare the scheme for calculations. Make calculations for Biot No., Fourier No. And constant C_1 as shown. Enter the initial temps also.

Biot | =h*deltax/k

	B	C	D
4	k	28	W/m.C
5	Ta	30	C
6	alpha	0.000012	m2/s
7	h	45	W/m2.C
8	qg	8.00E+05	W/m3
9	deltax	0.1	m2/s
10	deltat	60	s
11	Fo	0.072	Fourier No.
12	Biot	0.160714	Biot No.
13	C_1	71.42857	
14	T_initial	20	C

$T1 = 4 \cdot Fo \cdot \left[Bi \cdot (Ta - T1_0) + (T2_0 - T1_0) + C1\right] + T1_0$ where $C1 := \frac{\Delta x^2 \cdot qg}{4 \cdot k}$

For Node 2:

$T2 = T2_0 + 2 \cdot Fo \cdot \left[Bi \cdot (Ta - T2_0) + (T1_0 - T2_0) + (T5_0 - T2_0) + 2 \cdot C1\right]$

For Node 5:

$T5 = T5_0 + 4 \cdot Fo \cdot (T2_0 - T5_0 + C1)$

			Temperatures								
18	TimeStep	Time (s)	T1	T2	T3	T4	T5	T6	T7	T8	T9
19	0	0	20	20	20	20	20	20	20	20	20

Initial temp.

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3. Now, enter the eqn for T1 in cell D20, for T2 in cell E20n and for T5 in cell H20. Corner temps T3, T7 and T9 are set equal to T1. Mid-face temps T4, T6 and T8 are set equal to T2. Eqn. for T5 can be seen in the Formula bar in the screen shot below:

16											
17			Temperatures								
18	TimeStep	Time (s)	T1	T2	T3	T4	T5	T6	T7	T8	T9	
19	0	0	20	20	20	20	20	20	20	20	20	Initial temp.
20	1	60	41.034	40.803	41.034	40.803	40.571	40.803	41.034	40.803	41.034	
21	2	120										

4. Now, select cells B19 to L19 and drag-copy for 20 time steps (i.e. 20 min. since each time step = 60 s). The temp values are immediately calculated:

17			Temperatures								
18	TimeStep	Time (s)	T1	T2	T3	T4	T5	T6	T7	T8	T9	
19	0	0	20	20	20	20	20	20	20	20	20	Initial temp.
20	1	60	41.034	40.803	41.034	40.803	**40.571**	40.803	41.034	40.803	41.034	
21	2	120	61.028	61.124	61.028	61.124	**61.210**	61.124	61.028	61.124	61.028	
22	3	180	80.191	80.974	80.191	80.974	**81.756**	80.974	80.191	80.974	80.191	
23	4	240	98.665	100.366	98.665	100.366	**102.102**	100.366	98.665	100.366	98.665	
24	5	300	116.548	119.314	116.548	119.314	**122.174**	119.314	116.548	119.314	116.548	
25	6	360	133.910	137.832	133.910	137.832	**141.921**	137.832	133.910	137.832	133.910	
26	7	420	150.801	155.932	150.801	155.932	**161.315**	155.932	150.801	155.932	150.801	
27	8	480	167.259	173.625	167.259	173.625	**180.336**	173.625	167.259	173.625	167.259	
28	9	540	183.311	190.922	183.311	190.922	**198.975**	190.922	183.311	190.922	183.311	
29	10	600	198.978	207.833	198.978	207.833	**217.227**	207.833	198.978	207.833	198.978	
30	11	660	214.279	224.367	214.279	224.367	**235.093**	224.367	214.279	224.367	214.279	
31	12	720	229.226	240.532	229.226	240.532	**252.575**	240.532	229.226	240.532	229.226	
32	13	780	243.832	256.337	243.832	256.337	**269.678**	256.337	243.832	256.337	243.832	
33	14	840	258.108	271.791	258.108	271.791	**286.407**	271.791	258.108	271.791	258.108	
34	15	900	272.062	286.901	272.062	286.901	**302.769**	286.901	272.062	286.901	272.062	
35	16	960	285.703	301.675	285.703	301.675	**318.771**	301.675	285.703	301.675	285.703	
36	17	1020	299.039	316.121	299.039	316.121	**334.419**	316.121	299.039	316.121	299.039	
37	18	1080	312.077	330.246	312.077	330.246	**349.720**	330.246	312.077	330.246	312.077	
38	19	1140	324.825	344.057	324.825	344.057	**364.683**	344.057	324.825	344.057	324.825	
39	20	1200	337.289	357.561	337.289	357.561	**379.314**	357.561	337.289	357.561	337.289	

Thus: Temp T5 after 20 min. is = 379.314 deg.C…. Ans.

This value matches very well with values obtained with Mathcad. (See Prob. 1ID.2)

5. Next, we would like to see when steady state is reached. We put a criterion that when the variation in temp T5 is within 0.05, we will say that steady state is reached. We can go on drag-copying the row 39 till we get this criterion fulfilled; but, it is laborious and the worksheet may become too long. So, we will write a small VBA program which will run the worksheet for desired no. of time steps. It will stop and give a message when the steady state is reached:
6. As a preparation to this, let us first write a Macro to make the calculations for the 'next step'. Following is the procedure to write the Macro:

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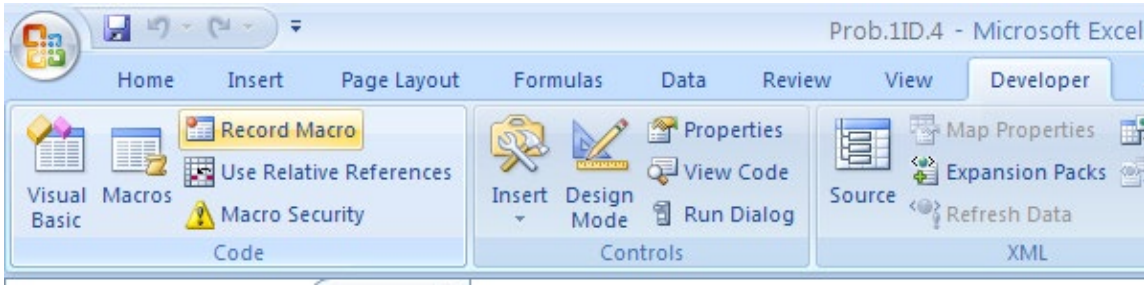
7. Go to Developer and click on: Record Macro:



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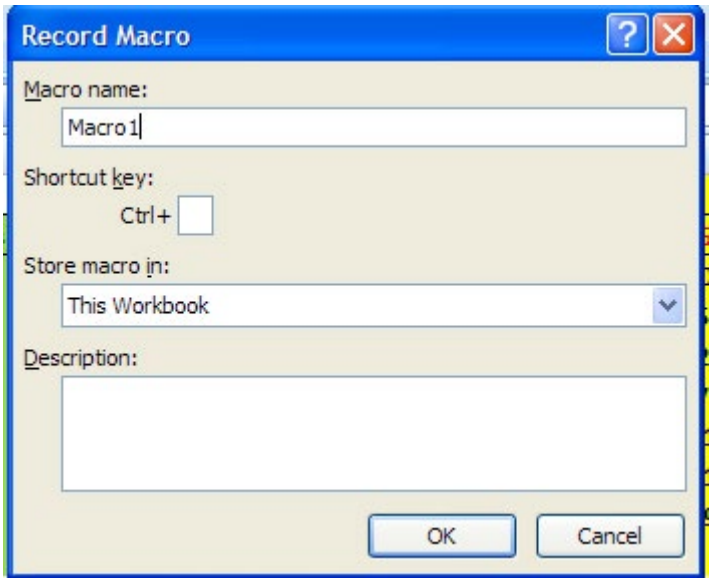
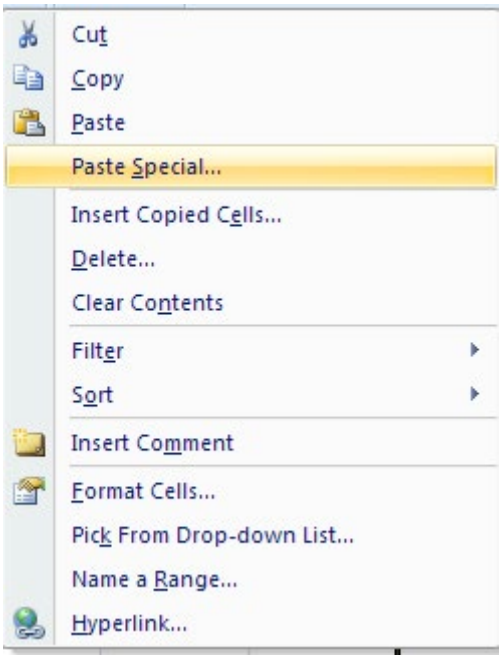
We get:



Enter the short cut key to what you like like, write a description what the Macro will do and press OK. Immediately in Developer tab, Record Macro button changes to Stop Recording, and the recording process starts. Now, go through these simple steps: keep the worksheet up to 6th time step (i.e. up to row 25) as it is; then copy this row (Ctrl+C), and 'Paste special' *only values* in the previous row.



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Press OK, Then, immediately row 25 gets updated.

Following is the VBA code produced:

```
Sub Macro1()
'
' Macro1 Macro
'
' Run for desired no. of time steps
'
' Keyboard Shortcut: Ctrl+Shift+C
'

'
    Range("B25:L25").Select
    Selection.Copy
    Range("B24").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
        :=False, Transpose:=False

End Sub
```

8. Above code advances the calculations by one time step. Now, let us modify the above code to up- date the calculations by the required no. of time steps. Let the time steps desired be entered in cell K12 in the worksheet. If the entered value for N is less than 7, we need an error message.Now, the VBA program should read the value of N, and run the code for that many no. of steps:

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9. VBA program to do this is shown below: (Read the comments given in code)

```
Sub Macro1()
'
' Macro1 Macro
' Run for desired no. of time steps
'
' Keyboard Shortcut: Ctrl+Shift+C
'
Dim N As Integer
Dim DIFF1 As Double
Dim i As Integer

N = Range("K12") 'Reads the value of N from cell K12

'Following portion gives an error message if N < 7 and ends the program

 If N < 7 Then
          MsgBox ("N should be equal to 7 or more !!")
          End
        End If

 N = N - 6 'Value of N reduced by 6 since 6 time steps are already calculated in the worksheet

  For i = 1 To N 'Start of (For...Next) loop to up=date temperatures

    Range("B25:L25").Select
    Selection.Copy
    Range("B24").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
        :=False, Transpose:=False

    'Check when steady state is reached, and give a message and end the program
    |
    DIFF1 = Abs(Range("H25").Value - Range("H24").Value)
         If DIFF1 < 0.01 Then
           MsgBox ("Steady State is reached! Now, Time step is: " & Str(Range("B25")))
           End
         End If

 Next i

End Sub
```

This program does the following:

Line 1, 2: declares the Subroutine Macro name

Line 3: says what it does

Line 4: gives keyboard short cut

Lines 5, 6 and 7: declaring Dimensions of variables used.

Line 8 to 12: reads value of cell K12, gives an error message if N < 7 and ends the program

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Line 13: deducts 6 from N, and assigns it as N. (Since 6 time steps are already there in the worksheet, we are deducting 6 from N here, so that in the output, the time step shown will be N. So, in K12, we should enter any value more than 7.

Line 14 to 19: For … Next loop to do calculations for N steps

Lines 15 to 19: are the same as obtained earlier by recording the Macro. *Thus, the heart of the program was recorded to get error free code.*

Line 21: calculates DIFF1, as absolute value of difference between two successive values of temp T5 (i.e. in cell H25 and H24).

Line 22 to 25: Apply the If condition to see if DIFF1 is < 0.01. If the condition is satisfied, give a message that the steady state is reached, and give the time step no.

Line 26: ends the For…Next loop

Line 27: ends the Sub statement.

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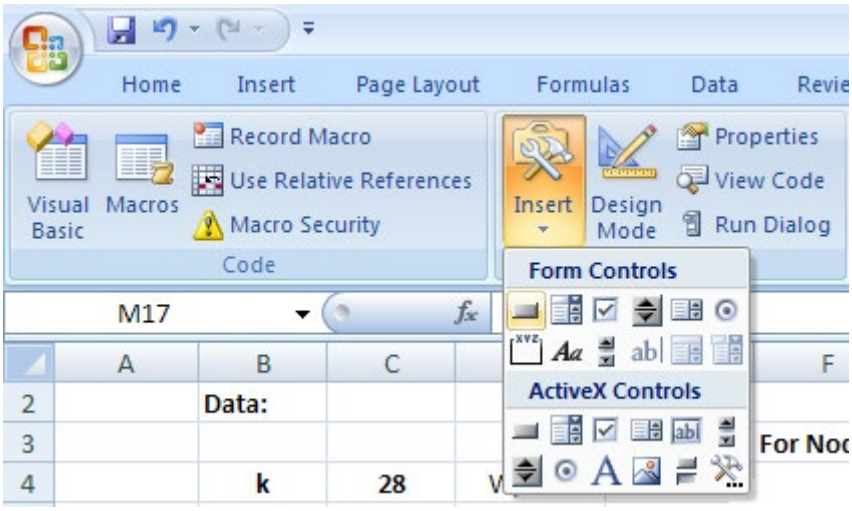
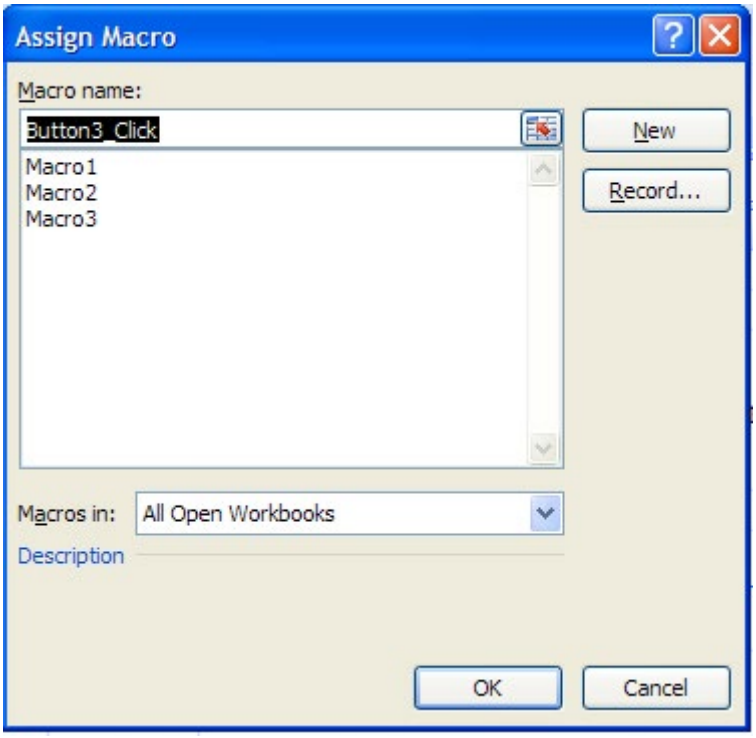
10. **For convenience, we *assign this Macro to a Form control button***:

Go to Developer – Insert – Form controls, and click on the first button in top row:



Create the button by dragging the mouse and we get the screen:



Select the Macro name and click OK. This means that when we press the button, the Macro will run.

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Now, edit the words in the button to Run:

2		Biot	0.160714	Biot No.		For Node 5:		Run	N =	20	Time steps

11. Now, if we click on Run, we get:

			Temperatures									
17												
18	TimeStep	Time (s)	T1	T2	T3	T4	T5	T6	T7	T8	T9	
19	0	0	20	20	20	20	**20**	20	20	20	20	Initial temp.
20	1	60	41.034	40.803	41.034	40.803	**40.571**	40.803	41.034	40.803	41.034	
21	2	120	61.028	61.124	61.028	61.124	**61.210**	61.124	61.028	61.124	61.028	
22	3	180	80.191	80.974	80.191	80.974	**81.756**	80.974	80.191	80.974	80.191	
23	4	240	98.665	100.366	98.665	100.366	**102.102**	100.366	98.665	100.366	98.665	
24	19	1140	324.825	344.057	324.825	344.057	**364.683**	344.057	324.825	344.057	324.825	
25	20	1200	337.289	357.561	337.289	357.561	**379.314**	357.561	337.289	357.561	337.289	
26												

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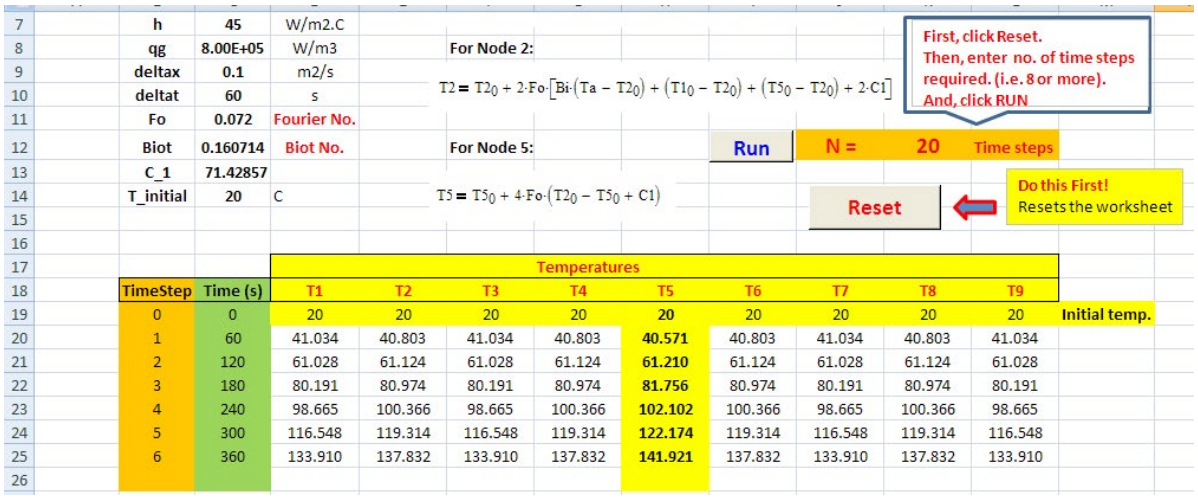
12. Another *important* thing is that now, if we change N to some other value, it will proceed from time step 20 shown in the above screen shot. So, we have to reset the worksheet to the original position before we enter the next trial. This is quite easy. Just create a Macro2 to copy the row 23 to row 24 (with Formulas), and assign this Macro to a Form control button, named as: **Reset**. When you click 'Reset' button, the worksheet is restored to original condition:



7		h	45	W/m2.C								
8		qg	8.00E+05	W/m3		For Node 2:						
9		deltax	0.1	m2/s								
10		deltat	60	s		$T2 = T2_0 + 2 \cdot Fo \cdot [Bi \cdot (Ta - T2_0) + (T1_0 - T2_0) + (T5_0 - T2_0) + 2 \cdot C1]$						
11		Fo	0.072	Fourier No.								
12		Biot	0.160714	Biot No.		For Node 5:						
13		C_1	71.42857									
14		T_initial	20	C		$T5 = T5_0 + 4 \cdot Fo \cdot (T2_0 - T5_0 + C1)$						
15												
16												
17						Temperatures						
18		TimeStep	Time (s)	T1	T2	T3	T4	T5	T6	T7	T8	T9
19		0	0	20	20	20	20	20	20	20	20	20
20		1	60	41.034	40.803	41.034	40.803	40.571	40.803	41.034	40.803	41.034
21		2	120	61.028	61.124	61.028	61.124	61.210	61.124	61.028	61.124	61.028
22		3	180	80.191	80.974	80.191	80.974	81.756	80.974	80.191	80.974	80.191
23		4	240	98.665	100.366	98.665	100.366	102.102	100.366	98.665	100.366	98.665
24		5	300	116.548	119.314	116.548	119.314	122.174	119.314	116.548	119.314	116.548
25		6	360	133.910	137.832	133.910	137.832	141.921	137.832	133.910	137.832	133.910
26												

Reset Macro is shown below:

```
Sub Macro2()
'
' Macro2 Macro
' Resets the worksheet
'
' Keyboard Shortcut: Ctrl+Shift+R
'
    Range("B23:L23").Select
    Selection.AutoFill Destination:=Range("B23:L24"), Type:=xlFillDefault
    Range("B23:L24").Select
    Range("M28").Select
End Sub
```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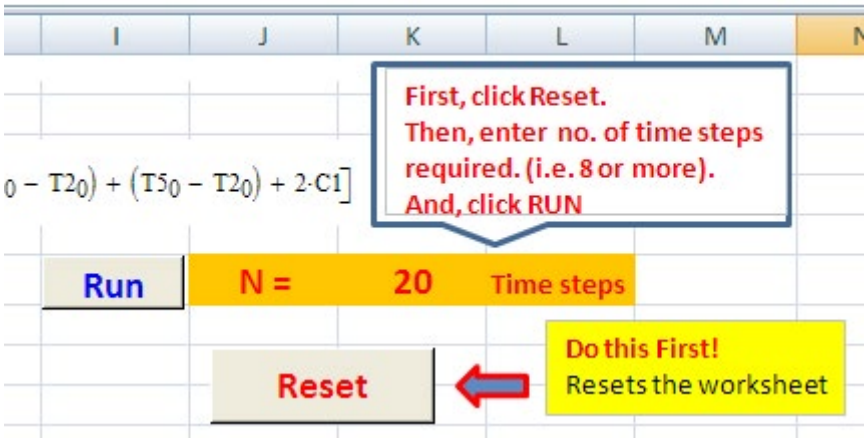
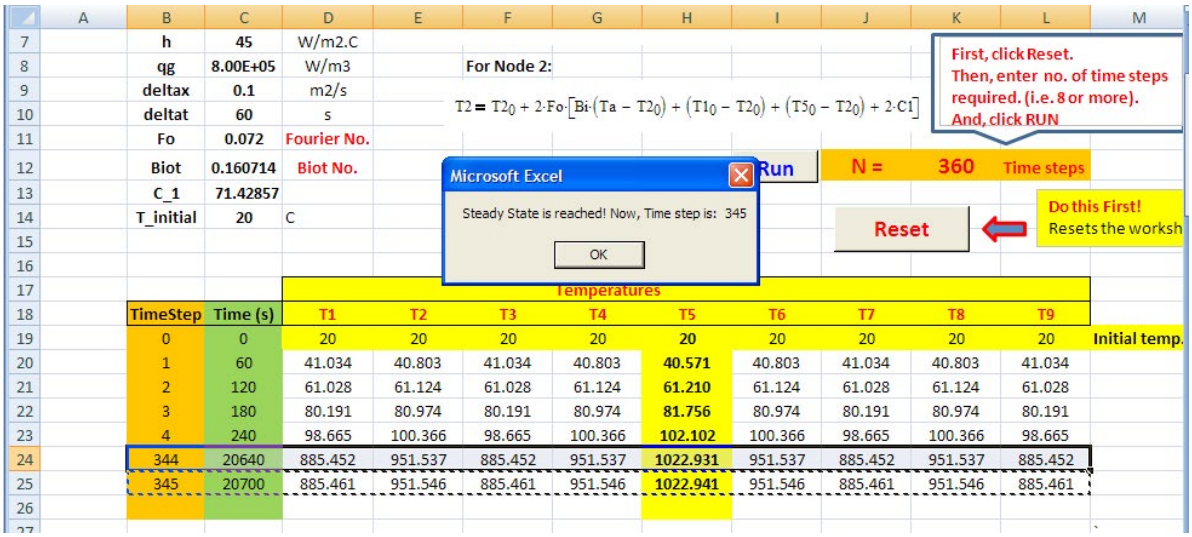
13. Add some comments/ notes on the procedure to be followed, for clarity:



14. Now, let us see when steady state is reached, by putting different values for N. Put N = 360 and click on Run, we get:



We observe that steady state is reached when time step of 345 is reached, and a message has appeared, i.e. difference between successive values of T5 in cells H25 and H24 is < 0.01. We can, of course, change this condition to some other value in the VBA code, if we so desire.

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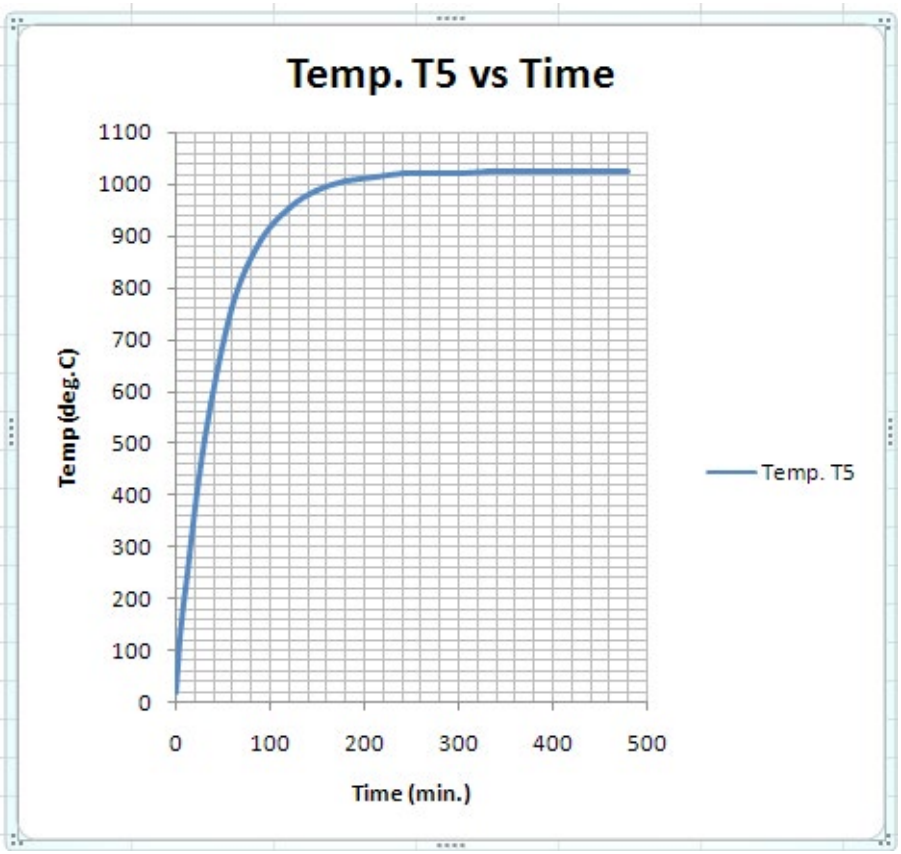
15. Now, comment out this condition in the VBA code, and run the program up to time step 480 (i.e. larger than step 345) and compile a Table by copying the 'values' separately, and draw the graph of Ts vs Time:

	A	B	C	D	E	F	G	H	I	J	K	L	M
31													
32	To plot temp T5 at various times:												
33													
34		TimeStep	Time (s)	T1	T2	T3	T4	T5	T6	T7	T8	T9	
35		0	0	20	20	20	20	**20**	20	20	20	20	**Initial temp.**
36		5	300	116.548	119.314	116.548	119.314	**122.174**	119.314	116.548	119.314	116.548	
37		10	600	198.978	207.833	198.978	207.833	**217.227**	207.833	198.978	207.833	198.978	
38		30	1800	447.661	477.154	447.661	477.154	**508.899**	477.154	447.661	477.154	447.661	
39		60	3600	662.499	709.949	662.499	709.949	**761.151**	709.949	662.499	709.949	662.499	
40		90	5400	772.001	828.602	772.001	828.602	**889.722**	828.602	772.001	828.602	772.001	
41		120	7200	827.813	889.079	827.813	889.079	**955.253**	889.079	827.813	889.079	827.813	
42		150	9000	856.260	919.904	856.260	919.904	**988.654**	919.904	856.260	919.904	856.260	
43		180	10800	870.759	935.615	870.759	935.615	**1005.678**	935.615	870.759	935.615	870.759	
44		210	12600	878.149	943.622	878.149	943.622	**1014.355**	943.622	878.149	943.622	878.149	
45		240	14400	881.915	947.704	881.915	947.704	**1018.778**	947.704	881.915	947.704	881.915	
46		270	16200	883.835	949.784	883.835	949.784	**1021.032**	949.784	883.835	949.784	883.835	
47		300	18000	884.814	950.845	884.814	950.845	**1022.181**	950.845	884.814	950.845	884.814	
48		330	19800	885.313	951.385	885.313	951.385	**1022.767**	951.385	885.313	951.385	885.313	
49		360	21600	885.567	951.660	885.567	951.660	**1023.065**	951.660	885.567	951.660	885.567	
50		390	23400	885.696	951.801	885.696	951.801	**1023.217**	951.801	885.696	951.801	885.696	
51		420	25200	885.762	951.872	885.762	951.872	**1023.295**	951.872	885.762	951.872	885.762	
52		450	27000	885.796	951.909	885.796	951.909	**1023.334**	951.909	885.796	951.909	885.796	
53		480	28800	885.813	951.927	885.813	951.927	**1023.354**	951.927	885.813	951.927	885.813	



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Now, solve the above problem in EXCEL, with *Implicit* finite difference equations:

Remember that with Implicit method *is inherently stable* and there is no restriction on the value of ΔT to be used. However, we shall use a ΔT = 60 s, as we used in the case of Explicit method.

In Implicit method, we use the Temp values in the next step; so, in each step, the finite difference equations have to be solved by iteration. This is extremely simple in EXCEL; all that we have to do is to ensure that Iteration Calculations are enabled .

For this problem, **Implicit finite difference eqns** are derived, as usual, by applying heat balance at the given node, considering all heat flow lines as going ***in to*** the node. Also, remember that, because of symmetry, all the corner temps are equal (i.e. T1 = T3 = T7 = T9), and mid-face temps are also equal (i.e. T2 = T4 = T6 = T8). The derivation of eqns is given below, briefly:

Implicit finite difference eqns:

Node 1:

$$2\cdot k\cdot\frac{\Delta y}{2}\cdot\frac{(T2-T1)}{\Delta x}+h\cdot\Delta x\cdot(Ta-T1)+\frac{q_g\cdot\Delta x^2}{4}=\frac{k}{\alpha}\cdot\frac{\Delta x^2}{4}\cdot\frac{(T1-T1\_old)}{\Delta t}\qquad \text{since } T2=T4$$

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i.e. $T2 - T1 + Bi\cdot(Ta - T1) + C\_1 = \frac{T1 - T1\_old}{4\cdot Fo}$ since $\Delta x = \Delta y$ and,

$$Bi = \frac{h\cdot\Delta x}{k} \qquad Fo = \frac{\alpha\cdot\Delta t}{\Delta x^2} \qquad \text{and} \qquad C\_1 = \frac{q_g\cdot\Delta x^2}{4\cdot k}$$

i.e. $T1 = \frac{(4\cdot T2\cdot Fo + T1\_old + 4\cdot Bi\cdot Ta\cdot Fo + 4\cdot C\_1\cdot Fo)}{(4\cdot Bi\cdot Fo + 4\cdot Fo + 1)}$

i.e. $T1 = \frac{[(4\cdot T2 + 4\cdot Bi\cdot Ta + 4\cdot C\_1)\cdot Fo + T1\_old]}{[(4\cdot Bi + 4)\cdot Fo + 1]}$

Node 2:

$$2\cdot k\cdot\frac{\Delta y}{2}\cdot\frac{T1 - T2}{\Delta x} + \left[k\cdot\Delta x\cdot\frac{(T5 - T2)}{\Delta y}\right] + h\cdot\Delta x\cdot(Ta - T2) + q_g\cdot\frac{\Delta x^2}{2} = \frac{k}{\alpha}\cdot\frac{\Delta x^2}{2}\cdot\frac{(T2 - T2\_old)}{\Delta t}$$

.....since T1 = T3

i.e. $T1 - 2\cdot T2 + T5 + Bi\cdot(Ta - T2) + 2\cdot C\_1 = \frac{T2 - T2\_old}{2\cdot Fo}$

i.e. $T2 = \frac{(2\cdot T1\cdot Fo + 4\cdot C\_1\cdot Fo + 2\cdot T5\cdot Fo + 2\cdot Bi\cdot Ta\cdot Fo + T2\_old)}{(2\cdot Bi\cdot Fo + 4\cdot Fo + 1)}$

i.e. $T2 = \frac{[(4\cdot C\_1 + 2\cdot T5 + 2\cdot Bi\cdot Ta + 2\cdot T1)\cdot Fo + T2\_old]}{[(2\cdot Bi + 4)\cdot Fo + 1]}$

Node 5:

$$4\cdot k\cdot\Delta x\cdot\frac{(T2 - T5)}{\Delta y} + \Delta x^2\cdot q_g = \frac{k}{\alpha}\cdot\Delta x^2\cdot\frac{(T5 - T5\_old)}{\Delta t}$$ ...since T2 = T4 = T6 =T8

i.e. $4\cdot(T2 - T5) + 4\cdot C\_1 = \frac{T5 - T5\_old}{Fo}$

i.e. $T5 = \frac{(4\cdot T2\cdot Fo + 4\cdot C\_1\cdot Fo + T5\_old)}{(4\cdot Fo + 1)}$

i.e. $T5 = \frac{[(4\cdot T2 + 4\cdot C\_1)\cdot Fo + T5\_old]}{(4\cdot Fo + 1)}$

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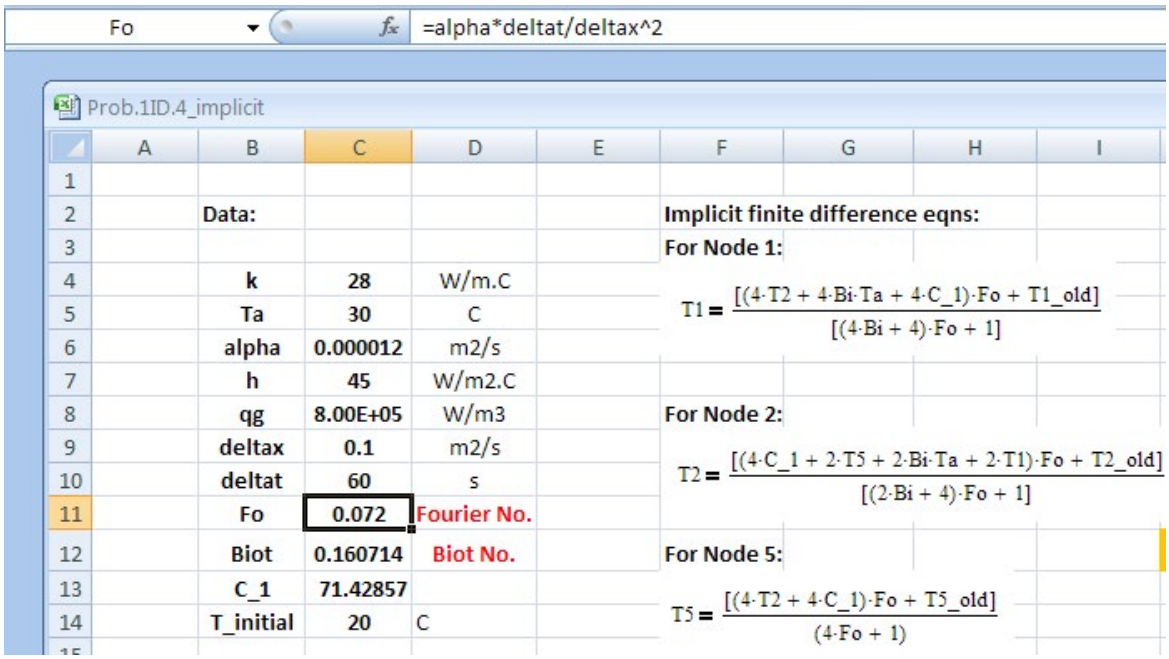
Following are the steps in EXCEL calculations:

1. We shall set up a worksheet to do the following:
   a) Calculate temperatures T1, T2 and T5 by Implicit method
   b) Have a Form control button driven Macro to continue the calculations till the desired time step
   c) Also, this button should build a Table of Time step vs Temp T5 up to the desired Time step, and draw a plot of the same
   d) And, a Form control button driven Reset to reset the calculations and plot to the original condition.
2. Set up the worksheet, enter data, name the cells:



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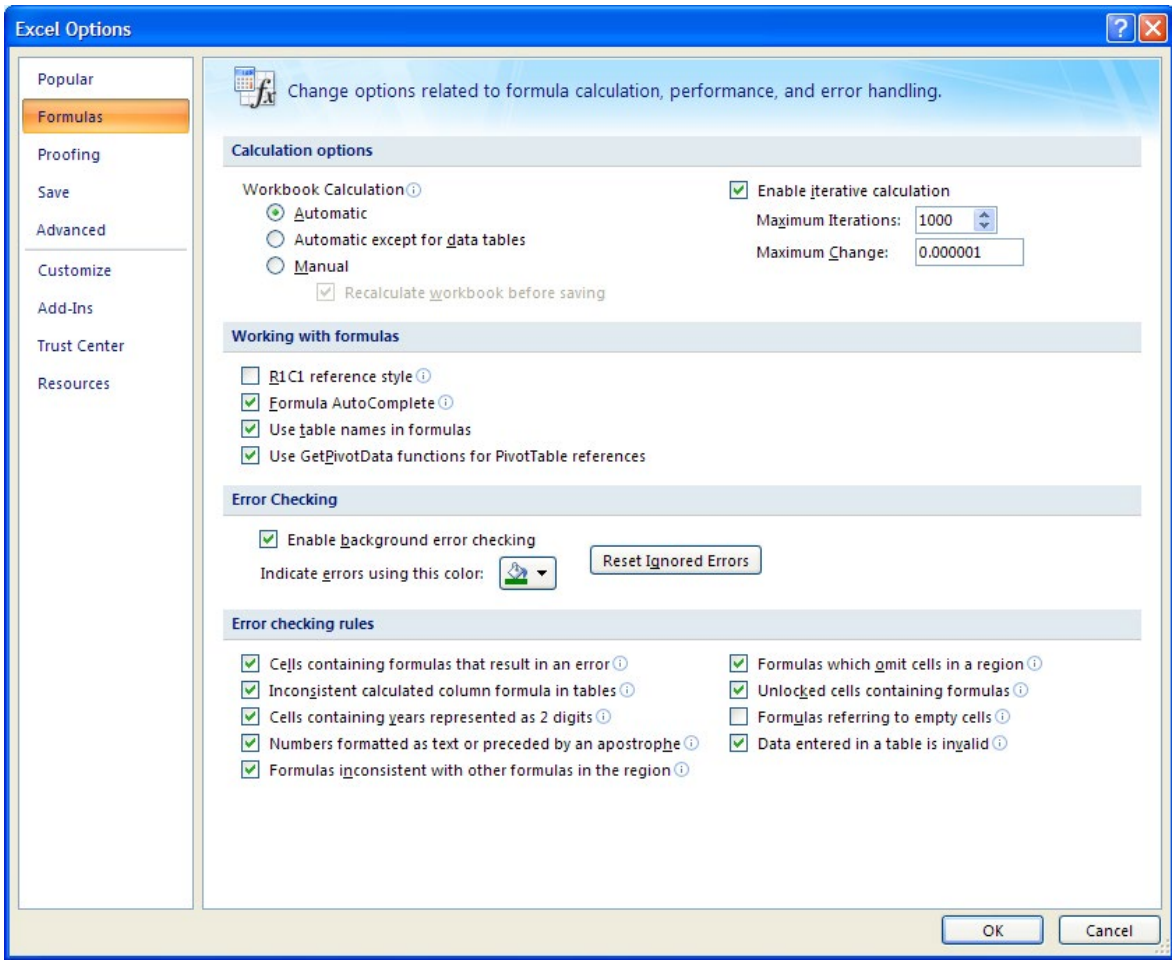
3. Set up the calculations: First enable Iterative calculations in EXCEL by going to: OFFICE button – EXCEL Options – Formulas:



Observe that we have put a check mark in front of Enable Iterative calculation, and changed the Max iterations to 1000 (from default value of 100) and Max change to 0.000001 (from default value of 0.001). Click OK.

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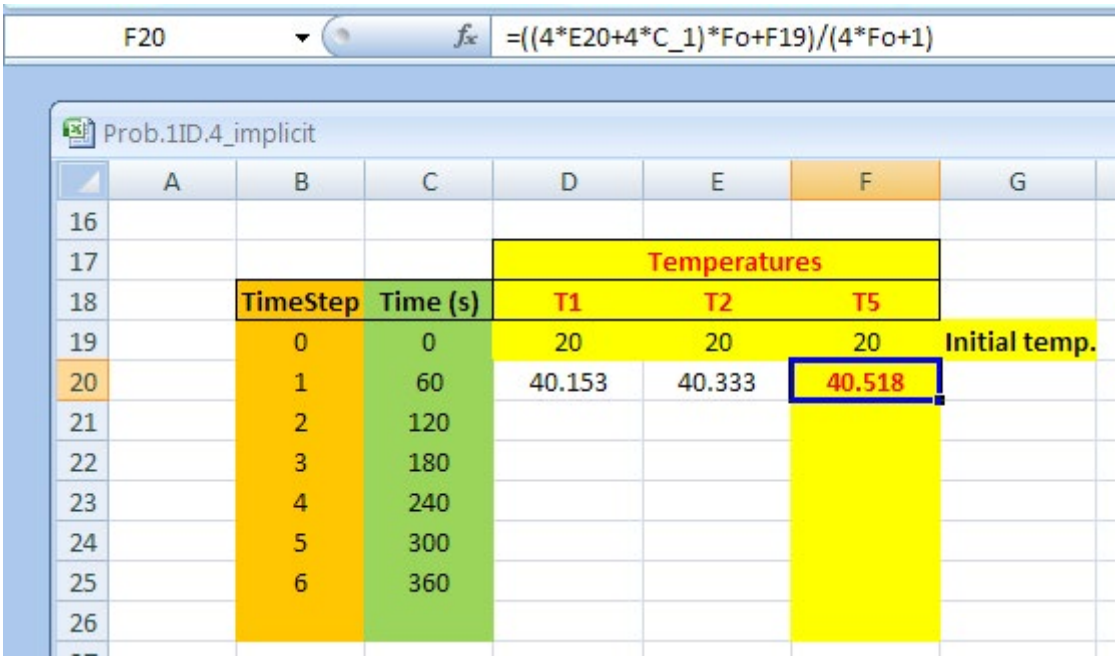
4. Set up the scheme as shown. Enter the eqns for T1, T2 and T5 in cells D20, E20 and F20 respectively. Eqn for T5 in cell F20 can be seen in the Formula bar:



	A	B	C	D	E	F	G
16							
17				Temperatures			
18		TimeStep	Time (s)	T1	T2	T5	
19		0	0	20	20	20	Initial temp.
20		1	60	40.153	40.333	40.518	
21		2	120				
22		3	180				
23		4	240				
24		5	300				
25		6	360				
26							

Note that the above scheme calculates only up to a time step of 6.

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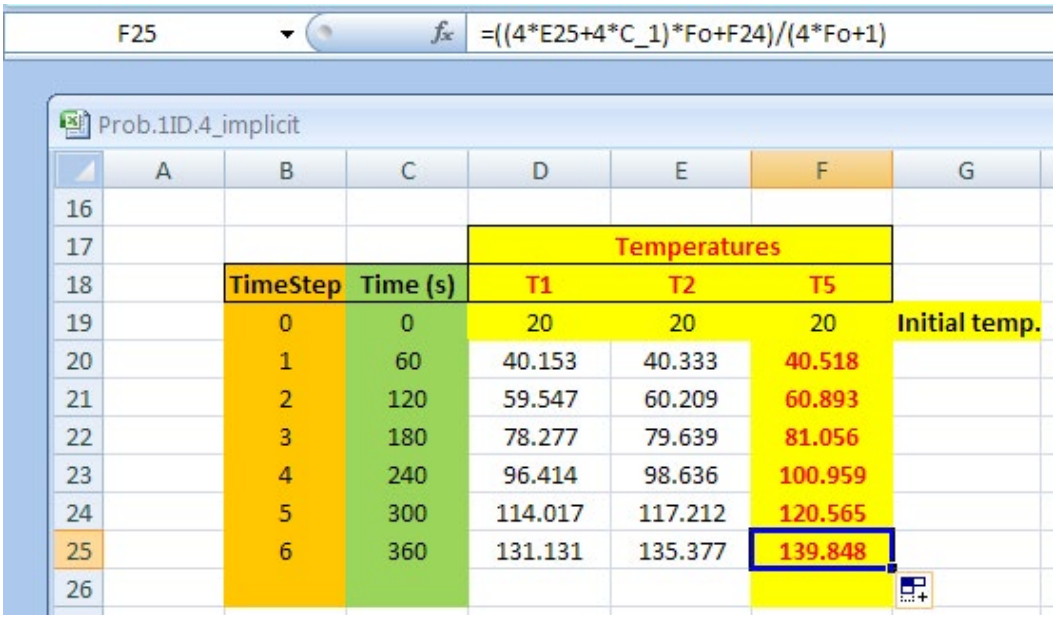
5. Now, select cells d20, E20 and F20 and drag-copy the entire line up to row 25. The calculations (with iterations) are immediately completed:



	A	B	C	D	E	F	G
16							
17				Temperatures			
18		TimeStep	Time (s)	T1	T2	T5	
19		0	0	20	20	20	Initial temp.
20		1	60	40.153	40.333	40.518	
21		2	120	59.547	60.209	60.893	
22		3	180	78.277	79.639	81.056	
23		4	240	96.414	98.636	100.959	
24		5	300	114.017	117.212	120.565	
25		6	360	131.131	135.377	139.848	
26							

6. Now, write a VBA code to perform these calculations up to the desired no. of time steps, N. Have a Form control button to run this Macro. (See the previous VBA code for Explicit method). This code also copies the calculated values of T5 against Time step and Time in to a separate Table starting from cell B30. Following is the code to do this: (Read the comments inside the code)

```
Sub Macro1()
'
' Macro1 Macro
' Run for desired no. of time steps
'
' Keyboard Shortcut: Ctrl+Shift+C
'
Dim N As Integer
Dim DIFF1 As Double
Dim i As Integer

 N = Range("K12") 'Reads the value of N from cell K12

 'Following portion gives a message and ends the program if N < 7

 If N < 7 Then
          MsgBox ("N should be equal to 7 or more !!")
          End
        End If

 N = N - 6 'Value of N reduced by 6 since 6 time steps are already calculated in the worksheet

 For i = 1 To N 'Start of (For ... Next) loop

 'Following portion calculates the temperatures for next time step:

    Range("B25:F25").Select
    Selection.Copy
    Range("B24").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
        :=False, Transpose:=False
```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```
'Following portion builds the Table of results as the calculations proceed:

    Cells(37 + i, 2) = Range("B25")
    Cells(37 + i, 3) = Range("C25")
    Cells(37 + i, 4) = Range("F25")

'Following portion stops the calculations and gives a Message, when the difference
'between successive values of T5 is < 0.01 deg.C:

    DIFF1 = Abs(Range("F25").Value - Range("F24").Value)
        If DIFF1 < 0.01 Then
          MsgBox ("Steady State is reached! Now, Time step is: " & Str(Range("B25")))
          End
        End If

 Next i

End Sub
```


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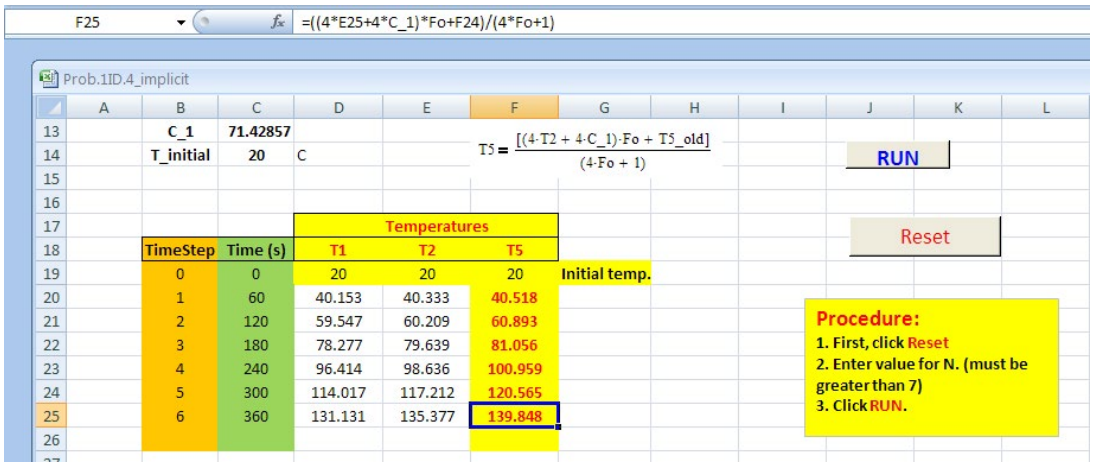
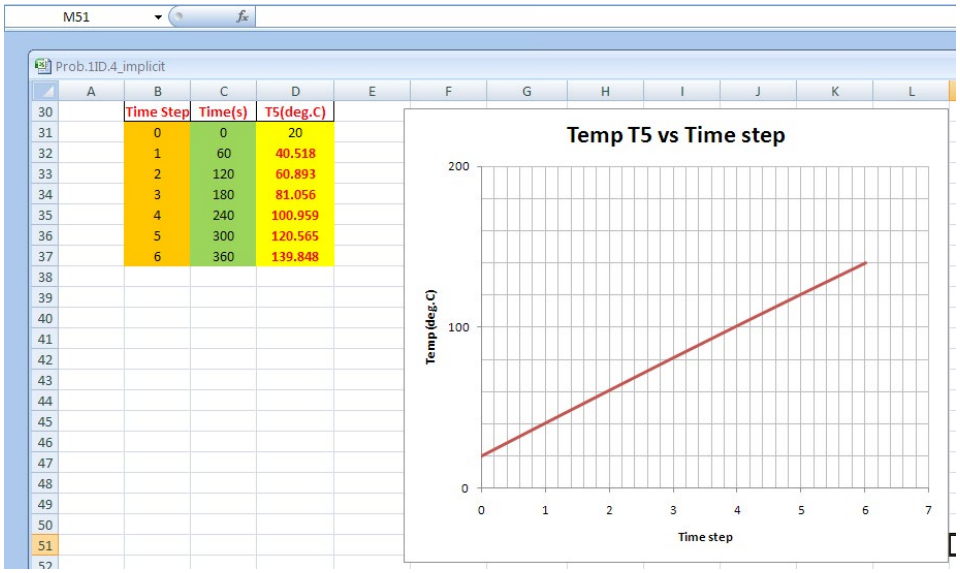
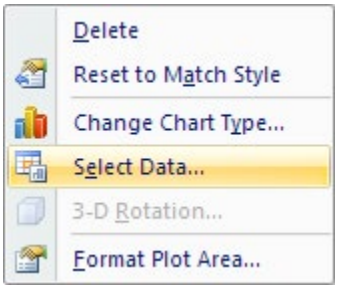
7. Have a Form control button "RUN" to run this code, as explained earlier:



8. Have the Table of results, to which results will be copied by the above code, set up. Also, draw the plot of T5 vs Time step, and format it with respect to grid lines, axes titles, plot title etc.:



9. Adjust the plot such that when the Table is filled up to, say, 400 Time steps, it should be accommodated in the plot. To do that, select the plot, and right click. We get:



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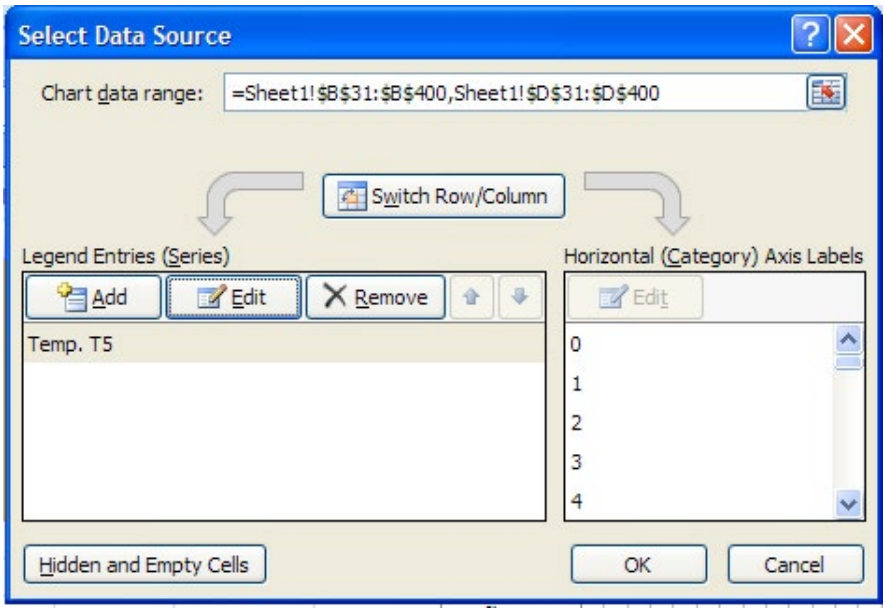
Click on Select Data:



On top of above fig., in Chart data range: change it to accommodate up to cell D400, as shown. This means that as we go on filling up the Table, up to cell D400, the values will automatically up dated in the plot. Click OK.

10. Next, *important thing is*: after one set of calculations are over for a given N, if we need to perform a new set of calculations for another N, we have to 'Reset' the calculations to the original values i.e. for 6 time steps only. Also, we have to delete the values after 6 time steps from the Table that we built up during the previous calculations, and the plot has to re-drawn. This is done by another Macro, connected to the Reset button. Its VBA code is shown below:

```
Sub Macro2()
'
' Macro2 Macro
' Resets the calculations and the plot
'
' Keyboard Shortcut: Ctrl+Shift+R
'
'This portion resets the alculations to the earlier values:

    Range("B23:F23").Select
    Selection.Copy
    Range("B24").Select
    ActiveSheet.Paste

'This portion clears the contents of all cells up to cell D400, starting from cell B38:


    Range("B38:D400").Select
    Selection.ClearContents
    Range("M54").Select

End Sub
```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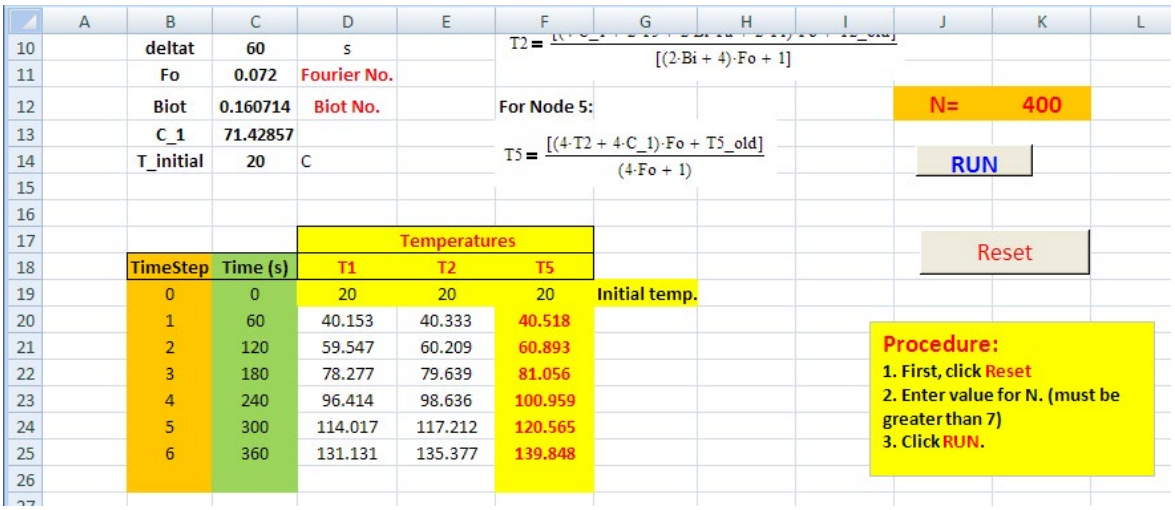
This Macro is also connected to a Form control Button, named as: 'Reset'. This is shown below:



deltat	60	s				
Fo	0.072	Fourier No.				
Biot	0.160714	Biot No.				
C_1	71.42857					
T_initial	20	C				

$$T2 = \frac{[(4 \cdot C\_1 + 2 \cdot Bi + 2 \cdot Bi \cdot Fo + 2 \cdot Fo) \cdot Fo + T2\_old]}{[(2 \cdot Bi + 4) \cdot Fo + 1]}$$

For Node 5:

$$T5 = \frac{[(4 \cdot T2 + 4 \cdot C\_1) \cdot Fo + T5\_old]}{(4 \cdot Fo + 1)}$$

N= 400

RUN

Reset

		Temperatures			
TimeStep	Time (s)	T1	T2	T5	
0	0	20	20	20	Initial temp.
1	60	40.153	40.333	40.518	
2	120	59.547	60.209	60.893	
3	180	78.277	79.639	81.056	
4	240	96.414	98.636	100.959	
5	300	114.017	117.212	120.565	
6	360	131.131	135.377	139.848	

Procedure:
1. First, click Reset
2. Enter value for N. (must be greater than 7)
3. Click RUN.

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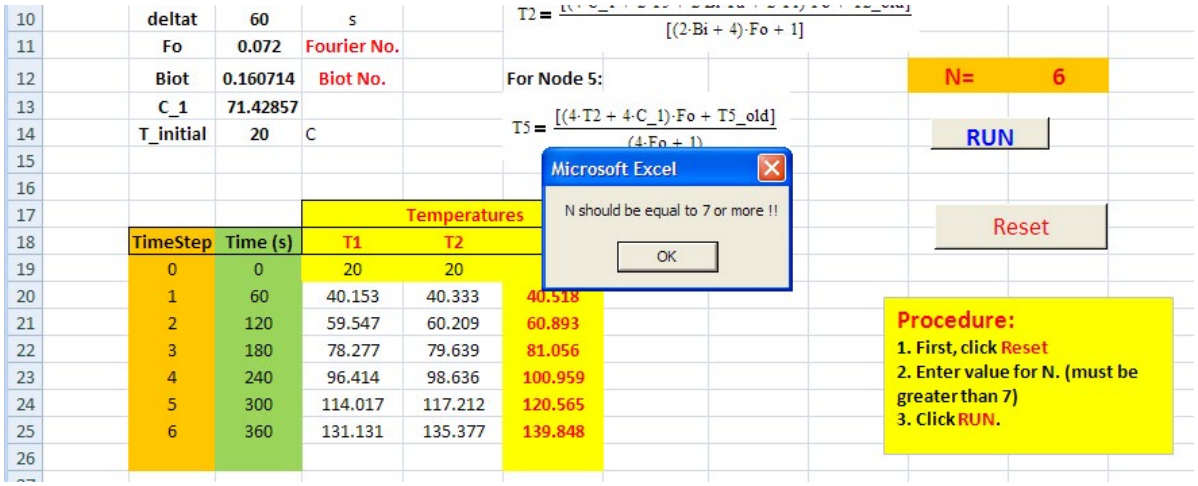
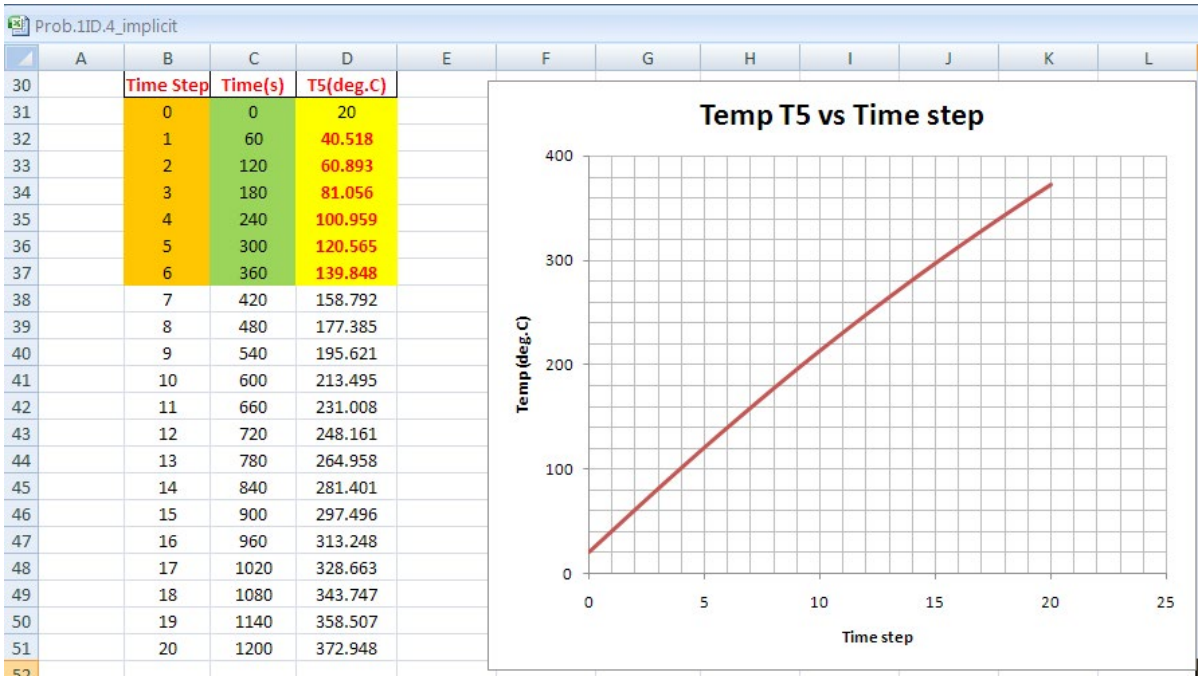
11. Now, as an example, let us enter N = 6 and click 'Run'. We get immediately a Error message:



TimeStep	Time (s)	Temperatures T1	T2	
0	0	20	20	
1	60	40.153	40.333	40.518
2	120	59.547	60.209	60.893
3	180	78.277	79.639	81.056
4	240	96.414	98.636	100.959
5	300	114.017	117.212	120.565
6	360	131.131	135.377	139.848

12. Click OK. Now, click 'Reset'. Now, Enter N = 20, and click 'Run'. We get:



Time Step	Time(s)	T5(deg.C)
0	0	20
1	60	40.518
2	120	60.893
3	180	81.056
4	240	100.959
5	300	120.565
6	360	139.848
7	420	158.792
8	480	177.385
9	540	195.621
10	600	213.495
11	660	231.008
12	720	248.161
13	780	264.958
14	840	281.401
15	900	297.496
16	960	313.248
17	1020	328.663
18	1080	343.747
19	1140	358.507
20	1200	372.948

Observe that the Table has been built for 20 Time steps, and the plot is also up-dated.

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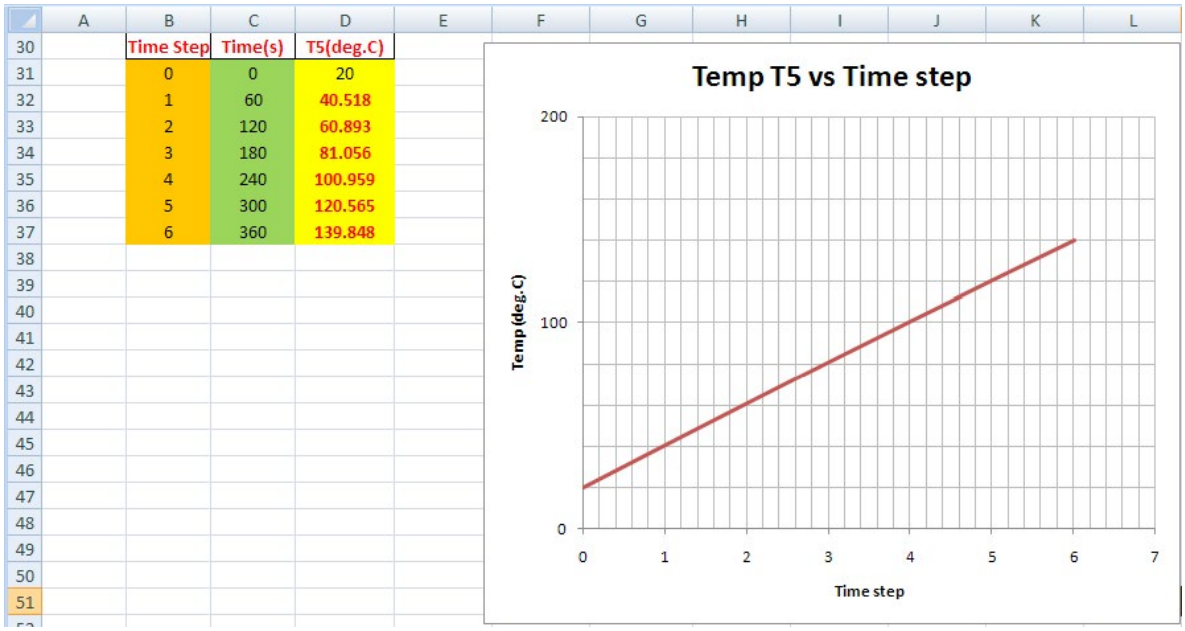
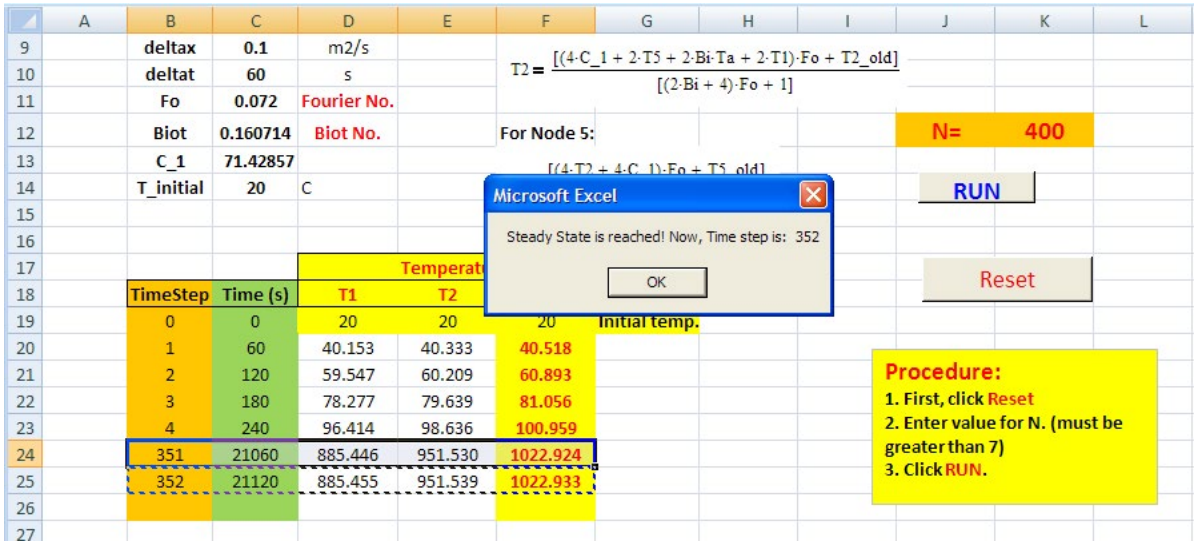
13. Now, let us run the program for 400 Time steps. Before doing so, click on 'Reset'. We get the original set up:



Time Step	Time(s)	T5(deg.C)
0	0	20
1	60	40.518
2	120	60.893
3	180	81.056
4	240	100.959
5	300	120.565
6	360	139.848

14. Now, enter N = 400, and click on 'Run':



deltax	0.1	m2/s		
deltat	60	s		
Fo	0.072	Fourier No.		
Biot	0.160714	Biot No.		
C_1	71.42857			
T_initial	20	C		

$$T2 = \frac{[(4 \cdot C\_1 + 2 \cdot T5 + 2 \cdot Bi \cdot Ta + 2 \cdot T1) \cdot Fo + T2\_old]}{[(2 \cdot Bi + 4) \cdot Fo + 1]}$$

For Node 5:

N= 400

RUN

Reset

Microsoft Excel

Steady State is reached! Now, Time step is: 352

OK

TimeStep	Time (s)	T1	T2	
0	0	20	20	20
1	60	40.153	40.333	40.518
2	120	59.547	60.209	60.893
3	180	78.277	79.639	81.056
4	240	96.414	98.636	100.959
351	21060	885.446	951.530	1022.924
352	21120	885.455	951.539	1022.933

Initial temp.

Procedure:
1. First, click Reset
2. Enter value for N. (must be greater than 7)
3. Click RUN.

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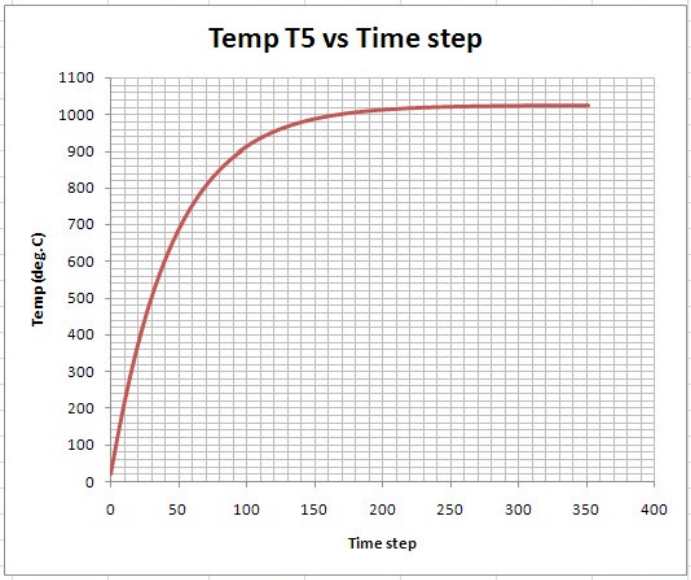
We see that the steady state is reached after 352 time steps. And, at this time, T5 = 1022.933 deg.C. Press OK. Now, we also see that the Results Table has been built up to this time and also the plot is updated:

	Time Step	Time(s)	T5(deg.C)
30			
31	0	0	20
32	1	60	40.518
33	2	120	60.893
34	3	180	81.056
35	4	240	100.959
36	5	300	120.565
37	6	360	139.848
38	7	420	158.792
39	8	480	177.385
40	9	540	195.621
41	10	600	213.495
42	11	660	231.008
43	12	720	248.161
44	13	780	264.958
45	14	840	281.401
46	15	900	297.496
47	16	960	313.248
48	17	1020	328.663
49	18	1080	343.747
50	19	1140	358.507
51	20	1200	372.948
52	21	1260	387.077



377	346	20760	1022.871
378	347	20820	1022.882
379	348	20880	1022.893
380	349	20940	1022.903
381	350	21000	1022.914
382	351	21060	1022.924
383	352	21120	1022.933
384			

Thus, we see that EXCEL, combined with the power of VBA programming and Macro recording, can solve heat transfer programs very effectively.

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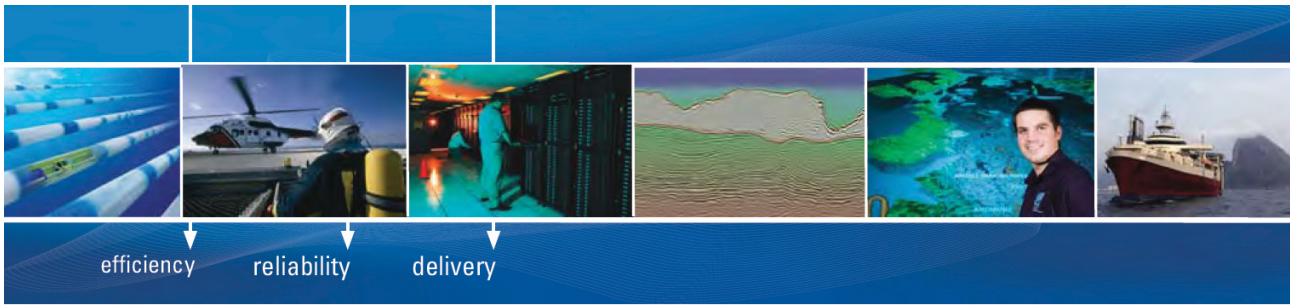

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