Software Solutions to Problems on Heat Transfer

Heat Exchangers

Dr. M. Thirumaleshwar

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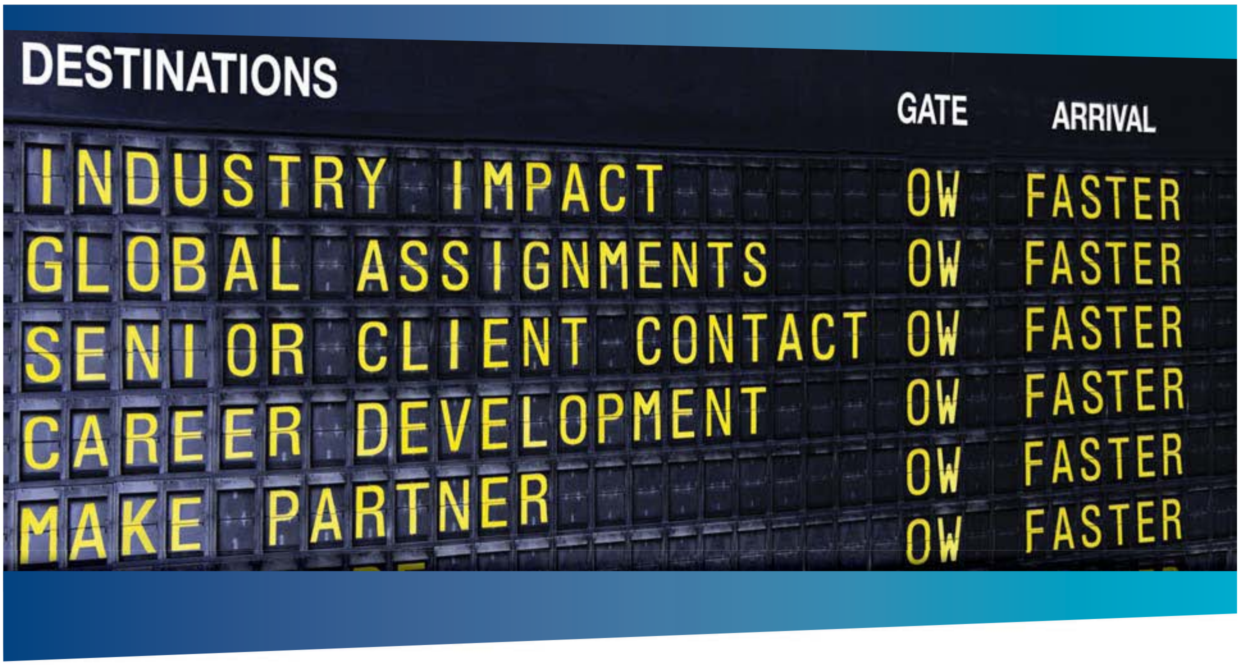
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PREFACE to Vol. 4

This is Vol. 4 of the book **"Software Solutions to Problems on Heat Transfer".**

In Vol. 1, problems on various aspects of CONDUCTION heat transfer were solved with Mathcad, EES, FEHT and EXCEL.

Vol. 2 contained solved problems on the topics of CONVECTION (i.e. Forced convection, Natural or Free convection).

In Vol. 3, we gave solved problems on the topics of BOILING and CONDENSATION.

Present Vol. viz. HEAT EXCHANGERS contains problems solved on following topics:

Vol. 4. HEAT EXCHANGERS:

Heat Exchangers – Equation summary

4.A. Overall heat transfer coeff., Fouling factors etc

Problems solved with Mathcad (Prob. 4A.1 to 4A.2)

Problems solved with EES (Prob. 4A.3 to 4A.5)

Problems solved with EXCEL (Prob. 4A.6 to 4A.7)

4.B. LMTD method of HX analysis, LMTD correction factors etc.:

Problems solved with Mathcad (Prob. 4B.1 to 4B.10)

Problems solved with EES (Prob. 4B.11 to 4B.19)

Problems solved with EXCEL (Prob. 4B.20 to 4B.23)

4.C. 'NTU-Effectiveness' method of HX design, compact HX:

Problems solved with Mathcad (Prob. 4C.1 to 4C.6)

Problems solved with EES (Prob. 4C.7 to 4C.14)

Problems solved with EXCEL (Prob. 4C.15 to 4C.19)

References

Here also, problems are solved using the popular software, viz. "Mathcad", "Engineering Equation Solver (EES)", and MS EXCEL spreadsheet. **Comments are included generously in the codes so that the logic behind the solutions is clear. An introductory chapter in Part-I gives a brief overview of the software used.**

When only graphs are available, but no equations (ex: LMTD correction factors, effectiveness or NTU for some type of HX, heat transfer and friction factor characteristics of cross flow HX), we have digitized the graphs using freely available digitizing software, curve-fitted, and produced Functions to get desired quantities by interpolation, so that calculations could be done without referring to the graphs.

As in Vol. I, II and III, emphasis is given not only to solving a given problem but also to parametric analysis and graphical representation of results. Advantage of using Software to solve a variety of problems thus becomes evident. Also, problems with EXCEL spreadsheet are solved in greater details with relevant screen shots for immediate help to students, since EXCEL is available in practically every Personal Computer.

Acknowledgements: Firstly, I thank my students, since it is they who inspired me and motivated me. Next, my thanks are due to the authorities at St. Joseph Engineering College, for their constant encouragement.

Also, my sincere thanks to **Bookboon.com** for publishing this book on the Internet. **Ms. Sophie** and her editorial staff have to be specially mentioned for their cooperation, suggestions and support.

Finally, my heart-felt appreciation to **my wife, Kala,** for her unfailing and thoughtful support and encouragement.

M. Thirumaleshwar

Author

August 2013

About the Author

Dr. M. Thirumaleshwar graduated in Mechanical Engineering from Karnataka Regional Engineering College, Surathkal, Karnataka, India, in the year 1965. He obtained M.Sc (cryogenics) from University of Southampton, U.K. and Ph.D.(cryogenics) from Indian Institute of Science, Bangalore, India.

He is a Fellow of Institution of Engineers (India), Life Member, Indian Society for Technical Education, and a Foundation Fellow of Indian Cryogenics Council.

He has worked in India and abroad on large projects in the areas involving heat transfer, fluid flow, vacuum system design, cryo-pumping etc.

He worked as Head of Cryogenics Dept. in Bhabha Atomic Research Centre (BARC), Bombay and Centre for Advanced Technology (CAT), Indore, from 1966 to 1992.

He worked as Guest Collaborator with Superconducting Super Collider Laboratory of Universities Research Association, in Dallas, USA from 1990 to 1993.

He also worked at the Institute of Cryogenics, Southampton, U.K. as a Visiting Research Fellow from 1993 to 1994.

He was Head of the Dept. of Mechanical Engineering, Fr. Conceicao Rodrigues Institute of Technology, Vashi, Navi Mumbai, India for eight years.

He also worked as Head of Dept. of Mechanical Engineering and Civil Engineering, and then as Principal, Vivekananda College of Engineering and Technology, Puttur (D.K.), India.

Presently, he is Professor and coordinator of Post-graduate program in the Dept. of Mechanical Engineering in St. Joseph Engineering College, Vamanjoor, Mangalore, India.

A book entitled **"Fundamentals of Heat and Mass Transfer"** authored by him and published by M/s Pearson Education, India (2006) **has been adopted as a Text book** for third year engineering students by the Visweswaraya Technological University (V.T.U.), Belgaum, India.

He has also written and published three book-lets entitled as follows:

1. Towards Excellence… How to Study (A Guide book to Students)
2. Towards Excellence… How to teach (A guide book to Teachers)
3. Towards Excellence… Seminars, GD's and Personal Interviews (A guide book to Professional and Management students)

Dr. M. Thirumaleshwar has attended several National and International conferences and has more than 50 publications to his credit.

About the Software used

Following three software are used while solving problems in this book:

1. Mathcad 2001 (Ref: www.ptc.com)
2. Engineering Equation Solver (EES) (Ref: www.fchart.com), and
3. Finite Element Heat Transfer (FEHT) (Ref: www.fchart.com)
4. MS EXCEL – (2007) Spreadsheet (Ref: Microsoft)

Trial versions of the first three software and detailed Instruction Manuals may be downloaded from the websites indicated. EXCEL is a very popular spreadsheet which comes bundled with MS OFFICE software, and is generally available in every computer.

See Part-I of this book for brief introduction to these four software.

While the information given there is enough to get going, for detailed instructions one should consult the respective Instruction manuals.

To the Student

Dear Student:

I would like to remind you that Heat Transfer is an important subject useful in many branches of engineering. It is also a subject in which you can score high marks in the examinations, since the question paper generally consists of derivations and numerical problems, almost in the ratio 50:50. Therefore, it requires that:

i) you are thorough with the derivations, and
ii) skillful in solving numerical problems.

To be thorough with derivations, you should refer to well known, standard Text books on the subject of Heat Transfer (See References at the end of this book). **And, to develop your skill in solving problems…. well, that is where I think that this book will help you.**

This book contains solutions to problems on heat transfer using four popular softwares, viz. Mathcad, Engineering Equation Solver (EES), Finite Element Heat Transfer (FEHT), and EXCEL spreadsheet. Trial versions of Mathcad, EES and FEHT can be downloaded from the websites indicated. EXCEL, which is a part of MS OFFICE, is generally pre-installed in most of the Personal Computers. Problems are chosen from the University question papers and standard heat transfer Text books.

Use of Software in solving problems has many advantages:

1. It helps in logical thinking
2. Problems are solved quickly and accurately
3. Parametric solutions (or 'what-if' solutions) are obtained easily
4. Solutions can be presented in tabular or graphical form, very easily and quickly
5. Once a particular type of problem is solved, solving a similar problem with different data input becomes very easy
6. Ease of getting solutions to problems in tabular or graphical form creates further interest and curiosity on the subject in the minds of students and encourages them to be creative and work further

How to use this Book?

You need not worry if you don't know about these softwares. Since each problem is solved systematically step by step, and is well commented, just reading through the solution will make the logic of the solution clear to you. That is the most important thing in solving the problems. ***Then, you must work out the problem yourself, by hand or using the software***. Of course, use of software has the above-mentioned advantages. *Simply reading the book won't do*. Have your favorite Text book nearby, in case you need to refer to it for any formulas or clarifications. There is no other 'easy method'.

As they say, *'there is no gain without pain!'*

Lastly, I would like to tell you how greatly I enjoyed solving the problems presented in this book using the softwares mentioned.

I hope that you too will enjoy as much as I did in solving these problems and get benefitted.

Good Luck!

Author

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4 Heat Exchangers

Learning objectives:

1. 'Heat Exchanger' is one of the most commonly used process equipments in industry and research.
2. Function of a heat exchanger is to transfer energy; this transfer of energy may occur to a single fluid (as in the case of a boiler where heat is transferred to water) or between two fluids that are a different temperatures (as in the case of an automobile radiator where heat is transferred from hot water to air).
3. Some typical examples of heat exchanger applications are:
    - i) Thermal power plants (boilers, super-heaters, steam condensers etc.)
    - ii) Refrigeration and Air-conditioning (evaporators, condensers, coolers)
    - iii) Automobile industry (radiators, all engine cooling and fuel cooling arrangements)
    - iv) Chemical process industry (variety of heat exchangers between different types of fluids, in combustors and reactors)
    - v) Cryogenic industry (condenser-re-boilers used in distillation columns, evaporators to produce gas from cryogenic liquids etc.)
    - vi) Research ('regenerators' used in Stirling engines, special ceramic heat exchangers used in ultra-low temperature devices, superconducting magnet systems etc.)
4. Important topics to be studied are: Overall heat transfer coefficient, Importance of 'Fouling factor', Analysis of heat exchangers by 'Logarithmic Mean Temp Difference (LMTD)' method, Correction factors for Cross-flow and Shell & Tube heat exchangers, Analysis of heat exchangers by 'No. of Transfer Units (NTU – Effectiveness (ε)' method, Compact heat exchangers etc.
5. Problems on above topics are worked out using Mathcad, EES and EXCEL software.

Formulas used:

Overall heat transfer coefficient:

In most of the practical cases of heat exchangers, temperature of the hot fluid (T_a) and that of the cold fluid (T_b) are known; then we would like to have the heat transfer given by a simple relation of the form

$$Q = U \cdot A \cdot \left(T_a - T_b\right) = U \cdot A \cdot \Delta T \qquad \ldots(4.21)$$

where Q is the heat transfer rate (W), A is the area of heat transfer perpendicular to the direction of heat transfer, and $(T_a - T_b) = \Delta T$ is the overall temperature difference between the temperature of hot fluid (T_a) and that of the cold fluid (T_b) and **U is the overall heat transfer coeff.**

In a normally used recuperative type of heat exchanger, the hot and cold fluids are separated by a solid wall. This may be a flat type of wall (as in the case of plate-fin type of heat exchangers), or, more often, a cylindrical wall (as in the case of a tube-in-tube type of heat exchangers).

Overall heat transfer coeff. is related to the total thermal resistance of the system, as follows:

$$U=\frac{1}{A\cdot\Sigma R_{th}} \qquad W(m^2.C)\ldots(4.23)$$

For plane wall:

Remember that for a plane wall, thermal resistance is L/(k.A), and convective resistance is 1/(h.A), and since the resistances are in series, we get:

$$U=\frac{1}{A\cdot\left(\frac{1}{h_i\cdot A}+\frac{L}{k\cdot A}+\frac{1}{h_o\cdot A}\right)}$$

i.e.

$$U=\frac{1}{\frac{1}{h_i}+\frac{L}{k}+\frac{1}{h_o}} \qquad W/(m^2.C)\ldots(12.1)$$

Now, if the thermal resistance of the wall is negligible compared to other resistances, we get:

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} \qquad \text{W/(m}^2\text{.C)}\ldots(12.2)$$

For cylindrical wall:

Remember that for a cylindrical wall, thermal resistance is:

$$\frac{\ln\left(\frac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L}$$

and convective resistance is 1/(h.A) and the resistances are in series. *However, the area to be considered has to be specified* since the inner surface area and the outer surface area of the cylinder are different. Now, we have, the general relation for U:

$$U = \frac{1}{A \cdot \Sigma R_{th}} \qquad \text{W/(m}^2\text{.C)}\ldots(4.23)$$

i.e.

$$U \cdot A = \frac{1}{\Sigma R_{th}}$$

We can also write:

$$U_i \cdot A_i = U_o \cdot A_o = \frac{1}{\Sigma R_{th}} \qquad \ldots(12.3)$$

Therefore, referred to outer surface area, U becomes:

$$U_o \cdot A_o = \frac{1}{\frac{1}{h_i \cdot A_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L} + \frac{1}{h_o \cdot A_o}} \qquad \ldots(12.4)$$

Now, for a cylindrical system, we have:

$$A_i = 2 \cdot \pi \cdot r_i \cdot L$$

and,

$$A_o = 2 \cdot \pi \cdot r_o \cdot L$$

Then,

$$U_o = \frac{1}{\frac{A_o}{h_i \cdot A_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L} \cdot A_o + \frac{1}{h_o \cdot A_o} \cdot A_o}$$

i.e. $U_o = \frac{1}{\frac{1}{h_i} \cdot \left(\frac{r_o}{r_i}\right) + \left(\frac{r_o}{k}\right) \cdot \ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o}}$ …(12.5)

Similarly, referred to inner surface area, U becomes:

$$U_i \cdot A_i = \frac{1}{\frac{1}{h_i \cdot A_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L} + \frac{1}{h_o \cdot A_o}}$$ …(12.6)

and,

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L} \cdot A_i + \frac{1}{h_o \cdot A_o} \cdot A_i}$$

i.e. $U_i = \frac{1}{\frac{1}{h_i} + \left(\frac{r_i}{k}\right) \cdot \ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o} \cdot \left(\frac{r_i}{r_o}\right)}$ …(12.7)

Again, *if the thermal resistance of the wall is negligible* compared to other resistances, (i.e. high value of thermal conductivity, k), or, *wall thickness of the tube is very small* (i.e. $(r_i/r_o) \approx 1$), we get:

$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}}$ W/(m^2.C)…(12.8)

For many practical situations, this simple eqn. gives a quick estimate of overall heat transfer coeff., U.

If fins are provided on a particular surface, then the total heat transfer area on that surface is:

$$A_{total} = A_{fin} + A_{unfinned} \qquad \ldots(12.9)$$

where A_{fin} is the surface area of the fins and $A_{unfinned}$ is the area of the un-finned portion of the tube.

For short fins of a material of high thermal conductivity, since there is practically no temperature drop along the length we can use the value of total area as given by eqn. (12.9) to calculate the convection resistance on the finned surface.

However, for long fins where there is a temperature drop along the length of fin, we should use the total or effective area, given by:

$$A_{total} = A_{unfinned} + \eta_{fin} \cdot A_{fin} \qquad \ldots(12.10)$$

where η_{fin} **is the 'fin efficiency'**.

Sometimes, an **'overall surface efficiency'** η_o is used. η_o is defined as:

$$\eta_o \cdot A_{total} = A_{unfinned} + \eta_{fin} \cdot A_{fin}$$

i.e. η_o tells us how much of the total surface area is really effective in transferring heat.

Then, since the effective surface area is also equal to the unfinned area plus the effective area of fin, we can get an expression for overall surface efficiency as follows:

$$\eta_o \cdot A_{total} = \left(A_{total} - A_{fin}\right) + \eta_{fin} \cdot A_{fin}$$

i.e.
$$\eta_o = 1 - \frac{A_{fin}}{A_{total}} + \frac{\eta_{fin} \cdot A_{fin}}{A_{total}}$$

i.e.
$$\eta_o = 1 - \frac{A_{fin}}{A_{total}} \cdot \left(1 - \eta_{fin}\right) \qquad \ldots(12.11)$$

Then, while determining U, we should use η_o .A_{total} for the finned surface, whether it is inner surface area, outer surface area or both.

Fouling factor:

Effect of fouling is accounted for by a term called, 'Fouling factor', (or, 'dirt factor'), defined as:

$$R_f = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}} \quad m^2.K/W \qquad \ldots(12.14)$$

While taking into account the effect of fouling, the 'fouling resistance' (= R_f/area) should be added to the other thermal resistances. For example, for a tube, we can write:

$$U_i \cdot A_i = U_o \cdot A_o = \frac{1}{\sum R_{th}} = \frac{1}{\frac{1}{h_i \cdot A_i} + \frac{R_{fi}}{A_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L} + \frac{1}{h_o \cdot A_o} + \frac{R_{fo}}{A_o}} \qquad \ldots(12.15)$$

where R_{fi} and R_{fo} are the fouling factors for the inside and outside surfaces respectively, and L is the length of tube. From eqn. (12.15), U_i or U_o can easily be calculated.

Based on experience, Tubular Exchanger Manufacturers' Association (TEMA) have given suggested values of fouling factors.

The LMTD method for heat exchanger analysis:

Parallel flow heat exchanger:

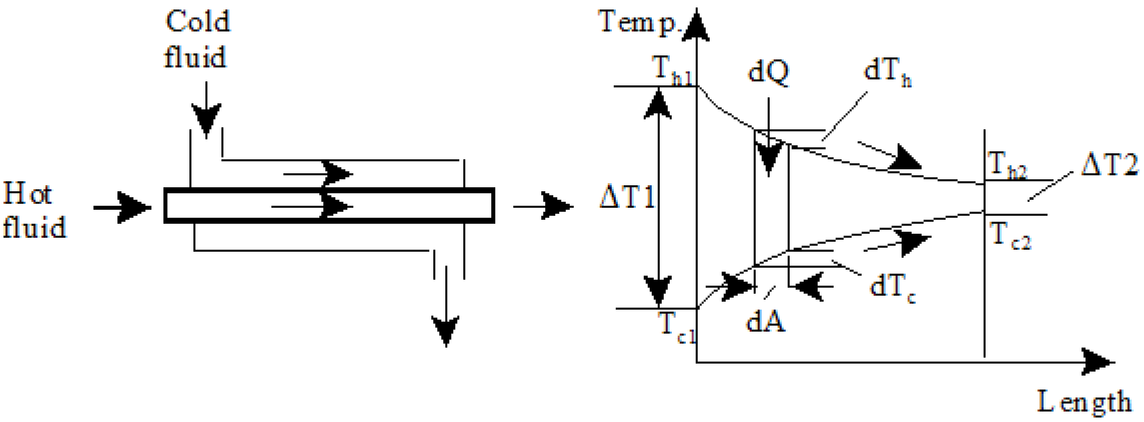


Fig.12.5: Parallel flow heat exchanger

$$\ln\left(\frac{T_{h2}-T_{c2}}{T_{h1}-T_{c1}}\right) = -U\cdot A\cdot\left(\frac{1}{m_h\cdot C_{ph}}+\frac{1}{m_c\cdot C_{pc}}\right) \qquad ....(12.21)$$

$$LMTD = \frac{(T_{h2}-T_{c2})-(T_{h1}-T_{c1})}{\ln\left(\frac{T_{h2}-T_{c2}}{T_{h1}-T_{c1}}\right)} \qquad ....(12.24)$$

$$LMTD = \frac{\Delta T2-\Delta T1}{\ln\left(\frac{\Delta T2}{\Delta T1}\right)} = \frac{\Delta T1-\Delta T2}{\ln\left(\frac{\Delta T1}{\Delta T2}\right)} \qquad ....(12.25)$$

Counter-flow heat exchanger:

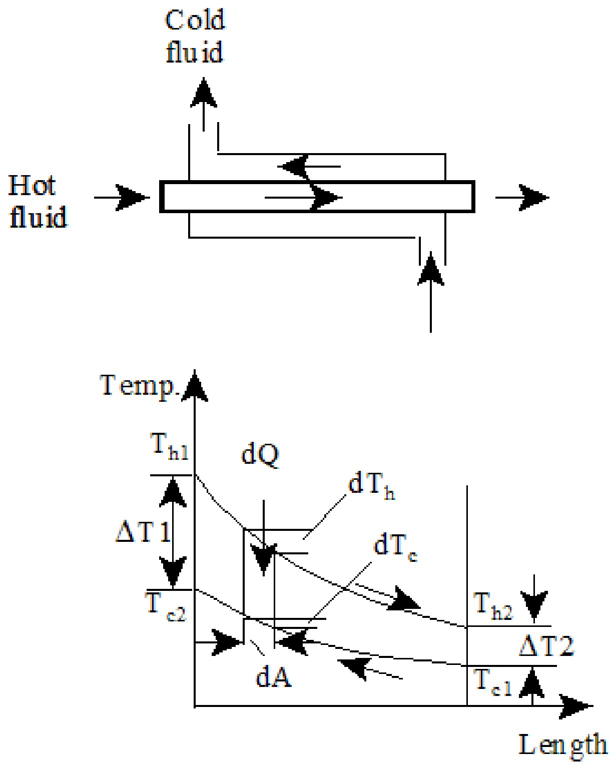
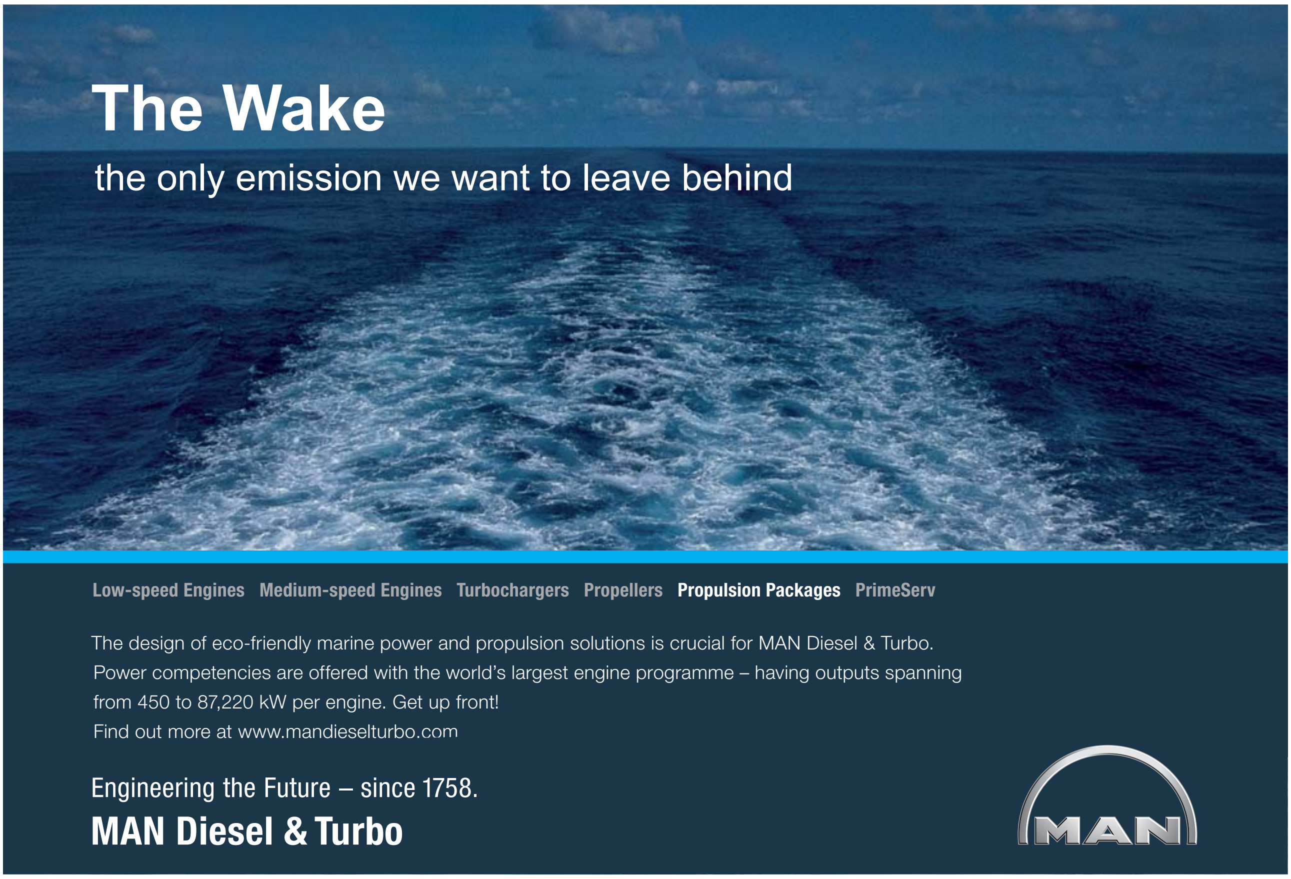


Fig.12.6: Counterflow heat exchanger



$$\ln\left(\frac{T_{h2}-T_{c1}}{T_{h1}-T_{c2}}\right)=-U\cdot A\cdot\left(\frac{1}{m_h\cdot C_{ph}}-\frac{1}{m_c\cdot C_{pc}}\right)\qquad\text{....(12.31)}$$

$$LMTD=\frac{\left(T_{h2}-T_{c1}\right)-\left(T_{h1}-T_{c2}\right)}{\ln\left(\frac{T_{h2}-T_{c1}}{T_{h1}-T_{c2}}\right)}\qquad\text{....(12.34)}$$

$$LMTD=\frac{\Delta T2-\Delta T1}{\ln\left(\frac{\Delta T2}{\Delta T1}\right)}=\frac{\Delta T1-\Delta T2}{\ln\left(\frac{\Delta T1}{\Delta T2}\right)}\qquad\text{....(12.35)}$$

Note that the LMTD expressions for the parallel flow and the counter-flow heat exchangers (i.e. eqns. (12.25) and (12.35)) are the same.

For Condensers and Evaporators:

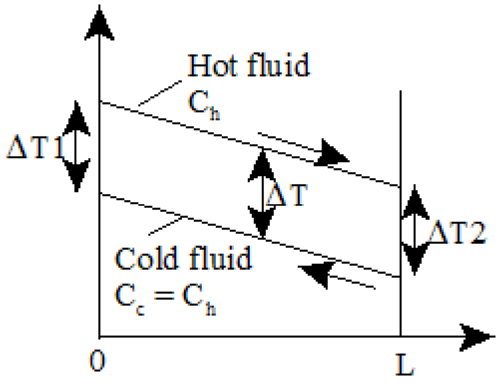


Fig.12.8(a) Both fluids have same capacity rates

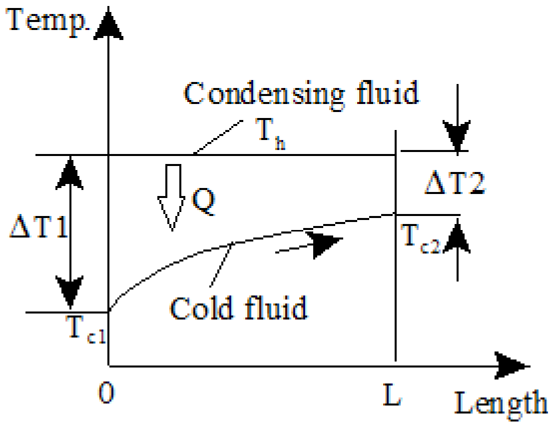


Fig.12.8(b) One of the fluids condensing (C_h => ∞)

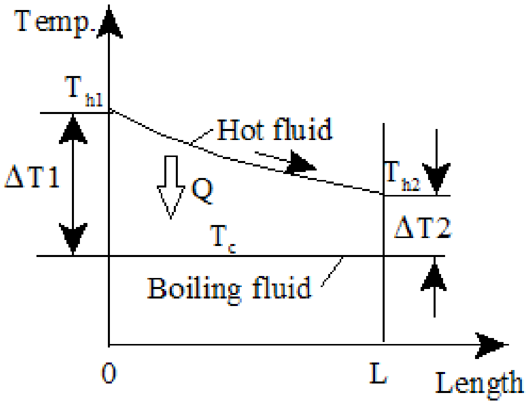


Fig.12.8(c) One of the fluids boiling ($C_c \Rightarrow \infty$)

Correction factors for multi-pass and cross-flow heat exchangers:

first, LMTD is calculated as if for a counter-flow heat exchanger with the inlet and exit temperatures for the two fluids as per the actual data, and next, a 'correction factor (F)' is applied to the calculated LMTD to get the mean temperature difference between the fluids. Now, heat transfer rate is calculated as:

$$Q = U \cdot A \cdot (F \cdot LMTD) \quad W \ldots (12.39)$$

where, A is the area of heat transfer, U is the overall heat transfer coefficient referred to that area, and F is the correction factor.

Values of correction factor (F) for a few selected heat exchangers are given in graphical representation in Fig. 12.9. F varies from 0 to 1. In these graphs, correction factor F is plotted as function of two parameters, viz. P and R, defined as:

$$P = \frac{t_2 - t_1}{T_1 - t_1} \quad \ldots .(12.40)$$

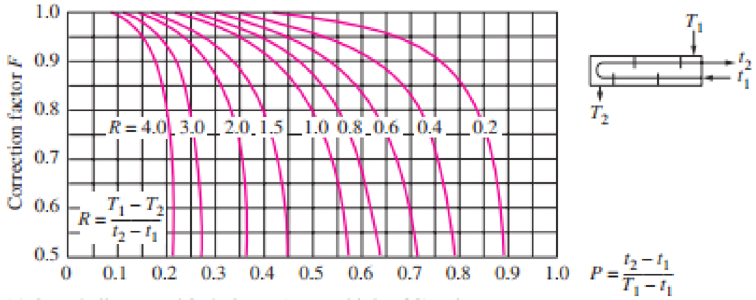
$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{C_{tube\_side}}{C_{shell\_side}} \quad \ldots .(12.41)$$

where C is the capacity rate = $m.C_p$.

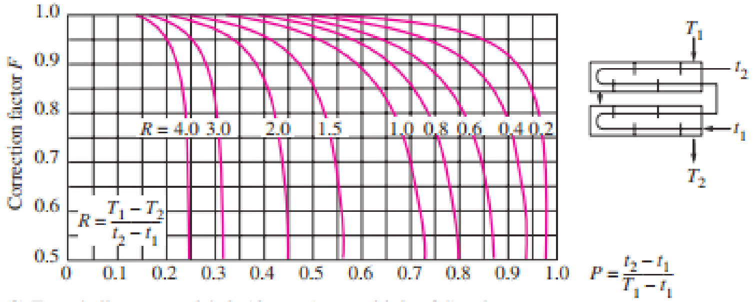
Also, for a Shell-and-tube heat exchanger, T and t represent the temperatures of fluids flowing through the Shell and tube sides, respectively. And, subscripts 1 and 2 refer to the inlet and exit, respectively.

For a condenser or boiler, F = 1.

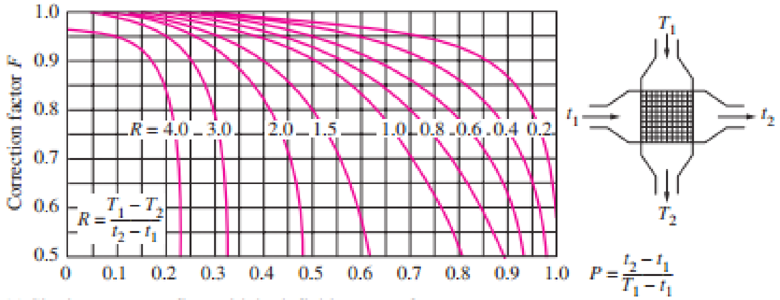
Following graphs for F are from Cengel (Ref. 2):



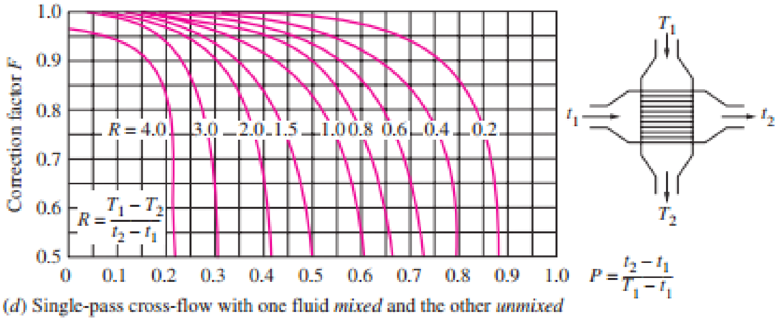
(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes



(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes



(*c*) Single-pass cross-flow with both fluids *unmixed*



(*d*) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

Note: To apply the correction factor F from these graphs, it is necessary that the end temperatures of both the fluids must be known.

The Effectiveness – NTU method for heat exchanger analysis:

Effectiveness of a heat exchanger (ε):

$$\varepsilon = \frac{Q}{Q_{max}} \qquad \ldots(12.42)$$

where Q = actual heat transferred in the heat exhanger

Q_{max} = max. possible heat transfer in the heat exchanger

Capacity Ratio (C):

$$C = \frac{C_{min}}{C_{max}} \qquad \ldots(12.43)$$

Number of Transfer Units (NTU):

$$NTU = \frac{U \cdot A}{C_{min}} \qquad \ldots(12.44)$$

where U is the overall heat transfer coeff. and A is the corresponding heat transfer area. For given value of A and flow conditions, NTU is a measure of the area (i.e. size) of the heat exchanger. Larger the NTU, larger the size of the heat exchanger.

Maximum possible heat transfer in a heat exchanger (Q_{max}):

If hot fluid has the minimum capacity rate, we write:

$$Q_{max} = C_h \cdot (T_{h1} - T_{c1})$$ ...if C_h is min. capacity rate, C_{min}

Instead, if cold fluid has the minimum capacity rate, we write:

$$Q_{max} = C_c \cdot (T_{h1} - T_{c1})$$ ...if C_c is min. capacity rate, C_{min}

Or, more generally, we write:

$$Q_{max} = C_{min} \cdot (T_{h1} - T_{c1})$$ ...(12.45)

Therefore, we can write for effectiveness:

$$\varepsilon = \frac{Q}{Q_{max}} = \frac{C_h \cdot (T_{h1} - T_{h2})}{C_{min} \cdot (T_{h1} - T_{c1})} = \frac{C_c \cdot (T_{c2} - T_{c1})}{C_{min} \cdot (T_{h1} - T_{c1})}$$ .....(12.46)

Now, if hot fluid is the 'minimum fluid' (i.e. $C_h < C_c$), we get from eqn. (12.46):

$$\varepsilon = \frac{(T_{c2} - T_{c1})}{(T_{h1} - T_{c1})}$$ ..for $C_c < C_h$.....(12.47, b)

And, if cold fluid is the 'minimum fluid' (i.e. C_c < Ch), we get from eqn. (12.46):

$$\varepsilon = \frac{(T_{c2} - T_{c1})}{(T_{h1} - T_{c1})}$$ ..for $C_c < C_h$.....(12.47, b)

Now, for any heat exchanger, effectiveness can be expressed as a function of the NTU and capacity ratio, C_{min}/C_{max}. i.e.

$$\varepsilon = f\left(NTU, \frac{C_{mn}}{C_{max}}\right)$$ ....(12.47,c)

Table 12.5 gives the Effectiveness relations for a few types of heat exchangers; and Table 12.6 gives the NTU relations:

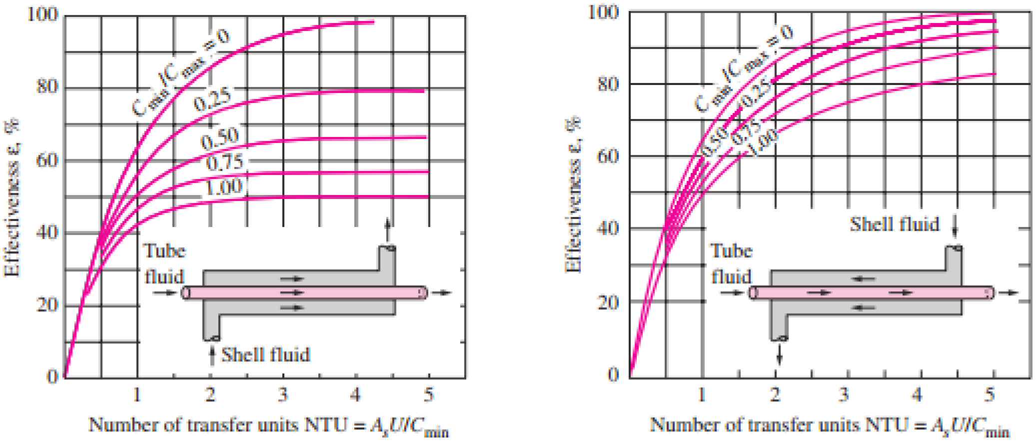
Flow geometry	**Relation**
Double pipe: parallel flow	$\varepsilon = \frac{1 - \exp(-N\cdot(1+C))}{1+C}$
Double pipe: counter flow	$\varepsilon = \frac{1 - \exp(-N\cdot(1-C))}{(1 - C\cdot\exp(-N\cdot(1-C)))}$
Counter flow, C = 1	$\varepsilon = \frac{N}{1+N}$
Cross flow: both fluids unmixed	$\varepsilon = 1 - \exp\left(\frac{\exp(-N\cdot C\cdot n) - 1}{C\cdot n}\right)$ where $n = N^{-0.22}$
Cross flow: both fluids mixed	$\varepsilon = \left(\frac{1}{1-\exp(-N)} + \frac{C}{1-\exp(-N\cdot C)} - \frac{1}{N}\right)^{-1}$
Cross flow: C_{max} mixed, C_{min} unmixed	$\varepsilon = \frac{1}{C}\cdot\left[1 - \exp\left[-C\cdot\left(1 - e^{-N}\right)\right]\right]$
Cross flow: C_{max} unmixed, C_{min} mixed	$\varepsilon = 1 - \exp\left[\frac{-1}{C}\cdot(1 - \exp(-N\cdot C))\right]$
Shell and Tube:	
One shell pass, 2,4,6 tube passes	$\varepsilon = 2\cdot\left[1 + C + \left(1+C^2\right)^{\frac{1}{2}}\cdot\frac{1+\exp\left[-N\cdot\left(1+C^2\right)^{\frac{1}{2}}\right]}{1-\exp\left[-N\cdot\left(1+C^2\right)^{\frac{1}{2}}\right]}\right]^{-1}$
Multiple shell passes, 2n, 4n, 6n tube passes (εp = effectiveness of each shell pass, n = no. of shell passes)	$\varepsilon = \frac{\left[\frac{\left(1-\varepsilon_p\cdot C\right)}{\left(1-\varepsilon_p\right)}\right]^n - 1}{\left[\frac{\left(1-\varepsilon_p\cdot C\right)}{\left(1-\varepsilon_p\right)}\right]^n - C}$
Special case for C= 1	$\varepsilon = \frac{n\cdot\varepsilon_p}{1 + (n-1)\cdot\varepsilon_p}$
All exchangers, with C = 0 (Condensers and Evaporators)	$\varepsilon = 1 - e^{-N}$

Table 12.5 Effectiveness relations for heat exchangers
[$N = NTU = U.A/C_{min}$, $C = C_{min}/C_{max}$]

Flow geometry	**Relation**
Double pipe: parallel flow	$N=\frac{-\ln(1-(1+C)\cdot\varepsilon)}{1+C}$
Double pipe: counter flow	$N=\frac{1}{C-1}\cdot\ln\left(\frac{\varepsilon-1}{C\cdot\varepsilon-1}\right)$
Counter flow, C = 1	$N=\frac{\varepsilon}{1-\varepsilon}$
Cross flow: C_{max} mixed, C_{min} unmixed	$N=-\ln\left(1+\frac{1}{C}\cdot\ln(1-C\cdot\varepsilon)\right)$
Cross flow: C_{max} unmixed, C_{min} mixed	$N=\frac{-1}{C}\cdot\ln(1+C\cdot\ln(1-\varepsilon))$
Shell and Tube: One shell pass, 2,4,6 tube passes	$N=-\left(1+C^2\right)^{\frac{-1}{2}}\cdot\ln\left[\frac{\frac{2}{\varepsilon}-1-C-\left(1+C^2\right)^{\frac{1}{2}}}{\frac{2}{\varepsilon}-1-C+\left(1+C^2\right)^{\frac{1}{2}}}\right]$
All exchangers, with C = 0 (Condensers and Evaporators)	$N=-\ln(1-\varepsilon)$

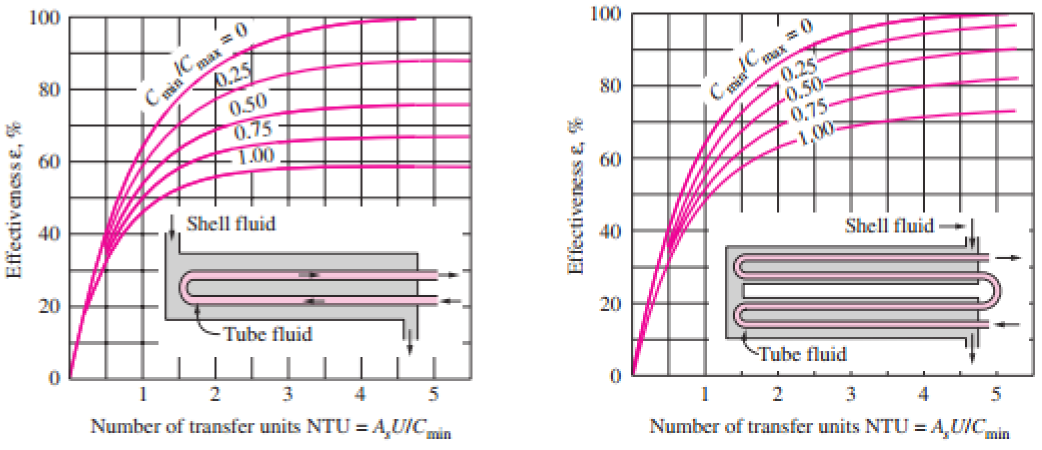
Table 12.6 NTU relations for heat exchangers
[N = NTU = U.A/C_{min}, C = C_{min}/C_{max} , ε = effectiveness]

NTU-Effectiveness graphs: (from Ref. 2)



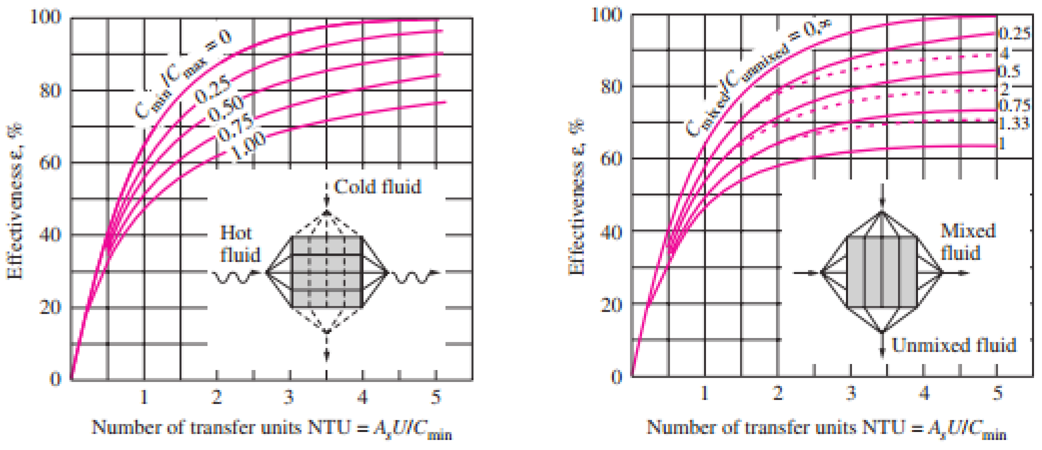
(*a*) Parallel-flow

(*b*) Counter-flow



(*c*) One-shell pass and 2, 4, 6, … tube passes

(*d*) Two-shell passes and 4, 8, 12, … tube passes



(*e*) Cross-flow with both fluids unmixed

(*f*) Cross-flow with one fluid mixed and the other unmixed

4A. Problems on Overall heat transfer coeff., Fouling factors etc.

Prob.4A.1. Water at a mean temperature of T_m = 107 C and a mean velocity of u_m = 3.5 m/s flows inside a 1.0 cm ID, 1.4 cm OD, 5 m long Stainless Steel (k = 14.2 W/m.C) tube. Outer surface of the tube where boiling occurs has a heat transfer coeff of 8400 W/m^2.C. Calculate the overall heat transfer coeff. based on inner surface of the tube.

(b) If there is a Fouling factor of 0.0005 m^2.C/W on the inner surface, what will be the value of U_i?

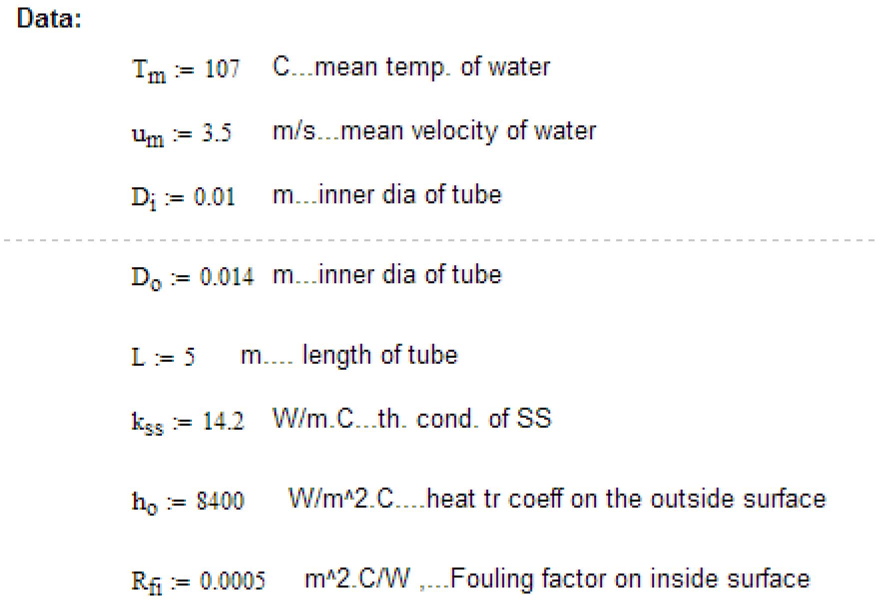
(c) Plot U_i for Fouling factors varying from 0.0001 to 0.0008 m^2.C/W ….. (Ref.2)

Mathcad Solution:

Note that while solving this problem, we will need the properties of Sat. Water.

But, we have already written Mathcad Functions for these properties.

So, we work out this problem using those Functions.



Data:

$T_m := 107$ C...mean temp. of water

$u_m := 3.5$ m/s...mean velocity of water

$D_i := 0.01$ m...inner dia of tube

$D_o := 0.014$ m...inner dia of tube

$L := 5$ m.... length of tube

$k_{ss} := 14.2$ W/m.C...th. cond. of SS

$h_o := 8400$ W/m^2.C....heat tr coeff on the outside surface

$R_{fi} := 0.0005$ m^2.C/W ,...Fouling factor on inside surface

Properties of water at mean temp. of 107 C:

Use the Mathcad Functions written earlier:

$$\rho := \frac{1}{vf\_H2O(TempK, H2Ov\_f, T_m + 273.15)}$$ kg/m³....density

i.e. $\rho = 953.166$ kg/m³....density

$$k := k\_f\_H2O(TempK, H2Ok\_f, T_m + 273.15)$$ W/(m.C)....thermal cond.

i.e. $k = 0.683$ W/(m.C)....thermal cond.

$$\mu := mu\_f\_H2O(TempK, H2Omu\_f, T_m + 273.15)$$ kg/(m.s)....dynamic viscosity

i.e. $\mu = 2.597 \times 10^{-4}$ kg/(m.s)....dynamic viscosity

$$Pr := Pr\_f\_H2O(TempK, H2OPr\_f, T_m + 273.15)$$ ..Prandl number

i.e. $Pr = 1.608$ ..Prandl number

Surface areas:

$A_i := \pi \cdot D_i \cdot L$ i.e. $A_i = 0.157$ m^2 ... inside surface area

$A_o := \pi \cdot D_o \cdot L$ i.e. $A_o = 0.22$ m^2 ... outside surface area

We need to calculate the heat transfer coefficients for the inner and outer surfaces:

For the water side (i.e. inner surface):

We have: $$Re := \frac{D_i \cdot u_m \cdot \rho}{\mu}$$ ...Reynolds number

i.e. $Re = 1.285 \times 10^5$ > 4000....Therefore, turbulent

Using Dittus-Boelter eqn. to determine heat transfer coeff. for inside surface:

$$Nu := 0.023 \cdot Re^{0.8} \cdot Pr^{0.3}$$

i.e. $Nu = 324.081$ ....Nusselts number

Therefore, $$h_i := Nu \cdot \frac{k}{D_i}$$

i.e. $h_i = 2.214 \times 10^4$ W/(m².C)....inside surface heat transfer coeff.

For the outer surface:

$h_o = 8.4 \times 10^3$ ....by data

Thermal Resistances:

$R_{conv1} := \frac{1}{h_i \cdot A_i}$ i.e. $R_{conv1} = 2.876 \times 10^{-4}$ C/W....conv. resist on inside

$R_{conv2} := \frac{1}{h_o \cdot A_o}$ i.e. $R_{conv2} = 5.413 \times 10^{-4}$ C/W....conv. resist on outside

$R_{cond} := \frac{\ln\left(\frac{D_o}{D_i}\right)}{2 \cdot \pi \cdot k_{ss} \cdot L}$ i.e. $R_{cond} = 7.542 \times 10^{-4}$ C/W....cond. resist of tube wall

$R_{total} := R_{conv1} + R_{cond} + R_{conv2}$ i.e. $R_{total} = 1.583 \times 10^{-3}$ C/W....Total thermal resist

And, Overall heat transfer coeff., U_i, based on inside surface:

We have: $U_i \cdot A_i = U_o \cdot A_o = \frac{1}{R_{total}}$

Therefore:

$$U_i := \frac{1}{A_i \cdot R_{total}}$$

i.e. $U_i = 4.021 \times 10^3$ **W/(m²**.**C)....overall heat transfer coeff....Ans.**

(b) When the Fouling factor on the inside surface is considered:

Now, first find out Total thermal resistance:

$R_{ci} := \frac{R_{fi}}{A_i}$ i.e. $R_{ci} = 3.183 \times 10^{-3}$ C/W....Fouling resist on the inside

Therefore total resistance:

$$R_{total} := R_{conv1} + R_{ci} + R_{cond} + R_{conv2}$$

i.e. $R_{total} = 4.766 \times 10^{-3}$ C/W....Total thermal resist

And, Overall heat transfer coeff., U_i, based on inside surface:

We have, when fouling resist on the inside surface is considered:

$$U_i := \frac{1}{A_i \cdot R_{total}}$$

i.e. $U_i = 1.336 \times 10^3$ W/(m².C)....overall heat transfer coeff....Ans.

To plot U_i against Fouling factor:

We write relevant quantities as functions of Fouling factor for convenience of plotting:

$R_{ci}(R_{fi}) := \frac{R_{fi}}{A_i}$ ....Fouling resist R_ci as a function of Fouling factor R_fi

Therefore total resistance:

$R_{total}(R_{fi}) := R_{conv1} + R_{ci}(R_{fi}) + R_{cond} + R_{conv2}$ ..Total resist as a function of Fouling factor

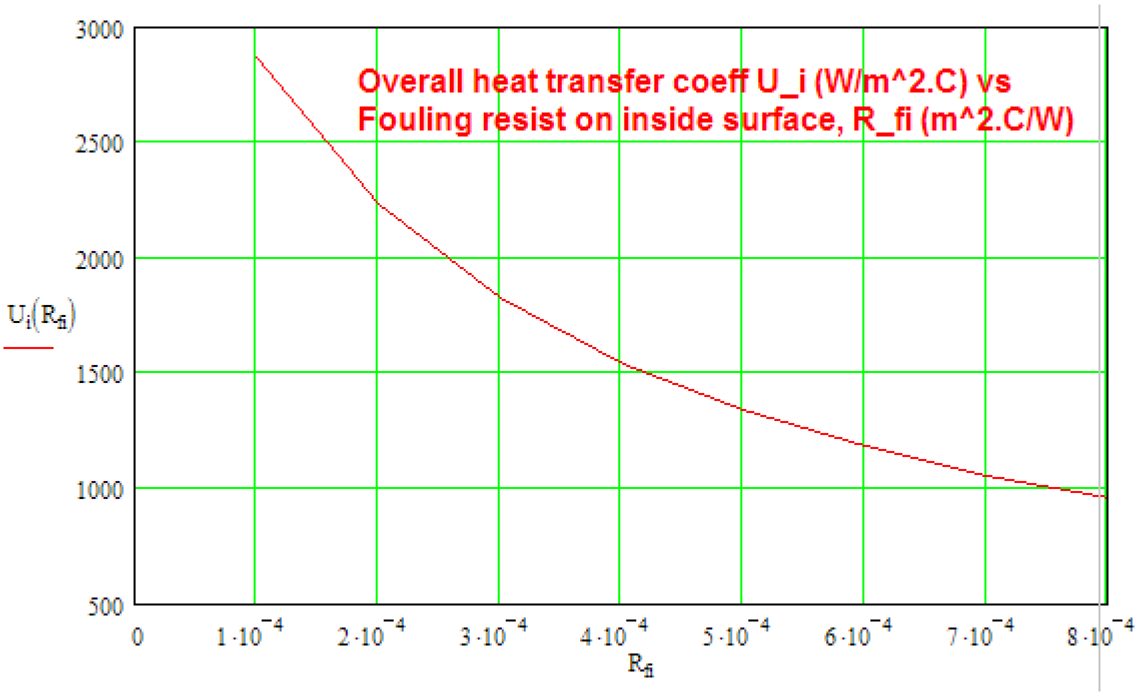
We have, when fouling resist on the inside surface is considered:

$$U_i(R_{fi}) := \frac{1}{A_i \cdot R_{total}(R_{fi})}$$ ..U_i as a function of Fouling factor

Now, plot the graph:

$R_{fi} := 0.0001, 0.0002 .. 0.0008$ ...define R_fi as a range variable

$R_{fi} =$	$U_i(R_{fi}) =$
$1 \cdot 10^{-4}$	$2.868 \cdot 10^{3}$
$2 \cdot 10^{-4}$	$2.229 \cdot 10^{3}$
$3 \cdot 10^{-4}$	$1.823 \cdot 10^{3}$
$4 \cdot 10^{-4}$	$1.542 \cdot 10^{3}$
$5 \cdot 10^{-4}$	$1.336 \cdot 10^{3}$
$6 \cdot 10^{-4}$	$1.178 \cdot 10^{3}$
$7 \cdot 10^{-4}$	$1.054 \cdot 10^{3}$
$8 \cdot 10^{-4}$	953.575



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Prob.4A.2. A steel tube (k = 50 W/m.K) of ID = 20 mm, OD = 26 mm is used to transfer heat from hot gases flowing over the tube (h_o = 200 W/m^2.K) to cold water flowing through the tube (h_i = 8000 W/m^2.K). What is the cold side overall heat transfer coeff. U_i?

(b) To enhance heat transfer, 16 straight fins of rectangular profile are installed longitudinally along the outer surface of the tube. The fins are equally spaced around the tube, fin thickness being 2 mm and length = 15 mm. What is the corresponding overall heat transfer coeff. U_i?

Mathcad Solution:

Data:

$D_i := 0.020$ m...inner dia of tube

$D_o := 0.026$ m...outer dia of tube

$h_i := 8000$ W/(m^2.C)....inside surface heat transfer coeff.

$h_o := 200$ W/(m^2.C)....outside surface heat transfer coeff.

$L := 0.015$ m....height of fins

$W := 1$ m...width of fins..i.e. along the length of cylinder....assumed

$t := 0.002$ m...thickness of fins

$N := 16$ ....no. of fins

$k := 50$ W/(m.K)...thermal cond. of fin material

$A_i := \pi \cdot D_i \cdot 1$ m^2/metre...inside surface area

i.e. $A_i = 6.2832 \times 10^{-2}$ m^2/metre.

$A_o := \pi \cdot D_o \cdot 1$ m^2/metre...inside surface area

i.e. $A_o = 8.1681 \times 10^{-2}$ m^2/metre.

a) **Overall heat transfer coeff. U_i, referred to the inside surface, when there are no fins:**

Considering the thermal resistance of tube wall, we write:

$$U_i \cdot A_i = \frac{1}{R_{total}} = \frac{1}{\frac{1}{h_i \cdot A_i} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{2 \cdot \pi \cdot k \cdot W} + \frac{1}{h_o \cdot A_o}}$$

First term in the denominator in RHS is the thermal resistance due to film coeff. on the inside, the second term is the thermal resistance of the tube material, and the third term is the thermal resistance due to film coeff. on the outside.

Then, we get:

$$R_{total} := \frac{1}{h_i \cdot A_i} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{2 \cdot \pi \cdot k \cdot W} + \frac{1}{h_o \cdot A_o}$$

i.e. $R_{total} = 6.4038 \times 10^{-2}$ C/W...Total thermal resist.

And, $U_i := \frac{1}{A_i \cdot R_{total}}$

i.e. $U_i = 2.4853 \times 10^{2}$ **W/m^2.K ..... Ans.**

b) **When there are fins on the outside surface:**

Now, we have:

$$U_i \cdot A_i = \frac{1}{R_{total}} = \frac{1}{\dfrac{1}{h_i \cdot A_i} + \dfrac{\ln\left(\dfrac{D_o}{D_i}\right)}{2 \cdot \pi \cdot k \cdot W} + \dfrac{1}{h_o \cdot \left(A_{unfinned} + \eta_{fin} \cdot A_{fins}\right)}}$$

First term in the denominator in RHS is the thermal resistance due to film coeff. on the inside, the second term is the thermal resistance of the tube material, and the third term is the thermal resistance due to film coeff. on the outside.

Unfinned surface (or the base surface) on the outside is at the wall temp. and is fully effective for heat transfer whereas the finned surface is not fully effective because of temp. drop along the length of fins; therefore, effective area of fins is obtained by multiplying the total area of fins by the fin effectiveness, η_{fin}.

Therefore, we need to find out the fin efficiency.

Fin efficiency:

For a rectangular fin with adiabatic tip, the fin efficiency is given by:

$$\eta_{fin} = \frac{\tanh(m \cdot L)}{m \cdot L}$$

where $m = \sqrt{\dfrac{h_o \cdot P}{k \cdot A_c}}$ 1/m....fin parameter

$P = 2 \cdot (W + t)$ ...perimeter, W = width of fin = 1 m

$A_c = W \cdot t$ ...area of cross-section of fin

Then, $\dfrac{P}{A_c} = \dfrac{2 \cdot (W + t)}{W \cdot t} = \dfrac{2}{t}$ ...for t << W

Therefore, $m := \sqrt{\dfrac{2 \cdot h_o}{k \cdot t}}$

i.e. $m = 6.3246 \times 10^{1}$ 1/m......Fin parameter

and, $m \cdot L = 9.4868 \times 10^{-1}$

Then:

$$\eta_{fin} := \frac{\tanh(m \cdot L)}{m \cdot L}$$

i.e. $\eta_{fin} = 7.7917 \times 10^{-1}$ ....fin efficiency

Areas:

$$A_{unfinned} := \pi \cdot D_o - N \cdot t$$

i.e. $A_{unfinned} = 4.9681 \times 10^{-2}$ m2....unfinned or prime (base) area

$A_{fins} := N \cdot (2 \cdot W \cdot L)$ m2....finned area of N fins (both upper and lower side of fins considered)

i.e. $A_{fins} = 4.8 \times 10^{-1}$ m2

Therefore, Overall heat transfer coeff. U_i, referred to the inside surface:

We have:

$$U_i \cdot A_i = \frac{1}{R_{total}} = \frac{1}{\frac{1}{h_i \cdot A_i} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{2 \cdot \pi \cdot k \cdot W} + \frac{1}{h_o \cdot \left(A_{unfinned} + \eta_{fin} \cdot A_{fins}\right)}}$$

Now:

$$\frac{1}{\frac{1}{h_i \cdot A_i} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{2 \cdot \pi \cdot k \cdot W} + \frac{1}{h_o \cdot \left(A_{unfinned} + \eta_{fin} \cdot A_{fins}\right)}} = 6.8372 \times 10^{1}$$

i.e. $U_i \cdot A_i = 68.372$

and, $U_i := \frac{68.372}{A_i}$

i.e. $U_i = 1.0882 \times 10^{3}$ **W/(m2.C)....overall heat transfer coeff. referred to inside area...Ans.**

Note: compare this to the earlier U value of 248.53 W/(m^2.C).

i.e. **There is great improvement in value of U by providing fins.**

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"**Prob. 4A.3:** A shell and tube counter-flow heat exchanger uses copper tubes (k = 380 W/(m.C)), 20 mm ID and 23 mm OD. Inside and outside film coefficients are 5000 and 1500 W/(m2.C) respectively. Fouling factors on the inside and outside may be taken as 0.0004 and 0.001 m^2.C/W respectively. Calculate the overall heat transfer coefficient based on: (i) outside surface, and (ii) inside surface."

EES Solution:

"Data:"

D_i = 0.02 [m]

D_o = 0.023 [m]

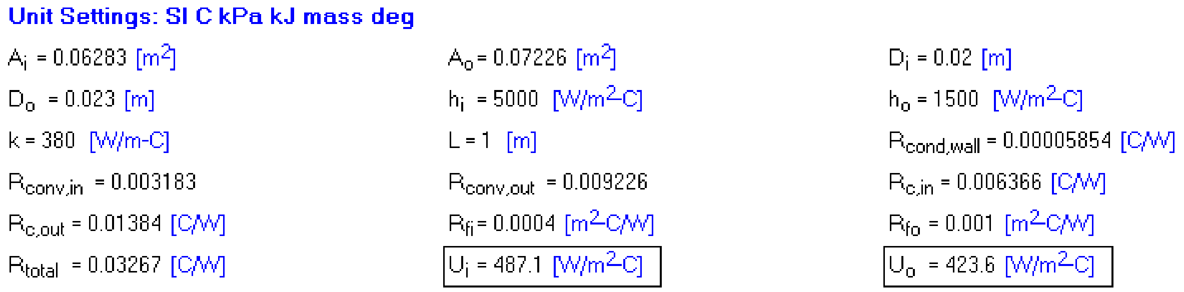
L = 1 [m]

k = 380 [W/m_C]

h_i = 5000 [W/m^2-C] "…heat tr coeff on the inside"

```
h_o = 1500 [W/m^2-C] "…heat tr coeff on the outside"

R_fi = 0.0004 [m^2-C/W] "…Fouling factor on the inside"

R_fo = 0.001 [m^2-C/W] "…Fouling factor on the outside"

"Calculations:"

"Areas:"

A_o = pi * D_o * L "[m^2]"

A_i = pi * D_i * L "[m^2]"

"We have:

U_i * A_i = U_o * A_o = 1 / R_total

where R_total = total thermal resistance,

And,

R_toal = R_conv_in + R_c_in + R_cond_wall + R_conv_out + R_c_out, where

R_conv_in = conv. resist. on the inside surface

R_c_in = Fouling resistance on inside

R_cond_wall = cond. resist of the tube wall

R_conv_out = conv. resist. on outside, and

R_c_out = Fouling resist on the outside"

R_conv_in = 1/(h_i * A_i) "[C/W]"

R_c_in = R_fi / A_i "[C/W]"

R_cond_wall = ln(D_o / D_i) / (2 * pi * k * L) "[C/W]"

R_conv_out = 1 / (h_o * A_o) "[C/W]"

R_c_out = R_fo / A_o "[C/W]"

R_total = R_conv_in + R_c_in + R_cond_wall + R_conv_out + R_c_out "[C/W]"

U_i * A_i = 1 / R_total "…determine U _i"

U_o * A_o = 1 / R_total "…determine U _o"
```

Results:



Unit Settings: SI C kPa kJ mass deg

A_i = 0.06283 [m^2]

A_o = 0.07226 [m^2]

D_i = 0.02 [m]

D_o = 0.023 [m]

h_i = 5000 [W/m^2-C]

h_o = 1500 [W/m^2-C]

k = 380 [W/m-C]

L = 1 [m]

$R_{cond,wall}$ = 0.00005854 [C/W]

$R_{conv,in}$ = 0.003183

$R_{conv,out}$ = 0.009226

$R_{c,in}$ = 0.006366 [C/W]

$R_{c,out}$ = 0.01384 [C/W]

R_{fi} = 0.0004 [m^2-C/W]

R_{fo} = 0.001 [m^2-C/W]

R_{total} = 0.03267 [C/W]

U_i = 487.1 [W/m^2-C]

U_o = 423.6 [W/m^2-C]

Thus:

U_i = 487.1 W/m^2.C …..Ans.

U_o = 423.6 W/m^2.C …. Ans.

Plot the variation of total thermal resistance, R_total as k varies from 10 to 400 W/m.C:

First, construct the parametric table:

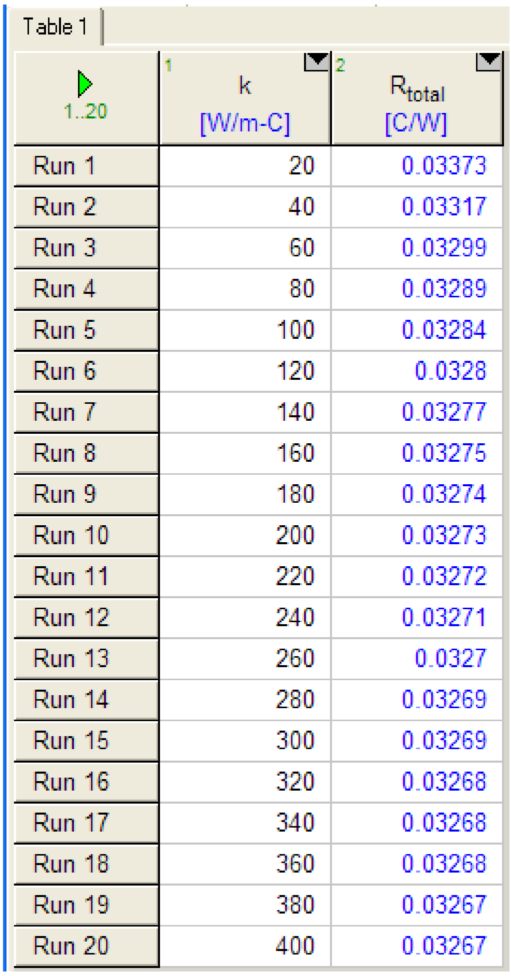
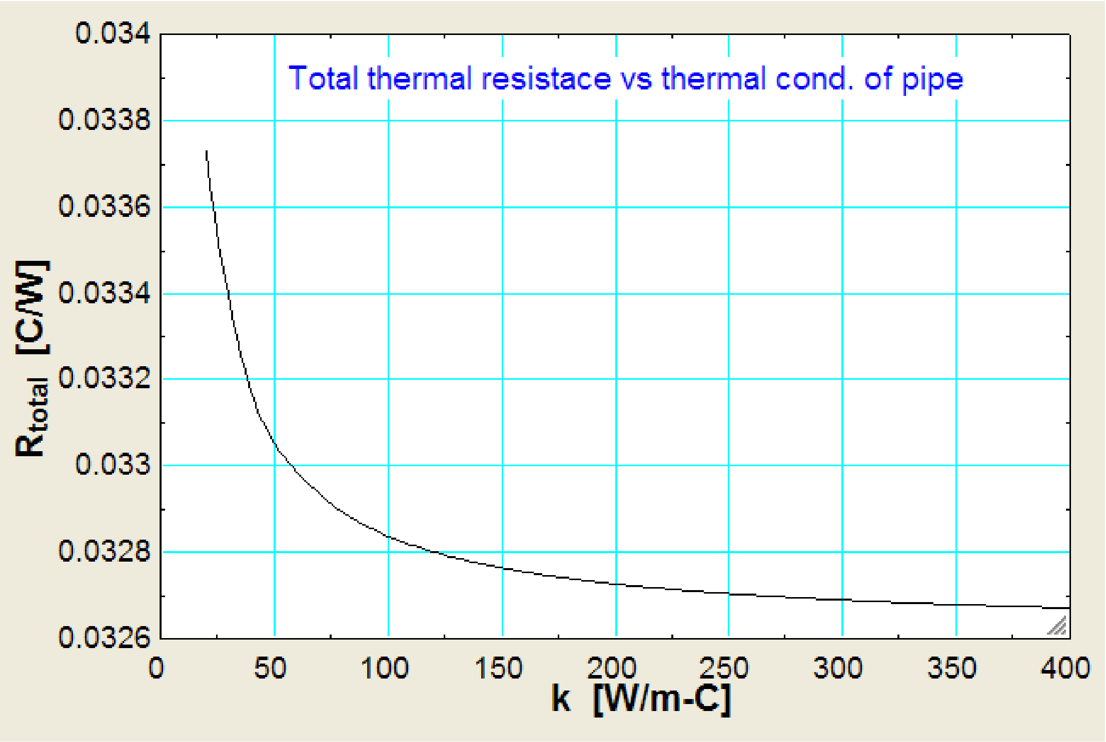


Table 1

1..20	k [W/m-C]	R_{total} [C/W]
Run 1	20	0.03373
Run 2	40	0.03317
Run 3	60	0.03299
Run 4	80	0.03289
Run 5	100	0.03284
Run 6	120	0.0328
Run 7	140	0.03277
Run 8	160	0.03275
Run 9	180	0.03274
Run 10	200	0.03273
Run 11	220	0.03272
Run 12	240	0.03271
Run 13	260	0.0327
Run 14	280	0.03269
Run 15	300	0.03269
Run 16	320	0.03268
Run 17	340	0.03268
Run 18	360	0.03268
Run 19	380	0.03267
Run 20	400	0.03267

Now, plot the result:



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"**Prob. 4A.4:** Consider a type 302 SS tube (k = 15.10 W/(m.C)), 22 mm ID and 27 mm OD, inside which water flows at a mean temp T_m = 75 C and velocity u_m = 0.5 m/s. Air at 15 C and at a velocity of V_o = 20 m/s flows across this tube. Fouling factors on the inside and outside may be taken as 0.0004 and 0.0002 m^2.C/W respectively. Determine the overall heat transfer coefficient based on the outside surface, U_o

(b) Plot U_o as a function of cross flow velocity, V_o in the range: 5 < V_o < 30 m/s (Ref. 3)."

EES Solution:

We note that h_i and h_o have to be found out.

Water flows inside the tube; find out the Reynolds No. and apply Dittus-Boelter eqn to get Nusselts No. (and, h_i therefrom) for turbulent flow, i.e. if Re > 4000.

Air flows across the cylinder. Apply Churchill_Bernstein eqn to get Nusselts No. and h_o therefrom.

However, we have to get properties of Air at film temp T_f = (T_s + T_m_air) / 2. But, T_s is not known. We calculate T_s by trial and error applying the concept of:

Heat Current = Temp Potential / Thermal Resistance, is the same through the circuit in steady state. It is very easy in EES, as will be seen below.

First, write a PROCEDURE in EES for calculations for cross flow of Air or any other fluid across a cylinder using Churchill – Bernstein eqn:

```
$UnitSystem SI Pa C J

PROCEDURE ForcedConv_AcrossCylinder (Fluid$,P_infinity, T_infinity, U_infinity, L, D, T_s: Re_D, Nusselt_D_bar, h_bar, Q)

"Ref: Incropera, 5th Ed. pp. 411, Eqn. (7.57)"

"Churchill and Bernstein eqn….for entire range of Re_D and a wide range of Pr"

"Finds various quantities for flow of Air or any Fluid across a cylinder:"

"Inputs: Pa, C, m/s, m"

"Outputs: W/m^2.C, W, W"

T_f := (T_infinity + T_s)/2 " mean film temp, C"
```

```
“Properties of Air (Ideal gas) or other Fluid at T_f :”

IF Fluid$ = ‘Air’ Then

rho:=Density(Fluid$,T=T_f,P=P_infinity)
mu:=Viscosity(Fluid$,T=T_f)
k:=Conductivity(Fluid$,T=T_f)
Pr:=Prandtl(Fluid$,T=T_f)
cp:=SpecHeat(Fluid$,T=T_f)

ELSE

rho:=Density(Fluid$,T=T_f,P=P_infinity)
mu:=Viscosity(Fluid$,T=T_f,P=P_infinity)
k:=Conductivity(Fluid$,T=T_f,P=P_infinity)
Pr:=Prandtl(Fluid$,T=T_f,P=P_infinity)
cp:=SpecHeat(Fluid$,T=T_f,P=P_infinity)

ENDIF

Re_D := D * U_infinity * rho/mu “Finds Reynolds No.”

 “To find h accurately: Use Churchill and Bernstein eqn.”

Nusselt_D_bar := 0.3 + ((0.62 * Re_D^0.5 * (Pr)^(1/3))/(1 + (0.4/Pr)^(2/3))^(1/4)) * (1 + (Re_D/282000)^(5/8))^(4/5)
h_bar :=Nusselt_D_bar * k / D “Finds h_bar”
Q := h_bar * (pi * D * L) * (T_s – T_infinity) “W…. heat tr”

END

“======================================================================”
```

Now, solve the above problem:

```
“EES Solution:”

“Data:”

D_i = 0.022 [m]

D_o = 0.027 [m]

L = 1 [m]
```

k_ss = 15.1 [W/m-C]

T_m_water = 75 [C]

U_m_water = 0.5 [m/s]

P_1 = 1.01325E05 [Pa]

T_m_air = 15 [C]

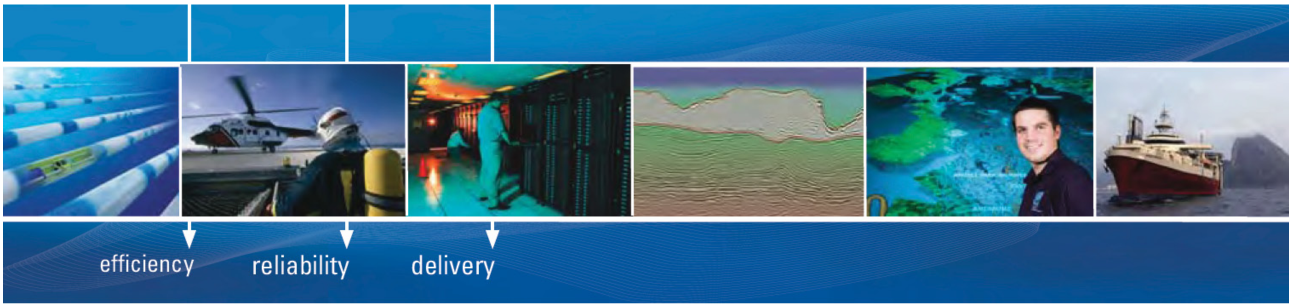
V_o_air = 20 [m/s]}

"Properties of Water at T_m:"

rho_w=Density(Steam_IAPWS,T=T_m_water,P=P_1)
mu_w=Viscosity(Steam_IAPWS,T=T_m_water,P=P_1)
cp_w=SpecHeat(Steam_IAPWS,T=T_m_water,P=P_1)
k_w=Conductivity(Steam_IAPWS,T=T_m_water,P=P_1)
Pr_w=Prandtl(Steam_IAPWS,T=T_m_water,P=P_1)

"Calculations:"

"To determine inside heat transfer coeff. h_i:"



```
Re_water = D_i * U_m_water * rho_w / mu_w "…finds Reynolds No. for water"

"Re_water = 28388 > 4000; So, apply Dittus – Boelter eqn to find out Nusselts No.:"

Nusselts_w = 0.023 * Re_water^0.8 * Pr_w^0.4 "…gives Nusselts No."

Nusselts_w = h_i * D_i /k_w "….finds h_i, heat tr coeff on the inside "

"To determine outside heat transfer coeff. h_o:"

"It is cross flow of air across a cylinder. So, use the EES PROCEDURE written above to find out h_o, using the Churchill – Bernstein eqn for cross flow of a fluid over a cylinder:"

Fluid$ = 'Air'

P_infinity = P_1
T_infinity = T_m_air
U_infinity = V_o_air
D = D_o

{T_s = 70 "[C] …. assumed, will be corrected later"}

CALL ForcedConv_AcrossCylinder (Fluid$,P_infinity, T_infinity, U_infinity, L, D, T_s: Re_D, Nusselt_D_bar, h_o, Q)

R_fi = 0.0004 [m^2-C/W] "…Fouling factor on the inside"

R_fo = 0.0002 [m^2-C/W] "…Fouling factor on the outside"

"Areas:"

A_o = pi * D_o * L "[m^2]"

A_i = pi * D_i * L "[m^2]"

"We have:

U_i * A_i = U_o * A_o = 1 / R_total

where R_total = total thermal resistance,

And,

R_toal = R_conv_in + R_c_in + R_cond_wall + R_conv_out + R_c_out, where

R_conv_in = conv. resist. on the inside surface
```

R_c_in = Fouling resistance on inside

R_cond_wall = cond. resist of the tube wall

R_conv_out = conv. resist. on outside, and

R_c_out = Fouling resist on the outside"

"Thermal resistances:"

R_conv_in = 1/(h_i * A_i) "[C/W]"

R_c_in = R_fi / A_i "[C/W]"

R_cond_wall = ln(D_o / D_i) / (2 * pi * k_ss * L) "[C/W]"

R_conv_out = 1 / (h_o * A_o) "[C/W]"

R_c_out = R_fo / A_o "[C/W]"

R_total = R_conv_in + R_c_in + R_cond_wall + R_c_out + R_conv_out "[C/W]…. total thermal resistance"

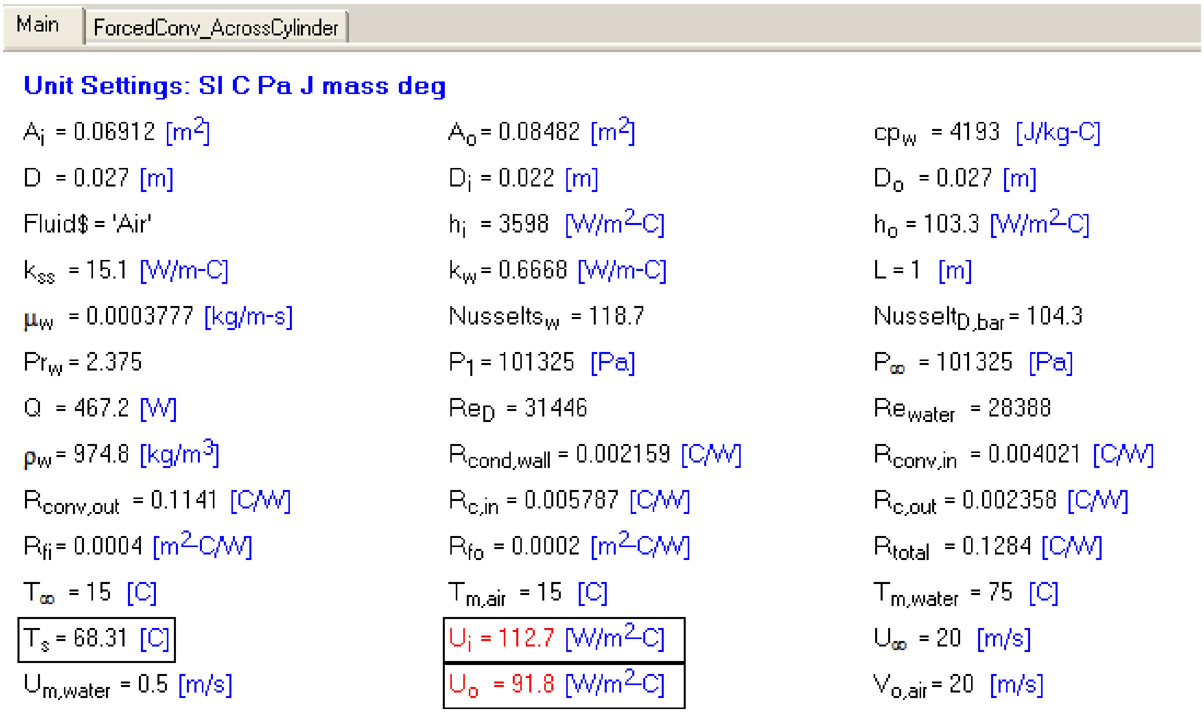
"To find T_s, the surface temp of cylinder:"

(T_s – T_m_air) / R_conv_out = (T_m_water – T_s) / (R_conv_in + R_c_in + R_cond_wall + R_c_out)

U_i * A_i = 1 / R_total "…determine U _i"

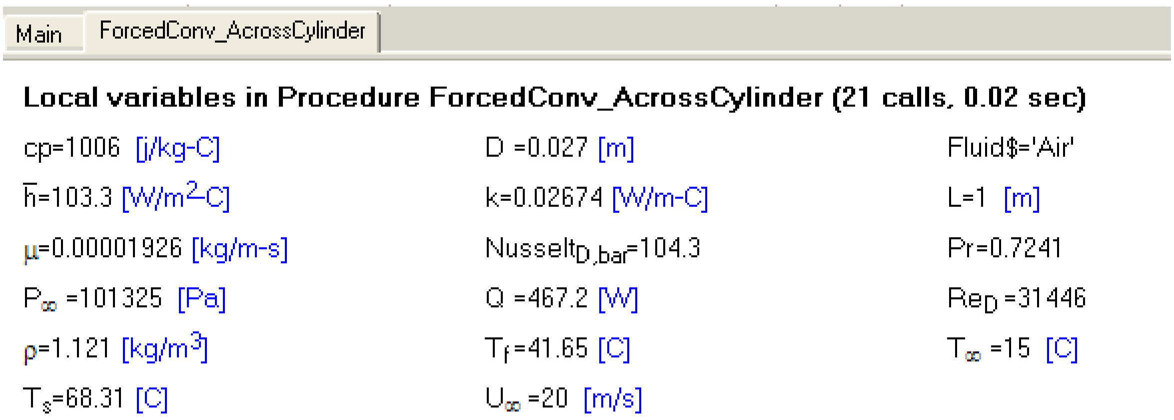
U_o * A_o = 1 / R_total "…determine U _o"

Results:



Unit Settings: SI C Pa J mass deg

A_i = 0.06912 [m^2]
A_o = 0.08482 [m^2]
cp_w = 4193 [J/kg-C]
D = 0.027 [m]
D_i = 0.022 [m]
D_o = 0.027 [m]
Fluid$ = 'Air'
h_i = 3598 [W/m^2-C]
h_o = 103.3 [W/m^2-C]
k_{ss} = 15.1 [W/m-C]
k_w = 0.6668 [W/m-C]
L = 1 [m]
μ_w = 0.0003777 [kg/m-s]
$Nusselts_w$ = 118.7
$Nusselt_{D,bar}$ = 104.3
Pr_w = 2.375
P_1 = 101325 [Pa]
P_∞ = 101325 [Pa]
Q = 467.2 [W]
Re_D = 31446
Re_{water} = 28388
ρ_w = 974.8 [kg/m^3]
$R_{cond,wall}$ = 0.002159 [C/W]
$R_{conv,in}$ = 0.004021 [C/W]
$R_{conv,out}$ = 0.1141 [C/W]
$R_{c,in}$ = 0.005787 [C/W]
$R_{c,out}$ = 0.002358 [C/W]
R_{fi} = 0.0004 [m^2-C/W]
R_{fo} = 0.0002 [m^2-C/W]
R_{total} = 0.1284 [C/W]
T_∞ = 15 [C]
$T_{m,air}$ = 15 [C]
$T_{m,water}$ = 75 [C]
T_s = 68.31 [C]
U_i = 112.7 [W/m^2-C]
U_∞ = 20 [m/s]
$U_{m,water}$ = 0.5 [m/s]
U_o = 91.8 [W/m^2-C]
$V_{o,air}$ = 20 [m/s]



Local variables in Procedure ForcedConv_AcrossCylinder (21 calls, 0.02 sec)

cp=1006 [j/kg-C]	D =0.027 [m]	Fluid$='Air'
$\bar{h}$=103.3 [W/m^2-C]	k=0.02674 [W/m-C]	L=1 [m]
μ=0.00001926 [kg/m-s]	$Nusselt_{D,bar}$=104.3	Pr=0.7241
P_{∞} =101325 [Pa]	Q =467.2 [W]	Re_D =31446
ρ=1.121 [kg/m^3]	T_f=41.65 [C]	T_{∞} =15 [C]
T_s=68.31 [C]	U_{∞} =20 [m/s]	

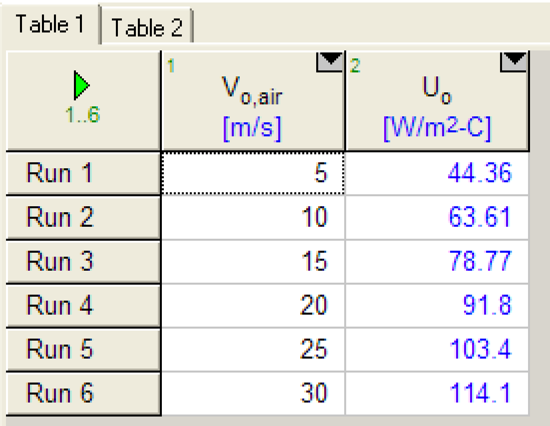
Thus:

T_s = 68.31 C ….surface temp on the fouling layer on the outer surface of the tube … Ans.

U_o = 91.8 W/m^2-C ….Overall heat tr coeff based on outer surface… Ans.

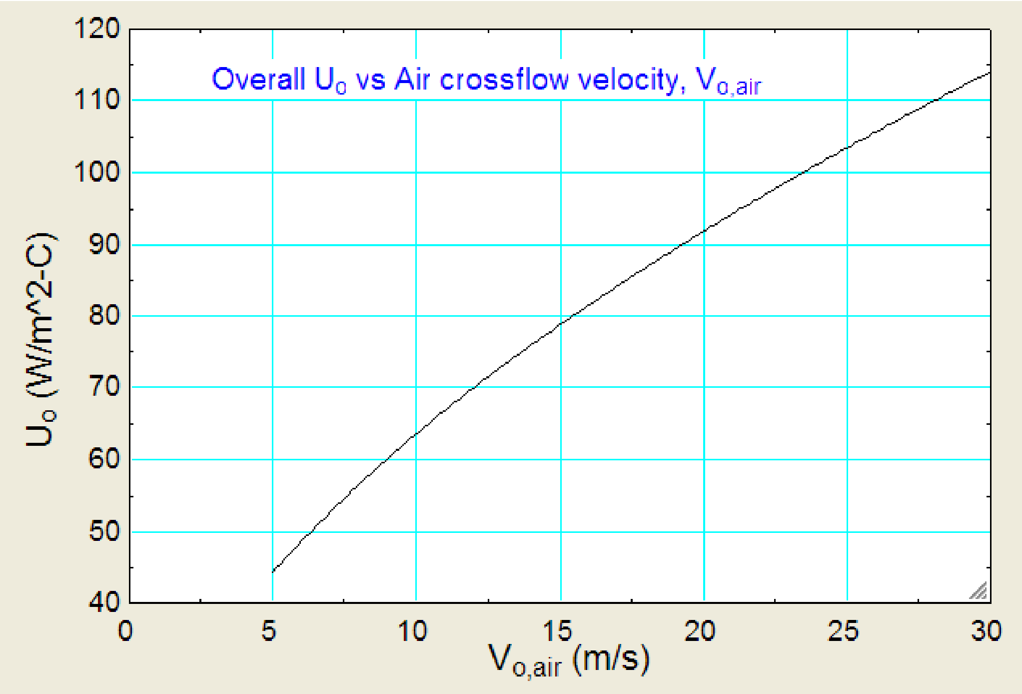
Plot U_o for various values of Air velocity across the cylinder:

First, construct the parametric table:



1..6	1 $V_{o,air}$ [m/s]	2 U_o [W/m2-C]
Run 1	5	44.36
Run 2	10	63.61
Run 3	15	78.77
Run 4	20	91.8
Run 5	25	103.4
Run 6	30	114.1

Now, draw the graph:



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"**Prob. 4A.5:** In Prob.4A.4 when the cross flow fluid is Water (instead of Air) flowing at a temp of 15 C and velocity of 1 m/s, determine the overall heat transfer coefficient based on the outside surface, U_o (b) Plot U_o as a function of mean water velocity, U_m_water, in the range: 0.5 < U_m_water < 2.5 m/s. (Ref. 3)"

EES Solution:

We will now use the EES PROCEDURE written above to calculate h_o, *using water as the Fluid.*

There is no change in the procedure to calculate h_i.

Following is the EES code:

"Data:"

D_i = 0.022 [m]

D_o = 0.027 [m]

L = 1 [m]

k_ss = 15.1 [W/m-C]

T_m_water = 75 [C]

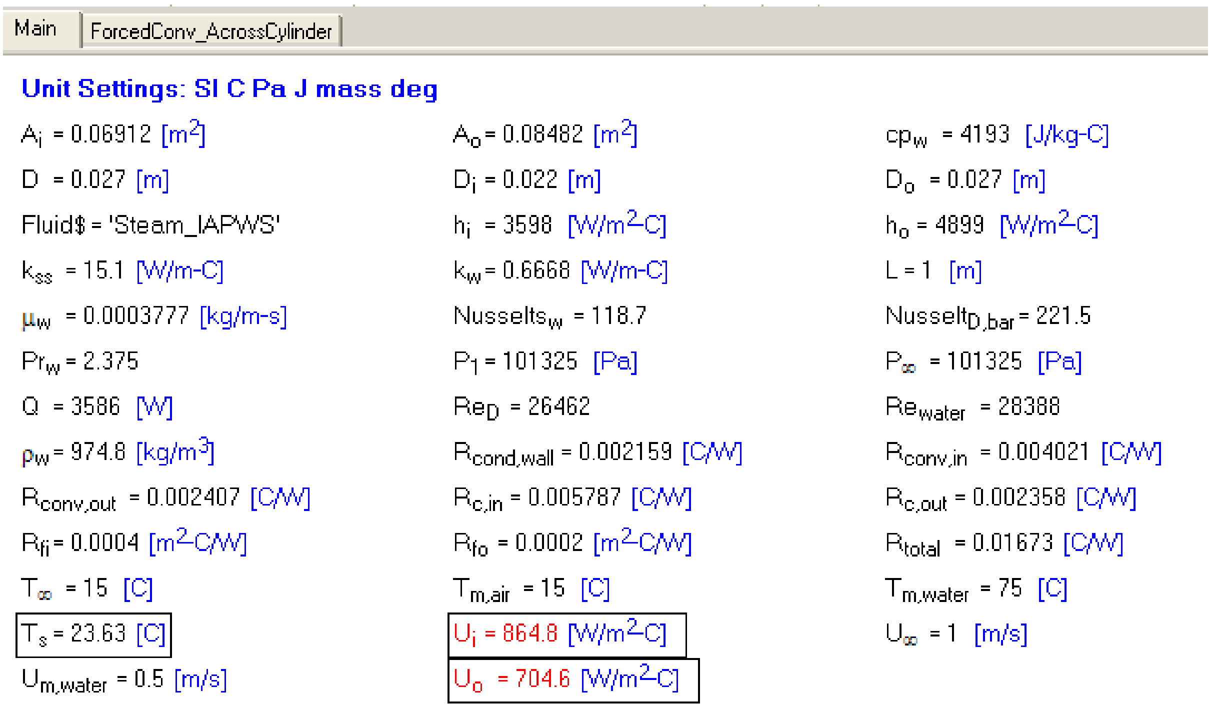
U_m_water = 0.5 [m/s]

```
P_1 = 1.01325E05 [Pa]

T_m_air = 15 [C]

{V_o_air = 20 [m/s]}

"Properties of Water at T_m:"

rho_w=Density(Steam_IAPWS,T=T_m_water,P=P_1)
mu_w=Viscosity(Steam_IAPWS,T=T_m_water,P=P_1)
cp_w=SpecHeat(Steam_IAPWS,T=T_m_water,P=P_1)
k_w=Conductivity(Steam_IAPWS,T=T_m_water,P=P_1)
Pr_w=Prandtl(Steam_IAPWS,T=T_m_water,P=P_1)

"Calculations:"

"To determine inside heat transfer coeff. h_i:"

Re_water = D_i * U_m_water * rho_w / mu_w "…finds Reynolds No. for water"

"Re_water = 28388 > 4000; So, apply Dittus – Boelter eqn to find out Nusselts No.:"

Nusselts_w = 0.023 * Re_water^0.8 * Pr_w^0.4 "…gives Nusselts No."

Nusselts_w = h_i * D_i /k_w "….finds h_i, heat tr coeff on the inside"
```

```
"To determine outside heat transfer coeff. h_o:"

"It is cross flow of air across a cylinder. So, use the EES PROCEDURE written above to find out h_o, using the Churchill – Bernstein eqn for cross flow of a fluid over a cylinder:"

Fluid$ = 'Steam_IAPWS'

{Fluid$ = 'Air'}

P_infinity = P_1
T_infinity = 15[C]
U_infinity = 1[m/s]
D = D_o

{T_s = 70 "[C] …. assumed, will be corrected later"}

CALL ForcedConv_AcrossCylinder (Fluid$,P_infinity, T_infinity, U_infinity, L, D, T_s: Re_D, Nusselt_D_bar, h_o, Q)

R_fi = 0.0004 [m^2-C/W] "…Fouling factor on the inside"
R_fo = 0.0002 [m^2-C/W] "…Fouling factor on the outside"

"Areas:"

A_o = pi * D_o * L "[m^2]"

A_i = pi * D_i * L "[m^2]"

"We have:

U_i * A_i = U_o * A_o = 1 / R_total

where R_total = total thermal resistance,

And,

R_toal = R_conv_in + R_c_in + R_cond_wall + R_conv_out + R_c_out, where

R_conv_in = conv. resist. on the inside surface

R_c_in = Fouling resistance on inside

R_cond_wall = cond. resist of the tube wall

R_conv_out = conv. resist. on outside, and

R_c_out = Fouling resist on the outside"
```

```
"Thermal resistances:"

R_conv_in = 1/(h_i * A_i) "[C/W]"

R_c_in = R_fi / A_i "[C/W]"

R_cond_wall = ln(D_o / D_i) / (2 * pi * k_ss * L) "[C/W]"

R_conv_out = 1 / (h_o * A_o) "[C/W]"

R_c_out = R_fo / A_o "[C/W]"

R_total = R_conv_in + R_c_in + R_cond_wall + R_c_out + R_conv_out "[C/W]…. total thermal resistance"

{(T_s – T_m_air) / R_conv_out = (T_m_water – T_s) / (R_conv_in + R_c_in + R_cond_wall + R_c_out) "….finds T_s, the surface temp of cylinder"}

(T_s – 15[C]) / R_conv_out = (T_m_water – T_s) / (R_conv_in + R_c_in + R_cond_wall + R_c_out)

U_i * A_i = 1 / R_total "…determine U _i"

U_o * A_o = 1 / R_total "…determine U _o"
```

Results:



Unit Settings: SI C Pa J mass deg

A_i = 0.06912 [m^2] A_o = 0.08482 [m^2] cp_w = 4193 [J/kg-C]

D = 0.027 [m] D_i = 0.022 [m] D_o = 0.027 [m]

Fluid$ = 'Steam_IAPWS' h_i = 3598 [W/m^2-C] h_o = 4899 [W/m^2-C]

k_{ss} = 15.1 [W/m-C] k_w = 0.6668 [W/m-C]   L = 1 [m]

μ_w = 0.0003777 [kg/m-s] $Nusselts_w$ = 118.7 $Nusselt_{D,bar}$ = 221.5

Pr_w = 2.375 P_1 = 101325 [Pa] P_∞ = 101325 [Pa]

Q = 3586 [W] Re_D = 26462 Re_{water} = 28388

ρ_w = 974.8 [kg/m^3] $R_{cond,wall}$ = 0.002159 [C/W] $R_{conv,in}$ = 0.004021 [C/W]

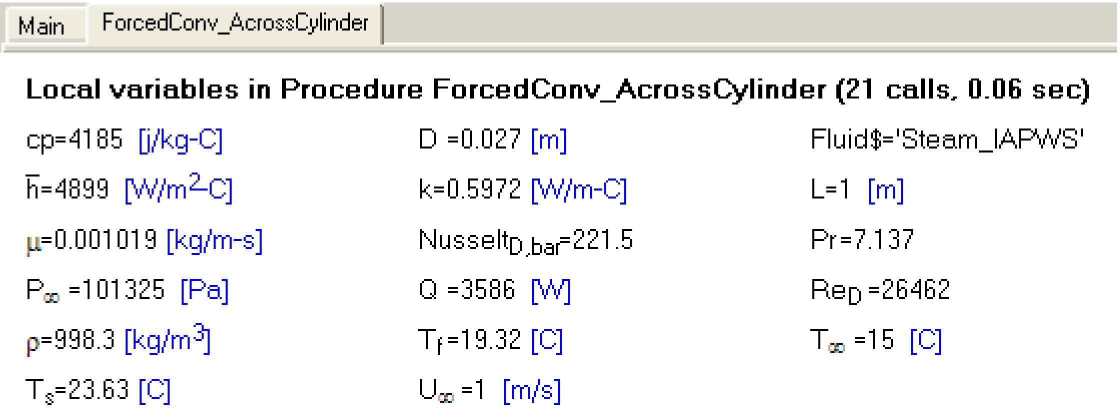
$R_{conv,out}$ = 0.002407 [C/W] $R_{c,in}$ = 0.005787 [C/W] $R_{c,out}$ = 0.002358 [C/W]

R_{fi} = 0.0004 [m^2-C/W] R_{fo} = 0.0002 [m^2-C/W] R_{total} = 0.01673 [C/W]

T_∞ = 15 [C] $T_{m,air}$ = 15 [C] $T_{m,water}$ = 75 [C]

T_s = 23.63 [C] U_i = 864.8 [W/m^2-C] U_∞ = 1 [m/s]

$U_{m,water}$ = 0.5 [m/s] U_o = 704.6 [W/m^2-C]



Local variables in Procedure ForcedConv_AcrossCylinder (21 calls, 0.06 sec)

cp=4185 [j/kg-C]
D =0.027 [m]
Fluid$='Steam_IAPWS'
$\bar{h}$=4899 [W/m^2-C]
k=0.5972 [W/m-C]
L=1 [m]
μ=0.001019 [kg/m-s]
$Nusselt_{D,bar}$=221.5
Pr=7.137
P_∞ =101325 [Pa]
Q =3586 [W]
Re_D =26462
ρ=998.3 [kg/m^3]
T_f=19.32 [C]
T_∞ =15 [C]
T_s=23.63 [C]
U_∞ =1 [m/s]

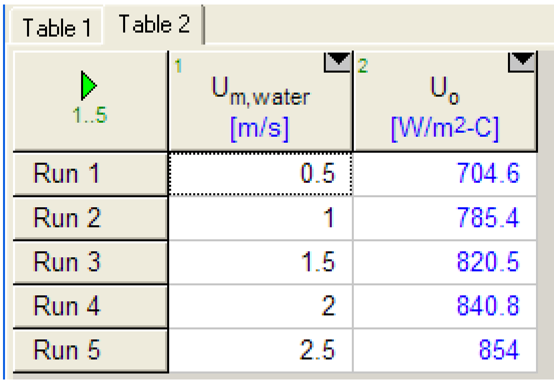
Thus:

T_s = 23.63 C ….surface temp on the fouling layer on the outer surface of the tube … Ans.

U_o = 704.6 W/m^2-C ….Overall heat tr coeff based on outer surface… Ans.

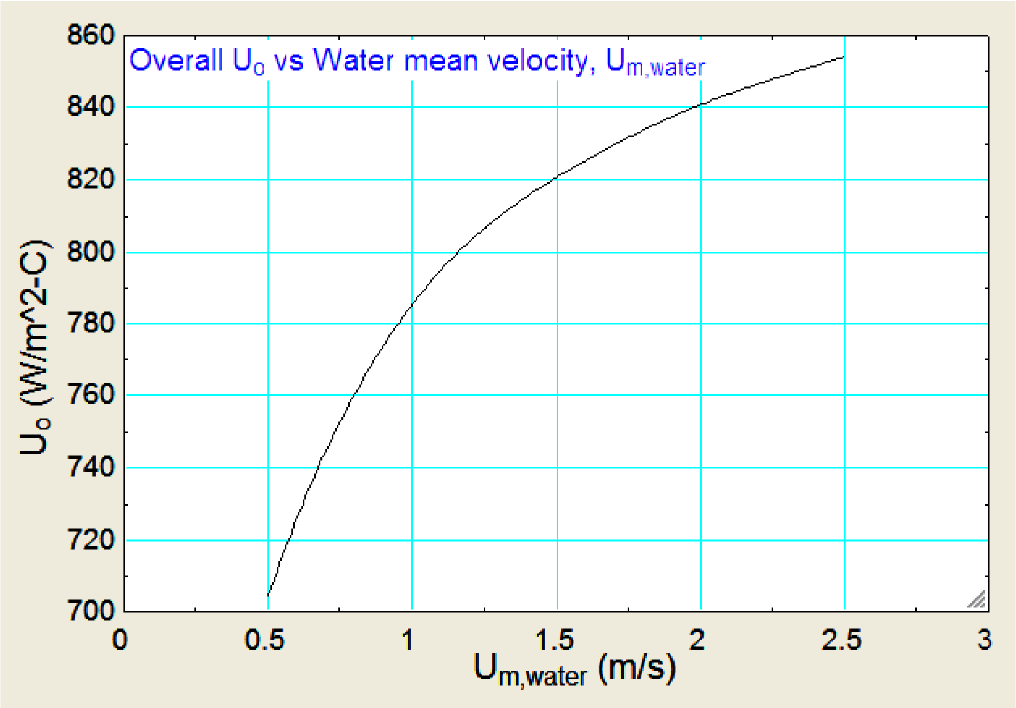
Plot U_o for various values of water velocity inside the cylinder:

Parametric Table:



1..5	1 $U_{m,water}$ [m/s]	2 U_o [W/m2-C]
Run 1	0.5	704.6
Run 2	1	785.4
Run 3	1.5	820.5
Run 4	2	840.8
Run 5	2.5	854

Plot:


Overall U_o vs Water mean velocity, $U_{m,water}$
U_o (W/m^2-C)
860
840
820
800
780
760
740
720
700
0
0.5
1
1.5
2
2.5
3
$U_{m,water}$ (m/s)

==

Prob.4A.6. Water at a mean temperature of T_m = 90 C and a mean velocity of u_m = 0.10 m/s flows inside a 2.5 cm ID, thin-walled copper tube. Outer surface of the tube dissipates heat to atmospheric air at T_a = 20 C, by free convection. Calculate the tube wall temperature (T_s), overall heat transfer coeff. and heat loss per metre length of tube. Use following simplified expression for air to determine heat transfer coeff. by free convection:

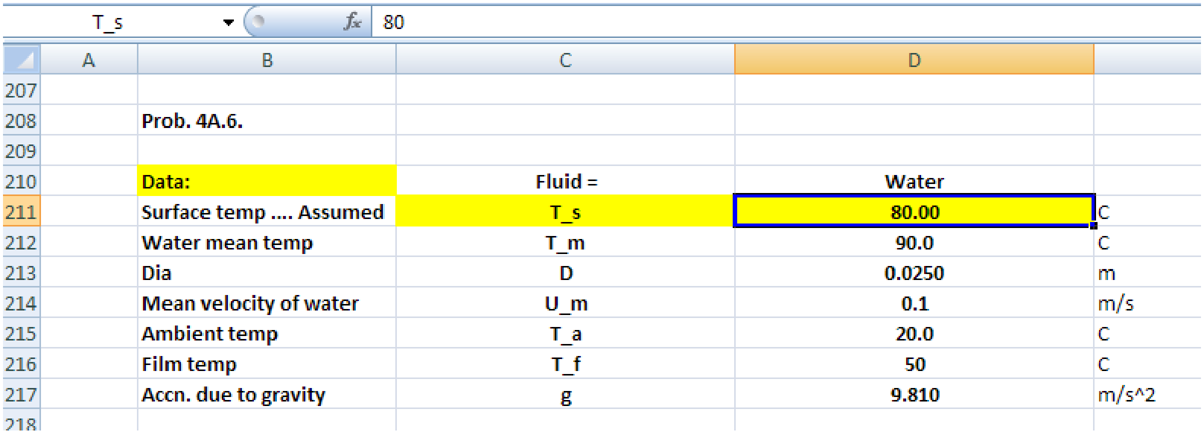
$$h_a = 1.32 \cdot \left(\frac{T_s - T_a}{D} \right)^{0.25}$$

EXCEL Solution:

To calculate the heat transfer coeff for flow inside the tube, we will apply the Dittus-Boelter eqn to get the Nusselts No. and h_i there-from. We need properties of Sat. water. But, we have already written VBA Functions for properties of sat. water. We shall use them.

Following are the steps in EXCEL Solution:

1. Set up the EXCEL worksheet, enter data and name the cells:

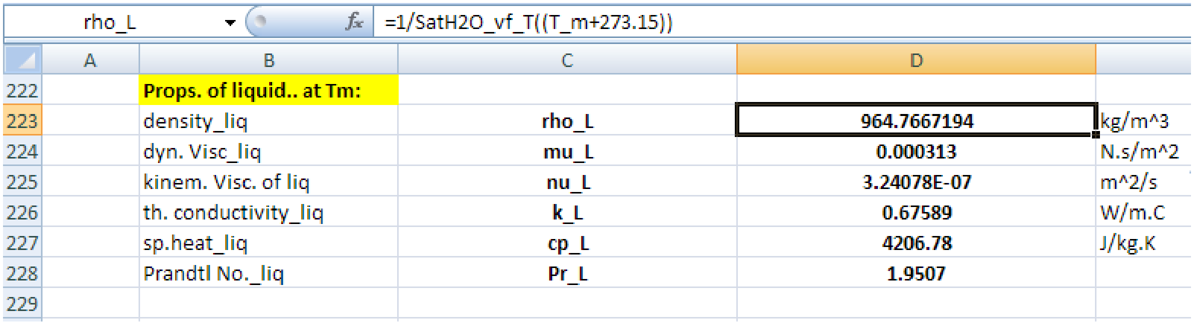


T_s 80

	A	B	C	D	
207					
208		**Prob. 4A.6.**			
209					
210		**Data:**	**Fluid =**	**Water**	
211		**Surface temp .... Assumed**	**T_s**	**80.00**	C
212		**Water mean temp**	**T_m**	**90.0**	C
213		**Dia**	**D**	**0.0250**	m
214		**Mean velocity of water**	**U_m**	**0.1**	m/s
215		**Ambient temp**	**T_a**	**20.0**	C
216		**Film temp**	**T_f**	**50**	C
217		**Accn. due to gravity**	**g**	**9.810**	m/s^2
218					

Note that we have assumed a value for T_s; it will be corrected later.

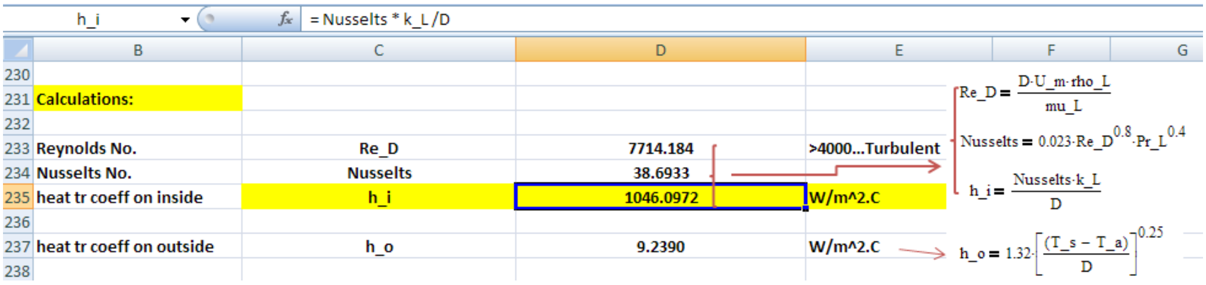
2. Calculate the fluid properties using the VBA Functions already written:



rho_L =1/SatH2O_vf_T((T_m+273.15))

	A	B	C	D	
222		**Props. of liquid.. at Tm:**			
223		density_liq	**rho_L**	**964.7667194**	kg/m^3
224		dyn. Visc_liq	**mu_L**	**0.000313**	N.s/m^2
225		kinem. Visc. of liq	**nu_L**	**3.24078E-07**	m^2/s
226		th. conductivity_liq	**k_L**	**0.67589**	W/m.C
227		sp.heat_liq	**cp_L**	**4206.78**	J/kg.K
228		Prandtl No._liq	**Pr_L**	**1.9507**	
229					

3. Now, perform the calculations. First, find out the inside heat transfer coeff h_i:



h_i = Nusselts * k_L / D

	B	C	D	E
230				
231	Calculations:			
232				
233	Reynolds No.	Re_D	7714.184	>4000...Turbulent
234	Nusselts No.	Nusselts	38.6933	
235	heat tr coeff on inside	h_i	1046.0972	W/m^2.C
236				
237	heat tr coeff on outside	h_o	9.2390	W/m^2.C
238				

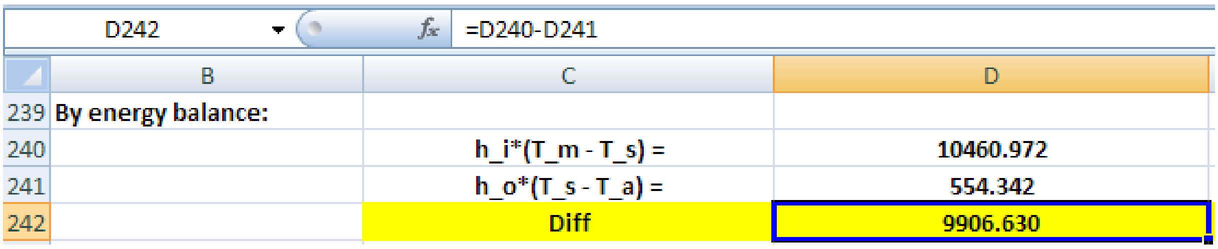
$$Re\_D = \frac{D \cdot U\_m \cdot rho\_L}{mu\_L}$$

$$Nusselts = 0.023 \cdot Re\_D^{0.8} \cdot Pr\_L^{0.4}$$

$$h\_i = \frac{Nusselts \cdot k\_L}{D}$$

$$h\_o = 1.32 \cdot \left[\frac{(T\_s - T\_a)}{D}\right]^{0.25}$$

4. Now, calculate the surface temp T_s by heat balance, i.e. heat lost by water = heat gained by air, thermal cond of wall being negligible. i.e. their difference should be equal to zero.

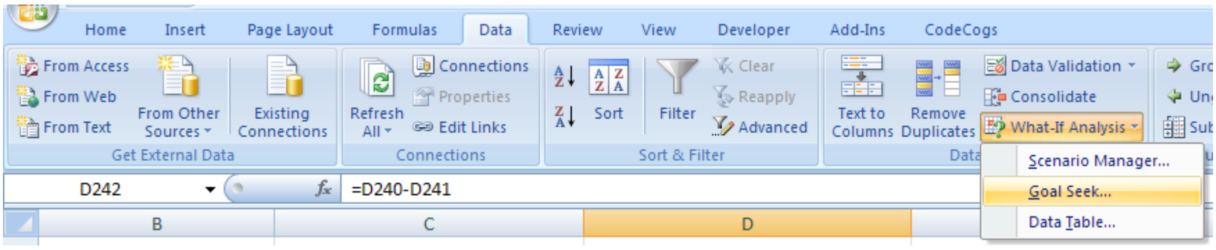


D242 =D240-D241

	B	C	D
239	By energy balance:		
240		h_i*(T_m - T_s) =	10460.972
241		h_o*(T_s - T_a) =	554.342
242		Diff	9906.630

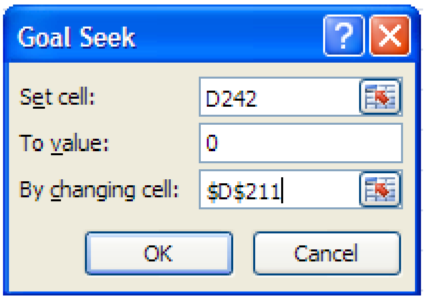
However, it will not be zero as it is, since initially we had assumed a value for T_s. Now, get the correct value of T_s by applying Goal Seek to to make cell D242 equal to zero by changing cell D211 (i.e. value of T_s):

Go to Data-WhatIf Analysis-Goal Seek:



D242 =D240-D241

Click on Goal Seek. Fill up the window that pops up as shown:



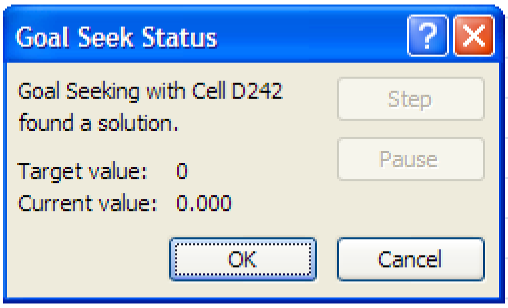
Goal Seek

Set cell: D242

To value: 0

By changing cell: $D$211

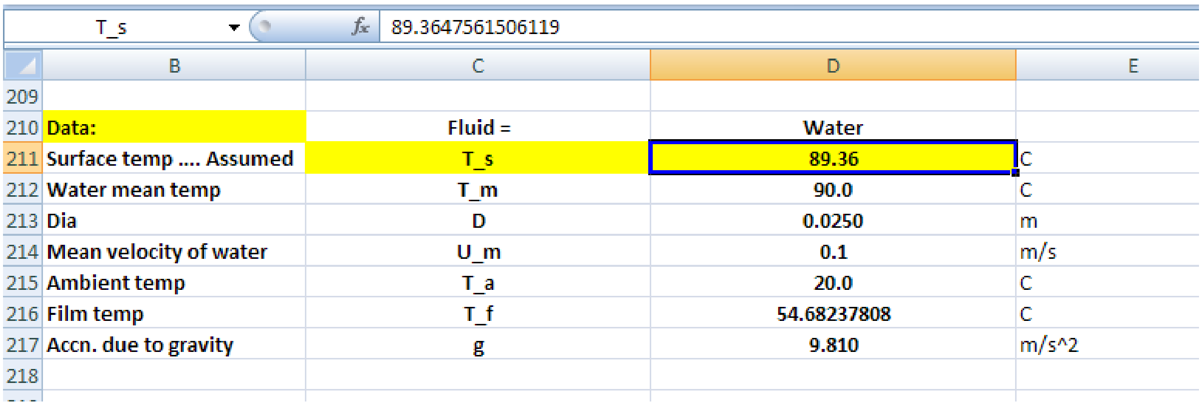
Click OK:



Goal Seeking with Cell D242
found a solution.

Target value: 0
Current value: 0.000

Goal Seek has found a solution. Again, click OK and see the value of T_s in cell D211:

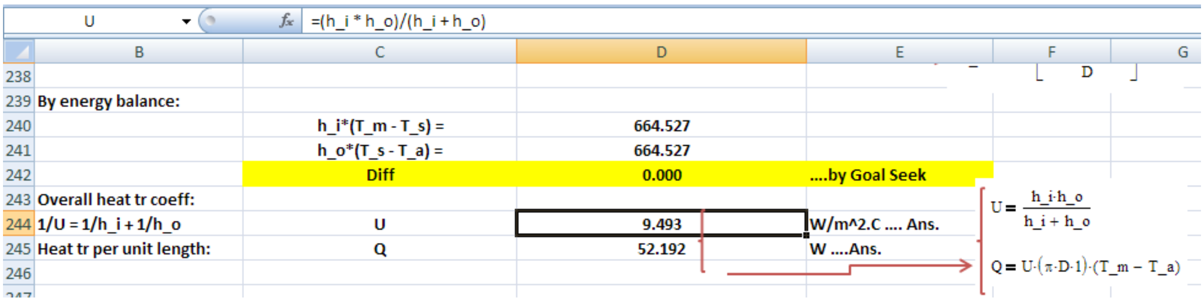


89.3647561506119

	B	C	D	E
209				
210	Data:	Fluid =	Water	
211	Surface temp .... Assumed	T_s	89.36	C
212	Water mean temp	T_m	90.0	C
213	Dia	D	0.0250	m
214	Mean velocity of water	U_m	0.1	m/s
215	Ambient temp	T_a	20.0	C
216	Film temp	T_f	54.68237808	C
217	Accn. due to gravity	g	9.810	m/s^2
218				

i.e. the surface temp is: 89.36 C … Ans.

5. Now, calculate overall heat transfer coeff. U and heat transferred, Q:



=(h_i * h_o)/(h_i + h_o)

	B	C	D	E	F	G
238					D	
239	By energy balance:					
240		h_i*(T_m - T_s) =	664.527			
241		h_o*(T_s - T_a) =	664.527			
242		Diff	0.000	....by Goal Seek		
243	Overall heat tr coeff:					
244	1/U = 1/h_i + 1/h_o	U	9.493	W/m^2.C .... Ans.		
245	Heat tr per unit length:	Q	52.192	W ....Ans.		
246						

$U = \frac{h\_i \cdot h\_o}{h\_i + h\_o}$

$Q = U \cdot (\pi \cdot D \cdot 1) \cdot (T\_m - T\_a)$

Note that formulas used are also shown in the above worksheets, for clarity.

Thus: U = 9.493 W/m^2. C …. Ans.

And, Q = 52.192 W … Ans.

==

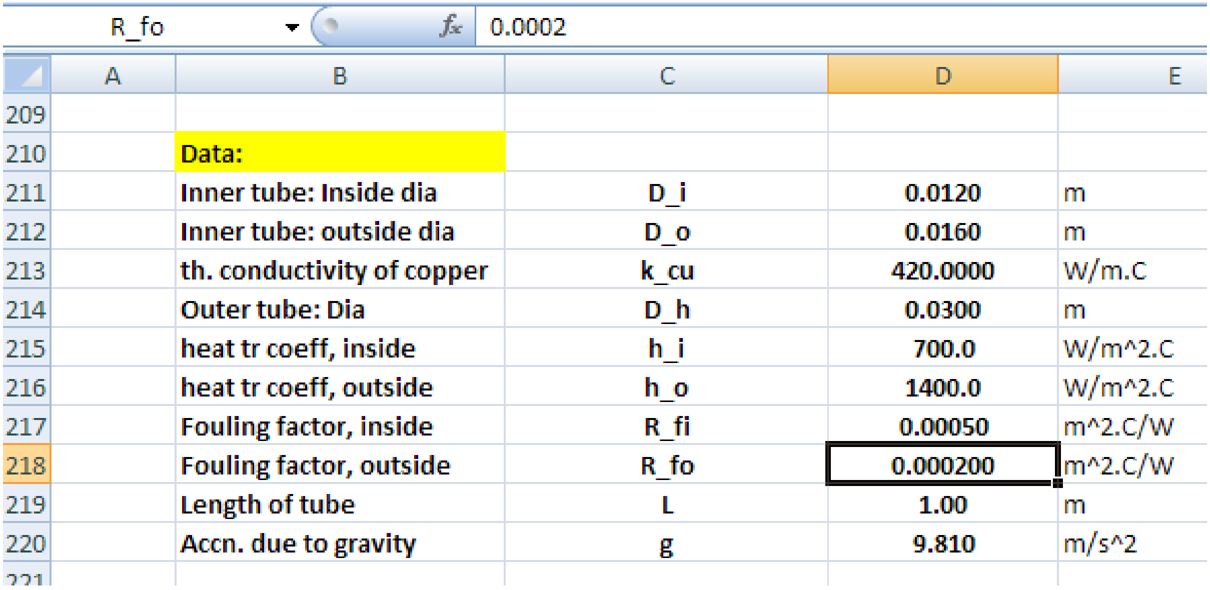
Prob.4A.7. A double pipe HX is made of copper (k = 380 W/m.C) inner tube of 1.2 cm ID, 1.6 cm OD and an outer tube of 3 cm ID. Heat transfer coeff on the inside and outside of inner tube are 700 and 1400 W/m^2.C respectively. Fouling factors on inside and outside are 0.0005 and 0.0002 m^2.C/W respectively. Determine: (a) thermal resistance of HX per unit length

(b) overall heat transfer coefficients U_i and U_o.

EXCEL Solution:

Following are the steps in EXCEL Solution:

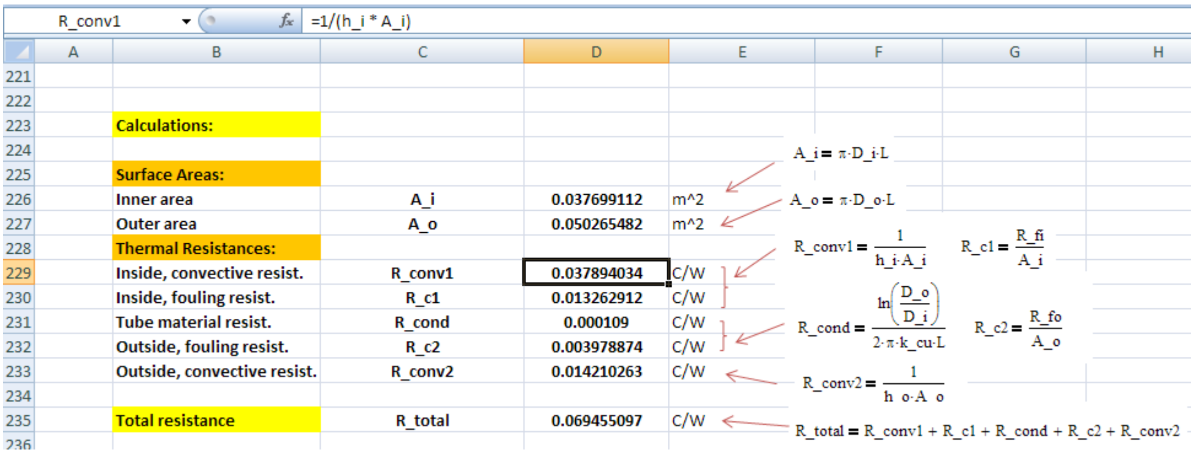
1. Set up the EXCEL worksheet, enter data and name the cells:



R_fo | f_x 0.0002

	A	B	C	D	E
209					
210		Data:			
211		Inner tube: Inside dia	D_i	0.0120	m
212		Inner tube: outside dia	D_o	0.0160	m
213		th. conductivity of copper	k_cu	420.0000	W/m.C
214		Outer tube: Dia	D_h	0.0300	m
215		heat tr coeff, inside	h_i	700.0	W/m^2.C
216		heat tr coeff, outside	h_o	1400.0	W/m^2.C
217		Fouling factor, inside	R_fi	0.00050	m^2.C/W
218		Fouling factor, outside	R_fo	0.000200	m^2.C/W
219		Length of tube	L	1.00	m
220		Accn. due to gravity	g	9.810	m/s^2

2. Do the calculations, as shown.



R_conv1 | f_x =1/(h_i * A_i)

	A	B	C	D	E
221					
222					
223		Calculations:			
224					
225		Surface Areas:			
226		Inner area	A_i	0.037699112	m^2
227		Outer area	A_o	0.050265482	m^2
228		Thermal Resistances:			
229		Inside, convective resist.	R_conv1	0.037894034	C/W
230		Inside, fouling resist.	R_c1	0.013262912	C/W
231		Tube material resist.	R_cond	0.000109	C/W
232		Outside, fouling resist.	R_c2	0.003978874	C/W
233		Outside, convective resist.	R_conv2	0.014210263	C/W
234					
235		Total resistance	R_total	0.069455097	C/W

$$A\_i = \pi \cdot D\_i \cdot L$$

$$A\_o = \pi \cdot D\_o \cdot L$$

$$R\_conv1 = \frac{1}{h\_i \cdot A\_i} \qquad R\_c1 = \frac{R\_fi}{A\_i}$$

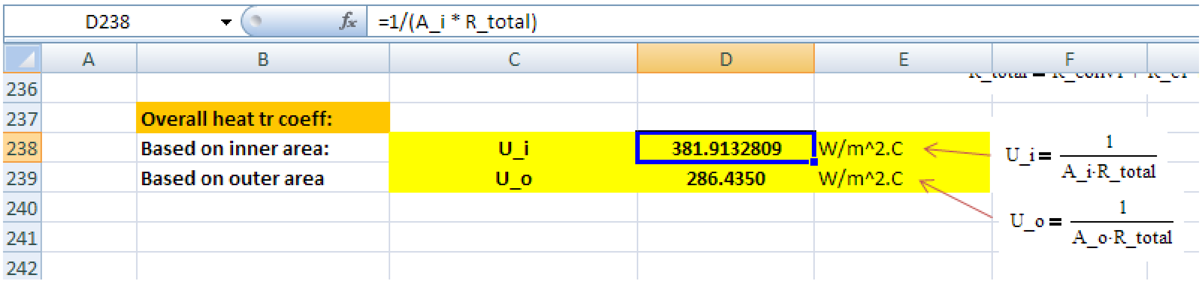
$$R\_cond = \frac{\ln\left(\frac{D\_o}{D\_i}\right)}{2 \cdot \pi \cdot k\_cu \cdot L} \qquad R\_c2 = \frac{R\_fo}{A\_o}$$

$$R\_conv2 = \frac{1}{h\_o \cdot A\_o}$$

$$R\_total = R\_conv1 + R\_c1 + R\_cond + R\_c2 + R\_conv2$$

Thus: Total thermal resistance per unit length = 0.06946 C/W … Ans.

3. After calculating the total thermal resistance, overall heat transfer coeffs based on inner and outer areas are easily calculated:



D238 =1/(A_i * R_total)

	A	B	C	D	E
236					
237		Overall heat tr coeff:			
238		Based on inner area:	U_i	381.9132809	W/m^2.C
239		Based on outer area	U_o	286.4350	W/m^2.C
240					
241					
242					

$$U\_i = \frac{1}{A\_i \cdot R\_total}$$

$$U\_o = \frac{1}{A\_o \cdot R\_total}$$

Note that formulas used are also shown in the worksheet.

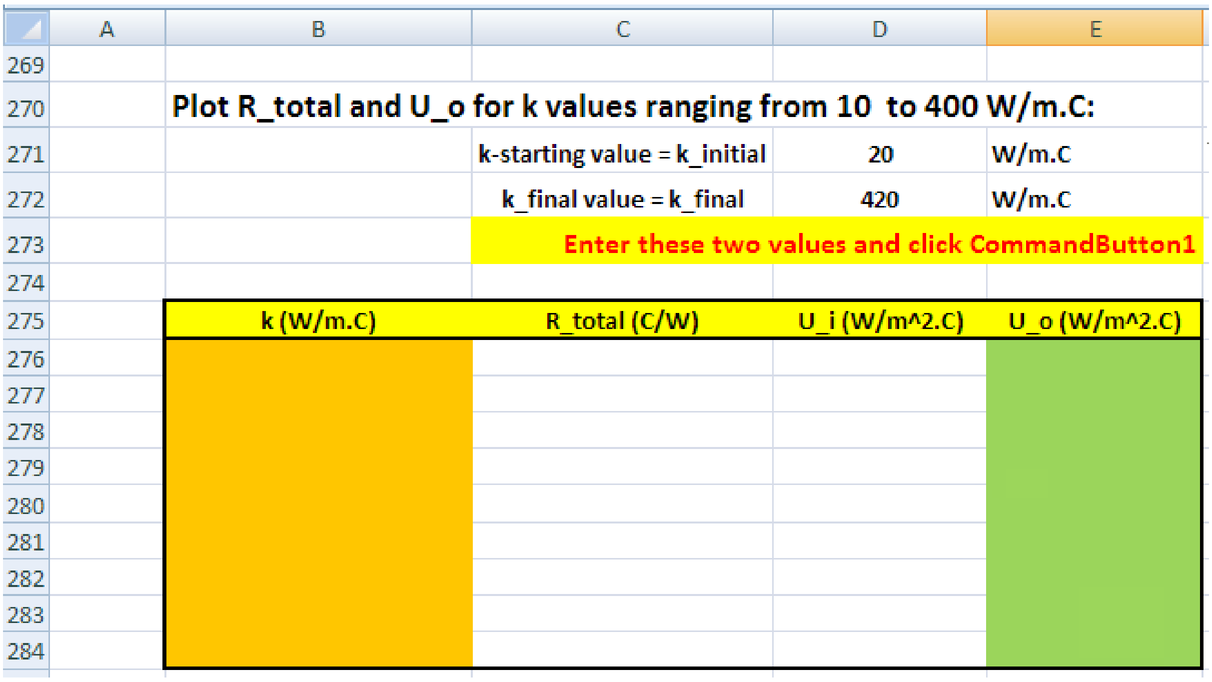
Thus:

U_i = 381.91 W/m^2.C …. Ans.

And, U_o = 286.44 W/m^2.C …. Ans.

4. **Now, plot R_total and U_i, U_o for different values of thermal conductivity, k (varying from 20 to 420 W/m.C):**

First, set up a Table as shown below:



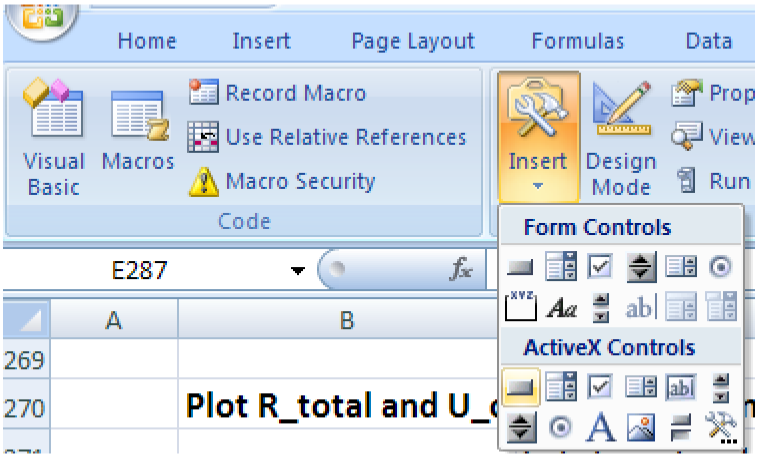
	A	B	C	D	E
269					
270		Plot R_total and U_o for k values ranging from 10 to 400 W/m.C:			
271			k-starting value = k_initial	20	W/m.C
272			k_final value = k_final	420	W/m.C
273			Enter these two values and click CommandButton1		
274					
275		k (W/m.C)	R_total (C/W)	U_i (W/m^2.C)	U_o (W/m^2.C)
276					
277					
278					
279					
280					
281					
282					
283					
284					

Here, we enter the range of k desired in cells D271 and D272 as shown.

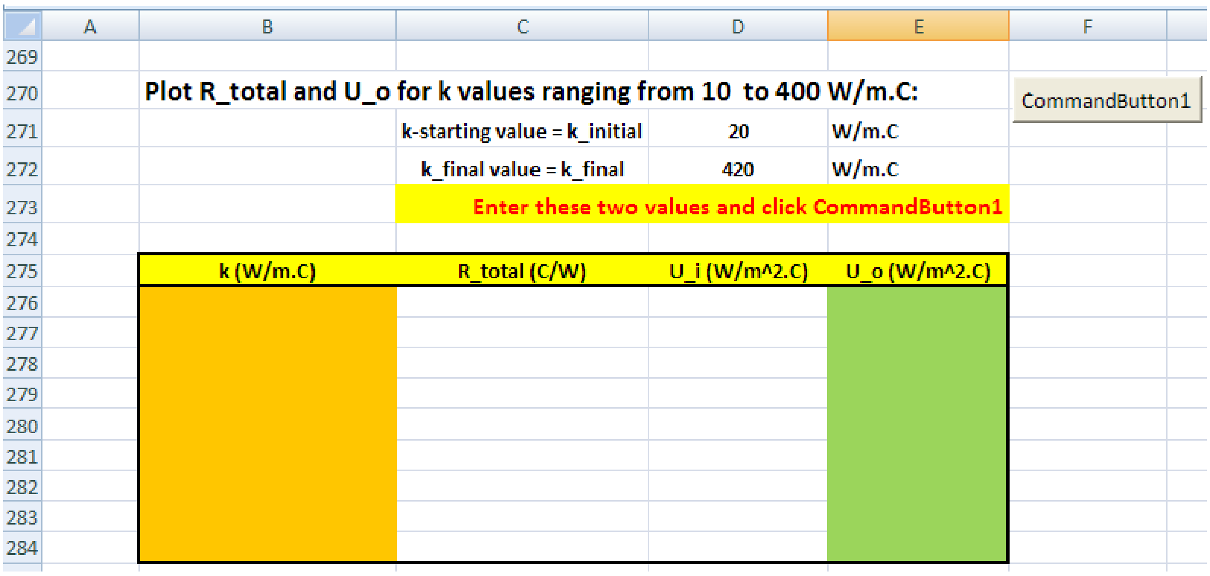
Table has 9 rows; so increment will be: Inc = (final value – initial value) / 8 = 5

Now, we will have a control button to operate our VBA program.

So, go to: Developer-Insert-ActiveX controls:

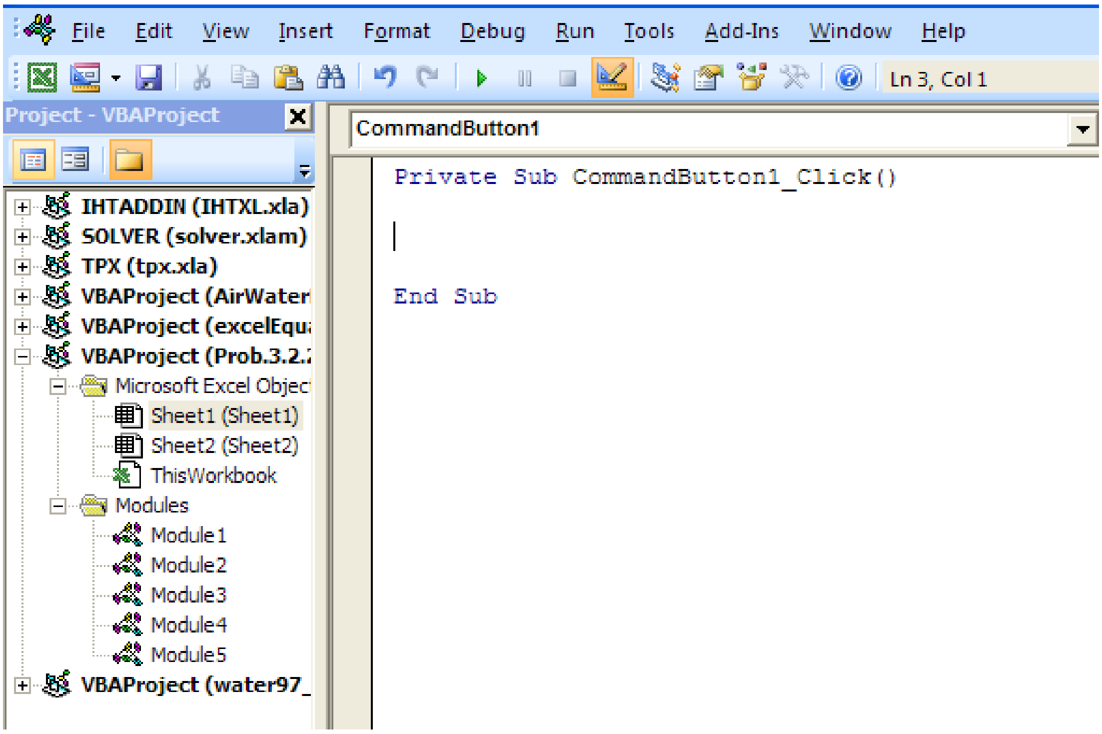


Click on first, left button under ActiveX Controls, and locate the button at the required place in the worksheet and adjust its size:



	A	B	C	D	E	F
269						
270		Plot R_total and U_o for k values ranging from 10 to 400 W/m.C:				CommandButton1
271			k-starting value = k_initial	20	W/m.C	
272			k_final value = k_final	420	W/m.C	
273			Enter these two values and click CommandButton1			
274						
275		k (W/m.C)	R_total (C/W)	U_i (W/m^2.C)	U_o (W/m^2.C)	
276						
277						
278						
279						
280						
281						
282						
283						
284						

Now, in Developer tab, click VisualBasic (extreme left) and we see under Sheet 1, the VBA program for this control button:



```
Private Sub CommandButton1_Click()


End Sub
```

Now, modify this program to:

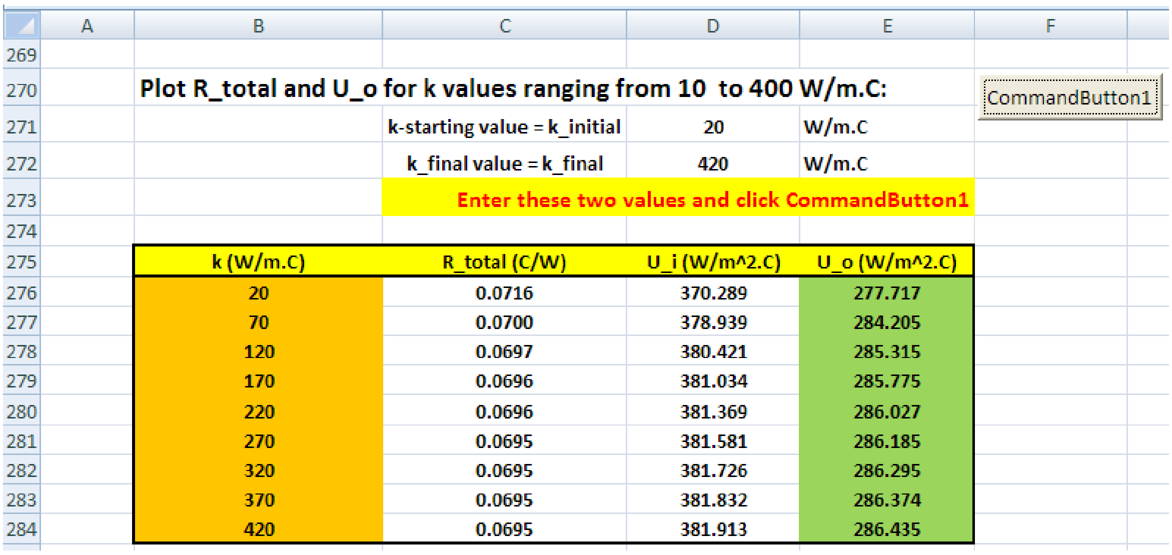
generate the valus of k within the desired range and get the values of R_total, U_i and U_o for each case, and write them all to appropriate places in the Table.

Following is the program:

```
Private Sub CommandButton1_Click()

Dim i As Integer
Dim k_initial As Double, k_final As Double, Inc As Double

k_initial = Range("D271") 'starting value for k
k_final = Range("D272") 'starting value for k

Inc = (k_final - k_initial) / 8

For i = 0 To 8

Range("D213") = k_initial + i * Inc

Cells(276 + i, 2) = k_initial + i * Inc 'Fills the first column of Table, i.e. k values

Cells(276 + i, 3) = Range("D235") 'Fills the second column of Table, i.e. R_totl values

Cells(276 + i, 4) = Range("D238") 'Fills the third column of Table, i.e. U_i values

Cells(276 + i, 5) = Range("D239") 'Fills the fourth column of Table, i.e. U_o values

Next i

End Sub
```

In the above program, read the comments given to see what each line does.

Now, click on the CommandButton1 and we get:



Plot R_total and U_o for k values ranging from 10 to 400 W/m.C:

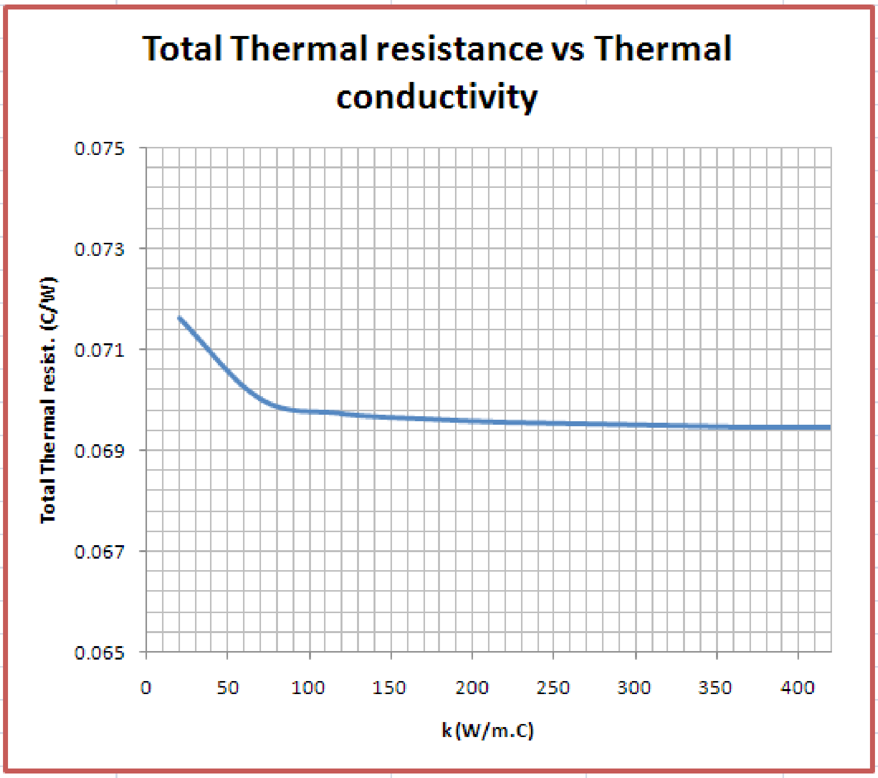
k-starting value = k_initial 20 W/m.C

k_final value = k_final 420 W/m.C

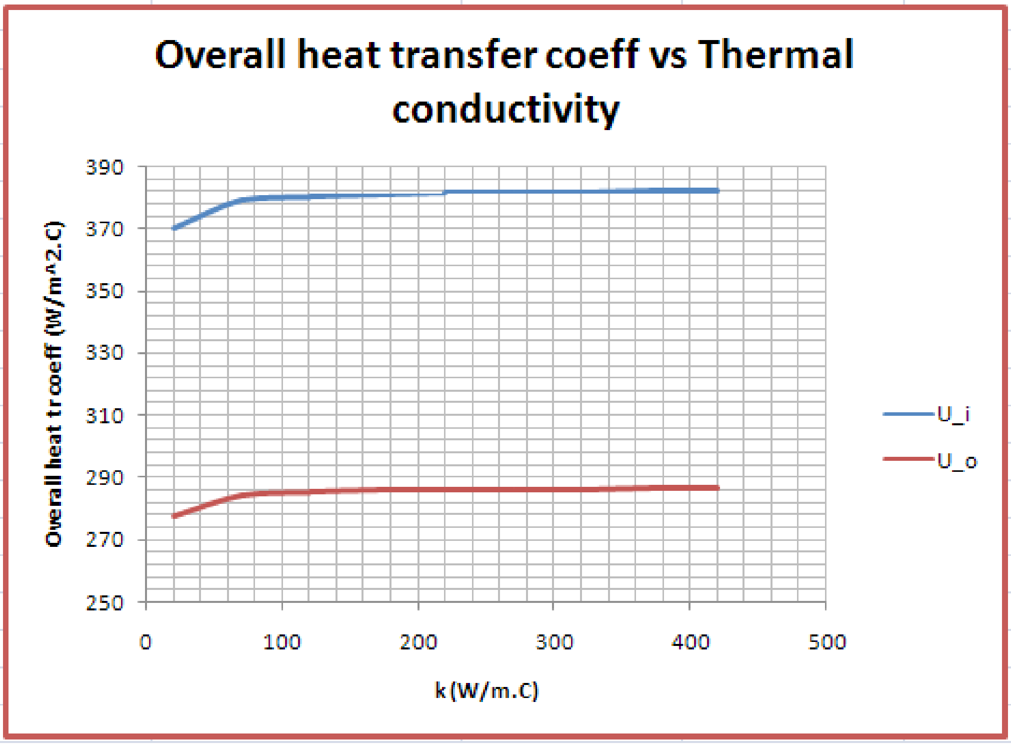
Enter these two values and click CommandButton1

k (W/m.C)	R_total (C/W)	U_i (W/m^2.C)	U_o (W/m^2.C)
20	0.0716	370.289	277.717
70	0.0700	378.939	284.205
120	0.0697	380.421	285.315
170	0.0696	381.034	285.775
220	0.0696	381.369	286.027
270	0.0695	381.581	286.185
320	0.0695	381.726	286.295
370	0.0695	381.832	286.374
420	0.0695	381.913	286.435

Now, plot the graph:



And,


Overall heat transfer coeff vs Thermal conductivity
Overall heat tr coeff (W/m^2.C)
390
370
350
330
310
290
270
250
0
100
200
300
400
500
k (W/m.C)
U_i
U_o

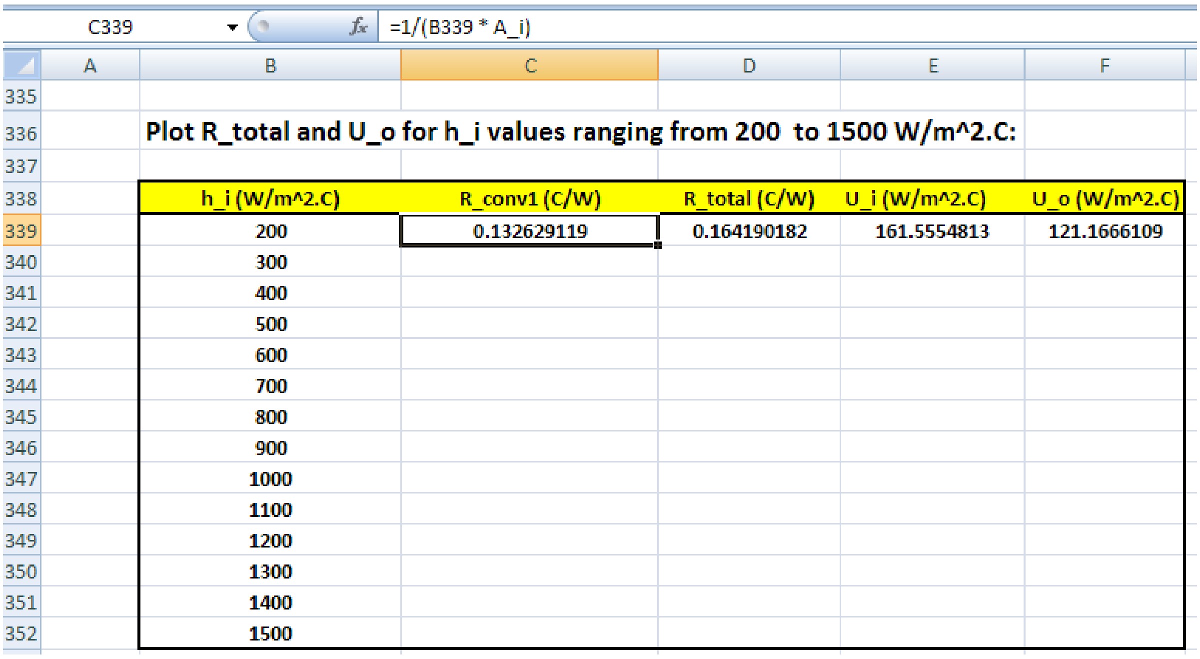
Note: If we need R_total, U_i and U_o values for some other range of k values, simply plug in the desired upper and lower values of k in the cells D271 and D272 and click on the commandButton1, and immediately the Table values get up-dated.

5. **Now, plot R_total and U_i, U_o for different values of h_i (varying from 200 to 1500 W/m^2.C):**

Again, we can write a VBA program to do this.

However, we can calculate the values in the usual way in the EXCEL spreadsheet without a VBA program.

First, set up a Table as shown below:



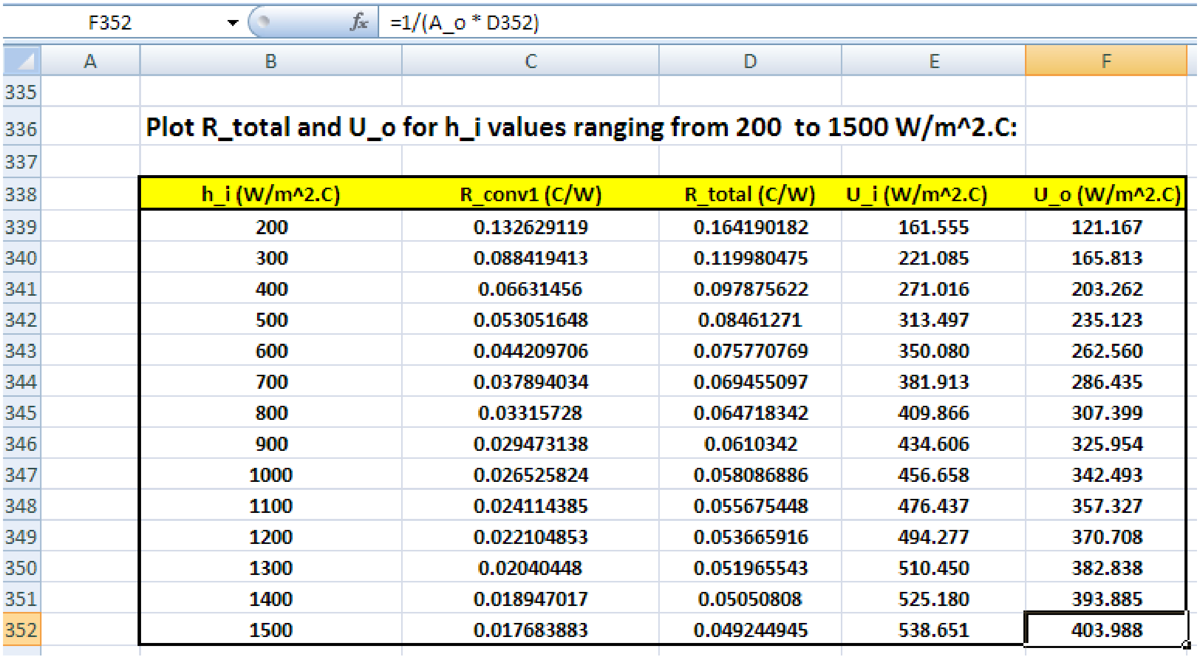
=1/(B339 * A_i)

Plot R_total and U_o for h_i values ranging from 200 to 1500 W/m^2.C:

h_i (W/m^2.C)	R_conv1 (C/W)	R_total (C/W)	U_i (W/m^2.C)	U_o (W/m^2.C)
200	0.132629119	0.164190182	161.5554813	121.1666109
300				
400				
500				
600				
700				
800				
900				
1000				
1100				
1200				
1300				
1400				
1500				

Enter the formulas for R_conv1, R_total, U_i and U_o in the first row, i.e. in row no. 339 as shown, remembering to use 'relative reference' to h_i (i.e. cell B339) in the equations. See in the Formula bar the eqn entered for R_conv1 in cell C339.

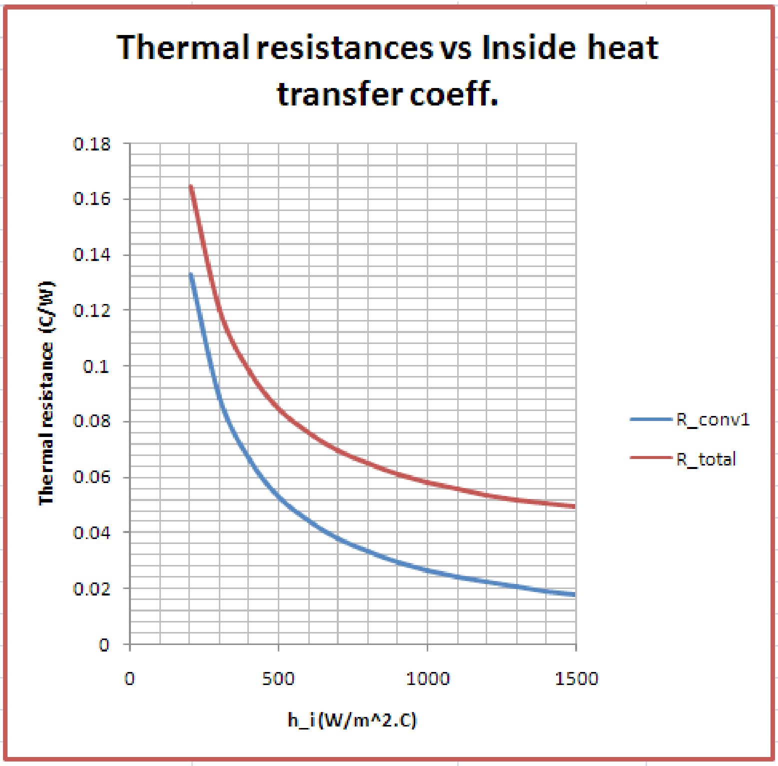
Now, simply select cells C339 to F339 and 'drag copy' to the end of the Table (i.e. up to cell F352) and immediately, the calculations are made and the Table is filled up:


F352 =1/(A_o * D352)

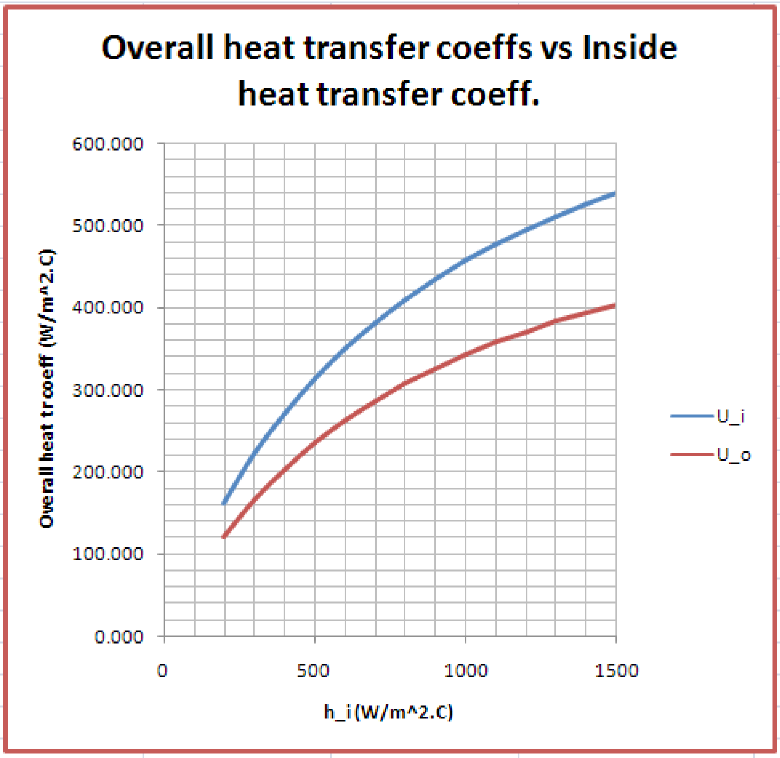
Plot R_total and U_o for h_i values ranging from 200 to 1500 W/m^2.C:

h_i (W/m^2.C)	R_conv1 (C/W)	R_total (C/W)	U_i (W/m^2.C)	U_o (W/m^2.C)
200	0.132629119	0.164190182	161.555	121.167
300	0.088419413	0.119980475	221.085	165.813
400	0.06631456	0.097875622	271.016	203.262
500	0.053051648	0.08461271	313.497	235.123
600	0.044209706	0.075770769	350.080	262.560
700	0.037894034	0.069455097	381.913	286.435
800	0.03315728	0.064718342	409.866	307.399
900	0.029473138	0.0610342	434.606	325.954
1000	0.026525824	0.058086886	456.658	342.493
1100	0.024114385	0.055675448	476.437	357.327
1200	0.022104853	0.053665916	494.277	370.708
1300	0.02040448	0.051965543	510.450	382.838
1400	0.018947017	0.05050808	525.180	393.885
1500	0.017683883	0.049244945	538.651	403.988

Now, plot the graphs a desired:

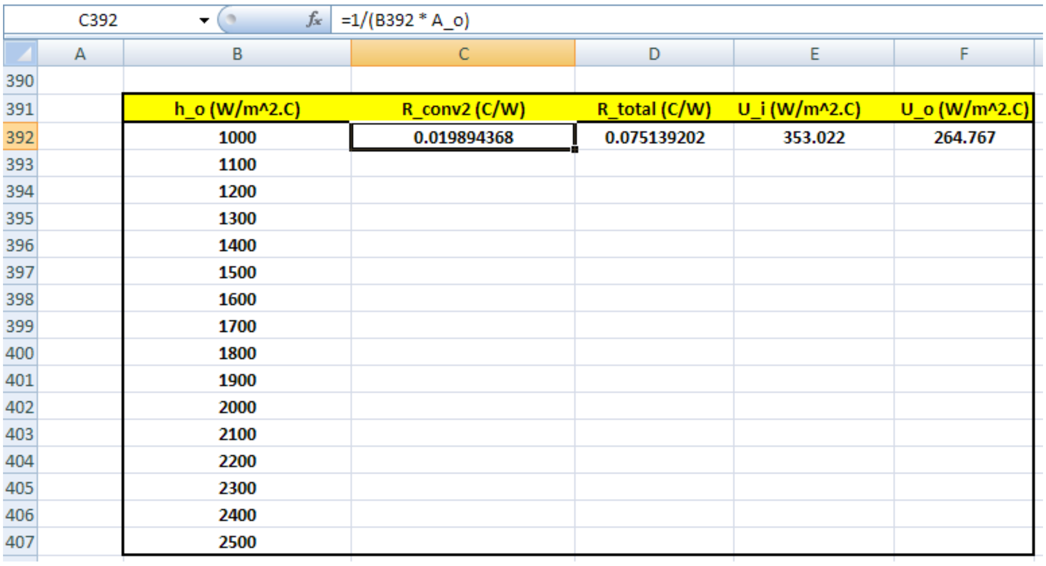


And,


Overall heat transfer coeffs vs Inside heat transfer coeff.
600.000
500.000
400.000
300.000
200.000
100.000
0.000
Overall heat tr coeff (W/m^2.C)
0
500
1000
1500
h_i (W/m^2.C)
U_i
U_o

6. **Also, plot R_total and U_i, U_o for different values of h_0 (varying from 1000 to 2500 W/m^2.C):**

First, set up a Table as shown below:

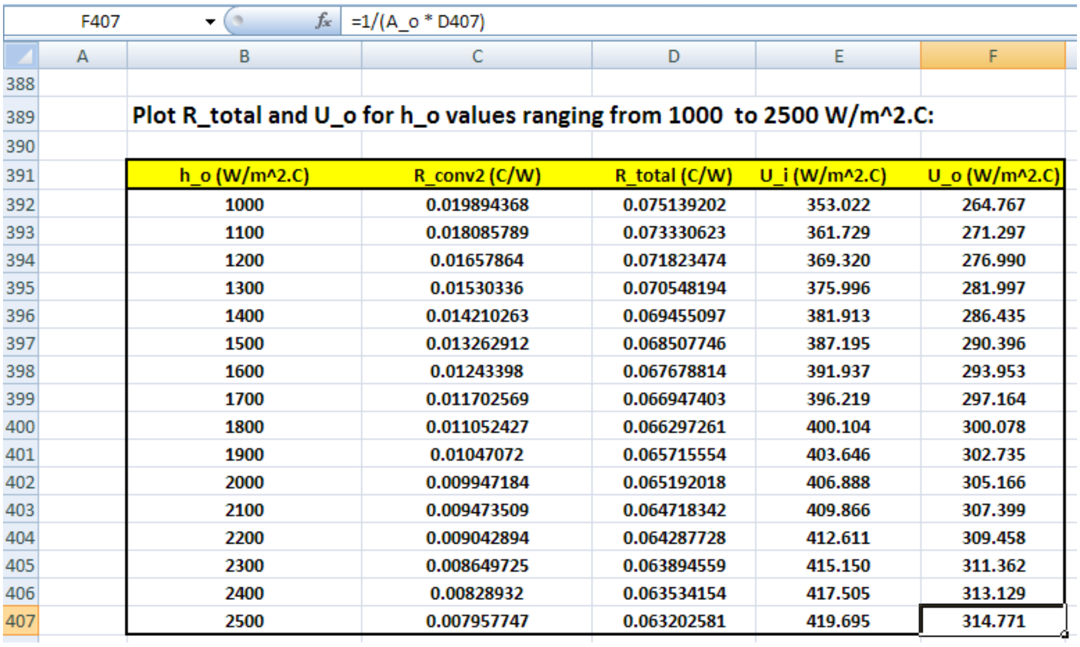


C392 =1/(B392 * A_o)

	h_o (W/m^2.C)	R_conv2 (C/W)	R_total (C/W)	U_i (W/m^2.C)	U_o (W/m^2.C)
392	1000	0.019894368	0.075139202	353.022	264.767
393	1100				
394	1200				
395	1300				
396	1400				
397	1500				
398	1600				
399	1700				
400	1800				
401	1900				
402	2000				
403	2100				
404	2200				
405	2300				
406	2400				
407	2500				

Enter the formulas for R_conv2, R_total, U_i and U_o in the first row, i.e. in row no. 392 as shown, remembering to use 'relative reference' to h_o (i.e. cell B392) in the equations. See in the Formula bar the eqn entered for R_conv2 in cell C392.

Now, simply select cells C392 to F392 and 'drag copy' to the end of the Table (i.e. up to cell F407) and immediately, the calculations are made and the Table is filled up:

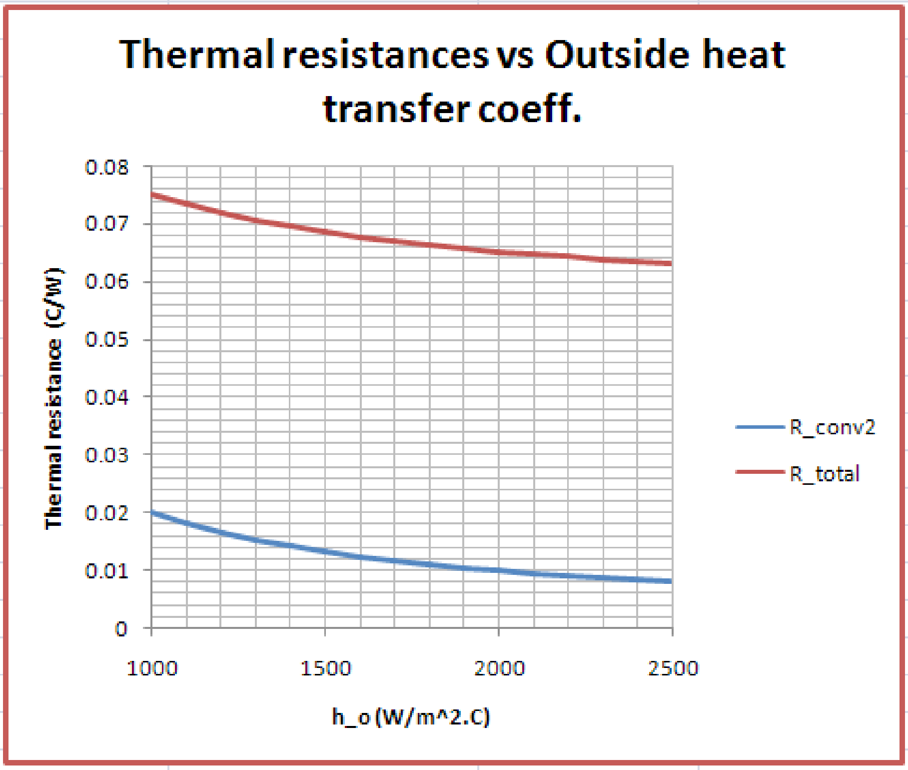


F407 =1/(A_o * D407)

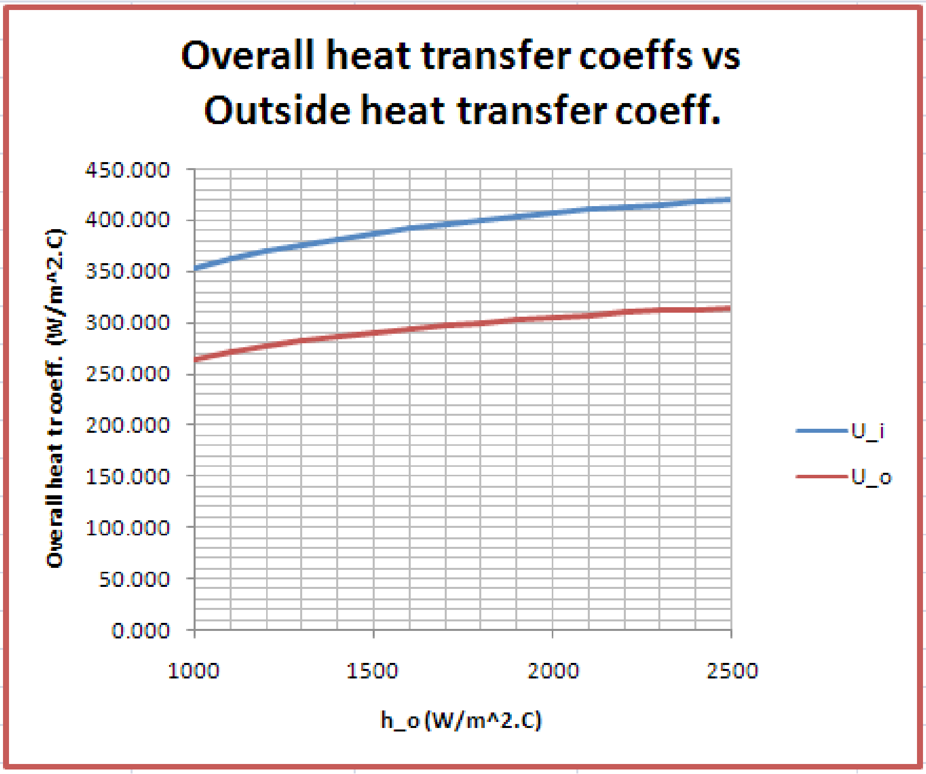
Plot R_total and U_o for h_o values ranging from 1000 to 2500 W/m^2.C:

	h_o (W/m^2.C)	R_conv2 (C/W)	R_total (C/W)	U_i (W/m^2.C)	U_o (W/m^2.C)
392	1000	0.019894368	0.075139202	353.022	264.767
393	1100	0.018085789	0.073330623	361.729	271.297
394	1200	0.01657864	0.071823474	369.320	276.990
395	1300	0.01530336	0.070548194	375.996	281.997
396	1400	0.014210263	0.069455097	381.913	286.435
397	1500	0.013262912	0.068507746	387.195	290.396
398	1600	0.01243398	0.067678814	391.937	293.953
399	1700	0.011702569	0.066947403	396.219	297.164
400	1800	0.011052427	0.066297261	400.104	300.078
401	1900	0.01047072	0.065715554	403.646	302.735
402	2000	0.009947184	0.065192018	406.888	305.166
403	2100	0.009473509	0.064718342	409.866	307.399
404	2200	0.009042894	0.064287728	412.611	309.458
405	2300	0.008649725	0.063894559	415.150	311.362
406	2400	0.00828932	0.063534154	417.505	313.129
407	2500	0.007957747	0.063202581	419.695	314.771

Now, plot the graphs a desired:



And,



==

4B. Problems on LMTD method of heat exchanger design, Use of Correction factor (F) etc.

Prob. 4B.1. Water at a rate of 4080 kg/h is heated from 35 C to 75 C by an oil of Cp = 1.9 kJ/(kg.K). The HX is of counter-flow, double pipe design. The oil enters at 110 C and leaves at 75 C. Determine:

(i) mass flow rate of oil (ii) area of HX necessary to handle this load, if overall heat transfer coeff., U = 320 W/(m^2.K). [M.U.]

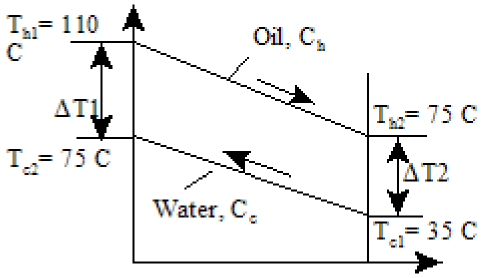
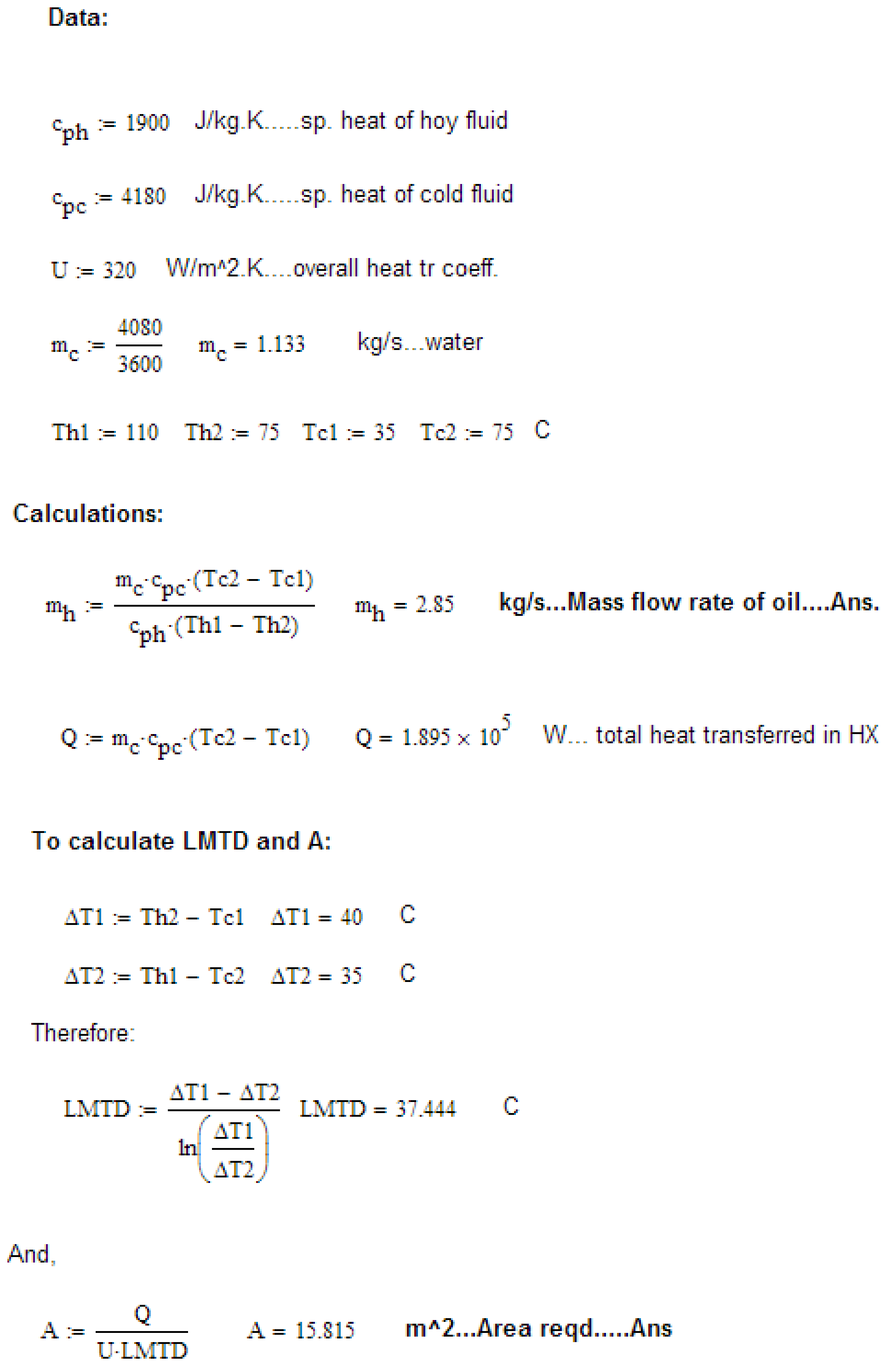


Fig. Prob.4B.1

Mathcad Solution:



Data:

$c_{ph} := 1900$ J/kg.K.....sp. heat of hoy fluid

$c_{pc} := 4180$ J/kg.K.....sp. heat of cold fluid

$U := 320$ W/m^2.K....overall heat tr coeff.

$m_c := \frac{4080}{3600}$ $m_c = 1.133$ kg/s...water

$Th1 := 110$ $Th2 := 75$ $Tc1 := 35$ $Tc2 := 75$ C

Calculations:

$m_h := \frac{m_c \cdot c_{pc} \cdot (Tc2 - Tc1)}{c_{ph} \cdot (Th1 - Th2)}$ $m_h = 2.85$ **kg/s...Mass flow rate of oil....Ans.**

$Q := m_c \cdot c_{pc} \cdot (Tc2 - Tc1)$ $Q = 1.895 \times 10^5$ W... total heat transferred in HX

To calculate LMTD and A:

$\Delta T1 := Th2 - Tc1$ $\Delta T1 = 40$ C

$\Delta T2 := Th1 - Tc2$ $\Delta T2 = 35$ C

Therefore:

$LMTD := \frac{\Delta T1 - \Delta T2}{\ln\left(\frac{\Delta T1}{\Delta T2}\right)}$ $LMTD = 37.444$ C

And,

$A := \frac{Q}{U \cdot LMTD}$ $A = 15.815$ **m^2...Area reqd.....Ans**

Consider following extension to this problem:

If after 3 years there is scale formation on the water side, and the outlet temp is 60 C for the same inlet temperatures and flow rates, find out: (i) heat transferred (ii) outlet temp of oil (iii) overall heat transfer coeff, and (iv) water side fouling factor, R_fw.

So, we have: $Tc2 := 60$ C

(i) Therefore: $Q := m_c \cdot c_{pc} \cdot (Tc2 - Tc1)$

i.e. $Q = 1.184 \times 10^5$ **W.....Ans.**

(ii) Outlet temp of oil:

We have: $Q = m_h \cdot c_{ph} \cdot (Th1 - Th2)$

and, $Th2 := Th1 - \dfrac{Q}{m_h \cdot c_{ph}}$

i.e. $Th2 = 88.125$ **C.....Ans.**

(iii) overall heat tr coeff, U:

To calculate LMTD and U:

$\Delta T1 := Th2 - Tc1$ $\Delta T1 = 53.125$ C

$\Delta T2 := Th1 - Tc2$ $\Delta T2 = 50$ C

Therefore:

$$LMTD := \frac{\Delta T1 - \Delta T2}{\ln\left(\dfrac{\Delta T1}{\Delta T2}\right)} \qquad LMTD = 51.547 \quad \text{C}$$

And,

$$U_{dirty} := \frac{Q}{A \cdot LMTD} \qquad U_{dirty} = 145.283$$ **W/m^2.C...Ans**

(iv) Water side Fouling factor:

By definition: $R_f = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}}$

Here: $U_{clean} := 320$ W/m^2.C

Therefore: $R_f := \frac{1}{U_{dirty}} - \frac{1}{U_{clean}}$

i.e. $R_f = 3.758 \times 10^{-3}$ **m^2.C/W......Ans.**

==

Prob. 4B.2. A water pre-heater of ID:3.2 cm, OD:3.52 cm, is heated by steam at 180 C. Water flows through pipe at a velocity of 1.2 m/s. 'h' on steam side:11000 $W/m^2.K$; water is heated from 25 C to 95 C. k of pipe material:59 W/m.K. Properties of water at 60 C are given. Calculate the length required. Use appropriate empirical relation.

Given: $\mu = 4.62 \times 10^{-4}$ kg/m.s; k = 0.653 W/m.K; Cp = 4200 J/kg.K. [M.U.]

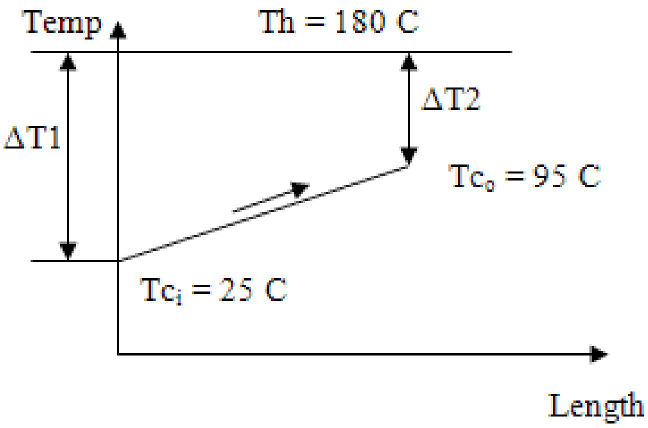


Fig. Prob.4B.2.

Mathcad Solution:

Data:

$Dc_i := 0.032$ m...ID of pipe $Dc_o := 0.0352$ m....OD of pipe $k := 59$ W/m.C...th. cond. of pipe

$Tc_i := 25$ C...Inlet temp $Tc_o := 95$ C....outlet temp $Th := 180$ C....condensing temp

$h_h := 11000$ W/m^2.C....on the condensing side

$V := 1.2$ m/s....water velocity in pipe

Properties of Water at bulk mean temp of 60 C:

$\mu_c := 4.62 \cdot 10^{-4}$ kg/m.s $k_c := 0.653$ W/m.C $\rho_c := 998$ kg/m^3

$cp_c := 4200$ J/kg.C

$Pr := \frac{\mu_c \cdot cp_c}{k_c}$ $Pr = 2.972$ ....Prandtl No.

Calculations:

LMTD:

$\Delta T1 := Th - Tc_i$ i.e. $\Delta T1 = 155$ C

$\Delta T2 := Th - Tc_o$ i.e. $\Delta T2 = 85$ C

$$LMTD := \frac{\Delta T1 - \Delta T2}{\ln\left(\frac{\Delta T1}{\Delta T2}\right)}$$

i.e. $LMTD = 116.516$ C

Area of cross-section:

$A_i := \frac{\pi \cdot Dc_i^{\,2}}{4}$ i.e. $A_i = 8.042 \times 10^{-4}$ m^2

Mass flow rate:

$m := A_i \cdot \rho_c \cdot V$ i.e. $m = 0.963$ kg/s

Mass velocity:

$G := \rho_c \cdot V$ i.e. $G = 1.198 \times 10^3$ kg/m^2.s

Reynolds No. on water side:

$Re_c := \frac{G \cdot Dc_i}{\mu_c}$ i.e. $Re_c = 8.295 \times 10^4$

Nusselts No...by Dittus-Boelter eqn.:

$Nu := 0.023 \cdot Re_c^{\,0.8} \cdot Pr^{0.4}$ i.e. $Nu = 306.179$

Heat tr coeff on water side:

$h_c := \frac{Nu \cdot k_c}{Dc_i}$ i.e. $h_c = 6.248 \times 10^3$ W/m^2.C

Total thermal resistance per metre length:

$$R_t := \frac{1}{h_c\cdot\pi\cdot Dc_i\cdot 1} + \frac{1}{h_h\cdot\pi\cdot Dc_o\cdot 1} + \frac{1}{2\cdot\pi\cdot k\cdot 1}\cdot\ln\left(\frac{Dc_o}{Dc_i}\right)$$

i.e. $R_t = 2.671\times 10^{-3}$ C/W...Total thermal resist/metre length

In the above expression for R_t, first term on the RHS is the *internal convective resistance*, second term is the *external convective resistance* and the third term is the *conductive resistance of the tube wall.*

Overall heat tr coeff:

$$U_i := \frac{1}{R_t\cdot\pi\cdot Dc_i\cdot 1}$$ i.e. $U_i = 3.724\times 10^{3}$ W/m^2.C

Total heat transfer:

$$Q_{tot} := m\cdot cp_c\cdot\left(Tc_o - Tc_i\right)$$ i.e. $Q_{tot} = 2.832\times 10^{5}$ W

Total heat transfer area:

$$A_{tot} := \frac{Q_{tot}}{U_i \cdot LMTD}$$

i.e. $A_{tot} = 0.653$ m^2

Length reqd:

$$L := \frac{A_{tot}}{\pi \cdot Dc_i}$$ i.e. $L = 6.492$ **m.....Ans.**

Consider the following extension of this problem:

Overall heat transfer coeff, U depends mostly on the smaller of the two heat transfer coeffs, i.e. on the water side heat transfer coeff, h_c. Plot the variation of L when h_c varies from 1000 to 10000 W/m^2.C:

Express relevant quantities as functions of h_c:

Total thermal resistance per metre length:

$$R_t(h_c) := \frac{1}{h_c \cdot \pi \cdot Dc_i \cdot 1} + \frac{1}{h_h \cdot \pi \cdot Dc_o \cdot 1} + \frac{1}{2 \cdot \pi \cdot k \cdot 1} \cdot \ln\left(\frac{Dc_o}{Dc_i}\right)$$

Overall heat tr coeff:

$$U_i(h_c) := \frac{1}{R_t(h_c) \cdot \pi \cdot Dc_i \cdot 1}$$

Total heat transfer:

$$Q_{tot} := m \cdot cp_c \cdot \left(Tc_o - Tc_i\right)$$

Total heat transfer area:

$$A_{tot}(h_c) := \frac{Q_{tot}}{U_i(h_c) \cdot LMTD}$$

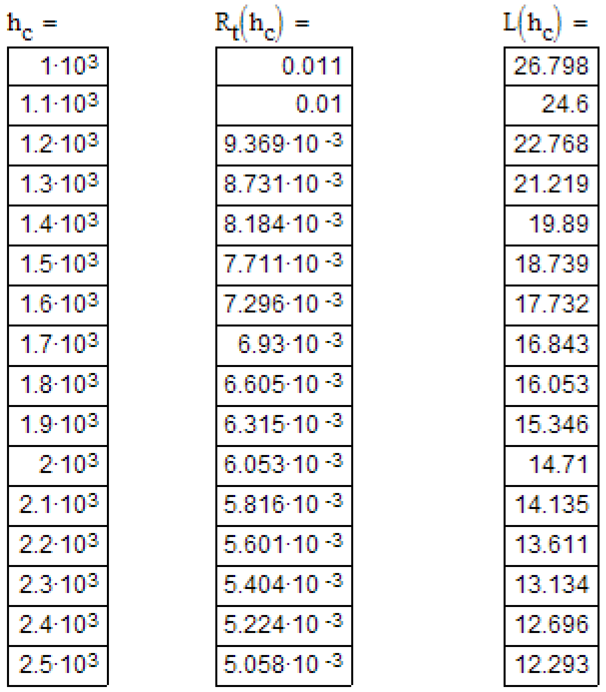
Length reqd:

$$L(h_c) := \frac{A_{tot}(h_c)}{\pi \cdot Dc_i}$$

To plot the graph:

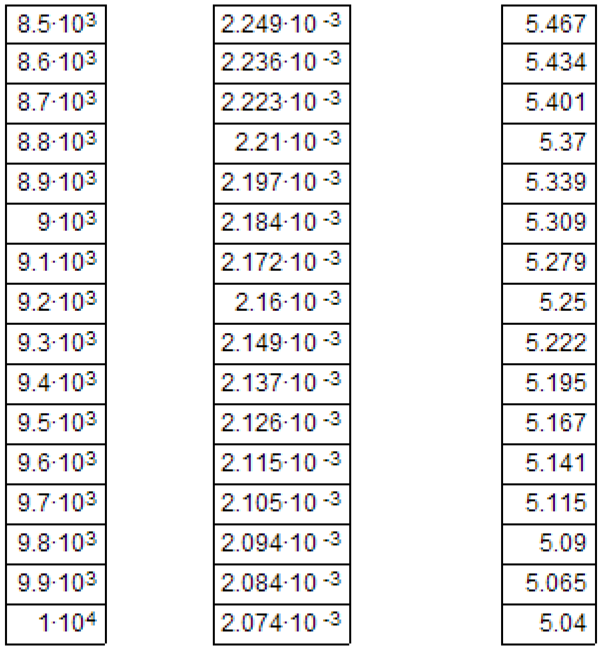
$h_c := 1000, 1100 .. 10000$ ...define a range variable hc

Part of the parametric table is shown below:

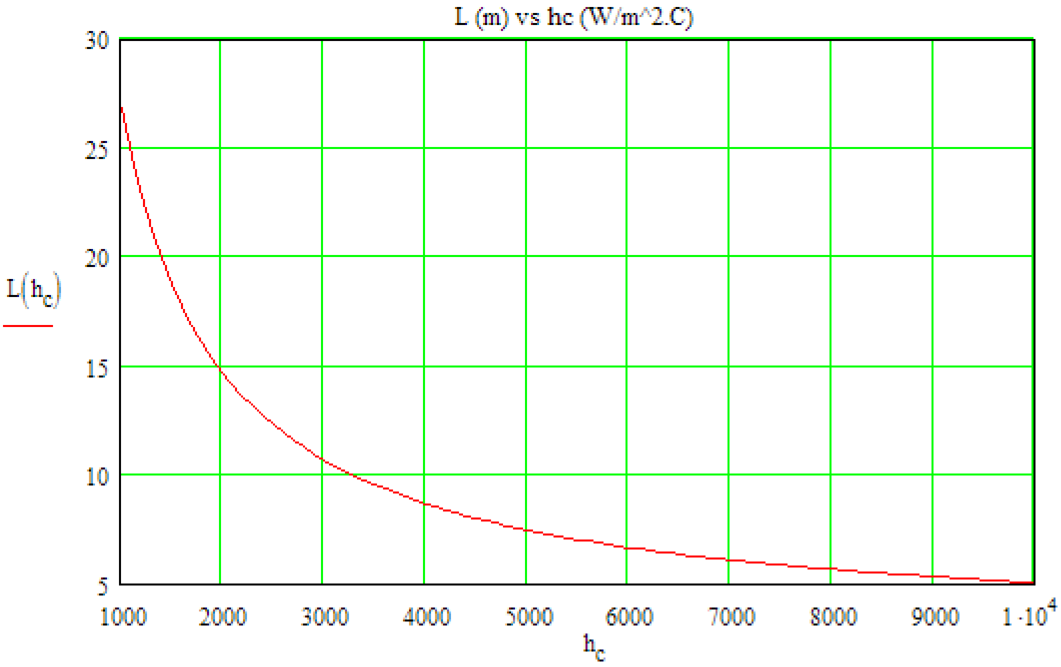


$h_c =$	$R_t(h_c) =$	$L(h_c) =$
$1\cdot 10^{3}$	0.011	26.798
$1.1\cdot 10^{3}$	0.01	24.6
$1.2\cdot 10^{3}$	$9.369\cdot 10^{-3}$	22.768
$1.3\cdot 10^{3}$	$8.731\cdot 10^{-3}$	21.219
$1.4\cdot 10^{3}$	$8.184\cdot 10^{-3}$	19.89
$1.5\cdot 10^{3}$	$7.711\cdot 10^{-3}$	18.739
$1.6\cdot 10^{3}$	$7.296\cdot 10^{-3}$	17.732
$1.7\cdot 10^{3}$	$6.93\cdot 10^{-3}$	16.843
$1.8\cdot 10^{3}$	$6.605\cdot 10^{-3}$	16.053
$1.9\cdot 10^{3}$	$6.315\cdot 10^{-3}$	15.346
$2\cdot 10^{3}$	$6.053\cdot 10^{-3}$	14.71
$2.1\cdot 10^{3}$	$5.816\cdot 10^{-3}$	14.135
$2.2\cdot 10^{3}$	$5.601\cdot 10^{-3}$	13.611
$2.3\cdot 10^{3}$	$5.404\cdot 10^{-3}$	13.134
$2.4\cdot 10^{3}$	$5.224\cdot 10^{-3}$	12.696
$2.5\cdot 10^{3}$	$5.058\cdot 10^{-3}$	12.293

……



$8.5\cdot 10^{3}$	$2.249\cdot 10^{-3}$	5.467
$8.6\cdot 10^{3}$	$2.236\cdot 10^{-3}$	5.434
$8.7\cdot 10^{3}$	$2.223\cdot 10^{-3}$	5.401
$8.8\cdot 10^{3}$	$2.21\cdot 10^{-3}$	5.37
$8.9\cdot 10^{3}$	$2.197\cdot 10^{-3}$	5.339
$9\cdot 10^{3}$	$2.184\cdot 10^{-3}$	5.309
$9.1\cdot 10^{3}$	$2.172\cdot 10^{-3}$	5.279
$9.2\cdot 10^{3}$	$2.16\cdot 10^{-3}$	5.25
$9.3\cdot 10^{3}$	$2.149\cdot 10^{-3}$	5.222
$9.4\cdot 10^{3}$	$2.137\cdot 10^{-3}$	5.195
$9.5\cdot 10^{3}$	$2.126\cdot 10^{-3}$	5.167
$9.6\cdot 10^{3}$	$2.115\cdot 10^{-3}$	5.141
$9.7\cdot 10^{3}$	$2.105\cdot 10^{-3}$	5.115
$9.8\cdot 10^{3}$	$2.094\cdot 10^{-3}$	5.09
$9.9\cdot 10^{3}$	$2.084\cdot 10^{-3}$	5.065
$1\cdot 10^{4}$	$2.074\cdot 10^{-3}$	5.04


L (m) vs hc (W/m^2.C)
$L(h_c)$
30
25
20
15
10
5
1000
2000
3000
4000
5000
6000
7000
8000
9000
$1\cdot10^4$
h_c

==

Prob. 4B.3. Calculate surface area required for a HX to cool 55000 kg/h of Alcohol from 66 C to 40 C using 40000 kg/h of water entering at 5 C. Assume U based on outside area of tubes as 570 W/m^2.K. cp of Alcohol is 3.8 kJ/kg.K and for water 4.187 kJ/kg.K, for the following arrangements:

(i) counter-flow tube & shell (ii) Parallel flow tube & shell (iii) Reversed current HX with 2 shell passes and 12 tube passes with Alcohol flow in the shell. Assume LMTD correction factor as 0.96 (iv) cross flow with one tube pass with shell side fluid assumed to be mixed with LMTD correction factor as 0.91. [M.U. 1997]

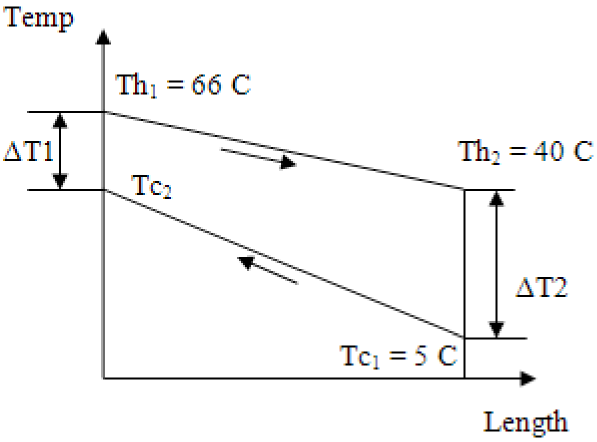


Fig. Prob.4B.3(a). Counter-flow arrangement

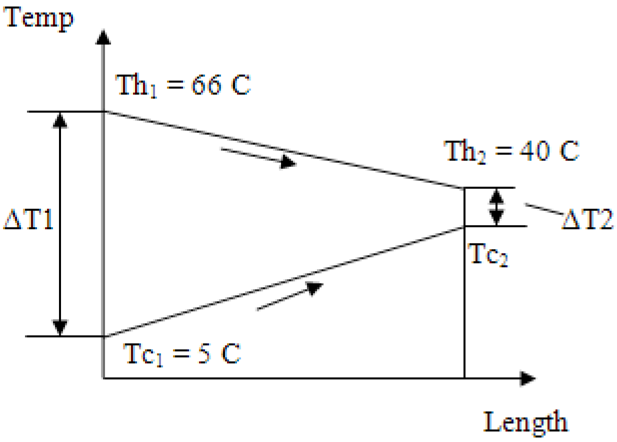


Fig. Prob.4B.3(b). Parallel flow arrangement

Mathcad Solution:

Data:

$mh := \frac{55000}{3600}$ i.e. $mh = 15.278$ kg/s

$mc := \frac{40000}{3600}$ i.e. $mc = 11.111$ kg/s

$Th1 := 66$ C $Th2 := 40$ C $Tc1 := 5$ C $U := 570$ W/m^2.K

$cpc := 4.187$ kJ/kg.K $cph := 3.8$ kJ/kg.K

$F1 := 0.96$ ...LMTD correction factor, case (iii)

$F2 := 0.91$ ...LMTD correction factor, case (iv)

Calculations:

Case (i): Counterflow HX:

$Q := mh \cdot cph \cdot (Th1 - Th2)$ i.e. $Q = 1.509 \times 10^3$ kW

$Tc2 := Tc1 + \frac{Q}{mc \cdot cpc}$ i.e. $Tc2 = 37.446$ C

$LMTD1 := \frac{(Th1 - Tc2) - (Th2 - Tc1)}{\ln\left(\left(\frac{Th1 - Tc2}{Th2 - Tc1}\right)\right)}$ i.e. $LMTD1 = 31.668$ C

Therefore:

$A1 := \frac{Q \cdot 10^3}{U \cdot LMTD1}$ i.e. $A1 = 83.622$ **m^2....Ans.**

Case (ii): Parallel flow HX:

$Q := mh \cdot cph \cdot (Th1 - Th2)$ i.e. $Q = 1.509 \times 10^3$ kW

$Tc2 := Tc1 + \frac{Q}{mc \cdot cpc}$ i.e. $Tc2 = 37.446$ C

$LMTD2 := \frac{(Th1 - Tc1) - (Th2 - Tc2)}{\ln\left(\left(\frac{Th1 - Tc1}{Th2 - Tc2}\right)\right)}$ i.e. $LMTD2 = 18.419$ C

Therefore:

$A2 := \frac{Q \cdot 10^3}{U \cdot LMTD2}$ i.e. $A2 = 143.771$ **m^2....Ans.**

Case (iii): Reversed current HX:

$A3 := \frac{Q \cdot 10^3}{F1 \cdot U \cdot LMTD1}$ i.e. $A3 = 87.107$ **m^2....Ans**

Case (iv): Cross flow current HX:

$A4 := \frac{Q \cdot 10^3}{F2 \cdot U \cdot LMTD1}$ i.e. $A4 = 91.893$ **m^2....Ans**

Mathcad Function for LMTD correction factor – F for Shell & Tube HX:

Shell &Tube heat exchangers are very commonly used in Industry.

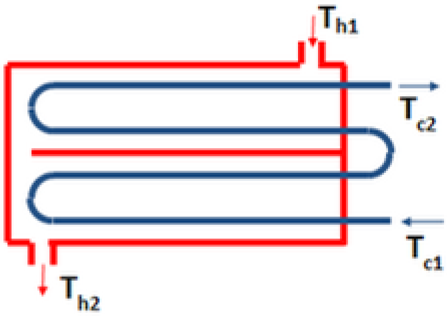
While designing Shell & Tube heat exchangers, we have to first calculate the LMTD as if it is a simple counter-flow HX and then apply a correction factor, F. F depends upon the types of HX, i.e. the no. of shell passes, no. of tube passes in the case of Shell & Tube HX, and whether it is a cross flow HX and if the fluids are 'mixed' (i.e. not confined in a channel) or 'unmixed' (i.e. confined to a channel) etc. F is given in graphs as a function of R and P, where P = (t2 – t1) / (T1 – t1) where T, t stand for Shell side and tube side flows, and 1, 2 stand for inlet and exit of the flows; R = (T1 – T2) / (t2 – t1) , i.e. ratio of temp drops of Shell side and tube side flows. F is generally read from graphs such as those given in the beginning of this chapter.

However, while designing with computer software, it is preferable to use equations to determine F, since it will be more accurate and avoid interpolating in the graphs.

In Shell & Tube HX, generally the hot fluid flows in the shell and the cold fluid, in the tubes.

We have the following formula to determine the LMTD correction Factor, F:

Schematic, with usual notations:



Equations:

$$LMTD = \frac{(T_{h1} - T_{c2}) - (T_{h2} - T_{c1})}{\ln\left(\frac{T_{h1} - T_{c2}}{T_{h2} - T_{c1}}\right)}$$

$$P = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} \qquad R = \frac{T_{h1} - T_{h2}}{T_{c2} - T_{c1}}$$

If R is not equal to 1:

$$X = \frac{1 - \left(\frac{R\,P - 1}{P - 1}\right)^{\frac{1}{N}}}{R - \left(\frac{R\,P - 1}{P - 1}\right)^{\frac{1}{N}}}$$

$$F = \frac{\left(\frac{\sqrt{R^2 + 1}}{R - 1}\right) ln\left(\frac{1 - X}{1 - R\,X}\right)}{\ln\left(\frac{\frac{2}{X} - 1 - R + \sqrt{R^2 + 1}}{\frac{2}{X} - 1 - R - \sqrt{R^2 + 1}}\right)}$$

If R = 1:

$$X = \frac{P}{(N - N \cdot P + P)}$$

$$F = \frac{X \cdot \sqrt{2}}{(1 - X) \cdot \ln\left[\frac{2 \cdot (1 - X) + X \cdot \sqrt{2}}{2 \cdot (1 - X) - X \cdot \sqrt{2}}\right]}$$

In the above, N is the no. of simple shells or no. of shell passes.

It should be noted that LMTD correction factors lower than 0.8 indicate inefficient heat exchanger design. Heat Exchanger Design Handbooks suggest that the minimum value should be 0.75.

Now, let us write a Mathcad Function to determine LMTD correction factor, F for Shell & Tube Heat Exchangers:

Following program takes care of the cases R = 1 and R other than 1.

In the Function given below:

Input: Tshell1 (Inlet temp of Shell side fluid, C), Tshell2 (exit temp of shell side fluid, C), Tube1 and Ttube2 (inlet and exit temp of rube side fluid, C), N is the no. of simple shells or no. of shell passes.

Output: LMTD correction Factor, F…..given in an array of LMTD for counter-flow, P, R and F.

```
$LMTDCorrectionFactor\_ShellTubeHX\_F(Tshell_1, Tshell_2, Ttube_1, Ttube_2, N) :=$
    $LMTD \leftarrow \dfrac{|Tshell_1 - Ttube_2| - |Tshell_2 - Ttube_1|}{\ln\left(\dfrac{Tshell_1 - Ttube_2}{Tshell_2 - Ttube_1}\right)}$
    if $Tshell_1 = Tshell_2 \lor Ttube_1 = Ttube_2$
        $F \leftarrow 1$
        return $\begin{pmatrix} \text{"LMTD\_CounterFlow"} & \text{"Correction\_Factor F"} \\ LMTD & F \end{pmatrix}$
    $P \leftarrow \dfrac{Ttube_2 - Ttube_1}{Tshell_1 - Ttube_1}$
    $R \leftarrow \dfrac{Tshell_1 - Tshell_2}{Ttube_2 - Ttube_1}$
    if $R = 1$
        $X \leftarrow \dfrac{P}{(N - N \cdot P + P)}$
        $F \leftarrow \dfrac{X \cdot \sqrt{2}}{(1 - X) \cdot \ln\left[\dfrac{2 \cdot (1 - X) + X \cdot \sqrt{2}}{2 \cdot (1 - X) - X \cdot \sqrt{2}}\right]}$
        return $\begin{pmatrix} \text{"LMTD\_CounterFlow"} & \text{"P"} & \text{"R"} & \text{"Correction\_Factor F"} \\ LMTD & P & R & F \end{pmatrix}$
    otherwise
        $X \leftarrow \dfrac{1 - \left(\dfrac{R \cdot P - 1}{P - 1}\right)^{\frac{1}{N}}}{R - \left(\dfrac{R \cdot P - 1}{P - 1}\right)^{\frac{1}{N}}}$
        $F \leftarrow \dfrac{\dfrac{\sqrt{R^2 + 1}}{R - 1} \cdot \ln\left(\dfrac{1 - X}{1 - R \cdot X}\right)}{\ln\left(\dfrac{\dfrac{2}{X} - 1 - R + \sqrt{R^2 + 1}}{\dfrac{2}{X} - 1 - R - \sqrt{R^2 + 1}}\right)}$
        return $\begin{pmatrix} \text{"LMTD\_CounterFlow"} & \text{"P"} & \text{"R"} & \text{"Correction\_Factor F"} \\ LMTD & P & R & F \end{pmatrix}$
```

To demonstrate the use of above Mathcad Function, let us work out the following problem:

Prob. 4B.4. A Shell & Tube HX has 2 shell passes and 12 tube passes. Water (cp = 4180 J/kg.C) is heated in the tubes from 20 C to 70 C at a rate of 4.5 kg/s. Heating is one by hot oil (cp = 2300 J/kg.C) which enters the shell at 170 C at a rate of 10 kg/s. On the tube side, h_c = 600 W/m^2.C. Determine the heat transfer area.

Mathcad Solution:

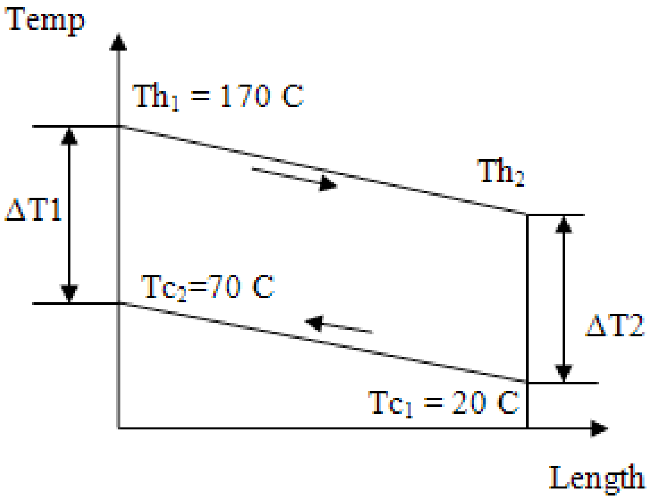


Fig. Prob.4B.4. Temp profile for Counter-flow arrangement

Data:

$mh := 10$ kg/s

$mc := 4.5$ kg/s

$cpc := 4180$ J/kg.K $\quad$ $cph := 2300$ J/kg.K

$Th1 := 170$ C $\quad$ $Tc1 := 20$ C $\quad$ $Tc2 := 70$ C

$U := 600$ W/m^2.C

Calculations:

$Q := mc \cdot cpc \cdot (Tc2 - Tc1)$ $\quad$ i.e. $\quad$ $Q = 9.405 \times 10^5$ W.....heat transferred

Therefore:

$Th2 := Th1 - \dfrac{Q}{mh \cdot cph}$ $\quad$ i.e. $\quad$ $Th2 = 129.109$ C....exit temp of hot fluid

Now, we have: $\quad$ $Q = U \cdot A \cdot \left(LMTD_{CF} \cdot F\right)$ $\quad$ **...where F is the LMTD correction factor**

To determine F:

Use the Mathcad Function written above:

We have:

$N := 2$ $\quad$ ...no. of shell passes

From the Mathcad Function:

$$LMTDCorrectionFactor\_ShellTubeHX\_F(Th1, Th2, Tc1, Tc2, N) = \begin{pmatrix} \text{"LMTD\_CounterFlow"} & \text{"P"} & \text{"R"} & \text{"Correction\_Factor F"} \\ 104.488 & 0.333 & 0.818 & 0.992 \end{pmatrix}$$

Therefore:

$LMTD_{CF} := 104.488$ C....LMTD for a counter-flow HX

$F := 0.992$ ....LMTD correction factor

And,

$$A1 := \frac{Q}{U \cdot LMTD_{CF} \cdot F}$$

i.e. $A1 = 15.123$ **m^2....Area of HX ..Ans.**

If m_C varies from 2 to 5 kg/s, plot the variation of Q and A with m_C:

Note that here, we are assuming Th1, Tc1, Tc2, mh and U to remain constant, and calculate Th2, LMTD_Counterflow, F and A as mc varies.

First, express the relevant quantities as functions of mc, so that it is convenient to plot the results:

$Q(mc) := mc \cdot cpc \cdot (Tc2 - Tc1)$ i.e. $Q(mc) = 9.405 \times 10^{5}$ W.....heat transferred

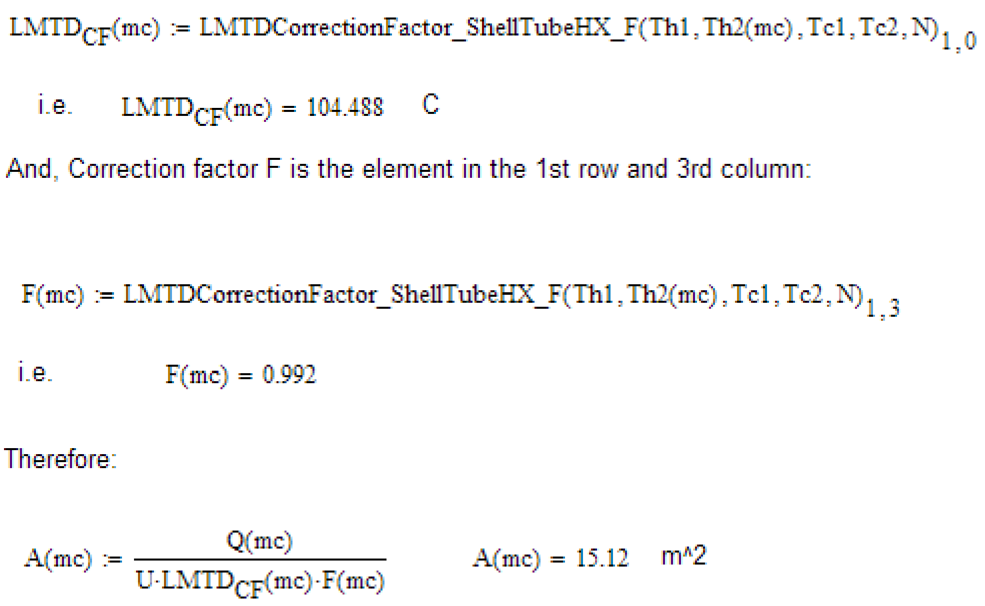
$Th2(mc) := Th1 - \frac{Q(mc)}{mh \cdot cph}$ i.e. $Th2(mc) = 129.109$ C....exit temp of hot fluid

Then,

$$LMTDCorrectionFactor\_ShellTubeHX\_F(Th1, Th2(mc), Tc1, Tc2, N) = \begin{pmatrix} \text{"LMTD\_CounterFlow"} & \text{"P"} & \text{"R"} & \text{"Correction\_Factor F"} \\ 104.488 & 0.333 & 0.818 & 0.992 \end{pmatrix}$$

If we need only the LMTD_Counterflow from the output, we extract it from the output matrix, remembering that in Mathcad, by default, Matrix rows and columns are numbered from zero. i.e. LMTD_Counterflow is the element in the 1st row and zeroth column:

i.e.



$$LMTD_{CF}(mc) := LMTDCorrectionFactor\_ShellTubeHX\_F(Th1, Th2(mc), Tc1, Tc2, N)_{1,0}$$

i.e. $LMTD_{CF}(mc) = 104.488$ C

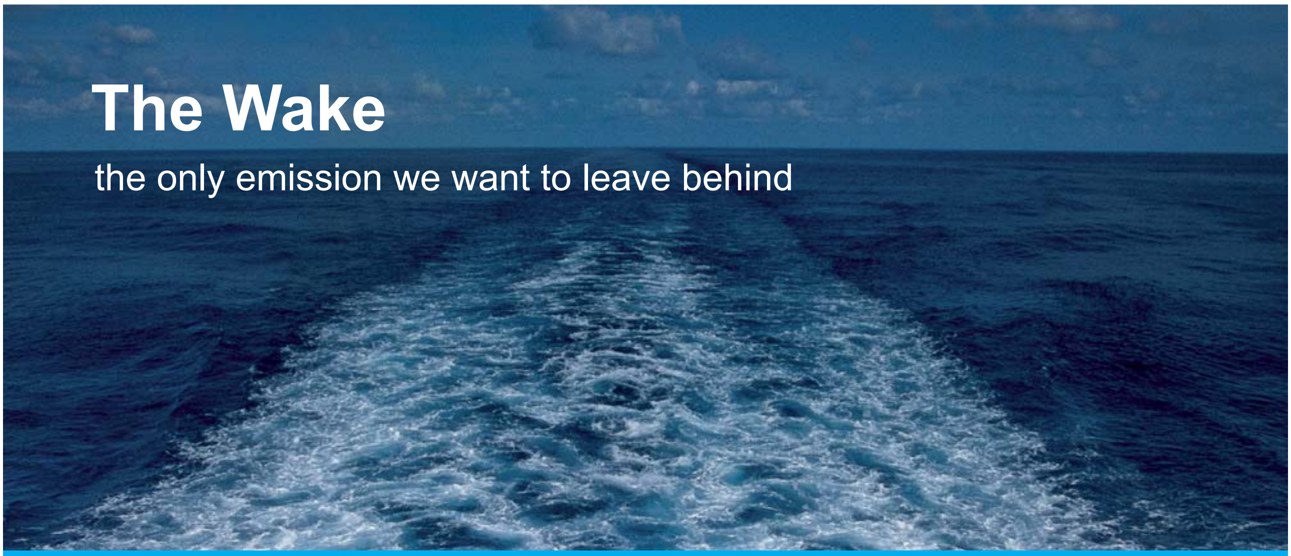
And, Correction factor F is the element in the 1st row and 3rd column:

$$F(mc) := LMTDCorrectionFactor\_ShellTubeHX\_F(Th1, Th2(mc), Tc1, Tc2, N)_{1,3}$$

i.e. $F(mc) = 0.992$

Therefore:

$$A(mc) := \frac{Q(mc)}{U \cdot LMTD_{CF}(mc) \cdot F(mc)} \qquad A(mc) = 15.12 \quad m\text{^}2$$

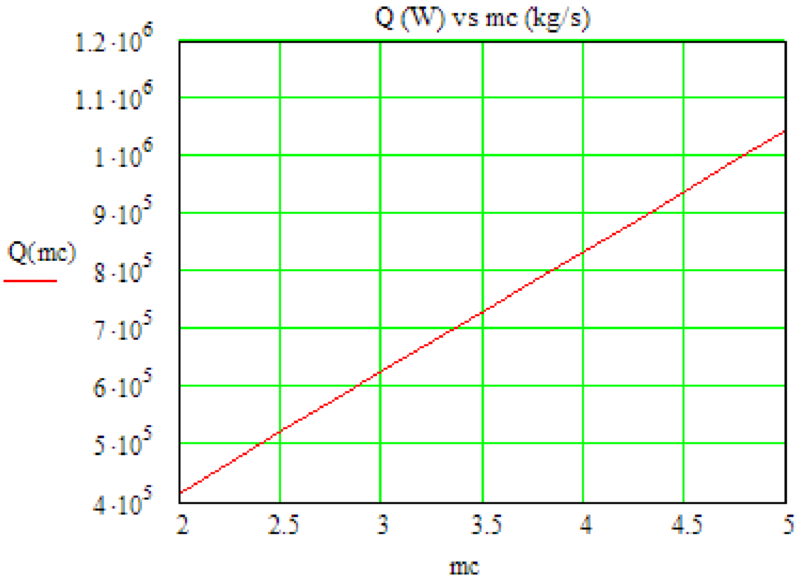


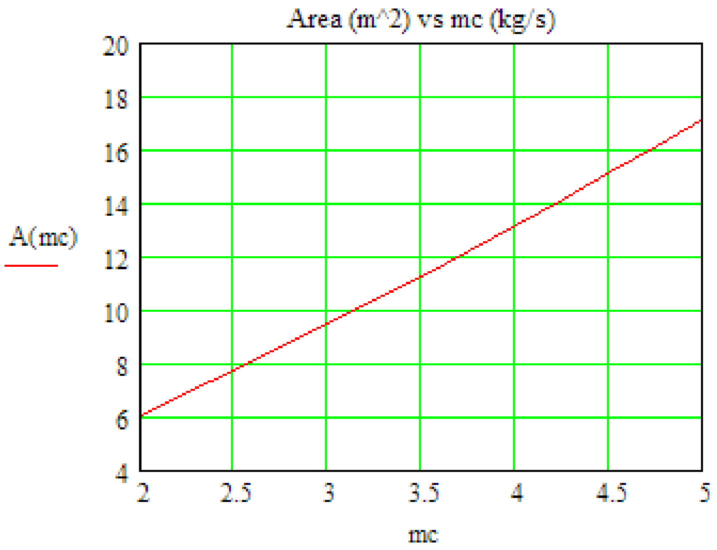
To plot the graph:

$mc := 2, 2.25 .. 5$ ....define a range variable mc

mc =	$LMTD_{CF}(mc)$ =	F(mc) =
2	115.181	0.997
2.25	114.14	0.997
2.5	113.093	0.996
2.75	112.041	0.996
3	110.982	0.995
3.25	109.916	0.995
3.5	108.844	0.994
3.75	107.766	0.994
4	106.681	0.993
4.25	105.588	0.993
4.5	104.488	0.992
4.75	103.381	0.992
5	102.266	0.991

Now, plot the graphs:





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Prob. 4B.5. A Shell & Tube HX has to be designed to heat 2.5 kg/s of water from 15 to 85 C, by passing hot engine oil (unused) at 160 C through the shell side of the HX. Average convection coeff on the oil side is h_h = 400 W/m^2.C on the outside of the tubes. Ten tubes pass the water and each tube is thin-walled of diameter 25 mm and makes 8 passes through the shell. If the oil leaves the exchanger at 100 C, what is its flow rate? How long must the tubes be to accomplish the desired heating? [VTU-M.Tech. May/June 2010]

Mathcad Solution:

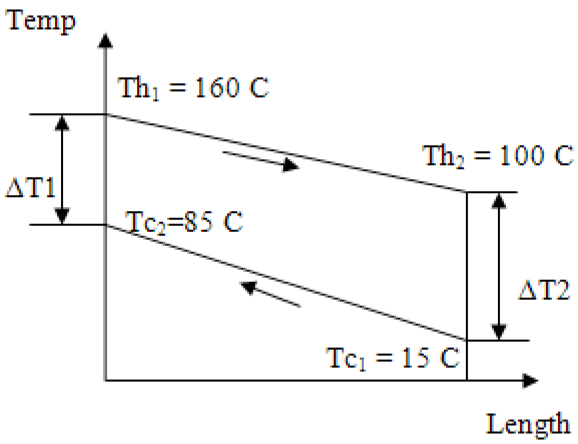
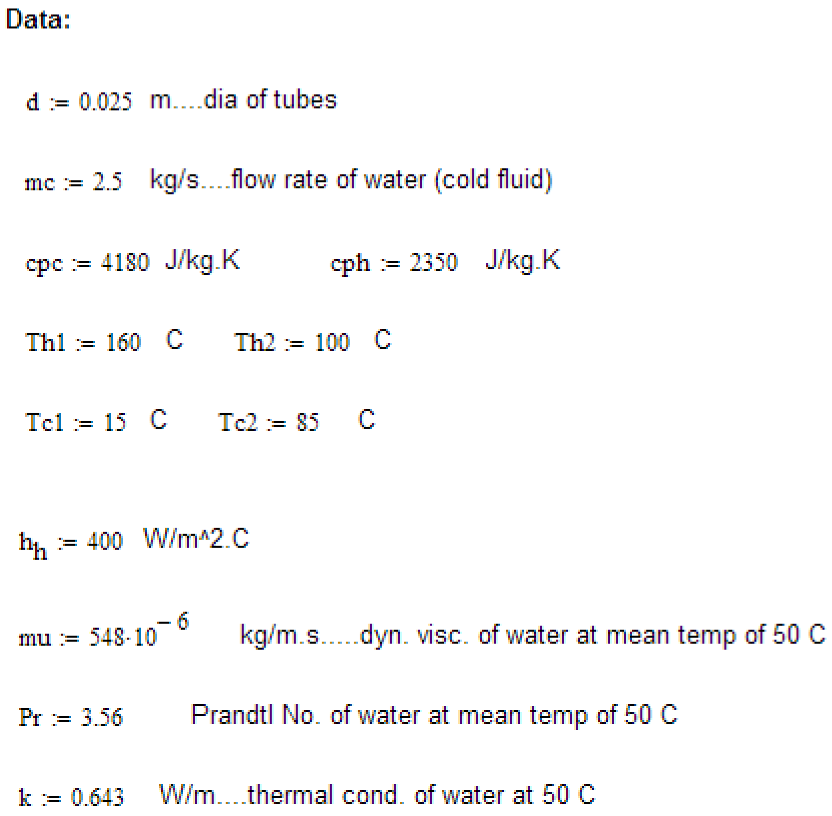


Fig. Prob.4B.5. Temp profile for Counter-flow arrangement



Data:

$d := 0.025$ m....dia of tubes

$mc := 2.5$ kg/s....flow rate of water (cold fluid)

$cpc := 4180$ J/kg.K $\quad cph := 2350$ J/kg.K

$Th1 := 160$ C $\quad Th2 := 100$ C

$Tc1 := 15$ C $\quad Tc2 := 85$ C

$h_h := 400$ W/m^2.C

$mu := 548 \cdot 10^{-6}$ kg/m.s.....dyn. visc. of water at mean temp of 50 C

$Pr := 3.56$ Prandtl No. of water at mean temp of 50 C

$k := 0.643$ W/m....thermal cond. of water at 50 C

Calculations:

$Q := mc \cdot cpc \cdot (Tc2 - Tc1)$ i.e. $Q = 7.315 \times 10^5$ W.....heat transferred

Therefore:

$mh := \frac{Q}{cph \cdot (Th1 - Th2)}$ i.e. $mh = 5.188$ **kg/s....mass flow rate of hot fluid ...Ans.**

To calculate overall heat transfer coeff. U:

$$U = \frac{1}{\frac{1}{h_h} + \frac{1}{h_c}}$$

So, we have to calculate hc, the heat transfer coeff inside the tubes:

Reynolds No. for flow inside tubes:

There are 10 tubes, and total flow rate of water is 2.5 kg/s.

Therefore, flow rate through each tube is:

$m_{tube} := \frac{mc}{10}$ kg/s i.e. $m_{tube} = 0.25$ kg/s

$A_c := \frac{\pi \cdot d^2}{4}$ i.e. $A_c = 4.909 \times 10^{-4}$ m^2....cross-sectional area of tube

$G := \frac{m_{tube}}{A_c}$ i.e. $G = 509.296$ kg/s.m^2.....mass velocity

$Re := \frac{G \cdot d}{mu}$ i.e. $Re = 2.323 \times 10^4$ **...Reynolds No. for water flow inside tubes**

Therefore:

$Nu := 0.023 \cdot Re^{0.8} \cdot Pr^{0.4}$ ...Dittus-Boelter eqn.

i.e. $Nu = 118.908$ **...Nusselts No.**

Therefore: $h_c := \dfrac{Nu \cdot k}{d}$ i.e. $h_c = 3.058 \times 10^3$ **W/m^2.C....heat tr coeff. on the inside of tubes.**

Therefore: $U := \dfrac{1}{\dfrac{1}{h_h} + \dfrac{1}{h_c}}$

i.e. $U = 353.735$ **W/m^2.C....Overall heat tr coeff.**

To calculate LMTD Correction Factor, F:

Use the Mathcad Function written earlier:

$N := 1$ ....No. of Shell passes

$$\text{LMTDCorrectionFactor\_ShellTubeHX\_F}(Th1, Th2, Tc1, Tc2, N) = \begin{pmatrix} \text{"LMTD\_CounterFlow"} & \text{"P"} & \text{"R"} & \text{"Correction\_Factor F"} \\ 79.896 & 0.483 & 0.857 & 0.878 \end{pmatrix}$$

i.e. $LMTD_{CF} := 79.896$ **C...= LMTD for counterflow HX**

$F := 0.878$ **...LMTD Correction Factor for 1 Shell pass, 8 tube passes**

Total area required, A_total:

$$Q = U \cdot A_{total} \cdot (LMTD_{CF} \cdot F)$$

And, $A_{total} = \pi \cdot d \cdot L \cdot 10 \cdot 8$ m^2...where 10 is the no. of tubes, 8 is the no. of passes of each tube, L is the length of each pass

Therefore: $A_{total} := \dfrac{Q}{\left[U \cdot (LMTD_{CF} \cdot F)\right]}$

i.e. $A_{total} = 29.479$ **m^2 .... total surface area**

And, $L := \dfrac{A_{total}}{(\pi \cdot d \cdot 10 \cdot 8)}$

i.e. $L = 4.692$ **m.... length of each pass = Length of Shell .... Ans.**

Plot the variation of oil flow rate and tube length L as water flow rate (mc) varies from 1 to 5 kg/s:

We keep the temperatures fixed, and, h_h also fixed.

Express all relavant quantities as functions of mc, for convenience in plotting:

$Q(mc) := mc \cdot cpc \cdot (Tc2 - Tc1)$ W....Q as a function of mc

Therefore:

$mh(mc) := \frac{Q(mc)}{cph \cdot (Th1 - Th2)}$ kg/s....mh as a function of mc

$m_{tube}(mc) := \frac{mc}{10}$ kg/s....m_{tube} as a function of mc

$G(mc) := \frac{m_{tube}(mc)}{A_c}$ kg/m^2.s....G as a function of mc

$Re(mc) := \frac{G(mc) \cdot d}{mu}$ ..Re as a function of mc

$Nu(mc) := 0.023 \cdot Re(mc)^{0.8} \cdot Pr^{0.4}$ ..Nu as a function of mc

Therefore: $h_c(mc) := \frac{Nu(mc) \cdot k}{d}$ W/m^2.C....hc as a function of mc

And: $U(mc) := \frac{1}{\frac{1}{h_h} + \frac{1}{h_c(mc)}}$ W/m^2.C....U as a function of mc

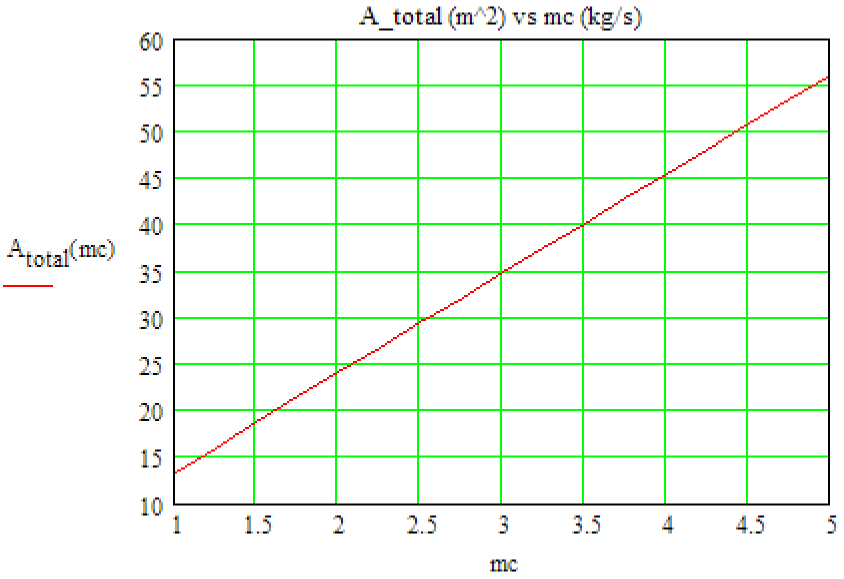
$A_{total}(mc) := \frac{Q(mc)}{\left[U(mc) \cdot \left(LMTD_{CF} \cdot F\right)\right]}$ m^2....Atotal as a function of mc

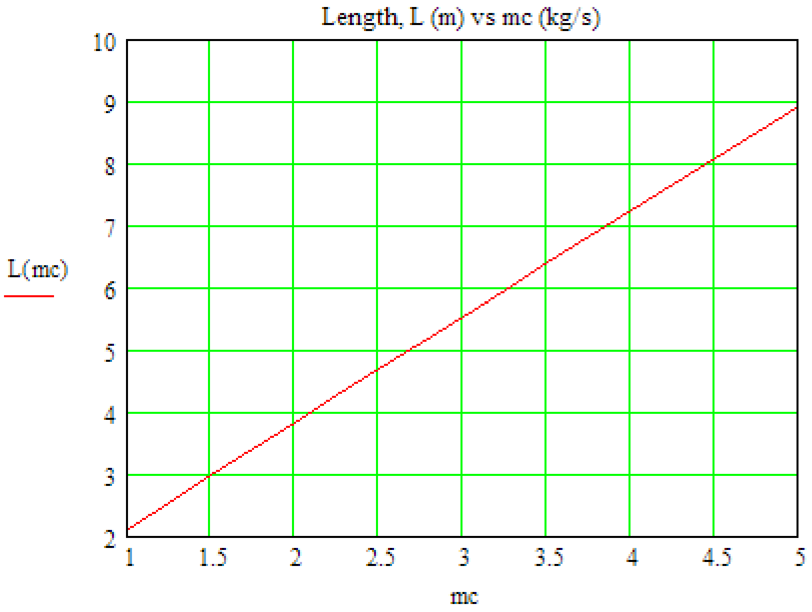
$L(mc) := \frac{A_{total}(mc)}{(\pi \cdot d \cdot 10 \cdot 8)}$ m....L as a function of mc

To plot the graphs:

$mc := 1, 1.25 .. 5$ kg/s.....define a range variable mc

mc =	Re(mc) =	$A_{total}(mc)$ =	L(mc) =
1	$9.294 \cdot 10^3$	13.267	2.111
1.25	$1.162 \cdot 10^4$	16.003	2.547
1.5	$1.394 \cdot 10^4$	18.72	2.979
1.75	$1.626 \cdot 10^4$	21.424	3.41
2	$1.859 \cdot 10^4$	24.117	3.838
2.25	$2.091 \cdot 10^4$	26.801	4.266
2.5	$2.323 \cdot 10^4$	29.479	4.692
2.75	$2.556 \cdot 10^4$	32.152	5.117
3	$2.788 \cdot 10^4$	34.82	5.542
3.25	$3.02 \cdot 10^4$	37.484	5.966
3.5	$3.253 \cdot 10^4$	40.144	6.389
3.75	$3.485 \cdot 10^4$	42.802	6.812
4	$3.717 \cdot 10^4$	45.457	7.235
4.25	$3.95 \cdot 10^4$	48.11	7.657
4.5	$4.182 \cdot 10^4$	50.76	8.079
4.75	$4.415 \cdot 10^4$	53.409	8.5





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Prob. 4B.6. A Shell & Tube HX has to be designed to heat 2 kg/s of air from 20 to 80 C, by passing 3 kg/s of hot oil (cp = 2100 J/kg.C) at 100 C through the tubes of the HX. There are 6 tube passes for oil and one shell pass for air. Overall heat transfer coeff U = 200 W/m^2.C. Calculate the area required.

Mathcad Solution:

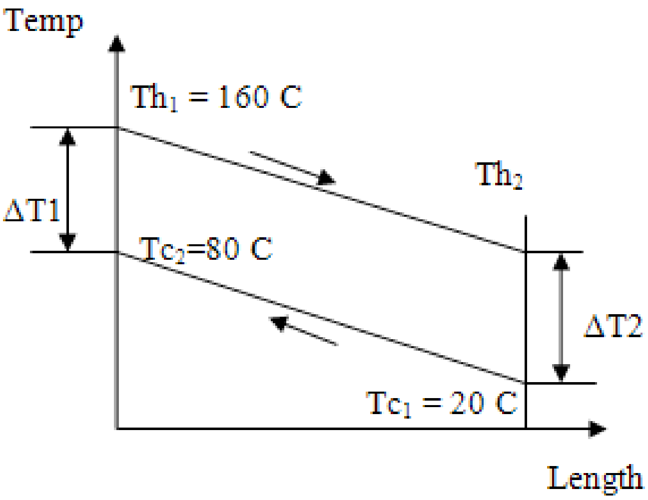
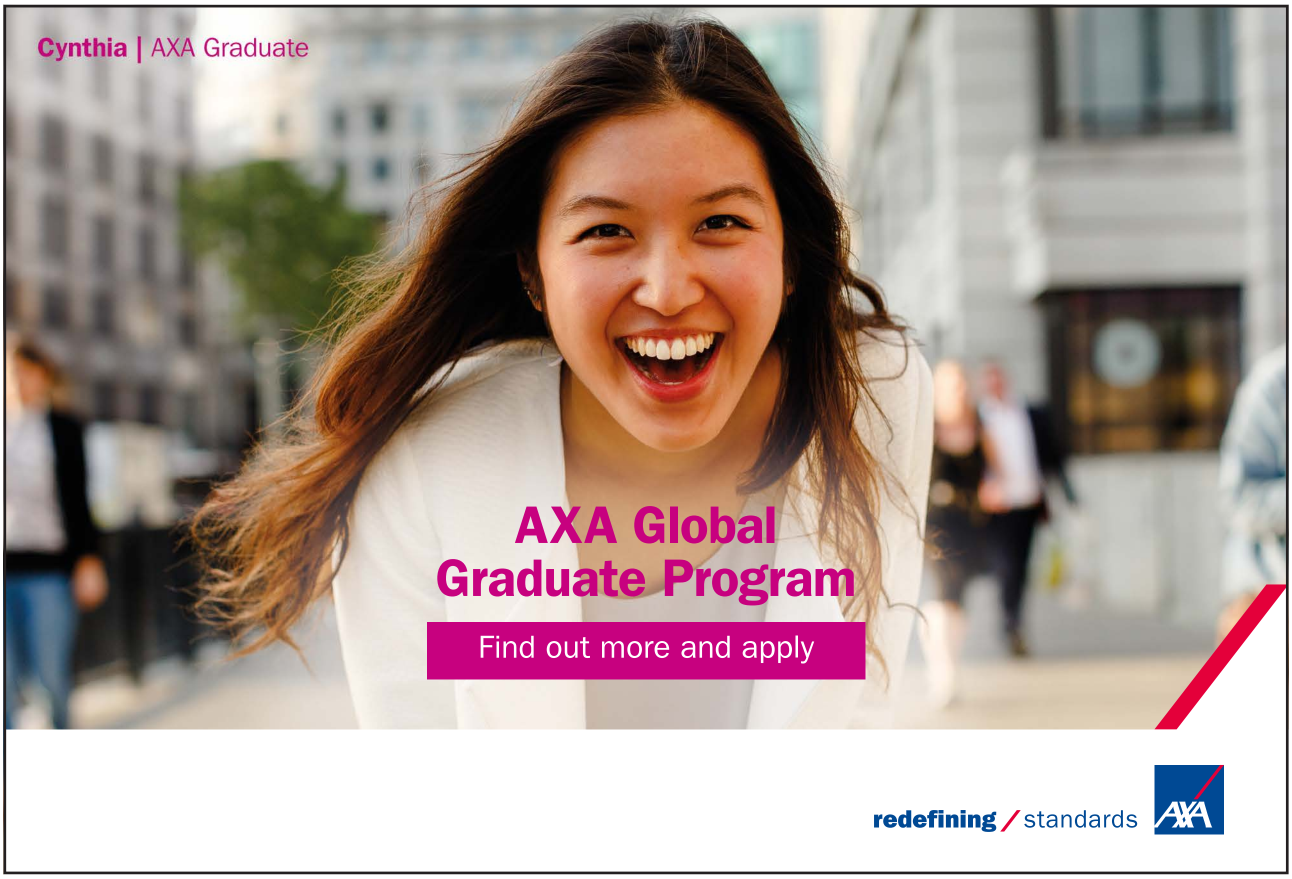


Fig. Prob.4B.6. Temp profile for Counter-flow arrangement



Data:

$mh := 3$ kg/s....mass flow rate of oil (hot fluid)

$mc := 2$ kg/s....mass flow rate of air (cold fluid)

$cpc := 1009$ J/kg.K $cph := 2100$ J/kg.K

$Th1 := 100$ C $Tc1 := 20$ C $Tc2 := 80$ C

$U := 200$ W/m^2.C

Calculations:

$Q := mc \cdot cpc \cdot (Tc2 - Tc1)$ i.e. $Q = 1.211 \times 10^{5}$ W.....heat transferred

Therefore:

$Th2 := Th1 - \frac{Q}{mh \cdot cph}$ i.e. $Th2 = 80.781$ **C....exit temp of hot fluid**

To determine F:

Use the Mathcad Function written above:

We have:

$N := 1$ ...no. of shell passes

$$\mathrm{LMTDCorrectionFactor\_ShellTubeHX\_F}(Th1, Th2, Tc1, Tc2, N) = \begin{pmatrix} \text{"LMTD\_CounterFlow"} & \text{"P"} & \text{"R"} & \text{"Correction\_Factor F"} \\ 36.689 & 0.75 & 0.32 & 0.822 \end{pmatrix}$$

Therefore:

$LMTD_{CF} := 36.689$ **C....LMTD for a counter-flow HX**

$F := 0.822$ **...LMTD Correction Factor for 1 Shell pass, 6 tube pass HX**

And,

$$A1 := \frac{Q}{U \cdot LMTD_{CF} \cdot F}$$

i.e. $A1 = 20.074$ **m^2....Area of HX ..Ans.**

===

Prob. 4B.7. A Shell & Tube HX is used to condense Ammonia vapours at 50 C. Water enters the tubes (single pass) at 20 C and leaves at 40 C. Overall heat transfer coeff is U = 100 W/m^2.C. If the surface area of the HX is 9 m^2, determine the water flow rate required and the Ammonia condensation rate.

Mathcad Solution:

Note that this is a condenser where condensing fluid (i.e. Ammonia) is at a constant temp.

So, LMTD correction factor, F = 1.

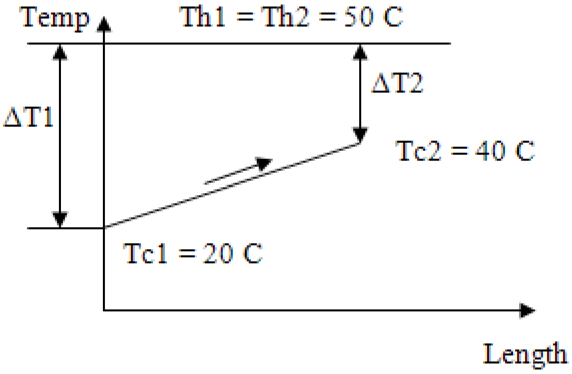


Fig. Prob.4B.7.

Data:

$Th1 := 50$ C $Th2 := 50$ **C....since condensation is at constant temp.**

$Tc1 := 20$ C $Tc2 := 40$ C

$cpc := 4180$ J/kg.K.... sp. heat of water

$U := 100$ W/m^2.C $A := 9$ m^2.... area of HX

$h_{fg} := 1050.5 \cdot 10^3$ J/kg.....heat of evaporation of Ammonia at 50 C

Calculations:

Now, we have: $Q = U \cdot A \cdot \left(LMTD_{CF} \cdot F\right)$ **...where F is the LMTD correction factor**

And, for a Condenser (or Evaporator), F = 1

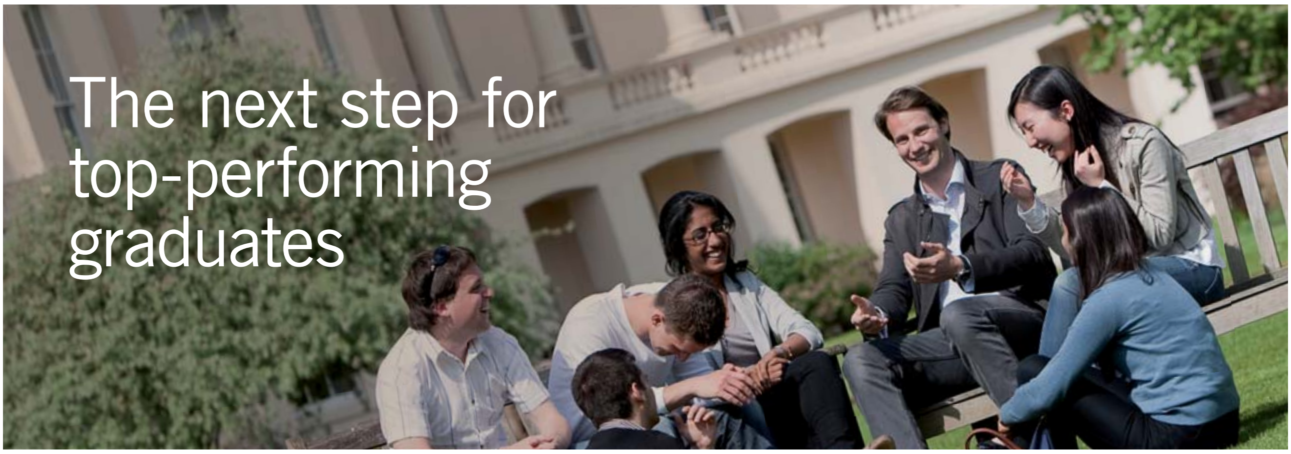
$F := 1$ ....LMTD Correction Factor

So, calculate LMTD:

$\Delta T1 := Th1 - Tc2$ i.e. $\Delta T1 = 10$ C

$\Delta T2 := Th2 - Tc1$ i.e. $\Delta T2 = 30$ C

$$LMTD_{CF} := \frac{\Delta T1 - \Delta T2}{\ln\left(\frac{\Delta T1}{\Delta T2}\right)}$$ i.e. $LMTD_{CF} = 18.205$ **C.....LMTD**



Therefore, water flow rate required, mc:

We have: $Q = mc \cdot cpc \cdot (Tc2 - Tc1)$

Then: $mc := \dfrac{Q}{cpc \cdot (Tc2 - Tc1)}$

i.e. $mc = 0.196$ **kg/s = 705.544 kg/h ....water flow rate required .... Ans.**

Condensation rate of Ammonia:

$m_{cond} := \dfrac{Q}{h_{fg}}$ kg/s

i.e. $m_{cond} = 0.016$ **kg/s = 56.148 kg/h ... condensation rate of Ammonia...Ans.**

Plot the variation of Ammonia condensation rate (m_{cond}) as water flow rate (m_c) varies from 500 to 1000 kg/h:

The water inlet and exit temps are maintained at 20 C and 40 C respectively.

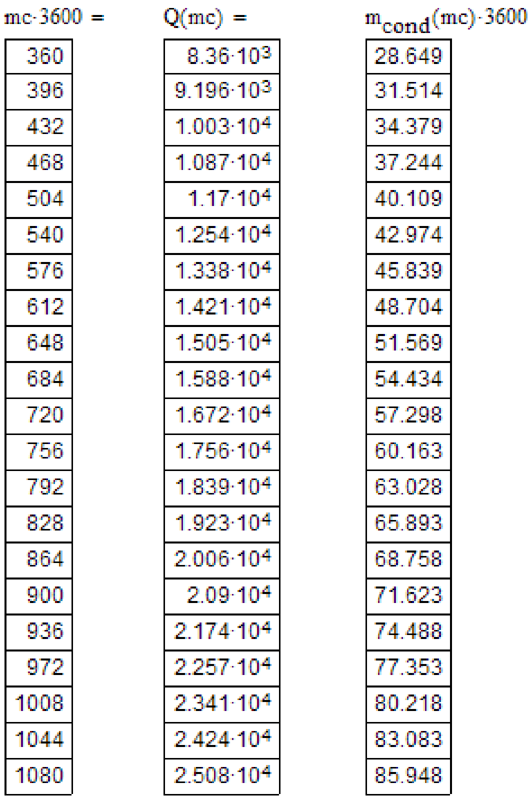
Therefore, LMTD does not change.

$Q(mc) := mc \cdot cpc \cdot (Tc2 - Tc1)$ W....Q as a function of mc

$m_{cond}(mc) := \dfrac{Q(mc)}{h_{fg}}$ kg/s....mcond as a function of mc

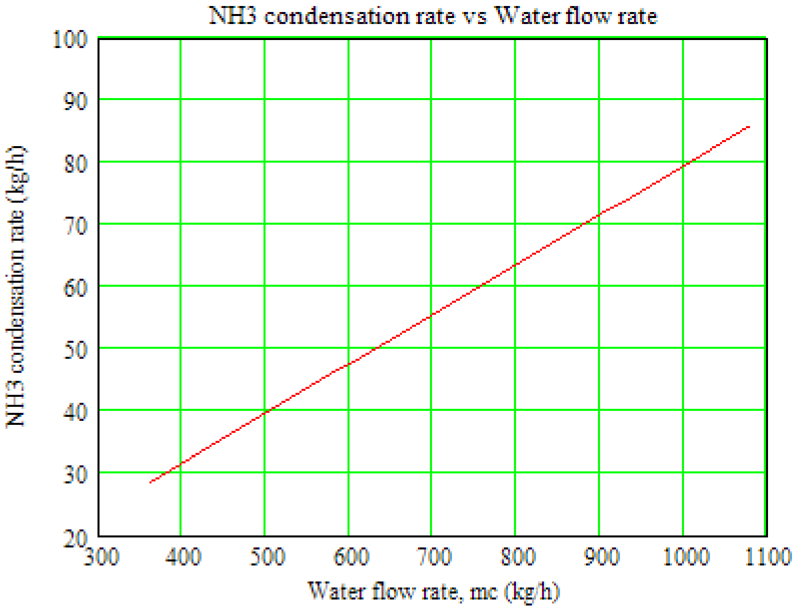
$mc := 0.1, 0.11 .. 0.3$ ...define a range variable mc

Prepare a Table:



$mc\cdot 3600 =$	$Q(mc) =$	$m_{cond}(mc)\cdot 3600$
360	$8.36\cdot 10^3$	28.649
396	$9.196\cdot 10^3$	31.514
432	$1.003\cdot 10^4$	34.379
468	$1.087\cdot 10^4$	37.244
504	$1.17\cdot 10^4$	40.109
540	$1.254\cdot 10^4$	42.974
576	$1.338\cdot 10^4$	45.839
612	$1.421\cdot 10^4$	48.704
648	$1.505\cdot 10^4$	51.569
684	$1.588\cdot 10^4$	54.434
720	$1.672\cdot 10^4$	57.298
756	$1.756\cdot 10^4$	60.163
792	$1.839\cdot 10^4$	63.028
828	$1.923\cdot 10^4$	65.893
864	$2.006\cdot 10^4$	68.758
900	$2.09\cdot 10^4$	71.623
936	$2.174\cdot 10^4$	74.488
972	$2.257\cdot 10^4$	77.353
1008	$2.341\cdot 10^4$	80.218
1044	$2.424\cdot 10^4$	83.083
1080	$2.508\cdot 10^4$	85.948

Now, plot the graph:



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LMTD Correction Factors for Cross-flow Heat Exchangers:

Here, each of the fluids may be ‘mixed’, or one fluid ‘mixed’ and the other ‘unmixed’ or both the fluids ‘unmixed’. Graphs are available to calculate the LMTD correction factors (F) for different types of cross-flow HX (see at the beginning of this chapter).

For example, an automobile radiator has water flowing through the tubes (i.e. flow unmixed) and air flowing across the tubes but confined between the fins (i.e. flow unmixed), i.e. it is a cross-flow HX with both flows ‘unmixed’.

Though charts are available to get F, it is preferable that we have Functions to calculate F while using a computer.

We have the following relations for LMTD correction factor and No. of Transfer Units (NTU).

(Ref: ‘Compact Heat Exchangers’ by Kays & London).

We shall use those relations to write Mathcad Functions to determine LMTD correction factor (F) for cross-flow HX:

$$F = \frac{A_{counterflow}}{A}$$

where, is the area of HX under consideration, $A_{counterflow}$ is the area of a true- reference counter-flow HX.

Also:

$$F = \frac{NTU_{counterflow\_for\_same\_\varepsilon}}{NTU_{actual}}$$

P and R (refer to the graphs at the beginning of this chapter) are related to effectiveness (ε) by:

$$P = \varepsilon \quad \text{for Cc = Cmin}$$

And,

$$P = \varepsilon \cdot \left(\frac{Ch}{Cc}\right) \quad \text{for Ch = Cmin}$$

$$\text{And,} \qquad R = \frac{C_c}{C_h} = \frac{C_{min}}{C_{max}} \quad \text{or,} \quad \frac{1}{\frac{C_{min}}{C_{max}}}$$

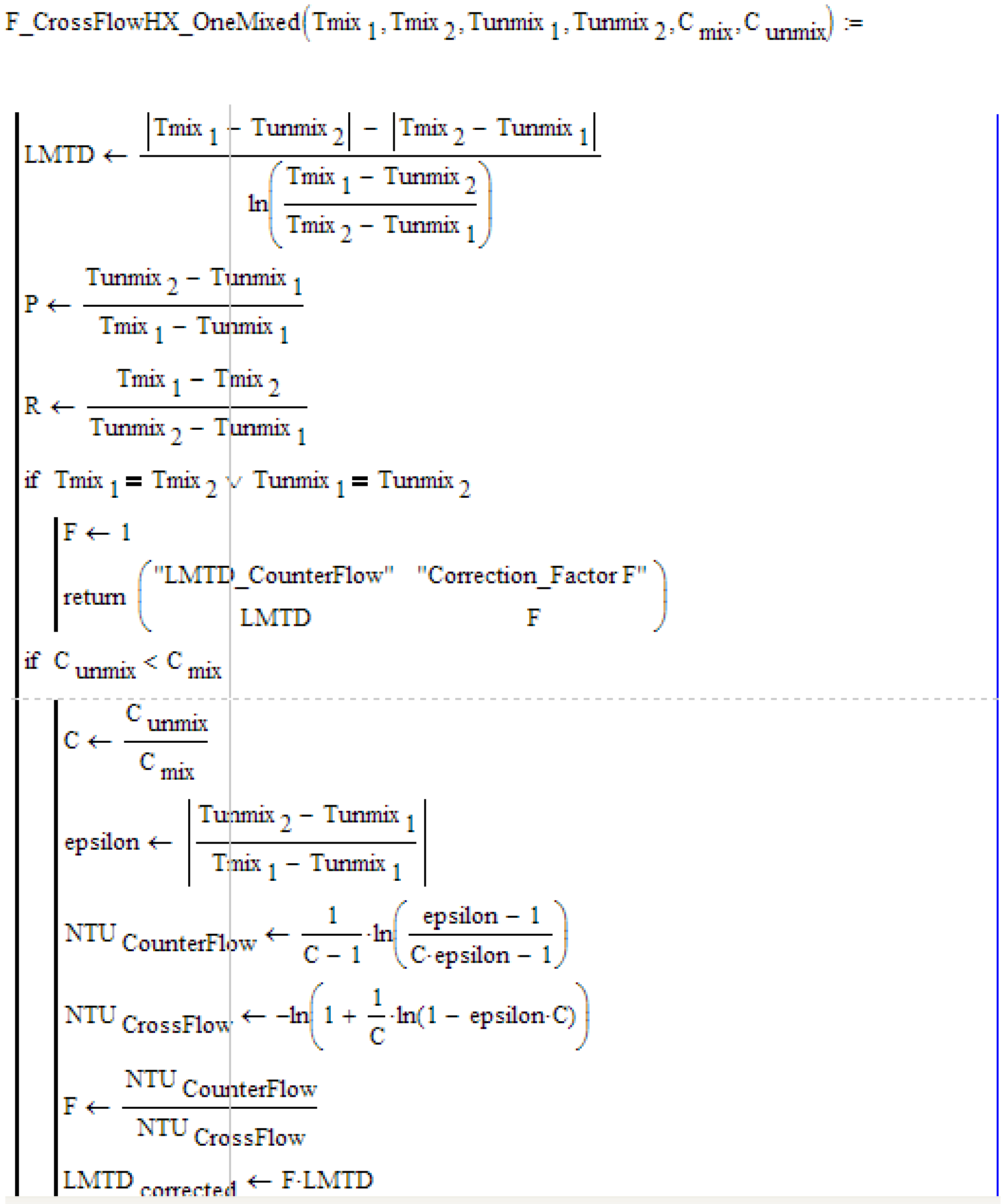
From Kays & London, for a Cross-flow HX with one fluid 'mixed', we have following relation for NTU:

$$NTU_{crossflow\_oneMixed} = \frac{-1}{C} \cdot \ln(C \cdot \ln(1 - epsilon) + 1)$$

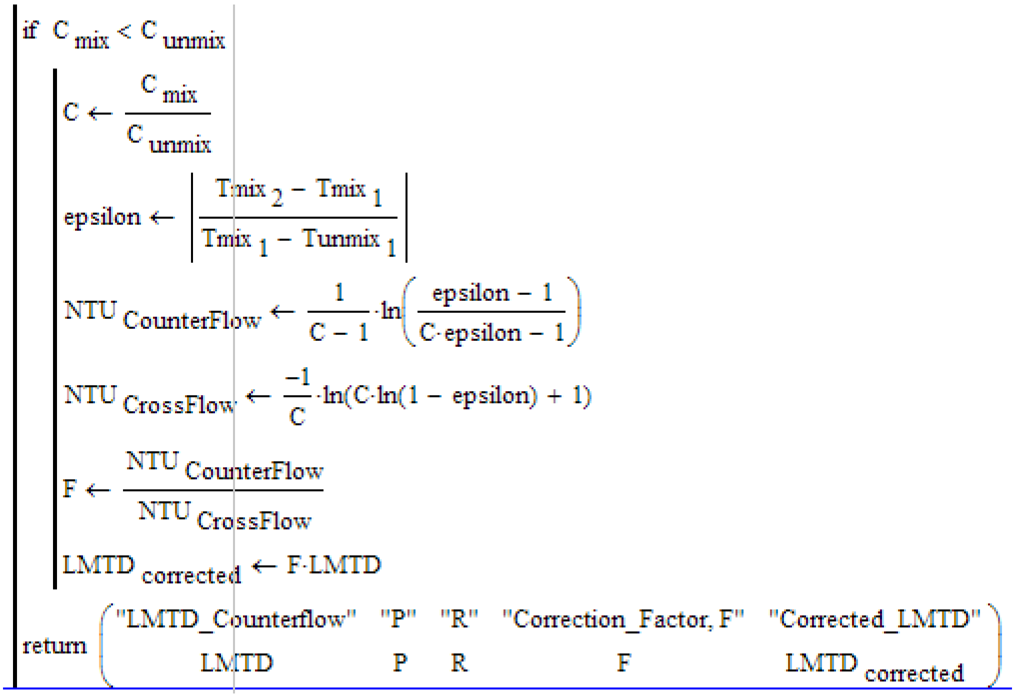
Also, from Kays & London, for a Counter-flow HX, we have following relation for NTU:

$$NTU_{counterflow} = \frac{1}{C - 1} \cdot \ln\left(\frac{epsilon - 1}{C \cdot epsilon - 1}\right)$$

Now, we shall write a Mathcad Function to determine F for a Cross-flow HX with One fluid mixed (and the other unmixed):



```
$F\_CrossFlowHX\_OneMixed(Tmix_1, Tmix_2, Tunmix_1, Tunmix_2, C_{mix}, C_{unmix}) :=$
    $LMTD \leftarrow \dfrac{|Tmix_1 - Tunmix_2| - |Tmix_2 - Tunmix_1|}{\ln\left(\dfrac{Tmix_1 - Tunmix_2}{Tmix_2 - Tunmix_1}\right)}$
    $P \leftarrow \dfrac{Tunmix_2 - Tunmix_1}{Tmix_1 - Tunmix_1}$
    $R \leftarrow \dfrac{Tmix_1 - Tmix_2}{Tunmix_2 - Tunmix_1}$
    if $Tmix_1 = Tmix_2 \vee Tunmix_1 = Tunmix_2$
        $F \leftarrow 1$
        return $\begin{pmatrix} \text{"LMTD\_CounterFlow"} & \text{"Correction\_Factor F"} \\ LMTD & F \end{pmatrix}$
    if $C_{unmix} < C_{mix}$
        $C \leftarrow \dfrac{C_{unmix}}{C_{mix}}$
        $epsilon \leftarrow \left|\dfrac{Tunmix_2 - Tunmix_1}{Tmix_1 - Tunmix_1}\right|$
        $NTU_{CounterFlow} \leftarrow \dfrac{1}{C - 1} \cdot \ln\left(\dfrac{epsilon - 1}{C \cdot epsilon - 1}\right)$
        $NTU_{CrossFlow} \leftarrow -\ln\left(1 + \dfrac{1}{C} \cdot \ln(1 - epsilon \cdot C)\right)$
        $F \leftarrow \dfrac{NTU_{CounterFlow}}{NTU_{CrossFlow}}$
        $LMTD_{corrected} \leftarrow F \cdot LMTD$
```



```
if $C_{mix} < C_{unmix}$
    $C \leftarrow \frac{C_{mix}}{C_{unmix}}$
    $epsilon \leftarrow \left|\frac{Tmix_2 - Tmix_1}{Tmix_1 - Tunmix_1}\right|$
    $NTU_{CounterFlow} \leftarrow \frac{1}{C-1}\cdot \ln\left(\frac{epsilon-1}{C\cdot epsilon-1}\right)$
    $NTU_{CrossFlow} \leftarrow \frac{-1}{C}\cdot \ln(C\cdot \ln(1-epsilon)+1)$
    $F \leftarrow \frac{NTU_{CounterFlow}}{NTU_{CrossFlow}}$
    $LMTD_{corrected} \leftarrow F\cdot LMTD$
return $\begin{pmatrix} \text{"LMTD\_Counterflow"} & \text{"P"} & \text{"R"} & \text{"Correction\_Factor, F"} & \text{"Corrected\_LMTD"} \\ LMTD & P & R & F & LMTD_{corrected} \end{pmatrix}$
```

In the above program:

Line 1: defines the Function. Here, the Inputs are: Inlet and exit Temps of mixed and Unmixed fluids, Capacity rates (C_{mix} and C_{unmix}) of the mixed and unmixed fluids (i.e. Capacity rate = mass flow rate × sp. heat). Also, note that rest of the program is to the right of this line, but is shown below, to split it and show clearly.

Line 2: Calculate LMTD for a Counter-flow HX

Lines 3, 4: Calculate P and R (see the graphs at the beginning of this chapter for definitions of P and R)

Lines 5, 6, 7: **If it is a condenser or Evaporator, then F = 1**

Lines 8 to 14: When $C_{unmix} < C_{mix}$, find out F using the formulas from Kays & London, given above

Lines 15 to 21: When $C_{unmix} > C_{mix}$, find out F using the formulas from Kays & London, given above

Line 22: Return the results in a 2 × 5 matrix. **It gives P and R values also, so that we can make a check with the graph provided.**

Now, let us use this Function in the following Problem:

Prob. 4B.8 . Consider a cross flow HX where oil flowing through the tubes is heated by steam flowing across the tubes. Oil (cp = 1900 J/kg.C) is heated from 15 C to 85 C and steam (cp = 1860 J/kg.C) enters at 130 C and leaves at 110 C with a mass flow rate of 5.2 kg/s. Overall heat transfer coeff U = 275 W/m^2.C. Calculate the surface area required for this HX.

Mathcad Solution:

This is a cross-flow HX.

Steam is the 'mixed' fluid and oil is the 'unmixed' fluid.

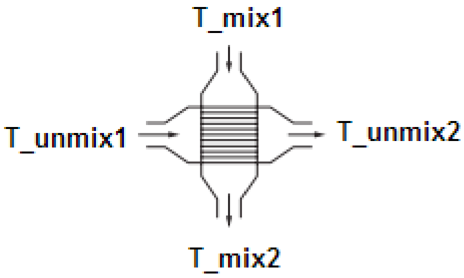


Fig. Prob.4B.8

Data:

$Tmix_1 := 130$ C $\quad$ $Tmix_2 := 110$ C $\quad$ $Tunmix_1 := 15$ C $\quad$ $Tunmix_2 := 85$ C

$cp_{oil} := 1900$ J/kg.C $\quad$ $cp_{steam} := 1860$ J/kg.C $\quad$ $m_{steam} := 5.2$ kg/s

$C_{mix} := m_{steam} \cdot cp_{steam}$

i.e. $C_{mix} = 9.672 \times 10^3$ W/K....Capacity rate of mixed fluid (steam)

$U := 275$ W/m^2.C

Calculations:

Capacity rate of un-mixed fluid (steam):

$Q := C_{mix} \cdot (Tmix_1 - Tmix_2)$

i.e. $Q = 1.934 \times 10^5$ **W ... total heat transferred**

Then,

$$C_{unmix} := \frac{Q}{Tunmix_2 - Tunmix_1}$$

i.e. $C_{unmix} = 2.763 \times 10^3$ W/K....Capacity rate of un-mixed fluid (steam)

Now, from the Mathcad Function for LMTD correction factor, F:

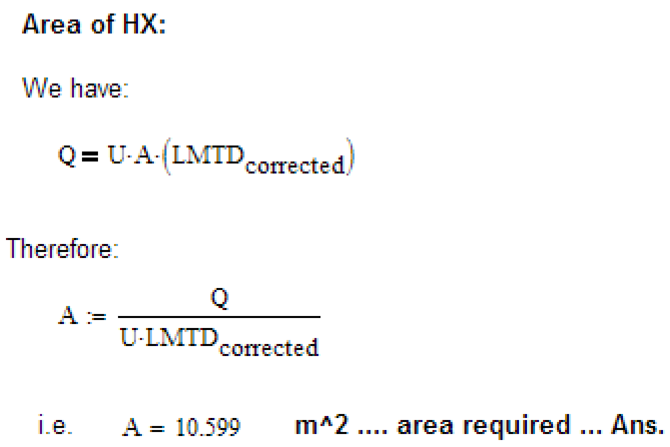
$$F\_CrossFlowHX\_OneMixed(Tmix_1, Tmix_2, Tunmix_1, Tunmix_2, C_{mix}, C_{unmix}) = \left(\begin{array}{c|cccc} \text{"LMTD\_Counterflow"} & \text{"P"} & \text{"R"} & \text{"Correction\_Factor, F"} & \text{"Corrected\_LMTD"} \\ 66.915 & 0.609 & 0.286 & 0.947 & 63.365 \end{array} \right)$$

And, we get:

$LMTD_{counterflow} := 66.915$ C

$F := 0.947$ ... LMTD correction factor

$LMTD_{corrected} := 66.365$ C



Area of HX:

We have:

$$Q = U \cdot A \cdot \left(LMTD_{corrected}\right)$$

Therefore:

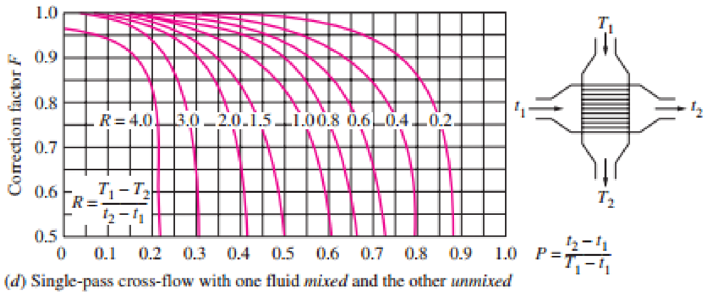
$$A := \frac{Q}{U \cdot LMTD_{corrected}}$$

i.e. $A = 10.599$ **m^2 .... area required ... Ans.**

Note: The Mathcad Function also returns parameters P and R.

We see that P = 0.609, and R = 0.286.

Then, see the graphs at the beginning of this chapter to get F from the graphs:



(*d*) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

For P = 0.609, and R = 0.286, we get: F = 0.95 approx.

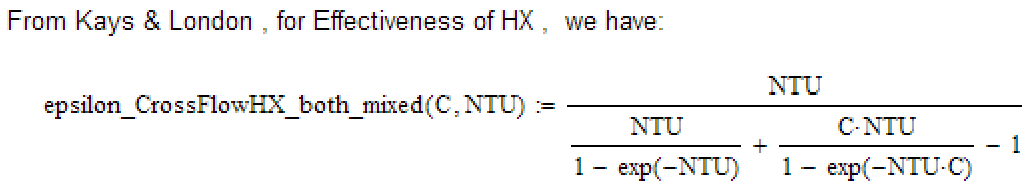
Note that it is more accurate to use the Mathcad Function than interpolate from the graph.

==

Mathcad Function to determine LMTD correction factor, F for Cross Flow HX, both fluids 'mixed':

This is not a very common type of arrangement.

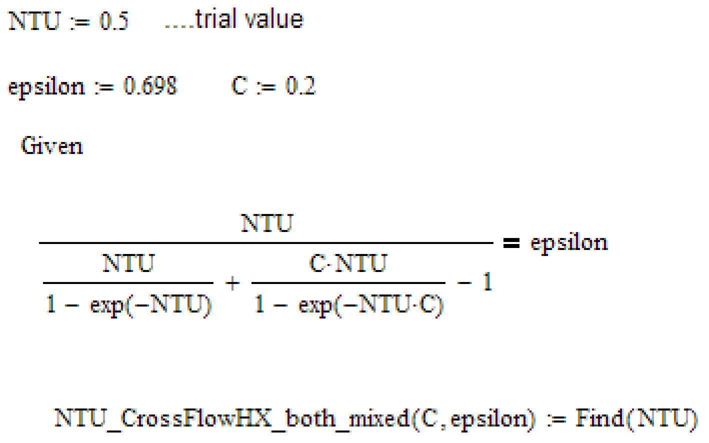
However, Kays & London give an equation to determine Effectiveness (ε) as a function of Capacity ratio ($C = C_{min} / C_{max}$). We use that eqn to write a Mathcad Function to get NTU and then use it in a Function to get F for Cross-flow HX with both the flows 'mixed'.



From Kays & London , for Effectiveness of HX , we have:

$$\text{epsilon\_CrossFlowHX\_both\_mixed}(C, NTU) := \frac{NTU}{\frac{NTU}{1 - \exp(-NTU)} + \frac{C \cdot NTU}{1 - \exp(-NTU \cdot C)} - 1}$$

Now, write a Function for NTU when epsilon is given:



NTU := 0.5 ....trial value

epsilon := 0.698 C := 0.2

Given

$$\frac{NTU}{\frac{NTU}{1 - \exp(-NTU)} + \frac{C \cdot NTU}{1 - \exp(-NTU \cdot C)} - 1} = \text{epsilon}$$

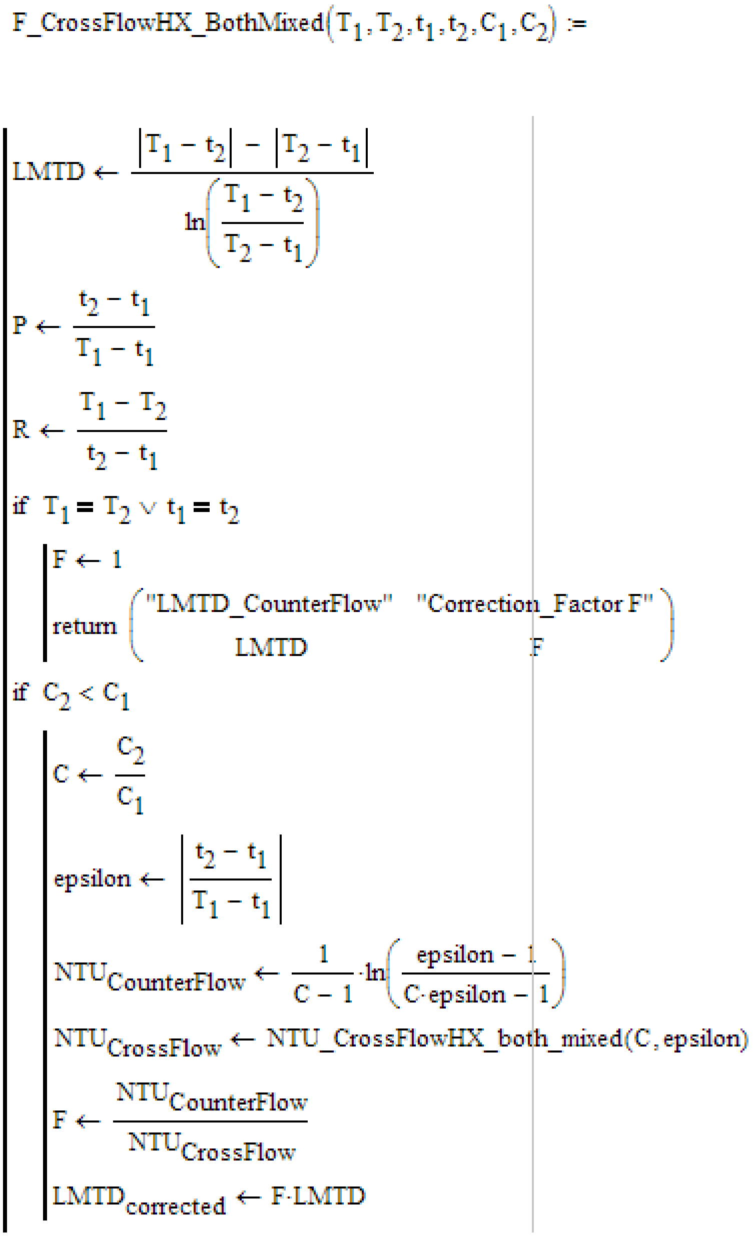
$$\text{NTU\_CrossFlowHX\_both\_mixed}(C, \text{epsilon}) := \text{Find}(NTU)$$

....

Above Function gives NTU as a function of C and epsilon.

Ex: NTU_CrossFlowHX_both_mixed(0.4, 0.703) = 1.797

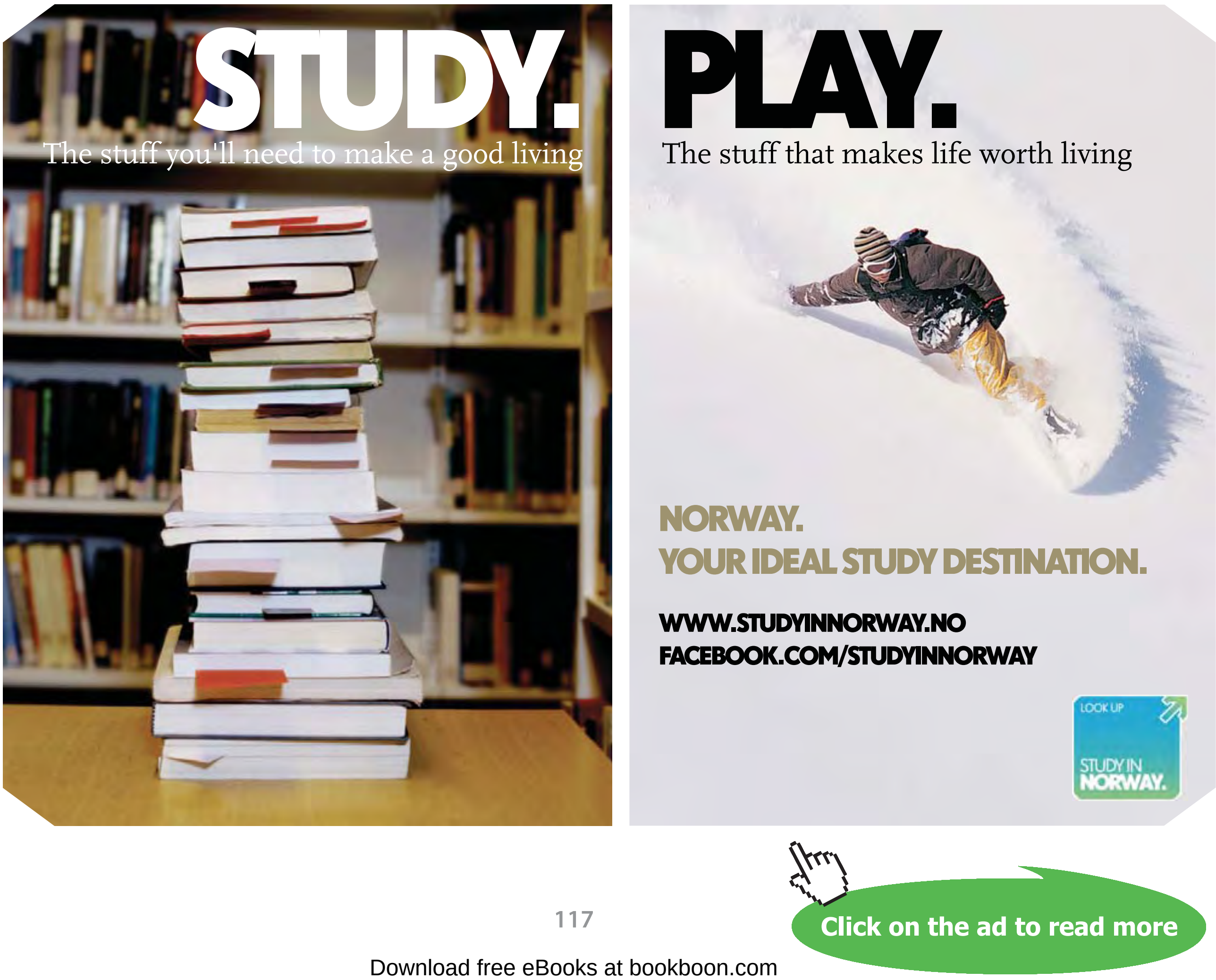
Now, use the above Function for NTU to write a Function to get LMTD Correction Factor, F for Cross Flow HX, both fluids 'mixed':



```
F_CrossFlowHX_BothMixed($T_1, T_2, t_1, t_2, C_1, C_2$) :=
    LMTD ← $\frac{|T_1 - t_2| - |T_2 - t_1|}{\ln\left(\frac{T_1 - t_2}{T_2 - t_1}\right)}$
    P ← $\frac{t_2 - t_1}{T_1 - t_1}$
    R ← $\frac{T_1 - T_2}{t_2 - t_1}$
    if $T_1 = T_2 \vee t_1 = t_2$
        F ← 1
        return $\begin{pmatrix} \text{"LMTD\_CounterFlow"} & \text{"Correction\_Factor F"} \\ \text{LMTD} & F \end{pmatrix}$
    if $C_2 < C_1$
        C ← $\frac{C_2}{C_1}$
        epsilon ← $\left|\frac{t_2 - t_1}{T_1 - t_1}\right|$
        $NTU_{CounterFlow}$ ← $\frac{1}{C - 1} \cdot \ln\left(\frac{\text{epsilon} - 1}{C \cdot \text{epsilon} - 1}\right)$
        $NTU_{CrossFlow}$ ← NTU_CrossFlowHX_both_mixed(C, epsilon)
        F ← $\frac{NTU_{CounterFlow}}{NTU_{CrossFlow}}$
        $LMTD_{corrected}$ ← $F \cdot LMTD$
```

```
if $C_1 < C_2$
    $C \leftarrow \frac{C_1}{C_2}$
    $epsilon \leftarrow \left|\frac{T_2 - T_1}{T_1 - t_1}\right|$
    $NTU_{CounterFlow} \leftarrow \frac{1}{C - 1} \cdot \ln\left(\frac{epsilon - 1}{C \cdot epsilon - 1}\right)$
    $NTU_{CrossFlow} \leftarrow NTU\_CrossFlowHX\_both\_mixed(C, epsilon)$
    $F \leftarrow \frac{NTU_{CounterFlow}}{NTU_{CrossFlow}}$
    $LMTD_{corrected} \leftarrow F \cdot LMTD$
return $\begin{pmatrix} \text{"LMTD\_Counterflow"} & \text{"P"} & \text{"R"} & \text{"Correction\_Factor, F"} & \text{"Corrected\_LMTD"} \\ LMTD & P & R & F & LMTD_{corrected} \end{pmatrix}$
```

==



As an example, work out the previous problem if both fluids are 'mixed':

We have:

$T_1 := 130$ C $\quad T_2 := 110$ C

$t_1 := 15$ C $\quad t_2 := 85$ C

$C_1 := 9672$ W/K $\quad C_2 := 2763$ W/K

Then, using the Mathcad Function,we get:

$$\text{F\_CrossFlowHX\_BothMixed}(T_1, T_2, t_1, t_2, C_1, C_2) = \begin{pmatrix} \text{"LMTD\_Counterflow"} & \text{"P"} & \text{"R"} & \text{"Correction\_Factor, F"} & \text{"Corrected\_LMTD"} \\ 66.915 & 0.609 & 0.286 & 0.944 & 63.135 \end{pmatrix}$$

i.e.

LMTD Correction Factor, F = 0.944 …. Ans.

===

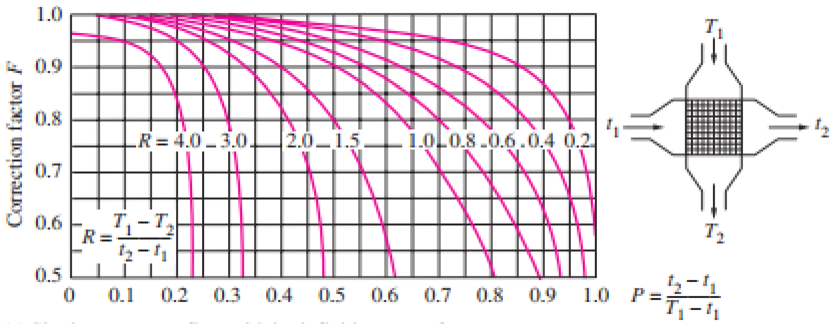
Mathcad Function to determine LMTD correction factor, F for Cross Flow HX, when both fluids are 'unmixed':

Well known automobile radiator falls in this category.

Here, we adopt another method:

We use the F vs P (for various values of R) given at the beginning of this chapter and digitize those graphs to get x, y coordinates, and then curve-fit them to get F vs P equations. We use those equations to write a Mathcad Program to calculate F for given P and R:

So, we have the following graph for LMTD correction Factor F:



(*c*) Single-pass cross-flow with both fluids *unmixed*

There are various curves for R = 4, 3, 2 …etc. x coordinate is P and y coordinate is F.

We digitize each of the curves for R = 4, 3 … etc. i.e. we get x-y coordinates for each curve. Then, it is an easy job to get curve-fit equations for each curve.

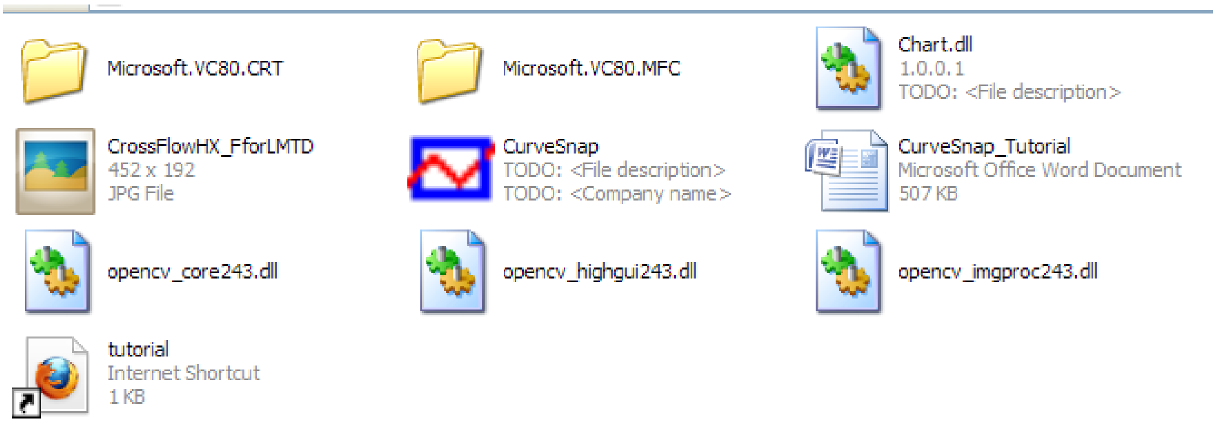
First, to digitize each curve:

We use the software 'CurveSnap', which is available for free from:

http://xoofee.com/2012/12/curvesnap/

Just download the zip file, unzip it and keep in a suitable folder. No installation is required.

Folder looks as follows:



Microsoft.VC80.CRT

Microsoft.VC80.MFC

Chart.dll
1.0.0.1
TODO: <File description>

CrossFlowHX_FforLMTD
452 x 192
JPG File

CurveSnap
TODO: <File description>
TODO: <Company name>

CurveSnap_Tutorial
Microsoft Office Word Document
507 KB

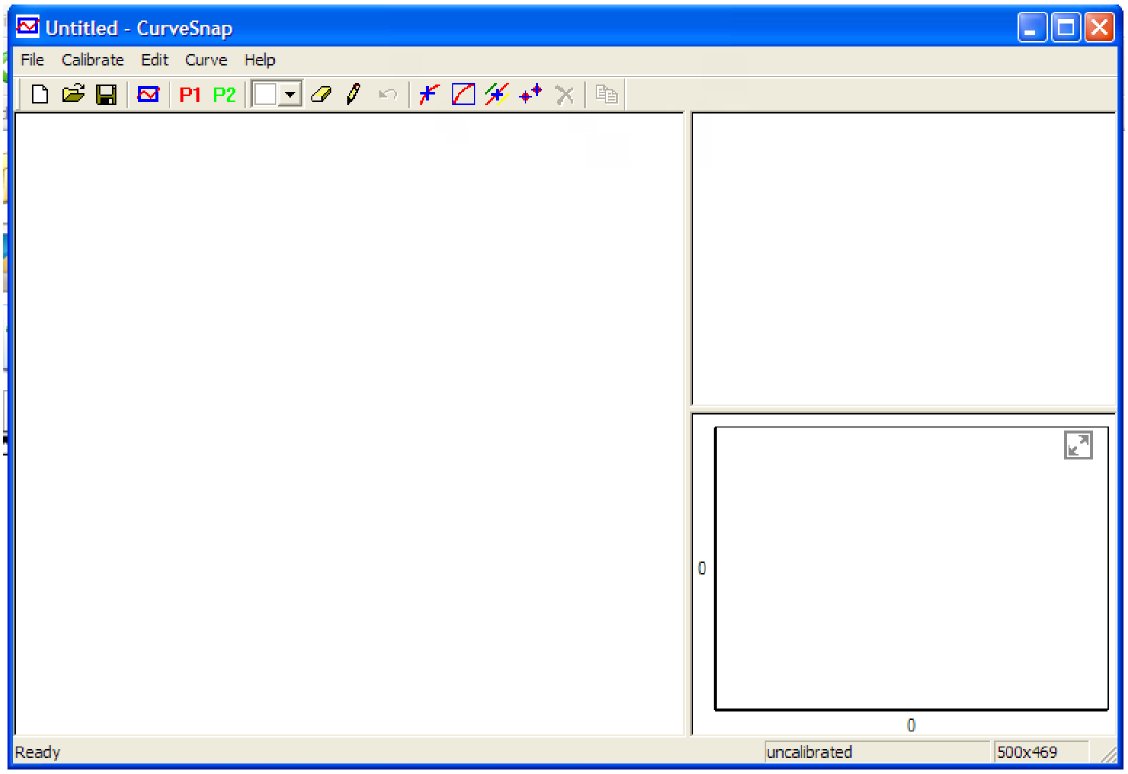
opencv_core243.dll

opencv_highgui243.dll

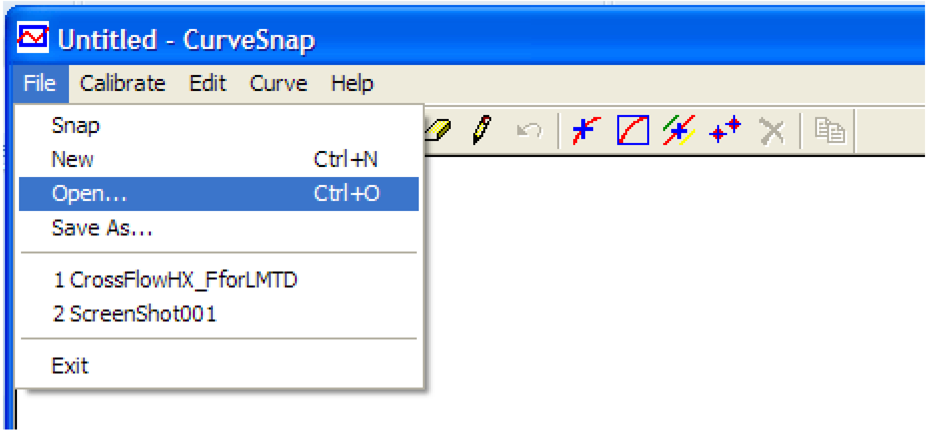
opencv_imgproc243.dll

tutorial
Internet Shortcut
1 KB

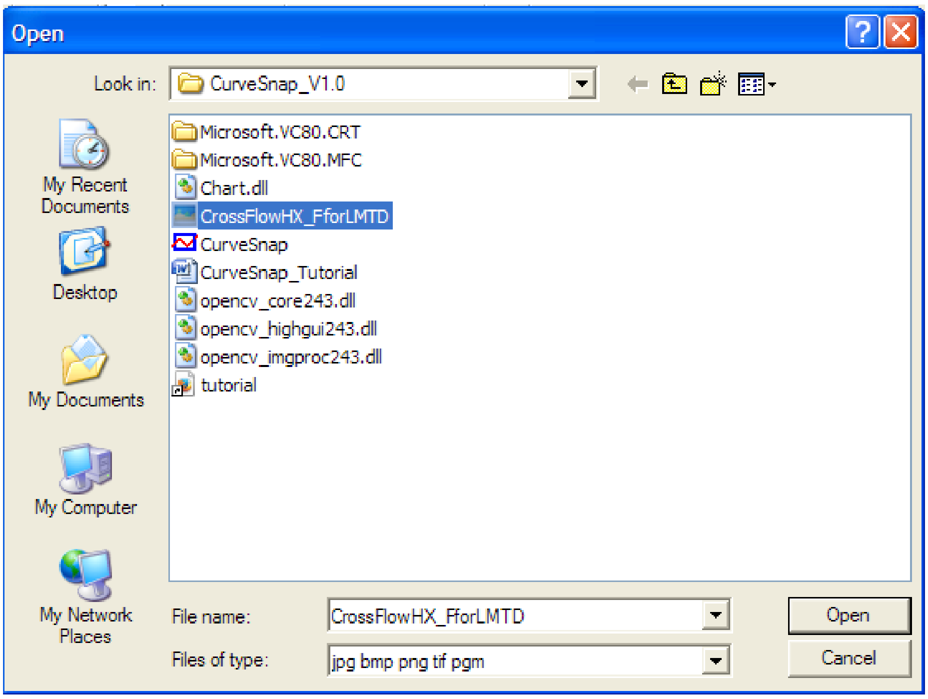
Double click on CurveSnap (blue rectangle in Fig. above). We get:



Now, open the required file (in jpg format) by going to: File-Open:



Click Open and choose the file to be opened:

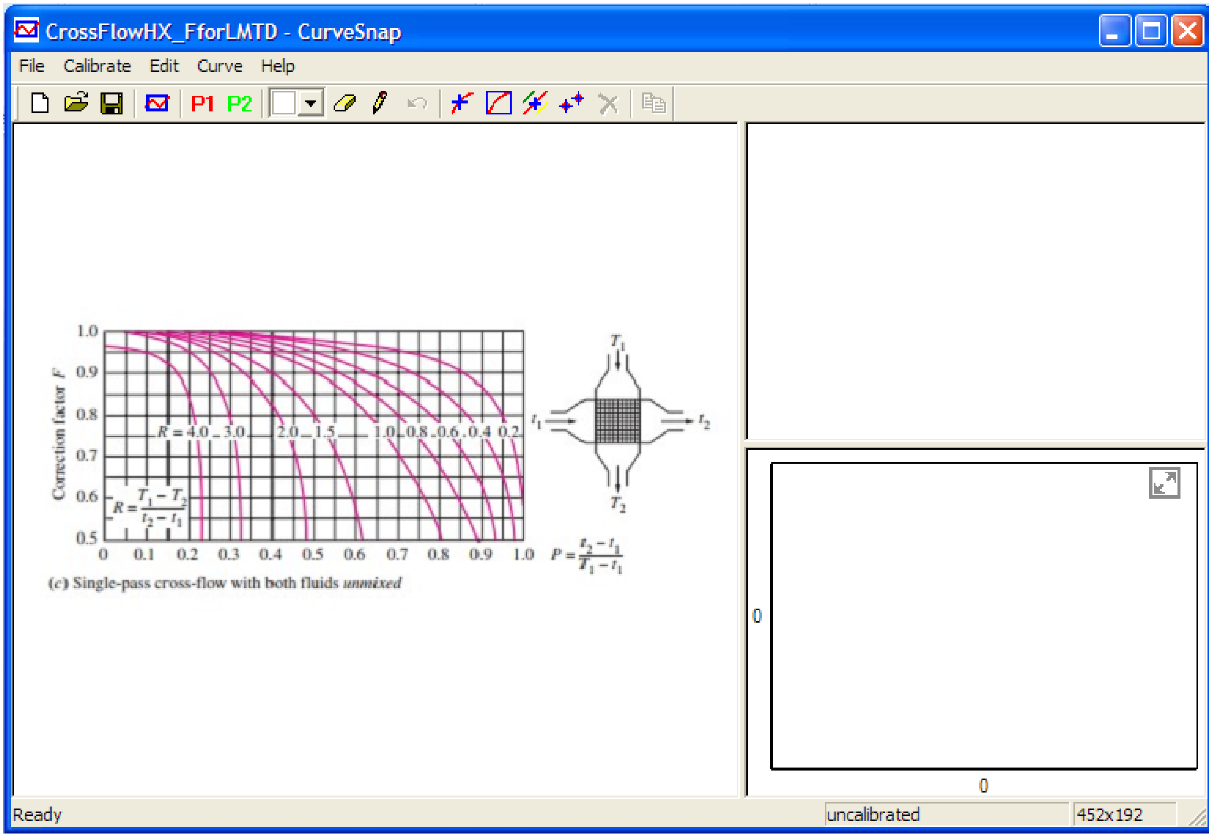


Look in: CurveSnap_V1.0

File name: CrossFlowHX_FforLMTD

Files of type: jpg bmp png tif pgm

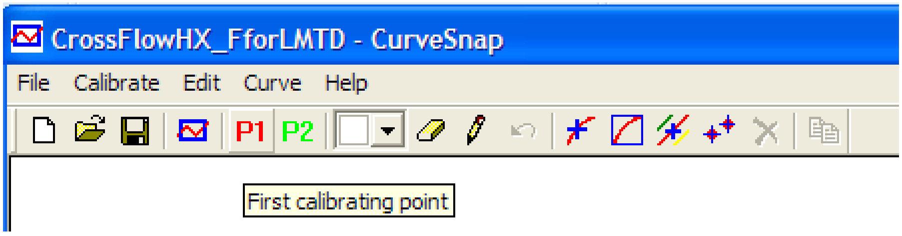
Click Open: We get:



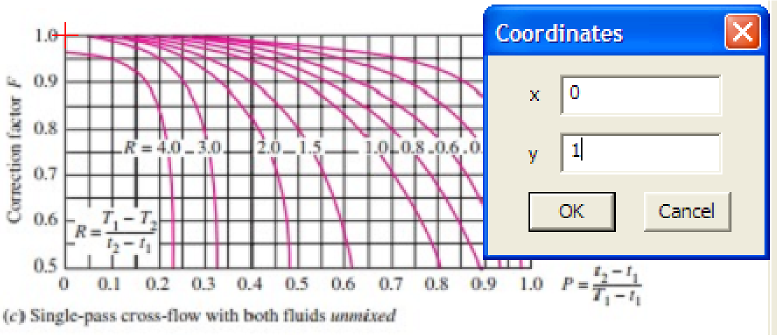
Now, we are ready to digitize.

First, calibrate. i.e. fix the origin and the scales of x, y axes.

Press P1 as shown below:



Then, cursor becomes a + sign, and click at the origin. See the red cross at the top of y-axis. A window pops up; fill the coordinates of top of y-axis as shown:



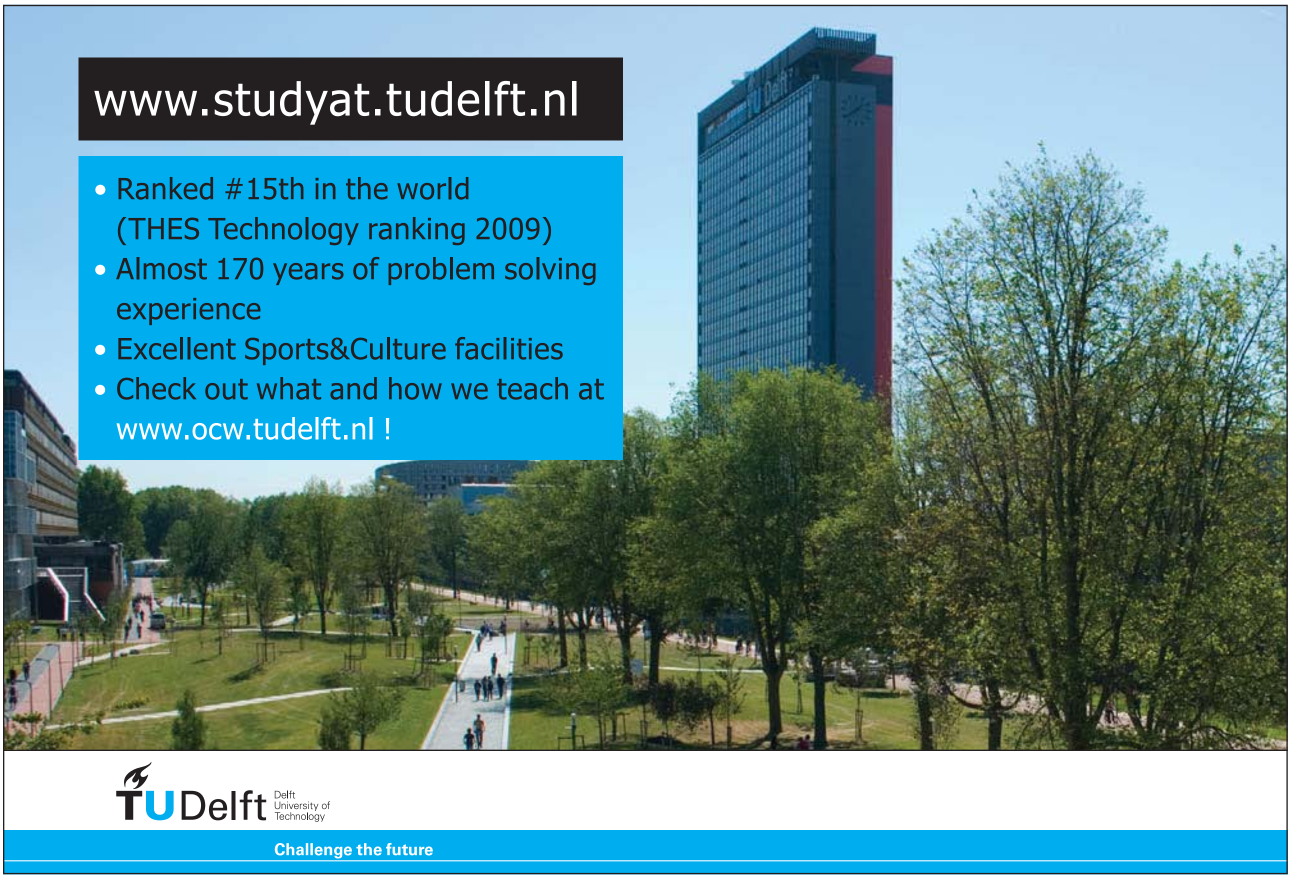
x: 0

y: 1

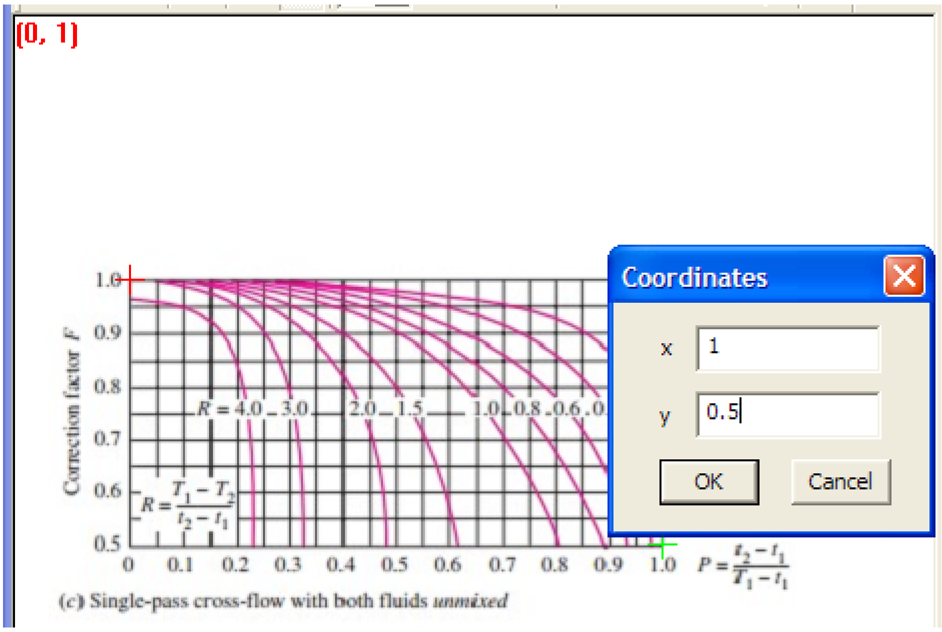
Press OK. Coordinates of origin are shown on the left top corner.

Now, press P2:

[0, 1]



Locate the cursor cross on the right extreme right of x-axis of the graph. Fill up the coordinates of that extreme point as shown below:

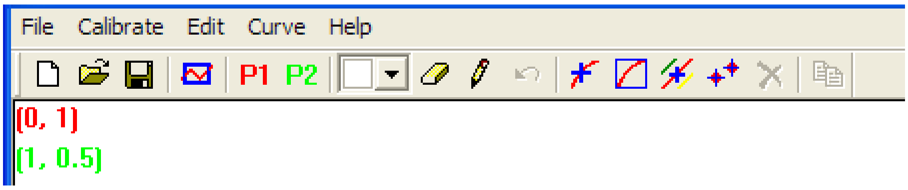

[0, 1]

Coordinates

x 1

y 0.5

Press OK. We get:



[0, 1]
[1, 0.5]

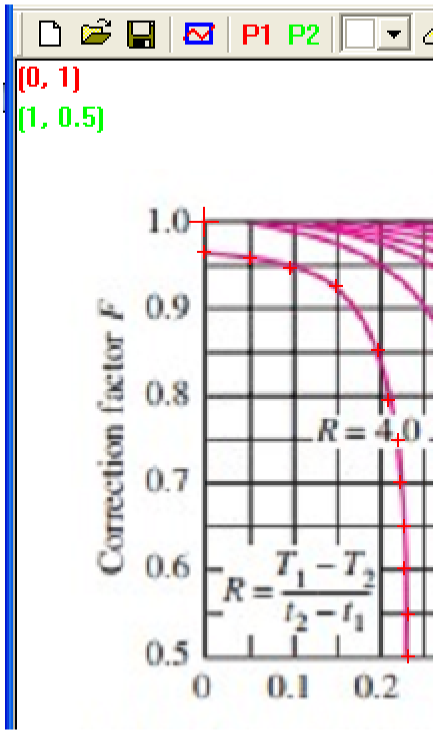
Now, we can digitize the graph,

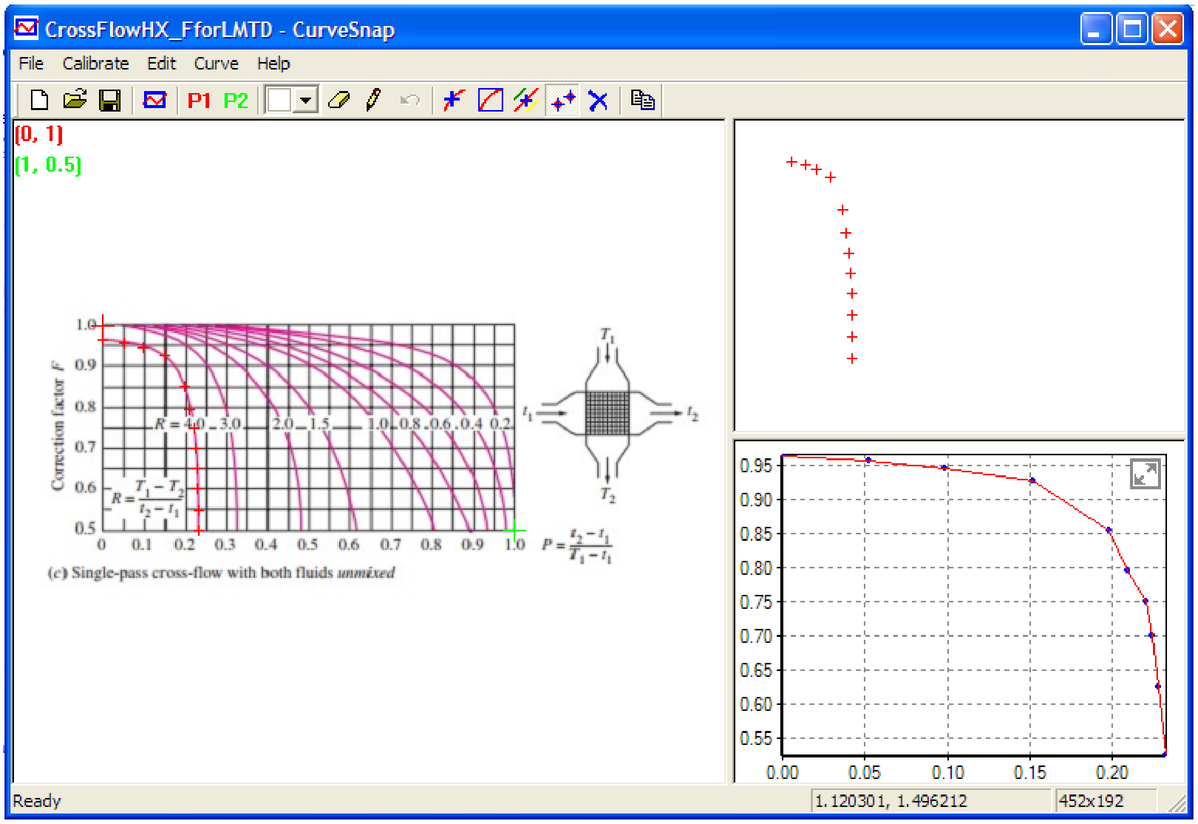
Start with the curve for R = 4:

Press 'Choose single data point' (i.e. button with '++') as shown below:

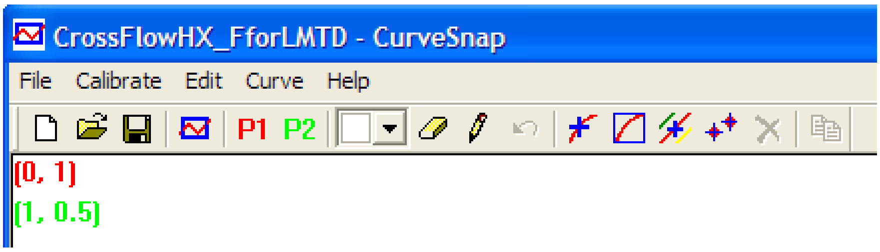
[0, 1]
[1, 0.5]

And go on clicking on the curve R = 4: We see the curve marked with red crosses:



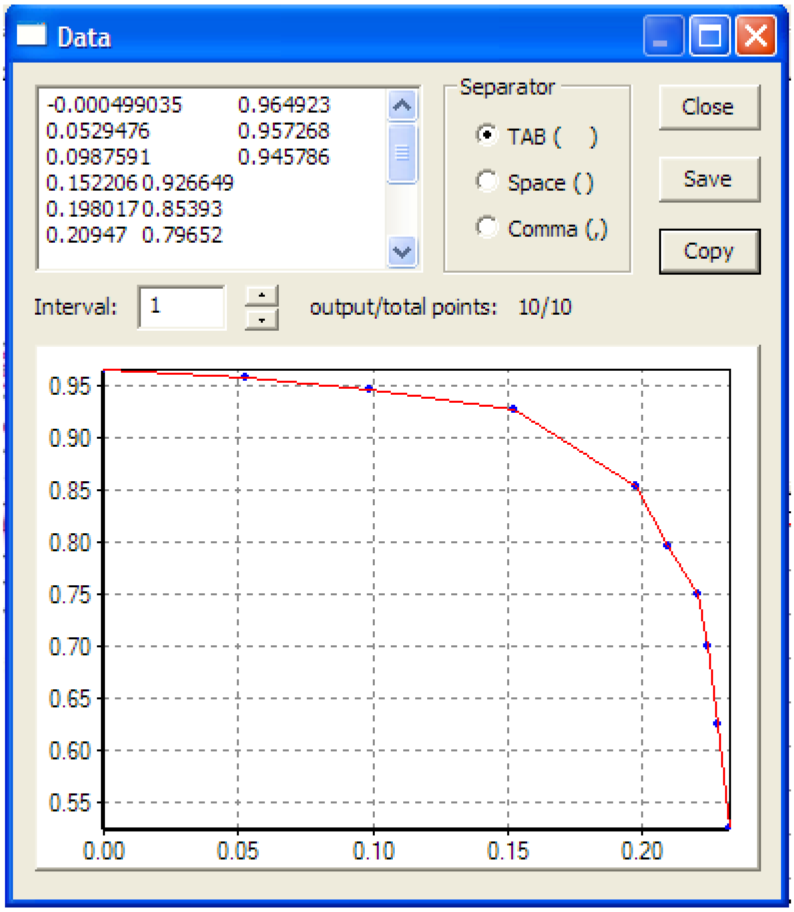


Now, click on the last button on tool bar:



(0, 1)
(1, 0.5)

We get:

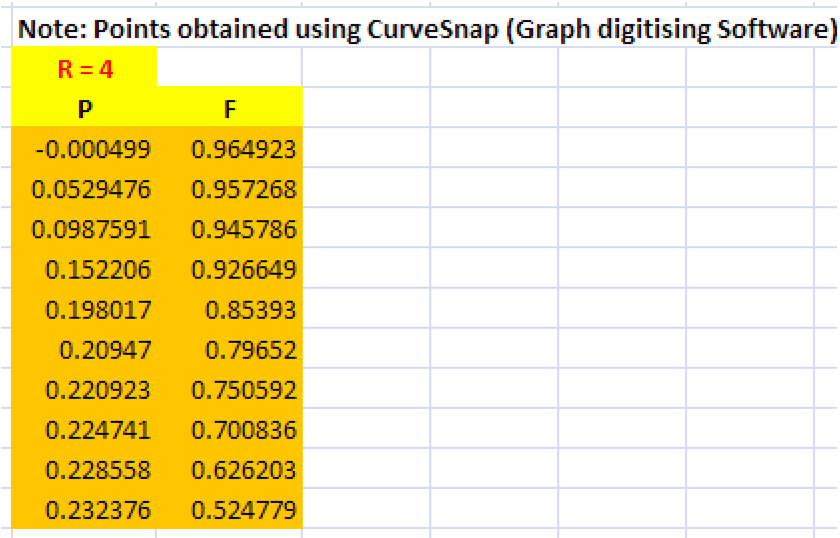


```
-0.000499035	0.964923
0.0529476	0.957268
0.0987591	0.945786
0.152206 0.926649
0.198017 0.85393
0.20947  0.79652
```

● TAB ( )

Interval: 1 output/total points: 10/10

Press ‘copy’ and paste the copied data to EXCEL:

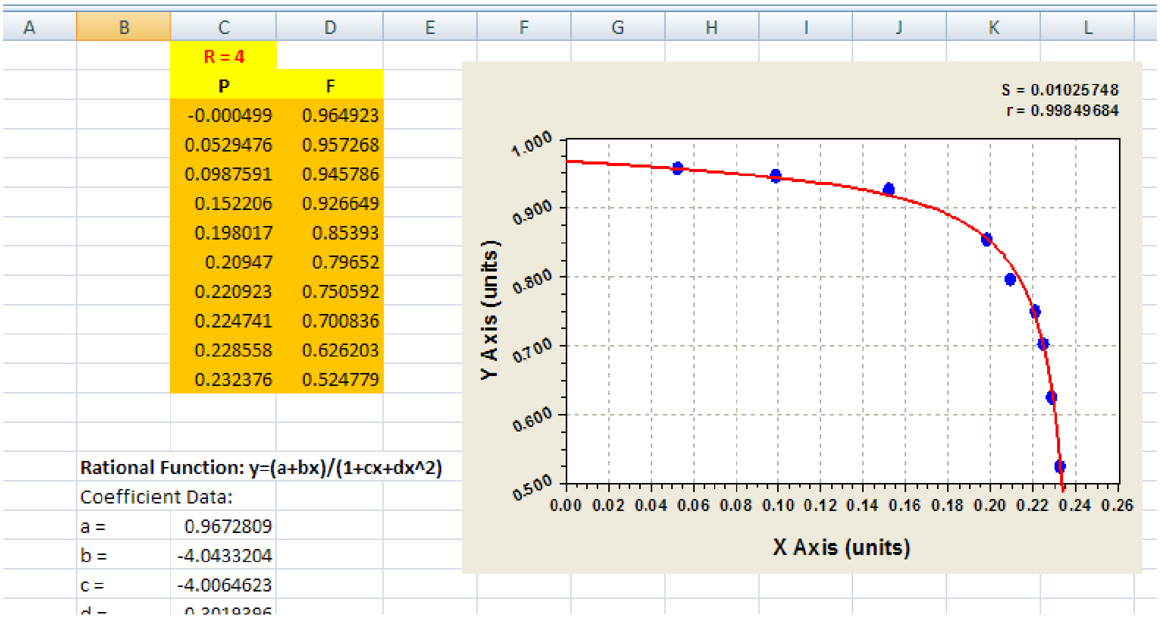


Note: Points obtained using CurveSnap (Graph digitising Software)

R = 4	
P	F
-0.000499	0.964923
0.0529476	0.957268
0.0987591	0.945786
0.152206	0.926649
0.198017	0.85393
0.20947	0.79652
0.220923	0.750592
0.224741	0.700836
0.228558	0.626203
0.232376	0.524779

Next, we copy the x-y data to CurveExpert software to get curve-fit equations:

The result, transferred to EXCEL is shown below:



R = 4

P	F
-0.000499	0.964923
0.0529476	0.957268
0.0987591	0.945786
0.152206	0.926649
0.198017	0.85393
0.20947	0.79652
0.220923	0.750592
0.224741	0.700836
0.228558	0.626203
0.232376	0.524779

Rational Function: y=(a+bx)/(1+cx+dx^2)

Coefficient Data:

a = 0.9672809

b = -4.0433204

c = -4.0064623

d = 0.3019296

i.e. At R = 4, the equation for F as a function of P is:

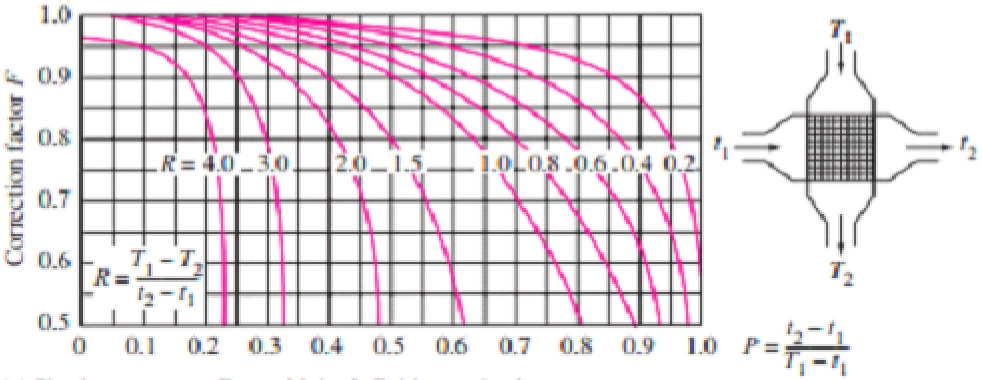
y=(a+bx)/(1+cx+dx^2) where y is F, in our case, and x is P.

a, b, c, and d are the coefficients in the equation.

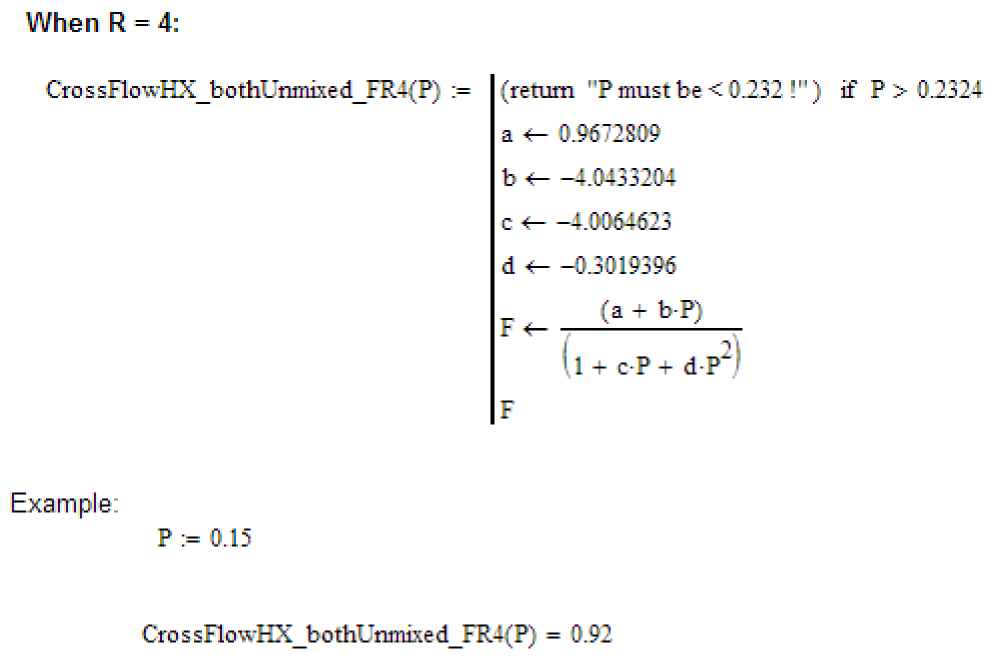
Now, use this coefficient data in Mathcad to write a Function for F:

Cross Flow HX with both fluids Unmixed:

LMTD Correction Factors for different R and P:



(c) Single-pass cross-flow with both fluids *unmixed*



When R = 4:

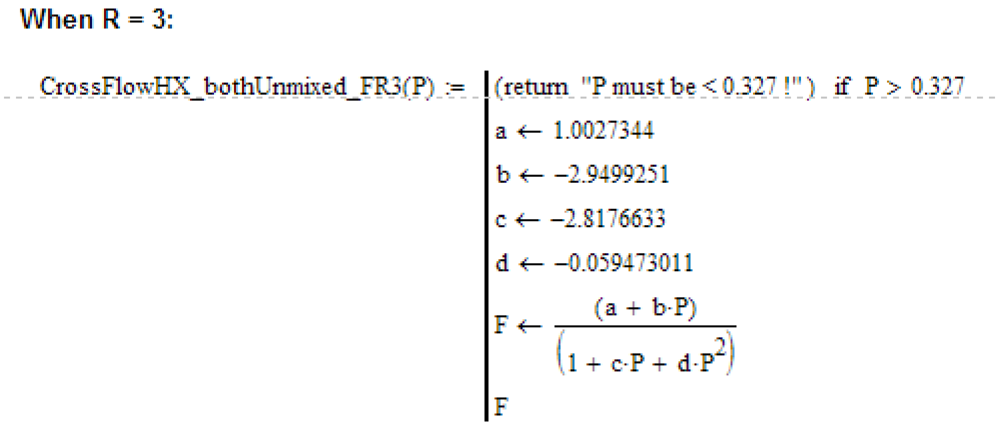
$$\mathrm{CrossFlowHX\_bothUnmixed\_FR4(P)} := \left| \begin{array}{l} (\mathrm{return}\ \text{"P must be < 0.232 !"})\ \ \mathrm{if}\ \ P > 0.2324 \\ a \leftarrow 0.9672809 \\ b \leftarrow -4.0433204 \\ c \leftarrow -4.0064623 \\ d \leftarrow -0.3019396 \\ F \leftarrow \dfrac{(a + b \cdot P)}{\left(1 + c \cdot P + d \cdot P^2\right)} \\ F \end{array} \right.$$

Example:

$P := 0.15$

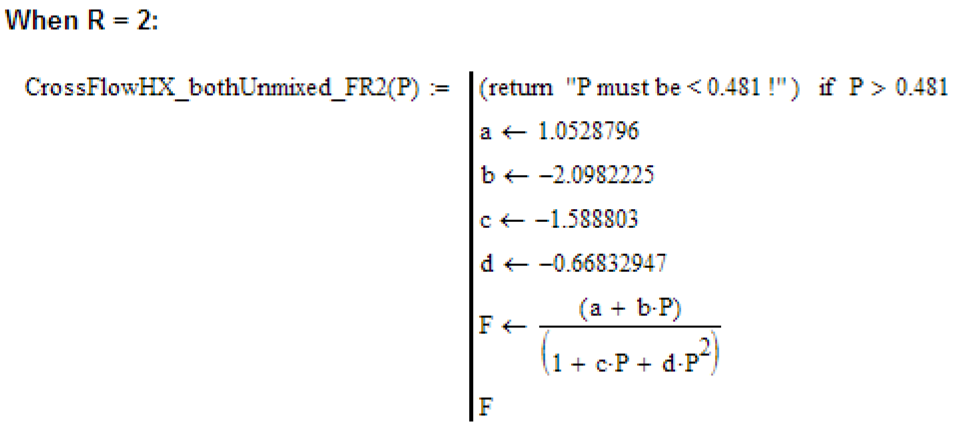
$\mathrm{CrossFlowHX\_bothUnmixed\_FR4(P)} = 0.92$

Repeat a similar procedure for other curves in the plot, i.e. for R = 3, 2, 1.5, 1, 0.8, 0.6, 0.4 and 0.2. And for these curves Mathcad Functions are:



When R = 3:

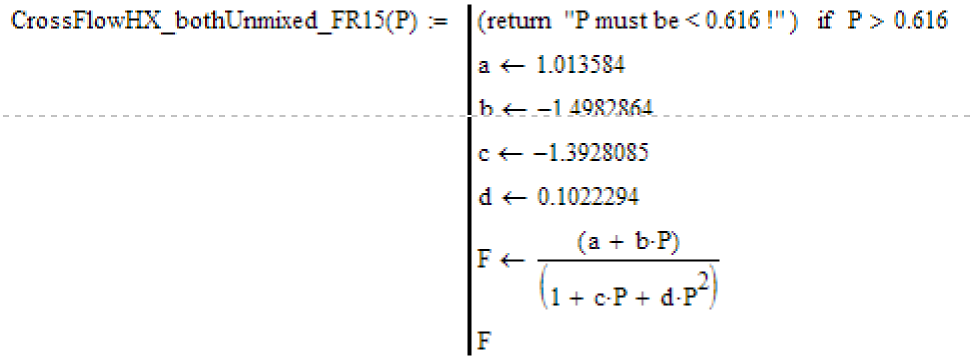
$$\mathrm{CrossFlowHX\_bothUnmixed\_FR3(P)} := \left| \begin{array}{l} (\mathrm{return}\ \text{"P must be < 0.327 !"})\ \ \mathrm{if}\ \ P > 0.327 \\ a \leftarrow 1.0027344 \\ b \leftarrow -2.9499251 \\ c \leftarrow -2.8176633 \\ d \leftarrow -0.059473011 \\ F \leftarrow \dfrac{(a + b \cdot P)}{\left(1 + c \cdot P + d \cdot P^2\right)} \\ F \end{array} \right.$$



When R = 2:

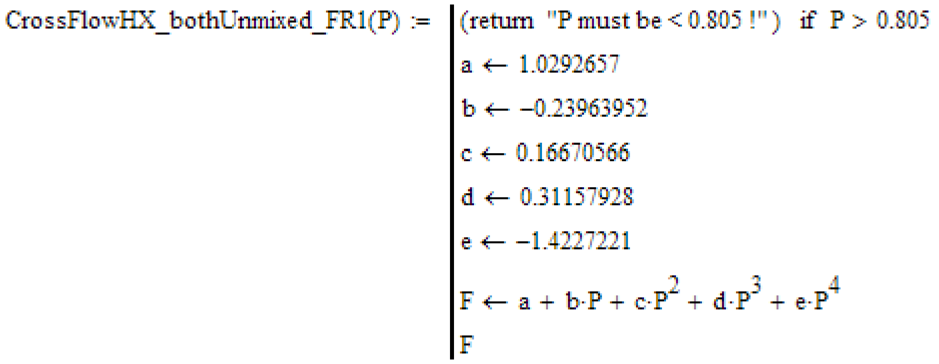
$$\mathrm{CrossFlowHX\_bothUnmixed\_FR2(P)} := \left| \begin{array}{l} (\mathrm{return}\ \text{"P must be < 0.481 !"})\ \ \mathrm{if}\ \ P > 0.481 \\ a \leftarrow 1.0528796 \\ b \leftarrow -2.0982225 \\ c \leftarrow -1.588803 \\ d \leftarrow -0.66832947 \\ F \leftarrow \dfrac{(a + b \cdot P)}{\left(1 + c \cdot P + d \cdot P^2\right)} \\ F \end{array} \right.$$

When R = 1.5:



```
CrossFlowHX_bothUnmixed_FR15(P) := | (return "P must be < 0.616 !") if P > 0.616
                                   | a ← 1.013584
                                   | b ← −1.4982864
                                   | c ← −1.3928085
                                   | d ← 0.1022294
                                   | F ← (a + b·P) / (1 + c·P + d·P^2)
                                   | F
```

When R = 1.0:



```
CrossFlowHX_bothUnmixed_FR1(P) := | (return "P must be < 0.805 !") if P > 0.805
                                  | a ← 1.0292657
                                  | b ← −0.23963952
                                  | c ← 0.16670566
                                  | d ← 0.31157928
                                  | e ← −1.4227221
                                  | F ← a + b·P + c·P^2 + d·P^3 + e·P^4
                                  | F
```

When R = 0.8:

$$\text{CrossFlowHX\_bothUnmixed\_FR08}(P) := \left| \begin{array}{l} (\text{return "P must be < 0.886 !"}) \ \ \text{if} \ \ P > 0.886 \\ a \leftarrow 1.007952 \\ b \leftarrow -0.99036246 \\ c \leftarrow -0.96731788 \\ d \leftarrow 0.14733358 \\ F \leftarrow \dfrac{(a + b \cdot P)}{\left(1 + c \cdot P + d \cdot P^2\right)} \\ F \end{array} \right.$$

When R = 0.6:

$$\text{CrossFlowHX\_bothUnmixed\_FR06}(P) := \left| \begin{array}{l} (\text{return "P must be < 0.932 !"}) \ \ \text{if} \ \ P > 0.932 \\ a \leftarrow 1.0510175 \\ b \leftarrow -1.0614768 \\ c \leftarrow -0.84847439 \\ d \leftarrow -0.099409186 \\ F \leftarrow \dfrac{(a + b \cdot P)}{\left(1 + c \cdot P + d \cdot P^2\right)} \\ F \end{array} \right.$$

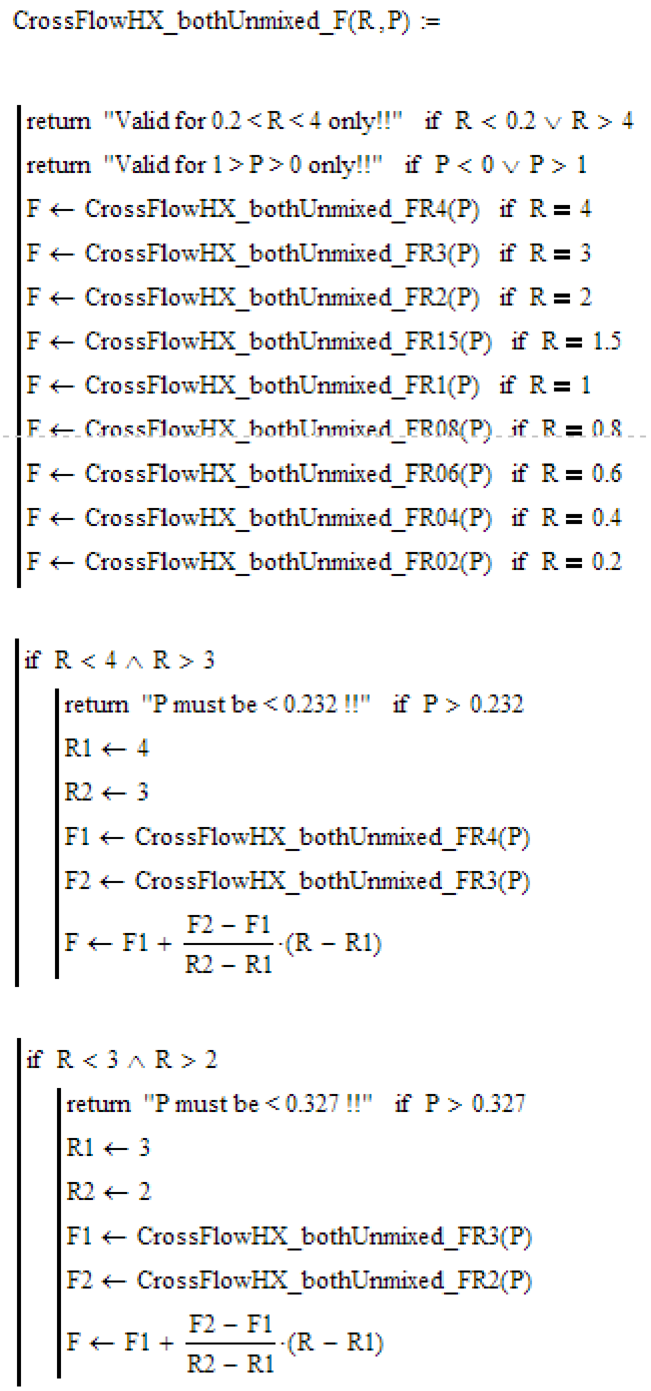
When R = 0.4:

$$\text{CrossFlowHX\_bothUnmixed\_FR04}(P) := \left| \begin{array}{l} (\text{return "P must be < 0.977 !"}) \ \ \text{if} \ \ P > 0.977 \\ a \leftarrow 1.0338023 \\ b \leftarrow -1.0158388 \\ c \leftarrow -0.87973396 \\ d \leftarrow -0.061943671 \\ F \leftarrow \dfrac{(a + b \cdot P)}{\left(1 + c \cdot P + d \cdot P^2\right)} \\ F \end{array} \right.$$

When R = 0.2:

$$\text{CrossFlowHX\_bothUnmixed\_FR02}(P) := \left| \begin{array}{l} (\text{return "P must be < 0.993 !"}) \ \ \text{if} \ \ P > 0.993 \\ a \leftarrow 1.016908 \\ b \leftarrow -0.99564685 \\ c \leftarrow -0.91765904 \\ d \leftarrow -0.043092966 \\ F \leftarrow \dfrac{(a + b \cdot P)}{\left(1 + c \cdot P + d \cdot P^2\right)} \\ F \end{array} \right.$$

Now, write a final Function to get F for any given P and R (within the range of this plot):

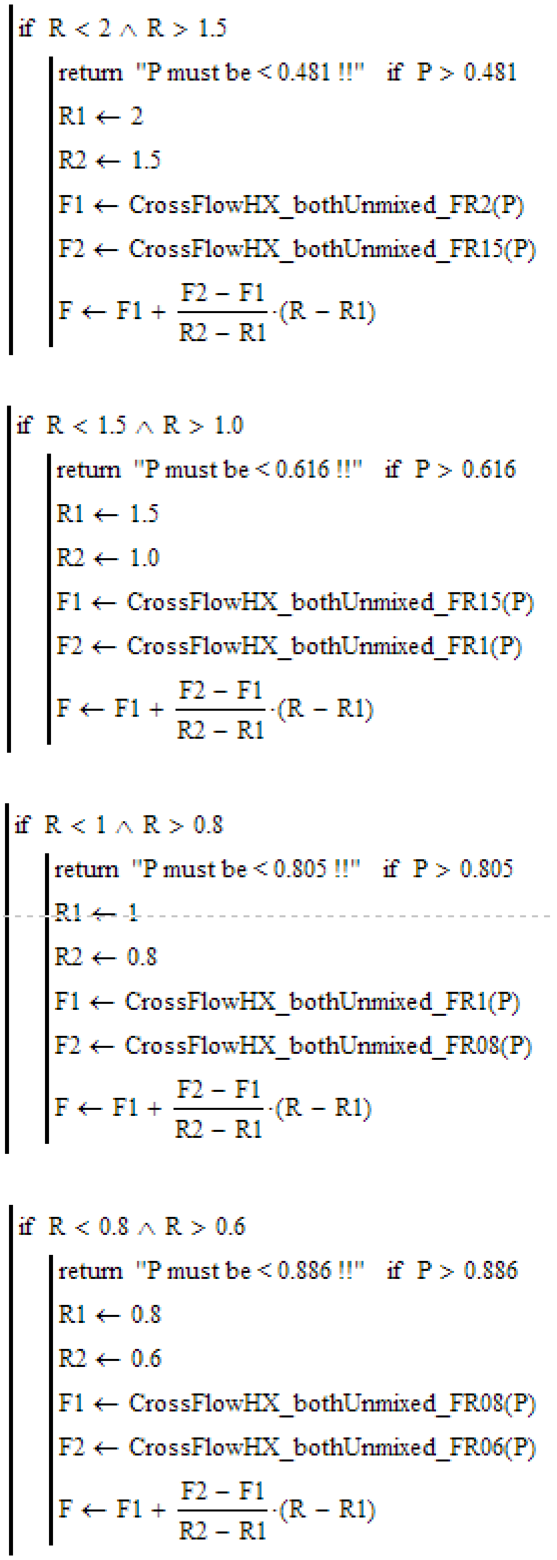


```
CrossFlowHX_bothUnmixed_F(R,P) :=

    return "Valid for 0.2 < R < 4 only!!"  if  R < 0.2 ∨ R > 4
    return "Valid for 1 > P > 0 only!!"  if  P < 0 ∨ P > 1
    F ← CrossFlowHX_bothUnmixed_FR4(P)  if  R = 4
    F ← CrossFlowHX_bothUnmixed_FR3(P)  if  R = 3
    F ← CrossFlowHX_bothUnmixed_FR2(P)  if  R = 2
    F ← CrossFlowHX_bothUnmixed_FR15(P)  if  R = 1.5
    F ← CrossFlowHX_bothUnmixed_FR1(P)  if  R = 1
    F ← CrossFlowHX_bothUnmixed_FR08(P)  if  R = 0.8
    F ← CrossFlowHX_bothUnmixed_FR06(P)  if  R = 0.6
    F ← CrossFlowHX_bothUnmixed_FR04(P)  if  R = 0.4
    F ← CrossFlowHX_bothUnmixed_FR02(P)  if  R = 0.2

    if  R < 4 ∧ R > 3
        return "P must be < 0.232 !!"  if  P > 0.232
        R1 ← 4
        R2 ← 3
        F1 ← CrossFlowHX_bothUnmixed_FR4(P)
        F2 ← CrossFlowHX_bothUnmixed_FR3(P)
        F ← F1 + (F2 − F1)/(R2 − R1)·(R − R1)

    if  R < 3 ∧ R > 2
        return "P must be < 0.327 !!"  if  P > 0.327
        R1 ← 3
        R2 ← 2
        F1 ← CrossFlowHX_bothUnmixed_FR3(P)
        F2 ← CrossFlowHX_bothUnmixed_FR2(P)
        F ← F1 + (F2 − F1)/(R2 − R1)·(R − R1)
```

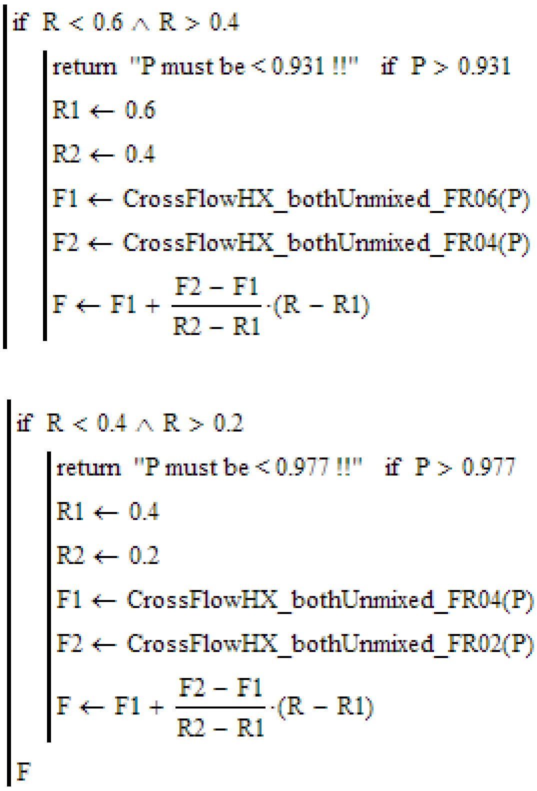


```
if  $R < 2 \wedge R > 1.5$
    return  "P must be < 0.481 !!"   if  $P > 0.481$
    $R1 \leftarrow 2$
    $R2 \leftarrow 1.5$
    $F1 \leftarrow$ CrossFlowHX_bothUnmixed_FR2(P)
    $F2 \leftarrow$ CrossFlowHX_bothUnmixed_FR15(P)
    $F \leftarrow F1 + \frac{F2 - F1}{R2 - R1} \cdot (R - R1)$

if  $R < 1.5 \wedge R > 1.0$
    return  "P must be < 0.616 !!"   if  $P > 0.616$
    $R1 \leftarrow 1.5$
    $R2 \leftarrow 1.0$
    $F1 \leftarrow$ CrossFlowHX_bothUnmixed_FR15(P)
    $F2 \leftarrow$ CrossFlowHX_bothUnmixed_FR1(P)
    $F \leftarrow F1 + \frac{F2 - F1}{R2 - R1} \cdot (R - R1)$

if  $R < 1 \wedge R > 0.8$
    return  "P must be < 0.805 !!"   if  $P > 0.805$
    $R1 \leftarrow 1$
    $R2 \leftarrow 0.8$
    $F1 \leftarrow$ CrossFlowHX_bothUnmixed_FR1(P)
    $F2 \leftarrow$ CrossFlowHX_bothUnmixed_FR08(P)
    $F \leftarrow F1 + \frac{F2 - F1}{R2 - R1} \cdot (R - R1)$

if  $R < 0.8 \wedge R > 0.6$
    return  "P must be < 0.886 !!"   if  $P > 0.886$
    $R1 \leftarrow 0.8$
    $R2 \leftarrow 0.6$
    $F1 \leftarrow$ CrossFlowHX_bothUnmixed_FR08(P)
    $F2 \leftarrow$ CrossFlowHX_bothUnmixed_FR06(P)
    $F \leftarrow F1 + \frac{F2 - F1}{R2 - R1} \cdot (R - R1)$
```



```
if R < 0.6 ∧ R > 0.4
    return "P must be < 0.931 !!" if P > 0.931
    R1 ← 0.6
    R2 ← 0.4
    F1 ← CrossFlowHX_bothUnmixed_FR06(P)
    F2 ← CrossFlowHX_bothUnmixed_FR04(P)
    $F \leftarrow F1 + \frac{F2 - F1}{R2 - R1} \cdot (R - R1)$

if R < 0.4 ∧ R > 0.2
    return "P must be < 0.977 !!" if P > 0.977
    R1 ← 0.4
    R2 ← 0.2
    F1 ← CrossFlowHX_bothUnmixed_FR04(P)
    F2 ← CrossFlowHX_bothUnmixed_FR02(P)
    $F \leftarrow F1 + \frac{F2 - F1}{R2 - R1} \cdot (R - R1)$
F
```

F is calculated by linear interpolation for intermediate R values i.e. at R values other than shown in the plot.

Example:

$R := 3.5 \qquad P := 0.15$

$\text{CrossFlowHX\_bothUnmixed\_F}(R, P) = 0.946$

===

Now, let us work out a problem to show the use of this Mathcad Function:

Prob. 4B.9. In an automobile radiator, hot water enters the tubes at 90 C at a rate of 0.6 kg/s and leaves at 60 C. Air flows across the radiator through the space between fins, and air is heated from 20 C to 40 C. Determine the overall heat transfer coeff. U(based on inner surface area of tubes), if the total inside area of tubes is 0.4 m^2.

Mathcad Solution:

This is a cross-flow HX with both fluids unmixed.

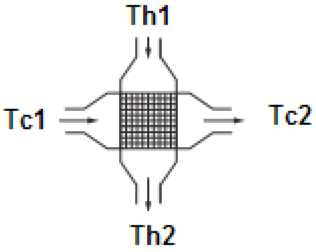


Fig. Prob.4B.9.

Data:

$Th_1 := 90$ C $\qquad Th_2 := 60$ C $\qquad Tc_1 := 20$ C $\qquad Tc_2 := 40$ C

$mh := 0.6$ kg/s.... mass flow rate of water

$cp_h := 4193$ J/kg.C.....sp. heat of water at mean temp of 75 C.

$A_i := 0.4$ m^2....inside surface area of tubes

Calculations:

Heat transferred, Q:

$Q := mh \cdot cp_h \cdot (Th_1 - Th_2)$ i.e. $Q = 7.547 \times 10^4$ W.

Now, Q = U * A * (LMTD * F), where F is the LMTD Correction factor

First, calculate LMTD for a counter flow HX:

$\Delta T1 := Th_1 - Tc_2$ i.e. $\Delta T1 = 50$ C

$\Delta T2 := Th_2 - Tc_1$ i.e. $\Delta T2 = 40$ C

$LMTD := \dfrac{\Delta T1 - \Delta T2}{\ln\left(\dfrac{\Delta T1}{\Delta T2}\right)}$ i.e. $LMTD = 44.814$ C

Now, find Correction factor F:

We have (see the graph):

$P = \dfrac{t_2 - t_1}{T_1 - t_1}$ and, $R = \dfrac{T_1 - T_2}{t_2 - t_1}$

i.e. $P := \dfrac{Tc_2 - Tc_1}{Th_1 - Tc_1}$ i.e. $P = 0.286$

$R := \dfrac{Th_1 - Th_2}{Tc_2 - Tc_1}$ i.e. $R = 1.5$

Then, using the Mathcad Function written above:

$F := \text{CrossFlowHX\_bothUnmixed\_F}(R, P)$

i.e. $F = 0.959$ **....LMTD Correction Factor**

Note: Verify the value of F by referring to the graph, with the calculated P and R values.

Therefore: Overall heat transfer coeff. based on inside area:

$$U_i := \frac{Q}{A_i \cdot LMTD \cdot F}$$

i.e. $U_i = 4.389 \times 10^3$ **m^2...overall heat tr. coeff. ....Ans.**

==

Prob. 4B.10. A cross flow HX in which both fluids are unmixed is used to heat water with an engine oil. Water enters at 30 C and leaves at 85 C at a rate of 1.5 kg/s, while the engine oil with cp = 2300 J/kg.C enters at 120 C with a mass flow rate of 3.5 kg/s. The heat transfer surface area is 30 m^2. Calculate the overall heat transfer coeff. using the LMTD method. [VTU – June/July 2009]

Mathcad Solution:

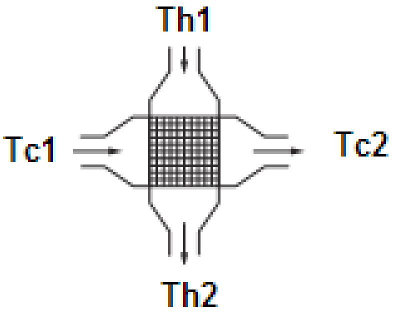


Fig. Prob.4B.10.

Data:

$Th_1 := 120$ C $Tc_1 := 30$ C $Tc_2 := 85$ C

$mc := 1.5$ kg/s.... mass flow rate of water (cold fluid)

$cp_c := 4183$ J/kg.C.....sp. heat of water at mean temp of 55 C.

$mh := 3.5$ kg/s.... mass flow rate of oil (hot fluid)

$cp_h := 2300$ J/kg.C.....sp. heat of oil.

$A := 30$ m^2.... surface area of HX

Calculations:

Heat transferred, Q:

$Q := mc \cdot cp_c \cdot (Tc_2 - Tc_1)$ i.e. $Q = 3.451 \times 10^5$ W.

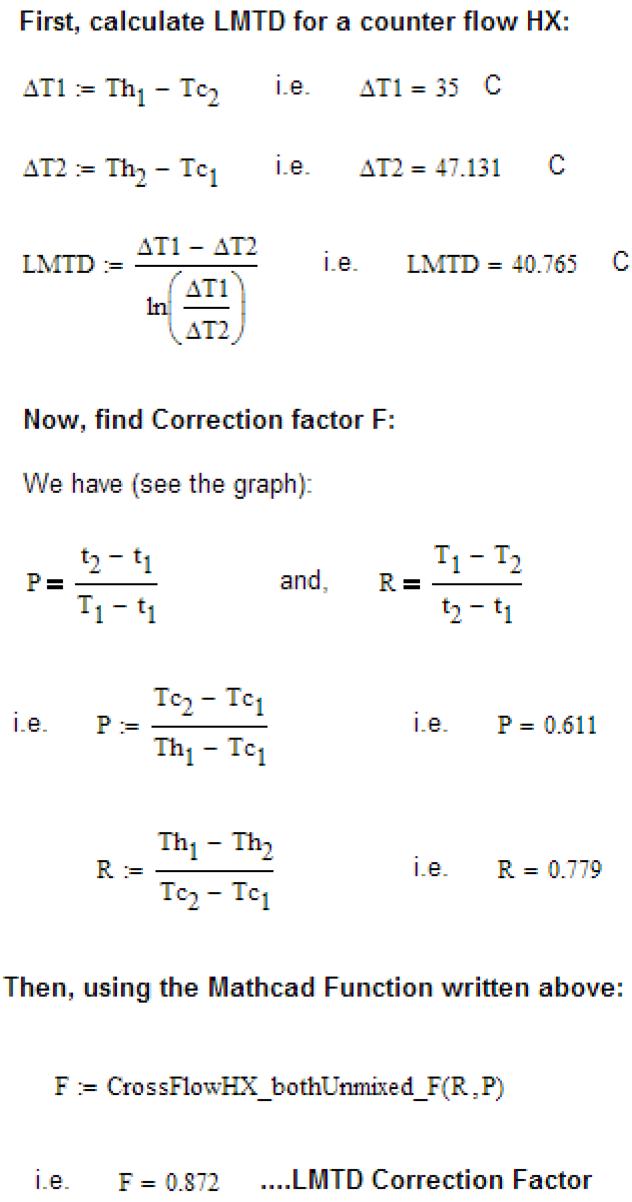
But, Q is also equal to:

$Q = mh \cdot cp_h \cdot (Th_1 - Th_2)$

Therefore:

$Th_2 := Th_1 - \dfrac{Q}{mh \cdot cp_h}$ i.e. $Th_2 = 77.131$ **C....exit temp of oil**

Now, Q = U * A * (LMTD * F), where F is the LMTD Correction factor



First, calculate LMTD for a counter flow HX:

$\Delta T1 := Th_1 - Tc_2$ i.e. $\Delta T1 = 35$ C

$\Delta T2 := Th_2 - Tc_1$ i.e. $\Delta T2 = 47.131$ C

$LMTD := \dfrac{\Delta T1 - \Delta T2}{\ln\left(\dfrac{\Delta T1}{\Delta T2}\right)}$ i.e. $LMTD = 40.765$ C

Now, find Correction factor F:

We have (see the graph):

$P = \dfrac{t_2 - t_1}{T_1 - t_1}$ and, $R = \dfrac{T_1 - T_2}{t_2 - t_1}$

i.e. $P := \dfrac{Tc_2 - Tc_1}{Th_1 - Tc_1}$ i.e. $P = 0.611$

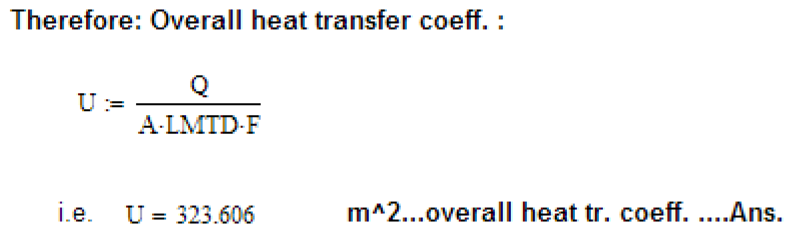
$R := \dfrac{Th_1 - Th_2}{Tc_2 - Tc_1}$ i.e. $R = 0.779$

Then, using the Mathcad Function written above:

$F := CrossFlowHX\_bothUnmixed\_F(R, P)$

i.e. $F = 0.872$ **....LMTD Correction Factor**

Note: check this value of F from the graph, with P = 0.611 and R = 0.779.



Therefore: Overall heat transfer coeff. :

$U := \dfrac{Q}{A \cdot LMTD \cdot F}$

i.e. $U = 323.606$ **m^2...overall heat tr. coeff. ....Ans.**

Consider following extension to the above problem:

If oil flow rate (m_h) varies from 2.5 kg/s to 5 kg/s, with the temperatures Th1, Tc1 and Tc2 remaining const., plot the variation of Th2, F and U with m_h:

As m_h varies, Th2 will vary; then, LMTD, F and, therefore, U will also vary.

To facilitate plotting the graph, express relevant quantities which depend on m_h as functions of m_h:

We have:

$$Th_2(mh) := Th_1 - \frac{Q}{mh \cdot cp_h}$$

Now, Q = U * A * (LMTD * F), where F is the LMTD Correction factor

$$\Delta T1 := Th_1 - Tc_2$$

$$\Delta T2(mh) := Th_2(mh) - Tc_1$$

$$LMTD(mh) := \frac{\Delta T1 - \Delta T2(mh)}{\ln\left(\frac{\Delta T1}{\Delta T2(mh)}\right)}$$

$$P = \frac{t_2 - t_1}{T_1 - t_1} \qquad \text{and,} \qquad R = \frac{T_1 - T_2}{t_2 - t_1}$$

i.e. $$P := \frac{Tc_2 - Tc_1}{Th_1 - Tc_1}$$

$$R(mh) := \frac{Th_1 - Th_2(mh)}{Tc_2 - Tc_1}$$

Then, using the Mathcad Function written above:

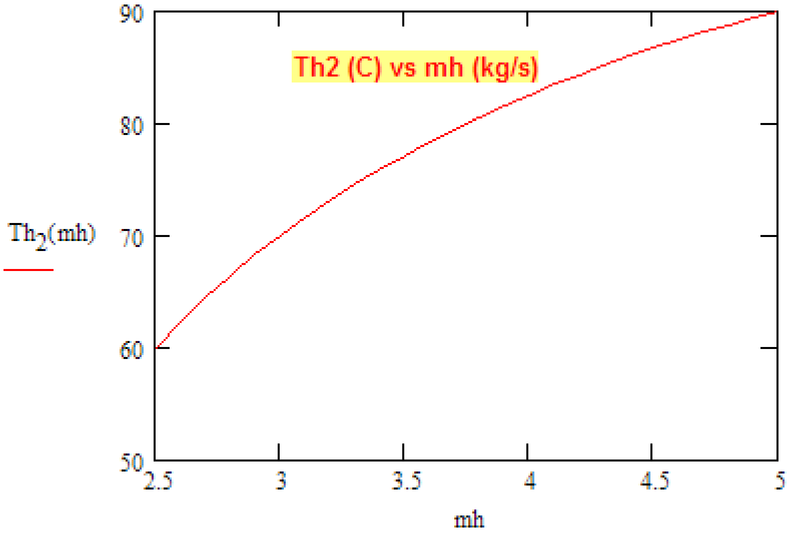
$$F(mh) := CrossFlowHX\_bothUnmixed\_F(R(mh), P)$$ **....LMTD Correction Factor**

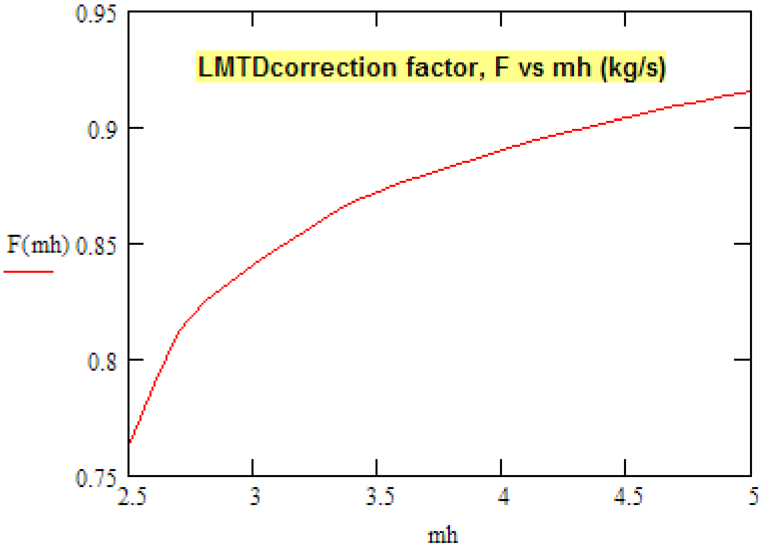
Therefore: Overall heat transfer coeff. :

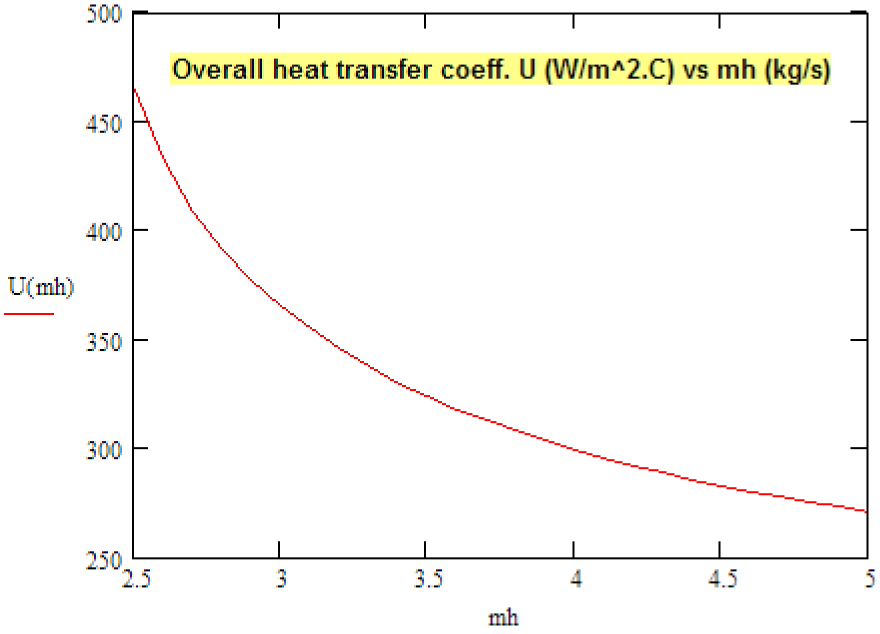
$$U(mh) := \frac{Q}{A \cdot LMTD(mh) \cdot F(mh)}$$

Now, plot the graphs:-

$mh := 2.5, 2.6 .. 5$ kg/s....define a range variable mh





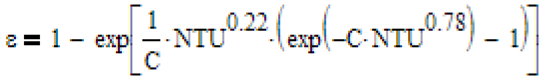

Overall heat transfer coeff. U (W/m^2.C) vs mh (kg/s)
500
450
400
350
300
250
U(mh)
2.5
3
3.5
4
4.5
5
mh

===

Another Mathcad Function to determine LMTD correction factor, F for Cross Flow HX, when both fluids are 'unmixed':

As we said earlier, to get LMTD correction factor, F, we have no analytic solution is available for NTU of such a heat exchanger.

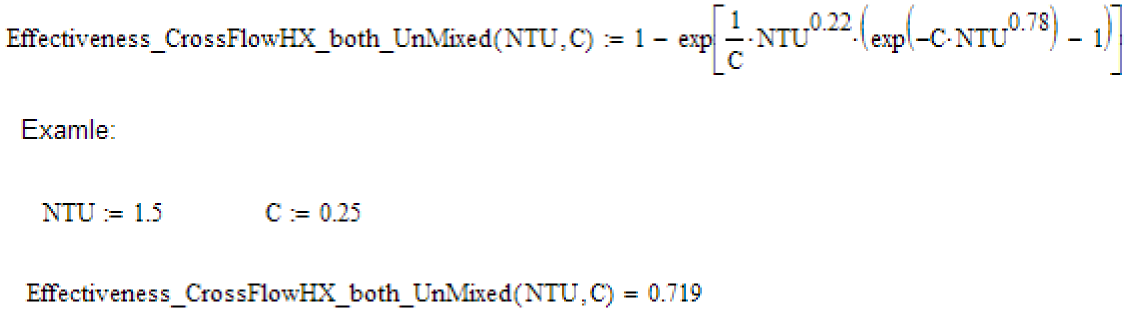
However, Incropera gives the following expression for Effectiveness (ε) of a **Cross Flow HX, when both fluids are 'unmixed':**



$$\varepsilon = 1 - \exp\left[\frac{1}{C}\cdot NTU^{0.22}\cdot\left(\exp\left(-C\cdot NTU^{0.78}\right) - 1\right)\right]$$

We shall use this eqn to get an expression for NTU when C and ε are given:

First, to get ε as a function of NTU and C:



$$\text{Effectiveness\_CrossFlowHX\_both\_UnMixed}(NTU, C) := 1 - \exp\left[\frac{1}{C}\cdot NTU^{0.22}\cdot\left(\exp\left(-C\cdot NTU^{0.78}\right) - 1\right)\right]$$

Examle:

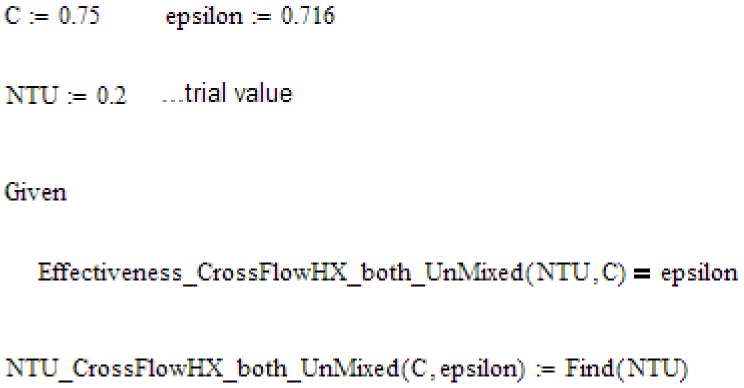
$NTU := 1.5$ $\quad C := 0.25$

$\text{Effectiveness\_CrossFlowHX\_both\_UnMixed}(NTU, C) = 0.719$

Now, use the above Function to write another function to get NTU when ε and C are given:

Cross Flow HX, with both fluids Unmixed:

Function to find NTU when epsilon and C are given:



$C := 0.75$ $\quad epsilon := 0.716$

$NTU := 0.2$ ...trial value

Given

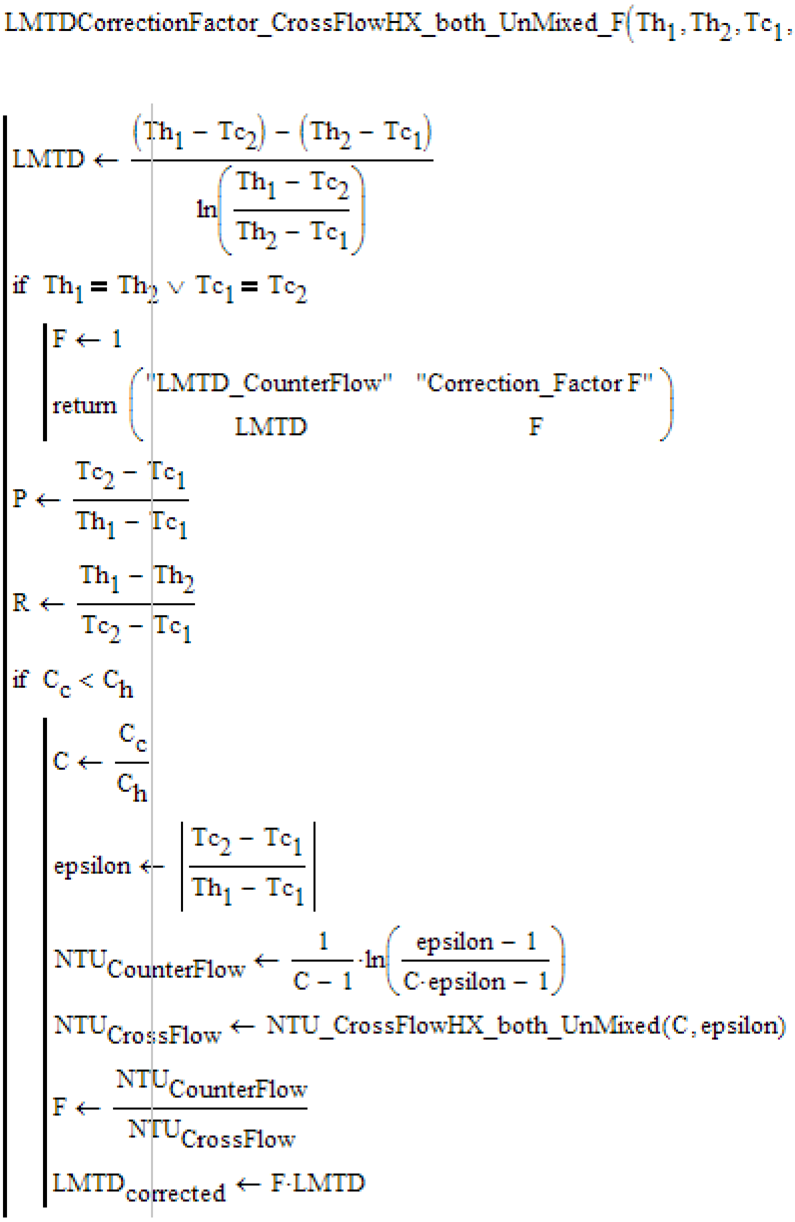
$\text{Effectiveness\_CrossFlowHX\_both\_UnMixed}(NTU, C) = epsilon$

$\text{NTU\_CrossFlowHX\_both\_UnMixed}(C, epsilon) := \text{Find}(NTU)$

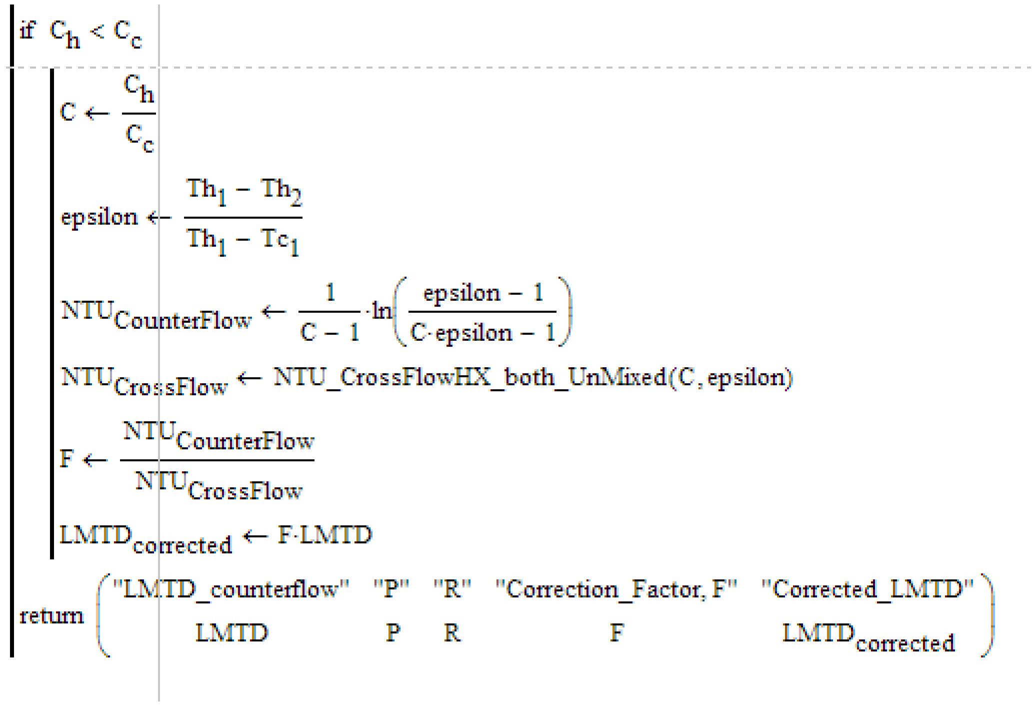
Ex:

$\mathrm{NTU\_CrossFlowHX\_both\_UnMixed}(C, epsilon) = 2.429$

Mathcad Function for LMTD correction factor, F for cross flow HX, both fluids unmixed:



```
$\mathrm{LMTDCorrectionFactor\_CrossFlowHX\_both\_UnMixed\_F}(Th_1, Th_2, Tc_1, Tc_2, C_h, C_c) :=$
    $LMTD \leftarrow \dfrac{(Th_1 - Tc_2) - (Th_2 - Tc_1)}{\ln\left(\dfrac{Th_1 - Tc_2}{Th_2 - Tc_1}\right)}$
    if $Th_1 = Th_2 \vee Tc_1 = Tc_2$
        $F \leftarrow 1$
        return $\begin{pmatrix} \text{"LMTD\_CounterFlow"} & \text{"Correction\_Factor F"} \\ LMTD & F \end{pmatrix}$
    $P \leftarrow \dfrac{Tc_2 - Tc_1}{Th_1 - Tc_1}$
    $R \leftarrow \dfrac{Th_1 - Th_2}{Tc_2 - Tc_1}$
    if $C_c < C_h$
        $C \leftarrow \dfrac{C_c}{C_h}$
        $epsilon \leftarrow \left|\dfrac{Tc_2 - Tc_1}{Th_1 - Tc_1}\right|$
        $NTU_{CounterFlow} \leftarrow \dfrac{1}{C - 1} \cdot \ln\left(\dfrac{epsilon - 1}{C \cdot epsilon - 1}\right)$
        $NTU_{CrossFlow} \leftarrow \mathrm{NTU\_CrossFlowHX\_both\_UnMixed}(C, epsilon)$
        $F \leftarrow \dfrac{NTU_{CounterFlow}}{NTU_{CrossFlow}}$
        $LMTD_{corrected} \leftarrow F \cdot LMTD$
```



```
if $C_h < C_c$
    $C \leftarrow \frac{C_h}{C_c}$
    $epsilon \leftarrow \frac{Th_1 - Th_2}{Th_1 - Tc_1}$
    $NTU_{CounterFlow} \leftarrow \frac{1}{C-1} \cdot \ln\left(\frac{epsilon - 1}{C \cdot epsilon - 1}\right)$
    $NTU_{CrossFlow} \leftarrow$ NTU_CrossFlowHX_both_UnMixed(C, epsilon)
    $F \leftarrow \frac{NTU_{CounterFlow}}{NTU_{CrossFlow}}$
    $LMTD_{corrected} \leftarrow F \cdot LMTD$
return $\begin{pmatrix} \text{"LMTD\_counterflow"} & \text{"P"} & \text{"R"} & \text{"Correction\_Factor, F"} & \text{"Corrected\_LMTD"} \\ LMTD & P & R & F & LMTD_{corrected} \end{pmatrix}$
```

Now, let us work out the Prob. 4B.10 with this Function:

Prob. 4B.10. A cross flow HX in which both fluids are unmixed is used to heat water with an engine oil. Water enters at 30 C and leaves at 85 C at a rate of 1.5 kg/s, while the engine oil with cp = 2300 J/kg.C enters at 120 C with a mass flow rate of 3.5 kg/s. The heat transfer surface area is 30 m^2. Calculate the overall heat transfer coeff. using the LMTD method. [VTU – June/July 2009]

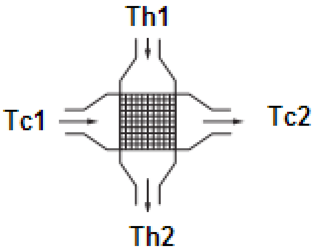
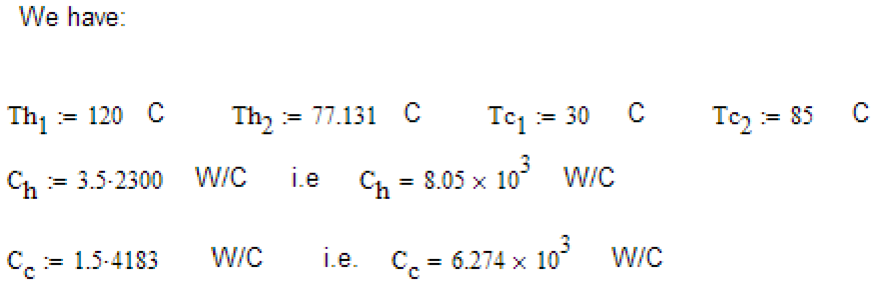


Fig. Prob.4B.10.

Here, we have (Refer to this problem worked out earlier):



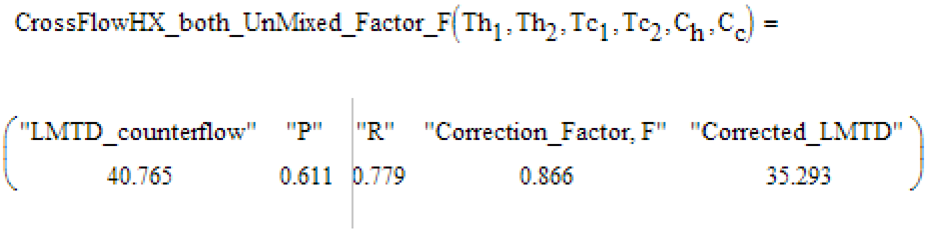
We have:

$Th_1 := 120$ C $\quad Th_2 := 77.131$ C $\quad Tc_1 := 30$ C $\quad Tc_2 := 85$ C

$C_h := 3.5 \cdot 2300$ W/C $\quad$ i.e $\quad C_h = 8.05 \times 10^3$ W/C

$C_c := 1.5 \cdot 4183$ W/C $\quad$ i.e. $\quad C_c = 6.274 \times 10^3$ W/C

And, using the new Mathcad Function:



$$\text{CrossFlowHX\_both\_UnMixed\_Factor\_F}(Th_1, Th_2, Tc_1, Tc_2, C_h, C_c) =$$

$$\begin{pmatrix} \text{"LMTD\_counterflow"} & \text{"P"} & \text{"R"} & \text{"Correction\_Factor, F"} & \text{"Corrected\_LMTD"} \\ 40.765 & 0.611 & 0.779 & 0.866 & 35.293 \end{pmatrix}$$

i.e. F = 0.866.

Note that earlier we got F = 0.872.

i.e. the difference is:

$$\frac{(0.872-0.866)*100}{0.872} = 0.688\% \text{ ....quite OK.}$$

==

Prob. 4B.11. Sat. steam at 120 C is condensing on the outer surface of a single pass HX. The overall heat transfer coeff is 1600 W/m^2.C. Determine the surface area of the HX required to heat 2000 kg/h of water from 20 C to 90 C. Also determine the rate of condensation of steam (kg/h). Assume latent heat of steam as 2195 kJ/kg. [VTU – Aug. 2001]

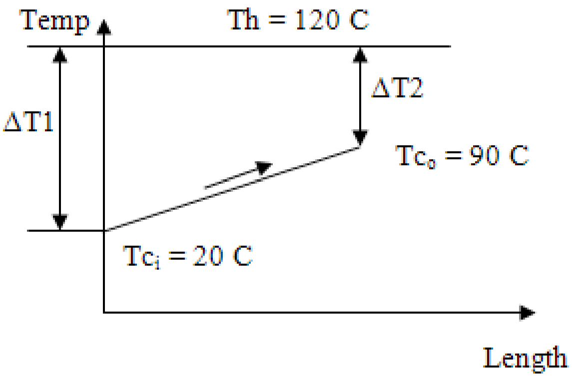


Fig. Prob.4B.11.

EES Solution:

"Data:"

m_c = 2000 [kg/h] * convert (kg/h, kg/s) "....cold fluid – water"
T_h = 120 [C]
T_c_i = 20 [C]
T_c_o = 90 [C]
cp_c = 4180 [J/kg-C]
U= 1600 [W/m^2-C]

"LMTD for a condenser:"

DELTAT_1 = T_h – T_c_i "Temp diff at inlet"
DELTAT_2 = T_h – T_c_o "Temp diff at exit"

LMTD = (DELTAT_1 – DELTAT_2)/ln(DELTAT_1/DELTAT_2) "C...determines LMTD"

Q = m_c * cp_c * (T_c_o – T_c_i) "W....finds heat tr, Q"

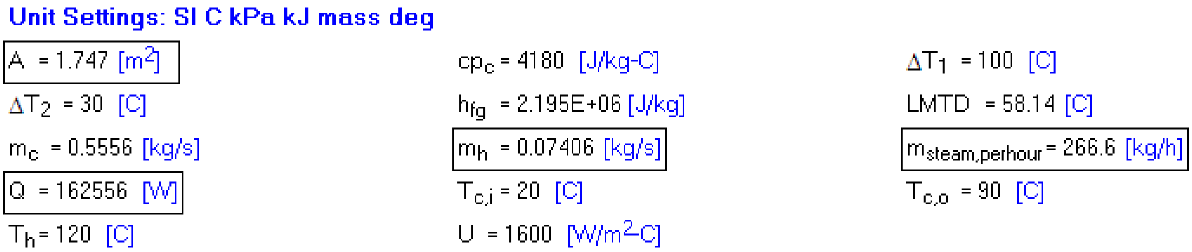
Q = U * A * LMTD "Finds Area, A for condenser"

h_fg = 2195000 [J/kg] "…latent heat for steam condensing"

m_h * h_fg = Q "kg/s…determines mass of steam condensed"

m_steam_perhour = m_h * convert (kg/s, kg/h)

Results:



Unit Settings: SI C kPa kJ mass deg

$\boxed{A = 1.747\ [m^2]}$ $cp_c = 4180\ [J/kg\text{-}C]$ $\Delta T_1 = 100\ [C]$

$\Delta T_2 = 30\ [C]$ $h_{fg} = 2.195E+06\ [J/kg]$ $LMTD = 58.14\ [C]$

$m_c = 0.5556\ [kg/s]$ $\boxed{m_h = 0.07406\ [kg/s]}$ $\boxed{m_{steam,perhour} = 266.6\ [kg/h]}$

$\boxed{Q = 162556\ [W]}$ $T_{c,i} = 20\ [C]$ $T_{c,o} = 90\ [C]$

$T_h = 120\ [C]$ $U = 1600\ [W/m^2\text{-}C]$

Thus:

Area of HX = A = 1.747 m^2 …. Ans.

Rate of condensation of steam = 266.6 kg/h … Ans.

===

"**Prob. 4B.12.** Following data pertains to an oil cooler of the form of tubular HX where oil is cooled by a large pool of stagnant water. Temp of stagnant water = 20 C. Inlet and outlet temps of oil are 80 C and 30 C. Inside dia and length of tube carrying oil are: 20 mm and 3 m. Sp. heat and sp. gravity of oil are: 2.5 kJ/kg.C and 0.85. Average velocity of oil = 0.55 m/s. Calculate the overall heat transfer coeff obtainable from the system. [VTU – July–Aug 2003]"

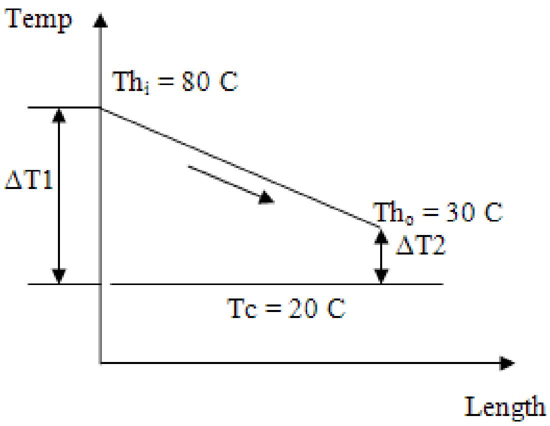


Fig. Prob.4B.12

EES Solution:

"Data:"

d = 0.02[m] "…dia of tube"
L = 3[m] "..length of tube"

T_c = 20 [C]
T_h_i = 80 [C]
T_h_o = 30 [C]
cp_h = 2500 [J/kg-C] "…sp. heat of oil"
rho = 0.85 * 1000 [kg/m^3] "…density of oil"
v = 0.55[m/s] "….velocity"
m_h = rho * (pi*d^2/4) * v "kg/s….mass flow rate of oil"
A = pi * d * L "m^2….area of heat transfer"

"LMTD for this HX:"

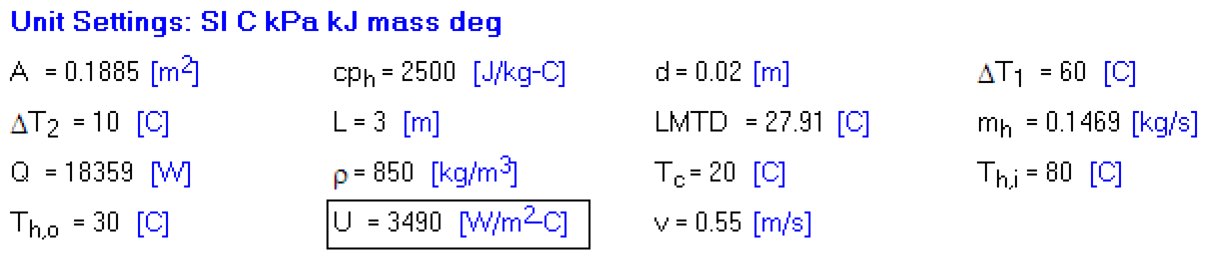
DELTAT_1 = T_h_i – T_c "Temp diff at inlet"
DELTAT_2 = T_h_o – T_c "Temp diff at exit"

LMTD = (DELTAT_1 – DELTAT_2)/ln(DELTAT_1/DELTAT_2) "C…determines LMTD"

Q = m_h * cp_h * (T_h_i – T_h_o) "W….finds heat tr, Q"

Q = U * A * LMTD "Finds U for HX"

Results:



Unit Settings: SI C kPa kJ mass deg

$A = 0.1885$ [m^2] $cp_h = 2500$ [J/kg-C] $d = 0.02$ [m] $\Delta T_1 = 60$ [C]

$\Delta T_2 = 10$ [C] $L = 3$ [m] $LMTD = 27.91$ [C] $m_h = 0.1469$ [kg/s]

$Q = 18359$ [W] $\rho = 850$ [kg/m^3] $T_c = 20$ [C] $T_{h,i} = 80$ [C]

$T_{h,o} = 30$ [C] $\boxed{U = 3490 \text{ [W/m}^2\text{-C]}}$ $v = 0.55$ [m/s]

Thus:

Overall heat tr coeff. = U = 3490 W/m^2.C …. Ans.

==

"**Prob. 4B.13.** A copper pipe (k = 350 W/m.K) of 17.5 mm ID and 20 mm OD conveys water and the oil flows through the annular passage between this pipe and a steel pipe. On the water side, the film coeff is 4600 W/m^2.K and the fouling factor is 0.00034 m^2.K/W. The corresponding values for the oil side are: 1200 W/m^2.K and 0.00086 m^2.K/W. Calculate the overall heat transfer coefficient between the water and oil based on outside surface area of inner pipe. [VTU – Feb. 2002]:"

EES Solution:

"Data:"

h_h = 4600[W/m^2-C] "…water side…flows inside the tube"

h_c = 1200[W/m^2-C] "…oil side… flows on annular side"

d_i = 0.0175[m]

d_o = 0.02[m]

k = 350 [W/m-C]

R_f_i = 0.00034[m^2-C/W]

R_f_o = 0.00086[m^2-C/W]

L = 1[m]"…assumed"

A_i = pi * d_i * L "[m^2]….inside surface area of pipe"

A_o = pi * d_o * L "[m^2]….outside surface area of pipe"

"Thermal resistances:"

R_conv_inner = 1/(h_h *A_i) "C/W….inner conv resistance"

R_conv_outer = 1/(h_c*A_o) "C/W….outer conv resistance"

R_cond = ln(d_o/d_i)/(2 * pi * k *L) "C/W….pipe wall conduction resistance"

R_fouling_in = R_f_i/A_i "C/W….inner fouling resistance"

R_fouling_out = R_f_o/A_o "C/W….outer fouling resistance"

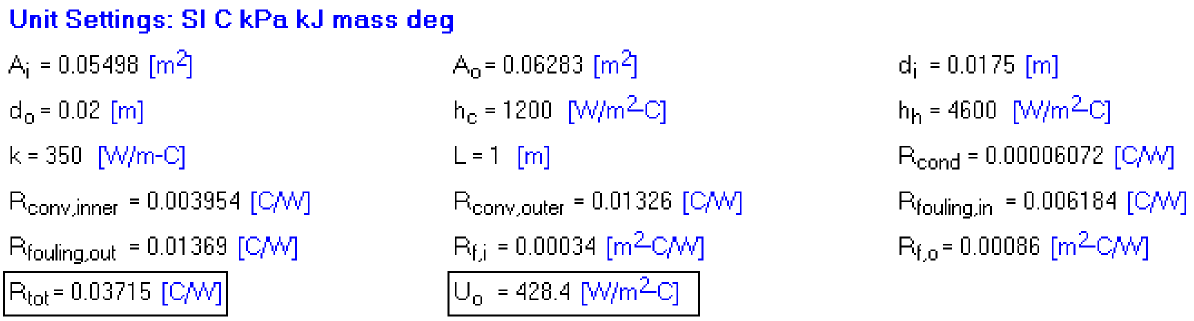
"Total thermal resistance:"

R_tot = R_conv_inner +R_fouling_in +R_cond + R_fouling_out + R_conv_outer "determines R_tot"

"Overall heat transfer coeff.:"

1/(U_o * A_o) = R_tot "determines U_o, overall U based on outer area"

Results:



Unit Settings: SI C kPa kJ mass deg

A_i = 0.05498 [m^2]	A_o = 0.06283 [m^2]	d_i = 0.0175 [m]
d_o = 0.02 [m]	h_c = 1200 [W/m^2-C]	h_h = 4600 [W/m^2-C]
k = 350 [W/m-C]	L = 1 [m]	R_{cond} = 0.00006072 [C/W]
$R_{conv,inner}$ = 0.003954 [C/W]	$R_{conv,outer}$ = 0.01326 [C/W]	$R_{fouling,in}$ = 0.006184 [C/W]
$R_{fouling,out}$ = 0.01369 [C/W]	$R_{f,i}$ = 0.00034 [m^2-C/W]	$R_{f,o}$ = 0.00086 [m^2-C/W]
R_{tot} = 0.03715 [C/W]	U_o = 428.4 [W/m^2-C]	

Thus:

Overall heat transfer coeff, Uo = 428.4 W/m^2.K …. Ans.

Plot the variation of U_o as R_{fi} varies from 0.0001 to 0.0008 m^2.C/W:

First, calculate the parametric table:

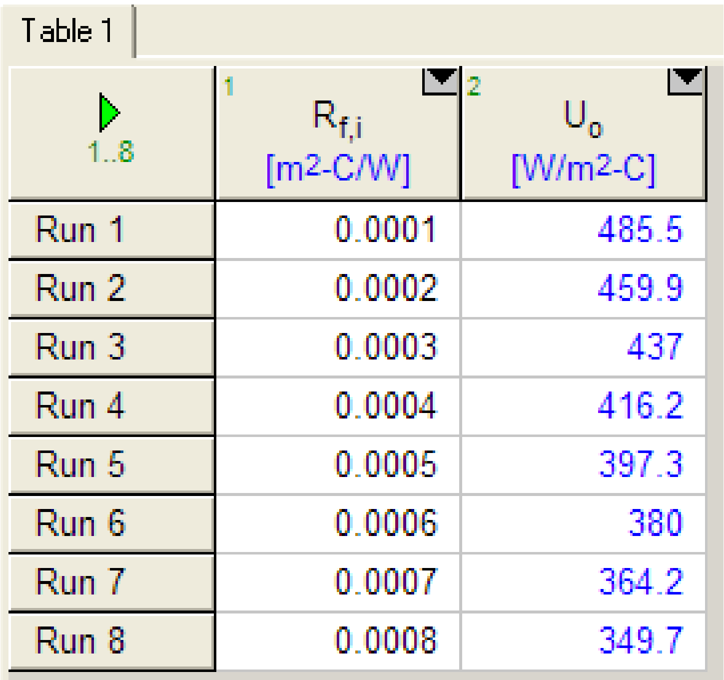
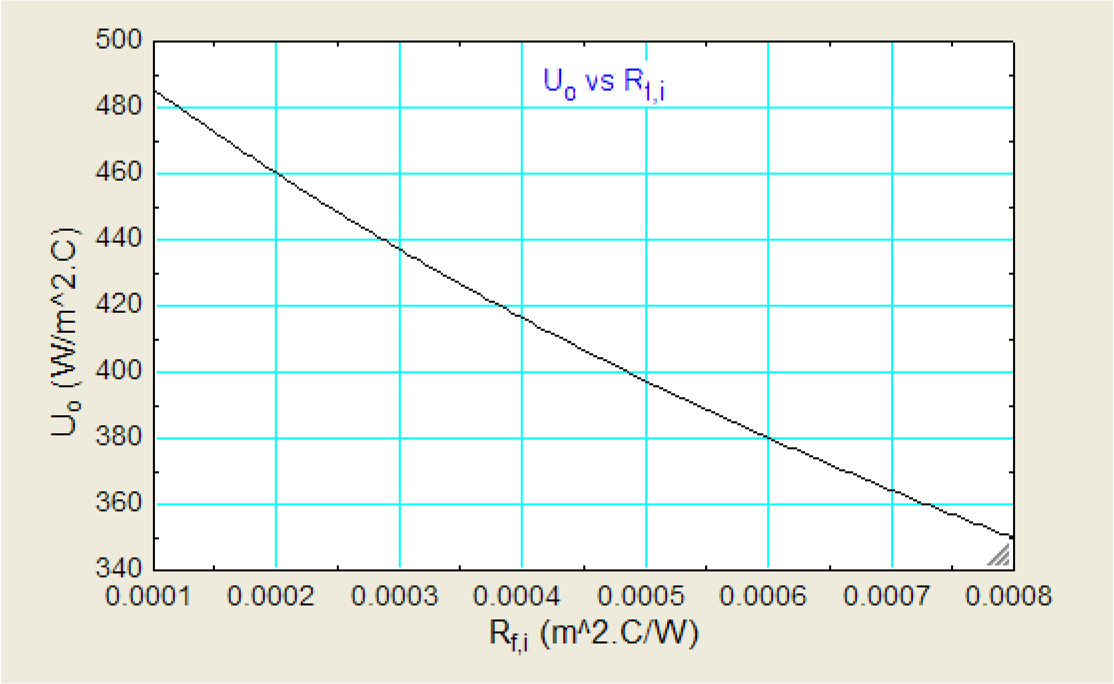


Table 1

1..8	1 $R_{f,i}$ [m2-C/W]	2 U_o [W/m2-C]
Run 1	0.0001	485.5
Run 2	0.0002	459.9
Run 3	0.0003	437
Run 4	0.0004	416.2
Run 5	0.0005	397.3
Run 6	0.0006	380
Run 7	0.0007	364.2
Run 8	0.0008	349.7

And, plot the results:



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"**Prob. 4B.14.** A simple HX consisting of two concentric passages is used for heating 1110 kg/h of oil (cp = 2.1 kJ/kg.K) from 27 C to 49 C. The oil flows through the inner pipe made of copper (OD = 2.86 cm, ID = 2.54 cm, k = 350 W/m.K) and the surface heat transfer coeff on the oil side is 635 W/m^2.K. The oil is heated by hot water supplied at a rate of 390 kg/h with an inlet temp of 93 C. The water side heat transfer coeff. is 1270 W/m^2.K. The fouling factors on the oil and water sides are 0.0001 and 0.0004 m^2.K/W respectively. What is the length of HX required for (i) parallel flow, and

(ii) counter-flow? [VTU – Jan–Feb. 2006]"

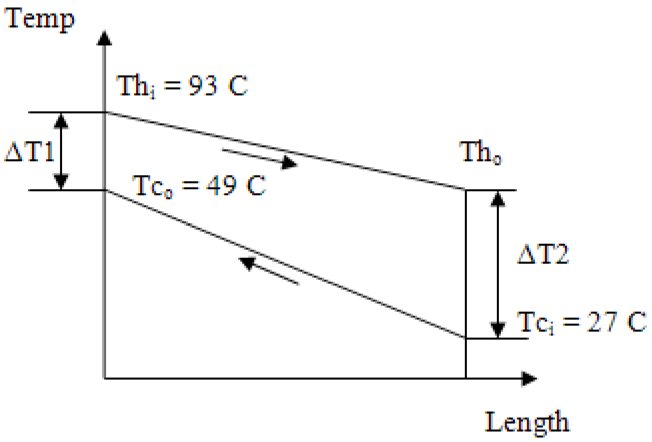


Fig. Prob.4B.14(a). Counter-flow arrangement

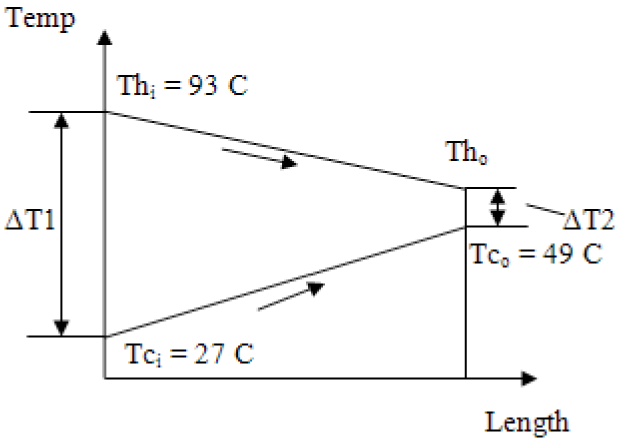


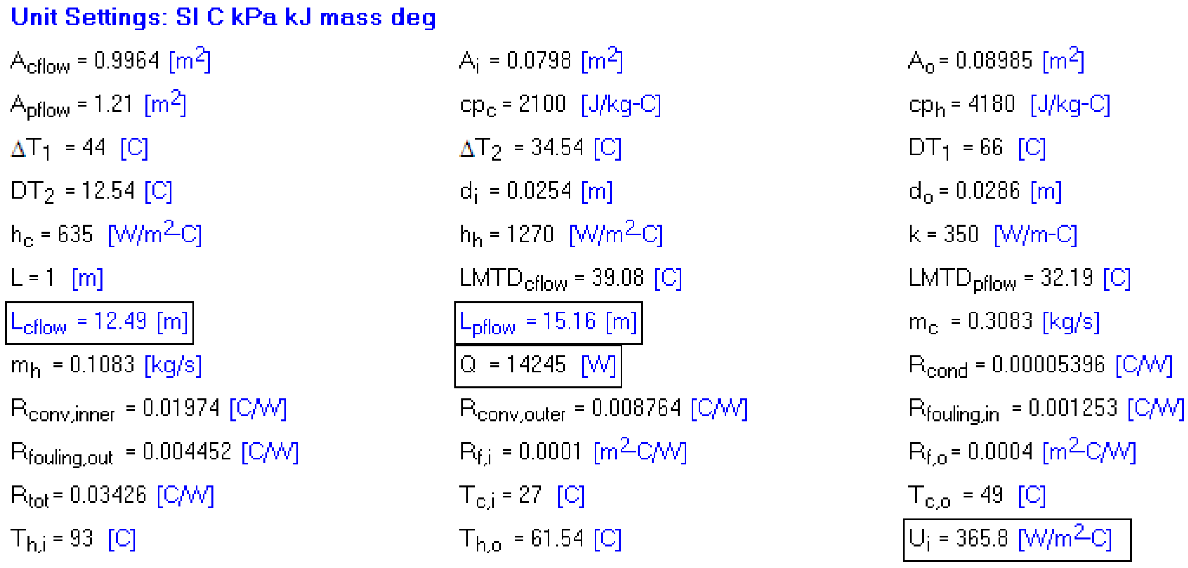
Fig. Prob.4B.14(b). Parallel flow arrangement

EES Solution:

```
“Data:”

m_h = 390 [kg/h] * convert (kg/h, kg/s) “…hot fluid---water”
m_c = 1110 [kg/h] * convert (kg/h, kg/s) “….cold fluid – oil – flows through inner pipe”
T_h_i = 93 [C]“…inlet temp of hot fluid”
T_c_i = 27 [C]“…inlet temp of cold fluid”
T_c_o = 49[C] “…outlet temp of cold fluid”
cp_h = 4180 [J/kg-C]
cp_c = 2100 [J/kg-C]
h_h = 1270[W/m^2-C]
h_c = 635[W/m^2-C]
d_i = 0.0254[m]
d_o = 0.0286[m]
k = 350 [W/m-C]
R_f_i = 0.0001[m^2-C/W]
R_f_o = 0.0004[m^2-C/W]
L = 1[m]”…assumed”
A_i = pi * d_i * L “[m^2]….inside surface area”
A_o = pi * d_o * L “[m^2]….outside surface area”
```

```
"Calculations:"

"Thermal resistances:"

R_conv_inner = 1/(h_c *A_i) "C/W….inner conv resistance"

R_conv_outer = 1/(h_h*A_o) "C/W….outer conv resistance"

R_cond = ln(d_o/d_i)/(2 * pi * k *L) "C/W….pipe wall conduction resistance"

R_fouling_in = R_f_i/A_i "C/W….inner fouling resistance"

R_fouling_out = R_f_o/A_o "C/W….outer fouling resistance"

"Total thermal resistance:"

R_tot = R_conv_inner +R_fouling_in +R_cond + R_fouling_out + R_conv_outer "determines R_tot"

"Overall heat tr coeff."

1/(U_i * A_i) = R_tot "determines U_i, overall U based on inner area"

m_h * cp_h * (T_h_i – T_h_o) = m_c * cp_c * (T_c_o – T_c_i) "C…determines T_h_o"

"LMTD for a counter-flow HX:"

DELTAT_1 = T_h_i – T_c_o "Temp diff at inlet of HX – for counter flow HX"

DELTAT_2 = T_h_o – T_c_i "Temp diff at exit of HX – for counter flow HX"

LMTD_cflow = (DELTAT_1 – DELTAT_2)/ln(DELTAT_1/DELTAT_2) "C…determines LMTD"

Q = m_h * cp_h * (T_h_i – T_h_o) "W…total heat tr."

Q = U_i * A_cflow * LMTD_cflow "Finds A_cflow for counter-flow HX"

A_cflow = pi* d_i * L_cflow "finds L_cflow, Length for cflow HX"

"LMTD for a parallel flow HX:"
DT_1 = T_h_i – T_c_i "Temp diff at inlet of HX – for parallel flow HX"
DT_2 = T_h_o – T_c_o "Temp diff at exit of HX – for parallel flow HX"
LMTD_pflow = (DT_1 – DT_2)/ln(DT_1/DT_2) "C…determines LMTD"
Q = U_i * A_pflow * LMTD_pflow "Finds A_pflow for parallel-flow HX"
A_pflow = pi* d_i * L_pflow "finds L_pflow, Length for pflow HX"
```

Results:



Unit Settings: SI C kPa kJ mass deg

A_{cflow} = 0.9964 [m2]
A_i = 0.0798 [m2]
A_o = 0.08985 [m2]

A_{pflow} = 1.21 [m2]
cp_c = 2100 [J/kg-C]
cp_h = 4180 [J/kg-C]

ΔT_1 = 44 [C]
ΔT_2 = 34.54 [C]
DT_1 = 66 [C]

DT_2 = 12.54 [C]
d_i = 0.0254 [m]
d_o = 0.0286 [m]

h_c = 635 [W/m2-C]
h_h = 1270 [W/m2-C]
k = 350 [W/m-C]

L = 1 [m]
$LMTD_{cflow}$ = 39.08 [C]
$LMTD_{pflow}$ = 32.19 [C]

$\boxed{L_{cflow} = 12.49 \text{ [m]}}$
$\boxed{L_{pflow} = 15.16 \text{ [m]}}$
m_c = 0.3083 [kg/s]

m_h = 0.1083 [kg/s]
$\boxed{Q = 14245 \text{ [W]}}$
R_{cond} = 0.00005396 [C/W]

$R_{conv,inner}$ = 0.01974 [C/W]
$R_{conv,outer}$ = 0.008764 [C/W]
$R_{fouling,in}$ = 0.001253 [C/W]

$R_{fouling,out}$ = 0.004452 [C/W]
$R_{f,i}$ = 0.0001 [m2-C/W]
$R_{f,o}$ = 0.0004 [m2-C/W]

R_{tot} = 0.03426 [C/W]
$T_{c,i}$ = 27 [C]
$T_{c,o}$ = 49 [C]

$T_{h,i}$ = 93 [C]
$T_{h,o}$ = 61.54 [C]
$\boxed{U_i = 365.8 \text{ [W/m}^2\text{-C]}}$

Thus:

Length of HX for parallel flow HX = 15.16 m …. Ans.

Length of HX for counter-flow HX = 12.49 m … Ans.

“**Prob. 4B.15.** A HX is required to cool 55000 kg/h of alcohol from 66 C to 40 C using 40000 kg/h of water entering at 5 C. Calculate the following: (i) the exit temp of water (ii) surface area required for parallel flow and counter-flow HXs. Take U = 580 W/m^2.K, cp for alcohol = 3760 J/kg.K, cp for water = 4180 J/kg.K. [VTU – May–June 2006]”

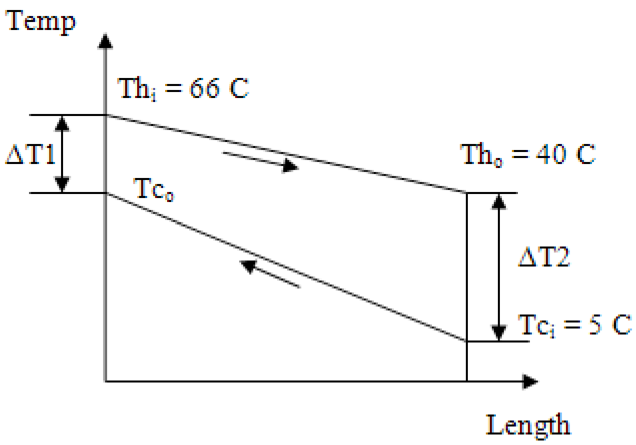


Fig. Prob.4B.15(a). Counter-flow arrangement

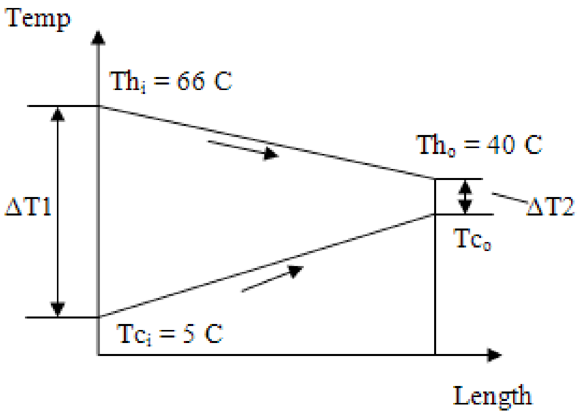


Fig. Prob.4B.15(b). Parallel flow arrangement

EES Solution:

"Data:"

m_h = 55000 [kg/h] * convert (kg/h, kg/s) "…hot fluid – alcohol"

m_c = 40000 [kg/h] * convert (kg/h, kg/s) "….cold fluid – water"

T_h_i = 66 [C]

T_c_i = 5 [C]

T_h_o = 40 [C]

cp_h = 3760 [J/kg-C]

cp_c = 4180 [J/kg-C]

U= 580 [W/m^2-C]

"Calculations:"

"Exit temp of cold fluid:"

m_h * cp_h * (T_h_i – T_h_o) = m_c * cp_c * (T_c_o – T_c_i) "C…determines T_c_o"

"LMTD for a counter-flow HX:"

DELTAT_1 = T_h_i – T_c_o "Temp diff at inlet of HX --- for counter flow HX"

DELTAT_2 = T_h_o – T_c_i "Temp diff at exit of HX --- for counter flow HX"

LMTD_cflow = (DELTAT_1 – DELTAT_2)/ln(DELTAT_1/DELTAT_2) "C…determines LMTD"

Q = m_h * cp_h * (T_h_i – T_h_o) "W…heat tr."

Q = U * A_cflow * LMTD_cflow "Finds A_cflow for counter-flow HX"

"LMTD for a parallel flow HX:"

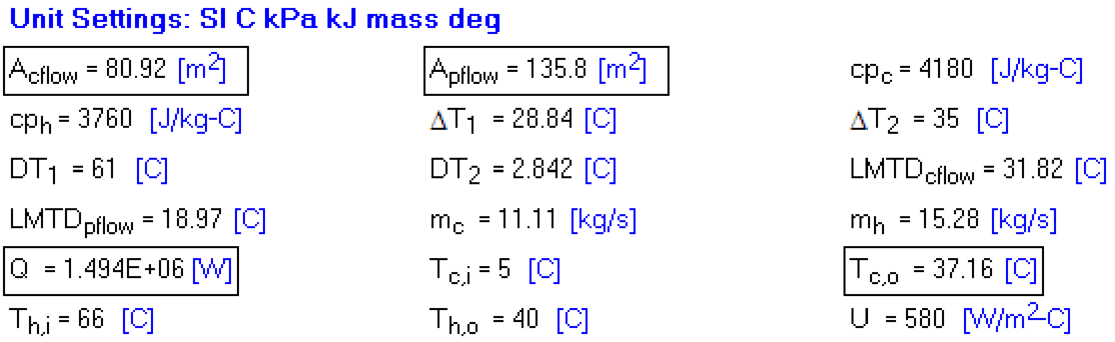
DT_1 = T_h_i – T_c_i "Temp diff at inlet of HX – for parallel flow HX"

DT_2 = T_h_o – T_c_o "Temp diff at exit of HX – for parallel flow HX"

LMTD_pflow = (DT_1 – DT_2)/ln(DT_1/DT_2) "C…determines LMTD"

Q = U * A_pflow * LMTD_pflow "Finds A_pflow for parallel-flow HX"

Results:



Unit Settings: SI C kPa kJ mass deg

$\boxed{A_{cflow} = 80.92\ [m^2]}$ $\boxed{A_{pflow} = 135.8\ [m^2]}$ $cp_c = 4180\ [J/kg\text{-}C]$

$cp_h = 3760\ [J/kg\text{-}C]$ $\Delta T_1 = 28.84\ [C]$ $\Delta T_2 = 35\ [C]$

$DT_1 = 61\ [C]$ $DT_2 = 2.842\ [C]$ $LMTD_{cflow} = 31.82\ [C]$

$LMTD_{pflow} = 18.97\ [C]$ $m_c = 11.11\ [kg/s]$ $m_h = 15.28\ [kg/s]$

$\boxed{Q = 1.494E+06\ [W]}$ $T_{c,i} = 5\ [C]$ $\boxed{T_{c,o} = 37.16\ [C]}$

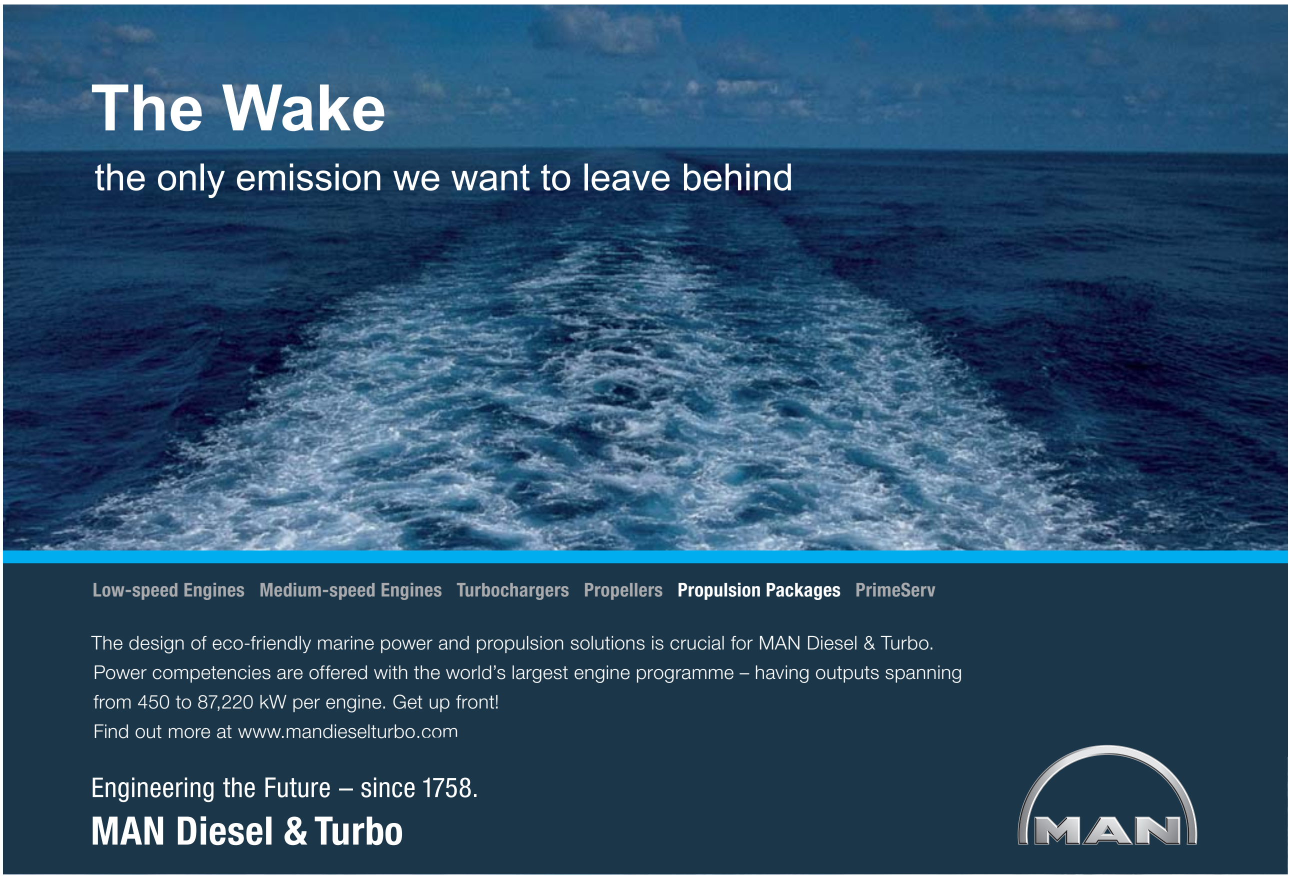
$T_{h,i} = 66\ [C]$ $T_{h,o} = 40\ [C]$ $U = 580\ [W/m^2\text{-}C]$

Thus:

Exit temp of water = 37.16 C … Ans.

Area for parallel flow HX = 135.8 m^2 …. Ans.

Area for counter-flow HX = 80.92 m^2 …. Ans.



"**Prob. 4B.16.** The flow rate of hot and cold fluid streams running through a parallel flow HX are 0.2 kg/s and 0.5 kg/s respectively. The inlet temps on the hot and cold sides are 75 C and 20 C respectively. The exit temp of hot water is 45 C. If the individual heat transfer coeffs on both sides are 650 W/m^2.C, calculate the area of heat transfer. [VTU – Dec. 2009–Jan. 2010]:"

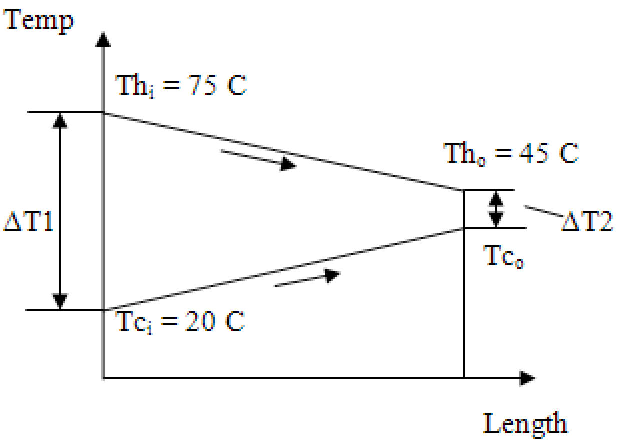


Fig. Prob.4B.16. Parallel flow arrangement

EES Solution:

"Data:"

m_h = 0.2 [kg/s]

m_c = 0.5 [kg/s]

T_h_i = 75 [C]

T_c_i = 20 [C]

T_h_o = 45 [C]

h_h = 650 [W/m^2-C]

h_c = 650 [W/m^2-C]

cp_h = 4180 [J/kg-C]

cp_c = 4180 [J/kg-C]

"Calculations:"

1/U = 1/h_h + 1/h_c "Finds Overall heat tr coeff. U"

m_h * (T_h_i – T_h_o) = m_c * (T_c_o – T_c_i) "…determines T_c_o; sp. heats are same for both streams…"

DELTAT_1 = T_h_i – T_c_i "Temp diff at inlet of HX – for parallel flow HX"

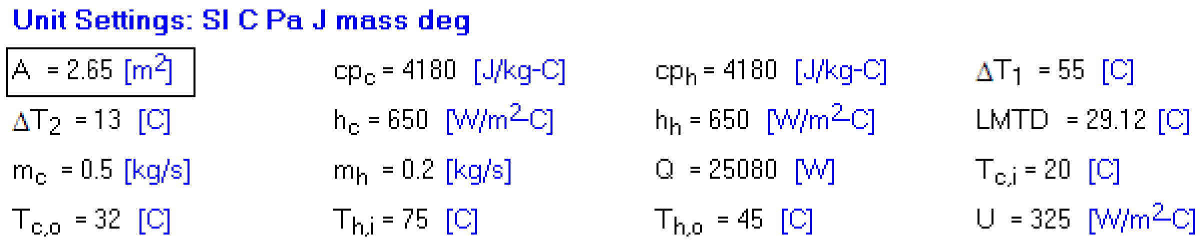
DELTAT_2 = T_h_o – T_c_o "Temp diff at exit of HX – for parallel flow HX"

LMTD = (DELTAT_1 – DELTAT_2)/ln(DELTAT_1/DELTAT_2) "C…determines LMTD"

Q = m_h * cp_h * (T_h_i – T_h_o) "W…heat tr."

Q = U * A * LMTD "…finds area, A"

Results:



Unit Settings: SI C Pa J mass deg

A = 2.65 [m2] cp_c = 4180 [J/kg-C] cp_h = 4180 [J/kg-C] ΔT_1 = 55 [C]
ΔT_2 = 13 [C] h_c = 650 [W/m2-C] h_h = 650 [W/m2-C]   LMTD = 29.12 [C]
m_c = 0.5 [kg/s] m_h = 0.2 [kg/s]   Q = 25080 [W] $T_{c,i}$ = 20 [C]
$T_{c,o}$ = 32 [C] $T_{h,i}$ = 75 [C] $T_{h,o}$ = 45 [C]   U = 325 [W/m2-C]

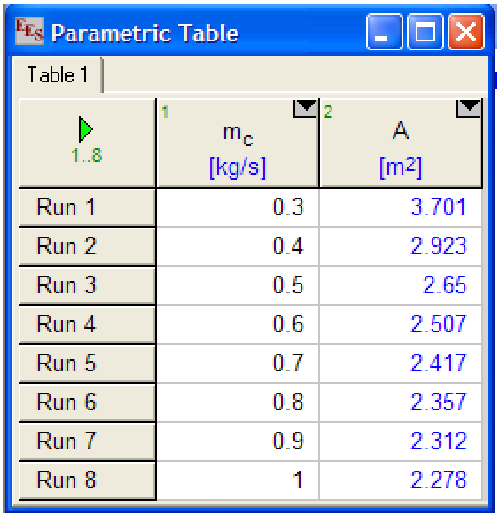
Thus:

Area required = A = 2.65 m^2 … Ans.

Consider the following extension to the above problem:

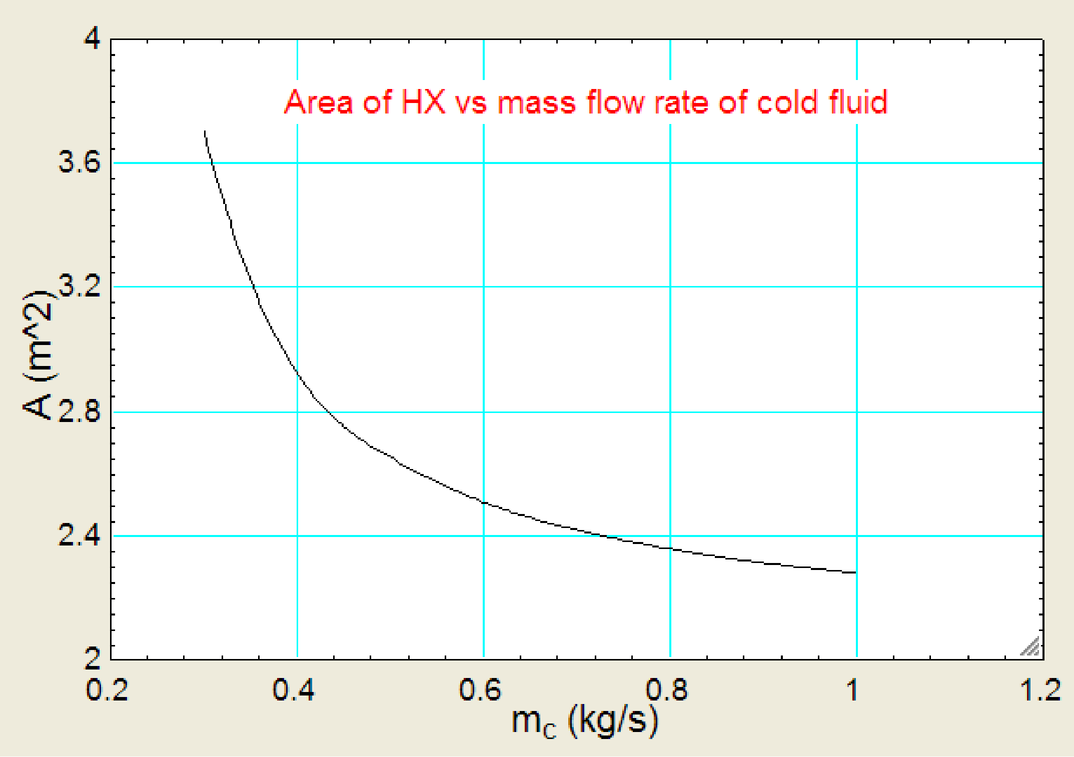
If the cold fluid flow rate (m_c) varies from 0.3 to 1 kg/s, plot the variation of Area of HX (A) with m_c:

First, prepare the Parametric Table:



▶ 1..8	1 m_c [kg/s]	2 A [m2]
Run 1	0.3	3.701
Run 2	0.4	2.923
Run 3	0.5	2.65
Run 4	0.6	2.507
Run 5	0.7	2.417
Run 6	0.8	2.357
Run 7	0.9	2.312
Run 8	1	2.278

And, now plot the graph:



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“**Prob. 4B.17**. A cross-flow HX in which both fluids are unmixed is used to heat water with engine oil. Water enter at 30 C and leaves at 85 C at a rate of 1.5 kg/s, while the engine oil with cp = 2.3 kJ/kg.C enters at 120 C with a mass flow rate of 3.5 kg/s. The heat transfer surface area is 30 m^2. Calculate the overall heat transfer coefficient by using the LMTD method. [VTU – June–July 2009]”

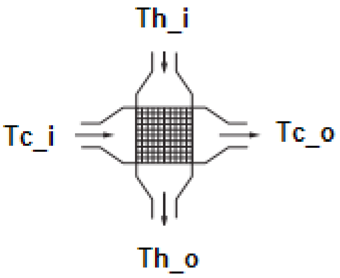


Fig. Prob.4B.17.

Note: This Prob. Is the same as Prob. 4B.10, which was solved with Mathcad.

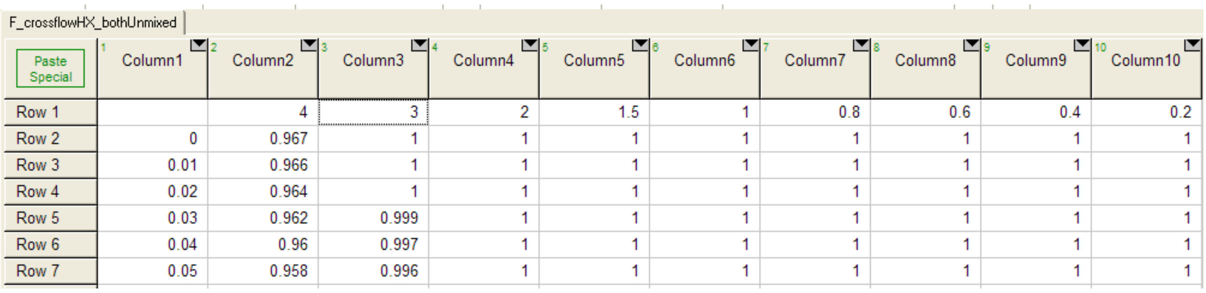
Now, we shall solve it with EES and demonstrate the use of 2D Interpolation in a Table:

Note: Here, since it is a cross flow HX, we need to apply the correction factor for LMTD.

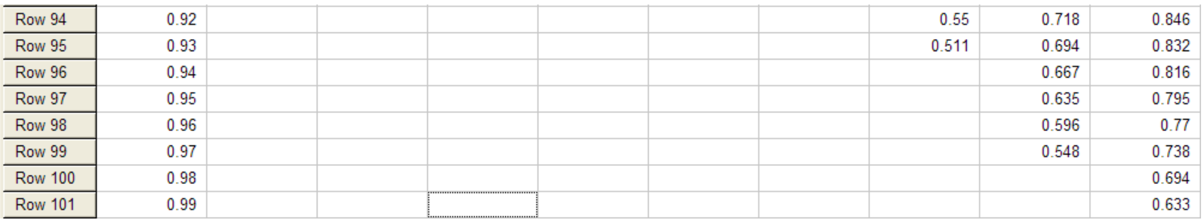
Recollect that prior to solving Problem 4B.9, we wrote a Mathcad program to determine F as a Function of P and R. Using that Function, generate a Table of F values for different R and P values and copy that Table to EES as a “Lookup Table”, named “F_crossFlowHX_bothUnmixed”.

Now, with this Look up Table, use the EES built-in 2D Interpolation Function Interpolate2DM(‘F_crossFlowHX_bothUnmixed’,R,P) to get value of at given R and P.

Part of the Lookup Table is shown below:



	Column1	Column2	Column3	Column4	Column5	Column6	Column7	Column8	Column9	Column10
Row 1		4	3	2	1.5	1	0.8	0.6	0.4	0.2
Row 2	0	0.967	1	1	1	1	1	1	1	1
Row 3	0.01	0.966	1	1	1	1	1	1	1	1
Row 4	0.02	0.964	1	1	1	1	1	1	1	1
Row 5	0.03	0.962	0.999	1	1	1	1	1	1	1
Row 6	0.04	0.96	0.997	1	1	1	1	1	1	1
Row 7	0.05	0.958	0.996	1	1	1	1	1	1	1



Row 94	0.92							0.55	0.718	0.846
Row 95	0.93							0.511	0.694	0.832
Row 96	0.94								0.667	0.816
Row 97	0.95								0.635	0.795
Row 98	0.96								0.596	0.77
Row 99	0.97								0.548	0.738
Row 100	0.98									0.694
Row 101	0.99									0.633

EES Solution:

"Data:"

m_h = 3.5 [kg/s] "…hot fluid – oil"

m_c = 1.5 [kg/s] "….cold fluid – water"

T_h_i = 120 [C]

T_c_i = 30 [C]

T_c_o = 85 [C]

cp_h = 2300 [J/kg-C]

cp_c = 4180 [J/kg-C]

A = 30 [m^2]

m_h * cp_h * (T_h_i – T_h_o) = m_c * cp_c * (T_c_o – T_c_i) "C…determines T_h_o"

"LMTD for a counter-flow HX:"

DELTAT_1 = T_h_i – T_c_o "Temp diff at inlet of HX – for counter flow HX"

DELTAT_2 = T_h_o – T_c_i "Temp diff at exit of HX – for counter flow HX"

LMTD = (DELTAT_1 – DELTAT_2)/ln(DELTAT_1/DELTAT_2) "C…determines LMTD"

"To find LMTD Correction factor F for a cross-flow HX….

either from the graph for a single pass HX with both fluids unmixed:, OR:

Use the Interpolation Function to read F value from the 'Look up Table':"

R = (T_h_i – T_h_o)/(T_c_o – T_c_i) "R = 0.7789, to be used in the graph to get F"

P = (T_c_o – T_c_i)/(T_h_i – T_c_i) "P = 0.6111, to be used in the graph to get F"

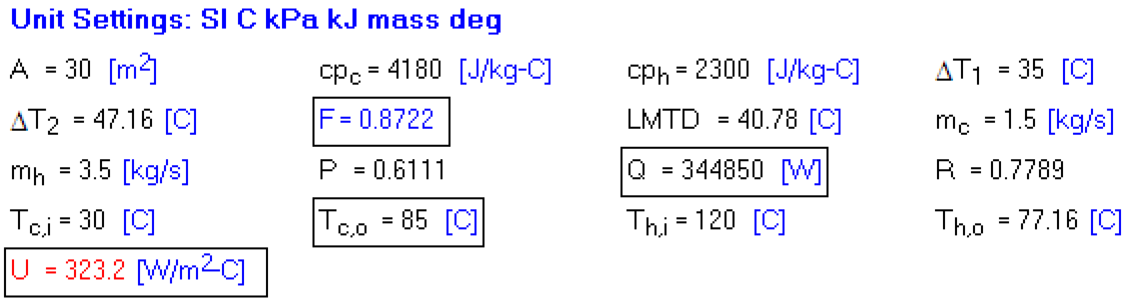
F=Interpolate2DM('F_crossFlowHX_bothUnmixed',R,P) "….finds F from the Lookup Table by 2 D Interpolation…we get: F = 0.8772"

{Note: F = 0.9 approx. "From graph, for above values of R and P"}

Q = m_h * cp_h * (T_h_i – T_h_o) "W…heat tr."

Q = U * A * LMTD * F "… Finds U"

Results:



Unit Settings: SI C kPa kJ mass deg

$A = 30$ [m^2]

$cp_c = 4180$ [J/kg-C]

$cp_h = 2300$ [J/kg-C]

$\Delta T_1 = 35$ [C]

$\Delta T_2 = 47.16$ [C]

$F = 0.8722$

LMTD = 40.78 [C]

$m_c = 1.5$ [kg/s]

$m_h = 3.5$ [kg/s]

$P = 0.6111$

$Q = 344850$ [W]

$R = 0.7789$

$T_{c,i} = 30$ [C]

$T_{c,o} = 85$ [C]

$T_{h,i} = 120$ [C]

$T_{h,o} = 77.16$ [C]

$U = 323.2$ [W/m^2-C]

Thus:

Overall heat transfer coeff. U = 323.2 W/m^2.C….Ans.

===

Consider following extension to the above problem:

If oil flow rate (m_h) varies from 2.5 kg/s to 5.25 kg/s, with the temperatures Th1, Tc1 and Tc2 remaining const., plot the variation of Th2, F and U with m_h:

First, construct the Parametric Table:

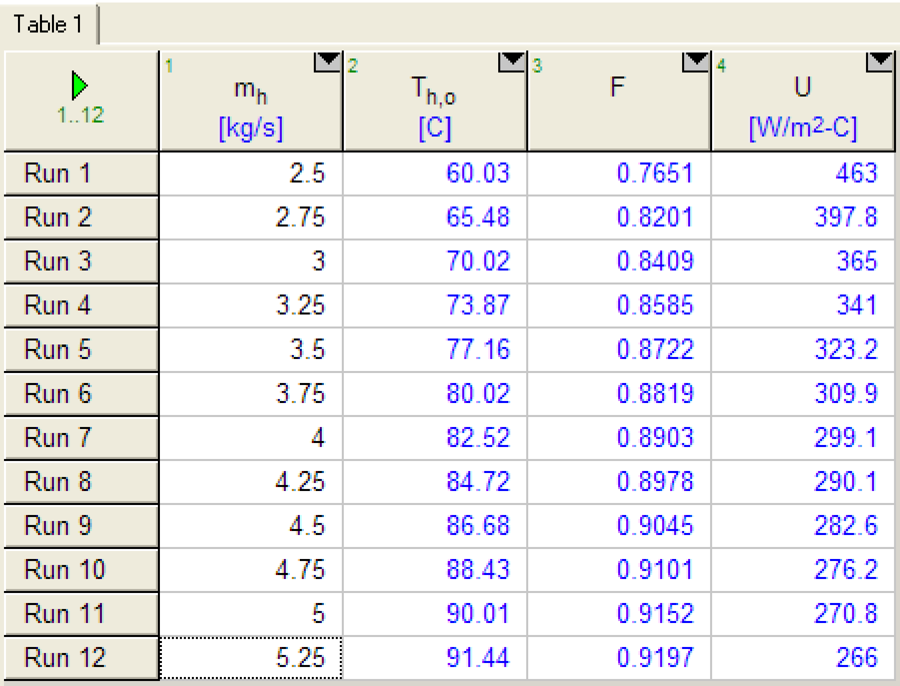
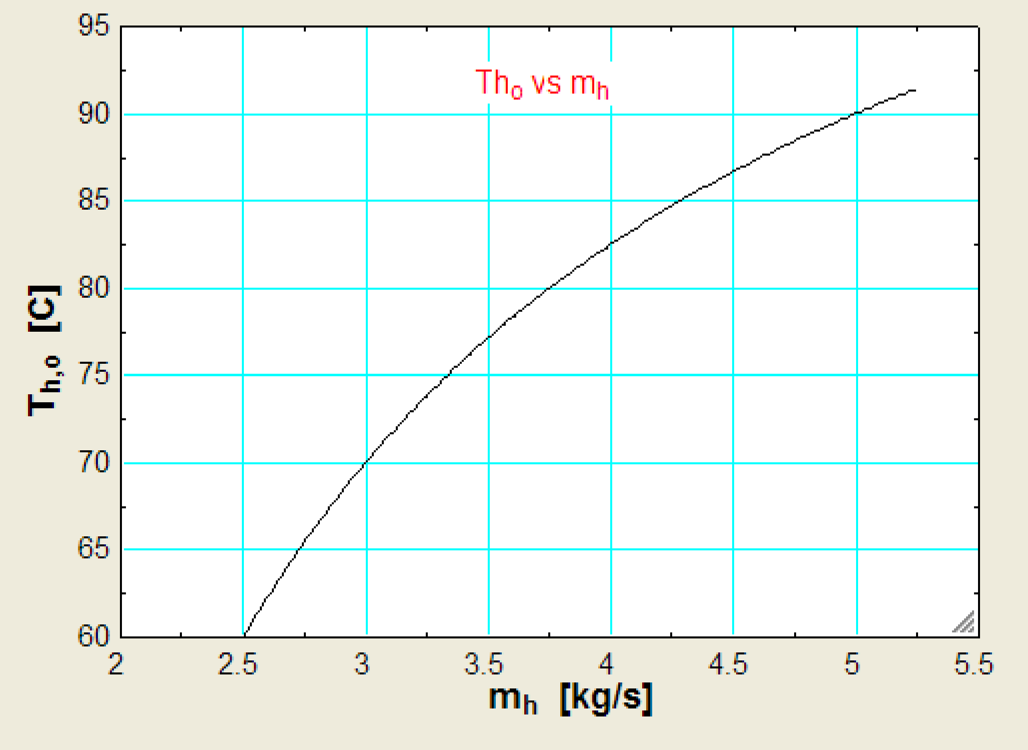
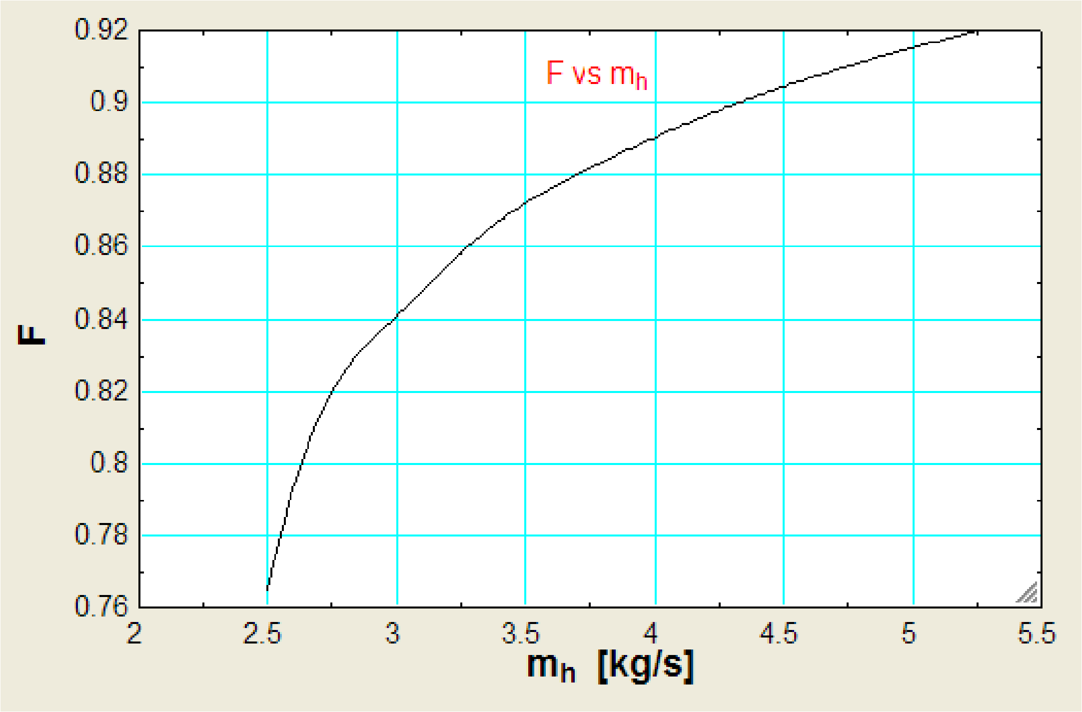


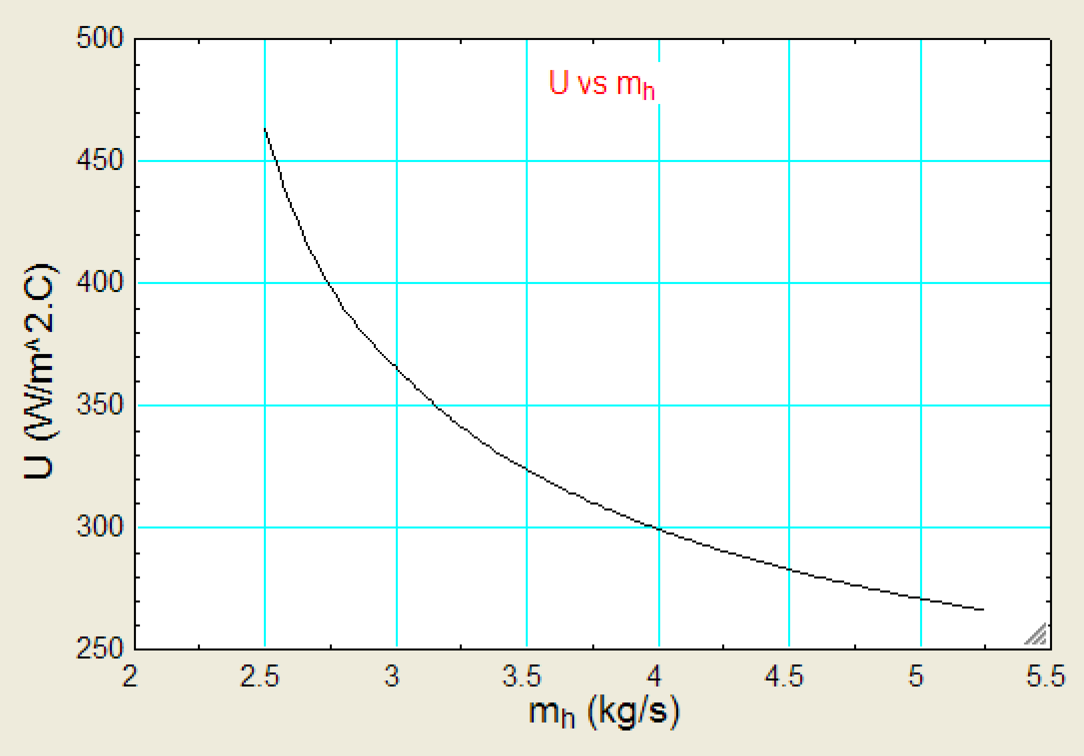
Table 1

1..12	1 m_h [kg/s]	2 $T_{h,o}$ [C]	3 F	4 U [W/m2-C]
Run 1	2.5	60.03	0.7651	463
Run 2	2.75	65.48	0.8201	397.8
Run 3	3	70.02	0.8409	365
Run 4	3.25	73.87	0.8585	341
Run 5	3.5	77.16	0.8722	323.2
Run 6	3.75	80.02	0.8819	309.9
Run 7	4	82.52	0.8903	299.1
Run 8	4.25	84.72	0.8978	290.1
Run 9	4.5	86.68	0.9045	282.6
Run 10	4.75	88.43	0.9101	276.2
Run 11	5	90.01	0.9152	270.8
Run 12	5.25	91.44	0.9197	266

Now, plot the graphs:







Note: Compare these values with those obtained in Prob. 4B.10, which was solved with Mathcad.

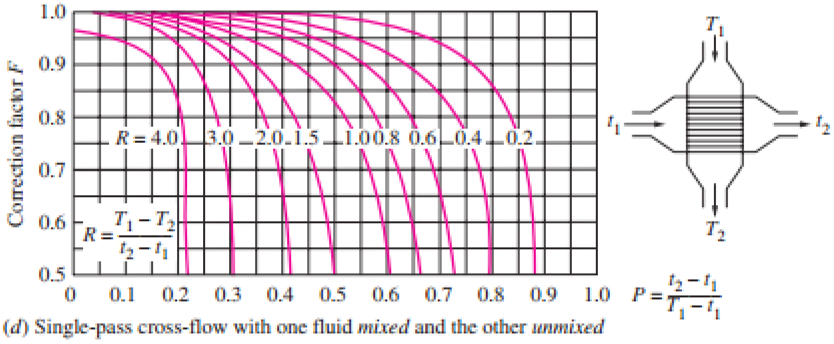
They match quite well.

==

LMTD correction factor F for a Cross-flow HX with one fluid 'mixed' and the other 'unmixed':

We follow the same method as we did for the case of cross-flow HX with both fluids 'unmixed'.

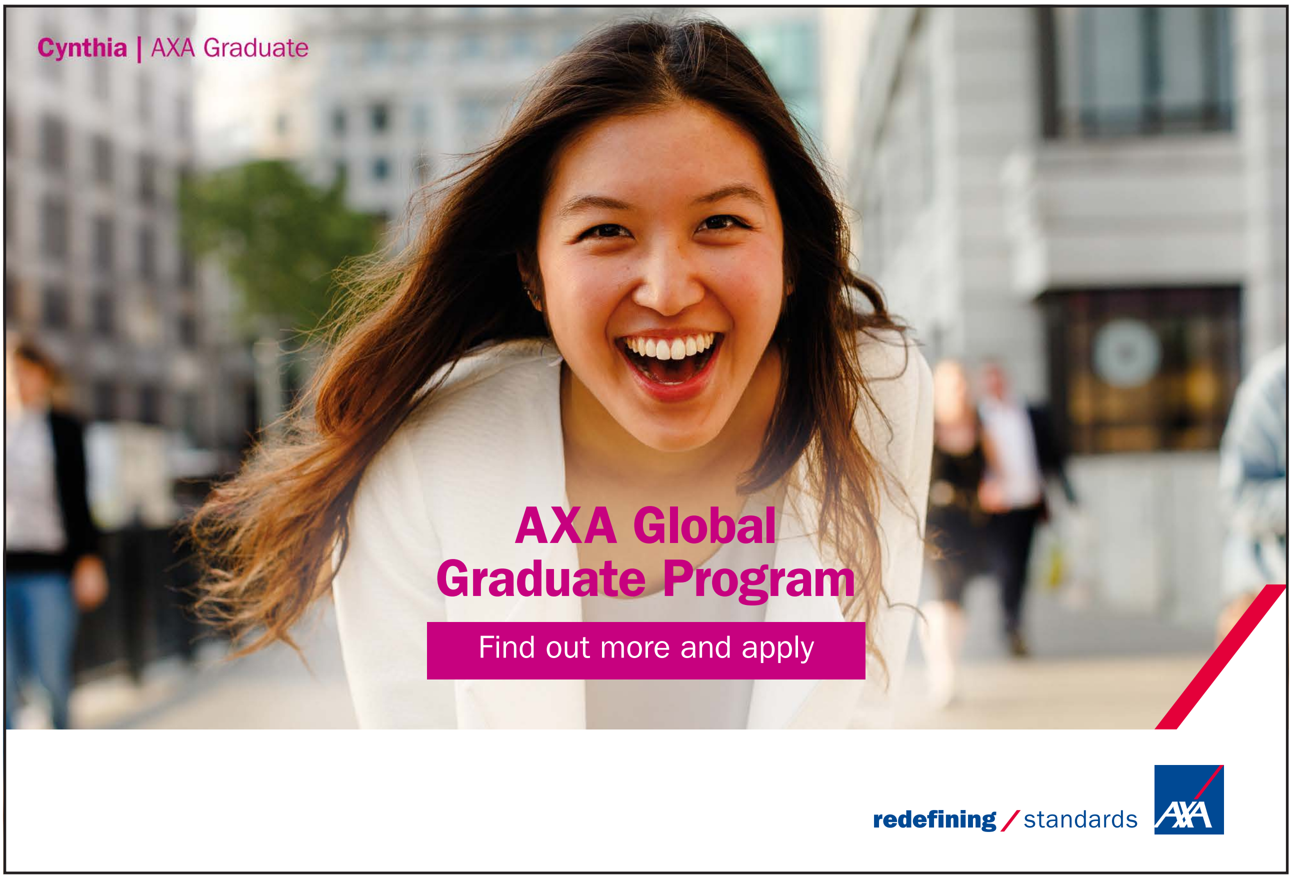
i.e. First, digitize the following graph (Ref: Cengel):



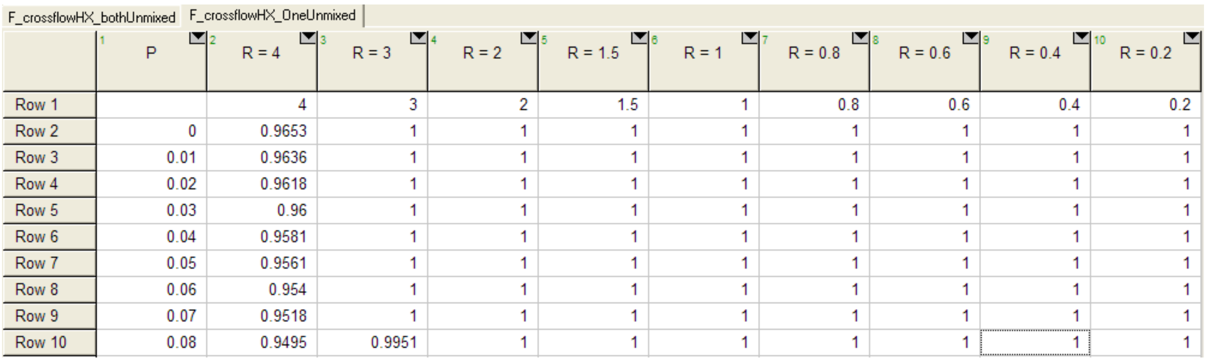
(*d*) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

Now, prepare a Table of F values for different R and P values and copy that Table to EES as a "Lookup Table", named "F_crossFlowHX_OneUnmixed".

Now, with this Look up Table, use the EES built-in 2D Interpolation Function Interpolate2DM('F_crossFlowHX_OneUnmixed',R,P) to get value of at given R and P.

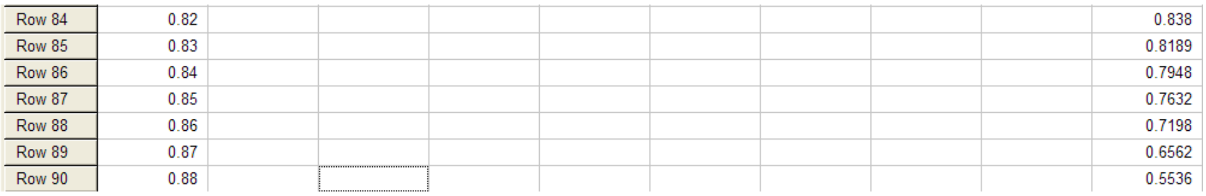


Part of the Lookup Table is shown below:



	P	R = 4	R = 3	R = 2	R = 1.5	R = 1	R = 0.8	R = 0.6	R = 0.4	R = 0.2
Row 1		4	3	2	1.5	1	0.8	0.6	0.4	0.2
Row 2	0	0.9653	1	1	1	1	1	1	1	1
Row 3	0.01	0.9636	1	1	1	1	1	1	1	1
Row 4	0.02	0.9618	1	1	1	1	1	1	1	1
Row 5	0.03	0.96	1	1	1	1	1	1	1	1
Row 6	0.04	0.9581	1	1	1	1	1	1	1	1
Row 7	0.05	0.9561	1	1	1	1	1	1	1	1
Row 8	0.06	0.954	1	1	1	1	1	1	1	1
Row 9	0.07	0.9518	1	1	1	1	1	1	1	1
Row 10	0.08	0.9495	0.9951	1	1	1	1	1	1	1

……



Row 84	0.82									0.838
Row 85	0.83									0.8189
Row 86	0.84									0.7948
Row 87	0.85									0.7632
Row 88	0.86									0.7198
Row 89	0.87									0.6562
Row 90	0.88									0.5536

……

==

Let us work out a problem to demonstrate the use of this Function for F for a cross flow HX with one fluid unmixed:

"**Prob. 4B.18**. Consider a cross flow HX in which oil (cp = 1900 J/kg.K) flowing inside tubes is heated from 15 C to 85 C by steam blowing across the tubes. Steam enters at 130 C and leaves at 110 C, with a mass flow rate of 5.2 kg/s. Overall heat transfer coeff, U is 275 W/m^2.K. For steam, cp = 1860 J/kg.K. Calculate the surface area of the HX."

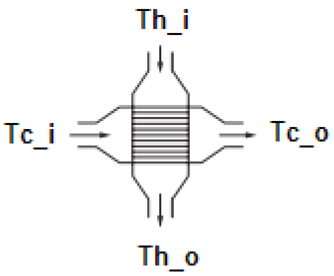
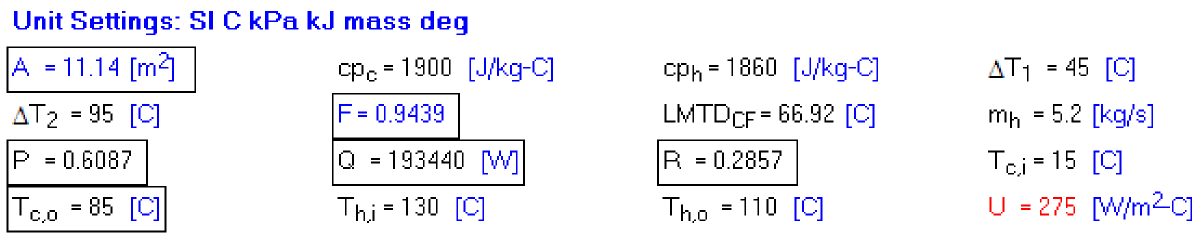


Fig. Prob.4B.18

EES Solution:

```
“Data:”

m_h = 5.2 [kg/s] “…hot fluid – steam”
T_h_i = 130 [C] “inlet temp of hot fluid – steam”
T_h_o = 110 [C] “exit temp of hot fluid – steam”
T_c_i = 15 [C] “inlet temp of cold fluid – oil”
T_c_o = 85 [C] “exitt temp of cold fluid – oil”
cp_h = 1860 [J/kg-C]
cp_c = 1900 [J/kg-C]
U = 275 [W/m^2-C]

“Calculations:”

m_h * cp_h * (T_h_i – T_h_o) = Q “[W]…determines Q = heat transferred”

“LMTD for a counter-flow HX:”

DELTAT_1 = T_h_i – T_c_o “Temp diff at inlet of HX – for counter flow HX”

DELTAT_2 = T_h_o – T_c_i “Temp diff at exit of HX – for counter flow HX”

LMTD_CF = (DELTAT_1 – DELTAT_2)/ln(DELTAT_1/DELTAT_2) “C…determines LMTD“

“To find LMTD Correction factor F for a crossflow HX….

either from the graph for a single pass HX with both fluids unmixed:, OR:

use the built-in Interpolation Function in EES to read F value from the ‘Look up Table’:”

R = (T_h_i – T_h_o)/(T_c_o – T_c_i) “R = 0.2857, to be used in the graph to get F”

P = (T_c_o – T_c_i)/(T_h_i – T_c_i) “P = 0.6087, to be used in the graph to get F”

F=Interpolate2DM(‘F_crossFlowHX_OneUnmixed’,R,P) “….finds F from the Lookup Table by 2 D Interpolation. We get: F = 0.9439 “

{F = 0.9 “ From graph, for above values of R and P”}

Q = U * A * LMTD_CF * F “… Finds A”
```

Results:



```
Unit Settings: SI C kPa kJ mass deg
A = 11.14 [m^2]      cpc = 1900 [J/kg-C]    cph = 1860 [J/kg-C]     ΔT1 = 45 [C]
ΔT2 = 95 [C]         F = 0.9439             LMTDCF = 66.92 [C]      mh = 5.2 [kg/s]
P = 0.6087           Q = 193440 [W]         R = 0.2857              Tc,i = 15 [C]
Tc,o = 85 [C]        Th,i = 130 [C]         Th,o = 110 [C]          U = 275 [W/m^2-C]
```

Thus:

LMTD correction factor, F = 0.9439

Area of HX, A = 11.14 m^2 …. Ans.

==

EES PROCEDURE to determine the LMTD correction factor F for Shell & Tube Heat Exchangers:

Let us write a EES PROCEDURE to find LMTD_CF and F for a Shell & Tube HX:

We recollect:

Equations:

$$LMTD = \frac{(T_{h1} - T_{c2}) - (T_{h2} - T_{c1})}{\ln\left(\frac{T_{h1} - T_{c2}}{T_{h2} - T_{c1}}\right)}$$

$$P = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} \qquad R = \frac{T_{h1} - T_{h2}}{T_{c2} - T_{c1}}$$

If R is not equal to 1:

$$X = \frac{1 - \left(\frac{R\,P - 1}{P - 1}\right)^{\frac{1}{N}}}{R - \left(\frac{R\,P - 1}{P - 1}\right)^{\frac{1}{N}}}$$

$$F = \frac{\left(\frac{\sqrt{R^2 + 1}}{R - 1}\right) ln\left(\frac{1 - X}{1 - R\,X}\right)}{\ln\left(\frac{\frac{2}{X} - 1 - R + \sqrt{R^2 + 1}}{\frac{2}{X} - 1 - R - \sqrt{R^2 + 1}}\right)}$$

If R = 1:

$$X = \frac{P}{(N - N \cdot P + P)}$$

$$F = \frac{X \cdot \sqrt{2}}{(1 - X) \cdot \ln\left[\frac{2 \cdot (1 - X) + X \cdot \sqrt{2}}{2 \cdot (1 - X) - X \cdot \sqrt{2}}\right]}$$

In the above, N is the no. of simple shells or no. of shell passes.

Following is the EES Procedure:

```
$UnitSystem SI Pa C J

PROCEDURE Shell_and_TubeHX_LMTD_F(Tshell_1,Tshell_2,Ttube_1,Ttube_2,N : R,P,F,LMTD_CF,LMTD_corrected)

“Gives R, P, F, LMTD_CF and LMTD_corrected as output:”

“Input: Inlet and exit temps of Shell side and Tube side fluids, and N is the no. of simple shells or no. of Shell passes”

DT1 := Tshell_1 – Ttube_2

DT2 := Tshell_2 – Ttube_1

LMTD_CF := ABS(DT1 – DT2) / ABS (ln (DT1/DT2))

P := (Ttube_2 – Ttube_1) / (Tshell_1 – Ttube_1)

R := (Tshell_1 – Tshell_2) / (Ttube_2 – Ttube_1)
```

```
IF (Tshell_1 = Tshell_2) OR (Ttube_1 = Ttube_2) THEN
        F := 1
        LMTD_corrected := F * LMTD_CF
        RETURN
ENDIF

IF ( R = 1) THEN
        X := P / (N – N * P + P)
        F := (X * sqrt(2)) / ((1 – X) * ln( (2 * (1 – X) + x * sqrt(2)) / (2 * (1 – X) – x * sqrt(2))))
ENDIF

IF ( R <> 1) THEN
        X :=(1 – ((R * P -1) / (P – 1))^ (1/N)) / (R – ((R * P -1)/(P – 1))^(1/N))
        F := (sqrt(R^2 + 1) / (R – 1)) * ln ((1 – X) / (1 – R * X)) / ln ((2/X – 1 – R + sqrt(R^2 + 1)) / (2/X – 1 – R – sqrt(R^2 + 1)))
ENDIF

LMTD_corrected := F * LMTD_CF

END
```

“ ”

Let us use this PROCEDURE to solve the following problem:

Prob. 4B.19. In a Shell & Tube HX, water, making one Shell pass, at a rate of 1 kg/s is heated from 35 to 75 C by an oil of sp. heat 1900 J/kg.C. Oil flows at a rate of 2.5 kg/s through the tubes making 2 passes and enters the Shell at 110 C. If the overall heat transfer coeff. U is 350 W/m^2.C, calculate the area required.”

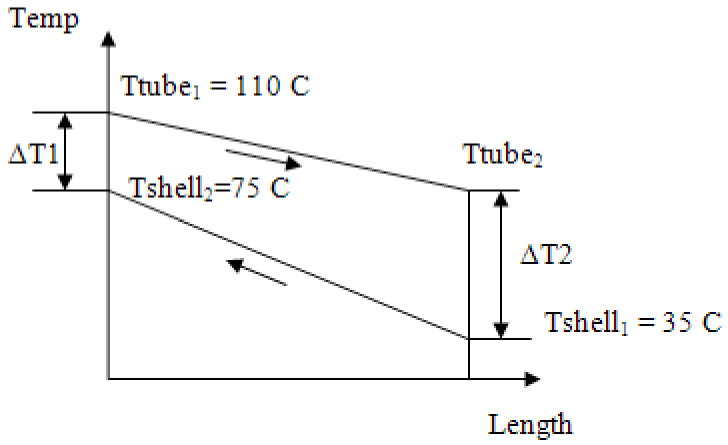


Fig. Prob.4B.19. Temp profile for Counter-flow arrangement

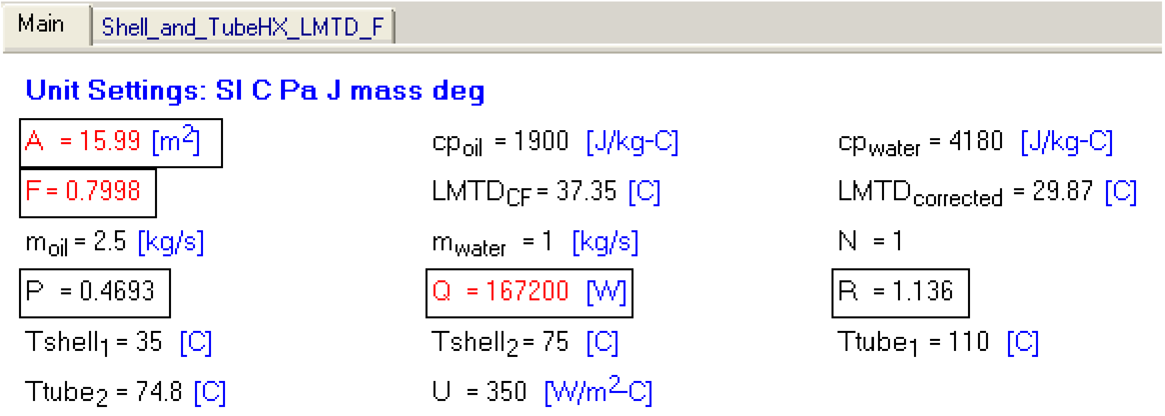
EES Solution:

“Data:”

Tshell_1 = 35[C]“….inlet temp of water”
Tshell_2 = 75 [C]“….exit temp of water”
Ttube_1 =110 [C]“….inlet temp of oil”

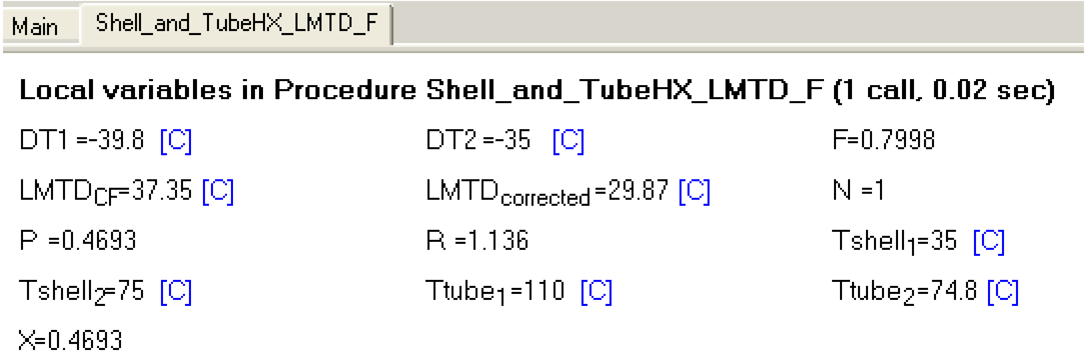
```
m_water = 1 [kg/s]
cp_water = 4180 [J/kg-C]

m_oil = 2.5 [kg/s]
cp_oil = 1900 [J/kg-C]

U = 350 [W/m^2-C]
N = 1

"Calculation:"

"Total heat transferred, Q:"

Q = m_water * cp_water * (Tshell_2 – Tshell_1) "[W]"

Q = m_oil * cp_oil * (Ttube_1 – Ttube_2)"[C]…finds exit temp of oil, Ttube_2"

"Also:"

Q = U * A * F * LMTD_CF "…where F is the LMTD correction factor, and LMTD_CF is the LMTD for a true counter-flow HX"

"Get F and LMTD by calling the EES PROCEDURE written above:"

CALL Shell_and_TubeHX_LMTD_F(Tshell_1,Tshell_2,Ttube_1,Ttube_2,N : R,P,F,LMTD_CF,LMTD_corrected)
```

Results:



Unit Settings: SI C Pa J mass deg

A = 15.99 [m^2]
cp_{oil} = 1900 [J/kg-C]
cp_{water} = 4180 [J/kg-C]
F = 0.7998
$LMTD_{CF}$ = 37.35 [C]
$LMTD_{corrected}$ = 29.87 [C]
m_{oil} = 2.5 [kg/s]
m_{water} = 1 [kg/s]
N = 1
P = 0.4693
Q = 167200 [W]
R = 1.136
$Tshell_1$ = 35 [C]
$Tshell_2$ = 75 [C]
$Ttube_1$ = 110 [C]
$Ttube_2$ = 74.8 [C]
U = 350 [W/m^2-C]



Local variables in Procedure Shell_and_TubeHX_LMTD_F (1 call, 0.02 sec)

DT1 =-39.8 [C]

DT2 =-35 [C]

F=0.7998

$LMTD_{CF}$=37.35 [C]

$LMTD_{corrected}$=29.87 [C]

N =1

P =0.4693

R =1.136

$Tshell_1$=35 [C]

$Tshell_2$=75 [C]

$Ttube_1$=110 [C]

$Ttube_2$=74.8 [C]

X=0.4693

Thus:

F = 0.8 … LMTD correction factor

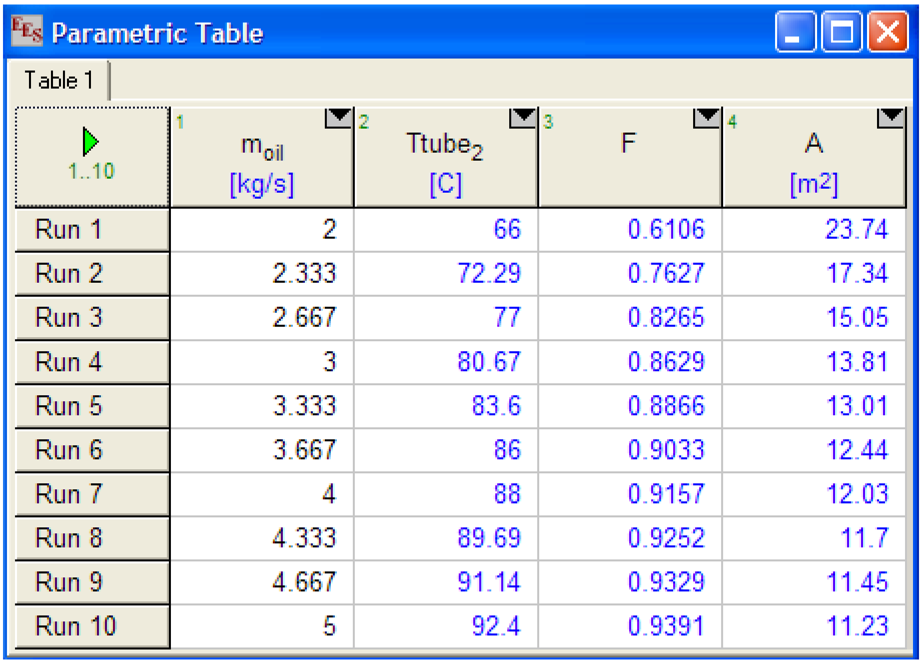
A = 15.99 m^2 …. Area required for the HX …. Ans.

Note: Since values of R and P are also returned by the program, check the value of F from the graph (given at the beginning of this chapter).

Consider the following variation:

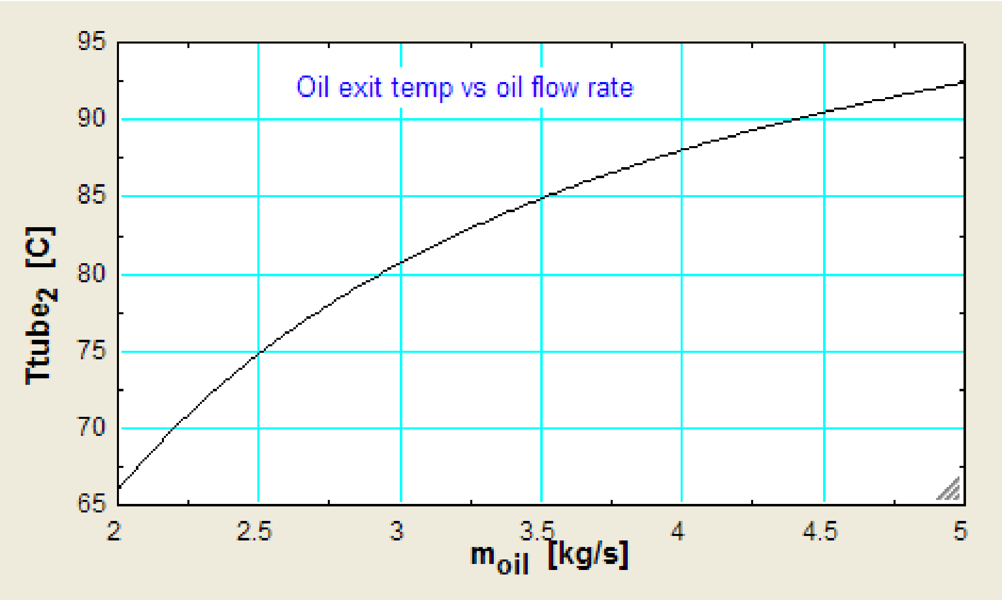
If the oil flow rate varies from 1 to 5 kg/s, plot the variation of Ttube_2 and A with m_oil:
Remember that in each case Ttube_2 will also change, i.e. LMTD_CF and F will also change.
This parametric calculation is done very easily in EES:

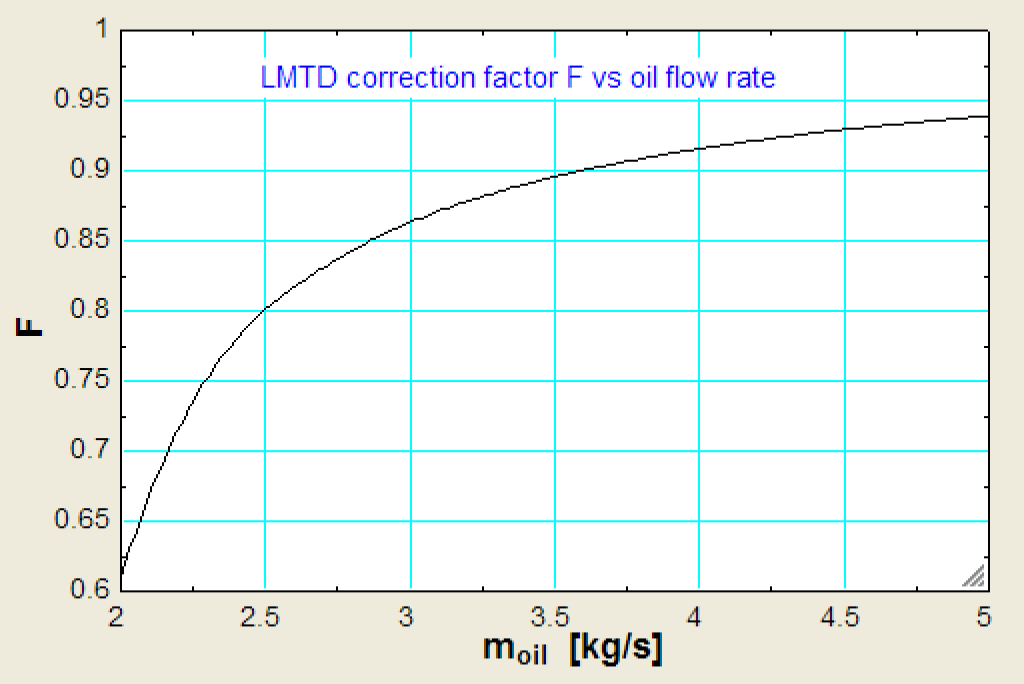
First, prepare the Parametric Table:

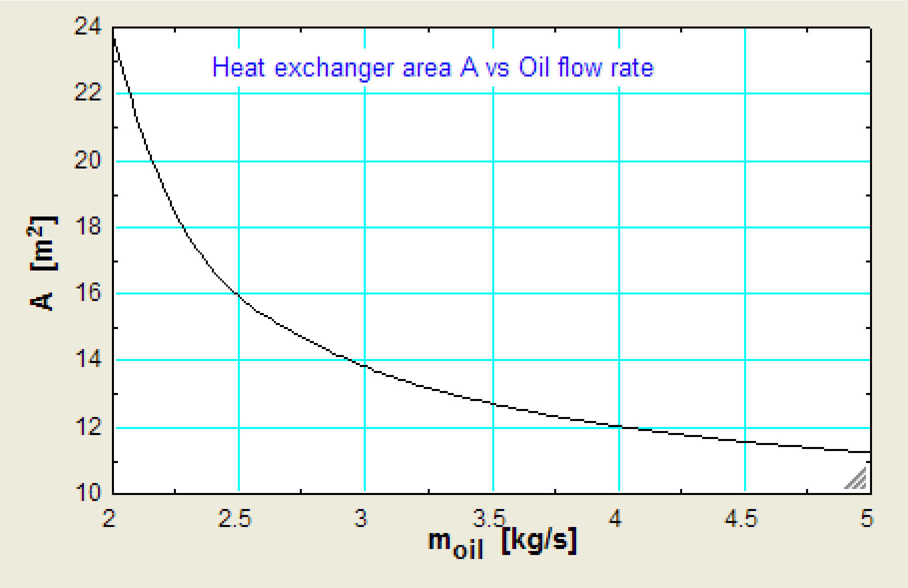


1..10	1 m_{oil} [kg/s]	2 $Ttube_2$ [C]	3 F	4 A [m2]
Run 1	2	66	0.6106	23.74
Run 2	2.333	72.29	0.7627	17.34
Run 3	2.667	77	0.8265	15.05
Run 4	3	80.67	0.8629	13.81
Run 5	3.333	83.6	0.8866	13.01
Run 6	3.667	86	0.9033	12.44
Run 7	4	88	0.9157	12.03
Run 8	4.333	89.69	0.9252	11.7
Run 9	4.667	91.14	0.9329	11.45
Run 10	5	92.4	0.9391	11.23

Now, plot the graphs:







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Prob. 4B.20. In a double pipe, parallel flow HX, water flowing at a rate of 5000 kg/h gets cooled from 95 C to 65 C while cooling water flowing at a rate of 50000 kg/h enters at 30 C. Overall heat transfer coefficient is 2270 W/m^2.C. Determine the heat transfer area required.

(b) Plot the variation of exit temp of cooling water and the area of HX required as the flow rate of cooling water varies.

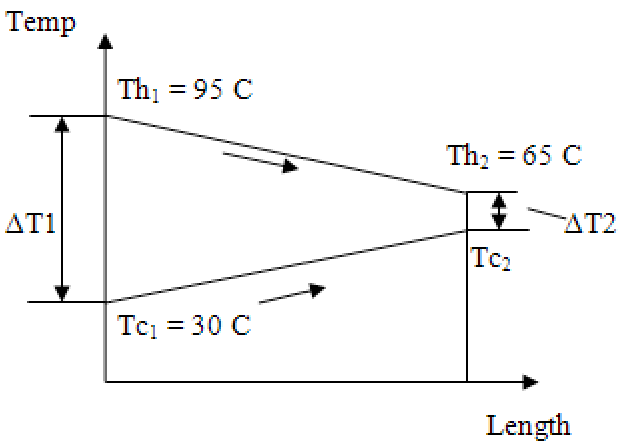


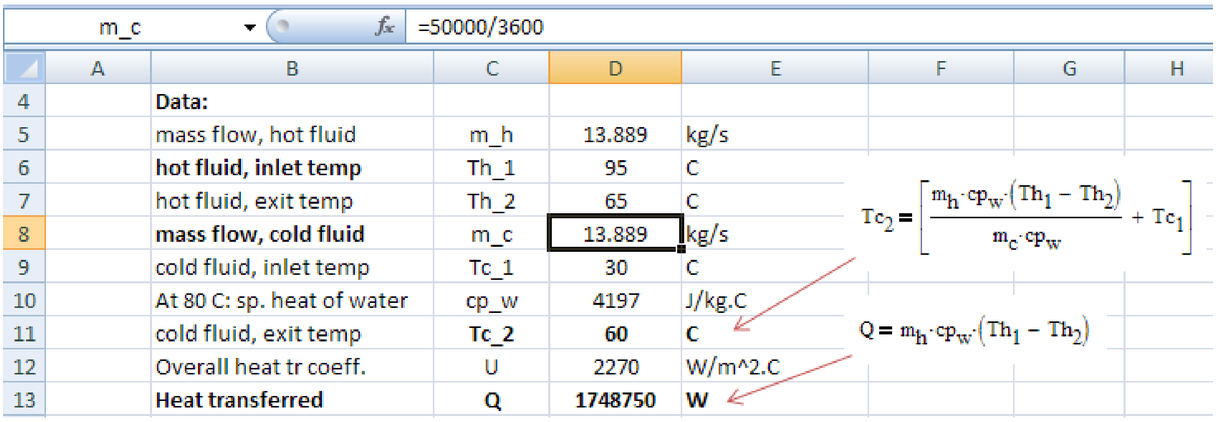
Fig. Prob.4B.20. Parallel flow arrangement

EXCEL Solution:

Let us solve this problem with EXCEL.

Following are the steps in EXCEL solution:

1. Set up the EXCEL worksheet, enter data and name the cells:



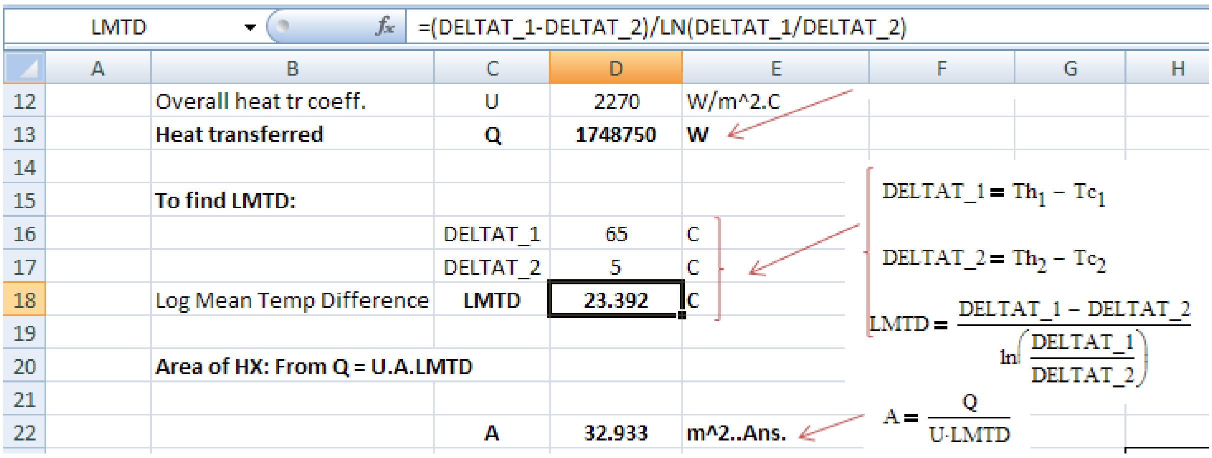
m_c =50000/3600

	A	B	C	D	E	F	G	H
4		**Data:**						
5		mass flow, hot fluid	m_h	13.889	kg/s			
6		**hot fluid, inlet temp**	Th_1	95	C			
7		hot fluid, exit temp	Th_2	65	C			
8		**mass flow, cold fluid**	m_c	13.889	kg/s			
9		cold fluid, inlet temp	Tc_1	30	C			
10		At 80 C: sp. heat of water	cp_w	4197	J/kg.C			
11		cold fluid, exit temp	**Tc_2**	**60**	**C**			
12		Overall heat tr coeff.	U	2270	W/m^2.C			
13		**Heat transferred**	**Q**	**1748750**	**W**			

$$Tc_2 = \left[\frac{m_h \cdot cp_w \cdot (Th_1 - Th_2)}{m_c \cdot cp_w} + Tc_1\right]$$

$$Q = m_h \cdot cp_w \cdot (Th_1 - Th_2)$$

2. Calculate LMTD and Area required for the HX:



LMTD =(DELTAT_1-DELTAT_2)/LN(DELTAT_1/DELTAT_2)

	A	B	C	D	E
12		Overall heat tr coeff.	U	2270	W/m^2.C
13		**Heat transferred**	**Q**	**1748750**	**W**
14					
15		**To find LMTD:**			
16			DELTAT_1	65	C
17			DELTAT_2	5	C
18		Log Mean Temp Difference	**LMTD**	23.392	C
19					
20		**Area of HX: From Q = U.A.LMTD**			
21					
22			**A**	**32.933**	**m^2..Ans.**

$$\mathrm{DELTAT\_1} = Th_1 - Tc_1$$

$$\mathrm{DELTAT\_2} = Th_2 - Tc_2$$

$$\mathrm{LMTD} = \frac{\mathrm{DELTAT\_1} - \mathrm{DELTAT\_2}}{\ln\left(\frac{\mathrm{DELTAT\_1}}{\mathrm{DELTAT\_2}}\right)}$$

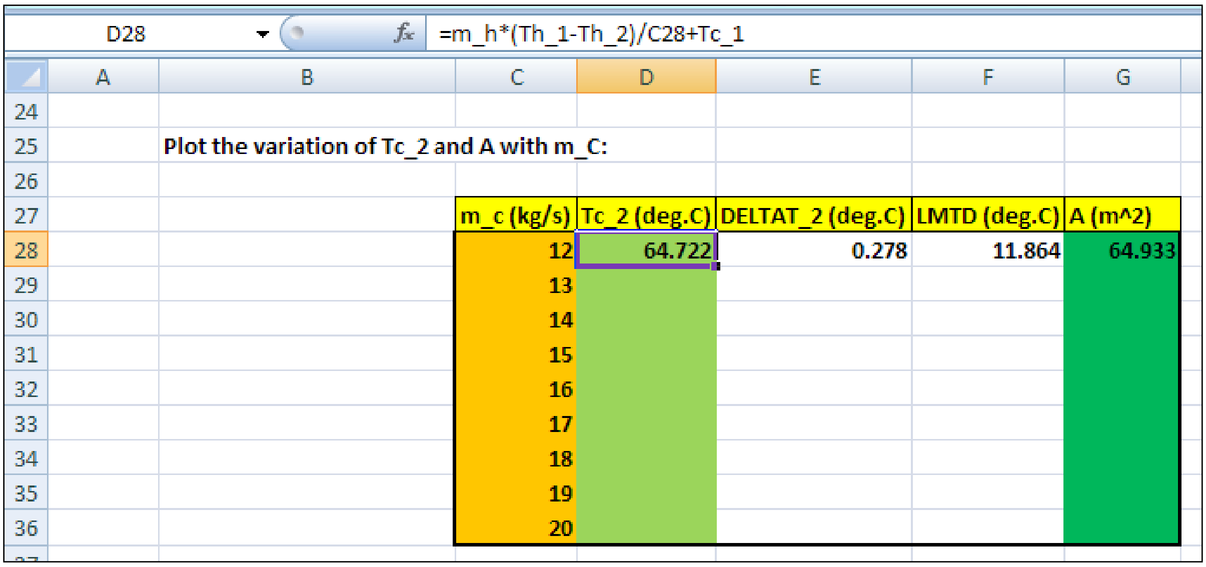
$$A = \frac{Q}{U \cdot \mathrm{LMTD}}$$

Note that in the above two screen shots, formulas used in calculations are also shown for clarity.

Thus, LMTD = 23.392 C, and Area of HX, A = 32.933 m^2 …. Ans.

3. **Now, to plot the variation of Tc_2 and A with m_c:**

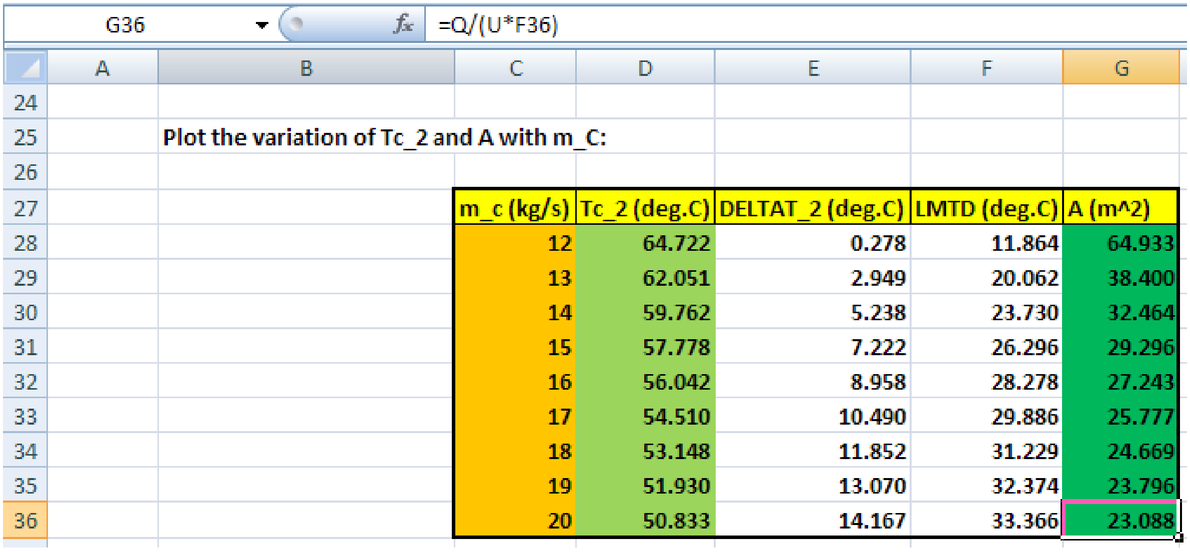
First, set up a Table, with m_c varying from 12 to 20 kg/s. Enter formulas for Tc_2, DELTAT_2, LMTD and A in the first row below the captions, taking care to see that m_c is referred to by relative reference. See the formula entered for Tc_2 in the Formula bar in the screen shot below. Similarly for DELTAT_2, LMTD and A:



D28 =m_h*(Th_1-Th_2)/C28+Tc_1

	A	B	C	D	E	F	G
24							
25		**Plot the variation of Tc_2 and A with m_C:**					
26							
27			**m_c (kg/s)**	**Tc_2 (deg.C)**	**DELTAT_2 (deg.C)**	**LMTD (deg.C)**	**A (m^2)**
28			**12**	**64.722**	**0.278**	**11.864**	**64.933**
29			**13**				
30			**14**				
31			**15**				
32			**16**				
33			**17**				
34			**18**				
35			**19**				
36			**20**				

Now, select cells from D28 to G28 and drag – copy up to the end of Table, i.e. up to cell G36. Immediately, all calculations are made and the Table is filled up:

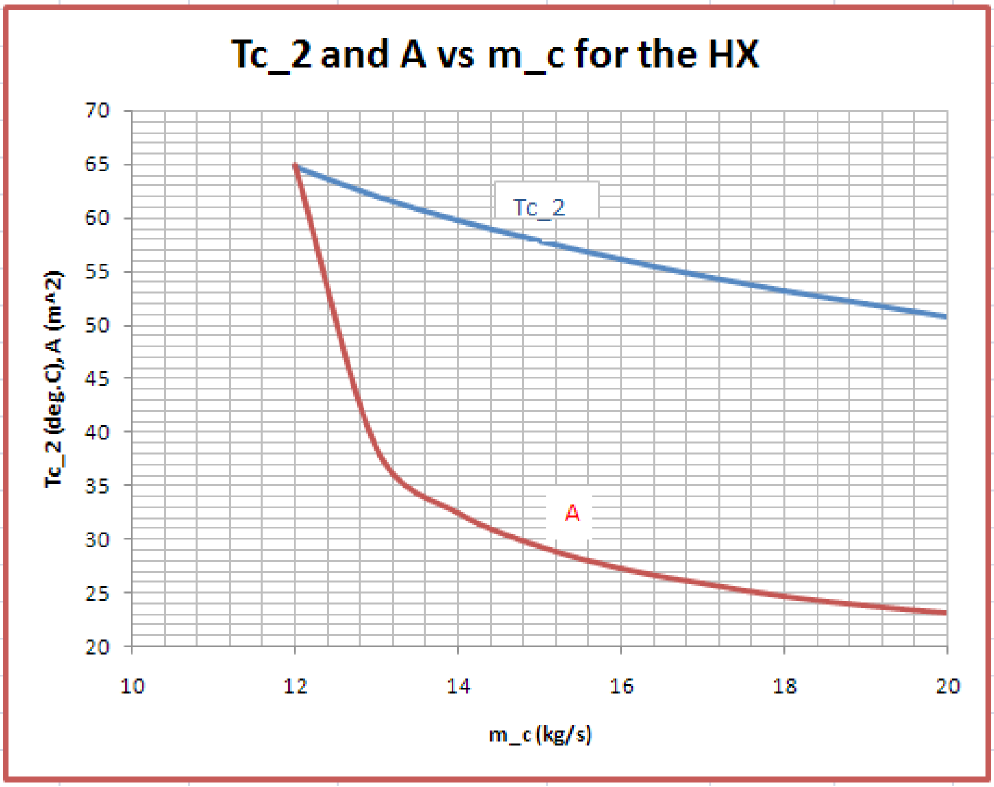


G36 =Q/(U*F36)

Plot the variation of Tc_2 and A with m_C:

m_c (kg/s)	Tc_2 (deg.C)	DELTAT_2 (deg.C)	LMTD (deg.C)	A (m^2)
12	64.722	0.278	11.864	64.933
13	62.051	2.949	20.062	38.400
14	59.762	5.238	23.730	32.464
15	57.778	7.222	26.296	29.296
16	56.042	8.958	28.278	27.243
17	54.510	10.490	29.886	25.777
18	53.148	11.852	31.229	24.669
19	51.930	13.070	32.374	23.796
20	50.833	14.167	33.366	23.088

4. Now, plot the graphs in EXCEL:



==

Prob. 4B.21. Saturated, dry **s**team at 10 bar at a flow rate of 800 kg/min enters the tubes of a counter-flow HX and leaves at 350 C. It is heated by gas entering at 650 C with a flow rate of 1350 kg/min. The tubes are 30 mm in dia and 3 m long. Determine the no. of tubes required. **Given**: T_sat of steam at 10 bar = 180 C, cp_steam = 2710 J/kg.C, and heat transfer coeff on steam side, h_s = 600 W/m^2.C. Also, cp_gas = 1000 J/kg.C and heat transfer coeff on gas side, h_g = 250 W/m^2.C.

(b) Plot the variation of exit temp of gas and the number of tubes required as the flow rate of gas varies.

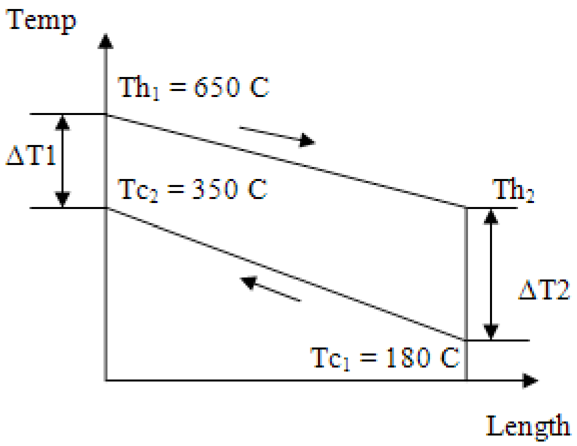
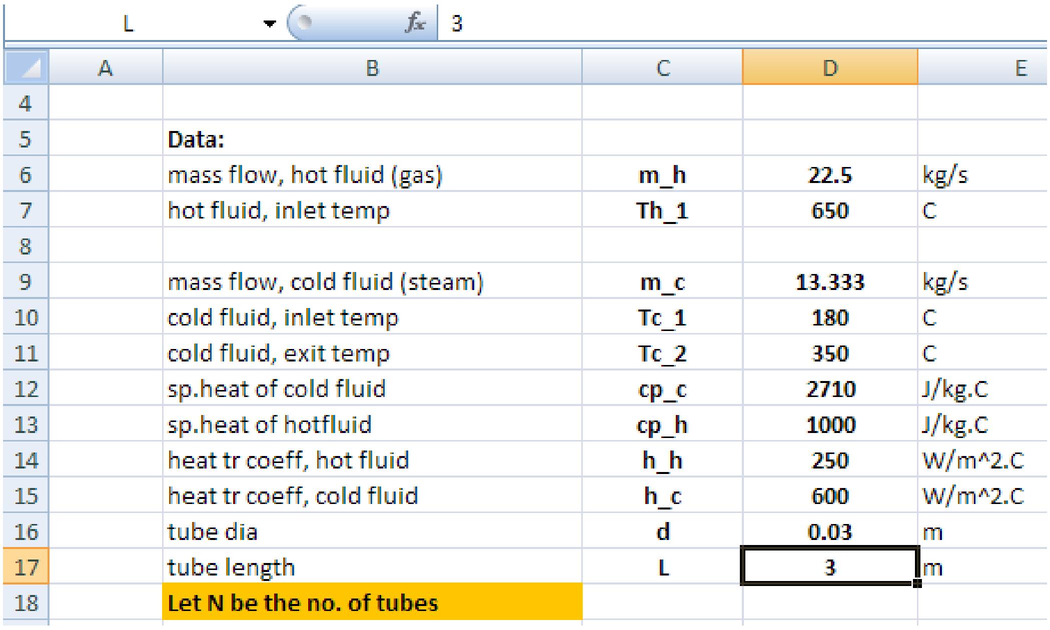


Fig. Prob.4B.21. Counter-flow arrangement

EXCEL Solution:

Following are the steps in EXCEL solution:

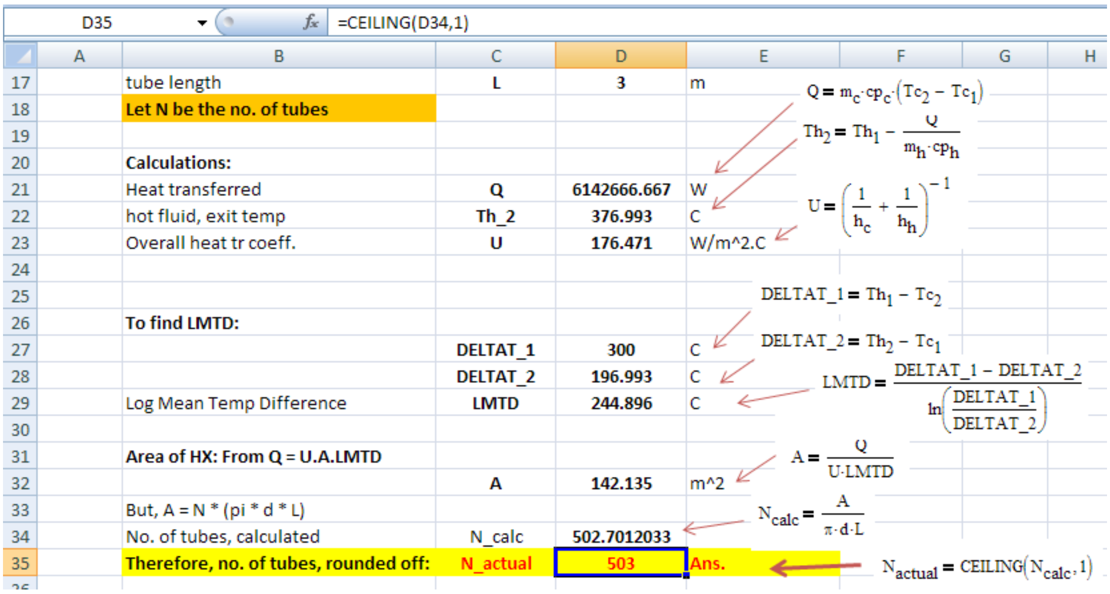
1. Set up the EXCEL worksheet, enter data and name the cells:



L | 3

	A	B	C	D	E
4					
5		**Data:**			
6		mass flow, hot fluid (gas)	**m_h**	**22.5**	kg/s
7		hot fluid, inlet temp	**Th_1**	**650**	C
8					
9		mass flow, cold fluid (steam)	**m_c**	**13.333**	kg/s
10		cold fluid, inlet temp	**Tc_1**	**180**	C
11		cold fluid, exit temp	**Tc_2**	**350**	C
12		sp.heat of cold fluid	**cp_c**	**2710**	J/kg.C
13		sp.heat of hotfluid	**cp_h**	**1000**	J/kg.C
14		heat tr coeff, hot fluid	**h_h**	**250**	W/m^2.C
15		heat tr coeff, cold fluid	**h_c**	**600**	W/m^2.C
16		tube dia	**d**	**0.03**	m
17		tube length	**L**	**3**	m
18		**Let N be the no. of tubes**			

2. Perform calculations as shown .i.e. first, calculate q, then Th_2 by heat balance, and then, LMTD; then get U, and then, area required, A. From area A, get the no. of tubes, N_calc; then, round it off to get an integer no. of tubes. Formulas are shown in worksheet, for clarity:



D35 | =CEILING(D34,1)

	A	B	C	D	E
17		tube length	**L**	**3**	m
18		**Let N be the no. of tubes**			
19					
20		**Calculations:**			
21		Heat transferred	**Q**	**6142666.667**	W
22		hot fluid, exit temp	**Th_2**	**376.993**	C
23		Overall heat tr coeff.	**U**	**176.471**	W/m^2.C
24					
25					
26		**To find LMTD:**			
27			**DELTAT_1**	**300**	C
28			**DELTAT_2**	**196.993**	C
29		Log Mean Temp Difference	**LMTD**	**244.896**	C
30					
31		**Area of HX: From Q = U.A.LMTD**			
32			**A**	**142.135**	m^2
33		But, A = N * (pi * d * L)			
34		No. of tubes, calculated	N_calc	**502.7012033**	
35		**Therefore, no. of tubes, rounded off:**	**N_actual**	**503**	**Ans.**

$$Q = m_c \cdot cp_c \cdot (Tc_2 - Tc_1)$$

$$Th_2 = Th_1 - \frac{Q}{m_h \cdot cp_h}$$

$$U = \left(\frac{1}{h_c} + \frac{1}{h_h}\right)^{-1}$$

$$DELTAT\_1 = Th_1 - Tc_2$$

$$DELTAT\_2 = Th_2 - Tc_1$$

$$LMTD = \frac{DELTAT\_1 - DELTAT\_2}{\ln\left(\frac{DELTAT\_1}{DELTAT\_2}\right)}$$

$$A = \frac{Q}{U \cdot LMTD}$$

$$N_{calc} = \frac{A}{\pi \cdot d \cdot L}$$

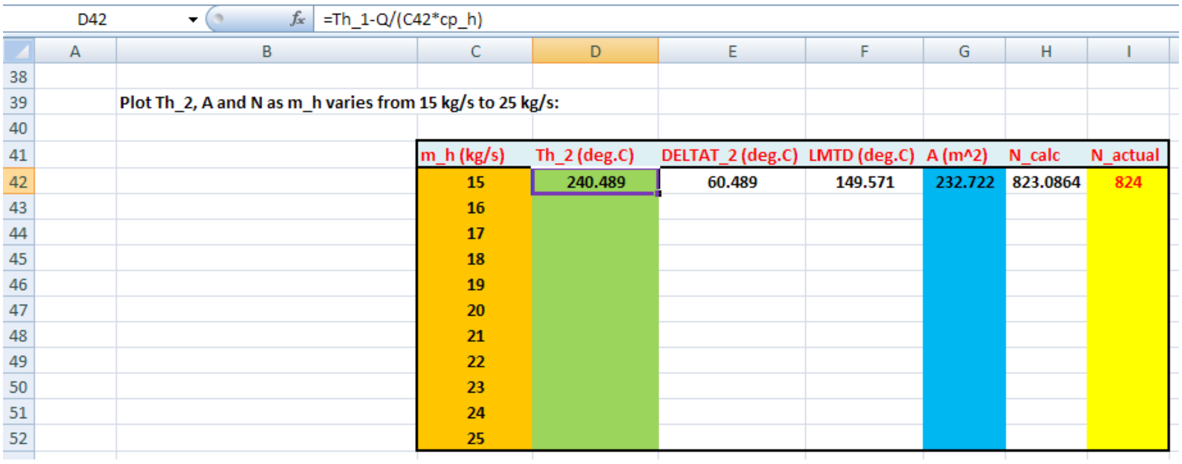
$$N_{actual} = CEILING(N_{calc}, 1)$$

Note the use of EXCEL built-in function CEILING to round off the no. of tubes.

Thus, No. of tubes required = N_actual = 503 …. Ans.

3. **Plot Th_2 and N as m_h varies from 15 to 25 kg/s:**

First, prepare a Table as shown, and fill up the first row below the captions. Remember to enter m_h by relative reference. See the Formula bar in the screen shot below, for the formula entered for Th_2 in cell D42. Similarly, for other items shown under respective captions:

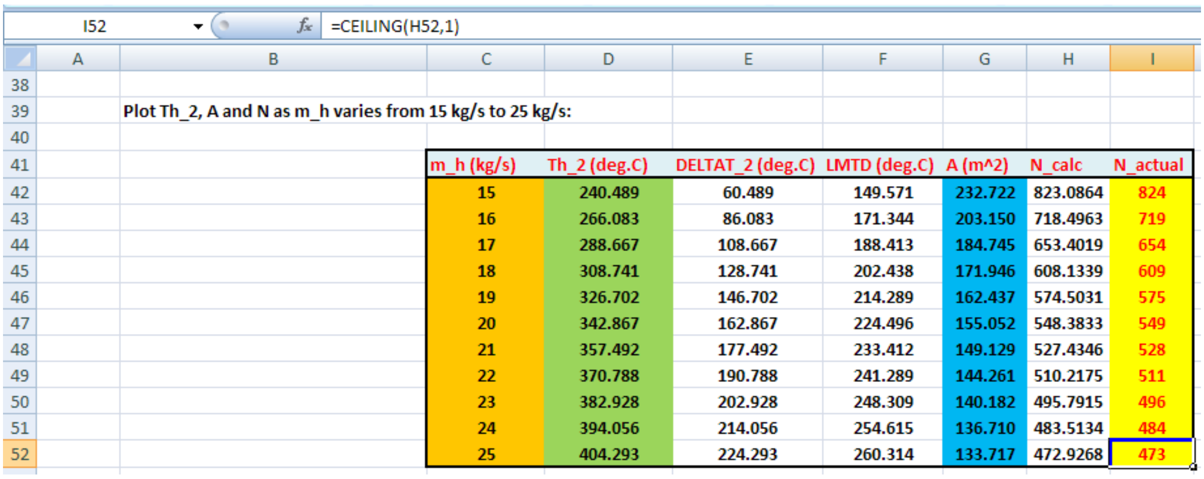


D42 =Th_1-Q/(C42*cp_h)

Plot Th_2, A and N as m_h varies from 15 kg/s to 25 kg/s:

m_h (kg/s)	Th_2 (deg.C)	DELTAT_2 (deg.C)	LMTD (deg.C)	A (m^2)	N_calc	N_actual
15	240.489	60.489	149.571	232.722	823.0864	824
16						
17						
18						
19						
20						
21						
22						
23						
24						
25						

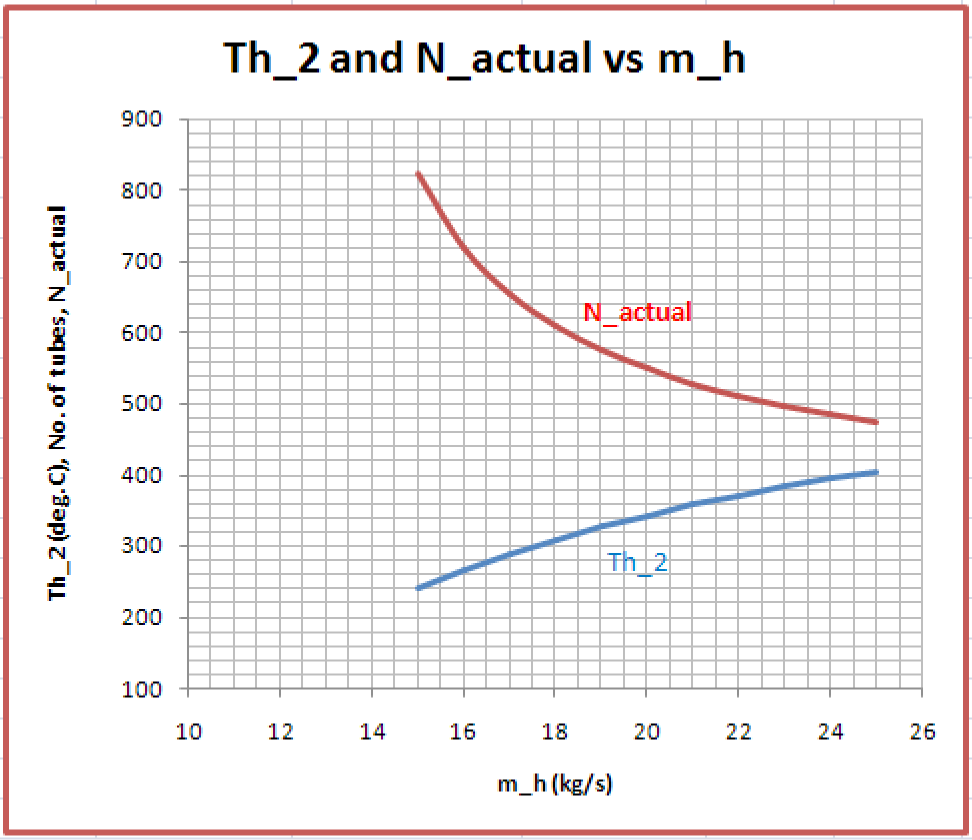
Now, select the cells D42 to I42, and drag-copy till the end of Table, i.e. up to cell I52. Immediately, all calculations are done and the Table gets filled up, as shown below:



Plot Th_2, A and N as m_h varies from 15 kg/s to 25 kg/s:

m_h (kg/s)	Th_2 (deg.C)	DELTAT_2 (deg.C)	LMTD (deg.C)	A (m^2)	N_calc	N_actual
15	240.489	60.489	149.571	232.722	823.0864	824
16	266.083	86.083	171.344	203.150	718.4963	719
17	288.667	108.667	188.413	184.745	653.4019	654
18	308.741	128.741	202.438	171.946	608.1339	609
19	326.702	146.702	214.289	162.437	574.5031	575
20	342.867	162.867	224.496	155.052	548.3833	549
21	357.492	177.492	233.412	149.129	527.4346	528
22	370.788	190.788	241.289	144.261	510.2175	511
23	382.928	202.928	248.309	140.182	495.7915	496
24	394.056	214.056	254.615	136.710	483.5134	484
25	404.293	224.293	260.314	133.717	472.9268	473

4. Now, plot the graphs in EXCEL:



==

Prob. 4B.22. In a Shell & Tube HX, hot oil (cp = 1900 J/kg.C) is used to heat water from 45 C to 85 C. Water flows in the Shell at a rate of 0.1 kg/s and oil enters the tubes at 110 C and leaves at 70 C. If the overall heat transfer coeff is U = 350 W.m^2.C, find the oil mass flow rate (m_h) and the heat transfer area, if oil makes two tube passes and water makes one shell pass.

(b) Also plot the variation of Th_2, F and A as m_h is varied from 0.15 to 0.3 kg/s.

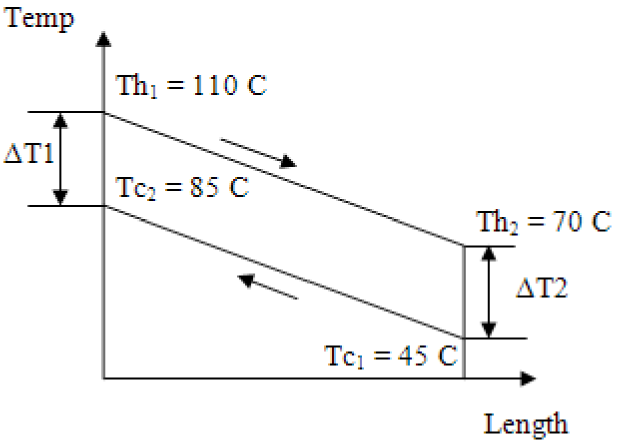


Fig. Prob.4B.22. Counter-flow arrangement

EXCEL Solution:

Note that this is a Shell & Tube HX. So, we need to use a LMTD correction Factor, F.

For a Shell & Tube HX:

We recollect:

Equations:

$$LMTD = \frac{(T_{h1} - T_{c2}) - (T_{h2} - T_{c1})}{\ln\left(\frac{T_{h1} - T_{c2}}{T_{h2} - T_{c1}}\right)}$$

$$P = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} \qquad R = \frac{T_{h1} - T_{h2}}{T_{c2} - T_{c1}}$$

If R is not equal to 1:

$$X = \frac{1 - \left(\frac{R\,P - 1}{P - 1}\right)^{\frac{1}{N}}}{R - \left(\frac{R\,P - 1}{P - 1}\right)^{\frac{1}{N}}}$$

$$F = \frac{\left(\frac{\sqrt{R^2 + 1}}{R - 1}\right) ln\left(\frac{1 - X}{1 - R\,X}\right)}{\ln\left(\frac{\frac{2}{X} - 1 - R + \sqrt{R^2 + 1}}{\frac{2}{X} - 1 - R - \sqrt{R^2 + 1}}\right)}$$

If R = 1:

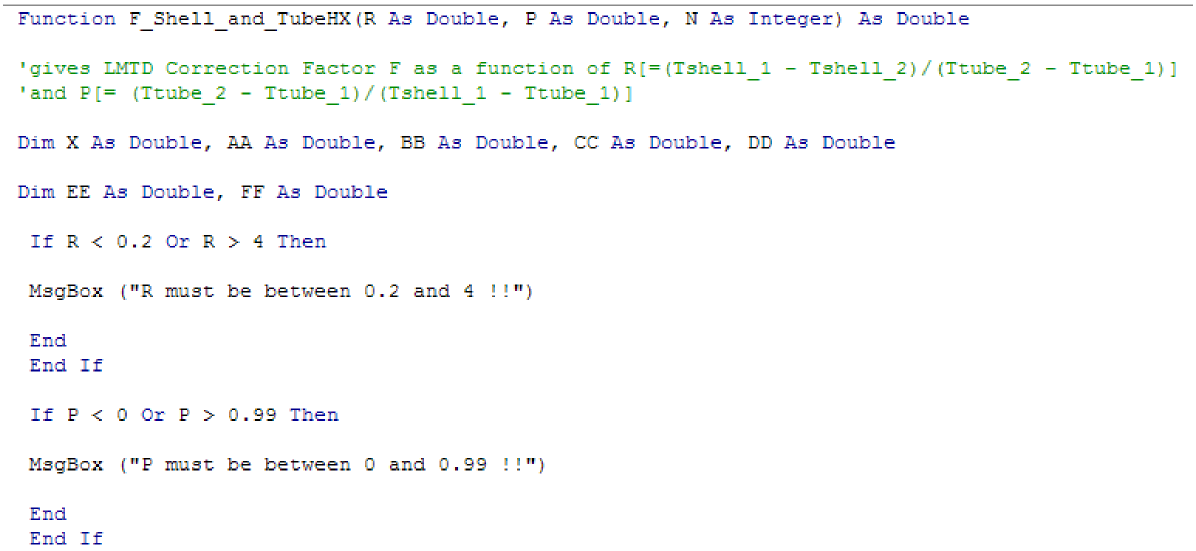
$$X = \frac{P}{(N - N \cdot P + P)}$$

$$F = \frac{X \cdot \sqrt{2}}{(1 - X) \cdot \ln\left[\frac{2 \cdot (1 - X) + X \cdot \sqrt{2}}{2 \cdot (1 - X) - X \cdot \sqrt{2}}\right]}$$

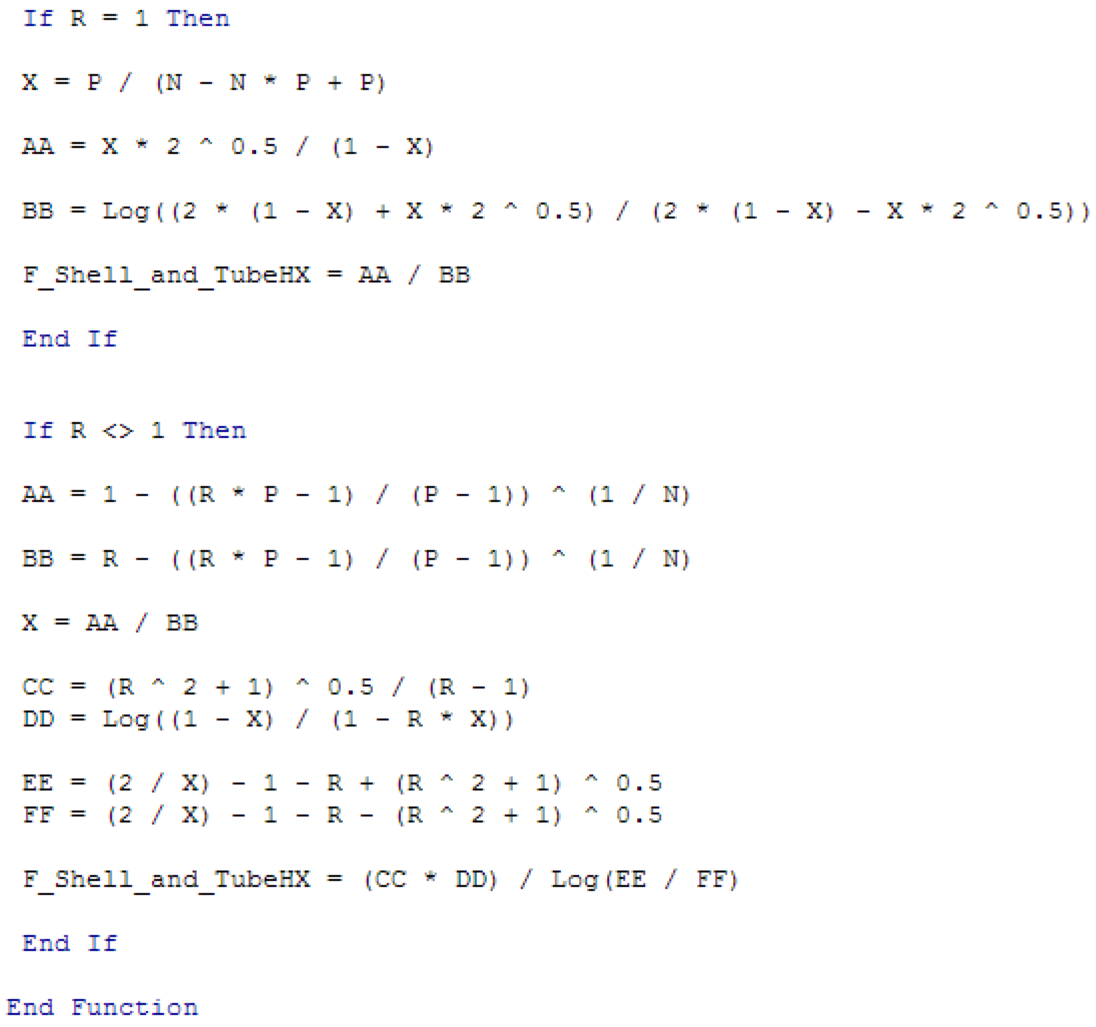
In the above, N is the no. of simple shells or no. of shell passes.

Let us write a VBA Function to calculate LMTD correction factor F when R and P are known:

In EXCEL, go to Developer – Visual Basic _ Module1, and write the following code:



```
Function F_Shell_and_TubeHX(R As Double, P As Double, N As Integer) As Double

'gives LMTD Correction Factor F as a function of R[=(Tshell_1 - Tshell_2)/(Ttube_2 - Ttube_1)]
'and P[= (Ttube_2 - Ttube_1)/(Tshell_1 - Ttube_1)]

Dim X As Double, AA As Double, BB As Double, CC As Double, DD As Double

Dim EE As Double, FF As Double

 If R < 0.2 Or R > 4 Then

 MsgBox ("R must be between 0.2 and 4 !!")

 End
 End If

 If P < 0 Or P > 0.99 Then

 MsgBox ("P must be between 0 and 0.99 !!")

 End
 End If
```



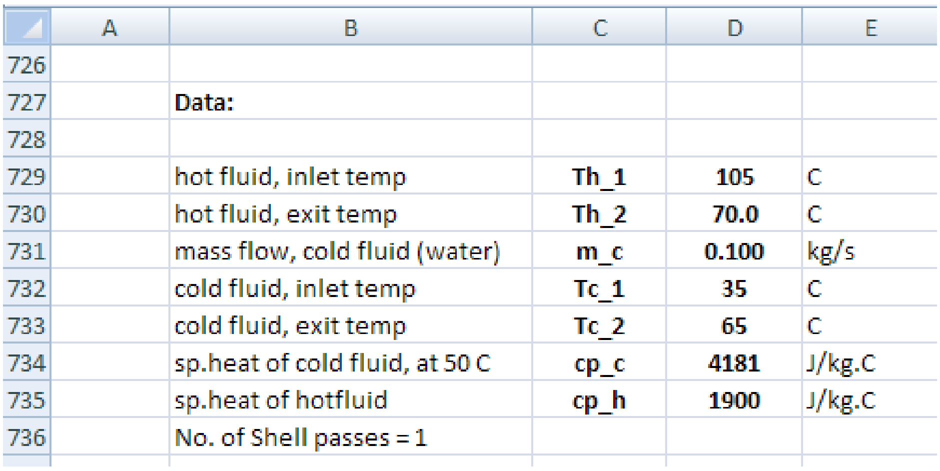
```
 If R = 1 Then

 X = P / (N - N * P + P)

 AA = X * 2 ^ 0.5 / (1 - X)

 BB = Log((2 * (1 - X) + X * 2 ^ 0.5) / (2 * (1 - X) - X * 2 ^ 0.5))

 F_Shell_and_TubeHX = AA / BB

 End If


 If R <> 1 Then

 AA = 1 - ((R * P - 1) / (P - 1)) ^ (1 / N)

 BB = R - ((R * P - 1) / (P - 1)) ^ (1 / N)

 X = AA / BB

 CC = (R ^ 2 + 1) ^ 0.5 / (R - 1)
 DD = Log((1 - X) / (1 - R * X))

 EE = (2 / X) - 1 - R + (R ^ 2 + 1) ^ 0.5
 FF = (2 / X) - 1 - R - (R ^ 2 + 1) ^ 0.5

 F_Shell_and_TubeHX = (CC * DD) / Log(EE / FF)

 End If

End Function
```

Now, the above Function will be available in EXCEL like any other built-in Function.

We shall use this Function in solving the above Problem:

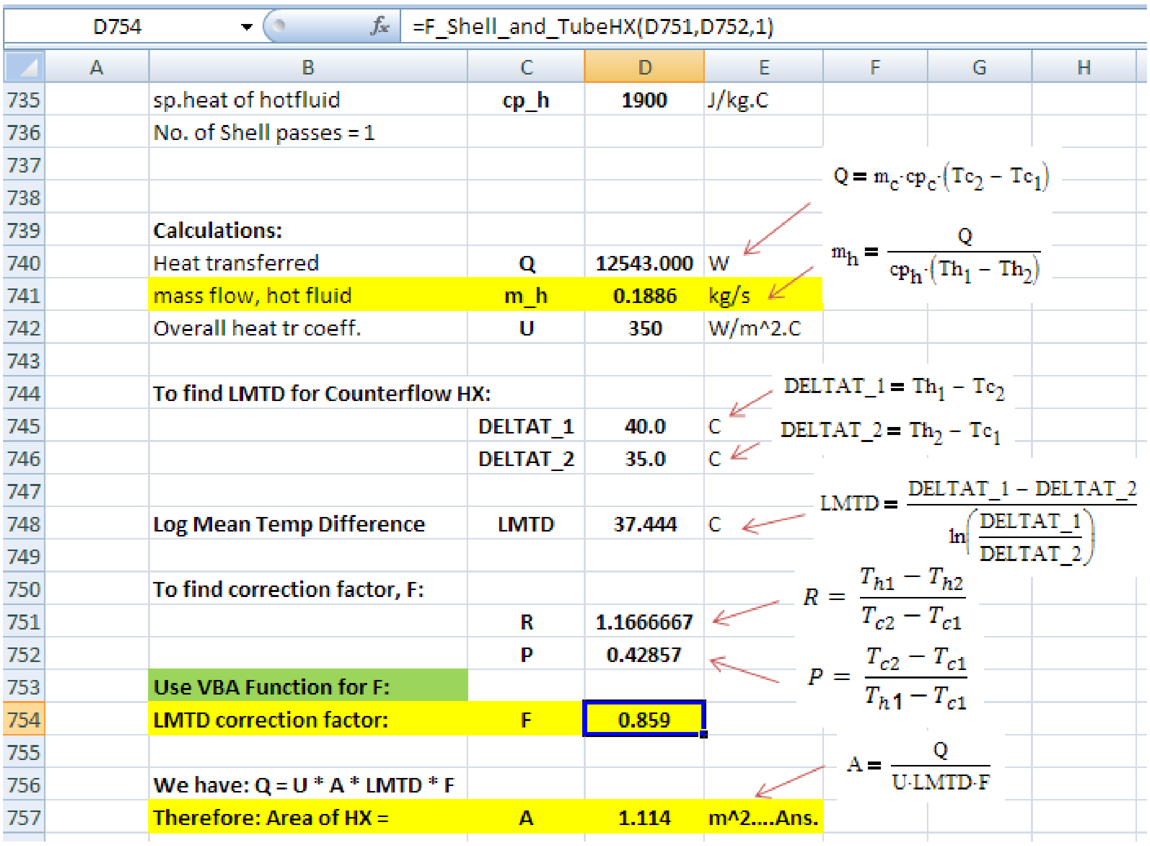
Following are the steps:

1. Set up the EXCEL worksheet, enter data:



	A	B	C	D	E
726					
727		**Data:**			
728					
729		hot fluid, inlet temp	**Th_1**	**105**	C
730		hot fluid, exit temp	**Th_2**	**70.0**	C
731		mass flow, cold fluid (water)	**m_c**	**0.100**	kg/s
732		cold fluid, inlet temp	**Tc_1**	**35**	C
733		cold fluid, exit temp	**Tc_2**	**65**	C
734		sp.heat of cold fluid, at 50 C	**cp_c**	**4181**	J/kg.C
735		sp.heat of hotfluid	**cp_h**	**1900**	J/kg.C
736		No. of Shell passes = 1			

2. Do the calculations as indicated. Formulas are shown in the worksheet:



D754 =F_Shell_and_TubeHX(D751,D752,1)

	A	B	C	D	E
735		sp.heat of hotfluid	**cp_h**	**1900**	J/kg.C
736		No. of Shell passes = 1			
737					
738					
739		**Calculations:**			
740		Heat transferred	**Q**	**12543.000**	W
741		mass flow, hot fluid	**m_h**	**0.1886**	kg/s
742		Overall heat tr coeff.	**U**	**350**	W/m^2.C
743					
744		**To find LMTD for Counterflow HX:**			
745			**DELTAT_1**	**40.0**	C
746			**DELTAT_2**	**35.0**	C
747					
748		**Log Mean Temp Difference**	**LMTD**	**37.444**	C
749					
750		**To find correction factor, F:**			
751			**R**	**1.1666667**	
752			**P**	**0.42857**	
753		**Use VBA Function for F:**			
754		**LMTD correction factor:**	**F**	**0.859**	
755					
756		**We have: Q = U * A * LMTD * F**			
757		**Therefore: Area of HX =**	**A**	**1.114**	**m^2....Ans.**

$$Q = m_c \cdot cp_c \cdot (Tc_2 - Tc_1)$$

$$m_h = \frac{Q}{cp_h \cdot (Th_1 - Th_2)}$$

$$DELTAT\_1 = Th_1 - Tc_2$$

$$DELTAT\_2 = Th_2 - Tc_1$$

$$LMTD = \frac{DELTAT\_1 - DELTAT\_2}{\ln\left(\frac{DELTAT\_1}{DELTAT\_2}\right)}$$

$$R = \frac{T_{h1} - T_{h2}}{T_{c2} - T_{c1}}$$

$$P = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}}$$

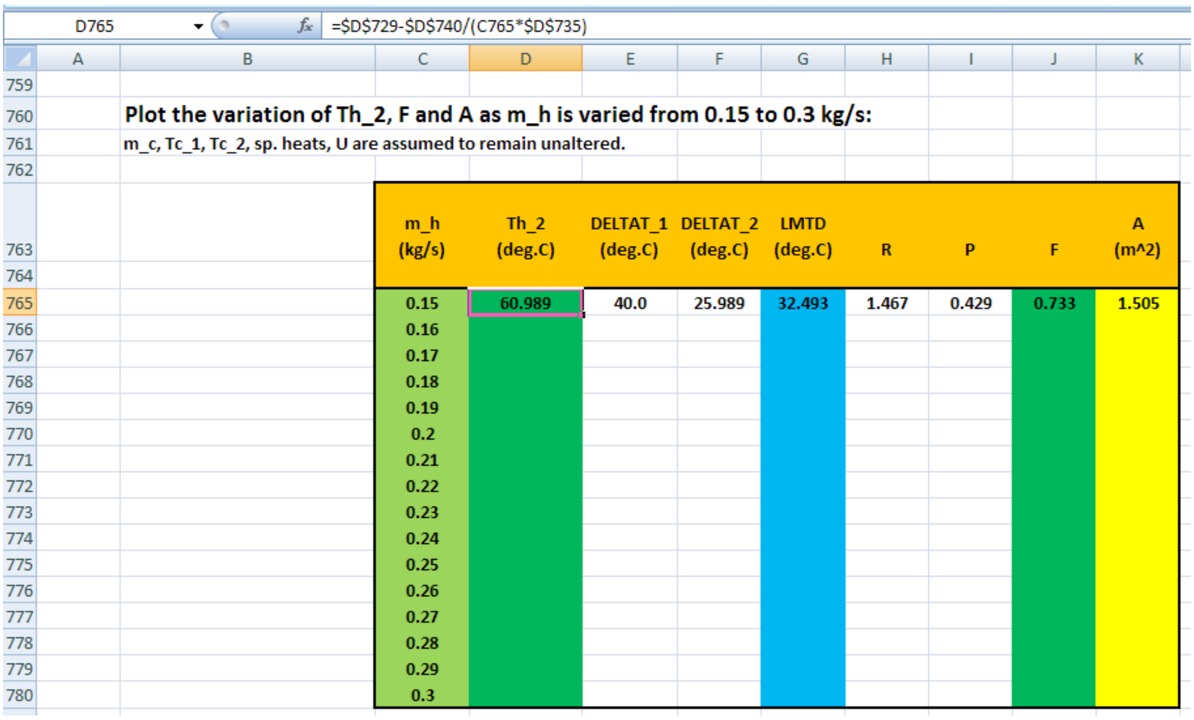
$$A = \frac{Q}{U \cdot LMTD \cdot F}$$

See the Function for calculation of F in the Formula bar, in the above screen shot. (Check the value of F from the graph.)

Thus: mass flow rate of hot fluid = 0.1886 kg/s and, Area of HX = A = 1.114 m^2 … Ans.

(b) Also plot the variation of Th_2, F and A as m_h is varied from 0.15 to 0.3 kg/s:

First, prepare a Table as shown, and fill up the first row below the captions. Remember to enter m_h by relative reference wherever it appears in the formula. See the Formula bar in the screen shot below, for the formula entered for Th_2 in cell D765. Similarly, for other items shown under respective captions:



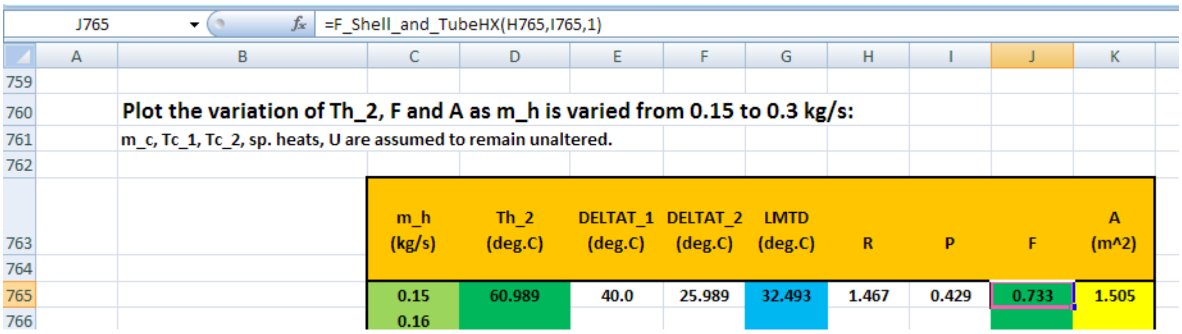
D765 =$D$729-$D$740/(C765*$D$735)

Plot the variation of Th_2, F and A as m_h is varied from 0.15 to 0.3 kg/s:

m_c, Tc_1, Tc_2, sp. heats, U are assumed to remain unaltered.

m_h (kg/s)	Th_2 (deg.C)	DELTAT_1 (deg.C)	DELTAT_2 (deg.C)	LMTD (deg.C)	R	P	F	A (m^2)
0.15	60.989	40.0	25.989	32.493	1.467	0.429	0.733	1.505
0.16								
0.17								
0.18								
0.19								
0.2								
0.21								
0.22								
0.23								
0.24								
0.25								
0.26								
0.27								
0.28								
0.29								
0.3								

LMTD Correction factor F is obtained by using the VBA Function for given R and P. See formula bar in the screen shot below:



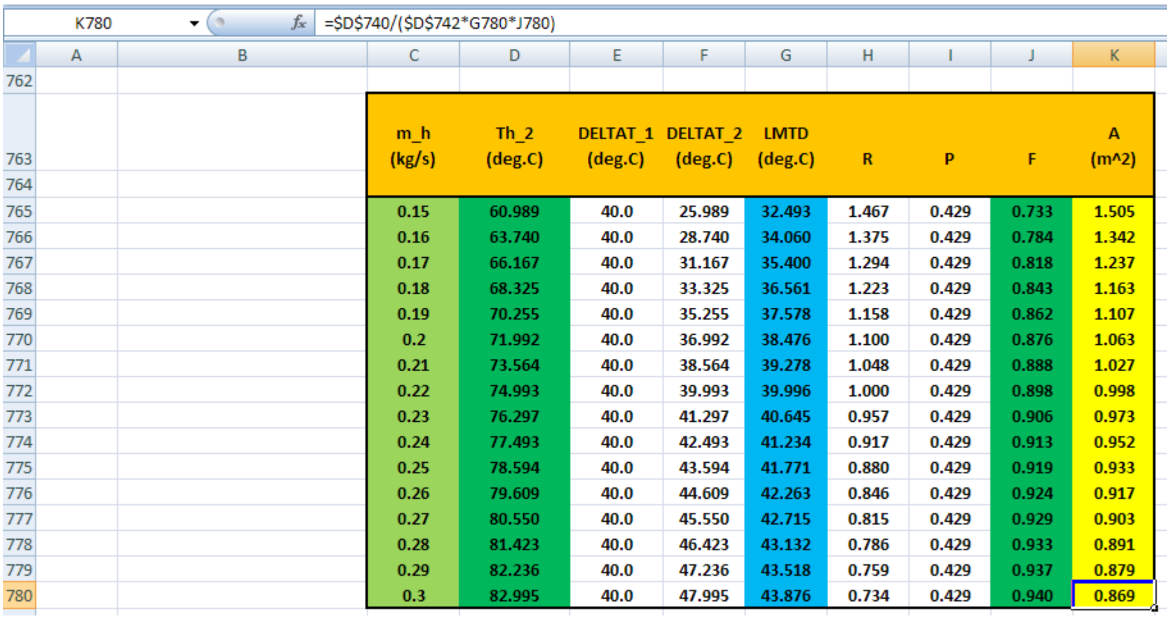
J765 =F_Shell_and_TubeHX(H765,I765,1)

Plot the variation of Th_2, F and A as m_h is varied from 0.15 to 0.3 kg/s:

m_c, Tc_1, Tc_2, sp. heats, U are assumed to remain unaltered.

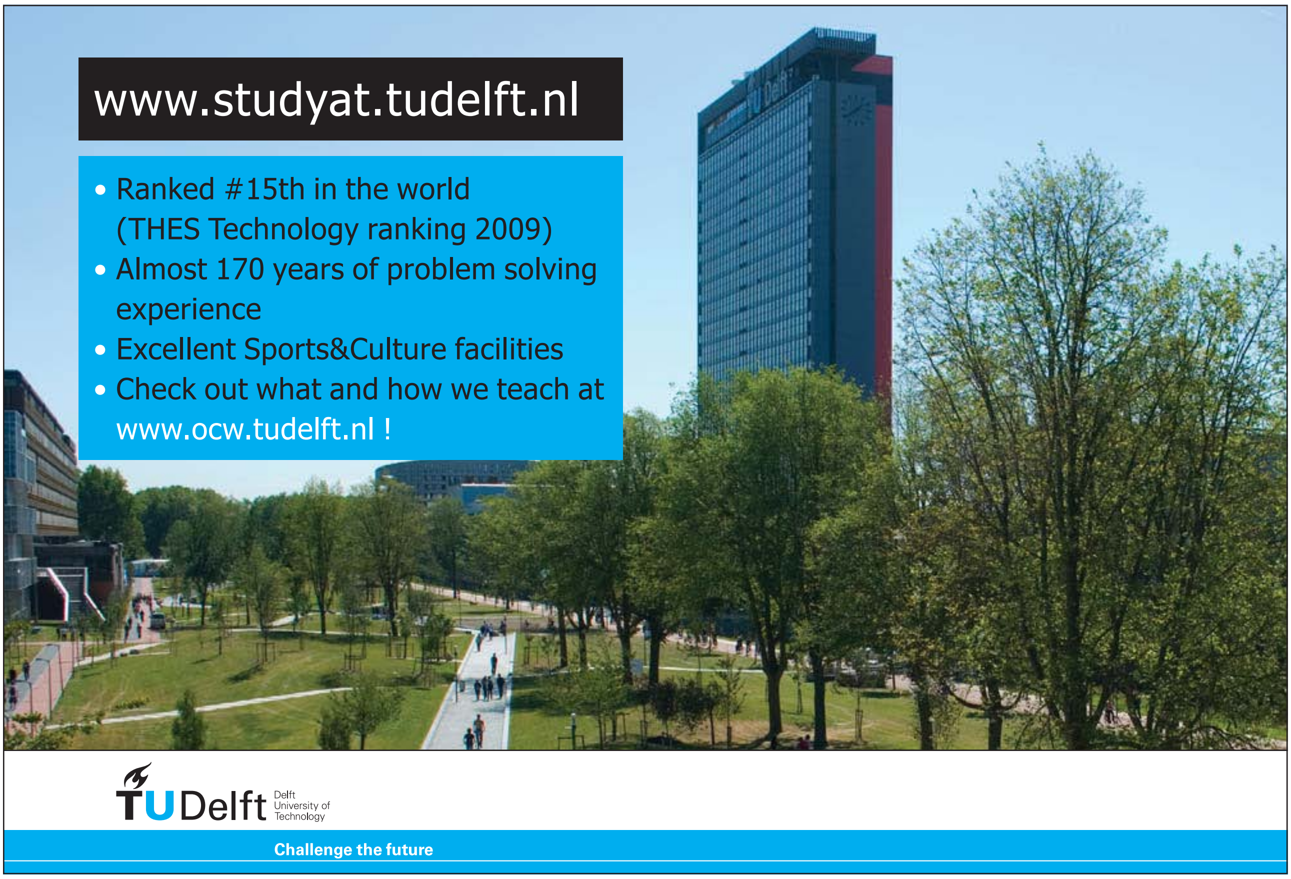
m_h (kg/s)	Th_2 (deg.C)	DELTAT_1 (deg.C)	DELTAT_2 (deg.C)	LMTD (deg.C)	R	P	F	A (m^2)
0.15	60.989	40.0	25.989	32.493	1.467	0.429	0.733	1.505
0.16								

Now, select the cells D765 to K765, and drag-copy till the end of Table, i.e. up to cell K780. Immediately, all calculations are done and the Table gets filled up, as shown below:

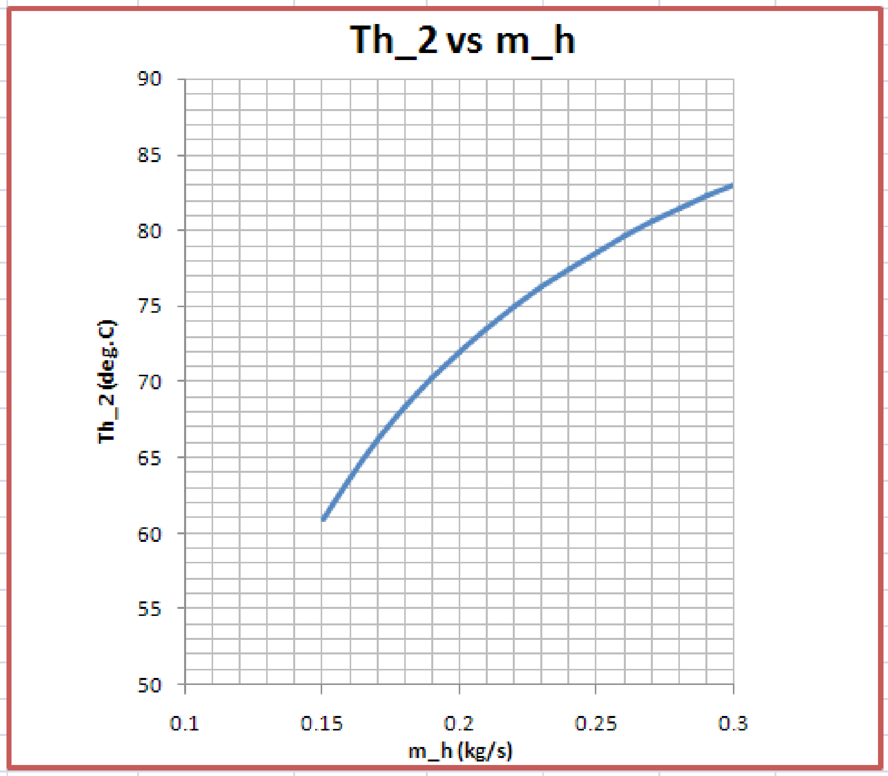


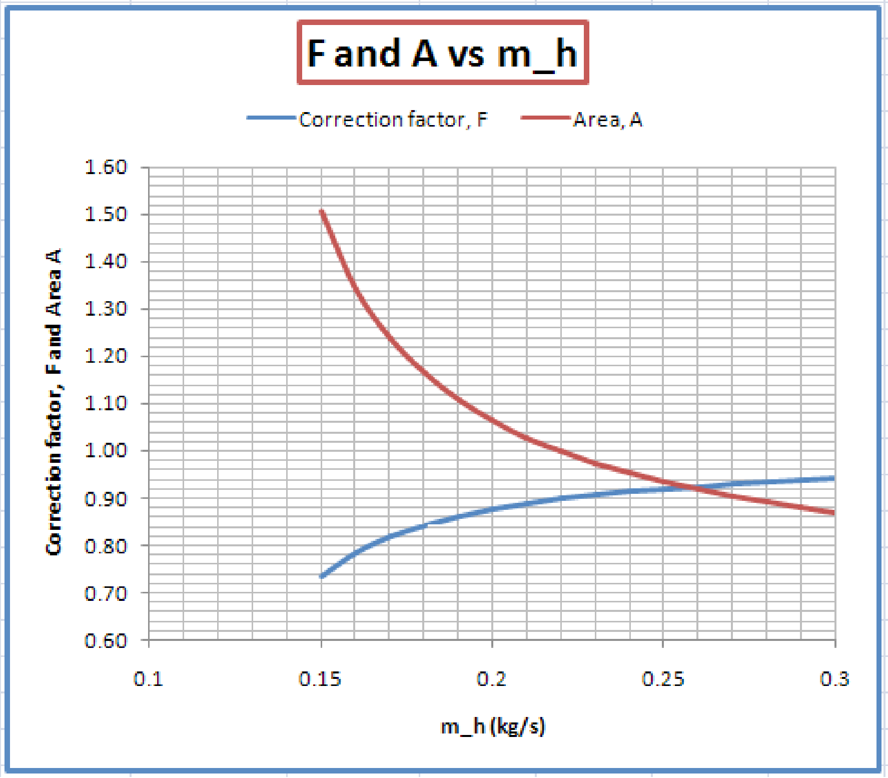
`K780` `=$D$740/($D$742*G780*J780)`

763	m_h (kg/s)	Th_2 (deg.C)	DELTAT_1 (deg.C)	DELTAT_2 (deg.C)	LMTD (deg.C)	R	P	F	A (m^2)
765	0.15	60.989	40.0	25.989	32.493	1.467	0.429	0.733	1.505
766	0.16	63.740	40.0	28.740	34.060	1.375	0.429	0.784	1.342
767	0.17	66.167	40.0	31.167	35.400	1.294	0.429	0.818	1.237
768	0.18	68.325	40.0	33.325	36.561	1.223	0.429	0.843	1.163
769	0.19	70.255	40.0	35.255	37.578	1.158	0.429	0.862	1.107
770	0.2	71.992	40.0	36.992	38.476	1.100	0.429	0.876	1.063
771	0.21	73.564	40.0	38.564	39.278	1.048	0.429	0.888	1.027
772	0.22	74.993	40.0	39.993	39.996	1.000	0.429	0.898	0.998
773	0.23	76.297	40.0	41.297	40.645	0.957	0.429	0.906	0.973
774	0.24	77.493	40.0	42.493	41.234	0.917	0.429	0.913	0.952
775	0.25	78.594	40.0	43.594	41.771	0.880	0.429	0.919	0.933
776	0.26	79.609	40.0	44.609	42.263	0.846	0.429	0.924	0.917
777	0.27	80.550	40.0	45.550	42.715	0.815	0.429	0.929	0.903
778	0.28	81.423	40.0	46.423	43.132	0.786	0.429	0.933	0.891
779	0.29	82.236	40.0	47.236	43.518	0.759	0.429	0.937	0.879
780	0.3	82.995	40.0	47.995	43.876	0.734	0.429	0.940	0.869



Now, plot the graphs in EXCEL:





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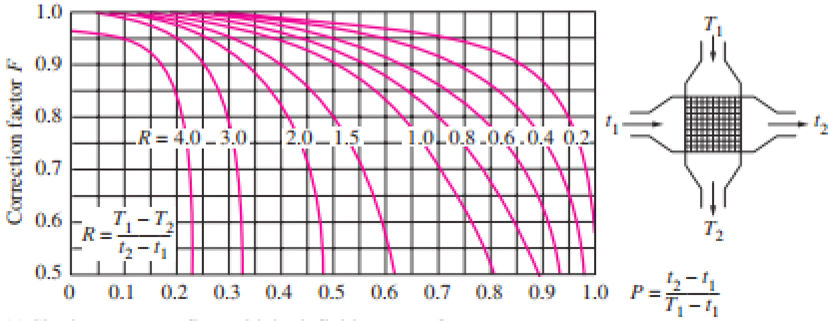
Prob. 4B.23. Consider a cross-flow HX with *both fluids unmixed*. Water (cp = 4181 J/kg.C), flowing at a rate of 1 kg/s, is heated from 40 C to 80 C. Hot engine oil (cp = 1900 J/kg.C) enters at a temp of 100 C and at a flow rate of 2.6 kg/s. If the overall heat transfer coeff. is 780 W/m^2.C, find out the exit temp of oil and the area required for the heat exchanger.(b) Also plot the variation of Th_2 and A as m_h is varied from 2 to 5 kg/s.

EXCEL Solution: Note that this is a **cross-flow HX with both fluids unmixed.**

So, we have to find out LMTD correction factor, F.

Following graph for F (from Cengel) as a function of R and P was given at the beginning of this chapter:

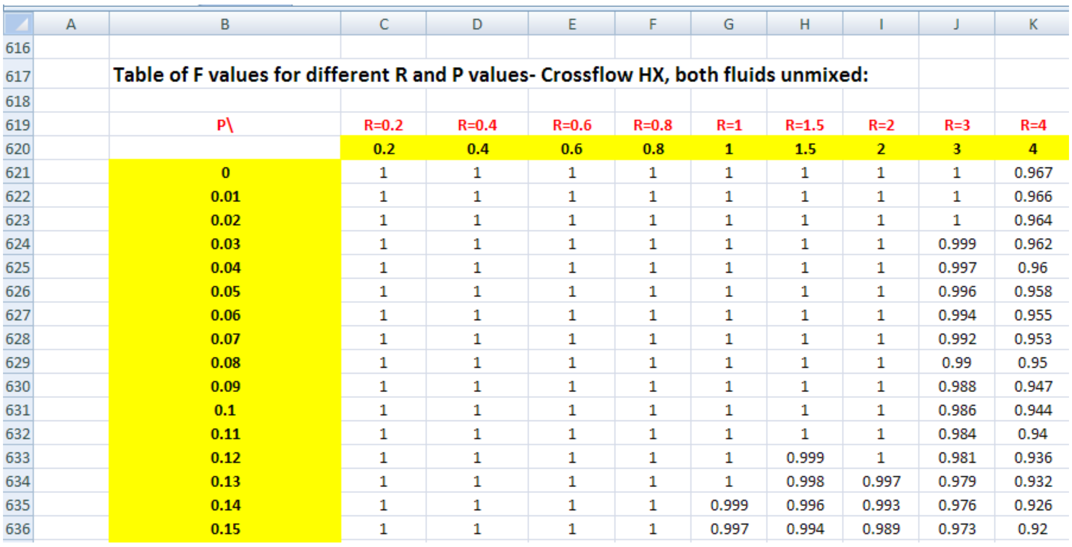


(*c*) Single-pass cross-flow with both fluids *unmixed*

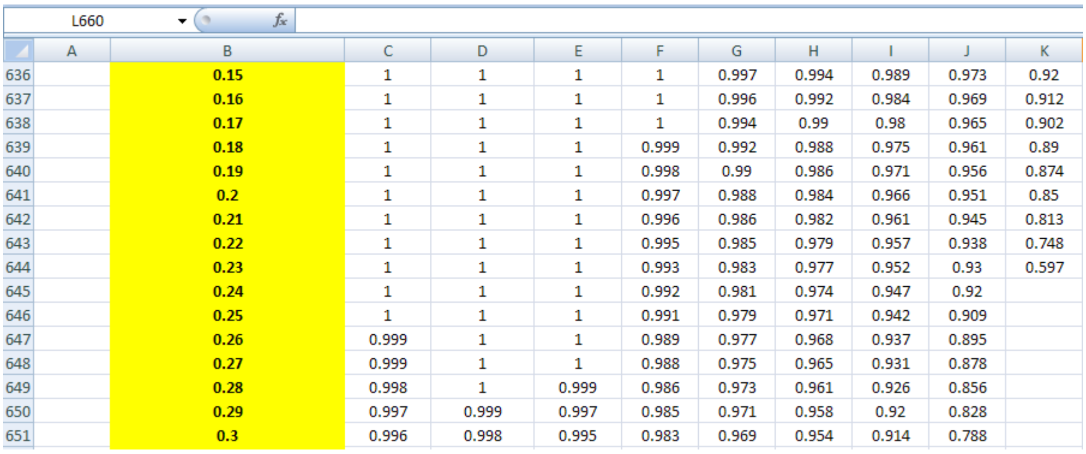
For computer solution, we can have the graphs in terms of curve-fit equations or Tables.

Here, we have digitized the various curves in the above graph, got the curve-fit equations and prepared the following Table. To get F when R and P are given, we have to do two-way interpolation in the Table. We shall write a VBA Function to perform this interpolation.

First, the Table of F as a function of R and P:

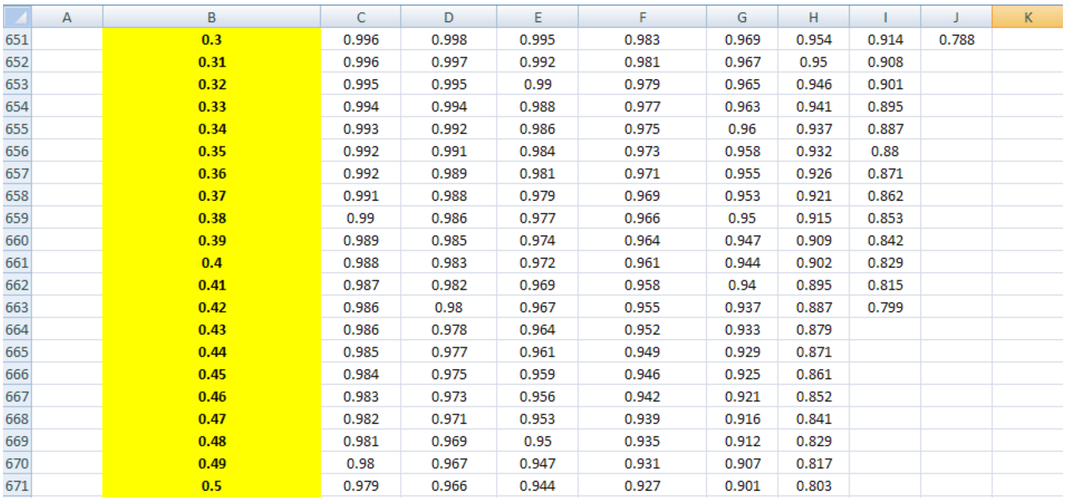


	A	B	C	D	E	F	G	H	I	J	K
616											
617		**Table of F values for different R and P values- Crossflow HX, both fluids unmixed:**									
618											
619		P\	R=0.2	R=0.4	R=0.6	R=0.8	R=1	R=1.5	R=2	R=3	R=4
620			**0.2**	**0.4**	**0.6**	**0.8**	**1**	**1.5**	**2**	**3**	**4**
621		**0**	1	1	1	1	1	1	1	1	0.967
622		**0.01**	1	1	1	1	1	1	1	1	0.966
623		**0.02**	1	1	1	1	1	1	1	1	0.964
624		**0.03**	1	1	1	1	1	1	1	0.999	0.962
625		**0.04**	1	1	1	1	1	1	1	0.997	0.96
626		**0.05**	1	1	1	1	1	1	1	0.996	0.958
627		**0.06**	1	1	1	1	1	1	1	0.994	0.955
628		**0.07**	1	1	1	1	1	1	1	0.992	0.953
629		**0.08**	1	1	1	1	1	1	1	0.99	0.95
630		**0.09**	1	1	1	1	1	1	1	0.988	0.947
631		**0.1**	1	1	1	1	1	1	1	0.986	0.944
632		**0.11**	1	1	1	1	1	1	1	0.984	0.94
633		**0.12**	1	1	1	1	1	0.999	1	0.981	0.936
634		**0.13**	1	1	1	1	1	0.998	0.997	0.979	0.932
635		**0.14**	1	1	1	1	0.999	0.996	0.993	0.976	0.926
636		**0.15**	1	1	1	1	0.997	0.994	0.989	0.973	0.92

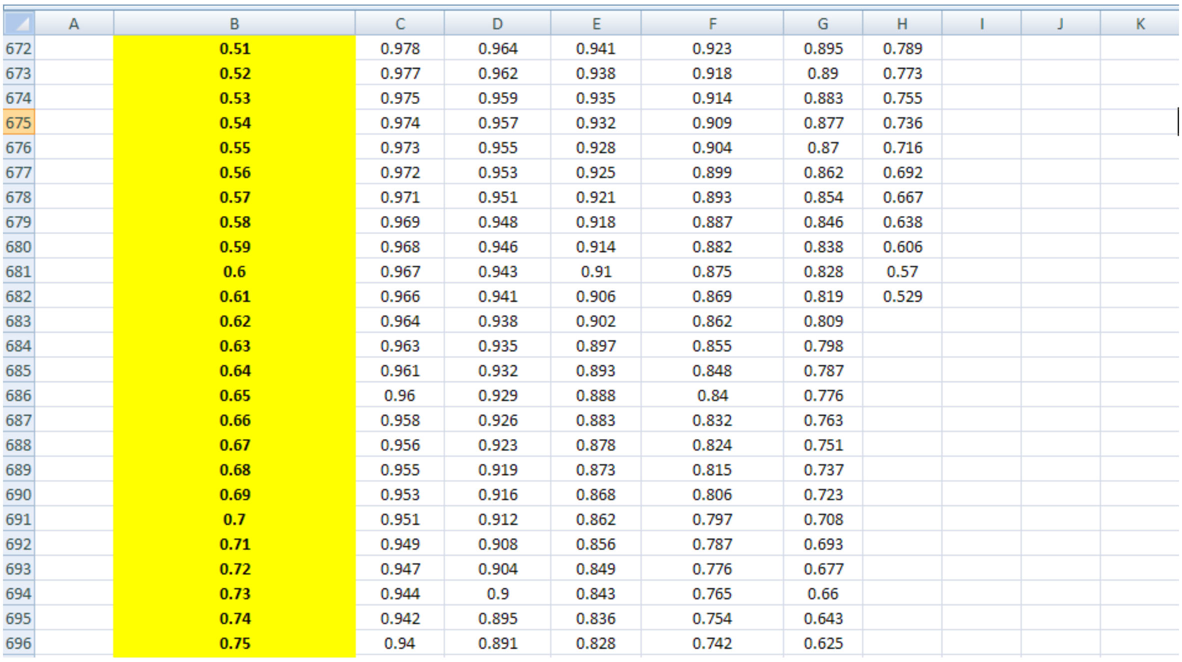


L660

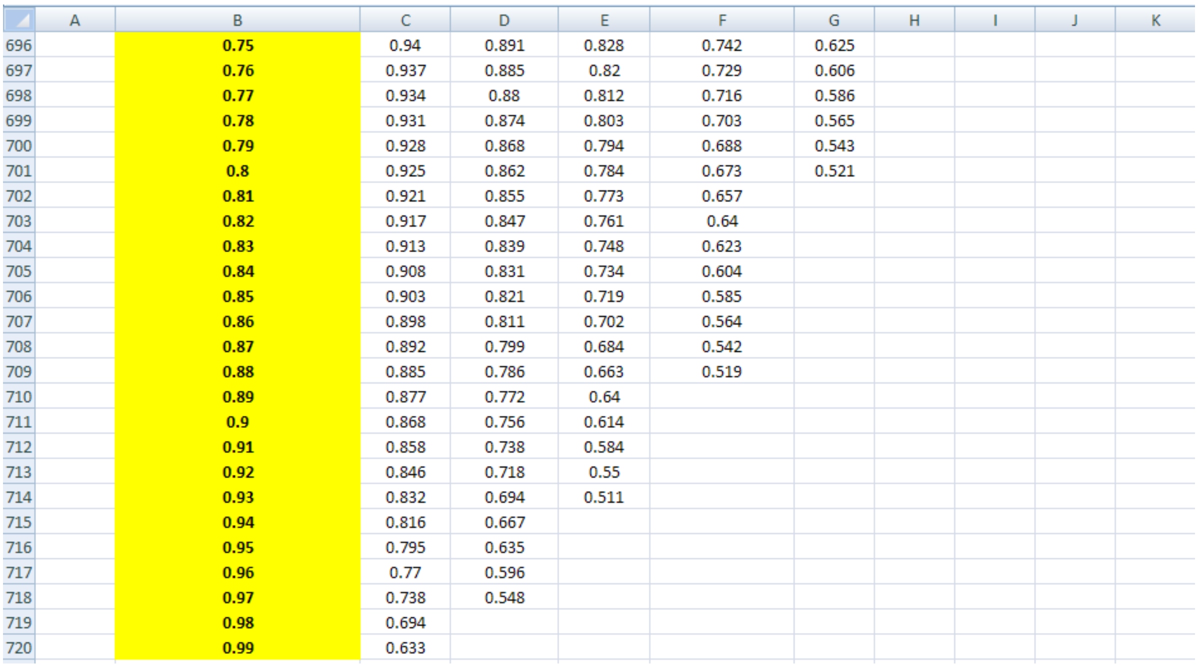
	A	B	C	D	E	F	G	H	I	J	K
636		**0.15**	1	1	1	1	0.997	0.994	0.989	0.973	0.92
637		**0.16**	1	1	1	1	0.996	0.992	0.984	0.969	0.912
638		**0.17**	1	1	1	1	0.994	0.99	0.98	0.965	0.902
639		**0.18**	1	1	1	0.999	0.992	0.988	0.975	0.961	0.89
640		**0.19**	1	1	1	0.998	0.99	0.986	0.971	0.956	0.874
641		**0.2**	1	1	1	0.997	0.988	0.984	0.966	0.951	0.85
642		**0.21**	1	1	1	0.996	0.986	0.982	0.961	0.945	0.813
643		**0.22**	1	1	1	0.995	0.985	0.979	0.957	0.938	0.748
644		**0.23**	1	1	1	0.993	0.983	0.977	0.952	0.93	0.597
645		**0.24**	1	1	1	0.992	0.981	0.974	0.947	0.92	
646		**0.25**	1	1	1	0.991	0.979	0.971	0.942	0.909	
647		**0.26**	0.999	1	1	0.989	0.977	0.968	0.937	0.895	
648		**0.27**	0.999	1	1	0.988	0.975	0.965	0.931	0.878	
649		**0.28**	0.998	1	0.999	0.986	0.973	0.961	0.926	0.856	
650		**0.29**	0.997	0.999	0.997	0.985	0.971	0.958	0.92	0.828	
651		**0.3**	0.996	0.998	0.995	0.983	0.969	0.954	0.914	0.788	



	A	B	C	D	E	F	G	H	I	J	K
651		**0.3**	0.996	0.998	0.995	0.983	0.969	0.954	0.914	0.788	
652		**0.31**	0.996	0.997	0.992	0.981	0.967	0.95	0.908		
653		**0.32**	0.995	0.995	0.99	0.979	0.965	0.946	0.901		
654		**0.33**	0.994	0.994	0.988	0.977	0.963	0.941	0.895		
655		**0.34**	0.993	0.992	0.986	0.975	0.96	0.937	0.887		
656		**0.35**	0.992	0.991	0.984	0.973	0.958	0.932	0.88		
657		**0.36**	0.992	0.989	0.981	0.971	0.955	0.926	0.871		
658		**0.37**	0.991	0.988	0.979	0.969	0.953	0.921	0.862		
659		**0.38**	0.99	0.986	0.977	0.966	0.95	0.915	0.853		
660		**0.39**	0.989	0.985	0.974	0.964	0.947	0.909	0.842		
661		**0.4**	0.988	0.983	0.972	0.961	0.944	0.902	0.829		
662		**0.41**	0.987	0.982	0.969	0.958	0.94	0.895	0.815		
663		**0.42**	0.986	0.98	0.967	0.955	0.937	0.887	0.799		
664		**0.43**	0.986	0.978	0.964	0.952	0.933	0.879			
665		**0.44**	0.985	0.977	0.961	0.949	0.929	0.871			
666		**0.45**	0.984	0.975	0.959	0.946	0.925	0.861			
667		**0.46**	0.983	0.973	0.956	0.942	0.921	0.852			
668		**0.47**	0.982	0.971	0.953	0.939	0.916	0.841			
669		**0.48**	0.981	0.969	0.95	0.935	0.912	0.829			
670		**0.49**	0.98	0.967	0.947	0.931	0.907	0.817			
671		**0.5**	0.979	0.966	0.944	0.927	0.901	0.803			

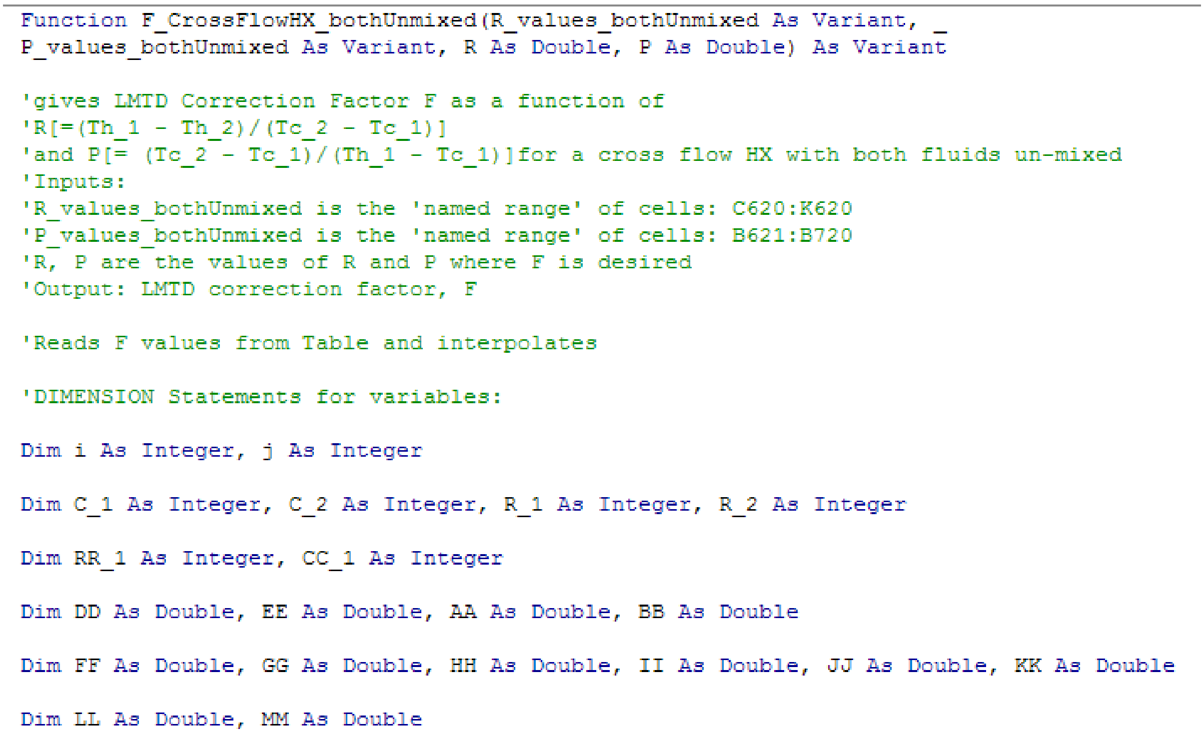


	A	B	C	D	E	F	G	H	I	J	K
672		0.51	0.978	0.964	0.941	0.923	0.895	0.789			
673		0.52	0.977	0.962	0.938	0.918	0.89	0.773			
674		0.53	0.975	0.959	0.935	0.914	0.883	0.755			
675		0.54	0.974	0.957	0.932	0.909	0.877	0.736			
676		0.55	0.973	0.955	0.928	0.904	0.87	0.716			
677		0.56	0.972	0.953	0.925	0.899	0.862	0.692			
678		0.57	0.971	0.951	0.921	0.893	0.854	0.667			
679		0.58	0.969	0.948	0.918	0.887	0.846	0.638			
680		0.59	0.968	0.946	0.914	0.882	0.838	0.606			
681		0.6	0.967	0.943	0.91	0.875	0.828	0.57			
682		0.61	0.966	0.941	0.906	0.869	0.819	0.529			
683		0.62	0.964	0.938	0.902	0.862	0.809				
684		0.63	0.963	0.935	0.897	0.855	0.798				
685		0.64	0.961	0.932	0.893	0.848	0.787				
686		0.65	0.96	0.929	0.888	0.84	0.776				
687		0.66	0.958	0.926	0.883	0.832	0.763				
688		0.67	0.956	0.923	0.878	0.824	0.751				
689		0.68	0.955	0.919	0.873	0.815	0.737				
690		0.69	0.953	0.916	0.868	0.806	0.723				
691		0.7	0.951	0.912	0.862	0.797	0.708				
692		0.71	0.949	0.908	0.856	0.787	0.693				
693		0.72	0.947	0.904	0.849	0.776	0.677				
694		0.73	0.944	0.9	0.843	0.765	0.66				
695		0.74	0.942	0.895	0.836	0.754	0.643				
696		0.75	0.94	0.891	0.828	0.742	0.625				

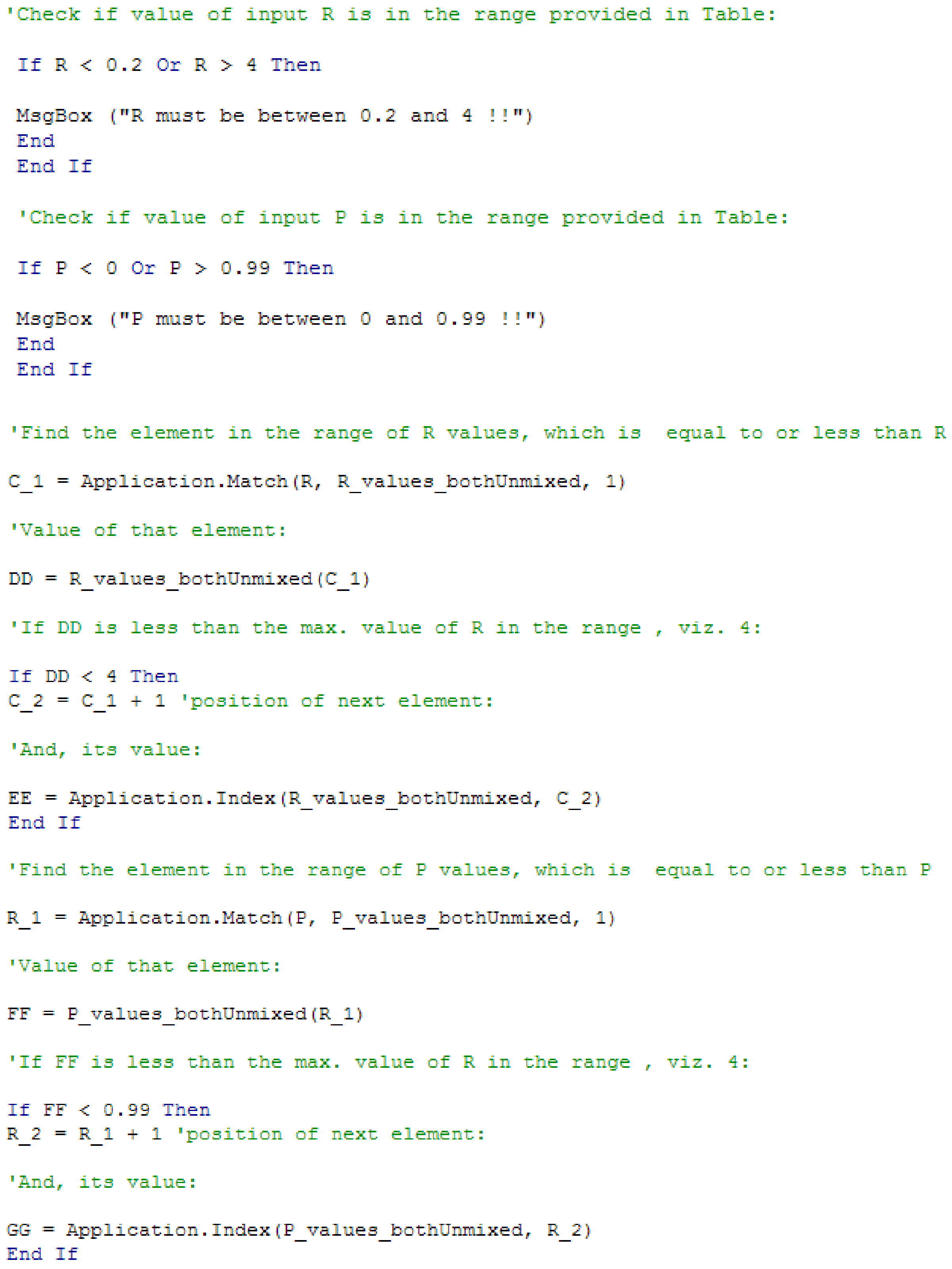


	A	B	C	D	E	F	G	H	I	J	K
696		0.75	0.94	0.891	0.828	0.742	0.625				
697		0.76	0.937	0.885	0.82	0.729	0.606				
698		0.77	0.934	0.88	0.812	0.716	0.586				
699		0.78	0.931	0.874	0.803	0.703	0.565				
700		0.79	0.928	0.868	0.794	0.688	0.543				
701		0.8	0.925	0.862	0.784	0.673	0.521				
702		0.81	0.921	0.855	0.773	0.657					
703		0.82	0.917	0.847	0.761	0.64					
704		0.83	0.913	0.839	0.748	0.623					
705		0.84	0.908	0.831	0.734	0.604					
706		0.85	0.903	0.821	0.719	0.585					
707		0.86	0.898	0.811	0.702	0.564					
708		0.87	0.892	0.799	0.684	0.542					
709		0.88	0.885	0.786	0.663	0.519					
710		0.89	0.877	0.772	0.64						
711		0.9	0.868	0.756	0.614						
712		0.91	0.858	0.738	0.584						
713		0.92	0.846	0.718	0.55						
714		0.93	0.832	0.694	0.511						
715		0.94	0.816	0.667							
716		0.95	0.795	0.635							
717		0.96	0.77	0.596							
718		0.97	0.738	0.548							
719		0.98	0.694								
720		0.99	0.633								

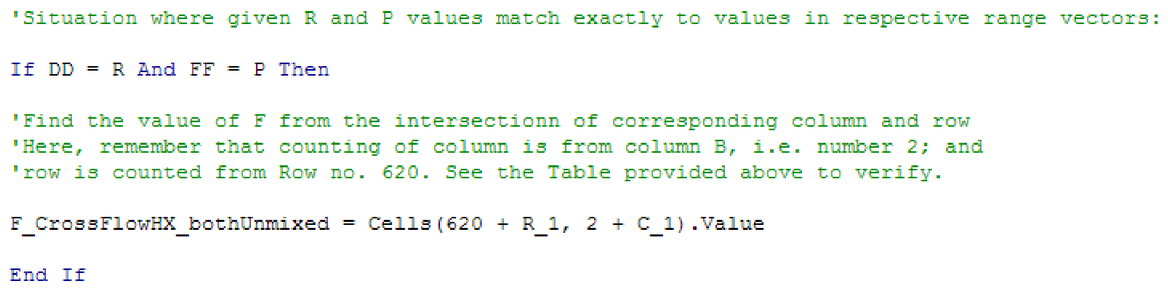
Following is the VBA program to do two-way interpolation to get F when R and P are given:



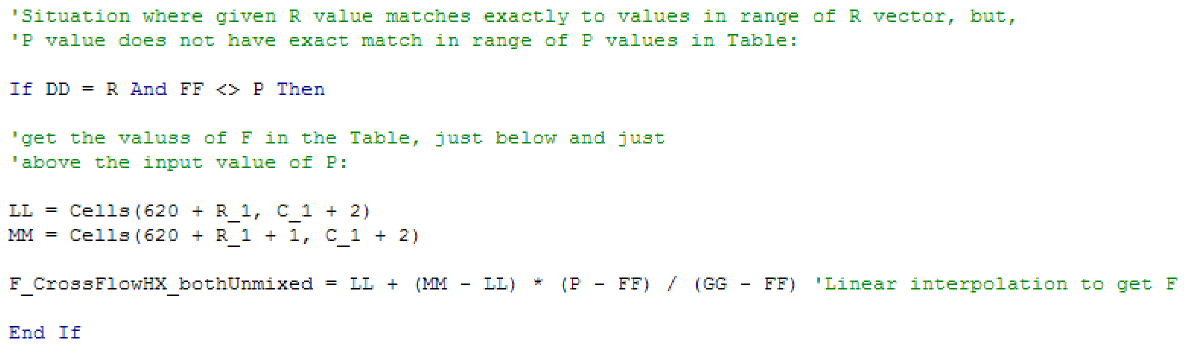
```
Function F_CrossFlowHX_bothUnmixed(R_values_bothUnmixed As Variant, _
P_values_bothUnmixed As Variant, R As Double, P As Double) As Variant

'gives LMTD Correction Factor F as a function of
'R[=(Th_1 - Th_2)/(Tc_2 - Tc_1)]
'and P[= (Tc_2 - Tc_1)/(Th_1 - Tc_1)]for a cross flow HX with both fluids un-mixed
'Inputs:
'R_values_bothUnmixed is the 'named range' of cells: C620:K620
'P_values_bothUnmixed is the 'named range' of cells: B621:B720
'R, P are the values of R and P where F is desired
'Output: LMTD correction factor, F

'Reads F values from Table and interpolates

'DIMENSION Statements for variables:

Dim i As Integer, j As Integer

Dim C_1 As Integer, C_2 As Integer, R_1 As Integer, R_2 As Integer

Dim RR_1 As Integer, CC_1 As Integer

Dim DD As Double, EE As Double, AA As Double, BB As Double

Dim FF As Double, GG As Double, HH As Double, II As Double, JJ As Double, KK As Double

Dim LL As Double, MM As Double
```



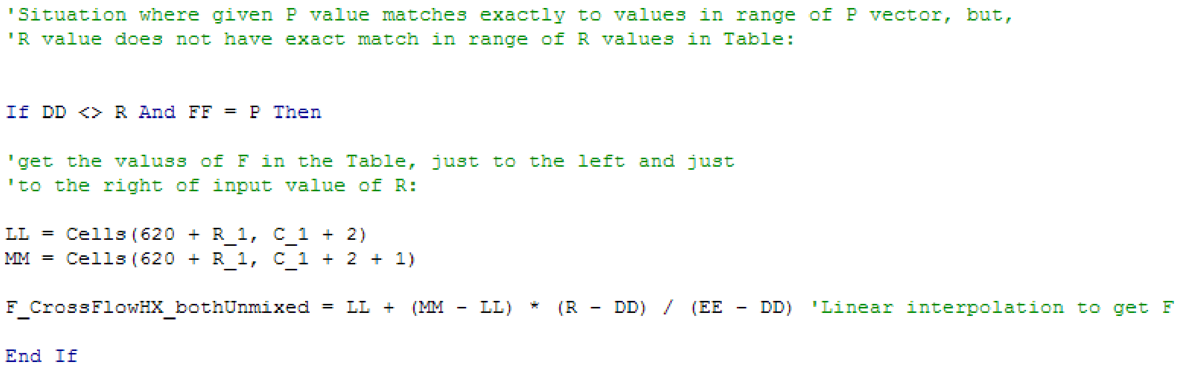
```
'Check if value of input R is in the range provided in Table:

 If R < 0.2 Or R > 4 Then

 MsgBox ("R must be between 0.2 and 4 !!")
 End
 End If

 'Check if value of input P is in the range provided in Table:

 If P < 0 Or P > 0.99 Then

 MsgBox ("P must be between 0 and 0.99 !!")
 End
 End If

'Find the element in the range of R values, which is  equal to or less than R

C_1 = Application.Match(R, R_values_bothUnmixed, 1)

'Value of that element:

DD = R_values_bothUnmixed(C_1)

'If DD is less than the max. value of R in the range , viz. 4:

If DD < 4 Then
C_2 = C_1 + 1 'position of next element:

'And, its value:

EE = Application.Index(R_values_bothUnmixed, C_2)
End If

'Find the element in the range of P values, which is  equal to or less than P

R_1 = Application.Match(P, P_values_bothUnmixed, 1)

'Value of that element:

FF = P_values_bothUnmixed(R_1)

'If FF is less than the max. value of R in the range , viz. 4:

If FF < 0.99 Then
R_2 = R_1 + 1 'position of next element:

'And, its value:

GG = Application.Index(P_values_bothUnmixed, R_2)
End If
```



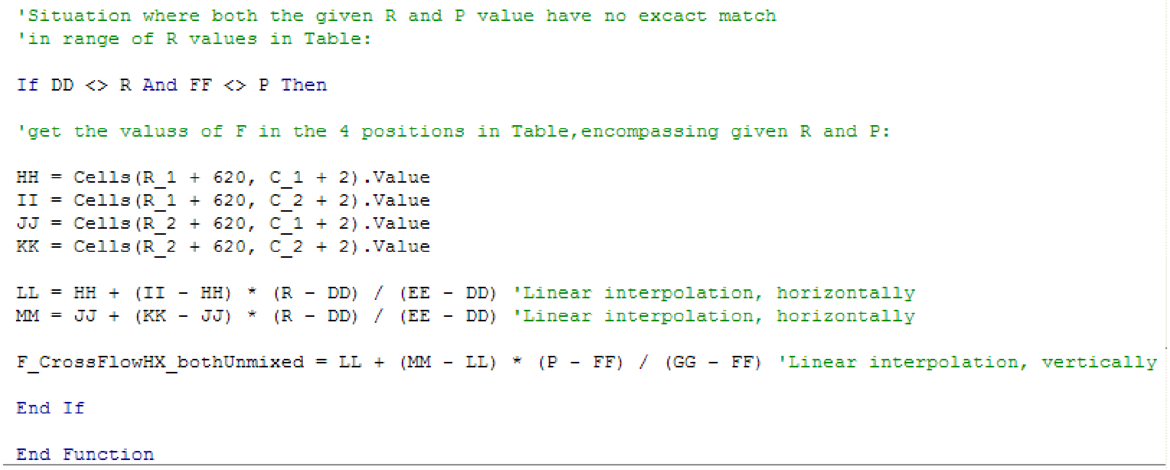
```
'Situation where given R and P values match exactly to values in respective range vectors:

If DD = R And FF = P Then

'Find the value of F from the intersectionn of corresponding column and row
'Here, remember that counting of column is from column B, i.e. number 2; and
'row is counted from Row no. 620. See the Table provided above to verify.

F_CrossFlowHX_bothUnmixed = Cells(620 + R_1, 2 + C_1).Value

End If
```



```
'Situation where given R value matches exactly to values in range of R vector, but,
'P value does not have exact match in range of P values in Table:

If DD = R And FF <> P Then

'get the valuss of F in the Table, just below and just
'above the input value of P:

LL = Cells(620 + R_1, C_1 + 2)
MM = Cells(620 + R_1 + 1, C_1 + 2)

F_CrossFlowHX_bothUnmixed = LL + (MM - LL) * (P - FF) / (GG - FF) 'Linear interpolation to get F

End If
```



```
'Situation where given P value matches exactly to values in range of P vector, but,
'R value does not have exact match in range of R values in Table:


If DD <> R And FF = P Then

'get the valuss of F in the Table, just to the left and just
'to the right of input value of R:

LL = Cells(620 + R_1, C_1 + 2)
MM = Cells(620 + R_1, C_1 + 2 + 1)

F_CrossFlowHX_bothUnmixed = LL + (MM - LL) * (R - DD) / (EE - DD) 'Linear interpolation to get F

End If
```



```
'Situation where both the given R and P value have no excact match
'in range of R values in Table:

If DD <> R And FF <> P Then

'get the valuss of F in the 4 positions in Table,encompassing given R and P:

HH = Cells(R_1 + 620, C_1 + 2).Value
II = Cells(R_1 + 620, C_2 + 2).Value
JJ = Cells(R_2 + 620, C_1 + 2).Value
KK = Cells(R_2 + 620, C_2 + 2).Value

LL = HH + (II - HH) * (R - DD) / (EE - DD) 'Linear interpolation, horizontally
MM = JJ + (KK - JJ) * (R - DD) / (EE - DD) 'Linear interpolation, horizontally

F_CrossFlowHX_bothUnmixed = LL + (MM - LL) * (P - FF) / (GG - FF) 'Linear interpolation, vertically

End If

End Function
```

Read the comments in the above program to see what each line does.

Now, let us use this Function in solving the above Problem.

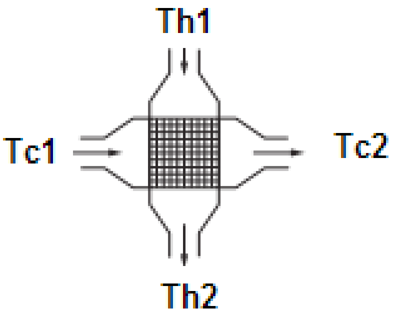
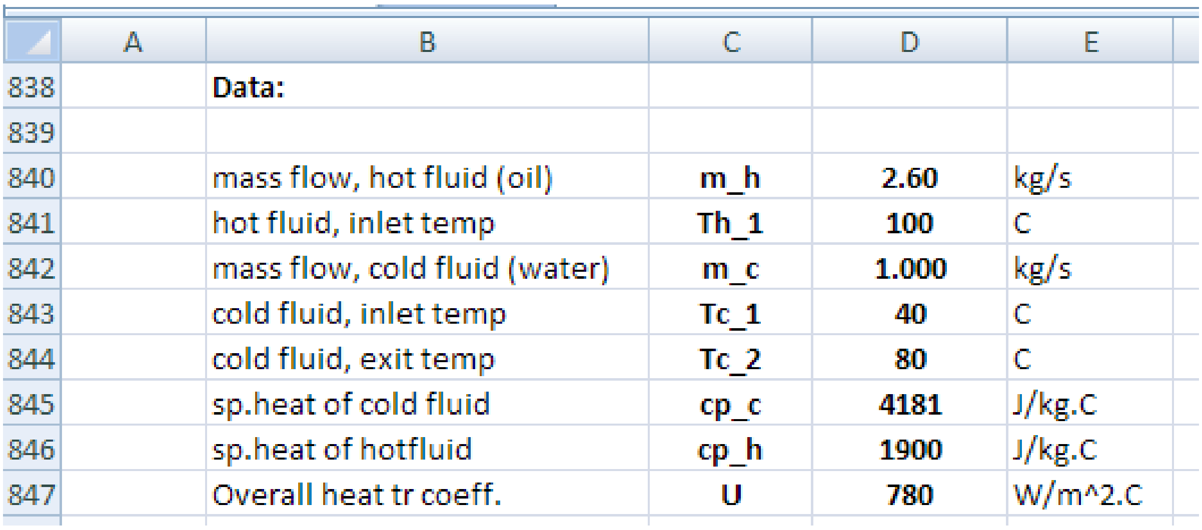


Fig. Prob.4B.23.

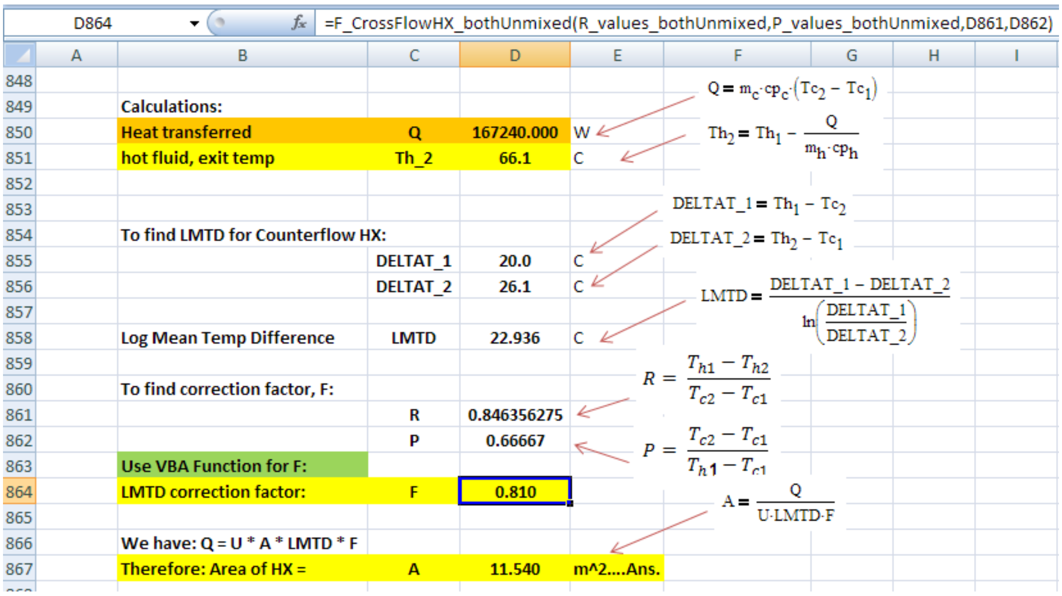
Following are the steps in EXCEL Solution:

1. Set up the EXCEL worksheet, enter data:



	A	B	C	D	E
838		**Data:**			
839					
840		mass flow, hot fluid (oil)	**m_h**	**2.60**	kg/s
841		hot fluid, inlet temp	**Th_1**	**100**	C
842		mass flow, cold fluid (water)	**m_c**	**1.000**	kg/s
843		cold fluid, inlet temp	**Tc_1**	**40**	C
844		cold fluid, exit temp	**Tc_2**	**80**	C
845		sp.heat of cold fluid	**cp_c**	**4181**	J/kg.C
846		sp.heat of hotfluid	**cp_h**	**1900**	J/kg.C
847		Overall heat tr coeff.	**U**	**780**	W/m^2.C

2. Do the calculations as indicated below. Formulas are shown in the worksheet:



D864 =F_CrossFlowHX_bothUnmixed(R_values_bothUnmixed,P_values_bothUnmixed,D861,D862)

	A	B	C	D	E
848					
849		Calculations:			
850		Heat transferred	Q	167240.000	W
851		hot fluid, exit temp	Th_2	66.1	C
852					
853					
854		To find LMTD for Counterflow HX:			
855			DELTAT_1	20.0	C
856			DELTAT_2	26.1	C
857					
858		Log Mean Temp Difference	LMTD	22.936	C
859					
860		To find correction factor, F:			
861			R	0.846356275	
862			P	0.66667	
863		Use VBA Function for F:			
864		LMTD correction factor:	F	0.810	
865					
866		We have: Q = U * A * LMTD * F			
867		Therefore: Area of HX =	A	11.540	m^2....Ans.

$$Q = m_c \cdot cp_c \cdot (Tc_2 - Tc_1)$$

$$Th_2 = Th_1 - \frac{Q}{m_h \cdot cp_h}$$

$$DELTAT\_1 = Th_1 - Tc_2$$

$$DELTAT\_2 = Th_2 - Tc_1$$

$$LMTD = \frac{DELTAT\_1 - DELTAT\_2}{\ln\left(\frac{DELTAT\_1}{DELTAT\_2}\right)}$$

$$R = \frac{T_{h1} - T_{h2}}{T_{c2} - T_{c1}}$$

$$P = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}}$$

$$A = \frac{Q}{U \cdot LMTD \cdot F}$$

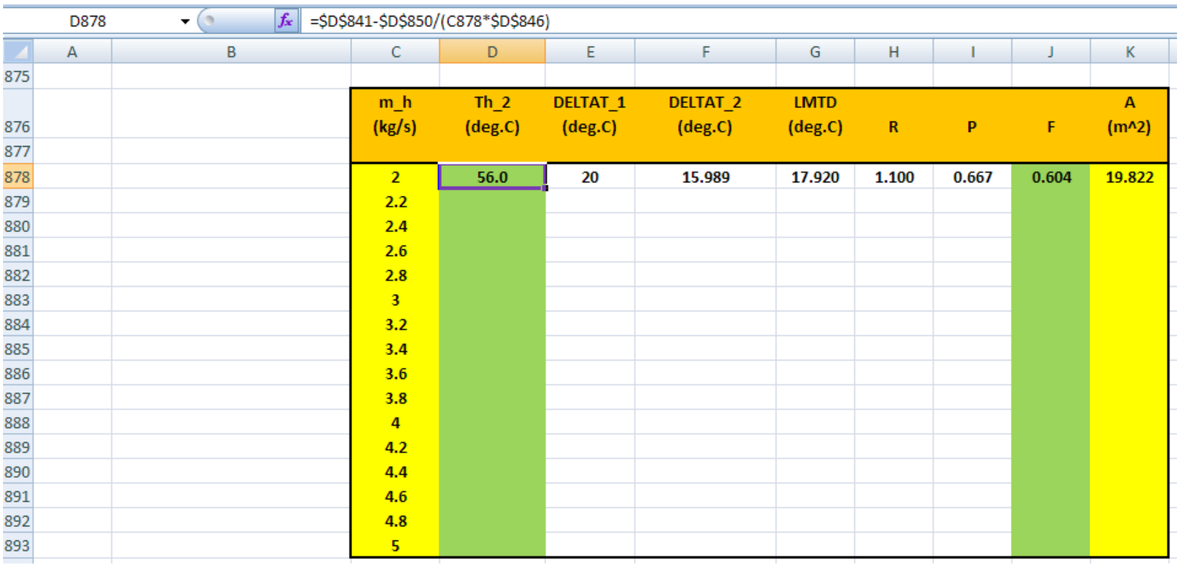
In the above screen shot, F is determined using the VBA Function written above; see the Formula bar.

(Check the value of F from the graph.)

Thus, exit temp of hot fluid, Th_2 = 66.1 C, Area of HX = A = 11.54 m^2 …. Ans.

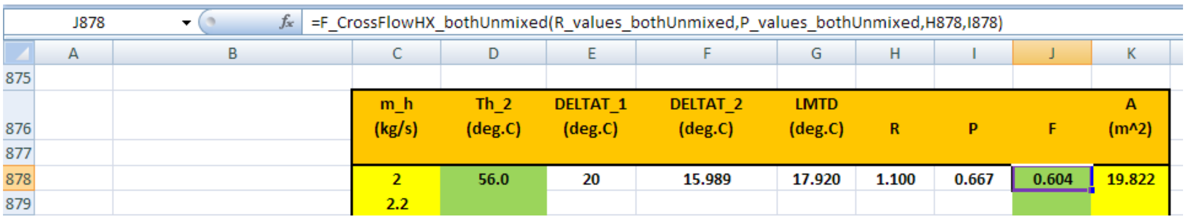
(b) Also plot the variation of Th_2, F and A as m_h is varied from 0.15 to 0.3 kg/s:

First, prepare a Table as shown, and fill up the first row below the captions. Remember to enter m_h by relative reference wherever it appears in the formula. See the Formula bar in the screen shot below, for the formula entered for Th_2 in cell D878. Similarly, for other items shown under respective captions:


D878
=$D$841-$D$850/(C878*$D$846)

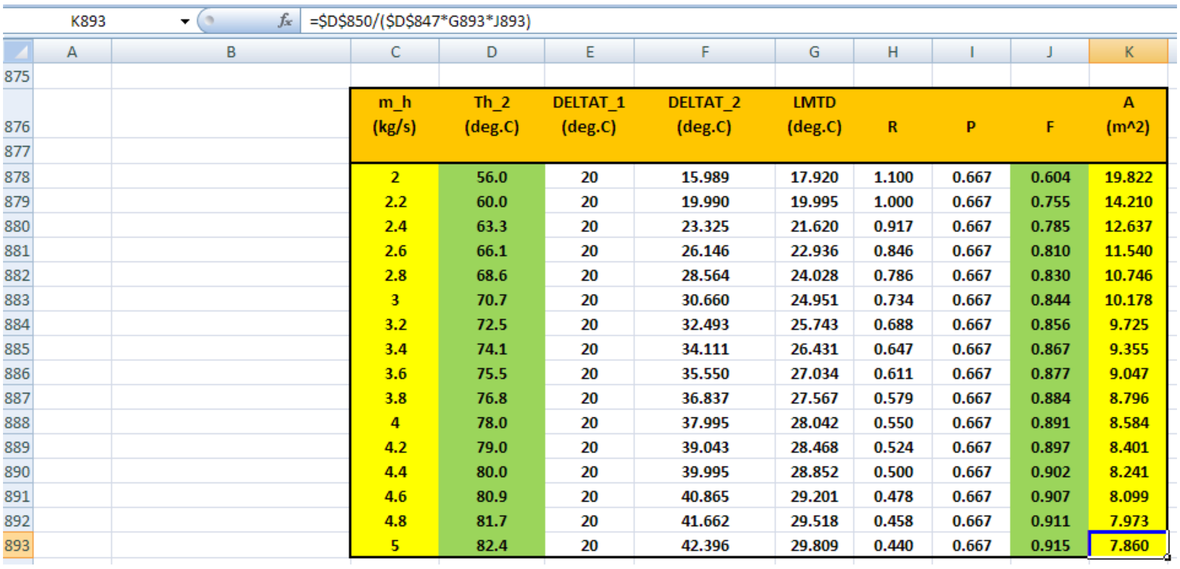
m_h (kg/s)	Th_2 (deg.C)	DELTAT_1 (deg.C)	DELTAT_2 (deg.C)	LMTD (deg.C)	R	P	F	A (m^2)
2	56.0	20	15.989	17.920	1.100	0.667	0.604	19.822
2.2								
2.4								
2.6								
2.8								
3								
3.2								
3.4								
3.6								
3.8								
4								
4.2								
4.4								
4.6								
4.8								
5								

LMTD Correction factor F is obtained by using the VBA Function for given R and P. See formula bar in the screen shot below:


J878
=F_CrossFlowHX_bothUnmixed(R_values_bothUnmixed,P_values_bothUnmixed,H878,I878)

m_h (kg/s)	Th_2 (deg.C)	DELTAT_1 (deg.C)	DELTAT_2 (deg.C)	LMTD (deg.C)	R	P	F	A (m^2)
2	56.0	20	15.989	17.920	1.100	0.667	0.604	19.822
2.2								

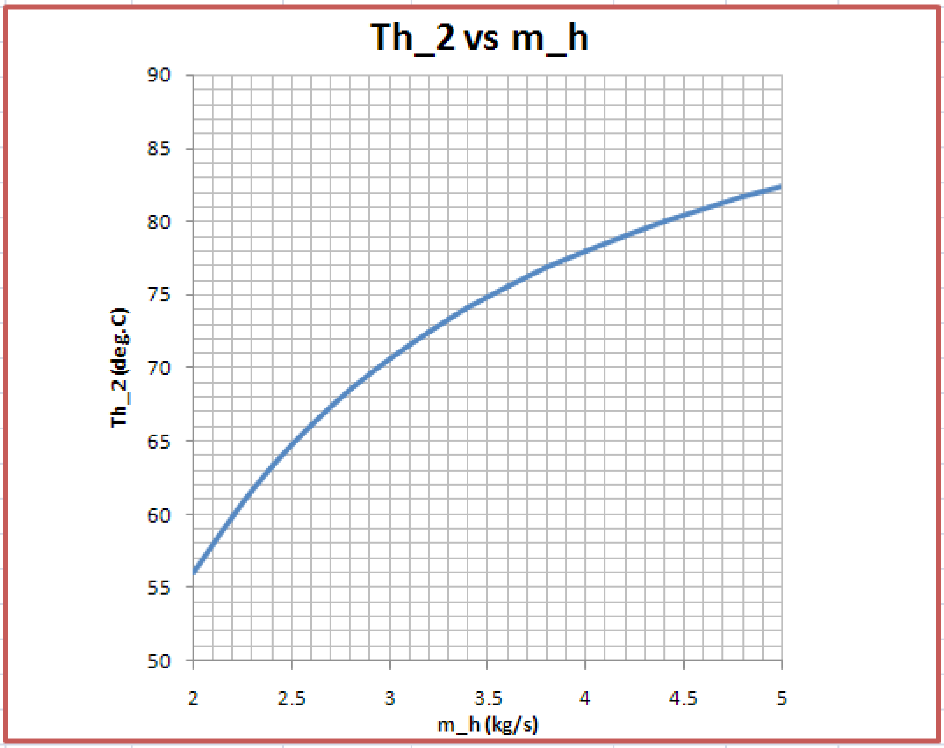
Now, select the cells D878 to K878, and drag-copy till the end of Table, i.e. up to cell K893. Immediately, all calculations are done and the Table gets filled up, as shown below:

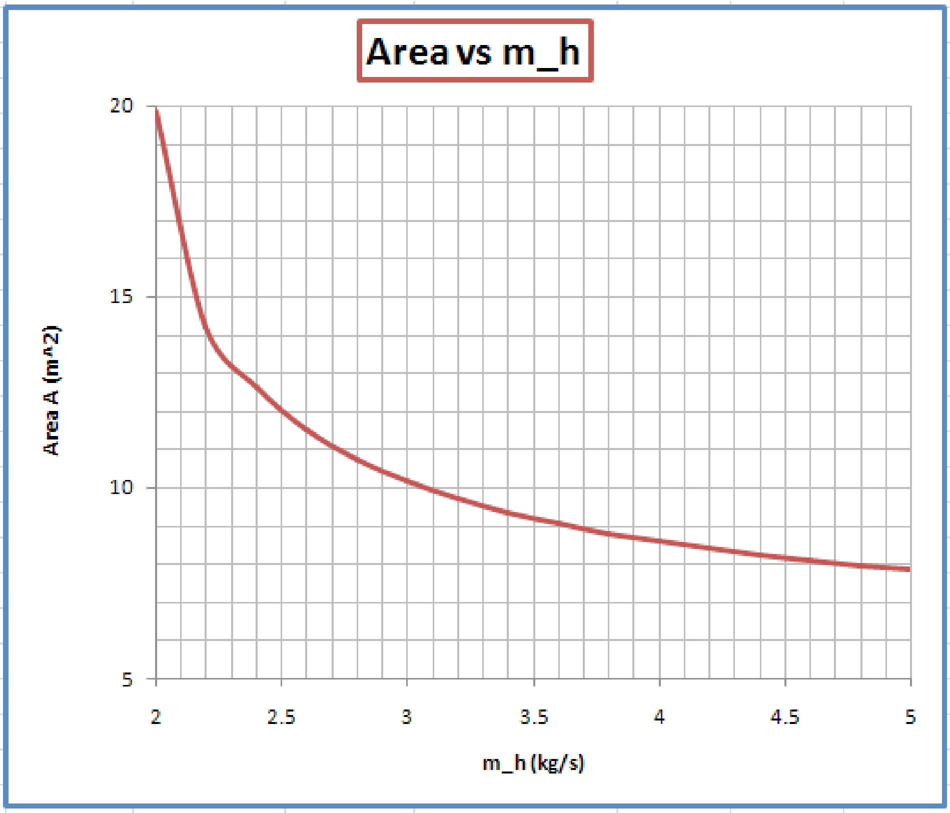


K893 =$D$850/($D$847*G893*J893)

m_h (kg/s)	Th_2 (deg.C)	DELTAT_1 (deg.C)	DELTAT_2 (deg.C)	LMTD (deg.C)	R	P	F	A (m^2)
2	56.0	20	15.989	17.920	1.100	0.667	0.604	19.822
2.2	60.0	20	19.990	19.995	1.000	0.667	0.755	14.210
2.4	63.3	20	23.325	21.620	0.917	0.667	0.785	12.637
2.6	66.1	20	26.146	22.936	0.846	0.667	0.810	11.540
2.8	68.6	20	28.564	24.028	0.786	0.667	0.830	10.746
3	70.7	20	30.660	24.951	0.734	0.667	0.844	10.178
3.2	72.5	20	32.493	25.743	0.688	0.667	0.856	9.725
3.4	74.1	20	34.111	26.431	0.647	0.667	0.867	9.355
3.6	75.5	20	35.550	27.034	0.611	0.667	0.877	9.047
3.8	76.8	20	36.837	27.567	0.579	0.667	0.884	8.796
4	78.0	20	37.995	28.042	0.550	0.667	0.891	8.584
4.2	79.0	20	39.043	28.468	0.524	0.667	0.897	8.401
4.4	80.0	20	39.995	28.852	0.500	0.667	0.902	8.241
4.6	80.9	20	40.865	29.201	0.478	0.667	0.907	8.099
4.8	81.7	20	41.662	29.518	0.458	0.667	0.911	7.973
5	82.4	20	42.396	29.809	0.440	0.667	0.915	7.860

Now, plot the graphs in EXCEL:




Area vs m_h
Area A (m^2)
20
15
10
5
2
2.5
3
3.5
4
4.5
5
m_h (kg/s)

==

4C. Problems on 'NTU – Effectiveness (ε)' method of heat exchanger design:

It should be noted that LMTD method is very convenient to use when all the four 'end temperatures' are known, or can easily be calculated. But, it becomes difficult to use and requires a trial and error solution when only inlet temperatures of the two fluids are known. Then, NTU-ε method is more convenient to use.

Also, when the performance of a given heat exchanger is to be assessed at off-design conditions, analysis is easier if we adopt the NTU – ε method.

Formulas are for ε as a function of NTU and 'Capacity ratio, C' (i.e. $C = C_{min} / C_{max}$) for different types of heat exchangers, and also for NTU as a function of ε and C are given at the beginning of this Chapter.

Prob. 4C.1. Consider a HX for cooling oil at 180 C, with water entering at 25 C. Mass flow rates of oil and water are: 2.5 and 1.2 kg/s. respectively. Area of HX: 16 m^2. Calculate the outlet temperatures of both the fluids for a (i) counter-flow HX, and (ii) for a parallel flow HX. Sp. heat data for oil and water are: 1900 J/kg.C and 4184 J/kg.C respectively. Overall U = 285 W/m^2.C. [M.U. 1995]

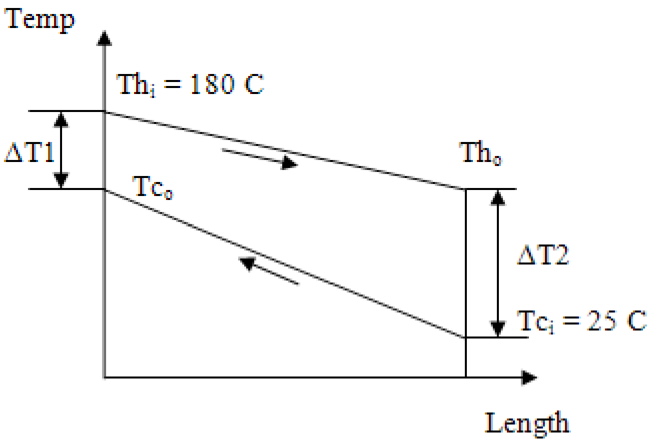


Fig. Prob.4C.1(a). Counter-flow arrangement

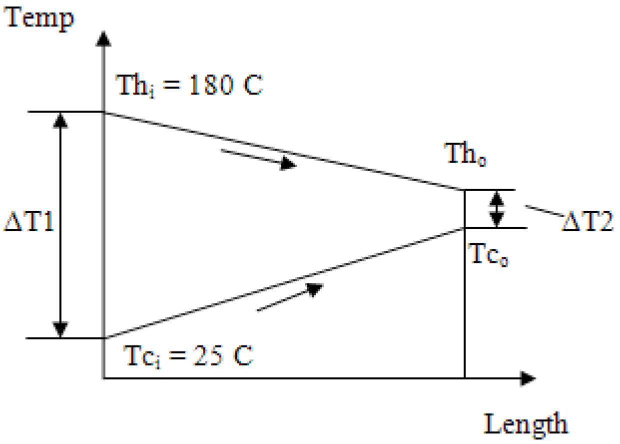


Fig. Prob.4C.1(b). Parallel flow arrangement

Mathcad Solution:

Data:

Hot fluid: Oil:

$cp_{oil} := 1900$ J/kg.C $m_{oil} := 2.5$ kg/s $Th_i := 180$ C

$C_{oil} := m_{oil} \cdot cp_{oil}$ i.e. $C_{oil} = 4.75 \times 10^3$ W/C....Capacity rate of hot fluid

Cold fluid: water:

$cp_{water} := 4184$ J/kg.C $m_{water} := 1.2$ kg/s $Tc_i := 25$ C

$C_{water} := m_{water} \cdot cp_{water}$ i.e. $C_{water} = 5.021 \times 10^3$ W/C....Capacity rate of cold fluid

$U := 285$ W/m^2.C .... overall heat tr coeff. $A := 16$ m^2.... area of HX

Then, To find Capacity ratio, NTU and effectiveness:

$C_{min} := \mathrm{if}\left(C_{oil} < C_{water}, C_{oil}, C_{water}\right)$ i.e. $C_{min} = 4.75 \times 10^3$ W/C .... min. capacity rate

$C_{max} := \mathrm{if}\left(C_{oil} < C_{water}, C_{water}, C_{oil}\right)$ i.e. $C_{max} = 5.021 \times 10^3$ W/C .... max. capacity rate

$C := \dfrac{C_{min}}{C_{max}}$ i.e. $C = 0.946$ **....Capacity ratio**

And: $NTU := \dfrac{U \cdot A}{C_{min}}$ i.e. $NTU = 0.96$ **..No. of Transfer Units**

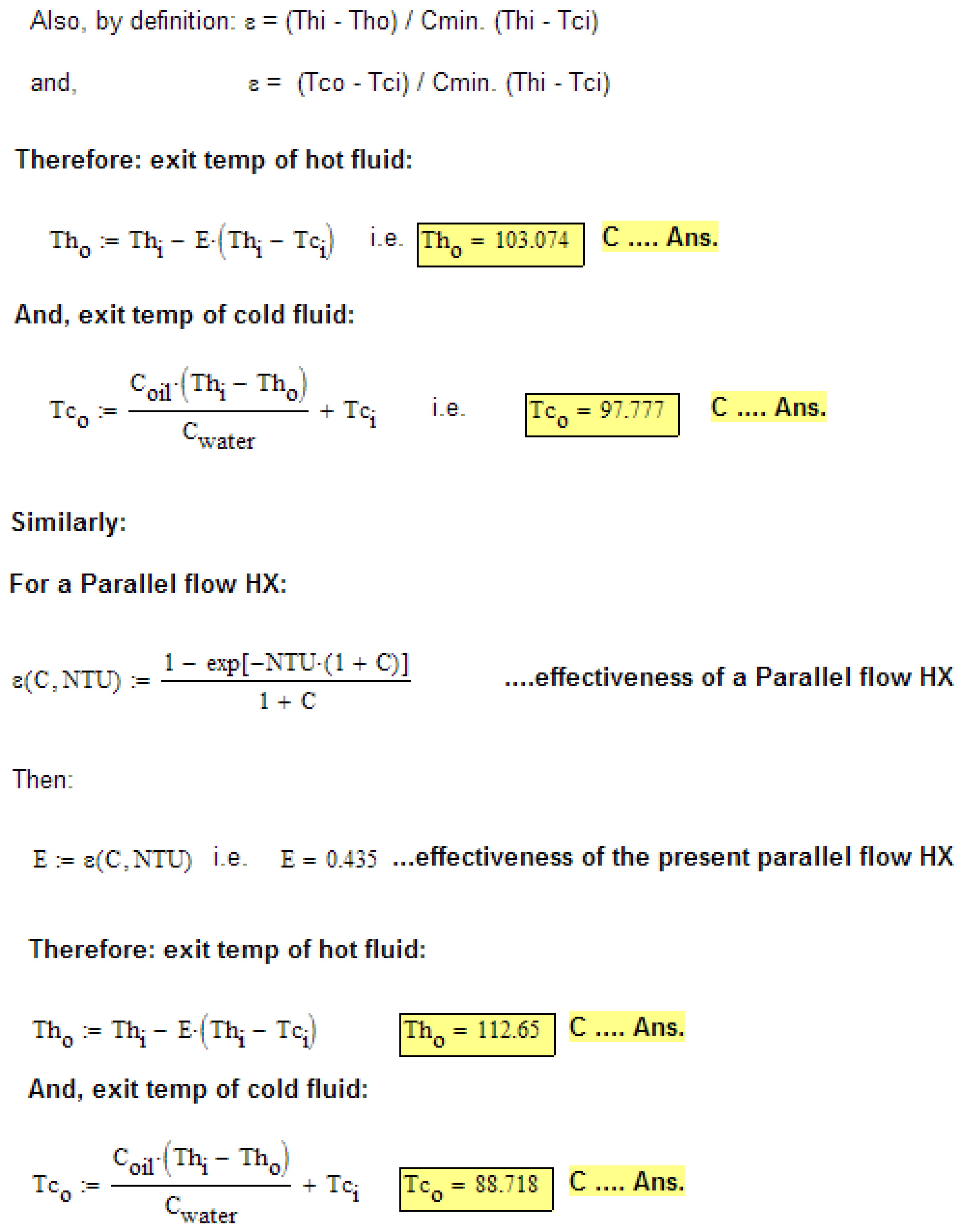
Now, for a Counterflow HX:

We have ε as a function of NTU and C: (see the formulas given at the beginning of this chapter)

$$\varepsilon(C, NTU) := \frac{1 - \exp[-NTU \cdot (1 - C)]}{1 - C \cdot \exp[-NTU \cdot (1 - C)]}$$ **....effectiveness of a Counterflow HX**

Then:

$E := \varepsilon(C, NTU)$ i.e. $E = 0.496$ **....effectiveness of the present counterflow HX**



Also, by definition: ε = (Thi - Tho) / Cmin. (Thi - Tci)

and, ε = (Tco - Tci) / Cmin. (Thi - Tci)

Therefore: exit temp of hot fluid:

$Th_o := Th_i - E \cdot (Th_i - Tc_i)$ i.e. $\boxed{Th_o = 103.074}$ **C .... Ans.**

And, exit temp of cold fluid:

$Tc_o := \frac{C_{oil} \cdot (Th_i - Th_o)}{C_{water}} + Tc_i$ i.e. $\boxed{Tc_o = 97.777}$ **C .... Ans.**

Similarly:

For a Parallel flow HX:

$\varepsilon(C, NTU) := \frac{1 - \exp[-NTU \cdot (1 + C)]}{1 + C}$ **....effectiveness of a Parallel flow HX**

Then:

$E := \varepsilon(C, NTU)$ i.e. $E = 0.435$ **...effectiveness of the present parallel flow HX**

Therefore: exit temp of hot fluid:

$Th_o := Th_i - E \cdot (Th_i - Tc_i)$ $\boxed{Th_o = 112.65}$ **C .... Ans.**

And, exit temp of cold fluid:

$Tc_o := \frac{C_{oil} \cdot (Th_i - Th_o)}{C_{water}} + Tc_i$ $\boxed{Tc_o = 88.718}$ **C .... Ans.**

Let us consider an extension to this problem:

Considering the HX to be of Counter-flow type:

Plot the variation of exit temps of two fluids and the effectiveness of HX against the mass flow rate of water (i.e. cold fluid) as it varies from 1.2 to 5 kg/s, assuming other conditions to remain the same:

First, write the related quantities as function of m_{water}:

$$C_{water}(m_{water}) := m_{water} \cdot cp_{water}$$

Then, To find Capacity ratio, NTU and effectiveness:

$$C_{min}(m_{water}) := \mathrm{if}(C_{oil} < C_{water}(m_{water}), C_{oil}, C_{water}(m_{water}))$$

$$C_{max}(m_{water}) := \mathrm{if}(C_{oil} < C_{water}(m_{water}), C_{water}(m_{water}), C_{oil})$$

$$C(m_{water}) := \frac{C_{min}(m_{water})}{C_{max}(m_{water})}$$

And: $$NTU(m_{water}) := \frac{U \cdot A}{C_{min}(m_{water})}$$

Now, for a Counterflow HX:

$$\varepsilon_{CF}(m_{water}) := \frac{1 - \exp\left[-NTU(m_{water}) \cdot (1 - C(m_{water}))\right]}{1 - C(m_{water}) \cdot \exp\left[-NTU(m_{water}) \cdot (1 - C(m_{water}))\right]}$$

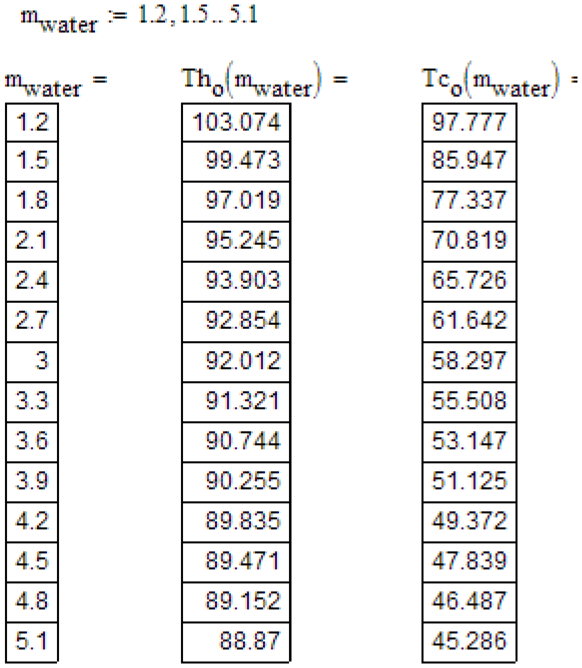
Therefore: exit temp of hot fluid:

$$Th_o(m_{water}) := Th_i - \varepsilon_{CF}(m_{water}) \cdot (Th_i - Tc_i)$$

And, exit temp of cold fluid:

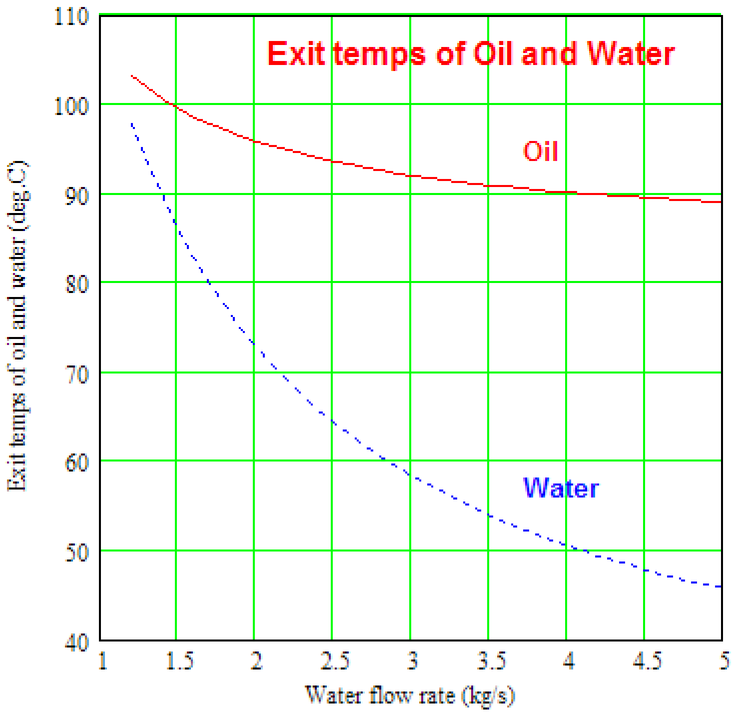
$$Tc_o(m_{water}) := \frac{C_{oil} \cdot (Th_i - Th_o(m_{water}))}{C_{water}(m_{water})} + Tc_i$$

To plot exit temps of fluids as a function of m_{water}:



$m_{water} := 1.2, 1.5 .. 5.1$

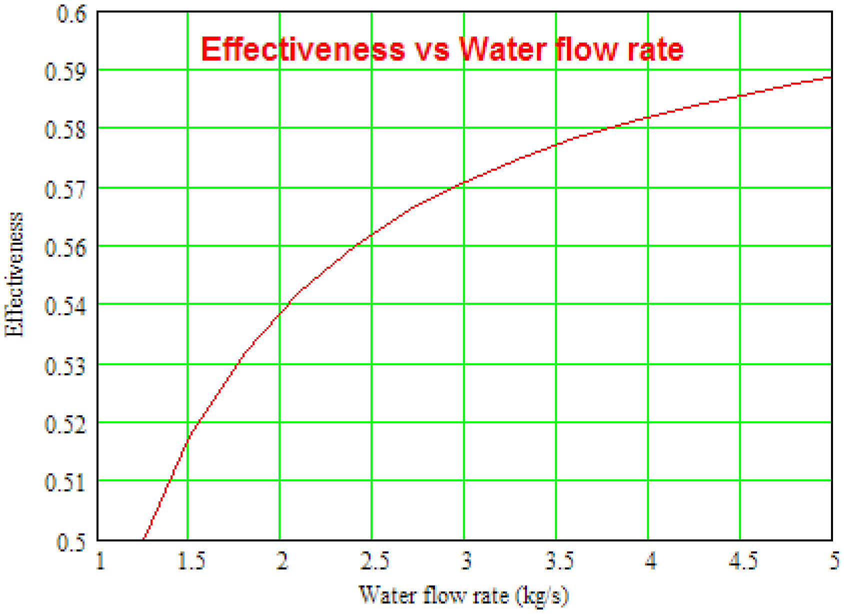
$m_{water} =$	$Th_o(m_{water}) =$	$Tc_o(m_{water}) =$
1.2	103.074	97.777
1.5	99.473	85.947
1.8	97.019	77.337
2.1	95.245	70.819
2.4	93.903	65.726
2.7	92.854	61.642
3	92.012	58.297
3.3	91.321	55.508
3.6	90.744	53.147
3.9	90.255	51.125
4.2	89.835	49.372
4.5	89.471	47.839
4.8	89.152	46.487
5.1	88.87	45.286



To plot Effectiveness as a function of m_{water}:

$m_{water} := 1.2, 1.5 .. 5.1$ ...define a range variable

$m_{water} =$	$\varepsilon_{CF}(m_{water})$
1.2	0.496
1.5	0.52
1.8	0.535
2.1	0.547
2.4	0.555
2.7	0.562
3	0.568
3.3	0.572
3.6	0.576
3.9	0.579
4.2	0.582
4.5	0.584
4.8	0.586
5.1	0.588


Effectiveness vs Water flow rate
Effectiveness
0.6
0.59
0.58
0.57
0.56
0.54
0.53
0.52
0.51
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Water flow rate (kg/s)

To demonstrate the ease with which we can draw Effectiveness – NTU graphs with Mathcad:

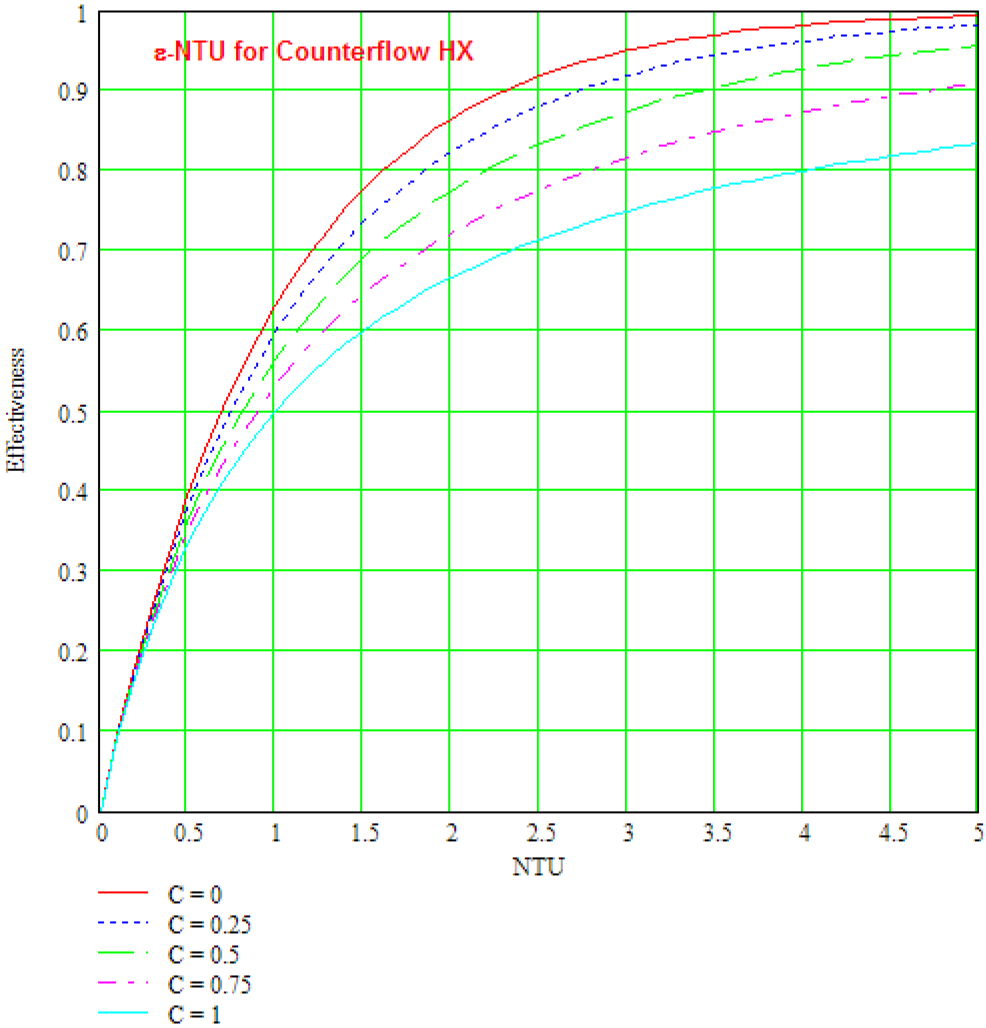
Draw Effectiveness vs NTU graph for Counter-flow HX:

We write a Mathcad Function for effectiveness of a Counter-flow HX:

$$\text{Epsilon\_CounterFlowHX}(NTU, C) := \left| \begin{array}{l} (\text{return } \text{"C must be less than or equal to 1 !!"}) \ \text{ if } \ C > 1 \\ (1 - \exp(-NTU)) \ \text{ if } \ C = 0 \\ \dfrac{NTU}{1 + NTU} \ \text{ if } \ C = 1 \\ \dfrac{1 - \exp[-NTU \cdot (1 - C)]}{1 - C \cdot \exp[-NTU \cdot (1 - C)]} \ \text{ if } \ C < 1 \end{array} \right.$$

And, now draw the graphs:

$NTU := 0, 0.1 .. 5$ ....define a range variable



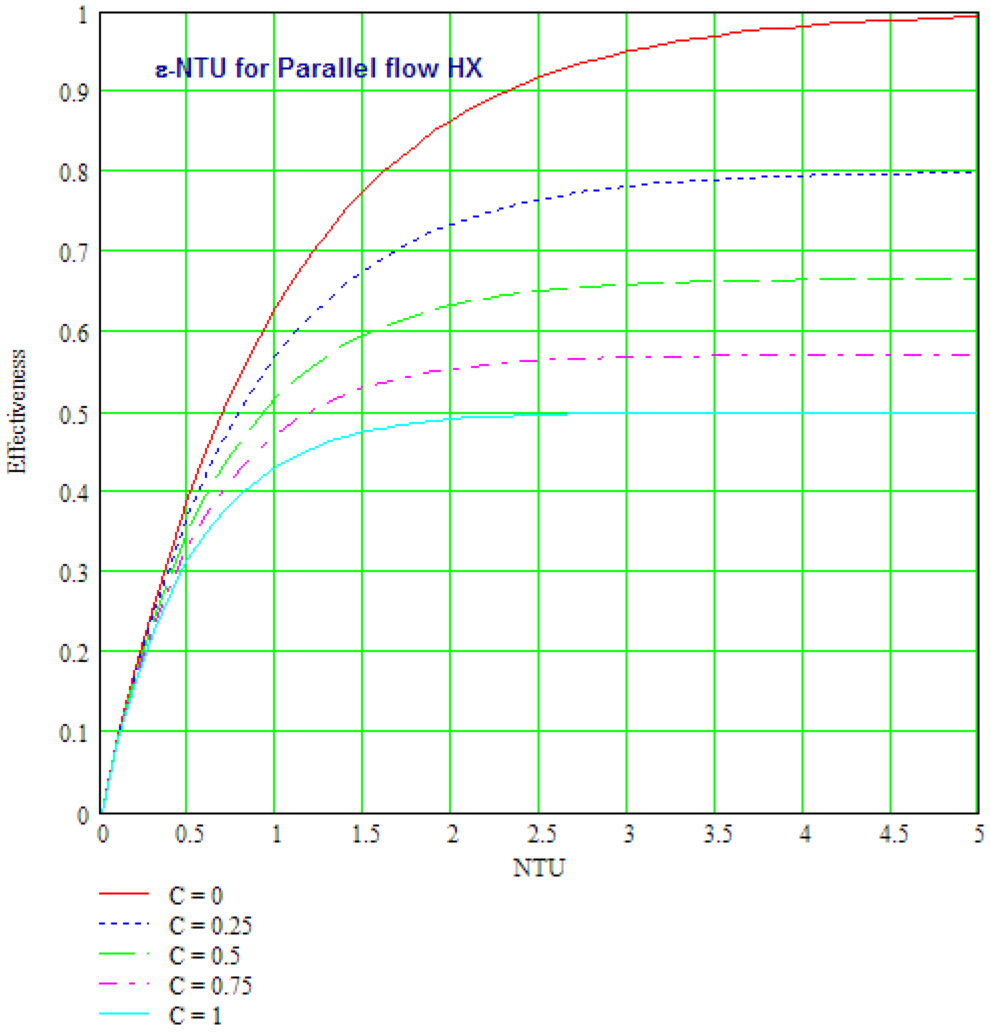
Also, draw Effectiveness vs NTU graph for Parallel flow HX:

We write a Mathcad Function for effectiveness of a Parallel flow HX:

$$\text{Epsilon\_ParallelFlowHX}(NTU, C) := \begin{array}{|l} (\text{return "C must be less than or equal to 1 !!"}) \quad \text{if } C > 1 \\ (1 - \exp(-NTU)) \quad \text{if } C = 0 \\ \dfrac{1 - \exp[-NTU \cdot (1 + C)]}{1 + C} \quad \text{otherwise} \end{array}$$

Now, draw the graphs:

$NTU := 0, 0.1 .. 5$ ....define a range variable



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Prob.4C.2. A One shell, 2 tube pass steam condenser has 2000 tubes of 20 mm dia. Cooling water enters the tubes at 20 C, with a flow rate of 3000 kg/s; Overall heat transfer coeff. U = 6890 W/m^2.K. Total heat to be transferred, Q=2.331*10^8 W. Steam condenses at 50 C. Determine tube length per pass using NTU method. [M.U. 1994]

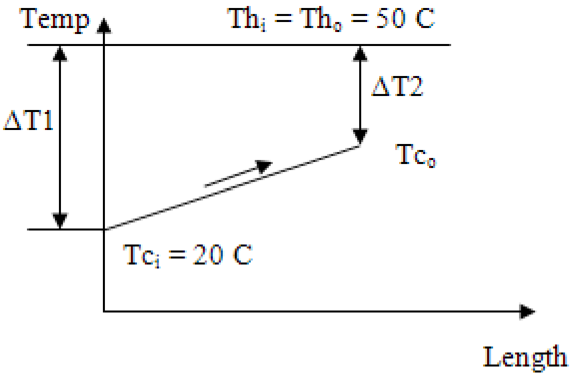


Fig. Prob.4C.2. Steam Condenser

Mathcad Solution:

Data:

Note that this is a steam condenser. So, T_h is constant at condensing temp of 50 C.

Point to be noted is that for condenser, condensing steam is the 'max. fluid' and water is the 'min. fluid' and Capacity ratio C = (Cmin / Cmax) = 0.

$Th_i := 50$ C $\quad Th_o := 50$ C $\qquad Q := 2.331 \cdot 10^8$ W... heat transferred

$Tc_i := 20$ C...water inlet temp $\qquad m := 3000$ kg/s...water flow rate

$D := 0.02$ m...dia of tubes $\qquad N := 2000$ ...no. of tubes $\qquad U := 6890$ W/m^2.C

$cp := 4170$ J/kg.C....sp. heat of water

Calculations:

$C_{min} := m \cdot cp$ W/C...water capacity rate $\qquad C_{max} := \infty$

i.e. $C_{min} = 1.251 \times 10^7$ W/C

$\Delta T1 := Th_i - Tc_i$ $\qquad$ i.e. $\Delta T1 = 30$ C

For exit temp of water:

$\Delta T := \frac{Q}{C_{min}}$ $\Delta T = 18.633$ C.... temp increase of water flow

Therefore: $Tc_o := Tc_i + \Delta T$

i.e. $Tc_o = 38.633$ **C...exit temp of water**

Efectiveness: = temp increase of water / max. temp differential

$$\varepsilon := \frac{\Delta T}{Th_i - Tc_i}$$

i.e. $\varepsilon = 0.621$

To find NTU:

For a condenser, i.e. when C = 0:

$NTU := -\ln(1 - \varepsilon)$ ....see the formulas at the beginning of this chapter.

i.e. $NTU = 0.9705$ **....No. of Transfer Units**

Therefore, total area of HX:

$A := \frac{NTU}{U} \cdot C_{min}$ ...from definition of NTU

i.e. $A = 1.762 \times 10^3$ **..Total area reqd., m^2..**

Area per tube, per metre length: $A_{tube} := \pi \cdot D \cdot 1$ i.e. $A_{tube} = 0.063$ m^2

Therefore:

Length of tube for 2 tube passes:

$$L := \frac{A}{A_{tube} \cdot N \cdot 2}$$

i.e. $\boxed{L = 7.011}$ **m...per pass...Ans.**

Plot variation of Tc_o, ε, and L against mass flow rate of water, other conditions remaining the same:

Write the relevant quantities as functions of mass flow rate of water, m:

$C_{min}(m) := m \cdot cp$ W/C...water capacity rate as a function of m $C_{max} := \infty$

$\Delta T1 := Th_i - Tc_i$

For exit temp of water:

$$\Delta T(m) := \frac{Q}{C_{min}(m)}$$ C.... temp increase of water flow, as a function of m

Therefore: $Tc_o(m) := Tc_i + \Delta T(m)$ **C...exit temp of water**

Efectiveness: = temp increase of water / max. temp differential

$$\varepsilon(m) := \frac{\Delta T(m)}{Th_i - Tc_i}$$ Effectiveness, as a function of m

To find NTU:

For a condenser, i.e. when C = 0:

$NTU(m) := -\ln(1 - \varepsilon(m))$ ....NTU as a function of m.

Therefore, area of HX:

$A(m) := \frac{NTU(m)}{U} \cdot C_{min}(m)$ A as function of m...from definition of NTU

Area per tube, per metre length: $A_{tube} := \pi \cdot D \cdot 1$ i.e. $A_{tube} = 0.063$ m^2

Therefore:

Length of tube for 2 tube passes:

$L(m) := \frac{A(m)}{A_{tube} \cdot N \cdot 2}$ ....L per pass, as a function of m

---

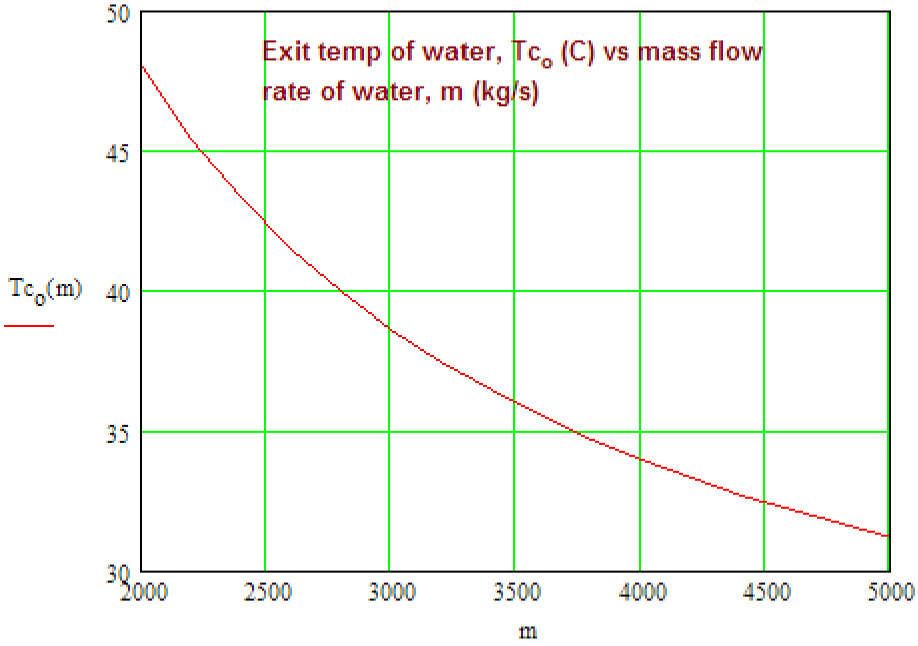
Now, we prepare a Table to show the variation of different parameters as m varies, and then plot the results:

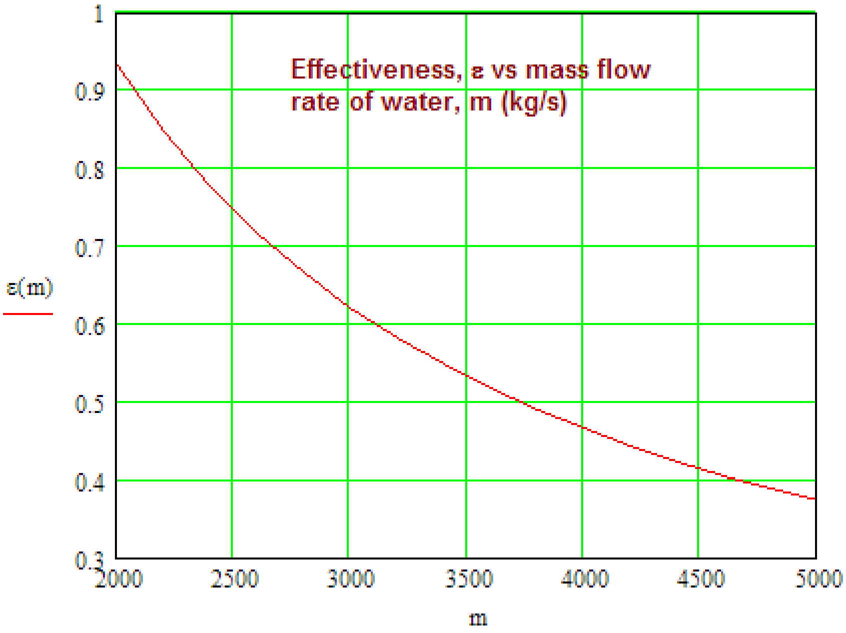
We get:

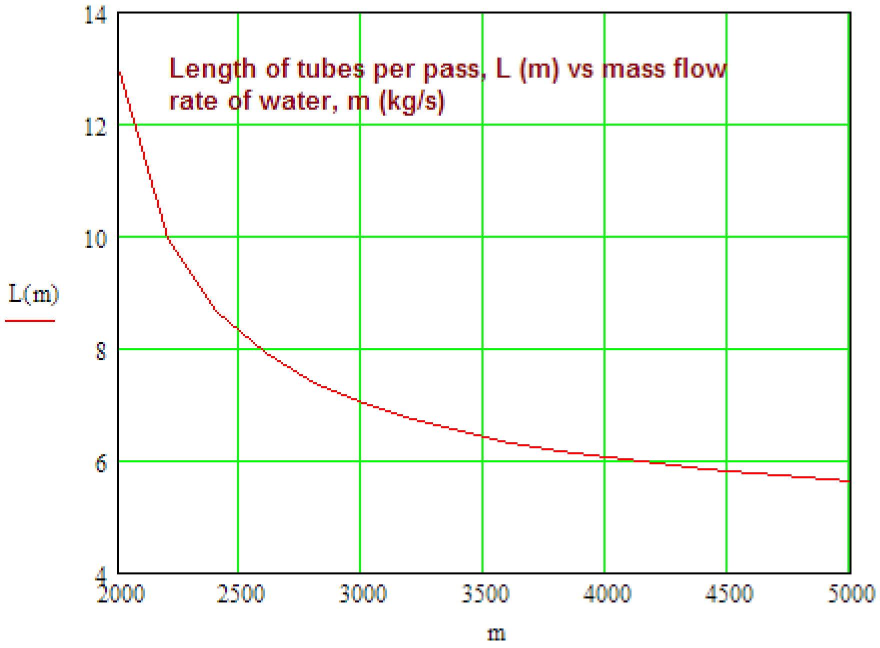
$m := 2000, 2200 .. 5000$ ....define a range variable for m

$m =$	$Tc_o(m) =$	$NTU(m) =$	$\varepsilon(m) =$	$A(m) =$	$L(m) =$
$2 \cdot 10^3$	47.95	2.683	0.932	$3.248 \cdot 10^3$	12.923
$2.2 \cdot 10^3$	45.409	1.877	0.847	$2.499 \cdot 10^3$	9.944
$2.4 \cdot 10^3$	43.291	1.498	0.776	$2.176 \cdot 10^3$	8.657
$2.6 \cdot 10^3$	41.5	1.261	0.717	$1.984 \cdot 10^3$	7.896
$2.8 \cdot 10^3$	39.964	1.095	0.665	$1.856 \cdot 10^3$	7.383
$3 \cdot 10^3$	38.633	0.97	0.621	$1.762 \cdot 10^3$	7.011
$3.2 \cdot 10^3$	37.469	0.873	0.582	$1.691 \cdot 10^3$	6.727
$3.4 \cdot 10^3$	36.441	0.794	0.548	$1.634 \cdot 10^3$	6.502
$3.6 \cdot 10^3$	35.528	0.729	0.518	$1.588 \cdot 10^3$	6.319
$3.8 \cdot 10^3$	34.71	0.674	0.49	$1.55 \cdot 10^3$	6.168
$4 \cdot 10^3$	33.975	0.627	0.466	$1.518 \cdot 10^3$	6.04
$4.2 \cdot 10^3$	33.309	0.586	0.444	$1.49 \cdot 10^3$	5.93
$4.4 \cdot 10^3$	32.704	0.551	0.423	$1.467 \cdot 10^3$	5.836
$4.6 \cdot 10^3$	32.152	0.519	0.405	$1.446 \cdot 10^3$	5.753
$4.8 \cdot 10^3$	31.646	0.491	0.388	$1.427 \cdot 10^3$	5.679
$5 \cdot 10^3$	31.18	0.466	0.373	$1.411 \cdot 10^3$	5.614

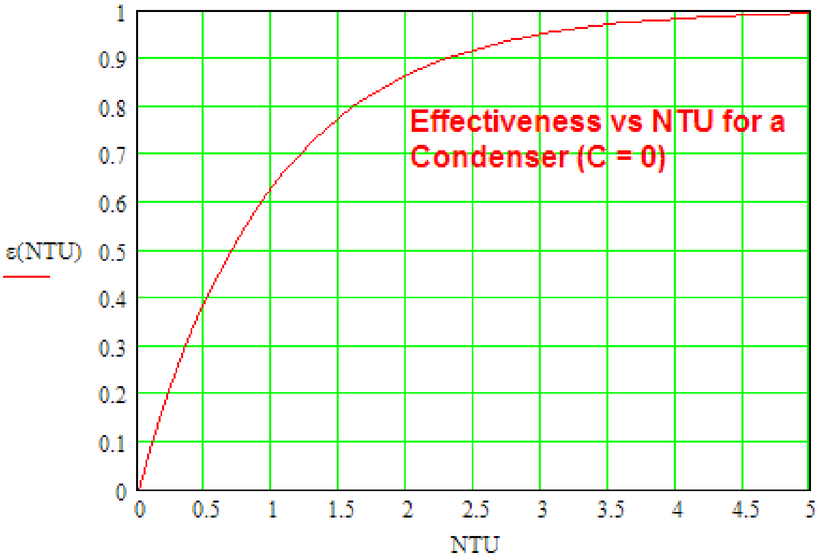
And, plot the results:






Length of tubes per pass, L (m) vs mass flow rate of water, m (kg/s)
L(m)
14
12
10
8
6
4
2000
2500
3000
3500
4000
4500
5000
m

In addition, plot Effectiveness vs NTU for a Condenser (C = 0):



For a Condenser, we have:

$\varepsilon(NTU) := (1 - \exp(-NTU))$ **....effectiveness of a condenser as a function of NTU**

$NTU := 0, 0.1 .. 5$ ....define a range variable for NTU

==

Prob.4C.3. A steam condenser, condensing at 70 C is to have a capacity of 100 kW. Water at 20 C is used and the outlet water temp is limited to 45 C. If the overall heat transfer coeff. is 3100 W/m^2.K, determine the area required.

(b) If the inlet water temp is increased to 30 C, determine the increased flow rate of water to maintain the same outlet temp. [M.U. Dec. 1998]

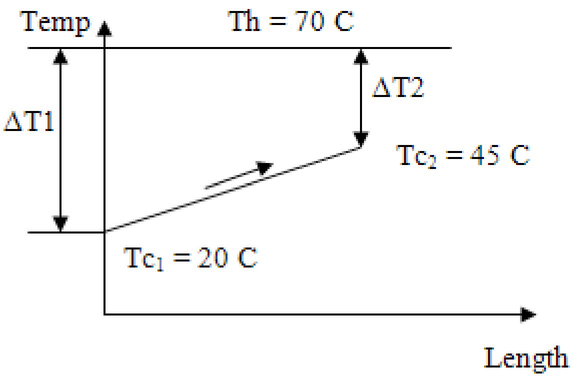


Fig. Prob.4B.11.

Mathcad Solution:

Data:

$T_h := 70$ C...condensing temp

$T_{c1} := 20$ C...water inlet temp $T_{c2} := 45$ C...water exit temp

$Q := 100 \cdot 10^3$ W... total heat transfer

$U := 3100$ W/m^2.K....overall heat tr coeff.

Calculations:

Note that this is a condenser. Therefore, condensing steam is the 'max. fluid' and water is the 'min. fluid', and Capacity ratio, C = 0.

Then, by definition, effectiveness is: temp rise of min. fluid (i.e. water) divided by the total temp differential ,(Th - Tc1)

i.e. $\varepsilon := \dfrac{T_{c2} - T_{c1}}{T_h - T_{c1}}$

i.e. $\varepsilon = 0.5$ Effectiveness

Also, for a condenser:

$NTU := -\ln(1 - \varepsilon)$

i.e. $NTU = 0.693$

And, $C_{min} := \dfrac{Q}{\varepsilon \cdot (T_h - T_{c1})}$ ...min. capacity rate

i.e. $C_{min} = 4 \times 10^3$ W/K

And, $A := \dfrac{NTU \cdot C_{min}}{U}$

i.e. $\boxed{A = 0.894}$ **m^2,...Area of HX....Ans.**

Case 2:

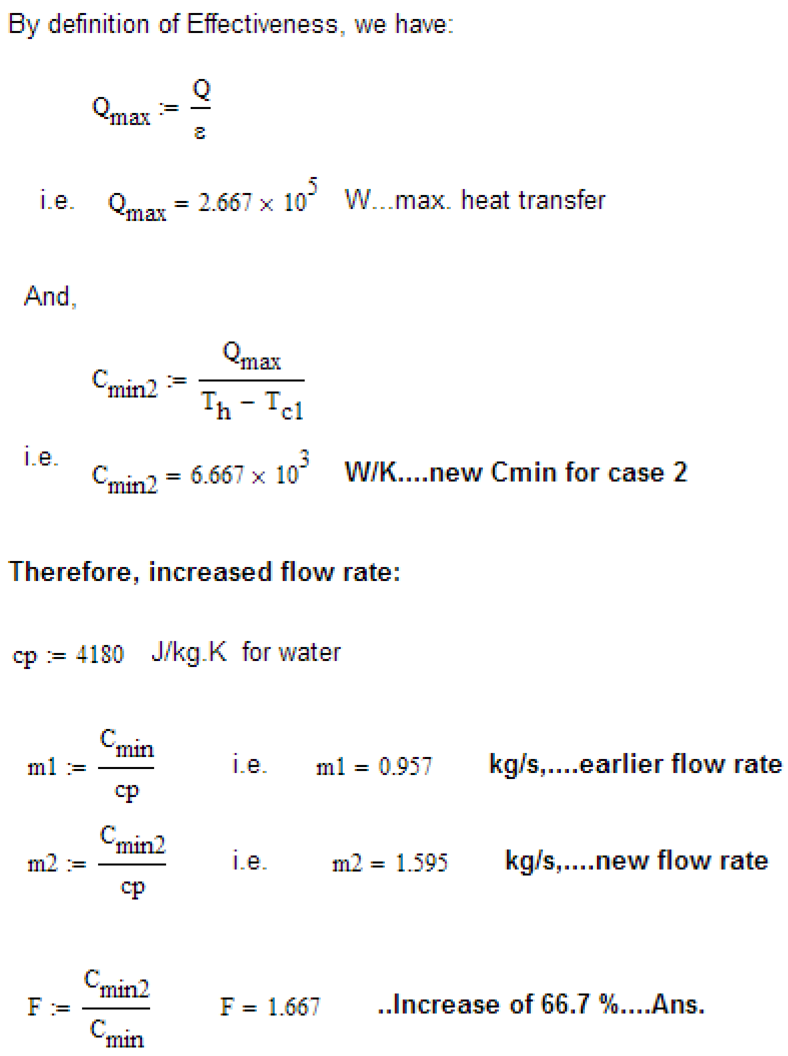
If Tc1 is increased to 30 C, and Tc2 maintained at 45 C, what is the increased flow rate?

$T_{c1} := 30$ C $\qquad T_{c2} := 45$ C

Then:

$$\varepsilon := \frac{T_{c2} - T_{c1}}{T_h - T_{c1}}$$

i.e. $\varepsilon = 0.375$ **...New effectiveness**



By definition of Effectiveness, we have:

$$Q_{max} := \frac{Q}{\varepsilon}$$

i.e. $Q_{max} = 2.667 \times 10^{5}$ W...max. heat transfer

And,

$$C_{min2} := \frac{Q_{max}}{T_h - T_{c1}}$$

i.e. $C_{min2} = 6.667 \times 10^{3}$ **W/K....new Cmin for case 2**

Therefore, increased flow rate:

$cp := 4180$ J/kg.K for water

$m1 := \frac{C_{min}}{cp}$ i.e. $m1 = 0.957$ **kg/s,....earlier flow rate**

$m2 := \frac{C_{min2}}{cp}$ i.e. $m2 = 1.595$ **kg/s,....new flow rate**

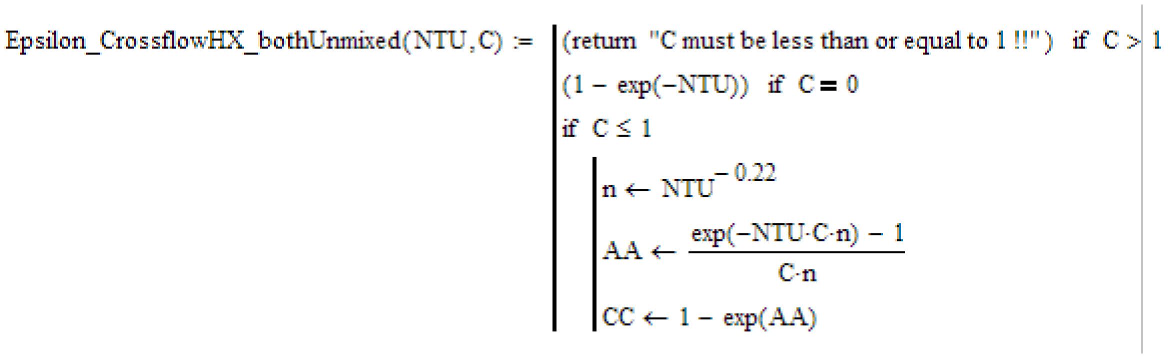
$F := \frac{C_{min2}}{C_{min}}$ $F = 1.667$ **..Increase of 66.7 %....Ans.**

==

Cross flow HX with both fluids unmixed:

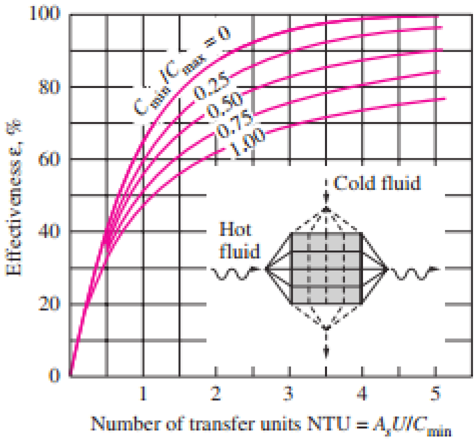
Mathcad Function for Effectiveness of a Cross-flow HX with *both fluids <u>unmixed</u>*:

Formula for effectiveness is given at the beginning of this chapter.



```
Epsilon_CrossflowHX_bothUnmixed(NTU,C) := | (return "C must be less than or equal to 1 !!") if C > 1
                                          | (1 - exp(-NTU)) if C = 0
                                          | if C ≤ 1
                                          |    | n ← NTU^(-0.22)
                                          |    | AA ← (exp(-NTU·C·n) - 1)/(C·n)
                                          |    | CC ← 1 - exp(AA)
```

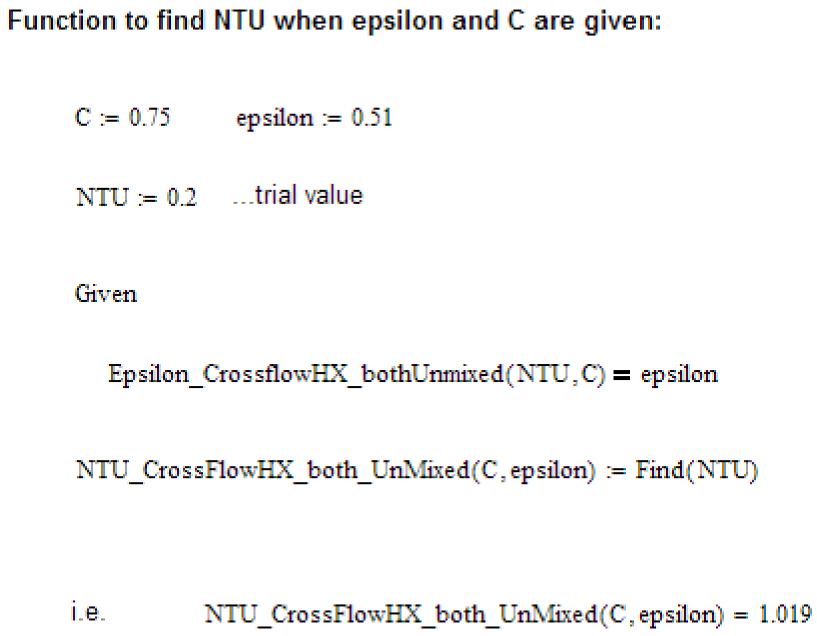
Following is the NTU-e graph for this HX: (Ref: Cengel)



(*e*) Cross-flow with both fluids unmixed

Mathcad Function for NTU of a Cross-flow HX with *both fluids* _*unmixed*_:

Since no explicit equation is available for NTU of a Cross flow HX when C and ε are known, let us write a Function for NTU using the Solve block of Mathcad:



Function to find NTU when epsilon and C are given:

C := 0.75 epsilon := 0.51

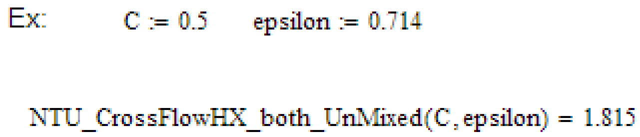
NTU := 0.2 ...trial value

Given

Epsilon_CrossflowHX_bothUnmixed(NTU, C) = epsilon

NTU_CrossFlowHX_both_UnMixed(C, epsilon) := Find(NTU)

i.e. NTU_CrossFlowHX_both_UnMixed(C, epsilon) = 1.019



Ex: C := 0.5 epsilon := 0.714

NTU_CrossFlowHX_both_UnMixed(C, epsilon) = 1.815

Now, let us solve a problem to demonstrate the use of these Mathcad Functions:

Prob.4C.4. An automobile radiator may be considered as a HX with both fluids unmixed. Water, with a flow rate of 0.5 kg/s, enters the radiator at 400 K and leaves at 330 K. Water is cooled by air which enters at 300 K at a rate of 0.75 kg/s. If the overall heat transfer coeff. is 200 W/m^2.K, what is the required heat transfer area? [Ref: 3]

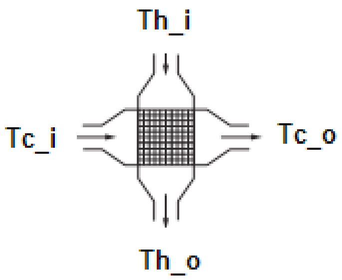


Fig. Prob.4C.4.

Mathcad Solution:

Data:

Hot fluid: Water:

$cp_h := 4209$ J/kg.C... at 365 K $m_h := 0.05$ kg/s $Th_i := 400$ K $Th_o := 330$ K

$C_h := m_h \cdot cp_h$ i.e. $C_h = 210.45$ W/K....Capacity rate of hot fluid, i.e. Water

Cold fluid: Air:

$cp_c := 1010$ J/kg.C... at 365 K $m_c := 0.75$ kg/s $Tc_i := 300$ K

$C_c := m_c \cdot cp_c$ i.e. $C_c = 757.5$ W/K....Capacity rate of cold fluid, i.e. Air

$U := 200$ W/m^2.C .... overall heat tr coeff.

Calculations:

$Q := m_h \cdot cp_h \cdot (Th_i - Th_o)$ i.e. $Q = 1.473 \times 10^4$ **W.... total heat transfer**

We also have: $Q = m_c \cdot cp_c \cdot (Tc_o - Tc_i)$ ....for cold fluid

Therefore, exit temp of cold fluid, i.e. air:

$Tc_o := Tc_i + \dfrac{Q}{m_c \cdot cp_c}$ i.e. $Tc_o = 319.448$ K

Then, To find Capacity ratio and effectiveness:

$C_{min} := \text{if}(C_h < C_c, C_h, C_c)$ i.e. $C_{min} = 210.45$ W/C .... min. capacity rate

$C_{max} := \text{if}(C_h < C_c, C_c, C_h)$ i.e. $C_{max} = 757.5$ W/C .... max. capacity rate

$C := \dfrac{C_{min}}{C_{max}}$ i.e. $C = 0.278$ **....Capacity ratio**

And, since Water is the 'min. fluid', we have, for effectiveness:

$$\varepsilon := \frac{Th_i - Th_o}{Th_i - Tc_i}$$

i.e. $\varepsilon = 0.7$ **...effectiveness of HX**

To find NTU:

Use the Mathcad Function written above:

We have: $\text{epsilon} := 0.7$ $C := 0.278$

And: $NTU := NTU\_CrossFlowHX\_both\_UnMixed(C, \text{epsilon})$

i.e. $NTU = 1.44$ **..No. of Transfer Units**

Then, by definition of NTU, we have:

$$NTU = \frac{U \cdot A}{C_{min}}$$

Therefore: $A := \frac{NTU \cdot C_{min}}{U}$ m^2.... area of HX

i.e. $\boxed{A = 1.515}$ **m^2.... area of HX....Ans.**

Plot the Air and water outlet temps against U:

Assume that U varies from 200 to 400 W.m^2.K, and all other parameters including the area of HX remaining the same:

i.e. Mass flow rates remain the same; so, Capacity ratio C remains the same.

But, NTU changes since U changes. As NTU changes, ε will change; correspondingly, calculate the exit temps.

Let us write the required parameters as functions of U:

$$NTU(U) := \frac{U \cdot A}{C_{min}}$$

$$Epsilon(U) := Epsilon\_CrossflowHX\_bothUnmixed(NTU(U), C)$$

But, we have, remembering that water is the **min. fluid**:

$$\varepsilon = \frac{(Th_i - Th_o)}{(Th_i - Tc_i)}$$ **..effectiveness**

Therefore, exit temp of water as a function of U is given by:

$$Th_o(U) := Th_i - Epsilon(U) \cdot (Th_i - Tc_i)$$

And, heat transferred, Q:

$$Q(U) := C_h \cdot (Th_i - Th_o(U))$$

And, by heat balance, exit temp of air is given by:

$$Q = m_c \cdot cp_c \cdot (Tc_o - Tc_i)$$ ....for cold fluid

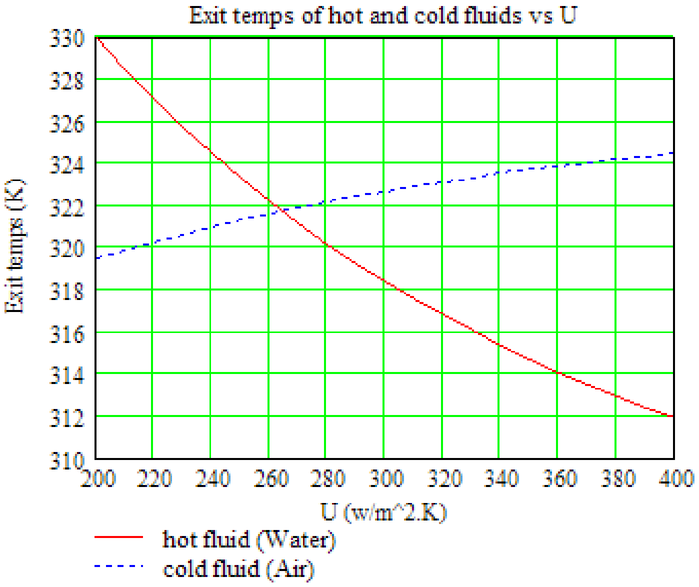
i.e. $$Tc_o(U) := Tc_i + \frac{Q(U)}{C_c}$$

To plot the graphs:

$U := 200, 210 .. 400$ ....define a range variable for U

U =	Epsilon(U) =	NTU(U) =	$Th_o(U)$ =	$Tc_o(U)$ =
200	0.7	1.44	330	319.448
210	0.715	1.512	328.487	319.868
220	0.729	1.584	327.068	320.262
230	0.743	1.656	325.736	320.632
240	0.755	1.728	324.483	320.98
250	0.767	1.8	323.306	321.307
260	0.778	1.872	322.198	321.615
270	0.788	1.944	321.154	321.905
280	0.798	2.016	320.171	322.178
290	0.808	2.088	319.244	322.436
300	0.816	2.16	318.369	322.679
310	0.825	2.232	317.544	322.908
320	0.832	2.304	316.764	323.125
330	0.84	2.376	316.026	323.33
340	0.847	2.448	315.329	323.524
350	0.853	2.52	314.669	323.707
360	0.86	2.592	314.044	323.881
370	0.865	2.664	313.452	324.045
380	0.871	2.736	312.891	324.201
390	0.876	2.808	312.358	324.349
400	0.881	2.88	311.853	324.489

And, the plot:



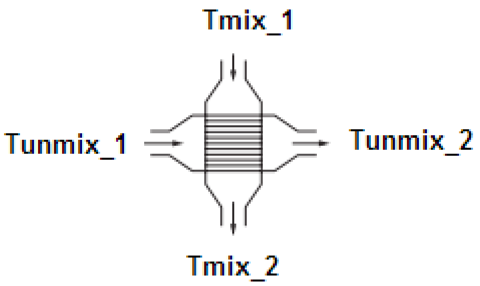
==

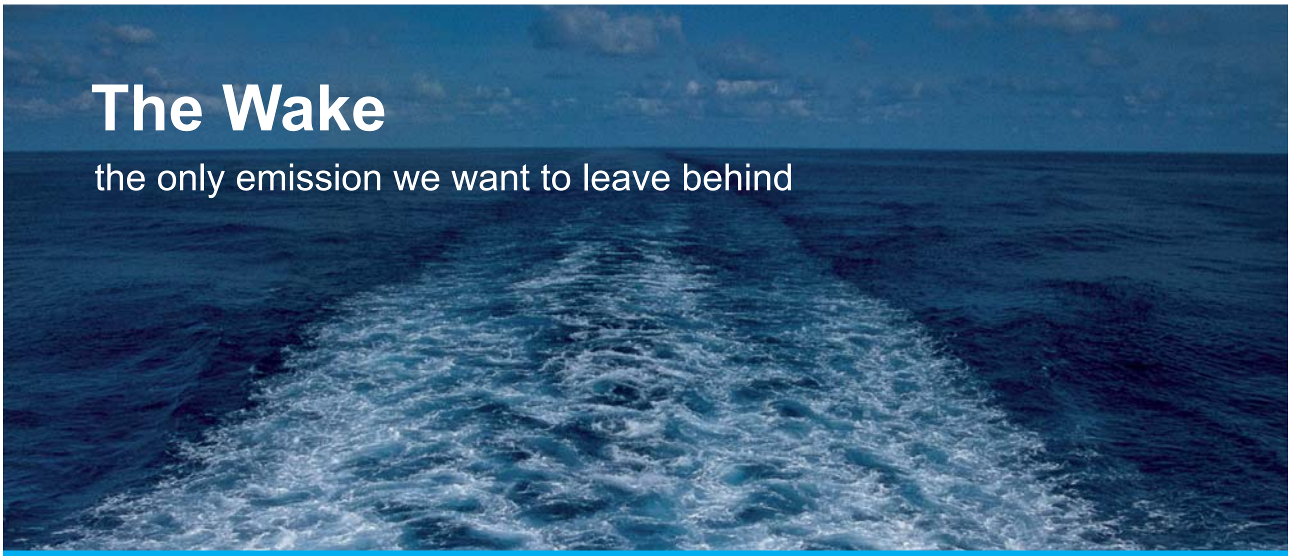
Prob. 4C.5. Consider a cross flow HX where oil flowing through the tubes is heated by steam flowing across the tubes. Oil (cp = 1900 J/kg.C) is heated from 15 C to 85 C and steam (cp = 1860 J/kg.C) enters at 130 C and leaves at 110 C with a mass flow rate of 5.2 kg/s. Overall heat transfer coeff U = 275 W/m^2.C. Calculate the surface area required for this HX.

Note: This problem is the same as Prob. 4B.18, which was solved by LMTD method.

Now, we shall solve it by NTU-ε method:

Steam is the 'mixed' fluid and oil is the 'unmixed' fluid.

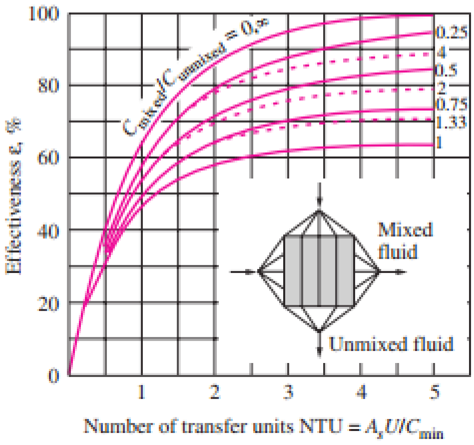




Mathcad Solution:

This is a cross-flow HX, with one fluid mixed and the other unmixed.

For such a HX, we have the graph: (Ref: Cengel)

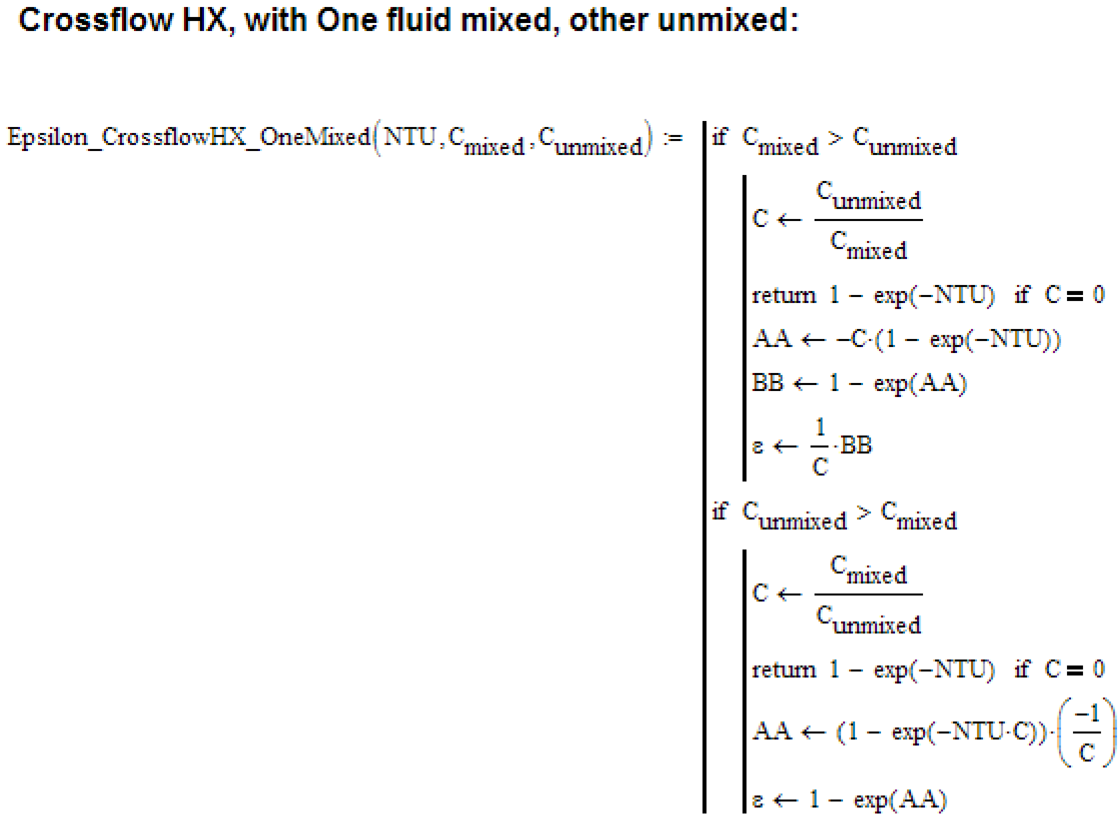


(*f*) Cross-flow with one fluid mixed and the other unmixed

First, let us write Mathcad Functions for Effectiveness and NTU.

Relevant equations are given at the beginning of this Chapter.

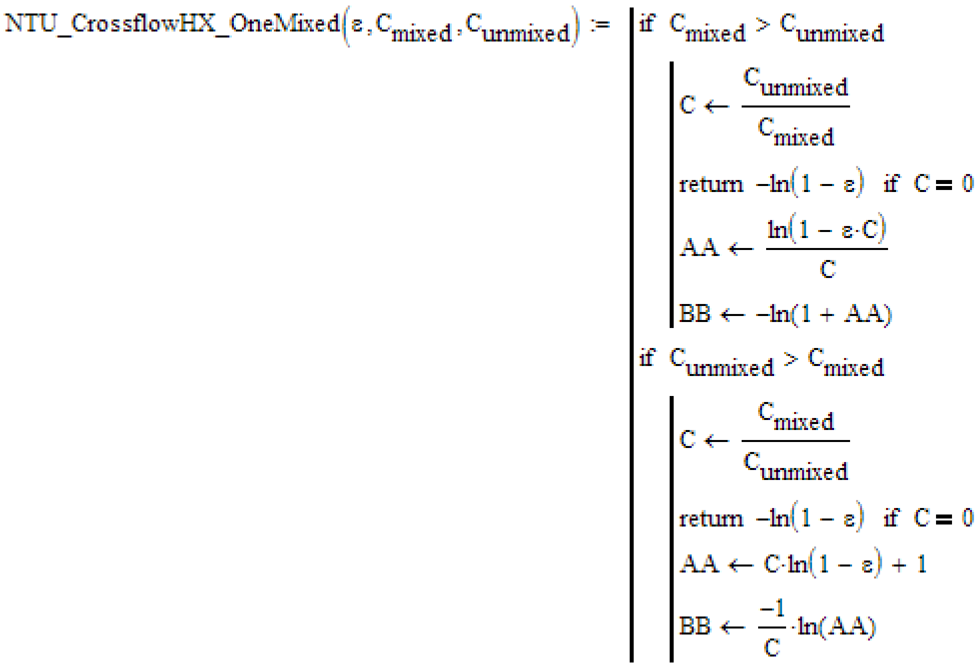
For Effectiveness:



Crossflow HX, with One fluid mixed, other unmixed:

```
Epsilon_CrossflowHX_OneMixed(NTU, C_mixed, C_unmixed) := | if C_mixed > C_unmixed
                                                          |    | C ← C_unmixed / C_mixed
                                                          |    | return 1 − exp(−NTU)  if C = 0
                                                          |    | AA ← −C·(1 − exp(−NTU))
                                                          |    | BB ← 1 − exp(AA)
                                                          |    | ε ← (1/C)·BB
                                                          | if C_unmixed > C_mixed
                                                          |    | C ← C_mixed / C_unmixed
                                                          |    | return 1 − exp(−NTU)  if C = 0
                                                          |    | AA ← (1 − exp(−NTU·C))·(−1/C)
                                                          |    | ε ← 1 − exp(AA)
```

For NTU:



```
$NTU\_CrossflowHX\_OneMixed(\varepsilon, C_{mixed}, C_{unmixed}) :=$
    if $C_{mixed} > C_{unmixed}$
        $C \leftarrow \frac{C_{unmixed}}{C_{mixed}}$
        return $-\ln(1-\varepsilon)$  if $C = 0$
        $AA \leftarrow \frac{\ln(1-\varepsilon \cdot C)}{C}$
        $BB \leftarrow -\ln(1+AA)$
    if $C_{unmixed} > C_{mixed}$
        $C \leftarrow \frac{C_{mixed}}{C_{unmixed}}$
        return $-\ln(1-\varepsilon)$  if $C = 0$
        $AA \leftarrow C \cdot \ln(1-\varepsilon) + 1$
        $BB \leftarrow \frac{-1}{C} \cdot \ln(AA)$
```

Now, let us solve the above problem:

Data:

Steam is 'mixed' fluid and oil is 'un-mixed' fluid.

$Tmix_1 := 130$ C $Tmix_2 := 110$ C $Tunmix_1 := 15$ C $Tunmix_2 := 85$ C

$cp_{oil} := 1900$ J/kg.C $cp_{steam} := 1860$ J/kg.C $m_{steam} := 5.2$ kg/s

$C_{mix} := m_{steam} \cdot cp_{steam}$

i.e. $C_{mix} = 9.672 \times 10^3$ W/K....Capacity rate of mixed fluid (steam)

$U := 275$ W/m^2.C

Calculations:

Capacity rate of un-mixed fluid (steam):

$Q := C_{mix} \cdot (Tmix_1 - Tmix_2)$

i.e. $Q = 1.934 \times 10^5$ **W ... total heat transferred**

Then,

$$C_{unmix} := \frac{Q}{Tunmix_2 - Tunmix_1}$$

i.e. $C_{unmix} = 2.763 \times 10^3$ W/K....Capacity rate of un-mixed fluid (steam)

$C_{min} := \text{if}(C_{unmix} < C_{mix}, C_{unmix}, C_{mix})$ ....find Cmin

i.e. $C_{min} = 2.763 \times 10^3$ W/C ... min. capacity rate

$C_{max} := \text{if}(C_{unmix} < C_{mix}, C_{mix}, C_{unmix})$ ....find Cmax

i.e. $C_{max} = 9.672 \times 10^3$ W/C ... max. capacity rate

To find effectiveness:

Remembering that oil is the 'min. fluid:

$$\varepsilon := \frac{Tunmix_2 - Tunmix_1}{Tmix_1 - Tunmix_1}$$

i.e. $\varepsilon = 0.609$ **....efectiveness of this HX**

Then, we find NTU:

Using the Mathcad Function:

$$NTU := NTU\_CrossflowHX\_OneMixed\left(\varepsilon, C_{mix}, C_{unmix}\right)$$

i.e. $NTU = 1.105$ **...No. of Transfer Units**

But: $NTU = \frac{U \cdot A}{C_{min}}$ ...by definition of NTU

Therefore:

$$A := \frac{NTU \cdot C_{min}}{U}$$

i.e. $A = 11.101$ **m^2 .... area of HX .... Ans.**

Note: compare this value of A with the value of A = 10.6 m^2, obtained in Prob. 4B.8, by LMTD method.

==

Prob.4C.6. A parallel flow HX has hot and cold water streams running through it and has following data: m_h = 10 kg/min. m_c = 25 kg/min, cp_h = cp_c = 4180 J/kg.K, Th_1 = 70 C, Th_2 = 50 C, Tc_1 = 25 C. Individual heat transfer coeff on both sides = 60 W/m^2.K. Calculate:

i) area of HX
ii) exit temps of hot and cold fluids if hot water flow is doubled. [M.U., May 2000]:

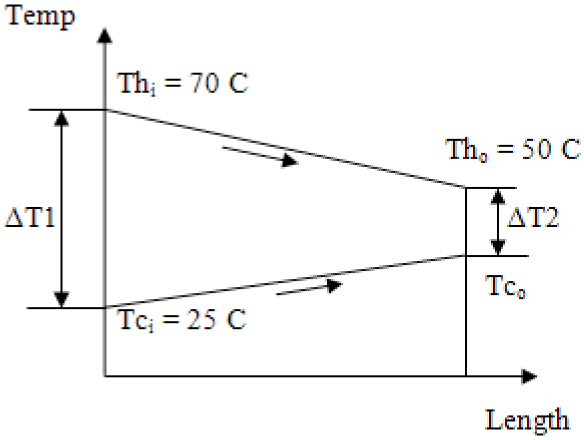


Fig. Prob.4C.6. Parallel flow arrangement

Mathcad Solution:

This is the case where calculations have to be made for an existing HX when operating conditions are changed. So, NTU method is more convenient to use:

Data:

$Th_1 := 70$ C $\quad Th_2 := 50$ C $\quad Tc_1 := 25$ C

$hh1 := 60$ W/m^2.K $\quad hc1 := 60$ W/m^2.K

$cp_h := 4180$ J/kg.K $\quad cp_c := 4180$ J/kg.K

$m_c := \frac{25}{60}$ i.e $m_c = 0.417$ kg/s

$m_h := \frac{10}{60}$ i.e. $m_h = 0.167$ kg/s

Therefore: overall heat transfer coeff, U:

$U := \frac{hh1 \cdot hc1}{hh1 + hc1}$ i.e. $U = 30$ W/m^2.C

Calculations:

Capacity rates:

$C_c := m_c \cdot cp_c$ i.e. $C_c = 1.742 \times 10^3$ W/C.... capacity rate of cold fluid

$C_h := m_h \cdot cp_h$ $\quad C_h = 696.667$ W/C.... capacity rate of hot fluid

$C_{min} := \mathrm{if}(C_c < C_h, C_c, C_h)$ i.e. $C_{min} = 696.667$ W/C...hot fluid is min. fluid

$C_{max} := \mathrm{if}(C_c < C_h, C_h, C_c)$ i.e. $C_{max} = 1.742 \times 10^3$ W/C

Exit temp of cold fluid:

$$Tc_2 := Tc_1 + \frac{C_h}{C_c} \cdot (Th_1 - Th_2)$$

i.e. $Tc_2 = 33$ **C....exit temp of cold fluid**

Capacity ratio, C = Cmin/Cmax:

$C := \frac{C_{min}}{C_{max}}$ i.e. $C = 0.4$ **...capacity ratio**

Effectiveness, ε:

$$\varepsilon := \frac{Th_1 - Th_2}{Th_1 - Tc_1}$$

i.e. $\varepsilon = 0.444$ **....since hot fluid is min. fluid, by definition of ε**

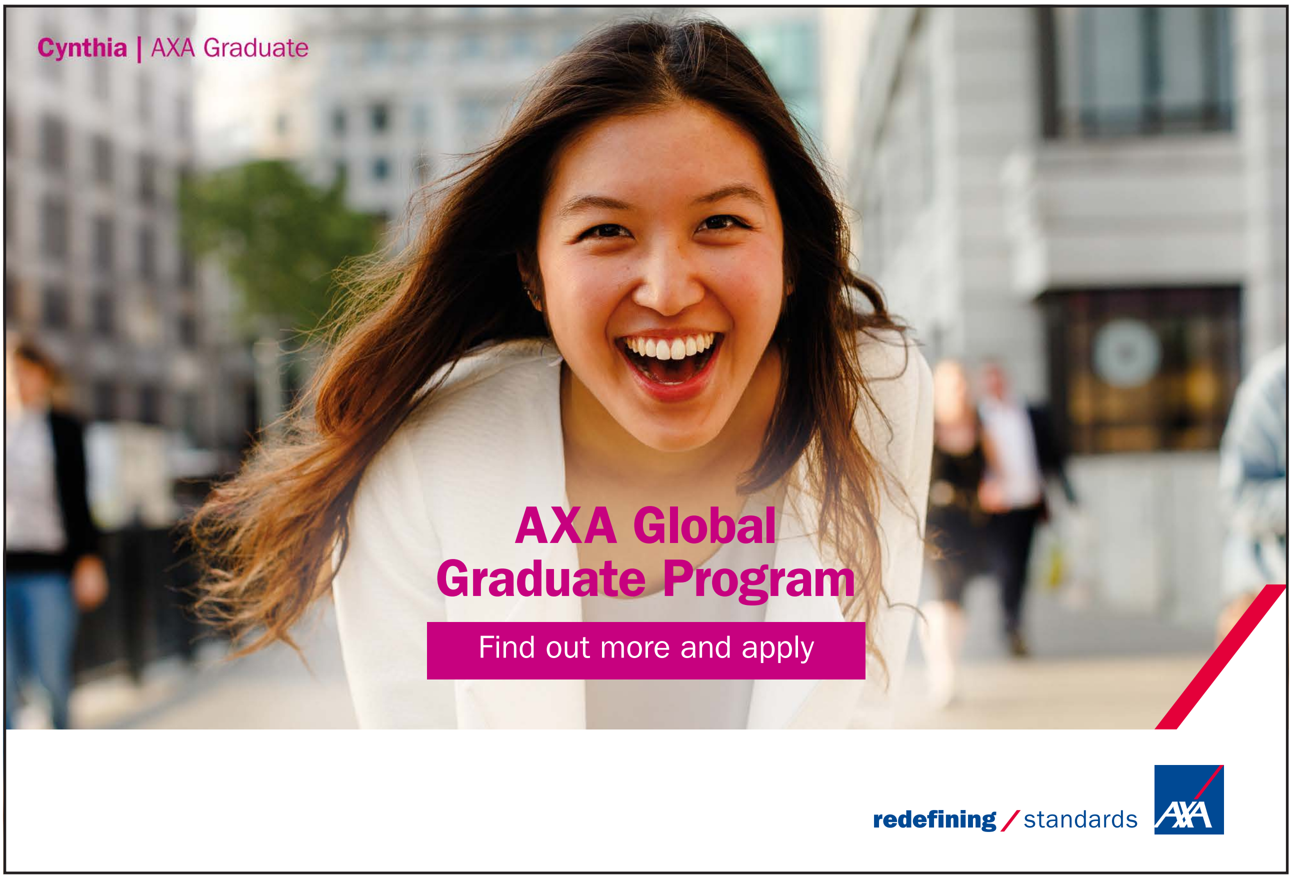
To calculate NTU:

Eqn for NTU for a parallel flow HX is given at the beginning of this chapter.

We have:

$$NTU := \frac{-\ln\left[1 - (1 + C)\cdot\varepsilon\right]}{1 + C}$$

i.e. $NTU = 0.695$ **...No. of Transfer Units**



Therefore, area of HX:

Since $\mathrm{NTU} = \dfrac{U \cdot A}{C_{min}}$ we have:

$$A := \frac{\mathrm{NTU} \cdot C_{min}}{U}$$

i.e. $\boxed{A = 16.147}$ **m^2 ....Area of HX ..Ans.**

(ii) Now, the mass flow of hot stream is doubled. So, hh1 will change:

$m_{h'} := \dfrac{20}{60}$ kg/s

$hh1' := hh1 \cdot \left(\dfrac{m_{h'}}{m_h}\right)^{0.8}$ ...since heat transfer coeff is proportional to Re^0.8 i.e. proportional to (mass flow)^0.8

i.e. $hh1' = 104.466$ W/m^2.C ... new heat transfer coeff. on hot side

So, new overall heat transfer coeff.:

$U := \dfrac{hh1' \cdot hc1}{hh1' + hc1}$ i.e. $U = 38.111$ **...W/m^2.C.....New overall heat tr. coeff.**

And, new capacity ratio, NTU, ε and exit temps:

$C_h := m_{h'} \cdot cp_h$ i.e. $C_h = 1.393 \times 10^3$ W/C....capacity rate, hot fluid

$C_c := m_c \cdot cp_c$ i.e. $C_c = 1.742 \times 10^3$ W/C....capacity rate, cold fluid

Therefore: $C_{min} := C_h$ W/C...min. capacity rate

$C := \dfrac{C_{min}}{C_{max}}$ i.e. $C = 0.8$ **...capacity ratio**

And:

$NTU := \frac{U \cdot A}{C_{min}}$ i.e. $NTU = 0.442$ **...No. of Transfer Units**

Therefore:

$$\varepsilon := \frac{1 - \exp[-NTU \cdot (1 + C)]}{1 + C}$$

i.e. $\varepsilon = 0.305$ **...effectiveness**

And,

$Th_2 := Th_1 - \varepsilon \cdot (Th_1 - Tc_1)$ **...exit temp of hot fluid**

i.e. $\boxed{Th_2 = 56.29}$ **C..Ans....since hot fluid is min. fluid**

Also,

$$Tc_2 := Tc_1 + \frac{\varepsilon \cdot C_{min} \cdot (Th_1 - Tc_1)}{C_c}$$

i.e. $\boxed{Tc_2 = 35.968}$ **C...exit temp of cold fluid....Ans.**

===

"**Prob. 4C.7.** A water to water HX of a counter-flow arrangement has a heating surface area of 2 m^2. Mass flow rates of hot and cold fluids are 2000 kg/h and 1500 kg/h respectively. Temperatures of hot and cold fluids at inlet are 85 C and 25 C respectively. Determine the amount of heat transferred from hot to cold water and their temps at the exit if the overall heat transfer coeff. U = 1400 W/m^2.K. [VTU – May–June 2010]:

(b) Plot the variation of Q, Th_o and Tc_o as the mass flow rate of cold fluid, m_c varies from 0.1 to 0.5 kg/s. Assume other conditions to remain the same."

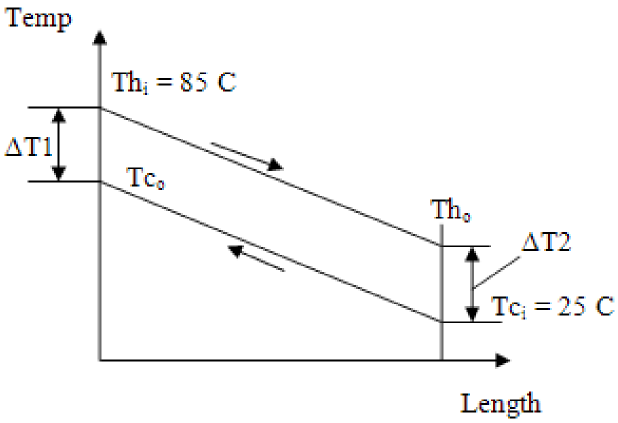


Fig. Prob.4C.7. Counter-flow arrangement

EES Solution:

"Data:"

m_h = 2000 [kg/h]* convert(kg/h, kg/s)

m_c = 1500 [kg/h] * convert(kg/h, kg/s)

T_h_i = 85 [C]

T_c_i = 25 [C]

cp_h = 4180 [J/kg-C]
cp_c = 4180 [J/kg-C]

U = 1400 [W/m^2-C]

A = 2 [m^2]

"Calculations:"

C_h = m_h * cp_h "W/C…=2322"

C_c = m_c * cp_c "W/C… = 1742"

C_min = C_c

C_max = C_h

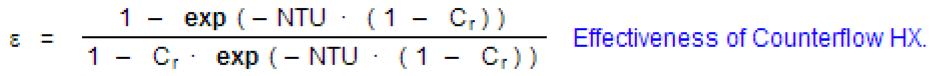
C_r = C_min/C_max "…capacity ratio"

epsilon = (T_c_o – T_c_i) / (T_h_i – T_c_i) "…by definition, taking the min fluid… calculates Tc_o"

"For counter-flow HX:"

NTU = U * A/C_min "…calculates NTU"

We have, for effectiveness of counter-flow HX:



$$\varepsilon = \frac{1 - \exp(-NTU \cdot (1 - C_r))}{1 - C_r \cdot \exp(-NTU \cdot (1 - C_r))}$$ Effectiveness of Counterflow HX.

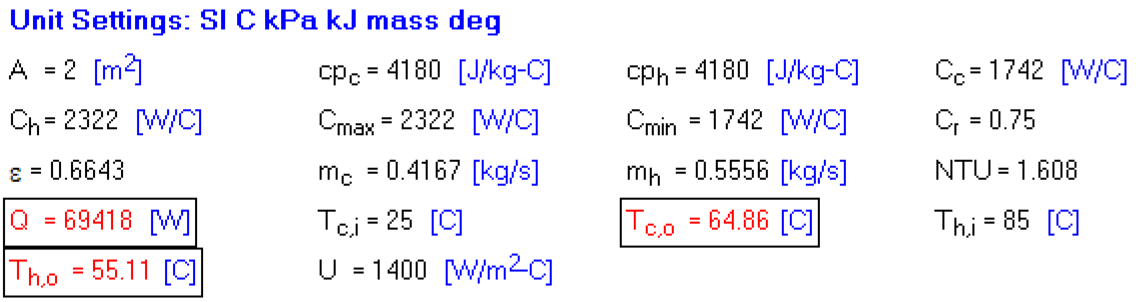
In EES, it is entered:

epsilon = (1 – exp (– NTU * (1 – C_r)))/(1 – C_r * exp(– NTU * (1 – C_r))) "Effectiveness of Counter-flow HX."

Q = m_h * cp_h * (T_h_i – T_h_o) "W…heat transferred… calculates Th_o"

Q= m_c * cp_c * (T_c_o – T_c_i) "W… calculates Q"

Results:



Unit Settings: SI C kPa kJ mass deg

$A = 2$ [m^2]
$cp_c = 4180$ [J/kg-C]
$cp_h = 4180$ [J/kg-C]
$C_c = 1742$ [W/C]

$C_h = 2322$ [W/C]
$C_{max} = 2322$ [W/C]
$C_{min} = 1742$ [W/C]
$C_r = 0.75$

$\varepsilon = 0.6643$
$m_c = 0.4167$ [kg/s]
$m_h = 0.5556$ [kg/s]
$NTU = 1.608$

$Q = 69418$ [W]
$T_{c,i} = 25$ [C]
$T_{c,o} = 64.86$ [C]
$T_{h,i} = 85$ [C]

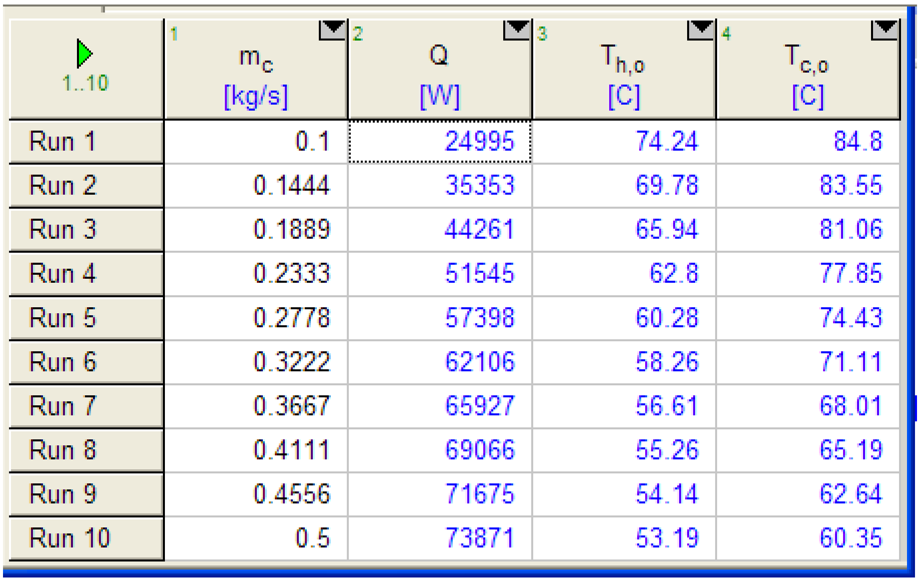
$T_{h,o} = 55.11$ [C]
$U = 1400$ [W/m^2-C]

Thus:

Q = 69418 W, Th_o = 55.11 C, Tc_o = 64.86 C .. Ans.

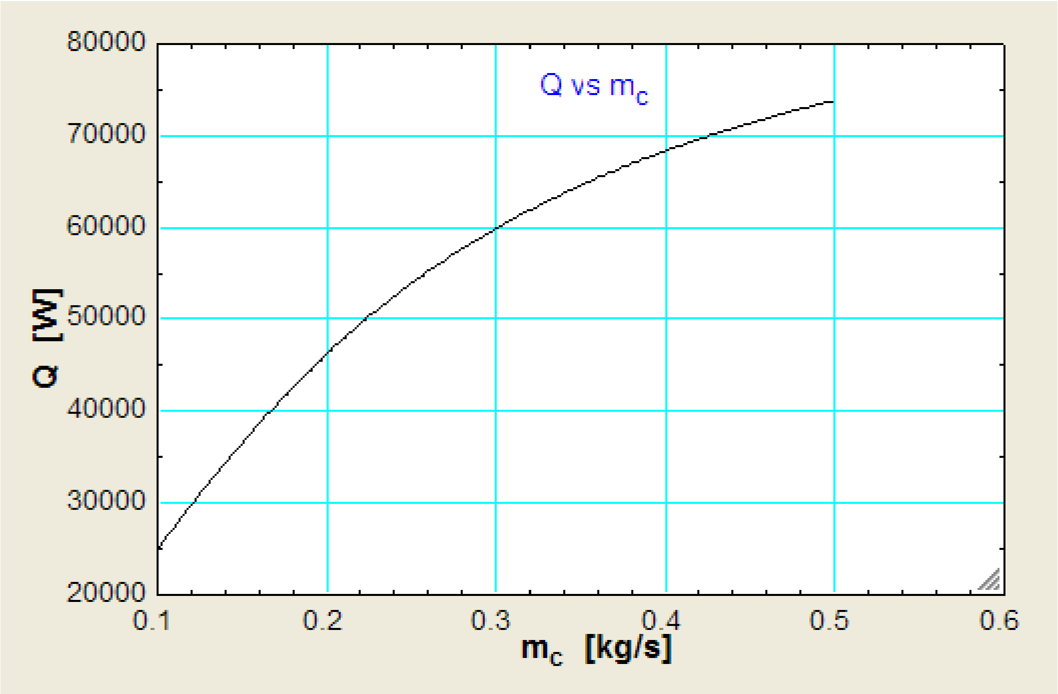
Plot the variation of Q, Th_o and Tc_o as the mass flow rate of cold fluid, m_c varies from 0.1 to 0.5 kg/s. Assume other conditions to remain the same:

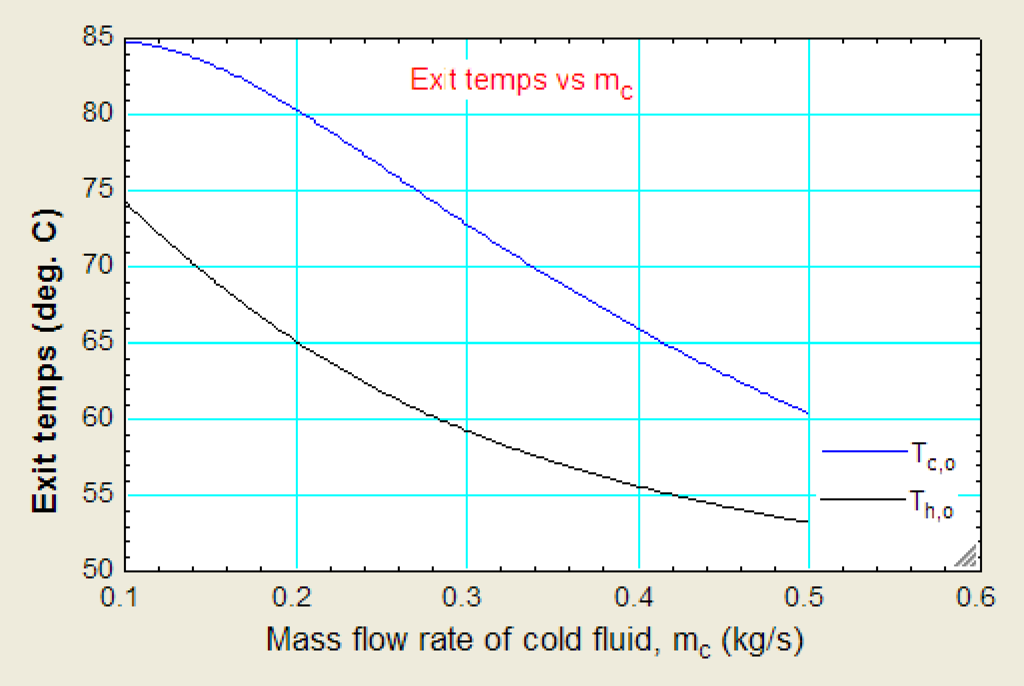
First, calculate the Parametric Table:



▶ 1..10	1 m_c [kg/s]	2 Q [W]	3 $T_{h,o}$ [C]	4 $T_{c,o}$ [C]
Run 1	0.1	24995	74.24	84.8
Run 2	0.1444	35353	69.78	83.55
Run 3	0.1889	44261	65.94	81.06
Run 4	0.2333	51545	62.8	77.85
Run 5	0.2778	57398	60.28	74.43
Run 6	0.3222	62106	58.26	71.11
Run 7	0.3667	65927	56.61	68.01
Run 8	0.4111	69066	55.26	65.19
Run 9	0.4556	71675	54.14	62.64
Run 10	0.5	73871	53.19	60.35

Now, plot the graphs:





==

"**Prob. 4C.8**. A counter-flow HX is employed to cool 0.55 kg/s (cp = 2.45 kJ/kg.K) of oil from 115 C to 40 C by the use of water. The inlet and outlet temps of cooling water are 15 C and 75 C respectively. The overall heat transfer coeff. is 1450 W/m^2.C. Using the NTU method, calculate the following:

i) mass flow rate of water
ii) effectiveness of HX, and
iii) surface area required. [VTU – Dec. 2010:]"

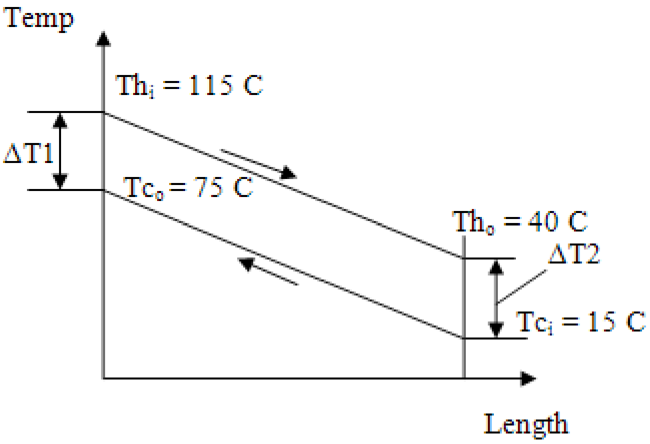


Fig. Prob.4C.8. Counter-flow arrangement

EES Solution:

```
"Data:"

m_h = 0.55 [kg/s]
T_h_i = 115 [C]
T_h_o = 40 [C]
T_c_i = 15 [C]
T_c_o = 75 [C]
cp_h = 2450 [J/kg-C]
cp_c = 4180 [J/kg-C]
U = 1450 [W/m^2-C]

"Calculations:"

Q = m_h * cp_h * (T_h_i – T_h_o) "W…determines heat tr."

Q= m_c * cp_c * (T_c_o – T_c_i) "W … determines m_c"

C_h = m_h * cp_h "W/C…capacity rate of hot fluid =1348"

C_c = m_c * cp_c "W/C…capacity rate of cold fluid = 1684"

C_min = C_h

C_max = C_c

C_r = C_min/C_max "…capacity ratio"

epsilon = (T_h_i – T_h_o) / (T_h_i – T_c_i) "…by definition, considering the fluid with min. capacity rate"

"For counter-flow HX:"
```

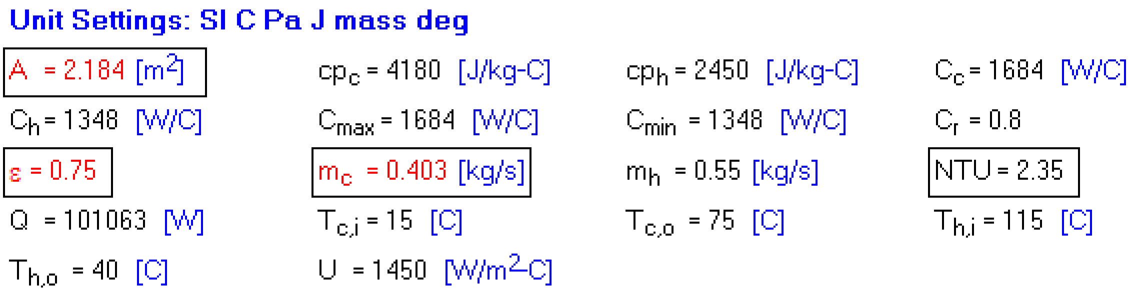
$$\varepsilon = \frac{1 - \exp\left(-NTU \cdot (1 - C_r)\right)}{1 - C_r \cdot \exp\left(-NTU \cdot (1 - C_r)\right)} \quad \text{Effectiveness of Counterflow HX .}$$

In EES, we enter:

```
epsilon = (1 – exp ( – NTU * (1 – C_r)))/(1 – C_r * exp( – NTU * (1 – C_r))) "Effectiveness of Counter-flow HX … determines NTU"

NTU = U * A/C_min "…determines A"
```

Results:



```
Unit Settings: SI C Pa J mass deg
A  = 2.184 [m^2]      cp_c = 4180  [J/kg-C]    cp_h = 2450  [J/kg-C]    C_c = 1684  [W/C]
C_h = 1348  [W/C]     C_max = 1684  [W/C]      C_min  = 1348  [W/C]     C_r  = 0.8
ε = 0.75              m_c  = 0.403 [kg/s]      m_h  = 0.55 [kg/s]       NTU = 2.35
Q  = 101063  [W]      T_c,i = 15  [C]          T_c,o  = 75  [C]         T_h,i = 115  [C]
T_h,o  = 40  [C]      U  = 1450  [W/m^2-C]
```

Thus:

Mass flow rate of water = m_c = 0.403 kg/s …. Ans.

Effectiveness of hX = 0.75 … Ans.

Area of HX = A = 2.184 m^2 … Ans.

"**Prob. 4C.9**. A certain HX has a total outside area of 15.82 m^2. It is to be operated for cooling oil at 110 C (cp = 1900 J/kg.K) flowing at a rate of 170.9 kg/min. Water at a rate of 68 kg/min is available at 35 C as a cooling agent. If the overall heat transfer coeff. is 320 W/m^2.K, calculate the outlet temp of oil and water for a counter-flow arrangement. [VTU – Dec. 06–Jan. 2007:]

(b) Plot the variation of ε, Th_o and Tc_o as the mass flow rate of cold fluid, m_c varies from 0.15to 1.2 kg/s. Assume other conditions to remain the same."

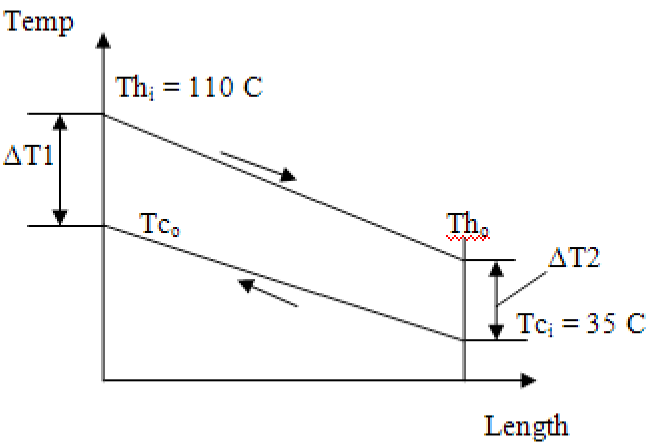


Fig. Prob.4C.9. Counter-flow arrangement

EES Solution:

"Data:"

m_h = 170.9 * convert(kg/min, kg/s)
m_c = 68 * convert(kg/min, kg/s)

T_h_i = 110 [C]
T_c_i = 35 [C]

cp_h = 1900 [J/kg-C]
cp_c = 4180 [J/kg-C]

U = 320 [W/m^2-C]

A = 15.82 [m^2]

"Calculations:"

C_h = m_h * cp_h "W/C…= 5412…. capacity rate of hot fluid"

C_c = m_c * cp_c "W/C… = 4737… capacity rate of cold fluid"

C_min = C_c "..min. capacity rate"

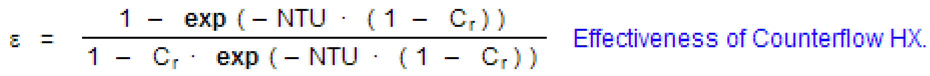
C_max = C_h "..max. capacity rate"

C_r = C_min/C_max "…capacity ratio"

"Therefore:"

NTU = U * A /C_min

"For counter-flow HX:"



$$\varepsilon = \frac{1 - \exp(-NTU \cdot (1 - C_r))}{1 - C_r \cdot \exp(-NTU \cdot (1 - C_r))}$$ Effectiveness of Counterflow HX.

In EES, we enter:

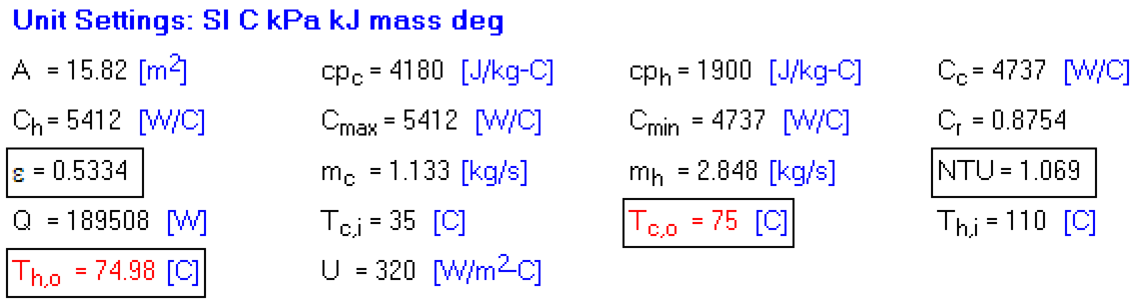
epsilon = (1 – exp (– NTU * (1 – C_r)))/(1 – C_r * exp(– NTU * (1 – C_r))) "Effectiveness of Counter-flow HX …"

epsilon = (T_c_o – T_c_i)/(T_h_i – T_c_i) "…determines T_c_o"

Q= m_c * cp_c * (T_c_o – T_c_i) "W … determines Q"

Q = m_h * cp_h * (T_h_i – T_h_o) "W…determines T_h_o."

Results:



Unit Settings: SI C kPa kJ mass deg

$A = 15.82$ [m^2] $cp_c = 4180$ [J/kg-C] $cp_h = 1900$ [J/kg-C] $C_c = 4737$ [W/C]

$C_h = 5412$ [W/C] $C_{max} = 5412$ [W/C] $C_{min} = 4737$ [W/C] $C_r = 0.8754$

$\varepsilon = 0.5334$ $m_c = 1.133$ [kg/s] $m_h = 2.848$ [kg/s] $NTU = 1.069$

$Q = 189508$ [W] $T_{c,i} = 35$ [C] $T_{c,o} = 75$ [C] $T_{h,i} = 110$ [C]

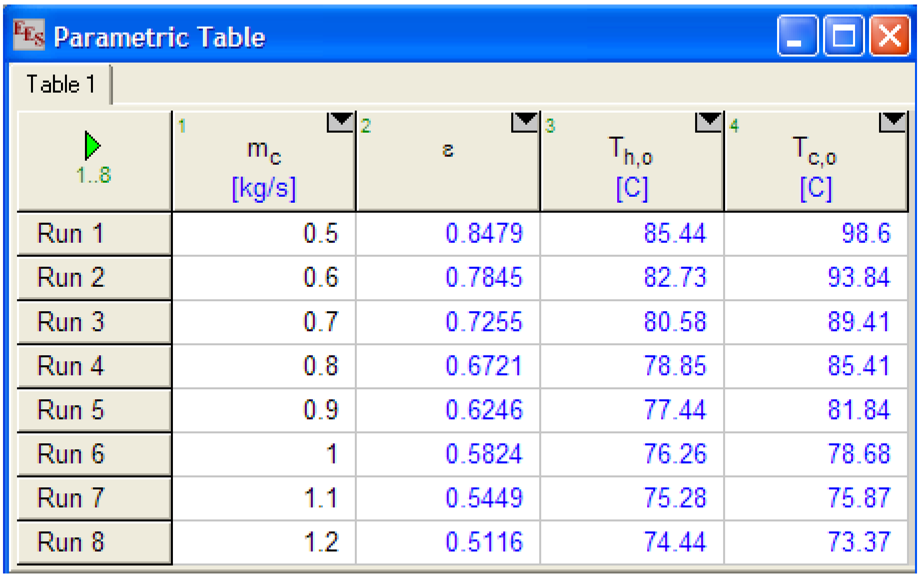
$T_{h,o} = 74.98$ [C] $U = 320$ [W/m^2-C]

Thus:

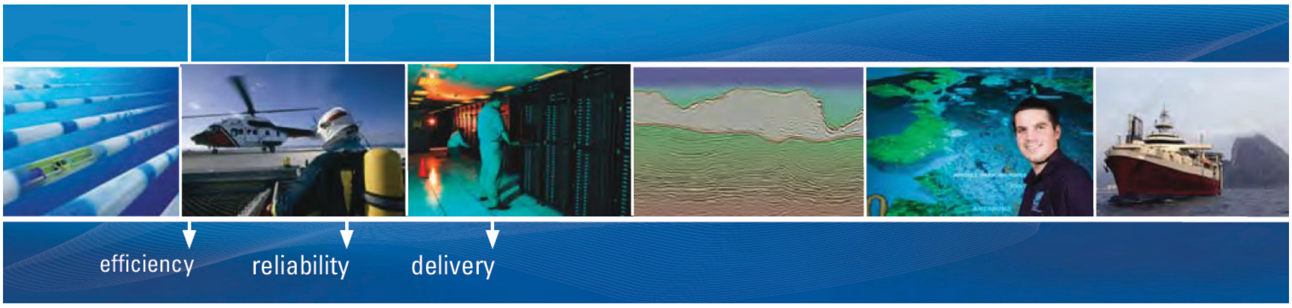
Th_o = 74.98 C, Tc_o = 75 C … Ans.

Plot the variation of ε, Th_o and Tc_o as the mass flow rate of cold fluid, m_c varies from 0.15to 1.2 kg/s. Assume other conditions to remain the same:

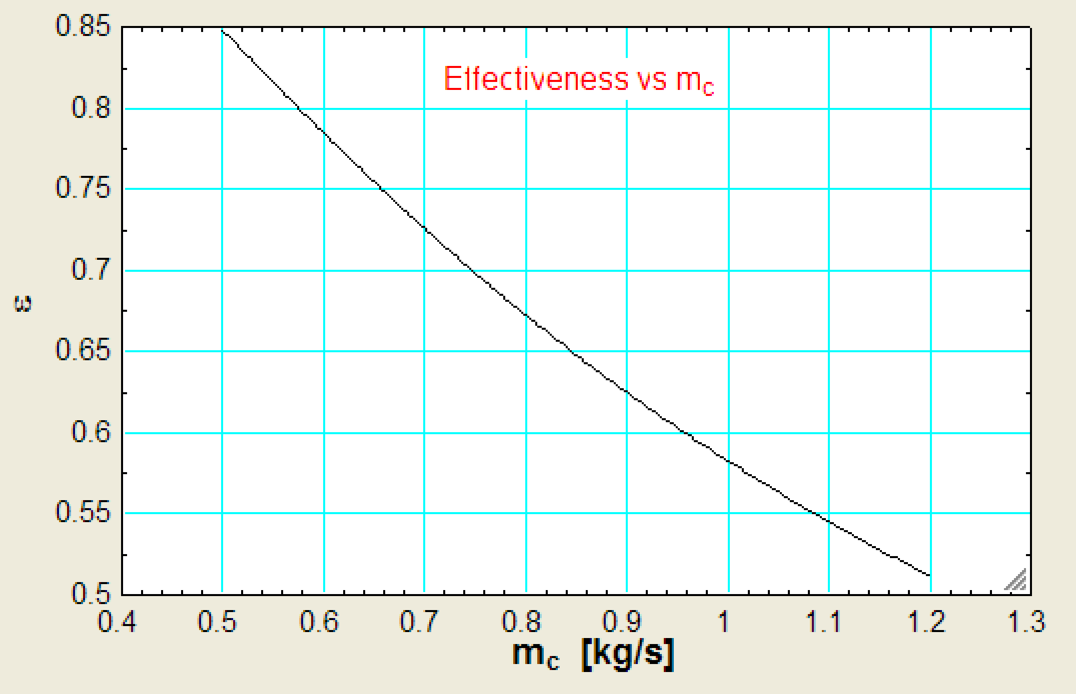
First, calculate the Parametric Table:

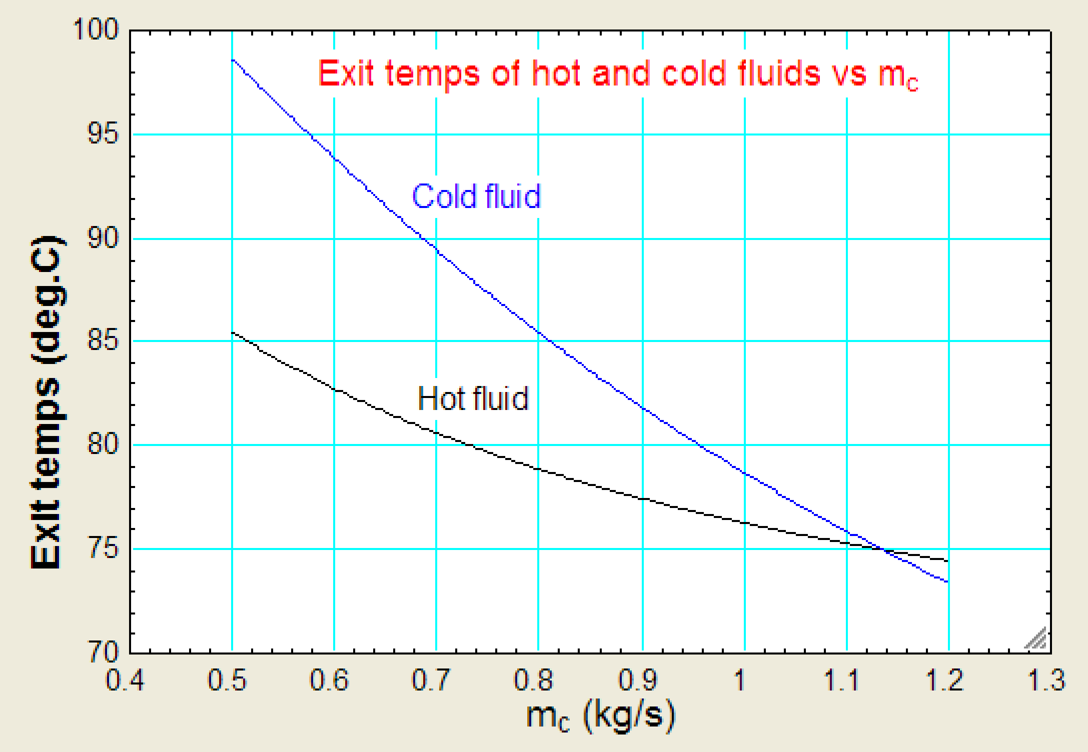


1..8	1 m_c [kg/s]	2 ε	3 $T_{h,o}$ [C]	4 $T_{c,o}$ [C]
Run 1	0.5	0.8479	85.44	98.6
Run 2	0.6	0.7845	82.73	93.84
Run 3	0.7	0.7255	80.58	89.41
Run 4	0.8	0.6721	78.85	85.41
Run 5	0.9	0.6246	77.44	81.84
Run 6	1	0.5824	76.26	78.68
Run 7	1.1	0.5449	75.28	75.87
Run 8	1.2	0.5116	74.44	73.37



Now, plot the results:





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"**Prob. 4C.10.** 3000 kg/h of furnace oil is heated from 10 C to 90 C in a Shell & Tube HX. The oil is to flow inside the tube while the steam at 120 C is to flow in the shell. Tubes of 1.65 cm ID and 1.9 cm OD are used. The heat transfer coefficients for oil and steam sides are 85 W/m^2.K and 7420 W/m^2.K respectively. Find the number of passes and no. of tubes in each pass, if the length of the tube is limited to 2.85 m due to space limitations. The velocity of oil is limited to 5 cm/s to keep the pressure drop low. Take rho = 900 kg/m^3, and cp (oil) = 1970 J/kg.K [VTU – Jan.–Feb. 2004]"

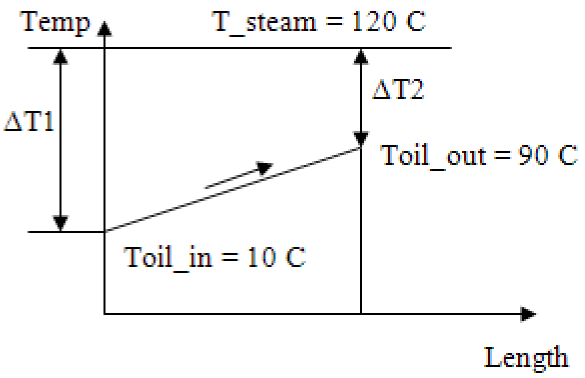


Fig. Prob.4C.10. Condenser

EES Solution:

"Data:"

m_oil = 3000*convert(kg/h, kg/s) "…mass flow rate of oil"

Toil_in = 10[C]"Oil…inlet temp"

Toil_out = 90 [C]"….oil … exit temp"

T_steam = 120"…Temp of condensing steam"

rho_oil = 900 [kg/m^3]"…density of oil"

cp_oil = 1970 [J/kg-C]"…sp. heat of oil"

d_i = 0.0165 [m]"….tube ID"

d_o = 0.019 [m]"…tube OD"

h_oil = 85 [W/m^2-C]"…heat transfer coeff on oil side"

h_steam = 7420 [W/m^2-C] "…heat transfer coeff on steam side"

vel_oil = 0.05 [m/s] "…velocity of oil"

L_tube = 2.85 [m]"…length of tube"

"Calculations:"

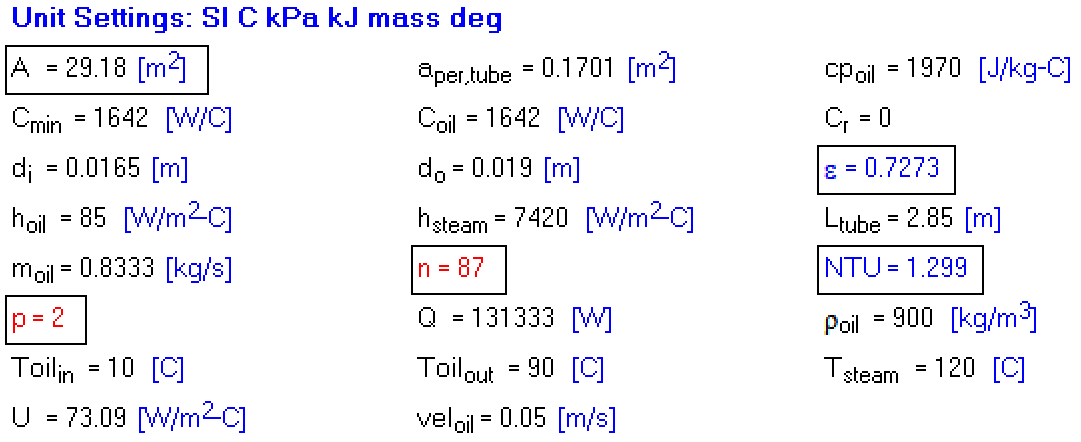
Q =m_oil * cp_oil * (Toil_out – Toil_in) "W….heat transferred"

U = 1/(1/h_steam + (d_o/d_i)*(1/h_oil)) "Finds overall heat transfer coeff. U, W/m^2.C"

```
“Since it is a condenser, steam is the ‘max. fluid’ with its capacity rate = infinity.

Oil, which is being heated, is the ‘min. fluid’, and the capacity ratio C_r = 0

First, find effectiveness, remembering that oil is the min. fluid:”

C_oil = m_oil * cp_oil “[W/C] … capacity rate of oil = C_min”

C_min = C_oil

C_r = 0 “….capacity ratio = (C_min/C_max)”

“Then: Effectiveness of a condenser, where oil is the ‘min. fluid’:”

epsilon = (Toil_out – Toil_in) / (T_steam – Toil_in) “.finds effectiveness….by definition of effectiveness”

“Therefore: NTU of the condenser, when C = 0: (see eqns given at the beginning of chapter).”

NTU = -ln(1 – epsilon) “…finds NTU, when epsilon is known”

“But:”

NTU = U * A / C_min “…by definition of NTU …. finds Area, A of HX”

“Also:”
```

```
n = ceil(m_oil / (pi * d_i^2/4 * vel_oil *rho_oil)) "…finds no. of tubes per pass, from limitation of velocity, rounded off to next higher integer value"

a_per_tube = pi * d_o * L_tube "m^2, surface area per tube, given the length L_tube:"

p = ceil (A / (n * a_per_tube))"….finds no. of passes, rounded off to next higher integer value"
```

Results:



Unit Settings: SI C kPa kJ mass deg

$\boxed{A = 29.18\ [m^2]}$ $a_{per,tube} = 0.1701\ [m^2]$ $cp_{oil} = 1970\ [J/kg\text{-}C]$

$C_{min} = 1642\ [W/C]$ $C_{oil} = 1642\ [W/C]$ $C_r = 0$

$d_i = 0.0165\ [m]$ $d_o = 0.019\ [m]$ $\boxed{\varepsilon = 0.7273}$

$h_{oil} = 85\ [W/m^2\text{-}C]$ $h_{steam} = 7420\ [W/m^2\text{-}C]$ $L_{tube} = 2.85\ [m]$

$m_{oil} = 0.8333\ [kg/s]$ $\boxed{n = 87}$ $\boxed{NTU = 1.299}$

$\boxed{p = 2}$ $Q = 131333\ [W]$ $\rho_{oil} = 900\ [kg/m^3]$

$Toil_{in} = 10\ [C]$ $Toil_{out} = 90\ [C]$ $T_{steam} = 120\ [C]$

$U = 73.09\ [W/m^2\text{-}C]$ $vel_{oil} = 0.05\ [m/s]$

Thus:

No. of tubes per pass = n = 87 …. Ans.

No. of passes = p = 2 …. Ans.

===

Consider the following variation to the above problem:

If m_oil varies from 0.5 to 1 kg/s, plot variation of Q and A:

As mass flow rate of oil changes, h_oil will change, and the U changes. C_min also changes with m_oil. However, effectiveness, and therefore NTU remain the same since temperature conditions are maintained the same as earlier. New values of NTU, n (i.e. no. of tubes per pass) and p (i.e. no. of passes) are calculated as shown below:

Add the following code at the end of the previous EES program:

```
"If m_oil varies from 0.5 to 1 kg/s, plot variation of Q and A:"

{m_oil_new = 1[kg/s]}
```

Q_new =m_oil_new * cp_oil * (Toil_out – Toil_in) "W….new heat transferred"

h_oil_new = h_oil * (m_oil_new / m_oil)^0.8 "…new heat transfer coeff on oil side, as mass flow rate changes"

"Therefore, new U:"

U_new = 1/(1/h_steam + (d_o/d_i)*(1/h_oil_new)) "Finds new overall heat transfer coeff. U, W/m^2.C"

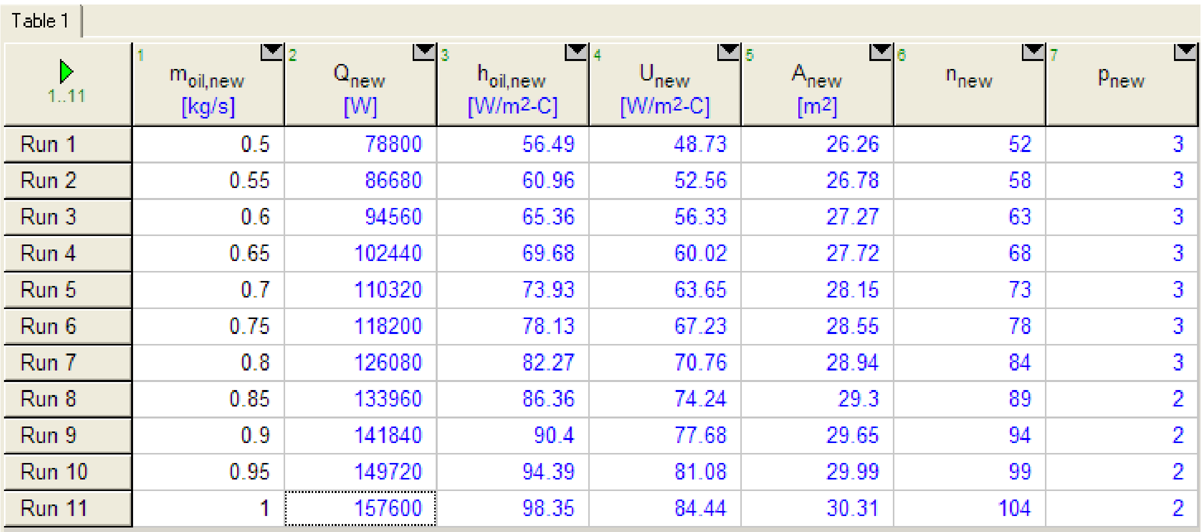
C_min_new = m_oil_new * cp_oil "[W/C] … capacity rate of oil = C_min_new"

NTU = U_new * A_new / C_min_new "…by definition of NTU …. finds Area, A_new of HX"

n_new = ceil(m_oil_new / (pi * d_i^2/4 * vel_oil *rho_oil)) "…finds no. of tubes per pass, from limitation of velocity, rounded off to next higher integer value"

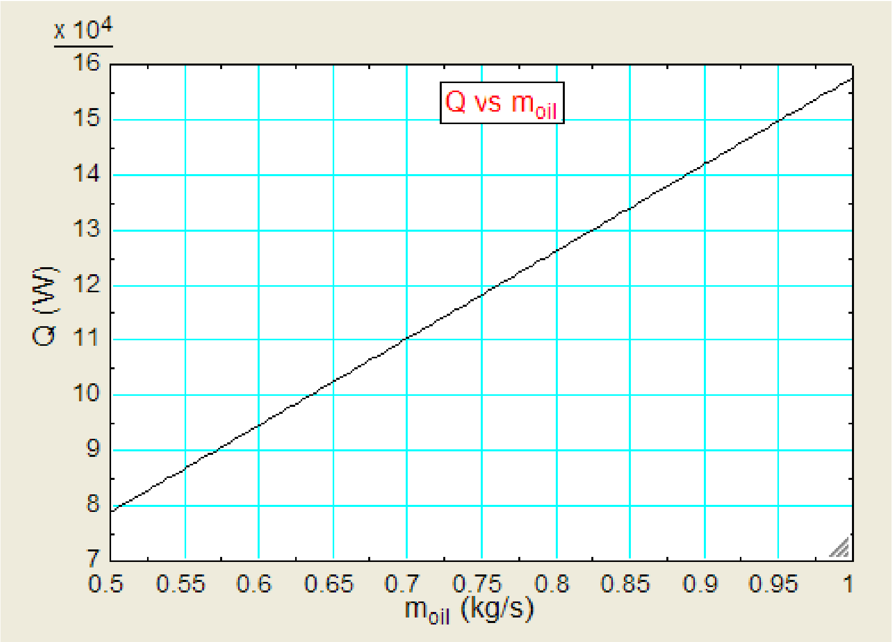
p_new = ceil (A_new / (n_new * a_per_tube))"….finds no. of passes, rounded off to next higher integer value"

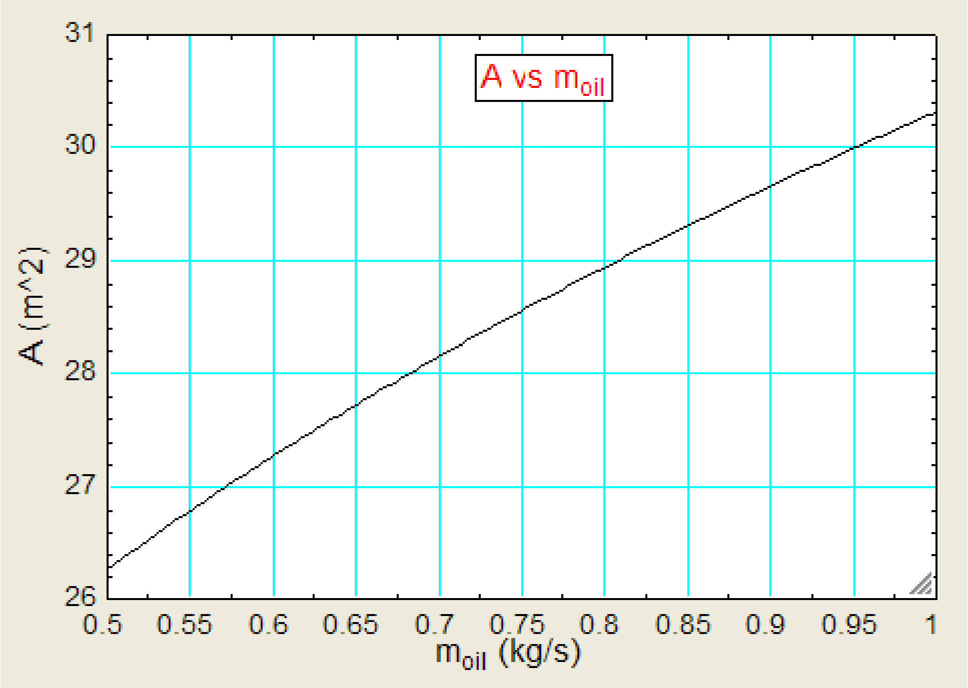
And, now, prepare the Parametric Table:



1..11	1 $m_{oil,new}$ [kg/s]	2 Q_{new} [W]	3 $h_{oil,new}$ [W/m2-C]	4 U_{new} [W/m2-C]	5 A_{new} [m2]	6 n_{new}	7 p_{new}
Run 1	0.5	78800	56.49	48.73	26.26	52	3
Run 2	0.55	86680	60.96	52.56	26.78	58	3
Run 3	0.6	94560	65.36	56.33	27.27	63	3
Run 4	0.65	102440	69.68	60.02	27.72	68	3
Run 5	0.7	110320	73.93	63.65	28.15	73	3
Run 6	0.75	118200	78.13	67.23	28.55	78	3
Run 7	0.8	126080	82.27	70.76	28.94	84	3
Run 8	0.85	133960	86.36	74.24	29.3	89	2
Run 9	0.9	141840	90.4	77.68	29.65	94	2
Run 10	0.95	149720	94.39	81.08	29.99	99	2
Run 11	1	157600	98.35	84.44	30.31	104	2

And, plot the results:





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"**Prob. 4C.11.** Water enters a counter-flow double pipe HX at 15 C, flowing at a rate of 1300 kg/h. It is heated by oil (cp = 2000 J/kg.K) flowing at a rate of 550 kg/h which has an inlet temp of 94 C. For an area of 1 m^2 and an overall heat transfer coeff of 1075 W/m^2.K, determine the total heat transfer and outlet temps of oil and water. Take cp of water as 4186 J/kg.K. [VTU – Feb. 2002]"

(b) Also, plot the variation of Q, Th_o and Tc_o as mass flow rate of water varies from 0.1 to 0.5 kg/s, other conditions remaining the same.

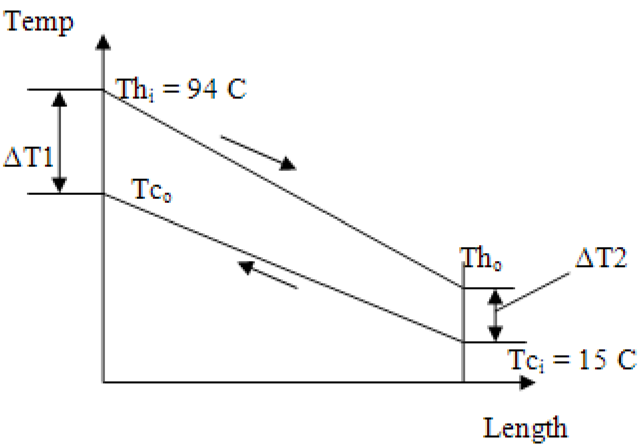


Fig. Prob.4C.11. Counter-flow arrangement

EES Solution:

```
“Data:”

m_h = 550[kg/h] * convert(kg/h, kg/s) “…oil, hot fluid”
m_c = 1300[kg/h] * convert(kg/h, kg/s) “….water, cold fluid”

T_h_i = 94 [C]
T_c_i = 15 [C]

cp_h = 2000 [J/kg-C]
cp_c = 4186 [J/kg-C]

U = 1075 [W/m^2-C]

A = 1 [m^2]

“Calculations:”

C_h = m_h * cp_h “W/C…capacity rate of hot fluid = 305.6”
C_c = m_c * cp_c “W/C…capacity rate of cold fluid = 1512”

“Therefore:”

C_min = C_h
C_max = C_c

C_r = C_min/C_max “…Capacity ratio”

“Calculate NTU:”

NTU = U * A /C_min “…by definition”

“Effectiveness for counter-flow HX:”
```

$$\varepsilon = \frac{1 - \exp\left(-NTU \cdot \left(1 - C_r \right) \right)}{1 - C_r \cdot \exp\left(-NTU \cdot \left(1 - C_r \right) \right)}$$

In EES, enter it as:

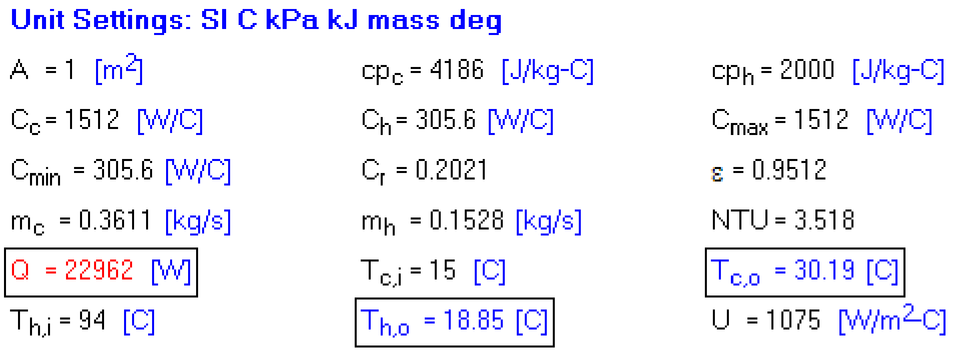
epsilon = (1 – exp (– NTU * (1 – C_r)))/(1 – C_r * exp(– NTU * (1 – C_r))) "Effectiveness of Counter-flow HX…"

epsilon = (T_h_i – T_h_o)/(T_h_i – T_c_i) "…determines exit temp of hot fluid, T_h_o"

Q = m_h * cp_h * (T_h_i – T_h_o) "W…determines Q"

Q= m_c * cp_c * (T_c_o – T_c_i) "W … determines exit temp of cold fluid, T_c_o"

Results:



Unit Settings: SI C kPa kJ mass deg

$A = 1$ [m^2]

$cp_c = 4186$ [J/kg-C]

$cp_h = 2000$ [J/kg-C]

$C_c = 1512$ [W/C]

$C_h = 305.6$ [W/C]

$C_{max} = 1512$ [W/C]

$C_{min} = 305.6$ [W/C]

$C_r = 0.2021$

$\varepsilon = 0.9512$

$m_c = 0.3611$ [kg/s]

$m_h = 0.1528$ [kg/s]

$NTU = 3.518$

$\boxed{Q = 22962 \text{ [W]}}$

$T_{c,i} = 15$ [C]

$\boxed{T_{c,o} = 30.19 \text{ [C]}}$

$T_{h,i} = 94$ [C]

$\boxed{T_{h,o} = 18.85 \text{ [C]}}$

$U = 1075$ [W/m^2-C]

Thus:

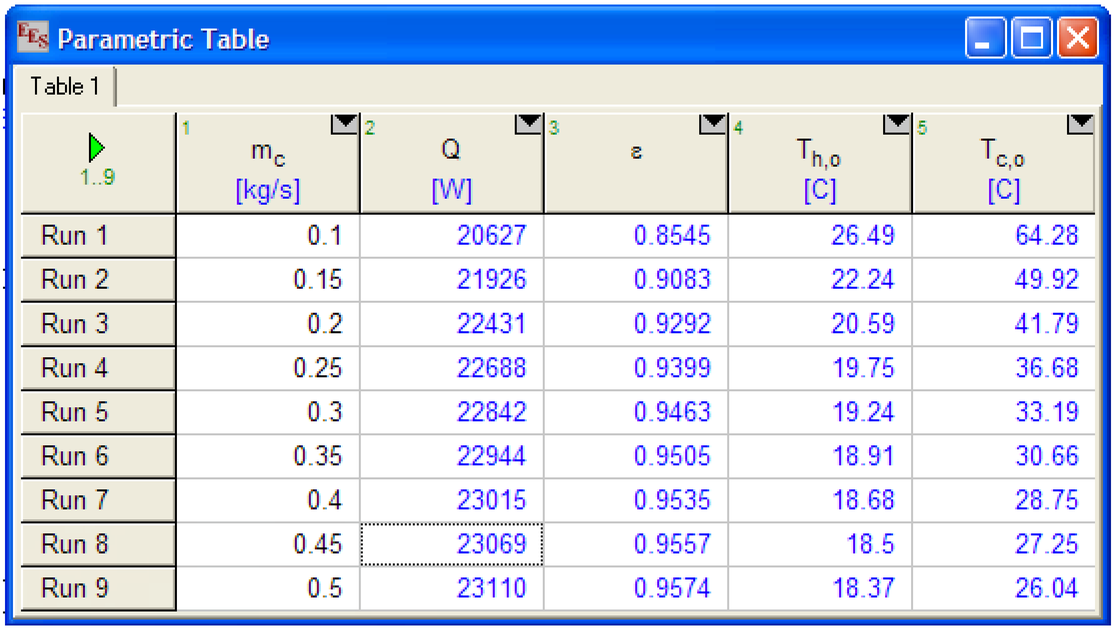
Q = 22962 W …Ans.

Th_o = 18.85 C …exit temp of oil… Ans.

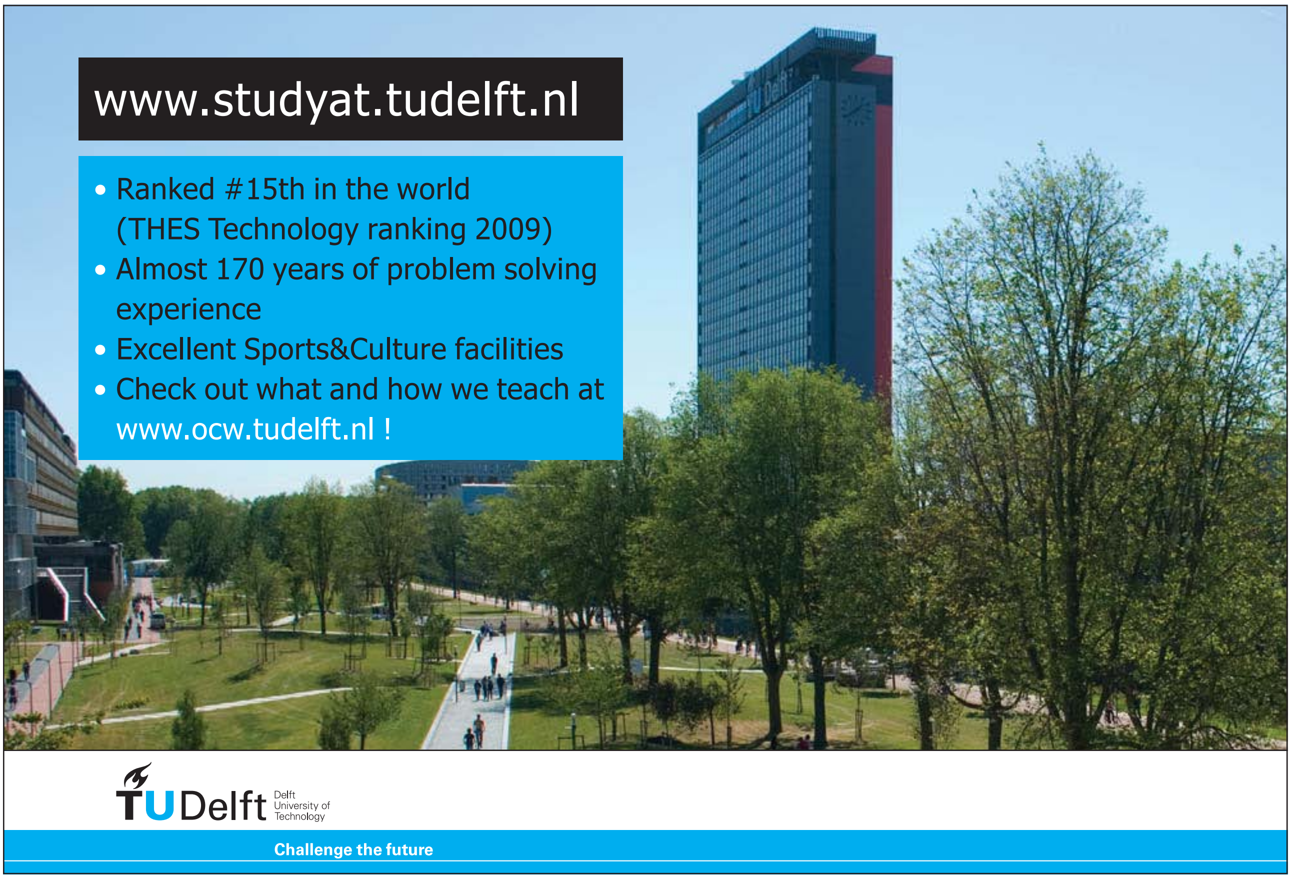
Tc_o = 30.19 C …exit temp of water… Ans.

(b) Also, plot the variation of Q, Th_o and Tc_o as m_c varies from 0.1 to 0.5 kg/s:

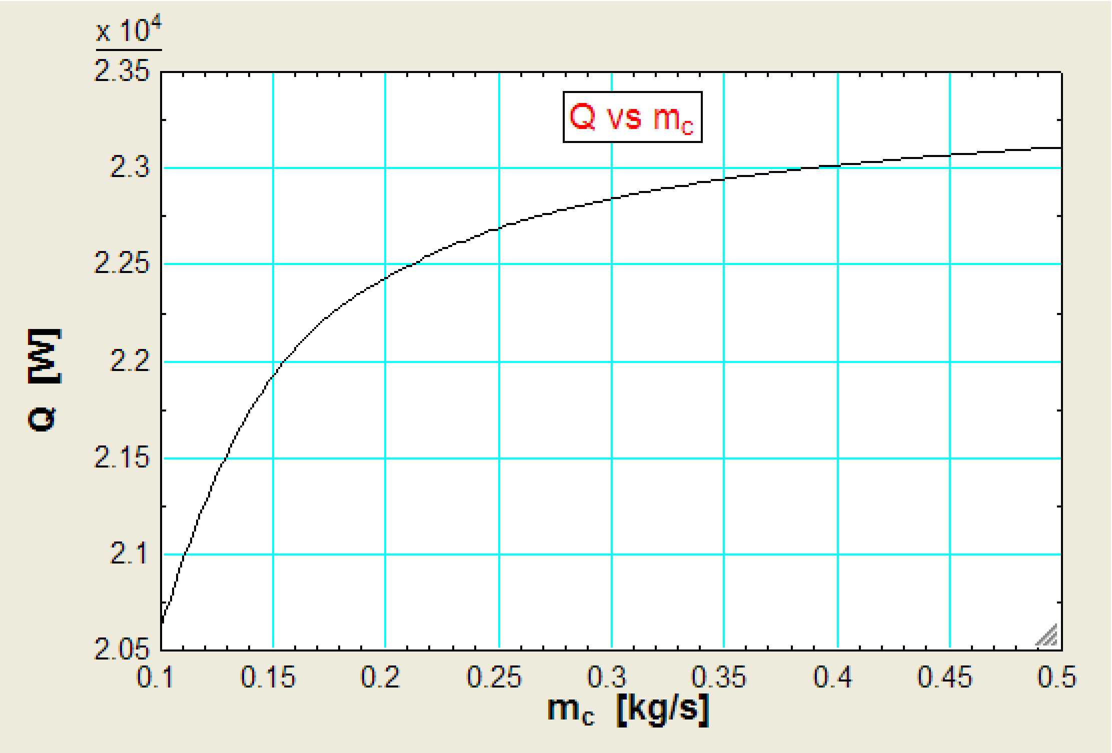
First, calculate and prepare the Parametric Table:

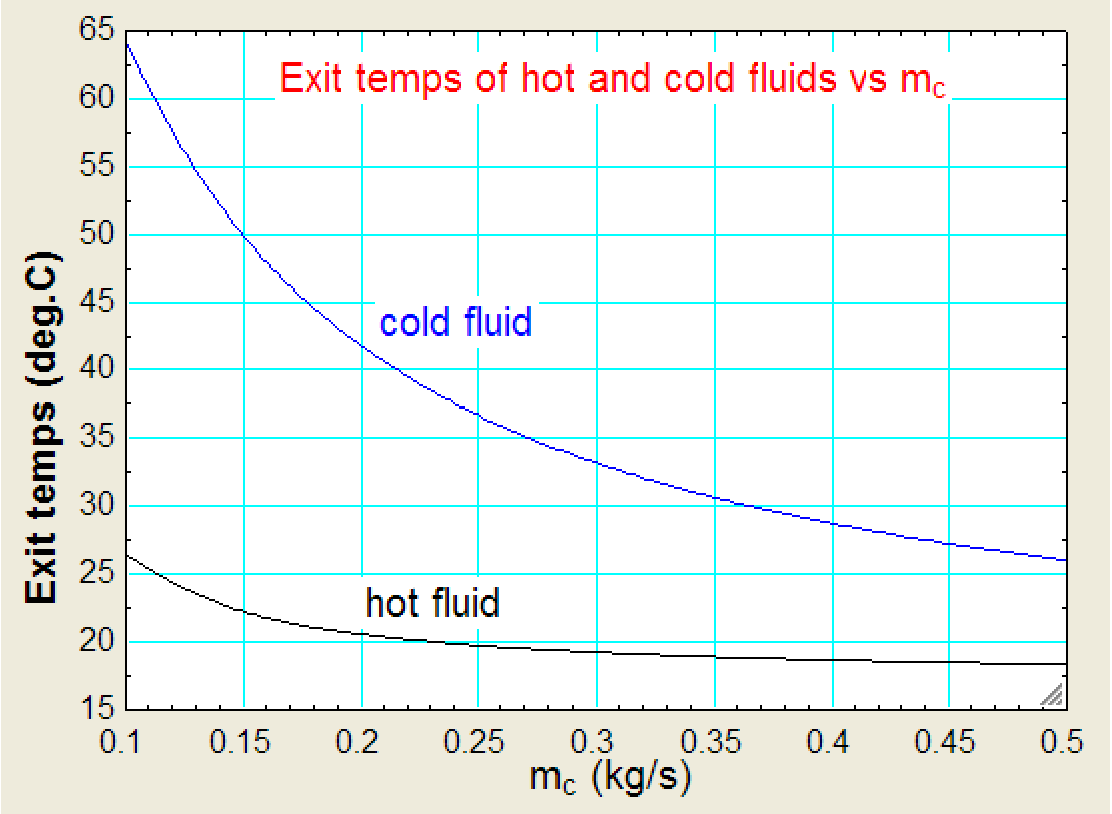


1..9	1 m_c [kg/s]	2 Q [W]	3 ε	4 $T_{h,o}$ [C]	5 $T_{c,o}$ [C]
Run 1	0.1	20627	0.8545	26.49	64.28
Run 2	0.15	21926	0.9083	22.24	49.92
Run 3	0.2	22431	0.9292	20.59	41.79
Run 4	0.25	22688	0.9399	19.75	36.68
Run 5	0.3	22842	0.9463	19.24	33.19
Run 6	0.35	22944	0.9505	18.91	30.66
Run 7	0.4	23015	0.9535	18.68	28.75
Run 8	0.45	23069	0.9557	18.5	27.25
Run 9	0.5	23110	0.9574	18.37	26.04



And, plot the results:





==

"**Prob. 4C.12.** A cross-flow HX (both fluids unmixed), having a heat transfer area of 8.4 m^2, is to heat air (cp = 1005 J/kg.K) with water (cp = 4180 J/kg.K). Air enters at 18 C with a mass flow rate of 2 kg/s while water enters at 90 C with a mass flow rate of 0.25 kg/s. Overall heat transfer coeff is 250 W/m^2.K Calculate the exit temps of the two fluids and the heat transfer rate. [VTU – July–Aug. 2004]"

(b) Plot the variation of Q and Th_o and Tc_o as air flow rate, m_c varies from 1.5 to 3 kg/s, all other conditions remaining the same as earlier:

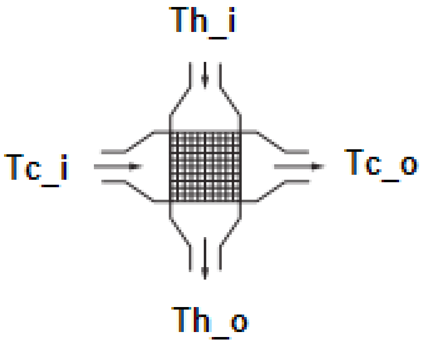


Fig. Prob.4C.12. Cross-flow arrangement

EES Solution:

"Data:"

m_h = 0.25 [kg/s] "….water is the hot fluid"
m_c = 2[kg/s] "….air is the cold fluid"

T_h_i = 90 [C]
T_c_i = 18 [C]
cp_c = 1005 [J/kg-C]
cp_h = 4180 [J/kg-C]
U = 250 [W/m^2-C]
A = 8.4 [m^2]

Calculations:

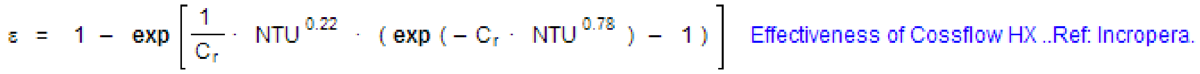
C_h = m_h * cp_h "W/C…= 1045 … capacity rate of hot fluid"
C_c = m_c * cp_c "W/C… = 2010 … capacity rate of cold fluid"

C_min = C_h "….min. capacity rate"
C_max = C_c "…max. capacity rate"

C_r = C_min/C_max "… Capacity ratio"

NTU = U * A /C_min "…NTU by definition"

"For cross-flow HX:"



$$\varepsilon = 1 - \exp\left[\frac{1}{C_r} \cdot NTU^{0.22} \cdot \left(\exp\left(-C_r \cdot NTU^{0.78}\right) - 1\right)\right]$$ Effectiveness of Cossflow HX ..Ref: Incropera.

In EES, enter it as:

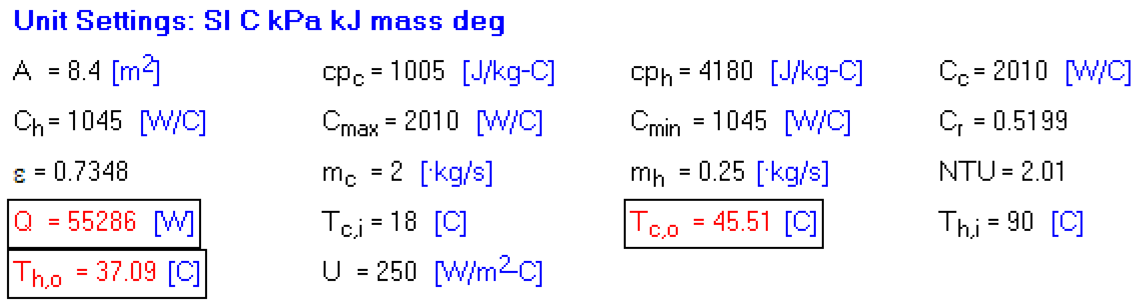
epsilon = 1 – exp ((1/C_r) * NTU^0.22 * (exp(– C_r * NTU^0.78) – 1)) "Effectiveness of Cross-flow HX ..Ref: Incropera."

epsilon = (T_h_i – T_h_o)/(T_h_i – T_c_i) "…determines exit temp of hot fluid, i.e. water, T_h_o"

Q= m_h * cp_h * (T_h_i – T_h_o) "W … determines Q"

Q = m_c * cp_c * (T_c_o – T_c_i) "W…determines exit temp of cold fluid, i.e. air, T_c_o."

Results:



Unit Settings: SI C kPa kJ mass deg

```
A  = 8.4 [m2]          cpc = 1005  [J/kg-C]    cph = 4180  [J/kg-C]    Cc = 2010  [W/C]
Ch = 1045  [W/C]       Cmax = 2010  [W/C]      Cmin  = 1045  [W/C]     Cr  = 0.5199
ε = 0.7348             mc  = 2  [·kg/s]        mh  = 0.25 [·kg/s]      NTU = 2.01
Q  = 55286  [W]        Tc,i = 18  [C]          Tc,o  = 45.51  [C]      Th,i = 90  [C]
Th,o  = 37.09 [C]      U  = 250  [W/m2-C]
```

Thus:

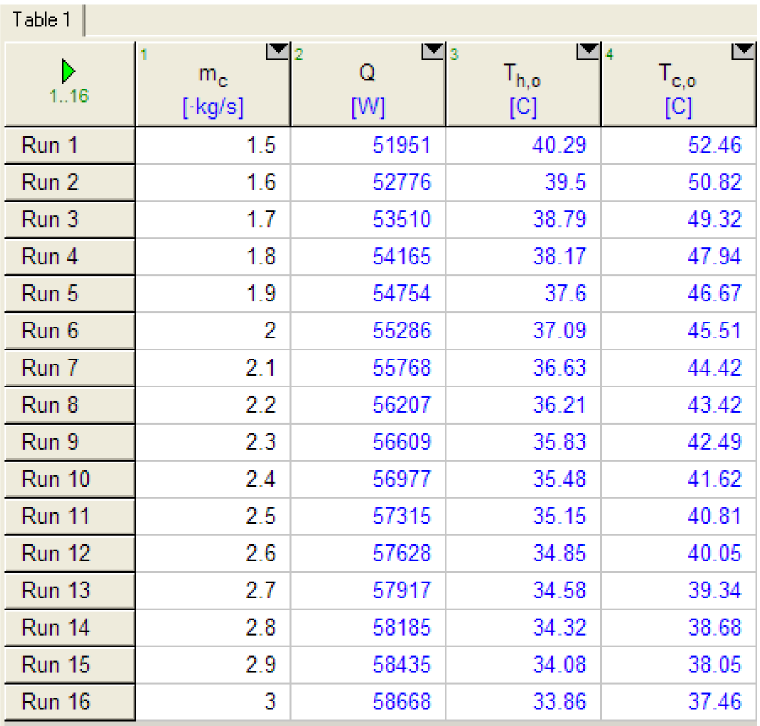
Q = 55286 W … heat transferred … Ans.

Th_o = 37.09 C … exit temp of hot fluid (water) …. Ans.

Tc_o = 45.51 C … exit temp of cold fluid (air) … Ans.

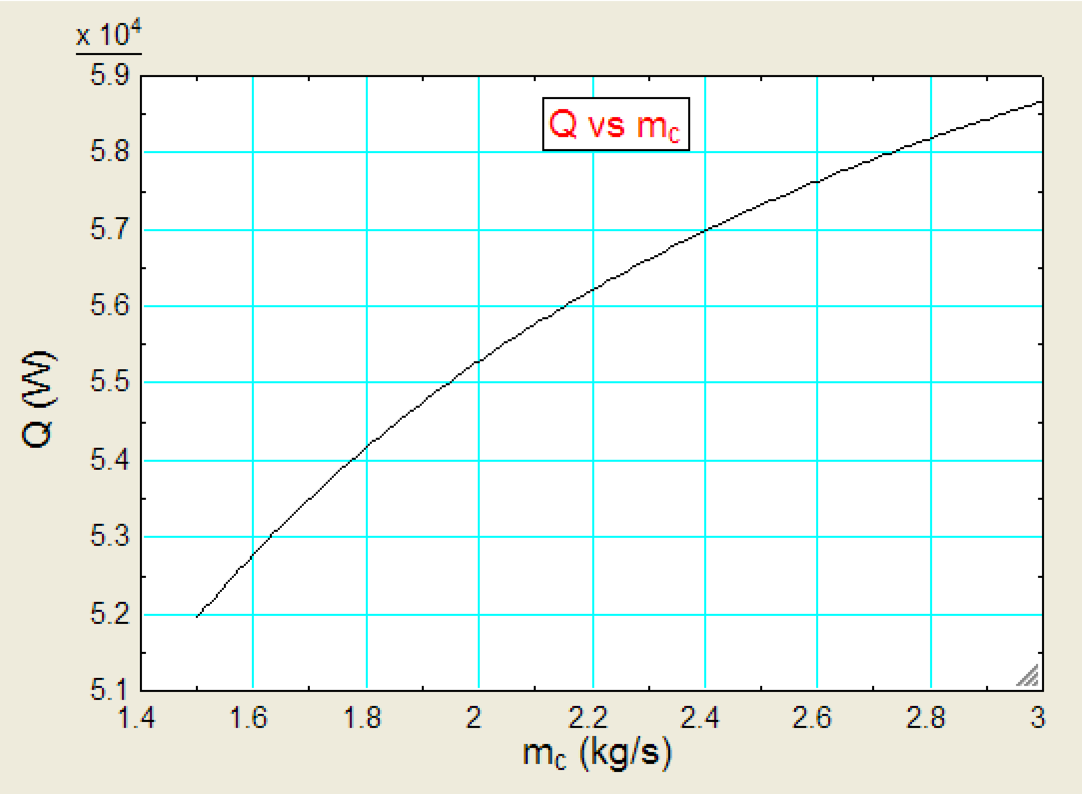
(b) Plot the variation of Q and Th_o and Tc_o as air flow rate, m_c varies from 1.5 to 3 kg/s, all other conditions remaining the same as earlier:

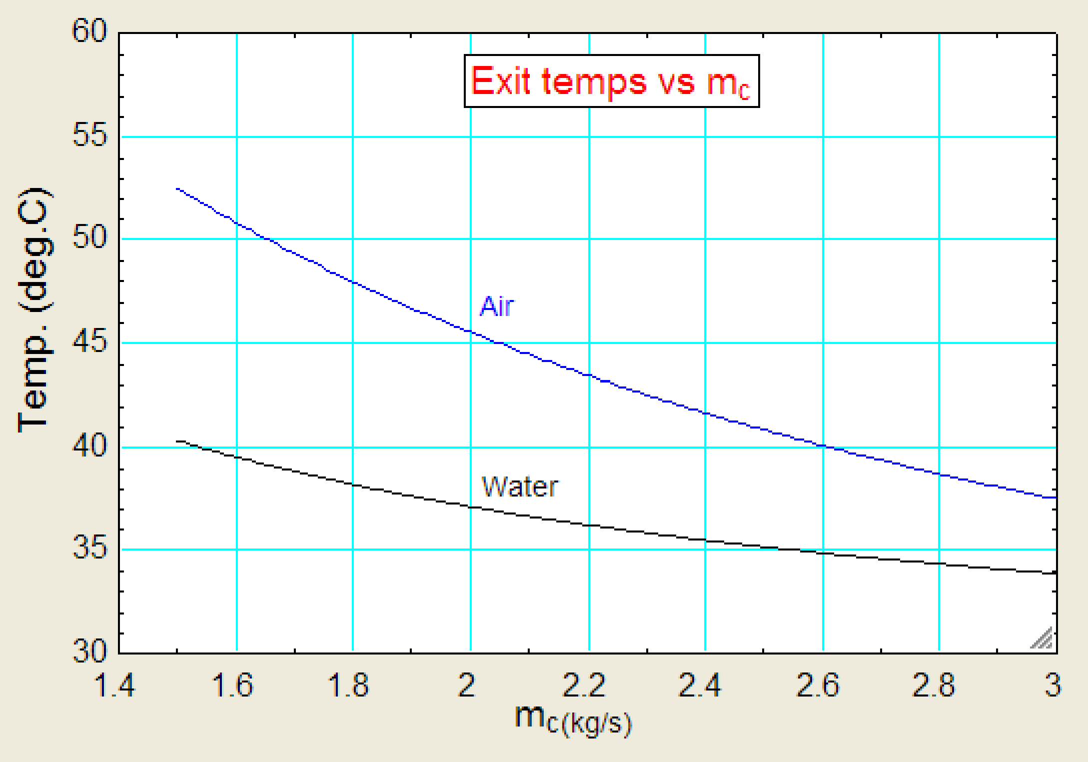
First, compute the Parametric Table:



1..16	1 m_c [kg/s]	2 Q [W]	3 $T_{h,o}$ [C]	4 $T_{c,o}$ [C]
Run 1	1.5	51951	40.29	52.46
Run 2	1.6	52776	39.5	50.82
Run 3	1.7	53510	38.79	49.32
Run 4	1.8	54165	38.17	47.94
Run 5	1.9	54754	37.6	46.67
Run 6	2	55286	37.09	45.51
Run 7	2.1	55768	36.63	44.42
Run 8	2.2	56207	36.21	43.42
Run 9	2.3	56609	35.83	42.49
Run 10	2.4	56977	35.48	41.62
Run 11	2.5	57315	35.15	40.81
Run 12	2.6	57628	34.85	40.05
Run 13	2.7	57917	34.58	39.34
Run 14	2.8	58185	34.32	38.68
Run 15	2.9	58435	34.08	38.05
Run 16	3	58668	33.86	37.46

Next, plot the results:





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"**Prob. 4C.13.** A HX has an effectiveness of 0.5 when the flow is counter-flow and the thermal capacity of one fluid is twice that of the other fluid. Calculate the effectiveness of the HX if the direction of flow of one of the fluids is reversed with the same mass flow rate as before. [VTU – June–July 2011]"

EES Solution:

"Data:"

C_r = 0.5 "…capacity ratio"

"For counter-flow HX:"

epsilon = 0.5 "…by data"

And:

$$\varepsilon = \frac{1 - \exp\left(-NTU_{cflow} \cdot (1 - C_r)\right)}{1 - C_r \cdot \exp\left(-NTU_{cflow} \cdot (1 - C_r)\right)}$$

i.e.

epsilon = (1 – exp (– NTU_cflow * (1 – C_r)))/(1 – C_r * exp(– NTU_cflow * (1 – C_r))) "Effectiveness of Counter-flow HX….Finds NTU"

"Now, for parallel flow HX:"

NTU_parallelflow = NTU_cflow "….since the A, U and Cmin remain same"

$$\varepsilon_{parallelflow} = \frac{1 - \exp\left(-NTU_{parallelflow} \cdot (1 + C_r)\right)}{1 + C_r}$$

i.e.

epsilon_parallelflow = (1 – exp (– NTU_parallelflow * (1 + C_r))) / (1 + C_r) "Effectiveness of parallel-flow HX….Finds epsilon_pflow"

Results:

Unit Settings: SI C kPa kJ mass deg

$C_r = 0.5$

$\varepsilon = 0.5$

$\boxed{\varepsilon_{parallelflow} = 0.4691}$

$NTU_{cflow} = 0.8109$

$NTU_{parallelflow} = 0.8109$

Thus: effectiveness, when it is parallel flow HX = 0.4691 …. Ans.

===

"**Prob. 4C.14.** A cross-flow HX (both fluids unmixed), having a heat transfer area of 30 m^2, is to heat water with engine oil (cp = 2300 J/kg.K). Water enters at 30 C and leaves at 85 C, with a mass flow rate of 1.5 kg/s while engine oil enters at 120 C with a mass flow rate of 3.5 kg/s. Calculate the overall heat transfer coeff. [VTU – June–July 2009]"

"(b) Plot the variation of NTU, U and Th_o as oil flow rate, m_h varies from 3 to 5 kg/s, all other conditions remaining the same as earlier:"

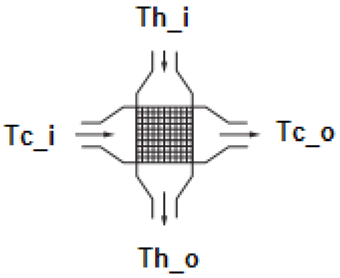


Fig. Prob.4C.14. Cross-flow arrangement

"EES Solution:"

"Data:"

m_h = 3.5 [kg/s] "….engine oil is the hot fluid"
m_c = 1.5[kg/s] "….water is the cold fluid"

Th_i = 120 [C]
Tc_i = 30 [C]
Tc_o = 85 [C]
cp_c = 4183 [J/kg-C]"…sp.heat of water at 55 C"
cp_h = 2300 [J/kg-C]
A = 30 [m^2]

"Calculations:"

Q = m_c * cp_c * (Tc_o – Tc_i) "W … determines heat transferred, Q"
Q= m_h * cp_h * (Th_i – Th_o) "W … determines Th_o"

C_h = m_h * cp_h "W/C…= 8050 …capacity rate of hot fluid"
C_c = m_c * cp_c "W/C… = 6275 …capacity rate of cold fluid"

C_min = C_c "min. capacity rate"
C_max = C_h "max. capacity rate"

C_r = C_min/C_max " Capacity ratio"

"For cross-flow HX:"

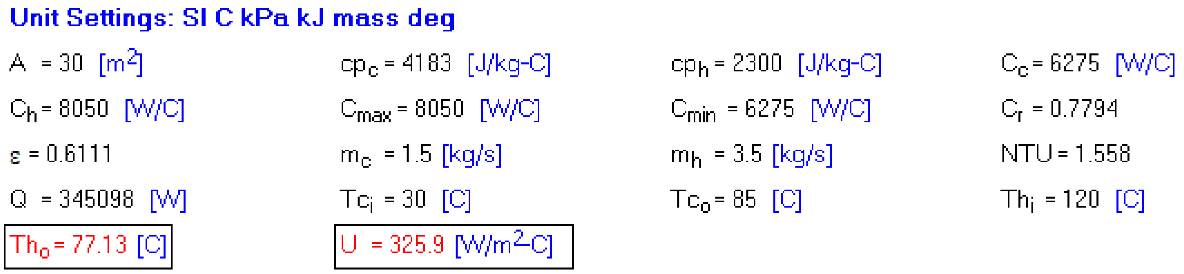
epsilon = (Tc_o – Tc_i) / (Th_i – Tc_i) "..by definition, considering the min. fluid (water)"

"But, we have, for a cross flow HX, with both fluids unmixed: Ref: Incropera"

epsilon = 1 – exp ((1/C_r) * NTU^0.22 * (exp(– C_r * NTU^0.78) – 1)) "Effectiveness of Cross flow HX …… determines NTU"

NTU = U * A /C_min "NTU by definition ….gives U"

Results:



Unit Settings: SI C kPa kJ mass deg

$A = 30$ [m^2] $cp_c = 4183$ [J/kg-C] $cp_h = 2300$ [J/kg-C] $C_c = 6275$ [W/C]

$C_h = 8050$ [W/C] $C_{max} = 8050$ [W/C] $C_{min} = 6275$ [W/C] $C_r = 0.7794$

$\varepsilon = 0.6111$ $m_c = 1.5$ [kg/s] $m_h = 3.5$ [kg/s] $NTU = 1.558$

$Q = 345098$ [W] $Tc_i = 30$ [C] $Tc_o = 85$ [C] $Th_i = 120$ [C]

$\boxed{Th_o = 77.13 \text{ [C]}}$ $\boxed{U = 325.9 \text{ [W/m}^2\text{-C]}}$

Thus:

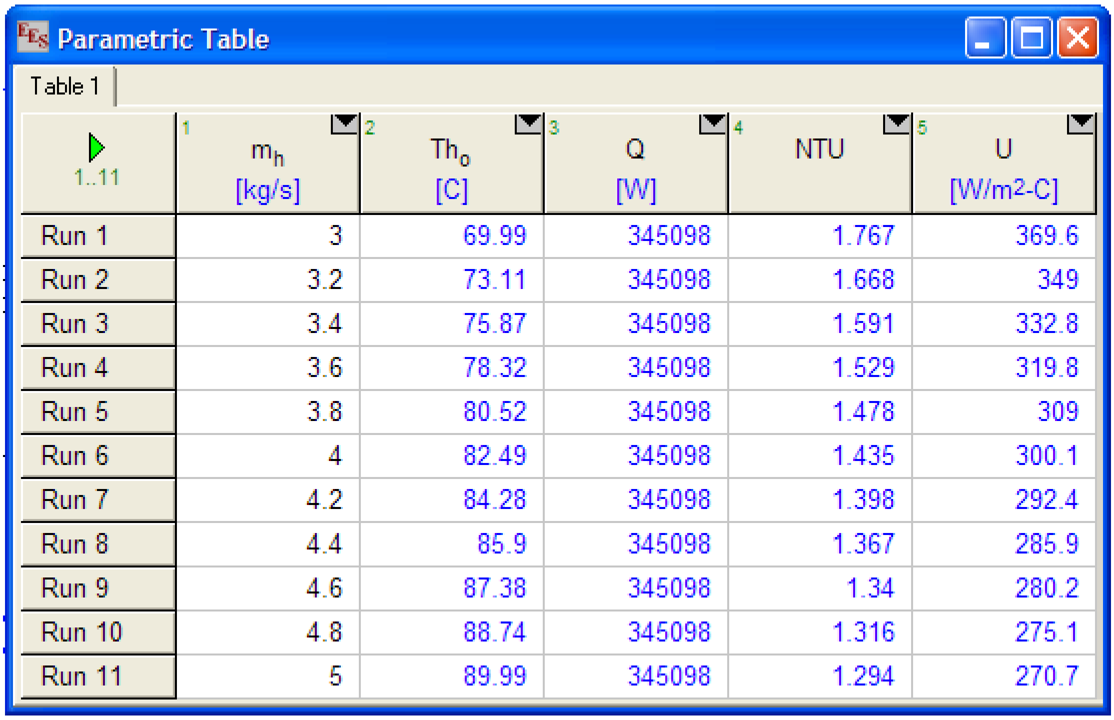
Exit temp of oil (hot fluid), Th_o = 77.13 C … Ans.

Overall heat transfer coeff. U = 325.9 W/m^2.C …. Ans.

=

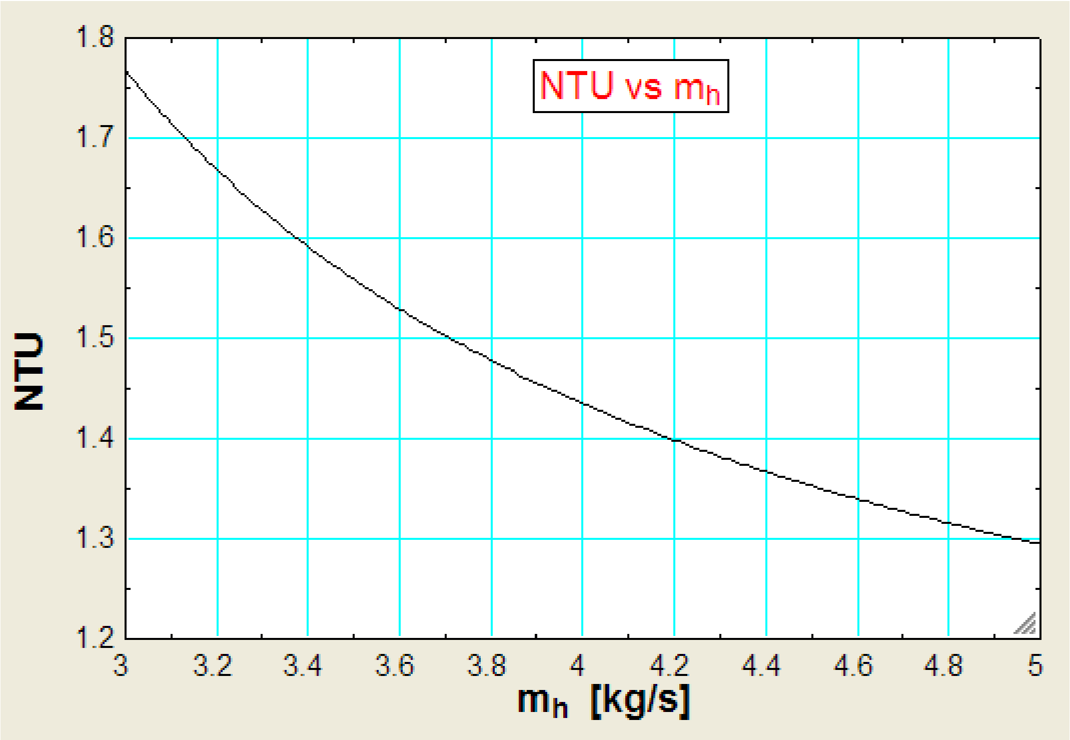
(b) Plot the variation of NTU, U and Th_o as oil flow rate, m_h varies from 3 to 5 kg/s, all other conditions remaining the same as earlier:

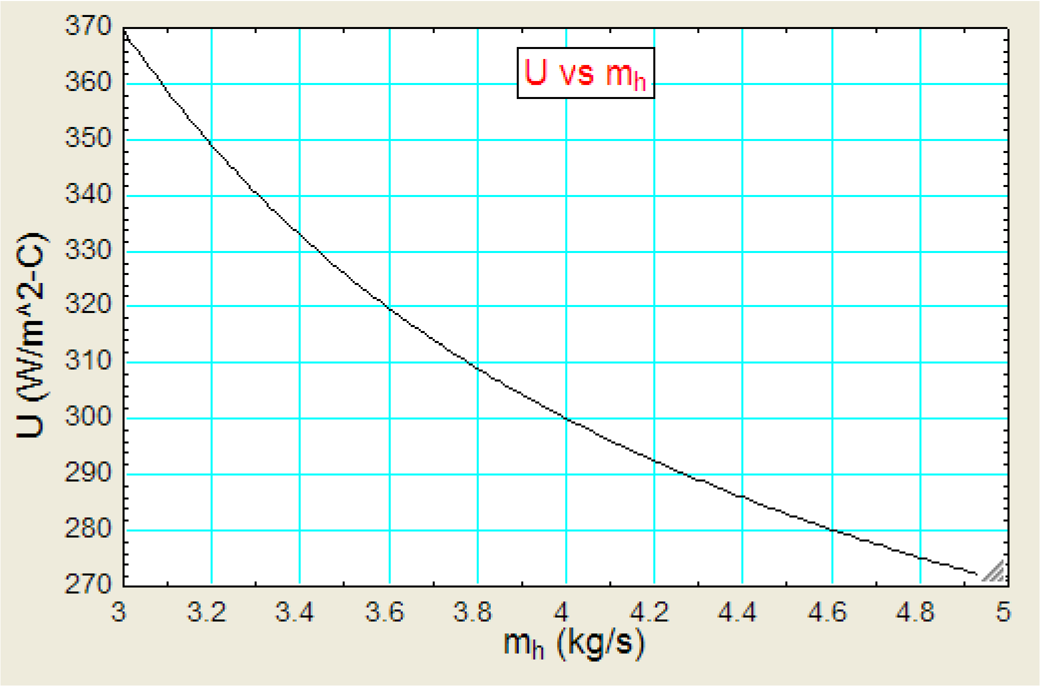
First, prepare the Parametric Table:

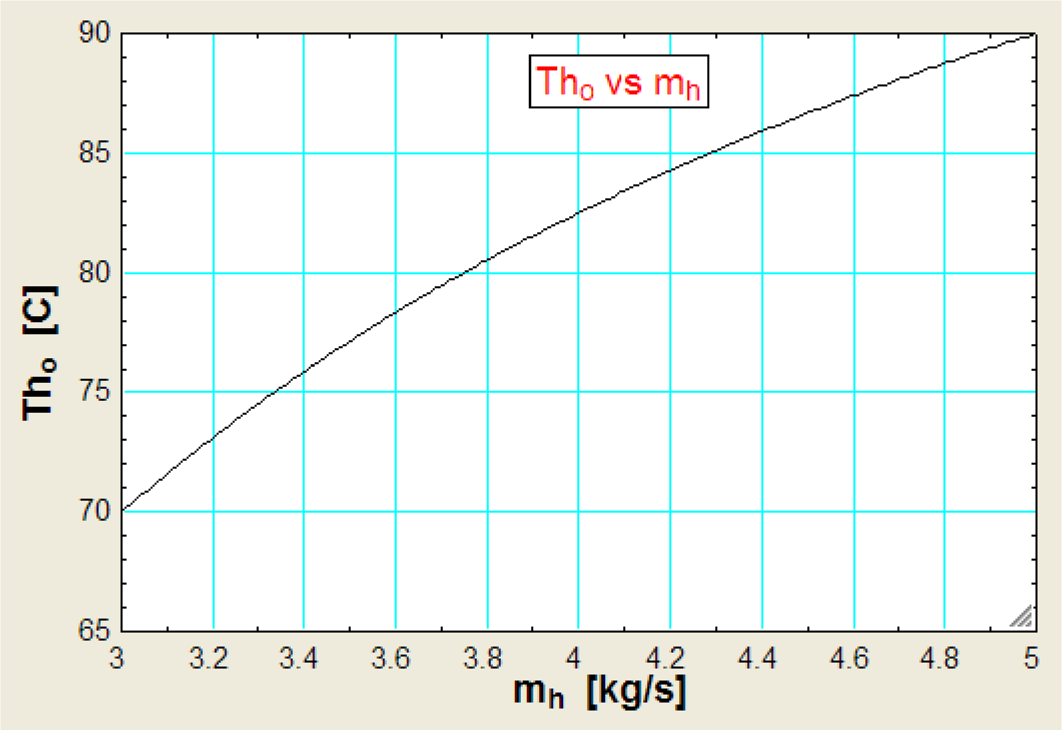


1..11	1 m_h [kg/s]	2 Th_o [C]	3 Q [W]	4 NTU	5 U [W/m2-C]
Run 1	3	69.99	345098	1.767	369.6
Run 2	3.2	73.11	345098	1.668	349
Run 3	3.4	75.87	345098	1.591	332.8
Run 4	3.6	78.32	345098	1.529	319.8
Run 5	3.8	80.52	345098	1.478	309
Run 6	4	82.49	345098	1.435	300.1
Run 7	4.2	84.28	345098	1.398	292.4
Run 8	4.4	85.9	345098	1.367	285.9
Run 9	4.6	87.38	345098	1.34	280.2
Run 10	4.8	88.74	345098	1.316	275.1
Run 11	5	89.99	345098	1.294	270.7

Now, plot the results:







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Prob. 4C.15. A parallel flow HX has following data: m_h = 10 kg/min, m_c = 25 kg/min, cp_h = cp_c = 4180 J/kg.C, Th_1 = 70 , Th_2 = 50 C, Tc_1 = 25 C. Individual heat transfer coeff on hot and cold side are both equal to 60 W/m^2.C. Find the area of the HX.

(b) If the hot water flow is doubled (inlet conditions and area remaining the same), plot the exit temps of the two fluids and the effectiveness of HX as m_h varies from 10 to 20 kg/min.

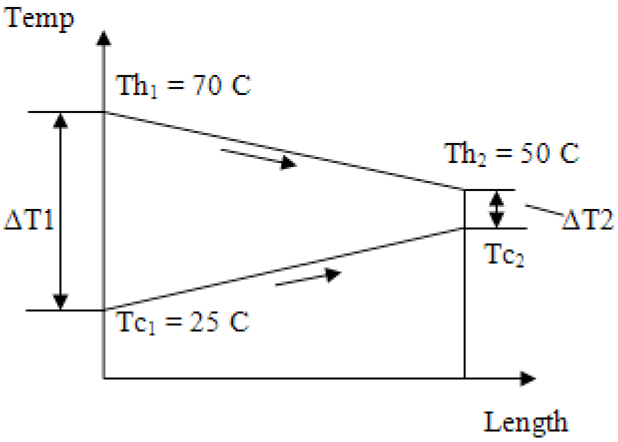
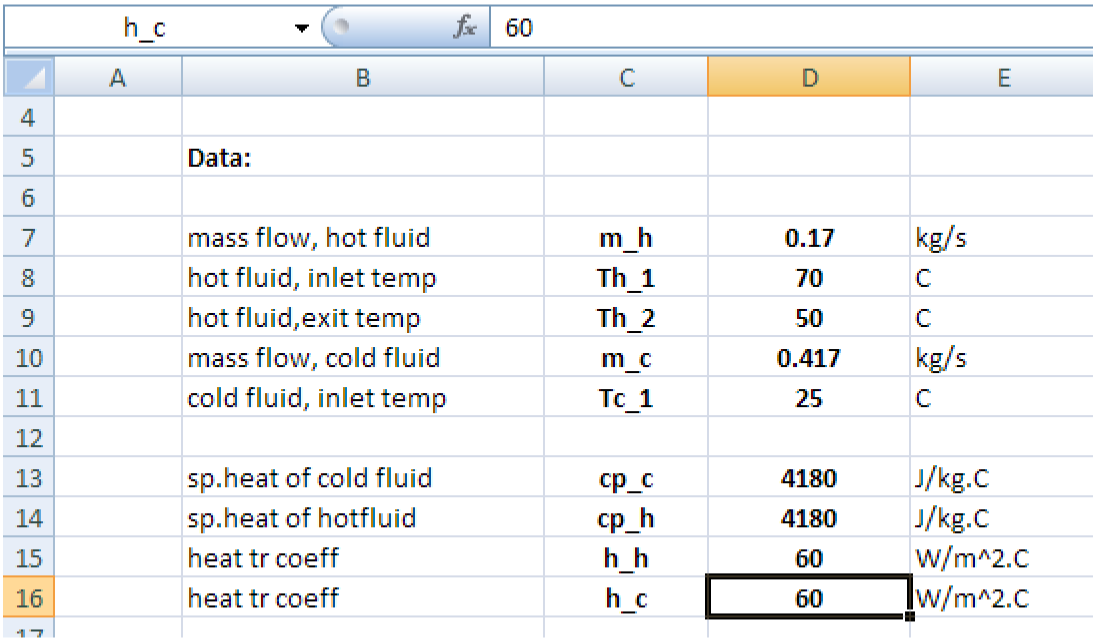


Fig. Prob.4C.15. Parallel flow arrangement

EXCEL Solution:

Following are the steps in EXCEL Solution:

1. Set up the EXCEL worksheet, enter data and name the cells:



	A	B	C	D	E
4					
5		**Data:**			
6					
7		mass flow, hot fluid	**m_h**	**0.17**	kg/s
8		hot fluid, inlet temp	**Th_1**	**70**	C
9		hot fluid,exit temp	**Th_2**	**50**	C
10		mass flow, cold fluid	**m_c**	**0.417**	kg/s
11		cold fluid, inlet temp	**Tc_1**	**25**	C
12					
13		sp.heat of cold fluid	**cp_c**	**4180**	J/kg.C
14		sp.heat of hotfluid	**cp_h**	**4180**	J/kg.C
15		heat tr coeff	**h_h**	**60**	W/m^2.C
16		heat tr coeff	**h_c**	**60**	W/m^2.C

2. Perform the calculations. Equations used are given below:

$U = \frac{1}{h_h} + \frac{1}{h_c}$ ...overall heat tr coeff.

$Q = m_h \cdot cp_h \cdot (Th_1 - Th_2)$ ....total heat transferred

$Tc_2 = Tc_1 + \frac{Q}{m_c \cdot cp_c}$ ...exit temp of cold fluid

$C_h = m_h \cdot cp_h$ ...capacity rate, hot fluid

$C_c = m_c \cdot cp_c$ ...capacity rate, cold fluid

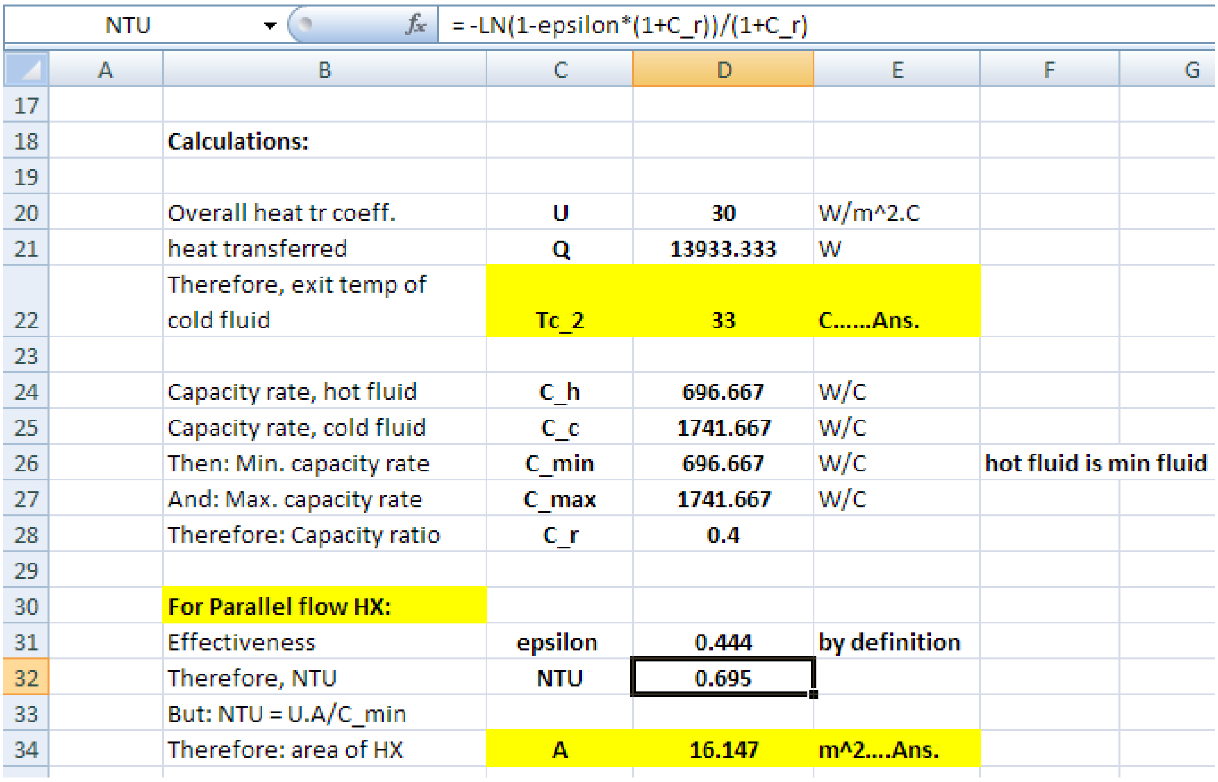
$C_r = \frac{C_{min}}{C_{max}}$ ...capacity ratio

$\varepsilon = \frac{Th_1 - Th_2}{Th_1 - Tc_1}$ ..since hot fluid is the 'min. fluid'.

$NTU = \frac{-\ln\left[1 - \varepsilon \cdot (1 + C_r)\right]}{1 + C_r}$ ...for a patrallel flow HX

$A = \frac{NTU \cdot C_{min}}{U}$ ...area of HX, by definition of NTU = U * A / C_{min}

Above equations are entered in calculations, as shown below:



NTU =-LN(1-epsilon*(1+C_r))/(1+C_r)

	A	B	C	D	E	F	G
17							
18		**Calculations:**					
19							
20		Overall heat tr coeff.	**U**	**30**	W/m^2.C		
21		heat transferred	**Q**	**13933.333**	W		
22		Therefore, exit temp of cold fluid	**Tc_2**	**33**	**C......Ans.**		
23							
24		Capacity rate, hot fluid	**C_h**	**696.667**	W/C		
25		Capacity rate, cold fluid	**C_c**	**1741.667**	W/C		
26		Then: Min. capacity rate	**C_min**	**696.667**	W/C	**hot fluid is min fluid**	
27		And: Max. capacity rate	**C_max**	**1741.667**	W/C		
28		Therefore: Capacity ratio	**C_r**	**0.4**			
29							
30		**For Parallel flow HX:**					
31		Effectiveness	**epsilon**	**0.444**	**by definition**		
32		Therefore, NTU	**NTU**	**0.695**			
33		But: NTU = U.A/C_min					
34		Therefore: area of HX	**A**	**16.147**	**m^2....Ans.**		

Note: Formula entered for NTU can be seen in the Formula bar in the above screen shot.

Thus:

exit temp of cold fluid = Tc_2 = 33 C … Ans.

Area of HX = A = 16.147 m^2 … Ans.

(b) If the hot water flow is doubled (inlet conditions and area remaining the same), plot the exit temps of the two fluids and the effectiveness of HX as m_h varies from 10 to 20 kg/min.

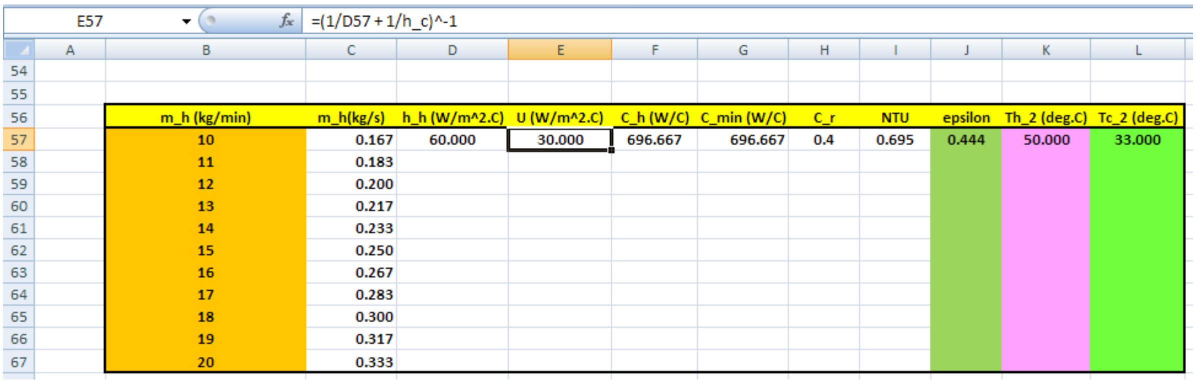
3. First, prepare a Table as shown below.

Two things must be kept in mind:

(a)When the flow of hot fluid changes, Reynolds No. will change, and therefore the heat transfer coeff. h_h will also change, and this is proportional to 0.8 power of mass flow.

i.e. new h_h will be h_h at m_h = 10 kg/min multiplied by (New mass flow rate / 10)^0.8

(b) all formulas entered must have reference to m_h by relative reference, so that we can easily extrapolate the calculations to other values of m_h, by 'drag-copy':

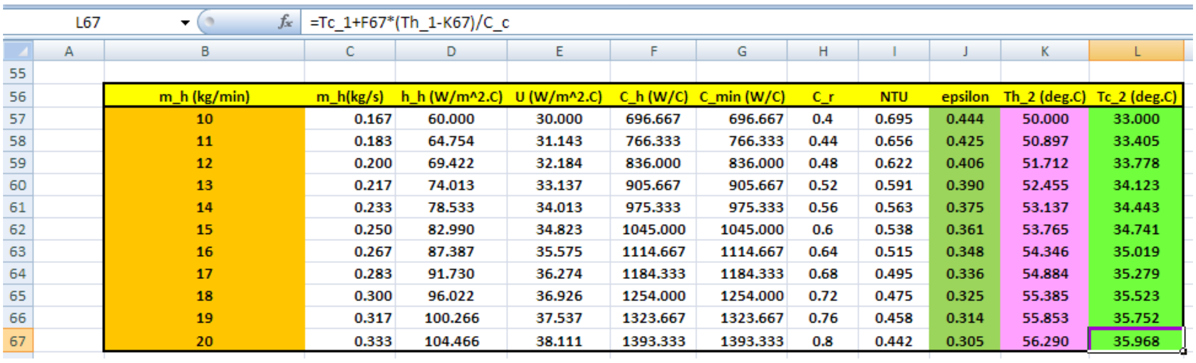


E57 =(1/D57 + 1/h_c)^-1

	A	B	C	D	E	F	G	H	I	J	K	L
54												
55												
56		m_h (kg/min)	m_h(kg/s)	h_h (W/m^2.C)	U (W/m^2.C)	C_h (W/C)	C_min (W/C)	C_r	NTU	epsilon	Th_2 (deg.C)	Tc_2 (deg.C)
57		10	0.167	60.000	30.000	696.667	696.667	0.4	0.695	0.444	50.000	33.000
58		11	0.183									
59		12	0.200									
60		13	0.217									
61		14	0.233									
62		15	0.250									
63		16	0.267									
64		17	0.283									
65		18	0.300									
66		19	0.317									
67		20	0.333									

In the above screen shot, note the formula entered in cell E57 for U in the formula bar. Similarly, for other quantities in the row 57, use relative reference to m_h.

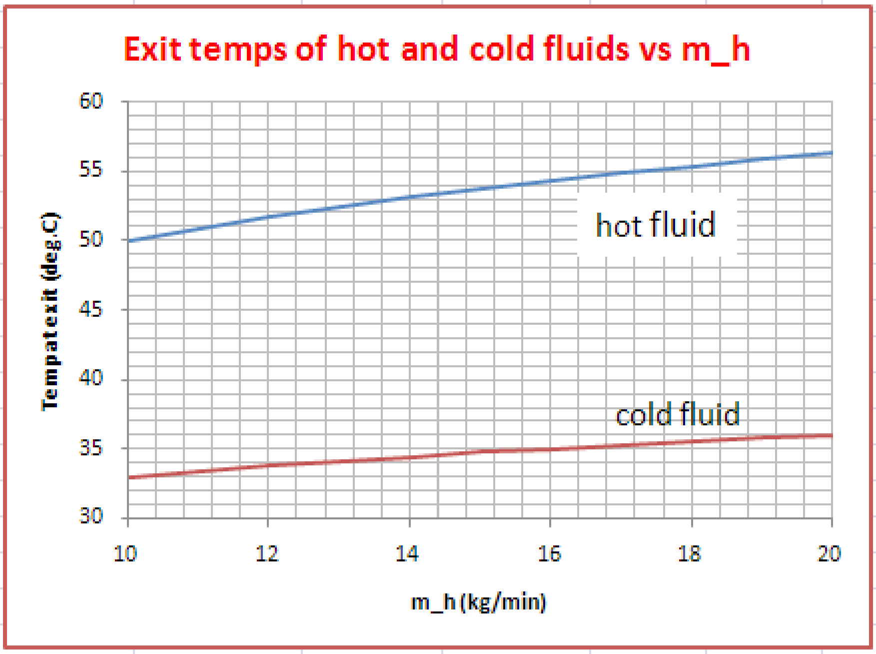
Now, select the cells D57 to L57 and 'drag-copy' to the end of Table, i.e. up to cell L67, and immediately, all calculations are made and the Table is filled up:

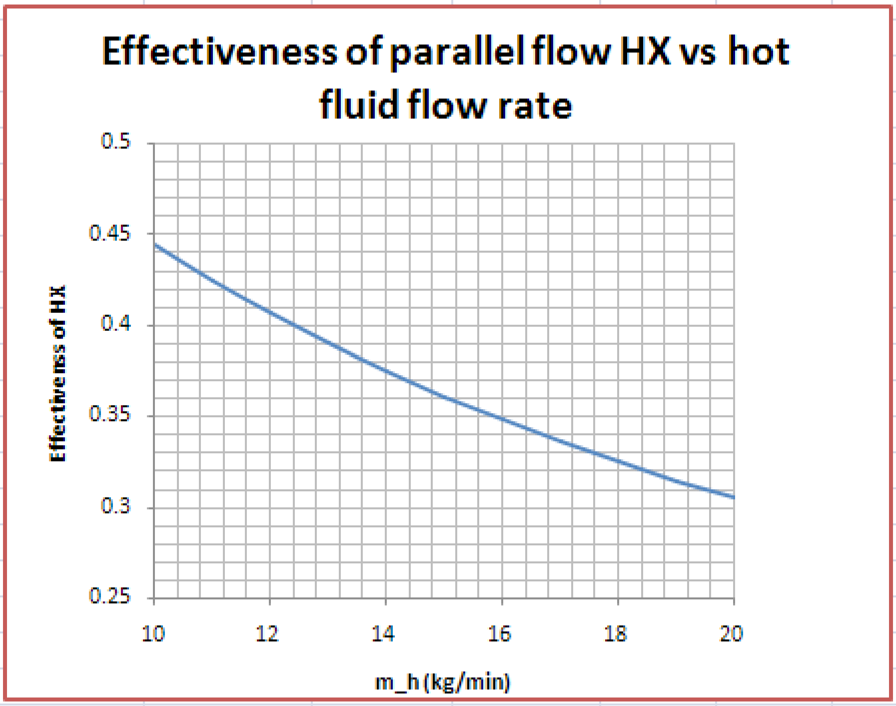


L67 =Tc_1+F67*(Th_1-K67)/C_c

	A	B	C	D	E	F	G	H	I	J	K	L
55												
56		m_h (kg/min)	m_h(kg/s)	h_h (W/m^2.C)	U (W/m^2.C)	C_h (W/C)	C_min (W/C)	C_r	NTU	epsilon	Th_2 (deg.C)	Tc_2 (deg.C)
57		10	0.167	60.000	30.000	696.667	696.667	0.4	0.695	0.444	50.000	33.000
58		11	0.183	64.754	31.143	766.333	766.333	0.44	0.656	0.425	50.897	33.405
59		12	0.200	69.422	32.184	836.000	836.000	0.48	0.622	0.406	51.712	33.778
60		13	0.217	74.013	33.137	905.667	905.667	0.52	0.591	0.390	52.455	34.123
61		14	0.233	78.533	34.013	975.333	975.333	0.56	0.563	0.375	53.137	34.443
62		15	0.250	82.990	34.823	1045.000	1045.000	0.6	0.538	0.361	53.765	34.741
63		16	0.267	87.387	35.575	1114.667	1114.667	0.64	0.515	0.348	54.346	35.019
64		17	0.283	91.730	36.274	1184.333	1184.333	0.68	0.495	0.336	54.884	35.279
65		18	0.300	96.022	36.926	1254.000	1254.000	0.72	0.475	0.325	55.385	35.523
66		19	0.317	100.266	37.537	1323.667	1323.667	0.76	0.458	0.314	55.853	35.752
67		20	0.333	104.466	38.111	1393.333	1393.333	0.8	0.442	0.305	56.290	35.968

Now, plot the graphs in EXCEL:





==

Prob. 4C.16. A Shell & Tube type of steam condenser has following data: Total heat transferred Q = 2100 MW, no. of shell passes = 1, no. of tube passes = 2, no. of tubes (thin walled) = 31500, tube dia = 25 mm, total mass flow rate of water through tubes = 3.4 × 10^4 kg/s, condensation temp of steam = 50 C, inlet temp of water = 20 C, heat transfer coeff on steam side = 11400 W/m^2.C. Find the exit temp of water and the length of tube per pass.

(b) Plot the variation of Tc_2, effectiveness and L_tube as mass flow rate of water, m_c varies from 20000 to 50000 kg/s:

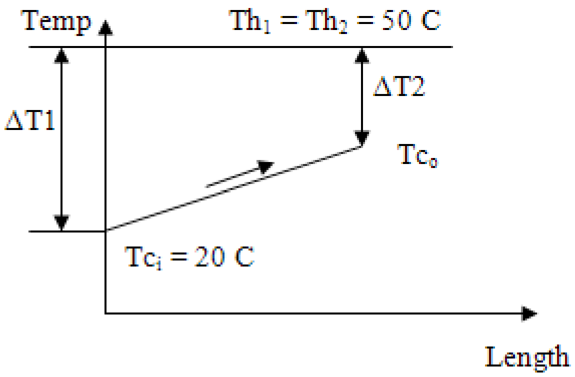
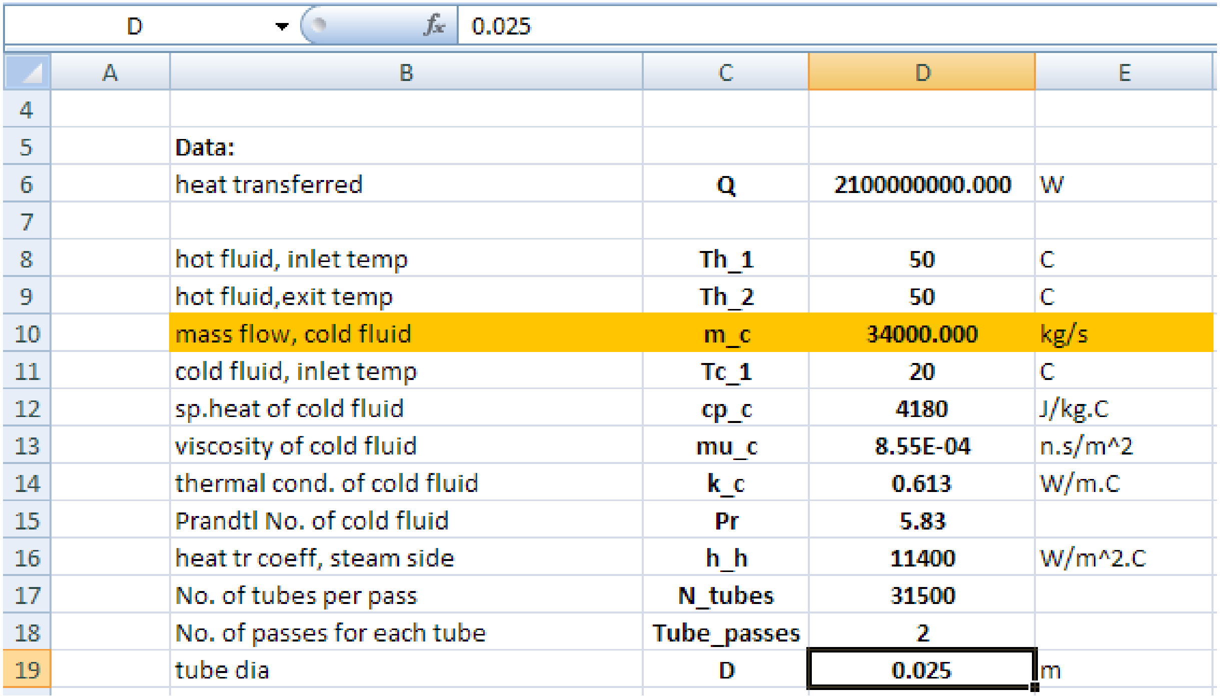


Fig. Prob.4C.16. Steam Condenser

EXCEL Solution:

Following are the steps in EXCEL Solution:

1. Set up the EXCEL worksheet, enter data and name the cells:



	A	B	C	D	E
4					
5		**Data:**			
6		heat transferred	**Q**	**2100000000.000**	W
7					
8		hot fluid, inlet temp	**Th_1**	**50**	C
9		hot fluid,exit temp	**Th_2**	**50**	C
10		mass flow, cold fluid	**m_c**	**34000.000**	kg/s
11		cold fluid, inlet temp	**Tc_1**	**20**	C
12		sp.heat of cold fluid	**cp_c**	**4180**	J/kg.C
13		viscosity of cold fluid	**mu_c**	**8.55E-04**	n.s/m^2
14		thermal cond. of cold fluid	**k_c**	**0.613**	W/m.C
15		Prandtl No. of cold fluid	**Pr**	**5.83**	
16		heat tr coeff, steam side	**h_h**	**11400**	W/m^2.C
17		No. of tubes per pass	**N_tubes**	**31500**	
18		No. of passes for each tube	**Tube_passes**	**2**	
19		tube dia	**D**	**0.025**	m

2. Perform the calculations. Equations used are shown below:

$$Tc_2 = Tc_1 + \frac{Q}{m_c \cdot cp_c}$$ ....exit temp of cold fluid (i.e. water)

$$m_{dot} = \frac{m_c}{N_{tubes}}$$ ...mass flow rate through each tube

$$A_c = \frac{\pi \cdot D^2}{4}$$ ....area of cross-section of tube

$$Re = \frac{m_{dot}}{A_c} \cdot \frac{D}{mu_c}$$ ...Reynolds No.

$$Nu = 0.023 \cdot Re^{0.8} \cdot Pr^{0.4}$$ .Nusselts No. by..Dittus-Boelter eqn.

$$h_c = \frac{Nu \cdot k_c}{D}$$ ....heat tr coeff on the water side

$$U = \left(\frac{1}{h_h} + \frac{1}{h_c}\right)^{-1}$$ ...Overall heat tr coeff.

$$C_c = m_c \cdot cp_c$$ ...capacity rate of cold fluid

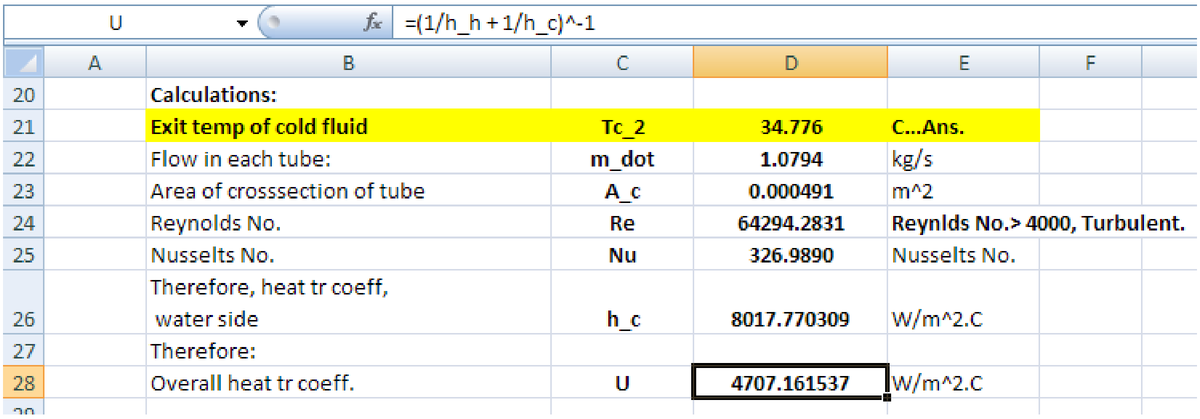
$$\varepsilon = \frac{Tc_2 - Tc_1}{Th_1 - Tc_1}$$ ...by definition, considering the 'min. fluid'

$$NTU = -\ln(1 - \varepsilon)$$ ....for a Condenser

$$A_{HX} = \frac{NTU \cdot C_{min}}{U}$$ ...total area of HX required

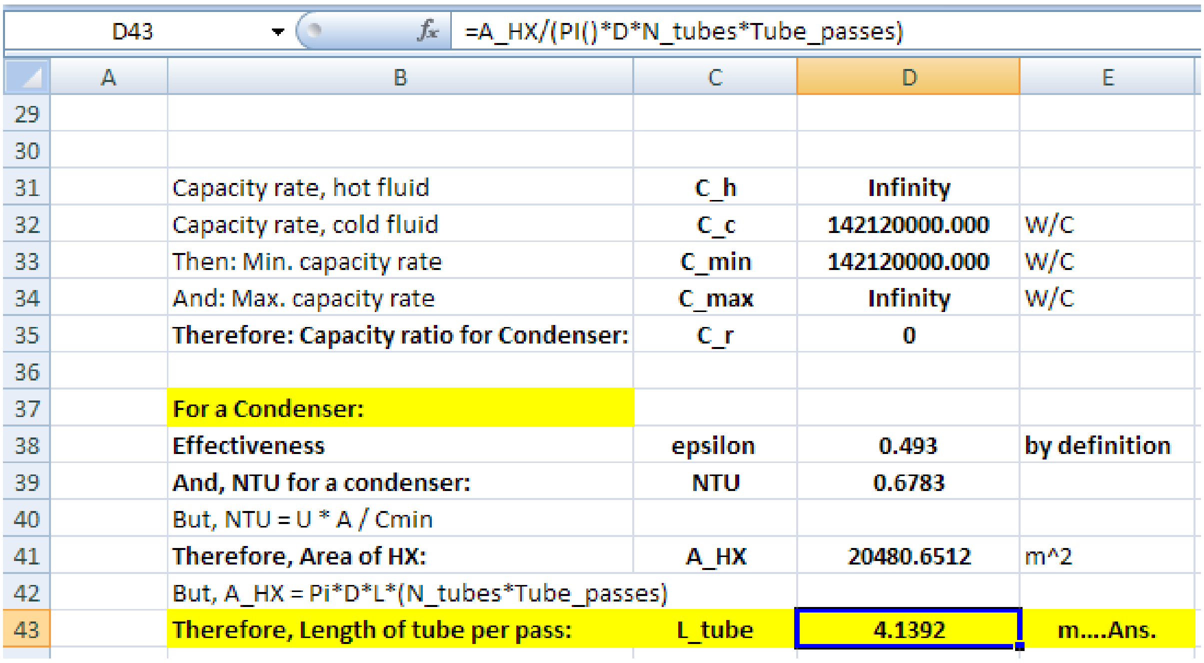
$$L_{tube} = \frac{A_{HX}}{\pi \cdot D \cdot N_{tubes} \cdot Tube\_passes}$$ ...Length of tube per pass

We get following results:



U =(1/h_h + 1/h_c)^-1

	A	B	C	D	E	F
20		**Calculations:**				
21		**Exit temp of cold fluid**	**Tc_2**	**34.776**	**C...Ans.**	
22		Flow in each tube:	**m_dot**	**1.0794**	kg/s	
23		Area of crosssection of tube	**A_c**	**0.000491**	m^2	
24		Reynolds No.	**Re**	**64294.2831**	**Reynlds No.> 4000, Turbulent.**	
25		Nusselts No.	**Nu**	**326.9890**	Nusselts No.	
26		Therefore, heat tr coeff, water side	**h_c**	**8017.770309**	W/m^2.C	
27		Therefore:				
28		Overall heat tr coeff.	**U**	**4707.161537**	W/m^2.C	



D43 =A_HX/(PI()*D*N_tubes*Tube_passes)

	A	B	C	D	E
29					
30					
31		Capacity rate, hot fluid	**C_h**	**Infinity**	
32		Capacity rate, cold fluid	**C_c**	**142120000.000**	W/C
33		Then: Min. capacity rate	**C_min**	**142120000.000**	W/C
34		And: Max. capacity rate	**C_max**	**Infinity**	W/C
35		**Therefore: Capacity ratio for Condenser:**	**C_r**	**0**	
36					
37		**For a Condenser:**			
38		**Effectiveness**	**epsilon**	**0.493**	**by definition**
39		**And, NTU for a condenser:**	**NTU**	**0.6783**	
40		But, NTU = U * A / Cmin			
41		**Therefore, Area of HX:**	**A_HX**	**20480.6512**	m^2
42		But, A_HX = Pi*D*L*(N_tubes*Tube_passes)			
43		**Therefore, Length of tube per pass:**	**L_tube**	**4.1392**	**m....Ans.**

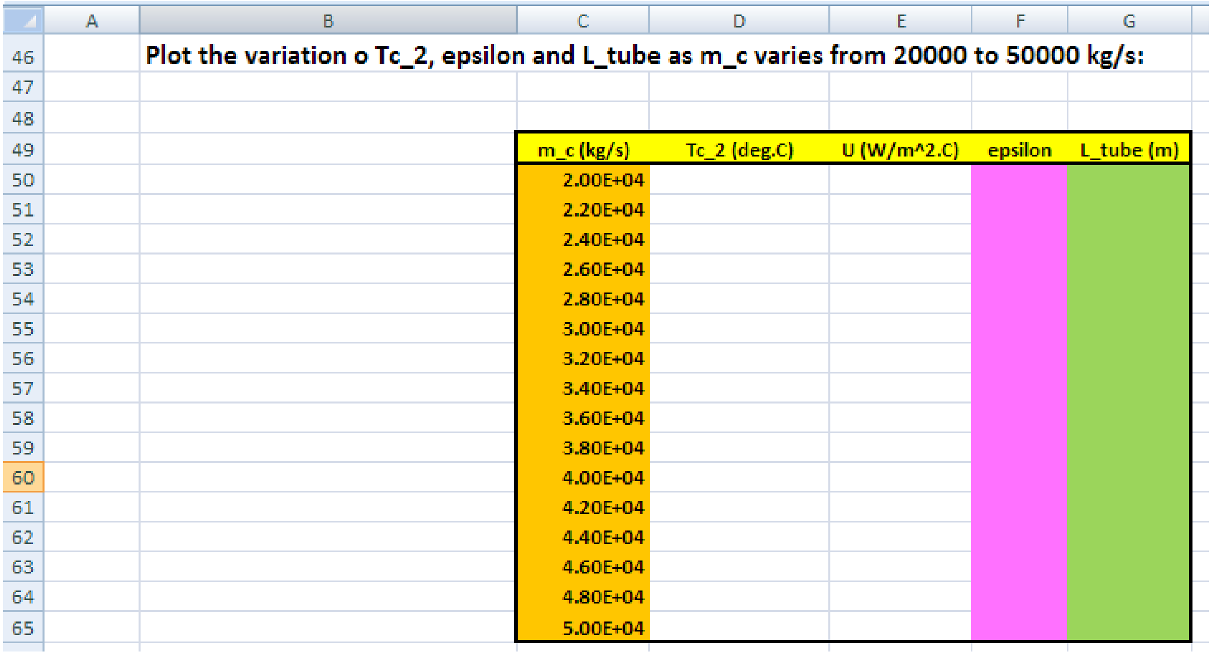
Thus:

Exit temp of water = Tc_2 = 34.776 C …Ans.

Length of tubes per pass + L_tube = 4.139 m … Ans.

(b) Plot the variation of Tc_2, effectiveness and L_tube as mass flow rate of water, m_c varies from 20000 to 50000 kg/s:

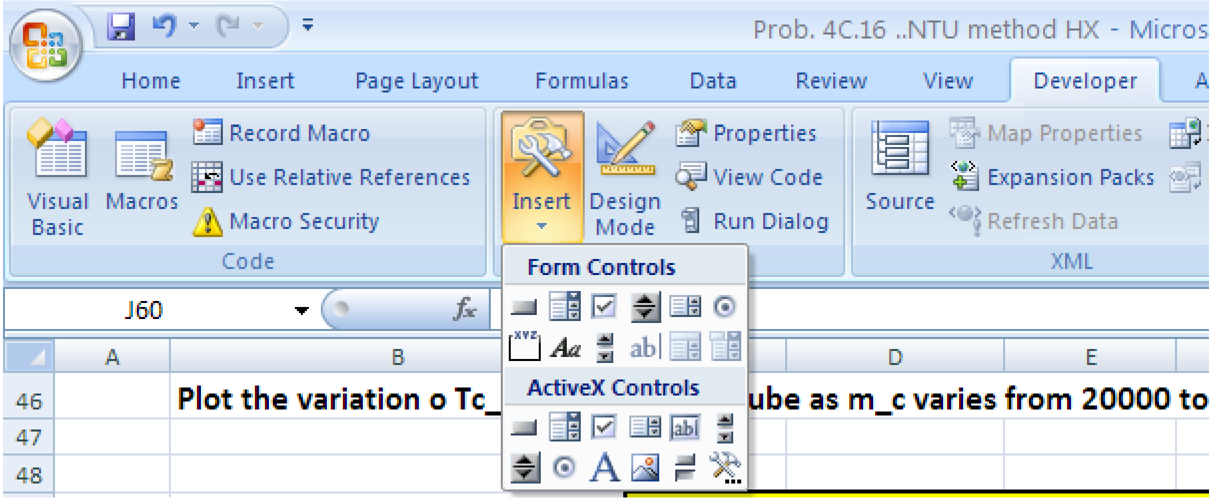
1. First, prepare a Table as shown:



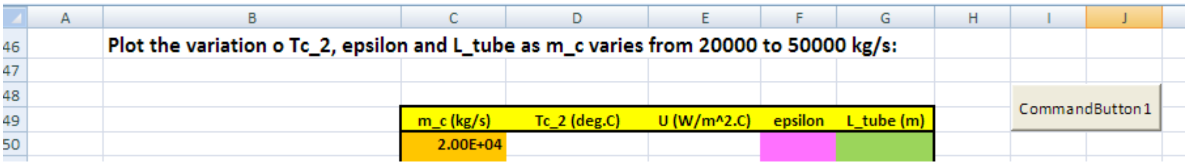
	A	B	C	D	E	F	G
46		Plot the variation o Tc_2, epsilon and L_tube as m_c varies from 20000 to 50000 kg/s:					
47							
48							
49			m_c (kg/s)	Tc_2 (deg.C)	U (W/m^2.C)	epsilon	L_tube (m)
50			2.00E+04				
51			2.20E+04				
52			2.40E+04				
53			2.60E+04				
54			2.80E+04				
55			3.00E+04				
56			3.20E+04				
57			3.40E+04				
58			3.60E+04				
59			3.80E+04				
60			4.00E+04				
61			4.20E+04				
62			4.40E+04				
63			4.60E+04				
64			4.80E+04				
65			5.00E+04				

2. Now, let us write a VBA program to read the values of m_c from the Table, one by one, and copy to cell D10 in the EXCEL worksheet just completed. As the value of m_c is changed, immediately, all other values will automatically update themselves in the worksheet. Now, for each value of m_c, copy the calculated values of Tc_2, U, epsilon and L_tube to the respective cells in the Table.
3. We proceed as follows to write the VBA program:

First, let us have a 'control button' to operate the program: Go to Developer – Insert – ActiveX Controls:



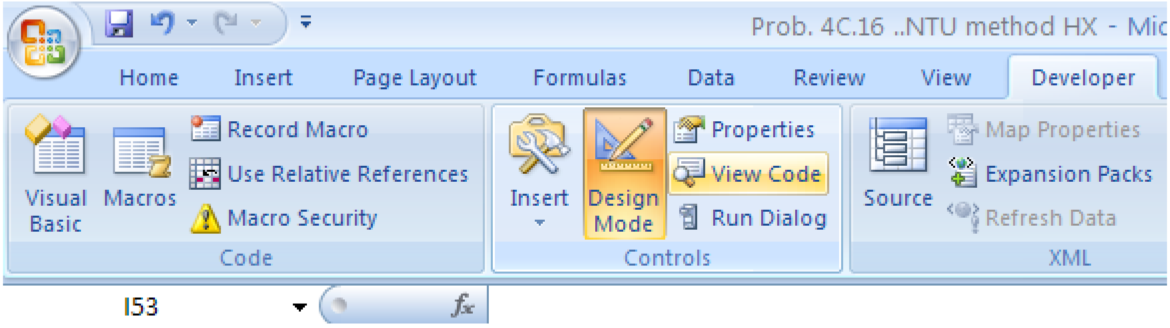
Click on the first, left top button under AciveX Controls. And, draw a command button at the required place to the required size:



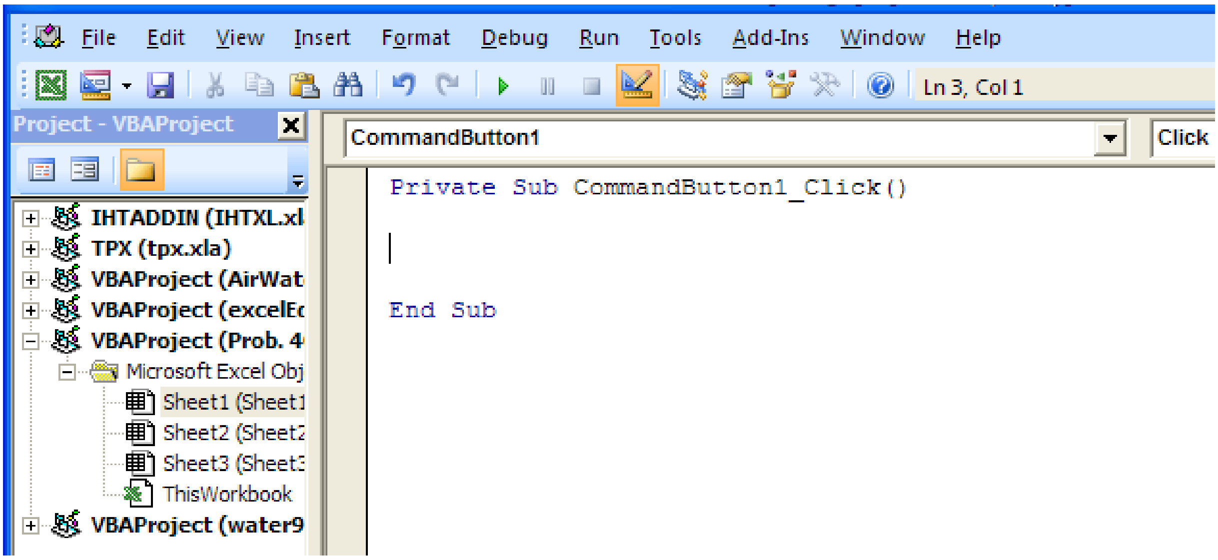
Plot the variation o Tc_2, epsilon and L_tube as m_c varies from 20000 to 50000 kg/s:

m_c (kg/s)	Tc_2 (deg.C)	U (W/m^2.C)	epsilon	L_tube (m)
2.00E+04				

Now, go to Design Mode, and press 'View Code':



We get:



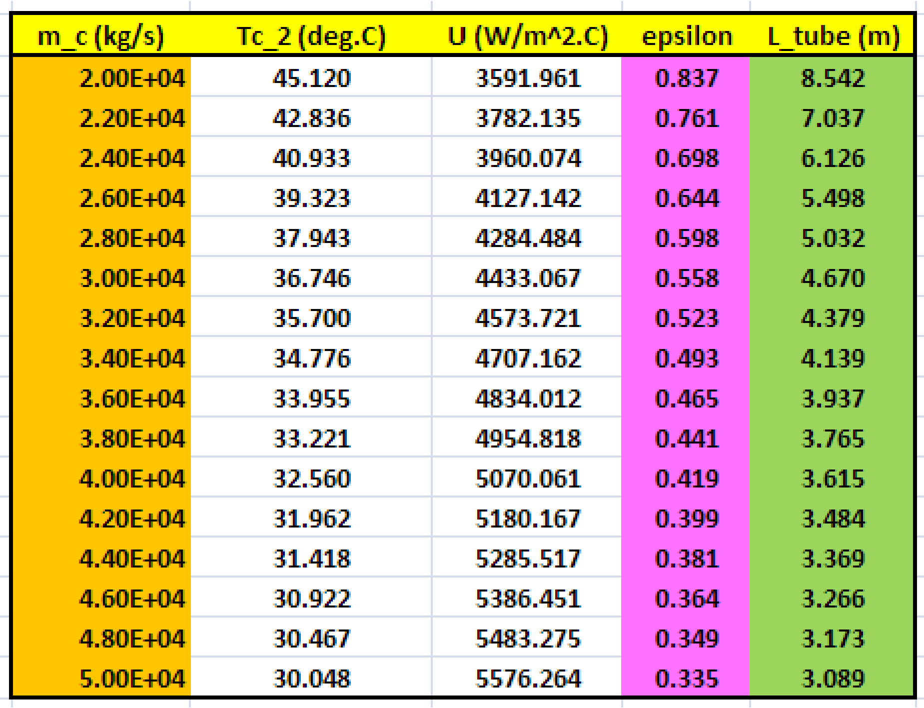
```
Private Sub CommandButton1_Click()


End Sub
```

4. Now, write the following code, which will do the desired job as explained above:

```
Private Sub CommandButton1_Click()

Dim i As Integer

For i = 0 To 15 '...there are 16 rows in the Table

   Range("D10") = Cells(50 + i, 3) 'copies first value of m_c to cell D10
   Cells(50 + i, 4) = Range("D21") 'copies the calculated value of Tc_2 to Table
   Cells(50 + i, 5) = Range("D28") 'copies the calculated value of U to Table
   Cells(50 + i, 6) = Range("D38") 'copies the calculated value of epsilon to Table
   Cells(50 + i, 7) = Range("D43") 'copies the calculated value of L_tube to Table

Next i

End Sub
```

Read the comments given in simple the program above.

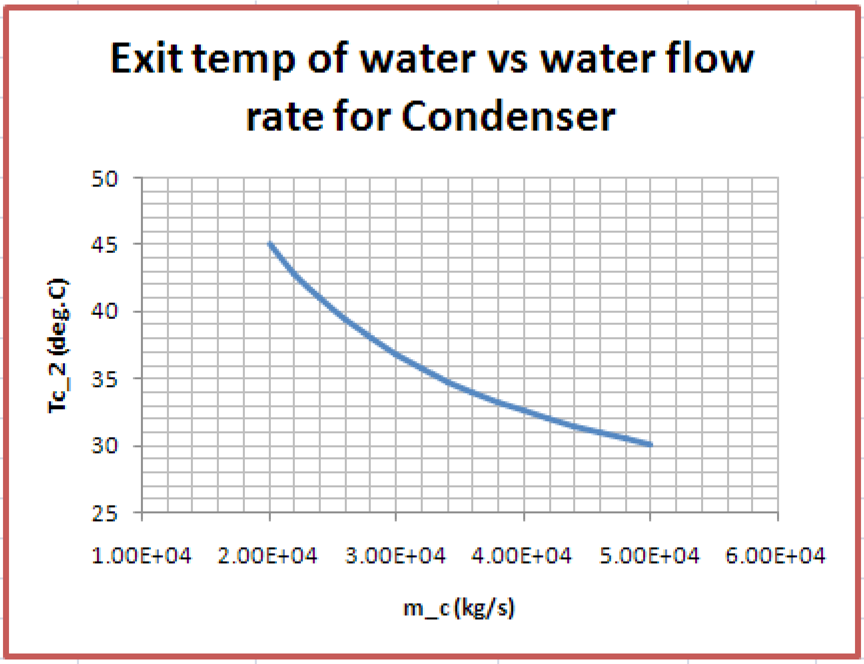
5. **Now, press the Command Button, and the Table immediately gets filled up:**

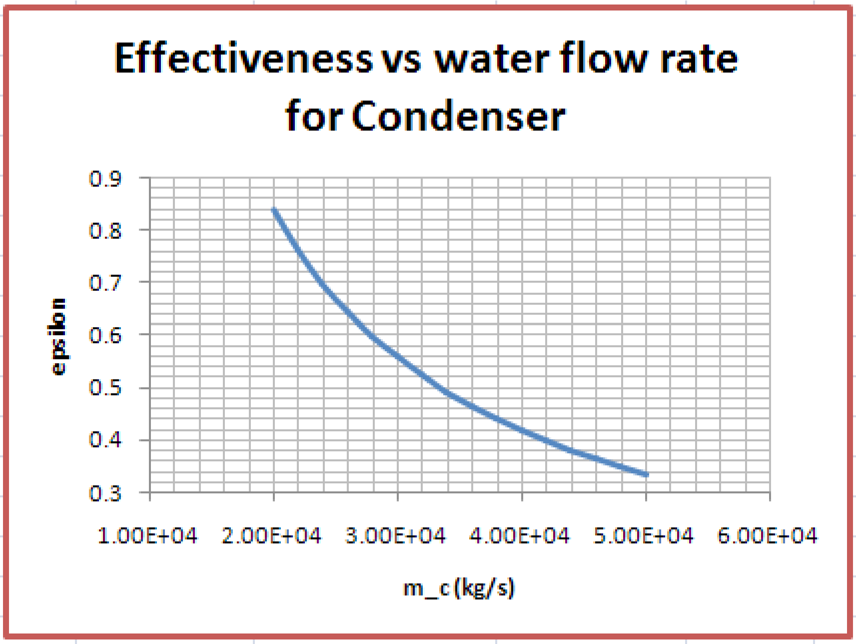


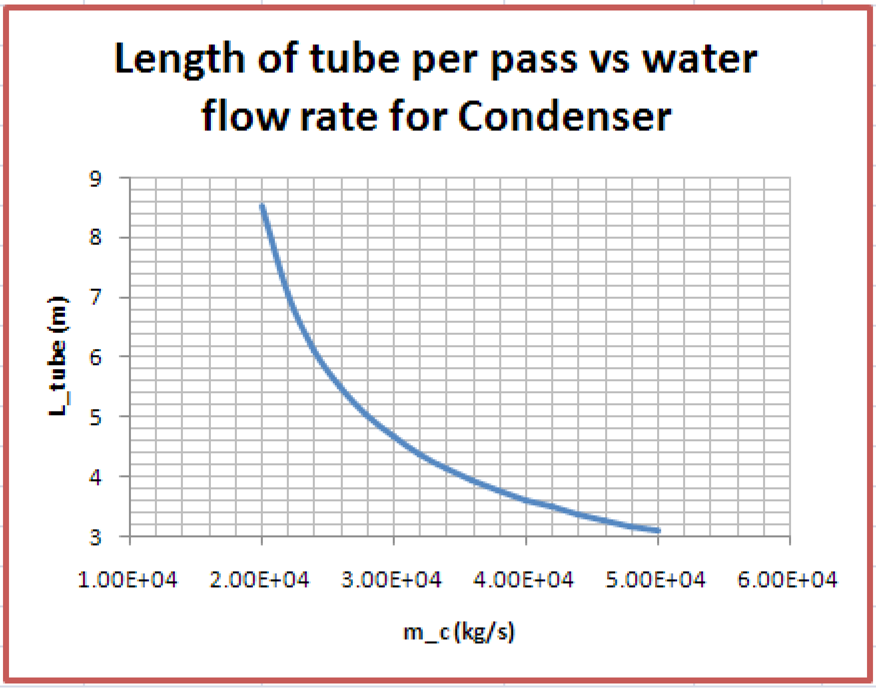
m_c (kg/s)	Tc_2 (deg.C)	U (W/m^2.C)	epsilon	L_tube (m)
2.00E+04	45.120	3591.961	0.837	8.542
2.20E+04	42.836	3782.135	0.761	7.037
2.40E+04	40.933	3960.074	0.698	6.126
2.60E+04	39.323	4127.142	0.644	5.498
2.80E+04	37.943	4284.484	0.598	5.032
3.00E+04	36.746	4433.067	0.558	4.670
3.20E+04	35.700	4573.721	0.523	4.379
3.40E+04	34.776	4707.162	0.493	4.139
3.60E+04	33.955	4834.012	0.465	3.937
3.80E+04	33.221	4954.818	0.441	3.765
4.00E+04	32.560	5070.061	0.419	3.615
4.20E+04	31.962	5180.167	0.399	3.484
4.40E+04	31.418	5285.517	0.381	3.369
4.60E+04	30.922	5386.451	0.364	3.266
4.80E+04	30.467	5483.275	0.349	3.173
5.00E+04	30.048	5576.264	0.335	3.089

Check: Observe from the Table that for m_c = 3400 kg/s, L_tube = 4.139 m. This is the same value we got in the main worksheet.

6. Now, plot the results:




Effectiveness vs water flow rate for Condenser
epsilon
0.9
0.8
0.7
0.6
0.5
0.4
0.3
1.00E+04
2.00E+04
3.00E+04
4.00E+04
5.00E+04
6.00E+04
m_c (kg/s)


Length of tube per pass vs water flow rate for Condenser
L_tube (m)
9
8
7
6
5
4
3
1.00E+04
2.00E+04
3.00E+04
4.00E+04
5.00E+04
6.00E+04
m_c (kg/s)

==

Prob. 4C.17. Consider a Shell & Tube type of HX which has 1 shell pass and 8 tube passes. Tubes are of copper, thin walled, dia = 1.4 cm. Length of tube in each pass is 5 m and overall U = 300 W/m^2.C. Water (cp = 4180 J/kg.C) flows through the tubes at a rate of 0.25 kg/s, and oil (cp = 2130 J/kg.C) flows through the shell at a rate of 0.35 kg/s. Water and oil enter the HX at 25 C and 150 C respectively. Find out the rate of heat transfer and exit temps of water and oil.

(b) Plot the variation of Q and exit temps of both the fluids as U varies from 200 to 500 W/m^2.C:

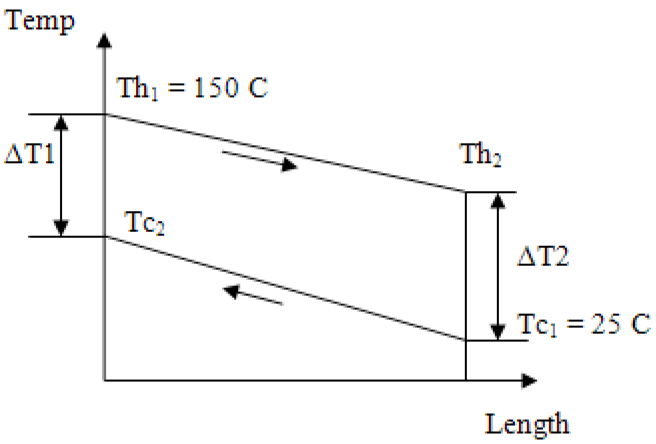
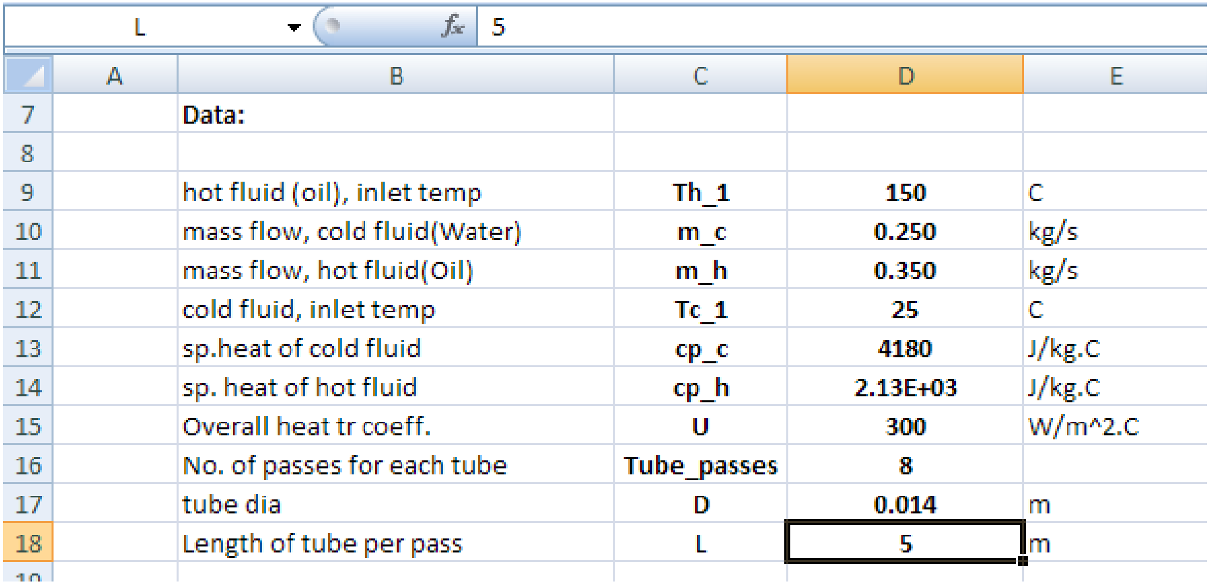


Fig. Prob.4C.17. Counter-flow arrangement

EXCEL Solution:

Following are the steps in EXCEL Solution:

1. Set up the EXCEL worksheet, enter data and name the cells:



	A	B	C	D	E
7		**Data:**			
8					
9		hot fluid (oil), inlet temp	**Th_1**	**150**	C
10		mass flow, cold fluid(Water)	**m_c**	**0.250**	kg/s
11		mass flow, hot fluid(Oil)	**m_h**	**0.350**	kg/s
12		cold fluid, inlet temp	**Tc_1**	**25**	C
13		sp.heat of cold fluid	**cp_c**	**4180**	J/kg.C
14		sp. heat of hot fluid	**cp_h**	**2.13E+03**	J/kg.C
15		Overall heat tr coeff.	**U**	**300**	W/m^2.C
16		No. of passes for each tube	**Tube_passes**	**8**	
17		tube dia	**D**	**0.014**	m
18		Length of tube per pass	**L**	**5**	m

2. Perform the calculations as shown. Equations used are:

$$A = \pi \cdot D \cdot L \cdot Tube\_passes$$ ...surface area of HX

$$C_h = m_h \cdot cp_h$$ ...capacity rate of hot fluid

$$C_c = m_c \cdot cp_c$$ ...capacity rate of cold fluid

$$C_r = \frac{C_{min}}{C_{max}}$$ ...capacity ratio

$$NTU = \frac{U \cdot A}{C_{min}}$$

$$AA = 1 + \exp\left(-NTU \cdot \sqrt{1 + C_r^{\,2}}\right)$$

$$BB = 1 - \exp\left(-NTU \cdot \sqrt{1 + C_r^{\,2}}\right)$$

$$epsilon = 2 \cdot \left(1 + C_r + \frac{\sqrt{1 + C_r^{\,2}} \cdot AA}{BB}\right)^{-1}$$ ...effectiveness

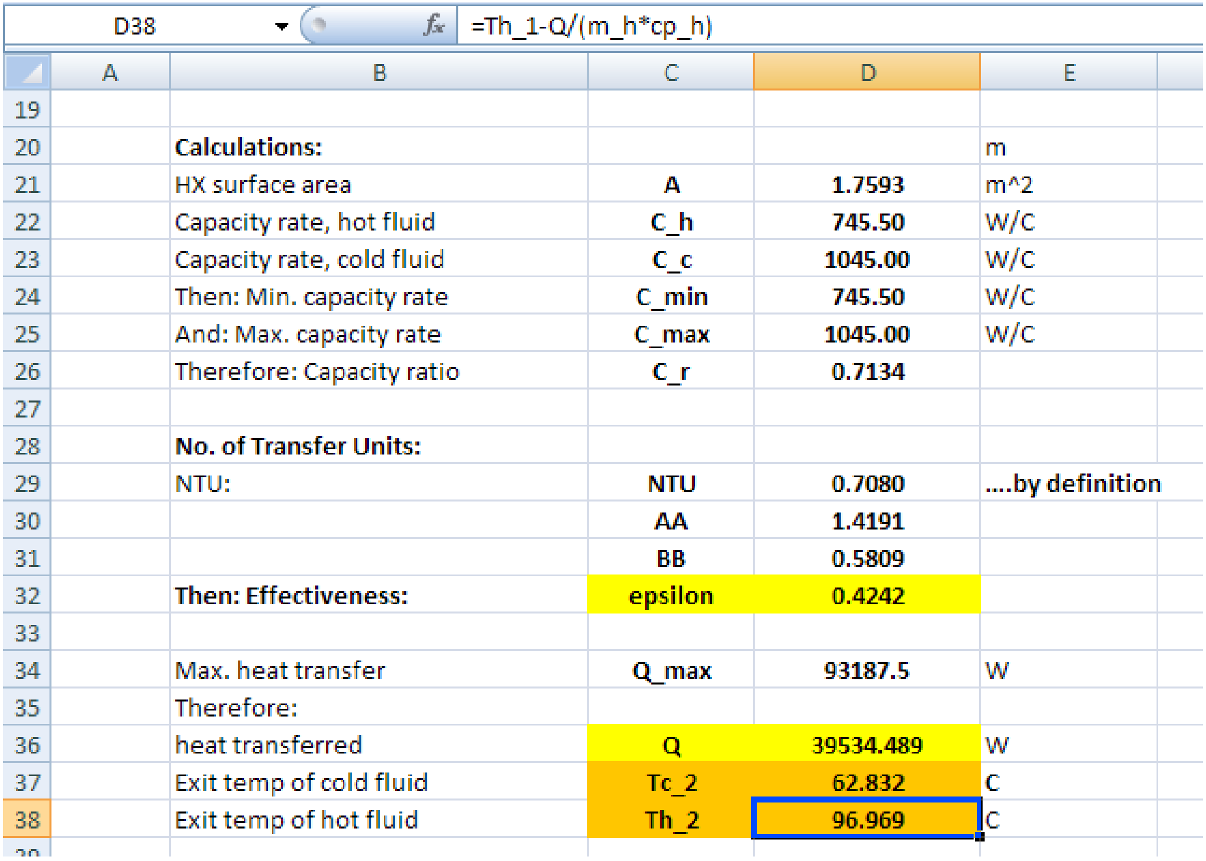
$Q_{max} = C_{min} \cdot (Th_1 - Tc_1)$ ..max. heat transfer

$Q = epsilon \cdot Q_{max}$ ..actual heat transfer

$Tc_2 = Tc_1 + \frac{Q}{m_c \cdot cp_c}$ ....exit temp of cold fluid (i.e. water)

$Th_2 = Th_1 - \frac{Q}{m_h \cdot cp_h}$ ....exit temp of hot fluid (i.e. oil)

The worksheet is shown below:



D38 =Th_1-Q/(m_h*cp_h)

	A	B	C	D	E
19					
20		**Calculations:**			m
21		HX surface area	**A**	**1.7593**	m^2
22		Capacity rate, hot fluid	**C_h**	**745.50**	W/C
23		Capacity rate, cold fluid	**C_c**	**1045.00**	W/C
24		Then: Min. capacity rate	**C_min**	**745.50**	W/C
25		And: Max. capacity rate	**C_max**	**1045.00**	W/C
26		Therefore: Capacity ratio	**C_r**	**0.7134**	
27					
28		**No. of Transfer Units:**			
29		NTU:	**NTU**	**0.7080**	**....by definition**
30			**AA**	**1.4191**	
31			**BB**	**0.5809**	
32		**Then: Effectiveness:**	**epsilon**	**0.4242**	
33					
34		Max. heat transfer	**Q_max**	**93187.5**	W
35		Therefore:			
36		heat transferred	**Q**	**39534.489**	W
37		Exit temp of cold fluid	**Tc_2**	**62.832**	C
38		Exit temp of hot fluid	**Th_2**	**96.969**	C

Thus:

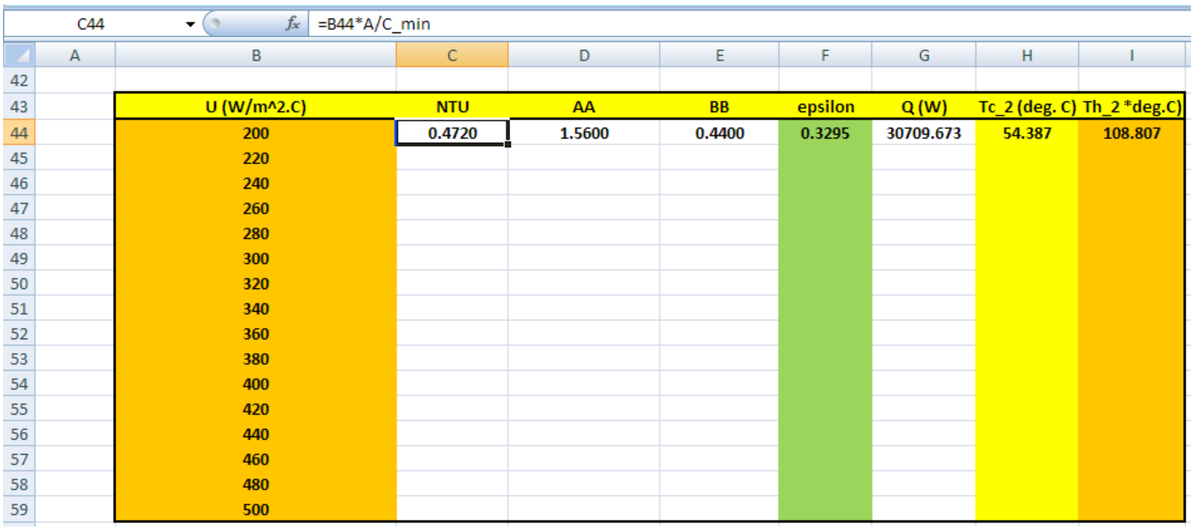
Rate of heat transfer = Q = 39534.5 W ….. Ans.

Exit temp of cold fluid = Tc_2 = 62.83 C …. Ans.

Exit temp of hot fluid = Th_2 = 96.97 C … Ans.

(b) Plot the variation of epsilon, Q and exit temps of both the fluids as U varies from 200 to 500 W/m^2.C:

First, prepare a Table as shown below:

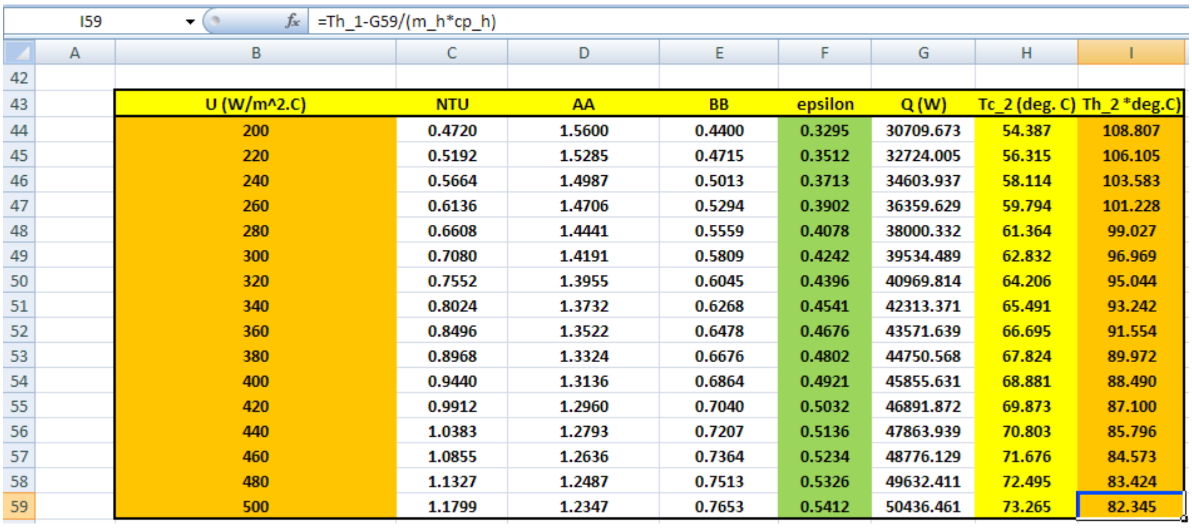


=B44*A/C_min

	U (W/m^2.C)	NTU	AA	BB	epsilon	Q (W)	Tc_2 (deg. C)	Th_2 *deg.C)
44	200	0.4720	1.5600	0.4400	0.3295	30709.673	54.387	108.807
45	220							
46	240							
47	260							
48	280							
49	300							
50	320							
51	340							
52	360							
53	380							
54	400							
55	420							
56	440							
57	460							
58	480							
59	500							

In the above screen shot, note the formula entered in cell C44 for NTU, in the formula bar. Similarly, for other quantities in the row 44, use relative reference to U.

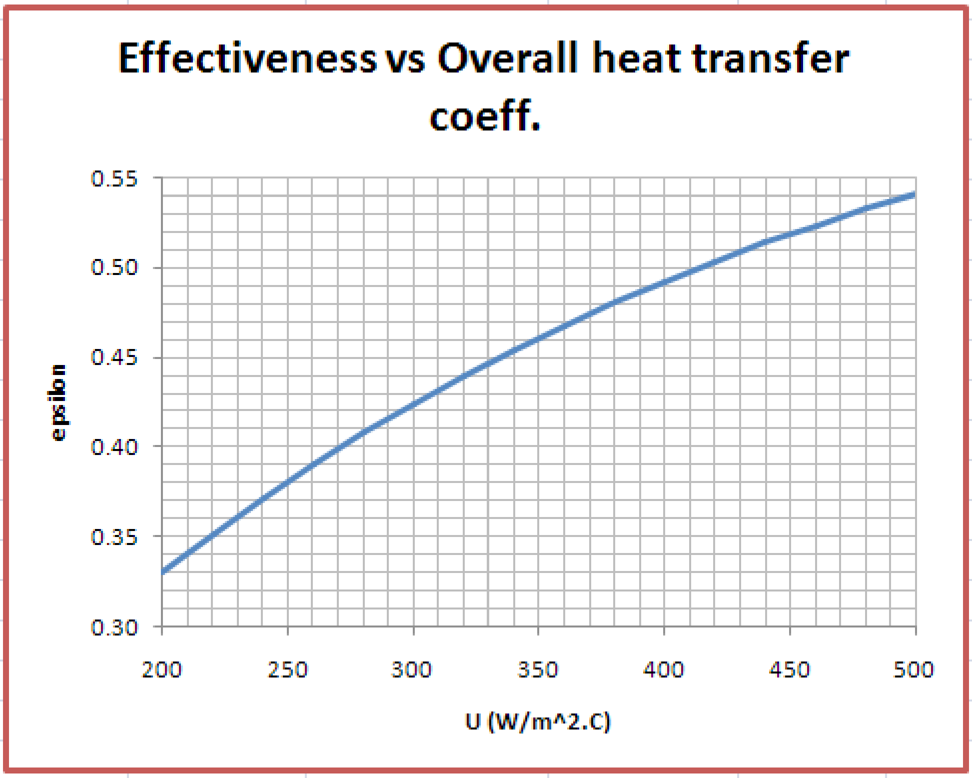
Now, select the cells C44 to I44 and 'drag-copy' to the end of Table, i.e. up to cell I59, and immediately, all calculations are made and the Table is filled up:

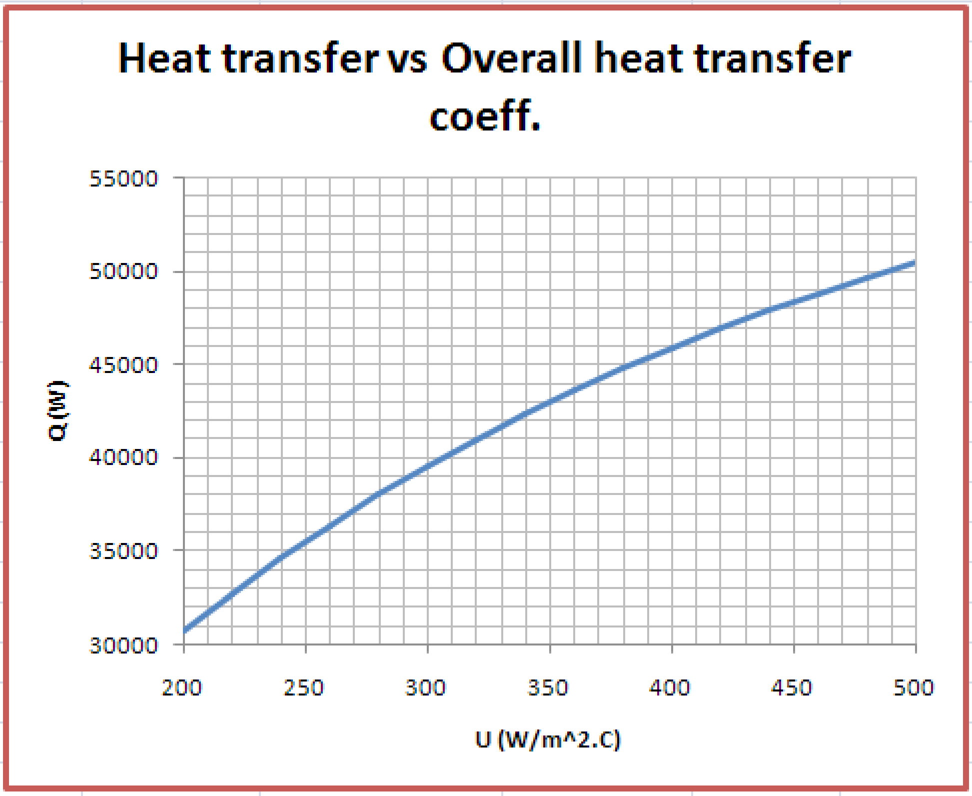


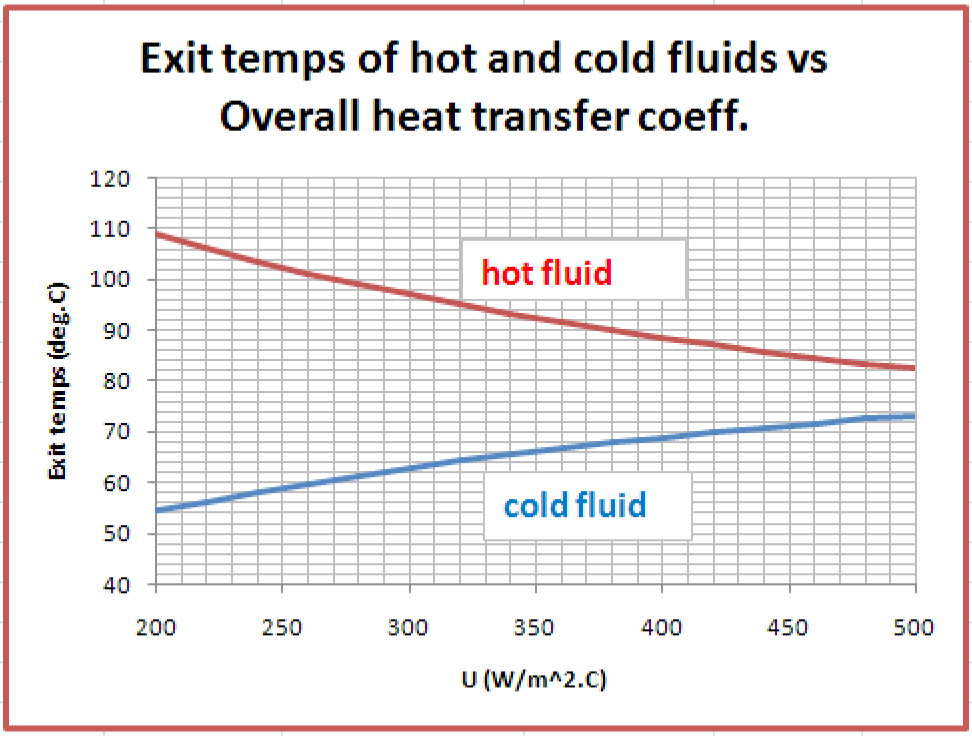
I59 =Th_1-G59/(m_h*cp_h)

	A	B	C	D	E	F	G	H	I
42									
43		U (W/m^2.C)	NTU	AA	BB	epsilon	Q (W)	Tc_2 (deg. C)	Th_2 *deg.C)
44		200	0.4720	1.5600	0.4400	0.3295	30709.673	54.387	108.807
45		220	0.5192	1.5285	0.4715	0.3512	32724.005	56.315	106.105
46		240	0.5664	1.4987	0.5013	0.3713	34603.937	58.114	103.583
47		260	0.6136	1.4706	0.5294	0.3902	36359.629	59.794	101.228
48		280	0.6608	1.4441	0.5559	0.4078	38000.332	61.364	99.027
49		300	0.7080	1.4191	0.5809	0.4242	39534.489	62.832	96.969
50		320	0.7552	1.3955	0.6045	0.4396	40969.814	64.206	95.044
51		340	0.8024	1.3732	0.6268	0.4541	42313.371	65.491	93.242
52		360	0.8496	1.3522	0.6478	0.4676	43571.639	66.695	91.554
53		380	0.8968	1.3324	0.6676	0.4802	44750.568	67.824	89.972
54		400	0.9440	1.3136	0.6864	0.4921	45855.631	68.881	88.490
55		420	0.9912	1.2960	0.7040	0.5032	46891.872	69.873	87.100
56		440	1.0383	1.2793	0.7207	0.5136	47863.939	70.803	85.796
57		460	1.0855	1.2636	0.7364	0.5234	48776.129	71.676	84.573
58		480	1.1327	1.2487	0.7513	0.5326	49632.411	72.495	83.424
59		500	1.1799	1.2347	0.7653	0.5412	50436.461	73.265	82.345

Now, plot the graphs:




Heat transfer vs Overall heat transfer coeff.
Q (W)
55000
50000
45000
40000
35000
30000
200
250
300
350
400
450
500
U (W/m^2.C)


Exit temps of hot and cold fluids vs Overall heat transfer coeff.
Exit temps (deg.C)
120
110
100
90
80
70
60
50
40
200
250
300
350
400
450
500
U (W/m^2.C)
hot fluid
cold fluid

==

Prob. 4C.18. Consider a cross flow HX to cool air by water, both fluids unmixed. Air (cp = 1000 J/kg.C) flows at a rate of 8000 kg/h, entering at 100 C, and water (cp = 4200 J/kg.C) enters the HX at 15 C at a rate of 7500 kg/h. Overall U = 150 W/m^2.C. Area of HX = 20 m^2. Find out the rate of heat transfer and exit temps of air and water.

(b) Plot the variation of Effectiveness, exit temps of both the fluids, and Q as U varies from 100 to 400 W/m^2.C:

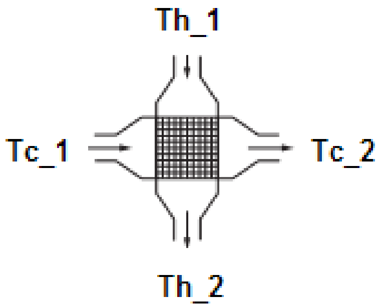
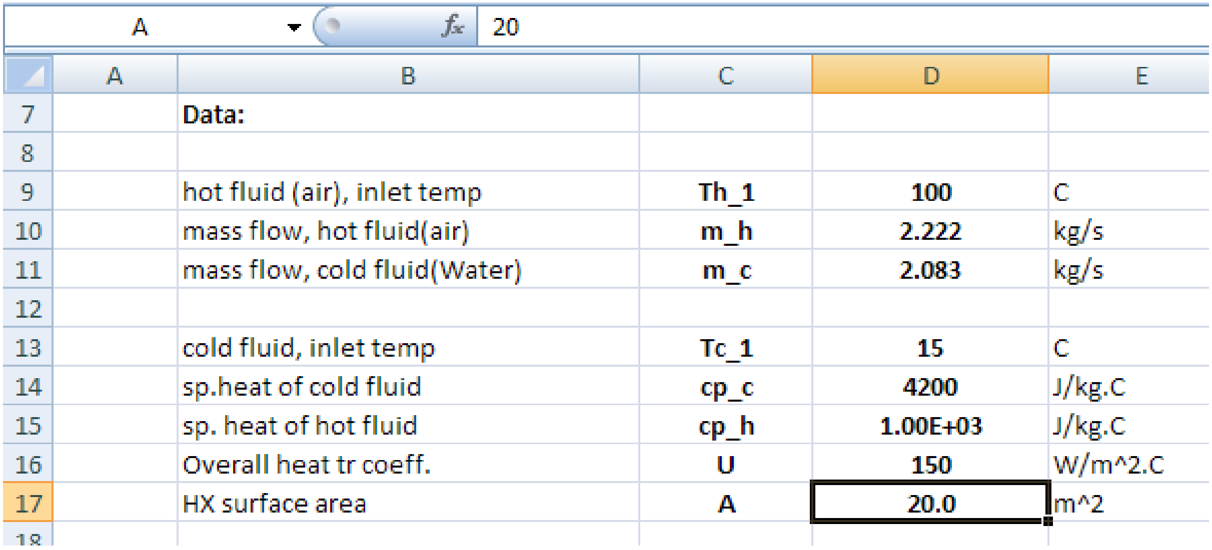


Fig. Prob.4C.18. Cross-flow arrangement

EXCEL Solution:

Following are the steps in EXCEL Solution:

1. Set up the EXCEL worksheet, enter data and name the cells:



	A	B	C	D	E
7		**Data:**			
8					
9		hot fluid (air), inlet temp	**Th_1**	**100**	C
10		mass flow, hot fluid(air)	**m_h**	**2.222**	kg/s
11		mass flow, cold fluid(Water)	**m_c**	**2.083**	kg/s
12					
13		cold fluid, inlet temp	**Tc_1**	**15**	C
14		sp.heat of cold fluid	**cp_c**	**4200**	J/kg.C
15		sp. heat of hot fluid	**cp_h**	**1.00E+03**	J/kg.C
16		Overall heat tr coeff.	**U**	**150**	W/m^2.C
17		HX surface area	**A**	**20.0**	m^2

2. Perform the calculations as shown. Equations used are:

$$C_h = m_h \cdot cp_h$$ ...capacity rate of hot fluid

$$C_c = m_c \cdot cp_c$$ ...capacity rate of cold fluid

$$C_r = \frac{C_{min}}{C_{max}}$$ ...capacity ratio

$$NTU = \frac{U \cdot A}{C_{min}}$$

$$AA = \exp\left(-C_r \cdot NTU^{0.78}\right) - 1$$

$$BB = \frac{1}{C_r} \cdot NTU^{0.22}$$

$$epsilon = 1 - \exp(BB \cdot AA)$$ ...effectiveness ... Ref: Incropera

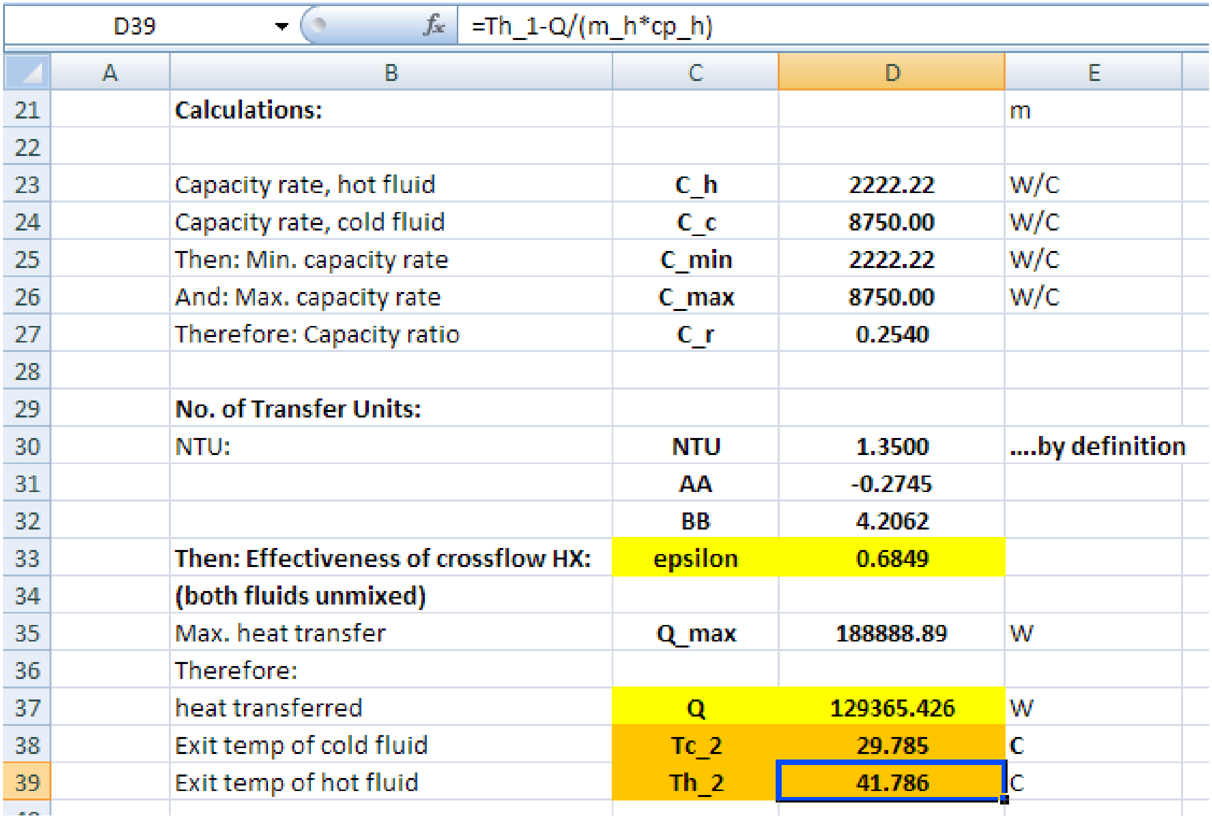
$$Q_{max} = C_{min} \cdot \left(Th_1 - Tc_1\right)$$ ..max. heat transfer

$$Q = epsilon \cdot Q_{max}$$ ..actual heat transfer

$$Tc_2 = Tc_1 + \frac{Q}{m_c \cdot cp_c}$$ ....exit temp of cold fluid (i.e. water)

$$Th_2 = Th_1 - \frac{Q}{m_h \cdot cp_h}$$ ....exit temp of hot fluid (i.e. oil)

The worksheet is shown below:



D39 =Th_1-Q/(m_h*cp_h)

	A	B	C	D	E
21		**Calculations:**			m
22					
23		Capacity rate, hot fluid	**C_h**	**2222.22**	W/C
24		Capacity rate, cold fluid	**C_c**	**8750.00**	W/C
25		Then: Min. capacity rate	**C_min**	**2222.22**	W/C
26		And: Max. capacity rate	**C_max**	**8750.00**	W/C
27		Therefore: Capacity ratio	**C_r**	**0.2540**	
28					
29		**No. of Transfer Units:**			
30		NTU:	**NTU**	**1.3500**	**....by definition**
31			**AA**	**-0.2745**	
32			**BB**	**4.2062**	
33		**Then: Effectiveness of crossflow HX:**	**epsilon**	**0.6849**	
34		**(both fluids unmixed)**			
35		Max. heat transfer	**Q_max**	**188888.89**	W
36		Therefore:			
37		heat transferred	**Q**	**129365.426**	W
38		Exit temp of cold fluid	**Tc_2**	**29.785**	C
39		Exit temp of hot fluid	**Th_2**	**41.786**	C

Thus:

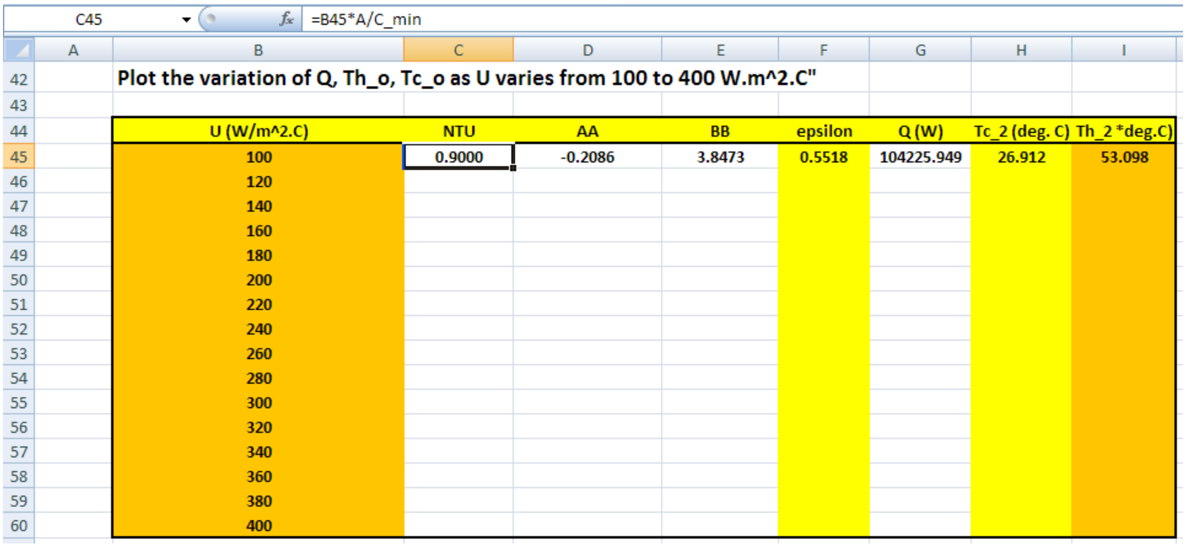
Rate of heat transfer = Q = 129365.4 W ….. Ans.

Exit temp of cold fluid = Tc_2 = 29.785 C …. Ans.

Exit temp of hot fluid = Th_2 = 41.786 C … Ans.

(b) Plot the variation of Effectiveness, exit temps of both the fluids, and Q as U varies from 100 to 400 W/m^2.C:

First, prepare a Table as shown below:

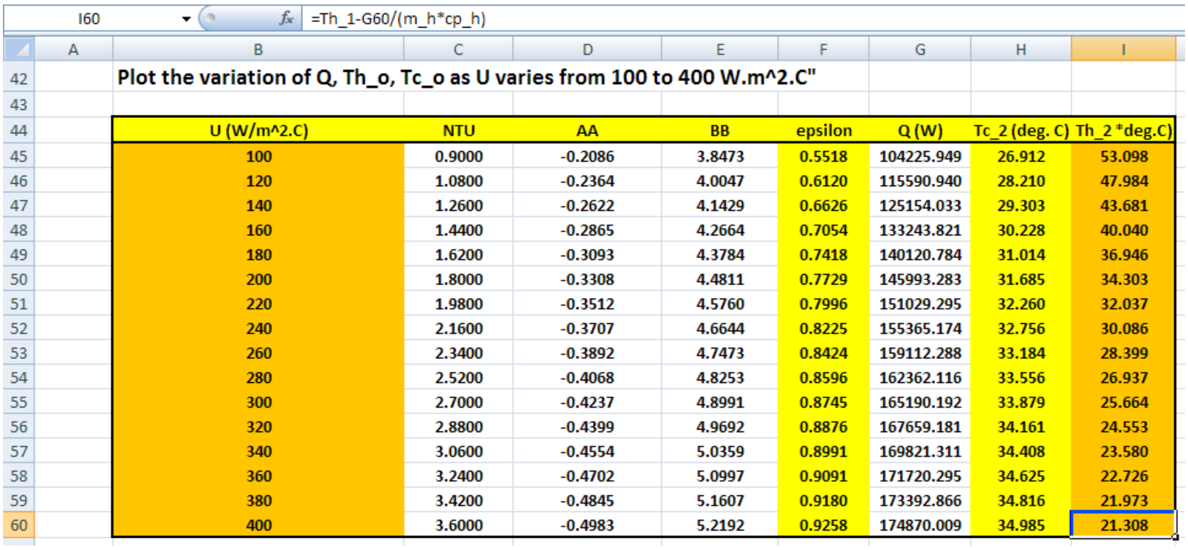


C45 =B45*A/C_min

	A	B	C	D	E	F	G	H	I
42		Plot the variation of Q, Th_o, Tc_o as U varies from 100 to 400 W.m^2.C"							
43									
44		U (W/m^2.C)	NTU	AA	BB	epsilon	Q (W)	Tc_2 (deg. C)	Th_2 *deg.C)
45		100	0.9000	-0.2086	3.8473	0.5518	104225.949	26.912	53.098
46		120							
47		140							
48		160							
49		180							
50		200							
51		220							
52		240							
53		260							
54		280							
55		300							
56		320							
57		340							
58		360							
59		380							
60		400							

In the above screen shot, note the formula entered in cell C45 for NTU, in the formula bar. Similarly, for other quantities in the row 45, use relative reference to U.

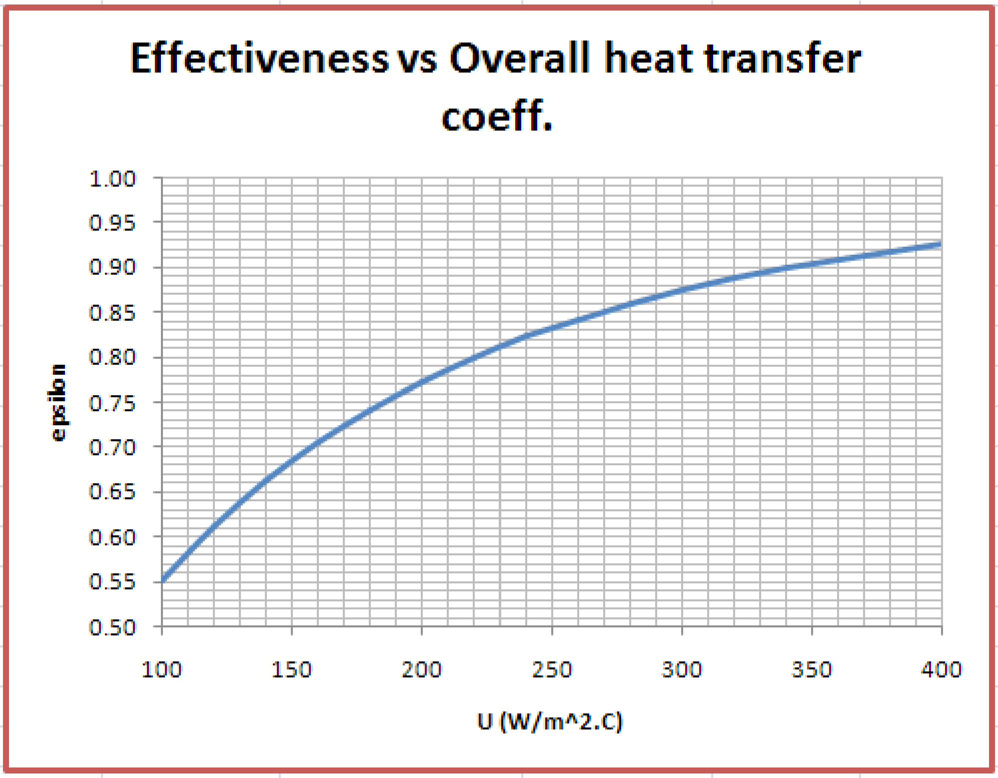
Now, select the cells C45 to I45 and 'drag-copy' to the end of Table, i.e. up to cell I60, and immediately, all calculations are made and the Table is filled up:

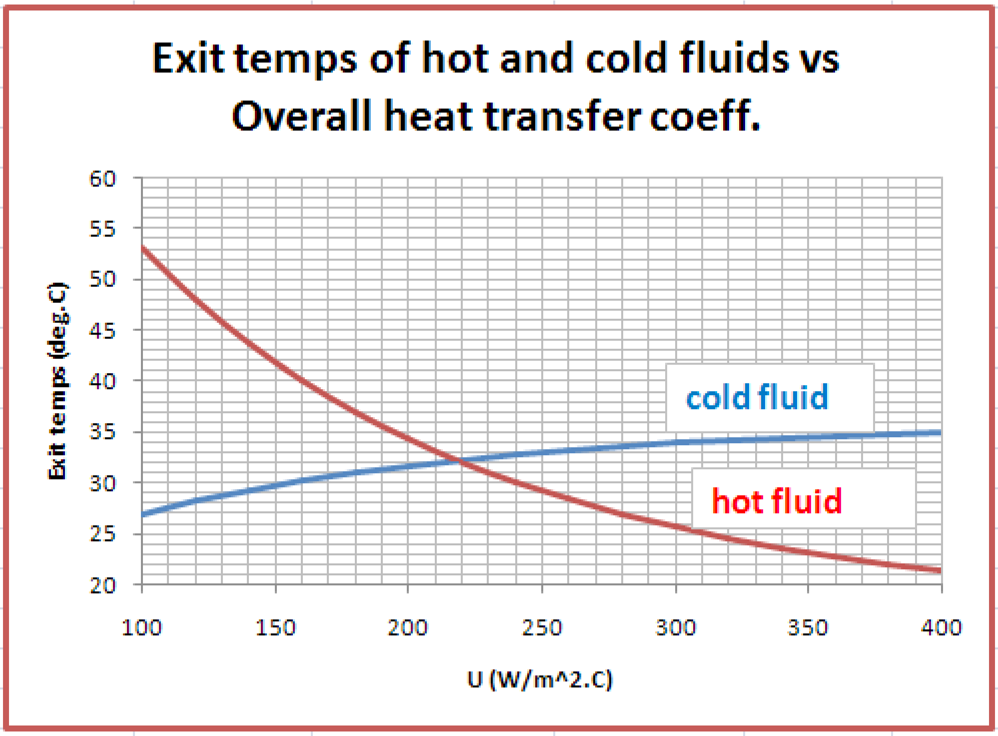


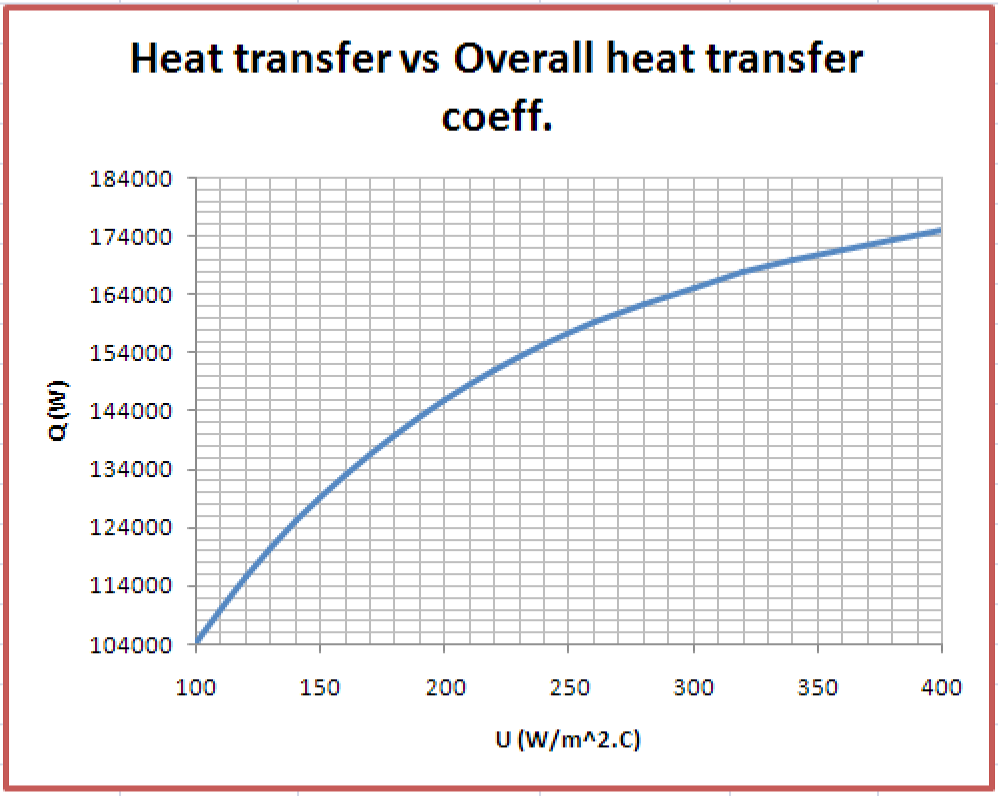
I60 =Th_1-G60/(m_h*cp_h)

	A	B	C	D	E	F	G	H	I
42		Plot the variation of Q, Th_o, Tc_o as U varies from 100 to 400 W.m^2.C"							
43									
44		U (W/m^2.C)	NTU	AA	BB	epsilon	Q (W)	Tc_2 (deg. C)	Th_2 *deg.C)
45		100	0.9000	-0.2086	3.8473	0.5518	104225.949	26.912	53.098
46		120	1.0800	-0.2364	4.0047	0.6120	115590.940	28.210	47.984
47		140	1.2600	-0.2622	4.1429	0.6626	125154.033	29.303	43.681
48		160	1.4400	-0.2865	4.2664	0.7054	133243.821	30.228	40.040
49		180	1.6200	-0.3093	4.3784	0.7418	140120.784	31.014	36.946
50		200	1.8000	-0.3308	4.4811	0.7729	145993.283	31.685	34.303
51		220	1.9800	-0.3512	4.5760	0.7996	151029.295	32.260	32.037
52		240	2.1600	-0.3707	4.6644	0.8225	155365.174	32.756	30.086
53		260	2.3400	-0.3892	4.7473	0.8424	159112.288	33.184	28.399
54		280	2.5200	-0.4068	4.8253	0.8596	162362.116	33.556	26.937
55		300	2.7000	-0.4237	4.8991	0.8745	165190.192	33.879	25.664
56		320	2.8800	-0.4399	4.9692	0.8876	167659.181	34.161	24.553
57		340	3.0600	-0.4554	5.0359	0.8991	169821.311	34.408	23.580
58		360	3.2400	-0.4702	5.0997	0.9091	171720.295	34.625	22.726
59		380	3.4200	-0.4845	5.1607	0.9180	173392.866	34.816	21.973
60		400	3.6000	-0.4983	5.2192	0.9258	174870.009	34.985	21.308

Now, plot the graphs:




Exit temps of hot and cold fluids vs Overall heat transfer coeff.
Exit temps (deg.C)
60
55
50
45
40
35
30
25
20
100
150
200
250
300
350
400
U (W/m^2.C)
cold fluid
hot fluid


Heat transfer vs Overall heat transfer coeff.
Q (W)
184000
174000
164000
154000
144000
134000
124000
114000
104000
100
150
200
250
300
350
400
U (W/m^2.C)

==

Compact heat exchangers:

Heat exchangers with an area density greater than about700 m^2/m^3 are classified as 'compact heat exchangers'. Generally, they are used for gases.

Compact heat exchangers are, typically, of three types:

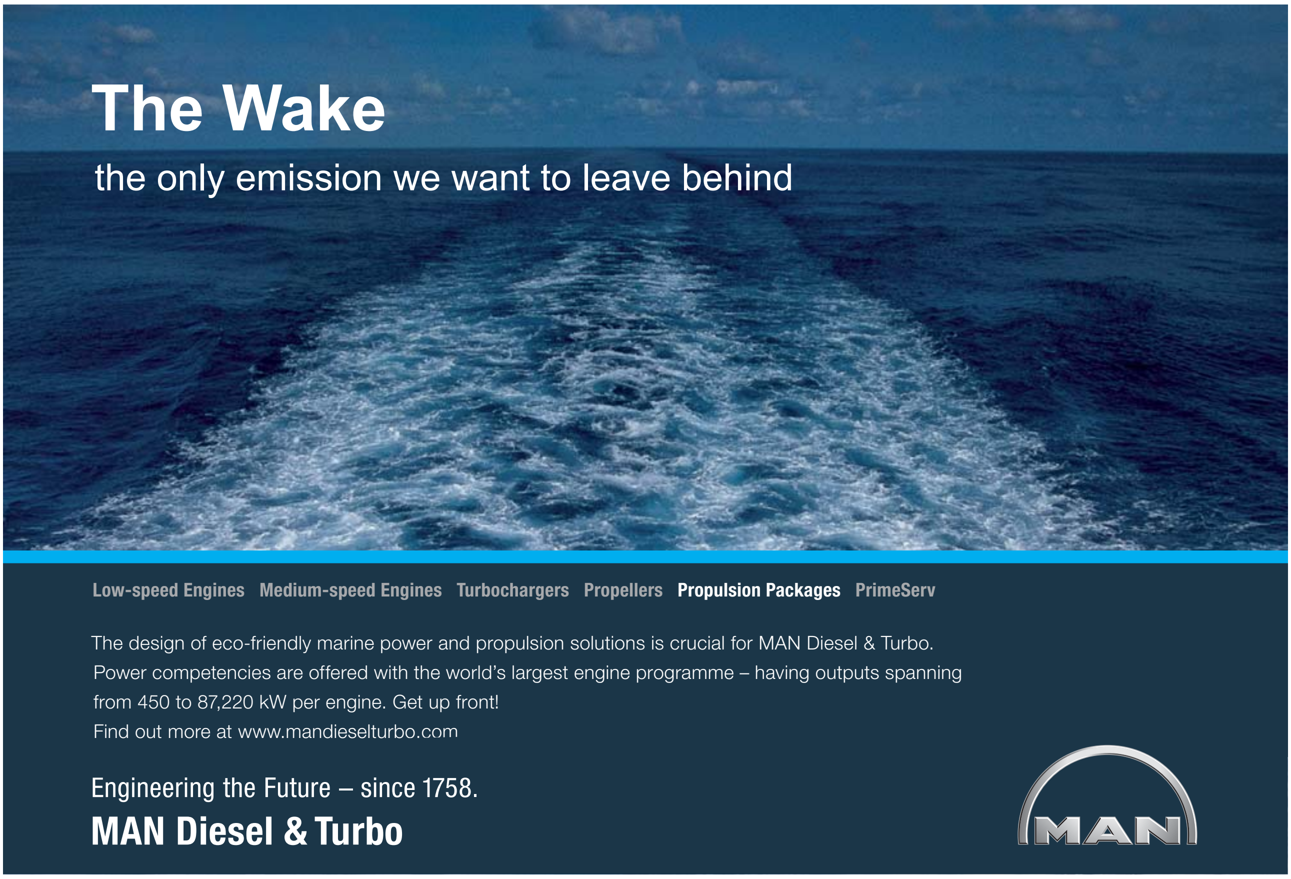
i) array of finned circular tubes
ii) array of plate-fin matrix, and
iii) array of finned flat-tube matrix

Kays and London have studied a large number of compact heat exchanger matrices and presented their experimental results in the form of generalized graphs. Heat transfer data is plotted as Colburn j-factor, $j_H = St.Pr^{2/3}$ against Re,

where, St = Stanton number = h/(G.Cp), Pr = Prandtl number = μ.Cp/k, and

Re = G.D_h/μ, G = mass velocity (= mass flow rate/Area of cross-section), kg/(s.m^2.)

In the same graphs, friction factor, f, is also plotted against Re.



One typical graph of characteristics for a **plate-finned circular tube matrix** (data of Trane Co.) given by Kays and London is shown below:

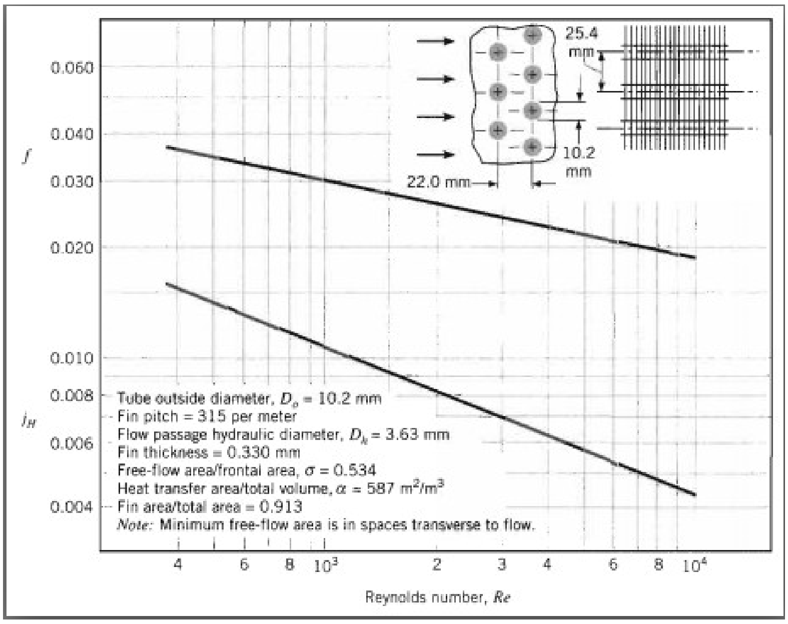


Fig.1. Heat transfer and friction factor for plate-finned circular tube matrix heat exchanger

In the above graph, Reynolds No. is shown on the x-axis.

Colburn j-factor, j_H is used to get heat transfer coeff. h.

Friction factor is used to get the frictional pressure drop:

$$\Delta P_f = f \cdot \frac{G^2}{2 \cdot \rho} \cdot \frac{A}{A_{min}} \quad \text{N/m}^2\text{....frictional pressure drop}$$

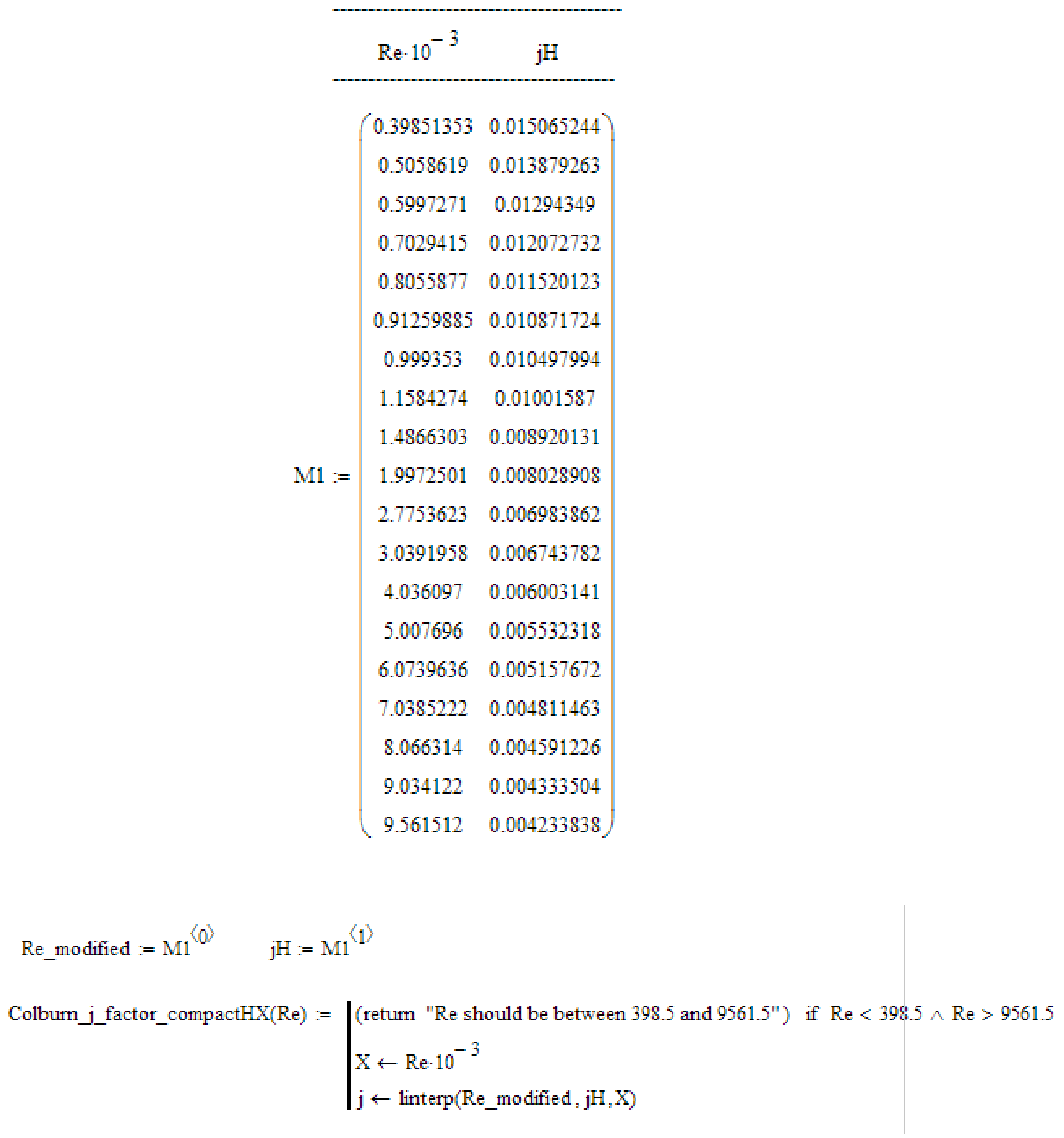
Now, to use the above graph in computer calculations, **we digitize the graphs for j_H and f, using PlotDigitizer, a freely available, java based software.**

(Ref: http://plotdigitizer.sourceforge.net)

Then, we write a Mathcad Functions for linear interpolation of j_H and f against Re.

Mathcad Functions are given below:

1. j-factor:



$$\mathrm{Re}\cdot 10^{-3} \qquad \mathrm{jH}$$

$$M1 := \begin{pmatrix} 0.39851353 & 0.015065244 \\ 0.5058619 & 0.013879263 \\ 0.5997271 & 0.01294349 \\ 0.7029415 & 0.012072732 \\ 0.8055877 & 0.011520123 \\ 0.91259885 & 0.010871724 \\ 0.999353 & 0.010497994 \\ 1.1584274 & 0.01001587 \\ 1.4866303 & 0.008920131 \\ 1.9972501 & 0.008028908 \\ 2.7753623 & 0.006983862 \\ 3.0391958 & 0.006743782 \\ 4.036097 & 0.006003141 \\ 5.007696 & 0.005532318 \\ 6.0739636 & 0.005157672 \\ 7.0385222 & 0.004811463 \\ 8.066314 & 0.004591226 \\ 9.034122 & 0.004333504 \\ 9.561512 & 0.004233838 \end{pmatrix}$$

$$\mathrm{Re\_modified} := M1^{\langle 0\rangle} \qquad \mathrm{jH} := M1^{\langle 1\rangle}$$

```
Colburn_j_factor_compactHX(Re) := | (return "Re should be between 398.5 and 9561.5")  if  Re < 398.5 ∧ Re > 9561.5
                                  | X ← Re·10^-3
                                  | j ← linterp(Re_modified, jH, X)
```

2. friction factor:

$$\begin{array}{cc} Re\cdot 10^{-3} & f \end{array}$$

$$M2 := \begin{pmatrix} 0.40814313 & 0.035778843 \\ 0.50655836 & 0.033722047 \\ 0.60094184 & 0.032894008 \\ 0.70470744 & 0.031732872 \\ 0.8171454 & 0.030963546 \\ 0.9153359 & 0.029555662 \\ 1.0026748 & 0.029188212 \\ 1.4932629 & 0.0268305 \\ 2.00746 & 0.025259914 \\ 3.0244725 & 0.023478124 \\ 4.0659337 & 0.022103779 \\ 5.047166 & 0.021068498 \\ 5.985631 & 0.020094514 \\ 7.01918 & 0.019385193 \\ 7.955443 & 0.01892125 \\ 9.120079 & 0.018465469 \\ 9.988675 & 0.018032158 \end{pmatrix}$$

$$Re\_modified := M2^{\langle 0 \rangle} \qquad ffactor := M2^{\langle 1 \rangle}$$

$$f\_factor\_compactHX(Re) := \left| \begin{array}{l} (\text{return "Re should be between 408 and 9988"}) \text{ if } Re < 408 \vee Re > 9988 \\ X \leftarrow Re\cdot 10^{-3} \\ ff \leftarrow linterp(Re\_modified, ffactor, X) \end{array} \right.$$

Now, let us solve a problem on compact heat exchangers:

Example 4C.19: Air at 2 atm and 400 K flows at a rate of 5 kg/s, across a finned circular tube matrix, for which heat transfer and friction factor characteristics are shown in Fig.1 above. Dimensions of the heat exchanger matrix are: 1 m (W) × 0.6 m (Deep) × 0.5 m (H), as shown in Fig.Prob.4C.19. Find: (a) the heat transfer coeff. (b) the friction factor, and (c) ratio of core friction pressure drop to the inlet pressure.

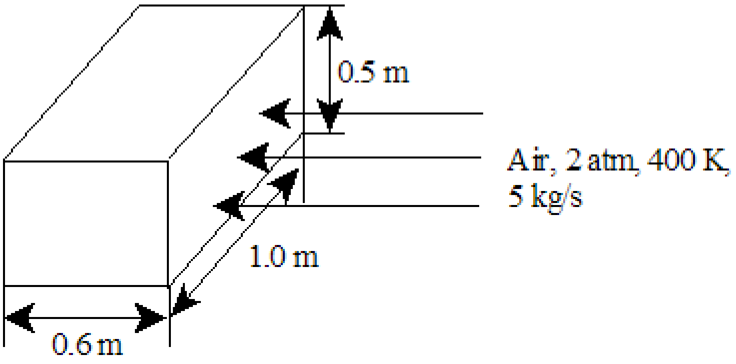


Fig. Prob.4C.19

Mathcad Solution:

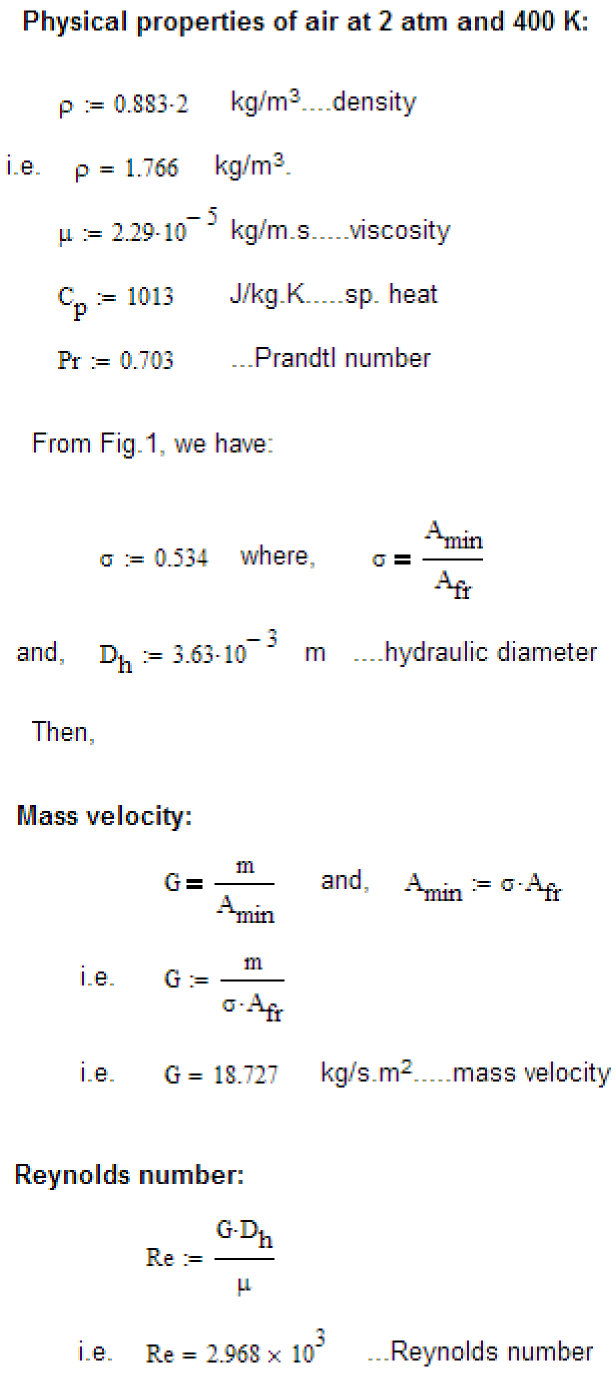
We shall use the Mathcad Functions written above.

Data:

$m := 5$ kg/s....mass flow rate

$A_{fr} := 0.5$ m^2....frontal area

$L := 0.6$ m....length of flow



Physical properties of air at 2 atm and 400 K:

$\rho := 0.883 \cdot 2$ kg/m^3....density

i.e. $\rho = 1.766$ kg/m^3.

$\mu := 2.29 \cdot 10^{-5}$ kg/m.s.....viscosity

$C_p := 1013$ J/kg.K.....sp. heat

$Pr := 0.703$ ...Prandtl number

From Fig.1, we have:

$\sigma := 0.534$ where, $\sigma = \frac{A_{min}}{A_{fr}}$

and, $D_h := 3.63 \cdot 10^{-3}$ m ....hydraulic diameter

Then,

Mass velocity:

$G = \frac{m}{A_{min}}$ and, $A_{min} := \sigma \cdot A_{fr}$

i.e. $G := \frac{m}{\sigma \cdot A_{fr}}$

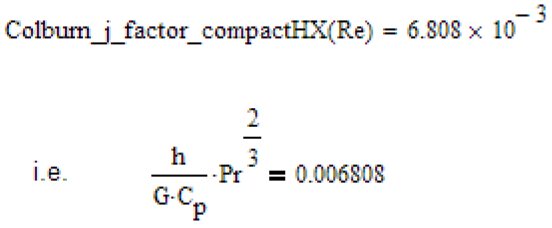
i.e. $G = 18.727$ kg/s.m^2.....mass velocity

Reynolds number:

$Re := \frac{G \cdot D_h}{\mu}$

i.e. $Re = 2.968 \times 10^3$ ...Reynolds number

Then, from the Mathcad Function written above, for Re = 2968, we get Colburn j-factor:



$\text{Colburn\_j\_factor\_compactHX}(Re) = 6.808 \times 10^{-3}$

i.e. $\frac{h}{G \cdot C_p} \cdot Pr^{\frac{2}{3}} = 0.006808$

(a) And, heat transfer coefficient:

$$h := \frac{0.006808 \cdot G \cdot C_p}{Pr^{\frac{2}{3}}}$$

i.e. $h = 163.349$ **W/(m^2.C)...heat transfer coeff. Ans.**

(b) Friction factor:

Again, using the Mathcad Function written abofe for f, for Re = 2968, we get:

$f\_factor\_compactHX(Re) = 0.024$

i.e. $f := 0.024$ **..friction factor...Ans.**

(c) Pressure drop:

$$\Delta P_f = f \cdot \frac{G^2}{2 \cdot \rho} \cdot \frac{A}{A_{min}}$$ N/m2....frictional pressure drop

Now, $$\frac{A}{A_{min}} = \frac{4 \cdot L}{D_h}$$

$$\frac{4 \cdot L}{D_h} = 661.157$$

i.e. $$\frac{A}{A_{min}} = 661.157$$

Therefore,

$$\Delta P_f := f \cdot \frac{G^2}{2 \cdot \rho} \cdot 661.157$$ N/m2.....core friction pressure drop

i.e. $\Delta P_f = 1.575 \times 10^3$ **N/m2.....core friction pressure drop...Ans.**

And, $$\frac{\Delta P_f}{P} = \frac{1575}{2 \cdot \left(1.013 \cdot 10^5\right)} \cdot 100 = 0.78\%$$

i.e. frictional pressure drop is 0.78% of the inlet pressure...Ans.

Plot the variation of h and DELTAP as mass flow rate varies from 1 to 8 kg/s:

Express related quantities as functions of mass flow rate:

Mass velocity:

$$G = \frac{m}{A_{min}}$$ and, $A_{min} := \sigma \cdot A_{fr}$

i.e. $$G(m) := \frac{m}{\sigma \cdot A_{fr}}$$ kg/s.m2.....mass velocity

Reynolds number:

$$Re(m) := \frac{G(m) \cdot D_h}{\mu}$$

Colburn j-factor:

Using the Mathcad Function written above for j_H:

$j_H(m) := \text{Colburn\_j\_factor\_compactHX}(\text{Re}(m))$

(a) And, heat transfer coefficient:

$$h(m) := \frac{j_H(m) \cdot G(m) \cdot C_p}{\Pr^{\frac{2}{3}}}$$ **...W/(m²·C)...heat transfer coeff.**

(b) Friction factor:

Again, using the Mathcad Function written above for f:

$ff(m) := \text{f\_factor\_compactHX}(\text{Re}(m))$ **..friction factor.**

(c) Pressure drop:

$$\Delta P_f(m) = ff(m) \cdot \frac{G(m)^2}{2 \cdot \rho} \cdot \frac{A}{A_{min}}$$ N/m²....frictional pressure drop

Now, $\frac{A}{A_{min}} = \frac{4 \cdot L}{D_h}$

$$\frac{4 \cdot L}{D_h} = 661.157$$

i.e. $\frac{A}{A_{min}} = 661.157$

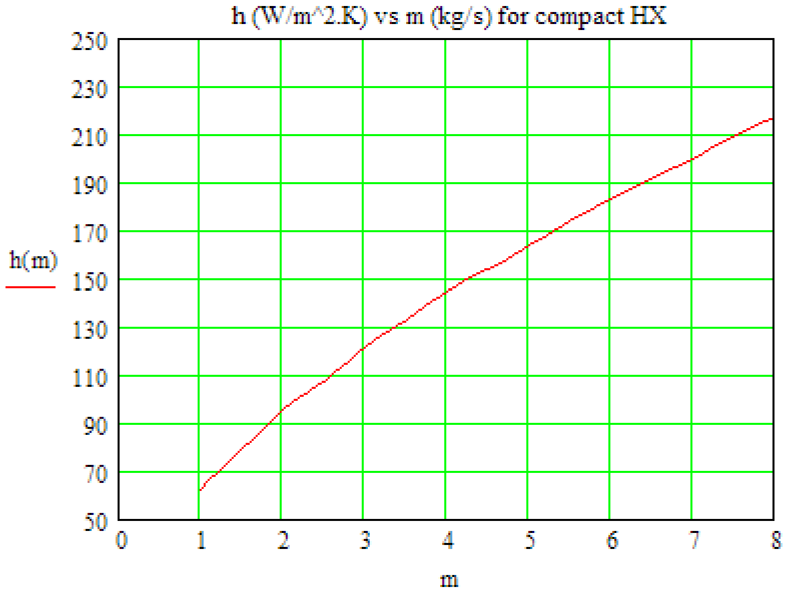
Therefore,

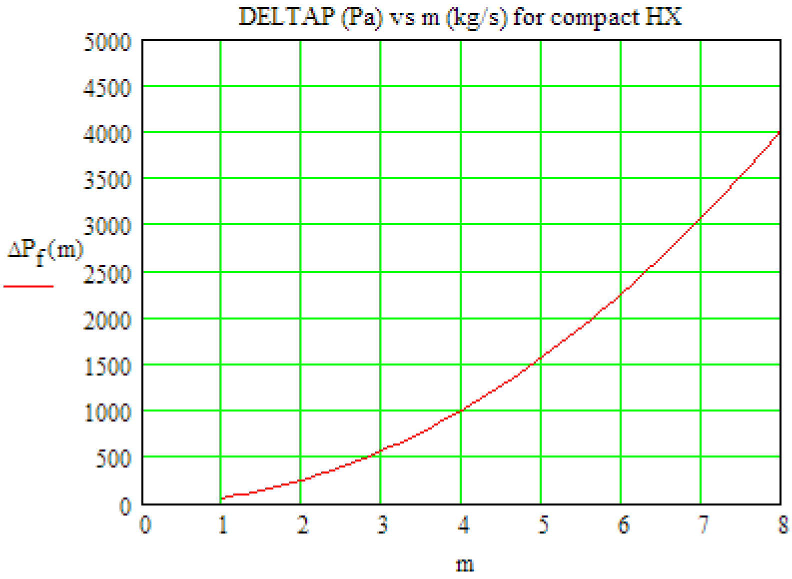
$$\Delta P_f(m) := ff(m) \cdot \frac{G(m)^2}{2 \cdot \rho} \cdot 661.157$$ N/m².....core friction pressure drop

Now, plot the graphs:

$m := 1, 1.5 .. 8$ ....define a range variable for m

$m =$	$Re(m) =$	$j_H(m) =$	$h(m) =$	$\Delta P_f(m) =$
1	593.69	0.013	62.401	63.019
1.5	890.535	0.011	79.218	141.793
2	$1.187\cdot10^{3}$	$9.919\cdot10^{-3}$	95.2	252.077
2.5	$1.484\cdot10^{3}$	$8.928\cdot10^{-3}$	107.11	393.87
3	$1.781\cdot10^{3}$	$8.406\cdot10^{-3}$	121.018	567.172
3.5	$2.078\cdot10^{3}$	$7.921\cdot10^{-3}$	133.031	771.985
4	$2.375\cdot10^{3}$	$7.522\cdot10^{-3}$	144.383	$1.008\cdot10^{3}$
4.5	$2.672\cdot10^{3}$	$7.123\cdot10^{-3}$	153.821	$1.276\cdot10^{3}$
5	$2.968\cdot10^{3}$	$6.808\cdot10^{-3}$	163.353	$1.575\cdot10^{3}$
5.5	$3.265\cdot10^{3}$	$6.576\cdot10^{-3}$	173.556	$1.906\cdot10^{3}$
6	$3.562\cdot10^{3}$	$6.355\cdot10^{-3}$	182.984	$2.269\cdot10^{3}$
6.5	$3.859\cdot10^{3}$	$6.135\cdot10^{-3}$	191.353	$2.663\cdot10^{3}$
7	$4.156\cdot10^{3}$	$5.945\cdot10^{-3}$	199.704	$3.088\cdot10^{3}$
7.5	$4.453\cdot10^{3}$	$5.801\cdot10^{-3}$	208.791	$3.545\cdot10^{3}$
8	$4.75\cdot10^{3}$	$5.657\cdot10^{-3}$	217.188	$4.033\cdot10^{3}$




DELTAP (Pa) vs m (kg/s) for compact HX
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
ΔP_f(m)
0
1
2
3
4
5
6
7
8
m

==

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