Software Solutions to Problems on Heat Transfer

Convection – Part I: Forced convection

Dr. M. Thirumaleshwar

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Convection – Part-I: Forced convection

(Boundary layer, flow over Flat plates, across cylinders, spheres and Tube banks)

Software Solutions to Problems on Heat Transfer: Convection – Part-I: Forced convection
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Preface To Vol. 2

This is Vol. 2 of the book **"Software Solutions to Problems on Heat Transfer".**

In Vol. 1, problems on various aspects of CONDUCTION heat transfer were solved with Mathcad, EES, FEHT and EXCEL.

Vol. 2 contains solved problems on the topics of CONVECTION (i.e. Forced convection, Natural or Free convection).

Part-I and II of CONVECTION contain problems solved on following subtopics:

Part-I:

2A1. Forced convection:
2A1. Convection equation summary
2A1.1. Boundary layer fundamentals, Flow over Flat plates, Momentum – heat transfer Analogy
2A1.2. Flow across Cylinders and Spheres
2A1.3. Flow across Tube banks
2A1.4. Flow inside Tubes and ducts
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Part-II:

2A2. Natural (or Free) convection:
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2A2.5. Natural convection from Finned surfaces
2A2.6. Combined Natural and Forced convection
References

Here also problems are solved using the popular software, viz. **"Mathcad"**, **"Engineering Equation Solver (EES)"**, **and MS EXCEL spreadsheet**. Comments are included generously in the codes so that the logic behind the solutions is clear. An introductory chapter in Part-I gives a brief overview of the software used.

As in Vol. 1, emphasis is given not only to solving a given problem but also to parametric analysis and graphical representation of results. Advantage of using Software to solve a variety of problems thus becomes evident. Also, problems with EXCEL spreadsheet are solved in greater details with relevant screen shots for immediate help to students, since EXCEL is available in practically every Personal Computer.

Acknowledgements: Firstly, I thank my students, since it is they who inspired me and motivated me.Next, my thanks are due to the authorities at St. Joseph Engineering College, for their constant encouragement.

Also, my sincere thanks to **Bookboon.com** for publishing this book on the Internet. **Ms. Sophie** and her editorial staff have to be specially mentioned for their cooperation, suggestions and support.

Finally, my heart-felt appreciation to **my wife, Kala,** for her unfailing and thoughtful support and encouragement.

M. Thirumaleshwar
Author
August 2013

About the Author

Dr. M. Thirumaleshwar graduated in Mechanical Engineering from Karnataka Regional Engineering College, Surathkal, Karnataka, India, in the year 1965. He obtained M.Sc (cryogenis) from University of Southampton, U.K. and Ph.D. (cryogenics) from Indian Institute of Science, Bangalore, India.

He is a Fellow of Institution of Engineers (India), Life Member, Indian Society for Technical Education, and a Foundation Fellow of Indian Cryogenics Council.

He has worked in India and abroad on large projects in the areas involving heat transfer, fluid flow, vacuum system design, cryo-pumping etc.

He worked as Head of Cryogenics Dept. in Bhabha Atomic Research Centre (BARC), Bombay and Centre for Advanced Technology (CAT), Indore, from 1966 to 1992.

He worked as Guest Collaborator with Superconducting Super Collider Laboratory of Universities Research Association, in Dallas, USA from 1990 to 1993.

He also worked at the Institute of Cryogenics, Southampton, U.K. as a Visiting Research Fellow from 1993 to 1994.

He was Head of the Dept. of Mechanical Engineering, Fr. Conceicao Rodrigues Institute of Technology, Vashi, Navi Mumbai, India for eight years.

He also worked as Head of Dept. of Mechanical Engineering and Civil Engineering, and then as Principal, Vivekananda College of Engineering and Technology, Puttur (D.K.), India.

Presently, he is Professor and coordinator of Post-graduate program in the Dept. of Mechanical Engineering in St. Joseph Engineering College, Vamanjoor, Mangalore, India.

A book entitled **"Fundamentals of Heat and Mass Transfer"** authored by him and published by M/s Pearson Education, India (2006) **has been adopted as a Text book** for third year engineering students by the Visweswaraya Technological University (V.T.U.), Belgaum, India.

He has also written and published three book-lets entitled as follows:

1. Towards Excellence… How to Study (A Guide book to Students)
2. Towards Excellence… How to teach (A guide book to Teachers)
3. Towards Excellence… Seminars, GD's and Personal Interviews
   (A guide book to Professional and Management students)

Dr. M. Thirumaleshwar has attended several National and International conferences and has more than 50 publications to his credit.

About the Softwares used

Following three softwares are used while solving problems in this book:

1. Mathcad 2001 (Ref: www.ptc.com)
2. Engineering Equation Solver (EES) (Ref: www.fchart.com), and
3. Finite Element Heat Transfer (FEHT) (Ref: www.fchart.com)
4. MS EXCEL – (2007) Spreadsheet (Ref: Microsoft)

Trial versions of the first three softwares and detailed Instruction Manuals may be down-loaded from the websites indicated. EXCEL is a very popular spreadsheet which comes bundled with MS OFFICE software, and is generally available in every computer.

See Part-I of this book for brief introduction to these four software.

While the information given there is enough to get going, for detailed instructions one should consult the respective Instruction manuals.

To the Student

Dear Student:

I would like to remind you that Heat Transfer is an important subject useful in many branches of engineering. It is also a subject in which you can score high marks in the examinations, since the question paper generally consists of derivations and numerical problems, almost in the ratio 50:50. Therefore, it requires that:

1) you are thorough with the derivations, and
2) skillful in solving numerical problems.

To be thorough with derivations, you should refer to well known, standard Text books on the subject of Heat Transfer (See References at the end of this book). **And, to develop your skill in solving problems… well, that is where I think that this book will help you.**

This book contains solutions to problems on heat transfer using four popular softwares, viz. Mathcad, Engineering Equation Solver (EES), Finite Element Heat Transfer (FEHT), and EXCEL spreadsheet. Trial versions of Mathcad, EES and FEHT can be downloaded from the websites indicated. EXCEL, which is a part of MS OFFICE, is generally pre-installed in most of the Personal Computers. Problems are chosen from the University question papers and standard heat transfer Text books.

Use of Software in solving problems has many advantages:

1. It helps in logical thinking
2. Problems are solved quickly and accurately
3. Parametric solutions (or 'what-if' solutions) are obtained easily
4. Solutions can be presented in tabular or graphical form, very easily and quickly
5. Once a particular type of problem is solved, solving a similar problem with different data input becomes very easy
6. Ease of getting solutions to problems in tabular or graphical form creates further interest and curiosity on the subject in the minds of students and encourages them to be creative and work further

How to use this Book?

You need not worry if you don't know about these softwares. Since each problem is solved systematically step by step, and is well commented, just reading through the solution will make the logic of the solution clear to you. That is the most important thing in solving the problems. ***Then, you must work out the problem yourself, by hand or using the software***. Of course, use of software has the above-mentioned advantages. *Simply reading the book won't do*. Have your favorite Text book nearby, in case you need to refer to it for any formulas or clarifications. There is no other 'easy method'.

As they say, *'there is no gain without pain!'*

Lastly, I would like to tell you how greatly I enjoyed solving the problems presented in this book using the softwares mentioned.

I hope that you too will enjoy as much as I did in solving these problems and get benefitted.

Good Luck!

Author

Chapter 2
Convection

2A.1 Forced Convection:

Learning objectives:

1. In convection heat transfer, there is a flow of fluid associated with heat transfer and the energy transfer is mainly due to bulk motion of the fluid.
2. When the flow of fluid is caused by an external agency such as a fan or pump or due to atmospheric disturbances, the resulting heat transfer is known as 'Forced convection heat transfer'.
3. When the flow of fluid is due to density differences caused by temperature differences, the heat transfer is said to be by 'Natural (or free) convection'.
4. Exact mathematical solutions, even for simple convection heat transfer cases, are rather complicated and it is common practice to resort to empirical relations for solutions of problems involving convection heat transfer.
5. In this chapter, we shall first solve problems on velocity and thermal boundary layers.
6. Our emphasis will be on practical solutions with the use *of empirical relations*. So, we present several empirical relations to determine friction and heat transfer coefficients for flow over different geometries such as a flat plate, cylinder and sphere for flow under laminar and turbulent conditions.

7. Finally, flow inside tubes will be considered and determination of heat transfer coefficient by analogy with the mechanism of fluid flow will be explained.

2A1 Equation Summary: [Ref: 1]

Summary of Basic equations for forced convection:

Geometry/details	Correlation	Restrictions
Flat Plate, laminar flow:		
Hydrodynamic boundary layer thickness	$\delta_{lam} = \frac{5\cdot x}{(Re_x)^{0.5}}$ ....(9.11)	Re < 5 x 10^5
Local friction coeff.	$C_{fx} = \frac{\tau}{\left(\frac{\rho\cdot U^2}{2}\right)} = \frac{0.664}{\sqrt{Re_x}}$ .....(9.31)	Re < 5 x 10^5
Local Nusselt number	$\frac{h\cdot x}{k} = Nu_x = 0.332\sqrt{Re_x}\cdot Pr^{0.333}$ ....(9.38)	Re < 5 x 10^5, Pr > 0.5
Avg. Friction coeff.	$C_{fa} = \frac{1}{L}\cdot\int_0^L C_{fx}\,dx = \frac{1.328}{\sqrt{Re_L}}$ ....(9.32)	Re < 5 x 10^5
Avg. Nusselt number	$Nu_a = 0.664\sqrt{Re_L}\cdot Pr^{0.333}$ .....(9.41)	Re < 5 x 10^5, Pr > 0.5
Local Nusselt number for liquid metals	$Nu_x = 0.565 Pe_x^{0.5}$ ....(Pr <.005).....(9.43)	Re < 5 x 10^5 Pe = Re. Pr
Flat Plate, turbulent flow:		
Hydrody. b.l. thickness	$\delta_{turb} = \frac{0.376x}{(Re_x)^{0.2}}$ ....(9.12)	Re_x > 5 x 10^5
Local friction coeff	$C_{fx} = 0.0576 Re_x^{\frac{-1}{5}}$ .....(9.74)	Re_x > 5 x 10^5, Pr > 0.5
Local Nusselt number	$Nu_x = \frac{h_x x}{k} = 0.0288 Re_x^{0.8} Pr^{\frac{1}{3}}$ .....(9.77)	Re_x > 5 x 10^5, Pr > 0.5
Avg. Friction coeff.	$C_{fa} = 0.072 Re_L^{\frac{-1}{5}}$ ...(9.75)	5 x 105 < ReL < 107

Geometry/details	Correlation	Restrictions
Avg. Friction coeff.	$C_{fa} = \frac{0.455}{\left(\log\left(Re_L\right)\right)^{2.58}}$ ...(9.76)	107 < ReL < 109
Flat Plate, mixed boundary layer:		
Avg. Friction coeff.	$C_{fa} = \frac{0.074}{Re_L^{\frac{1}{5}}} - \frac{1742}{Re_L}$ ...(9.184, a)	5 x 105 < ReL < 107 $Re_{x,c}$ = 5 x 105
Avg. Nusselt number	$Nu_{avg} = \frac{h \cdot L}{k} = \left(0.036 Re_L^{\frac{4}{5}} - 836\right) \cdot Pr^{\frac{1}{3}}$ ...(9.185, a)	0.6 < Pr < 60, 5 x 105 < ReL < 107
Cylinder in cross flow: Average Nusselt number	$Nu_{cyl} = \frac{h \cdot D}{k} = 0.3 + \frac{0.62 \cdot Re^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}} \cdot \left[1 + \left(\frac{Re}{28200}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$ .....(9.90)	Re.Pr > 0.2
Cylinder in liquid metal flow	$Nu_{cyl} = 1.125(Re \cdot Pr)^{0.413}$ ....for 1 < Re.Pr < 100....(9.93)	
Flow across a sphere: Comprehensive eqn. of Whitaker. Average Nusselt number	$Nu_{sph} = 2 + \left(0.4 Re^{\frac{1}{2}} + 0.06 Re^{\frac{2}{3}}\right) Pr^{0.4} \left(\frac{\mu_a}{\mu_w}\right)$ ....(9.97)	For gases & liquids. $3.5<Re<7.6.10^4$ 0.71<Pr<380, $1<\mu/\mu_s<3.2$
Falling drop: Avg. Nusselt no.	$Nu_{avg} = 2 + 0.6 Re^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}}$ .....(9.97, a)	
Flow across Tube bank: Turbulent flow	$Nu_a = 0.021 \cdot Re_D^{0.84} \cdot Pr^{0.36} \cdot \left(\frac{Pr}{Pr_w}\right)^{0.25}$ ..for In - line tubes, ...(9.113) $Nu_a = 0.022 Re_D^{0.84} \cdot Pr^{0.36} \cdot \left(\frac{Pr}{Pr_w}\right)^{0.25}$ ..for staggered tubes, Pr > 1 ...(9.114) $Nu_a = 0.019 Re_D^{0.84}$ ...for staggered tubes, Pr = 0.7.....(9.115)	For definition of Re_D, see text. N > 20, and 0.7 < Pr < 500, $1000 < Re_{D\_max} < 2 \times 10^6$

Geometry/details	Correlation	Restrictions
Flow across Tube banks: **Pressure drop**	$\Delta p = \frac{2 \cdot f \cdot G_{max}^{2} \cdot N}{\rho} \cdot \left(\frac{\mu_w}{\mu_b}\right)^{0.14}$ Pa......(9.118).	
Friction factor in eqn. (9.118)	$f = \left[0.25 + \frac{0.118}{\left[\frac{(S_T - D)}{D}\right]^{1.08}}\right] \cdot Re_D^{-0.16}$ ....for staggered tubes....(9.119)	
Friction factor in eqn. (9.118)	$f = \left[0.044 + \frac{0.08\left(\frac{S_L}{D}\right)}{\left[\frac{(S_T - D)}{D}\right]^{0.43 + 1.13 \cdot \frac{D}{S_L}}}\right] \cdot Re_D^{-0.15}$ ....for in-line tubes....(9.120)	
Flow through packed beds: Heat transfer between gas and packings	$\frac{h_a \cdot D_p}{k} = \frac{1 - \varepsilon}{\varepsilon} \cdot \left(0.5 Re_{Dp}^{\frac{1}{2}} + 0.2 Re_{Dp}^{\frac{2}{3}}\right) \cdot Pr^{\frac{1}{3}}$ ....(9.101)	$20 < Re_{Dp} < 10{,}000$, $0.34 < \varepsilon < 0.78$. see text for definition of Re_{Dp} and ε

Geometry/details	Correlation	Restrictions
Flow through packed beds: Heat transfer between walls of bed and gas	$\frac{h_a \cdot D_p}{k} = 2.58 Re_{Dp}^{\frac{1}{3}} \cdot Pr^{\frac{1}{3}} + 0.094 Re_{Dp}^{0.8} \cdot Pr^{0.4}$ ……(9.102)	For particles like cylinders, see text for definition of Re_{Dp}
Flow through packed beds: Heat transfer between walls of bed and gas	$\frac{h_a \cdot D_p}{k} = 0.203 Re_{Dp}^{\frac{1}{3}} \cdot Pr^{\frac{1}{3}} + 0.220 Re_{Dp}^{0.8} \cdot Pr^{0.4}$ ……(9.103)	For particles like spheres, see text for definition of Re_{Dp}
Flow inside tubes: Hydrodynamic and thermal entry lengths	$L_{h\_lam} = 0.05 \cdot Re \cdot D$ ....(9,121, a) $L_{t\_lam} = 0.05 \cdot Re \cdot Pr \cdot D$ ....(9.121, b) $L_{h\_turb} = L_{t\_turb} = 10 \cdot D$ ....(9,122)	Re < 2300…laminar Re > 4000…turb.
Darcy-Weisbach eqn. for pressure drop	$\frac{\Delta p}{L} = \frac{f}{D} \cdot \frac{\rho \cdot u_m^2}{2}$ .....(9.128)	
Friction factor	$f = \frac{64}{Re_D}$ ....(9.129)	Laminar flow in tubes
Flow inside tubes: Nusselt no. for fully developed lam. flow, const. wall heat flux	$Nu_D = \frac{h \cdot D}{k} = 4.364$ ....(9.145)	Pr>0.6
Flow inside tubes: Nusselt no. for fully developed lam. flow, const. wall temp.	$Nu_D = \frac{h \cdot D}{k} - 3.66$	Pr > 0.6
Flow inside short tubes: Nusselt no. for fully developed velocity profile, lam. flow, const. wall temp.	$Nu_{avg} = 3.66 + \frac{0.0668 \left(\frac{D}{L}\right) \cdot Re \cdot Pr}{1 + 0.04 \left[\left(\frac{D}{L}\right) \cdot Re \cdot Pr\right]^{\frac{2}{3}}}$ ..Pr > 0.7...(9.150, a)	L/D < 60
Flow inside short tubes: Nusselt no. for fully developed velocity profile, lam. flow, const. wall temp. Sieder & Tate relation.	$Nu_{avg} = 1.86 \left(\frac{Re \cdot Pr}{\frac{L}{D}}\right)^{\frac{1}{3}} \cdot \left(\frac{\mu}{\mu_s}\right)^{0.14}$ ....(9.150, c)	$\left(\frac{Re \cdot Pr}{\frac{L}{D}}\right)^{\frac{1}{3}} \cdot \left(\frac{\mu}{\mu_s}\right)^{0.14} \ge 2$ 0.48< Pr <16,700 $0.0044<(\mu/\mu_s)<9.75$

Geometry/details	Correlation	Restrictions
Flow inside short tubes: Local Nusselt no. for fully developed velocity profile, lam. flow, const. wall heat flux.	$\mathrm{Nu}=4.36+\dfrac{0.023\left(\dfrac{D}{L}\right)\cdot \mathrm{Re}\cdot \mathrm{Pr}}{1+0.0012\left[\left(\dfrac{D}{L}\right)\cdot \mathrm{Re}\cdot \mathrm{Pr}\right]}$ ..Pr > 0.7...(9.151, a)	
Flow inside tubes: Friction factor for smooth pipes	$f=0.316\cdot \mathrm{Re}^{-0.25}$ ....for 2 x 10^4 < Re < 8 x 10^4 .....(9.154) $f=0.184\cdot \mathrm{Re}^{-0.2}$ ....for 10^4 < Re < 10^5 .....(9.155) $f=(0.79\cdot \ln(\mathrm{Re})-1.64)^{-2}$ ....for 3000 < Re < 5 x 10^6 .....(9.156)	
Flow inside tubes: Friction factor for rough pipes	$f=\dfrac{1.325}{\left(\ln\left(\dfrac{\varepsilon}{D}\right)+\dfrac{5.74}{\mathrm{Re}^{0.9}}\right)^2}$ ....(9.156, b)	Relative roughness, (ε/D) is known
Reynold's analogy	$\mathrm{St}=\dfrac{h}{\rho\cdot C_p\cdot u_m}=\dfrac{\mathrm{Nu}_D}{\mathrm{Re}_d\cdot \mathrm{Pr}}=\dfrac{f}{8}$ ...(9.165)	

Geometry/details	Correlation	Restrictions
Colburn analogy	$St \cdot Pr^{\frac{2}{3}} = \frac{f}{8}$ ....(9.166)	
Flow inside tubes: **Turbulent flow:** Nusselt number	$Nu = 0.023 Re^{0.8} \cdot Pr^{n}$ ...for 0.7 < Pr <160, Re > 10,000..........(9.170) n = 0.4 when fluid is being heated, and n = 0.3 when fluid is being cooled	Dittus-Boelter eqn. 0.6<Pr<160 Re > 10,000 L/D > 10
Flow inside tubes: **Turbulent flow:** Nusselt number, when there is property variation	$Nu = 0.027 Re^{0.8} \cdot Pr^{\frac{1}{3}} \cdot \left(\frac{\mu_b}{\mu_s}\right)^{0.14}$ ....(9.171)	Sieder-Tate eqn. 0.7 < Pr < 16,700, 6000< Re < 107 L/D >10
Flow inside tubes: **Turbulent flow:** Nusselt number	$Nu = \frac{\left(\frac{f}{8}\right) \cdot Re \cdot Pr}{1.07 + 12.7 \cdot \left(\frac{f}{8}\right)^{0.5} \cdot \left(Pr^{0.67} - 1\right)} \cdot \left(\frac{\mu_b}{\mu_s}\right)^{n}$ ...(9.171, a)	Fits the experimental data better; n = 0.11 for heating of fluids, n = 0.25 for cooling of fluids, n = 0 for constant heat flux, $\mu_b/\mu_s = T_s/T_b$, temp. in Kelvin
Flow of liquid metals inside smooth pipes: const. surface heat flux.	$Nu = 4.82 + 0.0185 Pe^{0.827}$ ....(9.174)	3600 < Re < 9.05 x 105, 100 < Pe < 10,000
Flow of liquid metals inside smooth pipes: const. surface heat flux.	$Nu = 6.3 + \left(0.0167 Re^{0.85} \cdot Pr^{0.93}\right)$ .....(9.175)	Recent correlation which fits experimental data better.
Flow of liquid metals inside smooth pipes: const. surface temperature.	$Nu = 5.0 + 0.025 Pe^{0.8}$ ....for T_s = const, Pe > 100......(9.176)	
Helically coiled tubes: **Turbulent flow:** Hausen's relation	$\frac{Nu_{a\_helical}}{Nu_{a\_straight}} = 1 + \left(\frac{21}{Re^{0.14}}\right) \cdot \left(\frac{D}{d_c}\right)$ .....(9.183)	D = dia. of tube d_c = dia. of helix

2A1.1 Boundary layer fundamentals, Flow over flat plates, Momentum – heat transfer analogy etc.:

Prob. 2A1.1.1. Air at 15 C and 1 atm is flowing over a 0.3 m long plate at 65 C at a velocity of 3 m/s. Plot the following on a combined graph for the range of x = 0 m to x = xcr:

a) the hydrodynamic boundary layer as a function of x
b) the thermal boundary layer as a function of x

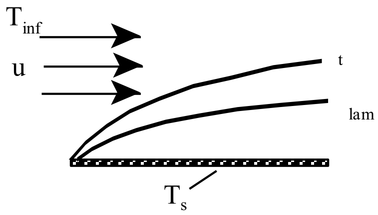


Fig. Prob.2A1.1.1

Mathcad Solution:

Data:

$T_{inf} := 15$ C $\quad T_s := 65$ C $\quad u := 3$ m/s $\quad L := 0.3$ m

$$T_f := \frac{T_{inf} + T_s}{2}$$ ....film temp.

i.e. $T_f = 40$ C

Properties of Air at 1 atm and 40 C:

$\rho := 1.1181$ kg/m^3

$cp := 1.008 \cdot 10^3$ J/kg.K

$\mu := 190.7 \cdot 10^{-7}$ N.s/m^2

$k := 27.3 \cdot 10^{-3}$ W/m.K

$Pr := 0.705$

Calculations:

For flow over a flat plate:

$Re_{cr} := 5 \cdot 10^5$ ...crit. Reynolds No.

And, $Re_{cr} = \frac{x_{cr} \cdot u \cdot \rho}{\mu}$

i.e. $x_{cr} := \frac{Re_{cr}}{(u \cdot \rho)} \cdot \mu$

i.e. $x_{cr} = 2.843$ m...

i.e. For the entire legth of 0.3m of the plate, the flow is laminar.

Therefore: Hydrodynamic Boundary layer thickness is given by:

$$\delta_{lam} = \frac{5 \cdot x}{\sqrt{Re_x}}$$

To plot the graph: let us write δ as a function of x :

$\mathrm{Re}_x(x) := \frac{x \cdot u \cdot \rho}{\mu}$ ....Re as a function of x

$\delta_{lam}(x) := \frac{5 \cdot x}{\sqrt{\mathrm{Re}_x(x)}}$ ... δ as a function of x

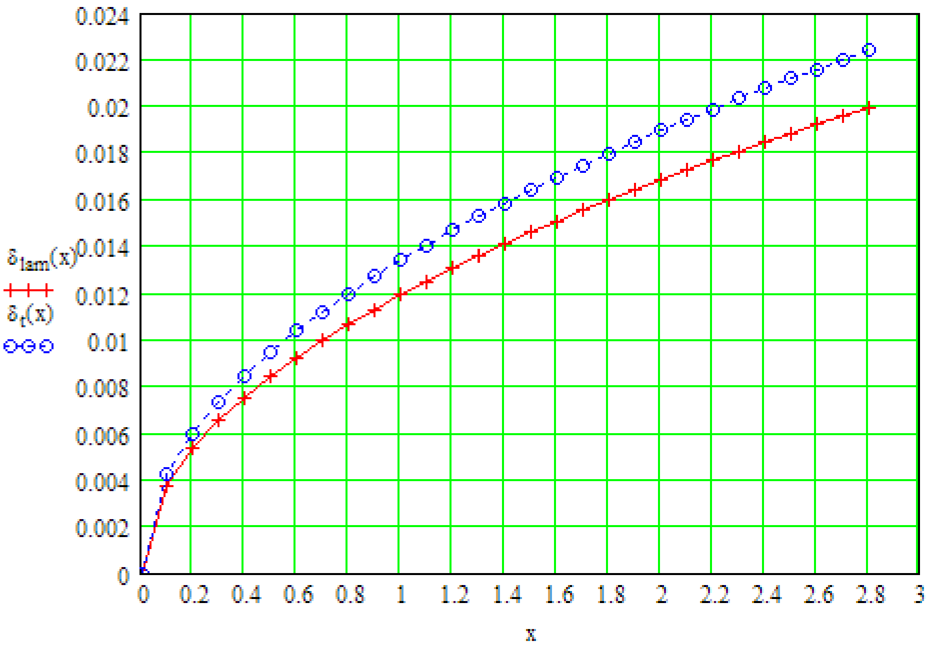
Also: Thermal boundary layer thickness is:

$\delta_t(x) := \frac{\delta_{lam}(x)}{\mathrm{Pr}^{\frac{1}{3}}}$ ... δ_t as a function of x

To plot the graphs for x = 0 to x = x_{cr}:

x	$\mathrm{Re}_x(x)$	$\delta_{lam}(x)$	$\delta_t(x)$
0	0	0	0
0.1	17589.407	0.0038	0.0042
0.2	35178.815	0.0053	0.006
0.3	52768.222	0.0065	0.0073
0.4	70357.63	0.0075	0.0085
0.5	87947.037	0.0084	0.0095
0.6	105536.445	0.0092	0.0104
0.7	123125.852	0.01	0.0112
0.8	140715.26	0.0107	0.012
0.9	158304.667	0.0113	0.0127
1	175894.074	0.0119	0.0134
1.1	193483.482	0.0125	0.014
1.2	211072.889	0.0131	0.0147
1.3	228662.297	0.0136	0.0153
1.4	246251.704	0.0141	0.0158
1.5	263841.112	0.0146	0.0164
1.6	281430.519	0.0151	0.0169
1.7	299019.927	0.0155	0.0175
1.8	316609.334	0.016	0.018
1.9	334198.741	0.0164	0.0185
2	351788.149	0.0169	0.0189
2.1	369377.556	0.0173	0.0194
2.2	386966.964	0.0177	0.0199
2.3	404556.371	0.0181	0.0203
2.4	422145.779	0.0185	0.0208
2.5	439735.186	0.0189	0.0212
2.6	457324.594	0.0192	0.0216
2.7	474914.001	0.0196	0.022
2.8	492503.408	0.0199	0.0224

x in metres, δ and δ_t in metres:



Prob. 2A1.1.2. A journal bearing can be idealized as a stationary flat plate and a moving flat plate that moves parallel to it. The space between the two plates is filled by an incompressible fluid. Consider such a bearing in which the stationary and moving plates are at 10 C and 20 C, respectively, the distance between them is 3 mm, the speed of the moving plate is 5 m/s, and there is engine oil between the plates. (a) Calculate the heat flux to the upper and lower plates (b) Determine the max. temp of the oil.

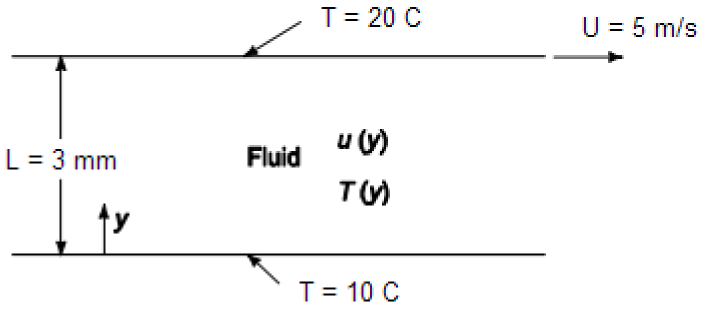


Fig.Prob.2A1.1.2

Mathcad Solution:

For steady, two dimensional flow of an incompressible fluid, we have the following equations for the conservation of mass, momentum and energy [Ref: 3]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\ldots(a)$$

$$\rho\cdot\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\cdot\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+X\ldots(b)$$

$$\rho\cdot\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\cdot\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+Y\ldots(c)$$

$$\rho\cdot cp\cdot\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\cdot\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\cdot\phi+q\ldots(d)$$

$$\mu\cdot\phi=\mu\cdot\left\{\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2+2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]\right\}\ldots(e)$$

In the above: eqn.(a) is the equation for conservation of mass.

Eqn. (b) is the eqn. for conservation of momentum, in x-direction, X is the net body forces in x-direction.

Eqn. (c) is conservation of momentum in y direction, Y is the net body forces in y-direction.

Eqn. (d) is the conservation of energy, with the viscous forces included.

Eqn. (e) gives expression for viscous dissipation.

Now, for this particular case:

Eqn. (a) for mass balance becomes, since v = 0:

$$\frac{\partial u}{\partial x} = 0$$

i.e. there is no variation of u with x; u changes only with y.

Along with the above eqn., **eqn. (b) for momentum balance becomes**, since v = 0, X = 0, and

$$\frac{\partial p}{\partial x} = 0:$$

$$\frac{\partial^2 u}{\partial y^2} = 0$$

Integrating twice to get velocity distribution:

$$u(y) = C_1\, y + C_2$$

Applying B.C's: u(0) = 0, and u(L) = U, we get C2 = 0, and C1 = U/L.

Therefore, velocity distribution is given by:

U(y) = (y/L).U ……….eqn. (1)

Next, the energy eqn. (d) becomes:

$$0 = k \cdot \frac{\partial^2 T}{\partial y^2} + \mu \cdot \left(\frac{\partial u}{\partial y}\right)^2$$

since v = 0, q = 0, and

$$\frac{\partial u}{\partial x} = 0, \quad and \quad \frac{\partial T}{\partial x} = 0$$

Therefore:

$$k \cdot \frac{\partial^2 T}{\partial y^2} = -\mu \cdot \left(\frac{\partial u}{\partial y}\right)^2 = -\mu \cdot \left(\frac{U}{L}\right)^2$$

Integrating twice, **we get Temp. distribution:**

$$T(y) = -(\mu/2k).(U/L)^2.y^2 + C_3.y + C_4$$

Applying the B.C'S: $T(0) = T_0$, $T(L) = T_L$, we get:

$$C_4 = T_0, \text{ and } C_3 = (T_L - T_0)/L + (\mu/2k).(U^2/L)$$

And the temp distribution becomes:

$$T(y) = T_0 + \left(\frac{\mu}{2 \cdot k}\right) \cdot U^2 \cdot \left[\left(\frac{y}{L}\right) - \left(\frac{y}{L}\right)^2\right] + (T_L - T_0) \cdot \left(\frac{y}{L}\right)$$

Once the temp. distribution is known, heat fluxes at the upper and lower surfaces can be obtained by applying Fourier's Law.

Mathcad Solution is given below:

Properties of Engine oil at mean temp of 15 C:

$\rho := 891.1$ kg/m^3

$cp := 1860$ J/kg.K

$\mu := 123.32 \cdot 10^{-2}$ N.s/m^2

$\nu := 1382 \cdot 10^{-6}$ m^2/s

$k := 0.1448$ W/m.K

$\alpha := 0.874 \cdot 10^{-7}$ m^2/s

$Pr := 15820$

Also:

$L := 0.003$ m

$T_0 := 10$ C

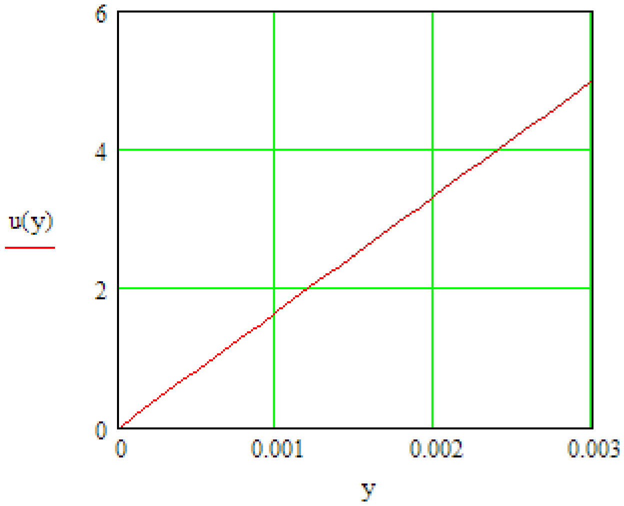
$T_L := 20$ C

$U := 5$ m/s

Velocity distribution is given by:

$$u(y) := \left(\frac{y}{L}\right) \cdot U$$ ....velocity as a function of y

This velocity distribution is plotted below:



Temp. distribution is given by:

$$T(y) := T_0 + \frac{\mu}{2 \cdot k} \cdot U^2 \cdot \left[\frac{y}{L} - \left(\frac{y}{L}\right)^2\right] + (T_L - T_0) \cdot \frac{y}{L}$$ ...Temp as a function of y

Check: $T(0) = 10$ $T(L) = 20$

Heat fluxes at the stationary and moving plates:

For Stationary plate: i.e. y = 0

Let: $T\_prime(y) := \frac{d}{dy}T(y)$

Therefore: $q\_stationary := -k \cdot T\_prime(0)$

i.e. $q\_stationary = -5.621 \cdot 10^{3}$ W/m^2...Ans.

And, for Moving plate:, i.e. y = L:

$q\_moving := -k \cdot T\_prime(L)$

i.e. $q\_moving = 4.656 \cdot 10^{3}$ W/m^2....Ans.

Mechanical power wasted by viscous dissipation in oil:

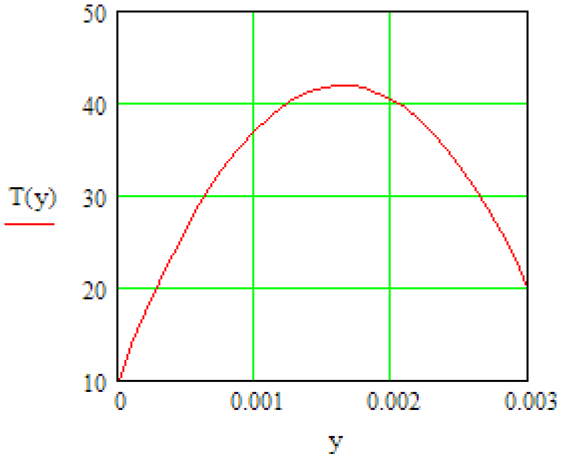
This is, obviously equal to the heat dissipated to the upper and lower plates, i.e.

$$q\_dissip := |q\_stationary| + |q\_moving|$$

i.e. $q\_dissip = 1.028 \cdot 10^4$ **W/m^2 .... Ans.**

Max. Temp. in oil:

First, draw the temp profile:



Max temp occurs when first derivative of T i.e. T'(y) = 0

Use the 'root function' of Mathcad to find ymax, where Tmax occurs:

$y := 0.0015$ ...guess value of ymax

$ymax := root(T\_prime(y), y)$

i.e. $ymax = 1.641 \cdot 10^{-3}$ **m... distance from bottom plate at which max temp occurs.**

i.e. T'(y) = 0 at y = ymax = 0.001641 m. See the graph above to confirm this.

Then, Tmax is obtained by substituting ymax in T(y):

$T(ymax) = 41.849$ **C.... Max. temp. in oil .... Ans.**

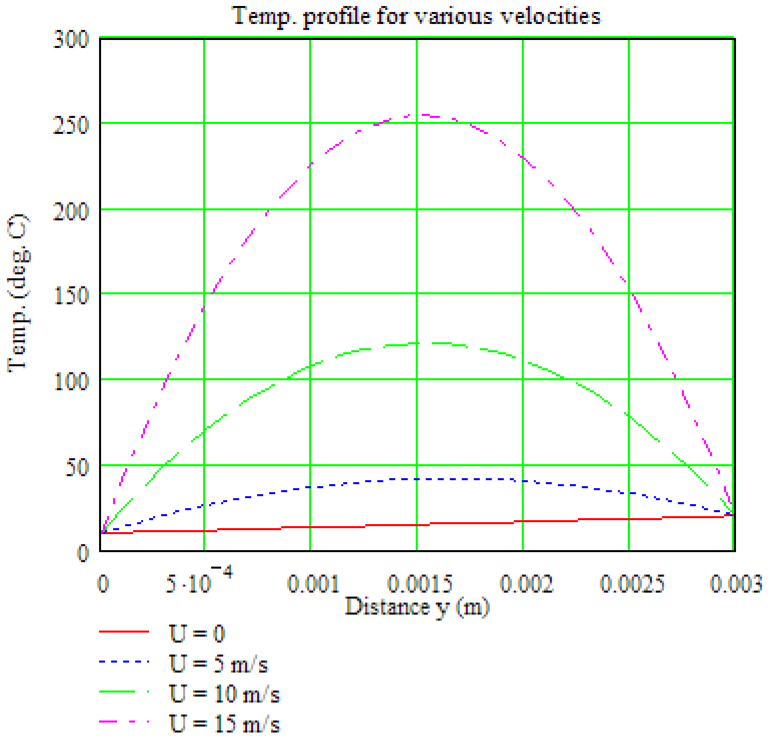
To draw Temp. distribution for different U values:

Let:

$$T(y,U) := T_0 + \frac{\mu}{2 \cdot k} \cdot U^2 \cdot \left[\frac{y}{L} - \left(\frac{y}{L} \right)^2 \right] + \left(T_L - T_0 \right) \cdot \frac{y}{L}$$ ...Temp as a function of y and U

Draw temp. profile for U = 0, 5, 10 and 15 m/s:

$y := 0, 0.0001 .. 0.003$ ...define a range variable y



Note that for U = 10 m/s, max temp in oil is about 120 C, and for U = 15 m/s, it is about 252 C. Now, remember that we have taken properties of oil at 15 C. Viscosity value is quite sensitive to temperature. Therefore, to be accurate, we have to take properties of oil at mean temp of (120 + 20) /2 or (252 + 20) / 2 and re-calculate. *This is left as an exercise to student.*

Plot the mechanical power wasted in dissipation in oil as a function of velocity U:

Write q_dissip as a function of U:

$$T(y,U) := T_0 + \frac{\mu}{2\cdot k}\cdot U^2\cdot\left[\frac{y}{L} - \left(\frac{y}{L}\right)^2\right] + \left(T_L - T_0\right)\cdot\frac{y}{L}$$ ...Temp as a function of y and U

Let: $T\_prime(y,U) := \frac{d}{dy}T(y,U)$

Therefore: $q\_stationary(U) := -k\cdot T\_prime(0,U)$

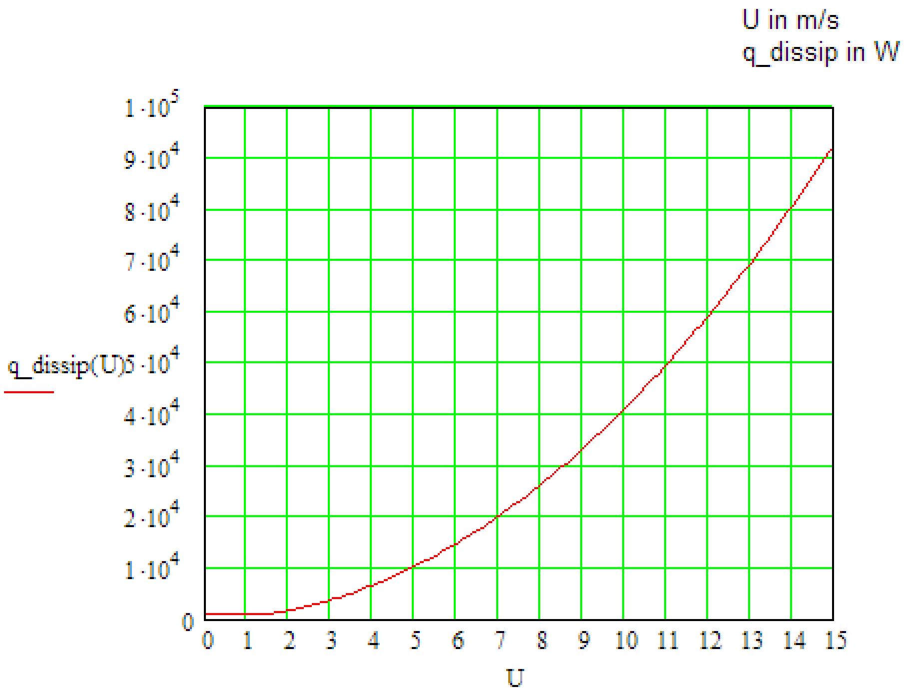
$q\_moving(U) := -k\cdot T\_prime(L,U)$

$$q\_dissip(U) := \left|q\_stationary(U)\right| + \left|q\_moving(U)\right|$$

Check: $q\_dissip(U) = 1.028\cdot10^4$ W, for $U = 5$ m/s

To draw the plot of q_dissip vs U for U = 0 to U = 15 m/s:

$U := 0, 0.1 .. 15$ range variable U



==

"**Prob. 2A1.1.3**. A journal bearing has a clearance of 0.5 mm. The journal has a diameter of 100 mm and rotates at 3600 rpm within the bearing. It is lubricated by an oil having a density of 800 kg/m^3, viscosity of 0.01 kg/m.s, and a thermal cond. of 0.14 W/m.K. If both the journal and bearing temps are maintained at 60 C, calculate the rate of heat transfer from the bearing and the power required for rotation per unit length."

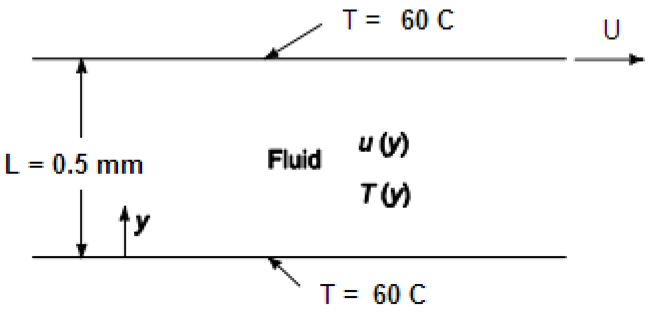


Fig.Prob.2A1.1.3

EES Solution:

```
“First write functions for temp distribution T(y) and first derivative of temp, i.e. dT/dy:”

Function T(y,mu,k,U,L,T_L,T_0)

T := T_0 + mu/(2 * k) * U^2 * ((y / L) – (y / L)^2) + (T_L – T_0) * (y / L)

end

Function dTbydy (y, mu, k, U, L, T_L, T_0)

dTbydy := (mu / (2 * k)) * U^2 * ( (1 / L) – 2 * y/ L^2) + (T_L – T_0) / L

end

“Data:”

D = 0.1 [m] “….dia of journal”
L = 0.5E-03 [m] “…distance between two parallel plates , idealizing the journal bearing as two plates, one stationary and the other one moving”
U = (3600 / 60) * (pi * D) “[m/s]”
T_0 = 60 [C]
T_L = 60 [C]

“Properties of oil:”

rho = 800 [kg/m^3]
mu = 0.01 [kg/m-s]
k = 0.14 [W/m-C]

“Calculations:”

“This problem is similar to the previous problem.

Heat transfer to the journal and bearing is given by Fourier’s Law:

Q_journal = – k * (pi * (D + 2.L) * 1) * dTbydy_y=0, per unit length of journal

Q_bearing = – k * (pi * (D + 2.L) * 1) * dTbydy_y=L, per unit length of bearing”
```

Q_journal = – k * (pi * (D + 2 * L) *1) * dTbydy(0, mu, k, U, L, T_L, T_0) "…. W per unit length of journal"

Q_bearing = – k * (pi * (D + 2 * L) *1) * dTbydy(L, mu, k, U, L, T_L, T_0) "…. W per unit length of bearing"

Q_tot = abs(Q_journal) + abs(Q_bearing) "W/m length"

"Power to turn the journal:

This is equal to the Drag force multiplied by the velocity.

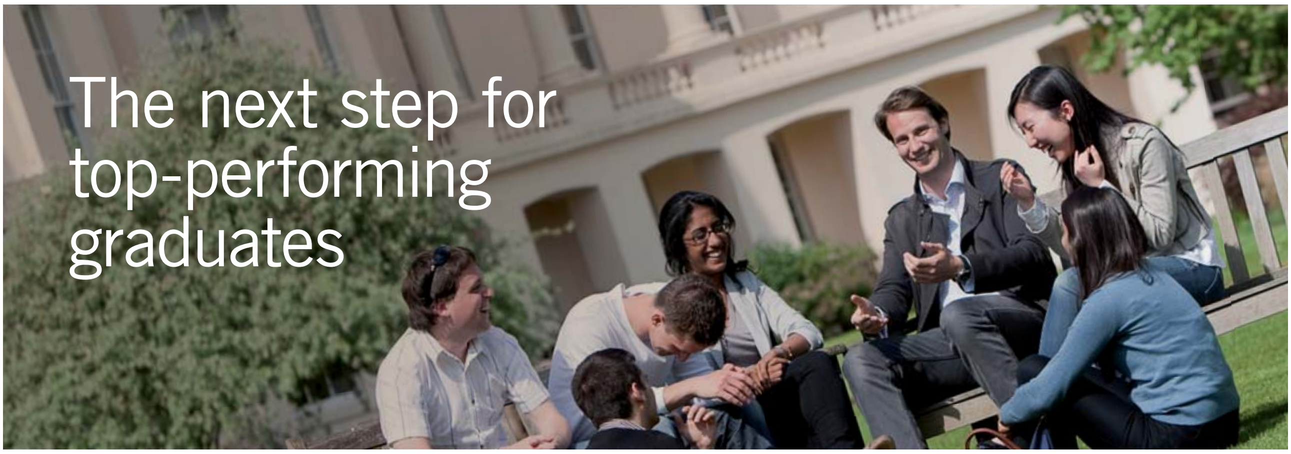
Drag force = shear stress * Area

Shear stress = tau = mu * dU/dy at y = 0

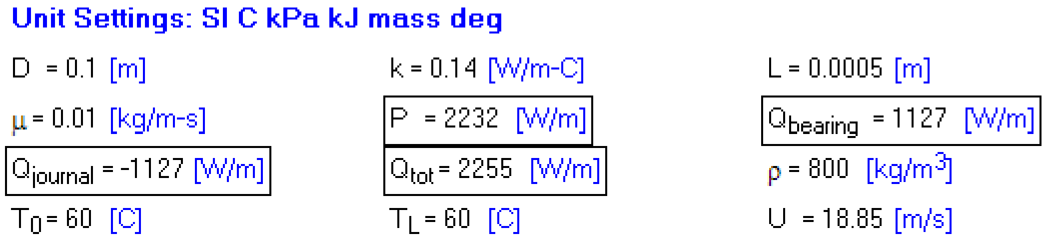
Area = p * D * L.

So, we get:"

P = (1/L) * pi * D * mu * U^2 "W per unit length of bearing"



Results:



```
Unit Settings: SI C kPa kJ mass deg
D  = 0.1 [m]                k = 0.14 [W/m-C]          L = 0.0005 [m]
μ = 0.01 [kg/m-s]           P  = 2232 [W/m]           Qbearing  = 1127 [W/m]
Qjournal = -1127 [W/m]      Qtot = 2255 [W/m]         ρ = 800 [kg/m3]
T0 = 60 [C]                 TL = 60 [C]               U  = 18.85 [m/s]
```

Thus:

Heat transfer to journal = -1127 W/m (-ve sign indicating heat flow in -ve y direction, i.e. into the bottom plate) …. Ans.

Heat transfer to bearing = 1127 W/m (+ve sign indicating heat flow in +ve y direction, i.e. into the top plate) …. Ans.

Total heat transfer, Qtot = 2255 W/m …. Ans.

Mech. Power required to turn the bearing, P = 2232 W.

Note that Qtotal = P, as it should be. Small difference is due to the difference in surface areas of journal and bearing.

===

"**Prob. 2A1.1.4.** Air at 20 C and atm pressure is flowing over a flat plate at a velocity of 3 m/s. If the plate is 30 cm wide and at a temp. of 60 C, calculate: (a) thickness of velocity and thermal boundary layers (b) local and average friction coeff. (c) local and average heat tr. coeff. (d) total drag force on the plate. Take the following properties of air at 313 K: [VTU – July/Aug. 2002]."

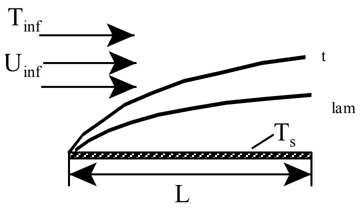


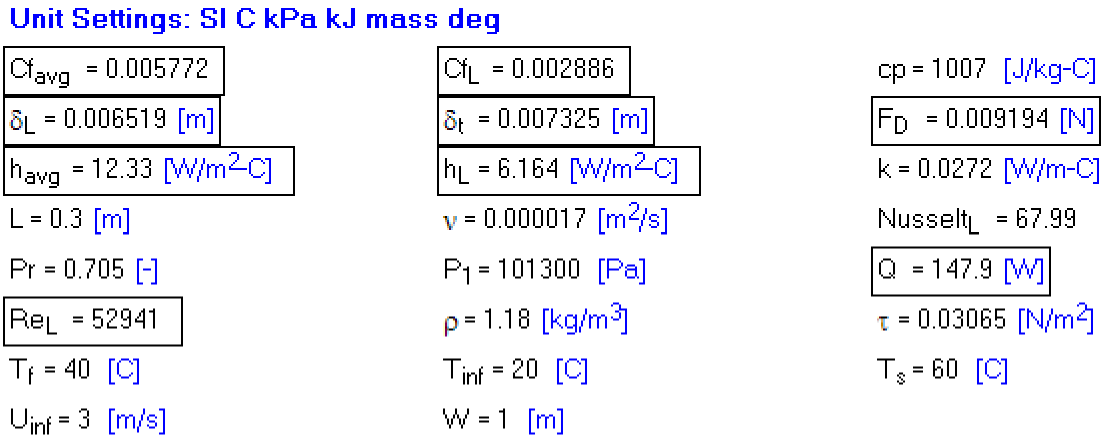
Fig.Prob.2A1.1.4

```
"Data:"

"Properties of Air at mean temp (T_f = 40 C = 313 K):"

rho = 1.18 [kg/m^3]
k = 0.0272 [W/m-C]
cp = 1007 [J/kg-C]
Pr = 0.705 [-]
nu = 17E-06 [m^2/s]

T_inf = 20[C]
T_s = 60 [C]
T_f = (T_inf + T_s) / 2
U_inf = 3[m/s]
L = 0.3[m]
W = 1[m]
P_1 = 1.013e05 [Pa]

"Calculations:"

Re_L = L * U_inf / nu "…Reynold's No. at the end of plate"

"We note that Re_L is 52941 < 5E05. Therefore, flow is laminar. Use correlations for laminar flow:"

delta_L = 5 * L / sqrt(Re_L) "thickness of velocity b.l."

delta_t = delta_L / Pr^(1/3) " thickness of thermal b.l."

Cf_L = 0.664 / sqrt(Re_L) "…local friction coeff. at end of plate"

Cf_avg = 2 * Cf_L "…average friction coeff. over the entire plate"

Nusselt_L = 0.332 * sqrt(Re_L) * Pr^(1/3) "…local Nusselt No. at end of plate"

Nusselt_L = h_L * L / k "…gives local heat tr coeff h_L at end of plate"

h_avg = 2 * h_L "gives avg heat tr coeff h_avg over entire plate"

Q = h_avg * ( L * W) * (T_s – T_inf) "[W] … total heat tr from surface of plate"
```

tau = Cf_avg * rho * U_inf^2 / 2 "..shear stress, [N/m^2]"

F_D = tau * (L * W) "[N]…total drag force"

Results:



```
Unit Settings: SI C kPa kJ mass deg
Cfavg = 0.005772          CfL = 0.002886              cp = 1007  [J/kg-C]
δL = 0.006519 [m]         δt = 0.007325 [m]           FD = 0.009194 [N]
havg = 12.33 [W/m^2-C]    hL = 6.164 [W/m^2-C]        k = 0.0272 [W/m-C]
L = 0.3 [m]               ν = 0.000017 [m^2/s]        NusseltL = 67.99
Pr = 0.705 [-]            P1 = 101300 [Pa]            Q = 147.9 [W]
ReL = 52941               ρ = 1.18 [kg/m^3]           τ = 0.03065 [N/m^2]
Tf = 40 [C]               Tinf = 20 [C]               Ts = 60 [C]
Uinf = 3 [m/s]            W = 1 [m]
```

Required results are shown boxed in the above Table.

"**Prob. 2A1.1.5**. Air at 20 C is flowing along a heated flat plate at 134 C at a velocity of 3 m/s. If the plate is 2 m long and 1.5 cm wide, calculate the thickness of hydrodynamic boundary layer and the skin friction coeff at 40 cm from the leading edge of the plate. The kinematic viscosity of air at 20 C is 15.06E-06 m^2/s. Also calculate the local heat transfer coeff at x = 0.4 m and the heat transferred from the first 40 cm of the plate [VTU – Dec.06/Jan. 2007]."

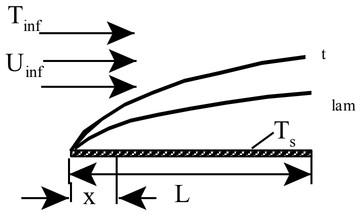


Fig.Prob.2A1.1.5

EES Solution:

"Data:"

"Take properties of Air at mean temp = (T_f = 77 C):"

T_inf = 20[C]
T_s = 134 [C]
T_f = (T_inf + T_s) / 2
U_inf = 3[m/s]

L = 2[m]
W = 0.015[m]
x = 0.4 [m]
P_1 = 1.013e05 [Pa]

"Properties of Air:"

rho = Density(Air,T=T_f,P=P_1) "[kg/m^3]"
k = Conductivity(Air,T=T_f) "[W/m-C]"
cp = Cp(Air,T=T_f) "[J/kg-C]"
mu = Viscosity(Air,T=T_f) "[kg/m-s]"
Pr=Prandtl(Air,T=T_f)
nu = mu / rho "[m^2/s]"

"**Note one great advantage with EES:** Properties of many fluids and substances of interest in Thermodynamics and Heat Transfer are available as built in functions in EES. Therefore, the labour of looking into Property Tables and the interpolation (with possibility of errors) is avoided.

In the above case, properties of Air are calculated from built in functions of EES."

"Calculations:"

Re_x = x * U_inf / nu "…Reynold's No. at x = 0.4 m"

"We note that Re_x is 58073 < 5E05. Therefore, flow is laminar. Use correlations for laminar flow:"

delta_x = 5 * x / sqrt(Re_x) "thickness of velocity b.l."

delta_t = delta_x / Pr^(1/3) " thickness of thermal b.l."

Cf_x = 0.664 / sqrt(Re_x) "…local friction coeff. at x = 0.4 m"

Cf_avg = 2 * Cf_x "…average friction coeff. upto x = 0.4 m of plate"

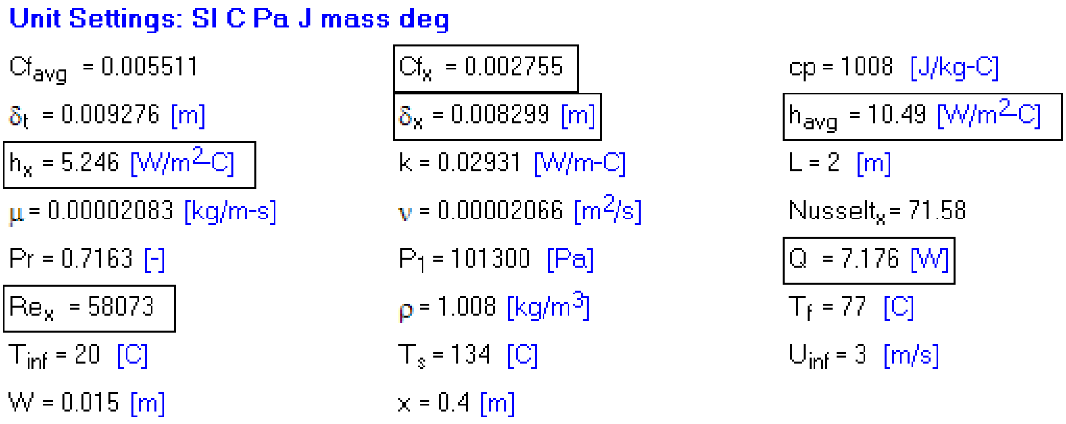
Nusselt_x = 0.332 * sqrt(Re_x) * Pr^(1/3) "…local Nusselt No. at x = 0.4 m of plate"

Nusselt_x = h_x * x / k "…gives local heat tr coeff Nu_x at x = 0.4 m of plate"

h_avg = 2 * h_x "gives avg heat tr coeff upto x = 0.4 m of plate"

Q = h_avg * (x * W) * (T_s – T_inf) "[W] … total heat tr from up to x = 0.4 m of plate"

Results:



Unit Settings: SI C Pa J mass deg

Cf_{avg} = 0.005511
δ_t = 0.009276 [m]
$\boxed{h_x = 5.246\ [W/m^2\text{-}C]}$
μ = 0.00002083 [kg/m-s]
Pr = 0.7163 [-]
$\boxed{Re_x = 58073}$
T_{inf} = 20 [C]
W = 0.015 [m]

$\boxed{Cf_x = 0.002755}$
$\boxed{\delta_x = 0.008299\ [m]}$
k = 0.02931 [W/m-C]
ν = 0.00002066 [m^2/s]
P_1 = 101300 [Pa]
ρ = 1.008 [kg/m^3]
T_s = 134 [C]
x = 0.4 [m]

cp = 1008 [J/kg-C]
$\boxed{h_{avg} = 10.49\ [W/m^2\text{-}C]}$
L = 2 [m]
$Nusselt_x$ = 71.58
$\boxed{Q = 7.176\ [W]}$
T_f = 77 [C]
U_{inf} = 3 [m/s]

The results are shown boxed in the above Table.

==

"**Prob. 2A.1.1.6.** Air at 200 C and velocity 5 m/s flows over a plate 1.5 m long. The plate is maintained at a uniform temp of 100 C. The avg. heat transfer coeff is 7.5 W/m^2.K. Calculate the drag force exerted on the plate per 0.75 m width by using Reynolds – Colburn analogy. – [VTU – May 2007]"

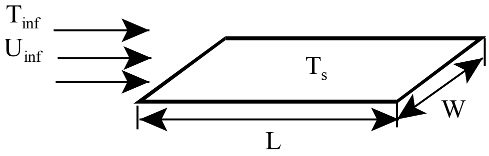


Fig.Prob.2A1.1.6

EES Solution:

"Data:"

T_s = 100[C]
T_inf = 200[C]
U_inf =5[m/s]
L = 1.5[m]
W = 0.75[m]
h_avg = 7.5[W/m^2-K]
T_f = (T_s + T_inf)/2

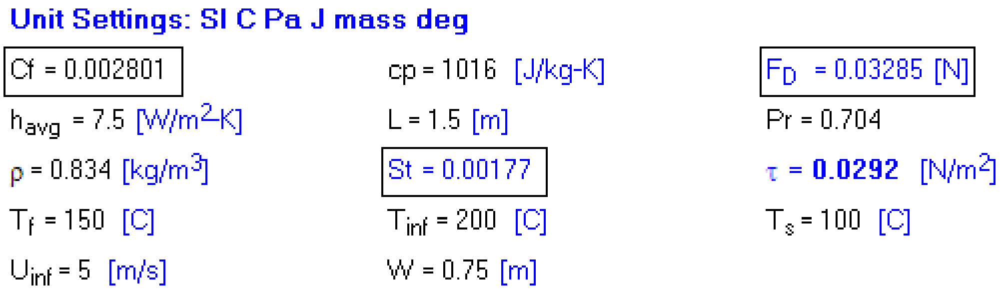
"Properties of Air at T_f:"

rho = Density(Air,T=T_f,P=1.013E05) "[kg/m^3]"
cp=Cp(Air,T=T_f) "[W/m-K]"
Pr=Prandtl(Air,T=T_f)

"Calculations:"

St = h_avg / (rho * U_inf * cp) "…Stanton No."
St * Pr^(2/3) = Cf / 2 "….Reynolds – Colburn Analogy …. finds Cf"
tau = Cf * rho * U_inf^2 / 2 "[N/m^2]..finds shear stress, tau"
F_D = tau * (L * W) "[N]…finds Drag force"

Results:



```
Unit Settings: SI C Pa J mass deg
Cf = 0.002801            cp = 1016  [J/kg-K]      FD  = 0.03285 [N]
havg = 7.5 [W/m2-K]      L = 1.5 [m]              Pr = 0.704
ρ = 0.834 [kg/m3]        St = 0.00177             τ = 0.0292  [N/m2]
Tf = 150  [C]            Tinf = 200  [C]          Ts = 100  [C]
Uinf = 5  [m/s]          W = 0.75 [m]
```

Thus:

The drag force = F_D = 0.03285 N … Ans.

===

"**Prob. 2A1.1.7.** Atmospheric air at T_inf = 400 K flows with a velocity of U_inf = 4 m/s along a flat plate of L = 1 m long, maintained at a uniform temp of T_s = 300 K. The avg. heat tr. coeff. is determined to be 7.75 W/m^2.C. Using Reynolds – Colburn analogy, estimate the drag force exerted on the plate per 1 m width. Take properties of Air as given below. [VTU – Dec.07–Jan.2008]"

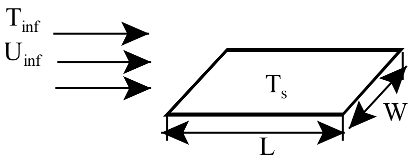


Fig.Prob.2A1.1.7

EES Solution:

"Data:"

T_s = 300[K]
T_inf = 400[K]
T_f = (T_s + T_inf) / 2
U_inf =4[m/s]
L = 1[m]
W = 1[m]
h_avg = 7.75[W/m^2-K]

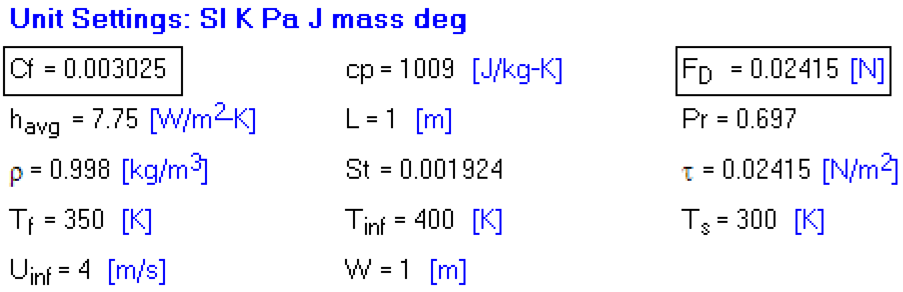
"Properties of Air at T_f:"

rho = 0.998[kg/m^3]
cp=1009[J/kg-K]
Pr=0.697

"Calculations:"

St = h_avg / (rho * U_inf * cp) "..finds Stanton No."
St * Pr^(2/3) = Cf / 2 " Reynolds – Colburn analogy … finds Cf"
tau = Cf * rho * U_inf^2 / 2 "[N/m^2]…finds shear stress, tau"
F_D = tau * (L*W) "[N]… finds Drag force"

Results:



Unit Settings: SI K Pa J mass deg

$\boxed{Cf = 0.003025}$ cp = 1009 [J/kg-K] $\boxed{F_D = 0.02415 \text{ [N]}}$
h_{avg} = 7.75 [W/m^2-K]   L = 1 [m]   Pr = 0.697
ρ = 0.998 [kg/m^3]   St = 0.001924 τ = 0.02415 [N/m^2]
T_f = 350 [K] T_{inf} = 400 [K] T_s = 300 [K]
U_{inf} = 4 [m/s]   W = 1 [m]

Thus:

Drag force = FD = 0.02415 N … Ans.

==

"**Prob. 2A1.1.8**. Air at 20 C flows over a thin plate with a velocity of 3 m/s. The plate is 2 m long and 1 m wide. Estimate the boundary layer thickness at the trailing edge of the plate and the total drag force experienced by the plate. [VTU – Dec. 2010]"

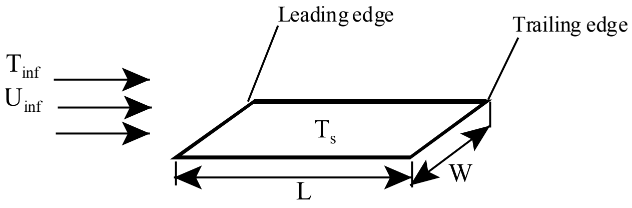
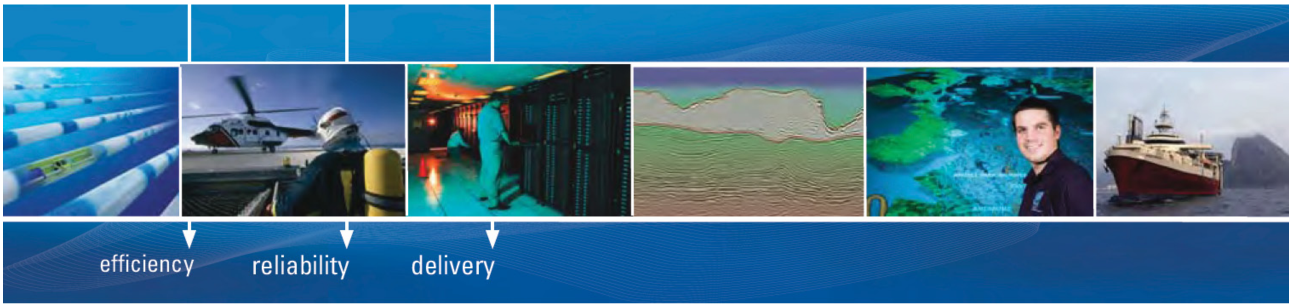


Fig.Prob.2A1.1.8

EES Solution:

"Data:"

T_inf = 20[C]
U_inf = 3[m/s]
L = 2[m]
W = 1[m]
P_1 = 1.013e05 [Pa]



"Properties of Air:"

mu=Viscosity(Air,T=T_inf)
rho = density(Air, T = T_inf, P = P_1)

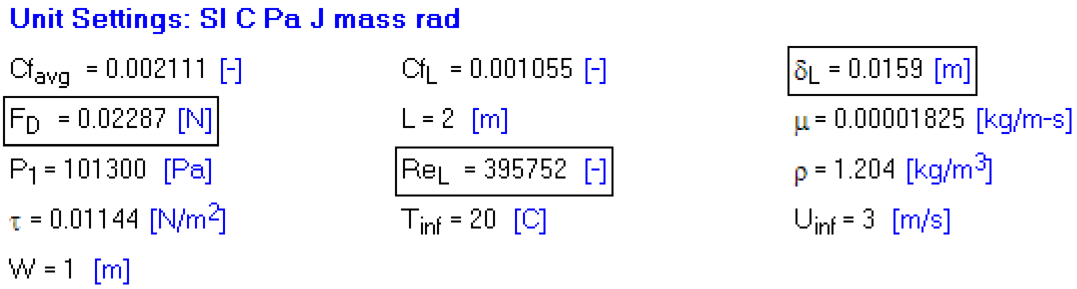
"Calculations:"

Re_L = L * U_inf * rho / mu "….Reynold's No."

**"We see that Re_L = 395752. This is less than 5E05; therefore, flow is laminar.
So, use laminar flow correlations:"**

delta_L = 5 * L / sqrt(Re_L) "..[m]….hydrodyn. b.l. thickness"
Cf_L = 0.664 / sqrt(Re_L) "…local friction coeff. at x = L"
Cf_avg = 2 * Cf_L "…avg. friction coeff. up to L"
tau = Cf_avg * rho * U_inf^2 / 2 "…[N/m^2]… shear stress"
F_D = tau * (L * W) "[N]…Drag force"

Results:



Unit Settings: SI C Pa J mass rad

Cf_{avg} = 0.002111 [-] Cf_L = 0.001055 [-] δ_L = 0.0159 [m]
F_D = 0.02287 [N]   L = 2 [m] μ = 0.00001825 [kg/m-s]
P_1 = 101300 [Pa] Re_L = 395752 [-] ρ = 1.204 [kg/m^3]
τ = 0.01144 [N/m^2] T_{inf} = 20 [C] U_{inf} = 3 [m/s]
W = 1 [m]

Thus:

Boundary later thickness = 0.0159 m …. Ans.

Drag force = 0.02287 N … Ans.

==

"Prob.2A1.1.9. The surface temp of a thin plate located parallel to air stream is 90 C. The free stream velocity is 60 m/s and the air temp is 10 C. The plate is 60 cm wide and 45 cm long in the direction of air stream. Assuming that transitional Reynolds No. is 4E05, determine: (i) the average heat transfer coeff in laminar and turbulent regions (ii) Rate of heat transfer for the entire plate considering both the sides of the plate. Given that the correlations for the local Nusselts No. are: 0.332 . Re_x^(1/2) . Pr^(1/3) for laminar flow and 0.028 . Re_x^(0.8) . Pr^(1/3) for turbulent flow. [Prob. 5 (b)-VTU – June–July 2009: Flow over flat plate: – *mixed boundary layer*]"

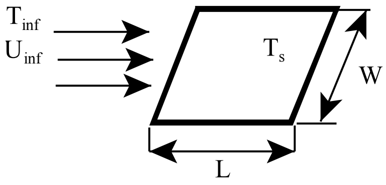


Fig.Prob.2A1.1.9

EES Solution:

"Data:"

T_s = 90[C]
T_inf = 10[C]
U_inf = 60[m/s]
L = 0.45[m]
W = 0.6[m]
T_f = (T_s + T_inf)/2
Re_cr = 4*10^5
P_1 = 101300 [Pa]

"Properties of Air:"

mu=Viscosity(Air,T=T_f)
rho = density(Air, T = T_f, P = P_1)
cp=Cp(Air,T=T_f)
k=Conductivity(Air,T=T_f)
Pr=Prandtl(Air,T=T_f)

"Calculations:"

Re_cr = U_inf * x_cr * rho / mu "finds x_cr…. the critical distance where transition from laminar to turb. flow occurs"

"heat tr coeff in laminar region:"

h_x * x_cr / k = 0.332 * sqrt(Re_cr) * Pr^(1/3) "finds h_x"
h_lam_avg = 2 * h_x "Avg h for lam region"
Q_lam = (h_lam_avg * (W * x_cr) * (T_s – T_inf)) * 2 "W … heat tr from lam region of plate; Multiplied by 2, since both sides of plate are to be considered."

"for turb region:"

Re_L = U_inf * L* rho / mu

"From the relation for local heat tr coeff for turb. Region:

$$Nu_{xturb}=0.0288\cdot Re_x^{0.8}\cdot Pr^{\frac{1}{3}}$$

Therefore, $h_{xturb}=\frac{k}{x}\cdot 0.0288\cdot\left(\frac{U\cdot x}{v}\right)^{0.8}\cdot Pr^{\frac{1}{3}}$ ...local value of turb. heat tr coeff

Then, avg value of turb heat tr coeff from x = xc to x = (L - xc) is given by:

$$h_{turbavg} = \frac{k \cdot 0.0288 \cdot \left(\frac{U}{v}\right)^{0.8} \cdot Pr^{\left(\frac{1}{3}\right)}}{L - x_c} \cdot \int_{x_c}^{L} x^{-0.2}\,dx$$

i.e. $$h_{turbavg} = \frac{0.0288 \cdot k \cdot Pr^{\frac{1}{3}} \cdot \left(\frac{U}{v}\right)^{0.8}}{L - x_c} \cdot \left(\frac{L^{0.8} - x_c^{\;0.8}}{0.8}\right)$$

i.e. $$h_{turbavg} = \frac{0.036 \cdot k \cdot Pr^{\frac{1}{3}}}{L - x_c} \cdot \left[\left(\frac{U \cdot L}{v}\right)^{0.8} - \left(\frac{U \cdot x_c}{v}\right)^{0.8}\right]$$

i.e. $$h_{turbavg} = \frac{0.036 \cdot k \cdot Pr^{\frac{1}{3}}}{L - x_c} \cdot \left(Re_L^{\;0.8} - Re_c^{\;0.8}\right)$$

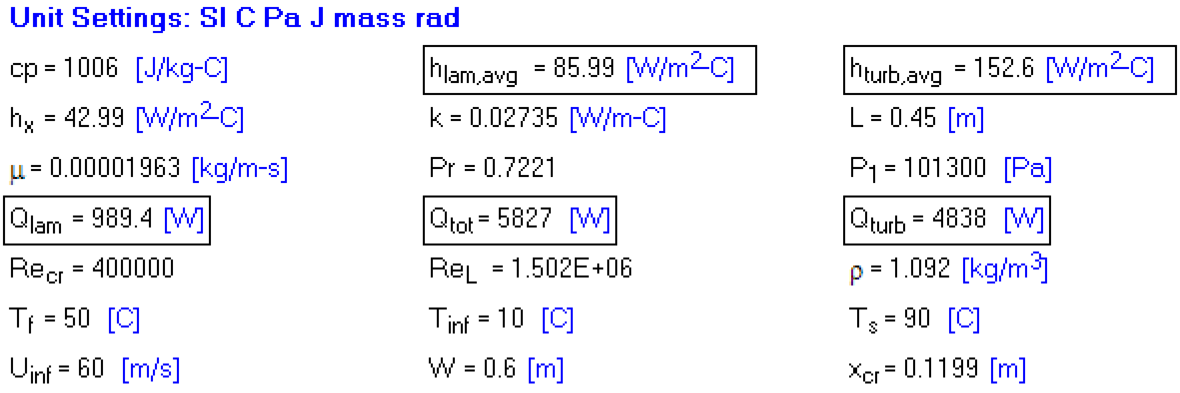
In EES, it is entered:

h_turb_avg = 0.036 * k * Pr^(1/3) * (Re_L^0.8 – Re_cr^0.8)/(L – x_cr) "…finds h_turb_avg"

Q_turb = (h_turb_avg*(W*(L – x_cr))*(T_s – T_inf)) * 2 "W … heat tr from turb region of plate; Multiplied by 2, since both sides of plate are to be considered."
Q_tot = Q_lam + Q_turb

{
For Mixed boundary layer, we can also use the following relations:
Nu_bar_L = Pr^(1/3)*(0.036*Re_L^0.8 – A), where
A =(0.036*Re_cr^0.8 – 0.664*Re_cr^0.5)
}

Results:



```
Unit Settings: SI C Pa J mass rad
cp = 1006  [J/kg-C]          hlam,avg = 85.99 [W/m2-C]     hturb,avg = 152.6 [W/m2-C]
hx = 42.99 [W/m2-C]          k = 0.02735 [W/m-C]           L = 0.45 [m]
μ = 0.00001963 [kg/m-s]      Pr = 0.7221                   P1 = 101300 [Pa]
Qlam = 989.4 [W]             Qtot = 5827 [W]               Qturb = 4838 [W]
Recr = 400000                ReL = 1.502E+06               ρ = 1.092 [kg/m3]
Tf = 50 [C]                  Tinf = 10 [C]                 Ts = 90 [C]
Uinf = 60 [m/s]              W = 0.6 [m]                   xcr = 0.1199 [m]
```

Thus:

Avg. heat tr coeff. in laminar region = 85.99 W/m^2.C …. Ans.

Avg. heat tr coeff. in turbulent region = 152.6 W/m^2.C … Ans.

Rate of heat tr for entire plate (for both sides) = 5827 W …. Ans.

==

"**Prob. 2A1.1.10.** Consider a rectangular fin that is used to cool a motor cycle engine. The fin is 0.15 m long and at a temp of 250 C, while the motor cycle is moving at 80 km/h in air at 27 C. The air is in parallel flow over both surfaces of the fin and turbulent flow conditions may be assumed to exist throughout. (a) What is the rate of heat removal per unit width of fin? (b) Plot the heat removal rate per unit width of fin for motor cycle speeds ranging from 10 to 100 km/h." [Ref:3]

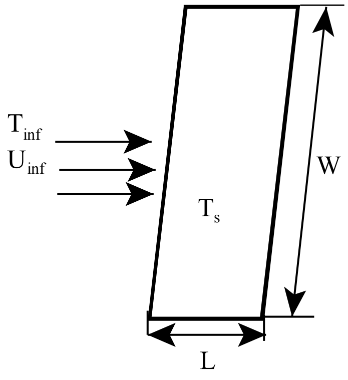


Fig.Prob.2A1.1.10

EES Solution:

"Data:"

```
T_inf = 27[C]
T_s = 250 [C]
T_f = (T_inf + T_s) / 2
U_inf_kmph = 80 [km/h]
U_inf_mps = U_inf_kmph*convert(km/h,m/s) "[m/s]"
L = 0.15[m]
W = 1[m]
P_1 = 1.013e05 [Pa]
```

"Properties of Air at mean temp (T_f = 277/2 = 138.5 C):"

```
rho=Density(Air,T=T_f,P=P_1) "[kg/m^3]"
k=Conductivity(Air,T=T_f) "[W/m-C]"
cp=Cp(Air,T=T_f) "[J/kg-C]"
Pr=Prandtl(Air,T=T_f)
mu=Viscosity(Air,T=T_f) "[kg/m-s]"
nu = mu / rho "[m^2/s]"
```

"Calculations:"

Re_L = L * U_inf_mps / nu "…Reynold's No. at the end of plate"

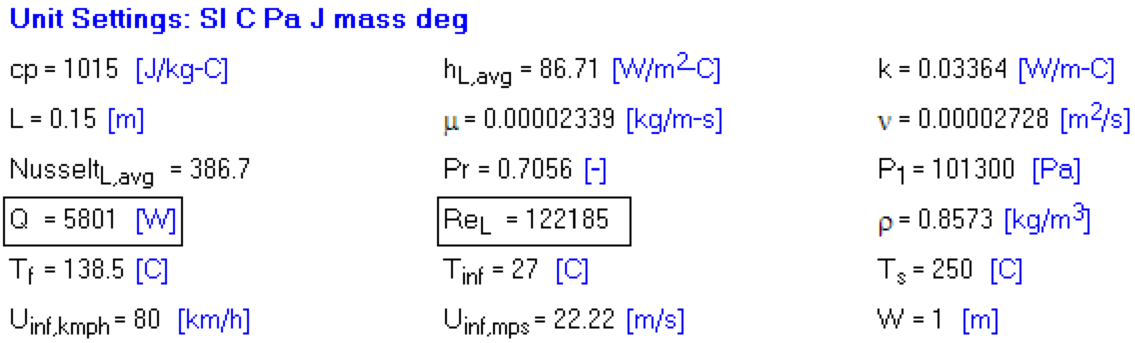
"Using correlations for turb. flow:"

Nusselt_L_avg = 0.037 * (Re_L)^0.8 * Pr^(1/3) "…avg. Nusselt No. for entire plate in turb. flow"

Nusselt_L_avg= h_L_avg * L / k "…gives avg. heat tr coeff for entire plate"

Q = 2 * (h_L_avg * (L * W) * (T_s – T_inf)) "[W] … total heat tr from both surfaces of plate"

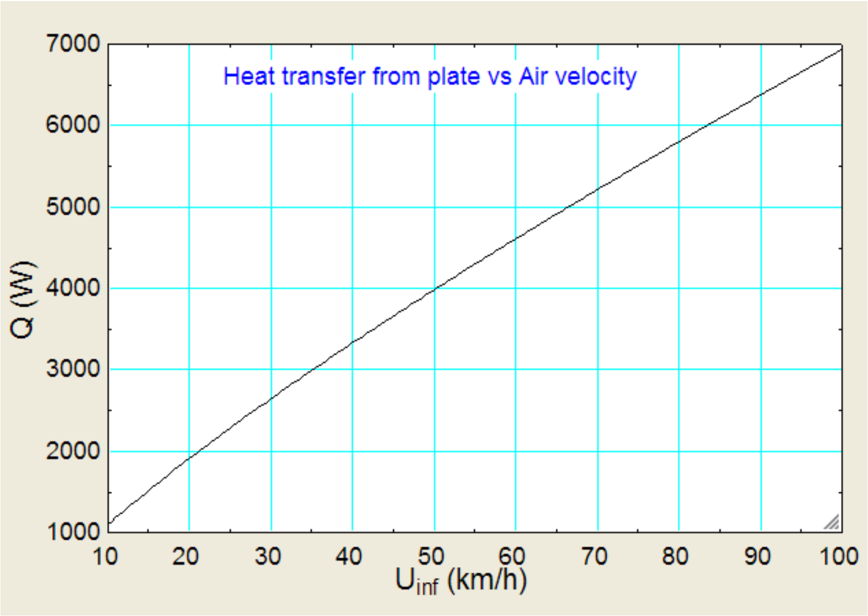
Results:



```
Unit Settings: SI C Pa J mass deg

cp = 1015  [J/kg-C]           hL,avg = 86.71  [W/m2-C]        k = 0.03364  [W/m-C]
L = 0.15  [m]                 μ = 0.00002339  [kg/m-s]        ν = 0.00002728  [m2/s]
NusseltL,avg = 386.7          Pr = 0.7056  [-]                P1 = 101300  [Pa]
Q = 5801  [W]                 ReL = 122185                    ρ = 0.8573  [kg/m3]
Tf = 138.5  [C]               Tinf = 27  [C]                  Ts = 250  [C]
Uinf,kmph = 80  [km/h]        Uinf,mps = 22.22  [m/s]         W = 1  [m]
```

Note that Q = 5801 W for both surfaces of fin… Ans.

To plot the graph of Q vs U_inf:



===

“**Prob. 2A1.1.11.** Air at 27 C an d at atm. pressure flows over a flat plate at a velocity of 2 m/s. If the plate is maintained at 93 C, calculate the heat transfer per unit width of plate, assuming the length of plate along the flow of air as 2 m. [VTU – June 2012]”

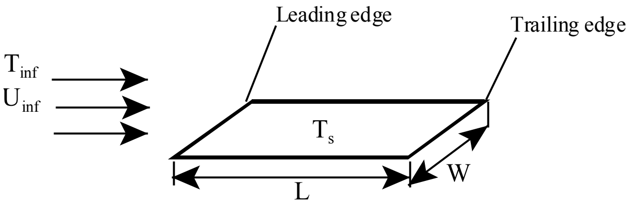


Fig.Prob.2A1.1.11

EES Solution:

“Data:”

T_inf = 27[C]
T_s = 93 [C]
T_f = (T_inf + T_s) / 2
U_inf = 2 “[m/s]”
L = 2[m]
W = 1[m]
P_1 = 1.013e05 [Pa]

“Properties of Air at mean temp (T_f = 60 C):”

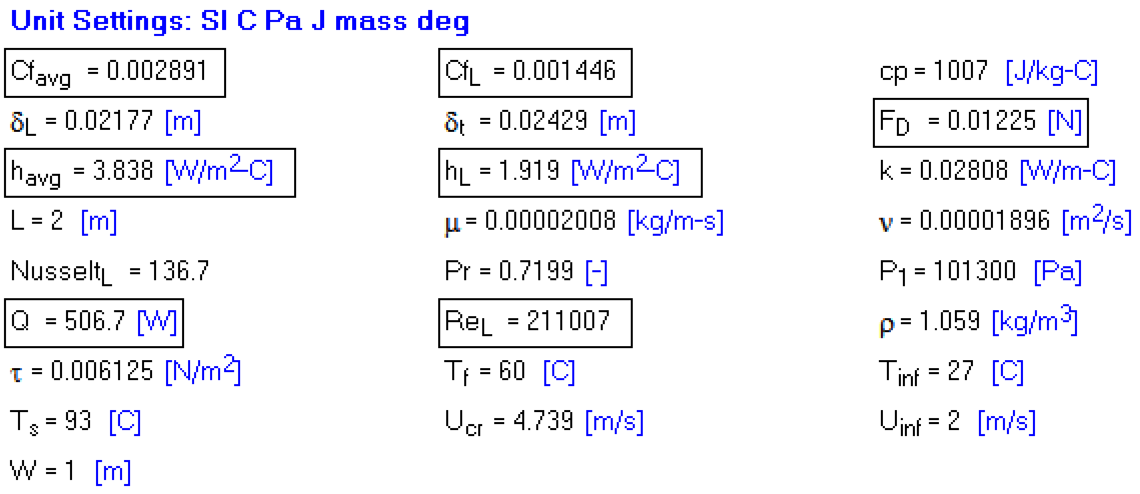
rho=Density(Air,T=T_f,P=P_1) “[kg/m^3]”
k=Conductivity(Air,T=T_f) “[W/m-C]”
cp=Cp(Air,T=T_f) “[J/kg-C]”
Pr=Prandtl(Air,T=T_f)
mu=Viscosity(Air,T=T_f) “[kg/m-s]”
nu = mu / rho “[m^2/s]”

“Calculations:”

Re_L = L * U_inf / nu “…Reynold’s No. at the end of plate”
5e05 = L * U_cr / nu “…finds critical velocity at which flow becomes turbulent”

```
"We note that Re_L is 211007 < 5E05. Therefore, flow is laminar. Use correlations for laminar flow:"

delta_L = 5 * L / sqrt(Re_L) "thickness of velocity b.l."
delta_t = delta_L / Pr^(1/3) "thickness of thermal b.l."
Cf_L = 0.664 / sqrt(Re_L) "…local friction coeff. at end of plate"
Cf_avg = 2 * Cf_L "…average friction coeff. over the entire plate"
Nusselt_L = 0.332 * sqrt(Re_L) * Pr^(1/3) "…local Nusselt No. at end of plate"
Nusselt_L = h_L * L / k "…gives local heat tr coeff Nu_L at end of plate"
h_avg = 2 * h_L "gives avg heat tr coeff over entire plate"
Q = h_avg * ( L * W) * (T_s – T_inf) "[W] … total heat tr from surface of plate"
tau = Cf_avg * rho * U_inf^2 / 2 "..shear stress, [N/m^2]"
F_D = tau * (L * W) "[N]…total drag force"
```

Results:



Unit Settings: SI C Pa J mass deg

$\boxed{Cf_{avg} = 0.002891}$
δ_L = 0.02177 [m]
$\boxed{h_{avg} = 3.838\ [W/m^2\text{-}C]}$
L = 2 [m]
$Nusselt_L$ = 136.7
$\boxed{Q = 506.7\ [W]}$
τ = 0.006125 [N/m^2]
T_s = 93 [C]
W = 1 [m]

$\boxed{Cf_L = 0.001446}$
δ_t = 0.02429 [m]
$\boxed{h_L = 1.919\ [W/m^2\text{-}C]}$
μ = 0.00002008 [kg/m-s]
Pr = 0.7199 [-]
$\boxed{Re_L = 211007}$
T_f = 60 [C]
U_{cr} = 4.739 [m/s]

cp = 1007 [J/kg-C]
$\boxed{F_D = 0.01225\ [N]}$
k = 0.02808 [W/m-C]
ν = 0.00001896 [m^2/s]
P_1 = 101300 [Pa]
ρ = 1.059 [kg/m^3]
T_{inf} = 27 [C]
U_{inf} = 2 [m/s]

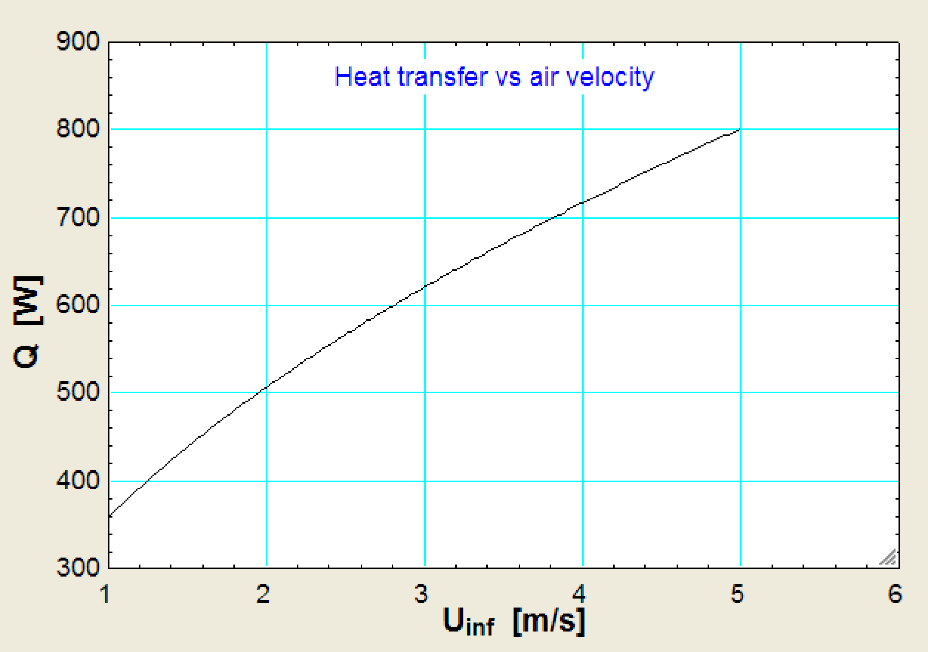
Thus:

Heat transferred from the surface of plate = Q = 506.7 W … Ans.

Plot Q as a function of Air velocity:

It is noted that the velocity required for the flow to become turbulent (i.e. for Re = 5E05) is 4.739 m/s.

So, plot Q vs U from U_inf = 1 m/s up to 5 m/s:



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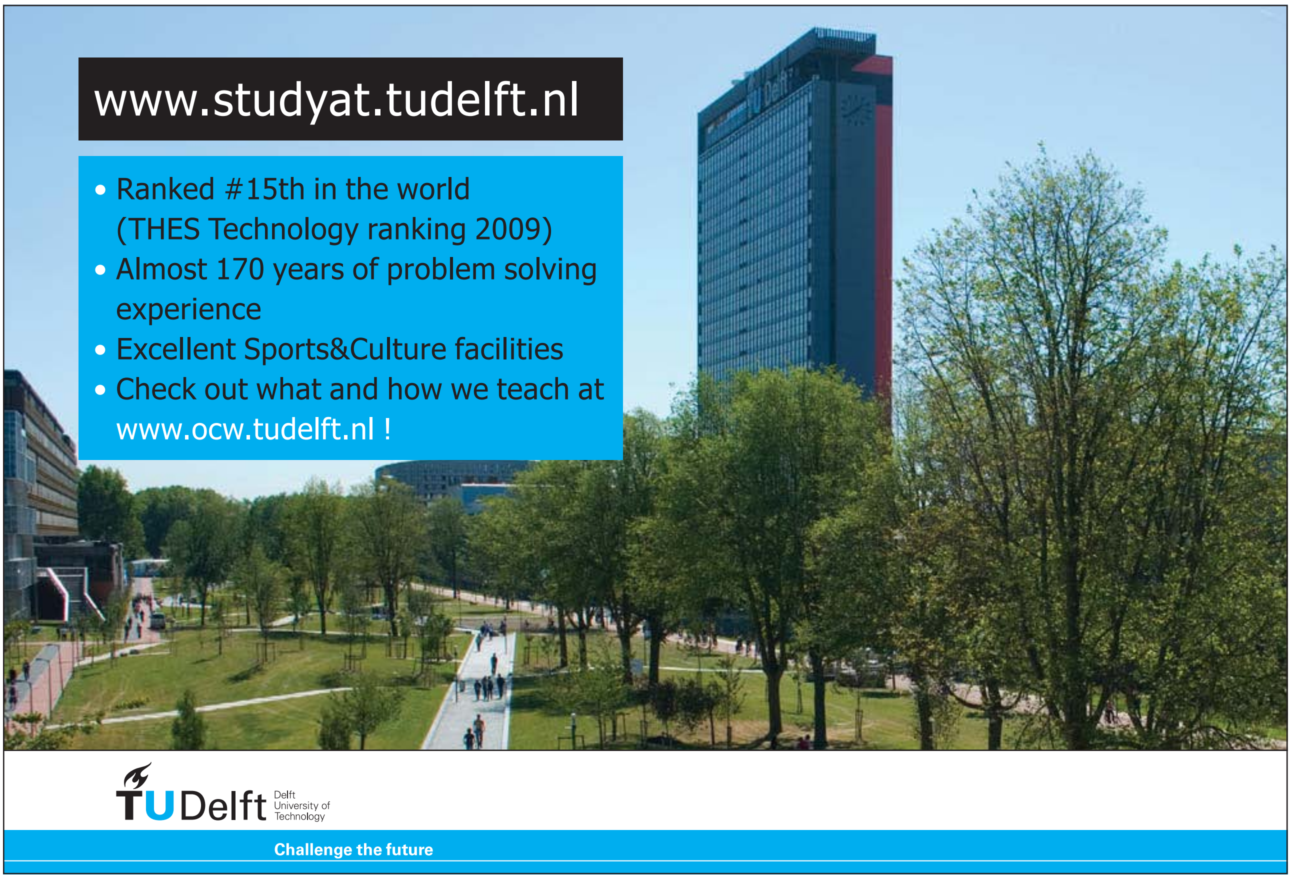


"**Prob. 2A1.1.12.** Air at 20 C flows over both sides of a surface of a flat plate measuring 0.2 m × 0.2 m. The drag force was 0.075 N. Determine the velocity gradient at the surface if kinematic viscosity has a value of 15.06E-06 m^2/s and density = 1.205 kg/m^3. Also, determine the drag coeff if the free stream velocity is 40 m/s. [VTU – May/June 2010]"

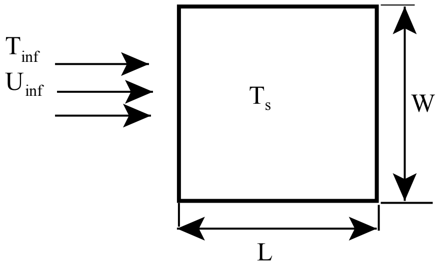


Fig.Prob.2A1.1.12

EES Solutions:

"Data:"

T_inf = 20[C]
U_inf = 40 [m/s]
L = 0.2[m]
W = 0.2[m]
F_D = 0.075 [N]

"Properties of Air at (T_inf = 20 C):"

rho=1.205 "[kg/m^3]"
nu = 15.06E-06 "[m^2/s]"

"Calculations:"

F_D = tau * (L * W) * 2 "…determines shear stress tau [N/m^2]"

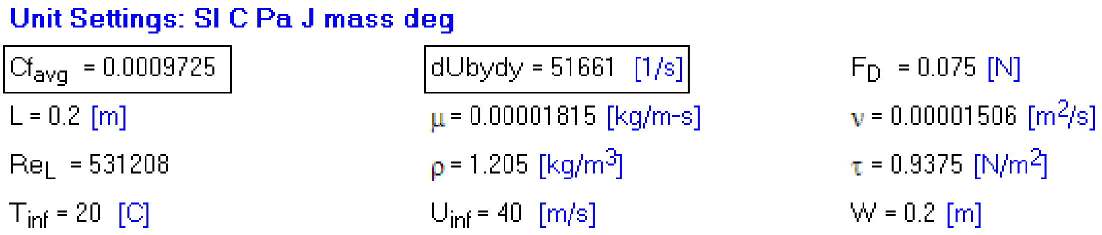
nu = mu / rho "…determine mu, dynamic viscosity [kg/m.s]"

tau = mu * dUbydy "…determines velocity gradient, dU/ dy, [1/s]"

tau = Cf_avg * rho * U_inf^2 / 2 "…determines Cf_avg"

Re_L = L * U_inf / nu "…Reynold's No. at the end of plate"

Results:



```
Unit Settings: SI C Pa J mass deg
Cfavg = 0.0009725        dUbydy = 51661  [1/s]      FD  = 0.075  [N]
L = 0.2  [m]             μ = 0.00001815  [kg/m-s]   ν = 0.00001506  [m2/s]
ReL  = 531208            ρ = 1.205  [kg/m3]         τ = 0.9375  [N/m2]
Tinf = 20  [C]           Uinf = 40  [m/s]           W = 0.2  [m]
```

Thus:

Velocity gradient, dU/dy = 51661 [1/s] ….. Ans.

Drag coeff. Cf_avg = 0.0009725 …. Ans.

===

"**Prob.2A1.1.13.** Air at 30 C flows at a velocity of 45 m/s past a flat plate 50 cm long. The plate is maintained at a uniform temp of 250 C. Find per metre width of plate: (i) the rate of heat transfer to the plate (ii) rate of heat transfer from laminar portion of the plate (iii) rate of heat transfer from the turb. portion of the plate. Assume a critical Reynolds No. of 5E05 (iv) What would be the error in the rate of heat transfer if the boundary layer is assumed to be turbulent from the leading edge? [VTU – July/ Aug. 2004]"

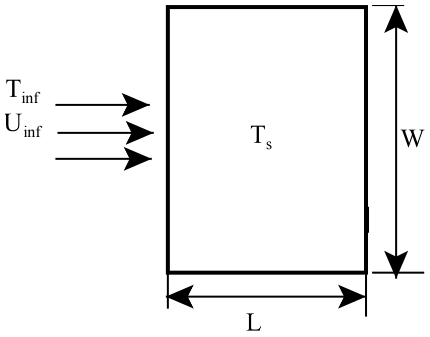


Fig.Prob.2A1.1.13

EES Solution:

"Data:"

T_s = 250[C]

T_inf = 30[C]

U_inf = 45[m/s]

L = 0.5[m]

W = 1[m]

T_f = (T_s + T_inf)/2

Re_cr = 5E05

P_1 = 101300 [Pa]

"Properties of Air:"

mu=Viscosity(Air,T=T_f)
rho = density(Air, T = T_f, P = P_1)
cp=Cp(Air,T=T_f)
k=Conductivity(Air,T=T_f)
Pr=Prandtl(Air,T=T_f)

"Calculations:"

Re_cr = U_inf * x_cr * rho / mu "finds x_cr…. the critical distance where transio from laminar to turb. flow occurs"

"We observe that x_cr = 0.305 m. i.e. from the leading edge, upto a distance of 0.305 m, the flow is laminar, and then from 0.305 up to 0.5 m the flow is turbulent. Use the relevant correlations to find heat transfer in the two regions:"

"heat tr coeff in laminar region:"

h_x * x_cr / k = 0.332 * sqrt(Re_cr) * Pr^(1/3) "finds h_x"

h_lam_avg = 2 * h_x “Avg h for lam region”

Q_lam = (h_lam_avg * (W * x_cr) * (T_s – T_inf)) “W … heat tr from lam region of plate; only top side of plate is considered.”

“heat transfer for turb region:”

“For Mixed boundary layer, we can also use the following relations for the entire plate:”

Re_L = U_inf * L * rho / mu

Nusselt_bar_L = (0.037 * Re_L^0.8 – A) * Pr^(1/3) “where….”

A =(0.037 * Re_cr^0.8 – 0.664 * Re_cr^0.5)

“Therefore:”

h_L_avg * L / k = Nusselt_bar_L “…finds h_L_avg for entire plate”

“And,”

Q_total = h_L_avg * (L * W) * (T_s – T_inf) “[W] … total heat tr rate from the entire plate”

“Therefore:”

Q_turb = Q_total – Q_lam “[W] …. heat tr from turb. region of plate”

“Alternatively: we can also use following eqns:”

{
h_turb_avg = 0.036 * k *(Re_L^0.8 – Re_cr^0.8) / (L – x_cr) “…finds h_turb_avg”

Q_turbulent = (h_turb_avg * (W * (L – x_cr)) * (T_s – T_inf)) “W … heat tr from turb region of plate considering only top surface of plate”

Q_tot = Q_lam + Q_turbulent
}

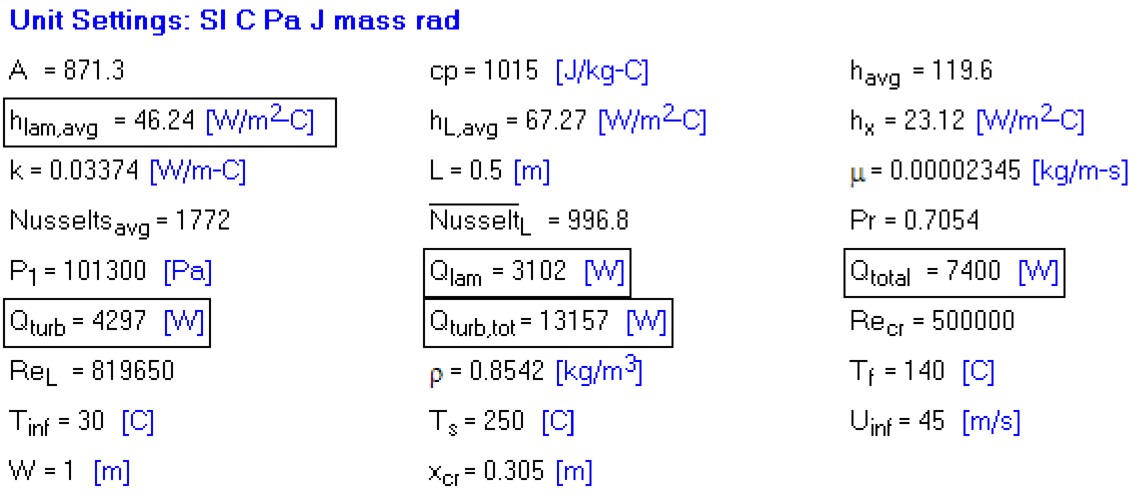
"If the entire boundary layer is turbulent:"

Nusselts_avg = 0.037 * Re_L^0.8 * Pr^(1/3) "….avg. Nusselts No."

h_avg * L / k = Nusselts_avg "…avg. heat tr coeff"

Q_turb_tot = h_avg * (L * W) * (T_s – T_inf) "total heat tr for the plate"

Results:



Unit Settings: SI C Pa J mass rad

$A = 871.3$ | $cp = 1015$ [J/kg-C] | $h_{avg} = 119.6$

$\boxed{h_{lam,avg} = 46.24 \text{ [W/m}^2\text{-C]}}$ | $h_{L,avg} = 67.27$ [W/m²-C] | $h_x = 23.12$ [W/m²-C]

$k = 0.03374$ [W/m-C] | $L = 0.5$ [m] | $\mu = 0.00002345$ [kg/m-s]

$Nusselts_{avg} = 1772$ | $\overline{Nusselt_L} = 996.8$ | $Pr = 0.7054$

$P_1 = 101300$ [Pa] | $\boxed{Q_{lam} = 3102 \text{ [W]}}$ | $\boxed{Q_{total} = 7400 \text{ [W]}}$

$\boxed{Q_{turb} = 4297 \text{ [W]}}$ | $\boxed{Q_{turb,tot} = 13157 \text{ [W]}}$ | $Re_{cr} = 500000$

$Re_L = 819650$ | $\rho = 0.8542$ [kg/m³] | $T_f = 140$ [C]

$T_{inf} = 30$ [C] | $T_s = 250$ [C] | $U_{inf} = 45$ [m/s]

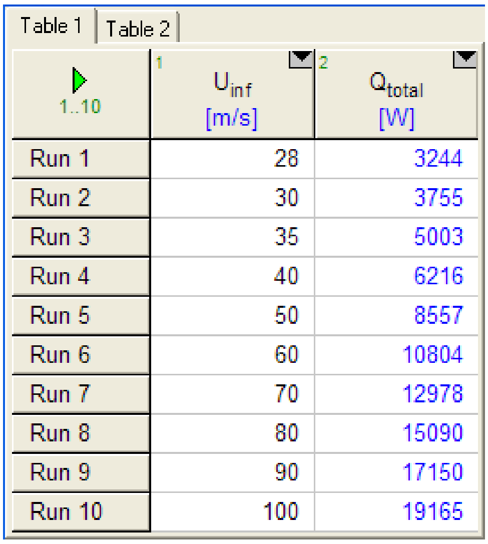
$W = 1$ [m] | $x_{cr} = 0.305$ [m]

Thus:

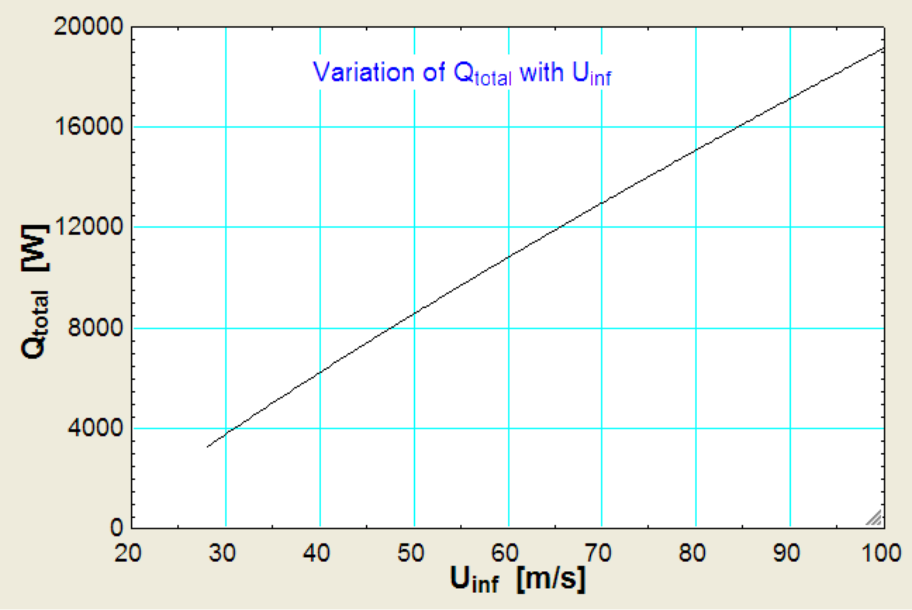
1) **Total heat transfer to plate = Q_total = 7400 W … Ans.**
2) **Heat transfer from laminar portion of plate = Q_lam = 3102 W …. Ans.**
3) **Heat transfer from turbulent portion of plate = Q_turb = 4297 W …. Ans.**
4) **Heat transfer from plate, if there is turb. flow over the entire plate = Q_tub_tot = 13157 W …. Ans.**

Plot the Q_total against the Air velocity U_inf:

Let Air velocity, U_inf vary from 28 m/s to 100 m/s. (At U_inf = 28 m/s, practically the entire plate is in laminar flow).



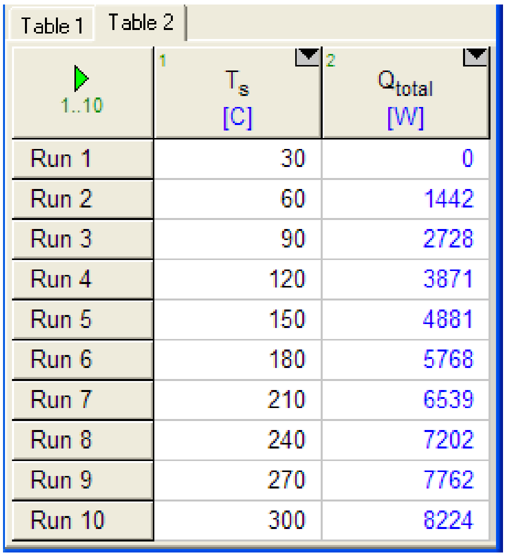
1..10	1 U_{inf} [m/s]	2 Q_{total} [W]
Run 1	28	3244
Run 2	30	3755
Run 3	35	5003
Run 4	40	6216
Run 5	50	8557
Run 6	60	10804
Run 7	70	12978
Run 8	80	15090
Run 9	90	17150
Run 10	100	19165



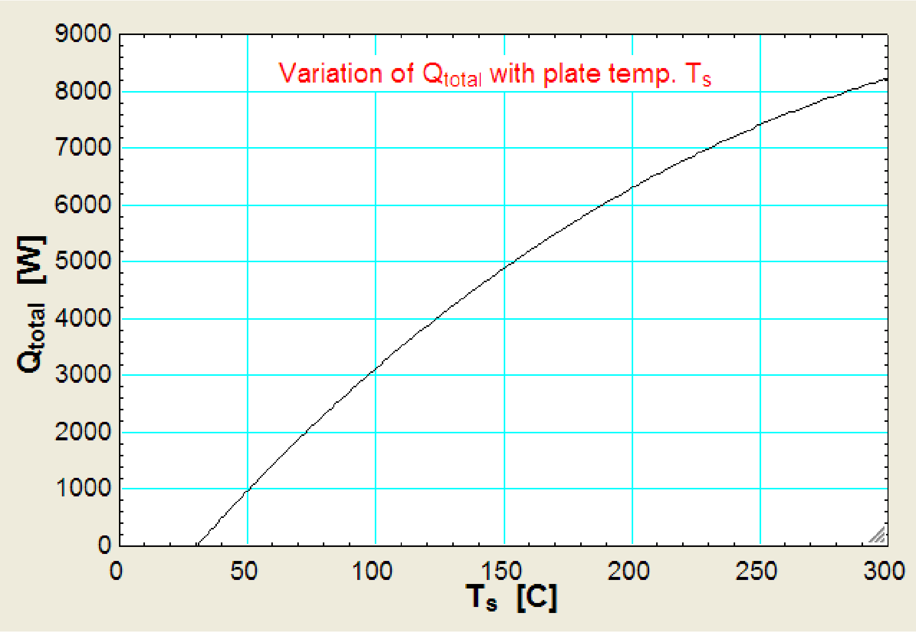
Plot the Q_total against the plate temp, T_s:

Let T_s vary from 30 C to 300 C, keeping U_inf = 45 m/s:

Note that now, the film temp. T_f varies with each trial. But, since the properties of air are obtained from the built I-in functions in EES, properties are evaluated at the correct T_f.



1..10	1 T_s [C]	2 Q_{total} [W]
Run 1	30	0
Run 2	60	1442
Run 3	90	2728
Run 4	120	3871
Run 5	150	4881
Run 6	180	5768
Run 7	210	6539
Run 8	240	7202
Run 9	270	7762
Run 10	300	8224



==

Prob. 2A1.1.14. In a certain chemical process, castor oil at 35 C flows over a flat plate at 6 cm/s. The length of plate is 6 m and its surface is maintained at a uniform temp of 95 C.

Calculate: (i) hydrodynamic and thermal boundary layer thickness at the trailing edge of the plate

(ii) total drag per unit width on one side of the plate, and

(iii) local heat transfer coeff. at the end of the plate.

Properties of the fluid at mean film temp. are given below.

Use Nu = 0.332 . Re^0.5 . Pr^0.33 [VTU – Aug. 2001]

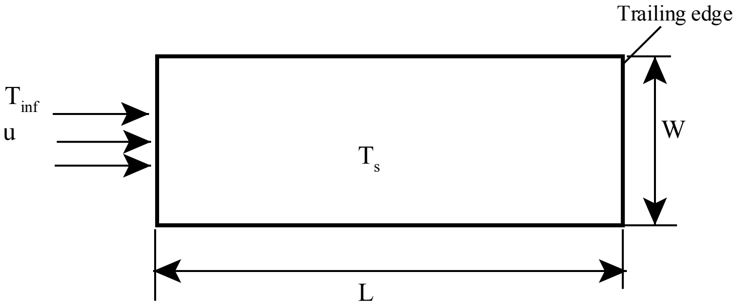


Fig.Prob.2A1.1.14

Mathcad Solution:

Data:

$T_{inf} := 35$ C $T_s := 95$ C

$u := 0.06$ m/s $L := 6$ m $W := 1$ m

$T_f := \frac{T_{inf} + T_s}{2}$ ....film temp.

i.e. $T_f = 65$ C

Properties of Castor oil at 65 C:

$\rho := 956.8$ kg/m^3

$\alpha := 7.2 \cdot 10^{-8}$ m^2/s

$\nu := 0.65 \cdot 10^{-4}$ m^2/s

$k := 0.213$ W/m.K

$Pr := \frac{\nu}{\alpha}$ i.e. $Pr = 902.778$ ...Prandtl No.

$cp := \frac{k}{\rho \cdot \alpha}$ i.e. $cp = 3.092 \cdot 10^3$ W/m.C

Calculations:

For flow over a flat plate:

$Re_{cr} := 5 \cdot 10^5$ ...crit. Reynolds No.

And, $Re_{cr} = \frac{x_{cr} \cdot u}{\nu}$

i.e. $x_{cr} := \frac{Re_{cr}}{u} \cdot \nu$

i.e. $x_{cr} = 541.667$ m...

i.e. For the entire legth of 6 m of the plate, the flow is laminar.

Hydrodynamic Boundary layer thickness:

$Re_L := \frac{u \cdot L}{\nu}$ ...Reynolds No. at the trailing edge i.e. at x = L

i.e. $Re_L = 5.538 \cdot 10^3$

Thickness of hydrodynamic boundary layer at x = L = 6 m:

$\delta_{lam} := \frac{5 \cdot L}{\sqrt{Re_L}}$

i.e. $\delta_{lam} = 0.403$ **m .. Ans.**

Thickness of thermal boundary layer:

$\delta_t := \frac{\delta_{lam}}{Pr^{\frac{1}{3}}}$

i.e. $\delta_t = 0.042$ **m.....Ans.**

To plot the graph: let us write δ_{lam} and δ_t as a functions of x:

$$Re_x(x) := \frac{x \cdot u}{\nu}$$ ....Re as a function of x

$$\delta_{lam}(x) := \frac{5 \cdot x}{\sqrt{Re_x(x)}}$$ ... δ as a function of x

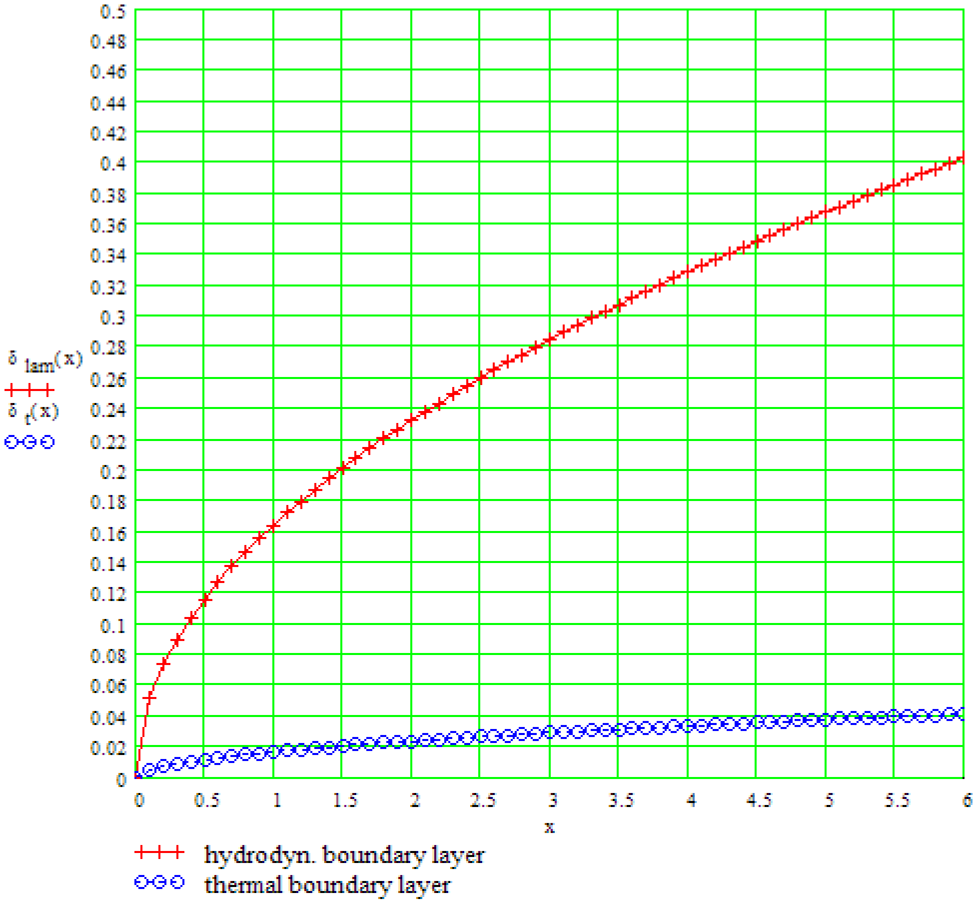
Also: Thermal boundary layer thickness is:

$$\delta_t(x) := \frac{\delta_{lam}(x)}{Pr^{\frac{1}{3}}}$$ ... δ_t as a function of x

To plot the graphs for x = 0 to x = L:

$x := 0, 0.1 .. 6$ ...define a range variable x

x in metres, δ and δ_t in metres:



Drag on one surface:

$F_D = \tau \cdot Area$ where τ is the shear stress

$\tau = C_{fa} \cdot \rho \cdot \frac{u^2}{2}$ where Cfa = avg. friction coeff.

Now, $C_{fa} := 1.328 \cdot Re_L^{-0.5}$

i.e. $C_{fa} = 0.018$ ...avg. friction coeff. (drag coeff.)

Therefore:

$\tau := C_{fa} \cdot \rho \cdot \frac{u^2}{2}$ i.e. $\tau = 0.031$ **N/m^2 .... shear stress**

And,

$F_D := \tau \cdot (L \cdot W)$ N....Drag force on one surface

i.e. $F_D = 0.184$ **N .... Ans.**

Local heat transfer coeff. at x = L:

Local Nusselt No. is given by:

$Nu_x = 0.332 \cdot Re_x^{0.5} \cdot Pr^{\frac{1}{3}}$

Also: $Nu_x = \frac{h \cdot x}{k}$

Therefore, at x = L = 6 m:

$Re_L = 5.538 \cdot 10^3$

$Nu_L := 0.332 \cdot Re_L^{0.5} \cdot Pr^{\frac{1}{3}}$

i.e. $Nu_L = 238.796$ ....Nusserlt No. at x = L

Therefore:

$$h := \frac{Nu_L \cdot k}{L}$$

i.e. $h = 8.477$ W/m^2.C .... local heat transfer coeff. at x = 6 m ... Ans.

==

"**Prob. 2A1.1.15.** Consider a rectangular fin that is used to cool a motor cycle engine. The fin is 0.15 m long and at a temp of 250 C, while the motor cycle is moving at 80 km/h in air at 27 C. The air is in parallel flow over both surfaces of the fin and turbulent flow conditions may be assumed to exist throughout. (a) What is the rate of heat removal per unit width of fin? (b) Plot the heat removal rate per unit width of fin for motor cycle speeds ranging from 10 to 100 km/h." [Ref:3]

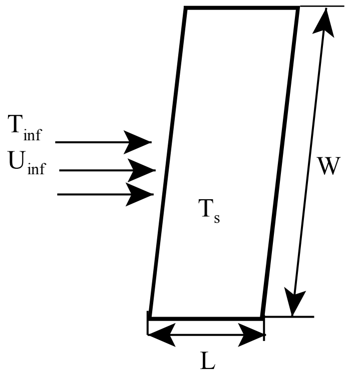


Fig.Prob.2A1.1.15

Note that this problem is the same as Prob.2A1.1.10.

But, we shall solve it with EXCEL.

EXCEL Solution:

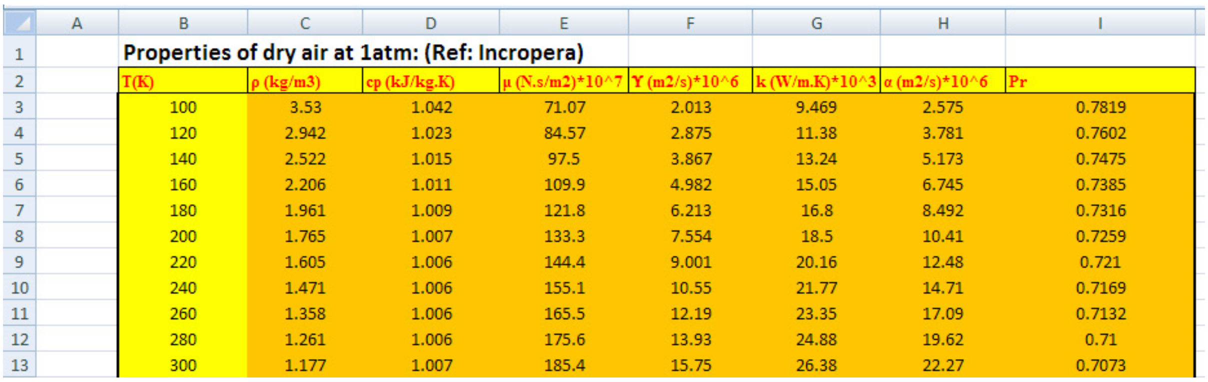
Here, we see that properties of Air are required at given temp. *But, EXCEL does not have built-in property functions for Air.*

These properties are available in Appendixes of many Heat Transfer Text books, (for ex. see Appendix of the well known text book "Fundamentals of Heat and Mass Transfer" by Incropera & DeWitt).

For our purpose, we shall use these Tables and write Functions in VBA to find out various properties of Air. The required properties are found at the given temp (in K) by interpolation.

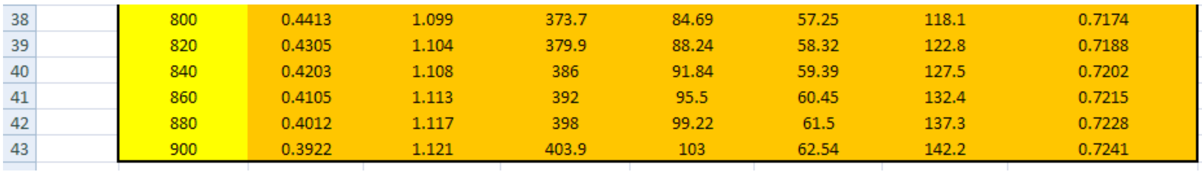
Following are the steps:

1. First, enter the data table in EXCEl as shown below. Only part of the Table is shown:



Properties of dry air at 1atm: (Ref: Incropera)

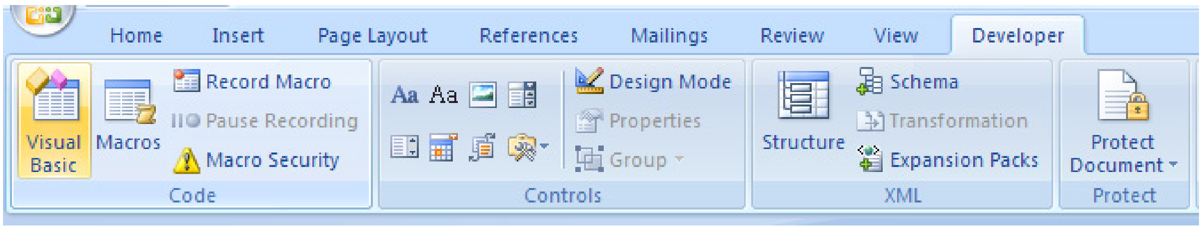
T(K)	ρ (kg/m3)	cp (kJ/kg.K)	μ (N.s/m2)*10^7	Υ (m2/s)*10^6	k (W/m.K)*10^3	α (m2/s)*10^6	Pr
100	3.53	1.042	71.07	2.013	9.469	2.575	0.7819
120	2.942	1.023	84.57	2.875	11.38	3.781	0.7602
140	2.522	1.015	97.5	3.867	13.24	5.173	0.7475
160	2.206	1.011	109.9	4.982	15.05	6.745	0.7385
180	1.961	1.009	121.8	6.213	16.8	8.492	0.7316
200	1.765	1.007	133.3	7.554	18.5	10.41	0.7259
220	1.605	1.006	144.4	9.001	20.16	12.48	0.721
240	1.471	1.006	155.1	10.55	21.77	14.71	0.7169
260	1.358	1.006	165.5	12.19	23.35	17.09	0.7132
280	1.261	1.006	175.6	13.93	24.88	19.62	0.71
300	1.177	1.007	185.4	15.75	26.38	22.27	0.7073



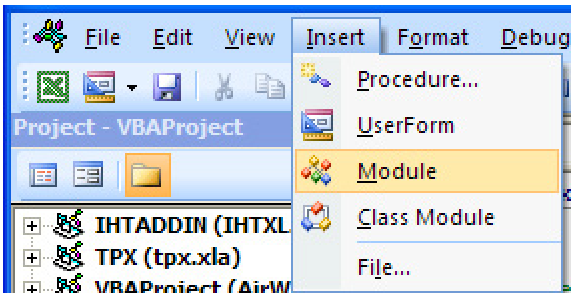
800	0.4413	1.099	373.7	84.69	57.25	118.1	0.7174
820	0.4305	1.104	379.9	88.24	58.32	122.8	0.7188
840	0.4203	1.108	386	91.84	59.39	127.5	0.7202
860	0.4105	1.113	392	95.5	60.45	132.4	0.7215
880	0.4012	1.117	398	99.22	61.5	137.3	0.7228
900	0.3922	1.121	403.9	103	62.54	142.2	0.7241

We have taken values up to T = 900 K only, since it is sufficient for our purpose.

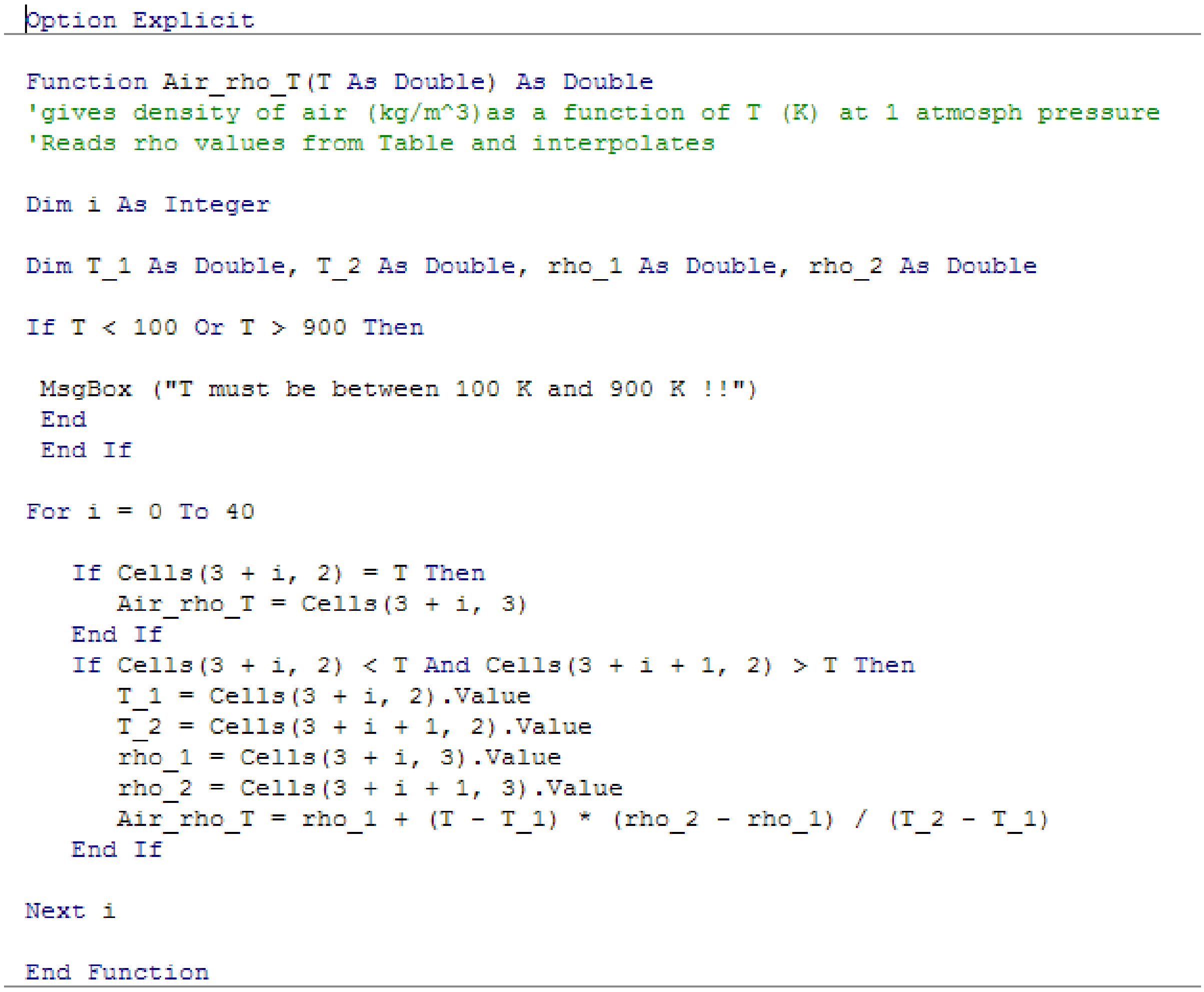
2. Now, the various properties are required as a function of T. So, the VBA program we write will search for the temp in the column B and if it finds the exact match, it will return the required property by reading the same in that row. If the exact temp match is not available (this is the more likely scenario), it will locate two consecutive cells where one temp is lower than required T, and the other temp is higher than T. Also, the corresponding values of the property at these two temperatures are noted. Then, a linear interpolation is made between these two values of the property and the same is returned.
3. Now, write the VBA code. Go to Developer – Visual Basic-



Click on Visual Basic-Insert-Module:



And, a blank window appears, and start typing the code. A very simple code for rho of Air as a function of T is shown below:



```
Option Explicit

Function Air_rho_T(T As Double) As Double
'gives density of air (kg/m^3)as a function of T (K) at 1 atmosph pressure
'Reads rho values from Table and interpolates

Dim i As Integer

Dim T_1 As Double, T_2 As Double, rho_1 As Double, rho_2 As Double

If T < 100 Or T > 900 Then

 MsgBox ("T must be between 100 K and 900 K !!")
 End
 End If

For i = 0 To 40

   If Cells(3 + i, 2) = T Then
      Air_rho_T = Cells(3 + i, 3)
   End If
   If Cells(3 + i, 2) < T And Cells(3 + i + 1, 2) > T Then
      T_1 = Cells(3 + i, 2).Value
      T_2 = Cells(3 + i + 1, 2).Value
      rho_1 = Cells(3 + i, 3).Value
      rho_2 = Cells(3 + i + 1, 3).Value
      Air_rho_T = rho_1 + (T - T_1) * (rho_2 - rho_1) / (T_2 - T_1)
   End If

Next i

End Function
```

In the above code:

1st line: says that all variables must be explicitly declared in the beginning

2nd line: declares name of the function, with dimensions of variables involved

Lines 3, 4: explains about the Function

Line 5, 6: declarations of dimensions of internal variables, i.e. variables available only inside the Function

Line 7 to 10: If statement to give an error message if value of T entered is beyond the values available in the Table

Line 11: Beginning of For…Next construct. Note that the construct will search in the 40 lines of the Table

Lines 12, 13, 14: If the exact match for T is found, Function returns the value of 'rho' in the column C

Lines 15 to 21: If he exact match of T is not there in the Table, then locate the values of T just below T and just above T, and name them as T_1 and T_2. Also, note the corresponding values of rho as rho_1 and rho_2. And, then, calculate the value of rho at the required T by interpolation.

Line 22: Go to the next I in the loop

Line 23: End statement of Function.

4. Similar Functions are written for other properties of Air, as functions of T. i.e.

   For density ….Air_rho_T(T)

   For th. conductivity ….Air_k_T(T)

   For sp. heat ….Air_cp_T(T)

   For Prandtl No. ….Air_Pr_T(T)

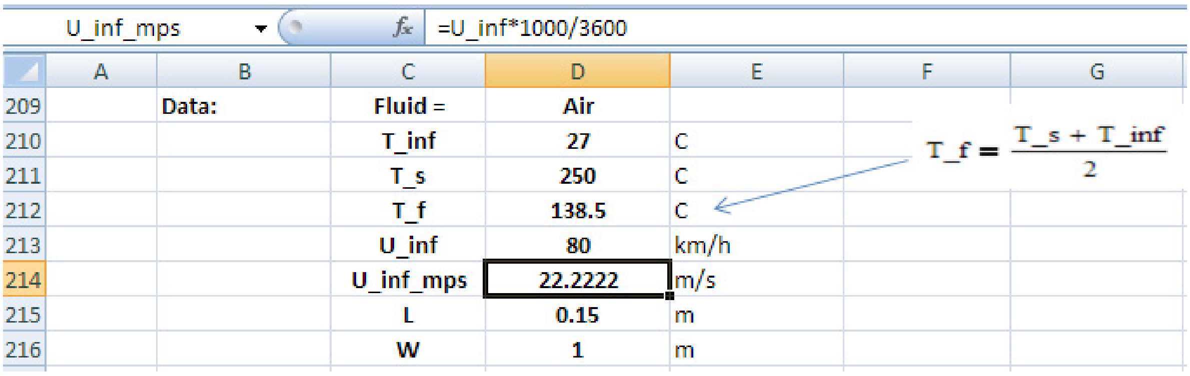
   For dynamic viscosity ….Air_mu_T(T), and

   For kinematic viscosity ….Air_nu_T(T)

5. Now, for calculations concerning this problem:

Following are the steps:

a) Set up the EXCEL worksheet, enter data and name the cells:

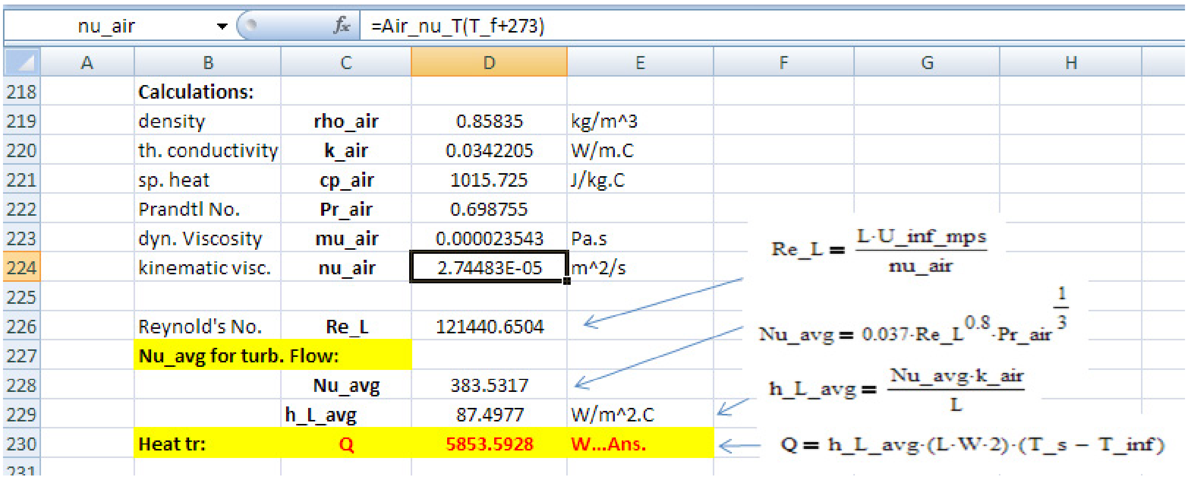


U_inf_mps
=U_inf*1000/3600

	A	B	C	D	E	F	G
209		Data:	Fluid =	Air			
210			T_inf	27	C		
211			T_s	250	C		
212			T_f	138.5	C		
213			U_inf	80	km/h		
214			U_inf_mps	22.2222	m/s		
215			L	0.15	m		
216			W	1	m		

$$T\_f = \frac{T\_s + T\_inf}{2}$$

b) **Find properties such as density, thermal conductivity, sp. heat, Prandtl No. etc using the Functions already written**. Then, calculate the Reynold's No., Nusselts No., heat transfer coeff. 'h', and finally, the heat transferred, Q. Formulas used are also shown in the worksheet for clarity:



=Air_nu_T(T_f+273)

	A	B	C	D	E
218		Calculations:			
219		density	rho_air	0.85835	kg/m^3
220		th. conductivity	k_air	0.0342205	W/m.C
221		sp. heat	cp_air	1015.725	J/kg.C
222		Prandtl No.	Pr_air	0.698755	
223		dyn. Viscosity	mu_air	0.000023543	Pa.s
224		kinematic visc.	nu_air	2.74483E-05	m^2/s
225					
226		Reynold's No.	Re_L	121440.6504	
227		Nu_avg for turb. Flow:			
228			Nu_avg	383.5317	
229			h_L_avg	87.4977	W/m^2.C
230		Heat tr:	Q	5853.5928	W...Ans.

$$\text{Re\_L} = \frac{\text{L}\cdot\text{U\_inf\_mps}}{\text{nu\_air}}$$

$$\text{Nu\_avg} = 0.037\cdot\text{Re\_L}^{0.8}\cdot\text{Pr\_air}^{\frac{1}{3}}$$

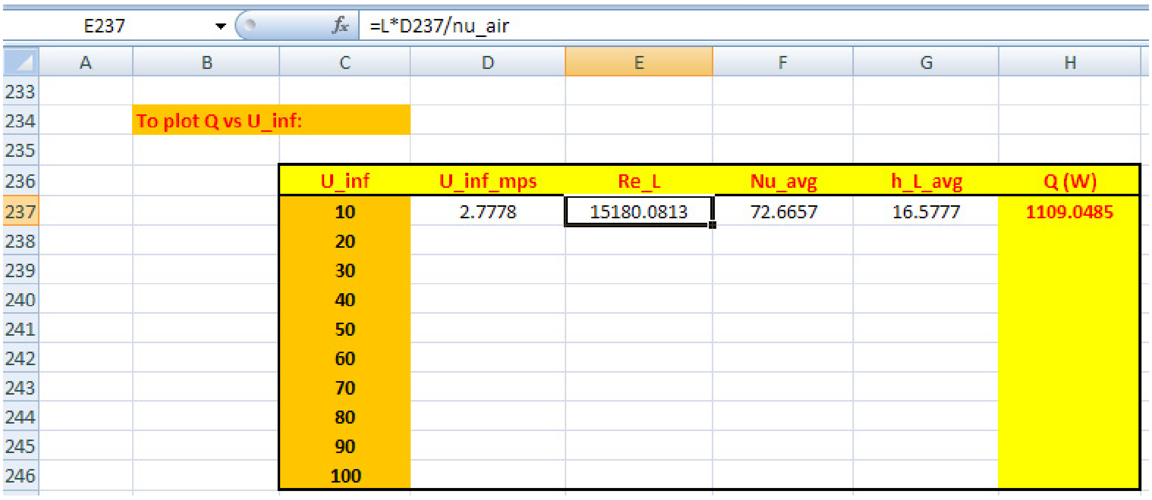
$$\text{h\_L\_avg} = \frac{\text{Nu\_avg}\cdot\text{k\_air}}{\text{L}}$$

$$\text{Q} = \text{h\_L\_avg}\cdot(\text{L}\cdot\text{W}\cdot2)\cdot(\text{T\_s} - \text{T\_inf})$$

In the above fig, **Function entered for kinematic viscosity of air can be seen in the Formula bar.**

Thus, the heat transfer, Q = 5853.59 W … Ans.

c) Now, let us plot the variation of Q as the air velocity changes from 10 mph to 100 mph. Set up the worksheet for calculations as shown:

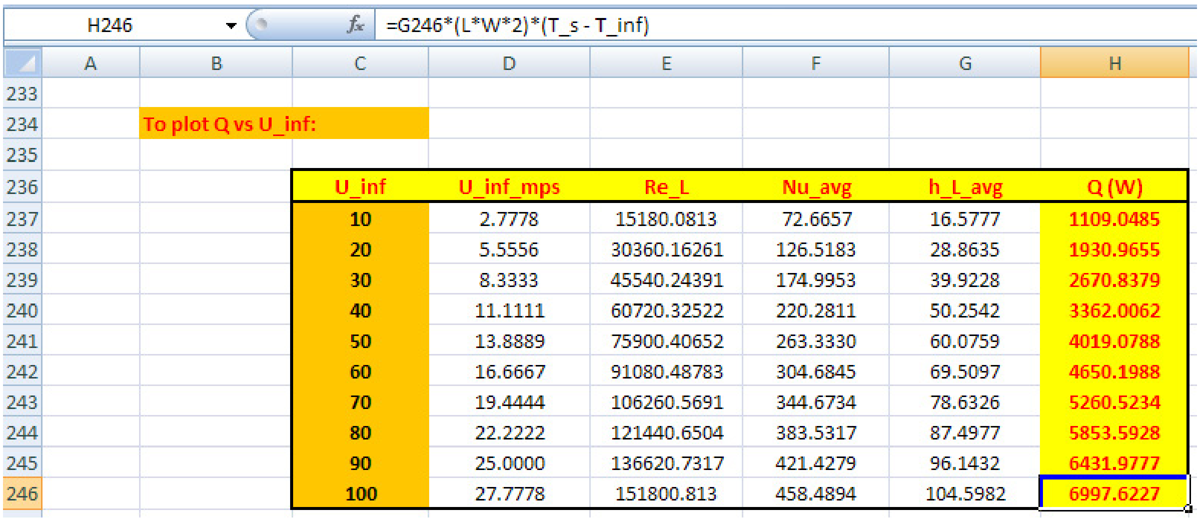


E237 =L*D237/nu_air

To plot Q vs U_inf:

U_inf	U_inf_mps	Re_L	Nu_avg	h_L_avg	Q (W)
10	2.7778	15180.0813	72.6657	16.5777	1109.0485
20					
30					
40					
50					
60					
70					
80					
90					
100					

Note that as U_inf changes, Reynold's No. will change. See the formula entered for Re_L in the Formula bar. Note that U_inf_mps (i.e. U_inf in metres/sec) is entered in 'relative reference', so that we can drag-copy for other values of U_inf. Similarly, enter formulas for other columns, again taking care to enter cells in relative reference as required. Now, select the cells D237 to H237, and drag-copy to the end of the Table. Immediately, all calculations are completed:



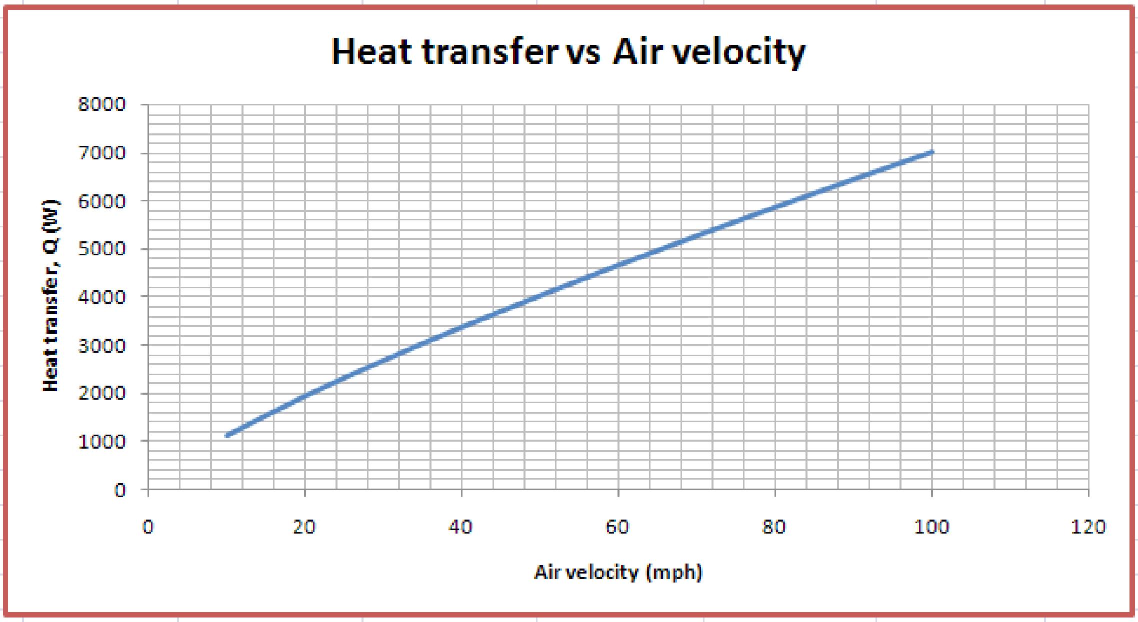
H246 =G246*(L*W*2)*(T_s - T_inf)

To plot Q vs U_inf:

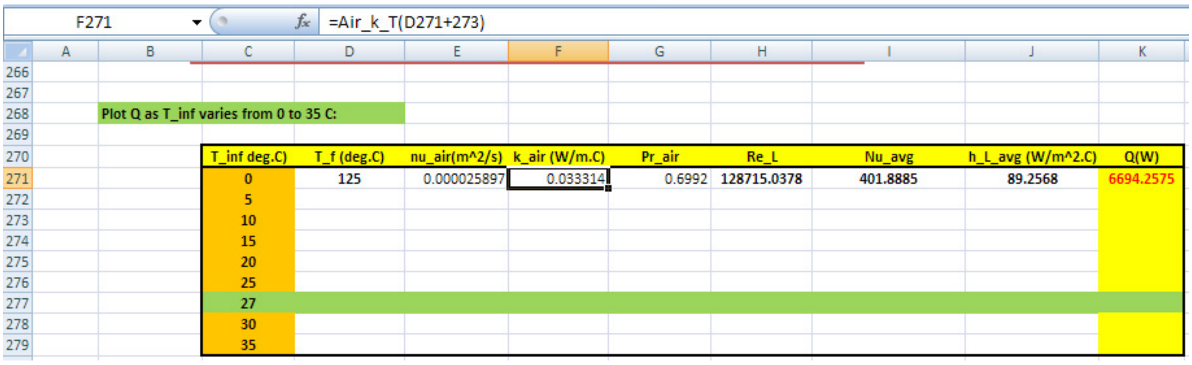
U_inf	U_inf_mps	Re_L	Nu_avg	h_L_avg	Q (W)
10	2.7778	15180.0813	72.6657	16.5777	1109.0485
20	5.5556	30360.16261	126.5183	28.8635	1930.9655
30	8.3333	45540.24391	174.9953	39.9228	2670.8379
40	11.1111	60720.32522	220.2811	50.2542	3362.0062
50	13.8889	75900.40652	263.3330	60.0759	4019.0788
60	16.6667	91080.48783	304.6845	69.5097	4650.1988
70	19.4444	106260.5691	344.6734	78.6326	5260.5234
80	22.2222	121440.6504	383.5317	87.4977	5853.5928
90	25.0000	136620.7317	421.4279	96.1432	6431.9777
100	27.7778	151800.813	458.4894	104.5982	6997.6227

It may be verified that for U_inf = 80 mph, the value of Q = 5853.59 W, thus confirming that we have entered the formulas correctly.

d) Now, plot the graph:



e) Next, let us also plot Q for various temperatures of atm. air, say T_inf varying from 0 to 35 deg. C. First, prepare a Table as shown:



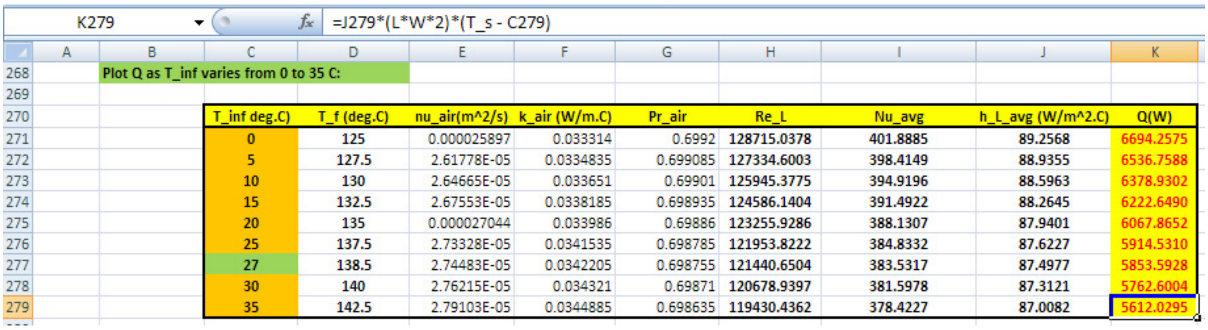
F271 =Air_k_T(D271+273)

Plot Q as T_inf varies from 0 to 35 C:

T_inf deg.C)	T_f (deg.C)	nu_air(m^2/s)	k_air (W/m.C)	Pr_air	Re_L	Nu_avg	h_L_avg (W/m^2.C)	Q(W)
0	125	0.000025897	0.033314	0.6992	128715.0378	401.8885	89.2568	6694.2575
5								
10								
15								
20								
25								
27								
30								
35								

Enter the formulas in the row 62 carefully, using 'relative references' for the cells which have to up-date themselves as we drag-copy the first row to the end of the Table.

f) Now, select cells D271 to K271 and then drag-copy the line till the end of the Table. Now, all calculations are completed:



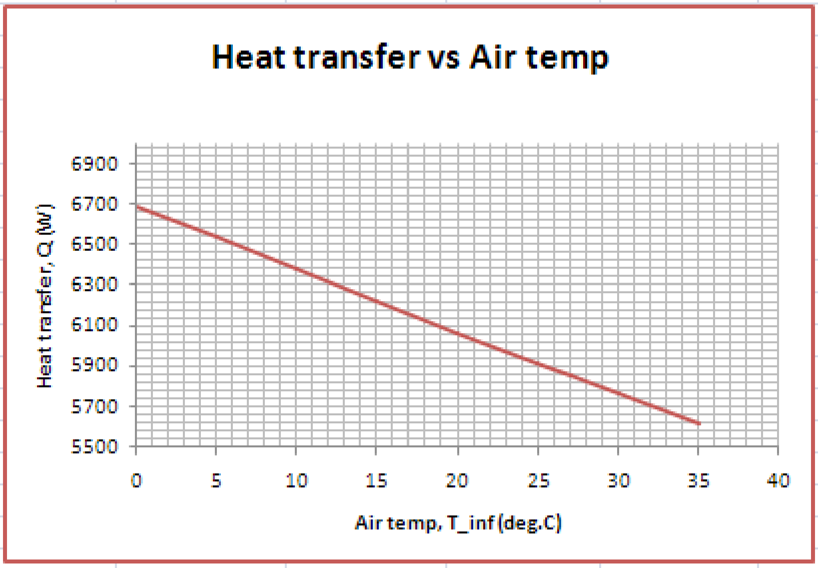
K279 =J279*(L*W*2)*(T_s - C279)

Plot Q as T_inf varies from 0 to 35 C:

T_inf deg.C)	T_f (deg.C)	nu_air(m^2/s)	k_air (W/m.C)	Pr_air	Re_L	Nu_avg	h_L_avg (W/m^2.C)	Q(W)
0	125	0.000025897	0.033314	0.6992	128715.0378	401.8885	89.2568	6694.2575
5	127.5	2.61778E-05	0.0334835	0.699085	127334.6003	398.4149	88.9355	6536.7588
10	130	2.64665E-05	0.033651	0.69901	125945.3775	394.9196	88.5963	6378.9302
15	132.5	2.67553E-05	0.0338185	0.698935	124586.1404	391.4922	88.2645	6222.6490
20	135	0.000027044	0.033986	0.69886	123255.9286	388.1307	87.9401	6067.8652
25	137.5	2.73328E-05	0.0341535	0.698785	121953.8222	384.8332	87.6227	5914.5310
27	138.5	2.74483E-05	0.0342205	0.698755	121440.6504	383.5317	87.4977	5853.5928
30	140	2.76215E-05	0.034321	0.69871	120678.9397	381.5978	87.3121	5762.6004
35	142.5	2.79103E-05	0.0344885	0.698635	119430.4362	378.4227	87.0082	5612.0295

As a check, see the Q for T_inf = 27 C. We find Q = 5853.59 W, same as we got earlier.

Now, plot the graph:



==

"**Prob. 2A1.1.16.** In a chemical processing plant glycerin flows over a 1 m long flat plate at free stream conditions U_inf = 3 m/s and T_inf = 15 C. If the plate is held at 40 C, determine the heat transfer per unit width, assuming Re_cr = 500,000. (b) Repeat for Ammonia.

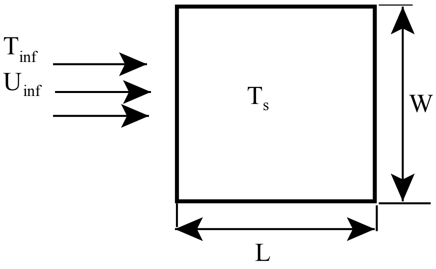


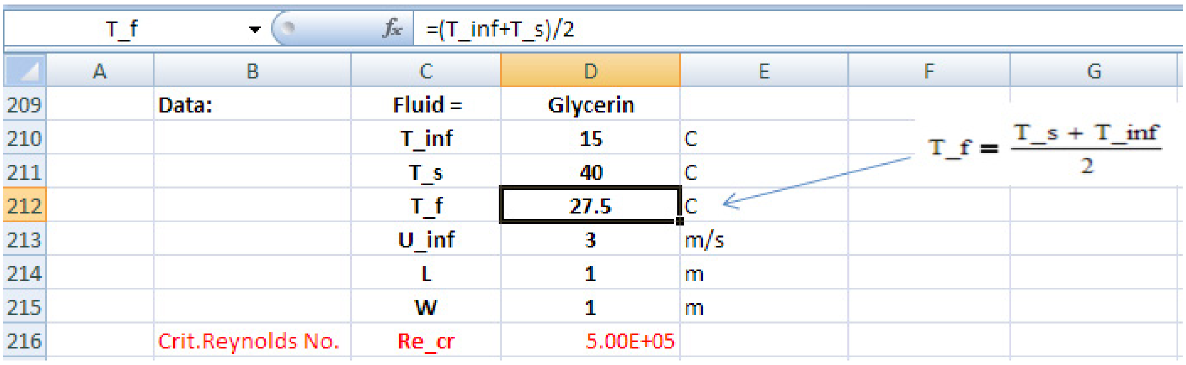
Fig. 2A.1.1.16

EXCEL Solution:

Again, since there are no built-in functions for properties of glycerin and Liquid Ammonia, we write the Functions in VBA as explained earlier in the previous problem.

Following are the calculation steps in EXCEL:

1. Set up the EXCEL worksheet, enter data and name the cells:

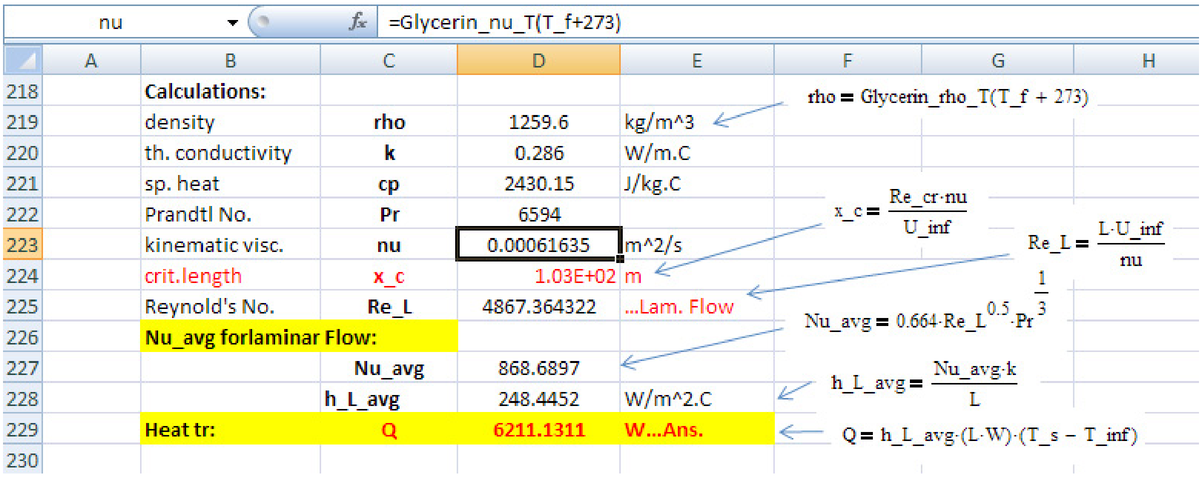


T_f =(T_inf+T_s)/2

	A	B	C	D	E	F	G
209		**Data:**	**Fluid =**	**Glycerin**			
210			**T_inf**	**15**	C		
211			**T_s**	**40**	C		
212			**T_f**	**27.5**	C		
213			**U_inf**	**3**	m/s		
214			**L**	**1**	m		
215			**W**	**1**	m		
216		Crit.Reynolds No.	**Re_cr**	5.00E+05			

$$T\_f = \frac{T\_s + T\_inf}{2}$$

2. Get the property data for Glycerin using the Functions already written. Also, find out critical length since Re_cr is given as 5E05. We find that x_cr = 103 m >> 1 m. **Therefore, fully Laminar flow exists on the entire plate.** We also find that Re_L = 4867.36 (i.e. Laminar flow). So, use Laminar flow eqn for Nusselts No. and then get heat transfer coeff. 'h' and finally, the heat transfer rate, Q. Eqns used are also shown in the worksheet for clarity. **See the screen shot given below:**

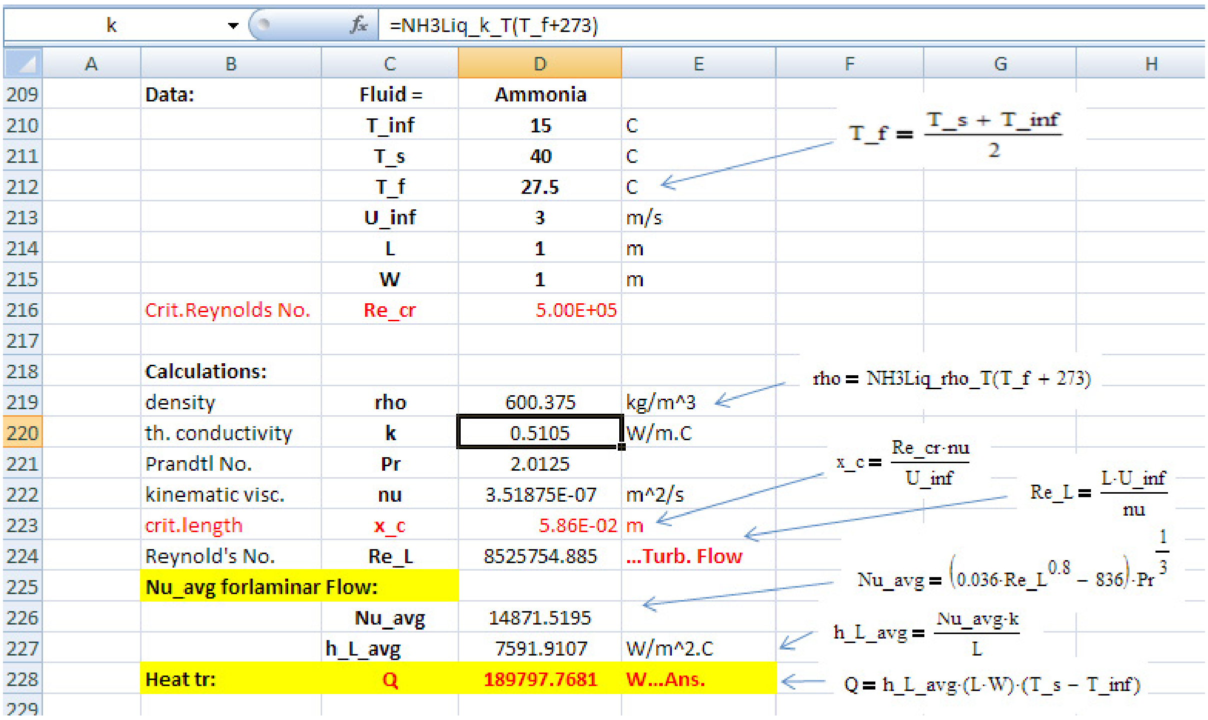


nu =Glycerin_nu_T(T_f+273)

	A	B	C	D	E	F	G	H
218		Calculations:				$rho = Glycerin\_rho\_T(T\_f + 273)$		
219		density	rho	1259.6	kg/m^3			
220		th. conductivity	k	0.286	W/m.C			
221		sp. heat	cp	2430.15	J/kg.C	$x\_c = \frac{Re\_cr \cdot nu}{U\_inf}$		
222		Prandtl No.	Pr	6594				
223		kinematic visc.	nu	0.00061635	m^2/s	$Re\_L = \frac{L \cdot U\_inf}{nu}$		
224		crit.length	x_c	1.03E+02	m			
225		Reynold's No.	Re_L	4867.364322	...Lam. Flow	$Nu\_avg = 0.664 \cdot Re\_L^{0.5} \cdot Pr^{\frac{1}{3}}$		
226		Nu_avg forlaminar Flow:						
227			Nu_avg	868.6897				
228			h_L_avg	248.4452	W/m^2.C	$h\_L\_avg = \frac{Nu\_avg \cdot k}{L}$		
229		Heat tr:	Q	6211.1311	W...Ans.	$Q = h\_L\_avg \cdot (L \cdot W) \cdot (T\_s - T\_inf)$		
230								

Thus, Q = 6211.13 W … Ans.

3. Now, repeat these calculations for Ammonia (liquid), using the property functions written for Ammonia:



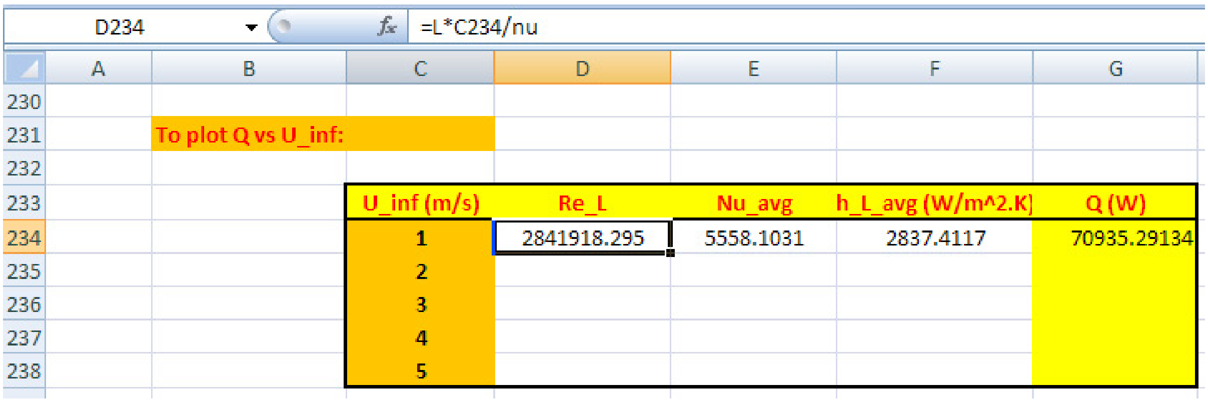
k =NH3Liq_k_T(T_f+273)

	A	B	C	D	E	F	G	H
209		Data:	Fluid =	Ammonia				
210			T_inf	15	C	$T\_f = \frac{T\_s + T\_inf}{2}$		
211			T_s	40	C			
212			T_f	27.5	C			
213			U_inf	3	m/s			
214			L	1	m			
215			W	1	m			
216		Crit.Reynolds No.	Re_cr	5.00E+05				
217								
218		Calculations:				$rho = NH3Liq\_rho\_T(T\_f + 273)$		
219		density	rho	600.375	kg/m^3			
220		th. conductivity	k	0.5105	W/m.C			
221		Prandtl No.	Pr	2.0125		$x\_c = \frac{Re\_cr \cdot nu}{U\_inf}$		
222		kinematic visc.	nu	3.51875E-07	m^2/s	$Re\_L = \frac{L \cdot U\_inf}{nu}$		
223		crit.length	x_c	5.86E-02	m			
224		Reynold's No.	Re_L	8525754.885	...Turb. Flow			
225		Nu_avg forlaminar Flow:				$Nu\_avg = \left(0.036 \cdot Re\_L^{0.8} - 836\right) \cdot Pr^{\frac{1}{3}}$		
226			Nu_avg	14871.5195				
227			h_L_avg	7591.9107	W/m^2.C	$h\_L\_avg = \frac{Nu\_avg \cdot k}{L}$		
228		Heat tr:	Q	189797.7681	W...Ans.	$Q = h\_L\_avg \cdot (L \cdot W) \cdot (T\_s - T\_inf)$		
229								

Also, find out critical length since Re_cr is given as 5E05. We find that x_cr = 5.86 cm << 1 m. **Therefore, fully Turb. flow is assumed over the entire plate.** We also find that Re_L = 8525754.9 (i.e. Turb. flow). So, use Turb. flow eqn for Nusselts No. and then get heat transfer coeff. 'h' and finally, the heat transfer rate, Q. Eqns used are also shown in the worksheet for clarity. **See the screen shot given above.**

Thus, Q = 189797.77 W … Ans.

4. Now, let us plot the variation of Q as the velocity changes from 1 m/s to 5 m/s. Set up the worksheet for calculations as shown:

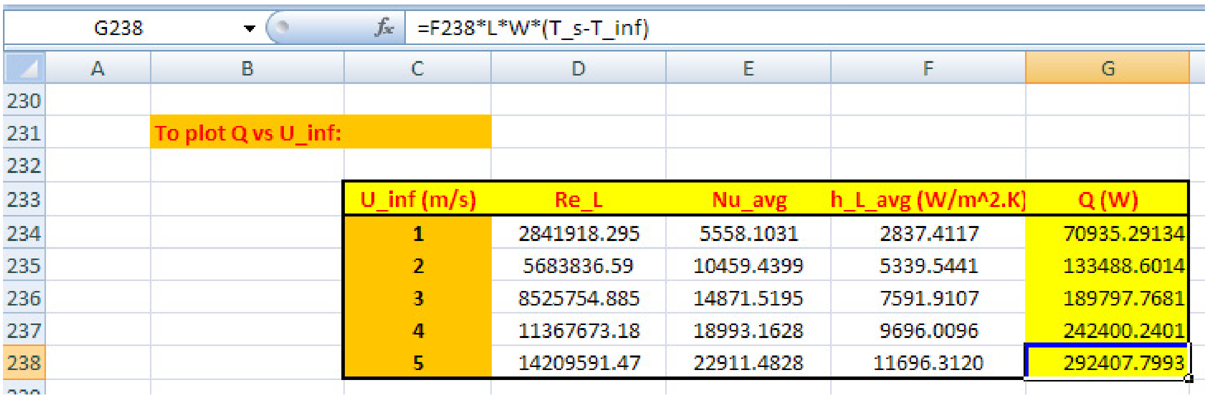


D234 =L*C234/nu

To plot Q vs U_inf:

U_inf (m/s)	Re_L	Nu_avg	h_L_avg (W/m^2.K)	Q (W)
1	2841918.295	5558.1031	2837.4117	70935.29134
2				
3				
4				
5				

Note that as U_inf changes, Reynold's No. will change. See the formula entered for Re_L in the Formula bar. Note that U_inf is entered in 'relative reference', so that we can drag-copy for other values of U_inf. Similarly, enter formulas for other columns, again taking care to enter cells in relative reference as required. Now, select the cells D234 to G234, and drag-copy to the end of the Table. Immediately, all calculations are completed:



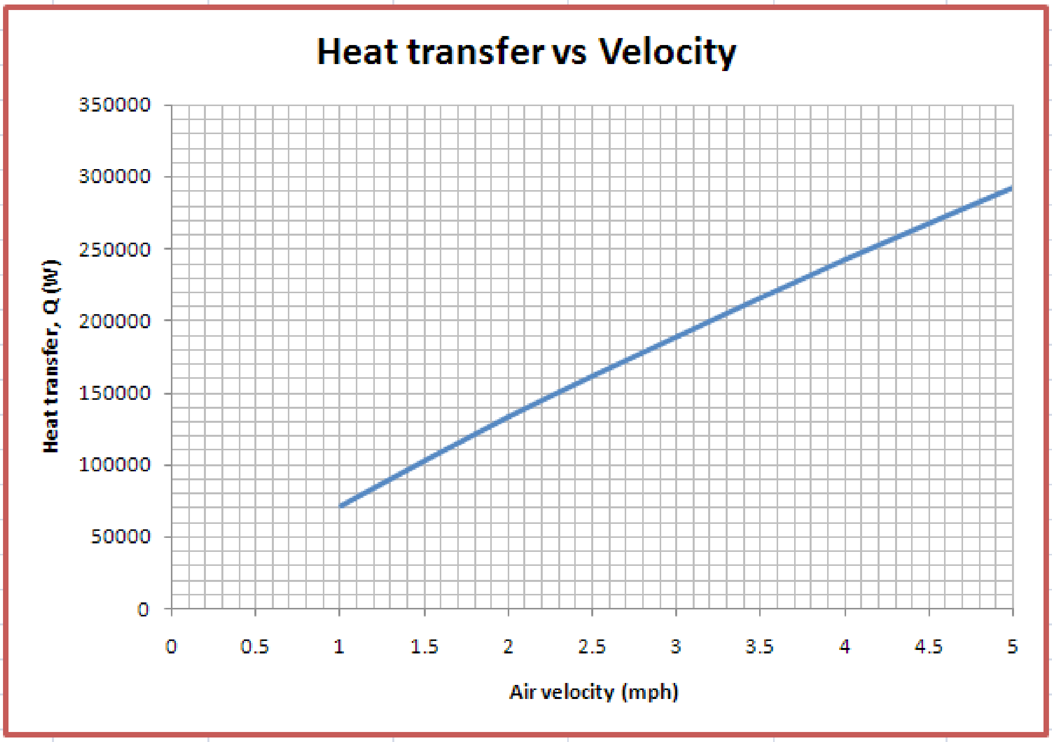
G238 =F238*L*W*(T_s-T_inf)

To plot Q vs U_inf:

U_inf (m/s)	Re_L	Nu_avg	h_L_avg (W/m^2.K)	Q (W)
1	2841918.295	5558.1031	2837.4117	70935.29134
2	5683836.59	10459.4399	5339.5441	133488.6014
3	8525754.885	14871.5195	7591.9107	189797.7681
4	11367673.18	18993.1628	9696.0096	242400.2401
5	14209591.47	22911.4828	11696.3120	292407.7993

It may be verified that for U_inf = 3 m/s, the value of Q = 189797.77 W, thus confirming that we have entered the formulas correctly.

5. Now, plot the graph:



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2A1.2 Flow across cylinders and spheres:

$UnitSystem SI Pa C J

"**Prob. 2A1.2.1**. Assuming that a man can be represented by a cylinder 30 cm in diameter and 1.7 m high with a surface temp of 30 C, calculate the heat he would lose while standing in a 36 km/h wind at 10 C. [VTU- Dec.06–Jan.07]"

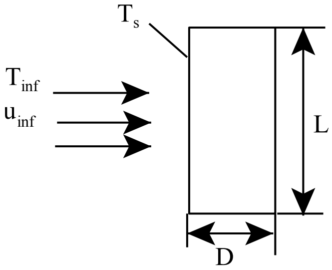


Fig.Prob.2A1.2.1

EES Solution:

"Data:"

D = 0.3 [m]
L = 1.7 [m]
u_inf = 10[m/s]
T_inf = 10[C]
T_s = 30[C]
P = 101300[Pa]

"Calculations:"

"This is a cylinder in cross flow. Use Churchill – Bernstein eqn."

T_f = (T_s + T_inf) / 2 "[C] … film tem."

"Properties of air at T_f:"

Pr = Prandtl(Air,T=T_f)
rho = Density(Air,T=T_f,P=P) "[kg/m^3]"
mu = Viscosity(Air,T=T_f) "[kg/m-s]"
k=Conductivity(Air,T=T_f) "[W/m-C]"

Re_D = D * u_inf * rho / mu "….Reynolds No."

"Churchill – Bernstein eqn:"

$$\text{Nusselt}_{cyl} = 0.3 + \frac{0.62 \cdot \text{Re}_D^{0.5} \cdot \text{Pr}^{(1/3)}}{\left[1 + \left(\frac{0.4}{\text{Pr}}\right)^{(2/3)}\right]^{(1/4)}} \cdot \left[1 + \left(\frac{\text{Re}_D}{282000}\right)^{(5/8)}\right]^{(4/5)}$$

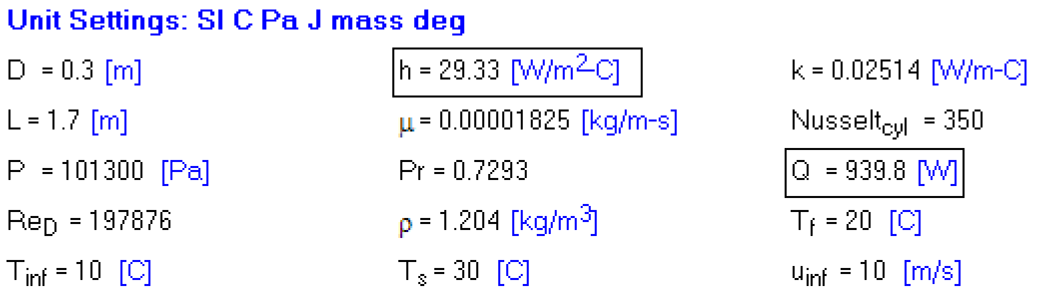
It is entered in EES:

Nusselt_cyl = 0.3 + ((0.62 * Re_D^0.5 * Pr^(1/3)) / (1 + (0.4 / Pr)^(2/3))^(1/4)) * (1 + (Re_D/282000)^(5/8))^(4/5) "…finds Nusselts No."

Nusselt_cyl = h * D / k "…finds heat tr. coeff."

Q = h * (pi * D * L) * (T_s – T_inf) "[W]… heat lost by man"

Results:



Unit Settings: SI C Pa J mass deg

$D = 0.3$ [m]
$L = 1.7$ [m]
$P = 101300$ [Pa]
$\text{Re}_D = 197876$
$T_{inf} = 10$ [C]

$\boxed{h = 29.33 \text{ [W/m}^2\text{-C]}}$
$\mu = 0.00001825$ [kg/m-s]
$\text{Pr} = 0.7293$
$\rho = 1.204$ [kg/m^3]
$T_s = 30$ [C]

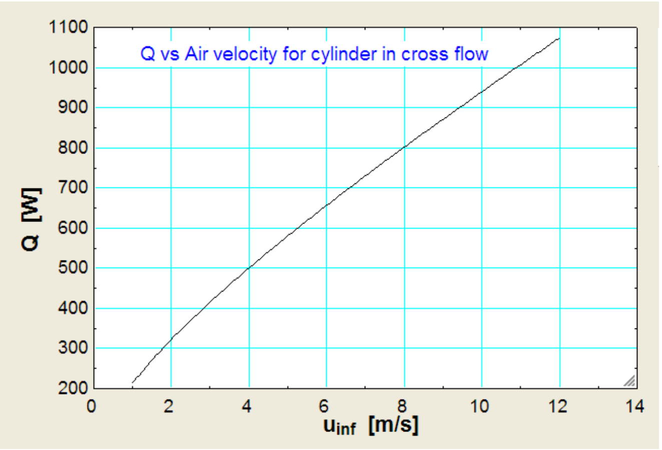
$k = 0.02514$ [W/m-C]
$\text{Nusselt}_{cyl} = 350$
$\boxed{Q = 939.8 \text{ [W]}}$
$T_f = 20$ [C]
$u_{inf} = 10$ [m/s]

Thus:

Heat transfer coeff = h = 29.33 W/m^2.C … Ans.

Heat lost by man = Q = 939.8 W … Ans.

Plot Q against air velocity, U_inf:



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$UnitSystem SI C Pa J

"**Prob. 2A1.2.2.** Air stream at 27 C is moving at 0.3 m/s across 100 W incandescent bulb glowing at 127 C. If the bulb is approximated by a 60 mm dia sphere, estimate the heat transfer rate and percentage of power lost due to convection. Use the correlation: Nu = 0.37. Re_D^0.6 [VTU – Dec. 2010]."

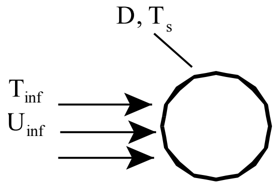


Fig.Prob.2A1.2.2

EES Solution:

"Data:"

D = 0.06[m]
T_s = 127[C]
T_inf = 27[C]
U_inf = 0.3[m/s]
T_f = (T_s+T_inf)/2 "[C]… mean film temp."

"Properties of Air:"

mu=Viscosity(Air,T=T_f) "[kg/m-s]"

rho=Density(Air,T=T_f,P=1.013e05) "[kg/m^3]"

nu = mu/rho "[m^2/s]"

cp=Cp(Air,T=T_f) "[J/kg-C]"

k=Conductivity(Air,T=T_f) "[W/m-C]"

Pr = mu*cp/k

"Calculations:"

Re_D = D*U_inf*rho/mu "finds Reynolds No., Re_D"

Nusselt = 0.37 * Re_D^0.6 "finds Nusselts No."

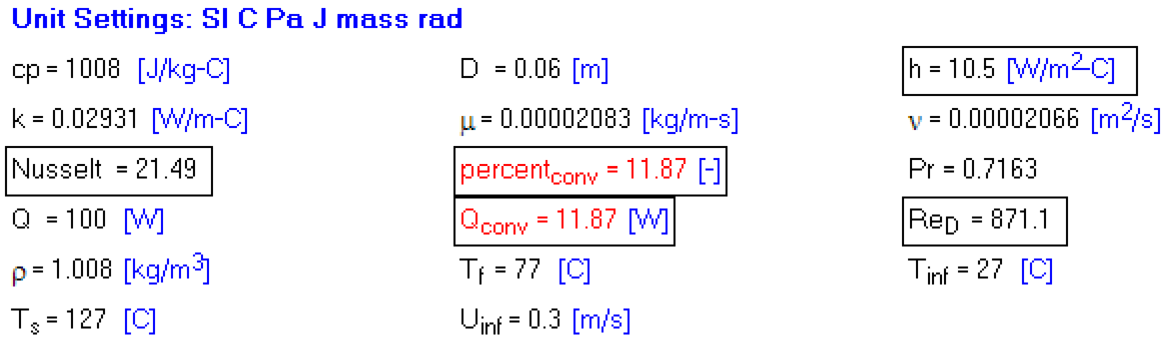
Nusselt = h*D/k "finds h"

Q_conv = h*(pi * D^2) * (T_s-T_inf) "W"

Q = 100[W] "…by data"

percent_conv = (Q_conv/Q) * 100

Results:



```
Unit Settings: SI C Pa J mass rad
cp = 1008  [J/kg-C]           D  = 0.06 [m]                 h = 10.5 [W/m2-C]
k = 0.02931 [W/m-C]           μ = 0.00002083 [kg/m-s]       ν = 0.00002066 [m2/s]
Nusselt = 21.49               percentconv = 11.87 [-]       Pr = 0.7163
Q = 100 [W]                   Qconv = 11.87 [W]             ReD = 871.1
ρ = 1.008 [kg/m3]             Tf = 77 [C]                   Tinf = 27 [C]
Ts = 127 [C]                  Uinf = 0.3 [m/s]
```

Thus:

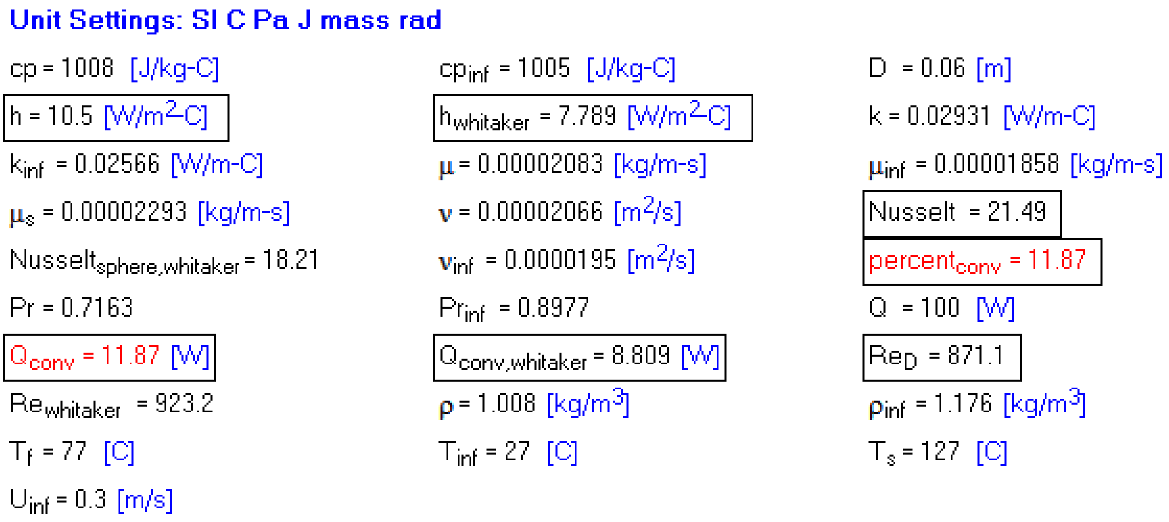
Heat transfer by convection = Q_conv = 11.87 W …. Ans.

Percentage of heat lost by convection = 11.87% …. Ans.

And, if we use the more accurate Whitaker's relation, viz.

$$Nusselt_{sphere,whitaker} = 2 + \left(0.4 \cdot Re_{whitaker}^{0.5} + 0.06 \cdot Re_{whitaker}^{(2/3)} \right) \cdot Pr_{inf}^{0.4} \cdot \left[\frac{\mu_{inf}}{\mu_s} \right]^{(1/4)}$$

We get:



Unit Settings: SI C Pa J mass rad

$cp = 1008$ [J/kg-C]
$cp_{inf} = 1005$ [J/kg-C]
$D = 0.06$ [m]
$\boxed{h = 10.5 \text{ [W/m}^2\text{-C]}}$
$\boxed{h_{whitaker} = 7.789 \text{ [W/m}^2\text{-C]}}$
$k = 0.02931$ [W/m-C]
$k_{inf} = 0.02566$ [W/m-C]
$\mu = 0.00002083$ [kg/m-s]
$\mu_{inf} = 0.00001858$ [kg/m-s]
$\mu_s = 0.00002293$ [kg/m-s]
$\nu = 0.00002066$ [m^2/s]
$\boxed{Nusselt = 21.49}$
$Nusselt_{sphere,whitaker} = 18.21$
$\nu_{inf} = 0.0000195$ [m^2/s]
$\boxed{percent_{conv} = 11.87}$
$Pr = 0.7163$
$Pr_{inf} = 0.8977$
$Q = 100$ [W]
$\boxed{Q_{conv} = 11.87 \text{ [W]}}$
$\boxed{Q_{conv,whitaker} = 8.809 \text{ [W]}}$
$\boxed{Re_D = 871.1}$
$Re_{whitaker} = 923.2$
$\rho = 1.008$ [kg/m^3]
$\rho_{inf} = 1.176$ [kg/m^3]
$T_f = 77$ [C]
$T_{inf} = 27$ [C]
$T_s = 127$ [C]
$U_{inf} = 0.3$ [m/s]

Note that now the Q_conv = 8.809 W.

===

Prob. 2A1.2.3. Air at 35 C flows across a cylinder of 50mm dia at a velocity of 50m/s. The cylinder surface is maintained at 145 C. Find the heat loss per unit length.

Take the following properties at the mean temp. of 90 C.

ρ =1 kg/m^3, μ = 20 × 10^-6 kg/ms, k = 0.0312 W/m C, cp = 1.0 kJ/kg C;

Use: Nu_D = 0.027. (Re)^0.805. (Pr)^(1/3)

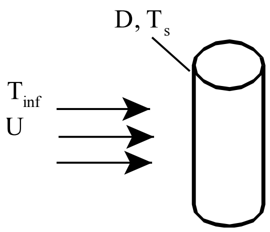


Fig.Prob.2A1.2.3

Mathcad Solution:

Data:

$T_s := 145$ C $\quad T_{inf} := 35$ C

$U := 50$ m/s $\quad L := 1$ m $\quad D := 0.05$ m

$A := \pi \cdot D \cdot L \quad A = 0.157$ m^2.... surface area

Now: $T_f := \dfrac{T_s + T_{inf}}{2}$

i.e. $T_f = 90$ C .... mean film temp.

Properties of fluid at Tf:

$\mu := 20 \cdot 10^{-6}$ kg/m.s $\quad k := 0.0312$ W/m.C

$cp := 1000$ J/kg.C $\quad \rho := 1$ kg/m^3

Calculations:

$Re_D := \frac{D \cdot U \cdot \rho}{\mu}$ $Re_D = 1.25 \cdot 10^5$ ....Reynolds No.

$Pr := \frac{cp \cdot \mu}{k}$ $Pr = 0.641$ ....Prandtl No.

$Nu_D := 0.027 \cdot Re_D^{0.805} \cdot Pr^{\frac{1}{3}}$ ....Nusselts No.... eqn.(A).... from Data

i.e. $Nu_D = 295.122$

Therefore: heat transfer coeff. is given by:

$$h := \frac{k \cdot Nu_D}{D}$$

i.e. $h = 184.156$ **...W/m^2.C....Ans.**

And, $Q := h \cdot A \cdot (T_s - T_{inf})$

i.e. $Q = 3.182 \cdot 10^3$ **...W.. Heat transferred ... Ans.**

Using Churchill and Bernstein's relation, which is more accurate:

$$Nu_{cyl} := 0.3 + \frac{0.62 \cdot Re_D^{0.5} \cdot Pr^{\frac{1}{3}}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}} \cdot \left[1 + \left(\frac{Re_D}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$

i.e. $Nu_{cyl} = 240.485$ ...compare this with earlier value of 295.1

Therefore:

$$h := \frac{k \cdot Nu_{cyl}}{D}$$

i.e. $h = 150.063$ **...W/m^2.C....Ans.**

And, $Q := h \cdot A \cdot (T_s - T_{inf})$

i.e. $Q = 2.593 \cdot 10^3$ **...W.. Heat transferred ... Ans.**

Comments:

Eqn. (A) given in data is a simpler alternative to Churchill and Bernstein's relation and is based on experimental results. It is due to Hilpert.

Eqn. (A) is presented as follows:

$Nu_{cyl} = \frac{h \cdot D}{k} = C \cdot Re_D{}^m \cdot Pr^n$ where constants C, m and n depend on the value of Reynolds No. Re_D. See following Table (Ref: Cengel)

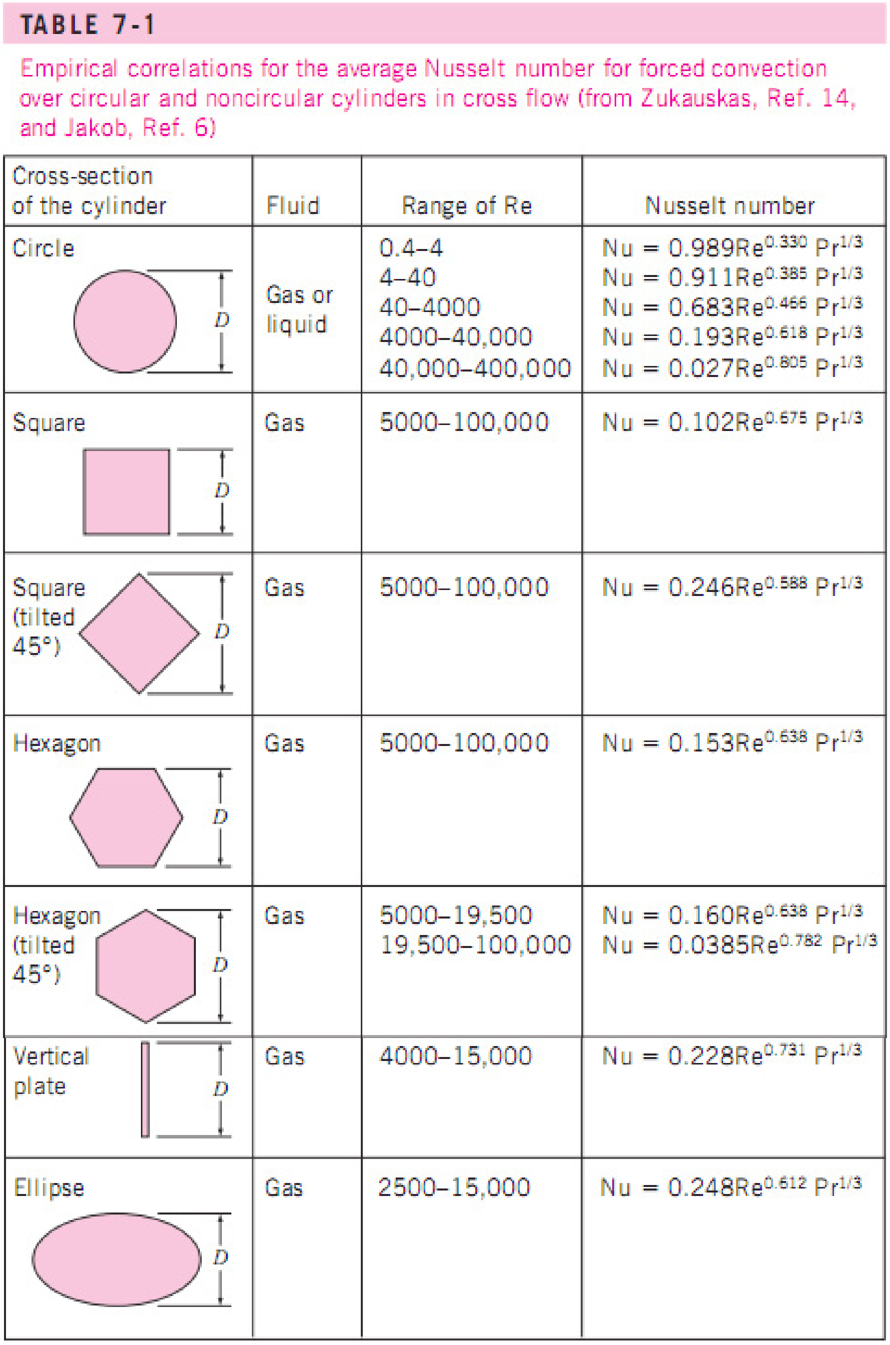


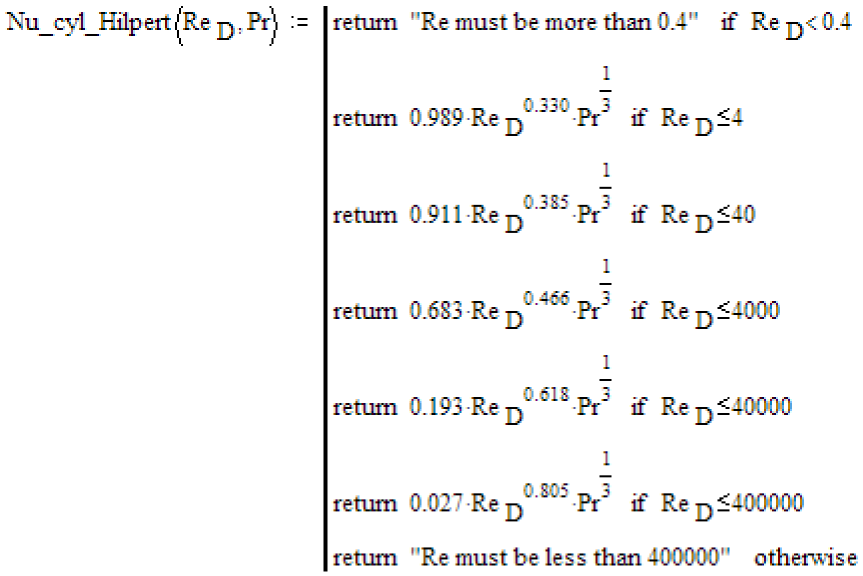
TABLE 7-1

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, Ref. 14, and Jakob, Ref. 6)

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\,Pr^{1/3}$ $Nu = 0.911Re^{0.385}\,Pr^{1/3}$ $Nu = 0.683Re^{0.466}\,Pr^{1/3}$ $Nu = 0.193Re^{0.618}\,Pr^{1/3}$ $Nu = 0.027Re^{0.805}\,Pr^{1/3}$
Square	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\,Pr^{1/3}$
Square (tilted 45°)	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\,Pr^{1/3}$
Hexagon	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\,Pr^{1/3}$
Hexagon (tilted 45°)	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638}\,Pr^{1/3}$ $Nu = 0.0385Re^{0.782}\,Pr^{1/3}$
Vertical plate	Gas	4000–15,000	$Nu = 0.228Re^{0.731}\,Pr^{1/3}$
Ellipse	Gas	2500–15,000	$Nu = 0.248Re^{0.612}\,Pr^{1/3}$

Following is a small Mathcad program to calculate Nusselts No. for cylinder in cross flow:

Nu is written as a function of Re and Pr. Depending on the value of Re, appropriate eqn is chosen to calculate Nu:



```
$Nu\_cyl\_Hilpert(Re_D, Pr) :=$
    return "Re must be more than 0.4"   if $Re_D < 0.4$
    return $0.989 \cdot Re_D^{0.330} \cdot Pr^{\frac{1}{3}}$   if $Re_D \le 4$
    return $0.911 \cdot Re_D^{0.385} \cdot Pr^{\frac{1}{3}}$   if $Re_D \le 40$
    return $0.683 \cdot Re_D^{0.466} \cdot Pr^{\frac{1}{3}}$   if $Re_D \le 4000$
    return $0.193 \cdot Re_D^{0.618} \cdot Pr^{\frac{1}{3}}$   if $Re_D \le 40000$
    return $0.027 \cdot Re_D^{0.805} \cdot Pr^{\frac{1}{3}}$   if $Re_D \le 400000$
    return "Re must be less than 400000"   otherwise
```

Example: $\text{Nu_cyl_Hilpert}(0.3, \text{Pr}) = $ "Re must be more than 0.4"

$\text{Nu\_cyl\_Hilpert}(30, \text{Pr}) = 2.91$

$\text{Nu\_cyl\_Hilpert}(400, \text{Pr}) = 9.607$

$\text{Nu\_cyl\_Hilpert}\left(1.25 \cdot 10^{5}, \text{Pr}\right) = 295.122$

$\text{Nu_cyl_Hilpert}(400001, \text{Pr}) = $ "Re must be less than 400000"

Let us draw the variation of Q with Air velocity, using the Churchill and Bernstein relation:

First, express Q as a function of air velocity, U:

$\text{Re}_{D}(U) := \dfrac{D \cdot U \cdot \rho}{\mu}$ ...Reynolds No. as a function of U

$$\text{Nu}_{\text{cyl}}(U) := 0.3 + \frac{0.62 \cdot \text{Re}_{D}(U)^{0.5} \cdot \text{Pr}^{\frac{1}{3}}}{\left[1 + \left(\frac{0.4}{\text{Pr}}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}} \cdot \left[1 + \left(\frac{\text{Re}_{D}(U)}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$

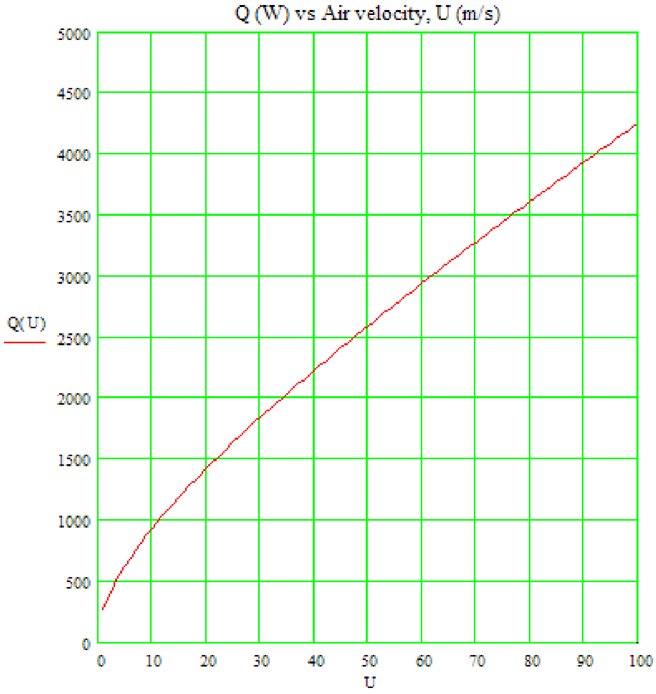
... Nusselts No. as a function of U

$h(U) := \dfrac{k \cdot \text{Nu}_{\text{cyl}}(U)}{D}$ ...heat transfer coeff. h as a function of U

And, $Q(U) := h(U) \cdot A \cdot \left(T_{s} - T_{\text{inf}}\right)$ ....Q as a function of U

To draw the graph:

$U := 1, 2 .. 100$ ...define a range variable U



==

Prob. 2A1.2.4. Air at 30 C flows across a 1.5 m long square duct (size: 20 cm × 20 cm) at a velocity of 200m/min. Electronic components located inside the duct generate heat and the duct surface should not exceed 65 C. Find the heat that can be generated by the electronic components. [Ref. 2]

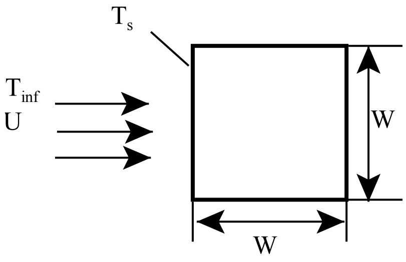


Fig.Prob.2A1.2.4

Mathcad Solution:

Data:

$T_s := 65$ C $\quad T_{inf} := 30$ C

$U := \frac{200}{60}$ m/s i.e. $U = 3.333$ m/s

$L := 1.5$ m $\quad W := 0.2$ m... side of the square cross-section

$A := 4 \cdot W \cdot L \quad A = 1.2$ m^2.... surface area

Now: $T_f := \frac{T_s + T_{inf}}{2}$

i.e. $T_f = 47.5$ C .... mean film temp.

Properties of air at Tf:

$\mu := 19.43 \cdot 10^{-6}$ kg/m.s $\quad k := 0.0278$ W/m.C

$cp := 1008$ J/kg.C $\quad \rho := 1.0932$ kg/m^3

Calculations:

Refer to the Table given in Prob. 2A1.2.3. Refer to duct of square cross-section.

To calculate Re, the side W is the characteristic dimension.

For the range of Re = 5000 to 100000, Nusselts No. is given by:

Nu = 0.102. Re^0.675 . Pr^(1/3)

$Re := \frac{W \cdot U \cdot \rho}{\mu}$ $Re = 3.751 \cdot 10^{4}$ ....Reynolds No.

$Pr := \frac{cp \cdot \mu}{k}$ $Pr = 0.705$ ....Prandtl No.

$Nu := 0.102 \cdot Re^{0.675} \cdot Pr^{\frac{1}{3}}$ ....Nusselts No..... from Table

i.e. $Nu = 111.029$

Therefore: heat transfer coeff. is given by:

$h := \frac{k \cdot Nu}{W}$

i.e. $h = 15.433$ **...W/m^2.C....Ans.**

And, $Q := h \cdot A \cdot \left(T_s - T_{inf}\right)$

i.e. $Q = 648.189$ **...W.. Heat transferred ... Ans.**

Let us draw the variation of Q with Air velocity:

First, express Q as a function of air velocity, U:

$Re(U) := \frac{W \cdot U \cdot \rho}{\mu}$ ...Reynolds No. as a function of U

$Nu(U) := 0.102 \cdot Re(U)^{0.675} \cdot Pr^{\frac{1}{3}}$ ....Nusselts No..... from Table

$h(U) := \frac{k \cdot Nu(U)}{W}$ ...heat transfer coeff. h as a function of U

And, $Q(U) := h(U) \cdot A \cdot (T_s - T_{inf})$ ....Q as a function of U

To draw the graph in the applicable range of Reynolds Numbers:

Note that:

At U = 1 m/s : $Re(1) = 1.125 \cdot 10^4$

At U = 8 m/s : $Re(8) = 9.002 \cdot 10^4$

$U := 1, 1.1 .. 8$ ...define a range variable U, from 1 m/s to 8 m/s

==

Prob. 2A1.2.5. A 6 mm dia electrical transmission line carries an electric current of 50 A and has a resistance of 0.002 ohm per metre length. Determine the surface temp of the wire during a windy day when the temp is 10 C and the wind is blowing across the transmission line at 40 km/h.

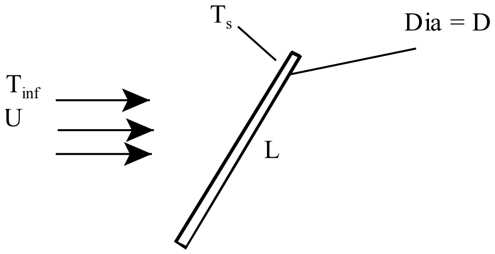


Fig.Prob.2A1.2.5

Mathcad Solution:

EES is most suitable to solve this problem since it has got properties of Air (and many other fluids) built into it.

However, let us solve this problem with Mathcad.

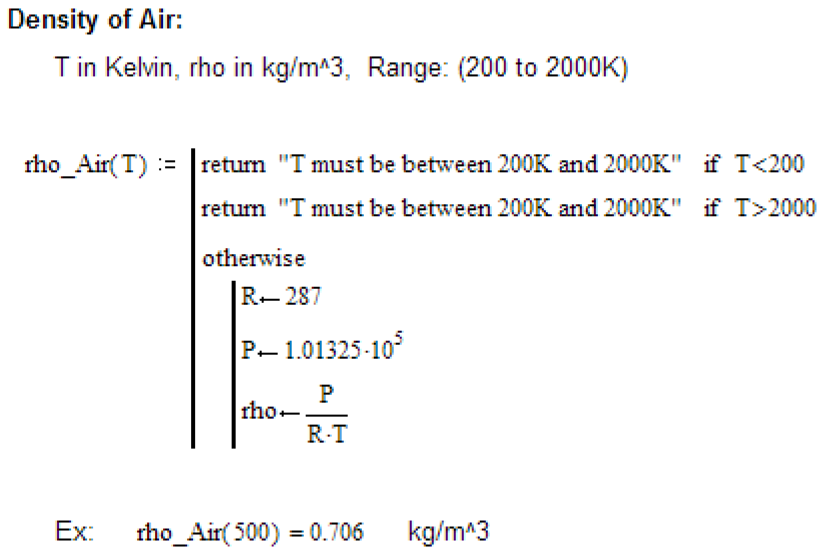
Mathcad does not have properties of Air built into it.

So, we will use the curve fit equations for Dyn. Viscosity (mu) and thermal cond. (k) of Air. The curve fit eqns were obtained using a curve-fit software, viz. CurveExpert.

Density of Air is calculated using Ideal gas eqn.

Prandtl No. is obtained in Mathcad by interpolation.

First, write the functions for Density, Dy. Viscosity,thermal cond. and Prandtl No. of Air:

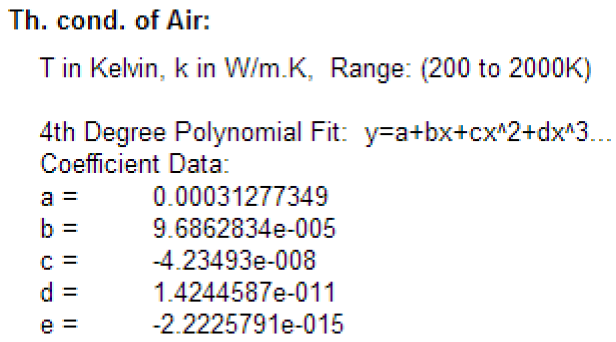


Density of Air:

T in Kelvin, rho in kg/m^3, Range: (200 to 2000K)

```
rho_Air(T) := | return "T must be between 200K and 2000K"  if T<200
              | return "T must be between 200K and 2000K"  if T>2000
              | otherwise
              |   | R ← 287
              |   | P ← 1.01325·10^5
              |   | rho ← P/(R·T)
```

Ex: rho_Air(500) = 0.706 kg/m^3

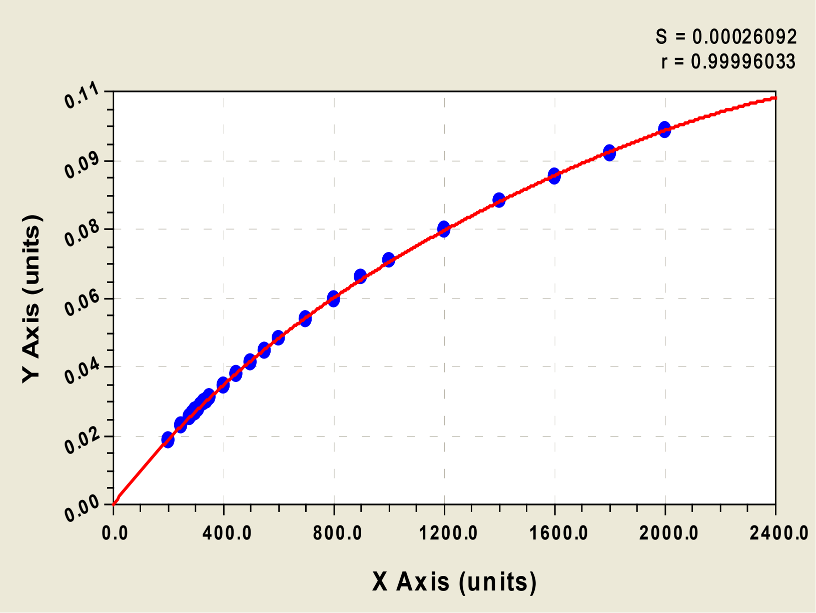


Th. cond. of Air:

T in Kelvin, k in W/m.K, Range: (200 to 2000K)

4th Degree Polynomial Fit: y=a+bx+cx^2+dx^3...
Coefficient Data:
a = 0.00031277349
b = 9.6862834e-005
c = -4.23493e-008
d = 1.4244587e-011
e = -2.2225791e-015

Above curve-fit coefficients are from CurveExpert software. Following is the graph of curve fit:



```
k_Air(T) :=  | return  "T must be between 200K and 2000K"  if  T<200
             | return  "T must be between 200K and 2000K"  if  T>2000
             | otherwise
             |    | a← 0.00031277349
             |    | b← 9.6862834·10^-5
             |    | c← - 4.23493·10^-8
             |    | d← 1.4244587·10^-11
             |    | e← - 2.2225791·10^-15
             |    | k← a + b·T + c·T^2 + d·T^3 + e·T^4
```

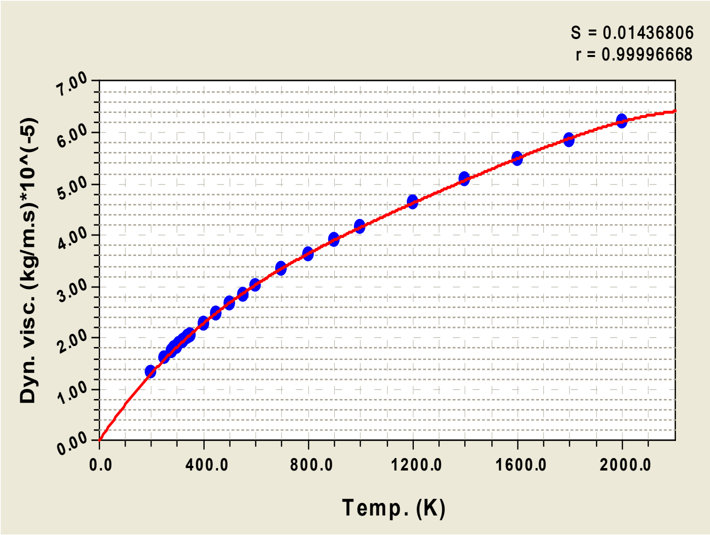
Ex: k_Air(500) = 0.04 W.m.K

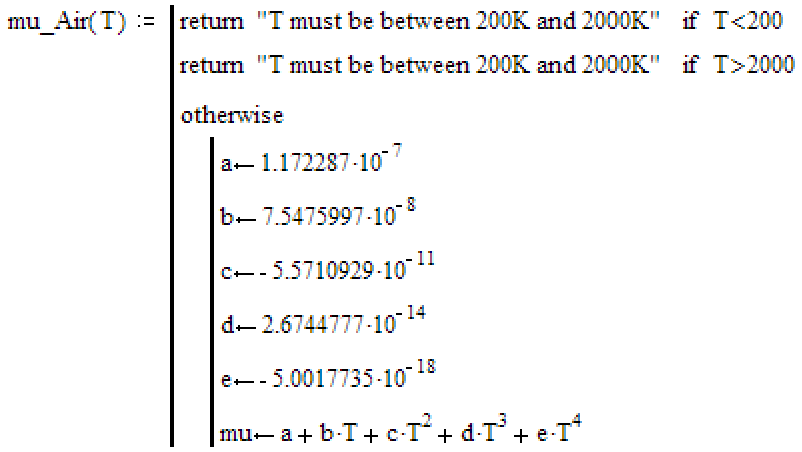
Dyn. Visc. of Air:

T in Kelvin, μ in kg/m.s, Range: (200 to 2000K)

4th Degree Polynomial Fit: y=a+bx+cx^2+dx^3...
Coefficient Data:
a = 1.172287e-007
b = 7.5475997e-008
c = -5.5710929e-011
d = 2.6744777e-014
e = -5.0017735e-018

Above curve-fit coefficients are from CurveExpert software. Following is the graph of curve fit:





```
mu_Air(T) := | return  "T must be between 200K and 2000K"  if  T<200
             | return  "T must be between 200K and 2000K"  if  T>2000
             | otherwise
             |   | a ← 1.172287·10^-7
             |   | b ← 7.5475997·10^-8
             |   | c ← -5.5710929·10^-11
             |   | d ← 2.6744777·10^-14
             |   | e ← -5.0017735·10^-18
             |   | mu ← a + b·T + c·T^2 + d·T^3 + e·T^4
```

Ex: mu_Air(550) $= 2.877 \cdot 10^{-5}$ kg/m.s

Temp (K) vs Prandtl No.

$$M := \begin{bmatrix} 100 & 0.786 \\ 150 & 0.758 \\ 200 & 0.737 \\ 250 & 0.720 \\ 300 & 0.707 \\ 350 & 0.700 \\ 400 & 0.69 \\ 450 & 0.686 \\ 500 & 0.684 \\ 550 & 0.683 \\ 600 & 0.685 \\ 650 & 0.69 \\ 700 & 0.695 \\ 750 & 0.702 \\ 800 & 0.709 \\ 850 & 0.716 \\ 900 & 0.72 \\ 950 & 0.723 \\ 1000 & 0.726 \end{bmatrix}$$

$$\left|\begin{matrix} 1100 & 0.728 \\ 1200 & 0.728 \\ 1300 & 0.719 \\ 1400 & 0.703 \\ 1500 & 0.685 \\ 1600 & 0.688 \\ 1700 & 0.685 \\ 1800 & 0.683 \\ 1900 & 0.677 \\ 2000 & 0.672 \end{matrix}\right]$$

Let: $Temp := M^{\langle 0 \rangle}$

$Prandtl := M^{\langle 1 \rangle}$

Then: $Pr\_Air(T) := linterp(Temp, Prandtl, T)$ ....gives Pr for given T(K)

Ex: $Pr\_Air(500) = 0.684$

Now, the Mathcad Solution:

Data:

$D := 0.006$ m $I := 50$ A $R_e := 0.002$ Ohm/m

$T_{inf} := 10$ C $U := \frac{40 \cdot 10^3}{3600}$ i.e. $U = 11.111$ m/s $L := 1$ m

Calculations:

$$T_f = \frac{T_s + T_{inf}}{2}$$ ....mean film temp in C

At mean film temp Tf, properties of Air and other parameters are determined from:

$$\mu = mu\_Air\left(\frac{T_s + T_{inf}}{2} + 273\right)$$

$$\rho = rho\_Air\left(\frac{T_s + T_{inf}}{2} + 273\right)$$

$$k=k\_Air\left(\frac{T_s+T_{inf}}{2}+273\right)$$

$$Pr=Pr\_Air\left(\frac{T_s+T_{inf}}{2}+273\right)$$

$$Re=\frac{D\cdot U\cdot rho\_Air\left(\frac{T_s+T_{inf}}{2}+273\right)}{mu\_Air\left(\frac{T_s+T_{inf}}{2}+273\right)} \qquad \text{...Reynolds No.}$$

Nusselt No. by Churchill and Bernstein eqn:

$$Nu_{cyl}:=0.3+\frac{0.62\cdot Re_D{}^{0.5}\cdot Pr^{\frac{1}{3}}}{\left[1+\left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}}\cdot\left[1+\left(\frac{Re_D}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$

Then, heat tr. coeff. h is given by:

$$h:=\frac{k\cdot Nu_{cyl}}{D}$$

And, heat transfer Q is given by:

$$Q:=h\cdot A\cdot\left(T_s-T_{inf}\right)$$

But, Q should be equal to Joule heat produced in the cable:

Q = I^2. R

These eqns are entered in the Solve block of Mathcad.

Start with a trial (guess) value for surface temp, Ts:

$T_s:=50$ ...guess value:

Given

$$I^2 \cdot (R_e \cdot L) = (\pi \cdot D \cdot L) \cdot (T_s - T_{inf}) \cdot \left[\frac{k\_Air\left(\frac{T_s + T_{inf}}{2} + 273\right)}{D} \cdot \left[0.3 + \frac{0.62 \cdot \left[\frac{D \cdot U \cdot rho\_Air\left(\frac{T_s + T_{inf}}{2} + 273\right)}{mu\_Air\left(\frac{T_s + T_{inf}}{2} + 273\right)} \right]^{0.5} \cdot Pr\_Air\left(\frac{T_s + T_{inf}}{2} + 273\right)^{\frac{1}{3}}}{\left[1 + \left(\frac{0.4}{Pr\_Air\left(\frac{T_s + T_{inf}}{2} + 273\right)} \right)^{\frac{2}{3}} \right]^{\frac{1}{4}}} \cdot \left[1 + \left[\frac{\frac{D \cdot U \cdot rho\_Air\left(\frac{T_s + T_{inf}}{2} + 273\right)}{mu\_Air\left(\frac{T_s + T_{inf}}{2} + 273\right)}}{282000} \right]^{\frac{5}{8}} \right]^{\frac{4}{5}} \right] \right]$$

$$T_{surface}(U) := Find(T_s)$$

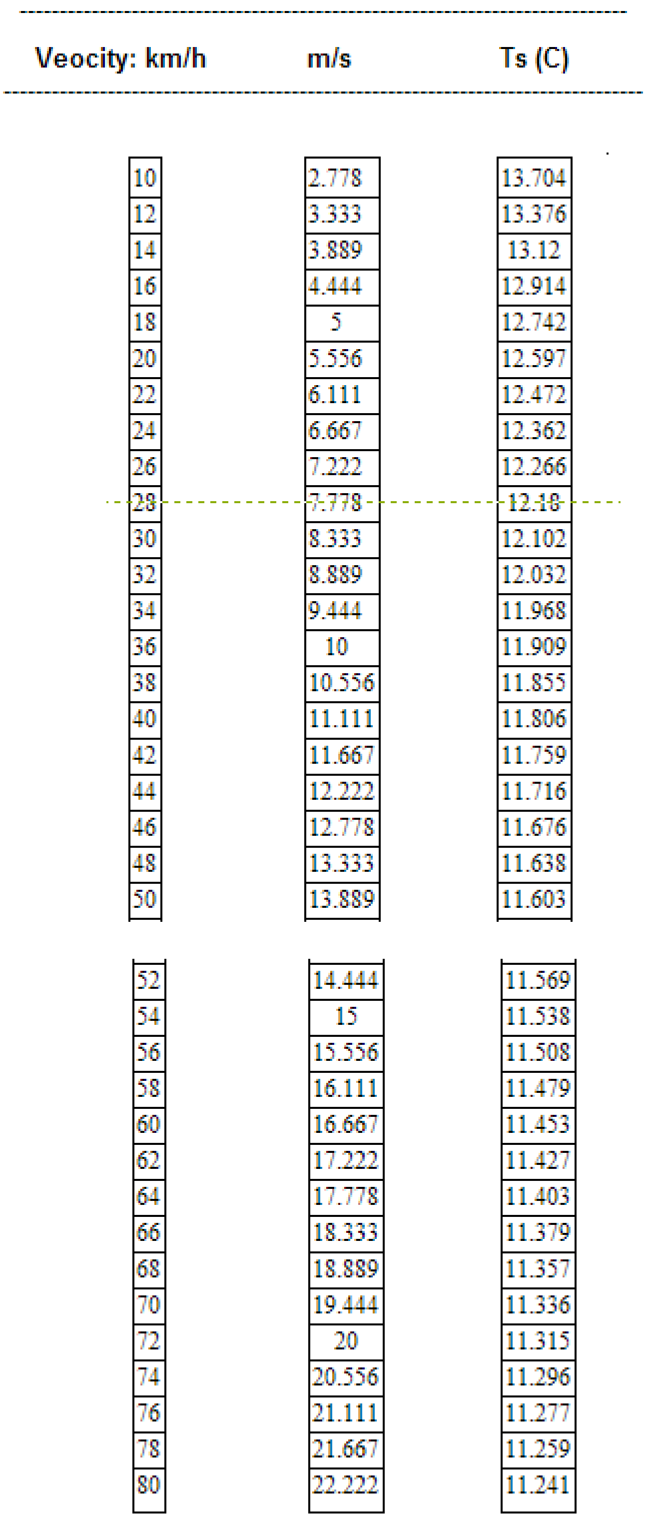
i.e.

$T_{surface}(U) = 11.806$ C...at $U = 11.111$ m/s

Note that, in the above, $T_{surface}$ is written as a function of Air velocity, U, so that plot of Ts vs U can be drawn easily:

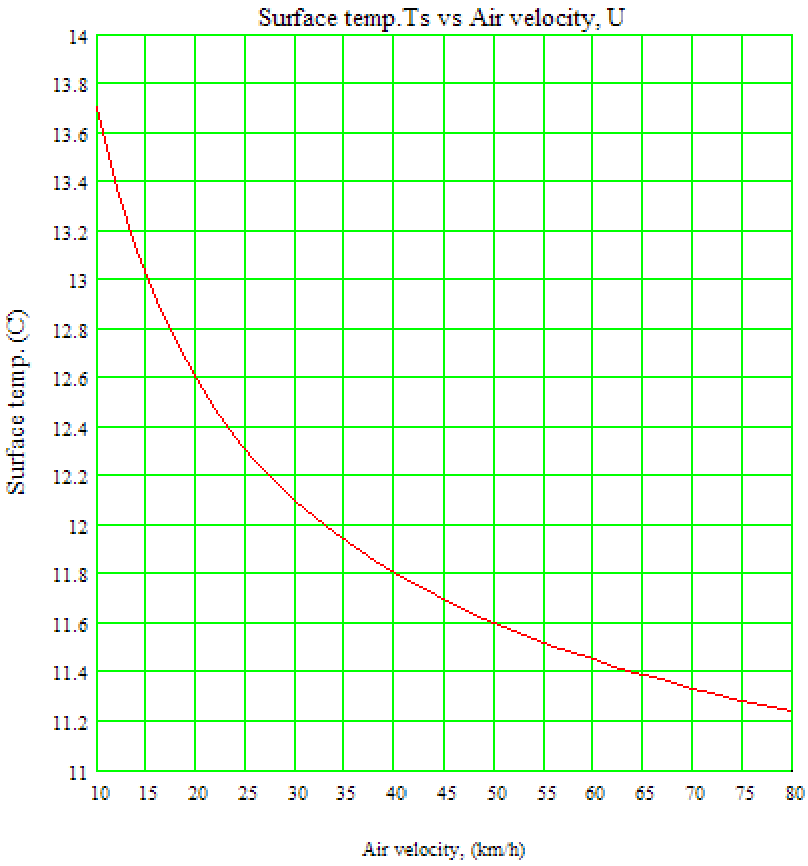
To draw graph of T_surface against U:

$U := 10, 12 .. 80$ ..km/h



Veocity: km/h	m/s	Ts (C)
10	2.778	13.704
12	3.333	13.376
14	3.889	13.12
16	4.444	12.914
18	5	12.742
20	5.556	12.597
22	6.111	12.472
24	6.667	12.362
26	7.222	12.266
28	7.778	12.18
30	8.333	12.102
32	8.889	12.032
34	9.444	11.968
36	10	11.909
38	10.556	11.855
40	11.111	11.806
42	11.667	11.759
44	12.222	11.716
46	12.778	11.676
48	13.333	11.638
50	13.889	11.603
52	14.444	11.569
54	15	11.538
56	15.556	11.508
58	16.111	11.479
60	16.667	11.453
62	17.222	11.427
64	17.778	11.403
66	18.333	11.379
68	18.889	11.357
70	19.444	11.336
72	20	11.315
74	20.556	11.296
76	21.111	11.277
78	21.667	11.259
80	22.222	11.241

Now, draw the plot:



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$UnitSystem SI C Pa J

"**Prob. 2A1.2.6.** A 25 mm dia high tension line has an electrical resistance of 10^-4 Ohm/m and is transmitting a current of 1000 A. If the ambient air is at 10 C, and is flowing at 5 m/s, what is the surface temp?

Plot the variation of surface temp for air velocities varying from 1 m/s to 10 m/s"

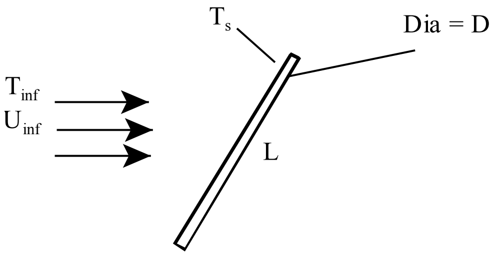
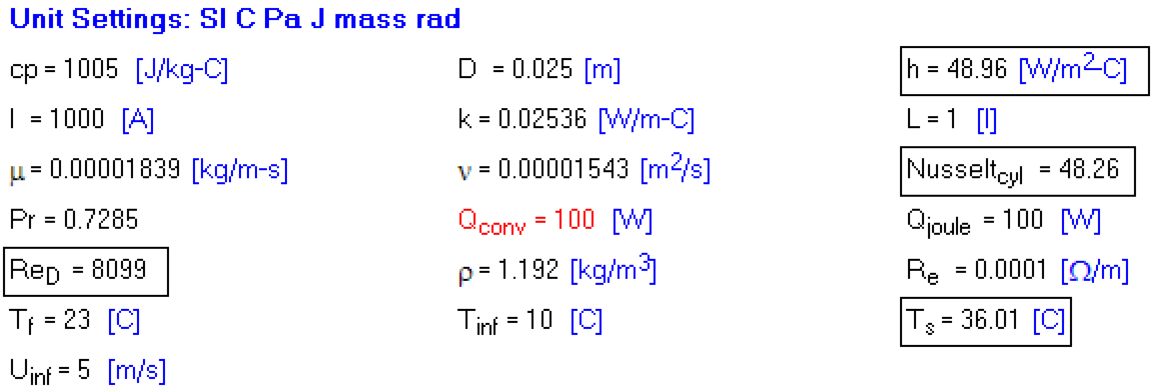


Fig.Prob.2A1.2.6

EES Solution:

```
"Data:"

D = 0.025[m]
L = 1 [l]
R_e = 1E-04 [Ohm/m]
I = 1000 [A]
{T_s = 127[C]}
T_inf = 10[C]
U_inf = 5[m/s]
T_f = (T_s+T_inf)/2 "[C]… mean film temp."

"Properties of Air:"

mu=Viscosity(Air,T=T_f) "[kg/m-s]"
rho=Density(Air,T=T_f,P=1.013e05) "[kg/m^3]"
nu = mu/rho "[m^2/s]"
cp=Cp(Air,T=T_f) "[J/kg-C]"
k=Conductivity(Air,T=T_f) "[W/m-C]"
Pr = mu*cp/k

"Calculations:"

Re_D = D * U_inf * rho / mu "finds Reynolds No., Re_D"

"This is a cylinder in cross flow. Use Churchill – Bernstein eqn."

Nusselt_cyl = 0.3 + ((0.62 * Re_D^0.5 * Pr^(1/3)) / (1 + (0.4 / Pr)^(2/3))^(1/4)) * (1 +
(Re_D/282000)^(5/8))^(4/5) "…finds Nusselts No."

Nusselt_cyl = h * D / k "finds h"
Q_conv = h * (pi * D * L) * (T_s-T_inf) "W"
Q_joule = I^2 * (R_e * L) "[W] …Joule heating ..by data"
Q_joule = Q_conv "…by heat balance"
```

Results:



Unit Settings: SI C Pa J mass rad

$cp = 1005$ [J/kg-C]

$I = 1000$ [A]

$\mu = 0.00001839$ [kg/m-s]

$Pr = 0.7285$

$\boxed{Re_D = 8099}$

$T_f = 23$ [C]

$U_{inf} = 5$ [m/s]

$D = 0.025$ [m]

$k = 0.02536$ [W/m-C]

$\nu = 0.00001543$ [m²/s]

$Q_{conv} = 100$ [W]

$\rho = 1.192$ [kg/m³]

$T_{inf} = 10$ [C]

$\boxed{h = 48.96}$ [W/m²-C]

$L = 1$ [l]

$\boxed{Nusselt_{cyl} = 48.26}$

$Q_{joule} = 100$ [W]

$R_e = 0.0001$ [Ω/m]

$\boxed{T_s = 36.01}$ [C]

Thus:

Surface temp of high tension line = T_s = 36.01 C … Ans.

To plot T_s against wind velocity, U_inf:

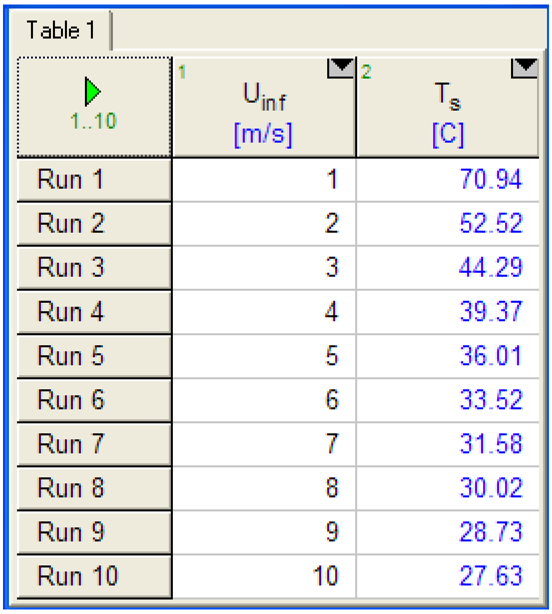
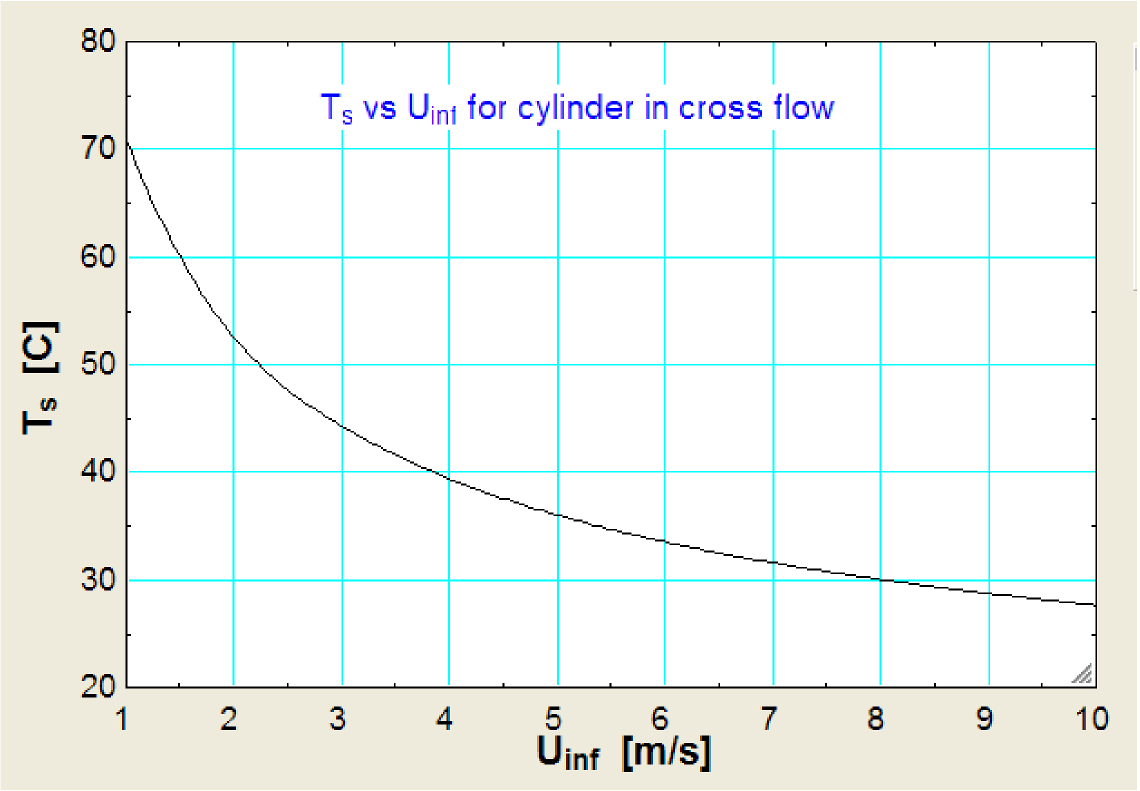
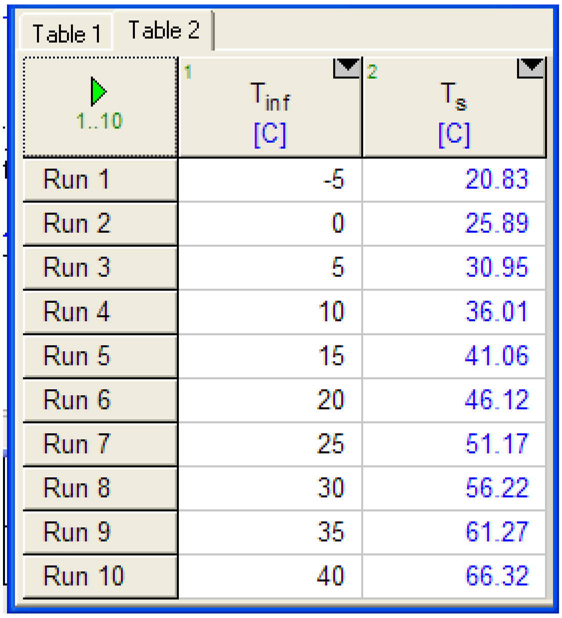


Table 1

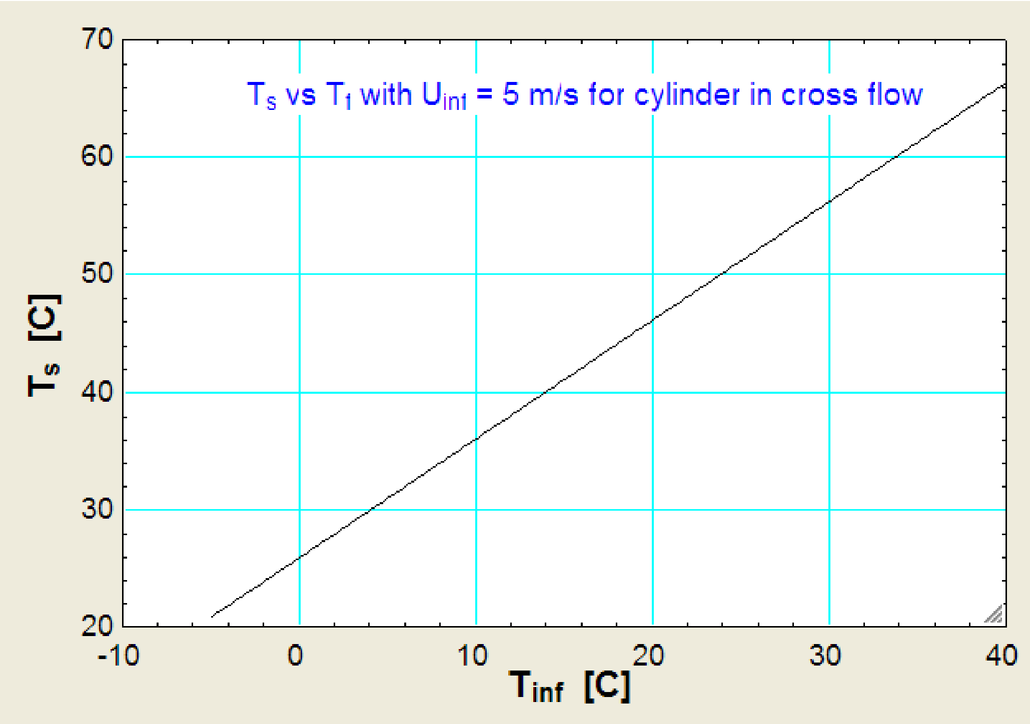
1..10	1 U_{inf} [m/s]	2 T_s [C]
Run 1	1	70.94
Run 2	2	52.52
Run 3	3	44.29
Run 4	4	39.37
Run 5	5	36.01
Run 6	6	33.52
Run 7	7	31.58
Run 8	8	30.02
Run 9	9	28.73
Run 10	10	27.63



To plot T_s against wind temp, T_inf, with U_inf = 5 m/s:



1..10	1 T_{inf} [C]	2 T_s [C]
Run 1	-5	20.83
Run 2	0	25.89
Run 3	5	30.95
Run 4	10	36.01
Run 5	15	41.06
Run 6	20	46.12
Run 7	25	51.17
Run 8	30	56.22
Run 9	35	61.27
Run 10	40	66.32



Note: EES is ideally suited to do these calculations since as ambient temp T_inf changes, T_f will also change and in each trial properties of air are calculated at the corresponding T_f by the built-in functions of EES.

===

$UnitSystem SI C Pa J

"**Prob. 2A1.2.7.** Consider a 100 W incandescent lamp of 10cm dia spherical shape, with a surface emissivity of 0.85. It is subjected to an air stream at 20 C, flowing at a velocity of 2 m/s. What will be the glass surface temp?

Plot the graph of surface temp against air velocities from 0.5 m/s to 5 m/s."

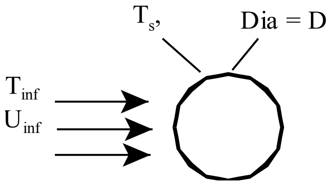
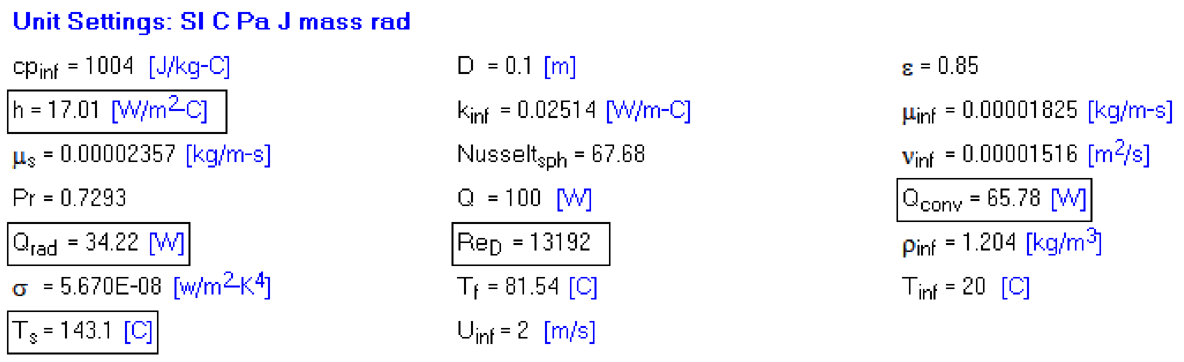


Fig.Prob.2A1.2.7

EES Solution:

```
“Data:”

D = 0.1[m]
{T_s = 127[C]}
T_inf = 20[C]
U_inf = 2[m/s]
epsilon = 0.85 “…emissivity”
sigma = 5.67e-08 [w/m^2-K^4]”….Stefan – Boltzmann const.”
T_f = (T_s+T_inf)/2 “[C]… mean film temp.”

“Properties of Air:”

mu_inf=Viscosity(Air,T=T_inf) “[kg/m-s]”
mu_s=Viscosity(Air,T=T_s) “[kg/m-s]”

rho_inf=Density(Air,T=T_inf,P=1.013e05) “[kg/m^3]”
nu_inf = mu_inf/rho_inf “[m^2/s]”
cp_inf=Cp(Air,T=T_inf) “[J/kg-C]”
k_inf=Conductivity(Air,T=T_inf) “[W/m-C]”
Pr = mu_inf*cp_inf/k_inf
Q = 100 [W]

“Calculations:”

Re_D = D * U_inf * rho_inf / mu_inf “finds Reynolds No., Re_D”

“This is a sphere in cross flow. Use Whitaker eqn.
Here, fluid prop. are taken at free stream temp T_inf except for mu_s which is evaluated at surface temp T_s.
Valid for 3.5 < Re_D < 80000 and 0.7 < Pr < 380”

Nusselt_sph = 2 + ( 0.4 * Re_D^0.5 + 0.06 * Re_D^(2/3)) * Pr^0.4 * (mu_inf / mu_s)^(1/4) “…finds Nusselts No.”

Nusselt_sph = h * D / k_inf “finds h”
Q_conv = h * (pi * D^2) * (T_s-T_inf) “W”
Q_rad = sigma * epsilon * (pi * d^2) * ((T_s + 273)^4 – (T_inf + 273)^4) “[W] …radition heat transfer.”
Q = Q_conv + Q_rad “…by heat balance”
```

Results:



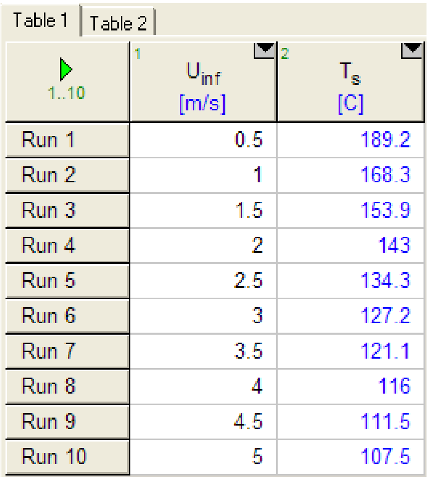
Unit Settings: SI C Pa J mass rad

cp_{inf} = 1004 [J/kg-C]
D = 0.1 [m]
ε = 0.85
$\boxed{h = 17.01\ [W/m^2\text{-}C]}$
k_{inf} = 0.02514 [W/m-C]
μ_{inf} = 0.00001825 [kg/m-s]
μ_s = 0.00002357 [kg/m-s]
$Nusselt_{sph}$ = 67.68
ν_{inf} = 0.00001516 [m^2/s]
Pr = 0.7293
Q = 100 [W]
$\boxed{Q_{conv} = 65.78\ [W]}$
$\boxed{Q_{rad} = 34.22\ [W]}$
$\boxed{Re_D = 13192}$
ρ_{inf} = 1.204 [kg/m^3]
σ = 5.670E-08 [W/m^2-K^4]
T_f = 81.54 [C]
T_{inf} = 20 [C]
$\boxed{T_s = 143.1\ [C]}$
U_{inf} = 2 [m/s]

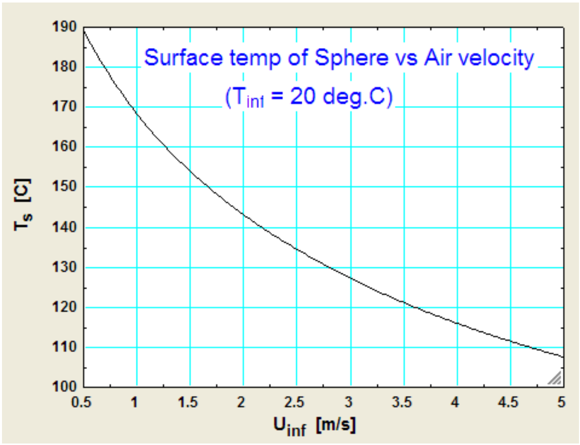
Thus:

The glass surface temp = T_s = 143.1 C … Ans.

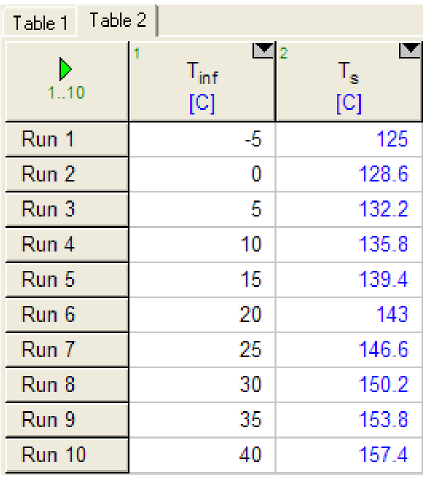
Plot T_s vs Air velocity, U_inf:



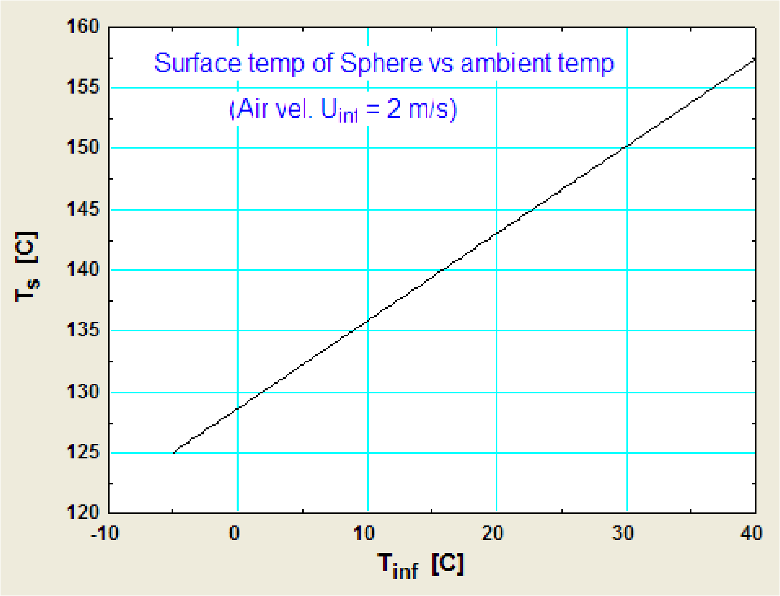
1..10	1 U_{inf} [m/s]	2 T_s [C]
Run 1	0.5	189.2
Run 2	1	168.3
Run 3	1.5	153.9
Run 4	2	143
Run 5	2.5	134.3
Run 6	3	127.2
Run 7	3.5	121.1
Run 8	4	116
Run 9	4.5	111.5
Run 10	5	107.5



Plot T_s vs Ambient temp, T_inf:



1..10	1 T_{inf} [C]	2 T_s [C]
Run 1	-5	125
Run 2	0	128.6
Run 3	5	132.2
Run 4	10	135.8
Run 5	15	139.4
Run 6	20	143
Run 7	25	146.6
Run 8	30	150.2
Run 9	35	153.8
Run 10	40	157.4



==

Prob. 2A1.2.8. Consider an un-insulated steam pipe, 0.5 m dia, with surface temp of 150 C and exposed to ambient air at -10 C, air moving in cross flow at a velocity of 5 m/s.

a) What is the heat lost per unit length of pipe, without considering the radiation heat loss?
b) What is the heat lost per unit length of pipe, considering the radiation heat loss? Take emissivity of surface = 0.9, radiation to an atmosphere of -10 C.
c) Plot the heat loss per unit length for different velocities, say, from U = 0.5 m/s to U = 5 m/s. while the ambient temp varies from -10 C to 20 C.

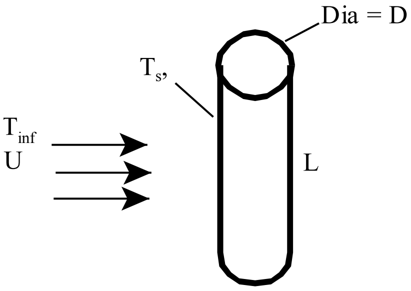


Fig.Prob.2A1.2.8

Mathcad Solution:

Data:

$D := 0.5$ m $L := 1$ m $T_s := 150$ C

$T_{inf} := -10$ C $U := 5$ m/s

$\varepsilon := 0.9$ ...emissivity $\sigma := 5.67 \cdot 10^{-8}$ W/m^2.K^4..... Stefan-Boltzmann const.

Calculations:

Case 1: Without including radiation loss, i.e. considering only the heat loss by forced convection:

$T_f := \dfrac{T_s + T_{inf}}{2}$ ....mean film temp in C

i.e. $T_f = 70$ C

To find Q1: first find h1, heat transfer coeff for case 1:

Use the Mathcad functions, already written, (see Prob. 2A1.2.5) to get properties of Air to calculate Reynolds No., Nusselts No. etc.

Properties of Air at T_f = 70 C:

$\rho := \mathrm{rho\_Air}(T_f + 273)$ i.e. $\rho = 1.029$ kg/m^3

$k := \mathrm{k\_Air}(T_f + 273)$ i.e. $k = 0.029$ W/m.K

$\mu := \mathrm{mu\_Air}(T_f + 273)$ i.e. $\mu = 2.046 \cdot 10^{-5}$ kg/m.s

$Pr := \mathrm{Pr\_Air}(T_f + 273)$ i.e. $Pr = 0.701$ ...Prandtl No.

Now: $Re := \dfrac{D \cdot U \cdot \rho}{\mu}$

i.e $Re = 1.258 \cdot 10^5$ ....Reynolds No.

Then, use Churchill – Bernstein eqn. to calculate Nusselts No.:

$$Nu_{cyl} := 0.3 + \frac{0.62 \cdot Re^{0.5} \cdot (Pr)^{\frac{1}{3}}}{\left[1+\left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}} \cdot \left[1+\left(\frac{Re}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$

i.e. $Nu_{cyl} = 250.329$

Then, heat transfer coeff. is given by:

$$h := \frac{Nu_{cyl} \cdot k}{D}$$

i.e. $h = 14.568$ W/m^2.C .... Ans.

And, heat transfer rate, Q1 is:

$Q_1 := h \cdot (\pi \cdot D \cdot L) \cdot (T_s - T_{inf})$ ... W

i.e. $Q_1 = 3.661 \cdot 10^3$ **W.....heat transfer, without considering the radiation heat loss ... Ans.**

Case 2: When radiation heat loss from surface of pipe is considered:

Let the heat loss rate be Q2:

Then,

$Q_2 = Q_1 + Q_{rad}$ where Q1 is the convection heat loss, already calculated, and Qrad is the radiation heat loss, given by:

$Q_{rad} := \sigma \cdot \varepsilon \cdot (\pi \cdot D \cdot L) \cdot \left[(T_s + 273)^4 - (T_{inf} + 273)^4 \right]$ W

i.e. $Q_{rad} = 2.183 \cdot 10^3$ **W... Note that radiation heat loss is considerable as compared to convection heat loss.**

Therefore: $Q_2 := Q_1 + Q_{rad}$

i.e. $Q_2 = 5.844 \cdot 10^3$ **W.... total heat loss ... Ans.**

To Plot total heat loss, Q_{tot} against variation of Air velocity, U, for different values of Air temp, T_{inf}:

Write all quantities as functions of U and T_{inf}:

$T_f(T_{inf}) := \dfrac{T_s + T_{inf}}{2}$ ....mean film temp as a function of T_inf

$Re(U, T_{inf}) := \dfrac{D \cdot U \cdot rho\_Air(T_f(T_{inf}) + 273)}{mu\_Air(T_f(T_{inf}) + 273)}$ ...Re as a function of U and T_inf

$$\mathrm{Nu}_{cyl}(U, T_{inf}) := 0.3 + \frac{0.62 \cdot \mathrm{Re}(U, T_{inf})^{0.5} \cdot \left(\mathrm{Pr\_Air}(T_f(T_{inf}) + 273)\right)^{\frac{1}{3}}}{\left[1 + \left(\frac{0.4}{\mathrm{Pr\_Air}(T_f(T_{inf}) + 273)}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}} \cdot \left[1 + \left(\frac{\mathrm{Re}(U, T_{inf})}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$

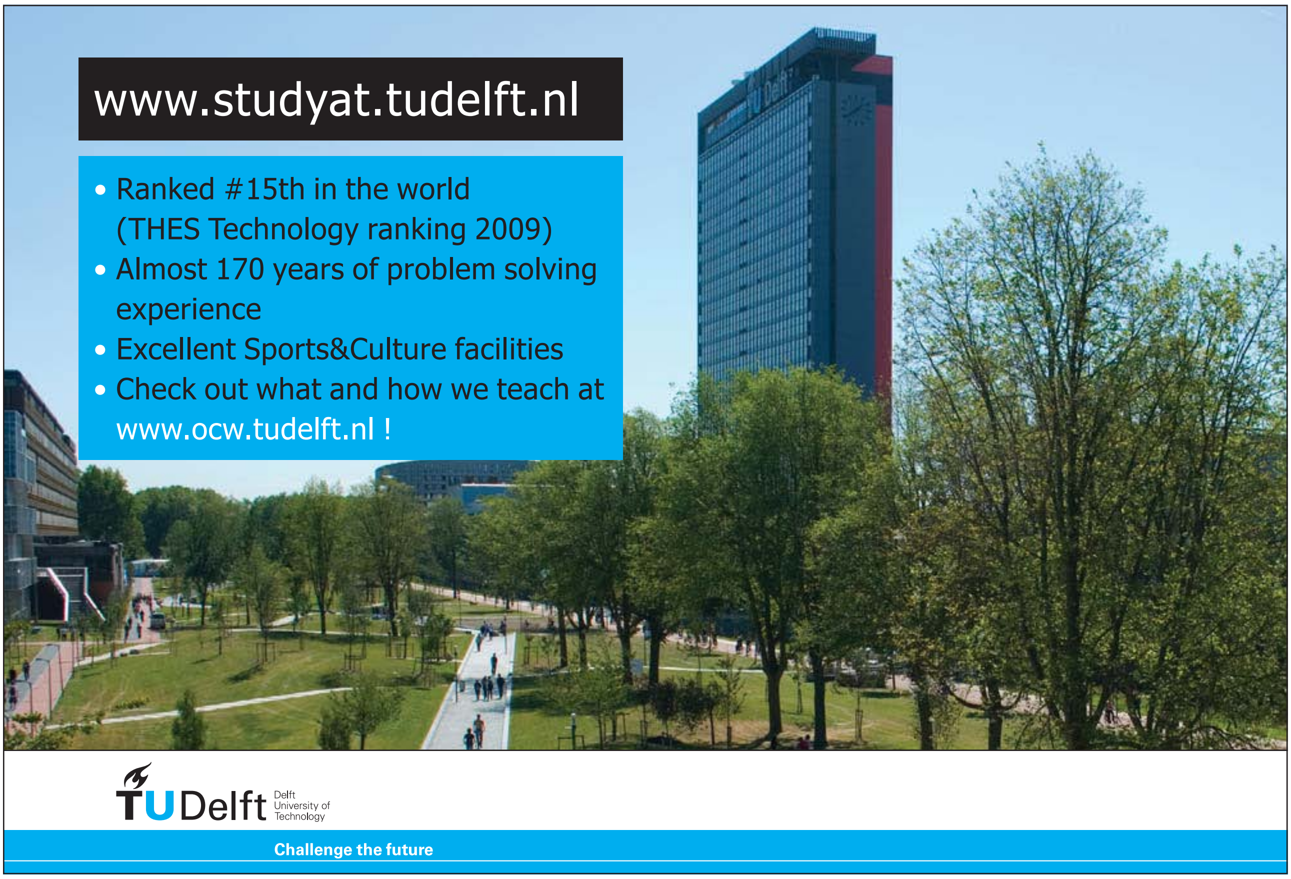
$$h(U, T_{inf}) := \frac{\mathrm{Nu}_{cyl}(U, T_{inf}) \cdot \mathrm{k\_Air}(T_f(T_{inf}) + 273)}{D}$$ ..heat tr. coeff. as a function of U and T_inf

$$Q_{conv}(U, T_{inf}) := h(U, T_{inf}) \cdot (\pi \cdot D \cdot L) \cdot (T_s - T_{inf})$$ .W ..Q_conv as a function of U and T_inf

$$Q_{rad}(T_{inf}) := \sigma \cdot \varepsilon \cdot (\pi \cdot D \cdot L) \cdot \left[(T_s + 273)^4 - (T_{inf} + 273)^4\right]$$. W..Q_rad as a function of U and T_inf

Therefore:

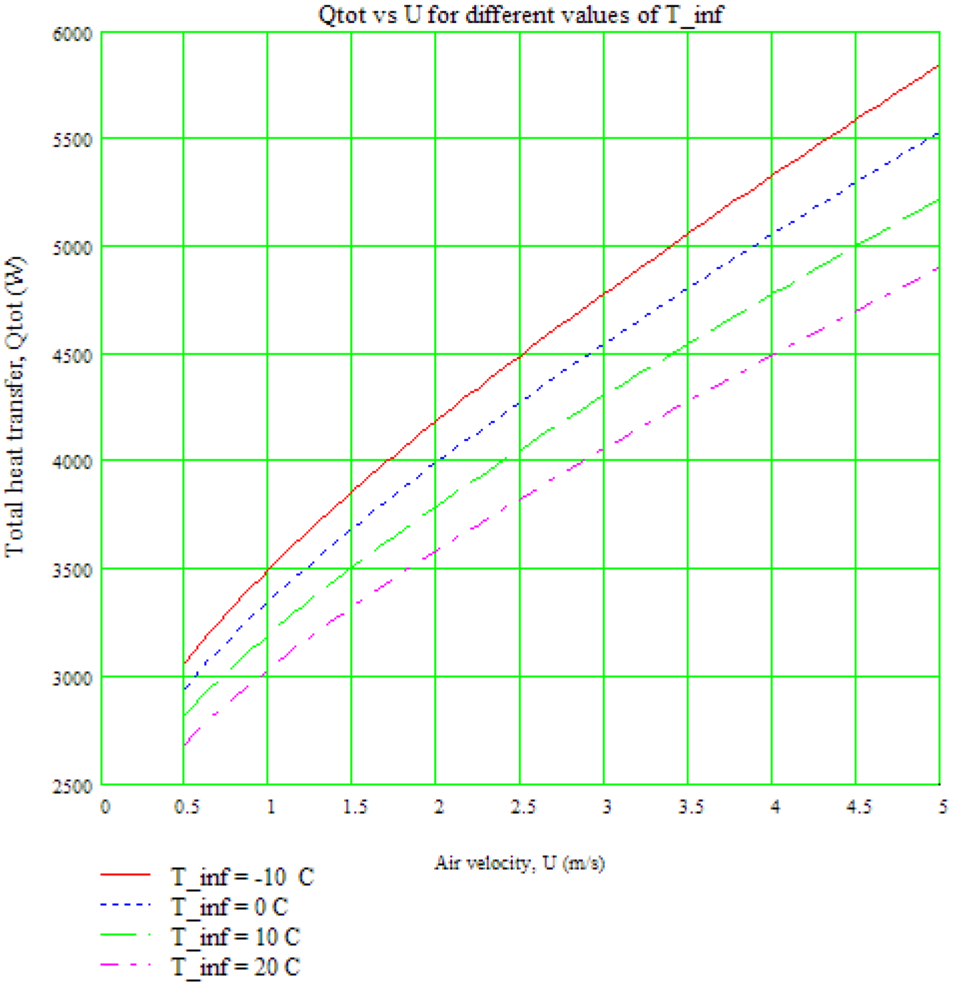
$$Q_{tot}(U, T_{inf}) := Q_{conv}(U, T_{inf}) + Q_{rad}(T_{inf})$$ W..Total heat tr.as a function of U and T_inf



Now, let us plot the graph of Q_{tot} vs U for different values of T_{inf}:

U := 0.5, 0.55.. 5 ..let U vary from 0.5 to 5 m/s

Qtot vs U for different values of T_inf

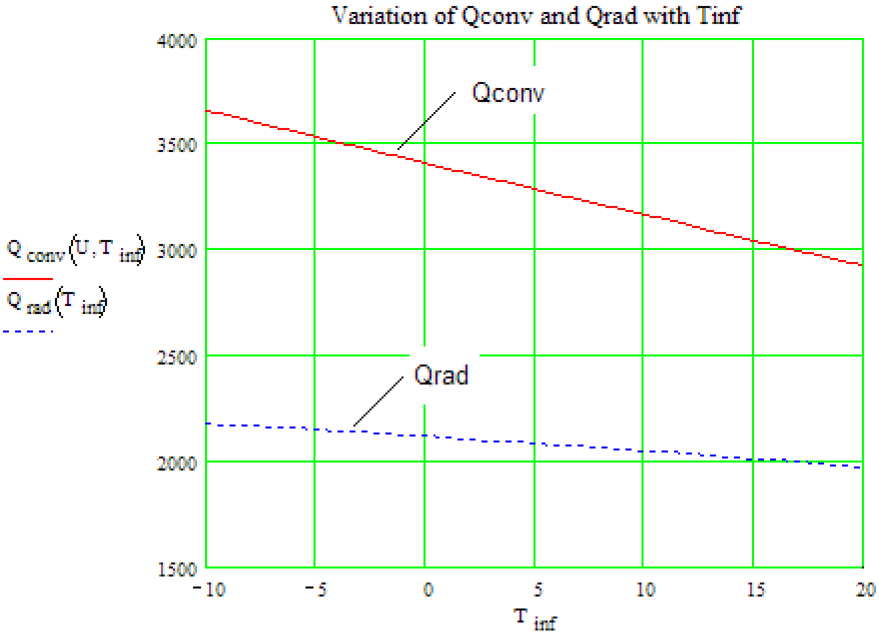


At a given air velocity, say, U = 5 m/s, how do Q_{conv} and Q_{rad} vary with T_{inf}?

We have: U := 5 m/s T_s = 150 C... cylinder surface temp.

Let: T_{inf} := - 10, - 9.. 20 ...T_inf varying from -10 C to 20 C

Note: In the graph below Q_{conv} and Q_{rad} in W, and T_{inf} in deg. C



===

Prob. 2A1.2.9: Air at 20 C flows across an elliptical tube 6 cm × 12 cm size, perpendicular to the minor axis with a velocity of 2 m/s. Tube surface is maintained at 60 C. Determine the value of convection coefficient.

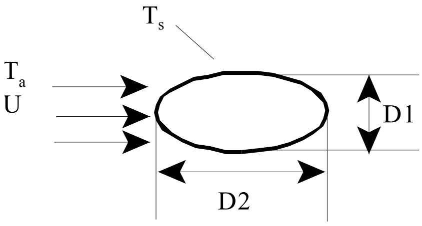


Fig.Prob.2A1.2.9

Mathcad Solution:

Data:

$T_s := 60$ C....tube surface temp.

$T_a := 20$ C...air temperature

$U := 2$ m/s....velocity of air

$D1 := 0.06$ m....minor axis of elliptical tube

$D2 := 0.12$ m....major axis of elliptical tube

$T_f := \frac{T_a + T_s}{2}$ i.e. $T_f = 40$ C...film temperature

Properties at Tf: Use the Mathcad functions for Air, already written. (See Prob. 2A1.2.5)

$\nu := nu\_Air(T_f + 273)$ $\nu = 1.689 \cdot 10^{-5}$ m^2/s....kinematic viscosity

$k := k\_Air(T_f + 273)$ $k = 0.027$ W/(m.K)

$Pr := Pr\_Air(T_f + 273)$ $Pr = 0.705$ ....Prandtl No.

Reynolds number:

See Table in Prob. 2A1.2.3 for the case of flow across an ellipse.

$Re := \frac{U \cdot D1}{\nu}$ i.e. $Re = 7.103 \cdot 10^3$

Then, we use the eqn. .

$$Nu = C \cdot Re^n \cdot Pr^{\frac{1}{3}}$$

where:

$C := 0.248$ and, $n := 0.612$

Therefore, $Nu := C \cdot Re^n \cdot Pr^{\frac{1}{3}}$

i.e. $Nu = 50.231$ ...Nusselts No.

Heat transfer coefficient:

Therefore, $h := \frac{Nu \cdot k}{D1}$

i.e. $h = 22.518$ **W/(m².K)...heat transfer coefficient...Ans.**

==

Prob.2A1.2.10. A long, 8 cm dia steam pipe has its external temp as 90 C and is passing through a room where the air is at 7 C and blowing at 50 km/h. Determine the rate of heat loss per unit length of pipe.

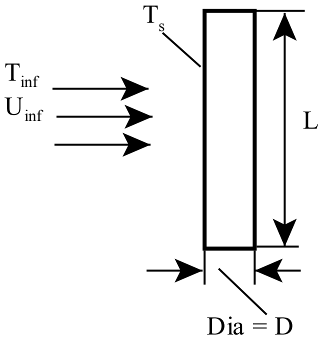


Fig.Prob.2A1.2.10

EES Solution:

Here, this is the case of cross flow across a cylinder. We use the Churchill-Bernstein eqn.

Now, when the eqn is complicated or the same type of problem has to be solved again and again, it is convenient to write a 'sub-routine' which can be called from the main program.

In EES, sub-routine is called as PROCEDURE.

While writing a PROCEDURE remember:

a) PROCEDURE must be placed in the beginning of the worksheet.
b) PROCEDURE should be given a name and then, within brackets, supply the Input parameters and output results. Input and outputs are separated by a colon.
c) While in a usual EES program, we can write the eqns in any order, in a PROCEDURE, we should write them *sequentially*. The reason is: in ordinary EES program all eqns are solved simultaneously, whereas in a PROCEDURE, the eqns are solved sequentially.
d) The assignment statement is ‘:= ‘, and *not* ’=’
e) Mention units of input and output variables, and add few comments for clarity

Therefore, first, Let us write an EES PROCEDURE to solve such a problem (i.e. cross flow of air on a cylinder), using Churchill-Bernstein eqn:

```
PROCEDURE ForcedConv_Air_AcrossCylinder (P_infinity, T_infinity, U_infinity, L, D, T_s: Re_D, Nusselt_D_bar, h_bar, Q)

“Ref: Incropera, 5th Ed. pp. 411, Eqn. (7.57)”
“Churchill and Bernstein eqn….for entire range of Re_D and a wide range of Pr”

“Finds various quantities for flow of Air across a cylinder:”
“Inputs: Pa, C, m/s, m”
“Outputs: W/m^2.C, W, W”

T_f := (T_infinity + T_s)/2 “mean film temp, C”

“Properties of Air (Ideal gas) at T_f:”

rho:=Density(Air,T=T_f,P=P_infinity)
{cp:=Cp(Air,T=T_f)}
mu:=Viscosity(Air,T=T_f)
k:=Conductivity(Air,T=T_f)
Pr:=Prandtl(Air,T=T_f)
cp:=SpecHeat(Air,T=T_f)

Re_D := D * U_infinity * rho/mu “Finds Reynolds No.”
“To find h accurately: Use Churchill and Bernstein eqn.”

Nusselt_D_bar := 0.3 + ((0.62 * Re_D^0.5 * (Pr)^(1/3))/(1 + (0.4/Pr)^(2/3))^(1/4)) * (1 + (Re_D/282000)^(5/8))^(4/5)
```

```
h_bar :=Nusselt_D_bar * k / D "Finds h_bar"
Q := h_bar * (pi * D * L) * (T_s – T_infinity) "W.... heat tr"

END
```

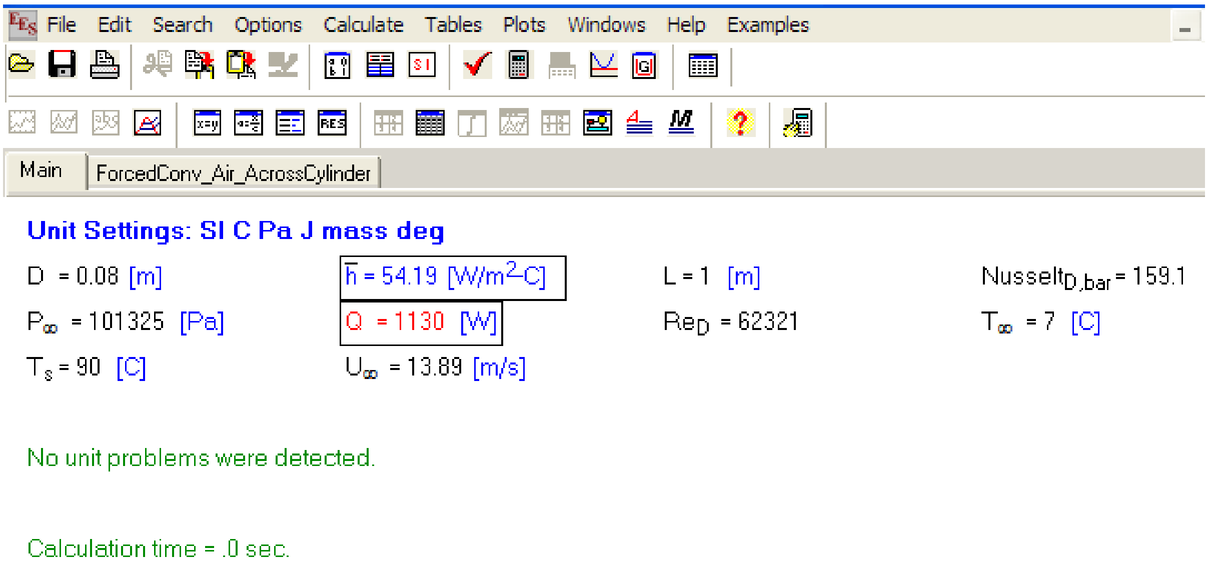
Now, to use the above PROCEDURE, in the main EES program, below the PROCEDURE, enter the data, and then simply CALL this PROCEDURE. This is shown below:

```
"Data:"

P_infinity = 1.01325E05 [Pa]
T_infinity = 7 [C]
U_infinity = 50 * convert(km/h,m/s) "[m/s] ...air velocity"
L = 1 [m]
D = 0.08 [m]
T_s = 90 [C]

CALL ForcedConv_Air_AcrossCylinder (P_infinity, T_infinity, U_infinity, L, D, T_s: Re_D, Nusselt_D_bar, h_bar, Q)
```

Now, simply press F2 and the solution appears in two windows: one, results of Main program, and the second, results of PROCEDURE:

See the Results below:

Results of Main program:



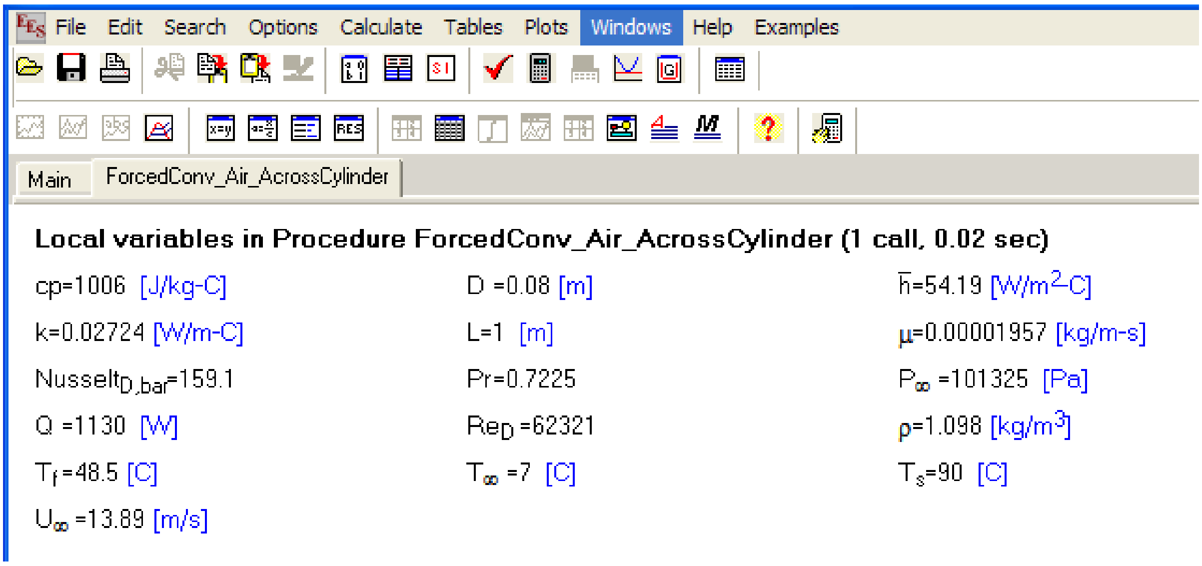
Unit Settings: SI C Pa J mass deg

D = 0.08 [m]
$\bar{h}$ = 54.19 [W/m^2-C]
L = 1 [m]
$Nusselt_{D,bar}$ = 159.1
P_∞ = 101325 [Pa]
Q = 1130 [W]
Re_D = 62321
T_∞ = 7 [C]
T_s = 90 [C]
U_∞ = 13.89 [m/s]

No unit problems were detected.

Calculation time = .0 sec.

And, Results of PROCEDURE:



```
Local variables in Procedure ForcedConv_Air_AcrossCylinder (1 call, 0.02 sec)
cp=1006  [J/kg-C]          D =0.08 [m]             h̄=54.19 [W/m2-C]
k=0.02724 [W/m-C]          L=1  [m]                μ=0.00001957 [kg/m-s]
NusseltD,bar=159.1         Pr=0.7225               P∞ =101325  [Pa]
Q =1130  [W]               ReD =62321              ρ=1.098 [kg/m3]
Tf=48.5 [C]                T∞ =7  [C]              Ts=90  [C]
U∞ =13.89 [m/s]
```

Thus, Q = 1130 W, with h_bar = 54.19 W/m^2.C … Ans.

Note that EES program is very useful when complicated eqns are involved, and, particularly when many property values have to be determined since EES has built-in Functions for properties of many substances.

To give one more example of using this PROCEDURE, let us visit Prob.2A1.2.1 again:

Prob.2A1.2.11.

"Assuming that a man can be represented by a cylinder 30 cm in diameter and 1.7 m high with a surface temp of 30 C, calculate the heat he would lose while standing in a 36 km/h wind at 10 C. [VTU – Dec.06–Jan.07]"

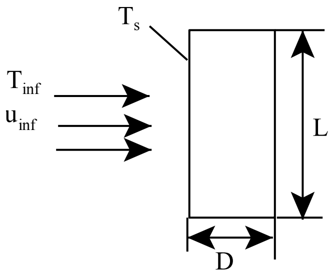


Fig.Prob.2A1.2.11

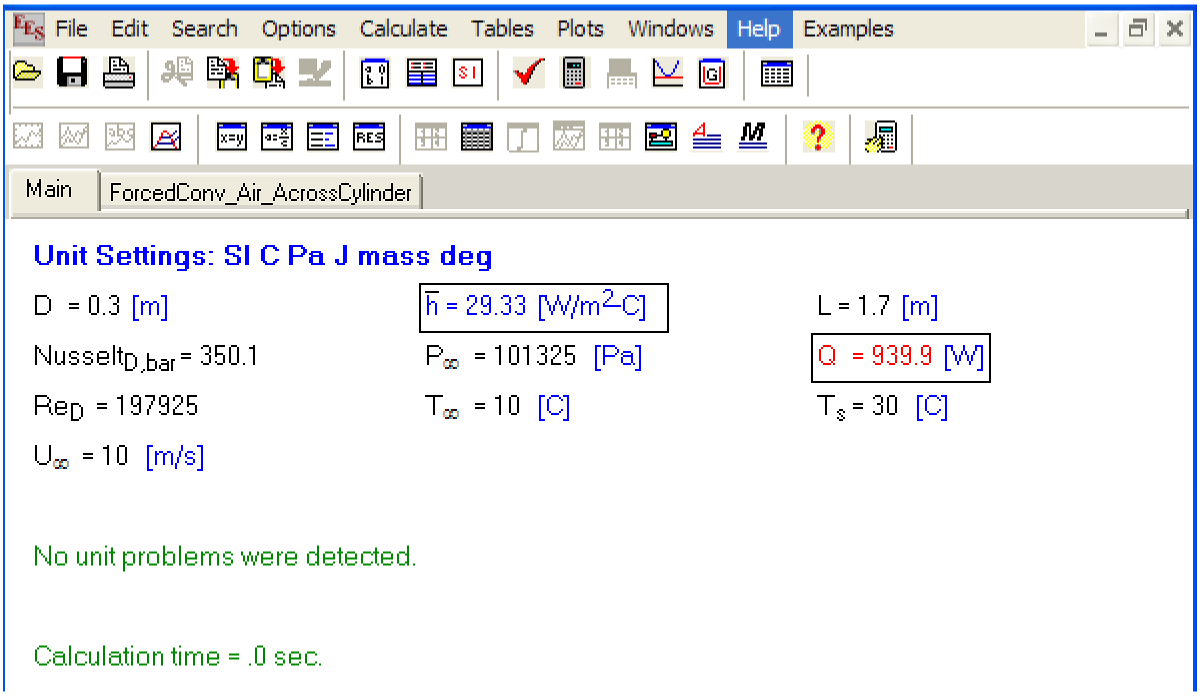
EES Solution:

In EES, ener:

```
"Data:"

D = 0.3 [m]
L = 1.7 [m]
U_infinity = 10[m/s]
T_infinity = 10[C]
T_s = 30[C]
P_infinity = 101325[Pa]

CALL ForcedConv_Air_AcrossCylinder (P_infinity, T_infinity, U_infinity, L, D, T_s: Re_D, Nusselt_D_bar, h_bar, Q)
```

And, press F2 to get the Solution:

Main Solution:



Unit Settings: SI C Pa J mass deg

$D = 0.3$ [m]

$\bar{h} = 29.33$ [W/m^2-C]

$L = 1.7$ [m]

$\text{Nusselt}_{D,bar} = 350.1$

$P_{\infty} = 101325$ [Pa]

$Q = 939.9$ [W]

$\text{Re}_D = 197925$

$T_{\infty} = 10$ [C]

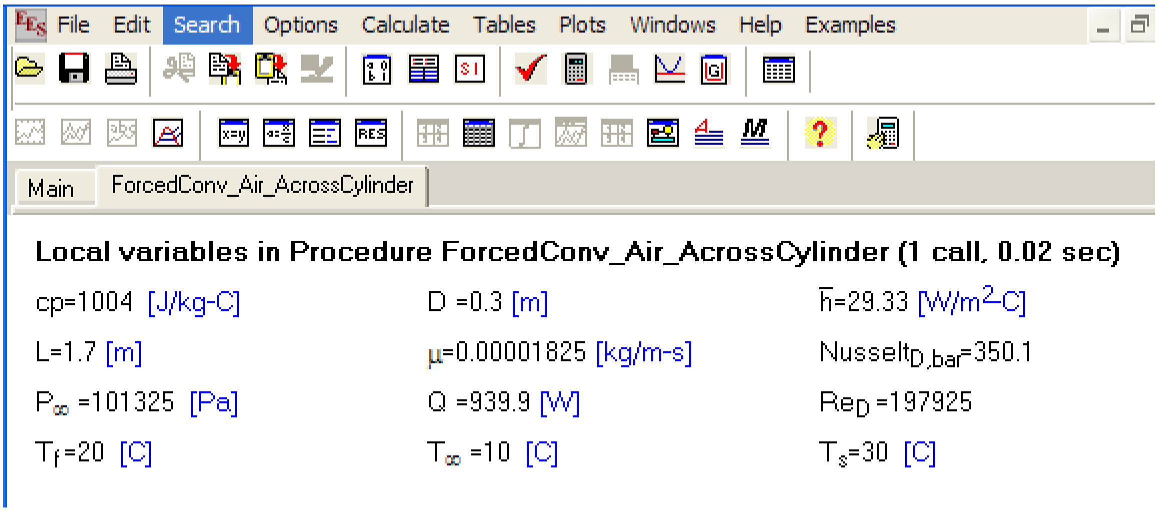
$T_s = 30$ [C]

$U_{\infty} = 10$ [m/s]

No unit problems were detected.

Calculation time = .0 sec.

Solution of PROCEDURE:



Local variables in Procedure ForcedConv_Air_AcrossCylinder (1 call, 0.02 sec)

$cp=1004$ [J/kg-C]

$D = 0.3$ [m]

$\bar{h}=29.33$ [W/m^2-C]

$L=1.7$ [m]

$\mu=0.00001825$ [kg/m-s]

$\text{Nusselt}_{D,bar}=350.1$

$P_{\infty} =101325$ [Pa]

$Q =939.9$ [W]

$\text{Re}_D =197925$

$T_f=20$ [C]

$T_{\infty} =10$ [C]

$T_s=30$ [C]

Thus, Q = 939.9 W, and h = 29.33 W/m^2.C …. Ans.

Of course, the results match with those obtained in Prob.2A1.2.1, as they should.

==

Prob.2A1.2.12. A copper sphere, 10 mm dia, and at 75 C is subjected to an air stream at 1 atm and 25 with a velocity of 10 m/s. Estimate the initial heat transfer rate and the heat transfer coeff.

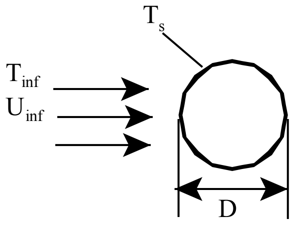


Fig.Prob.2A1.2.12

EES Solution:

Once again, let us write a PROCEDURE to find h and Q for the case of a sphere in cross flow of air:

```
PROCEDURE FC_Air_AcrossSphere_Whitaker (P_infinity, T_infinity, U_infinity, D, T_s: Re_D, Nusselt_D_bar, h_bar, Q)

"Ref: Incropera, 5th Ed. pp. 415, Eqn. (7.59)"
"Whitaker eqn….for 0.71 < Pr < 380 and 3.5 < Re_D < 7.6 e04"

"Finds various quantities for Forced Conv (FC) of Air across a Sphere:"
"Inputs: Pa, C, m/s, m"
"Outputs: W/m^2.C. W, W"

"Properties of Air (Ideal gas) at T_infinity:"

rho:=Density(Air,T=T_infinity,P=P_infinity)
{cp:=Cp(Air,T=T_f)}
mu:=Viscosity(Air,T=T_infinity)
mu_s := Viscosity(Air,T=T_s) "….mu_s at T_s"
k:=Conductivity(Air,T=T_infinity)
Pr:=Prandtl(Air,T=T_infinity)
cp:=SpecHeat(Air,T=T_infinity)

Re_D := D * U_infinity * rho/mu "Finds Reynolds No."

If (Re_D < 3.5) or (Re_D > 7.6e04) Then CALL WARNING ('The results may not be accurate since 3.5 < Re_D < 7.6e04 does not hold. Re_d = XXXA1', Re_D)
```

```
If (Pr < 0.71) or (Pr > 380) Then CALL WARNING ('The results may not be accurate since 0.71 < Pr < 380 does not hold. Pr = XXXA1', Pr)

"To find h_bar: Use Whitaker eqn."

Nusselt_D_bar := 2 + (0.4 * Re_D^(1/2) + 0.06 * Re_D^ (2/3)) * Pr^0.4 * (mu/mu_s)^(1/4) "Finds avg. Nusselts No."

h_bar :=Nusselt_D_bar * k / D "Finds h_bar"
Q := h_bar * (pi * D^2) * (T_s – T_infinity) "W.... heat transfer"

END
```

Note: In the above program, we have included WARNINGS also, to ensure that Re_D and Pr are within the permissible limits for the Whitaker eqn to be applied.

Now, to solve the problem in EES, enter the data, and call the PROCEDURE as shown below:

"Data:"

D = 0.01[m]

T_s = 75[C]

T_infinity = 25[C]

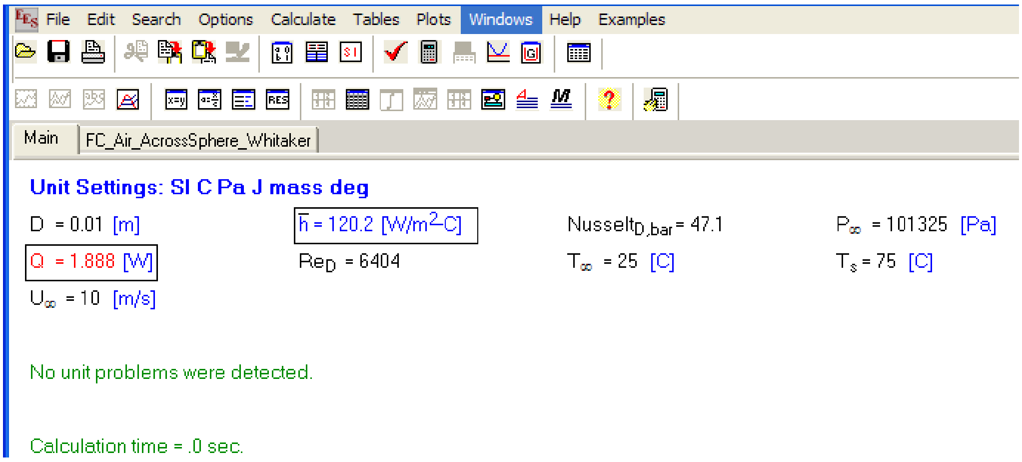
P_infinity = 1.01325e05[Pa]

U_infinity = 10[m/s]

CALL FC_Air_AcrossSphere_Whitaker (P_infinity, T_infinity, U_infinity, D, T_s: Re_D, Nusselt_D_bar, h_bar, Q)

Now, press F2 and the Results are presented in two windows, one for Main program and the other, for the PROCEDURE:

We get:



Unit Settings: SI C Pa J mass deg

D = 0.01 [m]

$\bar{h}$ = 120.2 [W/m^2-C]

$Nusselt_{D,bar}$ = 47.1

P_∞ = 101325 [Pa]

Q = 1.888 [W]

Re_D = 6404

T_∞ = 25 [C]

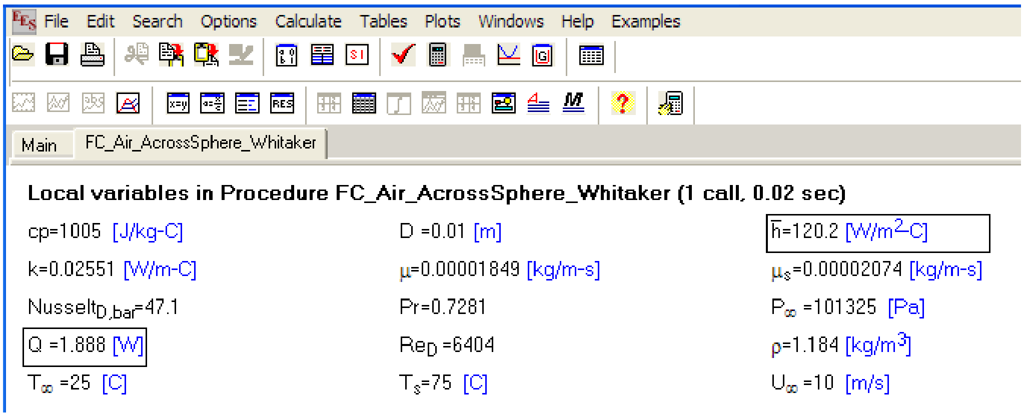
T_s = 75 [C]

U_∞ = 10 [m/s]

No unit problems were detected.

Calculation time = .0 sec.

And, Results in PROCEDURE window:



Local variables in Procedure FC_Air_AcrossSphere_Whitaker (1 call, 0.02 sec)

cp=1005 [J/kg-C]

D =0.01 [m]

$\bar{h}$=120.2 [W/m^2-C]

k=0.02551 [W/m-C]

μ=0.00001849 [kg/m-s]

μ_s=0.00002074 [kg/m-s]

$Nusselt_{D,bar}$=47.1

Pr=0.7281

P_∞ =101325 [Pa]

Q =1.888 [W]

Re_D =6404

ρ=1.184 [kg/m^3]

T_∞ =25 [C]

T_s=75 [C]

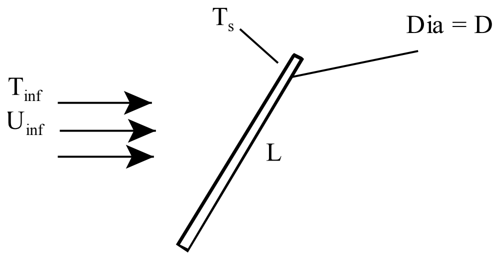
U_∞ =10 [m/s]

Thus: Q = 1.888 W, and h = 120.2 W/m^2.C …. Ans.

===

Prob. 2A1.2.13. A 25 mm dia high tension line has an electrical resistance of 10^-4 Ohm/m and is transmitting a current of 1000 A. If the ambient air is at 10 C, and is flowing at 5 m/s, what is the surface temp?

Plot the variation of surface temp for air velocities varying from 1 m/s to 10 m/s.



Prob.2A1.2.13.

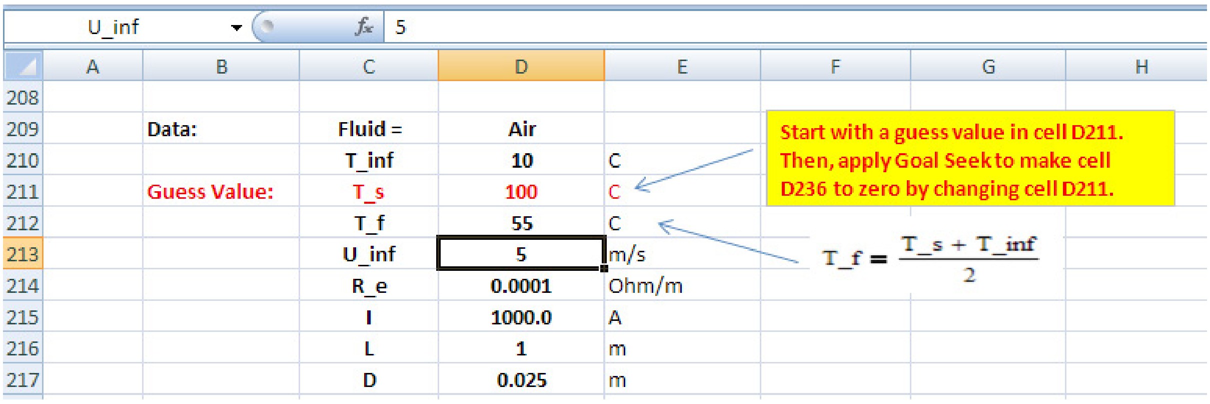
Note that this problem is the same as Prob.2A1.2.6.

But, we will now solve it with EXCEL, using the VBA Functions we wrote for properties of Air.

EXCEL Solution:

Following are the steps:

1. Set up the EXCEL worksheet, enter data and name the cells:



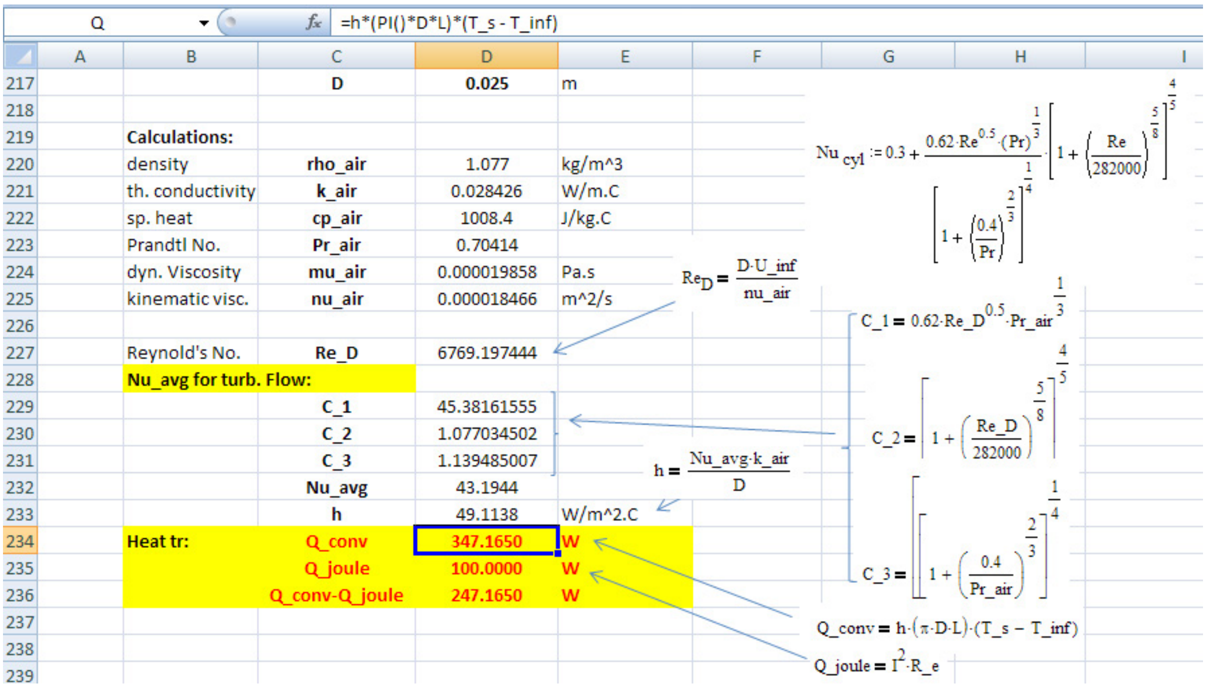
	A	B	C	D	E	F	G	H
208								
209		Data:	Fluid =	Air				
210			T_inf	10	C			
211		Guess Value:	T_s	100	C			
212			T_f	55	C			
213			U_inf	5	m/s			
214			R_e	0.0001	Ohm/m			
215			I	1000.0	A			
216			L	1	m			
217			D	0.025	m			

Start with a guess value in cell D211. Then, apply Goal Seek to make cell D236 to zero by changing cell D211.

$$T\_f = \frac{T\_s + T\_inf}{2}$$

Note that since T_s is not yet known, we have assumed a value of 100 C and we will correct it later by applying Goal Seek in EXCEL.

2. Get the property values for Air, using the VBA Functions already written and explained earlier. Do the calculations as shown:



Q | =h*(PI()*D*L)*(T_s - T_inf)

	A	B	C	D	E
217			D	0.025	m
218					
219		Calculations:			
220		density	rho_air	1.077	kg/m^3
221		th. conductivity	k_air	0.028426	W/m.C
222		sp. heat	cp_air	1008.4	J/kg.C
223		Prandtl No.	Pr_air	0.70414	
224		dyn. Viscosity	mu_air	0.000019858	Pa.s
225		kinematic visc.	nu_air	0.000018466	m^2/s
226					
227		Reynold's No.	Re_D	6769.197444	
228		Nu_avg for turb. Flow:			
229			C_1	45.38161555	
230			C_2	1.077034502	
231			C_3	1.139485007	
232			Nu_avg	43.1944	
233			h	49.1138	W/m^2.C
234		Heat tr:	Q_conv	347.1650	W
235			Q_joule	100.0000	W
236			Q_conv-Q_joule	247.1650	W
237					
238					
239					

$$Nu_{cyl} := 0.3 + \frac{0.62\cdot Re^{0.5}\cdot(Pr)^{\frac{1}{3}}}{\left[1+\left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}}\cdot\left[1+\left(\frac{Re}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$

$$Re_D = \frac{D\cdot U\_inf}{nu\_air}$$

$$C\_1 = 0.62\cdot Re\_D^{0.5}\cdot Pr\_air^{\frac{1}{3}}$$

$$C\_2 = \left[1+\left(\frac{Re\_D}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$

$$C\_3 = \left[\left[1+\left(\frac{0.4}{Pr\_air}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}\right.$$

$$h = \frac{Nu\_avg\cdot k\_air}{D}$$

$$Q\_conv = h\cdot(\pi\cdot D\cdot L)\cdot(T\_s - T\_inf)$$

$$Q\_joule = I^2\cdot R\_e$$

Here, first, get Re_D, the Reynold's No. Then, apply Churchill-Bernstein eqn to get Nu_cyl. Then, get the heat transfer coeff, h. And, calculate the heat transfer by forced convection as:

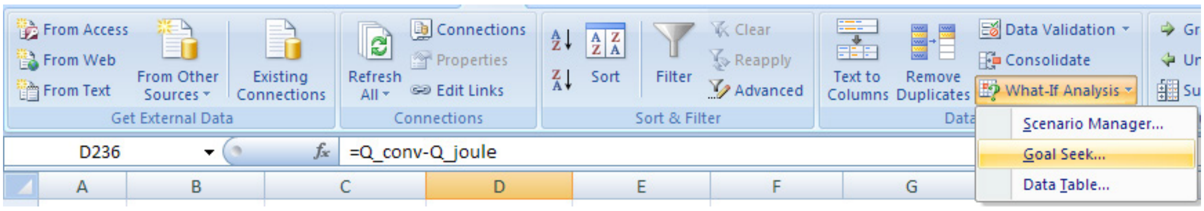
Q_conv = h * (π * D *L) * (T_s – T_inf). This should be equal to the Joule heating in the

cylinder = Q_joule = I^2 * R_e.

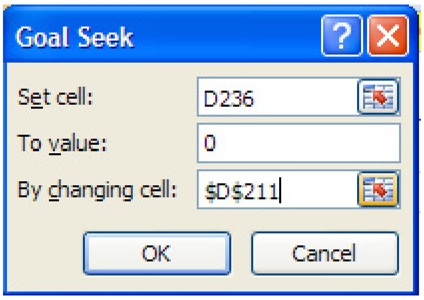
So, in cell D236, we have (Q_conv – Q_joule).

We apply Goal Seek to make cell D236 = 0 by changing T_s, i.e. cell D211.

3. Go to Data-What If Analysis – Goal Seek:

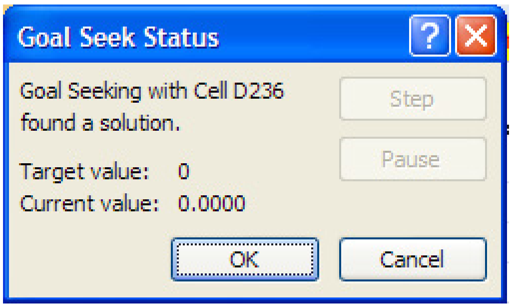

D236 =Q_conv-Q_joule

Click on Goal Seek. We get the following window, fill it up as shown:


Goal Seek

Set cell: D236
To value: 0
By changing cell: D211

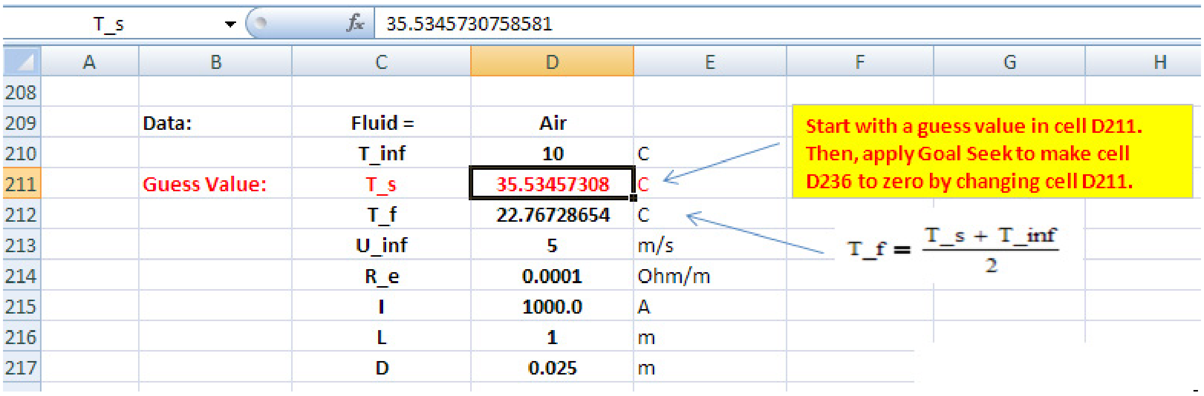
Click OK:


Goal Seek Status

Goal Seeking with Cell D236 found a solution.

Target value: 0
Current value: 0.0000

4. Again, click OK and see the result for T_s:



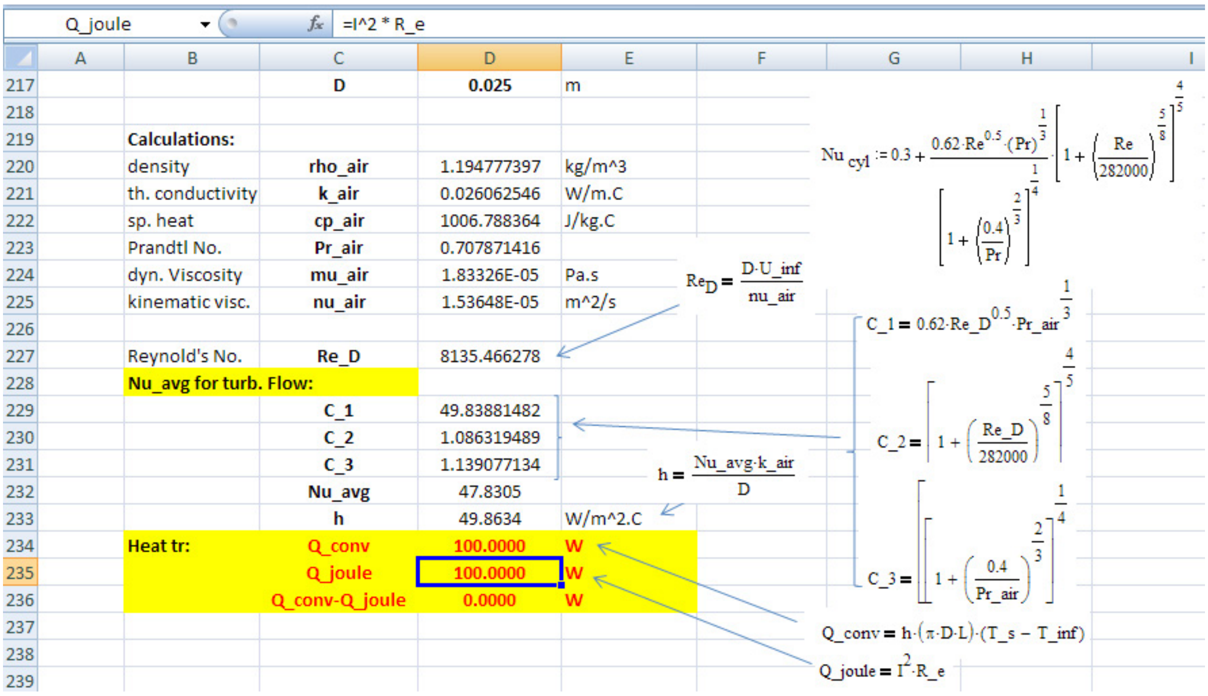
T_s | f_x 35.5345730758581

	A	B	C	D	E
208					
209		Data:	Fluid =	Air	
210			T_inf	10	C
211		Guess Value:	T_s	35.53457308	C
212			T_f	22.76728654	C
213			U_inf	5	m/s
214			R_e	0.0001	Ohm/m
215			I	1000.0	A
216			L	1	m
217			D	0.025	m

Start with a guess value in cell D211. Then, apply Goal Seek to make cell D236 to zero by changing cell D211.

$$T\_f = \frac{T\_s + T\_inf}{2}$$

5. Observe that correspondingly, all calculations are up-dated:



Q_joule | f_x =I^2 * R_e

	A	B	C	D	E
217			D	0.025	m
218					
219		Calculations:			
220		density	rho_air	1.194777397	kg/m^3
221		th. conductivity	k_air	0.026062546	W/m.C
222		sp. heat	cp_air	1006.788364	J/kg.C
223		Prandtl No.	Pr_air	0.707871416	
224		dyn. Viscosity	mu_air	1.83326E-05	Pa.s
225		kinematic visc.	nu_air	1.53648E-05	m^2/s
226					
227		Reynold's No.	Re_D	8135.466278	
228		Nu_avg for turb. Flow:			
229			C_1	49.83881482	
230			C_2	1.086319489	
231			C_3	1.139077134	
232			Nu_avg	47.8305	
233			h	49.8634	W/m^2.C
234		Heat tr:	Q_conv	100.0000	W
235			Q_joule	100.0000	W
236			Q_conv-Q_joule	0.0000	W
237					
238					
239					

$$Nu_{cyl} := 0.3 + \frac{0.62 \cdot Re^{0.5} \cdot (Pr)^{\frac{1}{3}}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}} \cdot \left[1 + \left(\frac{Re}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$

$$Re_D = \frac{D \cdot U\_inf}{nu\_air}$$

$$C\_1 = 0.62 \cdot Re\_D^{0.5} \cdot Pr\_air^{\frac{1}{3}}$$

$$C\_2 = \left[1 + \left(\frac{Re\_D}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$

$$C\_3 = \left[\left[1 + \left(\frac{0.4}{Pr\_air}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}\right]$$

$$h = \frac{Nu\_avg \cdot k\_air}{D}$$

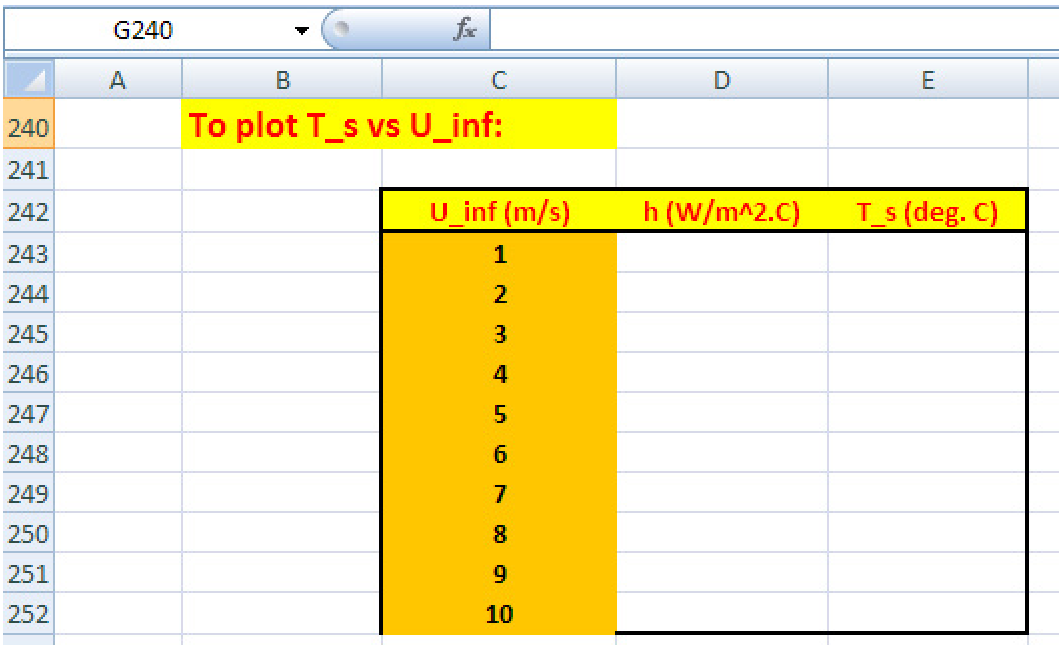
$$Q\_conv = h \cdot (\pi \cdot D \cdot L) \cdot (T\_s - T\_inf)$$

$$Q\_joule = I^2 \cdot R\_e$$

Thus: surface temp of cylinder = 35.53 deg. C … Ans.

Note that if this trial and error calculation has to be done by hand, it will be very tedious; also, having VBA Functions to get properties of Air has been of immense help in these calculations.

6. Now, plot the variation of T_s for air velocities varying from 1 m/s to 10 m/s: Set up the worksheet as shown:



To plot T_s vs U_inf:

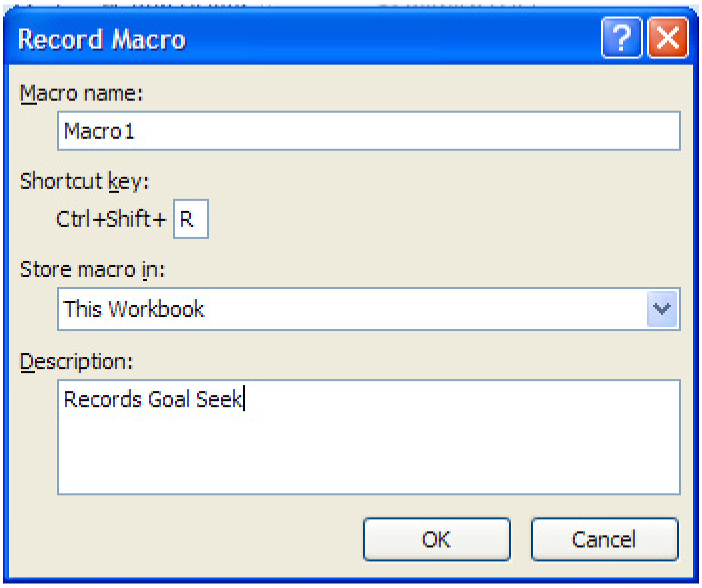
U_inf (m/s)	h (W/m^2.C)	T_s (deg. C)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Now, as U_inf changes, Re_D will change; so, h will also change. Equating Q_joule to Q_conv, we will apply Goal Seek, for each value of U_inf, to make (Q_conv – Q_joule) (i.e. cell D236) equal to zero by changing T_s.

So, we will write a VBA program that will each U_inf value from the Table above, copy it to cell D213, start with the existing T_s value in cell D211 as 'Guess value', complete the calculations after applying Goal Seek, and copy the resulting values of h and T_s from cells D233 and D211 to the respective positions in the Table.

7. As a first step, let us record a Macro to apply Goal Seek to make cell D236 to zero by changing cell D211:

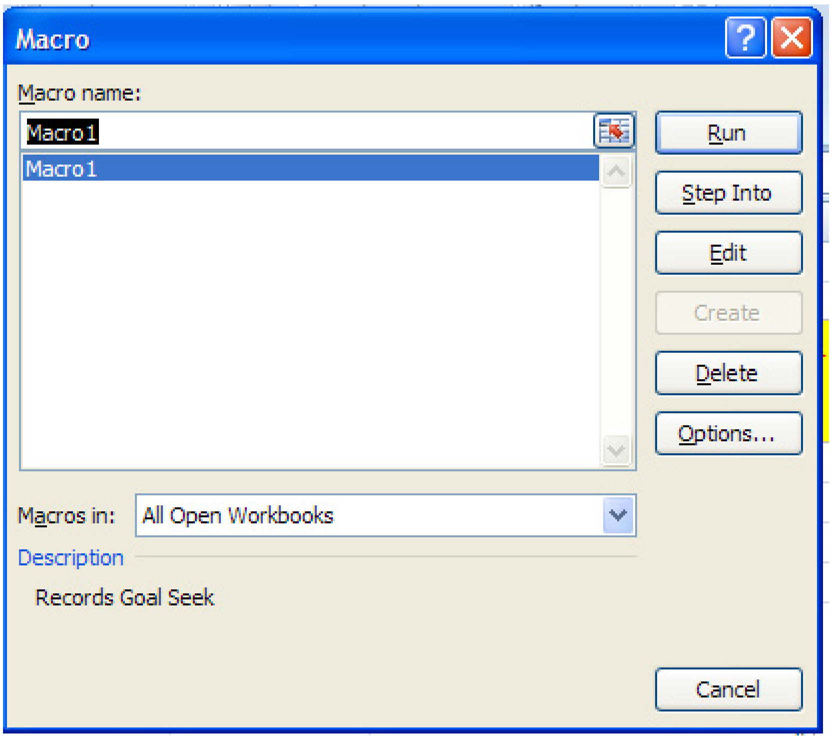
Click on Developer – Record Macro. Following window appears. Fill it up as shown:



Macro name:
Macro1
Shortcut key:
Ctrl+Shift+ R
Store macro in:
This Workbook
Description:
Records Goal Seek

Click OK. Recording starts. And, 'Record Macro' changes to 'Stop Recording'. After recording the Macro, click on 'Stop recording'.

Now, view the Macro recorded. For this, click on Developer – Macros. We get:



Macro name: Macro1

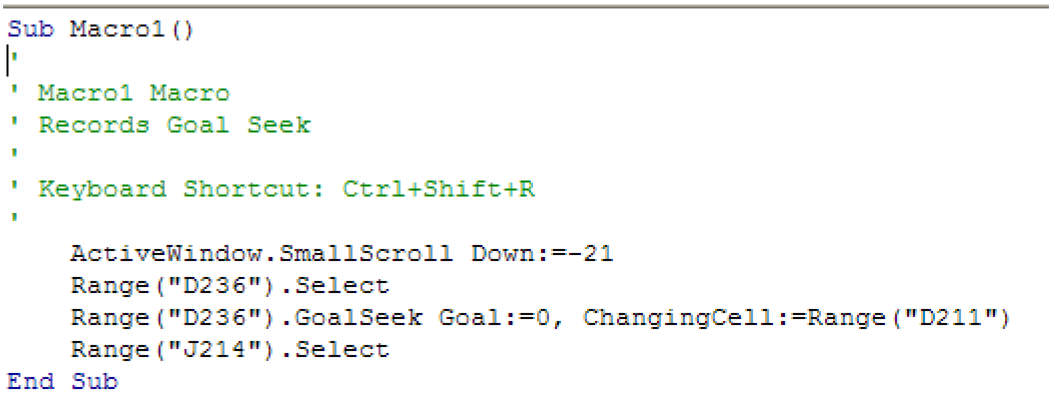
Macro1

Macros in: All Open Workbooks

Description

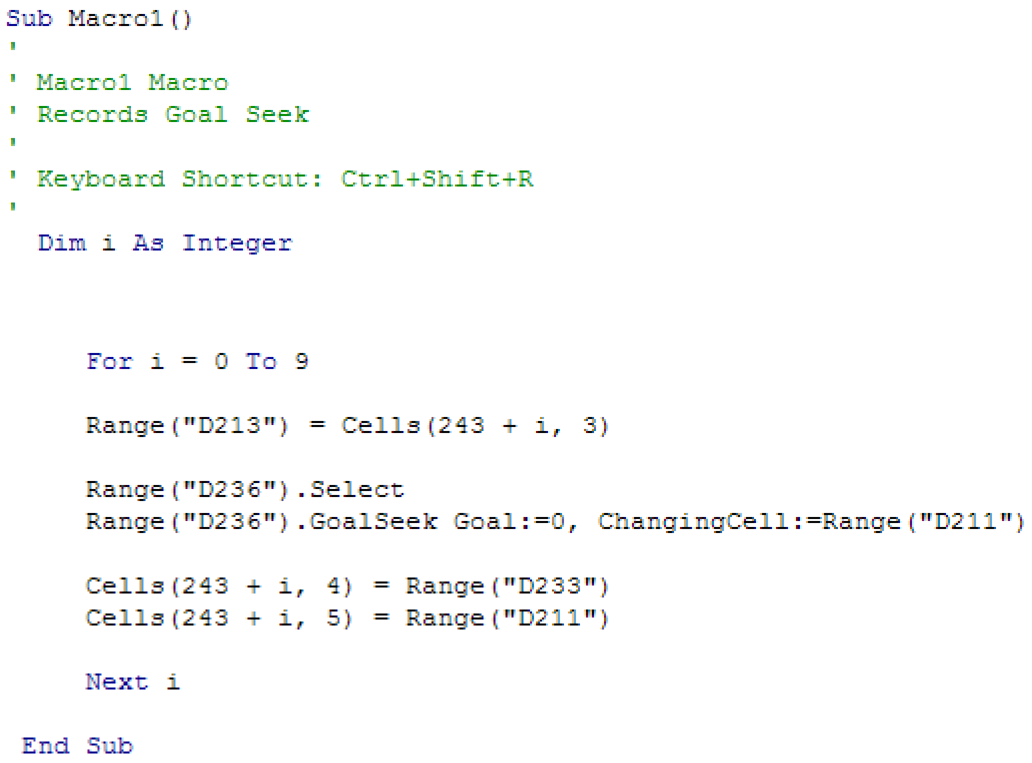
Records Goal Seek

Click on Edit. We see the code recorded:



```
Sub Macro1()
'
' Macro1 Macro
' Records Goal Seek
'
' Keyboard Shortcut: Ctrl+Shift+R
'
    ActiveWindow.SmallScroll Down:=-21
    Range("D236").Select
    Range("D236").GoalSeek Goal:=0, ChangingCell:=Range("D211")
    Range("J214").Select
End Sub
```

8. Now, we will modify this Macro to perform actions we explained in step 6 above. Following is the modified code:



```
Sub Macro1()
'
' Macro1 Macro
' Records Goal Seek
'
' Keyboard Shortcut: Ctrl+Shift+R
'
  Dim i As Integer


     For i = 0 To 9

     Range("D213") = Cells(243 + i, 3)

     Range("D236").Select
     Range("D236").GoalSeek Goal:=0, ChangingCell:=Range("D211")

     Cells(243 + i, 4) = Range("D233")
     Cells(243 + i, 5) = Range("D211")

     Next i

 End Sub
```

In the above code:

Line 1: Starts the Sub (Macro_Name)

Lines 2, 3: Name and function of this Macro

Line 4: Keyboard shortcut

Line 5: Dimension of counter I as Integer

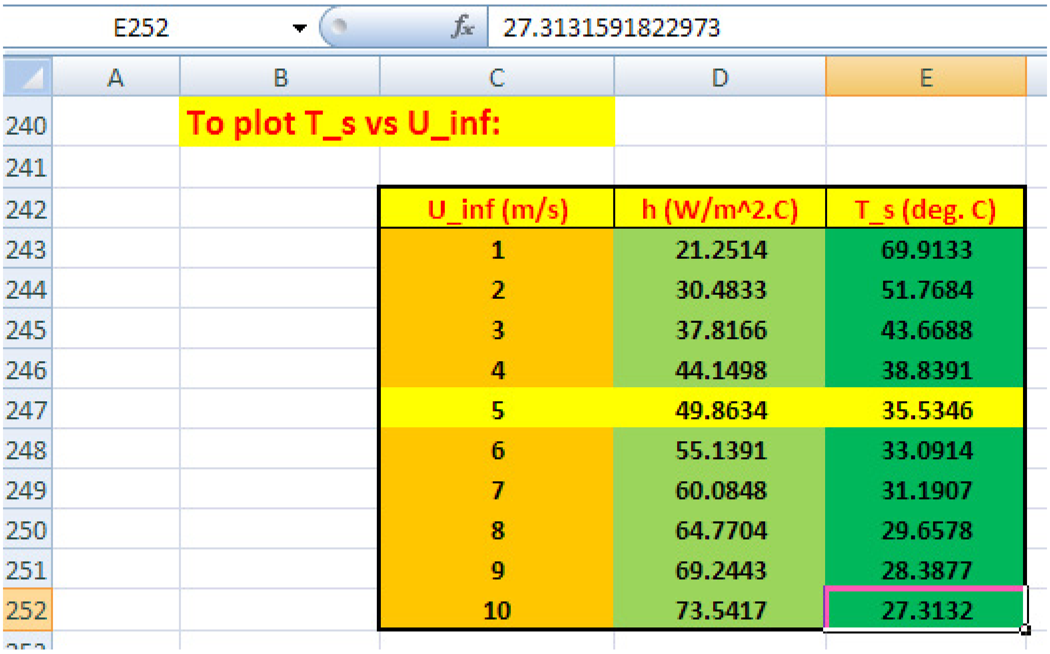
Line 6: For .. Next construct begins. i = 9 since there are 10 lines in our Table.

Line 7: First value of U_inf is picked up from the Table and is copied to cell D213. (Immediately, other values downwards from cell D213 will change).

Lines 8, 9: Applies Goal Seek. New value of T_s will be recorded in cell D211. (Immediately, all other values downwards will update themselves).

Line 10, 11: copy values of h and T_s from cells D233 and D211 to respective positions in the Table.

9. Now, we can run this program by clicking on keyboard shortcut, viz. Ctrl+Shift+R. We get:



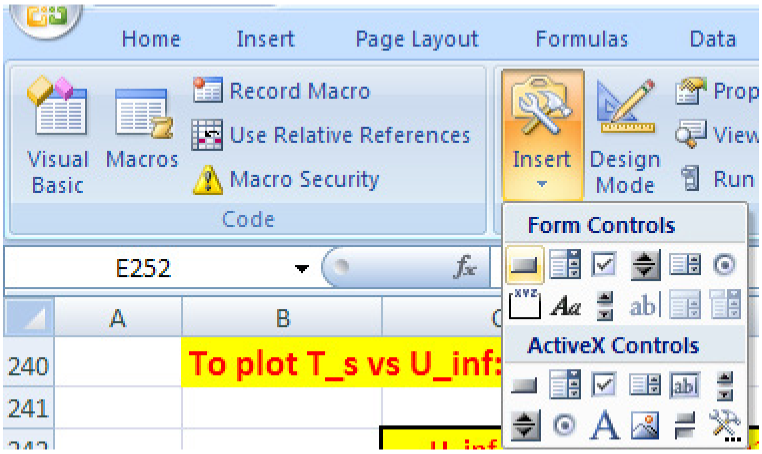
E252 27.3131591822973

	A	B	C	D	E
240		To plot T_s vs U_inf:			
241					
242			U_inf (m/s)	h (W/m^2.C)	T_s (deg. C)
243			1	21.2514	69.9133
244			2	30.4833	51.7684
245			3	37.8166	43.6688
246			4	44.1498	38.8391
247			5	49.8634	35.5346
248			6	55.1391	33.0914
249			7	60.0848	31.1907
250			8	64.7704	29.6578
251			9	69.2443	28.3877
252			10	73.5417	27.3132

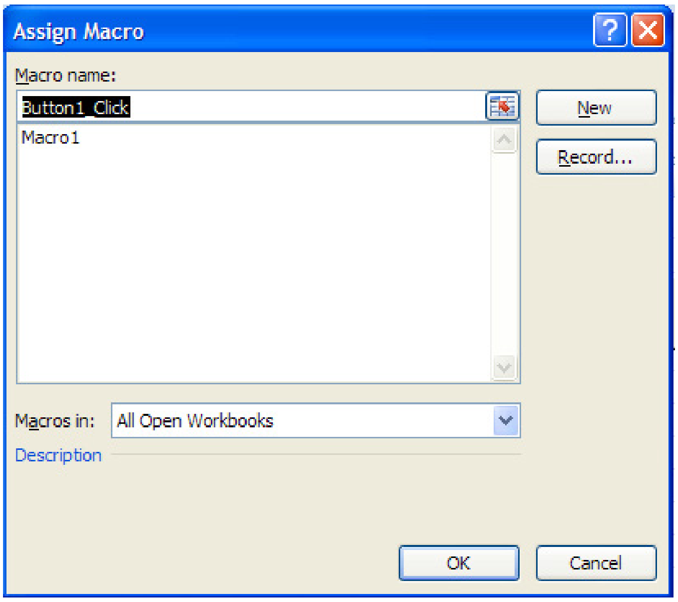
Note that when U_inf = 5 m/s, we have T_s = 35.53 deg. C and h = 49.86 W/m^2.C , the same values we got earlier.

10. It is more convenient to run this program from a button in the worksheet. To insert a 'Form control button' and connect it to the Macro, we proceed as follows:

Click on Developer- Insert. We get:



Now, click on first, top left button in Form Controls and draw a button on the worksheet to a suitable size. Immediately, following window appears:

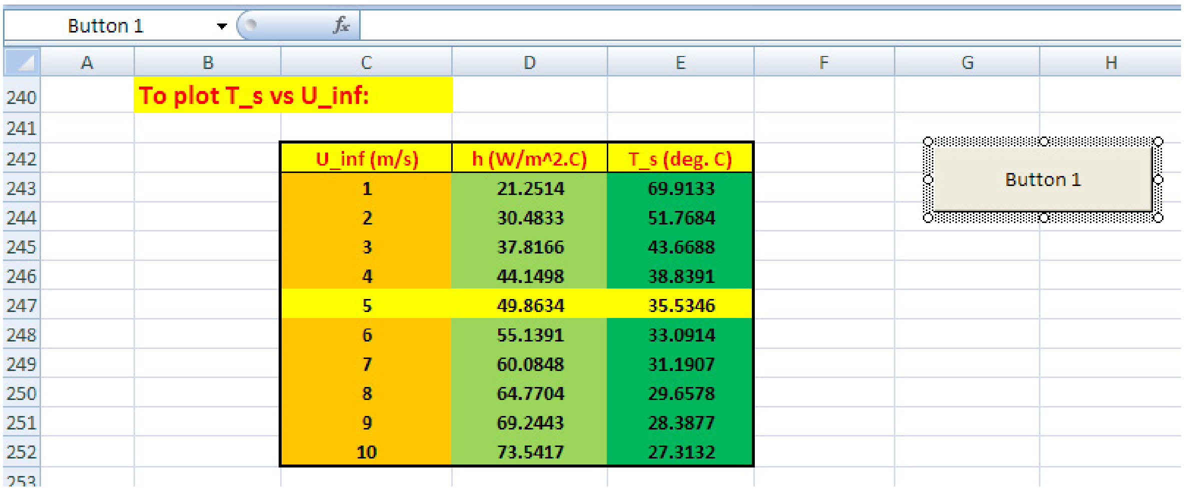


Macro name: Button1_Click

Macro1

Macros in: All Open Workbooks

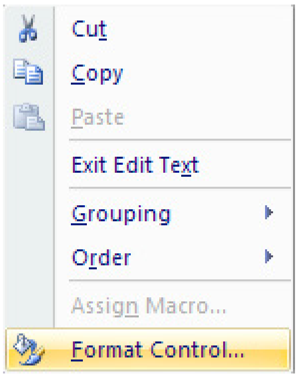
Click on Macro1 to Assign that Macro to the button in the Worksheet. Click OK. We get:



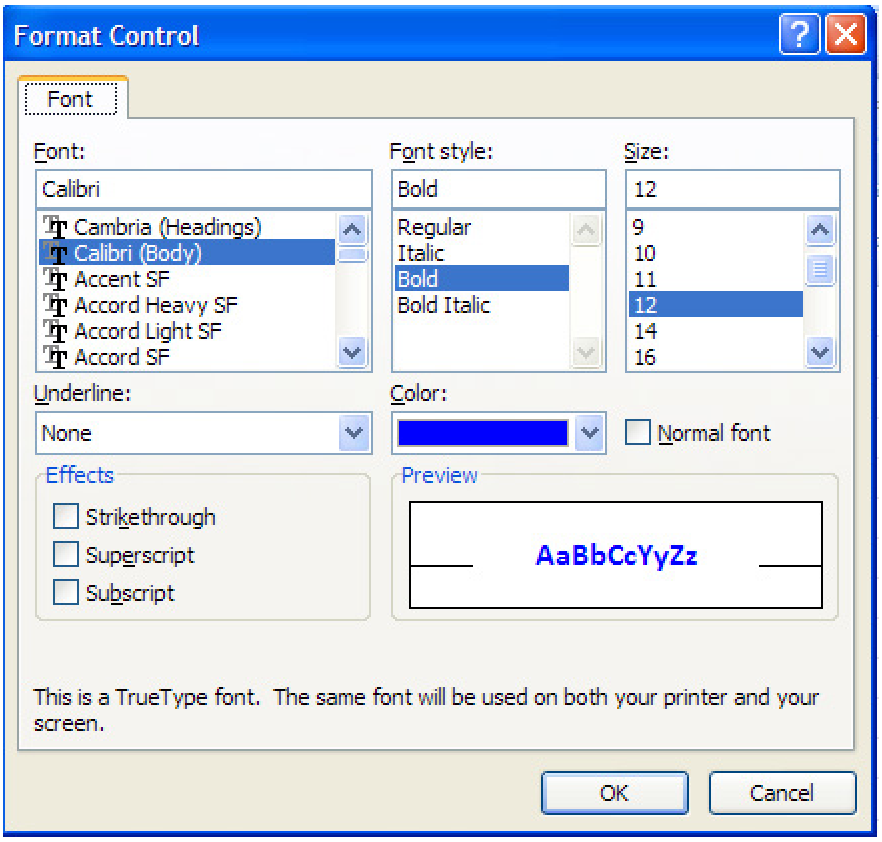
To plot T_s vs U_inf:

U_inf (m/s)	h (W/m^2.C)	T_s (deg. C)
1	21.2514	69.9133
2	30.4833	51.7684
3	37.8166	43.6688
4	44.1498	38.8391
5	49.8634	35.5346
6	55.1391	33.0914
7	60.0848	31.1907
8	64.7704	29.6578
9	69.2443	28.3877
10	73.5417	27.3132

Now, we can edit the name of the button, and format its size, colour etc by right-clicking on the button:



Click on Format control:



Font: Calibri

Font style: Bold

Size: 12

Underline: None

Color:

☐ Normal font

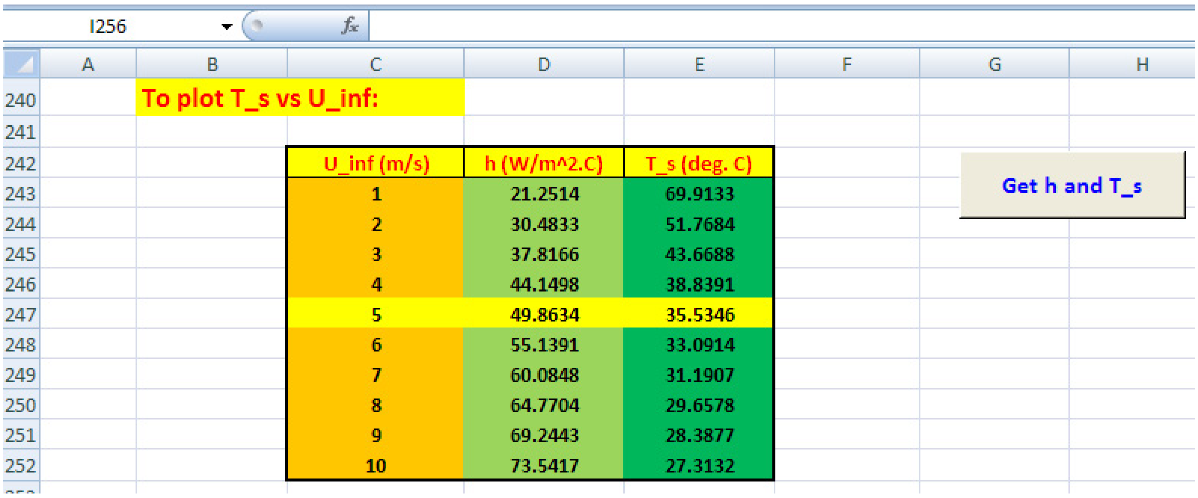
☐ Strikethrough

☐ Superscript

☐ Subscript

This is a TrueType font. The same font will be used on both your printer and your screen.

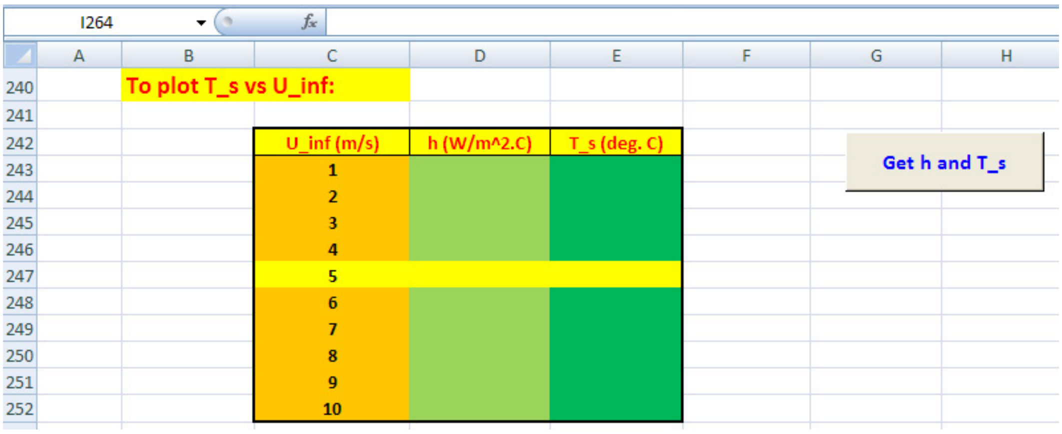
Now, you can control the size, Font style, colour etc. as shown.



To plot T_s vs U_inf:

U_inf (m/s)	h (W/m^2.C)	T_s (deg. C)
1	21.2514	69.9133
2	30.4833	51.7684
3	37.8166	43.6688
4	44.1498	38.8391
5	49.8634	35.5346
6	55.1391	33.0914
7	60.0848	31.1907
8	64.7704	29.6578
9	69.2443	28.3877
10	73.5417	27.3132

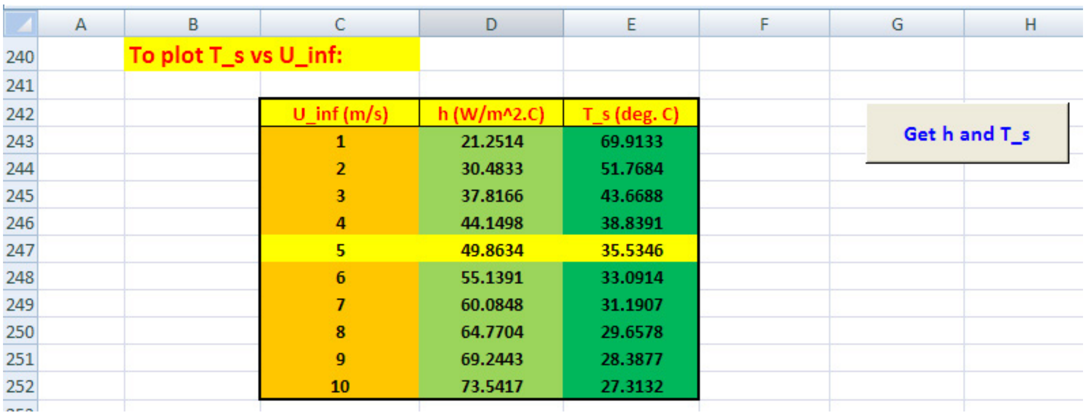
Now, delete cells D243 to E252; we get:



To plot T_s vs U_inf:

U_inf (m/s)	h (W/m^2.C)	T_s (deg. C)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

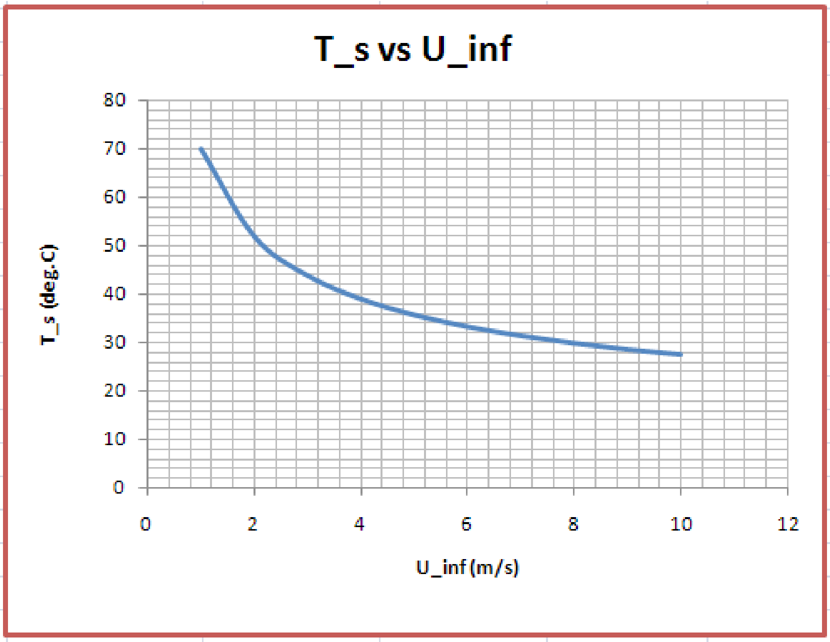
And, click the button, and immediately the calculations are complted:



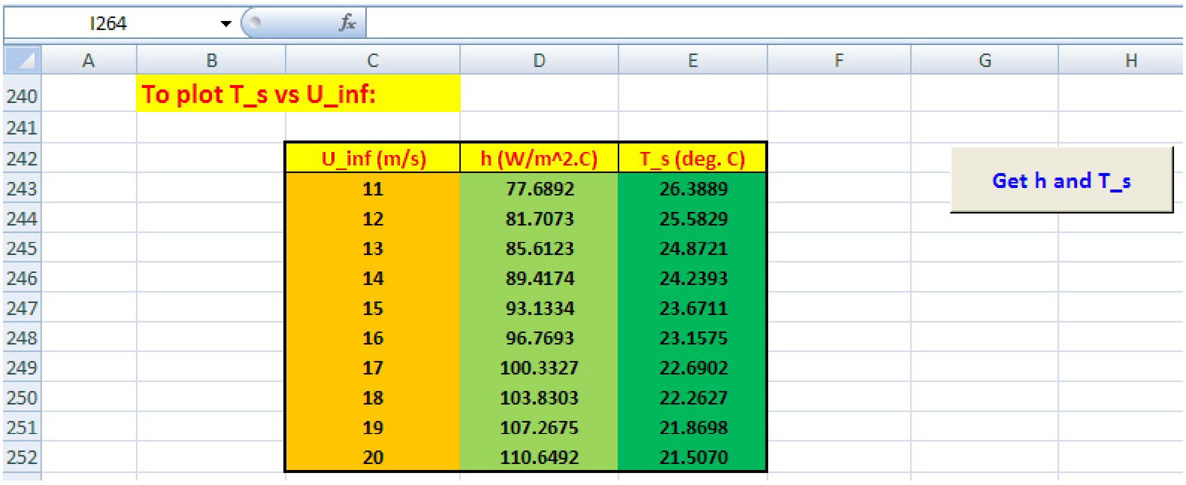
To plot T_s vs U_inf:

U_inf (m/s)	h (W/m^2.C)	T_s (deg. C)
1	21.2514	69.9133
2	30.4833	51.7684
3	37.8166	43.6688
4	44.1498	38.8391
5	49.8634	35.5346
6	55.1391	33.0914
7	60.0848	31.1907
8	64.7704	29.6578
9	69.2443	28.3877
10	73.5417	27.3132

11. Now, draw the plot of T_s vs T_inf:



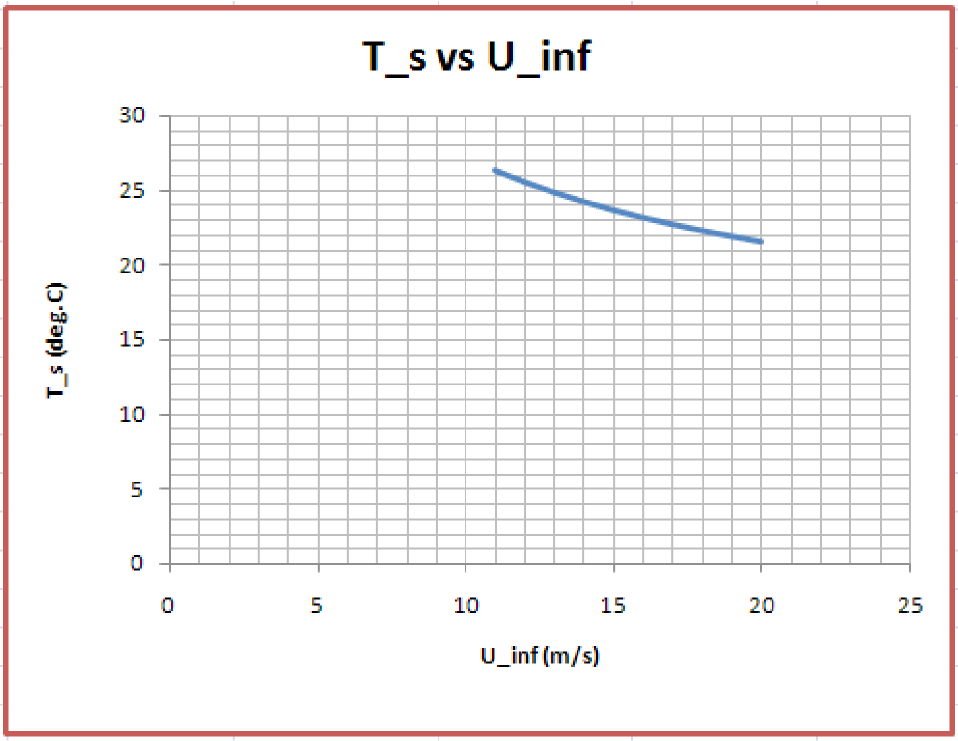
12. Now, in the above Table, U_inf values in the left column are the variables which we can change. Then, if we click on the button, values of h and T_s will be calculated and entered in the Table. *Of course, as the VBA program is written for 10 rows of U_inf only, any values of U_inf entered below the cell C252 will not be considered for calculations.* As an example, delete all cells from D243 to E252, and enter new values of U_inf from 11 m/s to 20 m/s. Then, click the button; we get:



To plot T_s vs U_inf:

U_inf (m/s)	h (W/m^2.C)	T_s (deg. C)
11	77.6892	26.3889
12	81.7073	25.5829
13	85.6123	24.8721
14	89.4174	24.2393
15	93.1334	23.6711
16	96.7693	23.1575
17	100.3327	22.6902
18	103.8303	22.2627
19	107.2675	21.8698
20	110.6492	21.5070

And, the plot is also automatically updated:



==

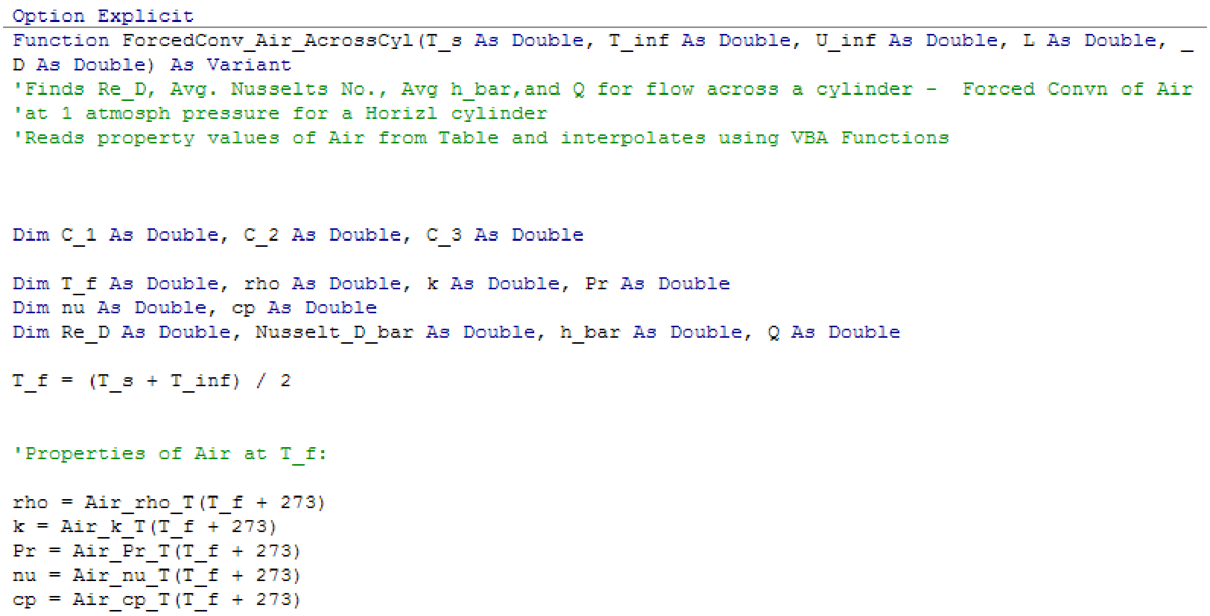
Prob. 2A1.2.14. Air flows at a velocity U_inf = 5 m/s and a temp t_inf = 20 C in cross-flow over a cylinder 10 mm dia, maintained at 50 C. Calculate the rate of heat transfer, Q per unit length.

(b) Plot Q as a function of fluid velocity, U_inf with U_inf varying from 0.5 to 10 m/s.

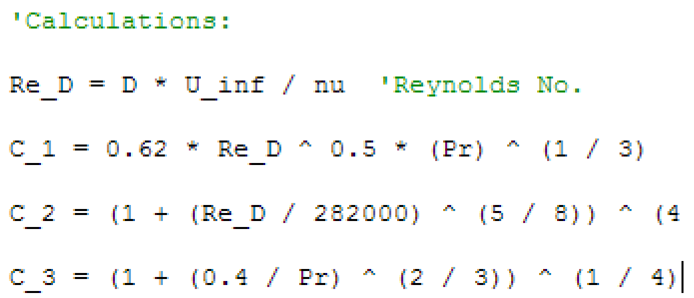
EXCEL Solution:

We use Churchill-Bernstein eqn to calculate average Nusselts No., Nu_D_bar, and then calculate avg heat transfer coeff., h_bar and then, the heat transfer, Q.

First, let us write a VBA Function to calculate Nusselts No. etc.



```
Option Explicit
Function ForcedConv_Air_AcrossCyl(T_s As Double, T_inf As Double, U_inf As Double, L As Double, _
D As Double) As Variant
'Finds Re_D, Avg. Nusselts No., Avg h_bar,and Q for flow across a cylinder -  Forced Convn of Air
'at 1 atmosph pressure for a Horizl cylinder
'Reads property values of Air from Table and interpolates using VBA Functions


Dim C_1 As Double, C_2 As Double, C_3 As Double

Dim T_f As Double, rho As Double, k As Double, Pr As Double
Dim nu As Double, cp As Double
Dim Re_D As Double, Nusselt_D_bar As Double, h_bar As Double, Q As Double

T_f = (T_s + T_inf) / 2


'Properties of Air at T_f:

rho = Air_rho_T(T_f + 273)
k = Air_k_T(T_f + 273)
Pr = Air_Pr_T(T_f + 273)
nu = Air_nu_T(T_f + 273)
cp = Air_cp_T(T_f + 273)
```



```
'Calculations:

Re_D = D * U_inf / nu  'Reynolds No.

C_1 = 0.62 * Re_D ^ 0.5 * (Pr) ^ (1 / 3)

C_2 = (1 + (Re_D / 282000) ^ (5 / 8)) ^ (4 / 5)

C_3 = (1 + (0.4 / Pr) ^ (2 / 3)) ^ (1 / 4)
```

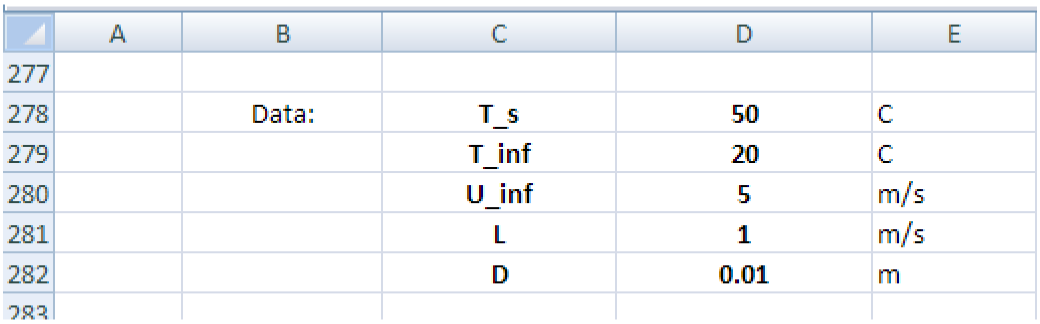
```
Nusselt_D_bar = 0.3 + (C_1 * C_2 / C_3) 'finds Avg. Nusselts No.

h_bar = Nusselt_D_bar * k / D 'Finds h_bar

Q = h_bar * (Application.Pi() * D * L) * (T_s - T_inf) 'W.... heat transfer

ForcedConv_Air_AcrossCyl = Application.Transpose(Array(Re_D, Nusselt_D_bar, h_bar, Q))

End Function
```

Note that above Function is an Array Function i.e. it returns an Vertical Array containing Re_D, Nu_D_bar, h_bar and Q, in that order from top downwards.

To use the Function, **we should first select four consecutive cells in a column, and then enter the Function and press Ctrl+Shift+Enter.**

Now, following are the steps in the EXCEL Worksheet for this problem:

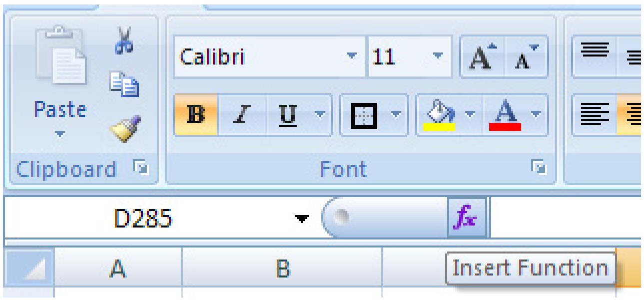
1. Set up the worksheet, enter data:



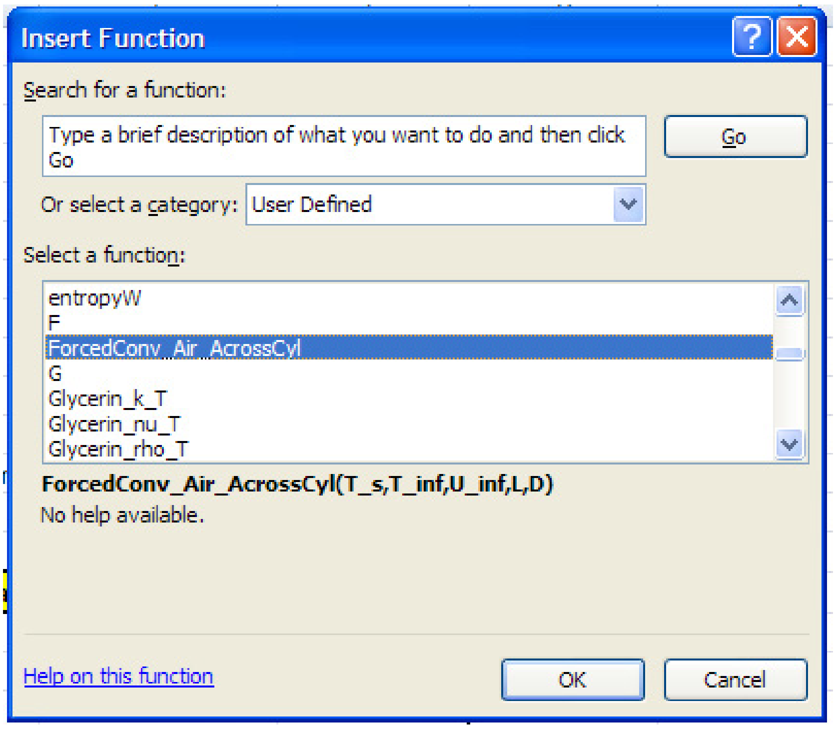
	A	B	C	D	E
277					
278		Data:	**T_s**	**50**	C
279			**T_inf**	**20**	C
280			**U_inf**	**5**	m/s
281			**L**	**1**	m/s
282			**D**	**0.01**	m
283					

2. Now, select 4 consecutive cells in column C and apply the Array Formula written above.

To do this, select cells D285 to D288 and click on the Insert Function symbol:



Then, you get the pop up window; choose the 'User Defined' category, and the Function as shown:



Type a brief description of what you want to do and then click Go

Or select a category: User Defined

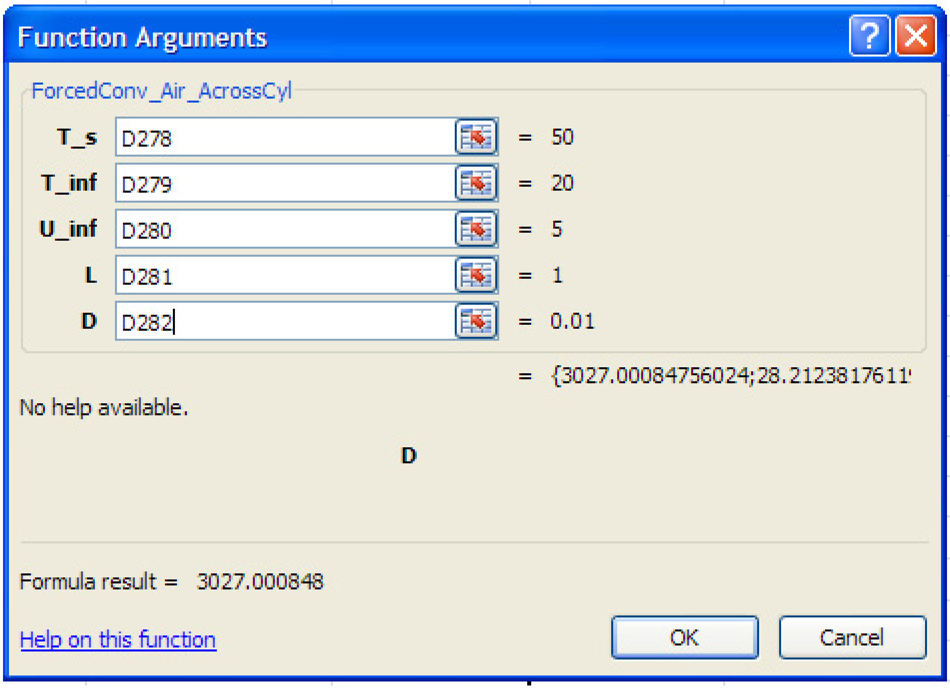
Select a function:

entropyW
F
ForcedConv_Air_AcrossCyl
G
Glycerin_k_T
Glycerin_nu_T
Glycerin_rho_T

ForcedConv_Air_AcrossCyl(T_s,T_inf,U_inf,L,D)

No help available.

Click OK. We get the following window. Fill it up as shown:



ForcedConv_Air_AcrossCyl

T_s D278 = 50
T_inf D279 = 20
U_inf D280 = 5
L D281 = 1
D D282 = 0.01

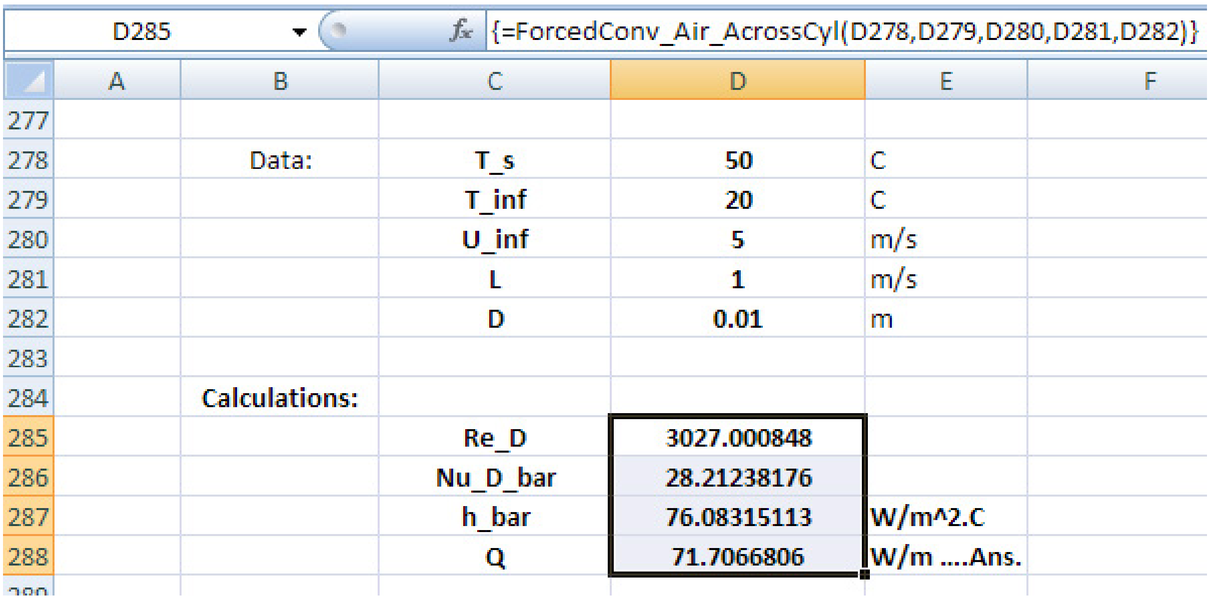
= {3027.00084756024;28.2123817611'

No help available.

D

Formula result = 3027.000848

Now, IMPORTANT: keeping Ctrl + Shift pressed, click OK, since we are using an ARRAY Function. We get:



D285 {=ForcedConv_Air_AcrossCyl(D278,D279,D280,D281,D282)}

	A	B	C	D	E	F
277						
278		Data:	T_s	50	C	
279			T_inf	20	C	
280			U_inf	5	m/s	
281			L	1	m/s	
282			D	0.01	m	
283						
284		Calculations:				
285			Re_D	3027.000848		
286			Nu_D_bar	28.21238176		
287			h_bar	76.08315113	W/m^2.C	
288			Q	71.7066806	W/m ....Ans.	

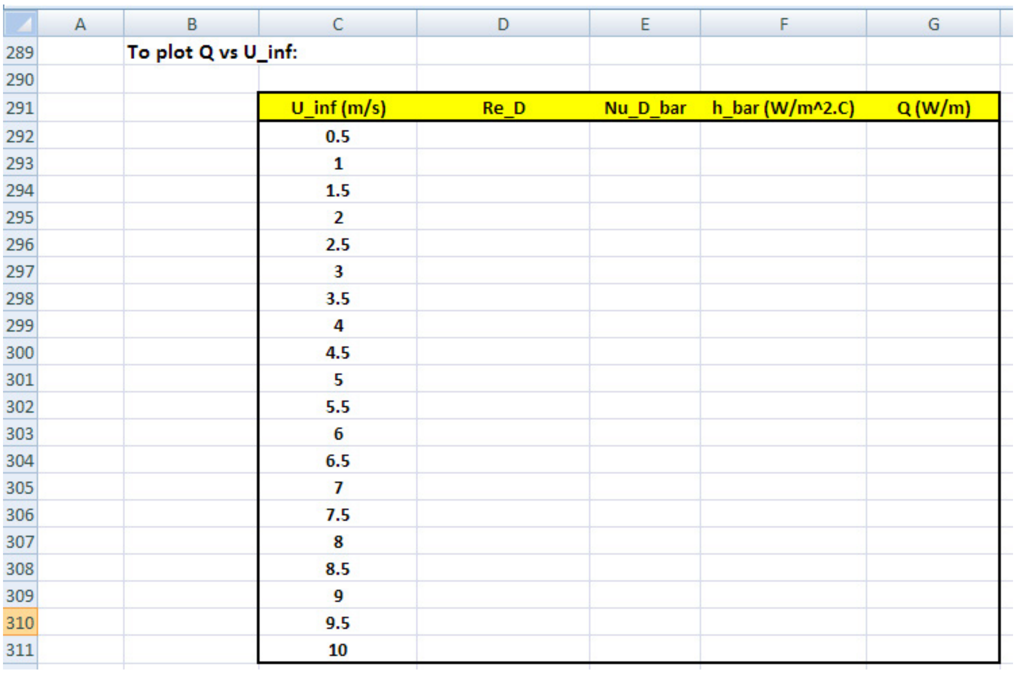
Note the Array Function in the Formula bar.

We observe that use of an Array Function makes the worksheet very compact.

(b) To plot Q against U_inf:

3. Set up the worksheet as shown.

Here, the method is to use the Array Function, and extract each element separately and place in the Table:



	A	B	C	D	E	F	G
289		To plot Q vs U_inf:					
290							
291			U_inf (m/s)	Re_D	Nu_D_bar	h_bar (W/m^2.C)	Q (W/m)
292			0.5				
293			1				
294			1.5				
295			2				
296			2.5				
297			3				
298			3.5				
299			4				
300			4.5				
301			5				
302			5.5				
303			6				
304			6.5				
305			7				
306			7.5				
307			8				
308			8.5				
309			9				
310			9.5				
311			10				

Now, in cell D292 we should have the Reynolds No, Re_D. We note that in the array returned by the Array Function, viz. ForcedConv_Air_AcrossCyl(T_s, T_inf, U_inf, L, D) we have:

Re_D as the first element. i.e. first row, first column.

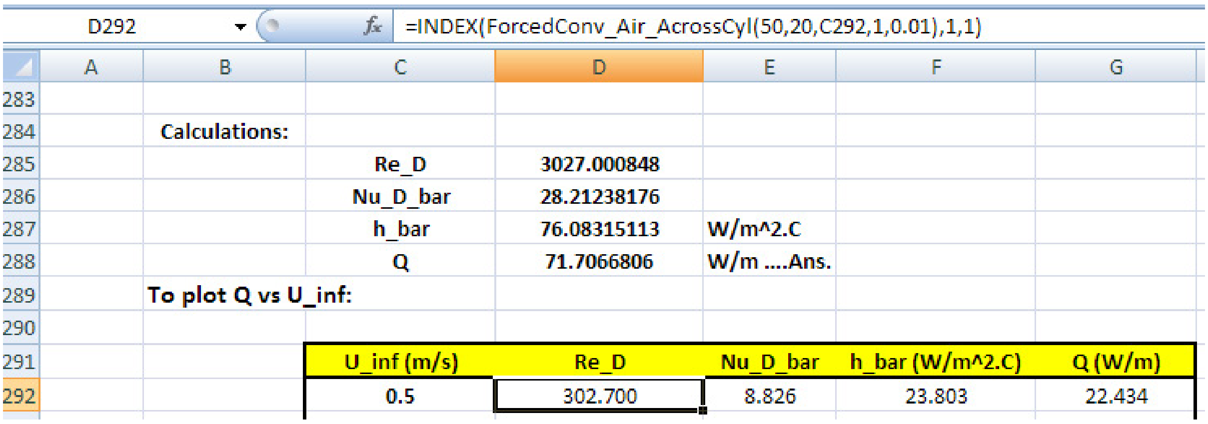
Nu_D_bar as the second element. i.e. second row, first column.

h_bar as the third element. i.e. third row, first column.

Q as the fourth element. i.e. fourth row, first column.

To extract elements from the Array output, we use the INDEX Function:

4. To get Re_D we enter in cell D292 the following (See the Formula bar):

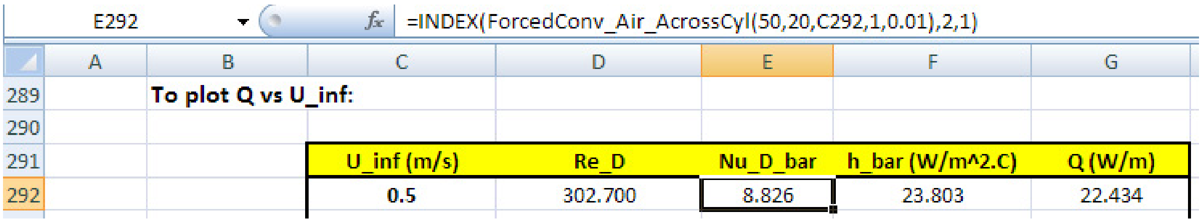


D292 =INDEX(ForcedConv_Air_AcrossCyl(50,20,C292,1,0.01),1,1)

	A	B	C	D	E	F	G
283							
284		Calculations:					
285			Re_D	3027.000848			
286			Nu_D_bar	28.21238176			
287			h_bar	76.08315113	W/m^2.C		
288			Q	71.7066806	W/m ....Ans.		
289		To plot Q vs U_inf:					
290							
291			U_inf (m/s)	Re_D	Nu_D_bar	h_bar (W/m^2.C)	Q (W/m)
292			0.5	302.700	8.826	23.803	22.434

Note in the Formula bar that reference to U_inf is by ‘relative reference’ to cell C292.

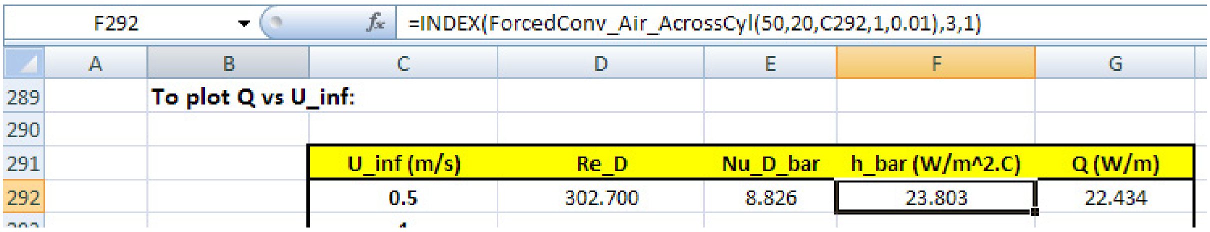
5. Similarly for Nu_D_bar, we have in cell E292 (See the Formula bar):



E292 =INDEX(ForcedConv_Air_AcrossCyl(50,20,C292,1,0.01),2,1)

	A	B	C	D	E	F	G
289		**To plot Q vs U_inf:**					
290							
291			**U_inf (m/s)**	**Re_D**	**Nu_D_bar**	**h_bar (W/m^2.C)**	**Q (W/m)**
292			**0.5**	302.700	8.826	23.803	22.434

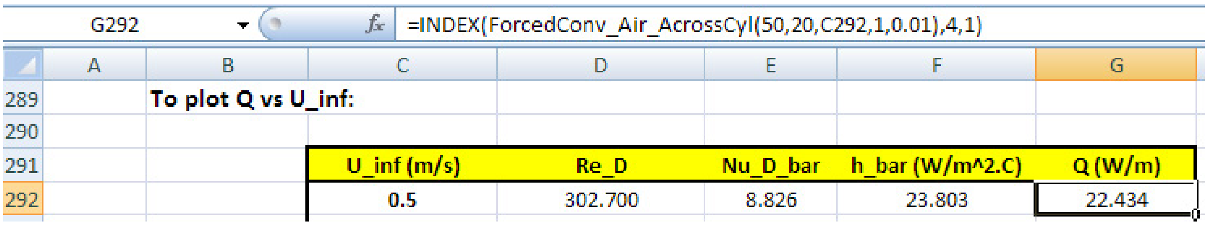
6. And, for h_bar (See the Formula bar):



F292 =INDEX(ForcedConv_Air_AcrossCyl(50,20,C292,1,0.01),3,1)

	A	B	C	D	E	F	G
289		**To plot Q vs U_inf:**					
290							
291			**U_inf (m/s)**	**Re_D**	**Nu_D_bar**	**h_bar (W/m^2.C)**	**Q (W/m)**
292			**0.5**	302.700	8.826	23.803	22.434

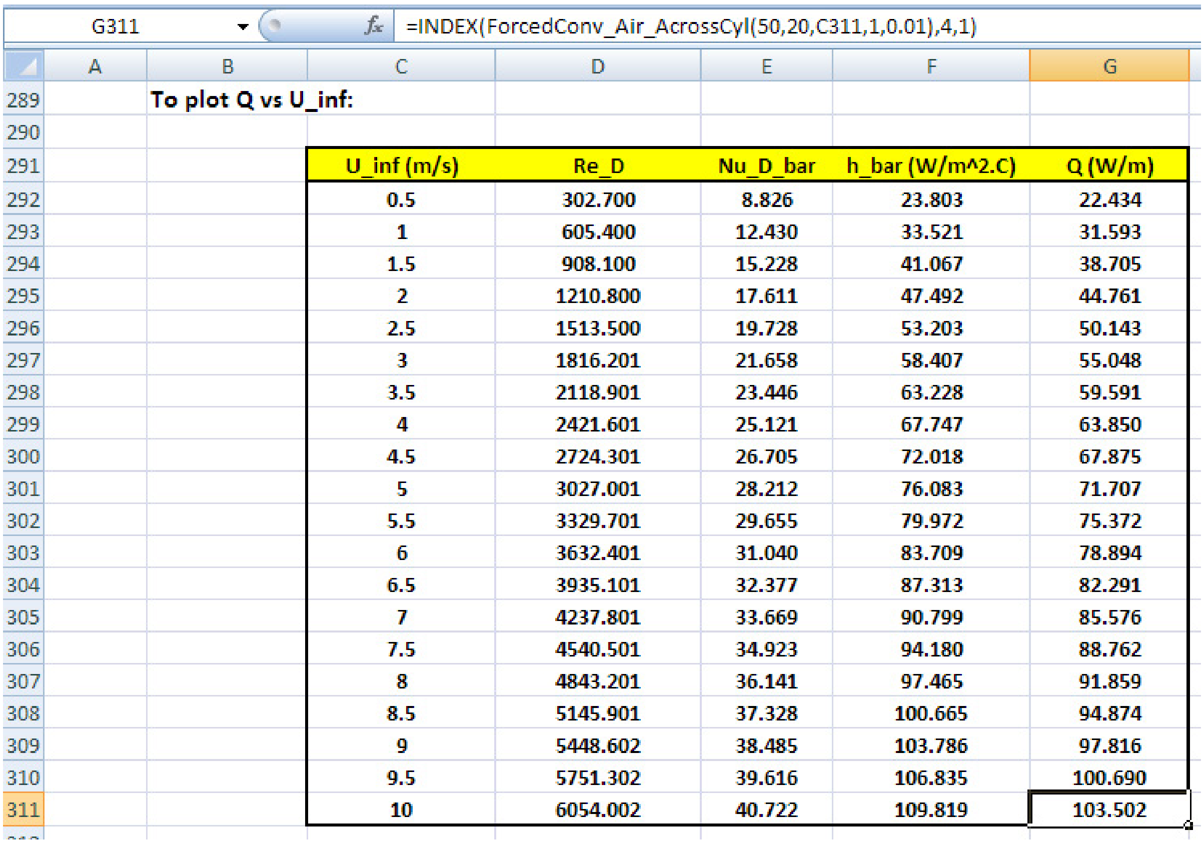
7. And, for Q:



G292 =INDEX(ForcedConv_Air_AcrossCyl(50,20,C292,1,0.01),4,1)

	A	B	C	D	E	F	G
289		**To plot Q vs U_inf:**					
290							
291			**U_inf (m/s)**	**Re_D**	**Nu_D_bar**	**h_bar (W/m^2.C)**	**Q (W/m)**
292			**0.5**	302.700	8.826	23.803	22.434

8. Now, select cells D292 to G292 and 'drag copy' to the end of the Table, i.e. up to cell G311. We see that the entire Table gets filled up:



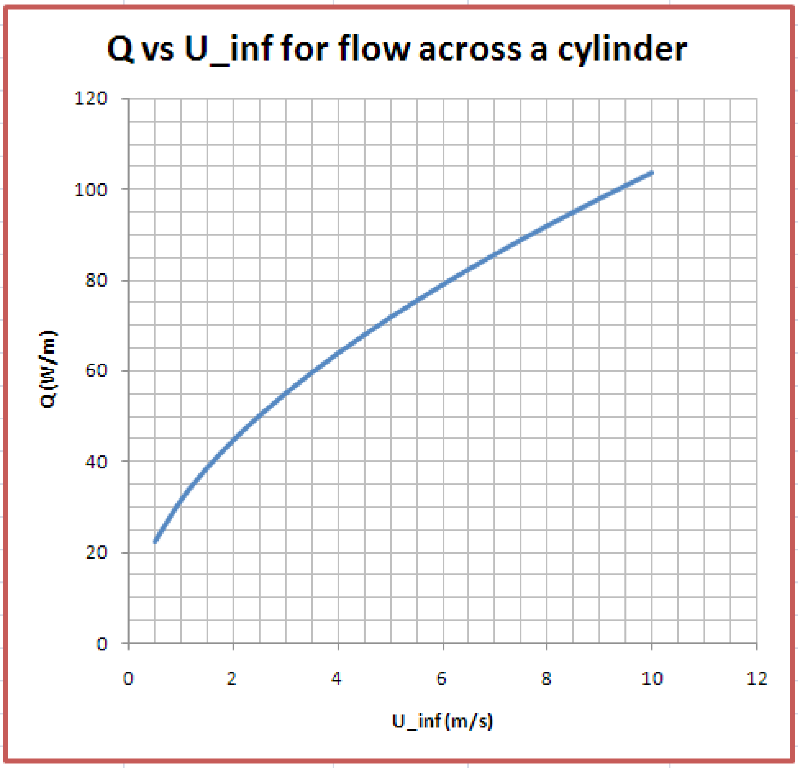
G311 =INDEX(ForcedConv_Air_AcrossCyl(50,20,C311,1,0.01),4,1)

289 **To plot Q vs U_inf:**

291	U_inf (m/s)	Re_D	Nu_D_bar	h_bar (W/m^2.C)	Q (W/m)
292	0.5	302.700	8.826	23.803	22.434
293	1	605.400	12.430	33.521	31.593
294	1.5	908.100	15.228	41.067	38.705
295	2	1210.800	17.611	47.492	44.761
296	2.5	1513.500	19.728	53.203	50.143
297	3	1816.201	21.658	58.407	55.048
298	3.5	2118.901	23.446	63.228	59.591
299	4	2421.601	25.121	67.747	63.850
300	4.5	2724.301	26.705	72.018	67.875
301	5	3027.001	28.212	76.083	71.707
302	5.5	3329.701	29.655	79.972	75.372
303	6	3632.401	31.040	83.709	78.894
304	6.5	3935.101	32.377	87.313	82.291
305	7	4237.801	33.669	90.799	85.576
306	7.5	4540.501	34.923	94.180	88.762
307	8	4843.201	36.141	97.465	91.859
308	8.5	5145.901	37.328	100.665	94.874
309	9	5448.602	38.485	103.786	97.816
310	9.5	5751.302	39.616	106.835	100.690
311	10	6054.002	40.722	109.819	103.502

Verify that for U_inf = 5 m/s (i.e. row 301), we get the same results that we got in the first part of this problem, confirming that the calculations are correct.

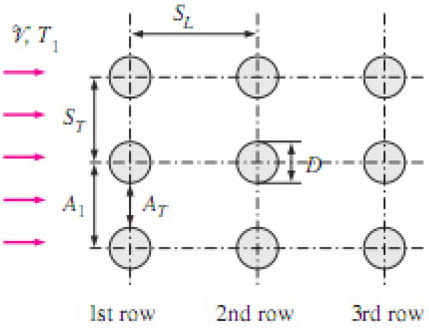
9. Now, plot the graph:



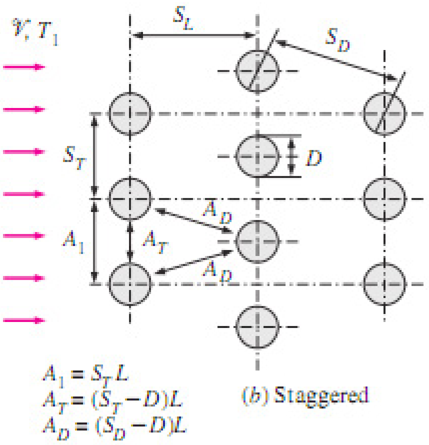
==

2A1.3 Flow across Tube banks:

Data from Ref.[2]:



(*a*) In-line



(*b*) Staggered

FIGURE 7–26
Arrangement of the tubes in in-line and staggered tube banks (A_1, A_T, and A_D are flow areas at indicated locations, and L is the length of the tubes).

Reynolds No. is based on Max. velocity, given by:

For In-line arrangement: $$\mathcal{V}_{\max} = \frac{S_T}{S_T - D}\mathcal{V}$$

Staggered and $S_D < (S_T + D)/2$: $$\mathcal{V}_{\max} = \frac{S_T}{2(S_D - D)}\mathcal{V}$$

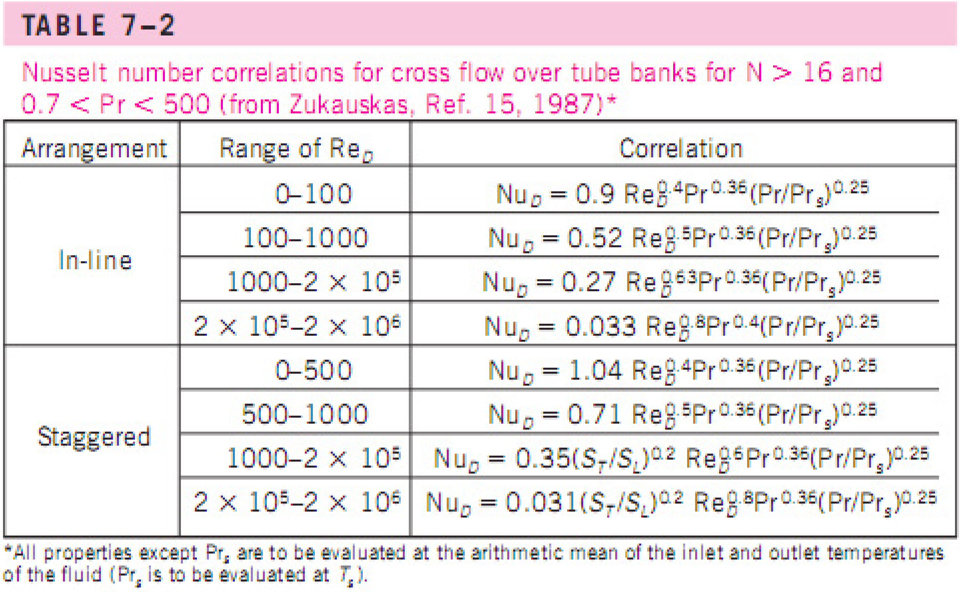


TABLE 7–2

Nusselt number correlations for cross flow over tube banks for N > 16 and 0.7 < Pr < 500 (from Zukauskas, Ref. 15, 1987)*

Arrangement	Range of Re_D	Correlation
In-line	0–100	$Nu_D = 0.9\ Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	100–1000	$Nu_D = 0.52\ Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	1000–2 × 10^5	$Nu_D = 0.27\ Re_D^{0.63} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	2 × 10^5–2 × 10^6	$Nu_D = 0.033\ Re_D^{0.8} Pr^{0.4} (Pr/Pr_s)^{0.25}$
Staggered	0–500	$Nu_D = 1.04\ Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	500–1000	$Nu_D = 0.71\ Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	1000–2 × 10^5	$Nu_D = 0.35 (S_T/S_L)^{0.2}\ Re_D^{0.6} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	2 × 10^5–2 × 10^6	$Nu_D = 0.031 (S_T/S_L)^{0.2}\ Re_D^{0.8} Pr^{0.36} (Pr/Pr_s)^{0.25}$

*All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid (Pr_s is to be evaluated at T_s).

Apply a Correction factor (F) to Nusselts No. when the No. of rows in flow direction, NL < 16:

$$Nu_{D,N_L} = F Nu_D$$

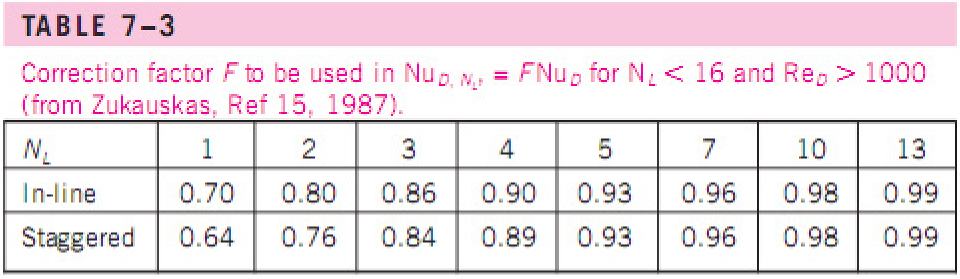


TABLE 7–3

Correction factor F to be used in $Nu_{D,N_L} = F Nu_D$ for $N_L < 16$ and $Re_D > 1000$ (from Zukauskas, Ref 15, 1987).

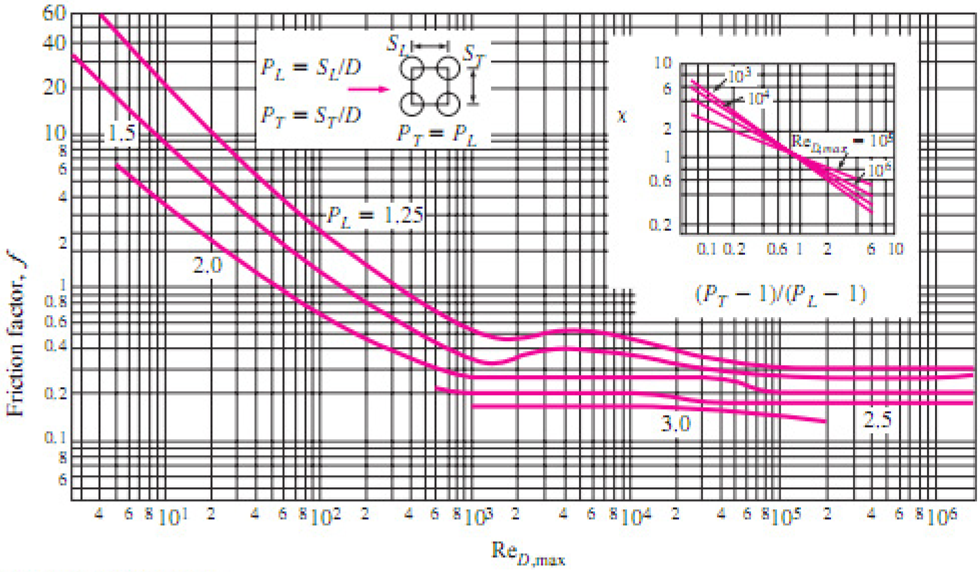
N_L	1	2	3	4	5	7	10	13
In-line	0.70	0.80	0.86	0.90	0.93	0.96	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.93	0.96	0.98	0.99

Correlations for Pressure drop:

$$\Delta P = N_L f \chi \frac{\rho V_{max}^2}{2}$$

where f = friction factor, X is the correction factor = 1 for In-line arrangement

Graphs for f and X for In-line and Staggered arrangement of tubes:



(a) In-line arrangement

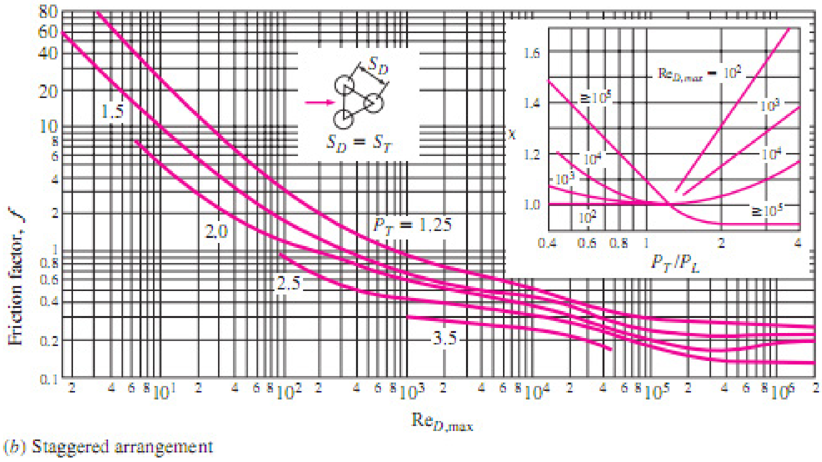
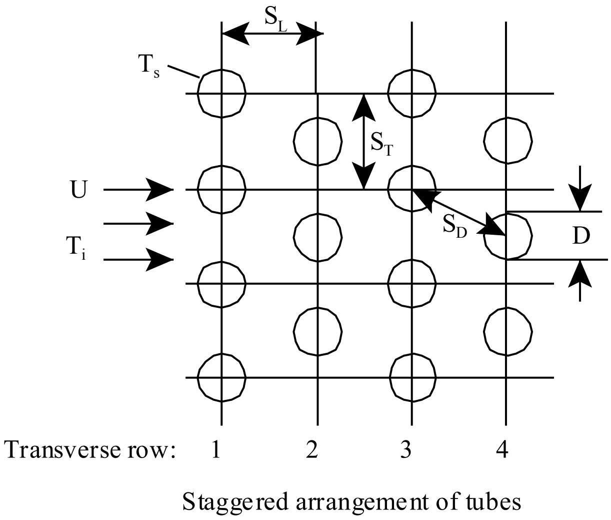


FIGURE 7–27
Friction factor f and correction factor χ for tube banks (from Zukauskas, Ref. 16, 1985).

===

Prob. 2A1.3.1: Air at 1 atm, 20 C and flowing at 5.2 m/s is to be heated by passing it over a bank of 1 m long tubes inside which steam is condensing at 100 C. OD of tubes is 1.6 cm. and arranged in a ***staggered manner*** with $S_L = S_T = 4$ cm. There are 20 rows in the flow direction and 10 tubes in each row. Determine: (a) the heat transfer rate (b) the pressure drop.



Prob.2A1.3.1.

Mathcad Solution:

First of all, write Mathcad functions for Nu for In-line and staggered arrangements of tubes:

For In-line arrangement of tubes:

```
$\text{Nu\_Tubebank\_Inline}(Re_D, Pr, Pr_s) :=$
    return "Re must be between 0 and 2*10^6"   if $Re_D > 2\cdot10^6$
    return "Pr must be > 0.7"   if $Pr < 0.7$
    return "Pr must be < 500  "   if $Pr > 500$
    return $0.9\cdot Re_D^{0.4}\cdot Pr^{0.36}\cdot\left(\frac{Pr}{Pr_s}\right)^{0.25}$   if $Re_D \le 100$
    return $0.52\cdot Re_D^{0.5}\cdot Pr^{0.36}\cdot\left(\frac{Pr}{Pr_s}\right)^{0.25}$   if $Re_D \le 1000$
    return $0.27\cdot Re_D^{0.63}\cdot Pr^{0.36}\cdot\left(\frac{Pr}{Pr_s}\right)^{0.25}$   if $Re_D \le 2\cdot10^5$
    return $0.033\cdot Re_D^{0.8}\cdot Pr^{0.4}\cdot\left(\frac{Pr}{Pr_s}\right)^{0.25}$   if $Re_D \le 2\cdot10^6$
```

For staggered arrangement of tubes:

```
$\text{Nu\_Tubebank\_Staggered}(Re_D, Pr, Pr_s, S_T, S_L) :=$
    return "Re must be between 0 and 2*10^6"   if $Re_D > 2\cdot10^6$
    return "Pr must be > 0.7"   if $Pr < 0.7$
    return "Pr must be < 500  "   if $Pr > 500$
    return $1.04\cdot Re_D^{0.4}\cdot Pr^{0.36}\cdot\left(\frac{Pr}{Pr_s}\right)^{0.25}$   if $Re_D \le 500$
    return $0.71\cdot Re_D^{0.5}\cdot Pr^{0.36}\cdot\left(\frac{Pr}{Pr_s}\right)^{0.25}$   if $Re_D \le 1000$
    return $0.35\cdot\left(\frac{S_T}{S_L}\right)^{0.2}\cdot Re_D^{0.6}\cdot Pr^{0.36}\cdot\left(\frac{Pr}{Pr_s}\right)^{0.25}$   if $Re_D \le 2\cdot10^5$
    return $0.031\cdot\left(\frac{S_T}{S_L}\right)^{0.2}\cdot Re_D^{0.8}\cdot Pr^{0.36}\cdot\left(\frac{Pr}{Pr_s}\right)^{0.25}$   if $Re_D \le 2\cdot10^6$
```

Now, the Solution:

Data:

$T_s := 100$ C....surface temp.

$T_i := 20$ C....air free stream temp.

$U := 5.2$ m/s....free stream velocity

$S_T := 0.04$ m...transverse length

$S_L := 0.04$ m...longitudinal length

$D := 0.016$ m....dia. of tubes

$L := 1$ m ... length of each tube

$N_L := 20$ ...No. of rows in flow direction

$N_T := 10$ ...No. of tubes in each row

Reynolds number:

This is based on U_{max}. We have, for staggered arrangement:

$$S_D := \left[S_L^2 + \left(\frac{S_T}{2}\right)^2\right]^{\frac{1}{2}} \qquad \text{i.e.} \qquad S_D = 0.045 \quad \text{m}$$

And, $\frac{S_T + D}{2} = 0.028$

..Therefore S_D is greater than $(S_T+D)/2$. So, U_{max} is given by:

$$U_{max} := \frac{S_T}{S_T - D} \cdot U$$

i.e. $U_{max} = 8.667$ m/s....max. velocity

We do not know the exit temp T_0 yet.

Let us assume T_0 as 70 deg. C.

Then, the mean fluid temp is (20 + 70)/2 = 45 C.

Take properties of air at T_f and then correct later if required.

$T_o := 70$ C ... **assumed**

$T_f(T_o) := \frac{(T_o + T_i)}{2}$ i.e. $T_f(T_o) = 45$ C.. T_f is written as a function of To for use later.

Taking properties of air at $T_f = 45$ C, using the Mathcad functions for properties of Air (See prob. 2A1.2.5):

Note that for the following functions for properties of Air, temp. must be in Kelvin:

$\rho := \text{rho\_Air}(T_f(T_o) + 273)$ i.e. $\rho = 1.11$ kg/m^3....density

$k := \text{k\_Air}(T_f(T_o) + 273)$ i.e. $k = 0.027$ W/(m.K)....thermal conductivity

$\nu := \text{nu\_Air}(T_f(T_o) + 273)$ i.e. $\nu = 1.738 \cdot 10^{-5}$ m^2/s....kinematic viscosity

$Pr := \text{Pr\_Air}(T_f(T_o) + 273)$ i.e. $Pr = 0.704$

$Pr_s := \text{Pr\_Air}(T_s + 273)$ i.e. $Pr_s = 0.695$

Therefore, $Re_D(T_o) := \frac{U_{max} \cdot D}{nu\_Air(T_f(T_o) + 273)}$

i.e. $Re_D(T_o) = 7.979 \cdot 10^3$ ....Reynolds number

Nusselts number:

Appropriate eqn. for average Nusselts number is automatically selected in the Mathcad function for Nu for Staggered Tube bank:

$$Nu_a(T_o) := Nu\_Tubebank\_Staggered(Re_D(T_o), Pr\_Air(T_f(T_o) + 273), Pr_s, S_T, S_L)$$

i.e. $Nu_a(T_o) = 67.903$

Therefore, avg. heat transfer coeff. is:

$$h_a(T_o) := \frac{Nu_a(T_o) \cdot k\_Air(T_f(T_o) + 273)}{D}$$ W/(m².C)....average heat transfer coeff.

i.e. $h_a(T_o) = 115.723$ W/(m^2.C)....average heat transfer coeff. (N > 20)

This is the value of heat transfer coeff. that would be obtained if there were 20 or more rows of tubes in the direction of flow. And, in the present case, there are 20 rows in the direction of flow. **So, no correction factor is required:**

Surface area for heat transfer for unit length of tubes is:

$A := (N_L \cdot N_T) \cdot (\pi \cdot D \cdot L)$ m^2/m....for 10 rows high, 20 rows deep

i.e. $A = 10.053$ m^2.

Total heat transfer rate, Q:

Now, total heat transfer rate is given by Newton's Law:

$$Q = h_a \cdot A \cdot \Delta T$$

Here ΔT is the average temp. difference between the wall and the air stream. However, temp. of air stream goes on changing from entry to exit in the heat exchanger. So, **we use a 'mean temperature difference' called LMTD (log. mean temp. difference).** Expression for LMTD is derived in the chapter on heat exchangers. For the present, let us take for LMTD:

$$\mathrm{LMTD}=\frac{\left(T_s-T_i\right)-\left(T_s-T_o\right)}{\ln\left(\frac{T_s-T_i}{T_s-T_o}\right)}$$

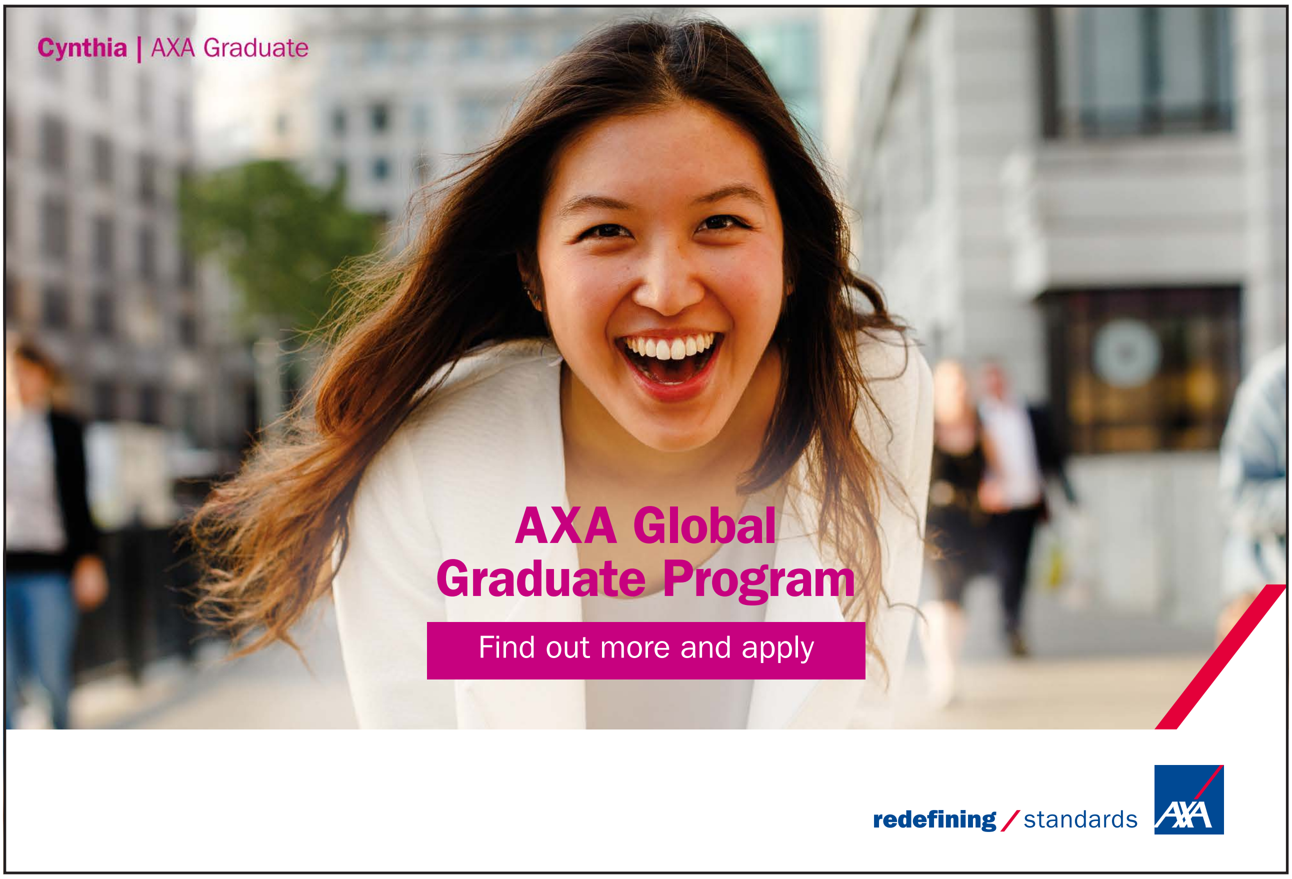
We need the exit temp. To of the air stream. This is calculated by a heat balance:

Q = ha.A.(LMTD)

$$\mathrm{mass\_flow} := \mathrm{rho\_Air}\left(T_i+273\right)\cdot U\cdot\left(N_T\cdot S_T\cdot L\right)$$

...kg/s...mass flow rate; 10 rows high, S_T is transverse dist.

i.e. $\mathrm{mass\_flow} = 2.506$ kg/s



Then, we can write the heat balance:

$$h \cdot A \cdot LMTD = mass\_flow \cdot cp \cdot (T_o - T_i)$$

Substitute for LMTD and solve for To:

Also:

$$cp\_Air(T_o) := \frac{k\_Air(T_f(T_o) + 273) \cdot Pr\_Air(T_f(T_o) + 273)}{mu\_Air(T_f(T_o) + 273)}$$ ..Mathcad function to find cp of Air

Example: $T_o = 70$ i.e. $cp\_Air(T_o) = 995.644$ J/kg.K

Use Solve block of Mathcad; assume a guess value for T_o to start with, say T_o = 70 C. Then type 'Given' and write the constraint; then type Find(T_o) and get the answer:

$T_o := 70$ ....guess value... we already assumed this value in the beginning

Given

$$h_a(T_o) \cdot A \cdot \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left(\frac{T_s - T_i}{T_s - T_o}\right)} = mass\_flow \cdot cp\_Air(T_o) \cdot (T_o - T_i)$$

$T_o := Find(T_o)$ ....Finds To

i.e. $T_o = 50.006$ **C...exit air temp.... Ans.**

Therefore, heat transfer rate, Q:

$$Q(T_o) := h_a(T_o) \cdot A \cdot \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left(\frac{T_s - T_i}{T_s - T_o}\right)}$$

i.e. $Q(T_o) = 74837.233$ **W/m = 74.837 kW/m.....Ans.**

Alternatively, we can use the arithmetic average value of air stream between the inlet and outlet temp; this is simpler to calculate and error involved will not be much:

Then, $$Q=h_a\cdot A\cdot\left[T_s-\left(\frac{T_o+T_i}{2}\right)\right]=mass\_flow\cdot C_p\cdot\left(T_o-T_i\right)$$

Using Solve block as earlier, to obtain To:

Given

$$h_a\left(T_o\right)\cdot A\cdot\left[T_s-\left(\frac{T_o+T_i}{2}\right)\right]=mass\_flow\cdot cp\_Air\left(T_o\right)\cdot\left(T_o-T_i\right)$$

$\mathrm{Find}\left(T_o\right)=50.447$

i.e. $T_o=50.447$ C...exit air temp.

i.e. we get practically the same value for T_o as compared to 50.006 C obtained earlier.

Pressure drop:

We have: $Re_D\left(T_o\right)=8447.683$ $T_o=50.006$ C

$$\Delta p=N_L\cdot X\cdot\left(\frac{\rho\cdot U_{max}^2}{2}\right)\cdot f$$ Pa......

$N_L=20$ ....no. of longitudinal rows

$P_T:=\frac{S_T}{D}$ i.e. $P_T=2.5$

$P_L:=\frac{S_L}{D}$ i.e. $P_L=2.5$

Therefore: $\frac{P_T}{P_L} = 1$

Then, X = 1, f = 0.31 from graph above

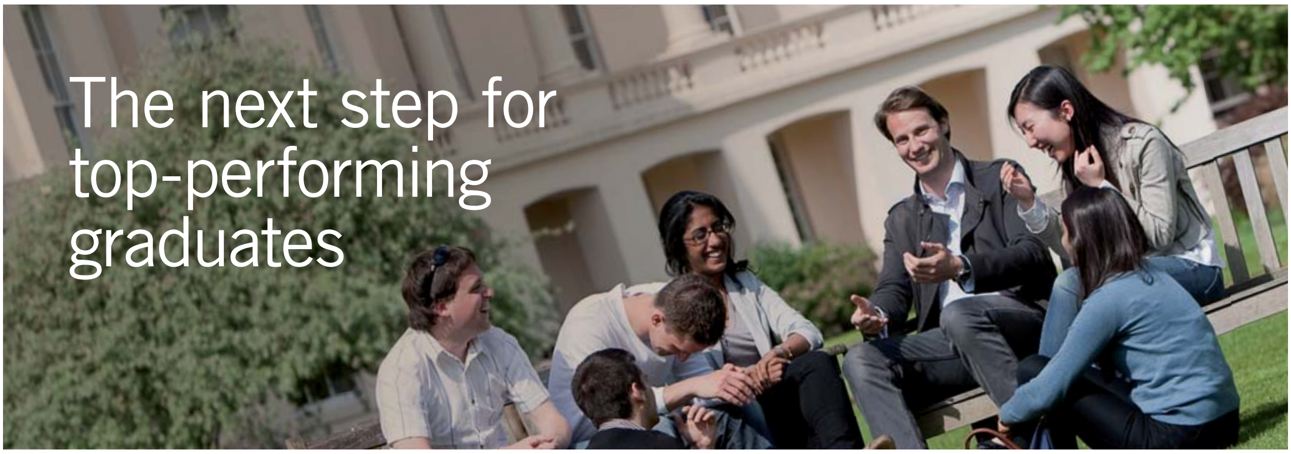
And, $f := 0.31$ $X := 1$

$\rho := \text{rho\_Air}\left(T_f\left(T_o\right) + 273\right)$ $\rho = 1.146$ kg/m^3 at when $T_o = 50.006$ C

Therefore:

$$\Delta P := N_L \cdot X \cdot \left(\frac{\rho \cdot U_{max}^2}{2}\right) \cdot f \quad \text{Pa......}$$

i.e. $\Delta P = 266.898$ **Pa... pressure drop...Ans.**



Alternatively:

Find Pressure drop from formulas given in Ref.[1]:

$$\text{DELTA\_P} = \frac{2 \cdot ff \cdot G_{max}{}^{2} \cdot N_L}{\rho_{in}} \cdot \left(\frac{mu\_w}{mu\_b}\right)^{0.14}$$

where ff = friction factor, given by:

$$ff = \left[0.25 + \frac{0.118}{\left(\frac{S_T - D}{D}\right)^{1.08}}\right] \cdot Re_D{}^{-0.16}$$ ....for staggered tube arrangement

Therefore:

$$ff := \left[0.25 + \frac{0.118}{\left(\frac{S_T - D}{D}\right)^{1.08}}\right] \cdot Re_D(T_o)^{-0.16}$$ i.e. $ff = 0.077$

$$U_{max}(U) := \frac{S_T}{S_T - D} \cdot U$$

$U = 5.2$ m/s $U_{max}(U) = 8.667$ m/s

$T_o = 50.006$ C

$T_f(T_o) = 35.003$ C

$\rho_{in} := rho\_Air(T_i + 273)$ ...density at free stream condition

i.e $\rho_{in} = 1.205$ kg/m^3

$Re_D(T_o) = 8447.683$

$G_{max} := \rho \cdot U_{max}(U)$ $G_{max} = 9.934$ kg/s.m^2

$mu\_w := mu\_Air(T_s + 273)$ $mu\_w = 2.181 \cdot 10^{-5}$ kg/m.s

$mu\_b := mu\_Air(T_f(T_o) + 273)$ $mu\_b = 1.884 \cdot 10^{-5}$ kg/m.s

Therefore:

$$DELTA\_P := \frac{2 \cdot ff \cdot G_{max}^2 \cdot N_L}{\rho_{in}} \cdot \left(\frac{mu\_w}{mu\_b}\right)^{0.14}$$

i.e. $DELTA\_P = 256.734$ **N/m^2.....Ans.**

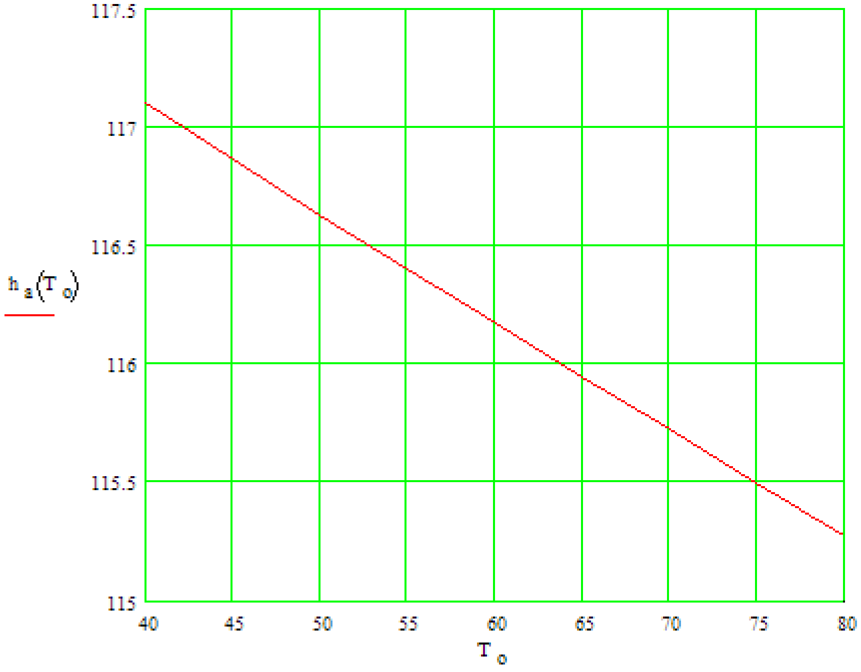
This value of pressure drop matches well with the one obtained earlier, using the graphs.

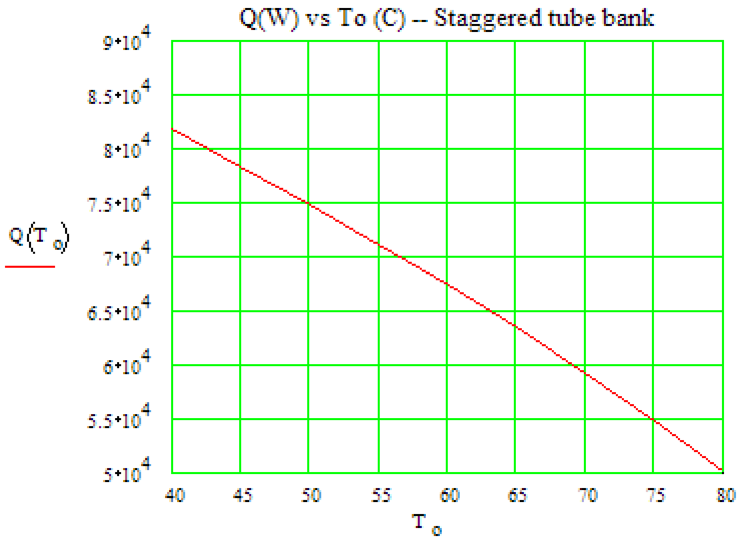
==

For given Tube bank, and given velocity of U = 5.2 m/s, let us plot the variation of h and Q with air exit temp. To:

$T_o := 40, 45 .. 80$ C....variation of air exit temp.

h (W/m^2.K) vs To (C)

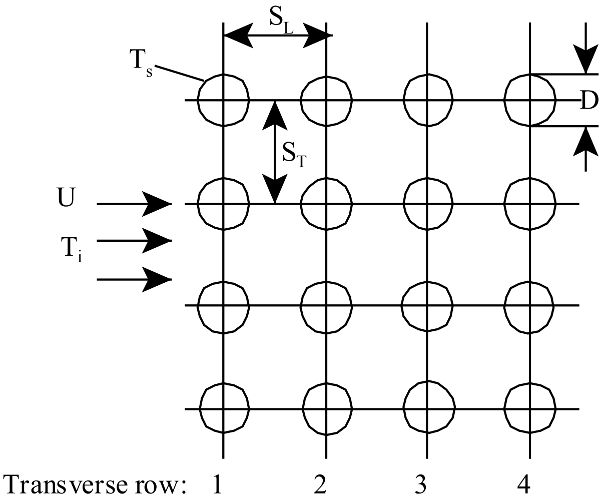



Q(W) vs To (C) -- Staggered tube bank
9•10^4
8.5•10^4
8•10^4
7.5•10^4
7•10^4
6.5•10^4
6•10^4
5.5•10^4
5•10^4
Q(T o)
40
45
50
55
60
65
70
75
80
T o

==

$UnitSystem SI Pa C J

"**Prob. 2A1.3.2**. A pre-heater for air has surface temp of tubes at 100 C and in the tube bank there are a total of 196 tubes arranged in a ***square, aligned array***. Tube dia = 10 mm, 1 m long and S_T = S_L = 15 mm. Air enters at 1 atm, 25 C and at 5 m/s. What is the total rate of heat transfer to air? Also, find the pressure drop."



Prob.2A1.3.2.

"EES Solution:"

"This problem is similar to the previous problem.

It is convenient to solve the problem with EES since the properties of air are available in EES as built-in functions."

"First, let us write functions for Nusselts No. for cross flow across Tube bank with In-line and Staggered arrangements:"

```
Function NUSSELT_TubeBank_In_line(Re_D, Pr, Pr_s)

   If (Re_D <= 100) Then
      NUSSELT_TubeBank_In_line:= 0.9 * Re_D^0.4 * Pr^0.36 * (Pr / Pr_s)^0.25
   Else
      If (Re_D > 100) And (Re_D <=1000) Then
         NUSSELT_TubeBank_In_line := 0.52 * Re_D^0.5 * Pr^0.36 * (Pr / Pr_s)^0.25
      Else
         If (Re_D > 1000) And (Re_D <=2E05) Then
```

```
                NUSSELT_TubeBank_In_line := 0.27 * Re_D^0.63 * Pr^0.36 * (Pr / Pr_s)^0.25
            Else
                If (Re_D > 2E05) And (Re_D <=2E06) Then
                    NUSSELT_TubeBank_In_line := 0.033 * Re_D^0.8 * Pr^0.4 * (Pr / Pr_s)^0.25
                Else
                    Call Error (' Re_D must be between 0 and 2E06 !!')
                EndIf
            EndIf
        EndIf
    EndIf
End

"                                                                                              "

Function NUSSELT_TubeBank_Staggered(Re_D, Pr, Pr_s,S_T,S_L)

    If (Re_D <= 500) Then
        NUSSELT_TubeBank_Staggered= 1.04 * Re_D^0.4 * Pr^0.36 * (Pr / Pr_s)^0.25
    Else
        If (Re_D > 500) And (Re_D <=1000) Then
            NUSSELT_TubeBank_Staggered:= 0.71 * Re_D^0.5 * Pr^0.36 * (Pr / Pr_s)^0.25
        Else
            If (Re_D > 1000) And (Re_D <=2E05) Then
                NUSSELT_TubeBank_Staggered:= 0.35 * (S_T / S_L)^0.2 * Re_D^0.6 * Pr^0.36 * (Pr /
                Pr_s)^0.25
            Else
                If (Re_D > 2E05) And (Re_D <=2E06) Then
                    NUSSELT_TubeBank_Staggered := 0.031 * (S_T / S_L)^0.2 * Re_D^0.8 * Pr^0.36 * (Pr
                    / Pr_s)^0.25
                Else
                    Call Error (' Re_D must be between 0 and 2E06 !!')
                EndIf
            EndIf
        EndIf
    EndIf
End

"                                                                                              "
```

"For In-line tube arrangement:
Correction Factor for N_L less than 14:

F_InLine=0.568730388 + 0.155621584*N_L – 0.0243568396*N_L^2 + 0.00178191834*N_L^3 – 0.0000490364034*N_L^4 "

"For Staggered tube arrangement:
Correction Factor for N_L less than 14:

F_staggered=0.47630486 + 0.191385976*N_L – 0.0289872621*N_L^2 + 0.00201365228*N_L^3 – 0.0000525013679*N_L^4"

"---"

```
“Correction Factors for In-Line tube banks, if the N_L, no. of rows in Longitudinal direction is less than 14:”

“Curve-fit from EES:”

Function F_InLine (N_L)

If (N_L < 14) Then

   F_InLine=0.568730388 + 0.155621584*N_L – 0.0243568396*N_L^2 + 0.00178191834*N_L^3 – 0.0000490364034*N_L^4

Else

   F_InLine = 1

EndIf

End
“------------------------------------------------------------------------------------------------------------------------------------------------”

“Correction Factors for Staggered tube banks, if the N_L, no. of rows in Longitudinal direction is less than 14:”

“Curve-fit from EES:”

Function F_Staggered (N_L)

If (N_L < 14) Then

   F_Staggered=0.47630486 + 0.191385976*N_L – 0.0289872621*N_L^2 + 0.00201365228*N_L^3 – 0.0000525013679*N_L^4

Else

   F_Staggered = 1

EndIf

End

“=================================================================”
```

```
"Data:"

T_s = 100 [C]
T_i = 25 [C]
{T_o = 70 [C] "…. assumed. Will be commented out later"}
U = 5 [m/s]
D = 0.01 [m]
L = 1 [m]
S_T = 0.015 [m]
S_L = 0.015 [m]
N_L = 14
N_T = 14
P = 1e05 [Pa]

"In-Line tube arrangement:"

"Calculations:"

T_f = (T_i + T_o) / 2 "..avg temp of air in the array"

"Properties of Air at T_f:"

rho=Density(Air,T=T_f,P=P) "[kg/m^3]"
rho_in = Density(Air,T=T_i,P=P) "[kg/m^3]"
cp=Cp(Air,T=T_f) "[J/kg-C]"
k=Conductivity(Air,T=T_f) "[W/m-K]"
mu=Viscosity(Air,T=T_f) "[kg/m-s]"
Pr=Prandtl(Air,T=T_f)"[-]"
Pr_s=Prandtl(Air,T=T_s)"[-]"
mu_s=Viscosity(Air,T=T_s) "[kg/m-s]"

"For In-Line arrangement:"

U_max = (S_T / (S_T – D)) * U "[m/s] …. max. velocity"
Re_D = U_max * D * rho / mu "…Reynolds No."

"Therefore: Nusselts No.:"

Nusselt_D = NUSSELT_TubeBank_In_line(Re_D, Pr, Pr_s)

"Note: No correction factor to Nusselts No. since N_L = 14"
```

"Therefore: heat transfer coeff.:"

h = Nusselt_D * k / D "..[W/m^2-K] … heat tr. coeff."

Outlet temp. of Air:

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left[-h \cdot \frac{\pi \cdot D \cdot L \cdot N_T \cdot N_L}{Mass_{flow} \cdot cp}\right]$$ ... determines air outlet temp, T_o

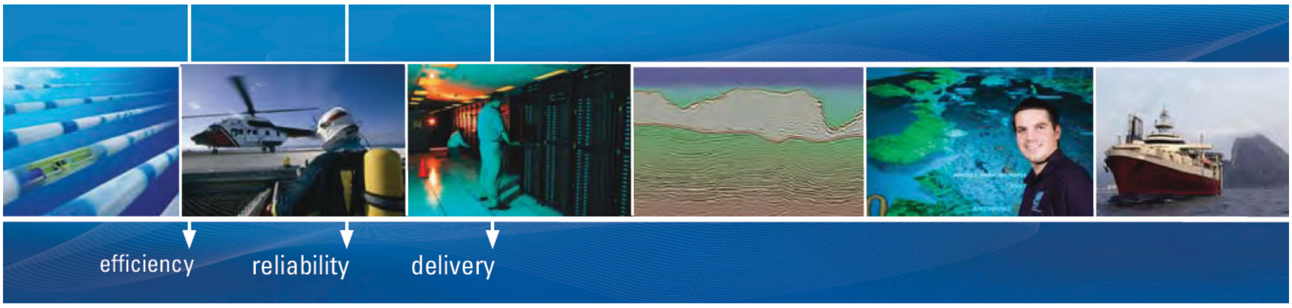
Enter it in EES:

(T_s – T_o) / (T_s – T_i) = exp(- h * (pi * D * L * N_T * N_L) / (Mass_flow * cp)) "… determines air outlet temp, T_o"

"Therefore: heat transferred:"

"Q is equal to heat gained by the air while passing through the array of tubes"

Mass_flow = rho_in * U * (N_T * S_T * L) "[kg/s]…mass flow rate through the array"



"And:"

Q = Mass_flow * cp * (T_o – T_i) "[W] … heat gained by air in the array"

"Pressure drop:[Ref:1]"

Friction factor for In-line arrangement is given by:

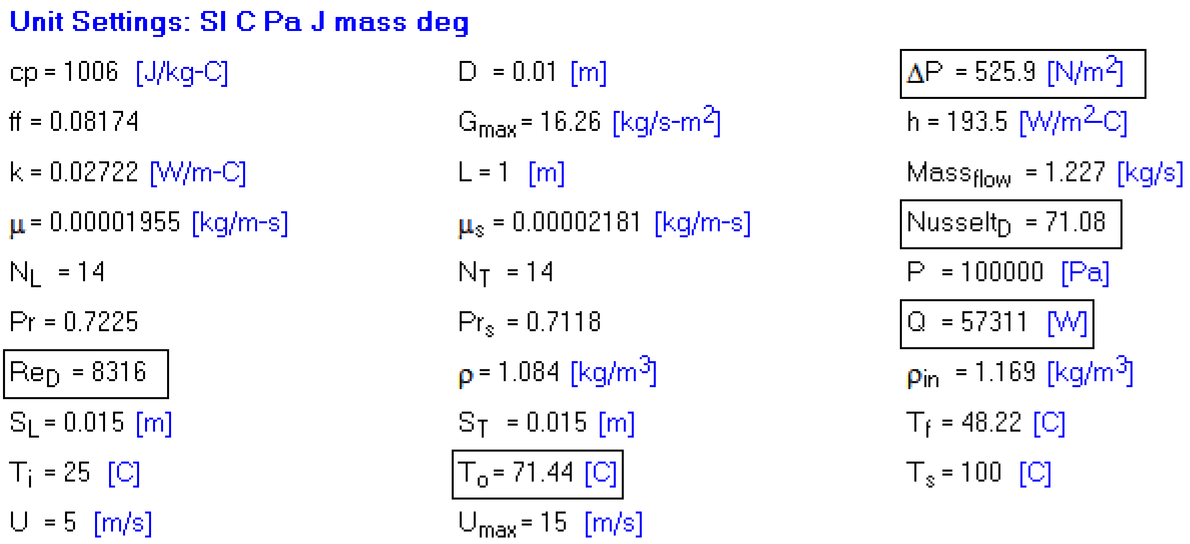
$$ff = \left[0.044 + \frac{0.08 \cdot \frac{S_L}{D}}{\left(\frac{S_T - D}{D} \right)^{\left(0.43 + 1.13 \cdot \frac{D}{S_L} \right)}} \right] \cdot Re_D^{-0.15} \quad \text{...friction factor}$$

i.e. in EES it is entered as:

ff = (0.044 + (0.08 * (S_L / D)) / ((S_T – D)/D)^(0.43 + 1.13 * D/S_L))*Re_D^(-0.15) "…friction factor"
G_max = rho * U_max "[kg/s-m^2] … mass velocity"

DELTAP = (2 * ff *G_max^2 * N_L / rho_in) * (mu_s / mu)^0.14 "[N/m^2] … pressure drop"

Results:



Unit Settings: SI C Pa J mass deg

$cp = 1006$ [J/kg-C]
$ff = 0.08174$
$k = 0.02722$ [W/m-C]
$\mu = 0.00001955$ [kg/m-s]
$N_L = 14$
$Pr = 0.7225$
$\boxed{Re_D = 8316}$
$S_L = 0.015$ [m]
$T_i = 25$ [C]
$U = 5$ [m/s]

$D = 0.01$ [m]
$G_{max} = 16.26$ [kg/s-m^2]
$L = 1$ [m]
$\mu_s = 0.00002181$ [kg/m-s]
$N_T = 14$
$Pr_s = 0.7118$
$\rho = 1.084$ [kg/m^3]
$S_T = 0.015$ [m]
$\boxed{T_o = 71.44 \text{ [C]}}$
$U_{max} = 15$ [m/s]

$\boxed{\Delta P = 525.9 \text{ [N/m}^2\text{]}}$
$h = 193.5$ [W/m^2-C]
$Mass_{flow} = 1.227$ [kg/s]
$\boxed{Nusselt_D = 71.08}$
$P = 100000$ [Pa]
$\boxed{Q = 57311 \text{ [W]}}$
$\rho_{in} = 1.169$ [kg/m^3]
$T_f = 48.22$ [C]
$T_s = 100$ [C]

Thus:

Exit temp. of Air = T_o = 71.44 C …Ans.

Heat transfer Q = 57311 W …. Ans.

Pressure drop DELTAP = 525.9 N/m^2 … Ans.

Plot the variation of exit temp of air (T_o) and heat transferred (Q) as Air velocity U varies from 1 to 10 m/s:

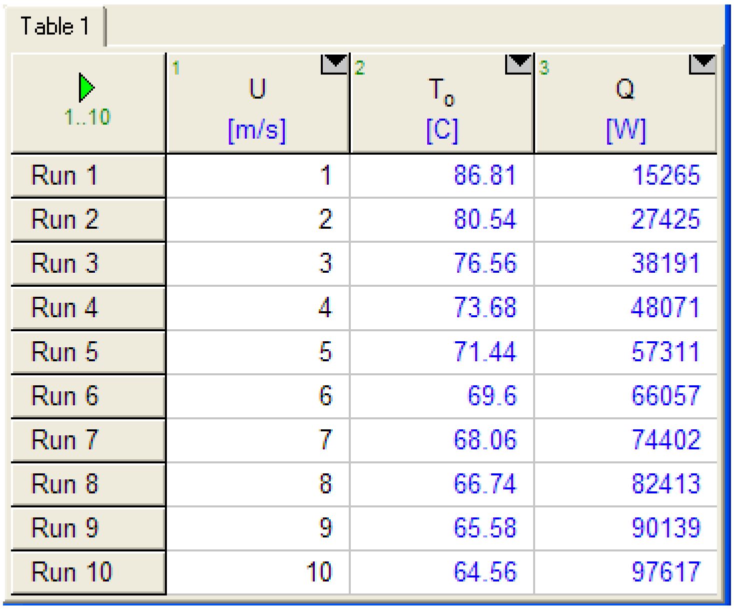
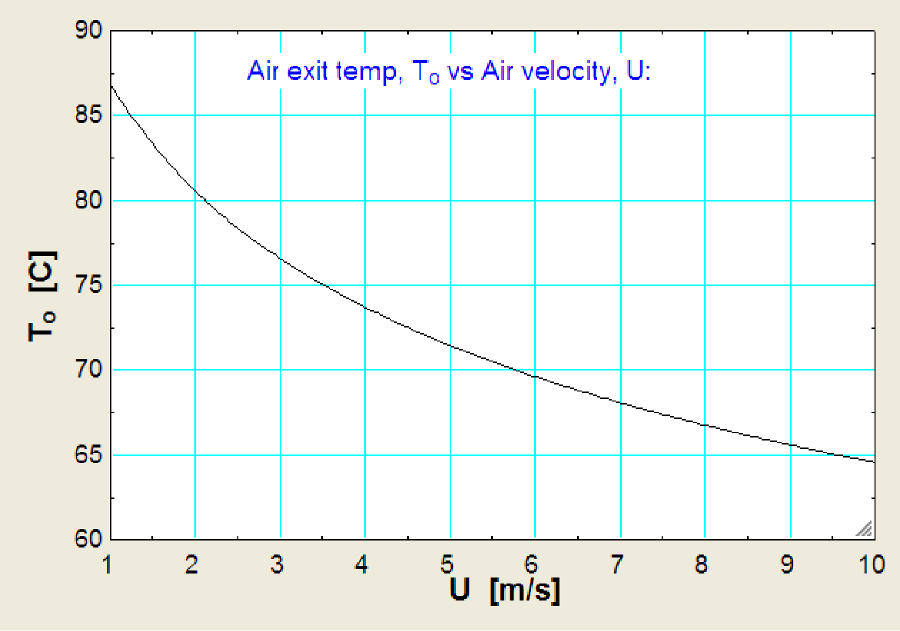
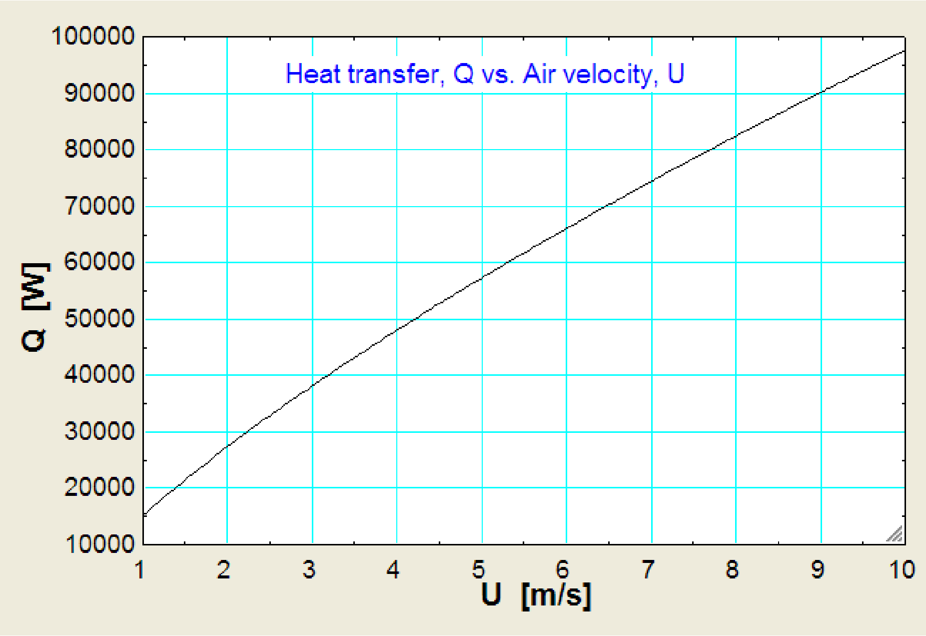


Table 1

1..10	U [m/s]	T_o [C]	Q [W]
Run 1	1	86.81	15265
Run 2	2	80.54	27425
Run 3	3	76.56	38191
Run 4	4	73.68	48071
Run 5	5	71.44	57311
Run 6	6	69.6	66057
Run 7	7	68.06	74402
Run 8	8	66.74	82413
Run 9	9	65.58	90139
Run 10	10	64.56	97617

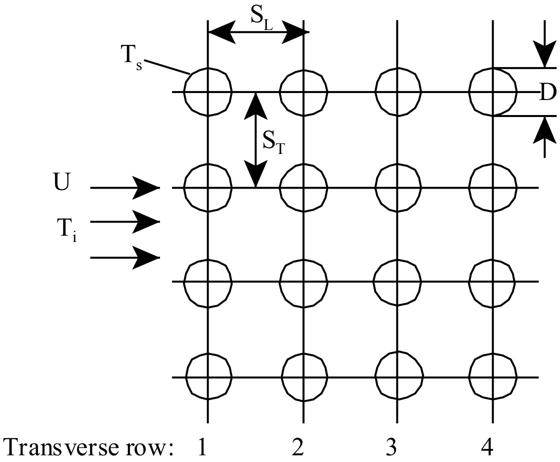



Heat transfer, Q vs. Air velocity, U
Q [W]
100000
90000
80000
70000
60000
50000
40000
30000
20000
10000
1
2
3
4
5
6
7
8
9
10
U [m/s]

===

$UnitSystem SI Pa C J

"**Prob. 2A1.3.3.** A tube bank has surface temp of tubes at 90 C and the tube bank has 6 rows of tubes, each stack 40 tubes high, in ***an in-line arrangement***. Tube dia = 6.33 mm mm, 1 m long and S_T = S_L = 1.9 cm. Air enters at 1 atm, 20 C and 4.5 m/s/. What is the total rate of heat transfer to air? Also, find the pressure drop."



Prob.2A1.3.3.

"EES Solution:"

"This problem is similar to the previous problem. See the EES Functions written in the previous problem for Nusselts No. and Correction factors for N_L < 14"

"Data:"

T_s = 90 [C]
T_i = 20 [C]
{T_o = 70 [C] "…. assumed. Will be commented out later"}
U = 4.5 [m/s]
D = 0.00633 [m]
L = 1 [m]
S_T = 0.019 [m]
S_L = 0.019 [m]
N_L = 6
N_T = 40
P = 1e05 [Pa]

```
"In-Line tube arrangement:"

"Calculations:"

T_f = (T_i + T_o) / 2 "..avg temp of air in the array"

"Properties of Air at T_f:"

rho=Density(Air,T=T_f,P=P) "[kg/m^3]"
rho_in = Density(Air,T=T_i,P=P) "[kg/m^3]"
cp=Cp(Air,T=T_f) "[J/kg-C]"
k=Conductivity(Air,T=T_f) "[W/m-K]"
mu=Viscosity(Air,T=T_f) "[kg/m-s]"
Pr=Prandtl(Air,T=T_f) "[-]"
Pr_s=Prandtl(Air,T=T_s) "[-]"
mu_s=Viscosity(Air,T=T_s) "[kg/m-s]"

"For In-Line arrangement:"

U_max = (S_T / (S_T – D)) * U "[m/s] …. max. velocity"
Re_D = U_max * D * rho / mu "…Reynolds No."

"Note: Correction factor to Nusselts No. has to be used since N_L =<14"

"This correction factor is obtained from the Table supplied in Text Books (ex. [2]), converted here into a EES curve fit eqn for convenience"

F = F_InLine (N_L) "…correction factor since there are less than 14 rows"

"Therefore: Nusselts No. (corrected):"

Nusselt_D = F * NUSSELT_TubeBank_In_line(Re_D, Pr, Pr_s)

"Therefore: heat transfer coeff.:"

h = Nusselt_D * k / D "..[W/m^2-K] … heat tr. coeff."

(T_s – T_o) / (T_s – T_i) = exp(- h * (pi * D * L * N_T * N_L) / (Mass_flow * cp)) "… determines air outlet temp, T_o"
```

"Therefore: heat transferred:"

"Q is also equal to heat gained by the air while passing through the array of tubes"

Mass_flow = rho_in * U * (N_T * S_T * L) "[kg/s]…mass flow rate through the array"

"And:"

Q = Mass_flow * cp * (T_o – T_i) "[W] … heat gained by air in the array"

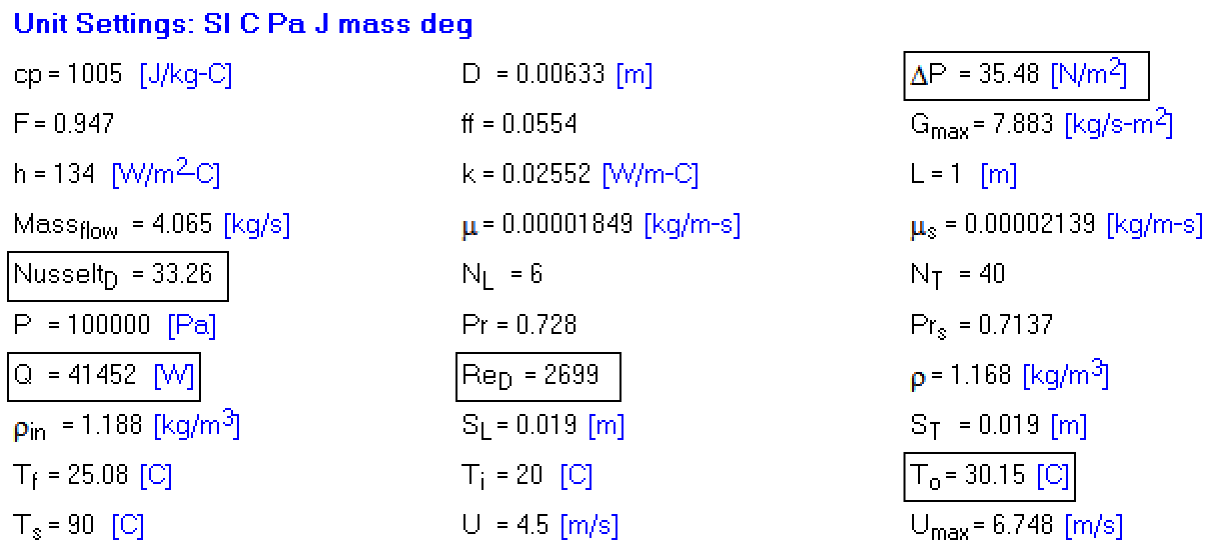
"Pressure drop:[Ref:1]"

ff = (0.044 + (0.08 * (S_L / D)) / ((S_T – D)/D)^(0.43 + 1.13 * D/S_L))*Re_D^(-0.15) "…friction factor"

G_max = rho * U_max "[kg/s-m^2] … mass velocity"

DELTAP = (2 * ff *G_max^2 * N_L / rho_in) * (mu_s / mu)^0.14 "[N/m^2] … pressure drop"

Results:



Unit Settings: SI C Pa J mass deg

$cp = 1005$ [J/kg-C]
$D = 0.00633$ [m]
$\boxed{\Delta P = 35.48 \text{ [N/m}^2\text{]}}$
$F = 0.947$
$ff = 0.0554$
$G_{max} = 7.883$ [kg/s-m^2]
$h = 134$ [W/m^2-C]
$k = 0.02552$ [W/m-C]
$L = 1$ [m]
$Mass_{flow} = 4.065$ [kg/s]
$\mu = 0.00001849$ [kg/m-s]
$\mu_s = 0.00002139$ [kg/m-s]
$\boxed{Nusselt_D = 33.26}$
$N_L = 6$
$N_T = 40$
$P = 100000$ [Pa]
$Pr = 0.728$
$Pr_s = 0.7137$
$\boxed{Q = 41452 \text{ [W]}}$
$\boxed{Re_D = 2699}$
$\rho = 1.168$ [kg/m^3]
$\rho_{in} = 1.188$ [kg/m^3]
$S_L = 0.019$ [m]
$S_T = 0.019$ [m]
$T_f = 25.08$ [C]
$T_i = 20$ [C]
$\boxed{T_o = 30.15 \text{ [C]}}$
$T_s = 90$ [C]
$U = 4.5$ [m/s]
$U_{max} = 6.748$ [m/s]

Thus:

Air exit temp. = T_o = 30.15 C… Ans.
Heat transferred = Q = 41452 W … Ans.
Pressure drop = DELTAP = 35.48 N/m^2 …. Ans.

Plot the variation of heat transferred (Q) and pressure drop DELTAP as Air velocity U varies from 1 to 10 m/s:

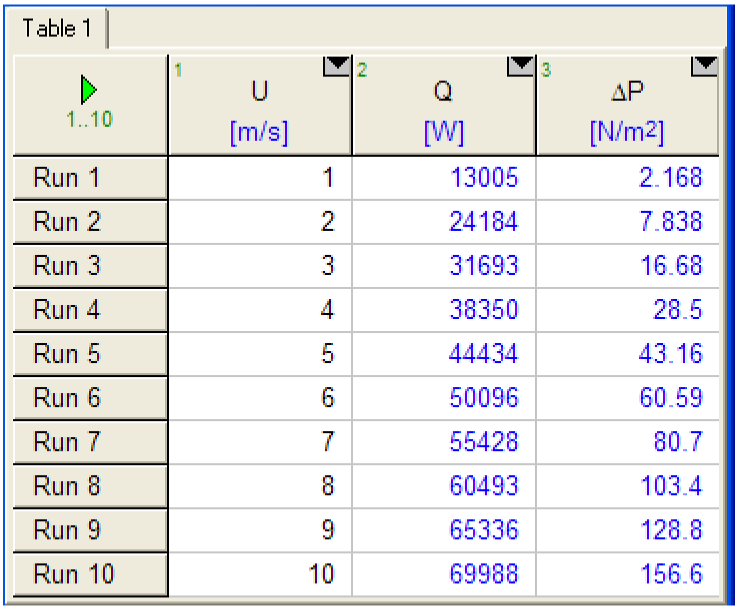
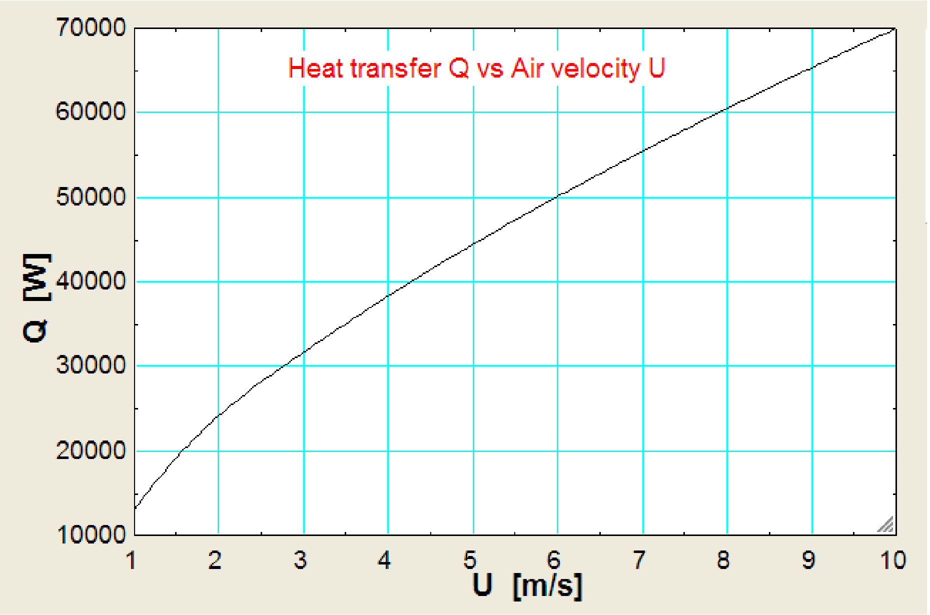
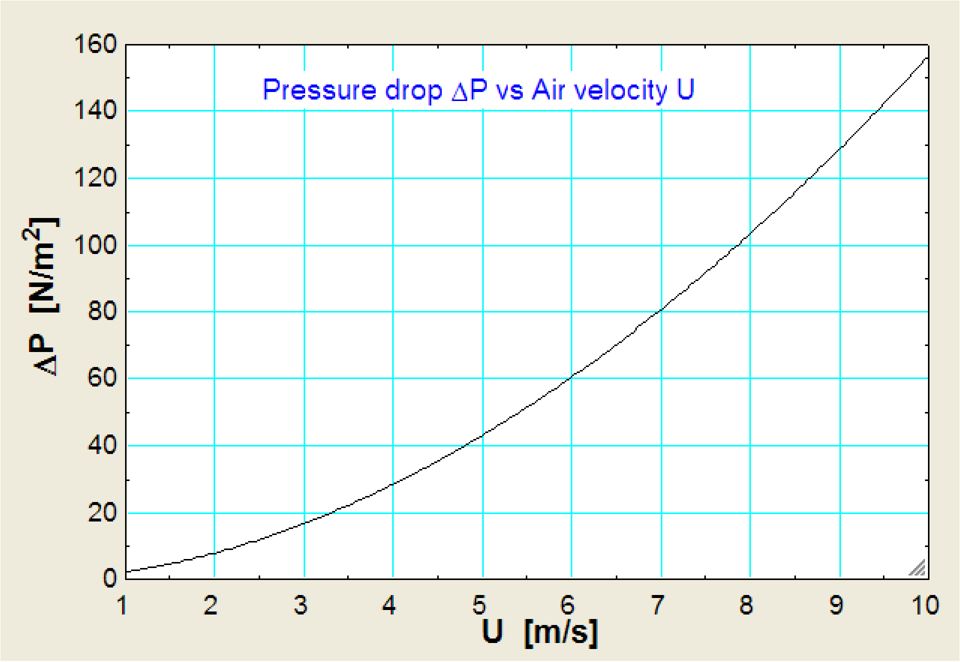


Table 1

1..10	1 U [m/s]	2 Q [W]	3 ΔP [N/m2]
Run 1	1	13005	2.168
Run 2	2	24184	7.838
Run 3	3	31693	16.68
Run 4	4	38350	28.5
Run 5	5	44434	43.16
Run 6	6	50096	60.59
Run 7	7	55428	80.7
Run 8	8	60493	103.4
Run 9	9	65336	128.8
Run 10	10	69988	156.6





===

2A1.4 Flow inside tubes and ducts:

"**Prob. 2A1.4.1.** Water at a velocity of 1.5 m/s enters a 2 cm dia heat exchanger tube at 40 C. The heat exchanger tube wall is maintained at a temp of 100 C. If the water is heated to a temp of 80 C, find the length of the length of the exchanger tube required. – [VTU – Dec. 09–Jan. 2010]"

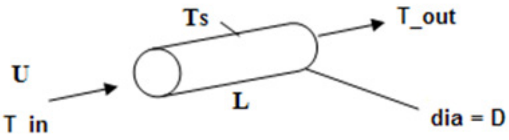


Fig.Prob.2A1.4.1

EES Solution:

This is a very common type of problem, often asked in the University exams.

So, let us write a Sub-routine (or, PROCEDURE) in EES. We will write the program such that the fluid can be chosen as Air or Water. Then, from the main program we call the PROCEDURE:

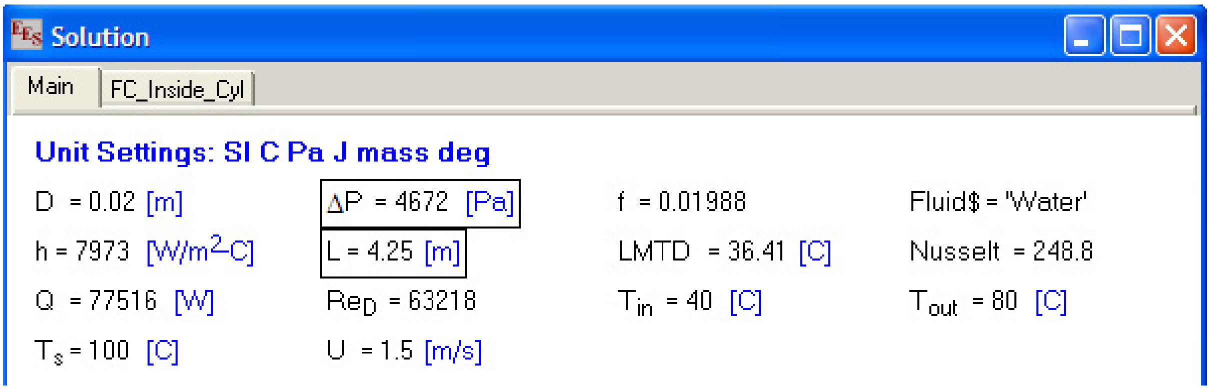
```
PROCEDURE FC_Inside_Cyl(Fluid$, T_s, T_in, T_out, D,U : Re_D, Nusselt, h,Q, LMTD, L, f, DELTAP )

"Forced convection (FC) inside Cylinders, with Fluid$: Water or Air, or Engine Oil"
"Inputs: T_s (C), T_in, T_out (C), D (m), U (m/s)"
"Outputs: Re_D, Nusselt, h (W/m^2-C), Q (W), LMTD (C), L (m),f, DELTAP (Pa)"
```

```
T_b := (T_out+T_inf)/2 "bulk mean temp"
A_c = pi * D^2/4 "Area of cross-section"

IF (Fluid$ = 'Water') Then
    k=Conductivity(Water,T=T_b,P=1.013e05)
    rho=Density(Water,T=T_b,P=1.013e05)
    mu=Viscosity(Water,T=T_b,P=1.013e05)
    cp=SpecHeat(Water,T=T_b,P=1.013e05)
    Pr=Prandtl(Water,T=T_b,P=1.013e05)
ELSE
    IF (Fluid$ = 'Air') Then
    beta := 1/(T_b + 273)
    mu:=Viscosity(Air,T=T_b)
    rho:=Density(Air,T=T_b,P=1e05)
    nu := mu/rho
    cp:=SpecHeat(Air,T=T_b)
    k:=Conductivity(Air,T=T_b)
    EndIF
EndIf
Re_D := D * U / nu
Pr := mu*cp/k

If (Re_D < 10000) Then CALL WARNING ('The results may not be accurate since Re_D > 10000 does
not hold. Re_D = XXXA1', Re_D)

If (Pr < 0.6) or (Pr > 160) Then CALL WARNING ('The results may not be accurate since 0.6 < Pr <
160 does not hold. Pr = XXXA1', Pr)

IF (T_in > T_out) Then
    n = 0.3
ELSE
        IF (T_in < T_out) Then
            n = 0.4
        ENDIF
ENDIF

Nusselt = 0.023 * Re^0.8 * Pr^n "finds Nusselts No."
h:= Nusselt * k / D "finds h"

DELTAT_in := T_s – T_in
DELTAT_out := T_s – T_out
```

```
LMTD: = (DELTAT_in – DELTAT_out) / ln(DELTAT_in/DELTAT_out) “Log Mean Temp Difference”
Q := rho * A_c * U * cp * (T_out – T_in) “finds Q, W”
L :=Q/ (h * (pi * d ) * LMTD) “by heat balance; finds L”

“Also, find the pressure drop:”

“Friction factor: Use the first Petukhov eqn, which is an explicit eqn. for f. Valid for Reynolds Number
range: Re = 3000 to 5E06”

f := (0.79 * ln(Re_D) – 1.64)^(-2) “…friction factor”

DELTAP := f * (L / D) * rho * U^2 / 2 “[N/m^2]”

END
“=========================================================================”
$UnitSystem SI Pa C J

“Prob.2A1.4.1”

Fluid$ = ‘Water’

T_in = 40[C]
T_out= 80[C]
T_s = 100[C]

D = 0.02[m]
U = 1.5[m/s]

CALL FC_Inside_Cyl(Fluid$, T_s, T_in, T_out, D,U : Re_D, Nusselt, h,Q, LMTD, L, f, DELTAP)

“------------------------------------------------------------------------------------------------------------------------------------”
```

Now, press F2, and the results appear:

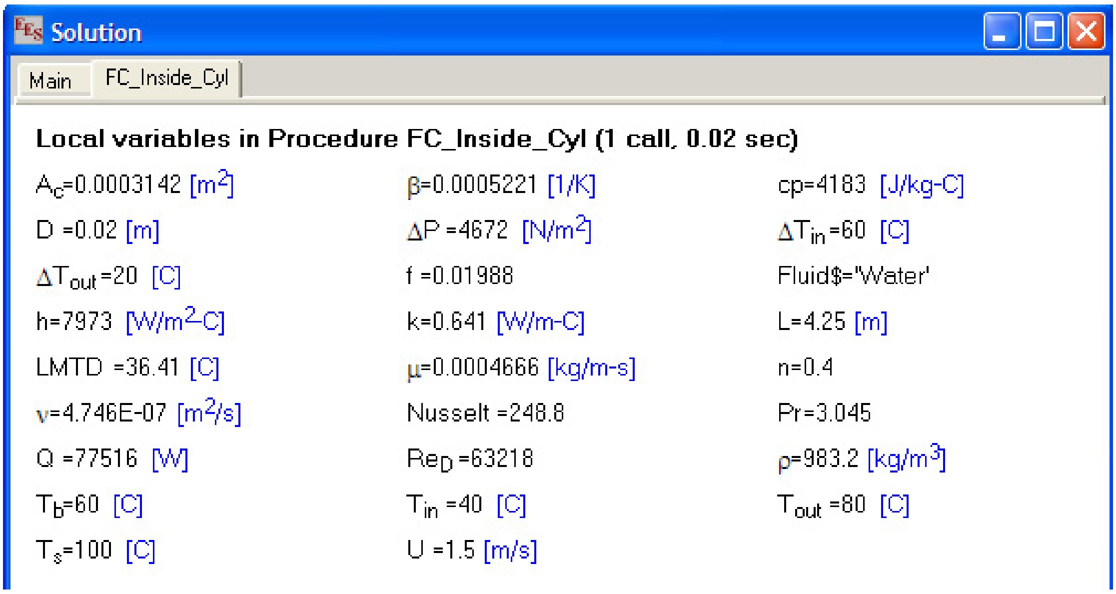
Main Results:



Unit Settings: SI C Pa J mass deg

D = 0.02 [m]
ΔP = 4672 [Pa]
f = 0.01988
Fluid$ = 'Water'
h = 7973 [W/m^2-C]
L = 4.25 [m]
LMTD = 36.41 [C]
Nusselt = 248.8
Q = 77516 [W]
Re$_D$ = 63218
T$_{in}$ = 40 [C]
T$_{out}$ = 80 [C]
T$_s$ = 100 [C]
U = 1.5 [m/s]

PROCEDURE Results:



Local variables in Procedure FC_Inside_Cyl (1 call, 0.02 sec)

A_c=0.0003142 [m^2]
β=0.0005221 [1/K]
cp=4183 [J/kg-C]
D =0.02 [m]
ΔP =4672 [N/m^2]
ΔT_{in} =60 [C]
ΔT_{out} =20 [C]
f =0.01988
Fluid$='Water'
h=7973 [W/m^2-C]
k=0.641 [W/m-C]
L=4.25 [m]
LMTD =36.41 [C]
μ=0.0004666 [kg/m-s]
n=0.4
ν=4.746E-07 [m^2/s]
Nusselt =248.8
Pr=3.045
Q =77516 [W]
Re_D =63218
ρ=983.2 [kg/m^3]
T_b=60 [C]
T_{in} =40 [C]
T_{out} =80 [C]
T_s=100 [C]
U =1.5 [m/s]

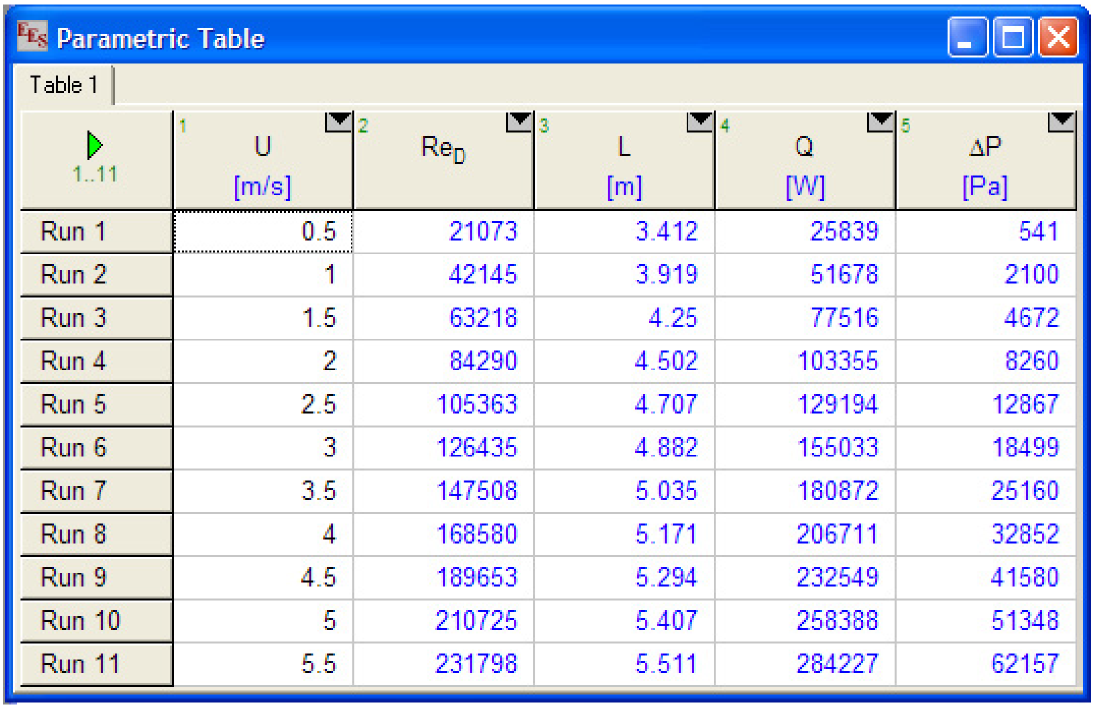
Thus:

L = 4.25 m … Length of heat exchanger tube required…. Ans.

ΔP = 4672 **N/m^2 … Pressure drop over length L …. Ans.**

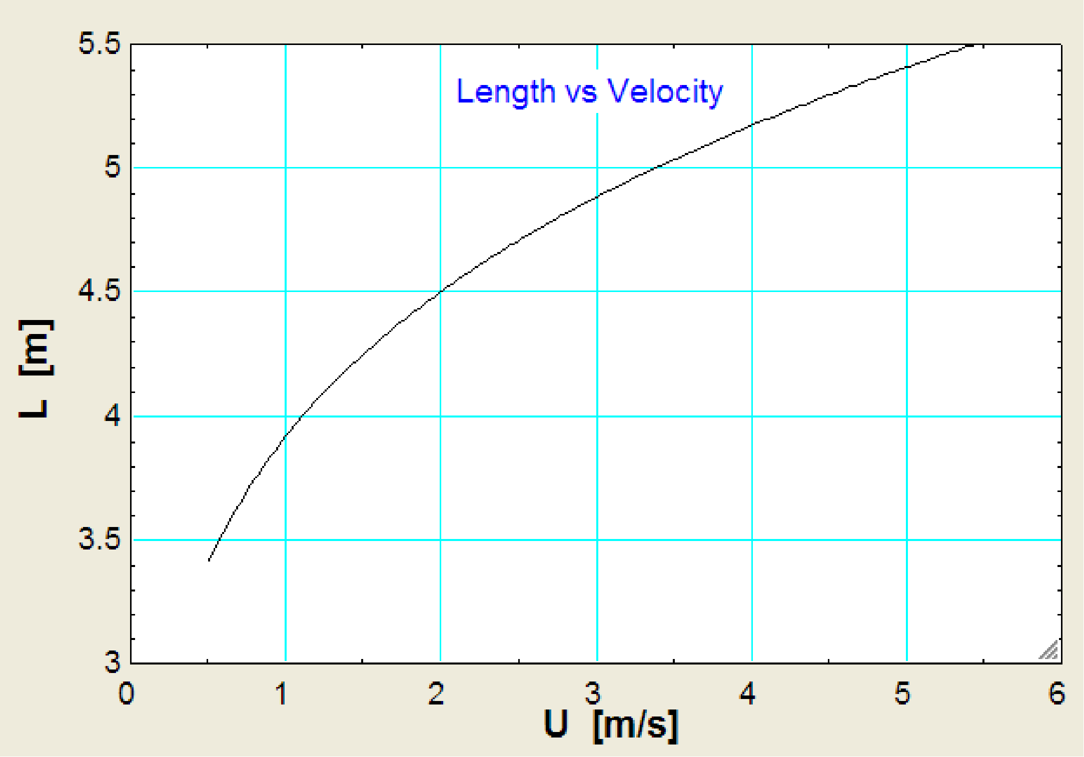
In addition, plot the variation of Length and DELTAP as the velocity changes:

Take the range for U as 0.5 to 5.5 m/s:

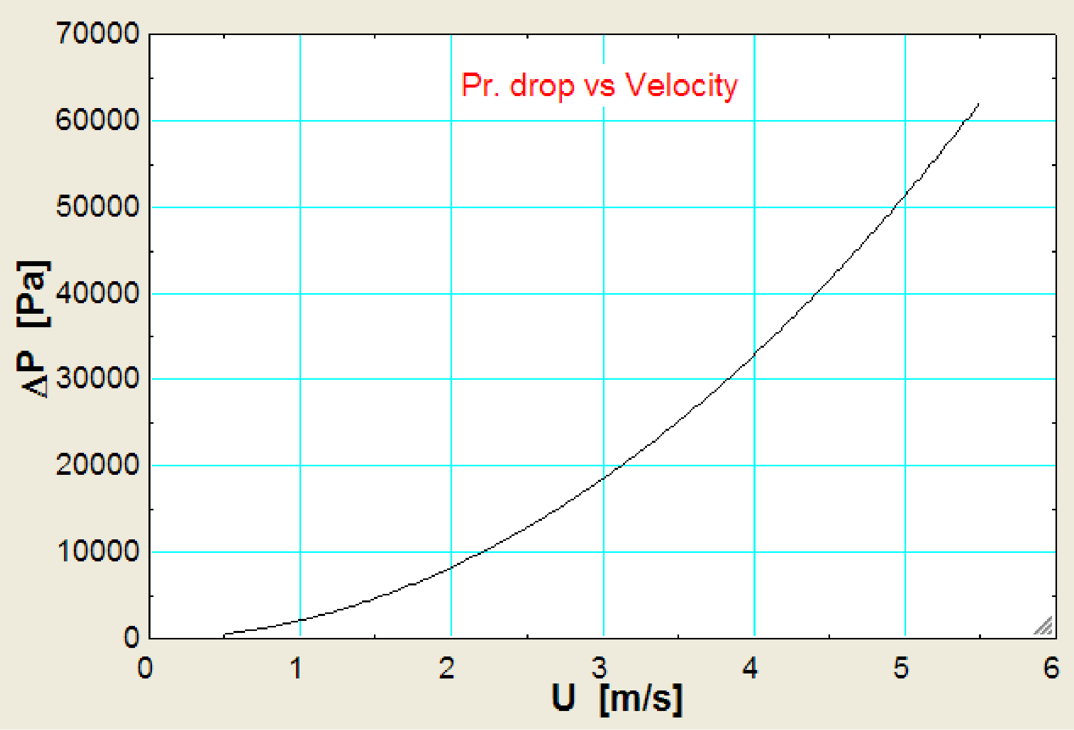


1..11	1 U [m/s]	2 Re_D	3 L [m]	4 Q [W]	5 ΔP [Pa]
Run 1	0.5	21073	3.412	25839	541
Run 2	1	42145	3.919	51678	2100
Run 3	1.5	63218	4.25	77516	4672
Run 4	2	84290	4.502	103355	8260
Run 5	2.5	105363	4.707	129194	12867
Run 6	3	126435	4.882	155033	18499
Run 7	3.5	147508	5.035	180872	25160
Run 8	4	168580	5.171	206711	32852
Run 9	4.5	189653	5.294	232549	41580
Run 10	5	210725	5.407	258388	51348
Run 11	5.5	231798	5.511	284227	62157

Plot Length against Velocity:



Plot pressure drop against Velocity:



“**Prob.2A1.4.2.** Engine Oil is heated by flowing through a circular tube of diameter d = 50 mm and length L = 25 m and whose surface is maintained at 150 C. (a) If the flow rate and inlet temp of oil are 0.5 kg/s and 20 C, what is the outlet temp and total heat transfer rate for the tube? (b) For flow rates in the range 0.5 < m_dot < 2.0 kg/s, compute and plot the variation of outlet temp, T_out and Q with m_dot. For what flow rate(s) are Q and T_out maximized?”

EES Solution:

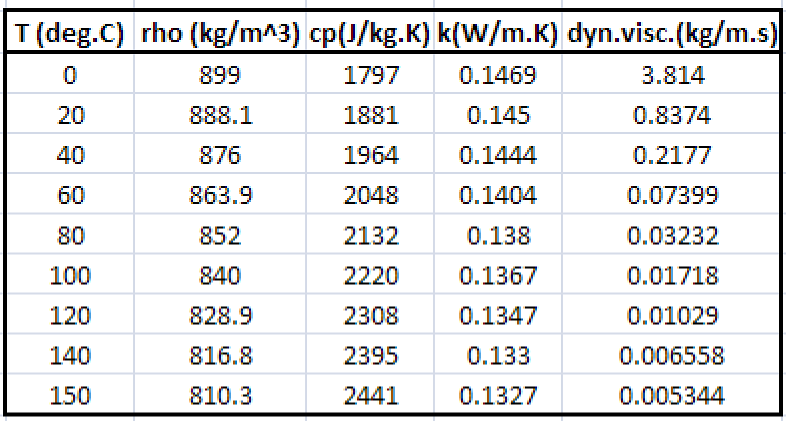
First, let us write functions for properties of Engine Oil.

They can be used subsequently to solve all problems where properties of Engine oil are required.

Properties of Engine oil are taken from Ref.[2], and the curve – fit equations were obtained using CurveExpert software.

“Functions for properties of Engine Oil in the Temp range: 0 to 150 C.
Ref: Data from HMT by Cengel, 3rd Ed.”

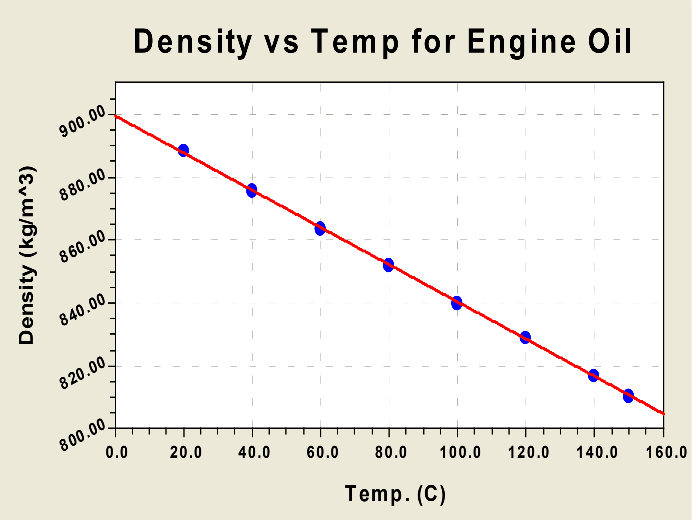
“ ”



T (deg.C)	rho (kg/m^3)	cp(J/kg.K)	k(W/m.K)	dyn.visc.(kg/m.s)
0	899	1797	0.1469	3.814
20	888.1	1881	0.145	0.8374
40	876	1964	0.1444	0.2177
60	863.9	2048	0.1404	0.07399
80	852	2132	0.138	0.03232
100	840	2220	0.1367	0.01718
120	828.9	2308	0.1347	0.01029
140	816.8	2395	0.133	0.006558
150	810.3	2441	0.1327	0.005344

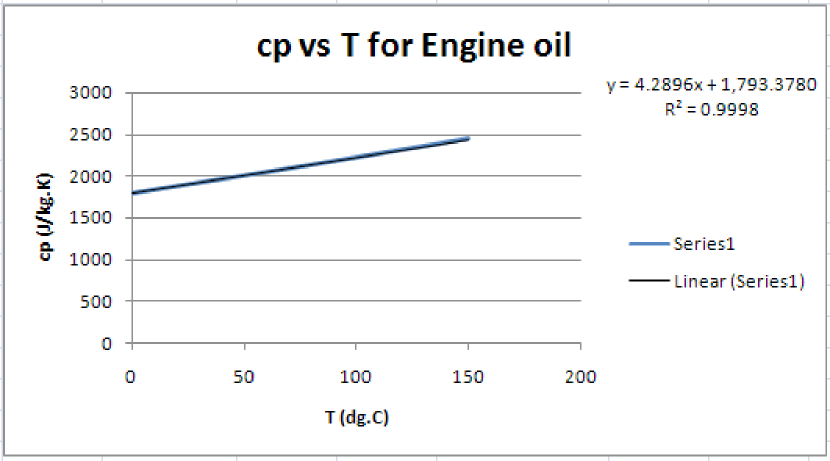
Density of Engine oil:

CurveExpert Curve fit:
Linear Fit: y=a+bx
r = 0.9999382
Coefficient Data:
a = 899.47228
b = -0.59190217


Density vs Temp for Engine Oil
Density (kg/m^3)
900.00
880.00
860.00
840.00
820.00
800.00
0.0
20.0
40.0
60.0
80.0
100.0
120.0
140.0
160.0
Temp. (C)

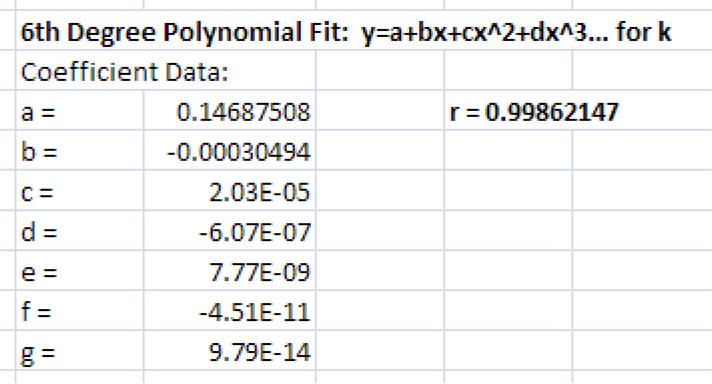
Sp. Heat of Engine Oil:

Curve-fit from Excel:

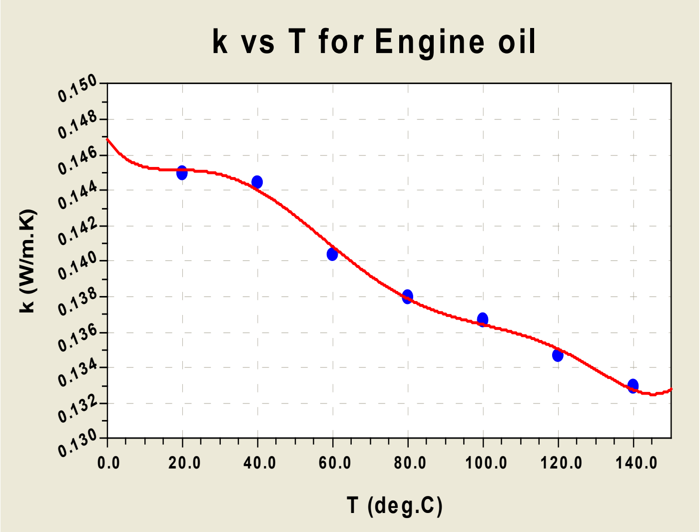


Th. cond. of Engine Oil:

Curve fit from CurvreExpert:



6th Degree Polynomial Fit: y=a+bx+cx^2+dx^3... for k			
Coefficient Data:			
a =	0.14687508	r = 0.99862147	
b =	-0.00030494		
c =	2.03E-05		
d =	-6.07E-07		
e =	7.77E-09		
f =	-4.51E-11		
g =	9.79E-14		

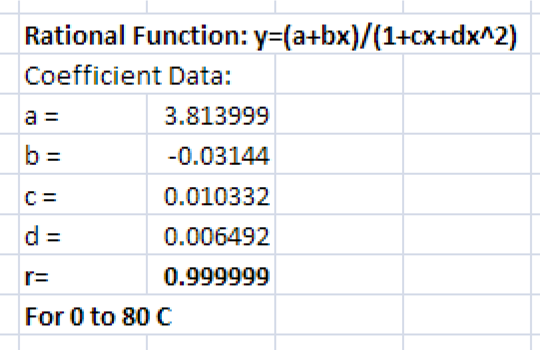


Dyn. viscosity of Engine Oil:

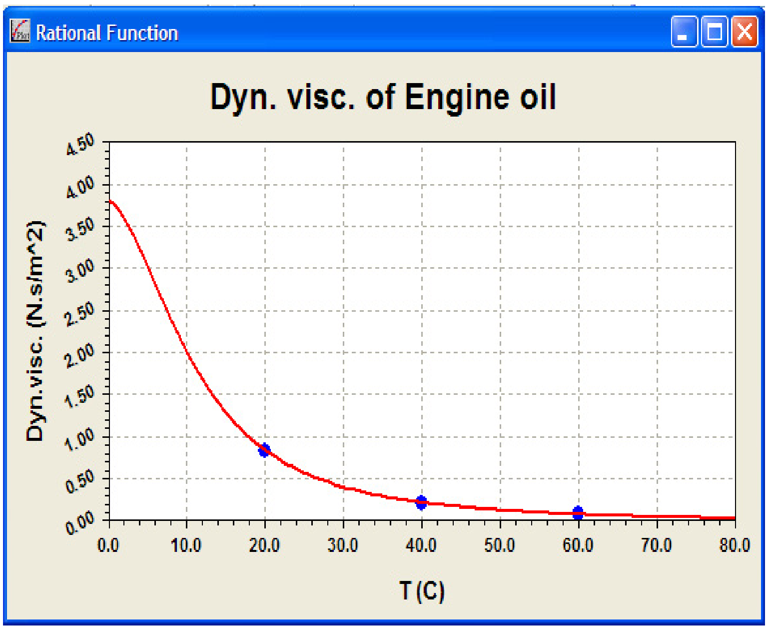
For better accuracy, curve fit equations are obtained in two temp. ranges, viz. from 0 to 80 deg.C and from 81 to 150 deg.C:

Curve fit from CurvreExpert:

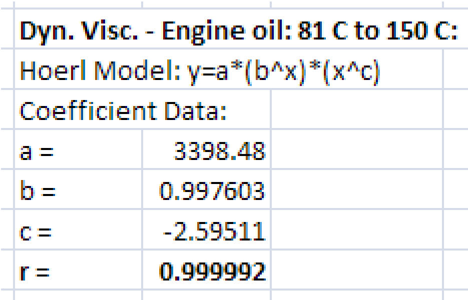
In the range 0 to 80 deg. C:



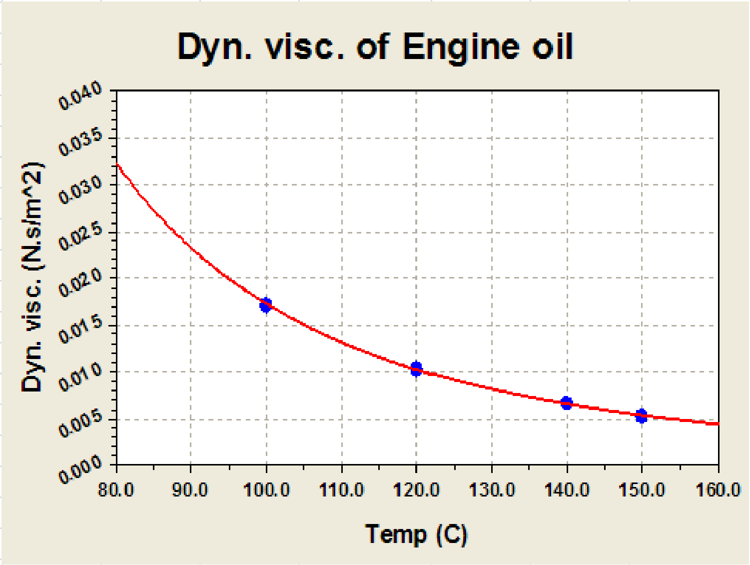
Rational Function: y=(a+bx)/(1+cx+dx^2)	
Coefficient Data:	
a =	3.813999
b =	-0.03144
c =	0.010332
d =	0.006492
r=	**0.999999**
For 0 to 80 C	



And, in the range 81 to 150 deg. C:



Dyn. Visc. - Engine oil: 81 C to 150 C:	
Hoerl Model: y=a*(b^x)*(x^c)	
Coefficient Data:	
a =	3398.48
b =	0.997603
c =	-2.59511
r =	**0.999992**


Dyn. visc. of Engine oil
Dyn. visc. (N.s/m^2)
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
80.0
90.0
100.0
110.0
120.0
130.0
140.0
150.0
160.0
Temp (C)

==

EES Functions:

```
Function rho_EngineOil(T)

{rho_EngineOil
This function returns the density (kg/m^3) of Engine oil as a function of Temp (deg.C) in the range:
0–150 C
}

If (T < 0) OR (T > 150) Then
CALL error('T must be between 0 and 150 deg. C !!')
EndIf
rho_EngineOil := 899.47228 – 0.59190217 * T
End

"-----------------------------------------------------------------------------------------------------------------------------------------"

Function cp_EngineOil(T)

{cp_EngineOil
This function returns the sp. heat (J/kg.C) of Engine oil as a function of Temp (deg.C) in the range:
0–150 C
}

If (T < 0) OR (T > 150) Then
CALL error('T must be between 0 and 150 deg. C !!')
EndIf
cp_EngineOil := 4.2896 * T + 1793.378
End

"-----------------------------------------------------------------------------------------------------------------------------------------"

Function k_EngineOil(T)

{k_EngineOil
This function returns the Th. cond. (W/m.C)) of Engine oil as a function of Temp (deg.C) in the range:
0–150 C
}
```

```
If (T < 0) OR (T > 150) Then
CALL error(‘T must be between 0 and 150 deg. C !!’)
EndIf
k_EngineOil := 0.14687508 – 0.00030494 * T + 2.03E-05 * T^2 – 6.07E-07 * T^3 + 7.77E-09 * T^4 –
4.51E-11 * T^5 + 9.79E-14 * T^6
End

“-----------------------------------------------------------------------------------------------------”

Function mu_EngineOil(T)

{mu_EngineOil
This function returns the Dyn. visc. (N.s/m^2)) of Engine oil as a function of Temp (deg.C) in the
range: 0–150 C
}

If (T < 0) OR (T > 150) Then
CALL error(‘T must be between 0 and 150 deg. C !!’)
EndIf
If (T >= 0) And (T <=80) Then
    mu_EngineOil := (3.813999 – 0.03144 * T) / (1 + 0.010332 * T + 0.006492 * T^2)
Else
    If (T > 80 ) And (T <= 150) Then
            mu_EngineOil := 3398.48 * 0.997603^T * T^(-2.59511)
        EndIf
    EndIf
End

“-----------------------------------------------------------------------------------------------------”

Function Pr_EngineOil(T)

{Pr_EngineOil
This function returns the Prandtl No. (= cp . mu /k) of Engine oil as a function of Temp (deg.C) in the
range: 0–150 C
}
```

```
If (T < 0) OR (T > 150) Then
CALL error('T must be between 0 and 150 deg. C !!')
EndIf
   Pr_EngineOil := cp_EngineOil(T) * mu_EngineOil(T) / k_EngineOil(T)
End

“------------------------------------------------------------------------------------------------”
```

And now, for the Solution of the problem:

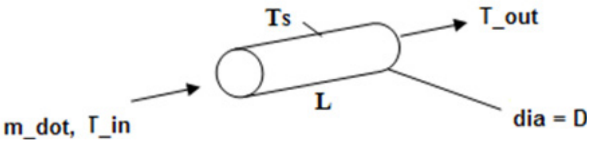


Fig.Prob.2A1.4.2

```
"Data:"

T_in = 20[C]
{T_out= 140[C]"..assumed…will be commented out later"}
T_s = 150[C]
T_b = (T_in + T_out)/2
d = 0.05[m]
L = 25[m]
m_dot = 0.5[kg/s]

"Properties of Engine oil at bulk mean temp, T_b:"

k=k_EngineOil(T_b) "[W/m-C]"
rho= rho_EngineOil(T_b) "[kg/m^3]"
mu= mu_EngineOil(T_b) "[kg/m-s]"
cp= cp_EngineOil(T_b) "[J/kg-C]"
Pr= Pr_EngineOil(T_b)

"Calculations:"

A = pi * d^2 / 4 "[m^2]… area of cross-section"
G = m_dot / A "[kg/s-m^2]"
Re = G * d /mu "Reynold's No."

"We get Re = 425.2 which is less than 2300; Therefore, flow is Laminar."

"Now, Thermal entry length L_t is given by:"

L_t = .05 * Re * Pr * d "[m] … thermal entry length"

"We get L_t = 493.6 m, which is much greater than the tube length of 25 m.
Therefore, the thermally boundary layer is still developing.
This is generally the case for high Prandtl No. fluids."
"Now, Nu = 3.66 for Laminar flow when the velocity and thermal boundary layers are fully developed.
In the present case, since the thermal boundary layer is still not fully developed, we use:"
```

$$\text{Nusselt} = 3.66 + \frac{0.065 \cdot \frac{d}{L} \cdot Re \cdot Pr}{1 + 0.04 \cdot \left[\frac{d}{L} \cdot Re \cdot Pr \right]^{(2/3)}} \quad \text{....Nusselts No.}$$

In EES, it is entered:

Nusselt = 3.66 + (0.065 * (d/L) * Re * Pr) / (1 + 0.04 * ((D / L) * Re * Pr)^(2/3)) "….Nusselts No."

"Therefore: heat transfer coeff. h:"

Nusselt = h * d / k "[W/m^2-C]…finds h"

"Now:"

A_s = pi * d * L "[m^2]…surface area"

"Heat transfer, Q:"

Q = m_dot * cp * (T_out – T_in) "[W]"

AMTD = ((T_s – T_out) + (T_s – T_in))/ 2

"Note: For use in Newton's eqn to calculate Q, we should strictly use LMTD. However, we have used here Arithmetic Mean Temp Diff (AMTD) = average of temp diff at the inlet and outlet of oil. We shall show later that values of LMTD and AMTD are practically the same."

"Now, Q is also equal to:"

Q = h * A_s * AMTD

"Pressure drop:"

U = G / rho "[m/s] …. mean velocity"

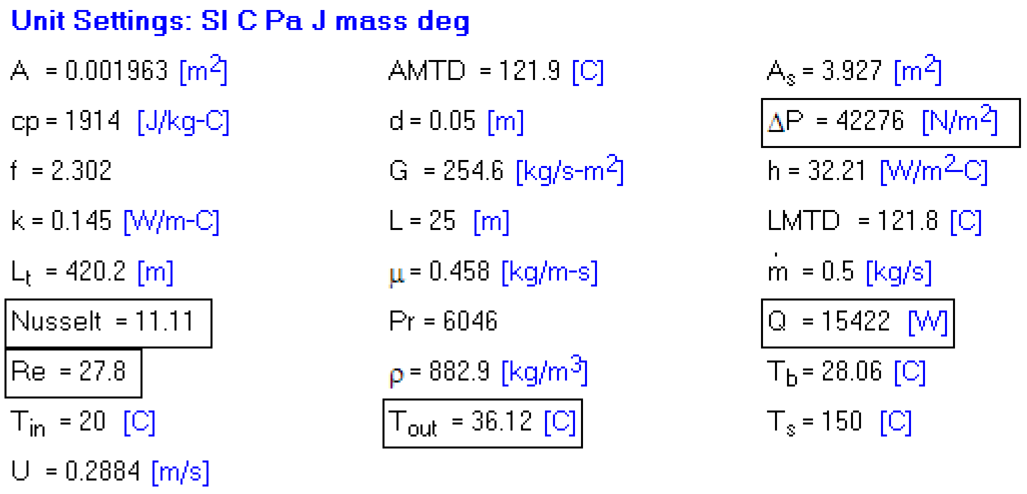
f = 64/Re "…friction factor for Laminar flow"

DELTAP = f * (L / d) * U^2 / 2 * rho "[N/m^2] … pressure drop"

"Check:"

LMTD = ((T_s – T_out) – (T_s – T_in)) / ln ((T_s – T_out) / (T_s – T_in))

Results:



Unit Settings: SI C Pa J mass deg

$A = 0.001963\ [m^2]$ $AMTD = 121.9\ [C]$ $A_s = 3.927\ [m^2]$

$cp = 1914\ [J/kg\text{-}C]$ $d = 0.05\ [m]$ $\boxed{\Delta P = 42276\ [N/m^2]}$

$f = 2.302$ $G = 254.6\ [kg/s\text{-}m^2]$ $h = 32.21\ [W/m^2\text{-}C]$

$k = 0.145\ [W/m\text{-}C]$ $L = 25\ [m]$ $LMTD = 121.8\ [C]$

$L_t = 420.2\ [m]$ $\mu = 0.458\ [kg/m\text{-}s]$ $\dot{m} = 0.5\ [kg/s]$

$\boxed{Nusselt = 11.11}$ $Pr = 6046$ $\boxed{Q = 15422\ [W]}$

$\boxed{Re = 27.8}$ $\rho = 882.9\ [kg/m^3]$ $T_b = 28.06\ [C]$

$T_{in} = 20\ [C]$ $\boxed{T_{out} = 36.12\ [C]}$ $T_s = 150\ [C]$

$U = 0.2884\ [m/s]$

Thus:

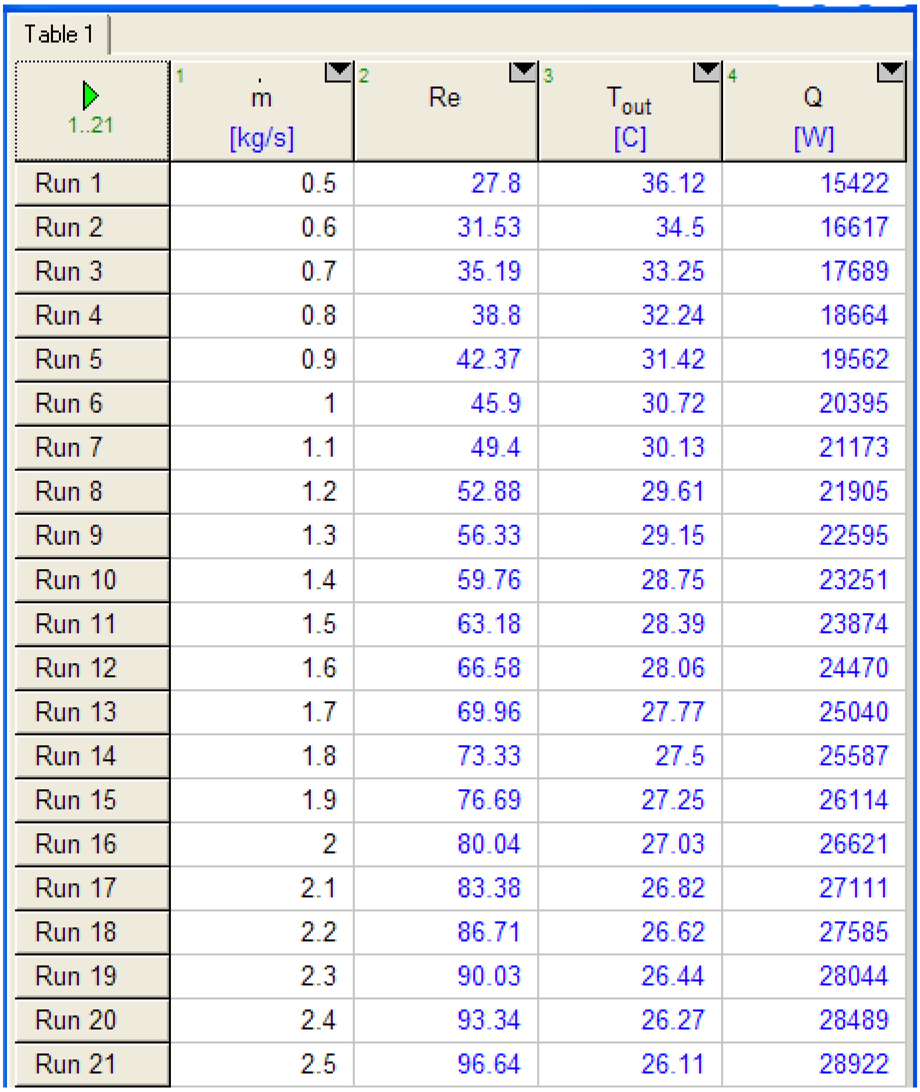
Oil outlet temp = T_out = 36.12 C …. Ans.

Heat transferred = Q = 15422 W … Ans.

Pressure drop DELTAP = 42276 Pa … Ans.

Note that AMTD = 121.9 C and LMTD = 121.8 C. i.e. they are practically the same.

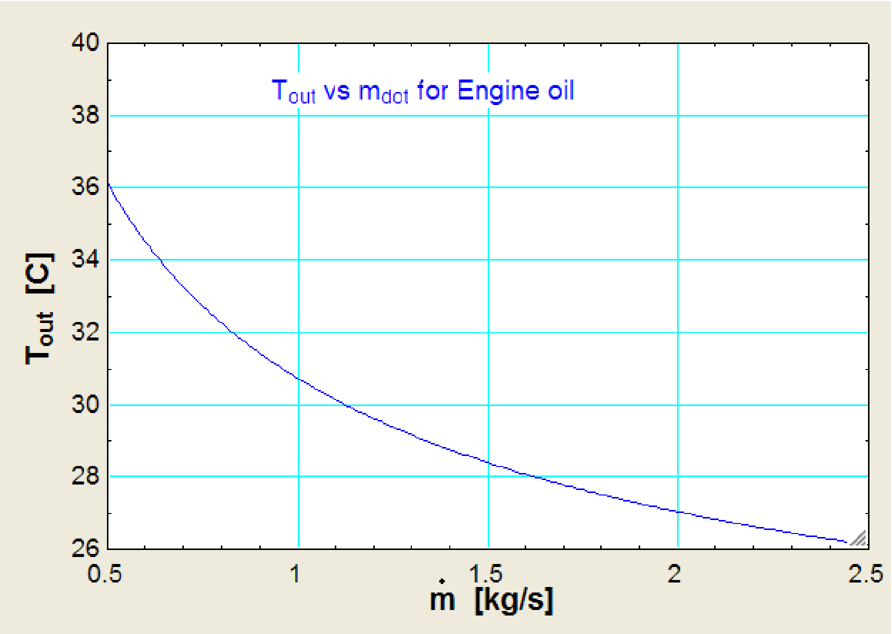
To plot T_out and Q against m_dot:



1..21	1 $\dot{m}$ [kg/s]	2 Re	3 T_{out} [C]	4 Q [W]
Run 1	0.5	27.8	36.12	15422
Run 2	0.6	31.53	34.5	16617
Run 3	0.7	35.19	33.25	17689
Run 4	0.8	38.8	32.24	18664
Run 5	0.9	42.37	31.42	19562
Run 6	1	45.9	30.72	20395
Run 7	1.1	49.4	30.13	21173
Run 8	1.2	52.88	29.61	21905
Run 9	1.3	56.33	29.15	22595
Run 10	1.4	59.76	28.75	23251
Run 11	1.5	63.18	28.39	23874
Run 12	1.6	66.58	28.06	24470
Run 13	1.7	69.96	27.77	25040
Run 14	1.8	73.33	27.5	25587
Run 15	1.9	76.69	27.25	26114
Run 16	2	80.04	27.03	26621
Run 17	2.1	83.38	26.82	27111
Run 18	2.2	86.71	26.62	27585
Run 19	2.3	90.03	26.44	28044
Run 20	2.4	93.34	26.27	28489
Run 21	2.5	96.64	26.11	28922

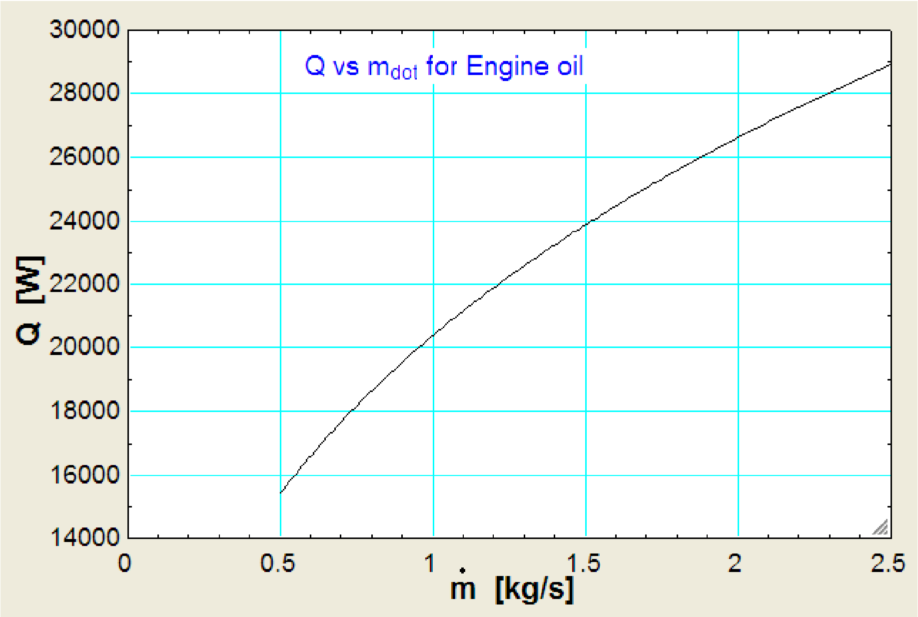
Note that flow is laminar for the entire range of mass flow rate.

T_out vs m_dot:



Note that T_out is maximum for minimum flow rate, i.e. for 0.5 kg/s.

Q vs m_dot:



Note that heat transfer, Q, goes on increasing with increasing flow rate.

==

$UnitSystem SI C Pa J

"**Prob. 2A1.4.3.** 50 kg of water per minute is heated from 30 C to 50 C by passing through a pipe of 2 cm dia. The pipe is heated by condensing the steam on its surface at 100 C. Find the length of pipe required. [VTU – June–July 2011]"

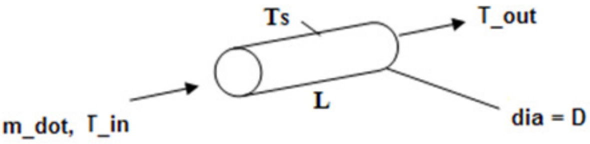
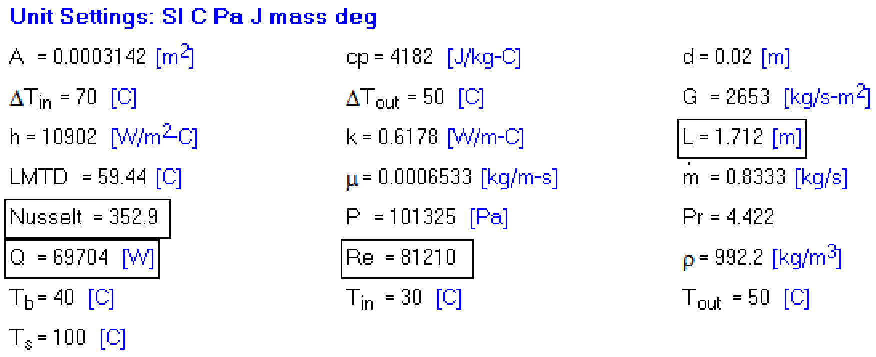


Fig.Prob.2A1.4.3

EES Solution:

```
“Data:”

m_dot = 50[kg/min]*convert (kg/min, kg/s)
T_in = 30[C]
T_out= 50[C]
T_s = 100[C]
d = 0.02[m]

T_b = (T_in + T_out) / 2 “[C]… bulk mean temp of water”
P = 1.01325E05 “[Pa] …. pressure of water”

“Properties of Water at T_b:”

rho=Density(Water,T=T_b,P=P) “[kg/m^3]”
cp=Cp(Water,T=T_b,P=P) “[J/kg-C]”
k=Conductivity(Water,T=T_b,P=P) “[W/m-C]”
mu=Viscosity(Water,T=T_b,P=P) “[kg/m-s]”
Pr=Prandtl(Water,T=T_b,P=P) “…Prandtl No.”

“Calculations:”

A = pi * d^2 / 4 “[m^2]…area of cross-section”
G = m_dot / A “[kg/s-m^2] … mass velocity”
Re = G * d / mu “finds Re = 166593”
Nusselt = 0.023 * Re^0.8 * Pr^0.4 “finds Nusselts No.”
Nusselt = h * d / k “finds h”

DELTAT_in = T_s – T_in
DELTAT_out = T_s – T_out
LMTD = (DELTAT_in -DELTAT_out) / ln(DELTAT_in / DELTAT_out)
Q = m_dot * cp * (T_out – T_in) “finds Q, W”
Q = h * (pi * d * L)* LMTD “finds L”
```

Results:



Unit Settings: SI C Pa J mass deg

$A = 0.0003142$ [m^2] $cp = 4182$ [J/kg-C] $d = 0.02$ [m]

$\Delta T_{in} = 70$ [C] $\Delta T_{out} = 50$ [C] $G = 2653$ [kg/s-m^2]

$h = 10902$ [W/m^2-C] $k = 0.6178$ [W/m-C] $\boxed{L = 1.712 \text{ [m]}}$

$LMTD = 59.44$ [C] $\mu = 0.0006533$ [kg/m-s] $\dot{m} = 0.8333$ [kg/s]

$\boxed{Nusselt = 352.9}$ $P = 101325$ [Pa] $Pr = 4.422$

$\boxed{Q = 69704 \text{ [W]}}$ $\boxed{Re = 81210}$ $\rho = 992.2$ [kg/m^3]

$T_b = 40$ [C] $T_{in} = 30$ [C] $T_{out} = 50$ [C]

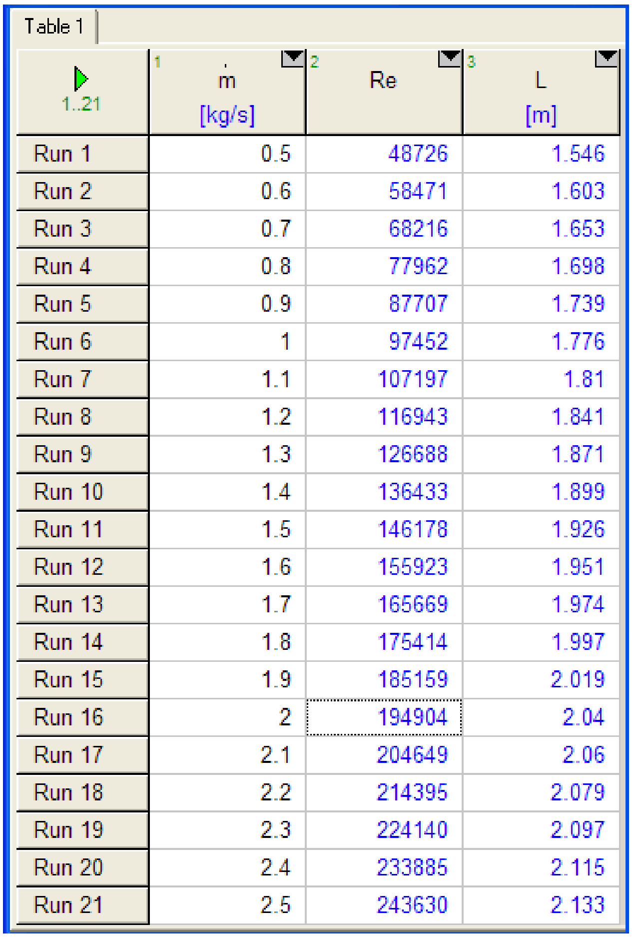
$T_s = 100$ [C]

Thus:

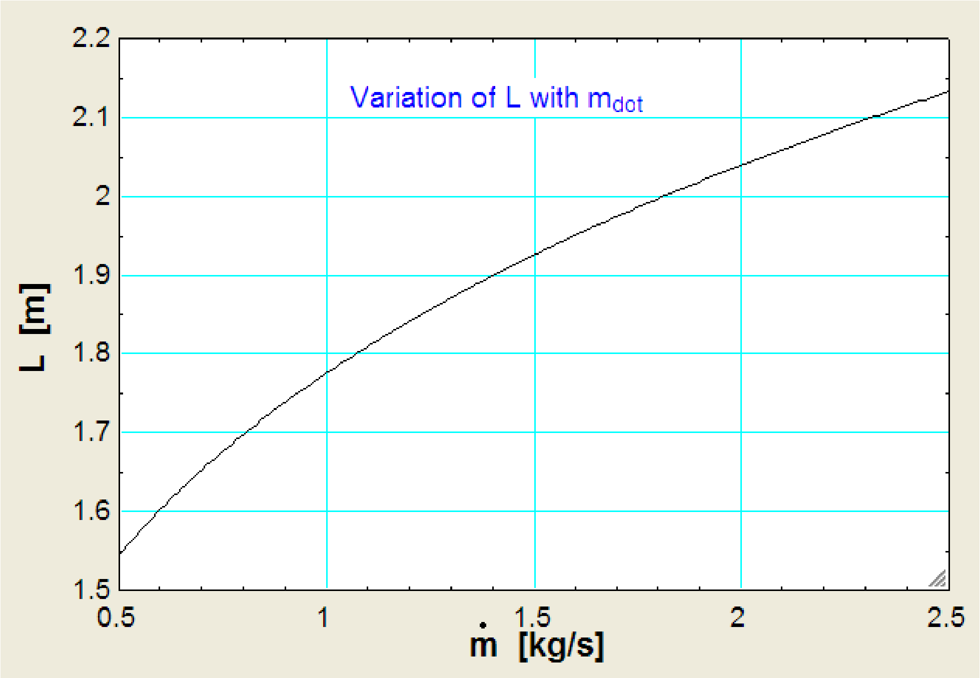
Length of tube required = L = 1.712 m …. Ans.

Heat transferred = Q = 69704 W … Ans.

Plot the variation of L with m_dot:



1..21	$\dot{m}$ [kg/s]	Re	L [m]
Run 1	0.5	48726	1.546
Run 2	0.6	58471	1.603
Run 3	0.7	68216	1.653
Run 4	0.8	77962	1.698
Run 5	0.9	87707	1.739
Run 6	1	97452	1.776
Run 7	1.1	107197	1.81
Run 8	1.2	116943	1.841
Run 9	1.3	126688	1.871
Run 10	1.4	136433	1.899
Run 11	1.5	146178	1.926
Run 12	1.6	155923	1.951
Run 13	1.7	165669	1.974
Run 14	1.8	175414	1.997
Run 15	1.9	185159	2.019
Run 16	2	194904	2.04
Run 17	2.1	204649	2.06
Run 18	2.2	214395	2.079
Run 19	2.3	224140	2.097
Run 20	2.4	233885	2.115
Run 21	2.5	243630	2.133



==

$UnitSystem SI C Pa J

"**Prob. 2A1.4.4.** Air at 2 atm and 200 C is heated as it flows through a tube with a diameter of 25 mm at a velocity of 10 m/s. Calculate the heat transfer per unit length of tube if a constant heat flux condition is maintained at the wall and the wall temp is 20 C above the air temp all along the length of the tube. How much would the bulk temp increase over a 3 m length of tube? [VTU – May–June 2010]"

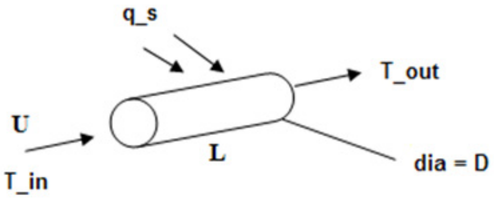


Fig.Prob.2A1.4.4

EES Solution:

"Data:"

T_in = 200[C]
T_out = 250 [C] "..assumed. Will be commented out later."
U = 10 [m/s]
d = 0.025[m]
L = 1 [m]
DELTAT = 20 [C]
"q_s is constant on the wall surface"
"When q_s is constant: we have:
q_s = h * DELTAT
i.e. DELTAT is constant, where DELTAT is the difference between the wall and bulk temp of fluid. Observe that DELTAT = LMTD in that case."

T_b = (T_in + T_out) / 2 "[C]… bulk mean temp of Air"
P = 2 [atm] * convert (atm, Pa) "[Pa] …. pressure of Air"

"Properties of Air at T_b:"

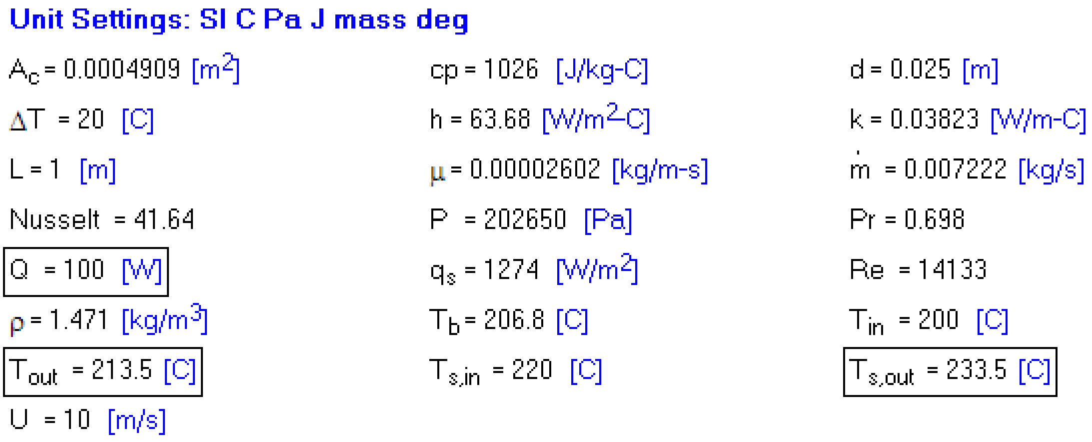
rho=Density(Air,T=T_b,P=P) "[kg/m^3]"
cp=Cp(Air,T=T_b) "[J/kg-C]"
k=Conductivity(Air,T=T_b) "[W/m-C]"
mu=Viscosity(Air,T=T_b) "[kg/m-s]"
Pr=Prandtl(Air,T=T_b) "…Prandtl No."

"Calculations:"

A_c = pi * d^2 / 4 "[m^2]…area of cross-section"
m_dot = rho * U * A_c "[kg/s] … mass flow rate"
Re = d * U * rho / mu "finds Re = 13273"
Nusselt = 0.023 * Re^0.8 * Pr^0.4 "finds Nusselts No."
Nusselt = h * d / k "finds h"

q_s = h * DELTAT "..finds q_s[W/m^2]"
DELTAT = T_s_in – T_in "…finds T_s_in, surface temp at inlet"
DELTAT = T_s_out – T_out "..finds T_s_out, surface temp at exit"
Q = q_s * pi * d * L "finds Q, W"

Results:



Unit Settings: SI C Pa J mass deg

A_c = 0.0004909 [m^2]
ΔT = 20 [C]
L = 1 [m]
Nusselt = 41.64
$\boxed{Q = 100 \text{ [W]}}$
ρ = 1.471 [kg/m^3]
$\boxed{T_{out} = 213.5 \text{ [C]}}$
U = 10 [m/s]

cp = 1026 [J/kg-C]
h = 63.68 [W/m^2-C]
μ = 0.00002602 [kg/m-s]
P = 202650 [Pa]
q_s = 1274 [W/m^2]
T_b = 206.8 [C]
$T_{s,in}$ = 220 [C]

d = 0.025 [m]
k = 0.03823 [W/m-C]
$\dot{m}$ = 0.007222 [kg/s]
Pr = 0.698
Re = 14133
T_{in} = 200 [C]
$\boxed{T_{s,out} = 233.5 \text{ [C]}}$

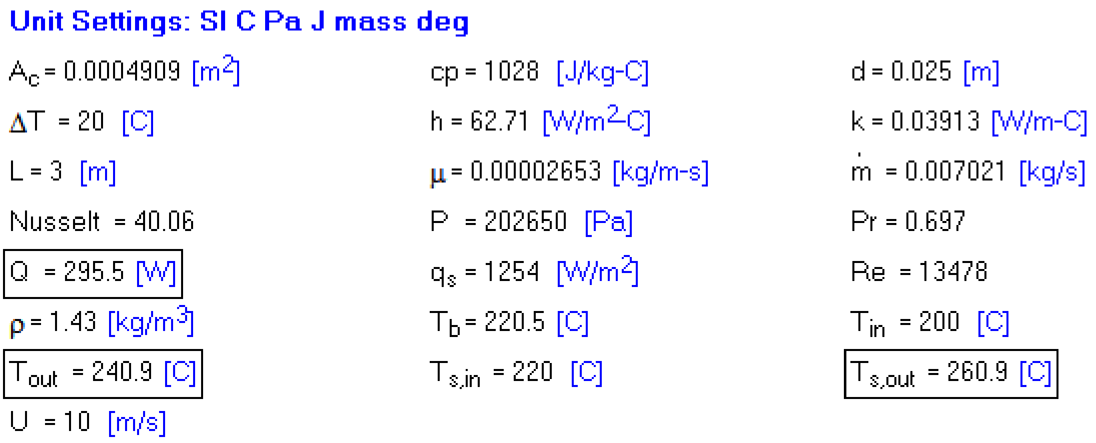
Thus:

Heat transfer per unit length = Q = 100 W …. Ans.

T_out for L = 1 m = 213.5 C …. Ans.

T_s_out = 233.5 C …. Ans.

When L = 3 m: we get the following results:



```
Unit Settings: SI C Pa J mass deg
A_c = 0.0004909 [m^2]      cp = 1028 [J/kg-C]        d = 0.025 [m]
ΔT = 20 [C]                h = 62.71 [W/m^2-C]       k = 0.03913 [W/m-C]
L = 3 [m]                  μ = 0.00002653 [kg/m-s]   ṁ = 0.007021 [kg/s]
Nusselt = 40.06            P = 202650 [Pa]           Pr = 0.697
Q = 295.5 [W]              q_s = 1254 [W/m^2]        Re = 13478
ρ = 1.43 [kg/m^3]          T_b = 220.5 [C]           T_in = 200 [C]
T_out = 240.9 [C]          T_s,in = 220 [C]          T_s,out = 260.9 [C]
U = 10 [m/s]
```

Thus:

Heat transfer per unit length = Q = 295.5 W …. Ans.

T_out for L = 3 m = 240.9 C …. Ans.

T_s_out = 260.9 C …. Ans.

==

Prob. 2A1.4.5. Engine oil at a rate of 0.02 kg/s flows through a 3 mm dia tube 30 m long. The oil has an inlet temp of 60 C while the tube wall temp is maintained at 100 C by condensing steam. (a) Estimate the average heat transfer coeff. for the flow (b) Determine the outlet temp of oil [Ref. 3]

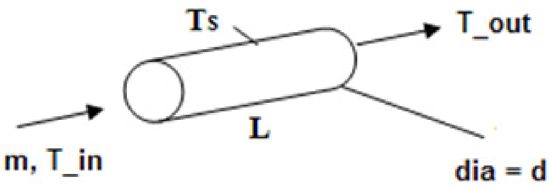


Fig.Prob.2A1.4.5

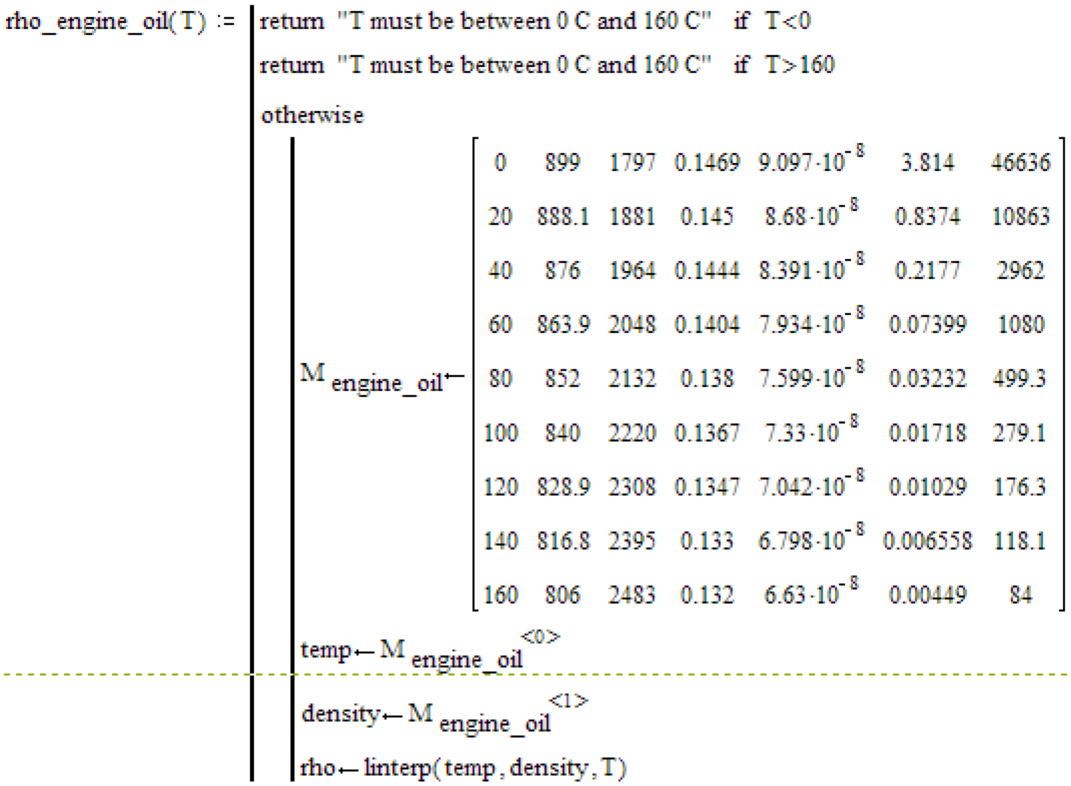
Mathcad Solution:

Let us solve this problem with Mathcad.
Since Mathcad does not have built-in functions for properties of Engine oil, let us write the functions ourselves:

Properties of Unused Engine oil: Data from Ref: Cengel-HMT-3rd Ed.

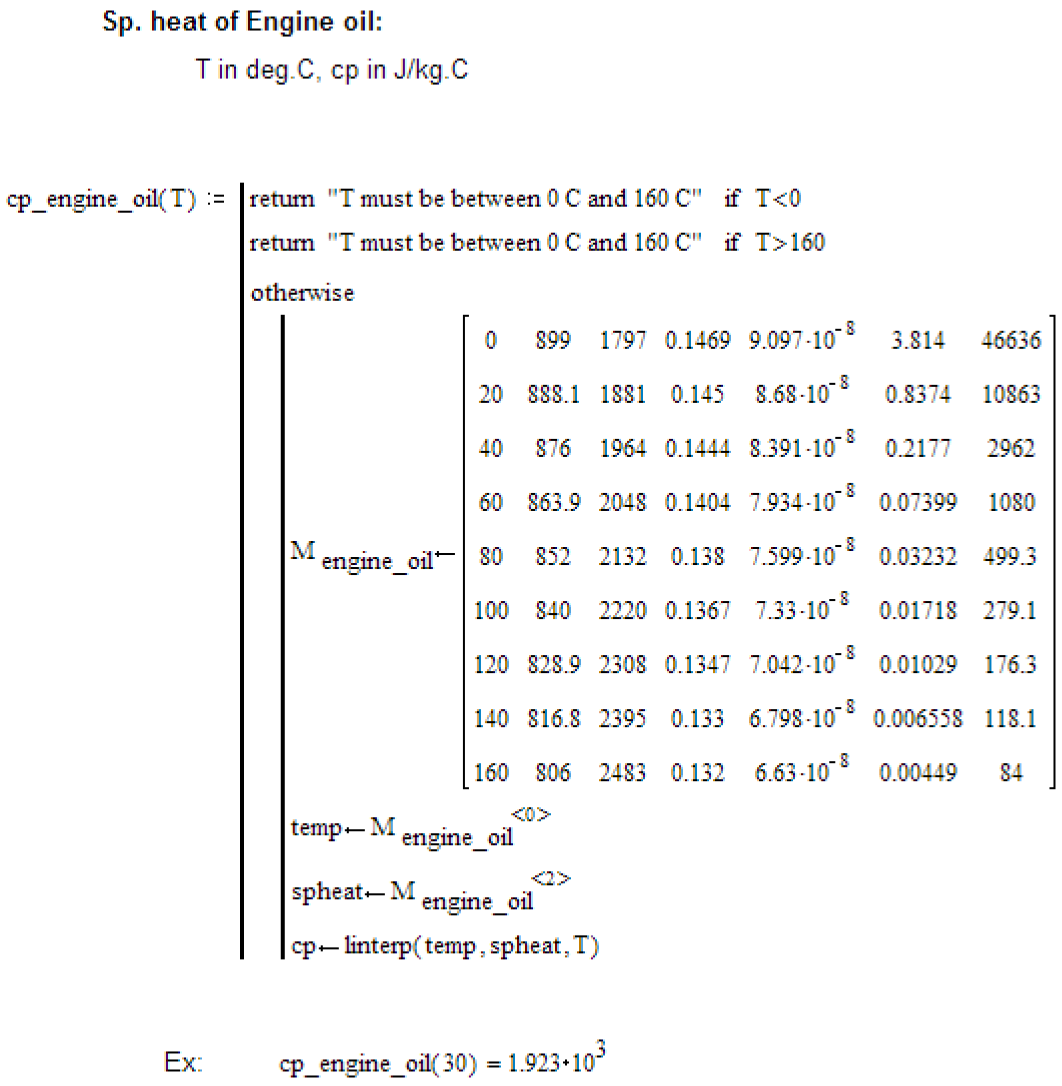
Density of Engine oil:

T in deg.C, rho in kg/m3



```
rho_engine_oil(T) := | return "T must be between 0 C and 160 C"  if  T<0
                     | return "T must be between 0 C and 160 C"  if  T>160
                     | otherwise
                     |  |                    [  0   899   1797  0.1469  9.097·10^-8  3.814     46636 ]
                     |  |                    [ 20   888.1 1881  0.145   8.68·10^-8   0.8374    10863 ]
                     |  |                    [ 40   876   1964  0.1444  8.391·10^-8  0.2177    2962  ]
                     |  |                    [ 60   863.9 2048  0.1404  7.934·10^-8  0.07399   1080  ]
                     |  | M_engine_oil ←     [ 80   852   2132  0.138   7.599·10^-8  0.03232   499.3 ]
                     |  |                    [100   840   2220  0.1367  7.33·10^-8   0.01718   279.1 ]
                     |  |                    [120   828.9 2308  0.1347  7.042·10^-8  0.01029   176.3 ]
                     |  |                    [140   816.8 2395  0.133   6.798·10^-8  0.006558  118.1 ]
                     |  |                    [160   806   2483  0.132   6.63·10^-8   0.00449   84    ]
                     |  | temp ← M_engine_oil^<0>
                     |  | density ← M_engine_oil^<1>
                     |  | rho ← linterp(temp, density, T)
```

Ex: rho_engine_oil(30) = 882.05

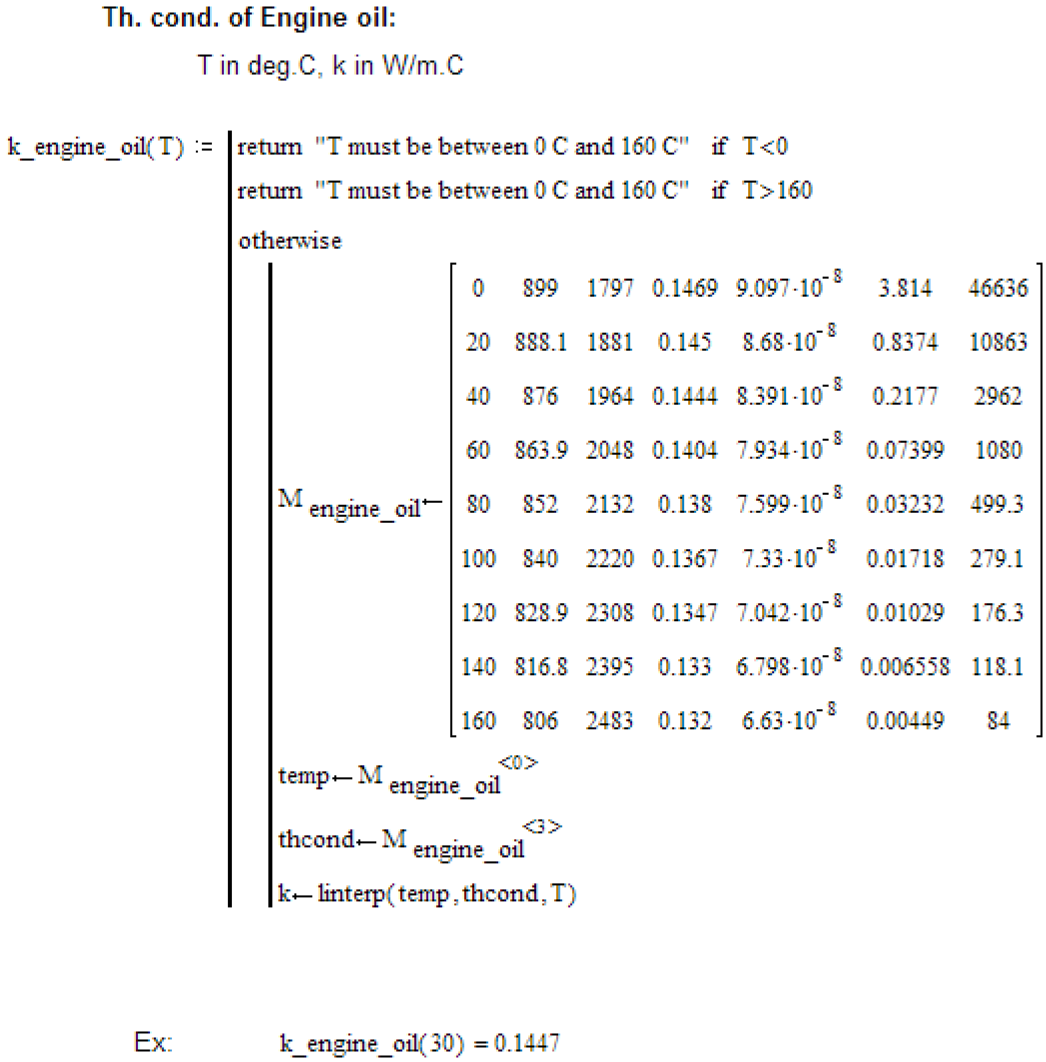


Sp. heat of Engine oil:

T in deg.C, cp in J/kg.C

```
cp_engine_oil(T) := | return "T must be between 0 C and 160 C"  if  T<0
                    | return "T must be between 0 C and 160 C"  if  T>160
                    | otherwise
                    |   M_engine_oil ← [  0   899    1797  0.1469  9.097·10^-8  3.814     46636 ]
                    |                  [ 20   888.1  1881  0.145   8.68·10^-8   0.8374    10863 ]
                    |                  [ 40   876    1964  0.1444  8.391·10^-8  0.2177    2962  ]
                    |                  [ 60   863.9  2048  0.1404  7.934·10^-8  0.07399   1080  ]
                    |                  [ 80   852    2132  0.138   7.599·10^-8  0.03232   499.3 ]
                    |                  [100   840    2220  0.1367  7.33·10^-8   0.01718   279.1 ]
                    |                  [120   828.9  2308  0.1347  7.042·10^-8  0.01029   176.3 ]
                    |                  [140   816.8  2395  0.133   6.798·10^-8  0.006558  118.1 ]
                    |                  [160   806    2483  0.132   6.63·10^-8   0.00449   84    ]
                    |   temp ← M_engine_oil^<0>
                    |   spheat ← M_engine_oil^<2>
                    |   cp ← linterp(temp, spheat, T)
```

Ex: $cp\_engine\_oil(30) = 1.923 \cdot 10^3$

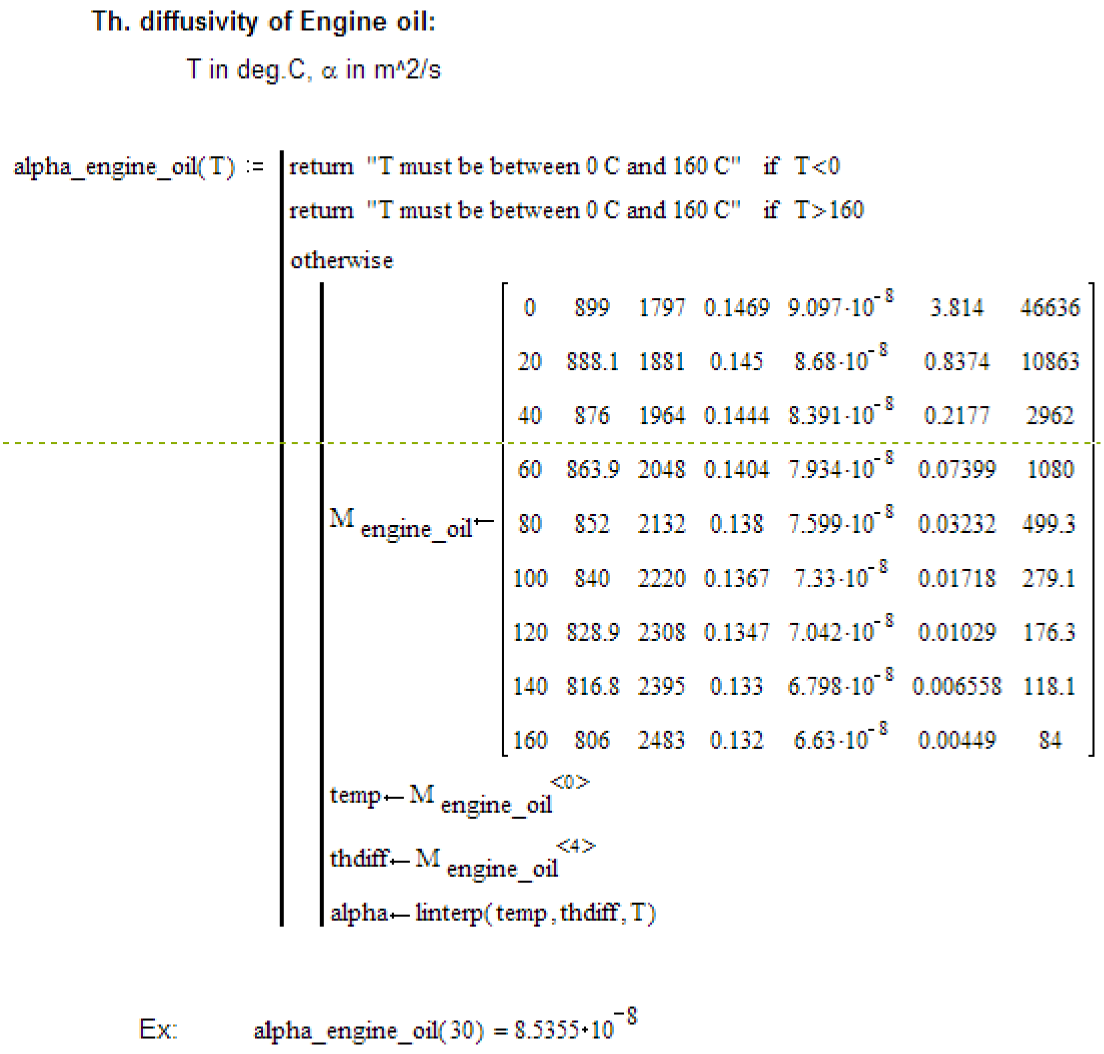


Th. cond. of Engine oil:

T in deg.C, k in W/m.C

```
k_engine_oil(T) := | return "T must be between 0 C and 160 C"  if  T<0
                   | return "T must be between 0 C and 160 C"  if  T>160
                   | otherwise
                   |   | M_engine_oil ←
                   |   |   [   0   899    1797  0.1469  9.097·10^-8  3.814     46636 ]
                   |   |   [  20   888.1  1881  0.145   8.68·10^-8   0.8374    10863 ]
                   |   |   [  40   876    1964  0.1444  8.391·10^-8  0.2177    2962  ]
                   |   |   [  60   863.9  2048  0.1404  7.934·10^-8  0.07399   1080  ]
                   |   |   [  80   852    2132  0.138   7.599·10^-8  0.03232   499.3 ]
                   |   |   [ 100   840    2220  0.1367  7.33·10^-8   0.01718   279.1 ]
                   |   |   [ 120   828.9  2308  0.1347  7.042·10^-8  0.01029   176.3 ]
                   |   |   [ 140   816.8  2395  0.133   6.798·10^-8  0.006558  118.1 ]
                   |   |   [ 160   806    2483  0.132   6.63·10^-8   0.00449   84    ]
                   |   | temp ← M_engine_oil^<0>
                   |   | thcond ← M_engine_oil^<3>
                   |   | k ← linterp(temp, thcond, T)
```

Ex: k_engine_oil(30) = 0.1447

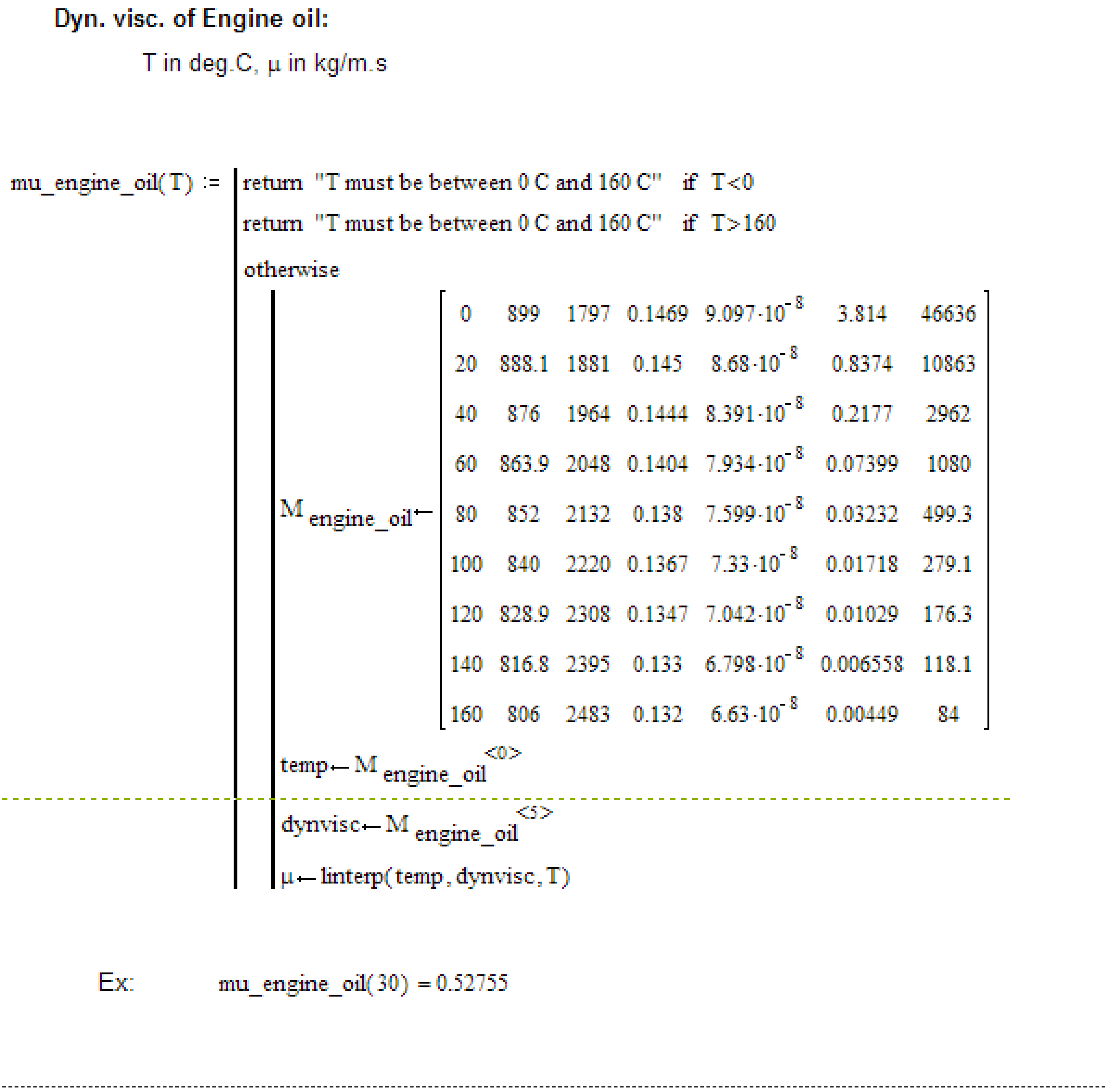


Th. diffusivity of Engine oil:

T in deg.C, α in m^2/s

$$\text{alpha\_engine\_oil}(T) := \left| \begin{array}{l} \text{return "T must be between 0 C and 160 C" if } T<0 \\ \text{return "T must be between 0 C and 160 C" if } T>160 \\ \text{otherwise} \\ \quad \left| \begin{array}{l} M_{\text{engine\_oil}} \leftarrow \begin{bmatrix} 0 & 899 & 1797 & 0.1469 & 9.097\cdot10^{-8} & 3.814 & 46636 \\ 20 & 888.1 & 1881 & 0.145 & 8.68\cdot10^{-8} & 0.8374 & 10863 \\ 40 & 876 & 1964 & 0.1444 & 8.391\cdot10^{-8} & 0.2177 & 2962 \\ 60 & 863.9 & 2048 & 0.1404 & 7.934\cdot10^{-8} & 0.07399 & 1080 \\ 80 & 852 & 2132 & 0.138 & 7.599\cdot10^{-8} & 0.03232 & 499.3 \\ 100 & 840 & 2220 & 0.1367 & 7.33\cdot10^{-8} & 0.01718 & 279.1 \\ 120 & 828.9 & 2308 & 0.1347 & 7.042\cdot10^{-8} & 0.01029 & 176.3 \\ 140 & 816.8 & 2395 & 0.133 & 6.798\cdot10^{-8} & 0.006558 & 118.1 \\ 160 & 806 & 2483 & 0.132 & 6.63\cdot10^{-8} & 0.00449 & 84 \end{bmatrix} \\ \text{temp} \leftarrow M_{\text{engine\_oil}}^{\langle 0 \rangle} \\ \text{thdiff} \leftarrow M_{\text{engine\_oil}}^{\langle 4 \rangle} \\ \text{alpha} \leftarrow \text{linterp}(\text{temp}, \text{thdiff}, T) \end{array} \right. \end{array} \right.$$

Ex: $\text{alpha\_engine\_oil}(30) = 8.5355\cdot10^{-8}$



Dyn. visc. of Engine oil:

T in deg.C, μ in kg/m.s

```
mu_engine_oil(T) := | return "T must be between 0 C and 160 C"  if  T<0
                    | return "T must be between 0 C and 160 C"  if  T>160
                    | otherwise
                    |   M_engine_oil ←
```

$$M_{engine\_oil} \leftarrow \begin{bmatrix} 0 & 899 & 1797 & 0.1469 & 9.097\cdot10^{-8} & 3.814 & 46636 \\ 20 & 888.1 & 1881 & 0.145 & 8.68\cdot10^{-8} & 0.8374 & 10863 \\ 40 & 876 & 1964 & 0.1444 & 8.391\cdot10^{-8} & 0.2177 & 2962 \\ 60 & 863.9 & 2048 & 0.1404 & 7.934\cdot10^{-8} & 0.07399 & 1080 \\ 80 & 852 & 2132 & 0.138 & 7.599\cdot10^{-8} & 0.03232 & 499.3 \\ 100 & 840 & 2220 & 0.1367 & 7.33\cdot10^{-8} & 0.01718 & 279.1 \\ 120 & 828.9 & 2308 & 0.1347 & 7.042\cdot10^{-8} & 0.01029 & 176.3 \\ 140 & 816.8 & 2395 & 0.133 & 6.798\cdot10^{-8} & 0.006558 & 118.1 \\ 160 & 806 & 2483 & 0.132 & 6.63\cdot10^{-8} & 0.00449 & 84 \end{bmatrix}$$

$$temp \leftarrow M_{engine\_oil}^{\langle 0 \rangle}$$

$$dynvisc \leftarrow M_{engine\_oil}^{\langle 5 \rangle}$$

$$\mu \leftarrow linterp(temp, dynvisc, T)$$

Ex: mu_engine_oil(30) = 0.52755

Kinematic visc. of Engine oil:

T in deg.C, ν in kg/m.s

```
nu_engine_oil(T) := | return "T must be between 0 C and 160 C"  if  T<0
                    | return "T must be between 0 C and 160 C"  if  T>160
                    | otherwise
                    |     | $\mu \leftarrow$ mu_engine_oil(T)
                    |     | $\rho \leftarrow$ rho_engine_oil(T)
                    |     | $\nu \leftarrow \frac{\mu}{\rho}$
```

Ex: $nu\_engine\_oil(30) = 5.98095 \cdot 10^{-4}$

Prandtl No. of Engine oil:

T in deg.C, Pr (dimensionless no.)

```
Pr_engine_oil(T) := | return "T must be between 0 C and 160 C"  if  T<0
                    | return "T must be between 0 C and 160 C"  if  T>160
                    | otherwise
                    |     | $M_{engine\_oil} \leftarrow \begin{bmatrix} 0 & 899 & 1797 & 0.1469 & 9.097 \cdot 10^{-8} & 3.814 & 46636 \\ 20 & 888.1 & 1881 & 0.145 & 8.68 \cdot 10^{-8} & 0.8374 & 10863 \\ 40 & 876 & 1964 & 0.1444 & 8.391 \cdot 10^{-8} & 0.2177 & 2962 \\ 60 & 863.9 & 2048 & 0.1404 & 7.934 \cdot 10^{-8} & 0.07399 & 1080 \\ 80 & 852 & 2132 & 0.138 & 7.599 \cdot 10^{-8} & 0.03232 & 499.3 \\ 100 & 840 & 2220 & 0.1367 & 7.33 \cdot 10^{-8} & 0.01718 & 279.1 \\ 120 & 828.9 & 2308 & 0.1347 & 7.042 \cdot 10^{-8} & 0.01029 & 176.3 \\ 140 & 816.8 & 2395 & 0.133 & 6.798 \cdot 10^{-8} & 0.006558 & 118.1 \\ 160 & 806 & 2483 & 0.132 & 6.63 \cdot 10^{-8} & 0.00449 & 84 \end{bmatrix}$
                    |     | $temp \leftarrow M_{engine\_oil}^{\langle 0 \rangle}$
                    |     | $Prandtl \leftarrow M_{engine\_oil}^{\langle 6 \rangle}$
                    |     | $Pr \leftarrow$ linterp(temp, Prandtl, T)
```

Ex: $Pr\_engine\_oil(30) = 6.9125 \cdot 10^{3}$

==

Now, let us solve the problem:

Data:

$m := 0.02$ kg/s $d := 0.003$ m $L := 30$ m $T_{in} := 60$ C $T_s := 100$ C

Let the outlet temp be assumed as T_{out} = 80 C…. Will be corrected later.

Therefore: $T_b(T_{out}) := \frac{T_{in} + T_{out}}{2}$ ...bulk mean temp as a function of T_{out}

Properties of Engine oil at T_b:

Use the functions written above.

$rho(T_{out}) := rho\_engine\_oil(T_b(T_{out}))$

$cp(T_{out}) := cp\_engine\_oil(T_b(T_{out}))$

$mu(T_{out}) := mu\_engine\_oil(T_b(T_{out}))$

$k(T_{out}) := k\_engine\_oil(T_b(T_{out}))$

$Pr(T_{out}) := Pr\_engine\_oil(T_b(T_{out}))$

Calculations:

$A_c := \frac{\pi \cdot d^2}{4}$ $A_c = 7.069 \cdot 10^{-6}$ m^2 .... cross-sectional area

$G := \frac{m}{A_c}$ $G = 2.829 \cdot 10^3$ kg/s.m^2.. mass velocity

$Re(T_{out}) := \frac{G \cdot d}{mu(T_{out})}$ i.e. $Re(T_{out}) = 159.689$

We see that the flow is laminar.

And the thermal entry length is:

$L_t(T_{out}) := 0.05 \cdot Re(T_{out}) \cdot Pr(T_{out}) \cdot d$ i.e. $L_t(T_{out}) = 18.915$ m

We see that Lt < L i.e. for part of the tube, the thermal boundary layer is fully developed. So, we use the eqn.from Ref. [3]:

$$Nu(T_{out}) := 1.86 \cdot \left(\frac{Re(T_{out}) \cdot Pr(T_{out})}{\frac{L}{d}} \right)^{\frac{1}{3}} \cdot \left(\frac{mu(T_{out})}{mu\_engine\_oil(T_s)} \right)^{0.14} \quad \text{....when } L_t < L$$

i.e. $Nu(T_{out}) = 5.071$

Then, heat transfer coeff. h is given by:

$$h(T_{out}) := \frac{Nu(T_{out}) \cdot k(T_{out})}{d} \qquad \text{i.e.} \qquad h(T_{out}) = 235.292$$

Therefore, heat transfer is given by: Q = h * As * LMTD

Now: $A_s(L) := \pi \cdot d \cdot L \quad A_s(L) = 0.283$ m^2....surface area

$$\mathrm{LMTD}(T_{out}) := \frac{(T_s - T_{in}) - (T_s - T_{out})}{\ln\left(\frac{T_s - T_{in}}{T_s - T_{out}}\right)} \qquad \text{i.e.} \qquad \mathrm{LMTD}(T_{out}) = 28.854$$

And: $Q(T_{out}, L) := h(T_{out}) \cdot A_s(L) \cdot \mathrm{LMTD}(T_{out})$

But, Q is also equal to: m * cp * (T_{out} – T_{in}).

Use the Solve block of Mathcad to find the value of T_{out} to satisfy the two eqns for Q simultaneously.

Start with a guess value for Tout: already taken as 80 C.

$T_{out} := 80$ C .... guess value

Given

$Q(T_{out}, L) = m \cdot cp(T_{out}) \cdot (T_{out} - T_{in})$

$T_{out}(L) := \mathrm{Find}(T_{out})$

i.e. $T_{out}(L) = 91.262$ **.... outlet temp of oil ... Ans.**

Note that T_{out} is written as a function of tube length, L. This is for the purpose of plotting T_{out} against L later.

Also:

$h(T_{out}(L)) = 227.43$ **W/m^2.C .... avg. heat transfer coeff.....Ans.**

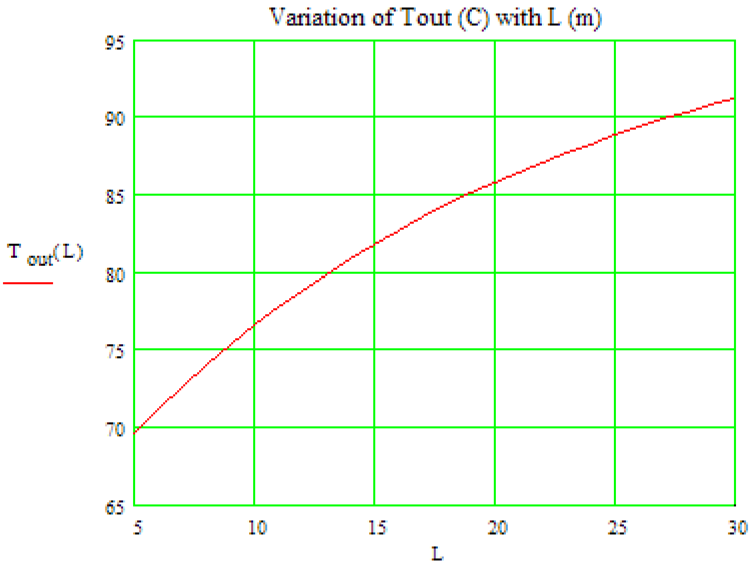
$Q(T_{out}(L), L) = 1.322 \cdot 10^3$ **W.... heat transfer rate....ans.**

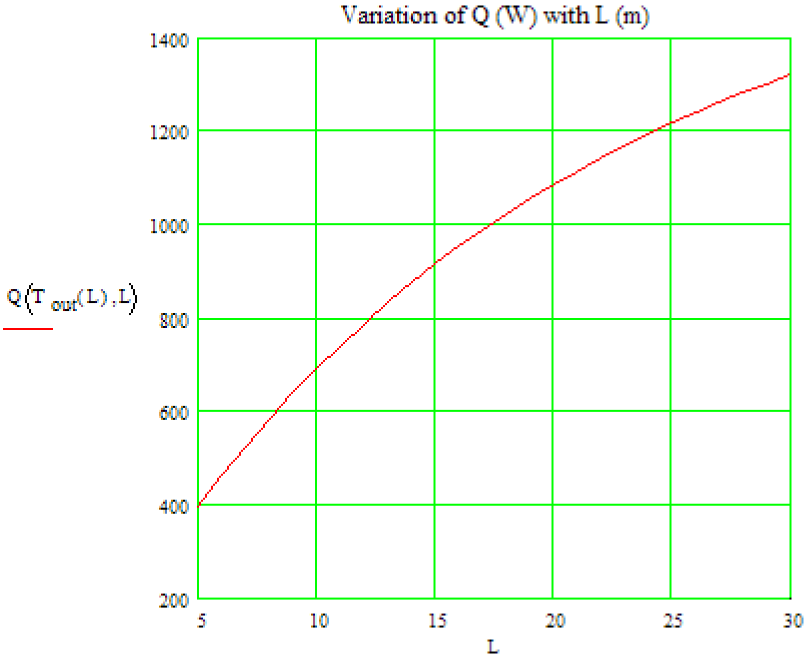
Also plot the variation of Tout and Q with L varying from 5 m to 30 m:

$L := 5, 6 .. 30$ ....variation of L

L	$T_{out}(L)$	$h(T_{out}(L))$	$Q(T_{out}(L),L)$
5	69.628	241.698	398.248
6	71.203	240.767	464.165
7	72.684	239.879	526.295
8	74.077	239.032	584.912
9	75.389	238.224	640.261
10	76.625	237.452	692.568
11	77.792	236.714	742.035
12	78.893	236.009	788.85
13	79.934	235.335	833.182
14	80.918	234.69	875.189
15	81.85	234.074	915.014
16	82.732	233.483	952.79
17	83.567	232.919	988.639
18	84.359	232.378	1022.675
19	85.11	231.861	1055.003
20	85.823	231.365	1085.72
21	86.5	230.891	1114.917
22	87.142	230.437	1142.678
23	87.752	230.002	1169.083
24	88.332	229.585	1194.203
25	88.884	229.186	1218.109
26	89.408	228.804	1240.864
27	89.906	228.438	1262.528
28	90.381	228.087	1283.159
29	90.832	227.752	1302.809
30	91.262	227.43	1321.528

Plots:




Variation of Q (W) with L (m)
1400
1200
1000
800
600
400
200
5
10
15
20
25
30
L
$Q(T_{out}(L), L)$

Instead, if we use the following eqn for Nusselts No:

$$Nu(T_{out}) := \left[3.66 + \frac{0.065 \cdot \left(\frac{d}{L}\right) \cdot Re(T_{out}) \cdot Pr(T_{out})}{1 + 0.04 \cdot \left[\left(\frac{d}{L}\right) \cdot \left(Re(T_{out}) \cdot Pr(T_{out})\right)\right]^{\frac{2}{3}}}\right] \quad \text{...when } L_t >> L$$

We get:

$$Nu(T_{out}) = 4.334$$

Then, heat transfer coeff. h is given by:

$$h(T_{out}) := \frac{Nu(T_{out}) \cdot k(T_{out})}{d} \qquad \text{i.e.} \qquad h(T_{out}) = 201.082$$

And, from Mathcad Solve block:

$T_{out}(L) = 89.571C$ **.... outlet temp of oil ... Ans.**

$h(T_{out}(L)) = 200.644$ **W/m^2.C .... avg. heat transfer coeff.....Ans.**

$Q(T_{out}(L), L) = 1.248 \cdot 10^3$ **W.... heat transfer rate....ans.**

==

Prob. 2A1.4.6. Engine oil flows through a 25 mm dia tube at a rate of 0.5 kg/s. The oil enters the tube at a temp of 25 C, while the tube surface temp is maintained at 100 C.

a) Determine the oil outlet temp for a 5 m long and for a 100 m long tube.
b) For 5 < L < 100 m, compute and plot the avg. Nusselt No. and the oil outlet temp as a function of L. [Ref. 3]

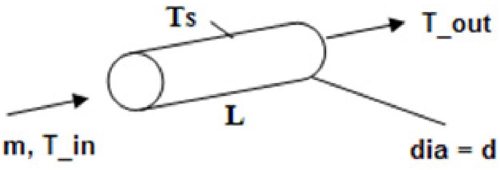


Fig.Prob.2A.1.4.6

Mathcad Solution:

While solving this problem, we shall use the Mathcad functions for properties of Engine oil which were developed in the previous problem.

Data:

$m := 0.5$ kg/s $\quad d := 0.025$ m $\quad L := 5$ m $\quad T_{in} := 25$ C $\quad T_s := 100$ C

Let the outlet temp be assumed as T_{out} = 80 C…. This will be corrected later.

Therefore: $T_b(T_{out}) := \dfrac{T_{in} + T_{out}}{2}$ ...bulk mean temp as a function of T_{out}

Properties of Engine oil at Tb:

$\text{rho}(T_{out}) := \text{rho\_engine\_oil}(T_b(T_{out}))$

$\text{cp}(T_{out}) := \text{cp\_engine\_oil}(T_b(T_{out}))$

$\text{mu}(T_{out}) := \text{mu\_engine\_oil}(T_b(T_{out}))$

$k(T_{out}) := \text{k\_engine\_oil}(T_b(T_{out}))$

$\text{Pr}(T_{out}) := \text{Pr\_engine\_oil}(T_b(T_{out}))$

Calculations:

$A_c := \dfrac{\pi \cdot d^2}{4} \qquad A_c = 4.909 \cdot 10^{-4}$ m^2 .... cross-sectional area

$G := \dfrac{m}{A_c} \qquad G = 1.019 \cdot 10^{3}$ kg/s.m^2.. mass velocity

$\text{Re}(T_{out}) := \dfrac{G \cdot d}{\text{mu}(T_{out})}$ i.e. $\text{Re}(T_{out}) = 199.128$

We see that Re < 2300; So,the flow is laminar.

And the thermal entry length is:

$L_t(T_{out}) := 0.05 \cdot Re(T_{out}) \cdot Pr(T_{out}) \cdot d$ i.e. $L_t(T_{out}) = 444.492$ m

We see that Lt >> L i.e. the thermal boundary layer is still developing.. So, we use the eqn.:

For Nusselts No. we use:

$$Nu(T_{out}) := \left[3.66 + \frac{0.065 \cdot \left(\frac{d}{L}\right) \cdot Re(T_{out}) \cdot Pr(T_{out})}{1 + 0.04 \cdot \left[\left(\frac{d}{L}\right) \cdot \left(Re(T_{out}) \cdot Pr(T_{out})\right) \right]^{\frac{2}{3}}} \right] \quad \text{...when } L_t >> L$$

i.e. $Nu(T_{out}) = 20.481$ **....Nusselts No.**

Then, heat transfer coeff. h is given by:

$h(T_{out}) := \frac{Nu(T_{out}) \cdot k(T_{out})}{d}$ i.e. $h(T_{out}) = 116.249$ W/m^2.C

Therefore, heat transfer is given by: Q = h * As * LMTD

Now: $A_s(L) := \pi \cdot d \cdot L \quad A_s(L) = 0.393$ m^2....surface area

$$LMTD(T_{out}) := \frac{(T_s - T_{in}) - (T_s - T_{out})}{\ln\left(\frac{T_s - T_{in}}{T_s - T_{out}}\right)} \qquad \text{i.e.} \quad LMTD(T_{out}) = 41.611 \quad C$$

And: $Q(T_{out}, L) := h(T_{out}) \cdot A_s(L) \cdot LMTD(T_{out})$

But, Q is also equal to: m * cp * (Tout – Tin).

Use the Solve block of Mathcad to find the value of T_{out} to satisfy these two eqns for Q simultaneously.

Start with a guess value for Tout: already taken as 80 C.

$T_{out} := 80$ C .... guess value

Given

$Q(T_{out}, L) = m \cdot cp(T_{out}) \cdot (T_{out} - T_{in})$

$T_{out}(L) := \text{Find}(T_{out})$

i.e. $T_{out}(L) = 28.489$C **.... outlet temp of oil ... Ans.**

Note that T_{out} is written as a function of L; this will be useful to draw graphs later.

And:

$h(T_{out}(L)) = 115.802$ **W/m^2.C .... avg. heat transfer coeff.....Ans.**

$Q(T_{out}(L), L) = 3.331 \cdot 10^3$ **W.... heat transfer rate....ans.**

When the length L is 100 m:

$T_{out}(100) = 70.022$ C .... outlet temp of oil ... Ans.

$h\left(T_{out}(100)\right) = 116.501$ W/m^2.C .... avg. heat transfer coeff.....Ans.

$Q\left(T_{out}(100), 100\right) = 4.492 \cdot 10^{4}$ W.... heat transfer rate....ans.

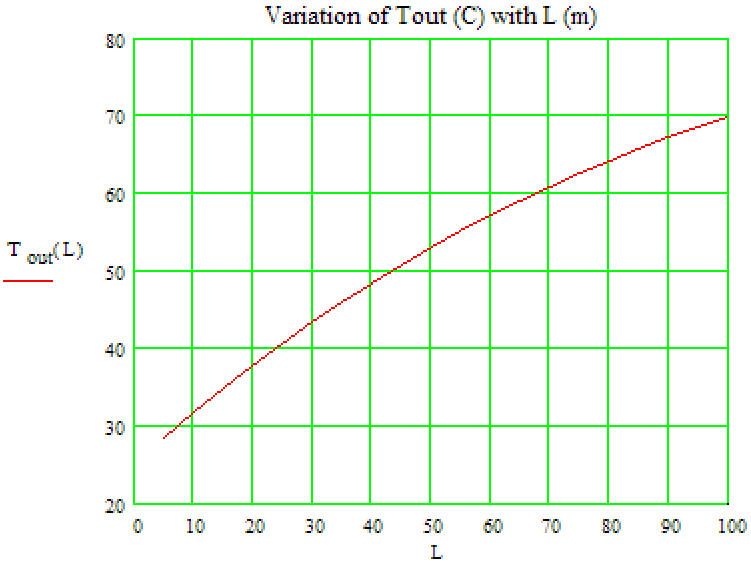
Also plot the variation of Tout and Q with L varying from 5 m to 100 m:

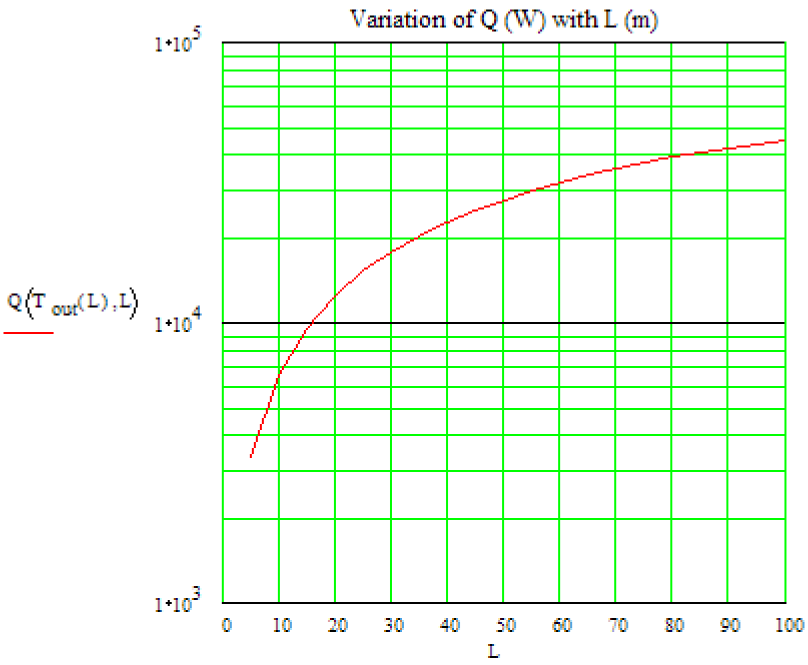
$L := 5, 10 .. 100$ ....variation of L

L	$T_{out}(L)$	$h\left(T_{out}(L)\right)$	$Q\left(T_{out}(L), L\right)$
5	28.489	115.802	3330.672
10	31.796	115.845	6509.782
15	34.932	115.9	9546.525
20	37.911	115.97	12449.476
25	40.743	116.057	15226.714
30	43.439	116.164	17885.952
35	46.009	116.296	20434.683
40	48.462	116.459	22880.363
45	50.807	116.663	25230.683
50	53.056	116.92	27493.996
55	55.208	117.203	29670.395
60	57.185	117.096	31679.744
65	59.069	116.996	33600.87
70	60.863	116.905	35438.29
75	62.574	116.821	37196.213
80	64.205	116.744	38878.559
85	65.762	116.674	40488.995
90	67.248	116.61	42030.954
95	68.667	116.553	43507.654
100	70.022	116.501	44922.12

Now, plot the graphs:

$L := 5, 10 .. 100$ ....variation of L





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Prob. 2A1.4.7. An oil pre-heater consists of a single tube of 9 mm dia and 5 m length, with its surface maintained at 160 C. The engine oil enters at 80 C. What flow rate must be supplied to maintain an oil outlet temp of 95 C? What is the corresponding heat transfer rate?

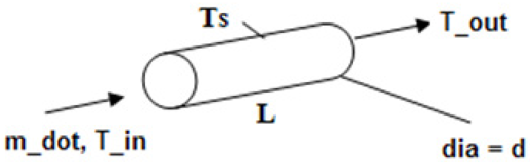


Fig.Prob.2A.1.4.7

Mathcad Solution:

While solving this problem, we shall use the Mathcad functions for properties of Engine oil which were developed in the previous problem.

Data:

$d := 0.009$ m $L := 5$ m $T_{in} := 80$ C $T_s := 160$ C $T_{out} := 95$ C

Let the mass flow rate be assumed as m_dot = 0.1 kg/s…. Will be corrected later.

$m\_dot := 0.1$ kg/s

Therefore: $T_b := \frac{T_{in} + T_{out}}{2}$ ...bulk mean temp

i.e. $T_b = 87.5$ C

Properties of Engine oil at Tb:

$rho := rho\_engine\_oil(T_b)$ $rho = 847.5$ kg/m^3

$cp := cp\_engine\_oil(T_b)$ $cp = 2.165 \cdot 10^3$ J/kg.C

$mu := mu\_engine\_oil(T_b)$ $mu = 0.027$ kg/m.s

$mu_s := mu\_engine\_oil(160)$ $mu_s = 4.49 \cdot 10^{-3}$ kg/m.s

$k := k\_engine\_oil(T_b)$ $k = 0.138$ W/m.C

$Pr := Pr\_engine\_oil(T_b)$ $Pr = 416.725$

Calculations:

$A_c := \frac{\pi \cdot d^2}{4}$ $A_c = 6.362 \cdot 10^{-5}$ m^2 .... cross-sectional area

$G(m\_dot) := \frac{m\_dot}{A_c}$ $G(m\_dot) = 1.572 \cdot 10^3$ kg/s.m^2.. mass velocity

$Re(m\_dot) := \frac{G(m\_dot) \cdot d}{mu}$ i.e. $Re(m\_dot) = 530.998$

Note that G and Re are written as functions of m_dot, so that we can determine the correct value of m_dot later, using the Solve block of Mathcad.

We see that the flow is laminar, since Re < 2300.

And the thermal entry length for laminar flow is:

$L_t := 0.05 \cdot Re(m\_dot) \cdot Pr \cdot d$ i.e. $L_t = 99.576$ m

We see that $L_t >> L$ i.e. the thermal boundary layer is still developing.. So, we use the eqn.:

For Nusselts No. we use:

$$Nu(m\_dot) := 1.86 \cdot \left(\frac{Re(m\_dot) \cdot Pr}{\frac{L}{d}} \right)^{\frac{1}{3}} \cdot \left(\frac{mu}{mu_s} \right)^{0.14}$$

...for laminar flow, combined entry length, ...(eqn. 8.57 ...Ref.[3]

i.e. $Nu(m\_dot) = 17.56$ **....Nusselts No.**

Then, heat transfer coeff. h is given by:

$$h(m\_dot) := \frac{Nu(m\_dot) \cdot k}{d}$$

i.e. $h(m\_dot) = 268.298$ W/m^2.C

Therefore, heat transfer is given by: Q = h * As * LMTD

Now: $A_s := \pi \cdot d \cdot L$ $A_s = 0.141$ m^2....surface area

$$LMTD := \frac{(T_s - T_{in}) - (T_s - T_{out})}{\ln\left(\frac{T_s - T_{in}}{T_s - T_{out}}\right)}$$

And: $Q(m\_dot) := h(m\_dot) \cdot A_s \cdot LMTD$

But, Q is also equal to: m_dot * cp * (T_{out} – T_{in}).

Use the Solve block of Mathcad to find the value of m_dot to satisfy the two eqns for Q simultaneously.

Start with a guess value for m_dot: already taken as 0.1 kg/s.

$m\_dot := 0.1$ kg/s .... guess value

Given

$Q(m\_dot) = m\_dot \cdot cp \cdot (T_{out} - T_{in})$

$m\_dot := Find(m\_dot)$

i.e. $m\_dot = 0.077503$ **kg/s .... mass flow rate ... Ans.**

Then: $Re(m\_dot) = 411.538$

$h(m\_dot) = 246.447$ **W/m^2.C .... avg. heat transfer coeff.....Ans.**

$Q(m\_dot) = 2.517 \cdot 10^3$ **W.... heat transfer rate....Ans.**

Prob. 2A1.4.8. Hot air at atm pressure and 85 C enters a 10 m long insulated square duct of size 0.15 m × 0.15 m that passes through the attic of a house at a rate of 0.1 m^3/s. The duct is at 70 C. Determine the exit temp of air and the rate of heat loss from the duct to the surroundings.

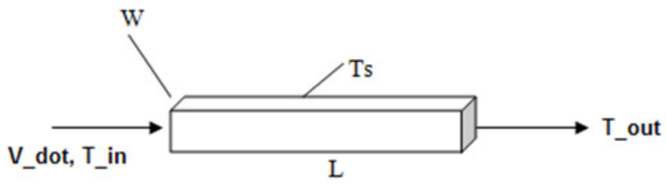


Fig.Prob.2A.1.4.8

Mathcad Solution:

While solving this problem, we shall use the Mathcad functions for properties of air which were developed in problem 2A1.2.5.

Data:

$W := 0.15$ m $\quad L := 10$ m $\quad T_{in} := 85$ C $\quad V\_dot := 0.1$ m^3/s

$T_s := 70$ C

Calculations:

Let the exit temp of air be 75 C…. will be corrected later

i.e. $T_{out} := 75$ C

Then:

$$T_b(T_{out}) := \frac{T_{in} + T_{out}}{2}$$ ....meanbulk temp in C

i.e. $T_b(T_{out}) = 80$ C

$\rho(T_{out}) := rho\_Air(T_b(T_{out}) + 273)$ i.e. $\rho(T_{out}) = 1$ kg/m^3

$k(T_{out}) := k\_Air(T_b(T_{out}) + 273)$ i.e. $k(T_{out}) = 0.03$ W/m.K

$\mu(T_{out}) := mu\_Air(T_b(T_{out}) + 273)$ i.e. $\mu(T_{out}) = 2.092 \cdot 10^{-5}$ kg/m.s

$Pr(T_{out}) := Pr\_Air(T_b(T_{out}) + 273)$ i.e. $Pr(T_{out}) = 0.699$ ...Prandtl No.

$$cp(T_{out}) := \frac{Pr(T_{out}) \cdot k(T_{out})}{\mu(T_{out})}$$ i.e. $cp(T_{out}) = 997.103$ J/kg.K

Hydraulic diameter:

$A_c := W \cdot W$ ....cross-sectional area

i.e. $A_c = 0.022$ m^2

$P := 4 \cdot W$ ....wetted perimeter

i.e. $P = 0.6$ m

Then: $D_h := \frac{4 \cdot A_c}{P}$

i.e. $D_h = 0.15$ m.... hydraulic dia

Reynolds No.:

$$Re(V\_dot, T_{out}) := \frac{\frac{V\_dot \cdot \rho(T_{out})}{A_c} \cdot D_h}{\mu(T_{out})}$$ ...Reynolds No...

i.e. $Re(V\_dot, T_{out}) = 3.188 \cdot 10^4$

Note that Re is written as a function of V_dot and T_{out}. This will be useful to draw graphs for various V_dot values, later.

Friction factor:

Use 'first Petukhov eqn' for f:

$f(V\_dot, T_{out}) := (0.79 \cdot \ln(Re(V\_dot, T_{out})) - 1.64)^{-2}$ ...for 3000 < Re < 5 x 10^6

i.e. $f(V\_dot, T_{out}) = 0.023$ ....friction factor

Nusselts No.:

We use Dittus Boelter eqn.

Write Nu as a function of V_dot and T_{out}:

$Nu(V\_dot, T_{out}) := 0.023 \cdot Re(V\_dot, T_{out})^{0.8} \cdot Pr(T_{out})^{0.3}$ ...Dittus Boelter eqn. for turb. flow, fluid being cooled

i.e. $Nu(V\_dot, T_{out}) = 82.779$

Heat transfer coeff. h:

$$h(V\_dot, T_{out}) := \frac{Nu(V\_dot, T_{out}) \cdot k(T_{out})}{D_h}$$

i.e. $h(V\_dot, T_{out}) = 16.457$ W/m^2.C

Heat transfer, Q:

$A_s := P \cdot L$ i.e. $A_s = 6$ m^2 .... surface area

$$LMTD(T_{out}) := \frac{(T_{in} - T_s) - (T_{out} - T_s)}{\ln\left(\frac{T_{in} - T_s}{T_{out} - T_s}\right)}$$ i.e. $LMTD(T_{out}) = 9.102$ C

$Q(V\_dot, T_{out}) := h(V\_dot, T_{out}) \cdot A_s \cdot LMTD(T_{out})$ i.e. $Q(V\_dot, T_{out}) = 898.773$ W

Note that h and Q are written as functions of V_dot and T_{out}.

Now, Q should also be equal to the heat lost by air as it passes through the duct. This is equal to:

m_dot * cp * (T_{in} – T_{out}).

Use the Solve block of Mathcad to determine the correct value of T_{out} to simultaneously satisfy the two eqns for Q:

Start with the guess value for T_{out}:

$T_{out} := 75$ C .... guess value

Given

$$Q(V\_dot, T_{out}) = V\_dot \cdot \rho(T_{out}) \cdot cp(T_{out}) \cdot (T_{in} - T_{out})$$

$$T_{out}(V\_dot) := \mathrm{Find}(T_{out})$$

i.e. $T_{out}(V\_dot) = 75.571$ **C ... exit temp of Air .... Ans.**

We see that correct value of Tout is 75.571 C.

Also:

$Re(V\_dot, T_{out}(V\_dot)) = 3.183 \cdot 10^4$ **....Reynolds No..... Ans.**

$Nu(V\_dot, T_{out}(V\_dot)) = 82.683$ **...Nusselts No......Ans.**

And: $Q(V\_dot, T_{out}(V\_dot)) = 939.527$ **W ... heat transfer... Ans.**

i.e. Heat transfer under these conditions is 939.527 W.

Pressure drop:

$U(V\_dot) := \frac{V\_dot}{A_c}$ $U(V\_dot) = 4.444$ m/s.... velocity of flow

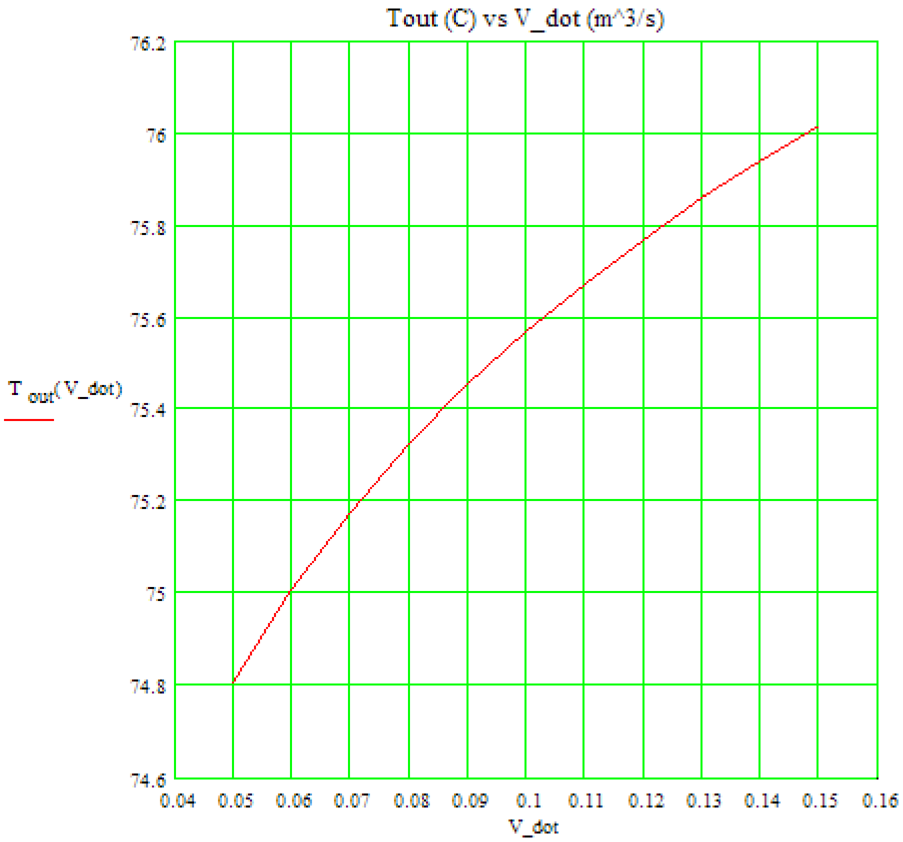
$$\Delta P(V\_dot) := f(V\_dot, T_{out}(V\_dot)) \cdot \left(\frac{L}{D_h}\right) \cdot \frac{U(V\_dot)^2}{2} \cdot \rho(T_{out}(V\_dot))$$

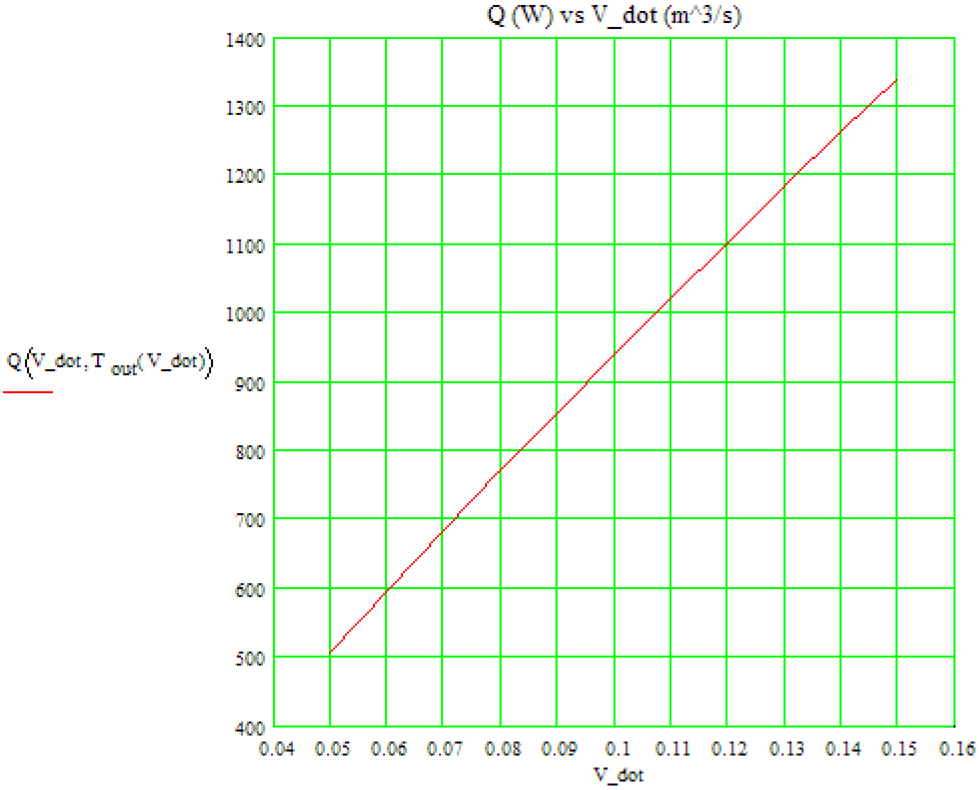
i.e. $\Delta P(V\_dot) = 15.333$ N/m^2

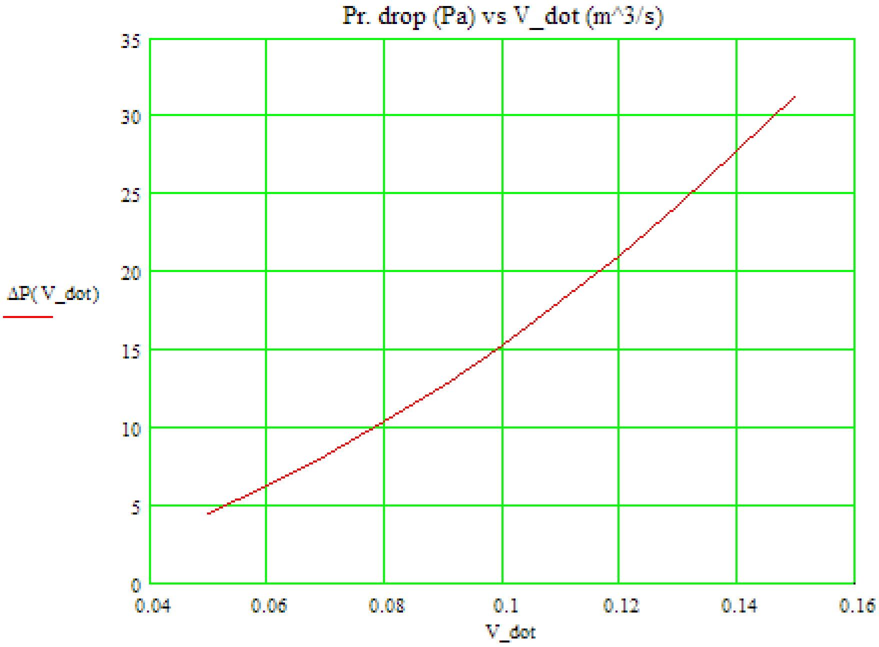
Plot T_{out} and Q as V_dot varies from 0.05 m^3/s to 0.15 m^3/s:

V_dot := 0.05, 0.06 .. 0.15   m^3/s ..... flow rate

V_dot	T_{out}(V_dot)	Q(V_dot, T_{out}(V_dot))	ΔP(V_dot)
0.05	74.811	508.205	4.567
0.06	75.01	597.747	6.271
0.07	75.179	685.405	8.206
0.08	75.326	771.456	10.365
0.09	75.455	856.11	12.743
0.1	75.571	939.527	15.333
0.11	75.676	1021.837	18.132
0.12	75.771	1103.145	21.136
0.13	75.859	1183.539	24.342
0.14	75.941	1263.093	27.746
0.15	76.016	1341.869	31.346




Q (W) vs V_dot (m^3/s)
Q(V_dot, T out(V_dot))
1400
1300
1200
1100
1000
900
800
700
600
500
400
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.11
0.12
0.13
0.14
0.15
0.16
V_dot


Pr. drop (Pa) vs V_dot (m^3/s)
ΔP(V_dot)
35
30
25
20
15
10
5
0
0.04
0.06
0.08
0.1
0.12
0.14
0.16
V_dot

==

Prob. 2A1.4.9. Air enters a 7 m long section of a rectangular duct (15 cm × 20 cm) at 50 C at an average velocity of 7 m/s. If the walls of the duct are maintained at 10 C, determine: (a) the outlet temp of air (b) rate of heat transfer, and (c) fan power needed to overcome the pressure losses.

Plot exit temp, heat transfer and fan power for velocities ranging from 1 m/s to 10 m/s.

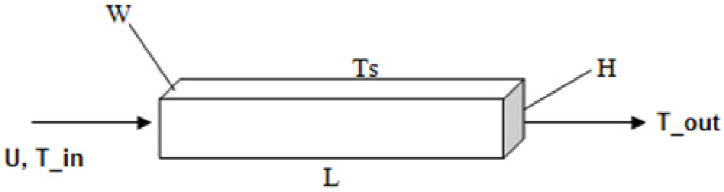


Fig.Prob.2A.1.4.9

Mathcad Solution:

While solving this problem, we shall use the Mathcad functions for properties of air which were developed in problem 2A1.2.5.

Data:

$W := 0.15$ m $H := 0.2$ m $L := 7$ m $T_{in} := 50$ C

$T_s := 10$ C $U := 7$ m/s

Calculations:

Let the exit temp of air be 20 C…. will be corrected later

i.e. $T_{out} := 20$ C

Then:

$$T_b(T_{out}) := \frac{T_{in} + T_{out}}{2}$$ ....mean bulk temp in C

i.e. $T_b(T_{out}) = 35$ C

Properties of Air at Tb:

Use the Mathcad functions already written for properties of Air.

See Prob. 2A1.2.5.

$\rho(T_{out}) := rho\_Air(T_b(T_{out}) + 273)$ i.e. $\rho(T_{out}) = 1.146$ kg/m^3

$k(T_{out}) := k\_Air(T_b(T_{out}) + 273)$ i.e. $k(T_{out}) = 0.027$ W/m.K

$\mu(T_{out}) := mu\_Air(T_b(T_{out}) + 273)$ i.e. $\mu(T_{out}) = 1.882 \cdot 10^{-5}$ kg/m.s

$Pr(T_{out}) := Pr\_Air(T_b(T_{out}) + 273)$ i.e. $Pr(T_{out}) = 0.706$ ...Prandtl No.

$cp(T_{out}) := \frac{Pr(T_{out}) \cdot k(T_{out})}{\mu(T_{out})}$ i.e. $cp(T_{out}) = 995.131$ J/kg.K

Hydraulic diameter:

$A_c := W \cdot H$ ....cross-sectional area

i.e. $A_c = 0.03$ m^2

$P := 2 \cdot (W + H)$ ....wetted perimeter

i.e. $P = 0.7$ m

Then: $D_h := \dfrac{4 \cdot A_c}{P}$

i.e. $D_h = 0.171$ m.... hydraulic dia

Reynolds No.:

$Re(U, T_{out}) := \dfrac{U \cdot D_h \cdot \rho(T_{out})}{\mu(T_{out})}$ ...Reynolds No...

i.e. $Re(U, T_{out}) = 7.311 \cdot 10^4$

Note that Re is written as a function of velocity U and Tout. This will be useful to draw graphs for various U values, later.

Friction factor:

Use 'first Petukhov eqn' for f:

$f(U, T_{out}) := (0.79 \cdot \ln(Re(U, T_{out})) - 1.64)^{-2}$ ...for 3000 < Re < 5 x 10^6

i.e. $f(U, T_{out}) = 0.019$ ....friction factor

Nusselts No.:

$Nu(U, T_{out}) := 0.023 \cdot Re(U, T_{out})^{0.8} \cdot Pr(T_{out})^{0.3}$ ...Dittus Boelter eqn. for turb. flow, fluid being cooled

i.e. $Nu(U, T_{out}) = 161.254$

Heat transfer coeff. h:

$h(U, T_{out}) := \dfrac{Nu(U, T_{out}) \cdot k(T_{out})}{D_h}$

i.e. $h(U, T_{out}) = 24.951$ W/m^2.C

Heat transfer, Q:

$A_s := P \cdot L$ i.e. $A_s = 4.9$ m^2 .... surface area

$$LMTD(T_{out}) := \frac{(T_{in} - T_s) - (T_{out} - T_s)}{\ln\left(\frac{T_{in} - T_s}{T_{out} - T_s}\right)}$$ i.e. $LMTD(T_{out}) = 21.64$ C

$Q(U, T_{out}) := h(U, T_{out}) \cdot A_s \cdot LMTD(T_{out})$ i.e. $Q(U, T_{out}) = 2.646 \cdot 10^3$ W

Now, Q should also be equal to the heat lost by air as it passes through the duct. This is equal to:

m_dot * cp * (T_{in} – T_{out}).

Use the Solve block of Mathcad to determine the correct value of T_{out} to simultaneously satisfy the two eqns for Q:

Start with the guess value for T_{out}.

$T_{out} := 20$ C .... guess value

Given

$Q(U, T_{out}) = U \cdot A_c \cdot \rho(T_{out}) \cdot cp(T_{out}) \cdot (T_{in} - T_{out})$

$T_{out}(U) := \mathrm{Find}(T_{out})$

i.e. $T_{out}(U) = 33.901$ **C ... exit temp of Air .... Ans.**

Note that T_{out} is written as a function of U.

Also:

$Re(U, T_{out}(U)) = 7.025 \cdot 10^4$ **....Reynolds No..... Ans.**

$Nu(U, T_{out}(U)) = 156.127$ **...Nusselts No......Ans.**

$h(U, T_{out}(U)) = 24.628$ **W/m^2.C ... heat transfer coeff.**

And: $Q(U, T_{out}(U)) = 3.773 \cdot 10^3$ **W ... heat transfer... Ans.**

Pressure drop:

$U = 7$ m/s.... velocity of flow

$$\Delta P(U) := f(U, T_{out}(U)) \cdot \left(\frac{L}{D_h}\right) \cdot \frac{U^2 \cdot \rho(T_{out}(U))}{2}$$

i.e. $\Delta P(U) = 21.776$ N/m^2

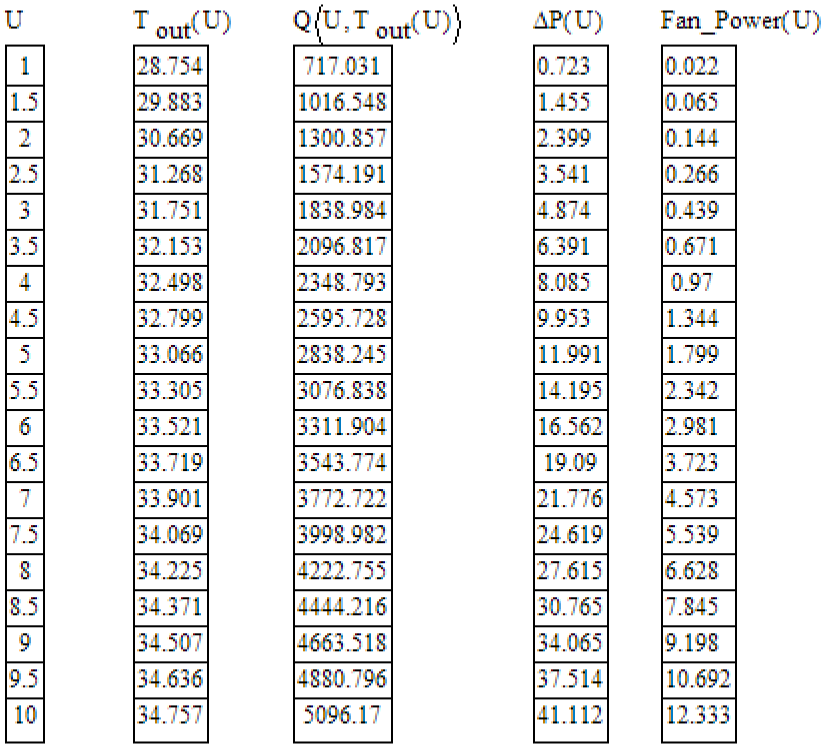
Fan Power:

$Fan\_Power(U) := \Delta P(U) \cdot A_c \cdot U$ W

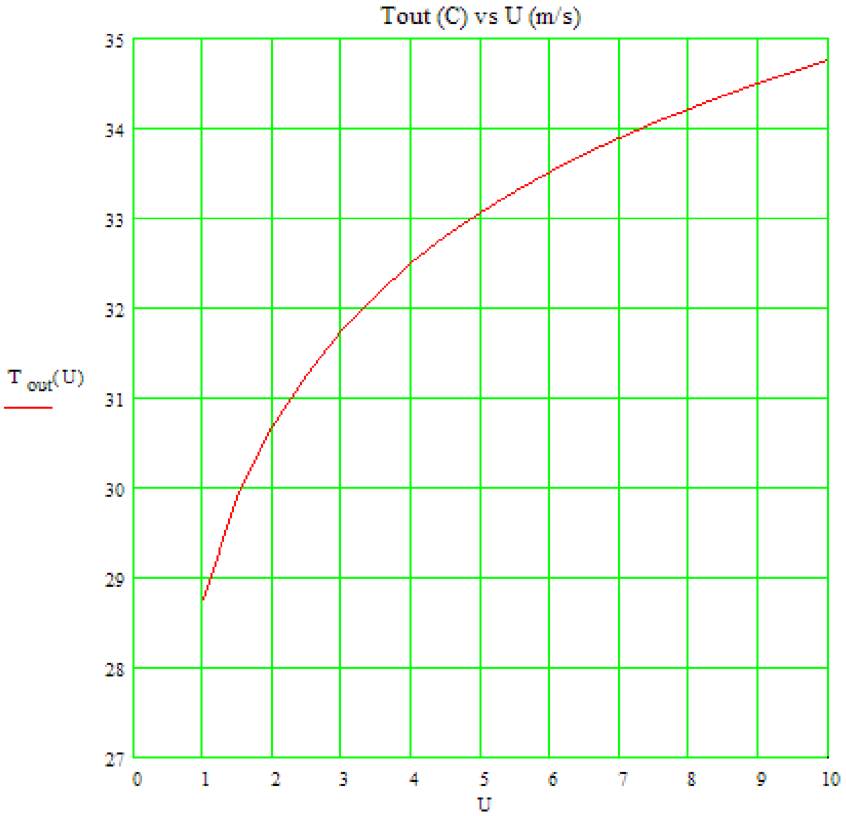
i.e. $Fan\_Power(U) = 4.573$ **W .... Ans.**

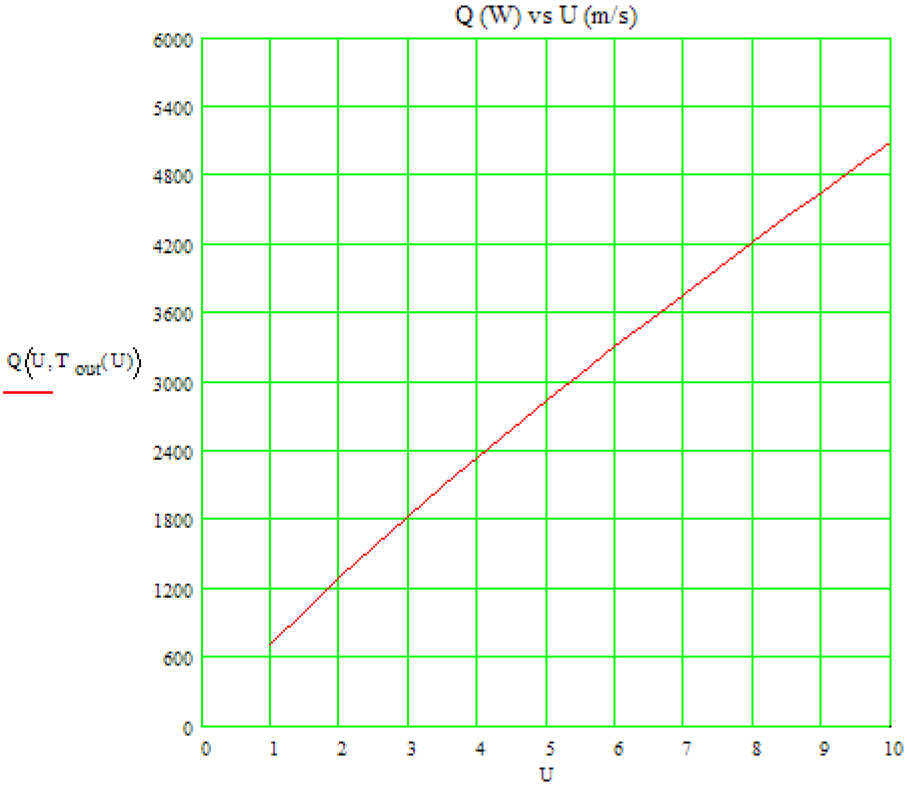
Plot T_{out} and Q as U varies from 1 m/s to 10 m/s:

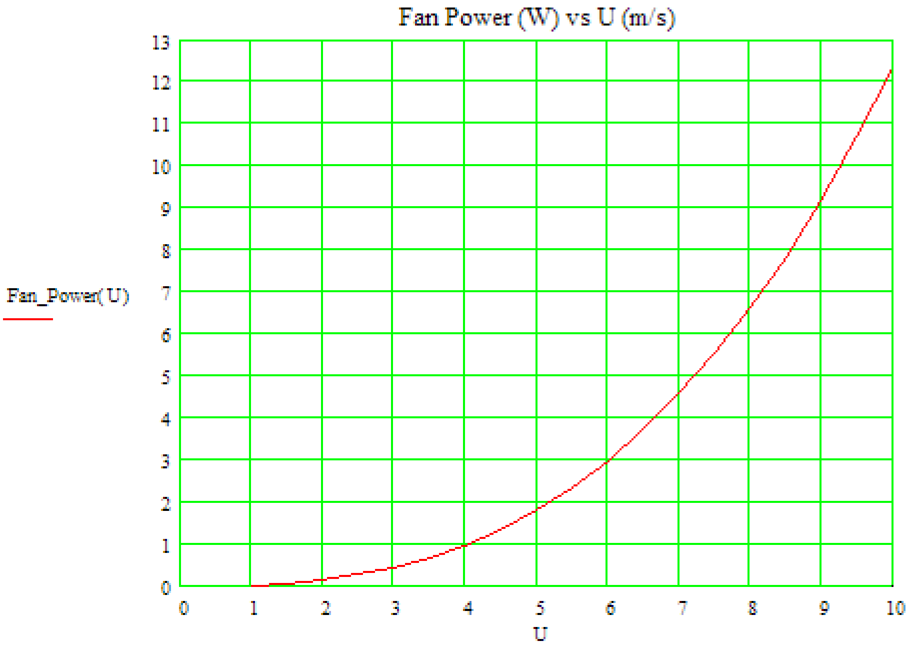
$U := 1, 1.5 .. 10$ m/s ..... velocity of flow



U	$T_{out}(U)$	$Q(U, T_{out}(U))$	$\Delta P(U)$	Fan_Power(U)
1	28.754	717.031	0.723	0.022
1.5	29.883	1016.548	1.455	0.065
2	30.669	1300.857	2.399	0.144
2.5	31.268	1574.191	3.541	0.266
3	31.751	1838.984	4.874	0.439
3.5	32.153	2096.817	6.391	0.671
4	32.498	2348.793	8.085	0.97
4.5	32.799	2595.728	9.953	1.344
5	33.066	2838.245	11.991	1.799
5.5	33.305	3076.838	14.195	2.342
6	33.521	3311.904	16.562	2.981
6.5	33.719	3543.774	19.09	3.723
7	33.901	3772.722	21.776	4.573
7.5	34.069	3998.982	24.619	5.539
8	34.225	4222.755	27.615	6.628
8.5	34.371	4444.216	30.765	7.845
9	34.507	4663.518	34.065	9.198
9.5	34.636	4880.796	37.514	10.692
10	34.757	5096.17	41.112	12.333


Tout (C) vs U (m/s)
35
34
33
32
31
30
29
28
27
$T_{out}(U)$
0
1
2
3
4
5
6
7
8
9
10
U


Q (W) vs U (m/s)
6000
5400
4800
4200
3600
3000
2400
1800
1200
600
0
$Q(U, T_{out}(U))$
0
1
2
3
4
5
6
7
8
9
10
U


Fan Power (W) vs U (m/s)
Fan_Power(U)
13
12
11
10
9
8
7
6
5
4
3
2
1
0
0
1
2
3
4
5
6
7
8
9
10
U

===

Prob. 2A1.4.10.: In a long annulus (3.125 cm ID, 5 cm OD), air is heated by maintaining the temperature of outer surface of the inner tube at 50 C. The air enters at 16 C and leaves at 32 C and its flow velocity is 30 m/s. Estimate the heat transfer coeff. between the air and the inner tube. Use Dittus – Boelter eqn, viz. NuD = 0.023.(ReD)0.8.Pr0.4; [M.U. 1999].

Also: Plot heat transfer and pressure drop for velocities ranging from 25 m/s to 35 m/s. for the same exit temp.

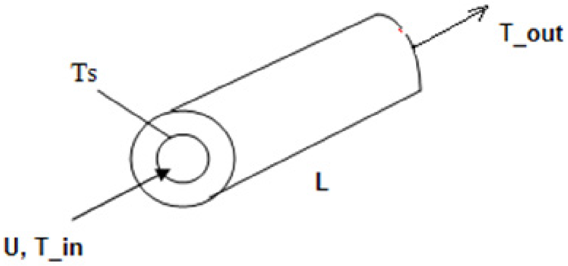


Fig.Prob.2A.1.4.10

Mathcad Solution:

While solving this problem, we shall use the Mathcad functions for properties of air which were developed in problem 2A1.2.5.

Data:

$T_{in} := 16$ C....inlet temp. of air

$T_{out} := 32$ C....exit temp. of air

$T_s := 50$ C....surface temp. of inner tube

$D_i := 0.03125$ m....inner dia of annulus

$D_o := 0.05$ m...outer dia of annulus

$L := 1$ m....length of annulus...assumed

$U := 30$ m/s....velocity of flow

Calculations:

Then:

$T_b := \frac{T_{in} + T_{out}}{2}$ ....mean bulk temp in C

i.e. $T_b = 24$ C

Properties of Air at T_b:

Use the Mathcad functions already written for properties of Air.

See Prob. 2A1.2.5.

$\rho := rho\_Air(T_b + 273)$ i.e. $\rho = 1.189$ kg/m^3

$k := k\_Air(T_b + 273)$ i.e. $k = 0.026$ W/m.K

$\mu := mu\_Air(T_b + 273)$ i.e. $\mu = 1.828 \cdot 10^{-5}$ kg/m.s

$Pr := Pr\_Air(T_b + 273)$ i.e. $Pr = 0.708$ ...Prandtl No.

$cp := \frac{Pr \cdot k}{\mu}$ i.e. $cp = 995.063$ J/kg.K

Hydraulic diameter:

$A_c := \pi \cdot \frac{D_o^2 - D_i^2}{4}$ i.e. $A_c = 1.197 \cdot 10^{-3}$ m^2 .... cross-sectional area

$P := \pi \cdot (D_o + D_i)$ i.e. $P = 0.255$ m .... perimeter

We have: $D_h = \frac{4 \cdot A_c}{P}$

i.e. $D_h := D_o - D_i$

i.e. $D_h = 0.019$ m.... hydraulic dia

Reynolds No.:

$$Re(U) := \frac{U \cdot D_h \cdot \rho}{\mu}$$

i.e. $Re(U) = 3.658 \cdot 10^4$ **...Reynolds No..Ans.**

Note that Re is written as a function of velocity U. This will be useful to draw graphs for various U values, later.

Friction factor:

Use 'first Petukhov eqn' for f:

$f(U) := (0.79 \cdot \ln(Re(U)) - 1.64)^{-2}$ ...for 3000 < Re < 5 x 10^6

i.e. $f(U) = 0.023$ ....friction factor

Nusselts No.:

$Nu(U) := 0.023 \cdot Re(U)^{0.8} \cdot Pr^{0.3}$ ...Dittus Boelter eqn. for turb. flow, fluid being cooled

i.e. $Nu(U) = 92.737$ **....Nusselts No.**

Heat transfer coeff. h:

$$h(U) := \frac{Nu(U) \cdot k}{D_h}$$

i.e. $h(U) = 127.119$ **W/m^2.C....heat transfer coeff. ...Ans.**

Heat transfer, Q:

$A_s := \pi \cdot D_i \cdot L$ i.e. $A_s = 0.098$ m^2 .... surface area

$$LMTD := \frac{(T_s - T_{in}) - (T_s - T_{out})}{\ln\left(\frac{T_s - T_{in}}{T_s - T_{out}}\right)}$$ i.e. $LMTD = 25.158$ C

$$Q(U) := h(U) \cdot A_s \cdot LMTD$$

i.e. $Q(U) = 313.964$ **W.... Ans.**

Pressure drop:

$U = 30$ m/s.... velocity of flow

$$\Delta P(U) := f(U) \cdot \left(\frac{L}{D_h}\right) \cdot \frac{U^2 \cdot \rho}{2}$$

i.e. $\Delta P(U) = 643.067$ N/m^2

Fan Power:

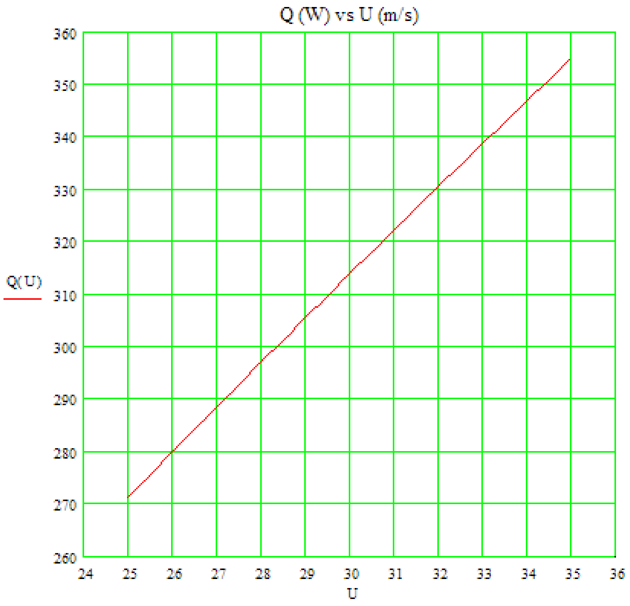
$Fan\_Power(U) := \Delta P(U) \cdot A_c \cdot U$ W

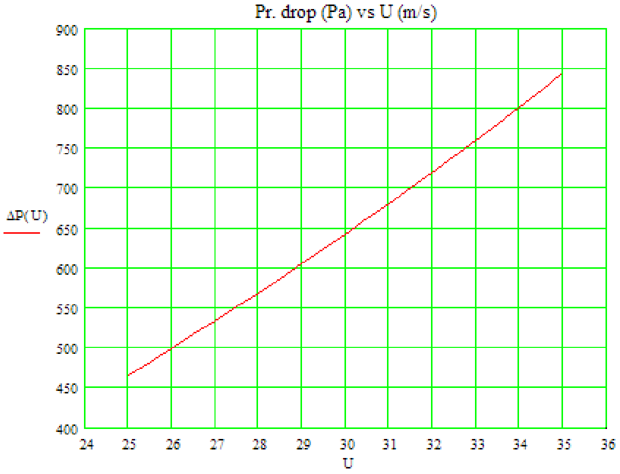
i.e. $Fan\_Power(U) = 23.083$ **W .... Ans.**

Plot Q and Fan_Power as U varies from 1 m/s to 10 m/s:

$U := 25, 25.5 .. 35$ m/s ..... velocity of flow

U	Q(U)	ΔP(U)	Fan_Power(U)
25	271.353	466.534	13.955
25.5	275.686	483.059	14.739
26	280.002	499.838	15.55
26.5	284.302	516.869	16.389
27	288.585	534.151	17.256
27.5	292.853	551.683	18.153
28	297.105	569.465	19.078
28.5	301.342	587.495	20.034
29	305.564	605.772	21.019
29.5	309.771	624.297	22.036
30	313.964	643.067	23.083
30.5	318.143	662.083	24.162
31	322.309	681.343	25.272
31.5	326.461	700.847	26.415
32	330.6	720.594	27.59
32.5	334.726	740.583	28.799
33	338.84	760.813	30.04
33.5	342.941	781.284	31.316
34	347.029	801.996	32.626
34.5	351.106	822.947	33.971
35	355.171	844.137	35.35





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$UnitSystem SI Pa C J

"**Prob. 2A1.4.11.** Water flows at 2 kg/s through a 40 mm dia tube 4 m long. The water enters the tube at 25 C, and the surface temp is 90 C.

a) What is the outlet temp of water? What is the rate of heat transfer to water?
b) Maintaining the outlet temp found in part (a), plot the tube length required as a function of tube surface temp when the surface temp varies from 85 to 95 C. All other conditions remain the same. [Ref. 3]"

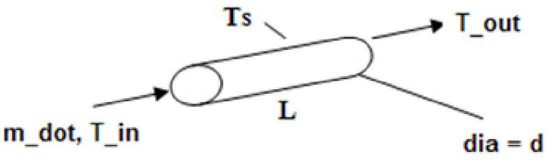


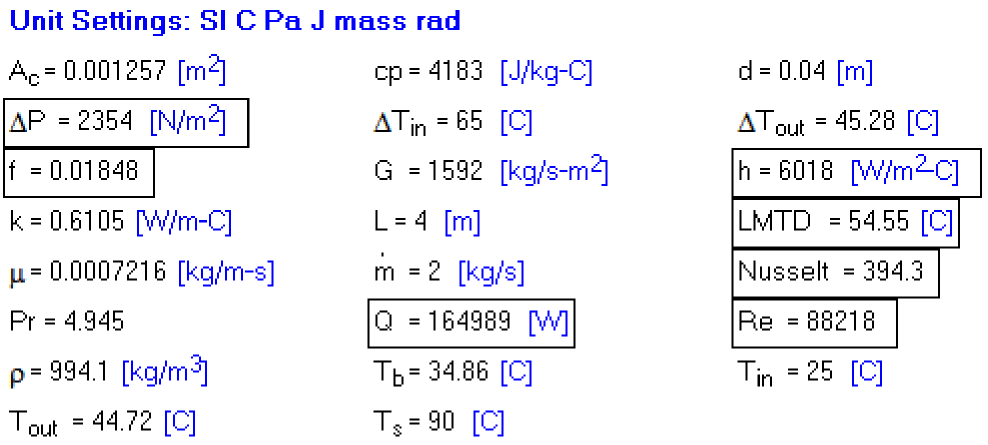
Fig.Prob.2A.1.4.11

EES Solution:

```
"Data:"

T_in = 25[C]
T_s = 90[C]
d = 0.04[m]
L = 4 [m]
m_dot = 2 [kg/s]

"Calculations:"
{T_out= 80[C]"...assumed .... will be commented out later"}

A_c = pi * d^2 / 4 "[m^2] .... area of cross-section"

T_b = (T_in + T_out)/2 "[C]....bulk mean temp of water"
```

```
“Properties of Water at bulk mean temp, T_b:”

k=Conductivity(Water,T=T_b,P=1.013e05)
rho=Density(Water,T=T_b,P=1.013e05)
mu=Viscosity(Water,T=T_b,P=1.013e05)
cp=SpecHeat(Water,T=T_b,P=1.013e05)
Pr=Prandtl(Water,T=T_b,P=1.013e05)

G = m_dot / A_c “[kg/s-m^2] … mass velocity”
Re = G * d /mu “finds Re > 10,000”

“Therefore: apply Dittus Boelter eqn.:”

Nusselt = 0.023 * Re^0.8 * Pr^0.4 “finds Nusselts No. for Re > 1000”
Nusselt = h * d / k “finds h”

DELTAT_in = T_s – T_in
DELTAT_out = T_s – T_out

LMTD = (DELTAT_in – DELTAT_out) / ln(DELTAT_in/DELTAT_out)
Q = h * (pi * d * L) * LMTD “finds Q”
Q = m_dot * cp * (T_out – T_in) “finds T_out”

“Also, find the pressure drop:”

“Friction factor: Use the first Petukhov eqn, which is an explicit eqn. for f. Re range: 3000 to 5E06”

f = (0.79 * ln(Re) – 1.64)^(-2)
DELTAP = f * (L / d) * rho * (G/rho)^2 / 2 “[N/m^2]… Pressure drop”
```

Results:



Unit Settings: SI C Pa J mass rad

A_c = 0.001257 [m^2]

ΔP = 2354 [N/m^2]

f = 0.01848

k = 0.6105 [W/m-C]

μ = 0.0007216 [kg/m-s]

Pr = 4.945

ρ = 994.1 [kg/m^3]

T_{out} = 44.72 [C]

cp = 4183 [J/kg-C]

ΔT_{in} = 65 [C]

G = 1592 [kg/s-m^2]

L = 4 [m]

$\dot{m}$ = 2 [kg/s]

Q = 164989 [W]

T_b = 34.86 [C]

T_s = 90 [C]

d = 0.04 [m]

ΔT_{out} = 45.28 [C]

h = 6018 [W/m^2-C]

LMTD = 54.55 [C]

Nusselt = 394.3

Re = 88218

T_{in} = 25 [C]

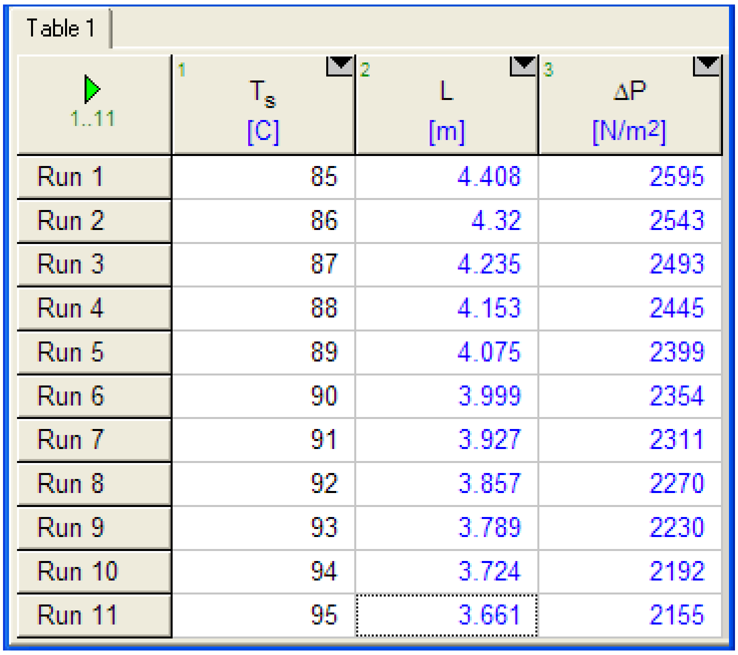
Thus:

T_out = 44.72 C …. Outlet temp of water …. Ans.

Q = 164989 W … heat transfer to water … Ans.

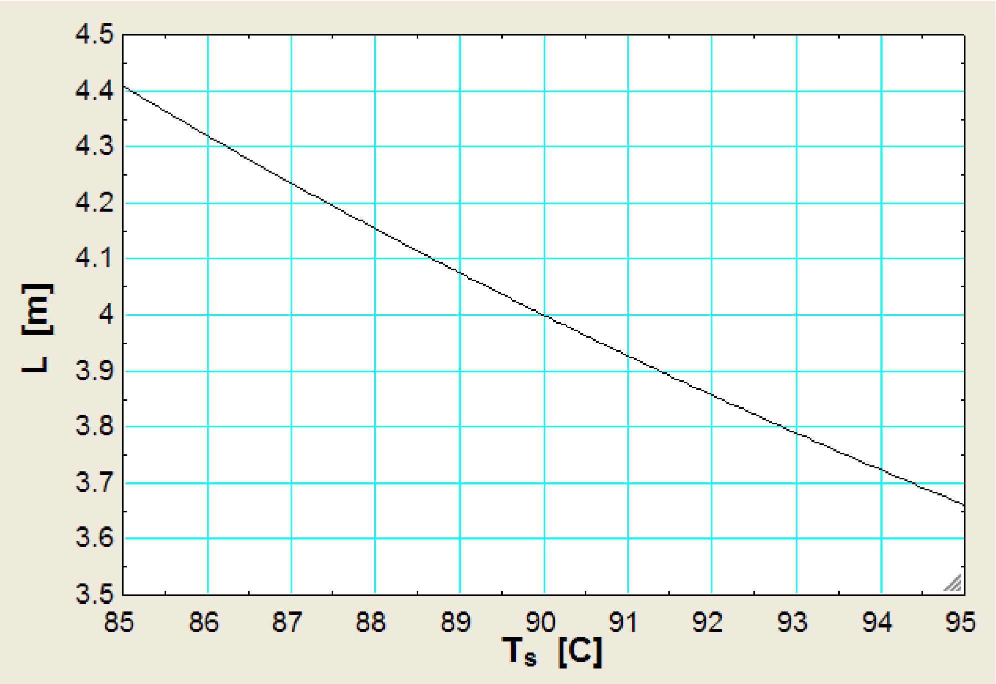
DELTAP = 2354 N/m^2 ….. Pressure drop …. Ans.

Plot L against various values of T_s, keeping T_out = 44.72 C:

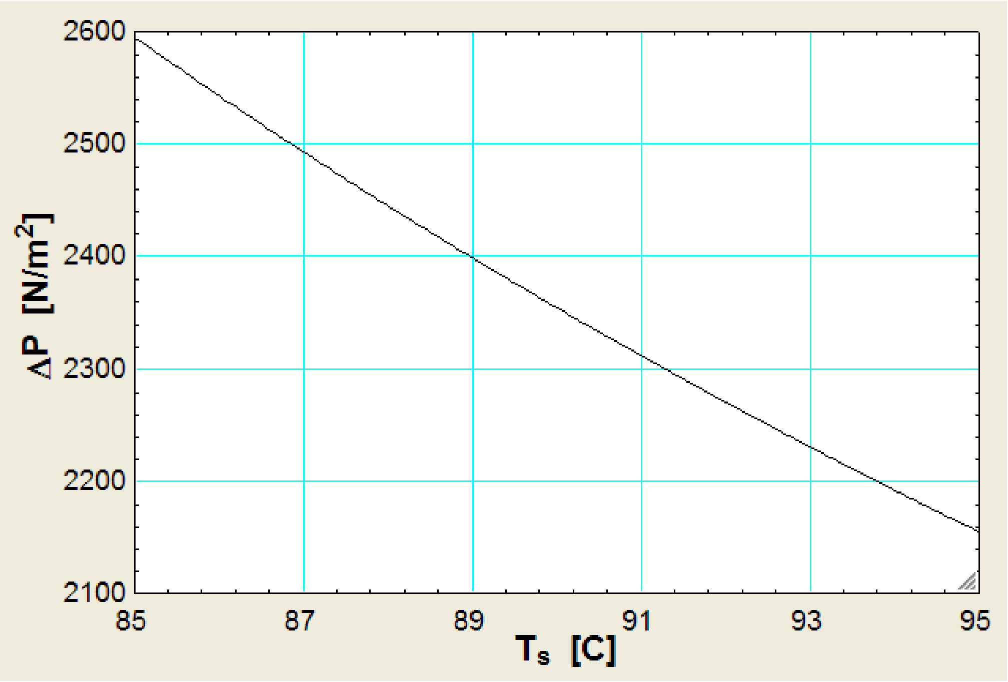


1..11	1 T_s [C]	2 L [m]	3 ΔP [N/m2]
Run 1	85	4.408	2595
Run 2	86	4.32	2543
Run 3	87	4.235	2493
Run 4	88	4.153	2445
Run 5	89	4.075	2399
Run 6	90	3.999	2354
Run 7	91	3.927	2311
Run 8	92	3.857	2270
Run 9	93	3.789	2230
Run 10	94	3.724	2192
Run 11	95	3.661	2155

Plot of L vs T_s:



Plot of DELTAP vs T_s:



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"**Prob.2A1.4.12.** Unused engine oil with a 100 C inlet temp flows at a rate of 250 g/s through a 5.1 cm ID pipe that is enclosed by a jacket containing condensing steam at 150 C. If the pipe is 9 m long, determine the outlet temp of oil [Ref. 5]"

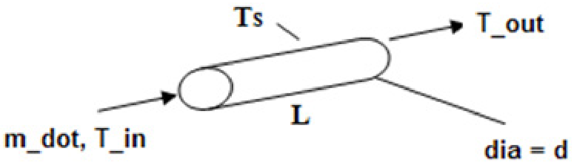


Fig.Prob.2A.1.4.12

EES Solution:

EES Functions for properties of Engine oil are already written.. See Prob. 2A1.4.2.

"Data:"

T_in = 100[C]
{T_out= 140[C]"..assumed…will be commented out later"}
T_s = 150[C]
T_b = (T_in + T_out)/2
d = 0.051[m]

L = 9[m]
m_dot = 0.25[kg/s]

"Properties of Engine oil at bulk mean temp, T_b:"

k=k_EngineOil(T_b) "[W/m-C]"
rho= rho_EngineOil(T_b) "[kg/m^3]"
mu= mu_EngineOil(T_b) "[kg/m-s]"
cp= cp_EngineOil(T_b) "[J/kg-C]"
Pr= Pr_EngineOil(T_b)

"Calculations:"

A = pi * d^2 / 4 "[m^2]… area of cross-section"
G = m_dot / A "[kg/s-m^2]"
Re = G * d /mu "Reynold's No."

"We get Re = 609.2 which is less than 2300; Therefore, flow is Laminar."

"Now, Thermal entry length L_t is given by:"

L_t = .05 * Re * Pr * d "[m] … thermal entry length"

"We get L_t = 272.1 m, which is much greater than the tube length of 9 m.

Therefore, the thermally boundary layer is still developing.

This is generally the case for high Prandtl No. fluids."

" Now, Nu = 3.66 for Laminar flow when the velocity and thermal boundary layers are fully developed.

In the present case, since the thermal boundary layer is still not fully developed, and the variation of viscosity of engine oil with temp is significant, we use:"

mu_s= mu_EngineOil(T_s) "[kg/m-s] …. Viscosity of engine oil at wall temp."

Nusselt = 1.86 * (Re * Pr * d / L)^(1/3) * (mu / mu_s)^0.14 "….Nusselts No."

"Therefore: heat transfer coeff. h:"

Nusselt = h * d / k "[W/m^2-C]…finds h"

"Now:"

A_s = pi * d * L "[m^2]…surface area"

"Heat transfer, Q:"

Q = m_dot * cp * (T_out – T_in) "[W]"

AMTD = ((T_s – T_out) + (T_s – T_in))/ 2 "…Arithmetic Mean Temp Difference"

"Now, Q is also equal to:"

Q = h * A_s * AMTD

"Simultaneous solution of above two eqns for Q gives the value of T_out"

"Pressure drop:"

U = G / rho "[m/s] …. mean velocity"

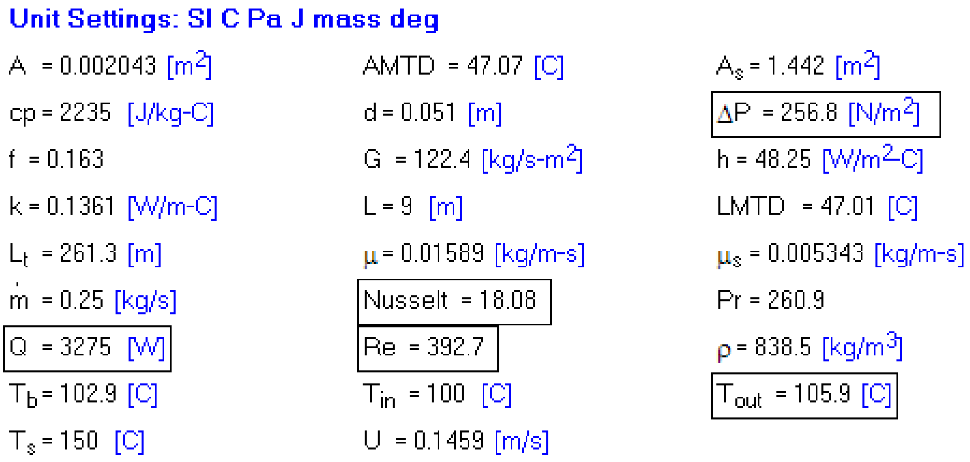
f = 64 / Re "…friction factor for Laminar flow"

DELTAP = f * (L / d) * U^2 / 2 * rho "[N/m^2] … pressure drop"

"Check:"

LMTD = ((T_s – T_out) – (T_s – T_in)) / ln ((T_s – T_out) / (T_s – T_in))

Results:



Unit Settings: SI C Pa J mass deg

$A = 0.002043$ [m^2]
$AMTD = 47.07$ [C]
$A_s = 1.442$ [m^2]
$cp = 2235$ [J/kg-C]
$d = 0.051$ [m]
$\boxed{\Delta P = 256.8 \text{ [N/m}^2\text{]}}$
$f = 0.163$
$G = 122.4$ [kg/s-m^2]
$h = 48.25$ [W/m^2-C]
$k = 0.1361$ [W/m-C]
$L = 9$ [m]
$LMTD = 47.01$ [C]
$L_t = 261.3$ [m]
$\mu = 0.01589$ [kg/m-s]
$\mu_s = 0.005343$ [kg/m-s]
$\dot{m} = 0.25$ [kg/s]
$\boxed{Nusselt = 18.08}$
$Pr = 260.9$
$\boxed{Q = 3275 \text{ [W]}}$
$\boxed{Re = 392.7}$
$\rho = 838.5$ [kg/m^3]
$T_b = 102.9$ [C]
$T_{in} = 100$ [C]
$\boxed{T_{out} = 105.9 \text{ [C]}}$
$T_s = 150$ [C]
$U = 0.1459$ [m/s]

Thus:

T_out = 105.9 C …. Ans.

Q = 3275 W …. Ans.

And, also note that by taking AMTD instead of LMTD, there is no significant error, since AMTD = 47.07 C and LMTD = 47.01 C, almost equal to AMTD.

Now, if the flow rate varies from 0.1 to 0.6 kg/s, plot the variation of T_out and Q, all other parameters remaining the same:

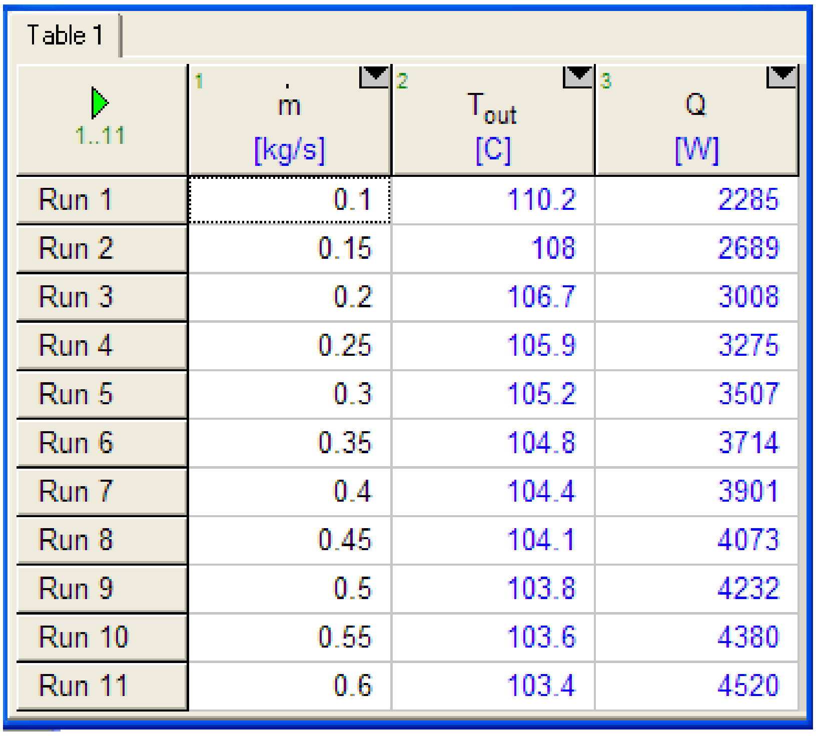
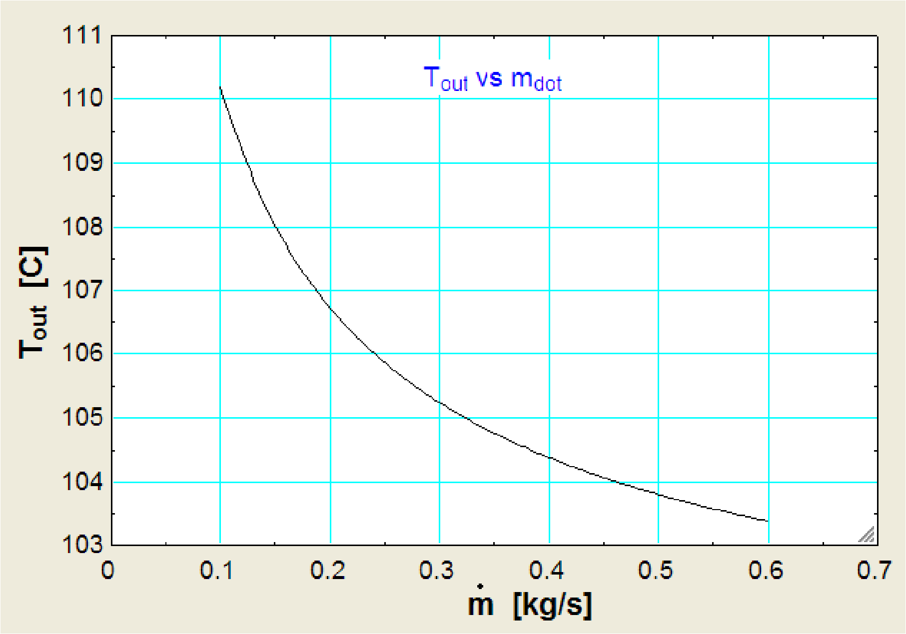
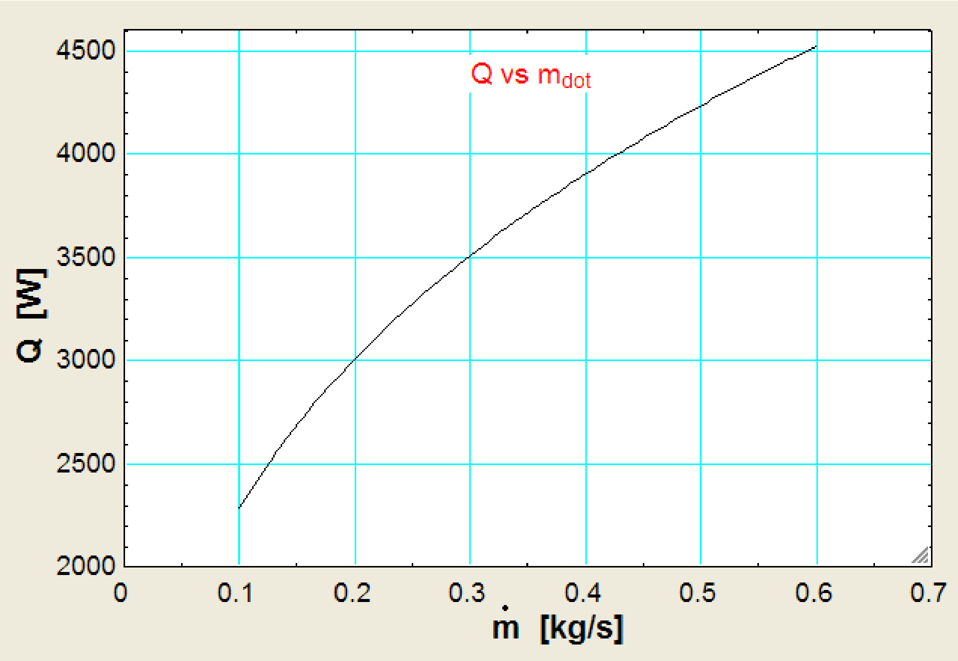


Table 1

1..11	1 $\dot{m}$ [kg/s]	2 T_{out} [C]	3 Q [W]
Run 1	0.1	110.2	2285
Run 2	0.15	108	2689
Run 3	0.2	106.7	3008
Run 4	0.25	105.9	3275
Run 5	0.3	105.2	3507
Run 6	0.35	104.8	3714
Run 7	0.4	104.4	3901
Run 8	0.45	104.1	4073
Run 9	0.5	103.8	4232
Run 10	0.55	103.6	4380
Run 11	0.6	103.4	4520




Q vs m_dot
Q [W]
4500
4000
3500
3000
2500
2000
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
ṁ [kg/s]

==

"**Prob. 2A1.4.13.** Mercury at an inlet bulk temp of 90 C flows through a 1.2 cm ID tube at a flow rate of 4535 kg/h. Tube surface is subjected to constant heat flux. Determine the length of tube required to raise the bulk temp of Hg to 230 C without generating any Hg-vapour and determine the corresponding heat flux. Boiling point of Hg is 355 C. [Ref: 5]"

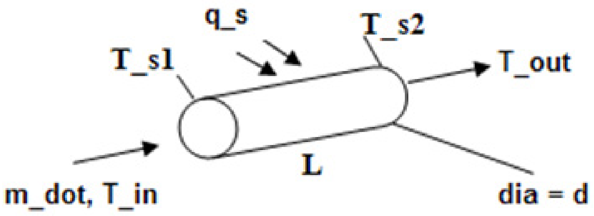


Fig.Prob.2A.1.4.13

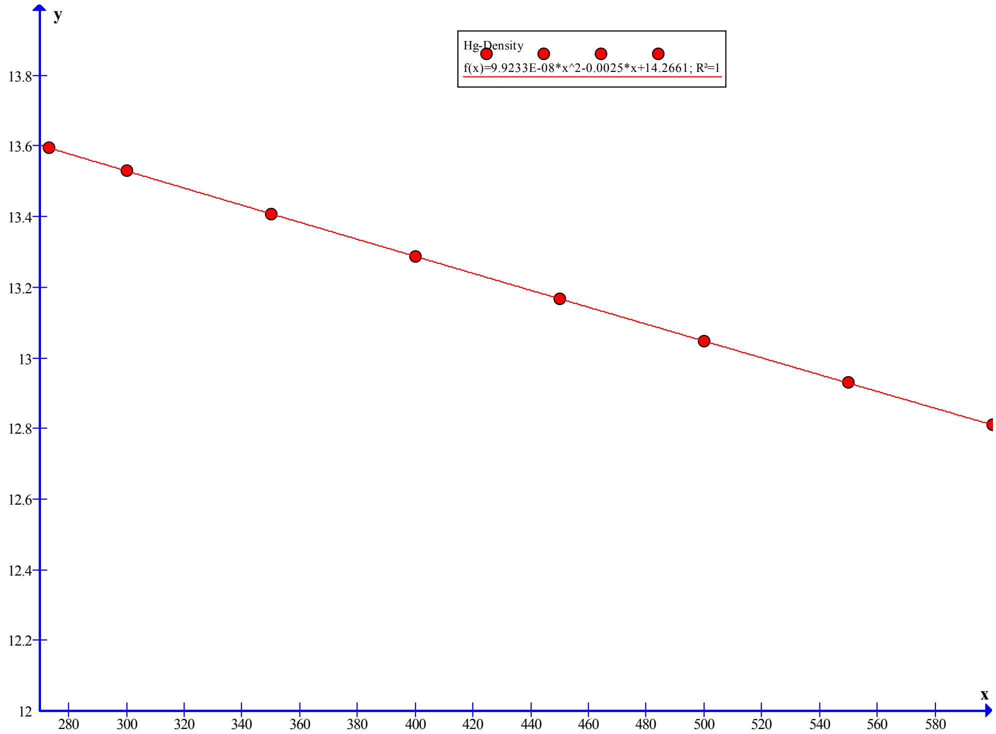
EES Solution:

Ref: Data for Mercury from Fundamentals of HMT by Incropera, 5th Ed."

Curve fit equations are obtained from the software GRAPH:

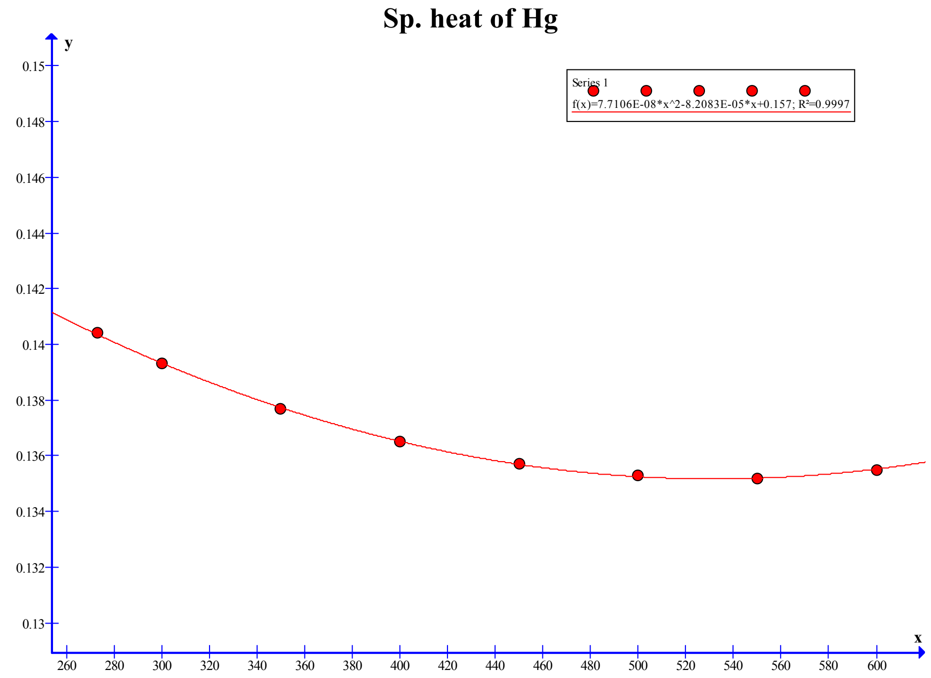
In the following graphs, x-axis is: Temp in K, y-axis is: concerned property of Hg

Density of Hg:



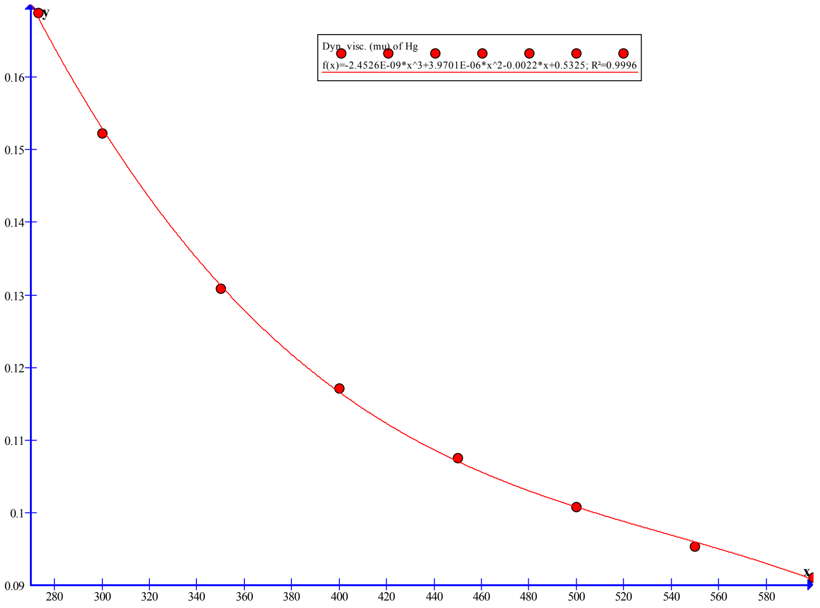
rho_Hg = (9.92325012E-08*T^2-0.00248688*T+14.26607138) * 1000 … kg/m^3

Sp. Heat of Hg:



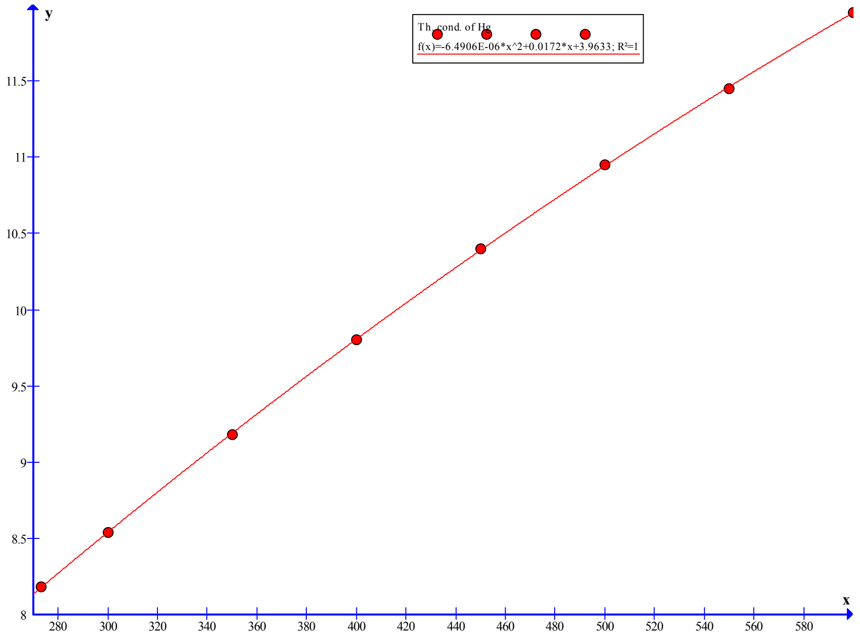
Cp_Hg = (7.71059577E-08*T^2-8.20830448E-05*T+0.15701607) * 1000 J/kg.K

Dyn. Viscosity (mu) of Hg:



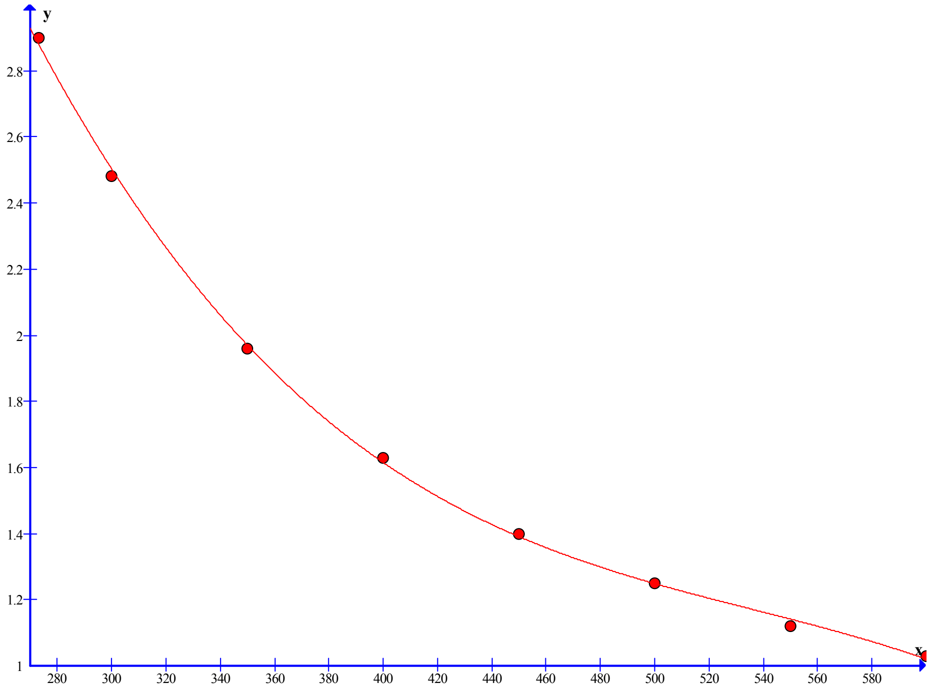
mu_Hg =(-2.45261803E-09*T^3+3.97008953E-06*T^2-0.00223533*T+0.53249793) / 100 N.s/m^2

Thermal cond. of Hg:



k_Hg = -6.49055603E-06*T^2+0.01720197*T+3.96326253 W/m.K

Prandtl No. of Hg:



Pr_Hg = (-6.40396723E-08*T^3+0.00010294*T^2-0.05724457*T+12.14092655) / 100

==

Now, let us first write functions for properties of Mercury in terms of temperature:

```
------------------------------------------------------------------------------------------------------------

$UnitSystem SI Pa C J

"Functions for properties of Hg in the Temp range: 273 to 600 K.

"------------------------------------------------------------------------------------------------------------"

Function rho_Hg(T)
{ rho_Hg
This function returns the density (kg/m^3) of Hg as a function of Temp (K) in the range: 273 to 600 K.
}

If (T < 273) OR (T > 600) Then
CALL error('T must be between 273 and 600 K !!')
EndIf
rho_Hg := (9.92325012E-08 * T^2 – 0.00248688 * T+14.26607138) * 1000
End

"------------------------------------------------------------------------------------------------------------"
```

```
Function cp_Hg(T)
{ cp_Hg
This function returns the cp (J/kg.K) of Hg as a function of Temp (K) in the range: 273 to 600 K.
}

If (T < 273) OR (T > 600) Then
CALL error('T must be between 273 and 600 K !!')
EndIf
cp_Hg := (7.71059577E-08*T^2-8.20830448E-05*T+0.15701607) * 1000
End
"-------------------------------------------------------------------------------------------------------"

Function k_Hg(T)

{k_Hg
This function returns the Th. cond. (W/m.K) of Hg as a function of Temp (K) in the range: 273 to 600 K.

}

If (T < 273) OR (T > 600) Then
CALL error('T must be between 273 and 600 K !!')
EndIf
k_Hg := -6.49055603E-06 * T^2 + 0.01720197 * T+3.96326253
End
"-------------------------------------------------------------------------------------------------------'

Function mu_Hg(T)

{mu_Hg
This function returns the Dyn. visc.. (N.s/m^2) of Hg as a function of Temp (K) in the range: 273 to 600 K.

}

If (T < 273) OR (T > 600) Then
CALL error('T must be between 273 and 600 K !!')
EndIf
mu_Hg := (-2.45261803E-09 * T^3+3.97008953E-06 * T^2-0.00223533 * T+0.53249793) / 100
End
"-------------------------------------------------------------------------------------------------------"
```

```
Function Pr_Hg(T)

{Pr_Hg
This function returns the Prandtl No. (= cp . mu /k) of Hg as a function of Temp (K) in the range: 273
to 600 K.

}

If (T < 273) OR (T > 600) Then
CALL error('T must be between 273 and 600 K !!')
EndIf
        Pr_Hg:= (-6.40396723E-08 * T^3 + 0.00010294 * T^2 – 0.05724457 * T+12.14092655) / 100
End

"----------------------------------------------------------------------------------------------------"
```

Now, let us solve the problem:

"Data:"

```
T_in = 90 [C]
T_out = 230 [C]
T_b = (T_in + T_out) / 2 "[C]….mean bulk temp."
d = 0.012[m]
m_dot = 4535 [kg/h] * convert (kg/h, kg/s) "[kg/s]"
T_s2 = 355 [C] "…surface temp at exit"
DELTAT2 = T_s2 – T_out "[C]… temp. difference between the surface and fluid at the exit"
DELTAT1 =T_s1 – T_in "[C]… temp. difference between the surface and fluid at the inlet"
DELTAT1 = DELTAT2 "….since for constant heat flux condition, DELTAT between the surface and fluid
is the same throughout"
T_s = (T_s1 + T_s2) / 2 "…mean surface temp"
```

"Properties of Hg at bulk mean temp, T_b:

Use the EES functions written above. Remember that T should be in Kelvin while using these functions:"

```
k=k_Hg(T_b+273) "[W/m-C]"

rho= rho_Hg(T_b+273) "[kg/m^3]"

mu= mu_Hg(T_b+273) "[kg/m-s]"

cp= cp_Hg(T_b+273) "[J/kg-C]"

Pr= Pr_Hg(T_b+273) "….Prandtl No. of fluid at bulk mean temp of fluid"

Pr_s = Pr_Hg(T_s + 273) "..Prandtl no. of fluid at surface temp"
```

"Calculations:"

```
A_c = pi * d^2 / 4 "[m^2]…. area of cross-section"

G = m_dot / A_c "[kg/s-m^2] …. mass velocity"

Re = G * d / mu "….Reynolds No."
```

"For Liquid metals:"

"When q_s is constant, we use the following correlation for Nusselt No.[Ref. 2]:"

Nusselt = 6.3 + 0.0167 * Re^0.85 * Pr_s^0.93 "….Nusselts No."

"Heat transfer coeff, h:"

Nusselt = h * d / k "[W/m^2-C]… finds h"

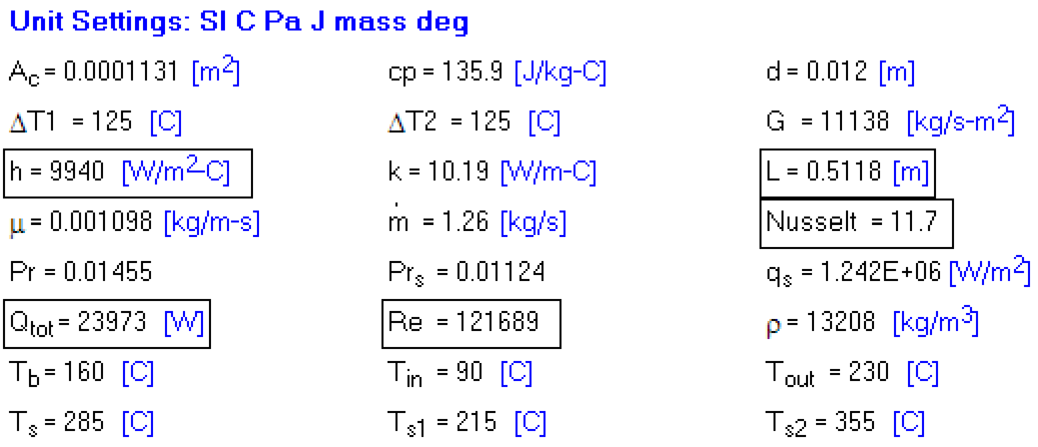
q_s = h * DELTAT2 "…[W/m^2] … constant surface heat flux"

"Length of tube required is calculated by equating the following two equations for total heat transferred, Q_tot:"

Q_tot = m_dot * cp * (T_out – T_in)

Q_tot = h * pi * d * L * DELTAT2

Results:



```
Unit Settings: SI C Pa J mass deg
A_c = 0.0001131  [m^2]       cp = 135.9  [J/kg-C]      d = 0.012  [m]
ΔT1 = 125  [C]               ΔT2 = 125  [C]            G = 11138  [kg/s-m^2]
h = 9940  [W/m^2-C]          k = 10.19  [W/m-C]        L = 0.5118  [m]
μ = 0.001098  [kg/m-s]       m_dot = 1.26  [kg/s]      Nusselt = 11.7
Pr = 0.01455                 Pr_s = 0.01124            q_s = 1.242E+06  [W/m^2]
Q_tot = 23973  [W]           Re = 121689               ρ = 13208  [kg/m^3]
T_b = 160  [C]               T_in = 90  [C]            T_out = 230  [C]
T_s = 285  [C]               T_s1 = 215  [C]           T_s2 = 355  [C]
```

Thus:

Length of tube required = L = 0.5118 m…..Ans.

Heat flux = q_s = 1.242E06 W/m^2 …. Ans.

Total heat transferred = Q_tot = 23973 W …. Ans.

"**Prob. 2A1.4.14.** Liquid Mercury at 0.5 kg/s is to be heated from 300 to 400 K by passing it through a 50 mm dia tube whose surface is maintained at 450 K. Calculate the required tube length by using an appropriate liqid metal heat transfer correlation. [Ref:3]"

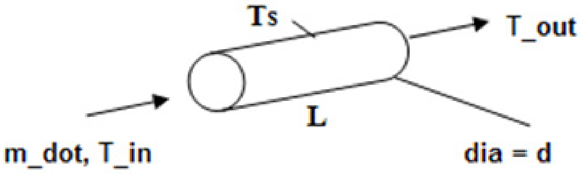


Fig.Prob.2A.1.4.14

EES Solution:

Use the EES functions for properties of Hg written in the previous problem.

"Data:"

T_in = 300 [K]
T_out = 400 [K]
T_b = (T_in + T_out) / 2 "[K]….mean bulk temp."
d = 0.05[m]
m_dot = 0.5 [kg/s]
T_s = 450 [K]

DELTAT1 = T_s – T_in

DELTAT2 = T_s – T_out

LMTD = (DELTAT1 – DELTAT2) / ln (DELTAT1 / DELTAT2)

"Properties of Hg at bulk mean temp, T_b:

Use the EES functions written above:"

k=k_Hg(T_b) "[W/m-K]"

rho= rho_Hg(T_b) "[kg/m^3]"

mu= mu_Hg(T_b) "[kg/m-s]"

cp= cp_Hg(T_b) "[J/kg-K]"

Pr= Pr_Hg(T_b) "….Prandtl No. of fluid at bulk mean temp of fluid"

Pr_s = Pr_Hg(T_s) "..Prandtl no. of fluid at surface temp"

"Calculations:"

A_c = pi * d^2 / 4 "[m^2]…. area of cross-section"

G = m_dot / A_c "[kg/s-m^2] …. mass velocity"

Re = G * d / mu "….Reynolds No."

"For Liquid metals:"

"When T_s is constant, we use the following correlation for Nusselt No.[Ref. 2]:"

Nusselt = 4.8 + 0.0156 * Re^0.85 * Pr_s^0.93 "….Nusselts No. for 10^4 < Re < 10^6, and, 0.004 < Pr < 0.01"

"Heat transfer coeff, h:"

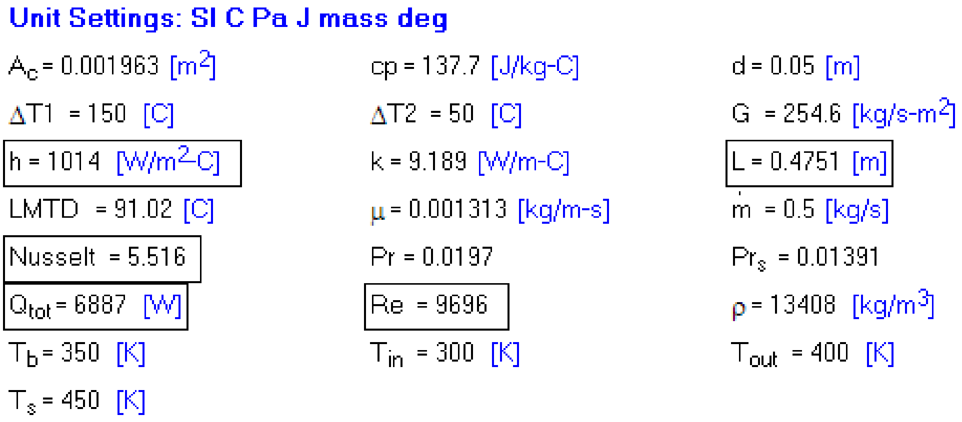
Nusselt = h * d / k "[W/m^2-C]… finds h"

"Length of tube required is calculated by equating the following two equations for total heat transferred, Q_tot:"

Q_tot = m_dot * cp * (T_out – T_in)

Q_tot = h * pi * d * L * LMTD

Results:



Unit Settings: SI C Pa J mass deg

$A_c = 0.001963$ [m^2]

cp = 137.7 [J/kg-C]

d = 0.05 [m]

$\Delta T1 = 150$ [C]

$\Delta T2 = 50$ [C]

G = 254.6 [kg/s-m^2]

$\boxed{h = 1014 \text{ [W/m}^2\text{-C]}}$

k = 9.189 [W/m-C]

$\boxed{L = 0.4751 \text{ [m]}}$

LMTD = 91.02 [C]

$\mu = 0.001313$ [kg/m-s]

$\dot{m} = 0.5$ [kg/s]

$\boxed{\text{Nusselt} = 5.516}$

Pr = 0.0197

$Pr_s = 0.01391$

$\boxed{Q_{tot} = 6887 \text{ [W]}}$

$\boxed{Re = 9696}$

$\rho = 13408$ [kg/m^3]

$T_b = 350$ [K]

$T_{in} = 300$ [K]

$T_{out} = 400$ [K]

$T_s = 450$ [K]

Thus:

Length of tube required = L = 0.4751 m … Ans.

Total heat transferred = Q_tot = 6887 W …. Ans.

Plot the variation of L with m_dot. Let m_dot vary from 0.5 to 1.5 kg/s, other conditions remaining the same:

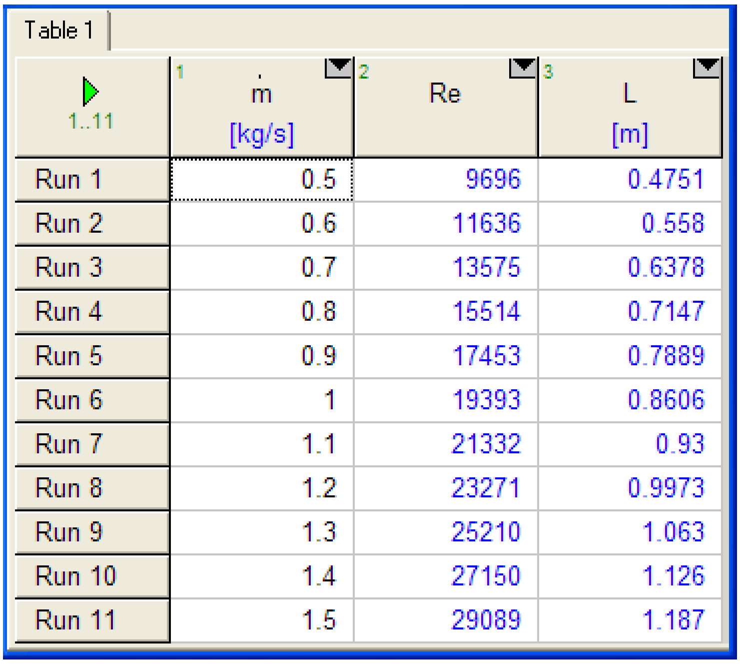
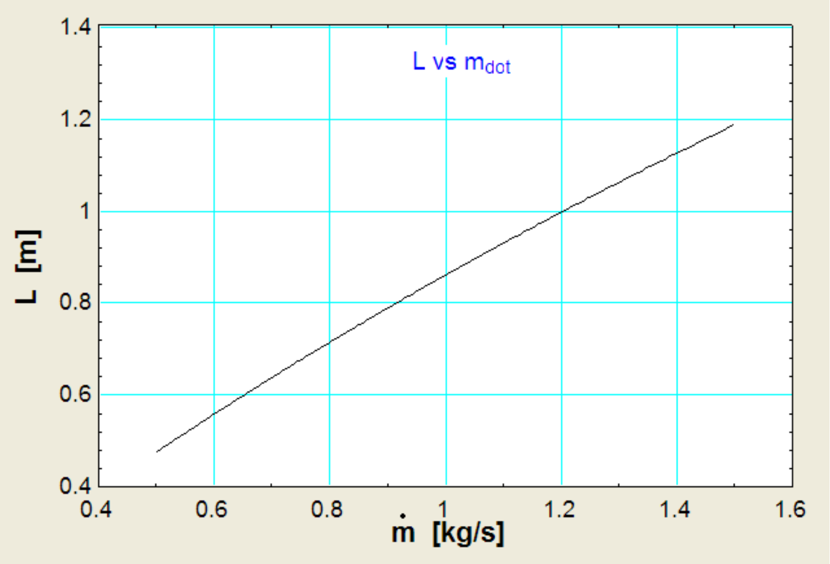


Table 1

1..11	1 $\dot{m}$ [kg/s]	2 Re	3 L [m]
Run 1	0.5	9696	0.4751
Run 2	0.6	11636	0.558
Run 3	0.7	13575	0.6378
Run 4	0.8	15514	0.7147
Run 5	0.9	17453	0.7889
Run 6	1	19393	0.8606
Run 7	1.1	21332	0.93
Run 8	1.2	23271	0.9973
Run 9	1.3	25210	1.063
Run 10	1.4	27150	1.126
Run 11	1.5	29089	1.187



==

"**Prob. 2A1.4.15**. The velocity of water flowing through a tube of 2.2 cm dia is 2 m/s. Steam condensing at 150 C on the outside surface of the tube heats the water from 15 C to 60 C over the length of the tube. Find the heat transfer coeff and the length of the tube neglecting the tube and steam side film resistance. – [VTU – June. 2012]"

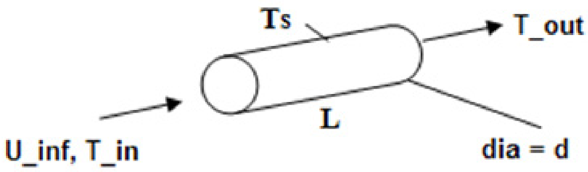


Fig.Prob.2A.1.4.15

EES Solution:

Let us use the EES PROCEDURE written while solving Prob. 2A1.4.1.

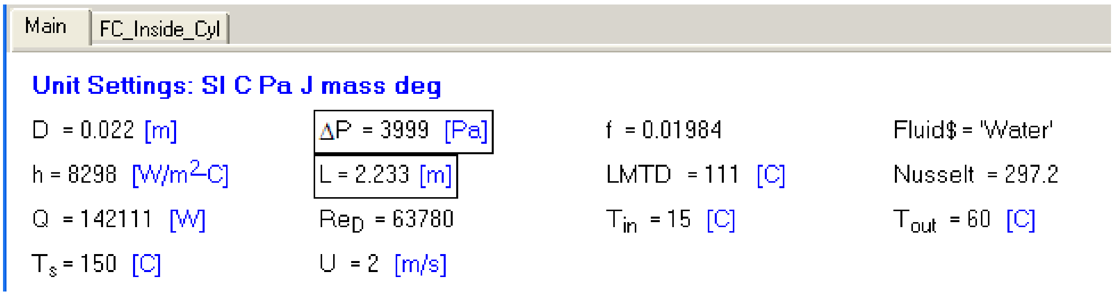
```
$UnitSystem SI Pa C J

PROCEDURE FC_Inside_Cyl(Fluid$, T_s, T_in, T_out, D,U : Re_D, Nusselt, h,Q, LMTD, L, f, DELTAP )

“Forced convection (FC) inside Cylinders, with Fluid$: Water or Air, or Engine Oil”
“Inputs: T_s (C), T_in, T_out (C), D (m), U (m/s)”
“Outputs: Re_D, Nusselt, h (W/m^2-C), Q (W), LMTD (C), L (m),f, DELTAP (Pa)”

T_b := (T_out+T_in)/2
A_c := pi * D^2 /4

IF (Fluid$ = ‘Water’) Then
    beta:=VolExpCoef(Water,T=T_b, P=1.013e05)
    mu:=Viscosity(Water,T=T_b, P = 1.013e05)
    rho:=Density(Water,T=T_b, P=1.013e05)
    nu := mu/rho
    cp:=SpecHeat(Water,T=T_b, P = 1.013e05)
    k:=Conductivity(Water,T=T_b, P = 1.013e05)
    Pr=Prandtl(Water,T=T_b,P=1.013e05)
ELSE
    IF (Fluid$ = ‘Air’) Then
    beta := 1/(T_b + 273)
    mu:=Viscosity(Air,T=T_b)
    rho:=Density(Air,T=T_b,P=1.01325e05)
    nu := mu/rho
    cp:=Cp(Air,T=T_b)
    k:=Conductivity(Air,T=T_b)
    Pr=Prandtl(Air,T=T_b)
    EndIF
EndIf
```

```
Re_D := D * U * rho/ mu

If (Re_D < 10000) Then CALL WARNING ('The results may not be accurate since Re_D > 10000 does not hold. Re_D = XXXA1', Re_D)

If (Pr < 0.6) or (Pr > 160) Then CALL WARNING ('The results may not be accurate since 0.6 < Pr < 160 does not hold. Pr = XXXA1', Pr)

IF (T_in > T_out) Then
    n = 0.3
ELSE
    IF (T_in < T_out) Then
        n = 0.4
    ENDIF
ENDIF
Nusselt := 0.023 * Re_D^0.8 * Pr^n "finds Nusselts No."
h:= Nusselt * k / D "finds h"

DELTAT_in := T_s – T_in
DELTAT_out := T_s – T_out
LMTD: = (DELTAT_in – DELTAT_out) / ln(DELTAT_in/DELTAT_out) "Log Mean Temp Difference"
Q := rho * A_c * U * cp * (T_out – T_in) "finds Q, W"
L :=Q/ (h * (pi * d ) * LMTD) "by heat balance; finds L"

"Also, find the pressure drop:"

"Friction factor: Use the first Petukhov eqn, which is an explicit eqn. for f. Valid for Reynolds Number range: Re_D = 3000 to 5E06"

f := (0.79 * ln(Re_D) – 1.64)^(-2) "friction factor"

DELTAP := f * (L / D) * rho * U^2 / 2 "[N/m^2]"

END

"=================================================================="
```

```
$UnitSystem SI Pa C J

"Prob.2A1.4.15"

Fluid$ = 'Water'

T_in = 15[C]
T_out= 60[C]
T_s = 150[C]

D = 0.022[m]

U = 2 "[m/s]"

CALL FC_Inside_Cyl(Fluid$, T_s, T_in, T_out, D,U : Re_D, Nusselt, h,Q, LMTD, L, f, DELTAP)
```

Now, press F2 to get the solution. We get:

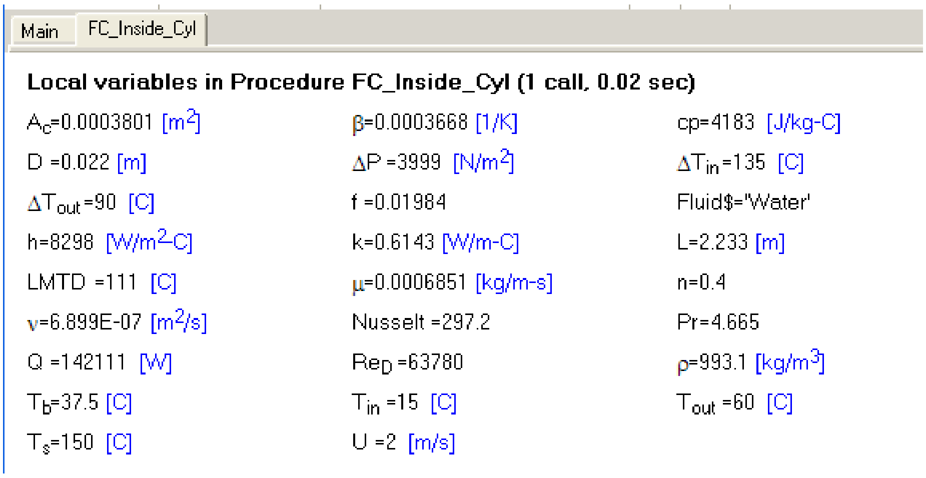
Results: Main:



Unit Settings: SI C Pa J mass deg

D = 0.022 [m]
ΔP = 3999 [Pa]
f = 0.01984
Fluid$ = 'Water'
h = 8298 [W/m^2-C]
L = 2.233 [m]
LMTD = 111 [C]
Nusselt = 297.2
Q = 142111 [W]
Re_D = 63780
T_{in} = 15 [C]
T_{out} = 60 [C]
T_s = 150 [C]
U = 2 [m/s]

Results: PROCEDURE:



Local variables in Procedure FC_Inside_Cyl (1 call, 0.02 sec)

```
Ac=0.0003801 [m2]       β=0.0003668 [1/K]       cp=4183 [J/kg-C]
D =0.022 [m]            ΔP =3999 [N/m2]         ΔTin=135 [C]
ΔTout=90 [C]            f =0.01984              Fluid$='Water'
h=8298 [W/m2-C]         k=0.6143 [W/m-C]        L=2.233 [m]
LMTD =111 [C]           μ=0.0006851 [kg/m-s]    n=0.4
ν=6.899E-07 [m2/s]      Nusselt =297.2          Pr=4.665
Q =142111 [W]           ReD =63780              ρ=993.1 [kg/m3]
Tb=37.5 [C]             Tin =15 [C]             Tout =60 [C]
Ts=150 [C]              U =2 [m/s]
```

Thus:

Heat transfer coeff. = h = 8298 W/m^2.C …. Ans.

Length of tube = L = 2.233 m … Ans.

Pressure drop = DELTAP = 3999 N/m^2 … Ans.

Plot L and DELTAP against velocity, U_inf:

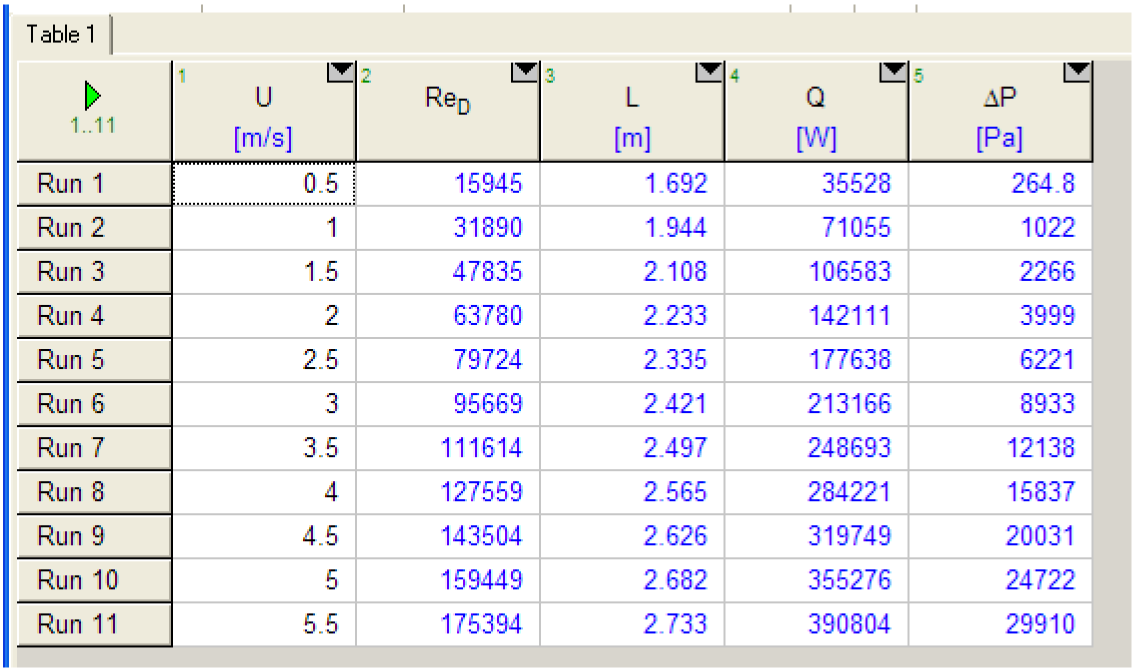
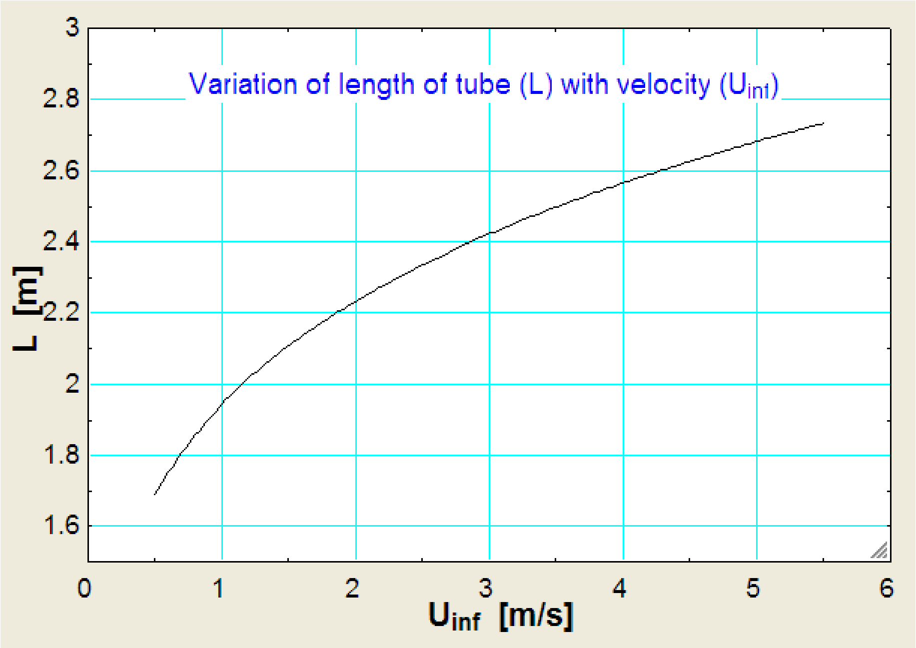
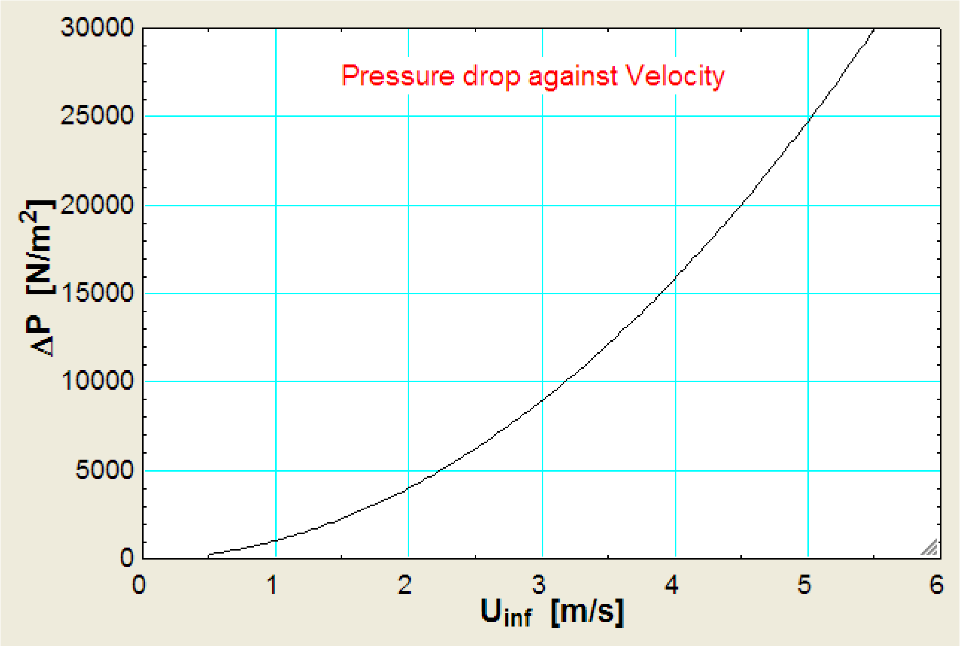


Table 1

1..11	1 U [m/s]	2 Re_D	3 L [m]	4 Q [W]	5 ΔP [Pa]
Run 1	0.5	15945	1.692	35528	264.8
Run 2	1	31890	1.944	71055	1022
Run 3	1.5	47835	2.108	106583	2266
Run 4	2	63780	2.233	142111	3999
Run 5	2.5	79724	2.335	177638	6221
Run 6	3	95669	2.421	213166	8933
Run 7	3.5	111614	2.497	248693	12138
Run 8	4	127559	2.565	284221	15837
Run 9	4.5	143504	2.626	319749	20031
Run 10	5	159449	2.682	355276	24722
Run 11	5.5	175394	2.733	390804	29910





==

"**Prob. 2A1.4.16.** A surface condenser consists of 200 thin-walled circular tubes (each tube is 22.5 mm dia and 5 m long) arranged in parallel, through which water flows. If the mass flow rate of water through the tube bank is 160 kg/s and its inlet and outlet temp are known to be 21 C and 29 C respectively, calculate the average heat transfer coeff associated with the flow of water. – [VTU – June. 2012]"

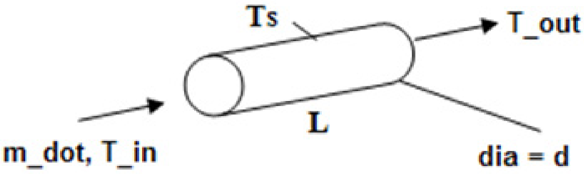


Fig.Prob.2A.1.4.16

EES Solution:

"Data:"

T_in = 21[C]
T_out= 29[C]
T_b = (T_in + T_out)/2
d = 0.0225[m]
L = 5 [m]
m_dot = 160/200 "[kg/s]… mass flow rate through each tube"
A = pi * d^2 / 4 "[m^2]"

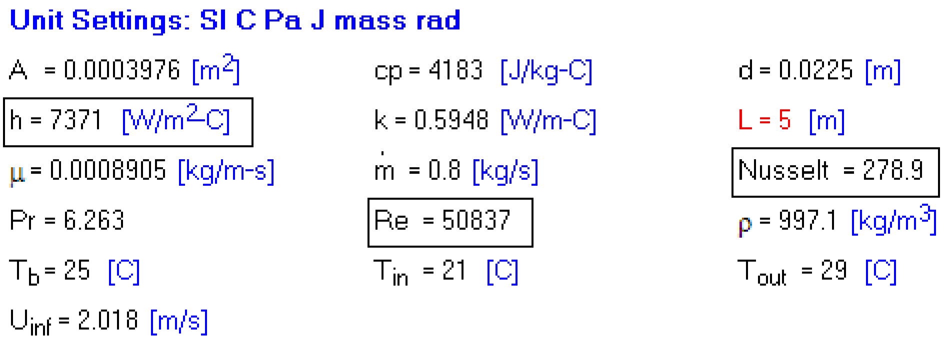
“Properties of Water at bulk mean temp, T_b:”

```
k=Conductivity(Water,T=T_b,P=1.013e05)
rho=Density(Water,T=T_b,P=1.013e05)
mu=Viscosity(Water,T=T_b,P=1.013e05)
cp=SpecHeat(Water,T=T_b,P=1.013e05)
Pr=Prandtl(Water,T=T_b,P=1.013e05)
```

“Calculations:”

```
m_dot = rho * A * U_inf "…finds the velocity, U_inf"

Re = d * U_inf * rho / mu "finds Re = 50837 > 10,000"
Nusselt = 0.023 * Re^0.8 * Pr^0.4 "finds Nusselts No. for Re > 10000"
Nusselt = h * d / k "finds h"
```

Results:



Unit Settings: SI C Pa J mass rad

A = 0.0003976 [m2]
cp = 4183 [J/kg-C]
d = 0.0225 [m]
h = 7371 [W/m2-C]
k = 0.5948 [W/m-C]
L = 5 [m]
μ = 0.0008905 [kg/m-s]
ṁ = 0.8 [kg/s]
Nusselt = 278.9
Pr = 6.263
Re = 50837
ρ = 997.1 [kg/m3]
Tb = 25 [C]
Tin = 21 [C]
Tout = 29 [C]
Uinf = 2.018 [m/s]

Thus:

Heat transfer coeff. = h = 7371 W/m^2.C …. Ans.

===

"**Prob. 2A1.4.17.** Water at 25 C flows through a tube of 50 mm dia. Determine the flow rate that will result in a Reynolds No. of 1600. The tube is provided with a nichrome heating element on its surface and receives a constant heat flux of 800 W/m length of the tube. Determine the average heat transfer coeff assuming fully developed conditions. Also, determine the length of the tube for the bulk temp of water to rise from 25 C to 50 C. [VTU – Dec. 2010:]"

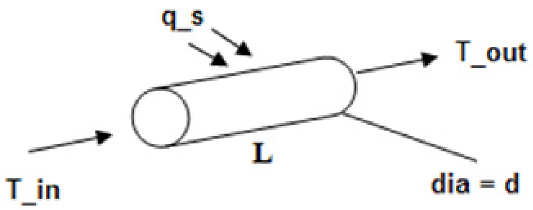


Fig.Prob.2A.1.4.17

EES Solution:

"Data:"

T_in = 25[C]
T_out= 50[C]
T_b = (T_in + T_out)/2
d = 0.05[m]
q_s = 800 [W/m]
A = pi * d^2 / 4
mu=Viscosity(Water,T=T_in,P=1.013e05) "..viscosity at temp T_in"
k=Conductivity(Water,T=T_in,P=1.013e05) "… th. cond. of water at T_in"
cp=SpecHeat(Water,T=T_b,P=1.013e05) "… sp. heat of water at mean temp T_b"

Re = 1600 "…given"
G = m_dot / A "[kg/m^2-s] .. mass velocity"
Re = G * d / mu "..determines mass flow rate reqd m_dot, to give Re = 1600"

"Since Re < 2300, it is laminar flow.

For fully developed laminar flow (by data), for constant heat flux conditions, Nusselts No is 4.36."

"Calculations:"

Nusselt = 4.36 "for Re < 2300, fully developed flow"
Nusselt = h * d / k "finds h"

Q = m_dot * cp * (T_out – T_in) "finds Q, W"

"But, Q is also equal to:"

Q = q_s * L "[W] … total heat in to the water …. finds L"

"Also, find the pressure drop:"

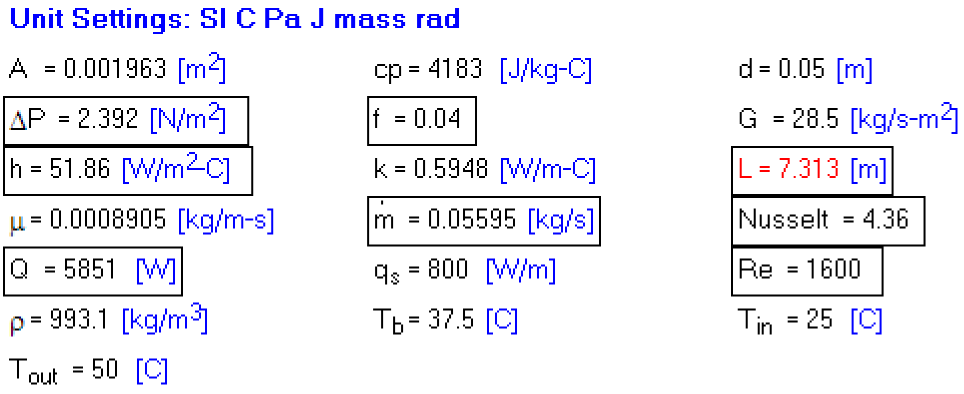
"Friction factor: for laminar flow, we have:"

f = 64 / Re "…finds f"

rho=Density(Water,T=T_b,P=1.013e05) "…[kg/m^3] … density of water at T_b"

DELTAP = f * (L / d) * rho * (G / rho)^2 / 2 "[N/m^2]"

Results:



Unit Settings: SI C Pa J mass rad

A = 0.001963 [m^2]
cp = 4183 [J/kg-C]
d = 0.05 [m]
ΔP = 2.392 [N/m^2]
f = 0.04
G = 28.5 [kg/s-m^2]
h = 51.86 [W/m^2-C]
k = 0.5948 [W/m-C]
L = 7.313 [m]
μ = 0.0008905 [kg/m-s]
$\dot{m}$ = 0.05595 [kg/s]
Nusselt = 4.36
Q = 5851 [W]
q_s = 800 [W/m]
Re = 1600
ρ = 993.1 [kg/m^3]
T_b = 37.5 [C]
T_{in} = 25 [C]
T_{out} = 50 [C]

Thus:

Flow rate to give a Re = 1600 is: m_dot = 0.05595 kg/s …. Ans.

Avg. heat transfer coeff. h = 51.86 W/m^2.C … Ans.

Length of tube required to raise the water temp from 25 C to 50 C = L = 7.313 m .. Ans.

Pressure drop under these conditions = DELTAP = 2.392 N/m^2 … Ans.

===

"**Prob. 2A1.4.18.** Lubricating oil at a temp of 60 C enters a 1 cm dia tube with a velocity of 2.5 m/s. The tube surface is maintained at 30 C. Calculate the length of the tube required to cool the oil to 45 C. Properties of oil at average temp are given. Use the relation: Nu = 0.023. Re^0.8 . Pr^0.3. – [VTU – Aug. 2001]:"

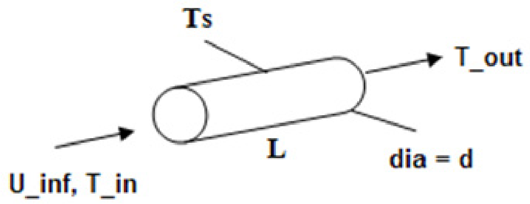
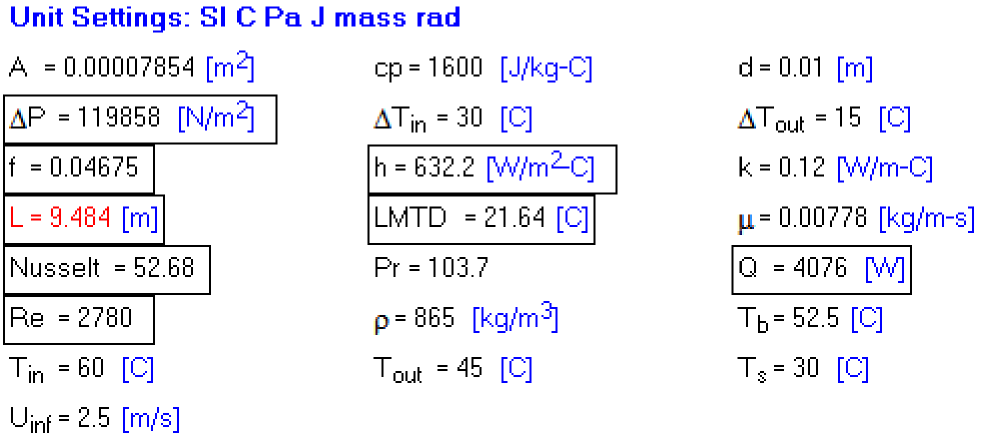


Fig.Prob.2A.1.4.18

EES Solution:

```
“Data:”

T_in = 60[C]
T_out= 45[C]
T_s = 30[C]
T_b = (T_in + T_out)/2
d = 0.01[m]
A = pi * d^2 / 4
U_inf = 2.5 [m/s]

“Properties of oil at bulk mean temp, T_b:”

k=0.12 [W/m-C]
rho=865 [kg/m^3]
mu=7.78e-03 [kg/m-s]
cp=1600 [J/kg-C]
Pr=mu * cp / k

“Calculations:”

Re = d * U_inf * rho / mu “finds Re = 2780 > 2300”
Nusselt = 0.023 * Re^0.8 * Pr^0.3 “finds Nusselts No.”
Nusselt = h * d / k “finds h”

DELTAT_in = T_in – T_s
DELTAT_out = T_out – T_s
LMTD = (DELTAT_in – DELTAT_out) / ln(DELTAT_in/DELTAT_out)
Q = rho * A * U_inf * cp * (T_in – T_out) “finds Q, W”
Q = h * (pi * d * L) * LMTD “finds L”

“Also, find the pressure drop:”

“Friction factor: Use the first Petukhov eqn, which is an explicit eqn. for f. Re range: 3000 to 5E06”

f = (0.79 * ln(Re) – 1.64)^(-2)

DELTAP = f * (L / D) * rho * U_inf^2 / 2 “[N/m^2]”
```

Results:



Unit Settings: SI C Pa J mass rad

A = 0.00007854 [m^2]
ΔP = 119858 [N/m^2]
f = 0.04675
L = 9.484 [m]
Nusselt = 52.68
Re = 2780
T_{in} = 60 [C]
U_{inf} = 2.5 [m/s]

cp = 1600 [J/kg-C]
ΔT_{in} = 30 [C]
h = 632.2 [W/m^2-C]
LMTD = 21.64 [C]
Pr = 103.7
ρ = 865 [kg/m^3]
T_{out} = 45 [C]

d = 0.01 [m]
ΔT_{out} = 15 [C]
k = 0.12 [W/m-C]
μ = 0.00778 [kg/m-s]
Q = 4076 [W]
T_b = 52.5 [C]
T_s = 30 [C]

Thus:

L = 9.484 m …. Length of tube required …. Ans.

DELTAP = 119858 N/m^2 …… Pressure drop …. Ans.

Also, plot the variation of L and DELTAP with velocity, U_inf:

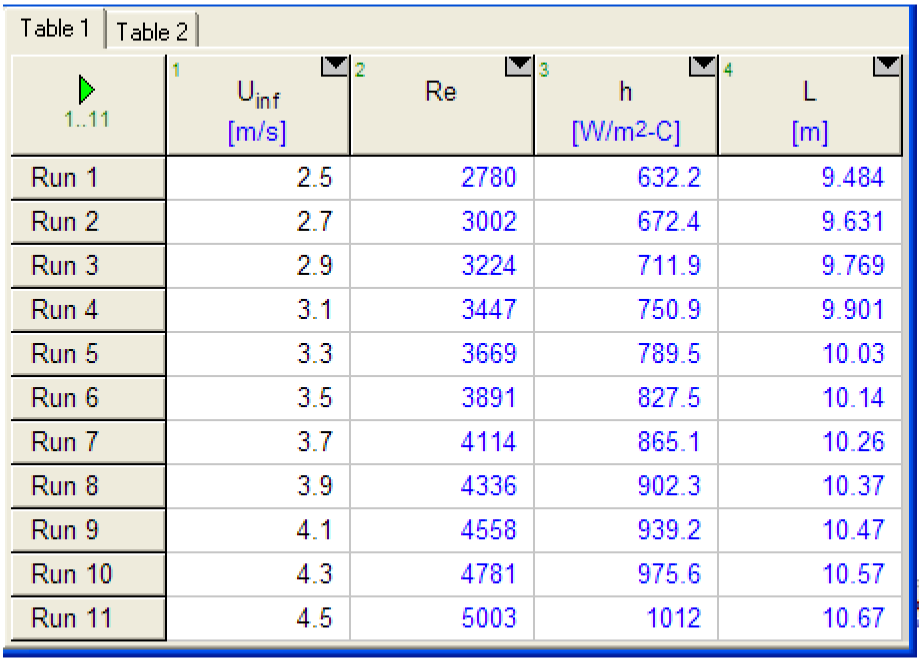
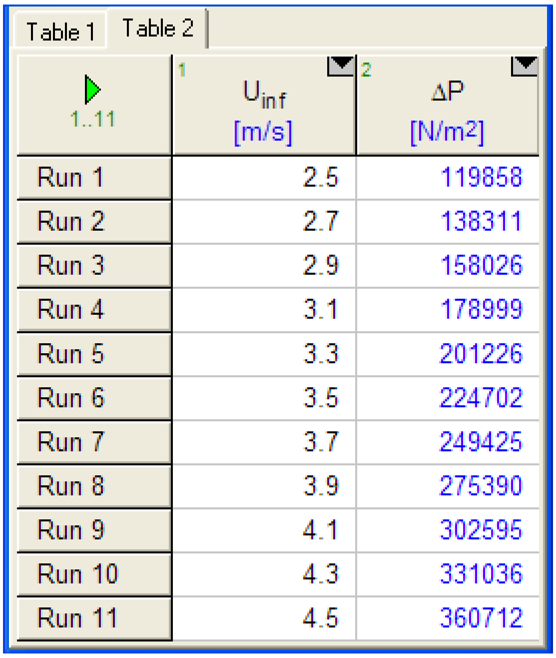
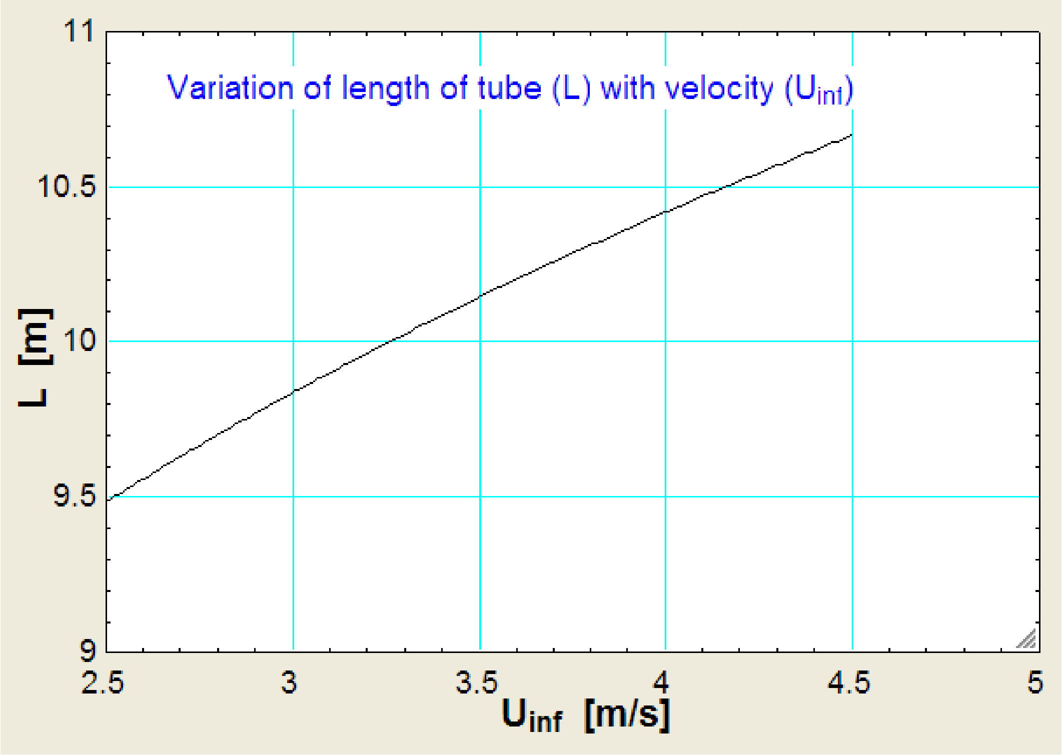


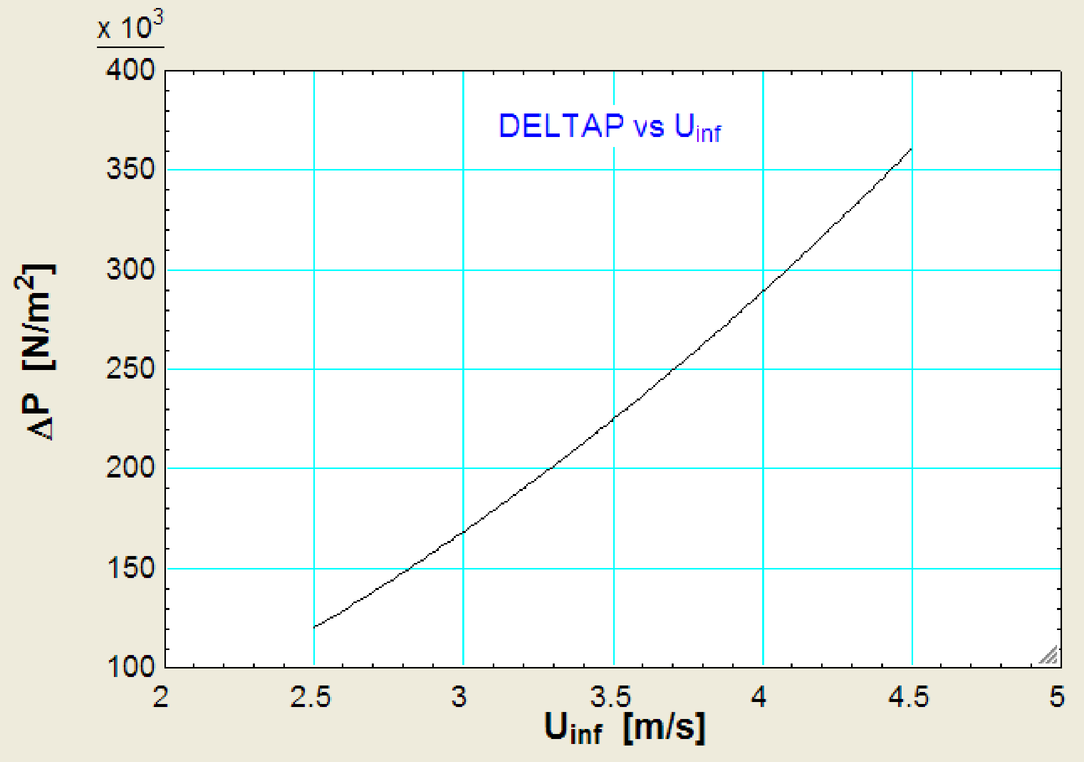
Table 1 | Table 2

1..11	1 U_{inf} [m/s]	2 Re	3 h [W/m2-C]	4 L [m]
Run 1	2.5	2780	632.2	9.484
Run 2	2.7	3002	672.4	9.631
Run 3	2.9	3224	711.9	9.769
Run 4	3.1	3447	750.9	9.901
Run 5	3.3	3669	789.5	10.03
Run 6	3.5	3891	827.5	10.14
Run 7	3.7	4114	865.1	10.26
Run 8	3.9	4336	902.3	10.37
Run 9	4.1	4558	939.2	10.47
Run 10	4.3	4781	975.6	10.57
Run 11	4.5	5003	1012	10.67



1..11	1 U_{inf} [m/s]	2 ΔP [N/m2]
Run 1	2.5	119858
Run 2	2.7	138311
Run 3	2.9	158026
Run 4	3.1	178999
Run 5	3.3	201226
Run 6	3.5	224702
Run 7	3.7	249425
Run 8	3.9	275390
Run 9	4.1	302595
Run 10	4.3	331036
Run 11	4.5	360712


Variation of length of tube (L) with velocity (U_inf)
L [m]
11
10.5
10
9.5
9
2.5
3
3.5
4
4.5
5
U_inf [m/s]


x 10^3
400
350
300
250
200
150
100
ΔP [N/m^2]
DELTAP vs U_inf
2
2.5
3
3.5
4
4.5
5
U_inf [m/s]

==

$UnitSystem C Pa J

"**Prob. 2A1.4.19.** Consider the flow of water at a rate of 0.015 kg/s through a square duct 2 cm × 2 cm whose walls are maintained at a uniform temp of 100 C. Assuming that the flow is hydrodynamically and thermally developed, determine the length of the duct required to heat water from 30 C to 70 C. [VTU – Jan./Feb. 2006]"

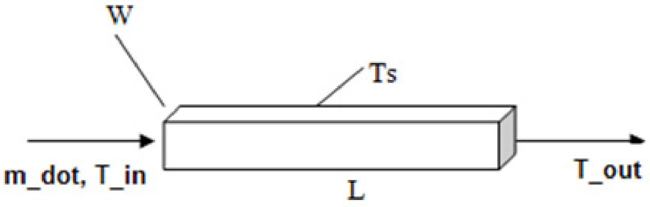


Fig.Prob.2A.1.4.19

EES Solution:

"Data:"

T_in = 30[C]
T_out= 70[C]
T_s = 100[C]
T_b = (T_in + T_out)/2
W = 0.02[m]
A_c = W^2 "[m^2]"
m_dot = 0.015 [kg/s]

"Properties of Water at bulk mean temp, T_b:"

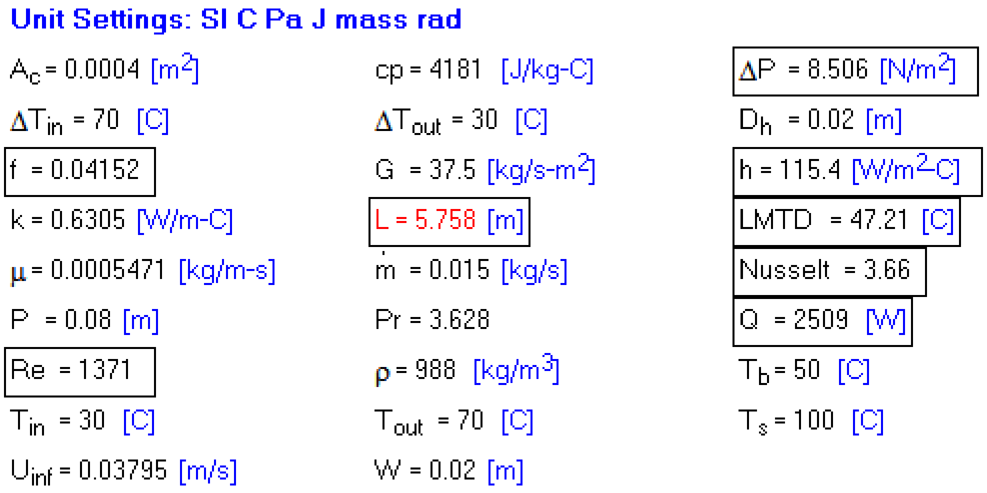
k=Conductivity(Water,T=T_b,P=1.013e05)
rho=Density(Water,T=T_b,P=1.013e05)
mu=Viscosity(Water,T=T_b,P=1.013e05)
cp=SpecHeat(Water,T=T_b,P=1.013e05)
Pr=Prandtl(Water,T=T_b,P=1.013e05)

"Calculations:"

G = m_dot / A_c "[kg/s-m^2]… mass velocity"
m_dot = rho * A_c * U_inf "….finds velocity, U_inf"
P = (W + W) * 2 "[m] …. perimeter"
D_h = 4 * A_c / P "[m] … hydraulic dia of duct"
Re = D_h * G / mu "finds Re = 1371 < 2300 …. Laminar flow"

```
Nusselt = 3.66 " Nusselts No. for Re < 2300 and fully developed flow"

Nusselt = h * D_h / k "finds h"

DELTAT_in = T_s – T_in
DELTAT_out = T_s – T_out
LMTD = (DELTAT_in – DELTAT_out) / ln(DELTAT_in/DELTAT_out)
Q = m_dot * cp * (T_out – T_in) "finds Q, W"
Q = h * (P * L) * LMTD "finds L"

"Also, find the pressure drop:"

"Friction factor: For laminar flow inside tubes: f = 56.92 / Re, for square section:"

f = 56.92 / Re

DELTAP = f * (L / D_h) * rho * U_inf^2 / 2 "[N/m^2]"
```

Results:



Unit Settings: SI C Pa J mass rad

$A_c = 0.0004$ [m^2]

$\Delta T_{in} = 70$ [C]

$\boxed{f = 0.04152}$

$k = 0.6305$ [W/m-C]

$\mu = 0.0005471$ [kg/m-s]

$P = 0.08$ [m]

$\boxed{Re = 1371}$

$T_{in} = 30$ [C]

$U_{inf} = 0.03795$ [m/s]

$cp = 4181$ [J/kg-C]

$\Delta T_{out} = 30$ [C]

$G = 37.5$ [kg/s-m^2]

$\boxed{L = 5.758 \text{ [m]}}$

$\dot{m} = 0.015$ [kg/s]

$Pr = 3.628$

$\rho = 988$ [kg/m^3]

$T_{out} = 70$ [C]

$W = 0.02$ [m]

$\boxed{\Delta P = 8.506 \text{ [N/m}^2\text{]}}$

$D_h = 0.02$ [m]

$\boxed{h = 115.4 \text{ [W/m}^2\text{-C]}}$

$\boxed{LMTD = 47.21 \text{ [C]}}$

$\boxed{Nusselt = 3.66}$

$\boxed{Q = 2509 \text{ [W]}}$

$T_b = 50$ [C]

$T_s = 100$ [C]

Thus:

L = 5.758 m …. Length of tube required…Ans.

DELTAP = 8.506 N/m^2 …. Ans.

==

Prob. 2A1.4.20. Atmospheric air at a mean temp of 300 K and bulk stream velocity of 10 m/s flows through a tube of 2.5 mm ID. Calculate the pressure drop for 100 m length of tube for (a) a smooth tube (b) commercial steel tube.[VTU – Jan/Feb. 2005]

Also: Plot the fan power required for velocities ranging from 5 m/s to 14 m/s.

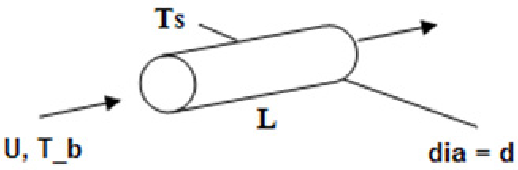


Fig.Prob.2A.1.4.20

Mathcad Solution:

Data:

$d := 0.0025$ m $\quad L := 100$ m $\quad U := 10$ m/s

$T_b := 27$ C $\quad A_c := \frac{\pi \cdot d^2}{4}$ i.e. $A_c = 4.909 \cdot 10^{-6}$ m^2 ... area of cross-section

Calculations:

Properties of Air at Tb:

Use the Mathcad functions already written for properties of Air.

See Prob. 2A1.2.5.

$\rho := \text{rho\_Air}(T_b + 273)$ i.e. $\rho = 1.177$ kg/m^3

$k := \text{k\_Air}(T_b + 273)$ i.e. $k = 0.026$ W/m.K

$\mu := \text{mu\_Air}(T_b + 273)$ i.e. $\mu = 1.843 \cdot 10^{-5}$ kg/m.s

$Pr := \text{Pr\_Air}(T_b + 273)$ i.e. $Pr = 0.707$ ...Prandtl No.

$cp := \frac{Pr \cdot k}{\mu}$ i.e. $cp = 994.714$ J/kg.K

Reynolds No.:

$Re(U) := \frac{U \cdot d \cdot \rho}{\mu}$ ...Reynolds No...

i.e. $Re(U) = 1.597 \cdot 10^3$ ...less than 2300. Therefore, laminar flow.

Note that Re is written as a function of velocity U. This will be useful to draw graphs for various U values, later.

Friction factor:

Use: f = 64 / Re for laminar flow in smooth tubes (and also for rough tubes):

$f(U) := \frac{64}{Re(U)}$ ...for fully developed laminar flow

i.e. $f(U) = 0.04$ ....friction factor

Pressure drop:

$U = 10$ m/s.... velocity of flow

$\Delta P(U) := f(U) \cdot \left(\frac{L}{d}\right) \cdot \frac{U^2 \cdot \rho}{2}$

i.e. $\Delta P(U) = 9.435 \cdot 10^4$ **.. N/m^2 for 100 m length .... Ans.**

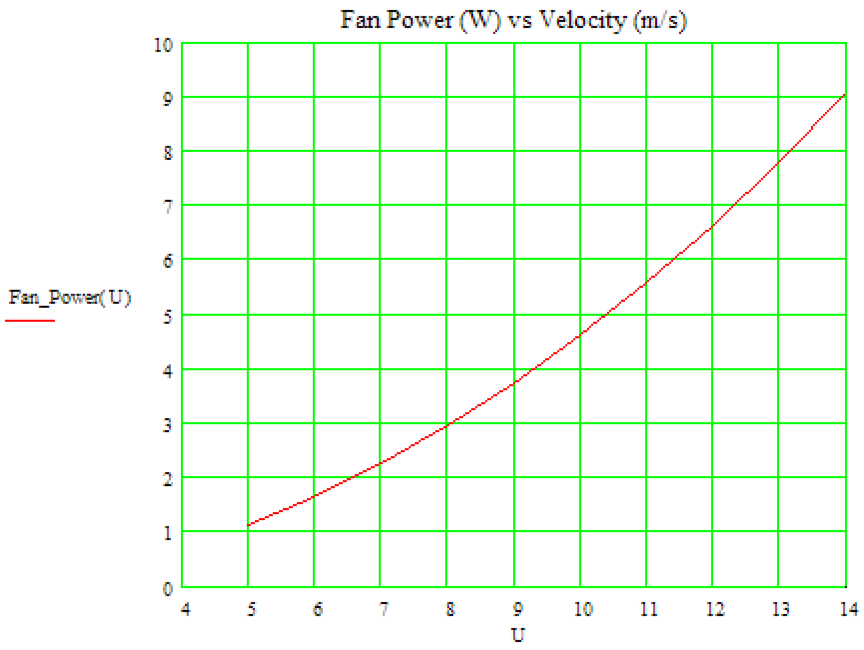
$Fan\_Power(U) := \Delta P(U) \cdot A_c \cdot U$ W

i.e. $Fan\_Power(U) = 4.631$ **W .... Ans.**

Plot fan power against U:

U := 5,6.. 14   m/s .... variation of velocity

U	Re(U)	f(U)	ΔP(U)	Fan_Power(U)
5	798.277	0.08	47174.755	1.158
6	957.933	0.067	56609.706	1.667
7	1117.588	0.057	66044.657	2.269
8	1277.244	0.05	75479.609	2.964
9	1436.899	0.045	84914.56	3.751
10	1596.555	0.04	94349.511	4.631
11	1756.21	0.036	103784.462	5.604
12	1915.866	0.033	113219.413	6.669
13	2075.521	0.031	122654.364	7.827
14	2235.176	0.029	132089.315	9.077



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"**Prob. 2A1.4.21.** Water flows with a velocity of 0.6 m/s through a tube of 6 mm ID and 3.5 m length. Find the heat transfer rate by convection if the mean water temp is 50 C and the tube surface temp is 70 C. Use the empirical relation: Nu = 0.023. Re ^0.8. Pr^0.4. – [VTU – Jan.–Feb. 2005]"

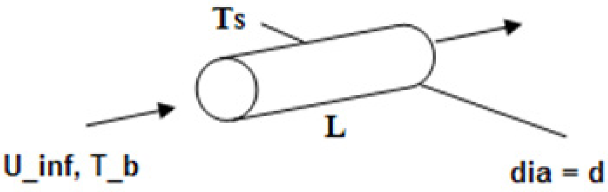


Fig.Prob.2A.1.4.21

EES Solution:

"Data:"

T_s = 70[C]
T_b = 50[C]
d = 0.006[m]
L = 3.5[m]
A = pi * d^2 / 4
U_inf = 0.6[m/s]

"Properties of Water at bulk mean temp, T_b:"

k=Conductivity(Water,T=T_b,P=1.013e05)
rho=Density(Water,T=T_b,P=1.013e05)
mu=Viscosity(Water,T=T_b,P=1.013e05)
cp=SpecHeat(Water,T=T_b,P=1.013e05)
Pr=Prandtl(Water,T=T_b,P=1.013e05)

"Calculations:"

Re = d*U_inf*rho/mu "finds Re = 6502 > 2300"
Nusselt = 0.023 * Re^0.8 * Pr^0.4 "finds Nusselts No. for Re > 2300"
Nusselt = h * d / k "finds h"

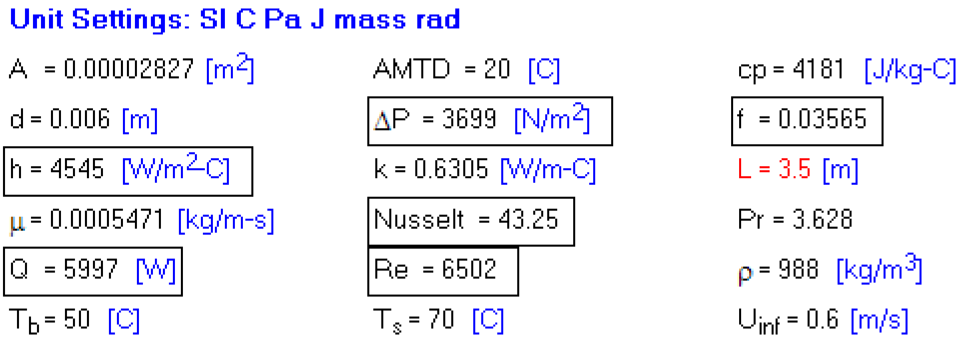
AMTD = T_s – T_b "[C]"
Q = h * (pi * d * L) * AMTD "finds Q"

"Also, find the pressure drop:"

"Friction factor: Use the first Petukhov eqn, which is an explicit eqn. for f. Re range: 3000 to 5E06"

f = (0.79 * ln(Re) – 1.64)^(-2)
DELTAP = f * (L / D) * rho * U_inf^2 / 2 "[N/m^2]"

Results:



Unit Settings: SI C Pa J mass rad

$A = 0.00002827$ [m^2]   AMTD = 20 [C]   cp = 4181 [J/kg-C]

$d = 0.006$ [m] $\boxed{\Delta P = 3699 \text{ [N/m}^2\text{]}}$ $\boxed{f = 0.03565}$

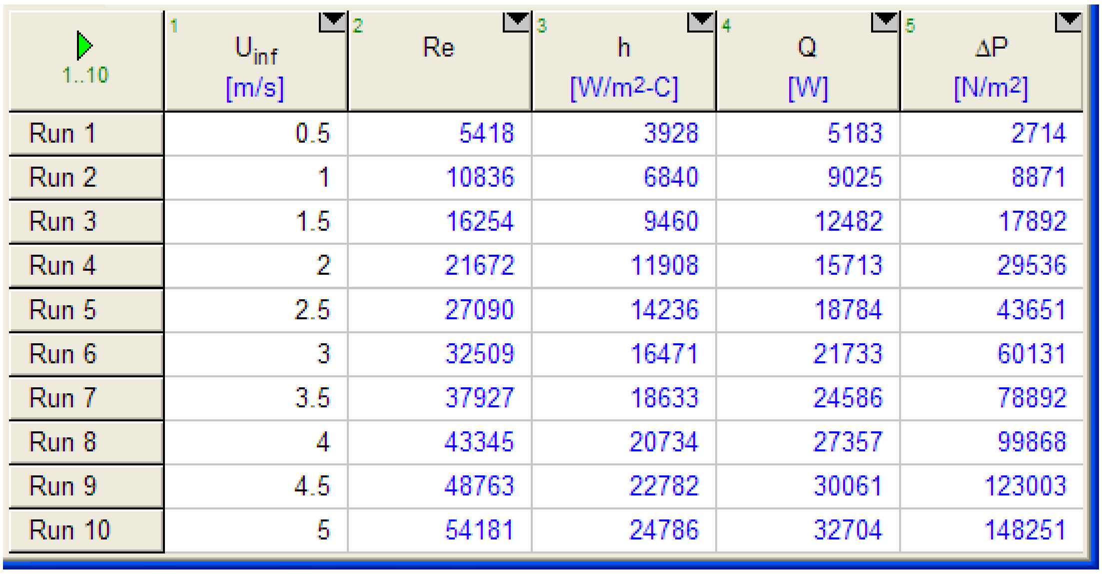
$\boxed{h = 4545 \text{ [W/m}^2\text{-C]}}$ $k = 0.6305$ [W/m-C] $L = 3.5$ [m]

$\mu = 0.0005471$ [kg/m-s] $\boxed{Nusselt = 43.25}$ Pr = 3.628

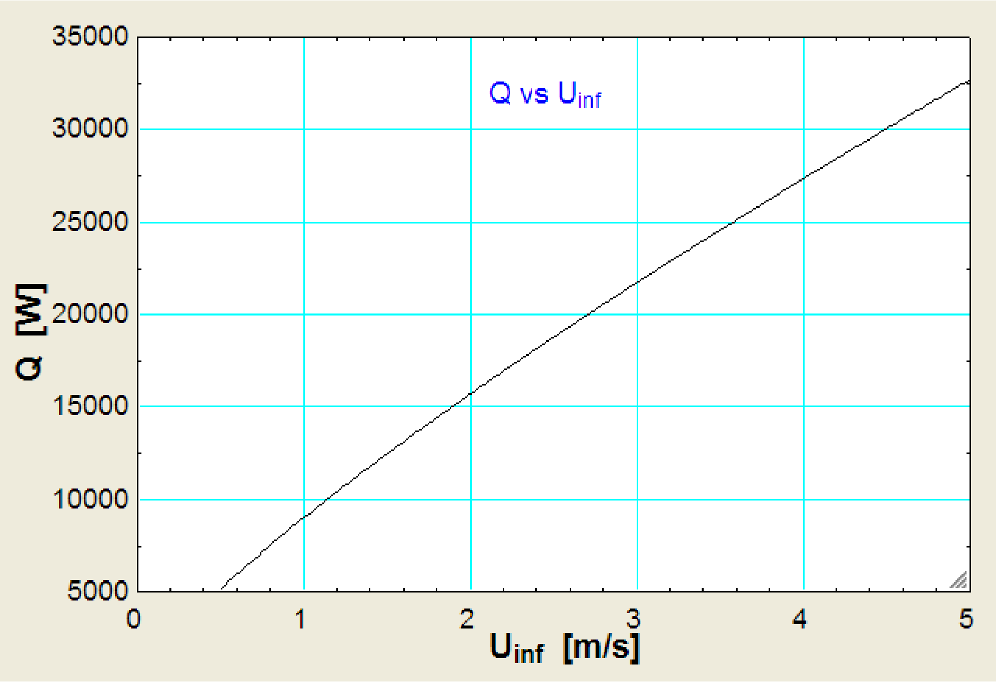
$\boxed{Q = 5997 \text{ [W]}}$ $\boxed{Re = 6502}$ $\rho = 988$ [kg/m^3]

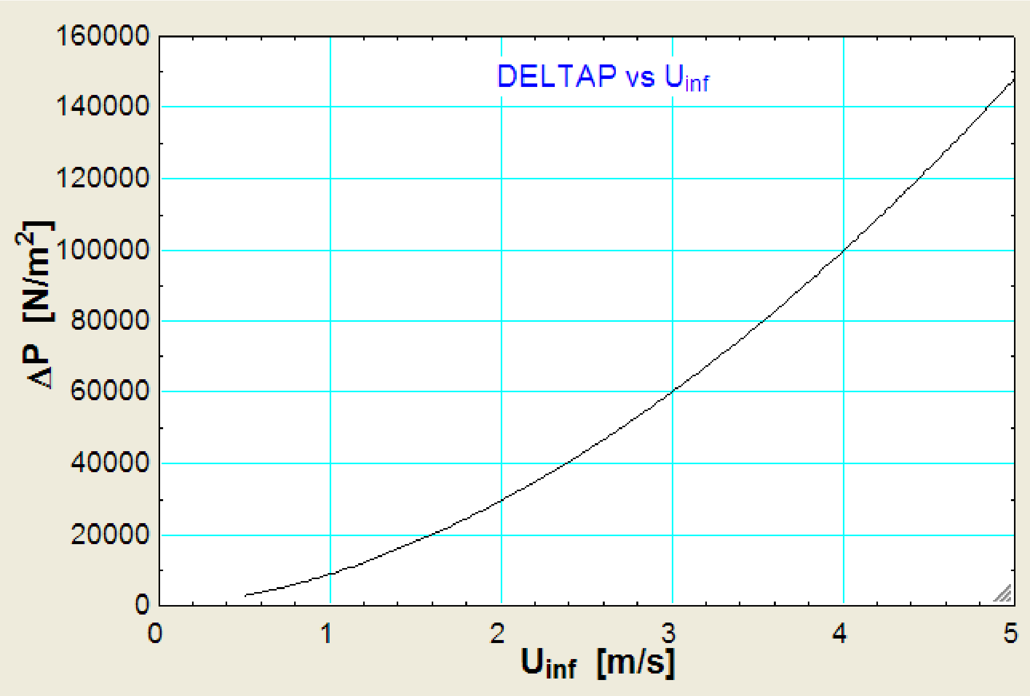
$T_b = 50$ [C] $T_s = 70$ [C] $U_{inf} = 0.6$ [m/s]

Plot Q and DELTAP against U_inf:



1..10	1 U_{inf} [m/s]	2 Re	3 h [W/m2-C]	4 Q [W]	5 ΔP [N/m2]
Run 1	0.5	5418	3928	5183	2714
Run 2	1	10836	6840	9025	8871
Run 3	1.5	16254	9460	12482	17892
Run 4	2	21672	11908	15713	29536
Run 5	2.5	27090	14236	18784	43651
Run 6	3	32509	16471	21733	60131
Run 7	3.5	37927	18633	24586	78892
Run 8	4	43345	20734	27357	99868
Run 9	4.5	48763	22782	30061	123003
Run 10	5	54181	24786	32704	148251





==

Prob.2A1.4.22. Engine Oil is heated by flowing through a circular tube of diameter d = 50 mm and length L = 25 m and whose surface is maintained at 150 C.

a) If the flow rate and inlet temp of oil are 0.5 kg/s and 20 C, what is the outlet temp and total heat transfer rate for the tube?
b) For flow rates in the range 0.5 < m_dot < 2.0 kg/s, compute and plot the variation of outlet temp, T_out and Q with m_dot.

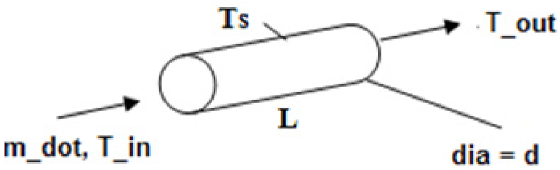


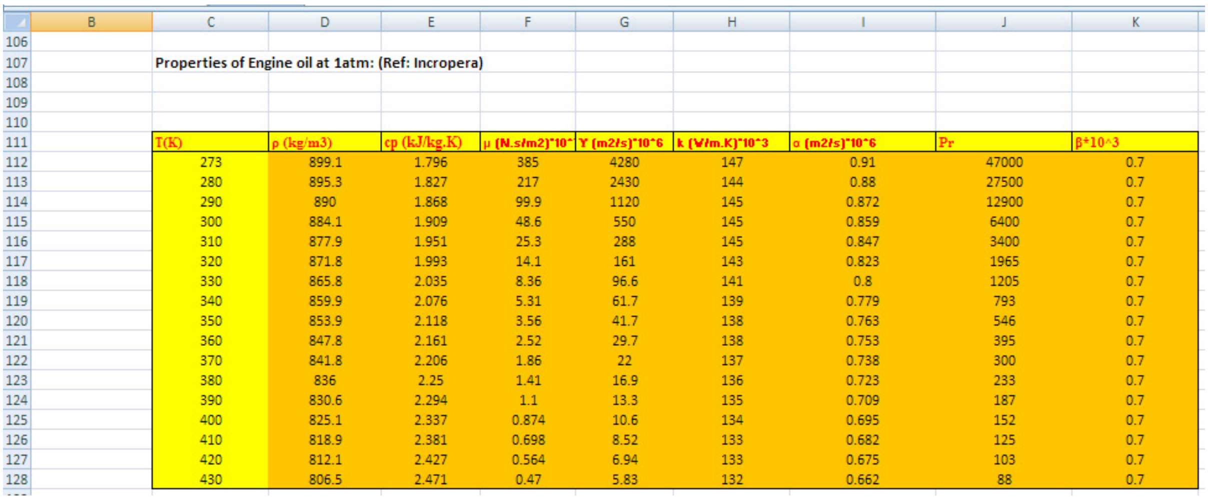
Fig.Prob.2A.1.4.22

Note that this is the same as Prob. 2A1.4.2.

But, now, we will solve it with EXCEL.

EXCEL Solution:

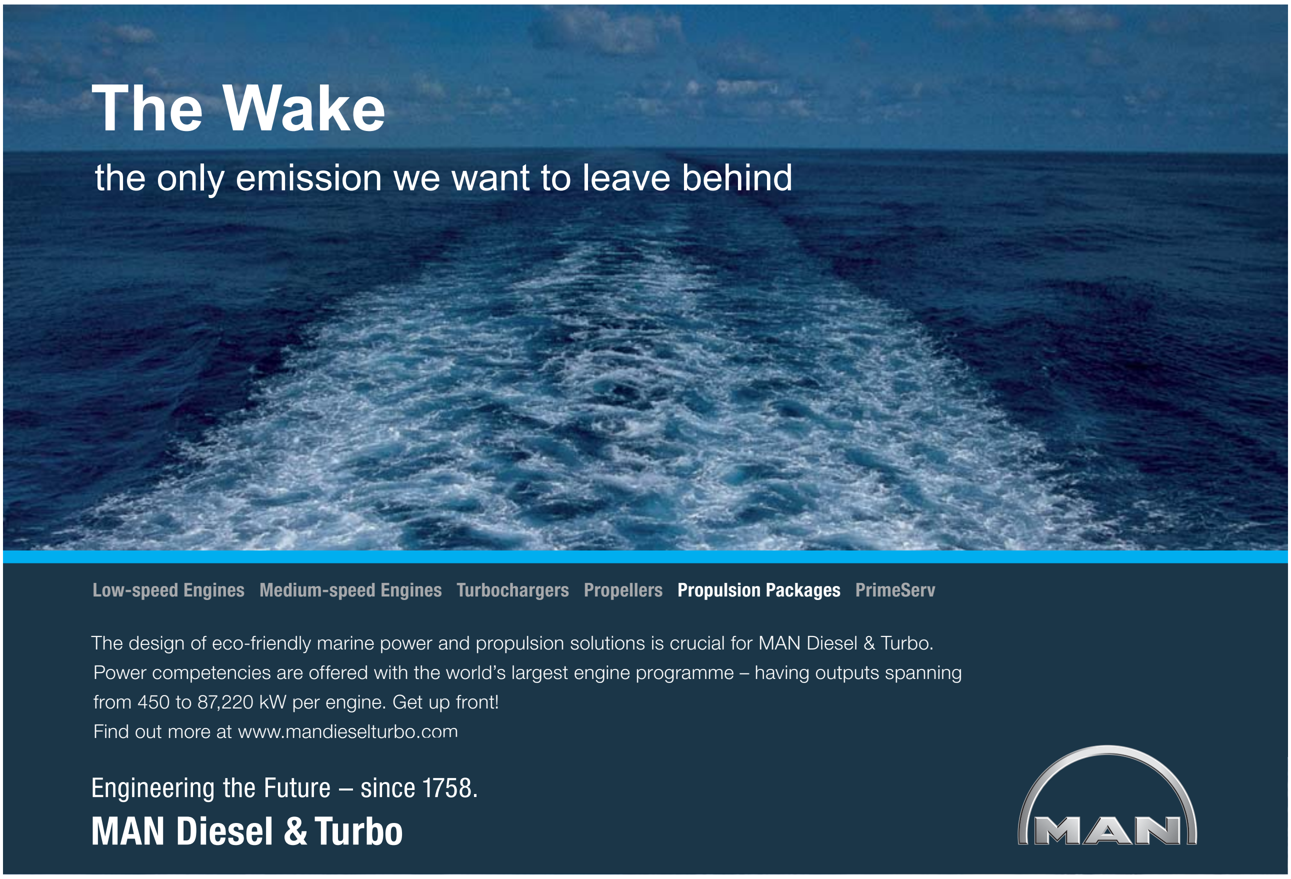
EXCEL does not have built-in functions for properties of Engine oil. So, we shall use the data table given in Ref.[3] and interpolate for results by writing suitable VBA Functions. The data Table in EXCEL is:



Properties of Engine oil at 1atm: (Ref: Incropera)

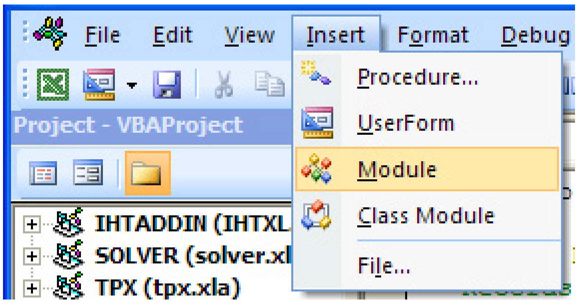
T(K)	ρ (kg/m3)	cp (kJ/kg.K)	μ (N.s/m2)*10^	ϒ (m2/s)*10^6	k (W/m.K)*10^3	α (m2/s)*10^6	Pr	β*10^3
273	899.1	1.796	385	4280	147	0.91	47000	0.7
280	895.3	1.827	217	2430	144	0.88	27500	0.7
290	890	1.868	99.9	1120	145	0.872	12900	0.7
300	884.1	1.909	48.6	550	145	0.859	6400	0.7
310	877.9	1.951	25.3	288	145	0.847	3400	0.7
320	871.8	1.993	14.1	161	143	0.823	1965	0.7
330	865.8	2.035	8.36	96.6	141	0.8	1205	0.7
340	859.9	2.076	5.31	61.7	139	0.779	793	0.7
350	853.9	2.118	3.56	41.7	138	0.763	546	0.7
360	847.8	2.161	2.52	29.7	138	0.753	395	0.7
370	841.8	2.206	1.86	22	137	0.738	300	0.7
380	836	2.25	1.41	16.9	136	0.723	233	0.7
390	830.6	2.294	1.1	13.3	135	0.709	187	0.7
400	825.1	2.337	0.874	10.6	134	0.695	152	0.7
410	818.9	2.381	0.698	8.52	133	0.682	125	0.7
420	812.1	2.427	0.564	6.94	133	0.675	103	0.7
430	806.5	2.471	0.47	5.83	132	0.662	88	0.7

Now, for a given value of T (K), the program should locate values of Temps just above and just below T, and interpolate the required property from this Table.



We give below one example of the Function to find density ‘rho’ for a given T:

First, go to Developer-Visual Basic – Insert_module:



Click on Module and a blank page appears. Start typing the code there:

```
Function EngineOil_rho_T(T As Double) As Double
'gives density of Engine oil (kg/m^3) as a function of T (K) at 1 atmosph pressure
'Reads rho values from Table and interpolates

Dim i As Integer

Dim T_1 As Double, T_2 As Double, rho_1 As Double, rho_2 As Double

If T < 273 Or T > 430 Then

 MsgBox ("T must be between 273 K and 430 K !!")

 End
 End If

For i = 0 To 16

   If Cells(112 + i, 3) = T Then
      EngineOil_rho_T = Cells(112 + i, 4)
   End If
   If Cells(112 + i, 3) < T And Cells(112 + i + 1, 3) > T Then
      T_1 = Cells(112 + i, 3).Value
      T_2 = Cells(112 + i + 1, 3).Value
      rho_1 = Cells(112 + i, 4).Value
      rho_2 = Cells(112 + i + 1, 4).Value
      EngineOil_rho_T = (rho_1 + (T - T_1) * (rho_2 - rho_1) / (T_2 - T_1))
   End If

Next i

End Function
```

In the above Function:

Line 1: gives the Function name, variables defined

Line 2, 3: details about the Function

Line 4: dimension of I, the counter in For… Next construct

Line 5: dimensions of local variables

Line 6, 7, 8, 9: Error message if temps are out of bound

Lines 10 to 21: For…Next construct

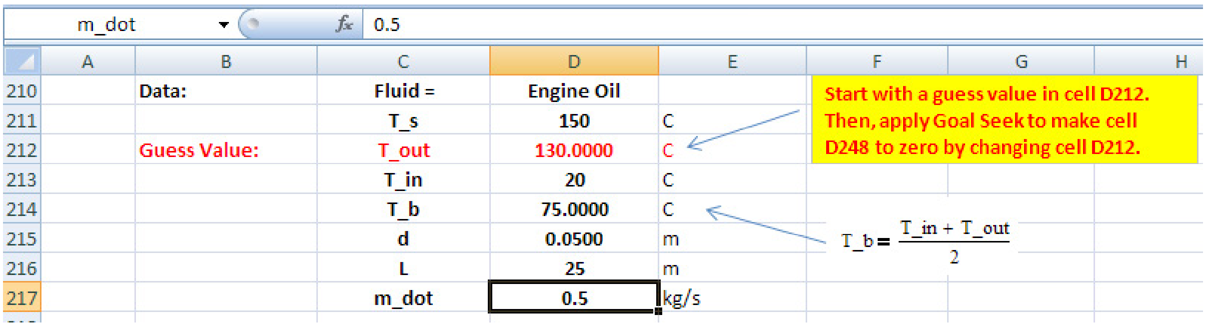
Lines 11, 12, 13: If an exact match of T is found in the Table, return the corresponding value of 'rho'

Lines 14 to 20: Otherwise, locate values of Temps just below and just above T, and the corresponding values of rho, and then return the interpolated value of rho for T

Line 22: End statement of Function

Now, let us proceed to solve the problem:

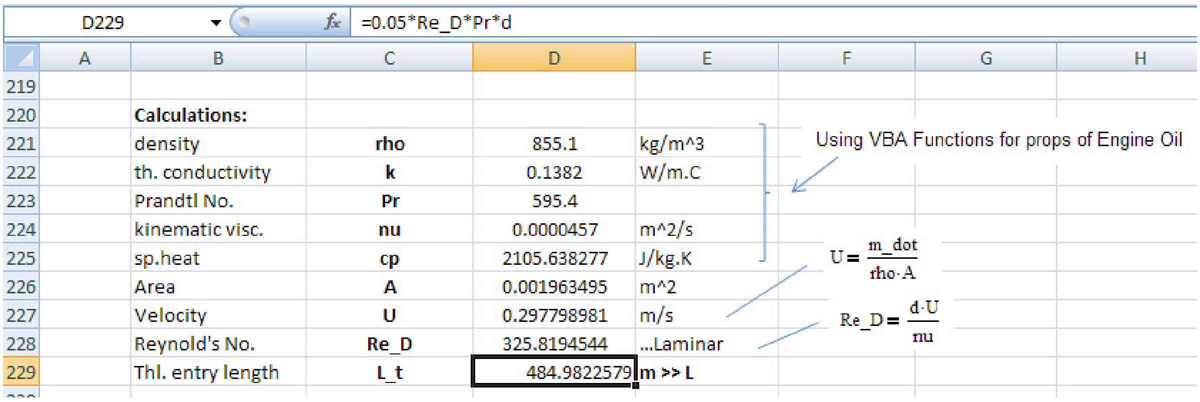
1. Set up the EXCEL worksheet, enter data and name the cells:



	A	B	C	D	E	F	G	H
210		**Data:**	**Fluid =**	**Engine Oil**		Start with a guess value in cell D212. Then, apply Goal Seek to make cell D248 to zero by changing cell D212.		
211			**T_s**	**150**	C			
212		Guess Value:	T_out	130.0000	C			
213			**T_in**	**20**	C			
214			**T_b**	**75.0000**	C			
215			**d**	**0.0500**	m	$T\_b = \frac{T\_in + T\_out}{2}$		
216			**L**	**25**	m			
217			**m_dot**	**0.5**	kg/s			

Here, T_out is not known; in fact, it is to be found out. So, we will start with a guess value as shown above.

2. Next, do the preliminary calculations. The formulas used are shown in the worksheet for clarity:



D229 =0.05*Re_D*Pr*d

	A	B	C	D	E
219					
220		**Calculations:**			
221		density	**rho**	855.1	kg/m^3
222		th. conductivity	**k**	0.1382	W/m.C
223		Prandtl No.	**Pr**	595.4	
224		kinematic visc.	**nu**	0.0000457	m^2/s
225		sp.heat	**cp**	2105.638277	J/kg.K
226		Area	**A**	0.001963495	m^2
227		Velocity	**U**	0.297798981	m/s
228		Reynold's No.	**Re_D**	325.8194544	...Laminar
229		Thl. entry length	**L_t**	484.9822579	**m >> L**

Using VBA Functions for props of Engine Oil

$$U = \frac{m\_dot}{rho \cdot A}$$

$$Re\_D = \frac{d \cdot U}{nu}$$

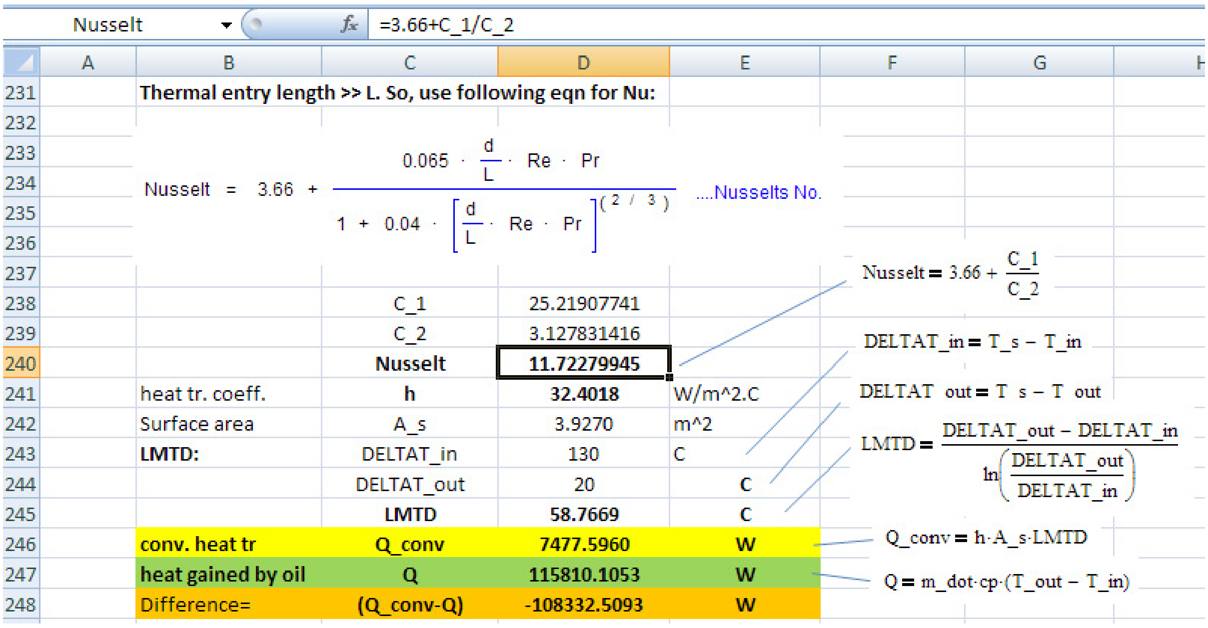
In the above, preliminary Re_D is less than 2300. So, Laminar flow. And, the thermal entry length =

Lt = (0.05 * Re_D * Pr * d) is calculated, and is found to be >> L.

3. Therefore, we use the eqn as shown, for Nusselts No.:

$$\text{Nusselt} = 3.66 + \frac{0.065 \cdot \frac{d}{L} \cdot \text{Re} \cdot \text{Pr}}{1 + 0.04 \cdot \left[\frac{d}{L} \cdot \text{Re} \cdot \text{Pr}\right]^{(2/3)}} \quad \text{....Nusselts No.}$$

This is shown in the following screen shot:



Nusselt | =3.66+C_1/C_2

	A	B	C	D	E
231		**Thermal entry length >> L. So, use following eqn for Nu:**			
232					
233–236		$\text{Nusselt} = 3.66 + \frac{0.065 \cdot \frac{d}{L} \cdot \text{Re} \cdot \text{Pr}}{1 + 0.04 \cdot \left[\frac{d}{L} \cdot \text{Re} \cdot \text{Pr}\right]^{(2/3)}}$			....Nusselts No.
237					
238			C_1	25.21907741	
239			C_2	3.127831416	
240			**Nusselt**	**11.72279945**	
241		heat tr. coeff.	**h**	**32.4018**	W/m^2.C
242		Surface area	A_s	3.9270	m^2
243		**LMTD:**	DELTAT_in	130	C
244			DELTAT_out	20	**C**
245			**LMTD**	**58.7669**	**C**
246		**conv. heat tr**	**Q_conv**	**7477.5960**	**W**
247		**heat gained by oil**	**Q**	**115810.1053**	**W**
248		Difference=	**(Q_conv-Q)**	**-108332.5093**	**W**

$$\text{Nusselt} = 3.66 + \frac{C\_1}{C\_2}$$

$$\text{DELTAT\_in} = T\_s - T\_in$$

$$\text{DELTAT out} = T \ s - T \ out$$

$$\text{LMTD} = \frac{\text{DELTAT\_out} - \text{DELTAT\_in}}{\ln\left(\frac{\text{DELTAT\_out}}{\text{DELTAT\_in}}\right)}$$

$$Q\_conv = h \cdot A\_s \cdot \text{LMTD}$$

$$Q = m\_dot \cdot cp \cdot (T\_out - T\_in)$$

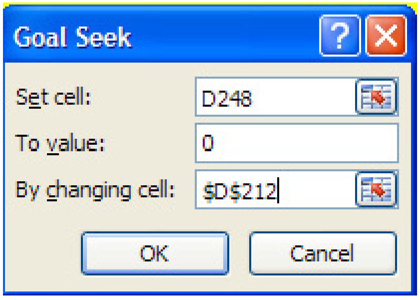
In the above fig., we see the calculation for heat transfer coeff, after Nusselts No. is calculated. And, the convective heat transfer between the fluid and the pipe surface is calculated using Newton's eqn i.e. Q_conv = h * A_s * LMTD, where A_s is the heat transfer surface area (= π * d * L), and LMTD is the Logarithmic Mean Temp Difference. These calculations, with the formulas used are shown.

Now, Q_conv must be equal to the heat gained by the oil, Q = m_dot * cp * (T_out – T_in).

It is calculated in cell D247. Cell D248 gives (Q_conv – Q). Of course, now it is not equal to zero, since we started with an assumed value for T_out.

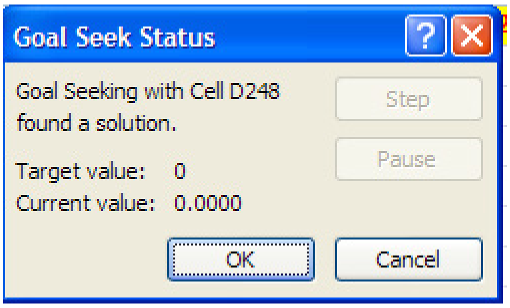
4. To find out T_out to satisfy the condition: (Q_conv – Q) = 0, we apply the Goal Seek in EXCEL:

Go to Data – What If Analysis – Goal Seek. We get the following pop up. Fill it up as shown:



Set cell: D248
To value: 0
By changing cell: D212

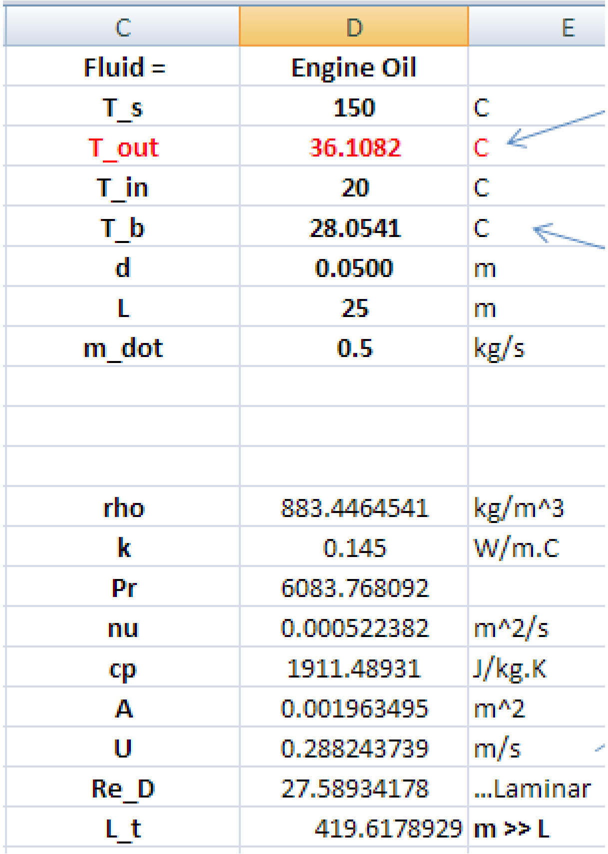
i.e. cell D248 will be reduced to zero by changing cell D212 (i.e. T_out). Click OK. We get:



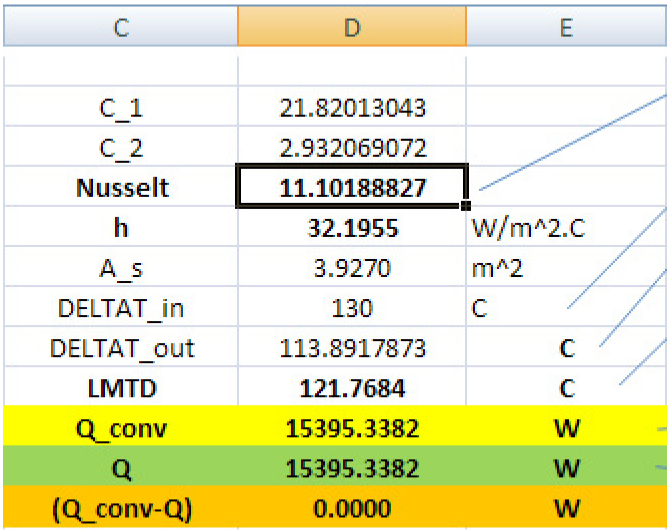
Goal Seeking with Cell D248
found a solution.

Target value: 0
Current value: 0.0000

Again click OK, and note the value of T_out in cell D212. Also, note that all other corresponding calculations have up dated themselves:

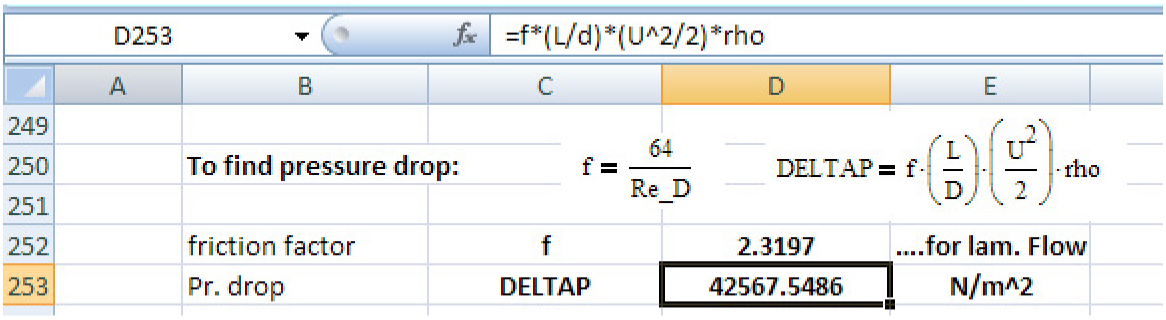


C	D	E
Fluid =	**Engine Oil**	
T_s	**150**	C
T_out	**36.1082**	C
T_in	**20**	C
T_b	**28.0541**	C
d	**0.0500**	m
L	**25**	m
m_dot	**0.5**	kg/s
rho	883.4464541	kg/m^3
k	0.145	W/m.C
Pr	6083.768092	
nu	0.000522382	m^2/s
cp	1911.48931	J/kg.K
A	0.001963495	m^2
U	0.288243739	m/s
Re_D	27.58934178	...Laminar
L_t	419.6178929	**m >> L**



C	D	E
C_1	21.82013043	
C_2	2.932069072	
Nusselt	**11.10188827**	
h	**32.1955**	W/m^2.C
A_s	3.9270	m^2
DELTAT_in	130	C
DELTAT_out	113.8917873	**C**
LMTD	**121.7684**	**C**
Q_conv	**15395.3382**	**W**
Q	**15395.3382**	**W**
(Q_conv-Q)	**0.0000**	**W**

5. Now, to find the pressure drop, use the eqn for friction factor 'f' for laminar flow, i.e. f = 64 / Re_D. And, pressure drop is calculated with the formula shown:



D253 =f*(L/d)*(U^2/2)*rho

	A	B	C	D	E
249					
250		**To find pressure drop:**	$f = \frac{64}{Re\_D}$		$DELTAP = f \cdot \left(\frac{L}{D}\right) \cdot \left(\frac{U^2}{2}\right) \cdot rho$
251					
252		friction factor	f	2.3197	....for lam. Flow
253		Pr. drop	DELTAP	42567.5486	N/m^2

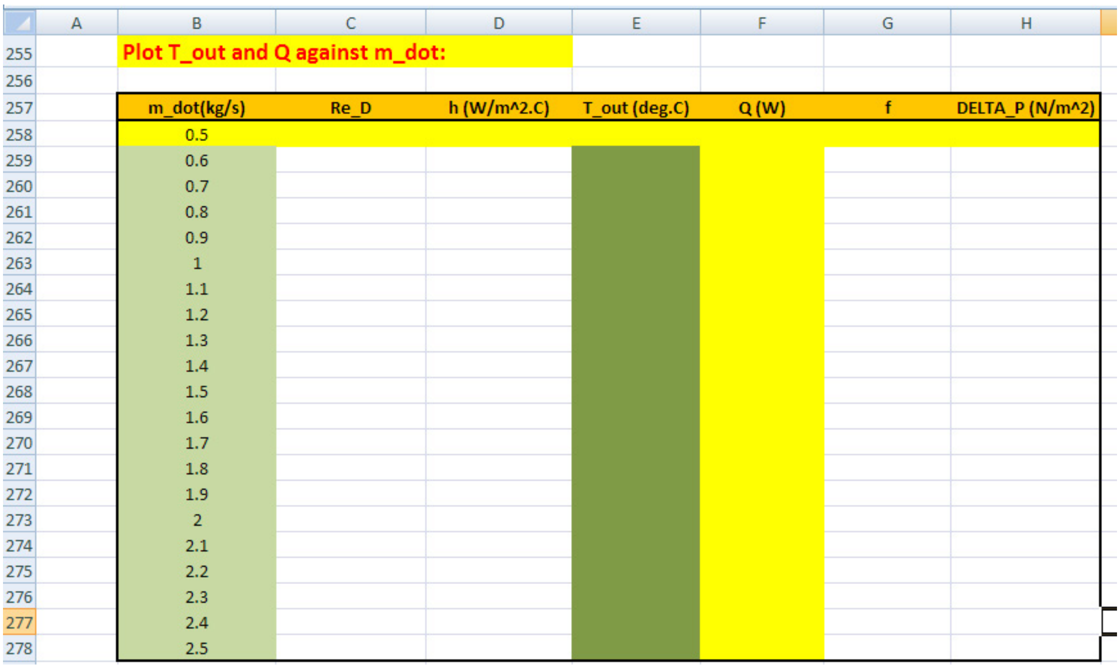
Thus, when m_dot = 0.5 kg/s, we have:

T_out = 36.1082 deg. C, DELTAP = 42567.55 N/m^2 … Ans.

6. Now, let us draw the plot of T_out and Q as m_dot varies from 0.5 kg/s to 2.5 kg/s:

For each value of m_dot, we will need to adopt Goal Seek to get T_out and other quantities. So, it is convenient to do this with a VBA program.

First, record a Macro for the Goal Seek operation and then change it to get the results in a Table as per following plan:



	A	B	C	D	E	F	G	H
255		Plot T_out and Q against m_dot:						
256								
257		m_dot(kg/s)	Re_D	h (W/m^2.C)	T_out (deg.C)	Q (W)	f	DELTA_P (N/m^2)
258		0.5						
259		0.6						
260		0.7						
261		0.8						
262		0.9						
263		1						
264		1.1						
265		1.2						
266		1.3						
267		1.4						
268		1.5						
269		1.6						
270		1.7						
271		1.8						
272		1.9						
273		2						
274		2.1						
275		2.2						
276		2.3						
277		2.4						
278		2.5						

Following is the modified VBA program of the recorded Macro 1 to fill up this Table:

```
Sub Macro1()
'
' Macro1 Macro
' Records Goal Seek
'
' Keyboard Shortcut: Ctrl+Shift+R
'
  Dim i As Integer

    For i = 0 To 20

    Range("D217") = Cells(258 + i, 2)


    Range("D248").GoalSeek Goal:=0, ChangingCell:=Range("D212")

    Cells(258 + i, 3) = Range("D228")
    Cells(258 + i, 4) = Range("D241")
    Cells(258 + i, 5) = Range("D212")
    Cells(258 + i, 6) = Range("D247")
    Cells(258 + i, 7) = Range("D252")
    Cells(258 + i, 8) = Range("D253")

    Next i

End Sub
```

In the above program:

Line 1: gives Macro name

Lines 2, 3: explain what it does

Line 4: gives key board short cut

Line 5: dimension of the counter i

Lines 6 to 15: For … Next construct

Line 7: Takes the first value of m_dot from the Table and copies it to cell D217 (i.e. m_dot)

Line 8: Goal Seek operation

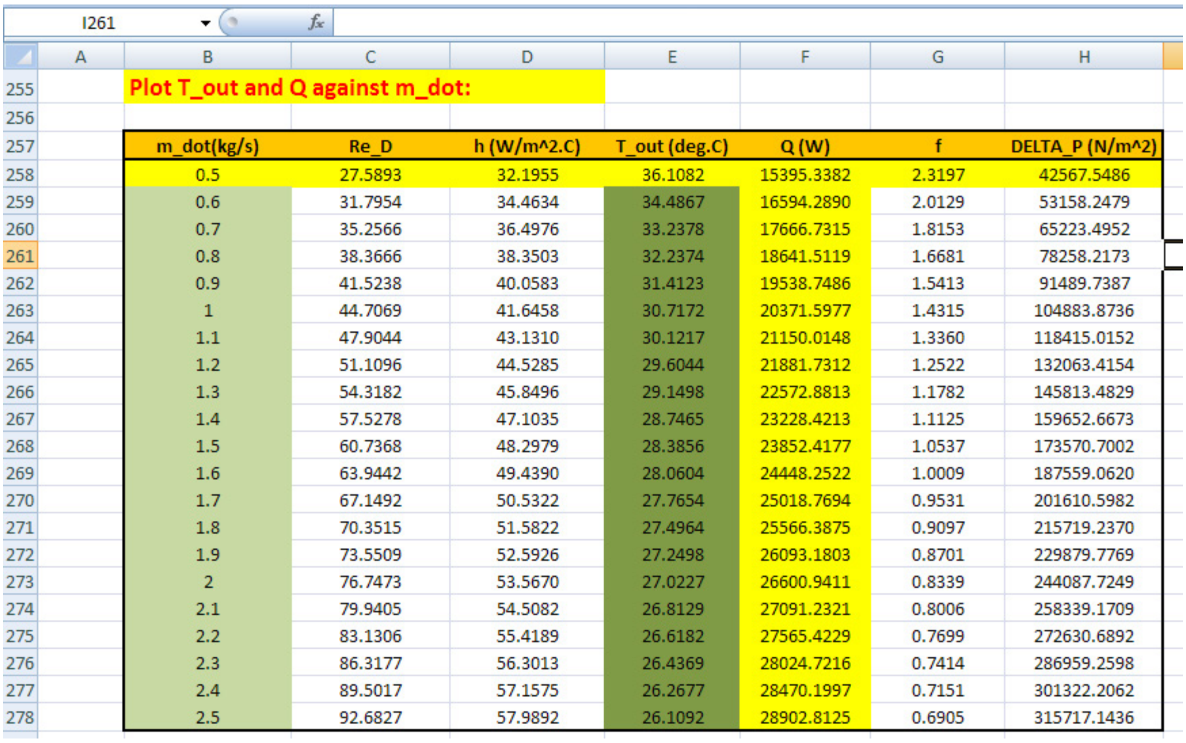
Lines 9 to 14: Immediately after Goal Seek is successfully completed, all the quantities would have up dated themselves. So, Re_d, h, T_out, Q, f and DELTAP values are copied into the respective positions in the Table.

Line 15: continue to the next value of m_dot in the Table

Line 16: End statement of Sub.

---

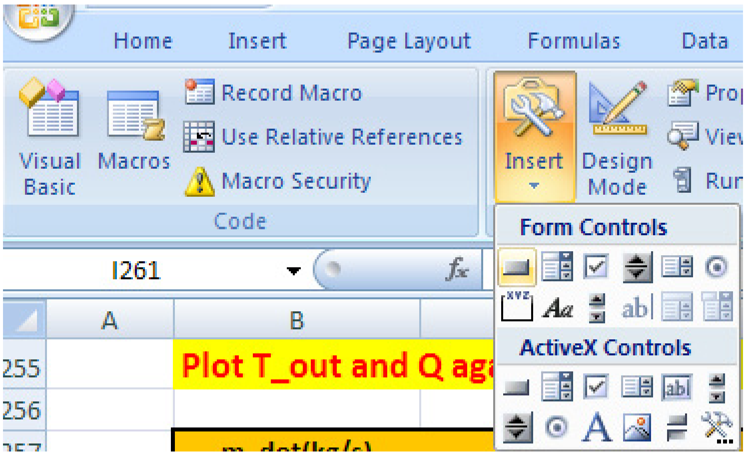
7. Now, pressing Ctrl+Shift+R will fill up the Table:



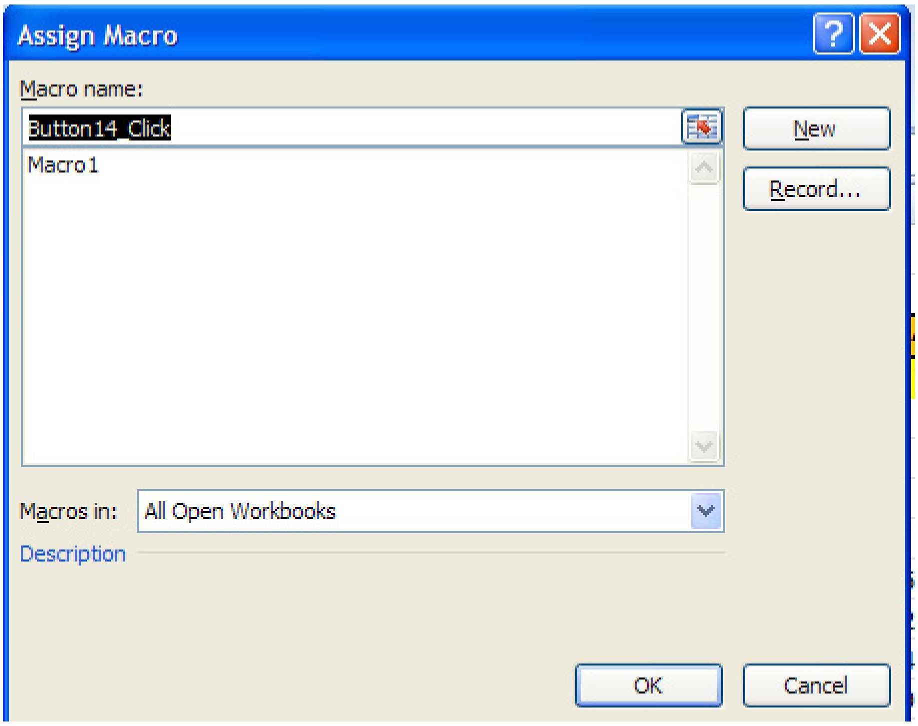
Plot T_out and Q against m_dot:

m_dot(kg/s)	Re_D	h (W/m^2.C)	T_out (deg.C)	Q (W)	f	DELTA_P (N/m^2)
0.5	27.5893	32.1955	36.1082	15395.3382	2.3197	42567.5486
0.6	31.7954	34.4634	34.4867	16594.2890	2.0129	53158.2479
0.7	35.2566	36.4976	33.2378	17666.7315	1.8153	65223.4952
0.8	38.3666	38.3503	32.2374	18641.5119	1.6681	78258.2173
0.9	41.5238	40.0583	31.4123	19538.7486	1.5413	91489.7387
1	44.7069	41.6458	30.7172	20371.5977	1.4315	104883.8736
1.1	47.9044	43.1310	30.1217	21150.0148	1.3360	118415.0152
1.2	51.1096	44.5285	29.6044	21881.7312	1.2522	132063.4154
1.3	54.3182	45.8496	29.1498	22572.8813	1.1782	145813.4829
1.4	57.5278	47.1035	28.7465	23228.4213	1.1125	159652.6673
1.5	60.7368	48.2979	28.3856	23852.4177	1.0537	173570.7002
1.6	63.9442	49.4390	28.0604	24448.2522	1.0009	187559.0620
1.7	67.1492	50.5322	27.7654	25018.7694	0.9531	201610.5982
1.8	70.3515	51.5822	27.4964	25566.3875	0.9097	215719.2370
1.9	73.5509	52.5926	27.2498	26093.1803	0.8701	229879.7769
2	76.7473	53.5670	27.0227	26600.9411	0.8339	244087.7249
2.1	79.9405	54.5082	26.8129	27091.2321	0.8006	258339.1709
2.2	83.1306	55.4189	26.6182	27565.4229	0.7699	272630.6892
2.3	86.3177	56.3013	26.4369	28024.7216	0.7414	286959.2598
2.4	89.5017	57.1575	26.2677	28470.1997	0.7151	301322.2062
2.5	92.6827	57.9892	26.1092	28902.8125	0.6905	315717.1436

8. It is convenient to have a Form control button to do this job. So, we connect this Macro program to a Form control button. To do this, go to: Developer_Insert_Form controls.



Click on first button in Form controls, and draw it to a suitable size on the worksheet. Immediately, following pop up appears:



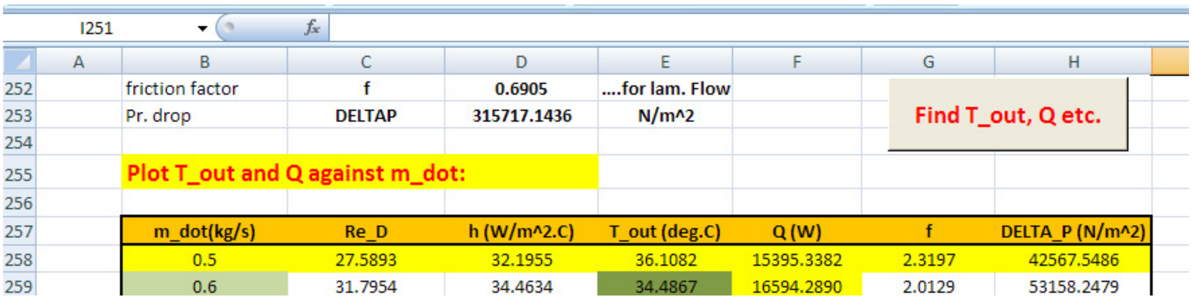
Macro name: Button14_Click

Macro1

Macros in: All Open Workbooks

Select Macro1 and click OK. Now, the Macro is connected to the button.

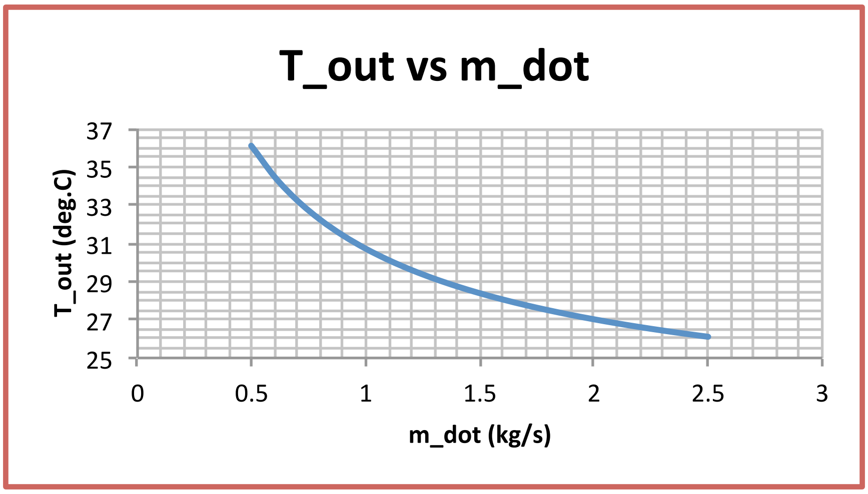
Right click on the button and click Format Control and format the Text in the button:

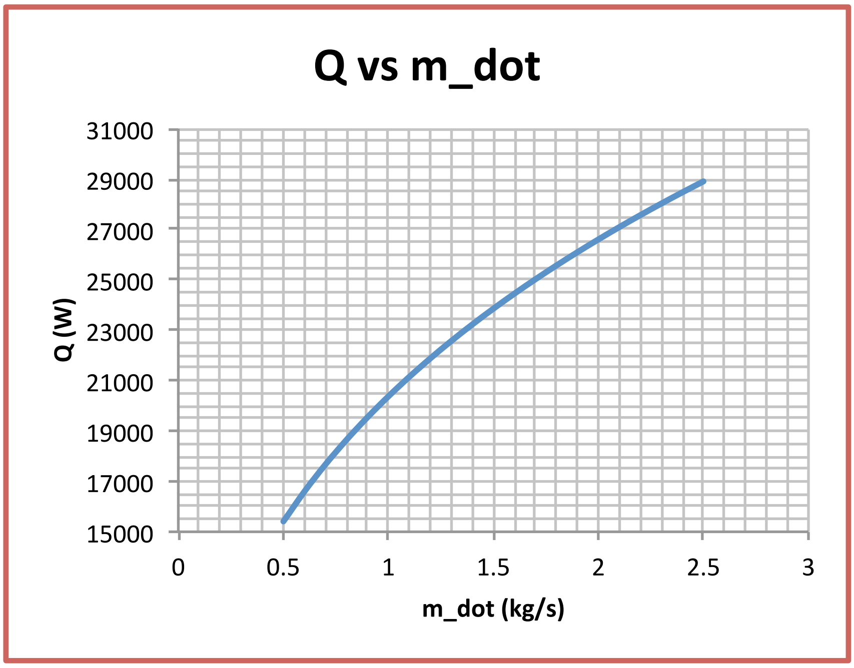


	A	B	C	D	E	F	G	H
252		friction factor	f	0.6905	....for lam. Flow			
253		Pr. drop	DELTAP	315717.1436	N/m^2			
254								
255		Plot T_out and Q against m_dot:						
256								
257		m_dot(kg/s)	Re_D	h (W/m^2.C)	T_out (deg.C)	Q (W)	f	DELTA_P (N/m^2)
258		0.5	27.5893	32.1955	36.1082	15395.3382	2.3197	42567.5486
259		0.6	31.7954	34.4634	34.4867	16594.2890	2.0129	53158.2479

Now, we can click on the button to operate the Macro and fill up the Table.

9. Now, plot the graphs in EXCEL:





Prob.2A1.4.23. Water at 50 C enters a 1.5 cm dia heat exchanger tube with a velocity of 1 m/s. The tube wall is maintained at a constant temp of 90 C. Calculate the length of the tube required if the exit temp of water is to be 65 C. Also, calculate the heat transfer rate. [M.Tech. – VTU – May/June 2010]

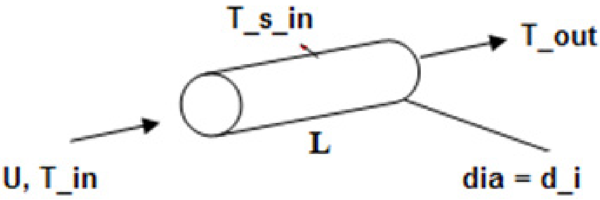


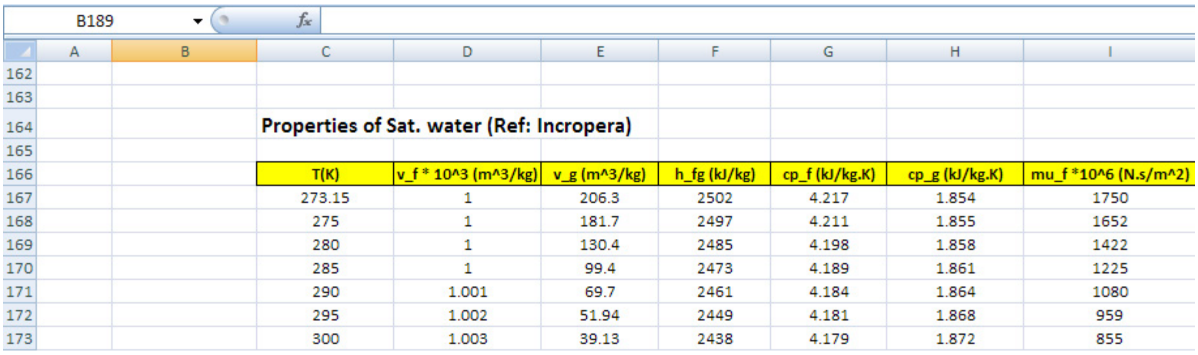
Fig.Prob.2A.1.4.23

EXCEL Solution:

Let us solve this problem with EXCEL.

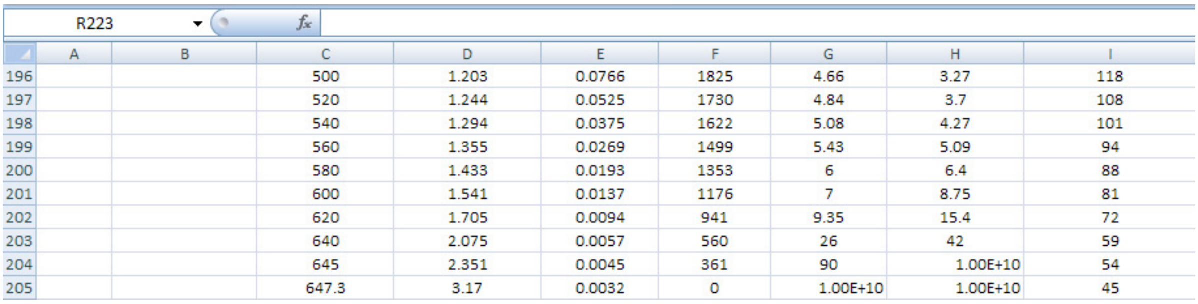
But, EXCEL does not have built-in Functions for properties of Water.

So, we shall use the data table given in Ref.[3] and interpolate for results by writing suitable VBA Functions. The data Table in EXCEL extends from T = 273.15 K to 647.3 K. It is shown partly below:



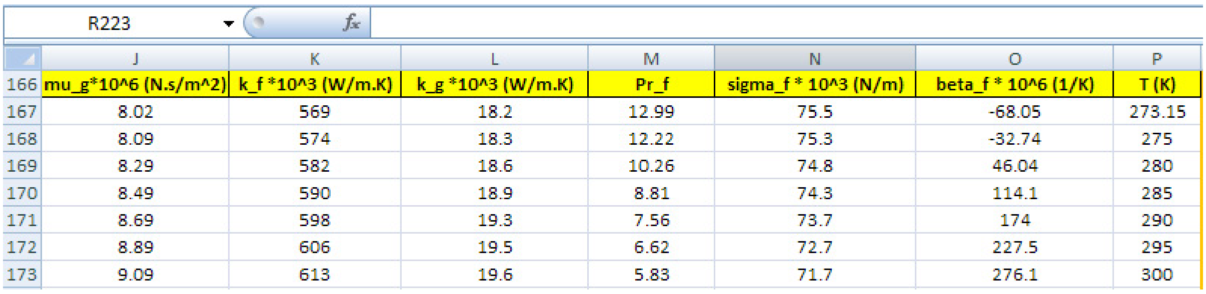
B189

	A	B	C	D	E	F	G	H	I
162									
163									
164			**Properties of Sat. water (Ref: Incropera)**						
165									
166			T(K)	v_f * 10^3 (m^3/kg)	v_g (m^3/kg)	h_fg (kJ/kg)	cp_f (kJ/kg.K)	cp_g (kJ/kg.K)	mu_f *10^6 (N.s/m^2)
167			273.15	1	206.3	2502	4.217	1.854	1750
168			275	1	181.7	2497	4.211	1.855	1652
169			280	1	130.4	2485	4.198	1.858	1422
170			285	1	99.4	2473	4.189	1.861	1225
171			290	1.001	69.7	2461	4.184	1.864	1080
172			295	1.002	51.94	2449	4.181	1.868	959
173			300	1.003	39.13	2438	4.179	1.872	855



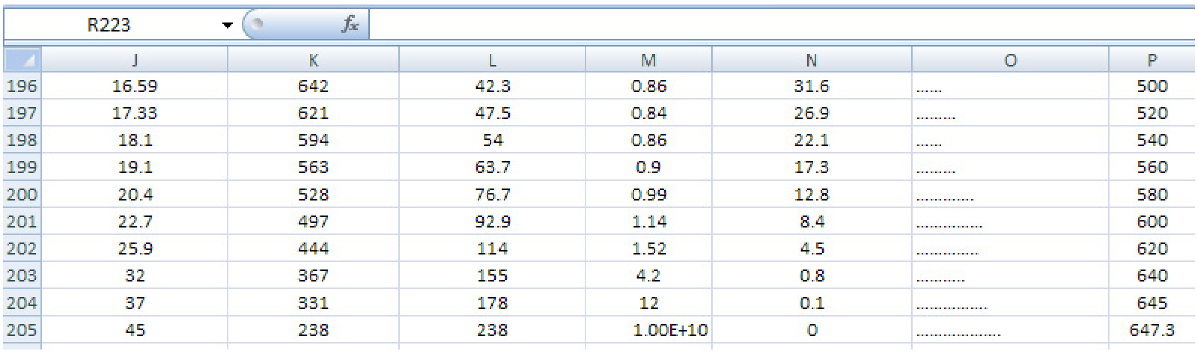
R223

	A	B	C	D	E	F	G	H	I
196			500	1.203	0.0766	1825	4.66	3.27	118
197			520	1.244	0.0525	1730	4.84	3.7	108
198			540	1.294	0.0375	1622	5.08	4.27	101
199			560	1.355	0.0269	1499	5.43	5.09	94
200			580	1.433	0.0193	1353	6	6.4	88
201			600	1.541	0.0137	1176	7	8.75	81
202			620	1.705	0.0094	941	9.35	15.4	72
203			640	2.075	0.0057	560	26	42	59
204			645	2.351	0.0045	361	90	1.00E+10	54
205			647.3	3.17	0.0032	0	1.00E+10	1.00E+10	45

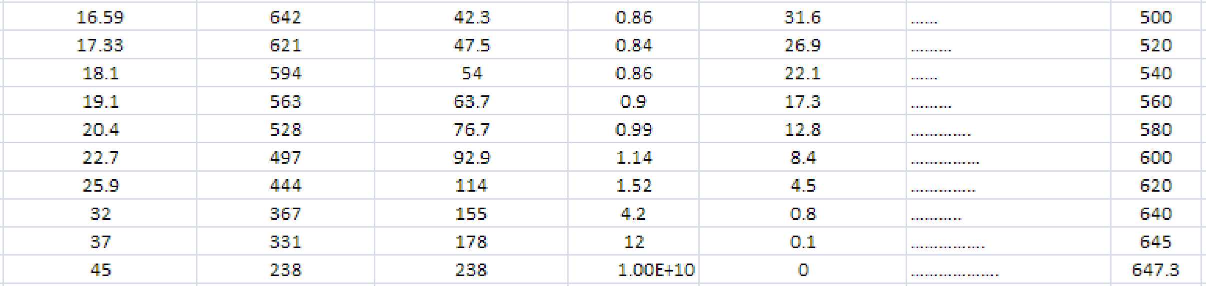


R223

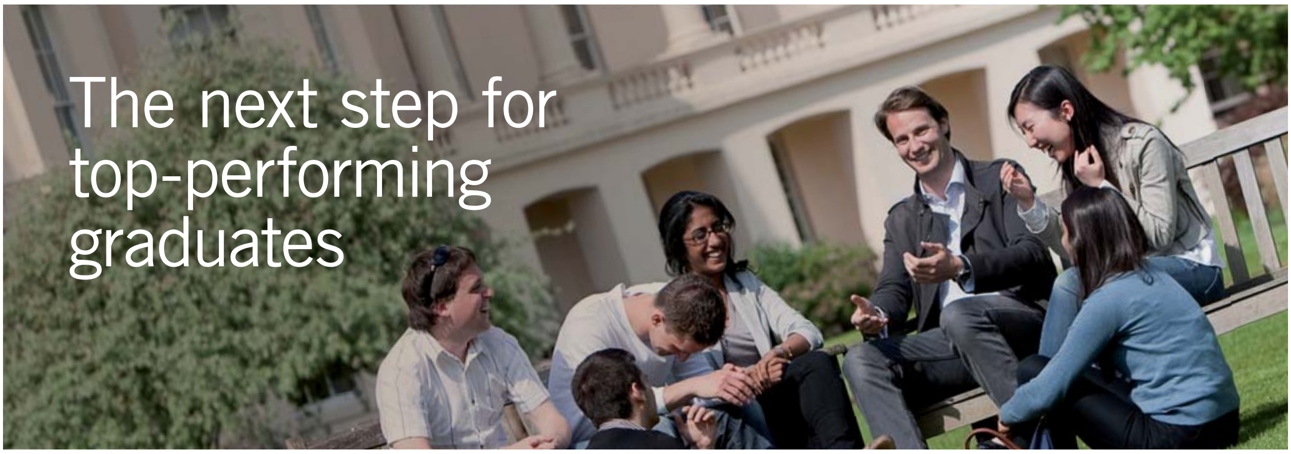
	J	K	L	M	N	O	P
166	mu_g*10^6 (N.s/m^2)	k_f *10^3 (W/m.K)	k_g *10^3 (W/m.K)	Pr_f	sigma_f * 10^3 (N/m)	beta_f * 10^6 (1/K)	T (K)
167	8.02	569	18.2	12.99	75.5	-68.05	273.15
168	8.09	574	18.3	12.22	75.3	-32.74	275
169	8.29	582	18.6	10.26	74.8	46.04	280
170	8.49	590	18.9	8.81	74.3	114.1	285
171	8.69	598	19.3	7.56	73.7	174	290
172	8.89	606	19.5	6.62	72.7	227.5	295
173	9.09	613	19.6	5.83	71.7	276.1	300



	J	K	L	M	N	O	P
196	16.59	642	42.3	0.86	31.6	......	500
197	17.33	621	47.5	0.84	26.9	....	520
198	18.1	594	54	0.86	22.1	......	540
199	19.1	563	63.7	0.9	17.3	....	560
200	20.4	528	76.7	0.99	12.8	....	580
201	22.7	497	92.9	1.14	8.4	....	600
202	25.9	444	114	1.52	4.5	....	620
203	32	367	155	4.2	0.8	....	640
204	37	331	178	12	0.1	....	645
205	45	238	238	1.00E+10	0	....	647.3



16.59	642	42.3	0.86	31.6	......	500
17.33	621	47.5	0.84	26.9	....	520
18.1	594	54	0.86	22.1	......	540
19.1	563	63.7	0.9	17.3	....	560
20.4	528	76.7	0.99	12.8	....	580
22.7	497	92.9	1.14	8.4	....	600
25.9	444	114	1.52	4.5	....	620
32	367	155	4.2	0.8	....	640
37	331	178	12	0.1	....	645
45	238	238	1.00E+10	0	....	647.3

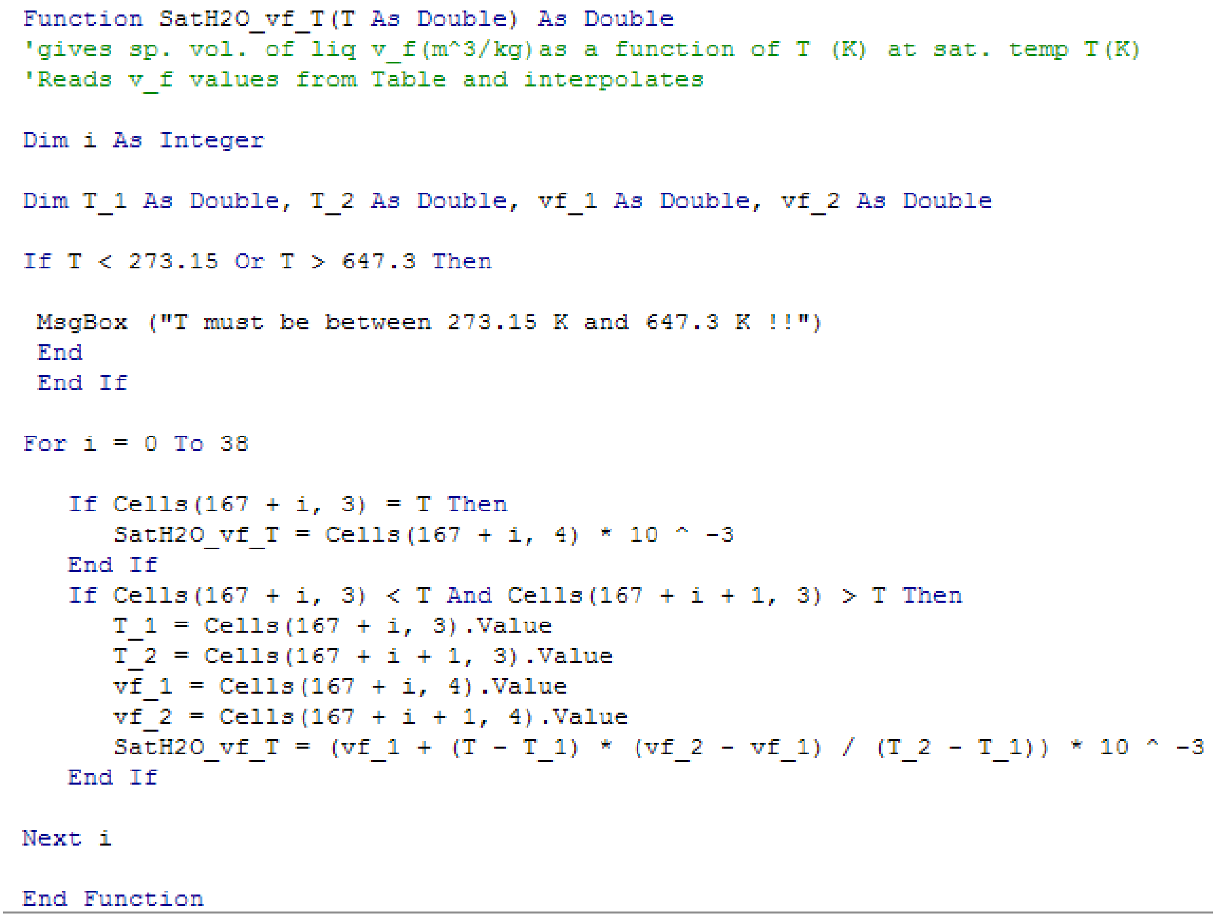


Now, for a given value of T (K), the program should locate values of Temps just above and just below T, and interpolate the required property from this Table.

We give below one example of the Function to find sp. volume of liquid, 'vf' for a given T.

Remember: density = (1 / specific volume)

First, click on Developer-Visual Basic – Insert_module, and a blank page appears. Start typing the code there:

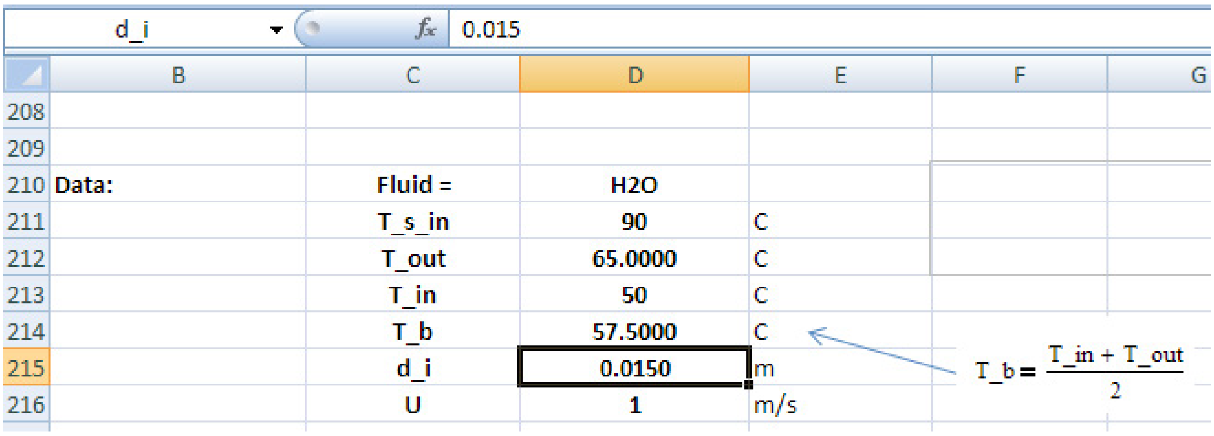


```
Function SatH2O_vf_T(T As Double) As Double
'gives sp. vol. of liq v_f(m^3/kg)as a function of T (K) at sat. temp T(K)
'Reads v_f values from Table and interpolates

Dim i As Integer

Dim T_1 As Double, T_2 As Double, vf_1 As Double, vf_2 As Double

If T < 273.15 Or T > 647.3 Then

 MsgBox ("T must be between 273.15 K and 647.3 K !!")
 End
 End If

For i = 0 To 38

   If Cells(167 + i, 3) = T Then
      SatH2O_vf_T = Cells(167 + i, 4) * 10 ^ -3
   End If
   If Cells(167 + i, 3) < T And Cells(167 + i + 1, 3) > T Then
      T_1 = Cells(167 + i, 3).Value
      T_2 = Cells(167 + i + 1, 3).Value
      vf_1 = Cells(167 + i, 4).Value
      vf_2 = Cells(167 + i + 1, 4).Value
      SatH2O_vf_T = (vf_1 + (T - T_1) * (vf_2 - vf_1) / (T_2 - T_1)) * 10 ^ -3
   End If

Next i

End Function
```

For detailed explanation of the code, see the previous example.

Now, let us proceed to solve the problem:

1. Set up the EXCEL worksheet, enter data and name the cells:

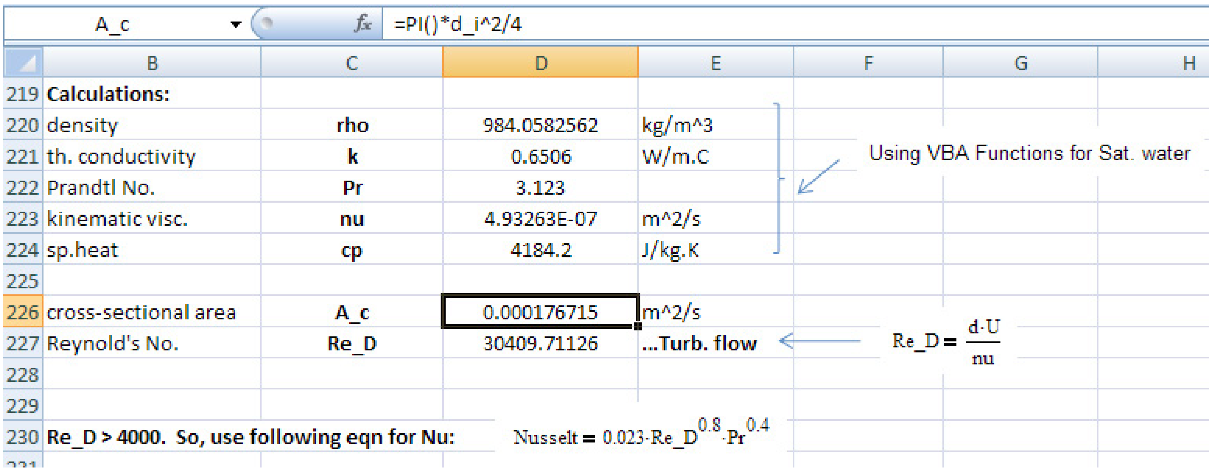


d_i | f_x | 0.015

	B	C	D	E
208				
209				
210	**Data:**	**Fluid =**	**H2O**	
211		**T_s_in**	**90**	C
212		**T_out**	**65.0000**	C
213		**T_in**	**50**	C
214		**T_b**	**57.5000**	C
215		**d_i**	**0.0150**	m
216		**U**	**1**	m/s

$$T\_b = \frac{T\_in + T\_out}{2}$$

2. Do the calculations. Eqns used are shown in the worksheet.



A_c | f_x | =PI()*d_i^2/4

	B	C	D	E
219	**Calculations:**			
220	density	**rho**	984.0582562	kg/m^3
221	th. conductivity	**k**	0.6506	W/m.C
222	Prandtl No.	**Pr**	3.123	
223	kinematic visc.	**nu**	4.93263E-07	m^2/s
224	sp.heat	**cp**	4184.2	J/kg.K
225				
226	cross-sectional area	**A_c**	0.000176715	m^2/s
227	Reynold's No.	**Re_D**	30409.71126	**...Turb. flow**
228				
229				
230	**Re_D > 4000. So, use following eqn for Nu:**			

Using VBA Functions for Sat. water

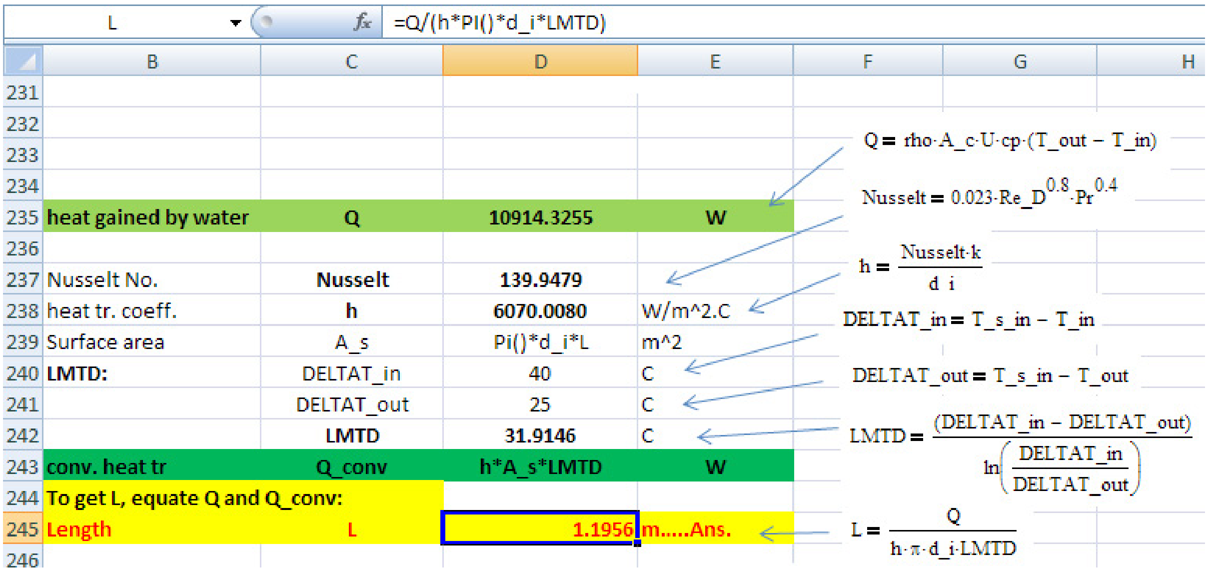
$$Re\_D = \frac{d \cdot U}{nu}$$

$$Nusselt = 0.023 \cdot Re\_D^{0.8} \cdot Pr^{0.4}$$

Note that properties of water are calculated using the VBA Functions already written.

For this problem, flow is turbulent. Reynold's No. is 24327.77 > 4000. So, use Dittus-Boelter eqn as shown.

Continuing the calculation: heat gained by water = heat transfer by forced convection from the fluid to the tube surface. To be accurate while calculating heat transfer by convection, we use LMTD for temp difference in the formula: Q_conv = h * A_s * LMTD. Here, A_s is the surface area for convection heat transfer = π * d_i * L, where L is the length of tube.



L =Q/(h*PI()*d_i*LMTD)

	B	C	D	E
231				
232				
233				
234				
235	heat gained by water	Q	10914.3255	W
236				
237	Nusselt No.	Nusselt	139.9479	
238	heat tr. coeff.	h	6070.0080	W/m^2.C
239	Surface area	A_s	Pi()*d_i*L	m^2
240	LMTD:	DELTAT_in	40	C
241		DELTAT_out	25	C
242		LMTD	31.9146	C
243	conv. heat tr	Q_conv	h*A_s*LMTD	W
244	To get L, equate Q and Q_conv:			
245	Length	L	1.1956	m.....Ans.
246				

$$\text{Q} = \text{rho}\cdot\text{A\_c}\cdot\text{U}\cdot\text{cp}\cdot(\text{T\_out} - \text{T\_in})$$

$$\text{Nusselt} = 0.023\cdot\text{Re\_D}^{0.8}\cdot\text{Pr}^{0.4}$$

$$\text{h} = \frac{\text{Nusselt}\cdot\text{k}}{\text{d\_i}}$$

$$\text{DELTAT\_in} = \text{T\_s\_in} - \text{T\_in}$$

$$\text{DELTAT\_out} = \text{T\_s\_in} - \text{T\_out}$$

$$\text{LMTD} = \frac{(\text{DELTAT\_in} - \text{DELTAT\_out})}{\ln\left(\frac{\text{DELTAT\_in}}{\text{DELTAT\_out}}\right)}$$

$$\text{L} = \frac{\text{Q}}{\text{h}\cdot\pi\cdot\text{d\_i}\cdot\text{LMTD}}$$

Thus, Length of tube required = 1.196 m … Ans.

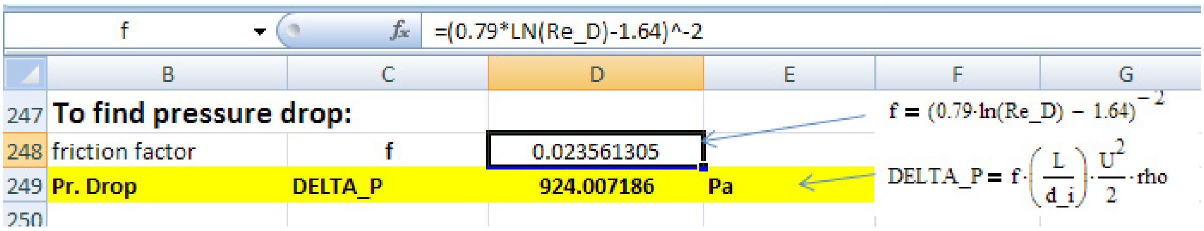
3. Find the pressure drop: First, find friction factor f, using the first Petukhov eqn. viz.

$$\text{f} = (0.79\cdot\ln(\text{Re\_D}) - 1.64)^{-2}$$

Then, find the pressure drop from the formula:

$$\text{DELTA\_P} = \text{f}\cdot\left(\frac{\text{L}}{\text{d\_i}}\right)\cdot\frac{\text{U}^2}{2}\cdot\text{rho}$$

Following part of worksheet shows this calculation:



f =(0.79*LN(Re_D)-1.64)^-2

	B	C	D	E
247	To find pressure drop:			
248	friction factor	f	0.023561305	
249	Pr. Drop	DELTA_P	924.007186	Pa
250				

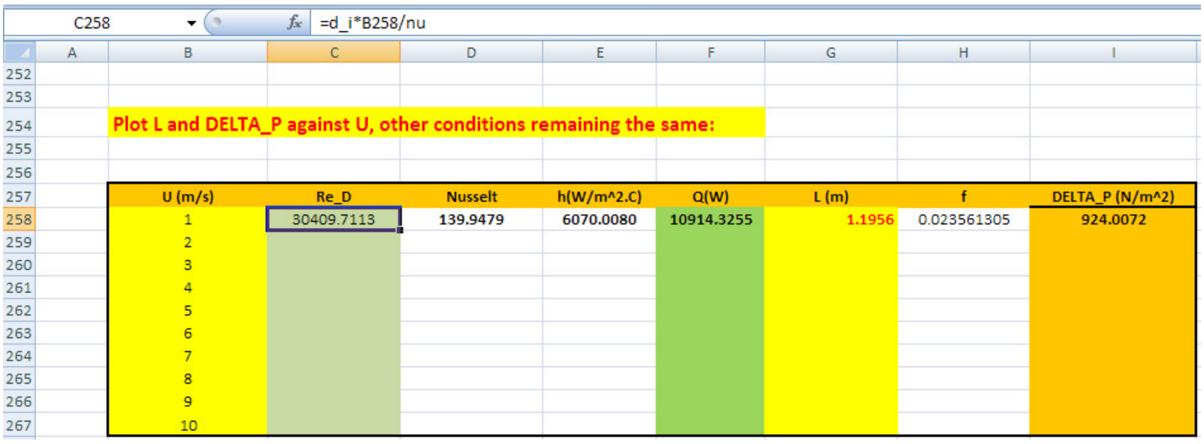
$$\text{f} = (0.79\cdot\ln(\text{Re\_D}) - 1.64)^{-2}$$

$$\text{DELTA\_P} = \text{f}\cdot\left(\frac{\text{L}}{\text{d\_i}}\right)\cdot\frac{\text{U}^2}{2}\cdot\text{rho}$$

So, pressure drop = 924.01 Pa …. Ans.

4. Now, plot the variation of h and DELTA_P as U varies from 1 to 10 m/s:

Remember that as U varies, Re_D will also vary; and therefore, Nusselts No. and heat transfer coeff. 'h' will also vary. However, properties of water will not change since T_in and T_out remain the same as earlier. Accordingly, set up the Table as follows:

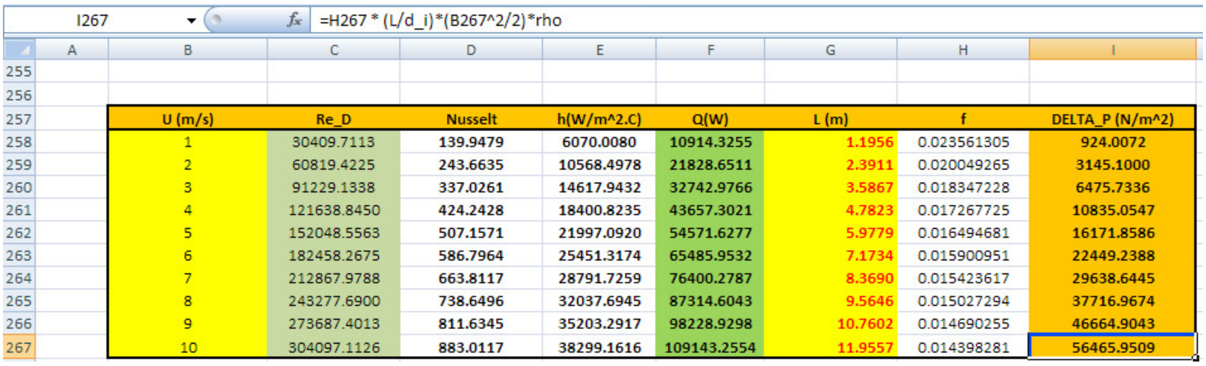


C258 =d_i*B258/nu

	A	B	C	D	E	F	G	H	I
252									
253									
254		Plot L and DELTA_P against U, other conditions remaining the same:							
255									
256									
257		U (m/s)	Re_D	Nusselt	h(W/m^2.C)	Q(W)	L (m)	f	DELTA_P (N/m^2)
258		1	30409.7113	139.9479	6070.0080	10914.3255	1.1956	0.023561305	924.0072
259		2							
260		3							
261		4							
262		5							
263		6							
264		7							
265		8							
266		9							
267		10							

In the above, note that U is the variable. Whilec calculating Re_D, take care to refer to U by 'relative reference' (i.e. to cell B258), so that while copying by drag-copy downwards, the cells adjust themselves. Similarly, refer to the changing cells in other formulas also by relative reference; i.e. for example, for Nusselts enter: ' =0.023*C258^0.8 * Pr^0.4' where Re_D is referred to by cell C258.

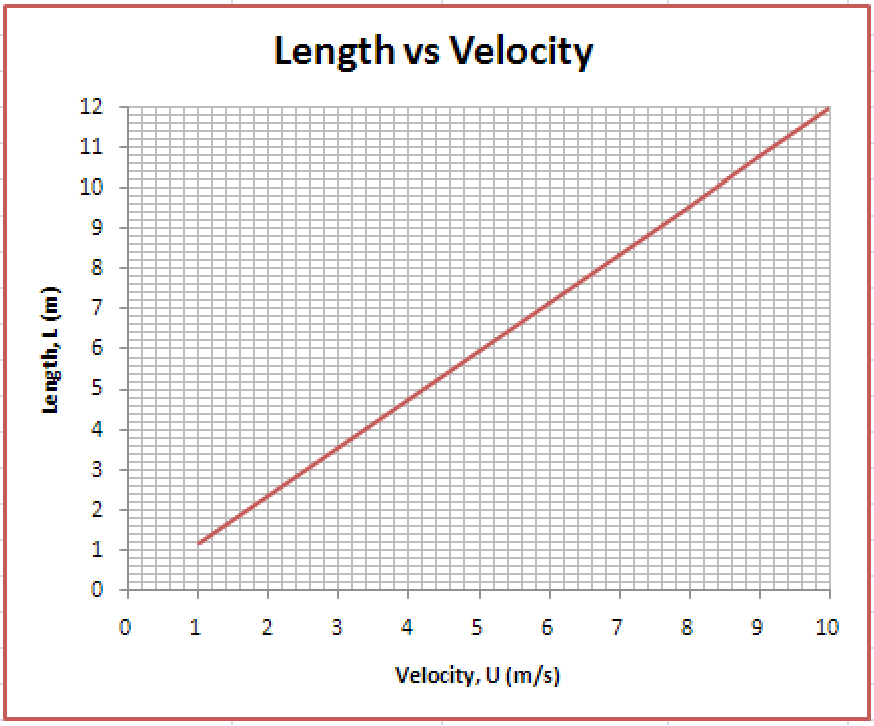
5. Now, select the cells C258 to I258 and drag-copy them up to the end of Table, and all calculations are immediately made:

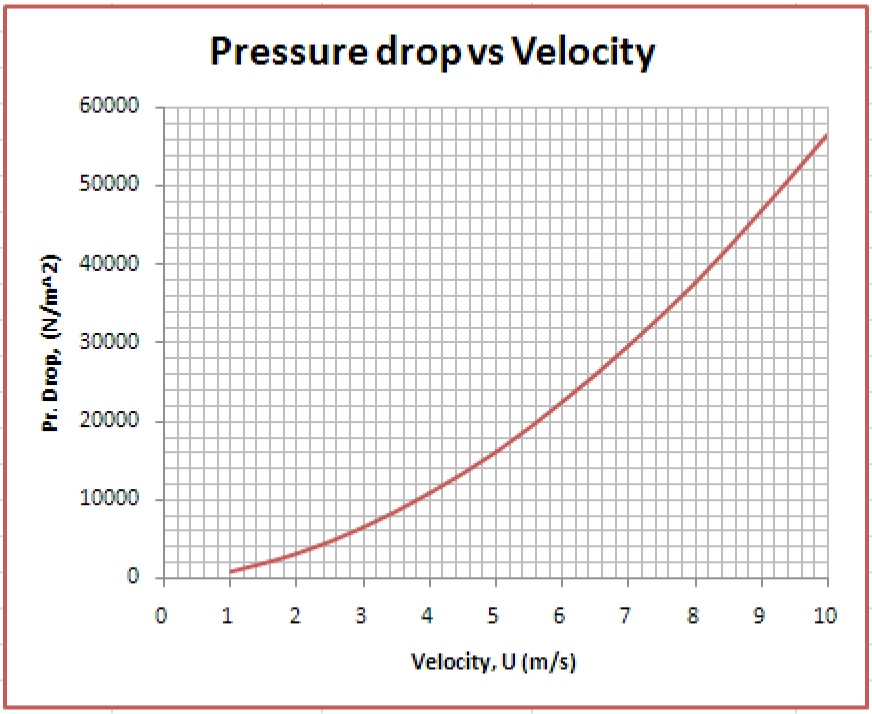


I267 =H267 * (L/d_i)*(B267^2/2)*rho

	A	B	C	D	E	F	G	H	I
255									
256									
257		U (m/s)	Re_D	Nusselt	h(W/m^2.C)	Q(W)	L (m)	f	DELTA_P (N/m^2)
258		1	30409.7113	139.9479	6070.0080	10914.3255	1.1956	0.023561305	924.0072
259		2	60819.4225	243.6635	10568.4978	21828.6511	2.3911	0.020049265	3145.1000
260		3	91229.1338	337.0261	14617.9432	32742.9766	3.5867	0.018347228	6475.7336
261		4	121638.8450	424.2428	18400.8235	43657.3021	4.7823	0.017267725	10835.0547
262		5	152048.5563	507.1571	21997.0920	54571.6277	5.9779	0.016494681	16171.8586
263		6	182458.2675	586.7964	25451.3174	65485.9532	7.1734	0.015900951	22449.2388
264		7	212867.9788	663.8117	28791.7259	76400.2787	8.3690	0.015423617	29638.6445
265		8	243277.6900	738.6496	32037.6945	87314.6043	9.5646	0.015027294	37716.9674
266		9	273687.4013	811.6345	35203.2917	98228.9298	10.7602	0.014690255	46664.9043
267		10	304097.1126	883.0117	38299.1616	109143.2554	11.9557	0.014398281	56465.9509

6. Now, plot the results in EXCEL:

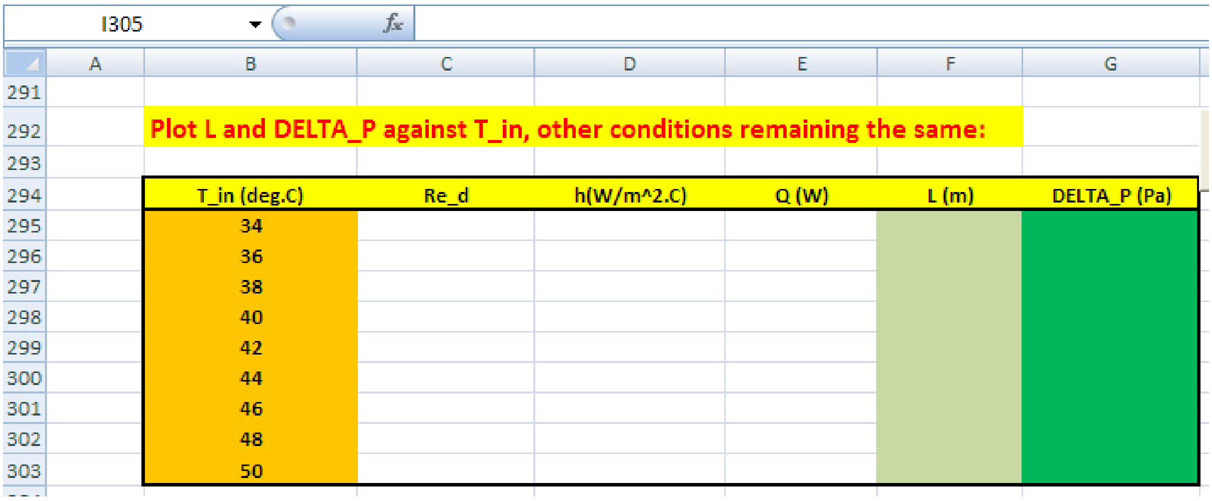




7. **Now, find the variation of L and DELTA_P as inlet temp of water changes from 34 deg.C to 50 deg.C, all other parameters remaining the same.**

Now, as T_in varies, T_b will vary, and all properties of water will also change. So, it is more convenient to write a simple VBA program that will take values of T_in one by one from the following Table, and copy it to cell D213. Then all other calculations in other cells will up-date themselves, and the VBA program should copy the required quantities from their respective cells in to the Table.

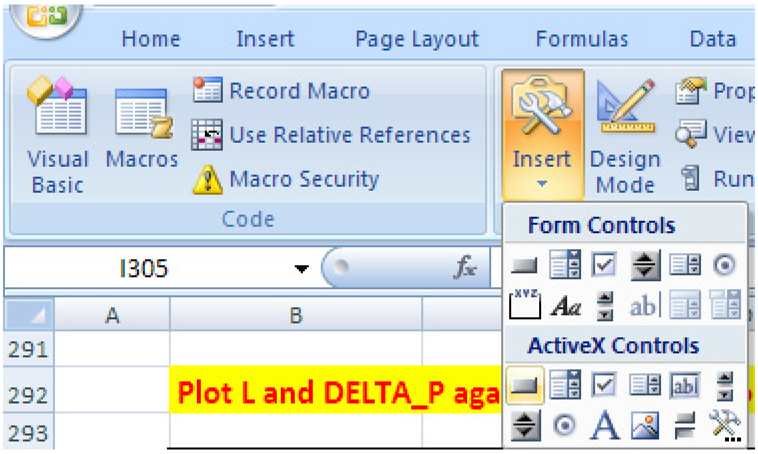
First, have the following scheme ready:



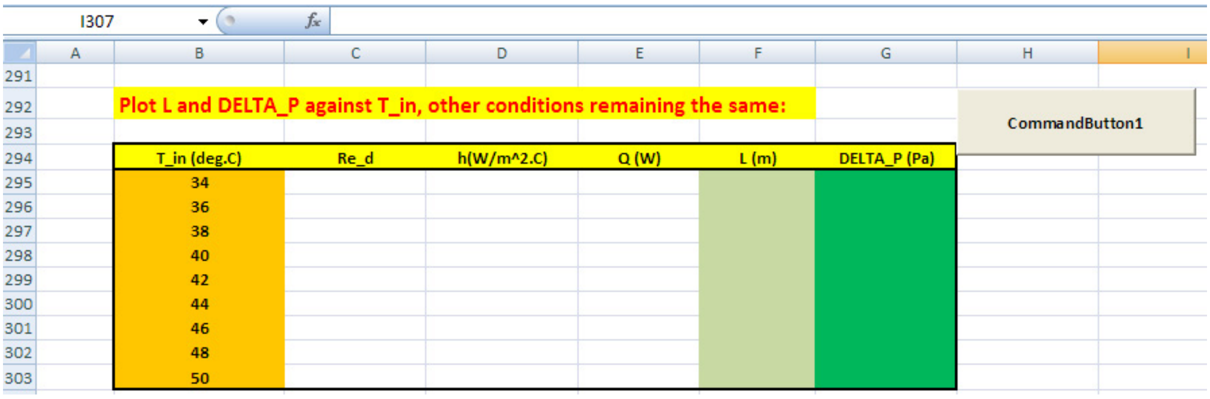
Plot L and DELTA_P against T_in, other conditions remaining the same:

T_in (deg.C)	Re_d	h(W/m^2.C)	Q (W)	L (m)	DELTA_P (Pa)
34					
36					
38					
40					
42					
44					
46					
48					
50					

8. Now, to write a VBA code that will be operated from a command button, go to: Developer-Insert-ActiveX controls-button:

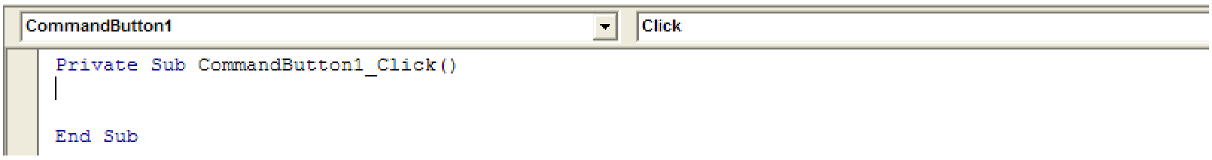


Click on the first, left button, and draw a Command Button1 in the worksheet, at the required location, to a suitable size, as shown below:



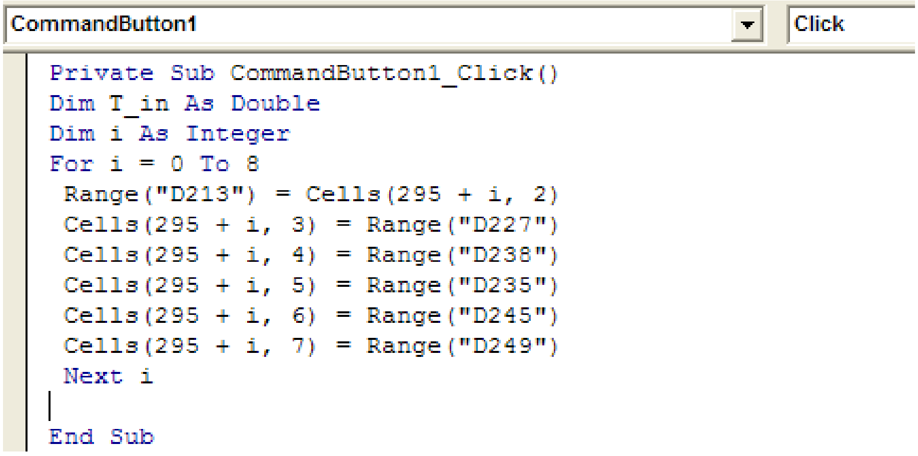
	A	B	C	D	E	F	G	H	I
291									
292		Plot L and DELTA_P against T_in, other conditions remaining the same:							
293									
294		T_in (deg.C)	Re_d	h(W/m^2.C)	Q (W)	L (m)	DELTA_P (Pa)		
295		34							
296		36							
297		38							
298		40							
299		42							
300		44							
301		46							
302		48							
303		50							

9. Then, click on Developer-View code. We get a partly written Sub …. End Sub construct for the command button, as shown:



```
Private Sub CommandButton1_Click()

End Sub
```

10. Now, complete the code to do the desired job as explained earlier in step 7:



```
Private Sub CommandButton1_Click()
Dim T_in As Double
Dim i As Integer
For i = 0 To 8
 Range("D213") = Cells(295 + i, 2)
 Cells(295 + i, 3) = Range("D227")
 Cells(295 + i, 4) = Range("D238")
 Cells(295 + i, 5) = Range("D235")
 Cells(295 + i, 6) = Range("D245")
 Cells(295 + i, 7) = Range("D249")
 Next i

End Sub
```

In this code:

Line 1: starts the Subroutine, with its name

Lines 2, 3: dimension statements for internal variables

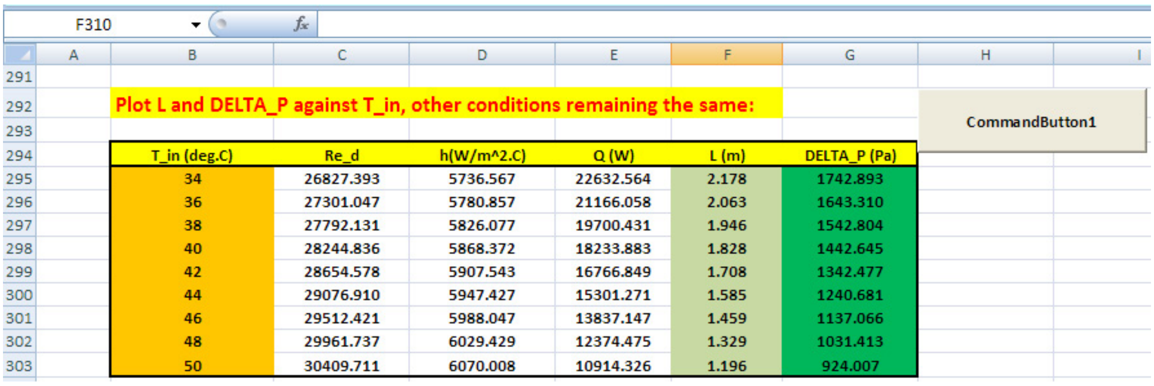
Line 4 to 11: For … Next loop

Line 5to 10: Cell D213 (i.e. temp T_in) in the worksheet is set equal to the first T_in value in the Table; Immediately, all other calculations are done and the up-dated values in respective cells for Re_D, h, Q, L and DELTA_P (i.e. cells D227, D238, D235, D245 and D249) are copied to their respective positions in the prepared Table, thus completing the first line.

Line 11: Next i : Now, repeat this procedure for the next value of T_in in the Table.

Line 12: End statement of Subroutine.

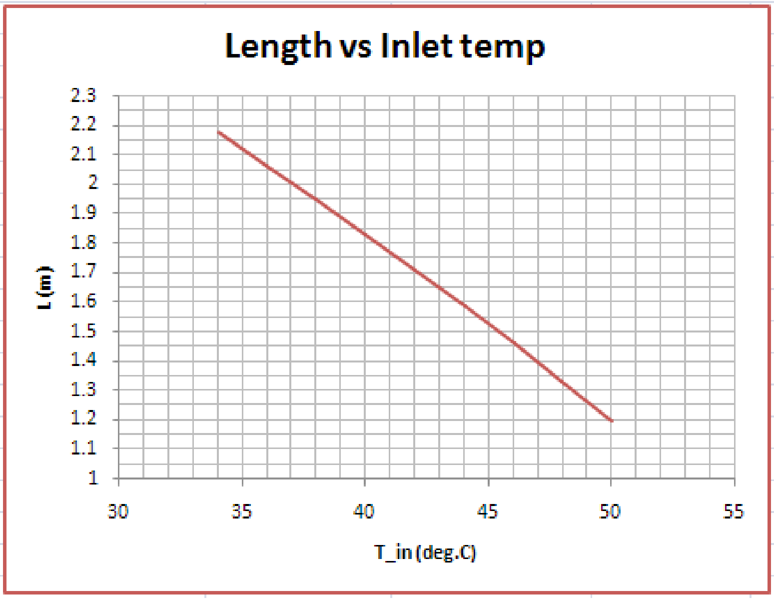
11. Now, click on the Command Button1, and immediately the Table gets filled up:

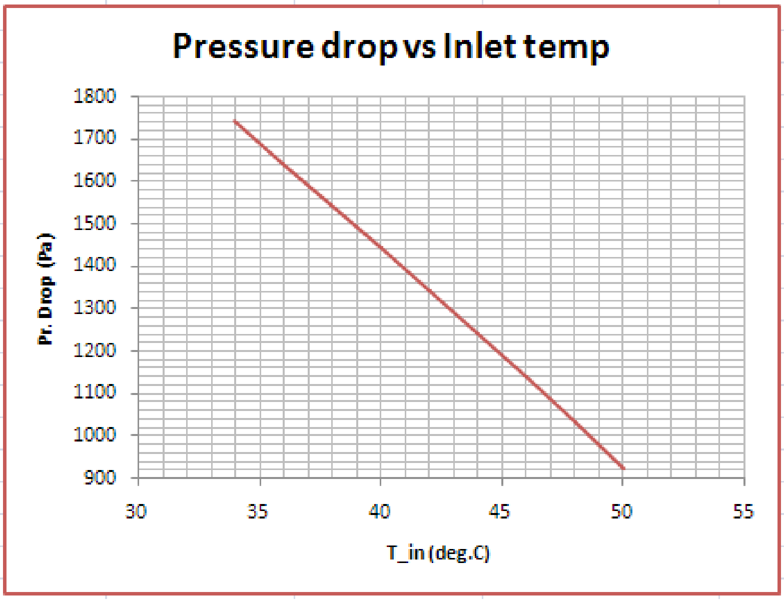


Plot L and DELTA_P against T_in, other conditions remaining the same:

T_in (deg.C)	Re_d	h(W/m^2.C)	Q (W)	L (m)	DELTA_P (Pa)
34	26827.393	5736.567	22632.564	2.178	1742.893
36	27301.047	5780.857	21166.058	2.063	1643.310
38	27792.131	5826.077	19700.431	1.946	1542.804
40	28244.836	5868.372	18233.883	1.828	1442.645
42	28654.578	5907.543	16766.849	1.708	1342.477
44	29076.910	5947.427	15301.271	1.585	1240.681
46	29512.421	5988.047	13837.147	1.459	1137.066
48	29961.737	6029.429	12374.475	1.329	1031.413
50	30409.711	6070.008	10914.326	1.196	924.007

12. Now, plot the results in EXCEL:




Pressure drop vs Inlet temp
Pr. Drop (Pa)
1800
1700
1600
1500
1400
1300
1200
1100
1000
900
30
35
40
45
50
55
T_in (deg.C)

===

References

1. M. Thirumaleshwar, *Fundamentals of Heat & Mass Transfer*, Pearson Education, India, 2006.
2. Yunus A Cengel, *Heat and Mass Transfer*, 3rd Ed., McGraw Hill Co.
3. F.P. Incropera and D.P.DeWitt, *Fundamentals of Heat and Mass Transfer*, 5th Ed., John Wiley & Sons.
4. Domkundwar et al, *A Course in Heat & Mass Transfer*, Dhanpat Rai & Co, 5th Ed, 1999.
5. Frank Kreith and Mark S Bohn, *Principles of Heat Transfer*, PWS Publ. Co. (Intl. Thomson Publ.), 5th Ed., 1997.