Software Solutions to Problems on Heat Transfer

Convection – Part II: Natural (or free) convection

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Contents

2A.2 Natural (Or, Free) Convection:

Learning objectives:

1. In 'Natural or free convection', fluid movement is caused because of density differences in the fluid due to temperature differences, under the influence of gravity. Density differences cause a 'buoyancy force' which in turn, causes the fluid circulation by 'convection currents'.
2. Obviously, fluid velocity in natural convection is low as compared to that in forced convection, and as a result, the heat transfer coefficient is also lower in the case of natural convection.
3. Still, natural convection is one of the important modes of heat transfer used in practice since there are no moving parts and as a result, there is an increased reliability.'
4. Natural convection heat transfer is extensively used in the following areas of engineering:
    1) cooling of transformers, transmission lines and rectifiers
    2) heating of houses by steam or electrical radiators
    3) heat loss from steam pipe lines in power plants and heat gain in refrigerant pipe lines in air-conditioning applications
    4) cooling of reactor core in nuclear power plants
    5) cooling of electronic devices (chips, transistors etc.) by finned heat sinks
5. We shall solve problems of Natural convection involving following geometries: Vertical plates and cylinders, horizontal plates, cylinders and spheres, different types of enclosed spaces, rotating cylinders, disks and spheres. We will also consider important problems of Natural convection from finned surfaces, and cases of combined Natural and Forced convection.
6. To give a few examples: Room heaters, Furnace doors etc are modeled as vertical plates in Natural convection. Steam pipes in plant rooms may be modeled as horizontal or vertical cylinders in Natural convection. Natural convection from Spherical storage tanks is quite common. Natural convection in inclined rectangular enclosures containing air has important relevance to Flat plate solar collectors. Natural convection from vertical/horizontal finned surfaces has important application in Natural convection cooling of Printed Circuit Boards.
7. Many problems are solved to illustrate these applications of Natural convection.

Equation Summary: [Ref: 1]

Summary of Basic equations for Natural convection:

Important correlations are summarized below:

Geometry	Correlation
Heated, vertical plate: Integral method:	**Temp. distribution:** $\frac{T - T_a}{T_s - T_a} = \left(1 - \frac{y}{\delta}\right)^2$ ….(10.11) Velocity distribution: $\frac{u}{u_x} = \frac{y}{\delta} \cdot \left(1 - \frac{y}{\delta}\right)^2$ ….(10.12) Max. velocity: $u_{max} = \frac{4}{27} \cdot u_x$ at y = δ/3…… ….(10.12, a) Mean velocity: $u_m = \frac{1}{12} \cdot u_x = \frac{27}{48} \cdot u_{max}$ ….(10.12, a) **Velocity function:** $u_x = 5.17 \cdot \nu \cdot (Pr + 0.952)^{-0.5} \cdot \left[\frac{\beta \cdot g \cdot (T_s - T_a)}{\nu^2}\right]^{0.5} \cdot x^{0.5}$ ….(10.16, a) Boundary layer thickness: $\frac{\delta}{x} = \frac{3.93 \cdot (0.952 + Pr)^{0.25}}{Gr_x^{0.25} \cdot Pr^{0.5}}$ ….(10.16, b) Total mass flow through the boundary: $m_{total} = 1.7 \cdot \rho \cdot \nu \cdot \left[\frac{Gr_L}{Pr^2 \cdot (Pr + 0.952)}\right]^{0.25}$ ….(10.18) **Avg. Nusselt number for laminar flow:** $Nu_{avg} = \frac{4}{3} \cdot Nu_L = \frac{0.667 \cdot Pr^{0.5} \cdot Gr_L^{0.25}}{(0.952 + Pr)^{0.25}}$ ….(10.20, b) **Avg. Nusselt number for turb. flow:** $Nu_{avg} = \frac{h_{avg} \cdot L}{k} = 0.0246 \left[\frac{Pr^{1.17} \cdot Gr_L}{1 + 0.495 \cdot Pr^{\frac{2}{3}}}\right]^{0.4}$ ..for turb. flow…(10.22)

Empirical relations:	
Vertical plate, **T_s = const.** For air and other gases:	Height L is the characteristic length. $Nu = 0.59 \cdot Ra^{\frac{1}{4}}$ ...$10^4 < Ra < 10^9$.......(10.23) $Nu = 0.13 \cdot Ra^{\frac{1}{3}}$ ...$10^9 < Ra < 10^{12}$.......(10.24)
For all Prandl numbers: 0 < Pr < ∞: 0.6 < Pr < ∞: 0 < Pr < 0.6: (Entire range of Ra)	$Nu = 0.68 + \frac{0.670 \cdot Ra^{\frac{1}{4}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}}$ ...$0 < Ra < 10^9$.......(10.25) $Nu = \frac{0.15 \cdot Ra^{\frac{1}{3}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{27}}}$ ... $Ra > 10^9$.......(10.26) $Nu = \left[0.825 + \frac{0.387 \cdot Ra^{\frac{1}{6}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right]^2$ ... $Ra > 10^9$.......(10.27)
Inclined plate, inclined at an angle θ to the vertical T_s = const.	Inclined height L is the characteristic length. Use vertical plate eqns. as a first approximation. Replace g by g.cos(θ).
Vertical cylinder	Height L is the characteristic length. Vertical cylinder can be treated as vertical plate, if the following relation is satisfied: $\frac{D}{L} \geq \frac{34}{Ra^{\frac{1}{4}}}$ ....(10.28)

Vertical plate, **q_s = const.**	**Eqns. (10.25) and (10.26) are still valid, with the modification that constant 0.492 is changed to 0.437.** **Alternatively:** **A modified Grashoff no. is defined:** $Gr' = Gr \cdot Nu_x = \frac{g \cdot \beta \cdot q_s \cdot x^4}{k \cdot \nu^2}$ ...(10.29) And following two relations for local Nusselt no.: $Nu_x = 0.60 \cdot (Gr' \cdot Pr)^{0.2}$ ..$10^5 < Gr'_x < 10^{11}$...(10.30) $Nu_x = 0.17 \cdot (Gr' \cdot Pr)^{0.25}$ ..$Gr'_x > 10^{11}$...(10.31) For Avg. Nu: $h = \frac{5}{4} \cdot h_L$ ...for laminar..(10.32) $h = h_L$ ....for turb.....(10.33)
Horizontal plate, **T_s= const.**	Characteristic Length: L_c = A/P **Upper surface of hot plate (or lower surface of cold plate):** $Nu = 0.54 \cdot Ra^{\frac{1}{4}}$ ...$10^4 < Ra < 10^7$.....(10.34) $Nu = 0.15 \cdot Ra^{\frac{1}{3}}$ ...$10^7 < Ra < 10^{11}$.....(10.35) Lower surface of hot plate (or upper surface of cold plate): $Nu = 0.27 \cdot Ra^{\frac{1}{4}}$ ...$10^5 < Ra < 10^{11}$.....(10.36)

Horizontal plate, **q_s= const.**	Characteristic Length: L_c = A/P All property values, except β, are evaluated at a temperature, T_e, defined by: $T_e = T_s - 0.25 \cdot (T_s - T_a)$ …(10.37) and, β is evaluated at T_a. Upper surface of hot plate (or lower surface of cold plate): $Nu = 0.13 \cdot Ra^{\frac{1}{3}}$ ...$Ra < 2 \times 10^8$..(10.39) $Nu = 0.16 \cdot Ra^{\frac{1}{3}}$ ... $2 \times 10^8 < Ra < 10^{11}$…..(10.40) For heated surface facing downward: $Nu = 0.58 \cdot Ra^{0.2}$ ... $10^6 < Ra < 10^{11}$…..(10.41)
Horizontal cylinder, **T_s= const.**	Dia. D is the characteristic length. For air: $Nu = C \cdot Ra^n$ …(10.42) C and n from Table 10.2. For (0 ≤ Pr ≤ ∞): $Nu = \left[0.60 + 0.387 \cdot \left[\frac{Ra}{\left[1 + \left(\frac{0.559}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{9}}}\right]^{\frac{1}{6}}\right]^2$ ....$10^{-5} < Ra < 10^{12}$.....(10.43). And, only for laminar range: $Nu = 0.36 + \frac{0.518 \cdot Ra^{\frac{1}{4}}}{\left[1 + \left(\frac{0.559}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}}$ ....$10^{-6} < Ra < 10^9$.....(10.44).

For thin wires: **(D = 0.2 to 1 mm)**	$Nu_D = 1.18 \cdot (Ra_D)^{\frac{1}{8}}$ .... Ra < 500.....(10.45)
From horizontal cylinders to liquid metals:	$Nu_D = 0.53 \cdot (Gr_D \cdot Pr^2)^{\frac{1}{4}}$ ....(10.46)
Spheres:	Dia. D is the characteristic length. $Nu = 2 + 0.43 \cdot (Ra)^{\frac{1}{4}}$ ...$1 < Ra < 10^5$, Pr = 1....(10.47) And, for higher range of Ra: $Nu = 2 + 0.50 \cdot (Ra)^{\frac{1}{4}}$ ...$3 \times 10^5 < Ra < 8 \times 10^8$.....(10.48)
Rectangular blocks:	Ch. Length: $L = \frac{L_H \cdot L_V}{L_H + L_V}$ .....(10.49, a) $Nu_L = 0.55 \cdot (Ra_L)^{\frac{1}{4}}$ ....$10^4 < Ra_L < 10^9$......(10.49)
Short cylinders **(D = H)**	$Nu = 0.775 \cdot (Ra)^{0.208}$ .....(10.50)
Simplified eqns. for air:	Refer to Table 10.3
Free convection in enclosed spaces: **For Horizontal enclosure:** For air: For liquids (water, silicone oils and mercury):	Space between the plates, 'b' is the characteristic dimension. $Gr_b = \frac{g \cdot \beta \cdot (T1 - T2) \cdot b^3}{\nu^2}$ .....(10.51) $Nu = 0.195 \cdot Gr^{\frac{1}{4}}$ ...$10^4 < Gr < 3.7 \times 10^5$.....(10.53) and, $Nu = 0.068 \cdot Gr^{\frac{1}{3}}$ ... $3.7 \times 10^5 < Gr < 10^7$.....(10.54) And, for Gr < 1700, we have Nu = 1. $Nu = 0.069 \cdot Ra^{\frac{1}{3}} \cdot Pr^{0.074}$ ... $1.5 \times 10^5 < Ra < 10^9$.....(10.55)

For Vertical enclosure: For Air: For fluids with Pr = 3 and 30,000 inside vertical enclosure: **Also:** For Vertical Enclosures: (Ref: Cengel)	For Gr (based on plate spacing 'b') < 1700, we have Nu = 1. $Nu = \frac{0.18 \cdot Gr^{\frac{1}{4}}}{\left(\frac{L}{b}\right)^{\frac{1}{9}}}$ ... $2 \times 10^4 < Gr < 2 \times 10^5$.....(10.56) and, $Nu = \frac{0.065 \cdot Gr^{\frac{1}{4}}}{\left(\frac{L}{b}\right)^{\frac{1}{9}}}$ ... $2 \times 10^5 < Gr < 10^7$.....(10.57) **Note** that for above two relations, aspect ratio, L/b > 3 . If L/b < 3, each vertical surface is treated independently. $Nu = 1$ ...for Ra < 1000......(10.58) and, $Nu = \frac{0.28 \cdot Ra^{\frac{1}{4}}}{\left(\frac{L}{b}\right)^{\frac{1}{4}}}$ ...for $1000 < Ra < 10^7$......(10.59) $Nu_b = 0.42 \cdot Ra_b^{\frac{1}{4}} \cdot Pr^{0.012} \cdot \left(\frac{L}{b}\right)^{-0.3}$ ...for 0 < L/b < 40, 1 < Pr < 2*10^4, 10^4 < Ra_b < 10^7) $Nu_b = 0.46 \cdot Ra_b^{\frac{1}{3}}$ ...for 1 < L/b < 40, 1 < Pr < 20 10^6 < Ra_b < 10^9

Free convection in inclined spaces: Flat plate solar collectors and double glazed windows)	**For (L/b > 12) and at tilt angles θ less than 70 deg.:** $$Nu_b = 1 + 1.44\cdot\left(1 - \frac{1708}{Ra_b\cdot\cos(\theta)}\right)\cdot\left(1 - \frac{1708\cdot\sin(1.8\cdot\theta)^{1.6}}{Ra_b\cdot\cos(\theta)}\right) + \left[\left(\frac{Ra_b\cdot\cos(\theta)}{5830}\right)^{\frac{1}{3}} - 1\right]$$ For L/b => 12, θ <=70 deg.: If the quantity in the first bracket and the last bracket is negative, then it must be set equal to zero. **When θ = 0, above eqn gives Nu_b for a Horizontal enclosure.** **For tilt angles between 70 deg. and 90 deg**. Catton recommends that the Nusselt number for a vertical enclosure (θ = 90 deg.) be multiplied by $(\sin\theta)^{1/4}$, i.e. $$Nu_b(\theta) = Nu_b(\theta = 90)\cdot\sin(\theta)^{\frac{1}{4}}$$ ...for 70 < θ <= 90 $$Nu_b = 1 + (Nu_b(\theta = 90) - 1)\cdot\sin\theta$$ ...for 90 < θ <= 180
Natural convection inside spherical cavities:	$$\frac{D\cdot h_{avg}}{k} = C\cdot(Gr_D\cdot Pr)^n$$ .....(10.62) For values of C and n, see table in text.
Concentric cylindrical annuli:	'b' is the gap or thickness of the enclosed fluid layer (i.e. b = $[D_o - D_i]/2$). $$\frac{Q}{L} = \frac{2\cdot\pi\cdot k_{eff}\cdot(T_i - T_o)}{\ln\left(\frac{D_o}{D_i}\right)} \qquad \frac{Q}{L} = \frac{2\cdot\pi\cdot k_{eff}\cdot(T_i - T_o)}{\ln\left(\frac{D_o}{D_i}\right)}$$ .....(10.63) $$\frac{k_{eff}}{k} = 0.386\cdot\left(\frac{Pr}{0.861 + Pr}\right)^{\frac{1}{4}}\cdot Ra_{cc}^{\frac{1}{4}}$$ ...100 < Ra_{cc} < 107.......(10.64) and, $$Ra_{cc} = \frac{\left(\ln\left(\frac{D_o}{D_i}\right)\right)^4\cdot Ra_b}{b^3\cdot\left[\frac{1}{D_i^{\frac{3}{5}}} + \frac{1}{D_o^{\frac{3}{5}}}\right]^5}$$ .....(10.65)

Concentric spherical annuli:	$$Q = \pi \cdot k_{eff} \left(\frac{D_i \cdot D_o}{b} \right) \cdot (T_i - T_o)$$ ......(10.66) $$\frac{k_{eff}}{k} = 0.74 \cdot \left(\frac{Pr}{0.861 + Pr} \right)^{\frac{1}{4}} \cdot Ra_{cs}^{\frac{1}{4}}$$ ...$10 < Ra_{cs} < 10^6$.......(10.67) and, $$Ra_{cs} = \frac{b \cdot Ra_b}{D_o^4 \cdot D_i^4 \cdot \left[\frac{1}{D_i^{\frac{7}{5}}} + \frac{1}{D_o^{\frac{7}{5}}} \right]^5}$$ ....(10.68)
Cooling of turbine blades: **(hole dia. D, hole length, L)**	$$Gr_L = \frac{(r_m \cdot \omega^2) \cdot \beta \cdot \Delta T \cdot L^3}{\nu^2}$$ Mostly, $Gr_L > 10^{12}$, and we use: $$Nu_a = \frac{h_a \cdot L}{k} = 0.0246 \left[\frac{Pr^{1.17} \cdot Gr_L}{1 + 0.495 \cdot Pr^{\frac{2}{3}}} \right]^{0.4}$$ .....(10.69) Total heat transferred: $$Q = h_a \cdot (\pi \cdot d \cdot L) \cdot (T_s - T_a)$$ .....(10.70)
Rotating cylinders:	Peripheral-speed Reynolds number: $$Re_\omega = \frac{\pi \cdot D^2 \cdot \omega}{\nu}$$ ......(10.71) For ($Re_\omega > 8000$ in air): Avg. Nusselt number: $$Nu_D = \frac{h_c \cdot D}{k} = 0.11 \cdot \left(0.5 \cdot Re_\omega^2 + Gr_D \cdot Pr \right)^{0.35}$$ .....(10.72)

Rotating disk:	For laminar regime, avg. Nu for a disk rotating in air: $$Nu_D = \frac{h_a \cdot D}{k} = 0.36 \cdot \left(\frac{\omega \cdot D^2}{\nu}\right)^{\frac{1}{2}}$$ ....for $\omega.D^2/\nu < 10^6$......(10.73) For laminar flow between r = 0 and r = r_c, and turbulent flow between r = r_c and r = r_0, average value of Nusselt number is given by: $$Nu_r = \frac{h_c \cdot r_o}{k} = 0.36 \cdot \left(\frac{\omega \cdot r_o^2}{\nu}\right)^{\frac{1}{2}} \cdot \left(\frac{r_c}{r_o}\right)^2 + 0.015 \cdot \left(\frac{\omega \cdot r_o^2}{\nu}\right)^{0.8} \cdot \left[1 - \left(\frac{r_c}{r_o}\right)^{2.6}\right]$$ ....for $r_c < r_o$......(10.75)
Rotating sphere:	For Pr > 0.7, in laminar flow regime, (i.e. $Re_\omega = \omega.D^2/\nu < 5 \times 10^4$), avg. Nusselt number is given by: $$Nu_D = 0.43 \cdot Re_\omega^{0.5} \cdot Pr^{0.4}$$ ...$Re_\omega < 5 \times 10^4$.....(10.76) and, $$Nu_D = 0.066 \cdot Re_\omega^{0.67} \cdot Pr^{0.4}$$ ... $5 \times 10^4 < Re_\omega < 7 \times 10^5$.....(10.77)
Rectangular fins on a vertical surface: **See Fig. 10.6.**	Optimum fin spacing: $$S_{opt} = 2.714 \cdot \frac{L}{Ra^{\frac{1}{4}}}$$ ......(10.78) $$h = 1.31 \cdot \frac{k}{S_{opt}}$$ .....(10.79) Rate of heat transfer: $$Q = h \cdot (2 \cdot n \cdot L \cdot H) \cdot (T_s - T_a)$$ .....(10.80)

<table>
<tr><td>Rectangular fins on a horizontal surface:

See Fig. 10.7</td><td>For fins facing upwards for $T_s > T_a$ (or facing downward for $T_s < T_a$):

$$Nu_s = \left[\left(\frac{1500}{Ra_s}\right)^2 + \left(0.081 \cdot Ra_s^{0.39}\right)^{-2}\right]^{\frac{-1}{2}}$$ ....(10.81)

Above eqn. is valid over the range:

$200 < Ra_s < 6 \times 10^5$, Pr = 0.71, 0.026 < H/W < 0.19, and 0.016 < S/W < 0.20, with the following definitions:

$$Nu_s = \frac{q \cdot S}{(T_s - T_a) \cdot k}$$

and,

$$Ra_s = \frac{g \cdot \beta \cdot (T_s - T_a) \cdot S^3}{\nu \cdot \alpha}$$</td></tr>
<tr><td>Combined Natural and Forced Convection</td><td>$Gr_L/(Re_L^2) << 1$ ….forced convection regime (negligible free convection)

$Gr_L/(Re_L^2) >> 1$ ….free convection regime (negligible forced convection)

$Gr_L/(Re_L^2) \approx 1$ ….mixed convection regime (both free and forced convection are important)

In the mixed convection regime, following eqn. is used to calculate the Nusselt number:

$Nu^m = Nu_{forced}^m \pm Nu_{free}^m$ …..(10.89)

A value of m = 3 is generally recommended. Positive or negative sign is taken if the free convection flow occurs in the same or opposite direction to that of forced convection.</td></tr>
</table>

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2A2.1 Natural convection from vertical plates and cylinders:

$UnitSystem SI Pa C J

"**Prob. 2A2.1.1.** Considering the body of a man as a vertical cylinder of 300 mm dia and 170 cm height, calculate the heat generated by the body in one day. Take the body temp as 36 C and atmospheric temp as 14 C. [VTU – June/July 2011:]"

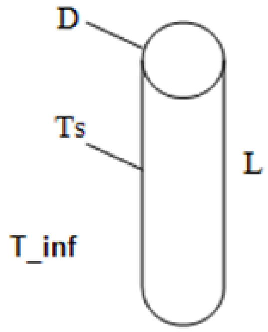


Fig. Prob.2A2.1.1

EES Solution:

This is the case of Natural convection from a vertical cylinder. This type of problem is often asked in the University exams.

So, let us write a EES PROCEDURE, so that we can call the PROCEDURE from the main program:

```
PROCEDURE NC_VertCyl_Air(T_s, T_inf, L, D : Gr_L, Ra_L, Nusselt_L_bar, h_bar, Q)

"Nat. convection (NC) on Vertical Cylinder, with Air"
"Inputs: T_s (C), T_inf (C), L (m), D (m)"
"Outputs: Gr_L, Ra_L, Nusselt_L_bar, h_bar (W/m^2-C), Q (W)"

T_f := (T_s+T_inf)/2
beta := 1/(T_f+273)
g := 9.81[m/s^2]
mu:=Viscosity(Air,T=T_f)
rho:=Density(Air,T=T_f,P=1.01325e05)
nu := mu/rho
cp:=Cp(Air,T=T_f)
Gr_L := g*beta*(T_s-T_inf)*L^3/nu^2
k:=Conductivity(Air,T=T_f)

Pr := mu*cp/k
Ra_L := Gr_L*Pr

"Verify:"
factor := 35*L/Gr_L^0.25 " Now if D > factor, OK to apply vertical plate eqn."

IF (D < factor) THEN Call WARNING ('Solution may not be accurate, since D < factor. factor 35*L/
Gr_L^0.25 = XXXA1', factor)

"Calculate Nu:"
A := 0.387*Ra_L^(1/6)
B := (0.492/Pr)^(9/16)
C := (1+B)^(8/27)
Nusselt_L_bar:= (0.825 + A/C)^2 "Finds Nu"
h_bar := Nusselt_L_bar*k/L "Finds h"
"Therefore:"
Q := h_bar * (pi*D*L)*(T_s – T_inf) "W"

END
"=================================================================="
```

"Data:"

```
D = 0.3[m]
L = 1.7[m]
T_s = 36[C]
T_inf = 14[C]
```

CALL NC_VertCyl_Air(T_s, T_inf, L, D : Gr_L, Ra_L, Nusselt_L_bar, h_bar, Q)

Qperday = Q * 3600 * 24 "Joules"

==

Results: Main:

Main | NC_VertCyl_Air

Unit Settings: SI C Pa J mass deg

D = 0.3 [m]	Gr_L = 1.459E+10	$\bar{h}$ = 3.873 [W/m^2-C]
L = 1.7 [m]	$Nusselt_{L,bar}$ = 258.1	Q = 136.5 [W]
Qperday = 1.179E+07 [J]	Ra_L = 1.063E+10	T_{inf} = 14 [C]
T_s = 36 [C]		

Results: PROCEDURE:

Main | NC_VertCyl_Air

Local variables in Procedure NC_VertCyl_Air (1 call, 0.03 sec)

A =18.15	B =0.8022	β=0.003356 [1/K]
C =1.191	cp=1005 [J/kg-C]	D =0.3 [m]
factor =0.1712	g=9.81 [m/s^2]	Gr_L=1.459E+10
$\bar{h}$=3.873 [W/m^2-C]	k=0.02551 [W/m-C]	L=1.7 [m]
μ=0.00001849 [kg/m-s]	ν=0.00001561 [m^2/s]	$Nusselt_{L,bar}$=258.1
Pr=0.7281	Q =136.5 [W]	Ra_L =1.063E+10
ρ=1.184 [kg/m^3]	T_f=25 [C]	T_{inf}=14 [C]
T_s=36 [C]		

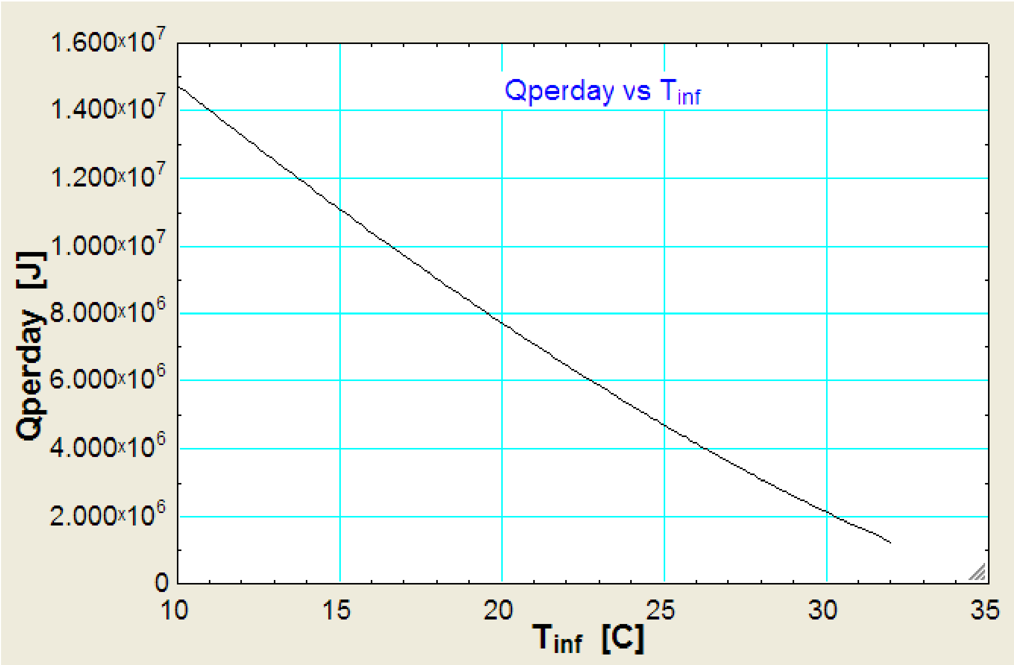
Thus:

Heat generated per day = Qperday = 1.179E07 J ….. Ans.

Plot Qperday as atmospheric temp varies from 10 C to 32 C:

Table 1

1..12	T_{inf} [C]	Qperday [J]	h [W/m2-C]
Run 1	10	1.476E+07	4.1
Run 2	12	1.325E+07	3.989
Run 3	14	1.179E+07	3.873
Run 4	16	1.038E+07	3.75
Run 5	18	9.021E+06	3.62
Run 6	20	7.711E+06	3.481
Run 7	22	6.456E+06	3.331
Run 8	24	5.262E+06	3.167
Run 9	26	4.133E+06	2.985
Run 10	28	3.077E+06	2.778
Run 11	30	2.105E+06	2.535
Run 12	32	1.235E+06	2.23



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$UnitSystem SI Pa C J

"**Prob. 2A2.1.2.** A nuclear reactor with its core constructed of parallel vertical plates 2.2 m high and 1.4 m wide has been designed on free convection heating of liquid Bismuth. The max temp of plate surface is limited to 960 C while the lowest allowable temp of bismuth is 340 C. Calculate the max possible heat dissipation from both sides of each plate. Use the following correlation: Nu = 0.13 (Gr. Pr)^0.33. Take the following physical properties for bismuth: [VTU – May/June 2006]"

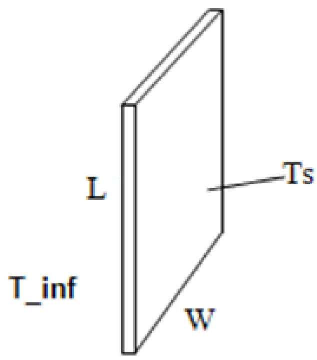

Fig. Prob.2A2.1.2

EES Solution:

"Data:"

W = 1.4[m]
L = 2.2[m]
T_s = 960[C]
T_inf = 340[C]
T_f = (T_s+T_inf)/2
g = 9.81[m/s^2]

"Properties of Bismuth at T_f:"

rho=10000[kg/m^3]
beta = 1.08e-03[1/K]
mu=3.66e-04[kg/m-s]
nu = mu/rho "[m^2 / s]"
cp=150.7[J/kg-C]
k= 13.02[W/m-C]

"Calculations:"

Gr = g * beta * (T_s-T_inf) * L^3 / nu^2 "…Grashoff No."
Pr = mu * cp /k "…Prandtl No."
Ra = Gr * Pr "…Rayleigh No."

Nusselt = 0.13 * Ra^0.33 "finds Nusselts No."
Nusselt = h * L / k "finds h"

Qperplate = 2 * h * (L * W) * (T_s – T_inf) "W …. heat tr from both sides of one plate"

Results:

Unit Settings: SI C Pa J mass deg

β = 0.00108 [1/K]	cp = 150.7 [J/kg-C]	g = 9.81 [m/s^2]
Gr = 5.221E+16	h = 41678 [W/m^2-C]	k = 13.02 [W/m-C]
L = 2.2 [m]	μ = 0.000366 [kg/m-s]	ν = 3.660E-08 [m^2/s]
Nusselt = 7042	Pr = 0.004236	Qperplate = 1.592E+08 [W]
Ra = 2.212E+14	ρ = 10000 [kg/m^3]	T_f = 650 [C]
T_{inf} = 340 [C]	T_s = 960 [C]	W = 1.4 [m]

Thus:

Max heat dissipation from each plate (both sides included) = Qperplate = 1.592E08 W … Ans.

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"**Prob. 2A2.1.3.** A tube of 0.036 m OD and 40 cm length is maintained at a uniform temp of 100 C. It is exposed to air at a temp of 20 C. Determine the rate of heat transfer from the surface of the tube when (a) the tube is vertical (b) the tube is horizontal. [VTU – June/July 2009]"

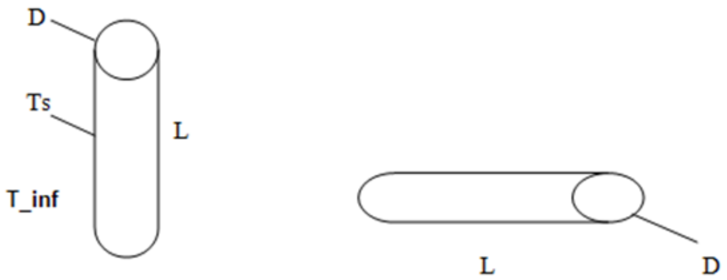


Fig. Prob.2A2.1.3 (a) and (b)

EES Solution:

"Data:"

D = 0.036[m]
L = 0.4[m]
T_s = 100[C]
T_inf = 20[C]
g = 9.81[m/s^2]

T_f = (T_s+T_inf)/2

"Calculations:"

beta = 1/(T_f+273)

"Properties of Air at T_f:"

mu=Viscosity(Air,T=T_f)"[kg/m-s]"
rho=Density(Air,T=T_f,P=1.01325E05) "[kg/m^3]"
nu = mu/rho "[m^2/s]"
cp=Cp(Air,T=T_f) "[J/kg-C]"
k=Conductivity(Air,T=T_f) "[W/m-C]"

Pr = mu * cp / k "…Prandtl No."
Ra_L = Gr_L * Pr "…Rayleigh No."

"Verify:"

factor = 35 * L / Gr_L^0.25 " = 0.0978; **Now D > factor, so, OK to apply vertical plate eqn.**"

"Case 1: cyl is vertical:"

Gr_L = g * beta * (T_s-T_inf) * L^3 / nu^2

"Calculate Nu:"

A = 0.387 * Ra_L^(1/6)
B = (0.492 / Pr)^(9/16)
C = (1+B)^(8/27)
Nusselt_vert = (0.825 + A / C)^2 "Finds Nu"
Nusselt_vert = h_vert * L / k "Finds h"

"Therefore:"

Q_vert = h_vert * (pi *D * L) * (T_s – T_inf) "W"

"Case 2: cyl is horizl:"

Gr_D = g * beta * (T_s-T_inf) * D^3 / nu^2
Ra_D = Gr_D * Pr

"Calculate Nu:"

AA = 0.387 * Ra_D^(1/6)
BB = (0.559 / Pr)^(9/16)
CC = (1+BB)^(8/27)
Nusselt_horizl = (0.6 + AA / CC)^2 "Finds Nu_horizl"
Nusselt_horizl = h_horizl * D/k "Finds h_horizl"

"Therefore:"

Q_horizl = h_horizl * (pi * D * L) * (T_s – T_inf) "W"

Results:

Unit Settings: SI C Pa J mass rad

A = 10.03	AA = 3.008	B = 0.8073
BB = 0.8674	β = 0.003003 [1/K]	C = 1.192
CC = 1.203	cp = 1007 [J/kg-C]	D = 0.036 [m]
factor = 0.0978 [m]	g = 9.81 [m^2/s]	Gr_D = 306135
Gr_L = 4.199E+08	h_{horizl} = 7.495 [W/m^2-C]	h_{vert} = 5.992 [W/m^2-C]
k = 0.02808 [W/m-C]	L = 0.4 [m]	μ = 0.00002008 [kg/m-s]
ν = 0.00001895 [m^2/s]	$Nusselt_{horizl}$ = 9.608	$Nusselt_{vert}$ = 85.35
Pr = 0.7199	**Q_{horizl} = 27.12** [W]	**Q_{vert} = 21.69** [W]
Ra_D = 220384	Ra_L = 3.023E+08	ρ = 1.06 [kg/m^3]
T_f = 60 [C]	T_{inf} = 20 [C]	T_s = 100 [C]

Thus:

When tube is vertical: Q_vert = 21.69 W …. Ans.

When tube is horizontal: Q_horizl = 27.12 W …. Ans.

===

$UnitSystem SI Pa C J

"**Prob. 2A2.1.4.** A 500 W cylindrical immersion heater (3 cm dia, 20 cm long) is placed vertically in stagnant water at 25 C. Calculate the average surface temp of heater. [VTU – July/Aug. 2004]"

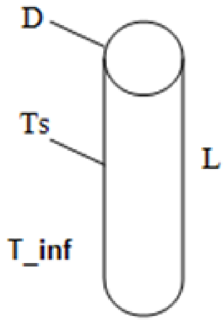


Fig. Prob.2A2.1.4

EES Solution:

"Data:"

D = 0.03[m]
L = 0.2[m]
T_inf = 25[C]
{T_s = 100 "[C] … assumed, will be corrected later"}
g = 9.81[m/s^2]
Q = 500 [W]
T_f = (T_s+T_inf)/2 "[C]… mean film temp."

"Calculations:"

P = 1.01325e05[N/m^2] "….Pressure"

"Properties of Water at T_f:"

beta=VolExpCoef(Water,T=T_f,P=P) "[1/K]"
mu=Viscosity(Water,T=T_f,P=P) "[kg/m-s]"
rho=Density(Water,T=T_f,P=P) "[kg/m^3]"
nu = mu/rho "[m^2/s]"
cp=Cp(Water,T=T_f,P=P) "[J/kg-C]"
k=Conductivity(Water,T=T_f,P=P)
Pr = mu * cp / k "…Prandtl No."
Ra_L = Gr_L * Pr "…Rayleigh No."

"Verify:"

factor = 35 * L / Gr_L^0.25 " = 0.033; Now (D > factor) is almost satisfied. So, OK to apply vertical plate eqn."

"Cyl. is vertical:"

Gr_L = g * beta * (T_s-T_inf) * L^3 / nu^2

"Calculate Nu:"

A = 0.387 * Ra_L^(1/6)
B = (0.492 / Pr)^(9/16)
C = (1+B)^(8/27)
Nusselt_vert = (0.825 + A / C)^2 "Finds Nu"
Nusselt_vert = h_vert * L / k "Finds h"

"Therefore:"

Q_vert = h_vert * (pi *D * L) * (T_s – T_inf) "W"

Q = Q_vert

Results:

Unit Settings: SI C Pa J mass rad

A = 17.61	B = 0.2894	β = 0.0003828 [1/K]
C = 1.078	cp = 4182 [J/kg-C]	D = 0.03 [m]
factor = 0.03313 [m]	g = 9.81 [m/s^2]	Gr_L = 1.993E+09
h_{vert} = 908.9 [W/m^2-C]	k = 0.6173 [W/m-C]	L = 0.2 [m]
μ = 0.0006583 [kg/m-s]	ν = 6.634E-07 [m^2/s]	$Nusselt_{vert}$ = 294.5
P = 101325 [N/m^2]	Pr = 4.46	Q = 500 [W]
Q_{vert} = 500 [W]	Ra_L = 8.888E+09	ρ = 992.4 [kg/m^3]
T_f = 39.59 [C]	T_{inf} = 25 [C]	T_s = 54.18 [C]

Thus:

The average surface temp of heater = T_s = 54.18 C … Ans.

==

$UnitSystem SI Pa C J

"**Prob. 2A2.1.5.** A tank contains water at 15 C. The water is heated by passing steam through a pipe placed in water. The pipe is 60 cm long and 4 cm dia and its surface is maintained at 85 C. Find the heat loss per hour from the pipe if (a) the pipe is kept horizontal (b) the pipe is kept vertical.

Following relation may be used: Nu = C. (Gr . Pr)^m where C = 0.53, m = 0.25 when 10^6 < (Gr. Pr) < 10^9 and C = 0.13, m = 1/3 when (Gr. Pr) > 10^9. Properties of water at average temp of 50 C are given. [M.U. 1998]"

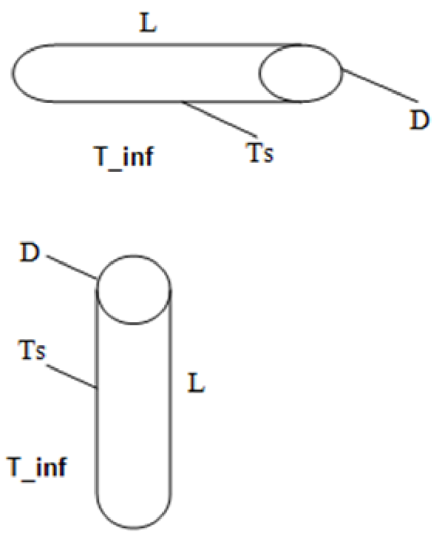


Fig. Prob.2A2.1.5 (a) and (b)

EES Solution:

"Data:"

D = 0.04[m]
L = 0.6[m]
T_inf = 15[C]
T_s = 85 [C]
g = 9.81[m/s^2]
T_f = (T_s+T_inf)/2 "[C]… mean temp."

"Calculations:"

P = 1.01325e05[N/m^2] "….Pressure"

"Properties of Water at T_f:"

beta=VolExpCoef(Water,T=T_f,P=P) "[1/K]"
mu=Viscosity(Water,T=T_f,P=P) "[kg/m-s]"
rho=Density(Water,T=T_f,P=P) "[kg/m^3]"
nu = mu/rho "[m^2/s]"
cp=Cp(Water,T=T_f,P=P) "[J/kg-C]"
k=Conductivity(Water,T=T_f,P=P)
Pr = mu * cp / k "…Prandtl No."

" Case 1: Cyl. is vertical: Therefore, L is the characteristic dimension:"

Gr_L = g * beta * (T_s-T_inf) * L^3 / nu^2 "…Grashoff No."
Ra_L = Gr_L * Pr "…Rayleigh No."

"Verify:"

factor = 35 * L / Gr_L^0.25 " = 0.03062 which is less than D; so, OK to apply vertical plate eqn."

"Calculate Nusselt_vert when cyl is vertical:"

"Note that Ra_L = 8.026E11 > 10^9"

"Therefore:"

Nusselt_vert = 0.13 * Ra_L^(1/3) "Finds Nu"
Nusselt_vert = h_vert * L / k "Finds h"

"Therefore:"

Q_vert = h_vert * (pi *D * L) * (T_s – T_inf) "W"

Q_vert_perhour = Q_vert * 3600 "[Joules/h]"

"Case 2: Cyl. is horizontal: Therefore, D is the characteristic dimension:"

Gr_D = g * beta * (T_s-T_inf) * D^3 / nu^2
Ra_D = Gr_D * Pr "…Rayleigh No."

"Calculate Nusselt_horizl when cyl is horizontal:"

"Note that Ra_D = 2.378E08 < 10^9"

"Therefore:"

Nusselt_horizl = 0.53 * Ra_D^0.25 "Finds Nu"

Nusselt_horizl = h_horizl * D / k "Finds h"

"Therefore:"

Q_horizl = h_horizl * (pi *D * L) * (T_s – T_inf) "W"

Q_horizl_perhour = Q_horizl * 3600 "[Joules/h]"

Results:

Unit Settings: SI C Pa J mass rad

β = 0.0004572 [1/K]	cp = 4181 [J/kg-C]	D = 0.04 [m]
factor = 0.03062 [m]	g = 9.81 [m/s^2]	Gr_D = 6.554E+07
Gr_L = 2.212E+11	h_{horizl} = 1037 [W/m^2-C]	h_{vert} = 1269 [W/m^2-C]
k = 0.6305 [W/m-C]	L = 0.6 [m]	μ = 0.0005471 [kg/m-s]
ν = 5.537E-07 [m^2/s]	$Nusselt_{horizl}$ = 65.82	$Nusselt_{vert}$ = 1208
P = 101325 [N/m^2]	Pr = 3.628	Q_{horizl} = 5475 [W]
$Q_{horizl,perhour}$ = 1.971E+07 [J]	Q_{vert} = 6700 [W]	$Q_{vert,perhour}$ = 2.412E+07 [J]
Ra_D = 2.378E+08	Ra_L = 8.026E+11	ρ = 988 [kg/m^3]
T_f = 50 [C]	T_{inf} = 15 [C]	T_s = 85 [C]

Thus:

Q_horizl per hour = 1.971E07 J /h …. Ans.

Q_vert per hour = 2.412E07 J /h …. Ans.

==

Prob. 2A2.1.6. In a heat exchanger, vertical pipes of 10 cm OD are used. The surface temp of these pipes is 100 C in a room where the air is at 20 C. The pipes are 3 m long. What is the rate of heat loss per metre length of the pipe? Properties of Air at 60 C are given. [M.U. May 1999]

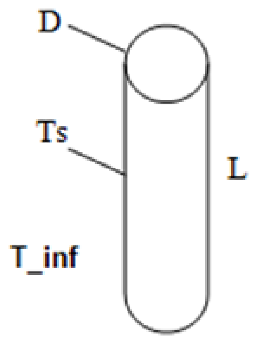


Fig. Prob.2A2.1.6

Mathcad Solution:

First, write a function for Nusselts No. in free convection over a vertical cylinder / vertical plate for different ranges of Rayleigh Numbers. **Use Churchill and Chu's equation:**

$$\text{Nusselt\_vertical\_plate\_cyl}(Ra,Pr) := \begin{array}{|l} \text{return "Rayleigh No. should be >10000 " if } Ra<10^4 \\ \text{return } \left[0.68+\dfrac{0.067\cdot Ra^{0.25}}{\left[1+\left(\dfrac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}}\right] \text{ if } Ra\le 10^9 \\ \text{return "Ra should be less than 10^12" if } Ra>10^{12} \\ \left[0.825+\dfrac{0.387\cdot Ra^{\frac{1}{6}}}{\left[1+\left(\dfrac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right]^2 \text{ if } Ra\le 10^{12} \end{array}$$

Data:

$L := 3.0$ m $D := 0.1$ m $T_s := 100$ C $T_{inf} := 20$ C $g := 9.81$ m/s^2

Then, mean film temp:

$T_f := \dfrac{T_s + T_{inf}}{2}$ i.e. $T_f = 60$ C

Properties of Air at T_f: Use the Mathcad Functions already written. See Prob. 2A1.2.5.

$\rho := \text{rho\_Air}(T_f + 273)$ i.e. $\rho = 1.06$ kg/m^3

$k := \text{k\_Air}(T_f + 273)$ i.e. $k = 0.028$ W/m.K

$\mu := \text{mu\_Air}(T_f + 273)$ i.e. $\mu = 2\cdot 10^{-5}$ kg/m.s

$Pr := \text{Pr\_Air}(T_f + 273)$ i.e. $Pr = 0.702$ ...Prandtl No.

$\nu := \dfrac{\mu}{\rho}$ i.e. $\nu = 1.886\cdot 10^{-5}$ m^2/s .... kinematic viscosity

Calculations:

$A := \pi\cdot D\cdot L$ i.e. $A = 0.942$ m^2... area for heat transfer

$\Delta T := T_s - T_{inf}$ $\Delta T = 80$ C

$\beta := \dfrac{1}{T_f + 273}$ $\beta = 3.003\cdot 10^{-3}$ 1/K....coeff of vol. expansivity

Grashoff No and Rayleigh No.:

$Gr_L := \frac{L^3 \cdot g \cdot \beta \cdot \Delta T}{\nu^2}$ i.e. $Gr_L = 6.623 \cdot 10^9$ ....Grashoff No.

$Ra_L := Gr_L \cdot Pr$ i.e. $Ra_L = 4.652 \cdot 10^9$ ...Rayleigh No.

Verify that Vertical plate relation is applicable i.e. D > (35 * L) / Gr_L^0.25:

Now, factor:: $\frac{35 \cdot L}{Gr_L^{0.25}} = 0.123$ and, $D = 0.1$ ...approxly equal to the factor.

Therefore:

Nusselts No....Use the function for Nu written above:

$Nu_L := Nusselt\_vertical\_plate\_cyl(Ra_L, Pr)$

i.e. $Nu_L = 197.947$ **....Nusselts No.**

Heat transfer coeff:

$h := \frac{k \cdot Nu_L}{L}$ i.e. $h = 5.616$ **W/m^2.K .... heat transfer coeff.**

Rate of heat transfer per metre length:

$Q := \frac{h \cdot A \cdot \Delta T}{L}$ i.e. $Q = 141.143$ **W/m, ...heat tr. per metre length**

==

Prob. 2A2.1.7. A hot plate of 100 cm height and 25 cm width is exposed to atmospheric air at 25 C. The surface temp of plate is 95 C. Find the heat loss from both surfaces of the plate.

(b) If the height of the plate is reduced to 50 cm and width is increased to 40 cm, what will be the change in heat loss?

Following empirical relation may be used: Nu = C * (Gr . Pr)^m where: C = 0.59 and m = 0.25 if (Gr. Pr) < 10^9, and C = 0.1, m = 1/3 if (Gr. Pr) > 10^9. Properties of Air are given. [M.U. – Dec. 1998]

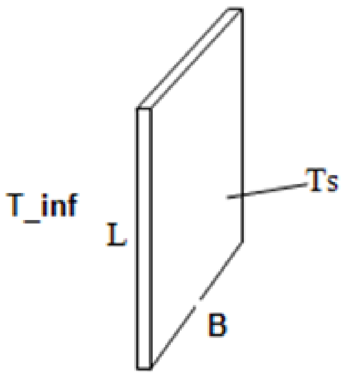


Fig. Prob.2A2.1.7

Mathcad Solution:

First, write a function for Nusselts No. in free convection over a vertical cylinder / vertical plate for different ranges of Rayleigh Numbers:

$$\text{Nusselt\_vertical\_plate\_cyl}(Ra) := \left| \begin{array}{l} \text{return "Rayleigh No. should be >10000 " if } Ra < 10^4 \\ \text{return } \left(0.59 \cdot Ra^{0.25}\right) \text{ if } Ra \le 10^9 \\ \text{return "Ra should be less than 10^12" if } Ra > 10^{12} \\ \left(0.1 \cdot Ra^{\frac{1}{3}}\right) \text{ if } Ra \le 10^{12} \end{array} \right.$$

Data:

$L := 1$ m $B := 0.25$ m $T_s := 95$ C $T_{inf} := 25$ C

$T_f := \frac{T_s + T_{inf}}{2}$ i.e. $T_f = 60$ C $g := 9.81$ m/s^2

$\beta := \frac{1}{T_f + 273}$ i.e. $\beta = 3.003 \cdot 10^{-3}$ **1/K... coeff of vol. expansivity**

Properties of Air:

Use the Mathcad functions for air, already written. See Prob. 2A1.2.5.

$\rho := \text{rho\_Air}(T_f + 273)$ i.e. $\rho = 1.06$ kg/m^3

$k := \text{k\_Air}(T_f + 273)$ i.e. $k = 0.028$ W/m.K

$\mu := \text{mu\_Air}(T_f + 273)$ i.e. $\mu = 2 \cdot 10^{-5}$ kg/m.s

$Pr := \text{Pr\_Air}(T_f + 273)$ i.e. $Pr = 0.702$ ...Prandtl No.

$\nu := \frac{\mu}{\rho}$ i.e. $\nu = 1.886 \cdot 10^{-5}$ m^2/s .... kinematic viscosity

Case 1: Plate held vertical: L is the characteristic dimension:

$A := L \cdot B$ i.e. $A = 0.25$ m^2 ... area

$\Delta T := T_s - T_{inf}$ i.e. $\Delta T = 70$ C

$Gr := \frac{L^3 \cdot g \cdot \beta \cdot \Delta T}{\nu^2}$ i.e. $Gr = 5.795 \cdot 10^9$ ....Grashoff No.

$Ra := Gr \cdot Pr$ i.e. $Ra = 4.071 \cdot 10^9$ ....Rayleigh No.

Nusselts No.: Use the function written above:

$Nu := \text{Nusselt\_vertical\_plate\_cyl}(Ra)$

i.e. $Nu = 159.668$ **....Nusselts No.**

Therefore, heat transfer coeff:

$Q := h \cdot A \cdot \Delta T \cdot 2$ i.e. $Q = 158.546$ **W, ...heat tr. from both surfaces... Ans.**

Heat transfer Q (from both surfaces):

$Q := h \cdot A \cdot \Delta T \cdot 2$ i.e. $Q = 158.546$ **W, ...heat tr. from both surfaces... Ans.**

Case 2: Plate held vertical, but L = 0.5 m and B = 0.4 m:

$L := 0.5$ m $B := 0.4$ m

$A := L \cdot B$ i.e. $A = 0.2$ m^2 ... area

$\Delta T := T_s - T_{inf}$ i.e. $\Delta T = 70$ C

Then: $Gr := \dfrac{L^3 \cdot g \cdot \beta \cdot \Delta T}{\nu^2}$ i.e. $Gr = 7.244 \cdot 10^8$ ....Grashoff No.

and, $Ra := Gr \cdot Pr$ i.e. $Ra = 5.088 \cdot 10^8$ ....Rayleigh No.

$Nu := Nusselt\_vertical\_plate\_cyl(Ra)$

i.e. $Nu = 88.612$ ...Nusselts No.

Heat transfer coeff:

$h := \frac{k \cdot Nu}{L}$ i.e. $h = 5.028$ W/m^2.C

Heat transfer Q:

$Q := h \cdot A \cdot \Delta T \cdot 2$ i.e. $Q = 140.783$ **W, ...heat tr. from both surfaces .... Ans.**

==

Prob. 2A2.1.8. An electric heating blanket is to dissipate 400 W on the high setting when hanging in air at 20 C. If the blanket is 1.3 m wide, (a) what is the length required if its average temp at the high setting is to be 40 C? (b) if the average temp at the low setting is to be 30 C, what is the rate of heat dissipation possible? [Ref. 5]

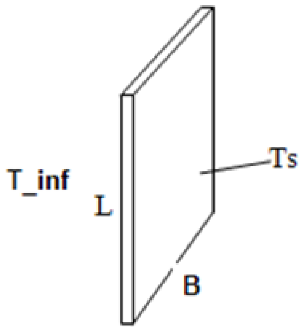


Fig. Prob.2A2.1.8

Mathcad Solution:

First write a function for Nusselts No. in free convection over a vertical cylinder / vertical plate for different ranges of Rayleigh Numbers. **Use Churchill and Chu's equation: Ref.**[3]

$$\text{Nusselt\_vertical\_plate\_cyl}(Ra, Pr) := \begin{array}{|l} \text{return "Rayleigh No. should be >10000 " if } Ra < 10^4 \\ \text{return } \left[0.68 + \frac{0.067 \cdot Ra^{0.25}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}}\right] \text{ if } Ra \le 10^9 \\ \text{return "Ra should be less than 10^12" if } Ra > 10^{12} \\ \left[0.825 + \frac{0.387 \cdot Ra^{\frac{1}{6}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right]^2 \text{ if } Ra \le 10^{12} \end{array}$$

Data:

$B := 1.3$ m $\quad T_s := 40$ C $\quad T_{inf} := 20$ C $\quad Q1 := 400$ W

$T_f := \frac{T_s + T_{inf}}{2}$ i.e. $T_f = 30$ C $\quad g := 9.81$ m/s^2

$\beta := \frac{1}{T_f + 273}$ i.e. $\beta = 3.3 \cdot 10^{-3}$ **1/K... coeff of vol. expansivity**

Properties of Air: Use the Mathcad functions for air, already written. See Prob. 2A1.2.5.

$\rho := rho\_Air(T_f + 273)$ i.e. $\rho = 1.165$ kg/m^3

$k := k\_Air(T_f + 273)$ i.e. $k = 0.026$ W/m.K

$\mu := mu\_Air(T_f + 273)$ i.e. $\mu = 1.857 \cdot 10^{-5}$ kg/m.s

$Pr := Pr\_Air(T_f + 273)$ i.e. $Pr = 0.707$ ...Prandtl No.

$\nu := \frac{\mu}{\rho}$ i.e. $\nu = 1.594 \cdot 10^{-5}$ m^2/s .... kinematic viscosity

Let the length of blanket be L m.

Then:

$A(L) := L \cdot B$ i.e. m^2 ... area

$\Delta T := T_s - T_{inf}$ i.e. $\Delta T = 20$ C

$Gr(L) := \frac{L^3 \cdot g \cdot \beta \cdot \Delta T}{\nu^2}$ ....Grashoff No. as a function of L

$Ra(L) := Gr(L) \cdot Pr$ ....Rayleigh No. as a function of L

Nusselts No.: Use the function written above:

$Nu(L) := Nusselt\_vertical\_plate\_cyl(Ra(L), Pr)$

Therefore, heat transfer coeff:

$h(L) := \frac{k \cdot Nu(L)}{L}$ W/m^2.C

Heat transfer Q (from both surfaces):

$Q(L) := h(L) \cdot A(L) \cdot \Delta T \cdot 2$ **W, ...heat tr. from both surfaces....**

This should be equal to Q1 = 400 W.

Apply the Solve Block of Mathcad to find L to meet this condition.

Start with a guess value for L:

$L := 2$ m .... guess value

Given

$Q(L) = Q1$

$L := \text{Find}(L)$

i.e. $L = 2.083$ **m .... Length of blanket.... Ans.**

Then:

$Gr(L) = 2.303 \cdot 10^{10}$ ...Grashoff No .... Ans.

$Ra(L) = 1.627 \cdot 10^{10}$ ....Rayleigh No..... Ans.

$Nu(L) = 294.142$ ....Nusselts No.... Ans.

$h(L) = 3.693$ W/m^2.C .... Ans.

$Q(L) = 400$ W ..... heat dissipated from both sides of blanket.

(b) For this blanket, now, if Ts = 30 C, what is the value of Q?

$T_s := 30$ C

Therefore:

$T_f := \frac{T_s + T_{inf}}{2}$ i.e. $T_f = 25$ C $g := 9.81$ m/s^2

$\beta := \frac{1}{T_f + 273}$ i.e. $\beta = 3.356 \cdot 10^{-3}$ **1/K... coeff of vol. expansivity**

$\Delta T := T_s - T_{inf}$ i.e. $\Delta T = 10$ C

Properties of Air: Use the Mathcad functions for air, already written. See Prob. 2A1.2.5.

$\rho := rho\_Air(T_f + 273)$ i.e. $\rho = 1.185$ kg/m^3

$k := k\_Air(T_f + 273)$ i.e. $k = 0.026$ W/m.K

$\mu := mu\_Air(T_f + 273)$ i.e. $\mu = 1.833 \cdot 10^{-5}$ kg/m.s

$Pr := Pr\_Air(T_f + 273)$ i.e. $Pr = 0.708$ ...Prandtl No.

$\nu := \frac{\mu}{\rho}$ i.e. $\nu = 1.547 \cdot 10^{-5}$ m^2/s .... kinematic viscosity

And:

$Gr(L) := \frac{L^3 \cdot g \cdot \beta \cdot \Delta T}{\nu^2}$ ....Grashoff No. as a function of L

$Ra(L) := Gr(L) \cdot Pr$ ....Rayleigh No. as a function of L

Nusselts No.: Use the function written above:

$Nu(L) := Nusselt\_vertical\_plate\_cyl(Ra(L), Pr)$

Therefore, heat transfer coeff:

$h(L) := \frac{k \cdot Nu(L)}{L}$ W/m^2.C

Heat transfer Q (from both surfaces):

$Q(L) := h(L) \cdot A(L) \cdot \Delta T \cdot 2$ **W, ...heat tr. from both surfaces....**

We get:

$Gr(L) = 1.243 \cdot 10^{10}$ ...Grashoff No .... Ans.

$Ra(L) = 8.794 \cdot 10^{9}$ ....Rayleigh No..... Ans.

$Nu(L) = 242.123$ ....Nusselts No.... Ans.

$h(L) = 2.996$ ... W/m^2.C .... Ans.

$Q(L) = 162.269$ **...W ..... heat dissipated from both sides of blanket... Ans.**

==

Plot the variation of Q with ambient temp T_{inf}, when the dimensions of the blanket remain the same and surface temp T_s remains at 40 C:

Use vectorize operator of Mathcad to calculate in parallel for various values of T_{inf}:

Data:

$T_{inf} := (0 \;\; 5 \;\; 10 \;\; 15 \;\; 20 \;\; 25)$ **C.....variation of ambient temp.... written as a vector**

$B := 1.3$ m $T_s := 40$ C $Q1 := 400$ W $L := 2.083$ m $g := 9.81$ m/s^2

Then:

$T_f := \frac{T_s + T_{inf}}{2}$ i.e. $T_f = [\,20 \;\; 22.5 \;\; 25 \;\; 27.5 \;\; 30 \;\; 32.5\,]$ C

$\beta := \overrightarrow{\frac{1}{T_f + 273}}$ i.e. $\beta = [\,3.413 \cdot 10^{-3} \;\; 3.384 \cdot 10^{-3} \;\; 3.356 \cdot 10^{-3} \;\; 3.328 \cdot 10^{-3} \;\; 3.3 \cdot 10^{-3} \;\; 3.273 \cdot 10^{-3}\,]$

1/K... coeff of vol. expansivity

Properties of Air: Use the Mathcad functions for air, already written. See Prob. 2A1.2.5.

$\rho := \overrightarrow{rho\_Air(T_f + 273)}$ i.e. $\rho = [\,1.205 \quad 1.195 \quad 1.185 \quad 1.175 \quad 1.165 \quad 1.156\,]$ kg/m^3

$k := \overrightarrow{k\_Air(T_f + 273)}$ i.e. $k = [\,0.025 \quad 0.026 \quad 0.026 \quad 0.026 \quad 0.026 \quad 0.026\,]$ W/m.K

$\mu := \overrightarrow{mu\_Air(T_f + 273)}$ i.e. $\mu = [\,1.808\cdot10^{-5} \quad 1.821\cdot10^{-5} \quad 1.833\cdot10^{-5} \quad 1.845\cdot10^{-5} \quad 1.857\cdot10^{-5} \quad 1.869\cdot10^{-5}\,]$ kg/m.s

$Pr := \overrightarrow{Pr\_Air(T_f + 273)}$ i.e. $Pr = [\,0.709 \quad 0.708 \quad 0.708 \quad 0.707 \quad 0.707 \quad 0.706\,]$ ...Prandtl No.

$\nu := \overrightarrow{\frac{\mu}{\rho}}$ i.e. $\nu = [\,1.501\cdot10^{-5} \quad 1.524\cdot10^{-5} \quad 1.547\cdot10^{-5} \quad 1.571\cdot10^{-5} \quad 1.594\cdot10^{-5} \quad 1.618\cdot10^{-5}\,]$ m^2/s .... kinematic viscosity

$\Delta T := T_s - T_{inf}$ i.e. $\Delta T = [\,40 \;\; 35 \;\; 30 \;\; 25 \;\; 20 \;\; 15\,]$ C

$Gr := \overrightarrow{\frac{B^3 \cdot g \cdot \beta \cdot \Delta T}{\nu^2}}$ ....Grashoff No.

i.e. $Gr = [\,1.306 \cdot 10^{10} \;\; 1.099 \cdot 10^{10} \;\; 9.064 \cdot 10^{9} \;\; 7.269 \cdot 10^{9} \;\; 5.599 \cdot 10^{9} \;\; 4.044 \cdot 10^{9}\,]$

$Ra := \overrightarrow{(Gr \cdot Pr)}$ ....Rayleigh No.

i.e. $Ra = [\,9.258 \cdot 10^{9} \;\; 7.784 \cdot 10^{9} \;\; 6.413 \cdot 10^{9} \;\; 5.139 \cdot 10^{9} \;\; 3.956 \cdot 10^{9} \;\; 2.856 \cdot 10^{9}\,]$

Nusselts No.: Use the function written above:

$Nu := \overrightarrow{Nusselt\_vertical\_plate\_cyl(Ra, Pr)}$

i.e. $Nu = [\,246.152 \;\; 233.009 \;\; 219.184 \;\; 204.415 \;\; 188.296 \;\; 170.04\,]$

Therefore, heat transfer coeff:

$h := \overrightarrow{\frac{k \cdot Nu}{L}}$ W/m^2.C

i.e. $h = [\,3.002 \;\; 2.862 \;\; 2.712 \;\; 2.548 \;\; 2.364 \;\; 2.15\,]$

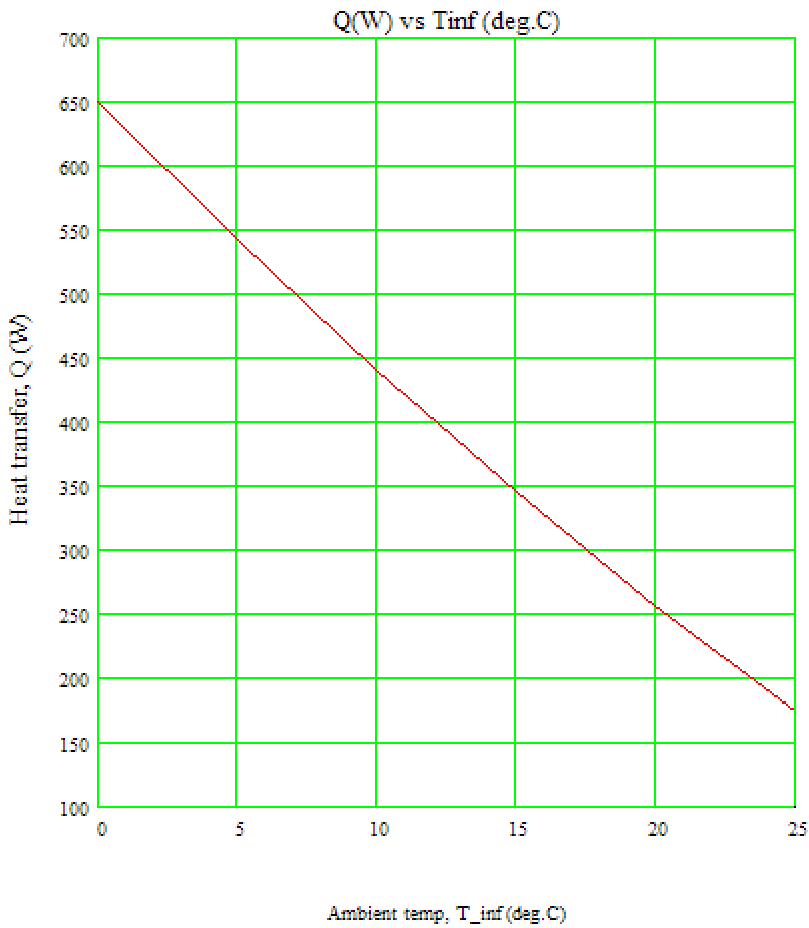
Heat transfer Q (from both surfaces):

$Q := \overrightarrow{(h \cdot L \cdot B \cdot \Delta T \cdot 2)}$ **W, ...heat tr. from both surfaces...**

i.e. $Q = [\,650.232 \;\; 542.572 \;\; 440.685 \;\; 344.988 \;\; 256.061 \;\; 174.667\,]$

Now, plot Q against T_{inf}:

$i := 0, 1 .. 5$



Plot the variation of T_s with possible heat transfer Q1, when ambient temp Ti_{nf} and the dimensions of the blanket remain the same:

Now, write the various quantities as functions of T_s:

Data:

$B := 1.3$ m $\quad T_s := 40$ C $\quad T_{inf} := 20$ C $\quad L := 2.083$ m

$T_f(T_s) := \dfrac{T_s + T_{inf}}{2}$ i.e. $T_f(T_s) = 30$ C $\quad g := 9.81$ m/s^2

$\beta(T_s) := \dfrac{1}{T_f(T_s) + 273}$ i.e. $\beta(T_s) = 3.3 \cdot 10^{-3}$ **1/K... coeff of vol. expansivity**

$Q1 := 400$ W

Properties of Air: Use the Mathcad functions for air, already written. See Prob. 2A1.2.5.

$\rho(T_s) := \text{rho\_Air}(T_f(T_s) + 273)$ i.e. $\rho(T_s) = 1.165$ kg/m^3

$k(T_s) := \text{k\_Air}(T_f(T_s) + 273)$ i.e. $k(T_s) = 0.026$ W/m.K

$\mu(T_s) := \text{mu\_Air}(T_f(T_s) + 273)$ i.e. $\mu(T_s) = 1.857 \cdot 10^{-5}$ kg/m.s

$Pr(T_s) := \text{Pr\_Air}(T_f(T_s) + 273)$ i.e. $Pr(T_s) = 0.707$ ...Prandtl No.

$\nu(T_s) := \dfrac{\mu(T_s)}{\rho(T_s)}$ i.e. $\nu(T_s) = 1.594 \cdot 10^{-5}$ m^2/s .... kinematic viscosity

Then:

$A := L \cdot B$ i.e. $A = 2.708$ m^2 ... area

$\Delta T(T_s) := T_s - T_{inf}$ i.e. $\Delta T(T_s) = 20$ C

$Gr(T_s) := \dfrac{L^3 \cdot g \cdot \beta(T_s) \cdot \Delta T(T_s)}{\nu(T_s)^2}$ ....Grashoff No. as a function of Ts

$Ra(T_s) := Gr(T_s) \cdot Pr(T_s)$ ....Rayleigh No. as a function of Ts

Nusselts No.: Use the function written above:

$Nu(T_s) := \text{Nusselt\_vertical\_plate\_cyl}(Ra(T_s), Pr(T_s))$

Therefore, heat transfer coeff:

$h(T_s) := \dfrac{k(T_s) \cdot Nu(T_s)}{L}$ W/m^2.C

Heat transfer Q (from both surfaces):

$Q(T_s) := h(T_s) \cdot A \cdot \Delta T(T_s) \cdot 2$ **W, ...heat tr. from both surfaces....**

This should be equal to Q1 = 400 W.

Apply the Solve Block of Mathcad to find T_s to meet this condition.

Start with a guess value for T_s:

$T_s := 30$ C .... guess value

Given

$Q(T_s) = Q1$

$T_s(Q1) := Find(T_s)$

i.e $T_s(Q1) = 40$ C

Note: T_s is written as a function of Q1 , to draw graphs with various options of Q1:

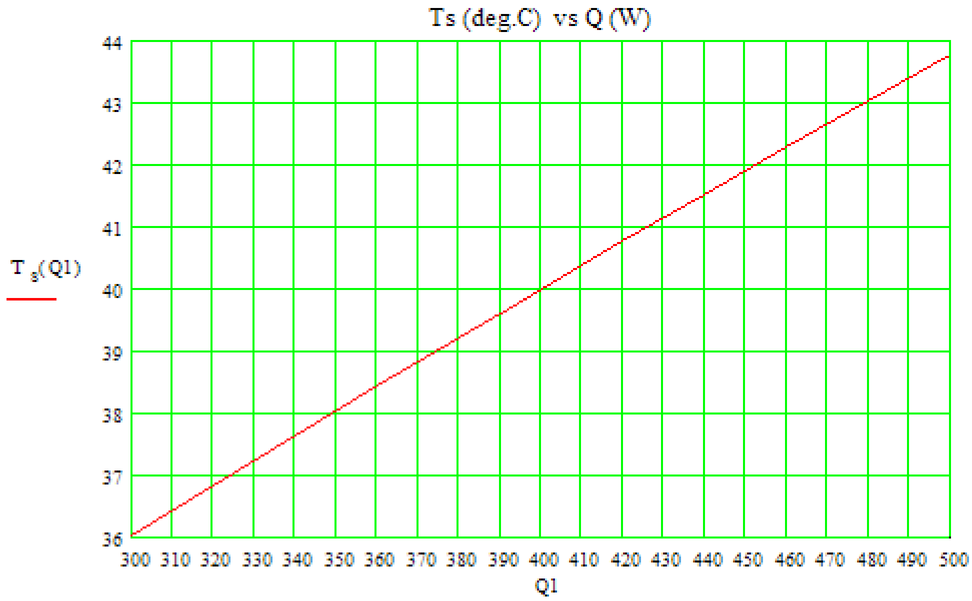
Values of T_s for different values of Q1:

$Q1 := 300, 320 .. 500$ ....variation of Q1

Table:

Q1	$T_s(Q1)$
300	36.027
320	36.842
340	37.647
360	38.441
380	39.225
400	40
420	40.767
440	41.526
460	42.278
480	43.022
500	43.76

Plot:



==

Prob. 2A2.1.9. A vertical pipe, 5 cm in dia carrying hot water is exposed to ambient air at 15 C. If the outer surface of the pipe is at 65 C, find the heat loss from 1 m length of pipe per hour. [VTU – July/Aug. 2003]

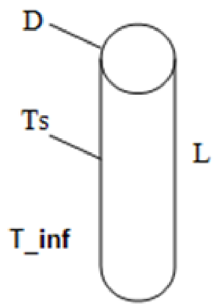


Fig. Prob.2A2.1.9

Mathcad Solution:

First write a function for Nusselts No. in free convection over a vertical cylinder / vertical plate for different ranges of Rayleigh Numbers. **Use Churchill and Chu's equation: Ref.[3]**

$$\text{Nusselt\_vertical\_plate\_cyl}(Ra,Pr) := \begin{array}{|l} \text{return "Rayleigh No. should be >10000 " if } Ra<10^4 \\ \text{return } \left[0.68 + \dfrac{0.067\cdot Ra^{0.25}}{\left[1+\left(\dfrac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}}\right] \text{ if } Ra\le 10^9 \\ \text{return "Ra should be less than 10^12" if } Ra>10^{12} \\ \left[0.825 + \dfrac{0.387\cdot Ra^{\frac{1}{6}}}{\left[1+\left(\dfrac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right]^2 \text{ if } Ra\le 10^{12} \end{array}$$

Data:

$D := 0.05$ m $\quad T_s := 65$ C $\quad T_{inf} := 15$ C $\quad L := 1$ m

$T_f := \dfrac{T_s + T_{inf}}{2}$ i.e. $T_f = 40$ C $\quad g := 9.81$ m/s^2

$\beta := \dfrac{1}{T_f + 273}$ i.e. $\beta = 3.195\cdot 10^{-3}$ **1/K... coeff of vol. expansivity**

Properties of Air: Use the Mathcad functions for air, already written. See Prob. 2A1.2.5.

$\rho := rho\_Air(T_f + 273)$ i.e. $\rho = 1.128$ kg/m^3

$k := k\_Air(T_f + 273)$ i.e. $k = 0.027$ W/m.K

$\mu := mu\_Air(T_f + 273)$ i.e. $\mu = 1.906\cdot 10^{-5}$ kg/m.s

$Pr := Pr\_Air(T_f + 273)$ i.e. $Pr = 0.705$ ...Prandtl No.

$\nu := \dfrac{\mu}{\rho}$ i.e. $\nu = 1.689\cdot 10^{-5}$ m^2/s .... kinematic viscosity

Then:

$A := L \cdot \pi \cdot D$ i.e. $A = 0.157$ m^2 ... area

$\Delta T := T_s - T_{inf}$ i.e. $\Delta T = 50$ C

$Gr_L := \frac{L^3 \cdot g \cdot \beta \cdot \Delta T}{\nu^2}$ ....Grashoff No.

i.e. $Gr_L = 5.491 \cdot 10^9$

$Ra_L := Gr_L \cdot Pr$ ....Rayleigh No.

i.e. $Ra_L = 3.872 \cdot 10^9$

Nusselts No.: Use the function written above:

$Nu_L := \text{Nusselt\_vertical\_plate\_cyl}(Ra_L, Pr)$

i.e. $Nu_L = 186.984$

Therefore, heat transfer coeff:

$h := \frac{k \cdot Nu_L}{L}$

i.e. $h = 5.029$ W/m^2.C

Heat transfer Q:

$Q := h \cdot A \cdot \Delta T$ **W, ...heat tr.**

i.e. $Q = 39.501$ **W....Ans.**

and: $Qperhour := Q \cdot 3600$ J..... per metre length

i.e. $Qperhour = 1.422 \cdot 10^{5}$ **J/h ... per metre length.....Ans**

If the ambient temp varies from 0 to 25 deg.C, with the surface temp remaining at 65 deg.C, plot the variation of Q with T_{inf}:

Use the vectorize operator of Mathcad:

$D := 0.05$ m $T_s := 65$ C $L := 1$ m $g := 9.81$ m/s^2

$T_{inf} := (0\ \ 5\ \ 10\ \ 15\ \ 20\ \ 25)$ C .... range of values for Tinf.

$T_f := \frac{T_s + T_{inf}}{2}$ i.e. $T_f = [\,32.5\ \ 35\ \ 37.5\ \ 40\ \ 42.5\ \ 45\,]$ C

$\beta := \overrightarrow{\frac{1}{T_f + 273}}$ i.e. $\beta = [\,3.273 \cdot 10^{-3}\ \ 3.247 \cdot 10^{-3}\ \ 3.221 \cdot 10^{-3}\ \ 3.195 \cdot 10^{-3}\ \ 3.17 \cdot 10^{-3}\ \ 3.145 \cdot 10^{-3}\,]$

1/K... coeff of vol. expansivity

Properties of Air: Use the Mathcad functions for air, already written. See Prob. 2A1.2.5.

$\rho := \overrightarrow{\text{rho\_Air}(T_f + 273)}$ i.e. $\rho = [\,1.156 \quad 1.146 \quad 1.137 \quad 1.128 \quad 1.119 \quad 1.11\,]$ kg/m^3

$k := \overrightarrow{\text{k\_Air}(T_f + 273)}$ i.e. $k = [\,0.026 \quad 0.027 \quad 0.027 \quad 0.027 \quad 0.027 \quad 0.027\,]$ W/m.K

$\mu := \overrightarrow{\text{mu\_Air}(T_f + 273)}$ i.e. $\mu = [\,1.869\cdot10^{-5} \quad 1.882\cdot10^{-5} \quad 1.894\cdot10^{-5} \quad 1.906\cdot10^{-5} \quad 1.917\cdot10^{-5} \quad 1.929\cdot10^{-5}\,]$ kg/m.s

$Pr := \overrightarrow{\text{Pr\_Air}(T_f + 273)}$ i.e. $Pr = [\,0.706 \quad 0.706 \quad 0.706 \quad 0.705 \quad 0.705 \quad 0.704\,]$ ...Prandtl No.

$\nu := \overrightarrow{\frac{\mu}{\rho}}$ i.e. $\nu = [\,1.618\cdot10^{-5} \quad 1.641\cdot10^{-5} \quad 1.665\cdot10^{-5} \quad 1.689\cdot10^{-5} \quad 1.714\cdot10^{-5} \quad 1.738\cdot10^{-5}\,]$

m^2/s .... kinematic viscosity

Then:

$A := L\cdot\pi\cdot D$ i.e. $A = 0.157$ m^2 ... area

$\Delta T := T_s - T_{inf}$ i.e. $\Delta T = [\,65 \quad 60 \quad 55 \quad 50 \quad 45 \quad 40\,]$ C

$Gr_L := \overrightarrow{\frac{L^3\cdot g\cdot\beta\cdot\Delta T}{\nu^2}}$ ....Grashoff No.

i.e. $Gr_L = [\,7.976\cdot10^{9} \quad 7.093\cdot10^{9} \quad 6.266\cdot10^{9} \quad 5.491\cdot10^{9} \quad 4.765\cdot10^{9} \quad 4.086\cdot10^{9}\,]$

$Ra_L := \overrightarrow{(Gr_L\cdot Pr)}$ ....Rayleigh No.

i.e. $Ra_L = [\,5.633\cdot10^{9} \quad 5.007\cdot10^{9} \quad 4.421\cdot10^{9} \quad 3.872\cdot10^{9} \quad 3.359\cdot10^{9} \quad 2.878\cdot10^{9}\,]$

Nusselts No.: Use the function written above:

$Nu_L := \overrightarrow{\text{Nusselt\_vertical\_plate\_cyl}(Ra_L, Pr)}$

i.e. $Nu_L = [\,210.368 \quad 202.706 \quad 194.925 \quad 186.984 \quad 178.831 \quad 170.398\,]$

Therefore, heat transfer coeff:

$$h := \overrightarrow{\frac{k \cdot Nu_L}{L}}$$

i.e. $h = [\ 5.541 \quad 5.377 \quad 5.207 \quad 5.029 \quad 4.843 \quad 4.646\ ]$ W/m^2.C

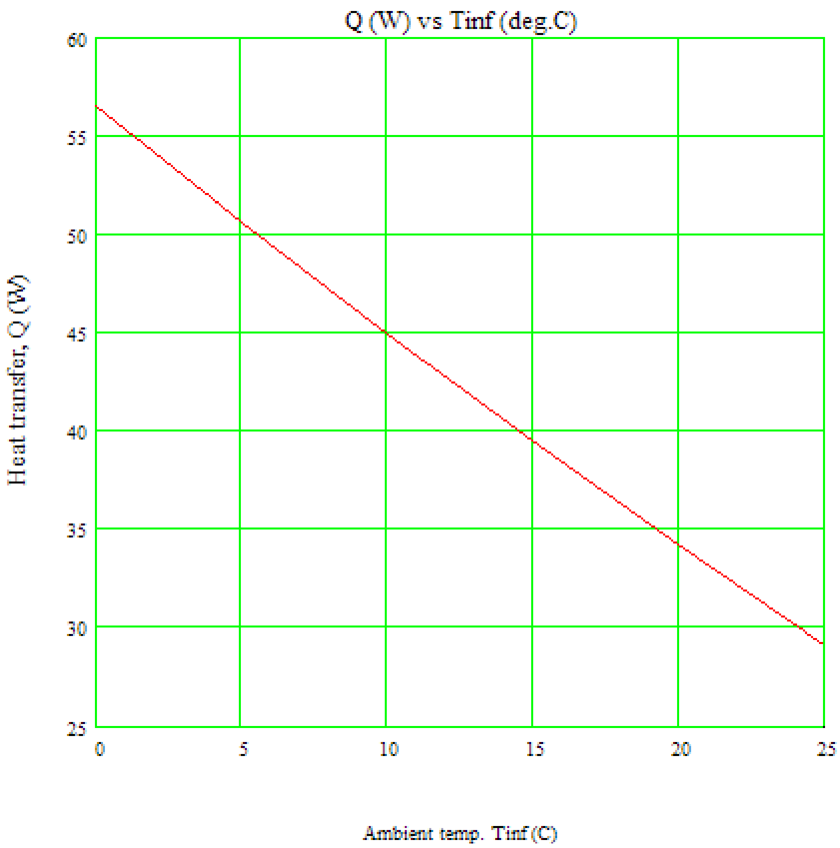
Heat transfer Q:

$Q := \overrightarrow{(h \cdot A \cdot \Delta T)}$ W, ...heat tr.

i.e. $Q = [\ 56.573 \quad 50.676 \quad 44.983 \quad 39.501 \quad 34.235 \quad 29.194\ ]$ W....Ans.

Now, plot Q against T_{inf}:

$i := 0, 1..5$



==

Prob. 2A2.1.10. A tube of 0.036 m OD and 40 cm length is maintained at a uniform temp of 100 C. It is exposed to air at a temp of 20 C. Determine the rate of heat transfer from the surface of the tube when (a) the tube is vertical (b) the tube is horizontal. [VTU – June/July 2009]

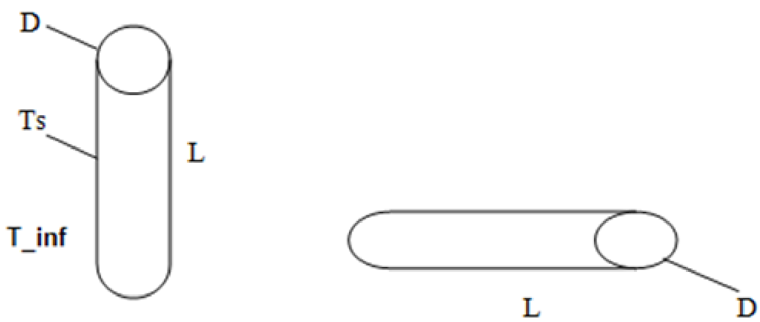


Fig.Prob.2A2.1.10 (a) and (b)

EXCEL Solution:

Note that this problem is the same as Prob.2A1.1.3
But, now we shall solve it with EXCEL.

We need properties of Air as a function of Temp.

VBA Functions for properties of Air were written while solving Prob.2A1.2.13. We will use those Functions.

Following are the steps in EXCEL Solution:

1. Set up the EXCEL worksheet, enter data and name the cells:

beta | f_x =1/(T_f+273)

	A	B	C	D	E	F
209						
210		Data:	Fluid =	Air		
211			T_s	100	C	
212			T_inf	20.0000	C	
213			T_f	42.0000	C	
214			D	0.036	m	$T\_f = \frac{T\_s + T\_inf}{2}$
215			L	0.4000	m	
216			g	9.8100	m/s^2	
217			beta	0.0032	1/K	

2. Do the calculations, use VBA Functions to get properties of Air:

cp | f_x =air_cp_T((T_f+273))

	A	B	C	D	E	F	G
218							
219		Calculations:				Using VBA Functions for Air	
220		density	rho	1.1215	kg/m^3		
221		th. conductivity	k	0.0274825	W/m.C		
222		Prandtl No.	Pr	0.7055			
223		kinematic visc.	nu	0.00001719	m^2/s		
224		sp.heat	cp	1007.75	J/kg.K		

And, further calculations:

h_vert f_x =Nusselt_vert * k/L

	A	B	C	D	E
225					
226		**Case 1. Cyl is vertical:**			
227		Grashof No.	Gr_L	539605864.8	
228		Verify:	factor	0.091856284	< D...so, OK to apply vertical plate eqn.
229		Rayleigh No.	Ra_L	380691937.6	
230		To calculate Nusselts No.			
231			A	10.41853316	
232			B	0.816489864	
233			C	1.193465232	
234		Nusselt No.	Nusselt_vert	91.291	
235		heat tr. coeff.	h_vert	6.272	W/m^2.C
236		heat transfer	Q_vert	22.700	W .... Ans.
237					
238		**Case 2. Cyl is horizontal:**			
239		Grashof No.	Gr_D	393372.6754	
240		Rayleigh No.	Ra_D	277524.4225	
241		To calculate Nusselts No.			
242			AA	3.1256	
243			BB	0.8773	
244			CC	1.205163114	
245			Nusselt_horizl	10.198	
246			h_horizl	7.785	W/m^2.C
247			Q_horizl	28.176	W
248					
249					
250					
251					
252					
253					

$$Gr\_L = \frac{g\cdot beta\cdot(T\_s - T\_inf)\cdot L^3}{nu^2}$$

$$factor = \frac{35\cdot L}{Gr\_L^{0.25}}$$

$$Ra\_L = Gr\_L\cdot Pr$$

$$A = 0.387\cdot Ra\_L^{\frac{1}{6}}$$

$$B = \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}$$

$$C\_ = (1 + B)^{\frac{8}{27}}$$

$$Nusselt\_vert = 0.825 + \left(\frac{A}{C\_}\right)^2$$

$$h\_vert = \frac{Nusselt\_vert\cdot k}{L}$$

$$Q\_vert = h\_vert\cdot(\pi\cdot D\cdot L)\cdot(T\_s - T\_inf)$$

$$Gr\_D = \frac{g\cdot beta\cdot(T\_s - T\_inf)\cdot D^3}{nu^2}$$

$$Ra\_D = Gr\_D\cdot Pr$$

$$AA = 0.387\cdot Ra\_D^{\frac{1}{6}}$$

$$BB = \left(\frac{0.559}{Pr}\right)^{\frac{9}{16}}$$

$$CC = (1 + BB)^{\frac{8}{27}}$$

$$Nusselt\_horizl = \left(0.6 + \frac{AA}{CC}\right)^2$$

$$h\_horizl = \frac{Nusselt\_horizl\cdot k}{D}$$

$$Q\_horizl = h\_horizl\cdot(\pi\cdot D\cdot L)\cdot(T\_s - T\_inf)$$

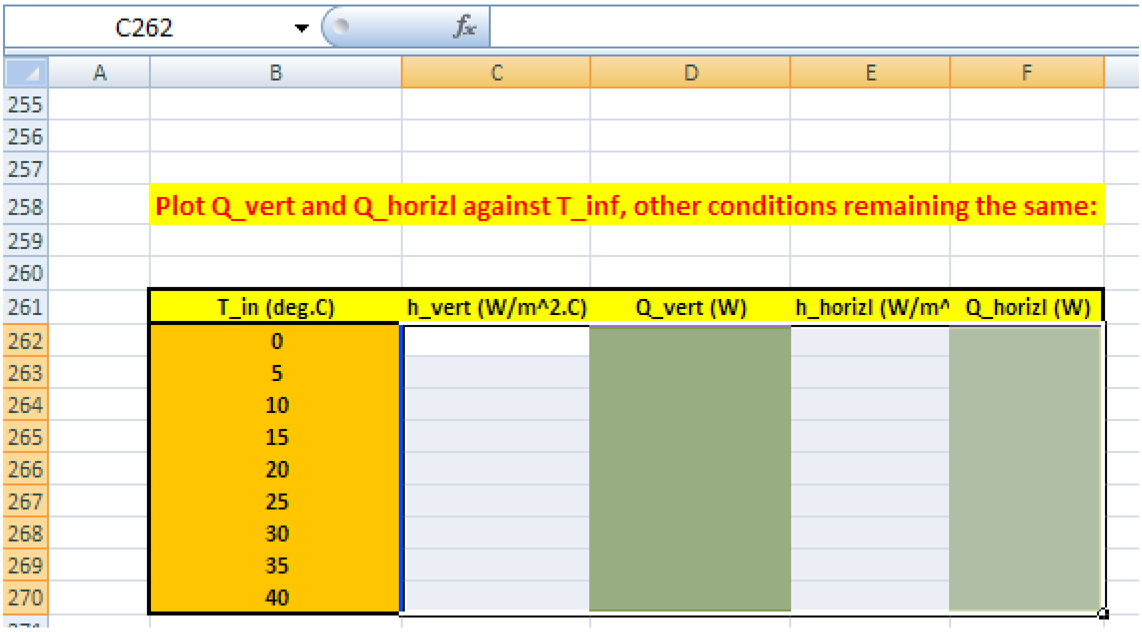
Note that in the above worksheet, formulas used are also shown, for clarity.

Thus:

Q_vert = 22.7 W, and Q_horizl = 28.176 W …. Ans.

3. Now, let us plot the variation of Q_vert and Q_horizl as the ambient temp varies from 0 to 40 deg.C.

Since T_b and the properties of Air also change as T_inf varies, it is convenient to write a VBA program to do this calculation. First, set up a Table as shown:

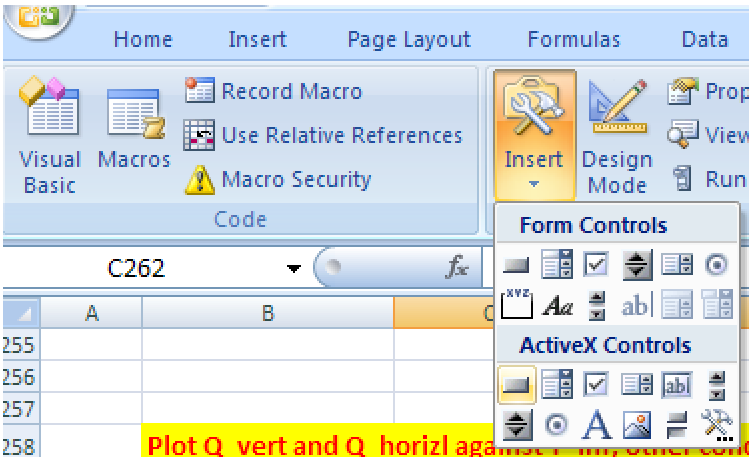


C262

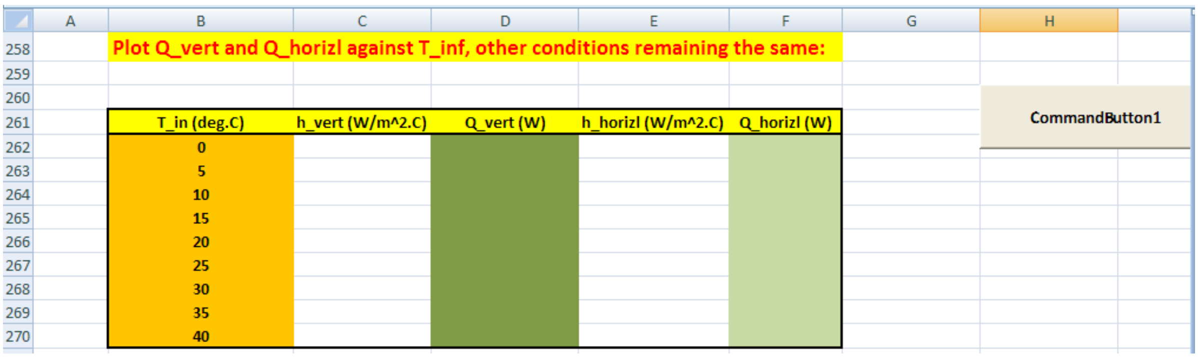
	A	B	C	D	E	F
255						
256						
257						
258		Plot Q_vert and Q_horizl against T_inf, other conditions remaining the same:				
259						
260						
261		T_in (deg.C)	h_vert (W/m^2.C)	Q_vert (W)	h_horizl (W/m^	Q_horizl (W)
262		0				
263		5				
264		10				
265		15				
266		20				
267		25				
268		30				
269		35				
270		40				

4. Now, we will write a VBA program to read the values of T_inf , one by one, from this Table and put in cell D212.; then, all other calculations in remaining cells get up-dated. The program selects the required quantity (i.e. h_vert, Q_vert, h_horizl and Q_horizl, from cells D235, D236, D246 and D247 respectively) and copies them to their respective places in the Table. And, we will have a Command Button to do this:

Go to Developer – Insert-Active X controls:



Click on the first, left button. And, draw the Command Button in the worksheet to the required size, at the required place:

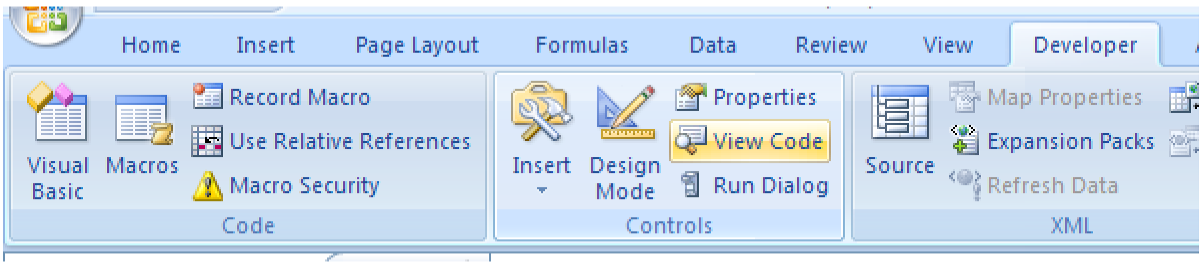


Plot Q_vert and Q_horizl against T_inf, other conditions remaining the same:

T_in (deg.C)	h_vert (W/m^2.C)	Q_vert (W)	h_horizl (W/m^2.C)	Q_horizl (W)
0				
5				
10				
15				
20				
25				
30				
35				
40				

CommandButton1

Click on Developer-View Code:



And, we get the code already written for the Command button:

```
Private Sub CommandButton1_Click()

End Sub
```

Now, complete the code to do the desired jobs as mentioned under point 4 above:

```
Private Sub CommandButton1_Click()

Dim i As Integer
For i = 0 To 8
 Range("D212") = Cells(262 + i, 2)
 Cells(262 + i, 3) = Range("D235")
 Cells(262 + i, 4) = Range("D236")
 Cells(262 + i, 5) = Range("D246")
 Cells(262 + i, 6) = Range("D247")

 Next i

End Sub
```

In the above code:

Line 1: defines the Sub

Line 2: dimension statement for I, the counter in For….Next construct

Line 3 to 9: For … Next slab

Line 4: Takes the first value of T_inf from the Table and copies it to cell D212 (i.e. T_inf in the original program)

Immediately, all other calculations in other cells are up-dated; and the rest of the program copies them to the respective places in the Table. i.e.

Line 5, 6, 7 and 8: copy values of h_vert, Q_vert, h_horizl and Q_horizl to respective cells in the Table.

Line 9: this is repeated for the next value of T_inf in the Table.

Line 10: End statement of Subroutine

---

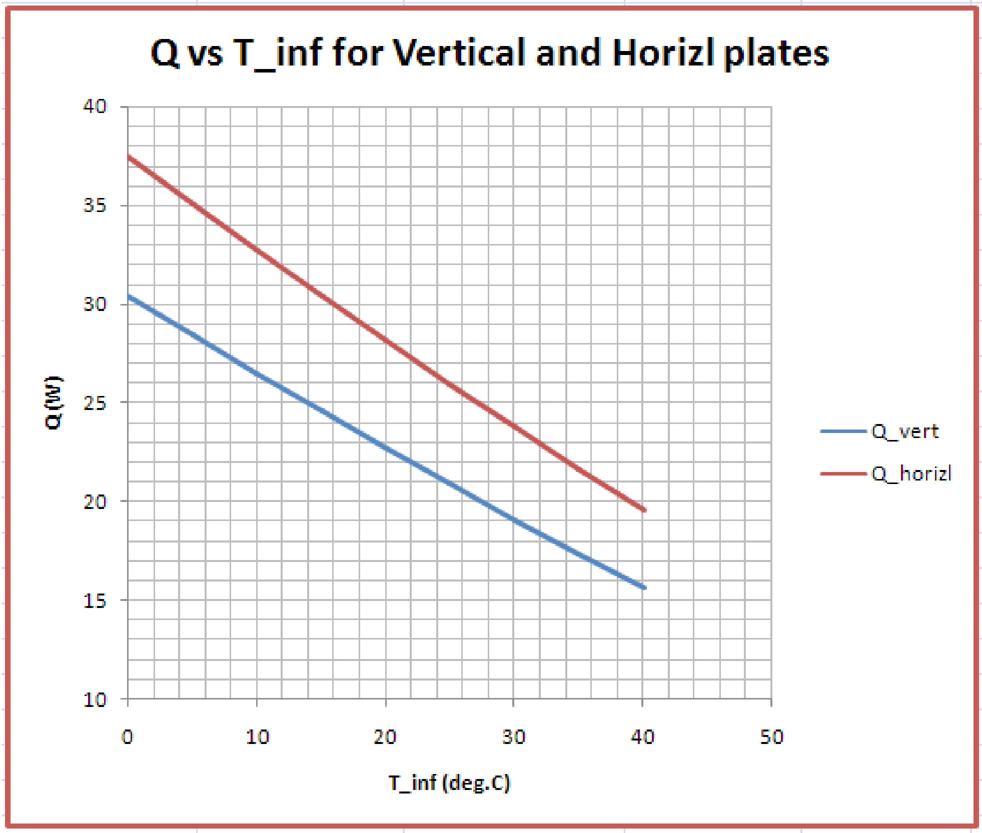
5. Now, click on the Command Button 1 and the Table gets filled up immediately:

Plot Q_vert and Q_horizl against T_inf, other conditions remaining the same:

CommandButton1

T_in (deg.C)	h_vert (W/m^2.C)	Q_vert (W)	h_horizl (W/m^2.C)	Q_horizl (W)
0	6.714	30.374	8.272	37.421
5	6.610	28.407	8.157	35.057
10	6.502	26.471	8.038	32.727
15	6.389	24.569	7.914	30.433
20	6.272	22.700	7.785	28.176
25	6.150	20.867	7.651	25.958
30	6.023	19.072	7.510	23.781
35	5.888	17.315	7.361	21.646
40	5.747	15.600	7.205	19.556

6. Plot the results in EXCEL:



2A2.2 Natural convection from horizontal plates, cylinders and spheres:

"**Prob. 2A2.2.1**. A horizontal plate, 1 m × 0.8 m is kept in a water tank, with the top surface at 60 C providing heat to warm stagnant water at 20 C. Determine the value of convection coeff. Repeat the problem for heating on the bottom surface. [VTU – May–June 2010]"

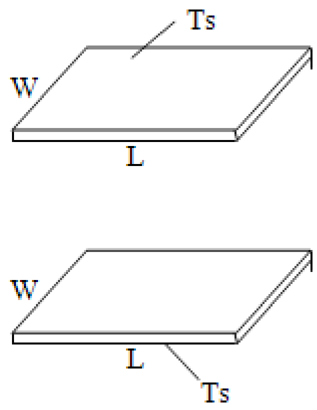


Fig.Prob.2A2.2.1(a) and (b).

EES Solution:

"Data:"

L = 1.0[m]

W = 0.8[m]

L_c = (L* W) / (2*(L+W)) "L_c for horizl plate"

T_s = 60[C]

T_inf = 20[C]

T_f = (T_s+T_inf)/2 "…film temp."

beta=VolExpCoef(Water,T=T_f, P=1e05)

g = 9.81[m/s^2]

"Properties of Water:"

mu=Viscosity(Water,T=T_f, P = 1e05) "[kg/m-s]"

rho=Density(Water,T=T_f, P=1e05) "[kg/m^3]"

nu = mu/rho "[m^2/s]"

cp=Cp(Water,T=T_f, P = 1e05) "[J/kg-C]"

Gr = g*beta*(T_s-T_inf)*L_c^3/nu^2

k=Conductivity(Water,T=T_f, P = 1e05) "[W/m-C]"

Pr = mu * cp / k "…Prandtl No."

Ra = Gr * Pr "…Grashoff No."

"We get: Ra = 1.695E10"

"Case 1: Upper surface heated:"

"Nu = 0.54 * Ra^(1/4) for 10^4 < Ra < 10^7;

Nu = 0.15 * Ra^(1/3) for 10^7 < Ra < 10^11"

"Therefore:"

Nusselt_upper = 0.15* Ra^(1/3) "For Ra > 10^7; here Ra = 1.695 * 10^10"

Nusselt_upper = h_upper*L_c/k "finds h_upper"

"Case 2: Lower surface heated:"

Nusselt_lower = 0.27* Ra^(1/4) "For 10^5 < Ra < 10^11; here Ra = 1.695 * 10^10"

Nusselt_lower = h_lower*L_c/k "finds h_lower"

Results:

Unit Settings: SI C Pa J mass deg

β = 0.0003859 [1/K]	cp = 4182 [J/kg-C]	g = 9.81 [m/s^2]
Gr = 3.834E+09	h_{lower} = 270.9 [W/m^2-C]	h_{upper} = 1071 [W/m^2-C]
k = 0.6178 [W/m-C]	L = 1 [m]	L_c = 0.2222 [m]
μ = 0.0006533 [kg/m-s]	ν = 6.584E-07 [m^2/s]	$Nusselt_{lower}$ = 97.43
$Nusselt_{upper}$ = 385.3	Pr = 4.422	Ra = 1.695E+10
ρ = 992.2 [kg/m^3]	T_f = 40 [C]	T_{inf} = 20 [C]
T_s = 60 [C]	W = 0.8 [m]	

Thus:

h_upper = 1071 W/m^2.C …. Convection coeff. when upper surface is heated….Ans.

h_lower = 270.9 W/m^2.C …. Convection coeff. when lower surface is heated….Ans.

===

"**Prob. 2A2.2.2.** A square plate (0.5 m × 0.5 m) with one surface insulated and the other surface maintained at a temp of 385 K is placed in ambient air at 315 K. Calculate the average heat transfer coeff for free convection for the following orientations of the hot surface: (a) The plate is horizontal and hot surface faces up (b) The plate is horizontal and the hot surface faces down. [VTU – Dec. 09–Jan. 2010]

(c) In addition, find h when plate is held vertical"

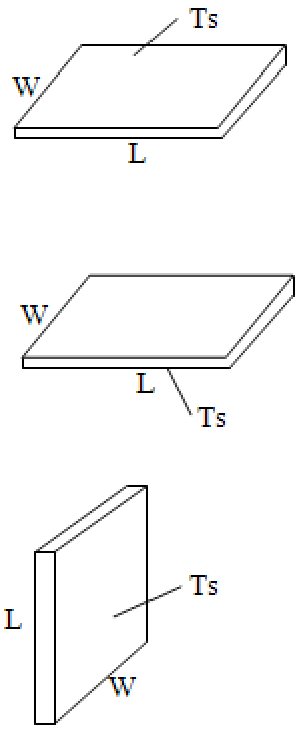


Fig.Prob.2A2.2.2(a),(b) and (c).

EES Solution:

For the horizontal plate, let us use an EES PROCEDURE. This PROCEDURE is suitable for either Air or Water as the fluid. Then, in the main program, we will call the PROCEDURE and get the desired parameters.

```
$UnitSystem SI Pa C J

PROCEDURE NC_HPlate(Fluid$, T_s, T_inf, L, W: Gr, Ra, Nusselt_upper, h_upper, Nusselt_lower, h_lower)

"Nat. convection (NC) on Horizl plate(HPlare), with Fluid$: Water or Air"
"Inputs: T_s (C), T_inf (C), L (m), W (m)"
"Outputs: Gr, Ra, Nusselt_bar, h_bar (W/m^2-C), Q (W)"
```

```
Lc := (L*W)/(2*(L+W)) "Lc for horizl plate"
T_f = (T_s+T_inf)/2
g := 9.81[m/s^2]

IF (Fluid$ = 'Water') Then
    beta:=VolExpCoef(Water,T=T_f, P=1.01325e05)
    mu:=Viscosity(Water,T=T_f, P = 1.01325e05)
    rho:=Density(Water,T=T_f, P=1.01325e05)
    nu := mu/rho
    cp:=Cp(Water,T=T_f, P = 1.01325e05)
    k:=Conductivity(Water,T=T_f, P = 1.01325e05)
ELSE
    IF (Fluid$ = 'Air') Then
    beta := 1/(T_f + 273)
    mu:=Viscosity(Air,T=T_f)
    rho:=Density(Air,T=T_f,P=1.01325e05)
    nu := mu/rho
    cp:=Cp(Air,T=T_f)
    k:=Conductivity(Air,T=T_f)
    EndIF
EndIf

Gr := g*beta*(T_s-T_inf)*Lc^3/nu^2
Pr := mu*cp/k
Ra := Gr*Pr

"Case 1: Upper surface heated:"

IF (Ra <= 10^7) Then
    Nusselt_upper = 0.54 * Ra^(1/4)
ENDIF
IF (Ra > 10^7) Then
Nusselt_upper = 0.15 * Ra^(1/3)
ENDIF

h_upper := Nusselt_upper * k/Lc "finds h_upper"
```

```
"Case 2: Lower surface heated:"
Nusselt_lower := 0.27* Ra^(1/4) "For 10^5 <Ra < 10^11"
h_lower := Nusselt_lower * k/Lc "finds h_lower"

END

"======================================================================="

$UnitSystem SI Pa C J

"Data:"

Fluid$='Air'

L = 0.5[m]
W = 0.5[m]
T_s = 385-273[C]
T_inf = 315-273[C]

CALL NC_HPlate(Fluid$, T_s, T_inf, L, W : Gr, Ra, Nusselt_upper, h_upper, Nusselt_lower, h_lower)
```

"When the plate is held vertical:"

"Now characteristic dimension is the vertical height = L"

"Then:"

```
T_f = (T_s+T_inf)/2 "…film temp."
beta = 1/(T_f+273) "[1/K] … coeff. of vol. expn"
g = 9.81[m/s^2]
k=Conductivity(Air,T=T_f) "[W/m-C]"
mu=Viscosity(Air,T=T_f) "[kg/m-s]"
rho=Density(Air,T=T_f,P=1.01325E05) "[kg/m^3]"
nu = mu/rho "[m^2/s]"
Pr=Prandtl(Air,T=T_f)
Gr_vert = g * beta * (T_s-T_inf) * L^3 / nu^2 "…Grashoff No."

Ra_vert = Gr_vert * Pr "…Rayleigh No."

"Calculate Nusselt_vert: Use eqn of Churchill and Chu for entire range of Ra_L"

AA = 0.387*Ra_vert^(1/6)
BB = (0.492/Pr)^(9/16)
CC = (1+BB)^(8/27)
Nusselt_vert = (0.825 + AA/CC)^2 "Finds Nu"
h_vert = Nusselt_vert * k / L "Finds h"
```

Now, press F2 and get the Results:

Results: Main:

Main | NC_HPlate

Unit Settings: SI C Pa J mass deg

AA = 10.56	BB = 0.8095	β = 0.002857 [1/K]
CC = 1.192	Fluid$ = 'Air'	g = 9.81 [m/s^2]
Gr = 8.979E+06	Gr_{vert} = 5.747E+08	h_{lower} = 3.188 [W/m^2-C]
h_{upper} = 6.377 [W/m^2-C]	h_{vert} = 5.492 [W/m^2-C]	k = 0.02931 [W/m-C]
L = 0.5 [m]	μ = 0.00002083 [kg/m-s]	ν = 0.00002066 [m^2/s]
$Nusselt_{lower}$ = 13.6	$Nusselt_{upper}$ = 27.19	$Nusselt_{vert}$ = 93.68
Pr = 0.7163	Ra = 6.432E+06	Ra_{vert} = 4.116E+08
ρ = 1.008 [kg/m^3]	T_f = 77 [C]	T_{inf} = 42 [C]
T_s = 112 [C]	W = 0.5 [m]	

Results: PROCEDURE:

Main | NC_HPlate

Local variables in Procedure NC_HPlate (1 call, 0.02 sec)

β=0.002857 [1/K]	cp=1008 [J/kg-C]	Fluid$='Air'
g=9.81 [m/s^2]	Gr=8.979E+06	h_{lower}=3.188 [W/m^2-C]
h_{upper}=6.377 [W/m^2-C]	k=0.02931 [W/m-C]	L=0.5 [m]
Lc=0.125 [m]	μ=0.00002083 [kg/m-s]	ν=0.00002066 [m^2/s]
$Nusselt_{lower}$=13.6	$Nusselt_{upper}$=27.19	Pr=0.7163
Ra=6.432E+06	ρ=1.008 [kg/m^3]	T_f=77 [C]
T_{inf}=42 [C]	T_s=112 [C]	W=0.5 [m]

Thus:

h_upper = 6.377 W/m^2.K …. Heat tr. coeff. when plate is horizontal and hot surface faces up….Ans.

h_lower = 3.188 W/m^2.K …. Heat tr. coeff. when plate is horizontal and hot surface faces down….Ans.

h_vert = 5.492 W/m^2.K …. Heat tr. coeff. when plate is held vertical….Ans.

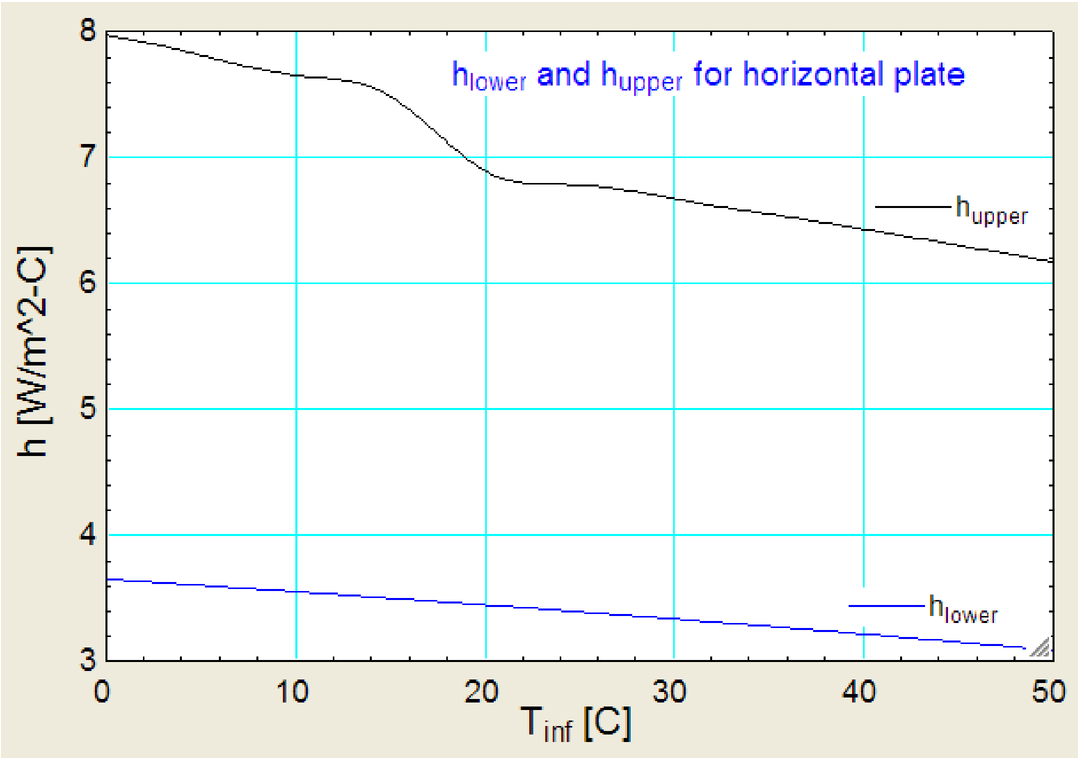
Plot h_upper, h_lower and h_vert as T_inf varies from 0 deg. C to 45 deg.C:

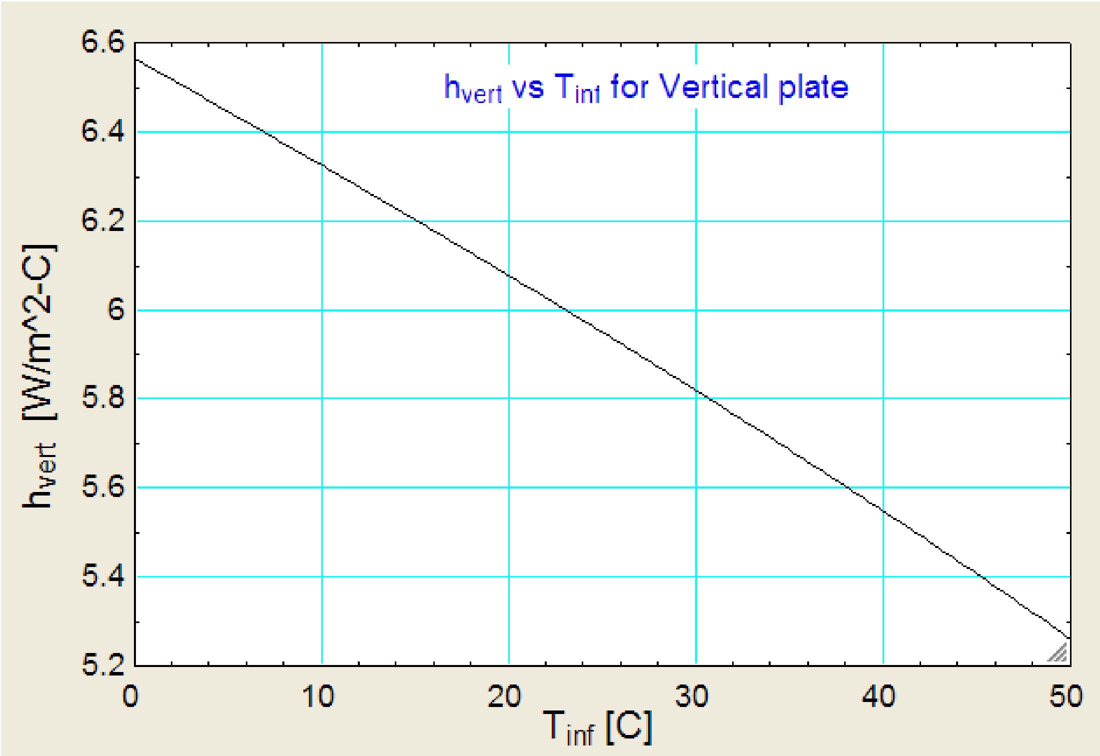
Parametric Table:

Table 1

1..11	T_{inf} [C]	h_{upper} [W/m2-C]	h_{lower} [W/m2-C]	h_{vert} [W/m2-C]
Run 1	0	7.97	3.649	6.564
Run 2	5	7.811	3.6	6.445
Run 3	10	7.649	3.549	6.324
Run 4	15	7.485	3.498	6.201
Run 5	20	6.889	3.445	6.076
Run 6	25	6.78	3.39	5.949
Run 7	30	6.666	3.333	5.819
Run 8	35	6.549	3.274	5.685
Run 9	40	6.427	3.213	5.548
Run 10	42	6.377	3.188	5.492
Run 11	50	6.166	3.083	5.26

And, the Plots:





"**Prob. 2A2.2.3**. A steam pipe, 100 mm in diameter, maintained at 170 C is exposed to air at 30 C. Length of the pipe is 2 m and is kept horizontal. Determine the heat lost by the pipe per hour. [VTU – May–2007]

Also calculate the heat lost by radiation if emissivity of surface is 0.9. Plot Q_total for ambient temperatures varying from 10 C to 30 C."

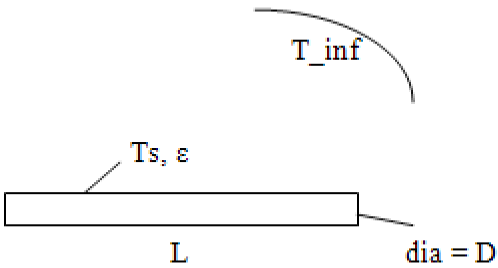


Fig.Prob.2A2.2.3

EES Solution:

"Data:"

L = 2[m]

D= 0.1[m]

T_s = 170[C]

T_inf = 30[C]

T_f = (T_s+T_inf)/2 "…film temp."

beta = 1/(T_f + 273) "[1/K] … coeff. of vol. expn"

g = 9.81[m/s^2]

"Properties of Air at Tf:"

mu=Viscosity(Air,T=T_f) "[kg/m-s]"

rho=Density(Air,T=T_f,P=1E05) "[kg/m^3]"

nu = mu/rho "[m^2/s]"

cp=Cp(Air,T=T_f) "[J/kg-C]"

k=Conductivity(Air,T=T_f) "[W/m-C]"

"Calculations:"

"Calculate Nusselts No.:"

"Characteristic dimension for horizontal pipe is the diameter, D"

"Therefore:"

Gr_D = g * beta * (T_s-T_inf) * D^3 / nu^2 "…Grashoff No."

Pr = mu * cp / k "Prandtl No."

Ra_D = Gr_D * Pr "…Rayleigh No."

"We get Ra_D = 4.804E06 i.e. < 10^12

Then, for Nusselts No. we use Churchill and Chu's relation:"

Nusselt = (0.6 + (0.387 * Ra_D^(1/6)) / (1 + (0.559 / Pr)^(9/16))^(8/27))^2 "…Finds Nusselts No."

Nusselt = h * D / k "….finds h"

Q_conv = h * (pi * D * L) * (T_s – T_inf) "[W] … heat lost by convection from the pipe surface"

Qperhour = Q_conv * 3600 "[Joules / h]"

"Heat loss by radiation:"

epsilon = 0.9 "emissivity"

sigma = 5.67E-08 "W/m^2-K^4 …. Stefan Boltzmann constant"

Q_rad = epsilon * sigma * (pi * D * L) * ((T_s + 273)^4 – (T_inf + 273)^4)

Q_tot = Q_conv + Q_rad "[W] … total heat loss"

Results:

Unit Settings: SI C Pa J mass deg

β = 0.002681 [1/K]	cp = 1010 [J/kg-C]	D = 0.1 [m]
ε = 0.9	g = 9.81 [m/s²]	Gr_D = 6.748E+06
h = 7.054 [W/m²-c]	k = 0.03095 [W/m-C]	L = 2 [m]
μ = 0.00002181 [kg/m-s]	ν = 0.00002336 [m²/s]	Nusselt = 22.79
Pr = 0.7118	Qperhour = 2.234E+06 [J]	Q_{conv} = 620.5 [W]
Q_{rad} = 964.6 [W]	Q_{tot} = 1585 [W]	Ra_D = 4.804E+06
ρ = 0.9337 [kg/m³]	σ = 5.670E-08 [W/m²-K⁴]	T_f = 100 [C]
T_{inf} = 30 [C]	T_s = 170 [C]	

Thus:

Q_conv = 620.5 W …. Heat loss by convection

Qperhour = 2.234E06 Joules…. Heat transfer by convection only, from the pipe surface per hour

Note that this is the heat loss only by convection.

Q_rad = 964.6 W.

Compare this to Q = 620.5 W by convection.

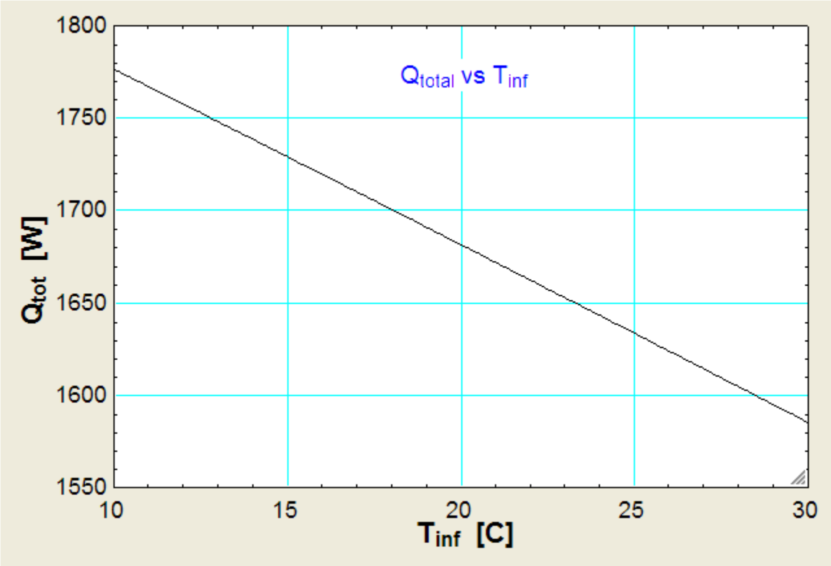
Then, total heat loss becomes: Q_tot = (964.6 + 620.5) W = 1585.1 W

==

Plot Q_total against T_inf:

Table 1

1..21	T_{inf} [C]	Q_{conv} [W]	Q_{rad} [W]	Q_{tot} [W]
Run 1	10	747.2	1029	1776
Run 2	11	740.6	1026	1767
Run 3	12	734.1	1023	1757
Run 4	13	727.6	1020	1748
Run 5	14	721.2	1017	1739
Run 6	15	714.7	1014	1729
Run 7	16	708.3	1011	1720
Run 8	17	701.9	1008	1710
Run 9	18	695.5	1005	1700
Run 10	19	689.1	1002	1691
Run 11	20	682.8	998.6	1681
Run 12	21	676.5	995.3	1672
Run 13	22	670.2	992	1662
Run 14	23	663.9	988.7	1653
Run 15	24	657.6	985.4	1643
Run 16	25	651.4	982	1633
Run 17	26	645.2	978.6	1624
Run 18	27	639	975.2	1614
Run 19	28	632.8	971.7	1604
Run 20	29	626.7	968.2	1595
Run 21	30	620.5	964.6	1585



==

Prob. 2A2.2.4. A horizontal rod 5 mm in dia is immersed in water maintained at 18 C. If the rod surface temp is 56 C, estimate the free convection heat transfer rate per unit length of the rod. [Ref: 3]

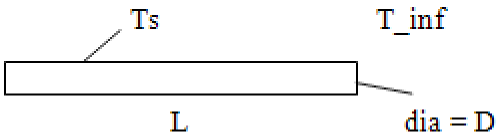


Fig.Prob.2A2.2.4

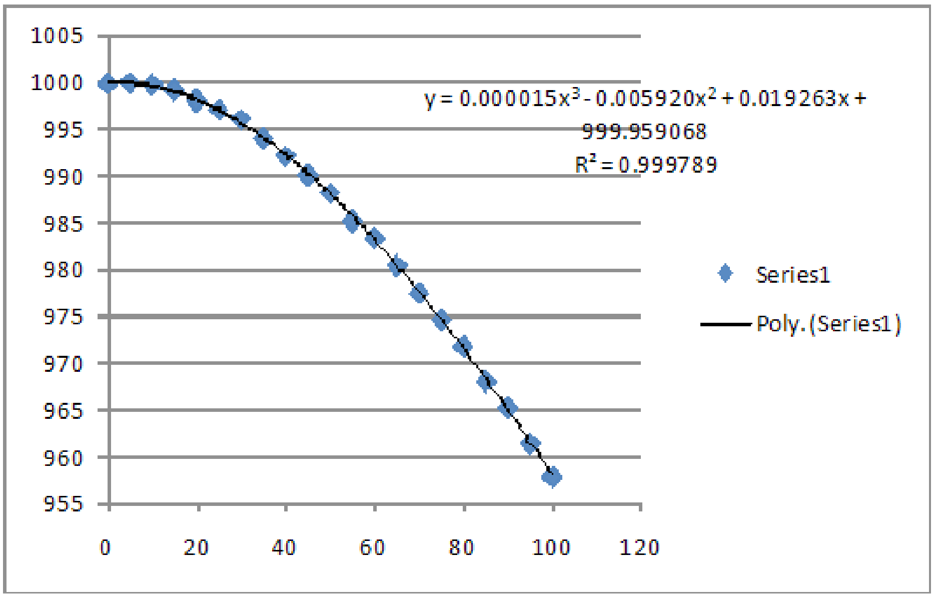
Mathcad Solution:

Let us solve this problem in Mathcad.
Mathcad does not have built in functions for properties of sat. water.
So, let us write our own functions in Mathcad for properties of water.
For this, we first find out curve fit equations and then write Mathcad functions:

Properties of Sat. water (Ref: HMT by Cengel, 3rd Ed.

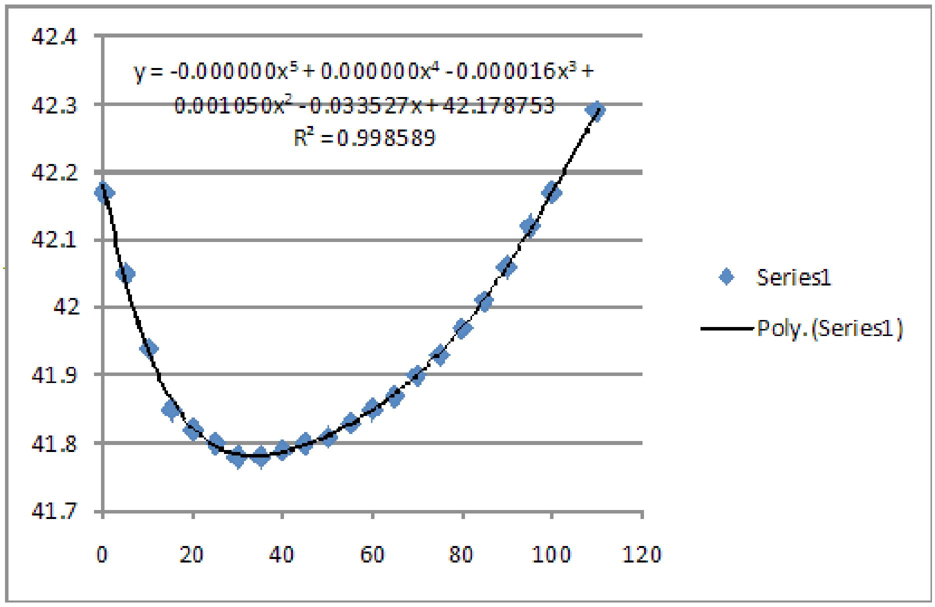
Curve fit eqns (Using Excel and curveExpert softwares):

1. Density in Range:0.01 to 100 deg.C:



---

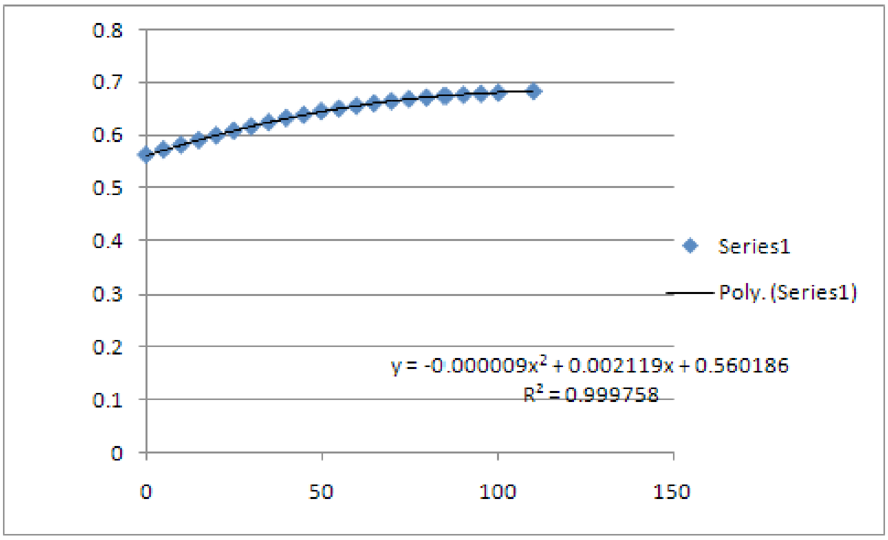
2. Sp. heat in range:0.01 to 100 deg.C:



In the above graph, multiply the y-values by 100 to get values of sp.heats.

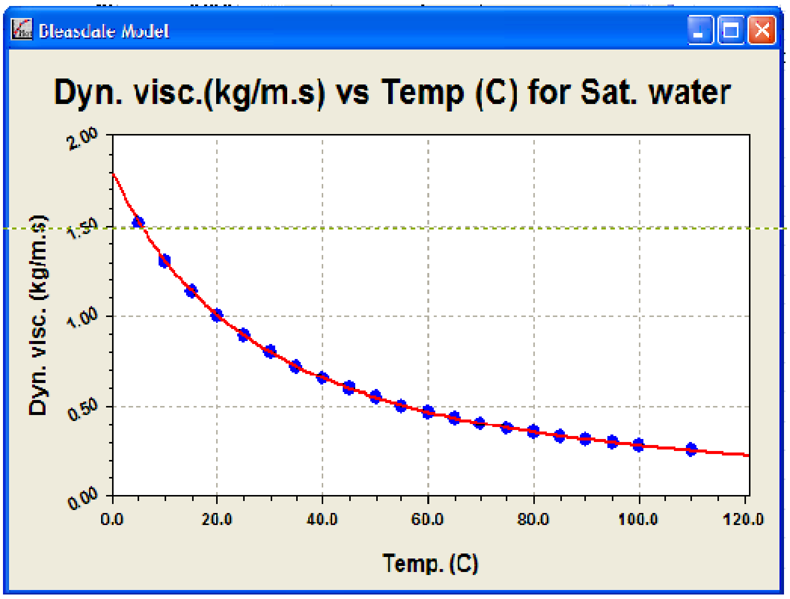
---

3. Th.cond. in range:0.01 to 100 deg.C:



---

4. Dyn. visc. in range:0.01 to 100 deg.C:

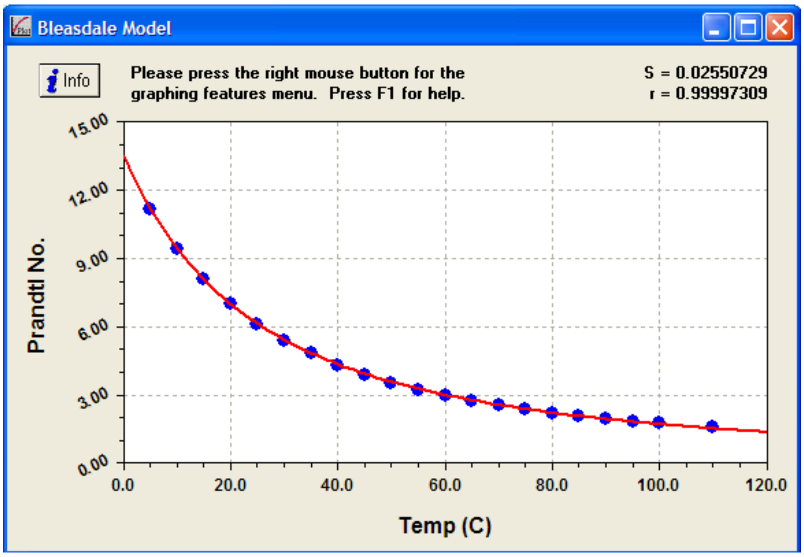


In the above graph, divide the y-values by 1000 to get values of Dyn. Visc.

Bleasdale Model: y=(a+bx)^(-1/c)	
Coefficient Data:	
a =	0.692354
b =	0.015338
c =	0.628922

---

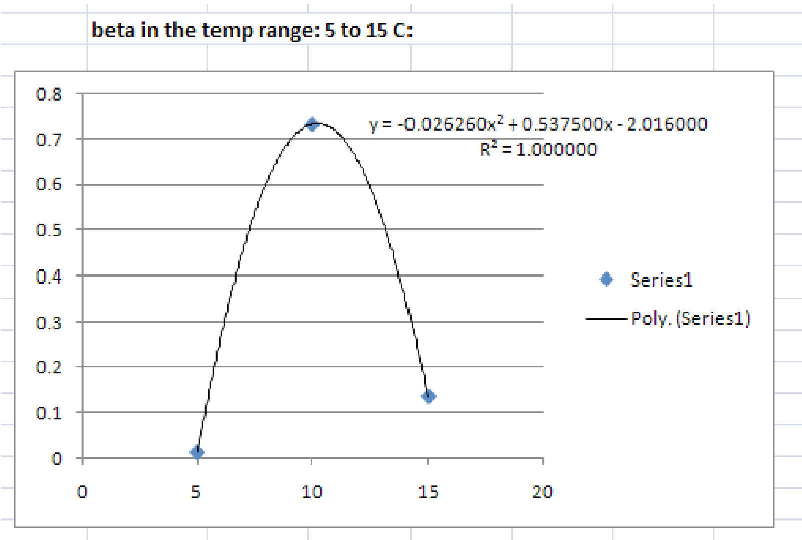
5. **Prandtl No. in the range: 0.01 C to 100 C:**



Bleasdale Model: y=(a+bx)^(-1/c)		
Coefficient Data:		
a =	0.21549637	
b =	0.00512374	
c =	0.58913204	

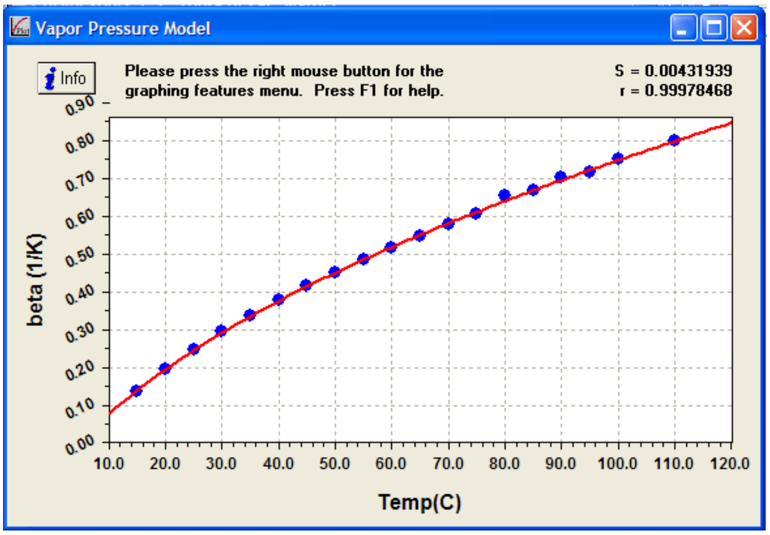
---

6. coeff of vol. expn. in range:0.01 to 100 deg.C:



In the above graph, divide the y-values by 1000 to get values of beta.

In the range: 15 C to 100 C:



In the above graph, divide the y-values by 1000 to get values of beta.

Vapor Pressure Model: y=exp(a+b/x+cln(x))				
Coefficient Data:				
a =	-2.89679			
b =	-9.96542			
c =	0.587896			

===

Mathcad functions:

1. **Density of Water:**

T in deg. C, rho in kg/m^3:

```
rho_water(T) := | return "T must be between 0.01  and 100 C !!"  if  T < 0.01
                | return "T must be between 0.01 C and 100 C !!"  if  T > 100
                | otherwise
                |   | a ← 0.000015
                |   | b ← 0.00592
                |   | c ← 0.019623
                |   | d ← 999.959068
                |   | rho ← (a·T^3 − b·T^2 + c·T + d)
```

Ex: rho_water(50) = 988.015

---

2. **Sp. Heat of Water:**

T in deg. C, cp in J/kg.K:

```
cp_water(T) := | return "T must be between 0.01  and 100 C !!"  if  T < 0.01
               | return "T must be between 0.01 C and 100 C !!"  if  T > 100
               | otherwise
               |   | a ← 0.0
               |   | b ← 0.00000012
               |   | c ← 0.00001568
               |   | d ← 0.00105007
               |   | e ← 0.03352666
               |   | f ← 42.17875262
               |   | cp ← (a·T^5 + b·T^4 − c·T^3 + d·T^2 − e·T + f)·100
```

Ex: cp_water(50) = $4.192 \cdot 10^{3}$

---

3. **Th. cond. of Water:**

T in deg. C, k in W/m.K:

```
k_water(T) := | return "T must be between 0.01  and 100 C !!"  if  T < 0.01
              | return "T must be between 0.01 C and 100 C !!"  if  T > 100
              | otherwise
              |   | a ← 0.000009
              |   | b ← 0.002119
              |   | c ← 0.560186
              |   | k ← (- a·T^2 + b·T + c)
```

Ex: k_water(10) = 0.58

---

4. **Dyn. Visc. Of Water:**

T in deg. C, mu in kg/m.s:

```
mu_water(T) := | return "T must be between 0.01  and 100 C !!"  if  T < 0.01
               | return "T must be between 0.01 C and 100 C !!"  if  T > 100
               | otherwise
               |   | a ← 0.692354
               |   | b ← 0.015338
               |   | c ← 0.628922
               |   | mu ← (a + b·T)^(-1/c) · 10^-3
```

Ex: $\text{mu\_water}(100) = 2.801 \cdot 10^{-4}$

---

5. **Prandtl No. of Water:**

T in deg. C, Prandtl No.:

```
Pr_water(T) := | return "T must be between 0.01  and 100 C !!"  if T < 0.01
               | return "T must be between 0.01 C and 100 C !!"  if T > 100
               | otherwise
               |   | a ← 0.21549637
               |   | b ← 0.00512374
               |   | c ← 0.58913204
               |   | Pr ← (a + b·T)^(-1/c)
```

$$Pr \leftarrow (a + b\cdot T)^{\frac{-1}{c}}$$

Ex: $Pr\_water(65) = 2.771$

---

6. **Coeff. of vol. expn for Water:**

T in deg. C, beta in 1/K:

```
beta_water(T) := | return "T must be between 5 and 100 C !!"  if T < 5
                 | return "T must be between 5 and 100 C !!"  if T > 100
                 | return (-0.02626·T^2 + 0.5375·T - 2.016)·10^-3  if T ≤ 15
                 | otherwise
                 |   | a ← -2.8967935
                 |   | b ← -9.9654212
                 |   | c ← 0.58789598
                 |   | beta ← (exp(a + b/T + c·ln(T)))·10^-3
```

$$beta \leftarrow \left(\exp\left(a + \frac{b}{T} + c\cdot \ln(T)\right)\right)\cdot 10^{-3}$$

Ex: $beta\_water(65) = 5.51\cdot 10^{-4}$

===

Now, solve the problem:

Prob. 2A2.2.4. A horizontal rod 5 mm in dia is immersed in water maintained at 18 C. If the rod surface temp is 56 C, estimate the free convection heat transfer rate per unit length of the rod.

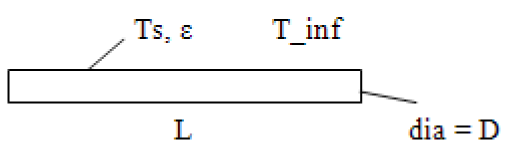


Fig.Prob.2A2.2.4

Mathcad Solution:

Data:

$D := 0.005$ m $L := 1$ m $T_{inf} := 18$ C $T_s := 56$ C

$T_f := \frac{T_{inf} + T_s}{2}$ i.e. $T_f = 37$ C $g := 9.81$ m/s^2

Calculations:

Properties of water at T_f:

$rho := rho\_water(T_f)$ i.e. $rho = 993.34$ kg/m^3

$cp := cp\_water(T_f)$ i.e. $cp = 4.181 \cdot 10^3$ K/kg.K

$mu := mu\_water(T_f)$ i.e. $mu = 6.926 \cdot 10^{-4}$ kg/m.s

$k := k\_water(T_f)$ i.e. $k = 0.626$ W/m.K

$Pr := Pr\_water(T_f)$ i.e. $Pr = 4.636$

$\beta := beta\_water(T_f)$ i.e. $\beta = 3.523 \cdot 10^{-4}$ 1/K

$\nu := \frac{mu}{rho}$ i.e. $\nu = 6.972 \cdot 10^{-7}$ m^2/s

Nusselts no. in free convection: We use Churchill and Chu's relation:

$\Delta T := T_s - T_{inf}$ i.e. $\Delta T = 38$ C

$Gr_D := \frac{g \cdot \beta \cdot \Delta T \cdot D^3}{\nu^2}$ i.e. $Gr_D = 3.377 \cdot 10^4$ ....Grashoff No.

$Ra_D := Gr_D \cdot Pr$ i.e. $Ra_D = 1.566 \cdot 10^5$ ....Rayleigh No.

Churchill and Chu's relation:

$$Nu_D := \left[0.6 + \frac{0.387 \cdot Ra_D^{\frac{1}{6}}}{\left[1 + \left(\frac{0.559}{Pr} \right)^{\frac{9}{16}} \right]^{\frac{8}{27}}} \right]^2$$ ...for wide range of Ra, Ra<=10^12

i.e. $Nu_D = 10.408$ ....Nusselts No.

Therefore: heat transfer coeff. h:

$h := \frac{Nu_D \cdot k}{D}$ i.e. $h = 1.304 \cdot 10^3$ W/m^2.K..... Ans.

Heat transfer rate, Q:

$Q := h \cdot \pi \cdot D \cdot L \cdot \Delta T$ i.e. $Q = 778.123$ W ..... Ans.

If the surface temp varies from 40 C to 70 C, plot the variation of Q with Ts:

Write all quantities as functions of T_s and T_{inf}, so that we can vary T_{inf} also, if required:

$$T_f(T_s, T_{inf}) := \frac{T_{inf} + T_s}{2}$$

Calculations:

Properties of water at T_f:

$$rho(T_s, T_{inf}) := rho\_water(T_f(T_s, T_{inf}))$$

$$cp(T_s, T_{inf}) := cp\_water(T_f(T_s, T_{inf}))$$

$$mu(T_s, T_{inf}) := mu\_water(T_f(T_s, T_{inf}))$$

$$k(T_s, T_{inf}) := k\_water(T_f(T_s, T_{inf}))$$

$$Pr(T_s, T_{inf}) := Pr\_water(T_f(T_s, T_{inf}))$$

$$\beta(T_s, T_{inf}) := beta\_water(T_f(T_s, T_{inf}))$$

$$\nu(T_s, T_{inf}) := \frac{mu(T_s, T_{inf})}{rho(T_s, T_{inf})}$$

Nusselts no. in free convection: We use Churchill and Chu's relation:

$$\Delta T(T_s, T_{inf}) := T_s - T_{inf}$$

$$Gr_D(T_s, T_{inf}) := \frac{g \cdot \beta(T_s, T_{inf}) \cdot \Delta T(T_s, T_{inf}) \cdot D^3}{\nu(T_s, T_{inf})^2}$$

i.e.

$$Ra_D(T_s, T_{inf}) := Gr_D(T_s, T_{inf}) \cdot Pr(T_s, T_{inf})$$

Churchill and Chu's relation:

$$Nu_D(T_s, T_{inf}) := \left[0.6 + \frac{0.387 \cdot Ra_D(T_s, T_{inf})^{\frac{1}{6}}}{\left[1 + \left(\frac{0.559}{Pr(T_s, T_{inf})}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right]^2$$

...for wide range of Ra, Ra<=10^12

Therefore: heat transfer coeff. h:

$$h(T_s, T_{inf}) := \frac{Nu_D(T_s, T_{inf}) \cdot k(T_s, T_{inf})}{D}$$

Heat transfer rate, Q:

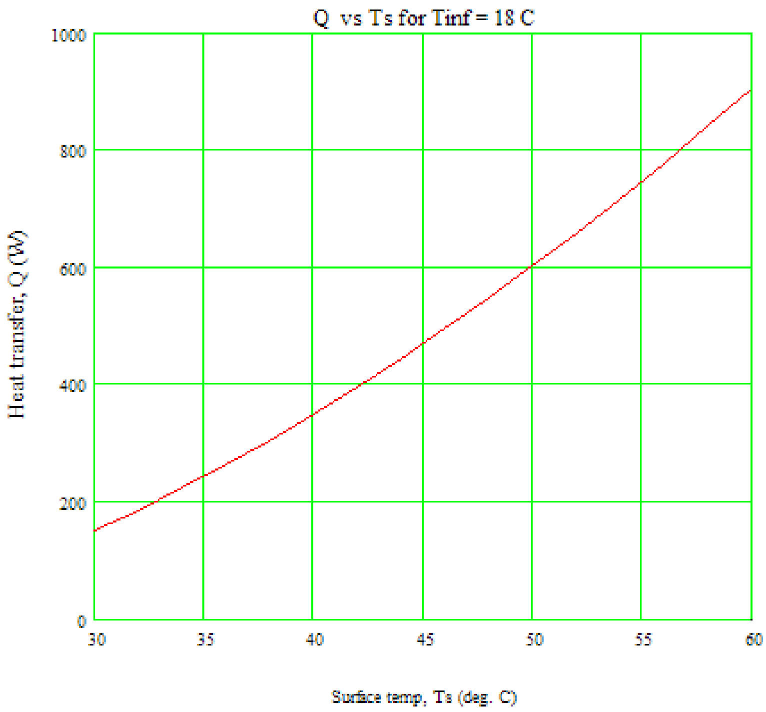
$$Q(T_s, T_{inf}) := h(T_s, T_{inf}) \cdot \pi \cdot D \cdot L \cdot \Delta T(T_s, T_{inf})$$

Now, plot the results for different values of Ts:

Let: $T_s := 30, 32 .. 60$ C

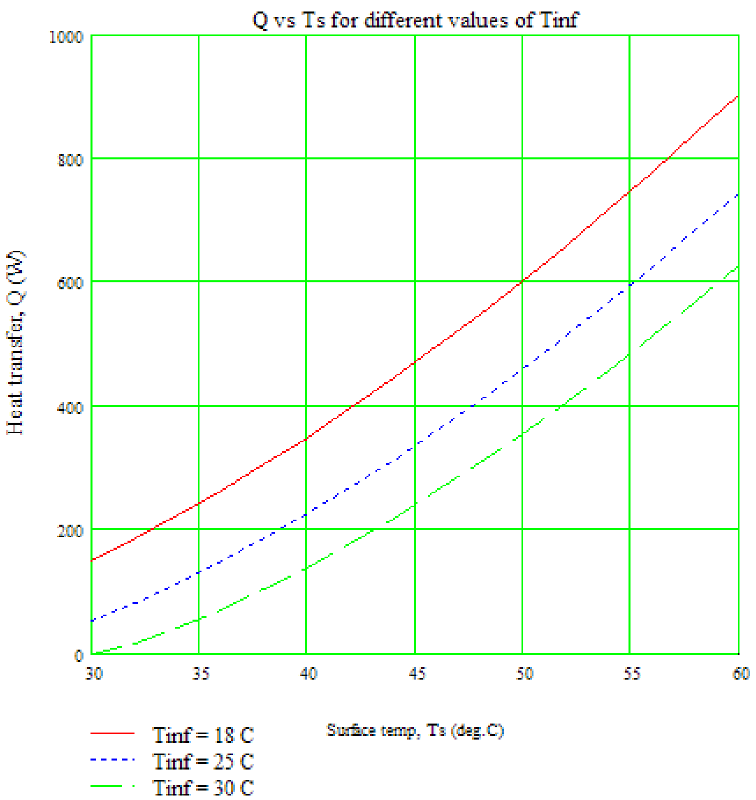
$T_{inf} := 18$ C

T_s	$Q(T_s, T_{inf})$
30	150.981
32	186.372
34	224.104
36	264.098
38	306.29
40	350.626
42	397.061
44	445.558
46	496.081
48	548.6
50	603.087
52	659.519
54	717.871
56	778.123
58	840.255
60	904.248


Q vs Ts for Tinf = 18 C
1000
800
600
400
200
0
30
35
40
45
50
55
60
Heat transfer, Q (W)
Surface temp, Ts (deg. C)

Plot Q vs Ts for different values of T_{inf}:

T_s	$Q(T_s, 18)$	$Q(T_s, 25)$	$Q(T_s, 30)$
30	150.981	53.33	0
32	186.372	82.41	17.884
34	224.104	114.545	42.903
36	264.098	149.443	72.131
38	306.29	186.914	104.758
40	350.626	226.82	140.368
42	397.061	269.059	178.704
44	445.558	313.551	219.589
46	496.081	360.228	262.893
48	548.6	409.037	308.516
50	603.087	459.931	356.378
52	659.519	512.869	406.414
54	717.871	567.816	458.569
56	778.123	624.74	512.795
58	840.255	683.612	569.051
60	904.248	744.405	627.3



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Prob. 2A2.2.5.: A 300 W cylindrical resistance heater is 0.75 m long and 0.5 cm in diameter. The resistance wire is placed horizontally in water at 20 C. Determine the outer surface temp of the heater wire. [Ref:2]

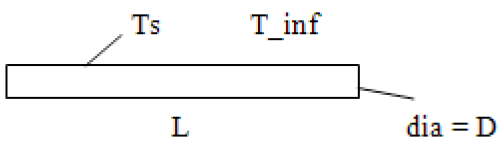


Fig.Prob.2A2.2.5

Mathcad Solution:

We use the functions for properties of water, written earlier in Prob. 2A2.2.4.

Data:

$D := 0.005$ m $L := 0.75$ m $T_{inf} := 20$ C $g := 9.81$ m/s^2

$Q_{heater} := 300$ W

Let the surface temp be T_s.

Then, we write all quantities as functions of T_s and T_{inf}, so that plots can be drawn conveniently, later:

$$T_f(T_s, T_{inf}) := \frac{T_{inf} + T_s}{2}$$ ...film temp.

Calculations:

Properties of water at T_f:

$$rho(T_s, T_{inf}) := rho\_water(T_f(T_s, T_{inf}))$$

$$cp(T_s, T_{inf}) := cp\_water(T_f(T_s, T_{inf}))$$

$$mu(T_s, T_{inf}) := mu\_water(T_f(T_s, T_{inf}))$$

$$k(T_s, T_{inf}) := k\_water(T_f(T_s, T_{inf}))$$

$$Pr(T_s, T_{inf}) := Pr\_water(T_f(T_s, T_{inf}))$$

$$\beta(T_s, T_{inf}) := beta\_water(T_f(T_s, T_{inf}))$$

$$\nu(T_s, T_{inf}) := \frac{mu(T_s, T_{inf})}{rho(T_s, T_{inf})}$$

Now:

$$\Delta T(T_s, T_{inf}) := T_s - T_{inf}$$

$$Gr_D(T_s, T_{inf}) := \frac{g \cdot \beta(T_s, T_{inf}) \cdot \Delta T(T_s, T_{inf}) \cdot D^3}{\nu(T_s, T_{inf})^2}$$ ....Grashoff No.

$$Ra_D(T_s, T_{inf}) := Gr_D(T_s, T_{inf}) \cdot Pr(T_s, T_{inf})$$ .....Rayleigh No.

Churchill and Chu's relation for Nusselts No.:

$$Nu_D(T_s, T_{inf}) := \left[0.6 + \frac{0.387 \cdot Ra_D(T_s, T_{inf})^{\frac{1}{6}}}{\left[1 + \left(\frac{0.559}{Pr(T_s, T_{inf})} \right)^{\frac{9}{16}} \right]^{\frac{8}{27}}} \right]^2$$ ...for wide range of Ra, Ra<=10^12

$$h(T_s, T_{inf}) := \frac{Nu_D(T_s, T_{inf}) \cdot k(T_s, T_{inf})}{D}$$

Heat transfer rate, Q:

$Q(T_s, T_{inf}) := h(T_s, T_{inf}) \cdot \pi \cdot D \cdot L \cdot \Delta T(T_s, T_{inf})$

This should be equal to Q = 300 W, from data.

Use Solve block of Mathcad to determine T_s. Start with a guess value for T_s:

Let: $T_s := 30$ C .... guess value

Given

$Q(T_s, T_{inf}) = Q_{heater}$

$T_s(T_{inf}, Q_{heater}) := \text{Find}(T_s)$

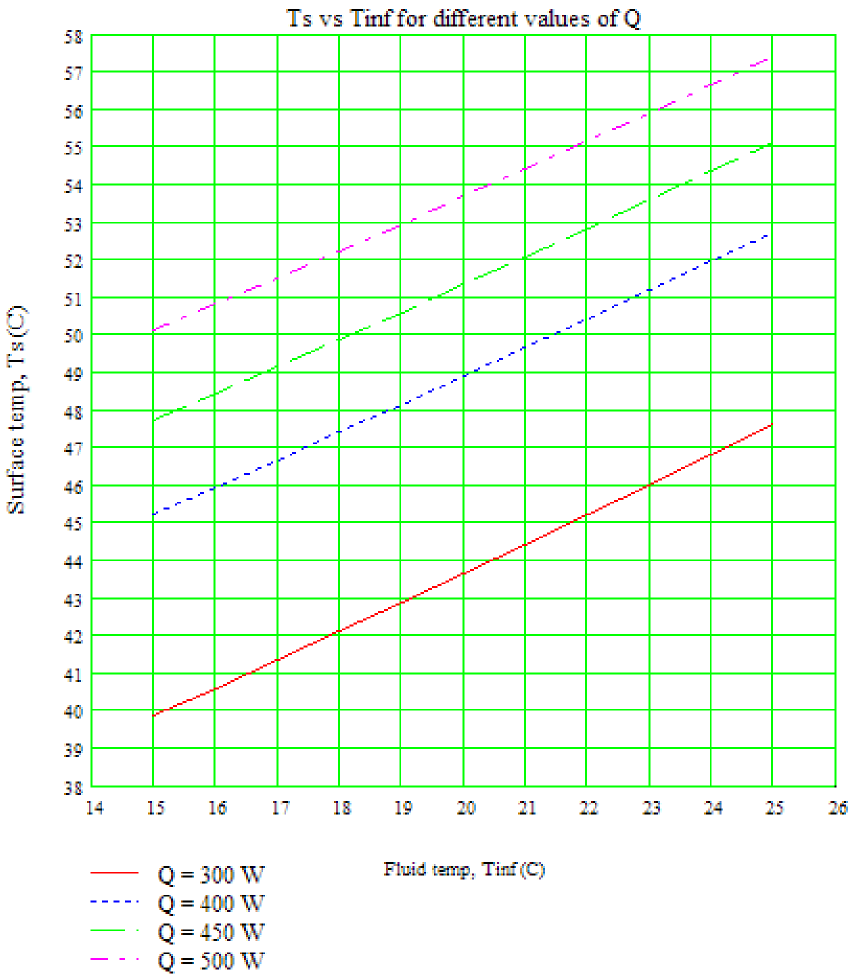
i.e. $T_s(T_{inf}, Q_{heater}) = 43.664$ **C.... temp of outer surface of heater .... Ans.**

Now, note that since Ts is obtained as a function of Tinf and Q_{heater}, it is easy to plot the graphs at different Tinf and Qheater values:

Plot T_s for T_{inf} varying from 15 to 25 deg.C:

$T_{inf} := 15, 16..25$ C....variation of T_{inf}

T_{inf}	$T_s(T_{inf}, 300)$	$T_s(T_{inf}, 400)$	$T_s(T_{inf}, 450)$	$T_s(T_{inf}, 500)$
15	39.874	45.256	47.75	50.138
16	40.615	45.971	48.454	50.833
17	41.365	46.694	49.167	51.537
18	42.124	47.426	49.889	52.249
19	42.89	48.166	50.618	52.968
20	43.664	48.914	51.354	53.695
21	44.446	49.669	52.098	54.429
22	45.234	50.43	52.849	55.17
23	46.029	51.198	53.606	55.917
24	46.829	51.973	54.37	56.671
25	47.636	52.753	55.139	57.431



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Prob. 2A2.2.6: Water in a 40 litre tank is heated from 15 C to 45 C by a spherical heater of 6 cm dia and the surface temp of heater is 85 C. Determine how long the heater should be kept on. [Ref: 2]

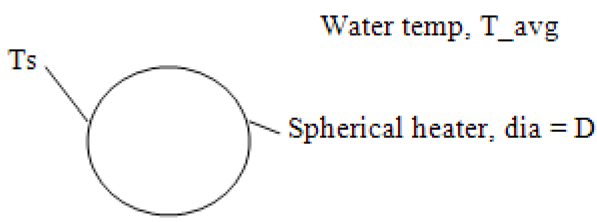


Fig.Prob.2A2.2.6

Mathcad Solution:

We use the functions for properties of water, written earlier in Prob. 2A2.2.4.

Data:

$D := 0.06$ m $\quad T_{ini} := 15$ C $\quad T_{final} := 45$ C $\quad T_s := 85$ C

$T_{avg} := \frac{T_{ini} + T_{final}}{2}$ i.e. $T_{avg} = 30$ C $\quad g := 9.81$ m/s^2 $\quad Vol := 40 \cdot 10^{-3}$ m^3

$T_f := \frac{T_{avg} + T_s}{2}$ i.e. $T_f = 57.5$ C .... film temp.

Calculations:

Properties of water at T_f:

$rho := rho\_water(T_f)$ i.e. $rho = 984.366$ kg/m^3

$cp := cp\_water(T_f)$ i.e. $cp = 4.205 \cdot 10^3$ K/kg.K

$mu := mu\_water(T_f)$ i.e. $mu = 4.86 \cdot 10^{-4}$ kg/m.s

$k := k\_water(T_f)$ i.e. $k = 0.652$ W/m.K

$Pr := Pr\_water(T_f)$ i.e. $Pr = 3.135$

$\beta := beta\_water(T_f)$ i.e. $\beta = 5.025 \cdot 10^{-4}$ 1/K

$\nu := \frac{mu}{rho}$ i.e. $\nu = 4.937 \cdot 10^{-7}$ m^2/s

Now:

$\Delta T := T_s - T_{avg}$ i.e. $\Delta T = 55$ C

$Gr_D := \frac{g \cdot \beta \cdot \Delta T \cdot D^3}{\nu^2}$ i.e. $Gr_D = 2.403 \cdot 10^8$ ....Grashoff No.

$Ra_D := Gr_D \cdot Pr$ i.e. $Ra_D = 7.532 \cdot 10^8$ ....Rayleigh No.

Relation for Nusselts No. for a sphere in free convection from Ref. [2]:

$$Nu_D := 2 + \frac{0.589 \cdot Ra_D^{\frac{1}{4}}}{\left[1 + \left(\frac{0.469}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}}$$

...for wide range of Ra, Ra<=10^12, Pr > = 0.7

i.e. $Nu_D = 87.576$ ....Nusselts No.

Therefore: heat transfer coeff. h:

$h := \frac{Nu_D \cdot k}{D}$ i.e. $h = 952.057$ **W/m^2.K..... Ans.**

Heat transfer rate, Q:

$Q := h \cdot \pi \cdot D^2 \cdot \Delta T$ i.e. $Q = 592.213$ **W ..... Ans.**

Heat transfer to water to raise the temp from T_{ini} to T_{final}:

$Q_{water} := rho \cdot Vol \cdot cp \cdot (T_{final} - T_{ini})$ J

i.e. $Q_{water} = 4.968 \cdot 10^6$ J

Therefore: time required to heat this water:

$$time := \frac{Q_{water}}{Q}$$

i.e. $time = 8.388 \cdot 10^{3}$ s.... = 2.33 hours .... Ans.

==

Prob. 2A2.2.7.: A horizontal hot water pipe, 6 cm in dia and 10 m long passes through a room whose temp is 27 C. Surface temp and emissivity of the pipe are 73 C and 0.8. Determine the heat loss from the pipe by free convection and radiation.

Also plot the total heat transfer for different ambient temperatures and different emissivity values.

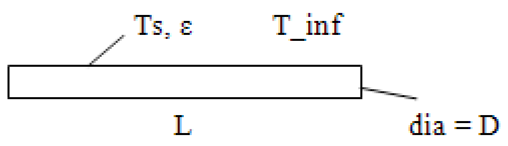


Fig.Prob.2A2.2.7

Mathcad Solution:

Use the Mathcad functions, already written, (see Prob. 2A1.2.5) to get properties of Air:

Data:

$D := 0.06$ m $L := 10$ m $T_s := 73$ C

$T_{inf} := 27$ C $g := 9.81$ m/s^2

$\varepsilon := 0.8$ ...emissivity $\sigma := 5.67 \cdot 10^{-8}$ W/m^2.K^4..... Stefan-Boltzmann const.

Calculations:

Write all quantities as functions of T_s, T_{inf} and e, so that we cal plot the results later by varying any of them:

$$T_f(T_s, T_{inf}) := \frac{T_s + T_{inf}}{2}$$ ....mean film temp in C

i.e. $T_f(T_s, T_{inf}) = 50$ C

Properties of Air at T_f:

$$\rho(T_s, T_{inf}) := rho\_Air(T_f(T_s, T_{inf}) + 273) \quad \text{i.e.} \quad \rho(T_s, T_{inf}) = 1.093 \text{ kg/m^3}$$

$$k(T_s, T_{inf}) := k\_Air(T_f(T_s, T_{inf}) + 273) \quad \text{i.e.} \quad k(T_s, T_{inf}) = 0.028 \quad \text{W/m.K}$$

$$\mu(T_s, T_{inf}) := mu\_Air(T_f(T_s, T_{inf}) + 273) \quad \text{i.e.} \quad \mu(T_s, T_{inf}) = 1.953 \cdot 10^{-5} \quad \text{kg/m.s}$$

$$Pr(T_s, T_{inf}) := Pr\_Air(T_f(T_s, T_{inf}) + 273) \quad \text{i.e.} \quad Pr(T_s, T_{inf}) = 0.704 \quad \text{...Prandtl No.}$$

$$\beta(T_s, T_{inf}) := \frac{1}{T_f(T_s, T_{inf}) + 273} \quad \text{i.e.} \quad \beta(T_s, T_{inf}) = 3.096 \cdot 10^{-3} \quad \text{1/K}$$

$$\nu(T_s, T_{inf}) := \frac{\mu(T_s, T_{inf})}{\rho(T_s, T_{inf})} \quad \text{i.e.} \quad \nu(T_s, T_{inf}) = 1.787 \cdot 10^{-5} \quad \text{m^2/s}$$

Now: $\Delta T(T_s, T_{inf}) := T_s - T_{inf}$

And: $$Gr_D(T_s, T_{inf}) := \frac{g \cdot \beta(T_s, T_{inf}) \cdot \Delta T(T_s, T_{inf}) \cdot D^3}{\nu(T_s, T_{inf})^2}$$

i.e $Gr_D(T_s, T_{inf}) = 9.452 \cdot 10^5$ ....Grashoff No.

Also: $Ra_D(T_s, T_{inf}) := Gr_D(T_s, T_{inf}) \cdot Pr(T_s, T_{inf})$

i.e. $Ra_D(T_s, T_{inf}) = 6.652 \cdot 10^5$ ...Rayleigh No.

Use Churchill – Chu's eqn. to get Nusselts No. for horizontal cylinder in free convection:

$$Nu_D(T_s, T_{inf}) := \left[0.6 + \frac{0.387 \cdot Ra_D(T_s, T_{inf})^{\frac{1}{6}}}{\left[1 + \left(\frac{0.559}{Pr(T_s, T_{inf})}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right]^2$$

...for wide range of Ra, Ra<=10^12

i.e. $Nu_D(T_s, T_{inf}) = 12.958$ ....Nusselts No.

Therefore: heat transfer coeff. h:

$$h(T_s, T_{inf}) := \frac{Nu_D(T_s, T_{inf}) \cdot k(T_s, T_{inf})}{D}$$

i.e. $h(T_s, T_{inf}) = 5.969$ W/m^2.C .... heat transfer coeff.

Heat transfer rate by convection, Q_{conv}:

$$Q_{conv}(T_s, T_{inf}) := h(T_s, T_{inf}) \cdot \pi \cdot D \cdot L \cdot \Delta T(T_s, T_{inf})$$

i.e. $Q_{conv}(T_s, T_{inf}) = 517.519$ **W .... heat transfer by convection.....Ans.**

Heat transfer rate by Radiation, Q_{rad}:

$$Q_{rad}(T_s, T_{inf}, \varepsilon) := \sigma \cdot \varepsilon \cdot (\pi \cdot D \cdot L) \cdot \left[(T_s + 273)^4 - (T_{inf} + 273)^4\right]$$

i.e. $Q_{rad}(T_s, T_{inf}, \varepsilon) = 532.839$ **W .... heat transfer by Radiation.....Ans.**

Therefore, Total heat transfer rate, Q_{tot}:

$$Q_{tot}(T_s, T_{inf}, \varepsilon) := Q_{conv}(T_s, T_{inf}) + Q_{rad}(T_s, T_{inf}, \varepsilon)$$

i.e. $Q_{tot}(T_s, T_{inf}, \varepsilon) = 1.05 \cdot 10^3$ **W .... Total heat transfer ....Ans.**

Now, plot Q_{total} for different values of T_{inf} and ε, T_s being held constant at 73 C:

$T_s := 73$ C

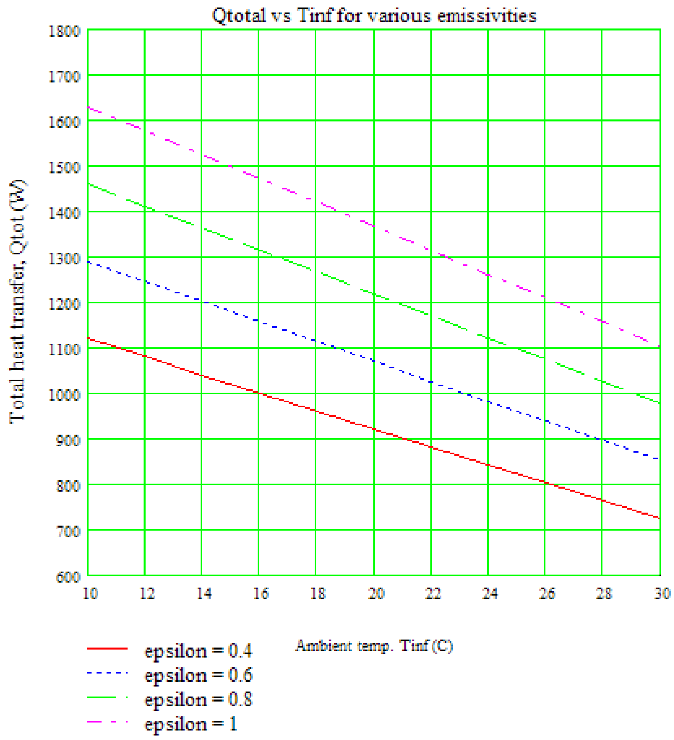
Let T_{inf} vary from 10 C to 30 C; ε from 0.4 to 1.0:

$T_{inf} := 10, 12 .. 30$

Sample Table for ε = 0.8:

T_{inf}	$Q_{conv}(T_s, T_{inf})$	$Q_{rad}(T_s, T_{inf}, 0.8)$	$Q_{tot}(T_s, T_{inf}, 0.8)$
10	783.005	676.974	1459.978
12	750.246	661.305	1411.551
14	717.882	645.303	1363.186
16	685.919	628.964	1314.883
18	654.362	612.281	1266.643
20	623.214	595.251	1218.465
22	592.483	577.868	1170.352
24	562.175	560.129	1122.304
26	532.295	542.027	1074.323
28	502.852	523.559	1026.411
30	473.854	504.718	978.572

Plot of Q_{total} against T_{inf} for different values of emissivity, ε:



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Prob. 2A2.2.8.: Consider a 8 cm dia, 60 W incandescent bulb in a room at 25 C. Emissivity of glass is 0.9. Assuming that 90% of the energy is converted to heat and dissipated by convection and radiation, determine the equilibrium temp of the bulb surface.

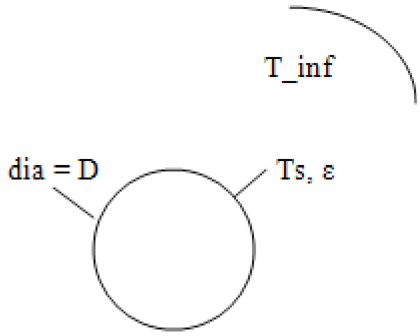


Fig.Prob.2A2.2.8

Mathcad Slution:

Use the Mathcad functions, already written, (see Prob. 2A1.2.5) to get properties of Air:

Data:

$D := 0.08$ m $\quad Q := 60$ W

$T_{inf} := 25$ C $\quad g := 9.81$ m/s^2

$\varepsilon := 0.9$ ...emissivity $\quad \sigma := 5.67 \cdot 10^{-8}$ W/m^2.K^4..... Stefan-Boltzmann const.

Let the surface temp be T_s.

Calculations:

Write all quantities as functions of T_s, T_{inf} and ε, so that we cal plot the results later by varying any of them:

$$T_f(T_s, T_{inf}) := \frac{T_s + T_{inf}}{2}$$ ....mean film temp in C

Properties of Air at T_f:

$\rho(T_s, T_{inf}) := rho\_Air(T_f(T_s, T_{inf}) + 273)$ kg/m^3

$k(T_s, T_{inf}) := k\_Air(T_f(T_s, T_{inf}) + 273)$ W/m.K

$\mu(T_s, T_{inf}) := mu\_Air(T_f(T_s, T_{inf}) + 273)$ kg/m.s

$$Pr(T_s, T_{inf}) := Pr\_Air(T_f(T_s, T_{inf}) + 273)$$ ...Prandtl No.

$$\beta(T_s, T_{inf}) := \frac{1}{T_f(T_s, T_{inf}) + 273}$$ 1/K

$$\nu(T_s, T_{inf}) := \frac{\mu(T_s, T_{inf})}{\rho(T_s, T_{inf})}$$ m^2/s

Now: $\Delta T(T_s, T_{inf}) := T_s - T_{inf}$ C

And: $$Gr_D(T_s, T_{inf}) := \frac{g \cdot \beta(T_s, T_{inf}) \cdot \Delta T(T_s, T_{inf}) \cdot D^3}{\nu(T_s, T_{inf})^2}$$ ....Grashoff No.

Also: $Ra_D(T_s, T_{inf}) := Gr_D(T_s, T_{inf}) \cdot Pr(T_s, T_{inf})$ ...Rayleigh No.

Then, use following eqn. to get Nusselts No. for a sphere in free convection:

$$Nu_D(T_s,T_{inf}) := 2 + \frac{0.589 \cdot Ra_D(T_s,T_{inf})^{\frac{1}{4}}}{\left[1 + \left(\frac{0.469}{Pr(T_s,T_{inf})}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}}$$ ...for Ra<=10^11, and Pr >=0.7

Therefore: heat transfer coeff. h:

$$h(T_s,T_{inf}) := \frac{Nu_D(T_s,T_{inf}) \cdot k(T_s,T_{inf})}{D}$$ W/m^2.C .... heat transfer coeff.

Heat transfer rate by convection, Q_{conv}:

$$Q_{conv}(T_s,T_{inf}) := h(T_s,T_{inf}) \cdot \pi \cdot D^2 \cdot \Delta T(T_s,T_{inf})$$ ....since surface area of sphere = π*D^2

Heat transfer rate by Radiation, Q_{rad}:

$$Q_{rad}(T_s,T_{inf},\varepsilon) := \sigma \cdot \varepsilon \cdot (\pi \cdot D^2) \cdot \left[(T_s + 273)^4 - (T_{inf} + 273)^4\right]$$

Therefore, Total heat transfer rate, Q_{tot}:

$$Q_{tot}(T_s,T_{inf},\varepsilon) := Q_{conv}(T_s,T_{inf}) + Q_{rad}(T_s,T_{inf},\varepsilon)$$

Now, this total heat transfer should be equal to 60 * 0.9 = 54 W.

Use Solve block of Mathcad to meet this condition:

Start with a guess value of Ts, say Ts = 150 C.

$T_s := 150$ C .... guess value

Given

$$Q_{tot}(T_s,T_{inf},\varepsilon) = 0.9 \cdot Q$$

$$T_s(T_{inf},Q,\varepsilon) := Find(T_s)$$

i.e. $T_s(T_{inf},Q,\varepsilon) = 169.298$ **C .... surface temp of bulb under given conditions .... Ans.**

Now, plot T_s for different values of T_{inf} and ε, and Q:

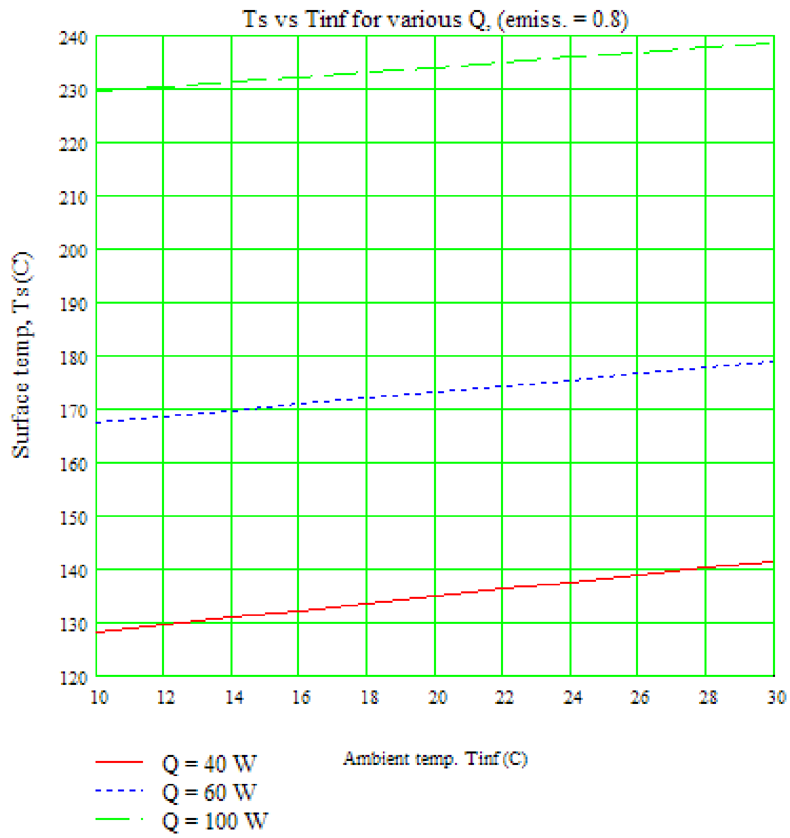
Let T_{inf} vary from 10 C to 30 C; ε from 0.4 to 1.0:

$T_{inf} := 10, 12 .. 30$

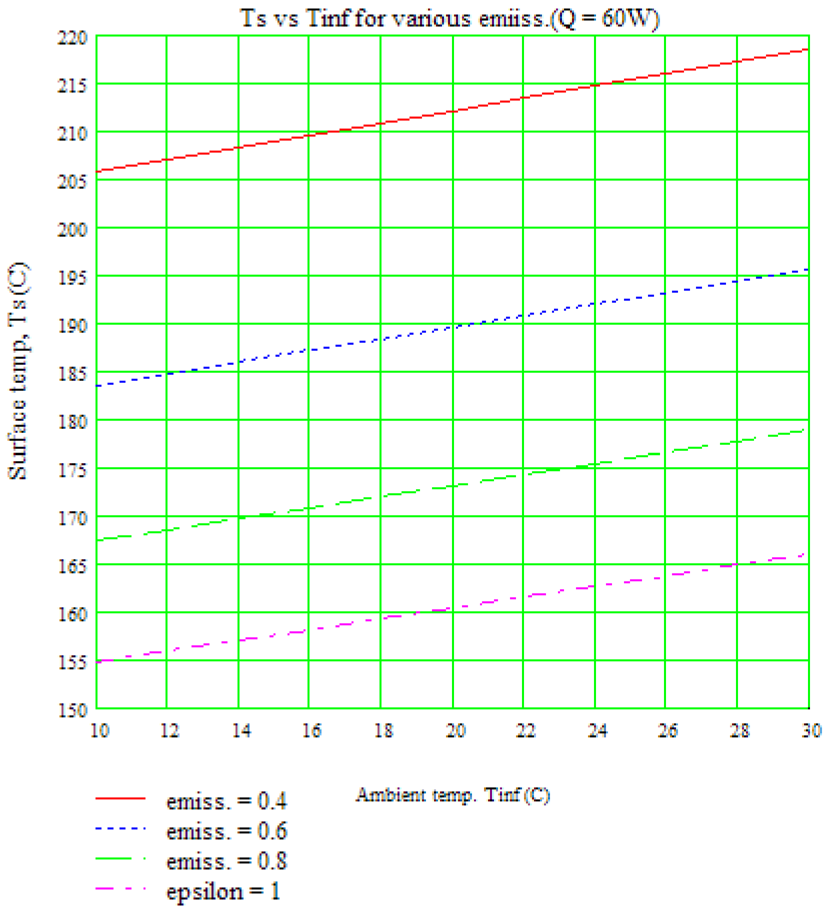
Sample Table for ε = 0.8, Q = 40 W, 60 W and 100 W:

T_{inf}	$T_s(T_{inf}, 40, 0.8)$	$T_s(T_{inf}, 60, 0.8)$	$T_s(T_{inf}, 100, 0.8)$
10	128.344	167.457	229.501
12	129.663	168.605	230.417
14	130.981	169.754	231.333
16	132.299	170.902	232.25
18	133.617	172.051	233.167
20	134.935	173.2	234.086
22	136.255	174.349	235.001
24	137.576	175.499	235.917
26	138.896	176.649	236.834
28	140.217	177.801	237.752
30	141.538	178.952	238.671

Plot of T_s against T_{inf} for different Q values, with emissivity = 0.8:



Plot of T_s against T_{inf} for different ε values, with Q = 60 W



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$UnitSystem SI Pa C J

"**Prob. 2A2.2.9.** A sphere 20 cm in diameter containing liquid air (-140 C) is covered with 5 cm thick glass wool (k = 0.037 W/m.K), with an emissivity of 0.8. Estimate the surface temp and the total heat transfer to liquid air (i.e. convection + radiation) if the surrounding air is at 20 C"

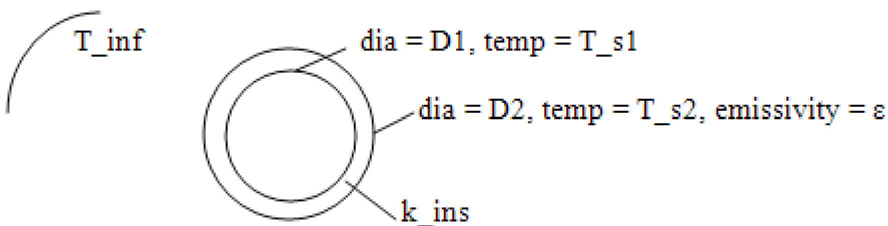


Fig.Prob.2A2.2.9

EES Solution:

"Data:"

D_1 = 0.2[m]"..dia of spherical container"
k_ins = 0.037 [W/m-C] "…thermal cond. of insulation"
t_ins = 0.05[m] "…thickness of insulation"
D_2 = D_1 + (t_ins * 2) "[m]..outer dia of sphere"
T_s1 = -140[C]
T_inf = 20 [C]
{T_s2 = 10 "[C]…. temp of outer surface of insulation …. assumed. Will be corrected later"}
T_f = (T_s2+T_inf)/2 "[C]…film temp."
beta = 1/(T_f + 273) "[1/K] … coeff. of vol. expn"
g = 9.81[m/s^2]
sigma = 5.67E-08 [W/m^2-K^4] "… Stefan Boltzmann constant"
epsilon = 0.8 "…emissivity"

"Properties of Air at Tf:"

mu=Viscosity(Air,T=T_f) "[kg/m-s]"
rho=Density(Air,T=T_f,P=1E05) "[kg/m^3]"
nu = mu/rho "[m^2/s]"
cp=Cp(Air,T=T_f) "[J/kg-K]"
k=Conductivity(Air,T=T_f) "[W/m-K]"

"Calculations:"

"Calculate Nusselts No.:"

Gr_D = g * beta * (T_inf-T_s2) * D_2^3 / nu^2 "…Grashoff No."
Pr = mu * cp / k "Prandtl No."
Ra_D = Gr_D * Pr "…Rayleigh No."

"We get Ra_D = 3.069E07 i.e. < 10^11"

"For a sphere in free convection, we have:"

Nusselt_D = 2 + ((0.589 * Ra_D^(1/4)) / (1 + (0.469 / Pr)^(9/16))^(4/9)) "Finds Nu_D for Ra <= 10^11, and Pr >= 0.7"

Nusselt_D = h * D_2 / k "Finds h"

Q_conv = h * (pi * D_2^2) * (T_inf – T_s2) "[W] … heat transfer by convection"

"Heat transfer by radiation:"

Q_rad = sigma * epsilon * (pi * D_2^2) * ((T_inf + 273)^4 – (T_s2 + 273)^4) "[W] …. heat tr by radiation"

Q_tot = Q_conv + Q_rad "[W] … total heat transfer"

"But this total heat transfer must be equal to heat tr by conduction through the insulation:"

R_sph = (D_2 – D_1) / (2 * pi * k_ins * D_2 * D_1) "[C/W] … Thermal resistance of spherical layer of insulation"

Q_tot = (T_s2 – T_s1) / R_sph "…finds T_s2"

Results:

Unit Settings: SI C Pa J mass deg

β = 0.003473 [1/K]	cp = 1004 [J/kg-K]	D_1 = 0.2 [m]
D_2 = 0.3 [m]	ε = 0.8	g = 9.81 [m/s^2]
Gr_D = 4.201E+07	h = 2.966 [w/m^2-C]	k = 0.02476 [W/m-C]
k_{ins} = 0.037 [W/m-C]	μ = 0.00001801 [kg/m-s]	ν = 0.00001489 [m^2/s]
$Nusselt_D$ = 35.93	Pr = 0.7305	Q_{conv} = 8.495 [W]
Q_{rad} = 12.41 [W]	Q_{tot} = 20.9 [W]	Ra_D = 3.069E+07
ρ = 1.209 [kg/m^3]	R_{sph} = 7.169 [C/W]	σ = 5.670E-08 [W/m^2-K^4]
T_f = 14.93 [C]	T_{inf} = 20 [C]	t_{ins} = 0.05 [m]
T_{s1} = -140 [C]	T_{s2} = 9.869 [C]	

Thus:

Surface temp of insulation = T_s2 = 9.869 C …. Ans.

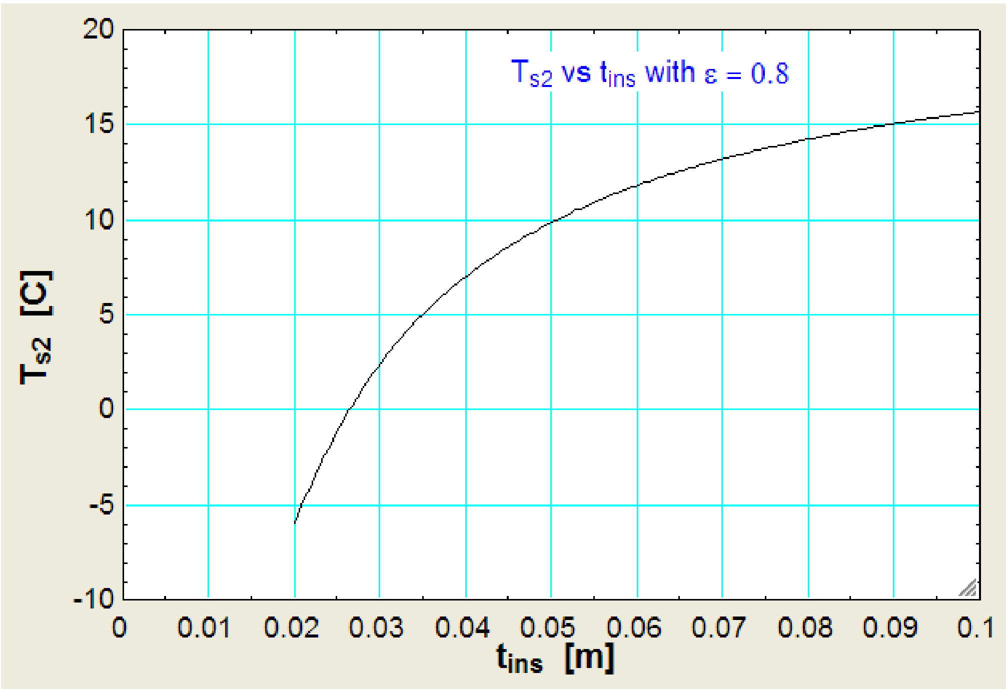
Total heat transfer = Q_tot = 20.9 W …. Ans.

Plot the variation of T_s2 and Q_tot as t_ins varies from 2 cm to 10 cm (with epsilon = 0.8):

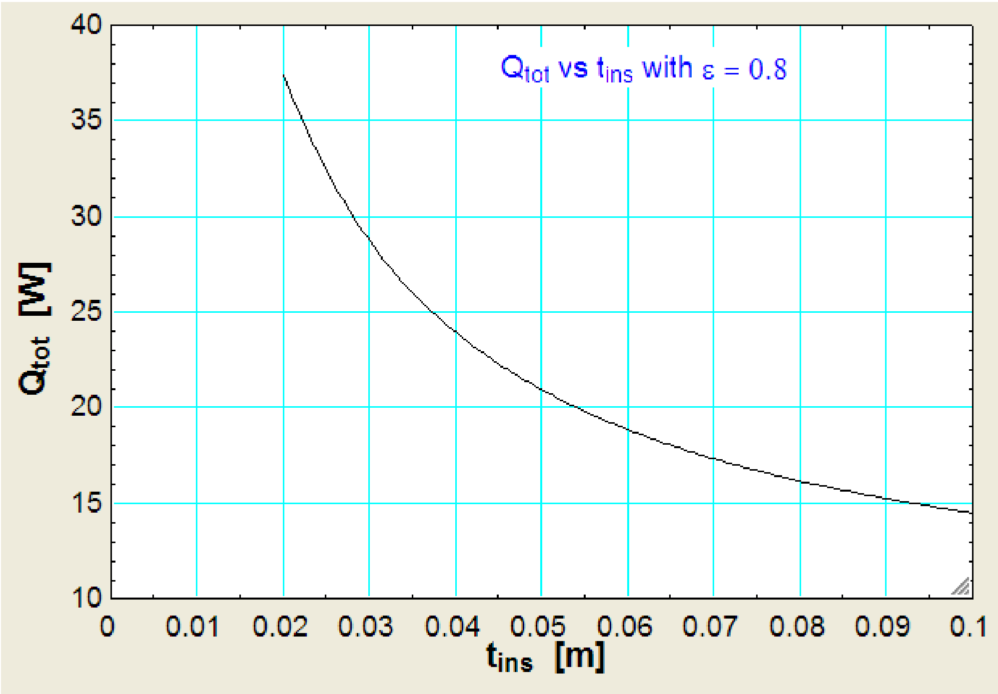
Table 1

1..9	t_{ins} [m]	T_{s2} [C]	Q_{tot} [W]
Run 1	0.02	-5.931	37.4
Run 2	0.03	2.451	28.7
Run 3	0.04	7.017	23.92
Run 4	0.05	9.869	20.9
Run 5	0.06	11.81	18.82
Run 6	0.07	13.2	17.3
Run 7	0.08	14.25	16.14
Run 8	0.09	15.05	15.22
Run 9	0.1	15.69	14.48

Surface temp (T_s2) vs thickness of insulation (t_ins) with epsilon = 0.8:



Total heat transfer (Q_tot) vs thickness of insulation (t_ins) with epsilon = 0.8:

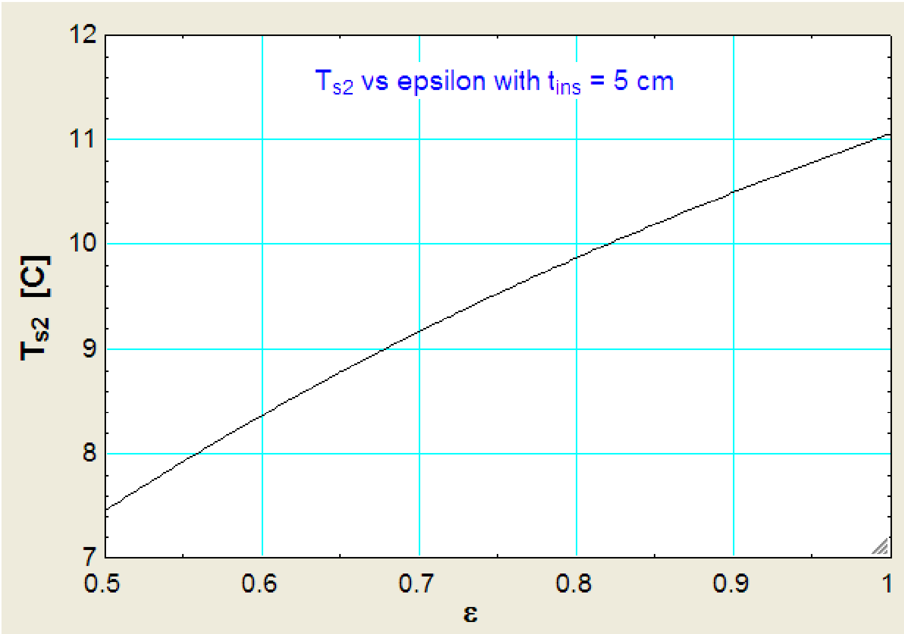


Plot the variation of T_s2 and Q_tot as epsilon varies from 0.4 to 1, (with t_ins = 5 cm):

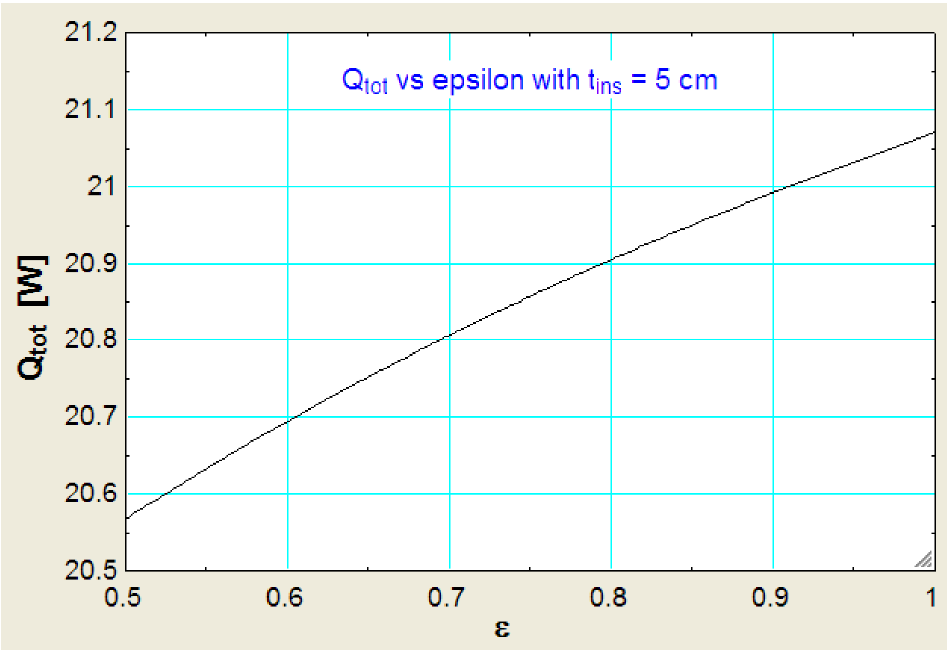
Table 1 | Table 2

1..11	ε	T_{s2} [C]	Q_{tot} [W]
Run 1	0.5	7.452	20.57
Run 2	0.55	7.923	20.63
Run 3	0.6	8.365	20.69
Run 4	0.65	8.777	20.75
Run 5	0.7	9.164	20.81
Run 6	0.75	9.528	20.86
Run 7	0.8	9.869	20.9
Run 8	0.85	10.19	20.95
Run 9	0.9	10.49	20.99
Run 10	0.95	10.78	21.03
Run 11	1	11.05	21.07

Surface temp (T_s2) vs emissivity (with t_ins = 5 cm):



Q_tot vs emissivity (with t_ins = 5 cm):



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Prob. 2A2.2.10. A wall mounted power transistor dissipates 0.18W in an environment of 35 C. The transistor is 0.45 cm long and 0.4 cm in diameter. The emissivity of outer surface of transistor is 0.1 and the average temp of surrounding surfaces is 25 C. Disregarding heat transfer from the base surface, determine the surface temp of transistor. Also, plot the surface temp for various values of environmental temperatures and emissivities.

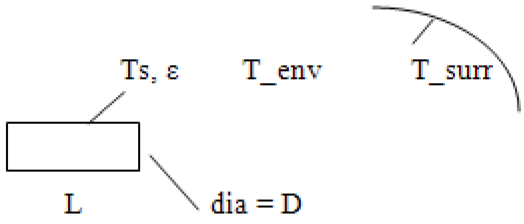


Fig.Prob.2A2.2.10

Mathcad Solution:

Use the Mathcad functions, already written, (see Prob. 2A1.2.5) to get properties of Air:

Data:

$L := 0.0045$ m...length of transistor $D := 0.004$ m...dia

$T_{env} := 35$ C.....ambient temp.

$Ts := 200$ C ... srface temp .... assumed

$T_{surr}(T_{env}) := T_{env} - 10$ C....surr. temp...written as a function of T_env

$\varepsilon := 0.1$ emissivity

$g := 9.81$ m/s^2....accn. due to gravity

We need properties of air at film temperature Tf:

$$T_f(Ts, T_{env}) := \frac{Ts + T_{env}}{2}$$ C....avg. temperature.

$\sigma := 5.67 \cdot 10^{-8}$ W/m^2.K^4....Stefan Boltzmann const.

Note: We will be writing T_surr and Tf….etc. as functions of T_env and epsilon since we need to vary T_env and epsilon to plot the graphs.

Properties of Air at Tf:

$$\rho(Ts, T_{env}) := rho\_Air(T_f(Ts, T_{env}) + 273) \qquad \text{kg/m^3}$$

$$k(Ts, T_{env}) := k\_Air(T_f(Ts, T_{env}) + 273) \qquad \text{W/m.K}$$

$$\mu(Ts, T_{env}) := mu\_Air(T_f(Ts, T_{env}) + 273) \qquad \text{kg/m.s}$$

$$Pr(Ts, T_{env}) := Pr\_Air(T_f(Ts, T_{env}) + 273) \qquad \text{...Prandtl No.}$$

$$\beta(Ts, T_{env}) := \frac{1}{T_f(Ts, T_{env}) + 273} \qquad \text{1/K}$$

$$\nu(Ts, T_{env}) := \frac{\mu(Ts, T_{env})}{\rho(Ts, T_{env})} \qquad \text{m^2/s}$$

Remember that here, the diameter 'D' is the characteristic dimension.

Then, $Gr_D(Ts, T_{env}) := \dfrac{g \cdot \beta(Ts, T_{env}) \cdot (Ts - T_{env}) \cdot D^3}{\nu(Ts, T_{env})^2}$ ....Grashof No. as a function of Ts

i.e. $Gr_D(Ts, T_{env}) = 425.616$ ....Grashof number

Then, Rayleigh No.:

$$Ra_D(Ts, T_{env}) := (Gr_D(Ts, T_{env}) \cdot Pr(Ts, T_{env}))$$

i.e. $Ra_D(Ts, T_{env}) = 294.484$ ....Rayleigh No.

Then, use Churchill and Chu's relation for horizl cylinder, to get Nusselts No. (Ref: 2):

$$Nu(Ts, T_{env}) := \left[0.6 + \frac{0.378 \cdot Ra_D(Ts, T_{env})^{\frac{1}{6}}}{\left[1 + \left(\frac{0.559}{Pr(Ts, T_{env})}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right]^2$$ ....for Ra < 10^12

i.e. $Nu(Ts, T_{env}) = 1.982$ ...Nusselts No.

And, $h(Ts, T_{env}) := \dfrac{Nu(Ts, T_{env}) \cdot k(Ts, T_{env})}{D}$

i.e. $h(Ts, T_{env}) = 16.091$ W/m^2.C .... heat tr coeff.

Heat transfer by convection:

Now, area for convective and radiative heat transfer is:

$A_s := \pi \cdot D \cdot L + \dfrac{\pi \cdot D^2}{4}$ i.e. $A_s = 6.912 \times 10^{-5}$ m^2 ... area

Therefore: $Q_{conv}(Ts, T_{env}) := [h(Ts, T_{env}) \cdot A_s \cdot (Ts - T_{env})]$

i.e. $Q_{conv}(Ts, T_{env}) = 0.184$ **W....heat transfer rate by convection**

Heat transfer by radiation:

$$Q_{rad}(Ts, T_{env}, \varepsilon) := \left[\sigma\cdot\varepsilon\cdot A_s\cdot\left[(Ts + 273)^4 - \left(T_{surr}(T_{env}) + 273\right)^4\right]\right]$$

i.e. $Q_{rad}(Ts, T_{env}, \varepsilon) = 0.017$ **W..... heat transfer by radiation**

To find Ts: Use Solve block:

Given

$$Q_{conv}(Ts, T_{env}) + Q_{rad}(Ts, T_{env}, \varepsilon) = 0.18$$

$Ts(T_{env}, \varepsilon) := Find(Ts)$ $Ts(T_{env}, \varepsilon) = 186.304$ **C ..... surface temp .... Ans.**

Note that in the Solve block, Ts is written as a function of T_env and epsilon.

So, now, it is convenient to plot the variation of Ts as a function of T_env and epsilon.

Thus:

Results are:

$\varepsilon = 0.1$ $T_{env} = 35$

$Ts(T_{env}, \varepsilon) = 186.304$ C .... ans.

$Gr_D(Ts(T_{env}, \varepsilon), T_{env}) = 422.568$

$Pr(Ts(T_{env}, \varepsilon), T_{env}) = 0.693$

$Ra_D(Ts(T_{env}, \varepsilon), T_{env}) = 292.954$

$Nu(Ts(T_{env}, \varepsilon), T_{env}) = 1.98$

$h(Ts(T_{env}, \varepsilon), T_{env}) = 15.841$ W/m^2.K

$Q_{conv}(Ts(T_{env}, \varepsilon), T_{env}) = 0.166$ W

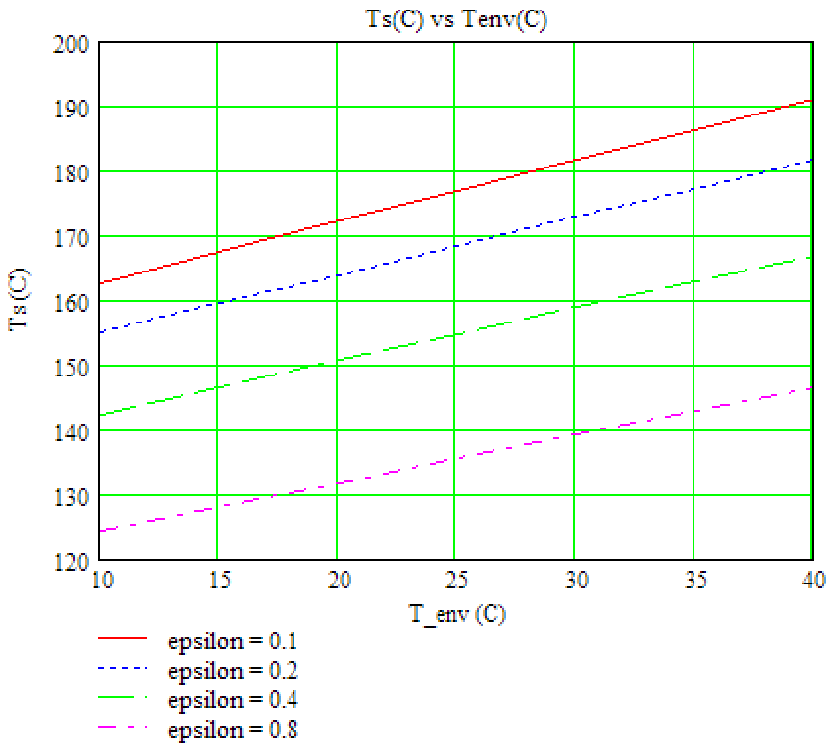
$Q_{rad}(Ts(T_{env}, \varepsilon), T_{env}, \varepsilon) = 0.014$ W

$Qtot = Q_{conv} + Q_{rad} = 0.18$ W

Now, plot T_s for various values of T_env, with different values of ε:

$T_{env} := 10, 15 .. 40$ $\qquad \varepsilon := 0.1$ ..etc.

T_{env} =	$Ts(T_{env}, 0.1)$	$Ts(T_{env}, 0.2)$	$Ts(T_{env}, 0.4)$	$Ts(T_{env}, 0.8)$
10	162.668	154.953	142.42	124.311
15	167.424	159.443	146.545	128.035
20	172.165	163.915	150.654	131.749
25	176.892	168.37	154.746	135.455
30	181.605	172.809	158.821	139.154
35	186.304	177.23	162.881	142.843
40	190.989	181.636	166.925	146.525



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Prob. 2A2.2.11. A steam pipe, 100 mm in diameter, maintained at 170 C is exposed to air at 30 C. Length of the pipe is 2 m and is kept horizontal. Determine the heat lost by the pipe per hour. [VTU – May 2007]

Also calculate the heat lost by radiation if emissivity of surface is 0.9. Plot Q_total for ambient temperatures varying from 0 to 40 C.

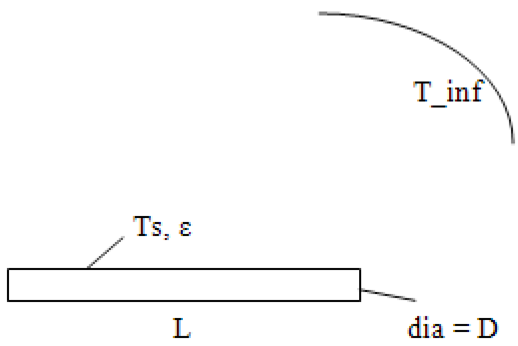


Fig.Prob.2A2.2.11

Note that this is the same problem as 2A2.2.3.

However, now we will solve it with EXCEL.

EXCEL Solution:

WE need properties of Air as temp varies.

We have already written VBA Functions in EXCEL to calculate the properties of Air, in Prob. 2A1.2.13. We will use those Functions.

Following are the steps:

1. Set up the EXCEL worksheet, enter data and name the cells:

L | fx 2

	A	B	C	D	E	F
210		Data:	Fluid =	Air		
211			T_s	170	C	
212			T_inf	30.0000	C	
213			T_f	100.0000	C	
214			D	0.1	m	
215			L	2.0000	m	
216			g	9.8100	m/s^2	
217			beta	0.0027	1/K	
218			epsilon	0.9000		
219			sigma	5.6700E-08	W/m^2-K^4	

$T\_f = \frac{T\_s + T\_inf}{2}$

2. Do the calculations, use VBA Functions to get properties of Air:

cp | fx =air_cp_T((T_f+273))

	A	B	C	D	E	F	G
220							
221		Calculations:					
222		density	rho	0.94706	kg/m^3		
223		th. conductivity	k	0.031607	W/m.C		
224		Prandtl No.	Pr	0.70052			
225		kinematic visc.	nu	2.31375E-05	m^2/s		
226		sp.heat	cp	1011.3	J/kg.K		

Using VBA Functions for Air

And, further calculations:

Q_horizl | =h_horizl*(PI()*D*L)*(T_s-T_inf)

Row	A	B	C	D	E
231		**For horizontal cylinder: use Churchill & Chu's eqn to find Nusselts No.:**			
232		Grashof No.	**Gr_D**	6877891.932	
233		Rayleigh No.	**Ra_D**	4818100.856	
234		**To calculate Nusselts No.**			
235			AA	5.0295	
236			BB	0.8808	
237			CC	1.205828931	
238			**Nusselt_horizl**	**22.762**	
239		convective heat tr. coeff.	**h_horizl**	**7.194**	W/m^2.C
240		**Conv. Heat transfer**	**Q_horizl**	**632.8576**	**W**
241		**Radiation Heat transfer:**			
242			**Q_rad**	**964.6111**	**W**
243		**Total heat transfer:**	**Q_tot**	**1597.4687**	**W**

$$Gr\_D = \frac{g \cdot beta \cdot (T\_s - T\_inf) \cdot D^3}{nu^2}$$

$$Ra\_D = Gr\_D \cdot Pr$$

$$AA = 0.387 \cdot Ra\_D^{\frac{1}{6}}$$

$$BB = \left(\frac{0.559}{Pr}\right)^{\frac{9}{16}}$$

$$CC = (1 + BB)^{\frac{8}{27}}$$

$$Nusselt\_horizl = \left(0.6 + \frac{AA}{CC}\right)^2$$

$$h\_horizl = \frac{Nusselt\_horizl \cdot k}{D}$$

$$Q\_horizl = h\_horizl \cdot (\pi \cdot D \cdot L) \cdot (T\_s - T\_inf)$$

$$Q\_rad = epsilon \cdot sigma \cdot (\pi \cdot D \cdot L) \cdot \left[(T\_s + 273)^4 - (T\_inf + 273)^4\right]$$

$$Q\_tot = Q\_horizl + Q\_rad$$

Thus:

Q_conv = Q_horizl = 632.86 W …. Heat loss by convection

Q_rad = 964.61 W…. Heat transfer by radiation

Q_tot = 1597.47 W … Total heat transfer

---

3. Now, let us plot the variation of Q_conv, Q_rad and Q_tot as the ambient temp varies from 0 to 40 deg.C.

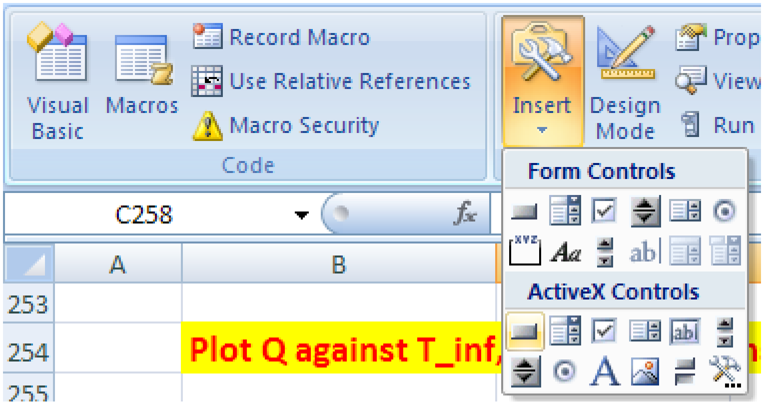
Since T_f and the properties of Air also change as T_inf varies, it is convenient to write a VBA program to do this calculation. First, set up a Table as shown:

C258

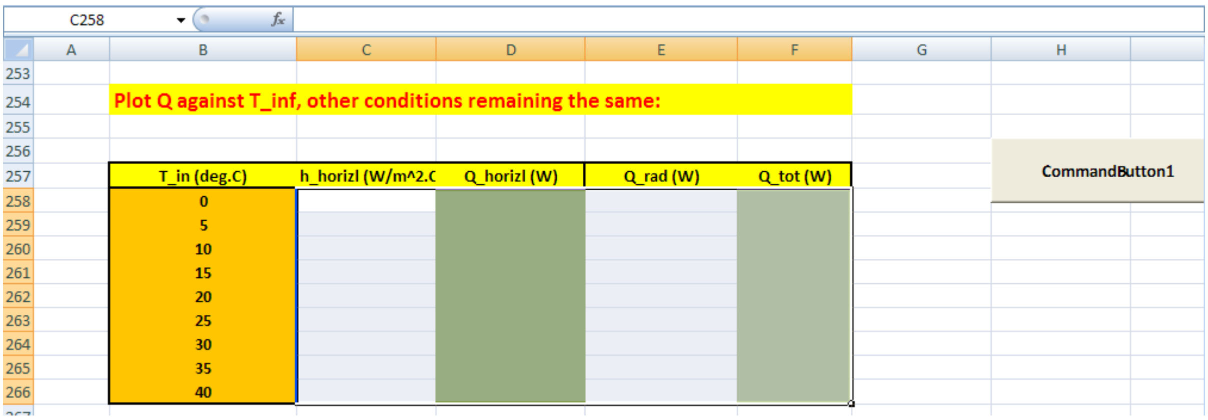
	A	B	C	D	E	F
253						
254		Plot Q against T_inf, other conditions remaining the same:				
255						
256						
257		T_in (deg.C)	h_horizl (W/m^2.C	Q_horizl (W)	Q_rad (W)	Q_tot (W)
258		0				
259		5				
260		10				
261		15				
262		20				
263		25				
264		30				
265		35				
266		40				

4. Now, we will write a VBA program to read the values of T_inf, one by one, from this Table and put in cell D212.; then, immediately, all other calculations in remaining cells get updated. The program selects the required quantity (i.e. h_horizl, Q_horizl, Q_rad and Q_tot, from cells D239, D240, D242 and D243 respectively) and copies them to their respective places in the Table. And, we will have a Command Button to do this:

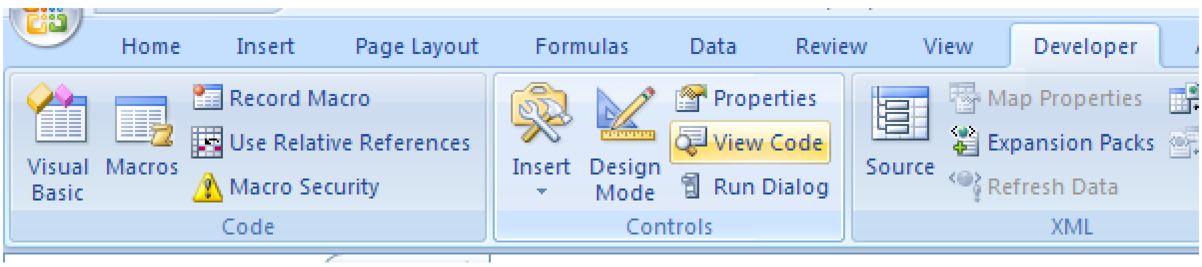
Go to Developer – Insert-Active X controls:



Click on the first, left button. And, draw the Command Button in the worksheet to the required size, at the required place:



Click on Developer-View Code:



And, we get the code already written for the Command button:

```
CommandButton1                                  Click
Private Sub CommandButton1_Click()

End Sub
```

Now, complete the code to do the desired jobs as mentioned under point 4 above:

```
CommandButton1                                  Click
Private Sub CommandButton1_Click()

Dim i As Integer
For i = 0 To 8
 Range("D212") = Cells(258 + i, 2)
 Cells(258 + i, 3) = Range("D239")
 Cells(258 + i, 4) = Range("D240")
 Cells(258 + i, 5) = Range("D242")
 Cells(258 + i, 6) = Range("D243")

 Next i

End Sub
```

In the above code:

Line 1: defines the Sub

Line 2: dimension statement for i, the counter in For….Next construct

Line 3 to 9: For … Next slab

Line 4: Takes the first value of T_inf from the Table and copies it to cell D212 (i.e. T_inf in the original program)

Immediately, all other calculations in other cells are up-dated; and the rest of the program copies them to the respective places in the Table. i.e.

Line 5, 6, 7 and 8: copy values of h_horizl, Q_horizl, Q_rad and Q_tot to respective cells in the Table.

Line 9: this is repeated for the next value of T_inf in the Table.

Line 10: End statement of Subroutine

---

5. Now, click on the Command Button 1 and the Table gets filled up immediately:

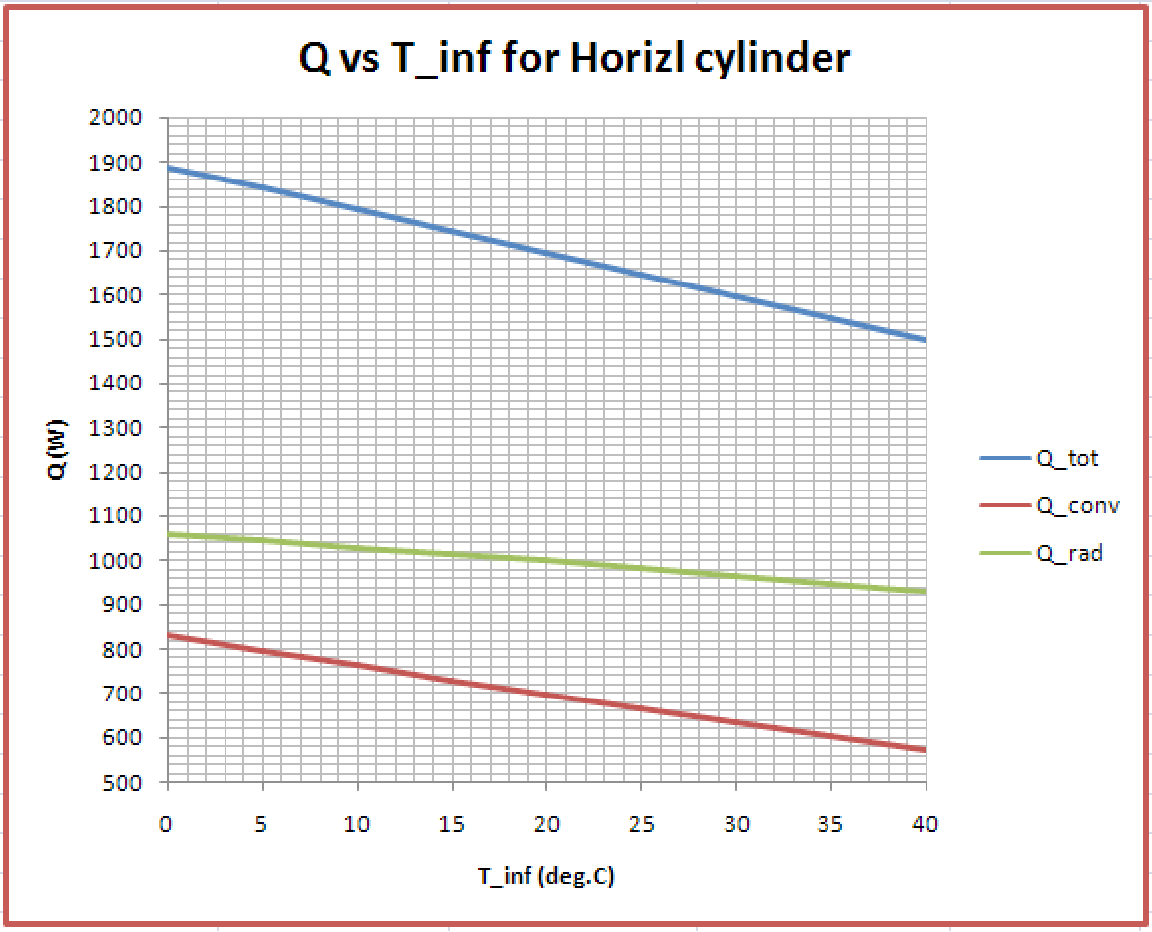
7.77844862681783

Plot Q against T_inf, other conditions remaining the same:

CommandButton1

T_in (deg.C)	h_horizl (W/m^2.C	Q_horizl (W)	Q_rad (W)	Q_tot (W)
0	7.778	830.848	1056.771	1887.619
5	7.684	796.583	1043.360	1839.944
10	7.586	762.674	1029.207	1791.881
15	7.489	729.355	1014.283	1743.637
20	7.391	696.617	998.561	1695.177
25	7.293	664.453	982.013	1646.466
30	7.194	632.858	964.611	1597.469
35	7.095	601.825	946.326	1548.151
40	6.995	571.351	927.128	1498.480

6. Plot the results in EXCEL:



===

VBA Functions for Nusselts No. for Natural convection in Air:

Natural convection in Air for various geometries is an very common occurrence.

It is very convenient in EXCEL to have VBA Functions to find Nusselts No. for these cases. Then, we can easily calculate heat transfer coefficient and the heat transfer rate.

We give below some VBA Functions for average Nusselts No. for different cases:

Remember that we have already written VBA Functions for properties of Air, NH3,…etc. (See, for ex. Prob. 2A1.2.13). In the same Worksheet, in another Module, we shall write these VBA Functions for Natural convection:

```
Option Explicit

Function NatConv_Air_HCyl_Nusselt_D(T_s As Double, T_inf As Double, L As Double, D As Double) As Double
'Finds Avg. Nusselts No. for Nat Convn in Air at 1 atmosph pressure for a Horizl cylinder
'Reads property values of Air from Table and interpolates using VBA Functions


Dim g As Double, beta As Double
Dim AA As Double, BB As Double, CC As Double


Dim T_f As Double, rho As Double, k As Double, Pr As Double
Dim nu As Double, cp As Double
Dim Gr_D As Double, Ra_D As Double

g = 9.81 'm/s^2....accn due to gravity

T_f = (T_s + T_inf) / 2

beta = 1 / (T_f + 273) 'vol. expansivity of Air


'Properties of Air:

rho = Air_rho_T(T_f + 273)
k = Air_k_T(T_f + 273)
Pr = Air_Pr_T(T_f + 273)
nu = Air_nu_T(T_f + 273)
cp = Air_cp_T(T_f + 273)

Gr_D = g * beta * Abs(T_s - T_inf) * D ^ 3 / nu ^ 2 'Grashof No.

Ra_D = Gr_D * Pr 'Rayleigh No.
```

```
AA = 0.387 * Ra_D ^ (1 / 6)
BB = (0.559 / Pr) ^ (9 / 16)
CC = (1 + BB) ^ (8 / 27)

NatConv_Air_HCyl_Nusselt_D = (0.6 + AA / CC) ^ 2 'For 10^-5 < Ra_D <= 10^12


End Function
```

```
Function NatConv_Air_VPlate_Nusselt_L(T_s As Double, T_inf As Double, L As Double) As Double
'Finds Avg. Nusselts No. for Nat Convn in Air at 1 atmosph pressure for a Vertical Plate (height L)
'Reads property values of Air from Table and interpolates using VBA Functions


Dim g As Double, beta As Double
Dim AA As Double, BB As Double, CC As Double


Dim T_f As Double, rho As Double, k As Double, Pr As Double
Dim nu As Double, cp As Double
Dim Gr_L As Double, Ra_L As Double

g = 9.81 'm/s^2....accn due to gravity

T_f = (T_s + T_inf) / 2

beta = 1 / (T_f + 273) 'vol. expansivity of Air


'Properties of Air:

rho = Air_rho_T(T_f + 273)
k = Air_k_T(T_f + 273)
Pr = Air_Pr_T(T_f + 273)
nu = Air_nu_T(T_f + 273)
cp = Air_cp_T(T_f + 273)

Gr_L = g * beta * Abs(T_s - T_inf) * L ^ 3 / nu ^ 2 'Grashof No.

Ra_L = Gr_L * Pr 'Rayleigh No.


AA = 0.387 * Ra_L ^ (1 / 6)
BB = (0.492 / Pr) ^ (9 / 16)
CC = (1 + BB) ^ (8 / 27)

NatConv_Air_VPlate_Nusselt_L = (0.825 + AA / CC) ^ 2 'For Ra_L <= 10^12


End Function
```

```
Function NatConv_Air_VCyl_Nusselt_L(T_s As Double, T_inf As Double, D As Double, L As Double) As Double
'Finds Avg. Nusselts No. for Nat Convn in Air at 1 atmosph pressure for a Vertical cyl (height L)
'Reads property values of Air from Table and interpolates using VBA Functions

Dim g As Double, beta As Double
Dim AA As Double, BB As Double, CC As Double

Dim T_f As Double, rho As Double, k As Double, Pr As Double
Dim nu As Double, cp As Double
Dim Gr_L As Double, Ra_L As Double

g = 9.81 'm/s^2....accn due to gravity

T_f = (T_s + T_inf) / 2

beta = 1 / (T_f + 273) 'vol. expansivity of Air

'Properties of Air:

rho = Air_rho_T(T_f + 273)
k = Air_k_T(T_f + 273)
Pr = Air_Pr_T(T_f + 273)
nu = Air_nu_T(T_f + 273)
cp = Air_cp_T(T_f + 273)

Gr_L = g * beta * Abs(T_s - T_inf) * L ^ 3 / nu ^ 2 'Grashof No.


If (D < 35 * L / Gr_L ^ 0.25) Then
  MsgBox (" D must be greater than or equal to Factor (35 * L/Gr_L^0.25)!!")
 End
 End If


Ra_L = Gr_L * Pr 'Rayleigh No.

AA = 0.387 * Ra_L ^ (1 / 6)
BB = (0.492 / Pr) ^ (9 / 16)
CC = (1 + BB) ^ (8 / 27)

NatConv_Air_VCyl_Nusselt_L = (0.825 + AA / CC) ^ 2 'For Ra_L <= 10^12


End Function
```

```
Function NatConv_Air_HPlate_HotUpper_Nusselt_Lc(T_s As Double, T_inf As Double, L As Double, _
W As Double) As Double
'Finds Avg. Nusselts No. for Nat Convn in Air at 1 atmosph pressure for a Horizl Plate (L x W)
'with hot surface facing upwards
'Reads property values of Air from Table and interpolates using VBA Functions


Dim g As Double, beta As Double

Dim Lc As Double

Dim T_f As Double, rho As Double, k As Double, Pr As Double
Dim nu As Double, cp As Double
Dim Gr_Lc As Double, Ra_Lc As Double

g = 9.81 'm/s^2....accn due to gravity

T_f = (T_s + T_inf) / 2

beta = 1 / (T_f + 273) 'vol. expansivity of Air


'Properties of Air:

rho = Air_rho_T(T_f + 273)
k = Air_k_T(T_f + 273)
Pr = Air_Pr_T(T_f + 273)
nu = Air_nu_T(T_f + 273)
cp = Air_cp_T(T_f + 273)

Lc = (L * W) / (2 * (L + W)) 'Characteristic dimension for flat plate

Gr_Lc = g * beta * (T_s - T_inf) * Lc ^ 3 / nu ^ 2 'Grashof No.

Ra_Lc = Gr_Lc * Pr 'Rayleigh No.


If Ra_Lc > 10 ^ 4 And Ra_Lc <= 10 ^ 7 Then

   NatConv_Air_HPlate_HotUpper_Nusselt_Lc = 0.54 * Ra_Lc ^ (1 / 4)

End If

If Ra_Lc > 10 ^ 7 And Ra_Lc <= 10 ^ 11 Then

    NatConv_Air_HPlate_HotUpper_Nusselt_Lc = 0.15 * Ra_Lc ^ (1 / 3)

End If


End Function
```

```
Function NatConv_Air_HPlate_HotLower_Nusselt_Lc(T_s As Double, T_inf As Double, L As Double, _
W As Double) As Double
'Finds Avg. Nusselts No. for Nat Convn in Air at 1 atmosph pressure for a Horizl Plate (L x W)
'with hot surface facing downwards
'Reads property values of Air from Table and interpolates using VBA Functions


Dim g As Double, beta As Double

Dim Lc As Double

Dim T_f As Double, rho As Double, k As Double, Pr As Double
Dim nu As Double, cp As Double
Dim Gr_Lc As Double, Ra_Lc As Double

g = 9.81 'm/s^2....accn due to gravity

T_f = (T_s + T_inf) / 2

beta = 1 / (T_f + 273) 'vol. expansivity of Air


'Properties of Air:

rho = Air_rho_T(T_f + 273)
k = Air_k_T(T_f + 273)
Pr = Air_Pr_T(T_f + 273)
nu = Air_nu_T(T_f + 273)
cp = Air_cp_T(T_f + 273)

Lc = (L * W) / (2 * (L + W)) 'Characteristic dimension for flat plate

Gr_Lc = g * beta * (T_s - T_inf) * Lc ^ 3 / nu ^ 2 'Grashof No.

Ra_Lc = Gr_Lc * Pr 'Rayleigh No.
```

```
If Ra_Lc >= 10 ^ 5 And Ra_Lc <= 10 ^ 11 Then

   NatConv_Air_HPlate_HotLower_Nusselt_Lc = 0.27 * Ra_Lc ^ (1 / 4)

End If


End Function
```

```
Function NatConv_Air_Sphere_Nusselt_D(T_s As Double, T_inf As Double, D As Double) As Double
'Finds Avg. Nusselts No. for Nat Convn in Air at 1 atmosph pressure for a Sphere of dia = D

'Reads property values of Air from Table and interpolates using VBA Functions


Dim g As Double, beta As Double

Dim AA As Double, BB As Double, CC As Double

Dim T_f As Double, rho As Double, k As Double, Pr As Double
Dim nu As Double, cp As Double
Dim Gr_D As Double, Ra_D As Double

g = 9.81 'm/s^2....accn due to gravity

T_f = (T_s + T_inf) / 2

beta = 1 / (T_f + 273) 'vol. expansivity of Air


'Properties of Air:

rho = Air_rho_T(T_f + 273)
k = Air_k_T(T_f + 273)
Pr = Air_Pr_T(T_f + 273)
nu = Air_nu_T(T_f + 273)
cp = Air_cp_T(T_f + 273)


Gr_D = g * beta * Abs(T_s - T_inf) * D ^ 3 / nu ^ 2 'Grashof No.

Ra_D = Gr_D * Pr 'Rayleigh No.


If Pr >= 0.7 And Ra_D <= 10 ^ 11 Then

   AA = 0.589 * Ra_D ^ (1 / 4)
   BB = (0.469 / Pr) ^ (9 / 16)
   CC = (1 + BB) ^ (4 / 9)

   NatConv_Air_Sphere_Nusselt_D = 2 + AA / CC

End If


End Function
```

==

Once these Functions are written, they are available for use like any other built-in Functions of EXCEL.

Let us demonstrate the use of these Functions with some examples:

Prob. 2A2.2.12. A square plate (0.5 m × 0.5 m) with one surface insulated and the other surface maintained at a temp of 385 K is placed in ambient air at 315 K. Calculate the average Nusselts No. and het transfer coeff. h for free convection for the following orientations of the hot surface: (a) The plate is horizontal and hot surface faces up (b) The plate is horizontal and the hot surface faces down. [VTU – Dec. 09–Jan. 2010] (c) In addition, find Nusselts No. and h when plate is held vertical

Note that this is the same as Prob. 2A2.2.2

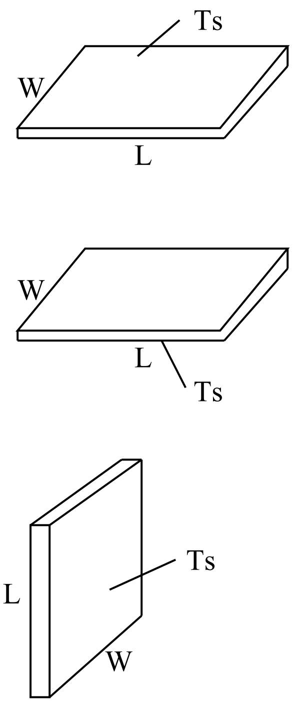


Fig. Prob. 2A2.2.12(a),(b) and (c).

EXCEL Solution steps:

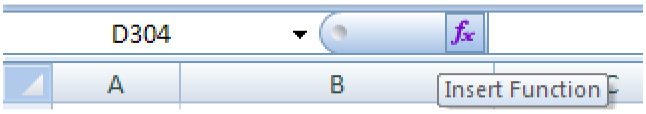
1. Go to the worksheet wherein we have written the VBA programs for properties of Air, and the programs for Natural convection given above. For this problem, we have: L = W = 0.5 m, T_s = (385-273) = 112 deg. C, and T_inf = (315 – 273) = 42 deg.C. Enter this data in the worksheet:

D302 =Air_k_T(D298+273)

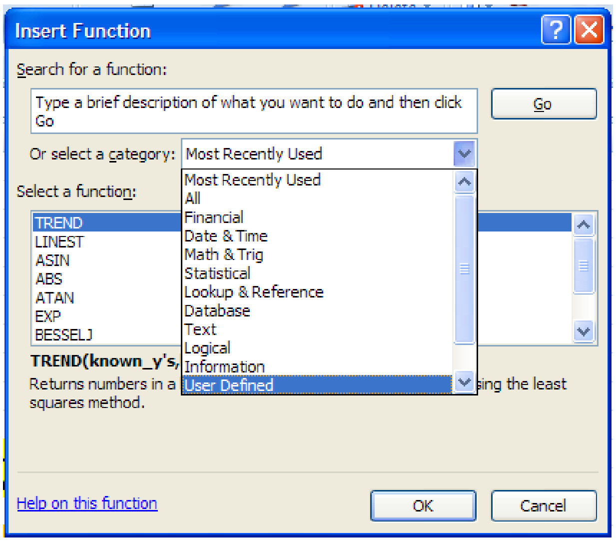
	A	B	C	D	E
294					
295		Data:	Fluid =	Air	
296			T_s	112	C
297			T_inf	42.0	C
298			T_f	77.0	C
299			L	0.5	m
300			W	0.5000	m
301			Lc	0.125	m
302			k	0.03	W/m.C

Note that characteristic dimension Lc = Area/Perimeter = (L * W) / ((2 * (L + W)), and k of Air is determined from the VBA function for properties of Air. See the formula bar for the function entered in cell D302.

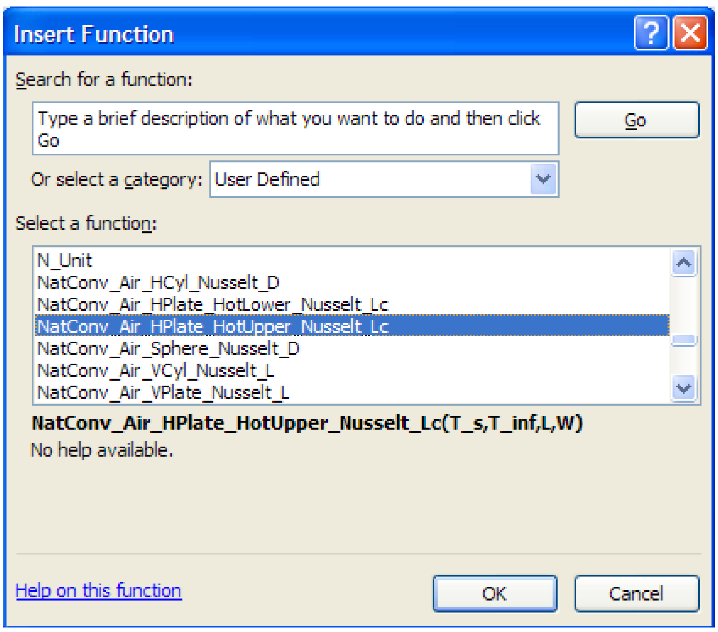
2. For the hot surface facing up, enter the VBA Function for Nusselts No. in cell D304. To do this, select cell D304 and click on Insert Function icon in EXCEL:



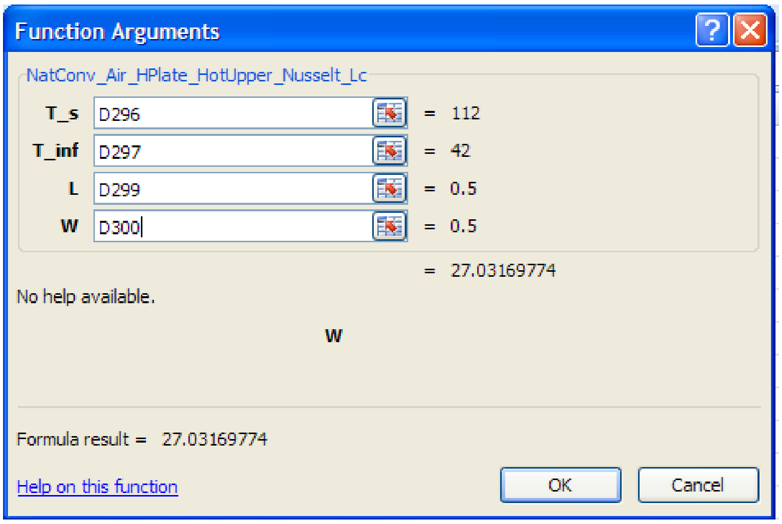
We get:



Select User Defined category, and select the required Function as shown below:



Press OK. We get the following screen. Fill up the values as shown:



Press OK and the calculated value is entered in cell D 304.

Then, h is calculated from the formula: h = Nusselts * k / Lc. These are shown in the following screen shot:

D304 =NatConv_Air_HPlate_HotUpper_Nusselt_Lc(D296,D297,D299,D300)

	A	B	C	D	E	F
300			W	0.5000	m	
301			Lc	0.125	m	
302			k	0.03	W/m.C	
303		Horizl plate:				
304		Hot surface facing up:	Nusslts_up	27.0317	.....Ans.	
305		heat tr coeff	h_up	6.4876	W/m^2.C...Ans.	

Note in the above the formula entered in cell D304, in the Formula bar.

3. Likewise, complete the calculation for the case of hot surface facing down:

D307 =NatConv_Air_HPlate_HotLower_Nusselt_Lc(D296,D297,D299,D300)

	A	B	C	D	E	F
303		Horizl plate:				
304		Hot surface facing up:	Nusslts_up	27.0317	.....Ans.	
305		heat tr coeff	h_up	6.4876	W/m^2.C...Ans.	
306						
307		Hot surface facing down:	Nusslts_down	13.5158	...Ans.	
308		heat tr coeff	h_down	3.2438	W/m^2.C ... Ans.	

Note how easy it becomes to get properties of Air, solve complicated eqns using VBA Functions.

Another example:

Prob. 2A2.2.13. A vertical radiator is 1 m high and 0.5 m wide, and maintained at a temp of 85 C in a room at 20 C. Find the convective heat loss.

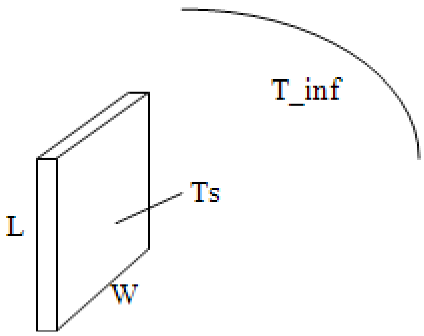


Fig.Prob.2A2.2.13

Following are the steps in EXCEL calculations:

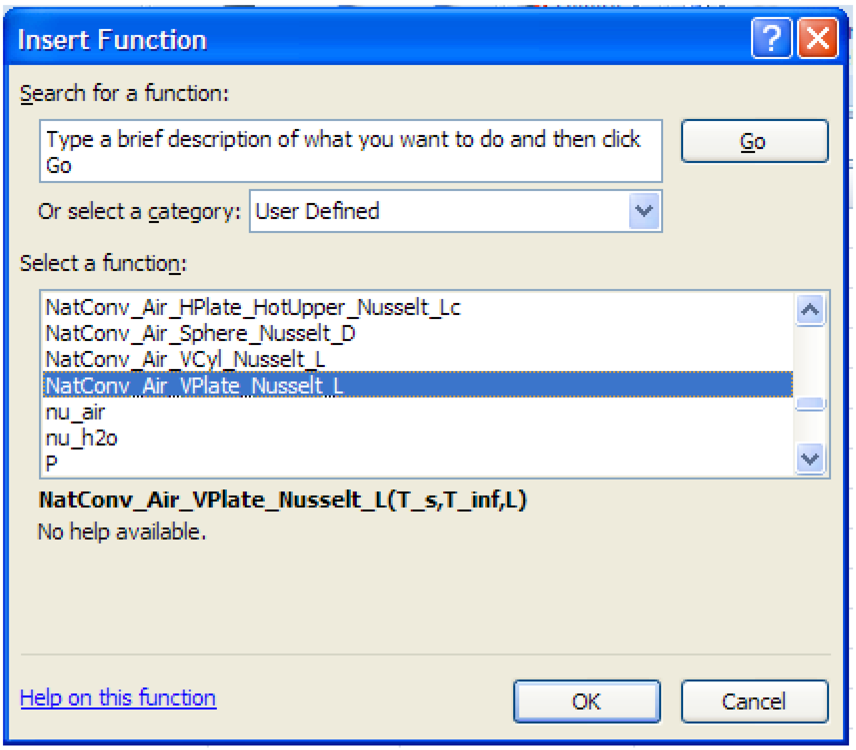
1. Set up the worksheet as shown:

D319 | =Air_k_T(D315+273)

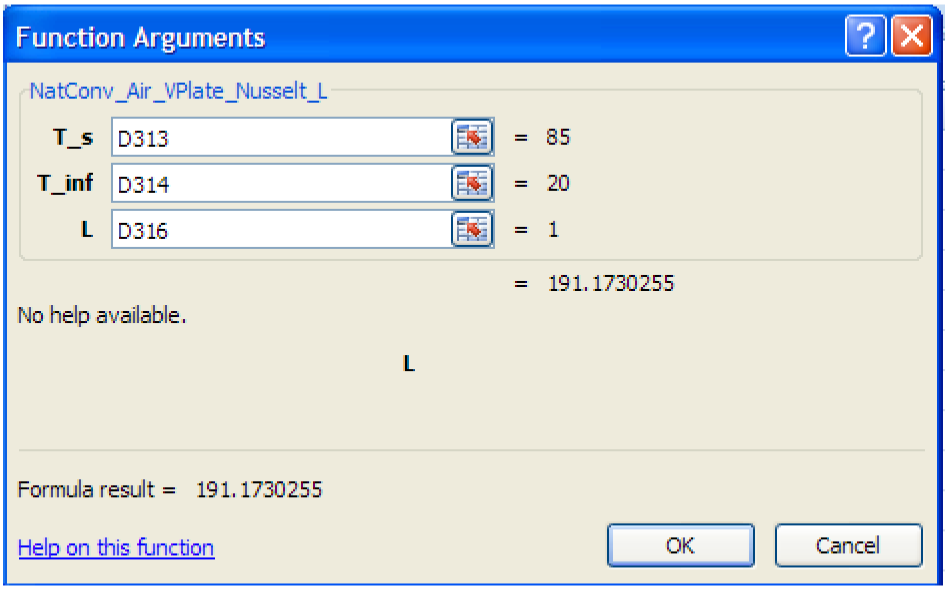
	A	B	C	D	
312		Data:	Fluid =	Air	
313			T_s	85	C
314			T_inf	20.0	C
315			T_f	52.5	C
316			L	1	m
317			W	0.50	m
318					
319		Th. cond. of Air at T_f	k	0.028246	W/m.C

In the above, note that k of Air is calculated using the VBA Function shown in the Formula bar.

2. Now, use the VBA Function for Avg. Nusselts No. for vertical plate: (see the previous example for procedure). Choose the Function as shown below:



Press OK, and following screen appears; fill it up appropriately:



Press OK and we get:

D321 =NatConv_Air_VPlate_Nusselt_L(D313,D314,D316)

	A	B	C	D	E
312		**Data:**	**Fluid =**	**Air**	
313			**T_s**	**85**	C
314			**T_inf**	**20.0**	C
315			**T_f**	**52.5**	C
316			**L**	**1**	m
317			**W**	**0.50**	m
318					
319		Th. cond. of Air at T_f	**k**	0.028246	W/m.C
320					
321		**Vert. plate:**	**Nusslts_vert**	**191.1730255**	.... Ans.

3. Now, calculate h and Q:

   h = Nusselts_vert * k / L

   Q = h * (L * W) * (T_s – T_inf)

See the following screen shot:

D323 | fx =D322*(D316*D317)*(D313-D314)

	A	B	C	D	E
315			T_f	52.5	C
316			L	1	m
317			W	0.50	m
318					
319		Th. cond. of Air at T_f	k	0.028246	W/m.C
320					
321		Vert. plate:	Nusslts_vert	191.1730	…. Ans.
322		heat tr coeff	h_vert	5.3999	W/m^2.C
323		Heat transfer	Q_vert	175.4959	W … Ans.

Another example of a horizl cylinder:

Prob. 2A2.2.14. A horizontal pipe, 10 cm OD is in an ambient air at 30 C. Surface temp is 170 C. Calculate the heat loss by convection only, per metre length of pipe.

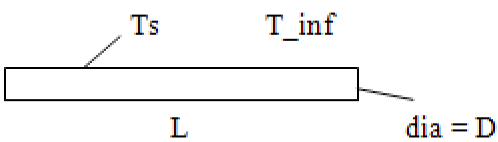


Fig.Prob.2A2.2.14

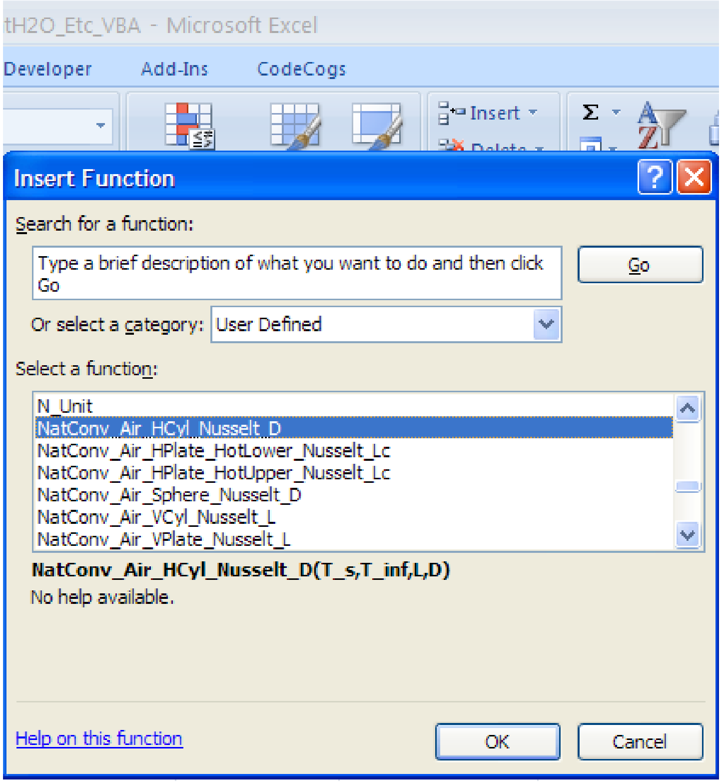
Following are the steps in EXCEL Solution:

1. Set up the worksheet:

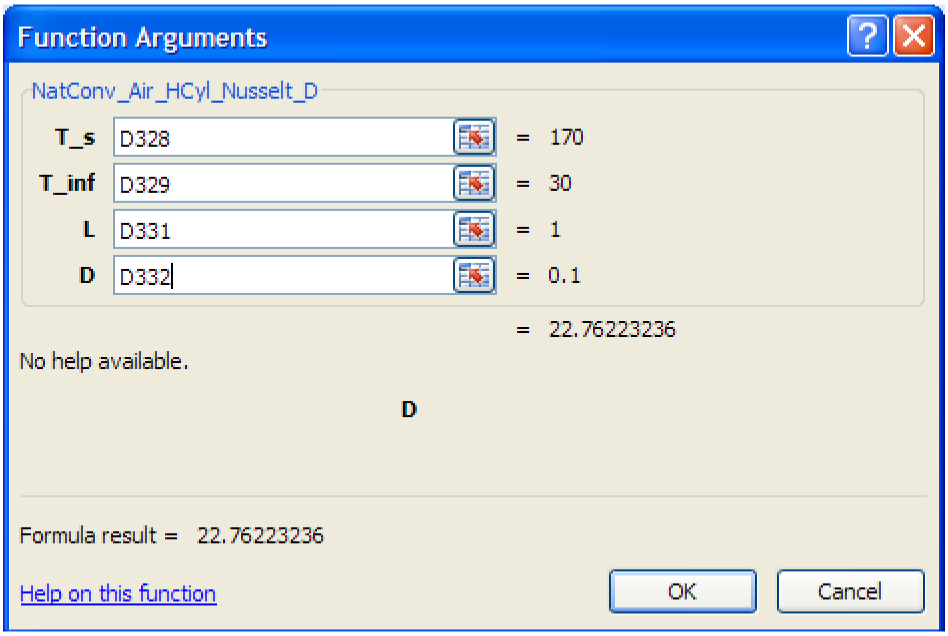
D334 | fx =Air_k_T(D330+273)

	A	B	C	D	
327		Data:	Fluid =	Air	
328			T_s	170	C
329			T_inf	30.0	C
330			T_f	100.0	C
331			L	1	m
332			D	0.10	m
333					
334		Th. cond. of Air at T_f	k	0.031607	W/m.C

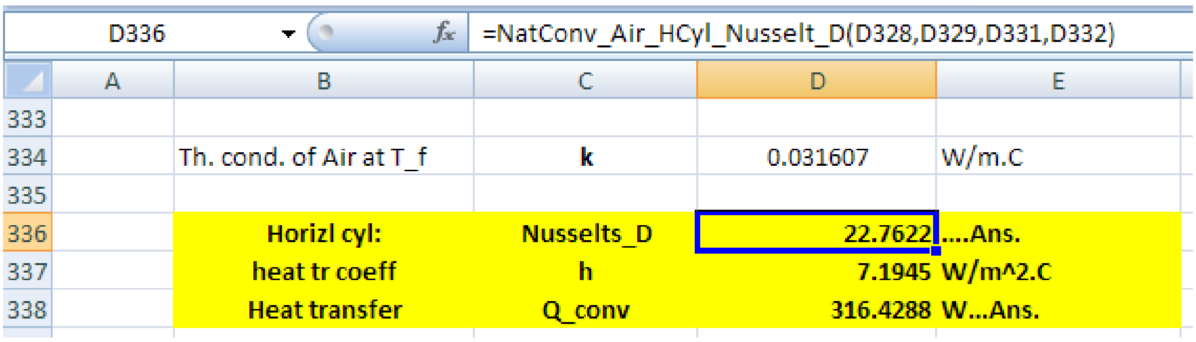
2. Now, calculate the Avg. Nusselts No. for Horizl cylinder using the appropriate VBA Function as shown below:



We get:



And,



In the above, h and Q_conv are calculated as:

h = Nusselts_D * k / D

Q_conv = h * (π * D * L) * (T_s – T_inf)

Another example of a vertical cylinder:

Prob. 2A2.2.15. A vertical pipe 15 cm OD, 1 m long has a surface temp of 90 C and is in a room where the Air is at 30 C. What is the rate of heat loss by Nat. convection?

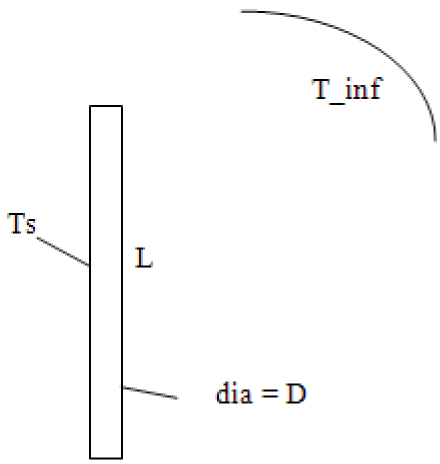


Fig.Prob.2A2.2.15

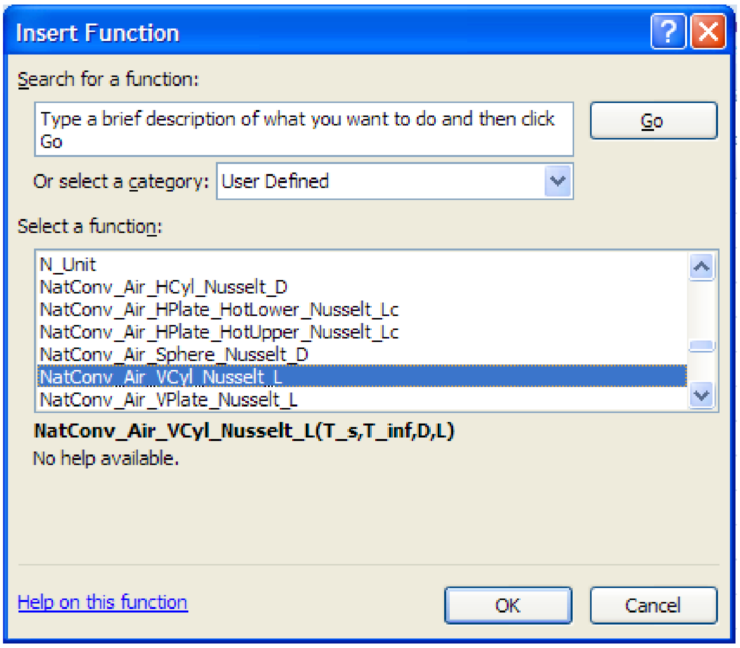
EXCEL Solution:

1. Set up the worksheet, enter data:

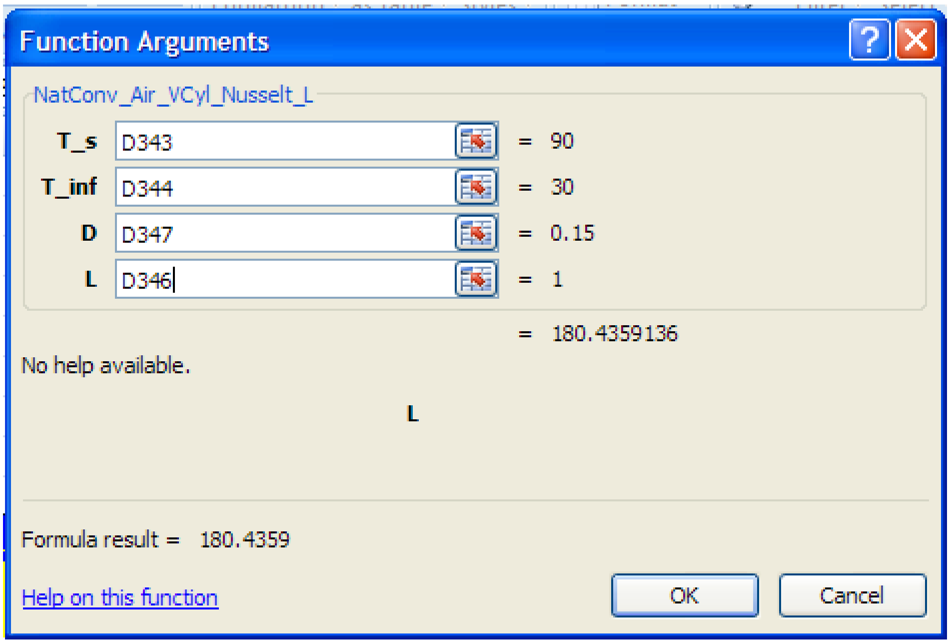
D349 | f_x =Air_k_T(D345+273)

	A	B	C	D	E
342		Data:	Fluid =	Air	
343			T_s	90	C
344			T_inf	30.0	C
345			T_f	60.0	C
346			L	1	m
347			D	0.15	m
348					
349		Th. cond. of Air at T_f	k	0.028786	W/m.C

2. Use the VBA Function for Nat. conv in Vert cyl to get Nusselts_L:



Press OK. Following screen appears; fill it up as shown:



Press OK. We get:

D351 | =NatConv_Air_VCyl_Nusselt_L(D343,D344,D347,D346)

	A	B	C	D	E
348					
349		Th. cond. of Air at T_f	k	0.028786	W/m.C
350					
351		Vertical cyl:	Nusselts_L	180.4359	....Ans.
352		heat tr coeff	h	5.1940	W/m^2.C
353		Heat transfer	Q_conv	146.8577	W...Ans.

In the above, h and Q_conv are calculated as:

h = Nusselts_L * k / L

Q_conv = h * (π * D * L) * (T_s – T_inf)

==

Prob. 2A2.2.16.: Consider a 8 cm dia, 60 W incandescent bulb in a room at 25 C. Emissivity of glass is 0.9. Assuming that 90% of the energy is converted to heat and dissipated by convection and radiation, determine the equilibrium temp of the bulb surface.

Also plot the surface temp T_s for different ambient temperatures and different emissivity values.

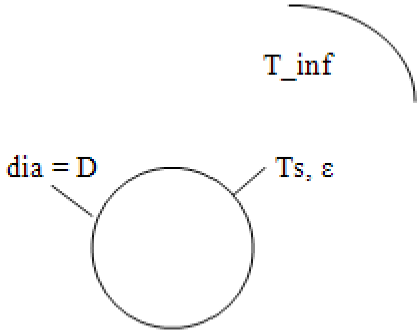


Fig.Prob.2A2.2.16

Note: This is the same as Prob. 2A2.2.8.

But, now, we shall solve it with EXCEL.

Following are the steps:

1. Set up the worksheet, enter data and name the cells:

Q_dissip | =0.9*Q

	A	B	C	D	E	F
210		Data:	Fluid =	Air		
211		Guess Value:	T_s	150	C	
212			T_inf	25.0	C	
213			T_f	87.50	C	
214		dia of sphere	D	0.08	m	$T\_f = \frac{T\_s + T\_inf}{2}$
215			Q	60.0000	W	
216		Heat dissipated by conv and radn	Q_dissip	54.0000	W	
217			g	9.8100	m/s^2	
218			beta	0.0028	1/K	
219			epsilon	0.9000		
220			sigma	5.6700E-08	W/m^2-K^4	

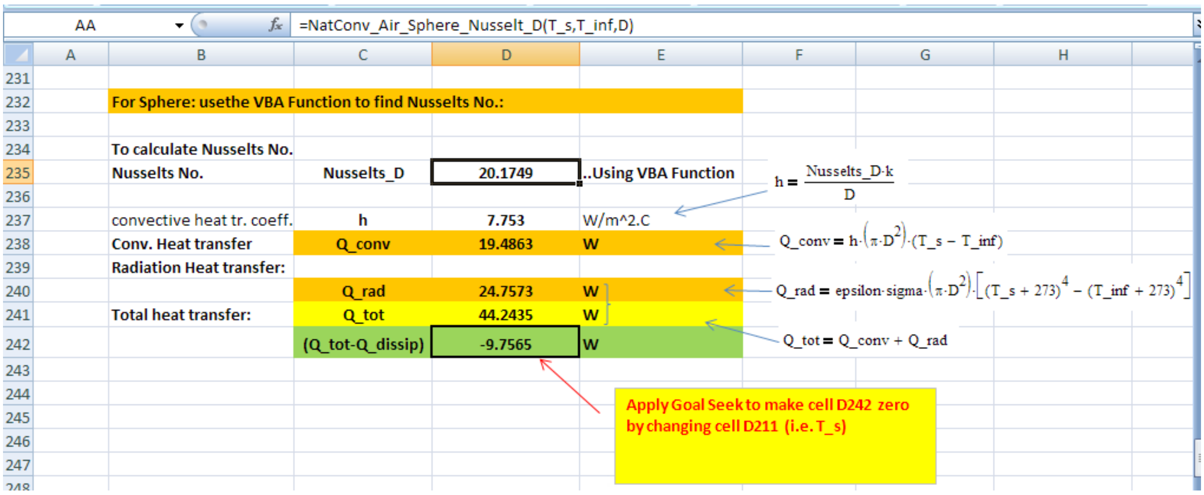
Note that temp of bulb surface is not yet known; we have assumed a value, using which we will calculate the convective heat transfer and radiative heat transfer from the bulb. Then, total of convective and radiative heat transfer should be equal to the actual heat dissipated from the bulb, Q_dissip = (0.9 * 60) = 54 W. So, we will apply Goal Seek in EXCEL to meet this condition by changing T_s.

2. Calculate the properties of Air using VBA Functions written earlier:

cp f_x =air_cp_T((T_f+273))

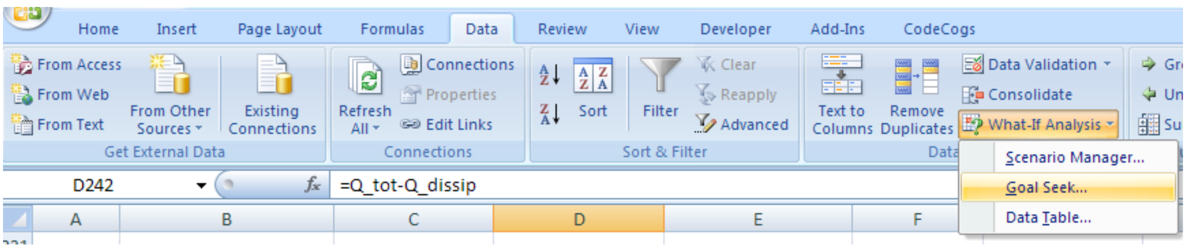
	A	B	C	D	E	F	G
222		**Calculations:**					
223		density	**rho**	0.97931	kg/m^3		
224		th. conductivity	**k**	0.0307445	W/m.C	Using VBA Functions for Air	
225		Prandtl No.	**Pr**	0.70127			
226		kinematic visc.	**nu**	2.17938E-05	m^2/s		
227		sp.heat	**cp**	1010.05	J/kg.K		
228							

3. Proceed with the calculations for Nusselts No. for sphere in Natural convection. Use the VBA Function written for Sphere in Nat. convection.

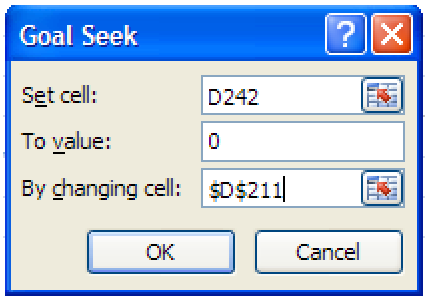


In the above worksheet, equations used are also shown, for clarity. We have to equate sum of (Q_conv + Q_rad) to Q_dissip. i.e. (Q_tot – Q_dissip) should be equal to zero. This is done in cell D242. And, apply Goal Seek to meet this condition by changing T_s, i.e. cell D211:

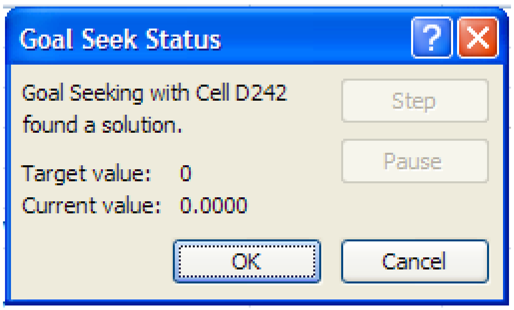
Go to Data-What If Analysis-Goal Seek:



Click on Goal Seek. We get the following screen. Fill it up as shown:



Press OK. We get the message:



Press OK. And read the value of T_s in cell D211:

G219

	A	B	C	D	E
210		Data:	Fluid =	Air	
211		Guess Value:	T_s	168.7916689	C
212			T_inf	25.0	C
213			T_f	96.90	C

Thus: Equilibrium temp of the bulb surface (when Q = 60 W, epsilon = 0.9) is: 168.79 deg.C….Ans.

Note: In Problem 2A2.2.8, with Mathcad, we obtained T_s = 169.298 C for the same value of Q, T_inf and epsilon. Slight difference in values of T_s is due to the fact that with Mathcad we had obtained the values of properties of Air from Curve fit eqns whereas with EXCEL, we used interpolation for properties of Air from the Data Table.

Also plot T_s for different ambient temperatures and different emissivity values:

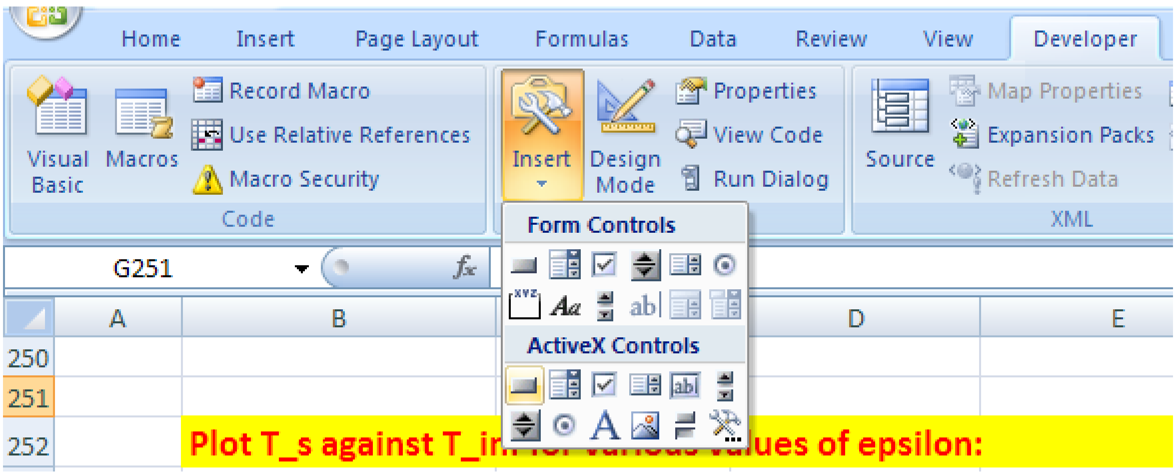
4. We will, first, set up a Table where T_inf is varied from 0 to 40 C, as shown below:

	A	B	C	D	E	F
250						
251						
252		Plot T_s against T_inf for various values of epsilon:				
253						
254		epsilon =	0.4	0.6	0.8	1
255		T_inf (deg.C)	T_s (deg.C)	T_s (deg.C)	T_s (deg.C)	T_s (deg.C)
256		0				
257		5				
258		10				
259		15				
260		20				
261		25				
262		30				
263		35				
264		40				

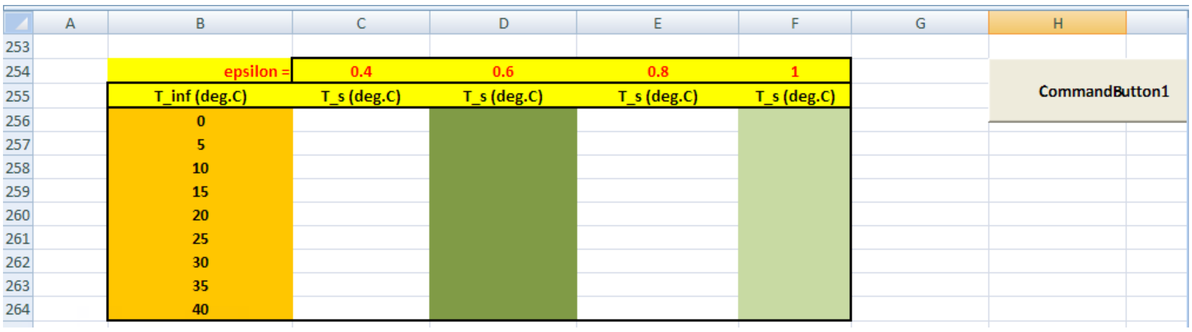
Now, we will write a VBA program which will first read the value of epsilon from cell C254, and then read value of T_inf one by one , starting from cell B256, and copy it to cell D212. Then, with the existing value for T_s, all other values will change. Then, apply Goal Seek to make cell D242 zero by changing T_s (i.e. cell D211). Then copy this value of T_s from cell 211 to cell C256. Now, proceed to the next value of T_inf, i.e. downwards from cell B256 up to cell B264, and repeat the calculations till the column under 'epsilon = 0.4' is completed. Next, set the value of epsilon = 0.6 and repeat the calculations under that column. Similarly, for epsilon = 0.8 and 1. And, we wish this program to be operated from a Command Button.

5. To write the VBA program to perform these tasks, proceed as follows:

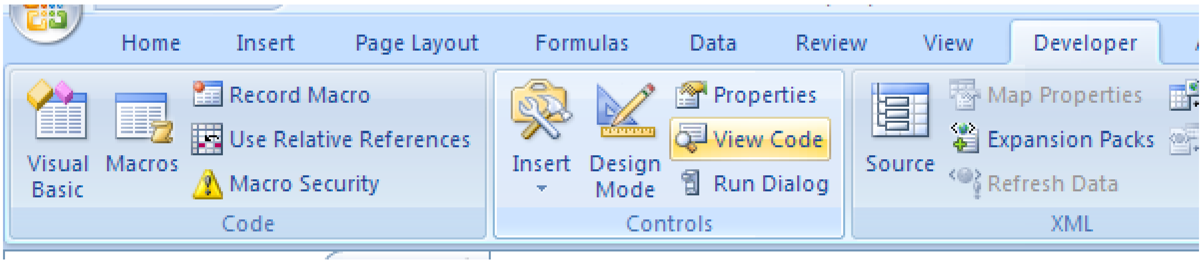
Go to Developer – Insert-Active X controls:



6. Click on the first, left button. And, draw the Command Button in the worksheet to the required size, at the required place:



7. Click on Developer-View Code:



8. And, we get the code already written for the Command button:

```
CommandButton1                                  Click

Private Sub CommandButton1_Click()

End Sub
```

9. Now, complete the code to do the desired jobs as mentioned under point 4 above:

```
CommandButton1                                  Click

Private Sub CommandButton1_Click()
Dim i As Integer
Dim j As Integer

For j = 0 To 3

Range("D219") = Cells(254, 3 + j)

For i = 0 To 8

  Range("D212") = Cells(256 + i, 2)

  Range("D242").GoalSeek Goal:=0, ChangingCell:=Range("D211")

  Cells(256 + i, 3 + j) = Range("D211")

Next i
Next j

End Sub
```

In the above code:

Line 1: defines the Sub

Line 2,3: dimension statement for I and j, the counters in For….Next constructs

Line 4 to 11: 'For … Next' slab

Line 5: Takes the first value of epsilon from the cell C254 in Table and copies it to cell D219 (i.e. 'epsilon' in the original program)

Immediately, some calculations in other cells are up-dated

Line 6 to 10: inner 'For….Next' loop to calculate T_s for different values of T_inf using Goal Seek.

Line 7: first value of T_inf from cell B256 is copied to cell D219 in the original program; now, some calculations get updated.

Line 8: This is the Goal Seek operation which sets cell D242 to zero and finds T_s by changing cell D211, which contains the guess value for T_s

Line 9: After the Goal Seek is over, the resulting value of T_s in cell D211 is copied to its respective position in the Table.

Line 10: go to next value of T_inf and repeat these calculations

Line 11: now, go to next value of epsilon in the Table and repeat the calculations

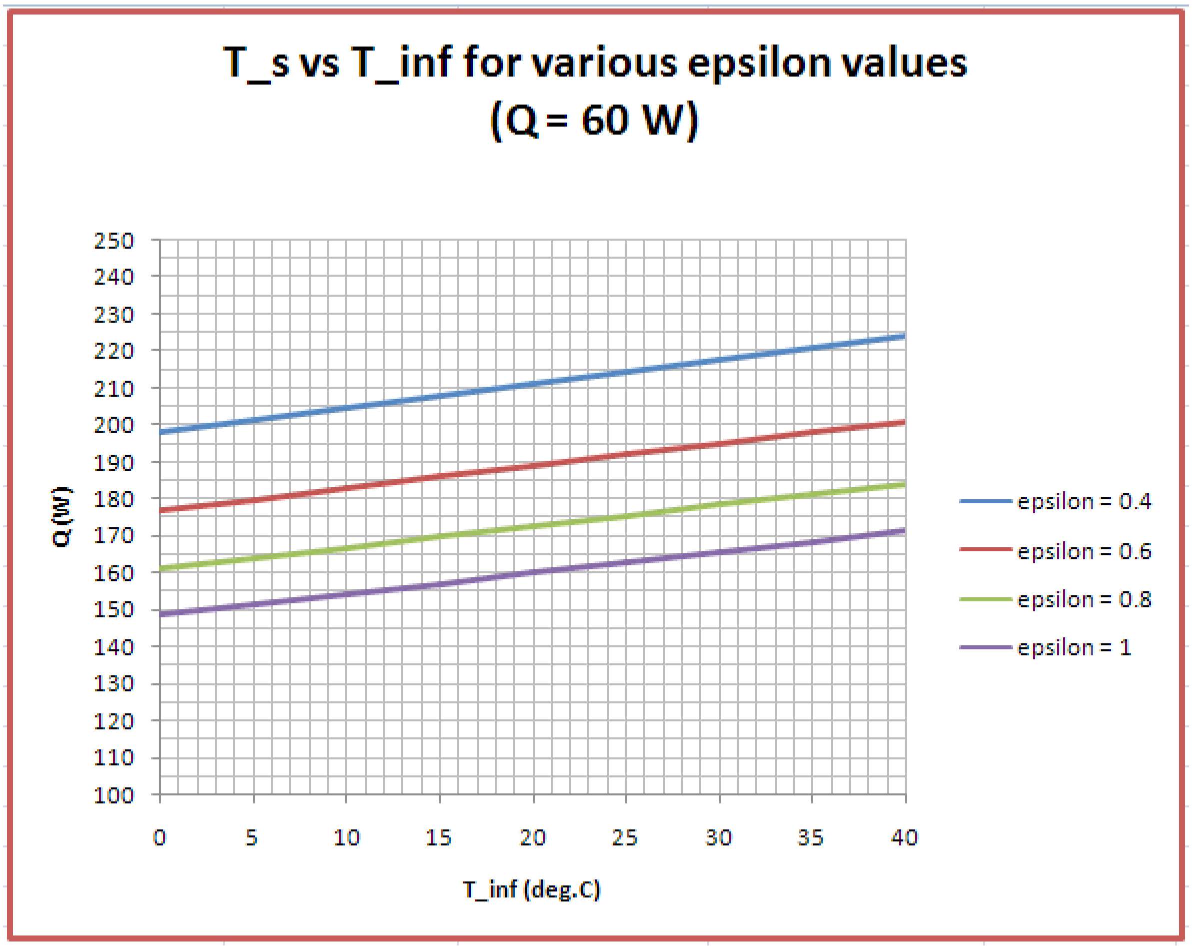
Line 12: End statement of Sub-routine

10. Now, click on the Command Button, and the Table gets filled up with the calculated values:

epsilon =	0.4	0.6	0.8	1
T_inf (deg.C)	T_s (deg.C)	T_s (deg.C)	T_s (deg.C)	T_s (deg.C)
0	198.198	176.819	161.061	148.683
5	201.471	179.859	163.945	151.494
10	204.724	182.885	166.832	154.306
15	207.986	185.897	169.730	157.120
20	211.226	188.898	172.624	159.952
25	214.445	191.898	175.516	162.793
30	217.646	194.905	178.407	165.639
35	220.836	197.904	181.304	168.492
40	224.032	200.896	184.215	171.353

CommandButton1

11. Plot the results in EXCEL:



12. **If we now need to get values of T_s for another value of Q,** just change the value of Q in cell D215, and click on the Command Button.

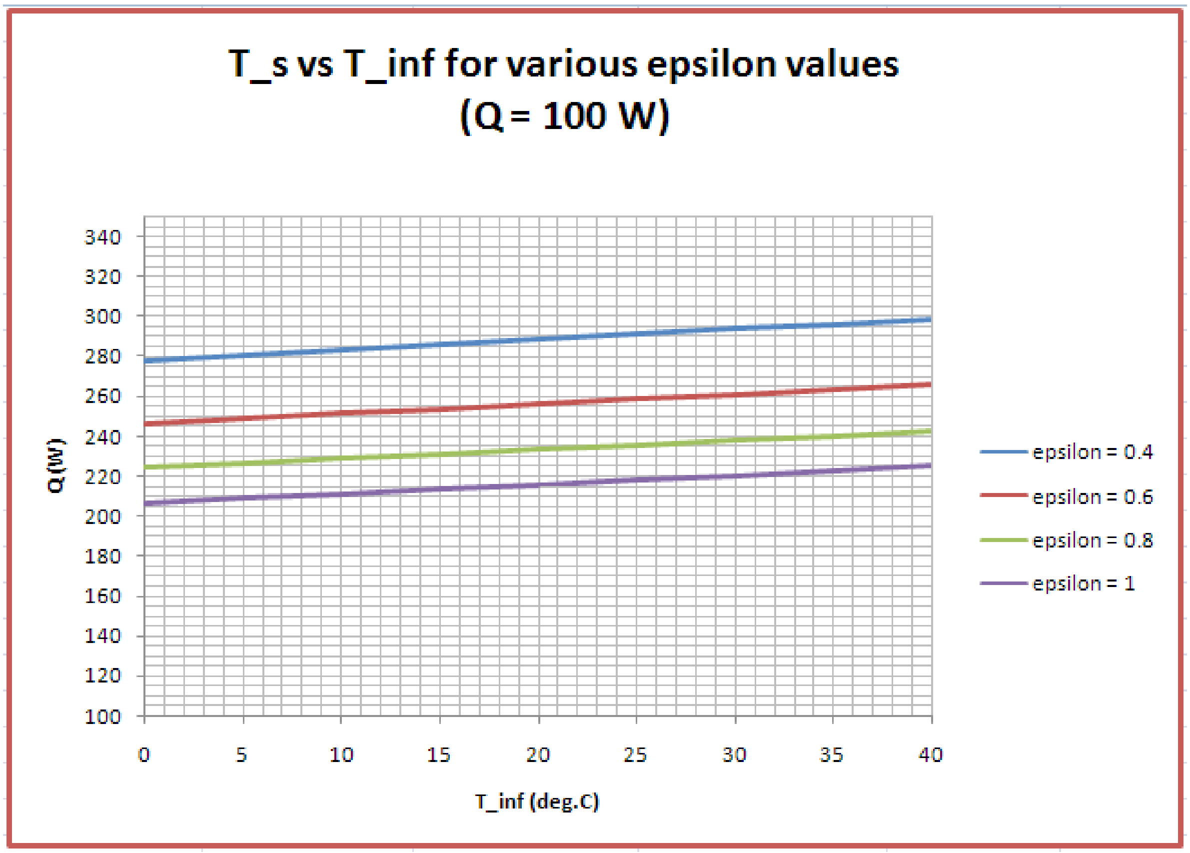
Thus, for Q = 100 W, we get:

Plot T_s against T_inf for various values of epsilon:

epsilon =	0.4	0.6	0.8	1
T_inf (deg.C)	T_s (deg.C)	T_s (deg.C)	T_s (deg.C)	T_s (deg.C)
0	278.085	246.624	224.211	206.978
5	280.721	249.034	226.517	209.207
10	283.341	251.460	228.820	211.457
15	285.971	253.878	231.123	213.711
20	288.588	256.289	233.426	215.971
25	291.191	258.695	235.748	218.238
30	293.782	261.097	238.074	220.514
35	296.361	263.509	240.403	222.806
40	298.945	265.926	242.737	225.116

CommandButton1

And, the plot is up-dated:



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2A2.3 Natural convection from enclosed spaces:

Prob. 2A2.3.1 : Air gap between the two glass panels of a double-pane window (1.8 m wide × 1.2 m high) is 2.5 cm. If the two glass surfaces are at 18 C and 4 C, determine the rate of heat transfer through the window by (a) free convection (b) radiation. Effective emissivity between the two plates is 0.82.

Plot these values for air gap thickness varying from 0.5 cm to 5 cm.

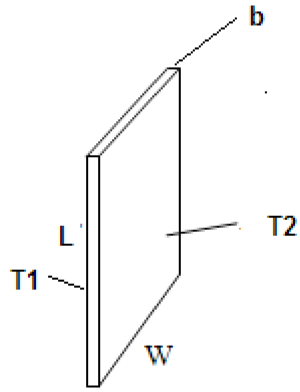


Fig.Prob.2A2.3.1

Mathcad solution:

Use the Mathcad functions, already written, (see Prob. 2A1.2.5) to get properties of Air.

Data:

$L := 1.2$ m...height of panels $W := 1.8$ m...width of panels

$b := 0.025$ m...distance between panels $T1 := 18$ C.....first surface temp.

$T2 := 4$ C....second surface temp. $g := 9.81$ m/s²....accn. due to gravity

We need properties of air at film temperature: $T_f := \frac{T1 + T2}{2}$

$T_f = 11$ C....avg. temperature

$\sigma := 5.67 \cdot 10^{-8}$ W/m^2.K^4....Stefan Boltzmann const.

Calculations:

Properties of Air at T_f:

$\rho := rho\_Air(T_f + 273)$ i.e. $\rho = 1.243$ kg/m^3

$k := k\_Air(T_f + 273)$ i.e. $k = 0.025$ W/m.K

$\mu := mu\_Air(T_f + 273)$ i.e. $\mu = 1.764 \cdot 10^{-5}$ kg/m.s

$Pr := Pr\_Air(T_f + 273)$ i.e. $Pr = 0.711$ ...Prandtl No.

$\beta := \frac{1}{T_f + 273}$ i.e. $\beta = 3.521 \cdot 10^{-3}$ 1/K

$\nu := \frac{\mu}{\rho}$ i.e. $\nu = 1.419 \cdot 10^{-5}$ m^2/s

$\varepsilon_{effective} := 0.82$

Remember that here, the distance between panels 'b' is the characteristic dimension.

Then, $Gr(b) := \frac{g \cdot \beta \cdot (T1 - T2) \cdot b^3}{\nu^2}$ ....Grashof No. as a function of air gap thickness b

i.e. $Gr(b) = 3.753 \cdot 10^4$ ....Grashoff number

Then, Rayleigh No.:

$Ra(b) := Gr(b) \cdot Pr$

i.e. $Ra(b) = 2.669 \cdot 10^4$ ....Rayleigh No.

Note that Grashof No. etc. are written as functions of air gap thickness 'b' so that we can draw the graphs later.

And, $\frac{L}{b} = 48$ > 3.....condition is satisfied.

Then, use the following eqn to get Nusselts No. (Ref: 2):

$Nu(b) := 0.42 \cdot Ra(b)^{\frac{1}{4}} \cdot Pr^{0.012} \cdot \left(\frac{L}{b}\right)^{-0.3}$ ....for 10 < L/b < 40, 1 < Pr < 2*10^4, 10^4 < Ra < 10^7

i.e. $Nu(b) = 1.674$ ...Nusselts No.

Heat transfer by convection:

Now, area for convective and radiative heat transfer is:

$A_s := L \cdot W$ i.e. $A_s = 2.16$ m^2 ... area

Therefore: $Q_{conv} = h \cdot A_s \cdot (T1 - T2) = k \cdot Nu \cdot A_s \cdot \frac{(T1 - T2)}{b}$

where, $k_{eff}(b) := Nu(b) \cdot k$

i.e. $k_{eff}(b) = 0.041$ W/(m.K)....effective thermal cond.

Therefore, $Q_{conv}(b) := k_{eff}(b) \cdot (A_s) \cdot \frac{(T1 - T2)}{b}$ W....heat transfer rate

i.e. $Q_{conv}(b) = 50.043$ **W....heat transfer rate by convection....Ans.**

Heat transfer by radiation:

For heat transfer between two parallel plates, we have:

$$Q_{rad} = \frac{\sigma \cdot A_s \cdot \left[(T1+273)^4 - (T2+273)^4\right]}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \varepsilon_{effective} \cdot \sigma \cdot A_s \cdot \left[(T1+273)^4 - (T2+273)^4\right]$$

Therefore:

$$Q_{rad} := \varepsilon_{effective} \cdot \sigma \cdot A_s \cdot \left[(T1+273)^4 - (T2+273)^4\right]$$

i.e. $Q_{rad} = 128.901$ W..... heat transfer by radiation .... Ans.

Note that Q_{rad} does not depend on the air gap thickness, b.

Therefore: Total heat transfer:

$$Q_{tot}(b) := Q_{conv}(b) + Q_{rad}$$

i.e. $Q_{tot}(b) = 178.944$ **W .... Total heat transfer...Ans.**

Prob. 2A2.3.2 : In a solar flat plate collector, the plate is of size 1.5 m high and 3m wide and is at a temperature of 80 C. The glass cover plate is at a distance of 2.5 cm from the collector surface and its temp. is 40 C. Space in between contains air at 1 atm. If the collector plate is inclined to the horizontal at θ deg., determine the heat transfer by free convection for θ = 0, 30 and 90 deg.

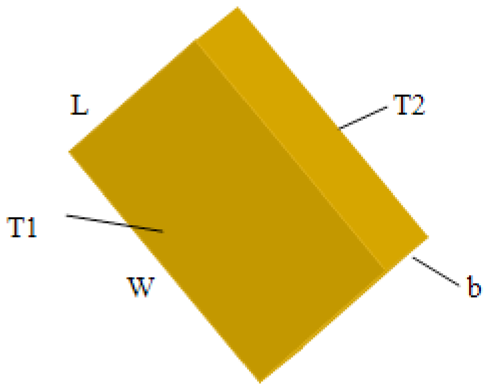


Fig.Prob.2A2.3.2

Mathcad Solution:

Use the Mathcad functions, already written, (see Prob. 2A1.2.5) to get properties of Air.

Data:

$L := 1.5$ m...height of panels $W := 3$ m...width of panels

$b := 0.025$ m...distance between panels $T1 := 80$ C.....first surface temp.

$T2 := 40$ C....second surface temp. $g := 9.81$ m/s^2....accn. due to gravity

We need properties of air at film temperature: $T_f := \frac{T1 + T2}{2}$

$T_f = 60$ C....avg. temperature

$\theta := 30$ deg.....angle of tilt (to horizontal)

But, while using Mathcad, *arguments for trigonometric functions must be in radians.*

Calculations:

Properties of Air at T_f:

$\rho := rho\_Air(T_f + 273)$ i.e. $\rho = 1.06$ kg/m^3

$k := k\_Air(T_f + 273)$ i.e. $k = 0.028$ W/m.K

$\mu := mu\_Air(T_f + 273)$ i.e. $\mu = 2 \cdot 10^{-5}$ kg/m.s

$Pr := Pr\_Air(T_f + 273)$ i.e. $Pr = 0.702$ ...Prandtl No.

$\beta := \frac{1}{T_f + 273}$ i.e. $\beta = 3.003 \cdot 10^{-3}$ 1/K

$\nu := \frac{\mu}{\rho}$ i.e. $\nu = 1.886 \cdot 10^{-5}$ m^2/s

Remember that here, the distance between the panels 'b' is the characteristic dimension.

And, $\frac{L}{b} = 60$ > 12.....condition is satisfied.

Then, $Gr_b := \dfrac{g \cdot \beta \cdot (T1 - T2) \cdot b^3}{\nu^2}$

i.e. $Gr_b = 5.174 \cdot 10^4$ ....Grashoff number

and, $Ra_b := Gr_b \cdot Pr$ ...Rayleigh number

i.e. $Ra_b = 3.634 \cdot 10^4$ ...Rayleigh number

Now, different relations are available for horizontal, tilted and vertical flat plate enclosures [Ref. 2].

To incorporate them let us write a Mathcad Function:

Note that for horizontal plates: tilt angle = 0 and for vertical plates, tilt angle = 90 deg. Following function calculates Nu_b for tilt angles from 0 to 180 deg.

$$Nu_b(L, b, Ra_b, \theta) := \begin{array}{|l} \text{return "L/b must be >=12 !" if } \frac{L}{b} < 12 \\ \text{return } \left[0.42 \cdot Ra_b^{\frac{1}{4}} \cdot Pr^{0.012} \cdot \left(\frac{L}{b}\right)^{-0.3}\right] \text{ if } \theta = 90 \\ \text{if } \theta \le 70 \\ \quad \begin{array}{|l} AA1 \leftarrow \left(1 - \frac{1708}{Ra_b \cdot \cos\left(\theta \cdot \frac{\pi}{180}\right)}\right) \\ BB1 \leftarrow \frac{\left(Ra_b \cdot \cos\left(\theta \cdot \frac{\pi}{180}\right)\right)^{\frac{1}{3}}}{18} - 1 \\ BB \leftarrow 0 \text{ if } BB1 \le 0 \\ BB \leftarrow BB1 \text{ if } BB1 > 0 \\ AA \leftarrow 0 \text{ if } AA1 \le 0 \\ AA \leftarrow AA1 \text{ if } AA1 > 0 \\ \text{return } 1 + 1.44 \cdot (AA) \cdot \left[1 - \frac{1708 \cdot \left[\sin\left[1.8 \cdot \left(\theta \cdot \frac{\pi}{180}\right)\right]\right]^{1.6}}{Ra_b \cdot \cos\left(\theta \cdot \frac{\pi}{180}\right)}\right] + BB \end{array} \\ \text{return } \left[0.42 \cdot Ra_b^{\frac{1}{4}} \cdot Pr^{0.012} \cdot \left(\frac{L}{b}\right)^{-0.3}\right] \cdot \left(\sin\left(\theta \cdot \frac{\pi}{180}\right)\right)^{\frac{1}{4}} \text{ if } \theta < 90 \\ \text{return } \left[1 + \left[0.42 \cdot Ra_b^{\frac{1}{4}} \cdot Pr^{0.012} \cdot \left(\frac{L}{b}\right)^{-0.3} - 1\right] \cdot \sin\left(\theta \cdot \frac{\pi}{180}\right)\right] \text{ if } \theta \le 180 \end{array}$$

Heat transfer Q, by convection:

Case 1: When the flat plate collector is horizontal, i.e. θ = 0:

$\theta := 0$ ....horizontal plates

Then: $Nu_b(L, b, Ra_b, \theta) = 3.213$ **..Nusselt Number**

$A_s := L \cdot W$ i.e. $A_s = 4.5$ m^2 .... area for heat transfer

$k_{eff}(L, b, Ra_b, \theta) := k \cdot Nu_b(L, b, Ra_b, \theta)$ **....effective thermal conductivity**

i.e. $k_{eff}(L, b, Ra_b, \theta) = 0.091$ **W/m.C .... effective thermal cond.**

$$Q_{conv}(L, b, Ra_b, \theta) := k_{eff}(L, b, Ra_b, \theta) \cdot A_s \cdot \frac{(T1 - T2)}{b}$$

i.e. $Q_{conv}(L, b, Ra_b, \theta) = 656.227$ **W ... heat transfer by natural convection ..... Ans.**

Case 2: When the flat plate collector is tilted to horizontal at θ = 30 deg.:

$\theta := 30$ ....tilted to horizontal at 30 deg.

Then: $Nu_b(L, b, Ra_b, \theta) = 3.063$ **..Nusselt Number**

And:

$k_{eff}(L, b, Ra_b, \theta) := k \cdot Nu_b(L, b, Ra_b, \theta)$ **....effective thermal conductivity**

i.e. $k_{eff}(L, b, Ra_b, \theta) = 0.087$ **W/m.C .... effective thermal cond.**

$$Q_{conv}(L, b, Ra_b, \theta) := k_{eff}(L, b, Ra_b, \theta) \cdot A_s \cdot \frac{(T1 - T2)}{b}$$

i.e. $Q_{conv}(L, b, Ra_b, \theta) = 625.736$ **W ... heat transfer by natural convection ..... Ans.**

Case 3: When the flat plate collector is vertical, i.e. θ = 90 deg.:

$\theta := 90$ ....vertical plates

Then: $Nu_b(L, b, Ra_b, \theta) = 1.691$ **..Nusselt Number**

And:

$k_{eff}(L, b, Ra_b, \theta) := k \cdot Nu_b(L, b, Ra_b, \theta)$ **....effective thermal conductivity**

i.e. $k_{eff}(L, b, Ra_b, \theta) = 0.048$ **W/m.C .... effective thermal cond.**

$$Q_{conv}(L, b, Ra_b, \theta) := k_{eff}(L, b, Ra_b, \theta) \cdot A_s \cdot \frac{(T1 - T2)}{b}$$

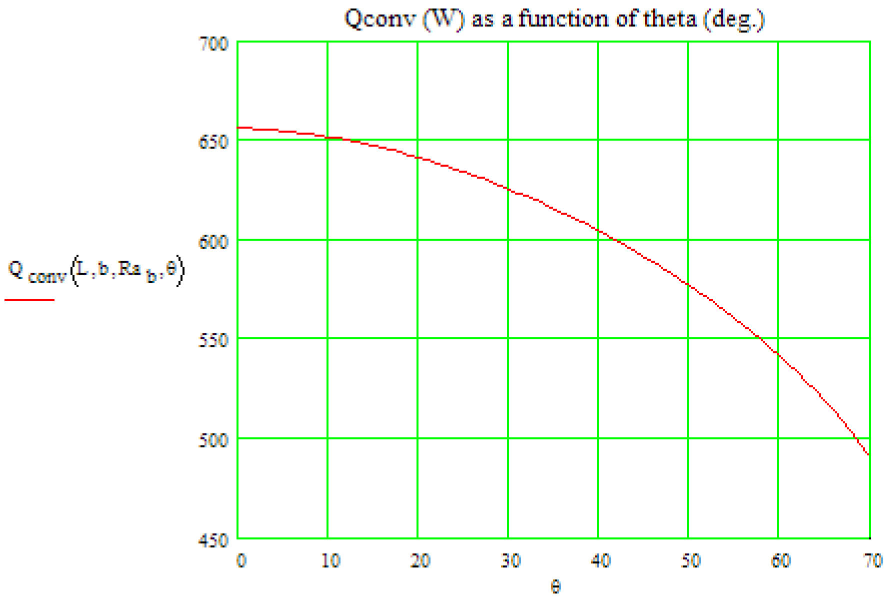
i.e. $Q_{conv}(L, b, Ra_b, \theta) = 345.362$ **W ... heat transfer by natural convection ..... Ans.**

Plot Q_{conv} for various values of θ: (For L/b > 12, critical angle of tilt is 70 deg.)

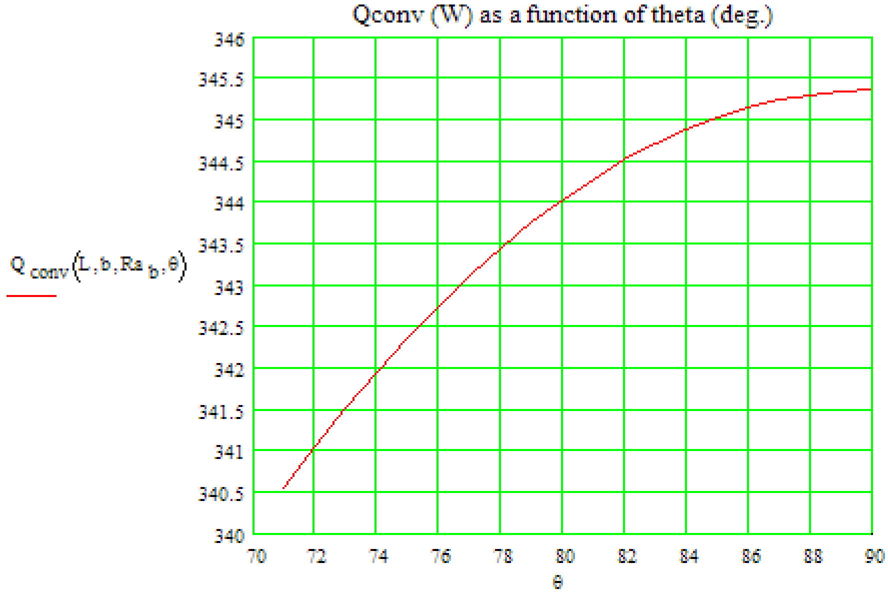
θ := 0, 10.. 180 deg ..... variation of θ

θ	$Nu_b(L, b, Ra_b, \theta)$	$Q_{conv}(L, b, Ra_b, \theta)$
0	3.213	656.227
10	3.192	652.059
20	3.141	641.654
30	3.063	625.736
40	2.959	604.42
50	2.825	577.122
60	2.652	541.739
70	2.407	491.775
80	1.684	344.043
90	1.691	345.362
100	1.68	343.219
110	1.649	336.853
120	1.598	326.459
130	1.529	312.353
140	1.444	294.962
150	1.345	274.816
160	1.236	252.526
170	1.12	228.77
180	1	204.269

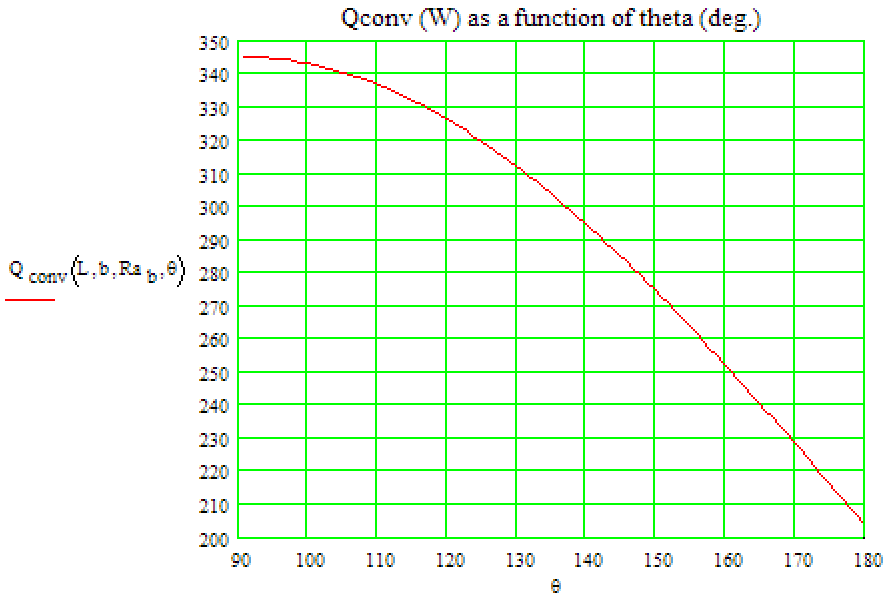
Q_{conv} between theta angles of 0 and 70 deg:



Q_{conv} between theta angles of 71 and 90 deg:



Q_{conv} between theta angles of 91 and 180 deg:



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Prob. 2A2.3.3 : A flat plate solar collector of 3 m × 5 m area has an absorber plate that is to operate at a temp of 70 C. The glass cover is placed 0.05 m from the absorber, and its operating temp is 35 C. Determine the rate of heat loss from the absorber if the 3 m edge is tilted at angles of 0, 30 and 60 deg. to horizontal.

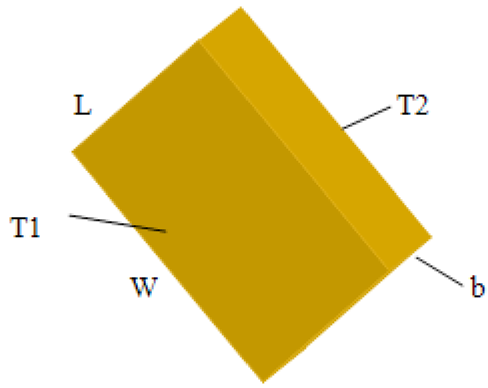


Fig.Prob.2A2.3.3

Mathcad Solution:

Use the Mathcad functions, already written, (see Prob. 2A1.2.5) to get properties of Air.

Data:

$L := 3$ m...height of panels $W := 5$ m...width of panels

$b := 0.05$ m...distance between panels $T1 := 70$ C.....first surface temp.

$T2 := 35$ C....second surface temp. $g := 9.81$ m/s^2....accn. due to gravity

We need properties of air at film temperature: $T_f := \frac{T1 + T2}{2}$

$T_f = 52.5$ C....avg. temperature

$\theta := 0$ deg.....angle of tilt (to horizontal)

But, while using Mathcad, remember that arguments for trigonometric functions *must be in radians.*

Calculations:

Properties of Air at T_f:

$\rho := \text{rho\_Air}(T_f + 273)$ i.e. $\rho = 1.085$ kg/m^3

$k := \text{k\_Air}(T_f + 273)$ i.e. $k = 0.028$ W/m.K

$\mu := \text{mu\_Air}(T_f + 273)$ i.e. $\mu = 1.965 \cdot 10^{-5}$ kg/m.s

$Pr := \text{Pr\_Air}(T_f + 273)$ i.e. $Pr = 0.703$ ...Prandtl No.

$\beta := \frac{1}{T_f + 273}$ i.e. $\beta = 3.072 \cdot 10^{-3}$ 1/K

$\nu := \frac{\mu}{\rho}$ i.e. $\nu = 1.812 \cdot 10^{-5}$ m^2/s

Remember that here, the distance between the panels 'b' is the characteristic dimension.

And, $\frac{L}{b} = 60$ > 12.....condition is satisfied.

Then, $Gr_b := \frac{g \cdot \beta \cdot (T1 - T2) \cdot b^3}{\nu^2}$

i.e. $Gr_b = 4.018 \cdot 10^5$ ....Grashoff number and, $Ra_b := Gr_b \cdot Pr$ ...Rayleigh number

i.e. $Ra_b = 2.826 \cdot 10^5$ ...Rayleigh number

Use the Mathcad function to calculate Nu for various conditions (θ between 0 and 70 deg.). Ref.[2]:

$$Nu_b(L, b, Ra_b, \theta) := \begin{array}{|l} \text{return "L/b must be >=12 !" if } \frac{L}{b} < 12 \\ \text{return } \left[0.42 \cdot Ra_b^{\frac{1}{4}} \cdot Pr^{0.012} \cdot \left(\frac{L}{b}\right)^{-0.3}\right] \text{ if } \theta = 90 \\ \text{if } \theta \le 70 \\ \quad \begin{array}{|l} AA1 \leftarrow \left(1 - \frac{1708}{Ra_b \cdot \cos\left(\theta \cdot \frac{\pi}{180}\right)}\right) \\ BB1 \leftarrow \frac{\left(Ra_b \cdot \cos\left(\theta \cdot \frac{\pi}{180}\right)\right)^{\frac{1}{3}}}{18} - 1 \\ BB \leftarrow 0 \text{ if } BB1 \le 0 \\ BB \leftarrow BB1 \text{ if } BB1 > 0 \\ AA \leftarrow 0 \text{ if } AA1 \le 0 \\ AA \leftarrow AA1 \text{ if } AA1 > 0 \\ \text{return } 1 + 1.44 \cdot (AA) \cdot \left[1 - \frac{1708 \cdot \left[\sin\left[1.8 \cdot \left(\theta \cdot \frac{\pi}{180}\right)\right]\right]^{1.6}}{Ra_b \cdot \cos\left(\theta \cdot \frac{\pi}{180}\right)}\right] + BB \end{array} \\ \text{return } \left[0.42 \cdot Ra_b^{\frac{1}{4}} \cdot Pr^{0.012} \cdot \left(\frac{L}{b}\right)^{-0.3}\right] \cdot \left(\sin\left(\theta \cdot \frac{\pi}{180}\right)\right)^{\frac{1}{4}} \text{ if } \theta < 90 \\ \text{return } \left[1 + \left[0.42 \cdot Ra_b^{\frac{1}{4}} \cdot Pr^{0.012} \cdot \left(\frac{L}{b}\right)^{-0.3} - 1\right] \cdot \sin\left(\theta \cdot \frac{\pi}{180}\right)\right] \text{ if } \theta \le 180 \end{array}$$

Therefore, $Nu_b(L, b, Ra_b, \theta) = 5.077$ .....Nusselts No.

Heat transfer Q, by convection:

Case 1: When the flat plate collector is horizontal, i.e. θ = 0:

$\theta := 0$ ....horizontal plates

Then: $Nu_b(L, b, Ra_b, \theta) = 5.077$ **..Nusselt Number**

$A_s := L \cdot W$ i.e. $A_s = 15$ m^2 .... area for heat transfer

$k_{eff}(L, b, Ra_b, \theta) := k \cdot Nu_b(L, b, Ra_b, \theta)$ **....effective thermal conductivity**

i.e. $k_{eff}(L, b, Ra_b, \theta) = 0.141$ **W/m.C .... effective thermal cond.**

$$Q_{conv}(L, b, Ra_b, \theta) := k_{eff}(L, b, Ra_b, \theta) \cdot A_s \cdot \frac{(T1 - T2)}{b}$$

i.e. $Q_{conv}(L, b, Ra_b, \theta) = 1.483 \cdot 10^3$ **W ... heat transfer by natural convection ..... Ans.**

Case 2: When the flat plate collector is tilted to horizontal at θ = 30 deg.:

$\theta := 30$ ....tilted to horizontal at 30 deg.

Then: $Nu_b(L, b, Ra_b, \theta) = 4.898$ **..Nusselt Number**

And:

$k_{eff}(L, b, Ra_b, \theta) := k \cdot Nu_b(L, b, Ra_b, \theta)$ **....effective thermal conductivity**

i.e. $k_{eff}(L, b, Ra_b, \theta) = 0.136$ **W/m.C .... effective thermal cond.**

$$Q_{conv}(L, b, Ra_b, \theta) := k_{eff}(L, b, Ra_b, \theta) \cdot A_s \cdot \frac{(T1 - T2)}{b}$$

i.e. $Q_{conv}(L, b, Ra_b, \theta) = 1.431 \cdot 10^3$ **W ... heat transfer by natural convection ..... Ans.**

Case 3: When the flat plate collector is at θ = 60 deg.:

$\theta := 60$ ....inclination to horizontal

Then: $Nu_b(L, b, Ra_b, \theta) = 4.3$ ..Nusselt Number

And:

$k_{eff}(L, b, Ra_b, \theta) := k \cdot Nu_b(L, b, Ra_b, \theta)$ ....effective thermal conductivity

i.e. $k_{eff}(L, b, Ra_b, \theta) = 0.12$ W/m.C .... effective thermal cond.

$$Q_{conv}(L, b, Ra_b, \theta) := k_{eff}(L, b, Ra_b, \theta) \cdot A_s \cdot \frac{(T1 - T2)}{b}$$

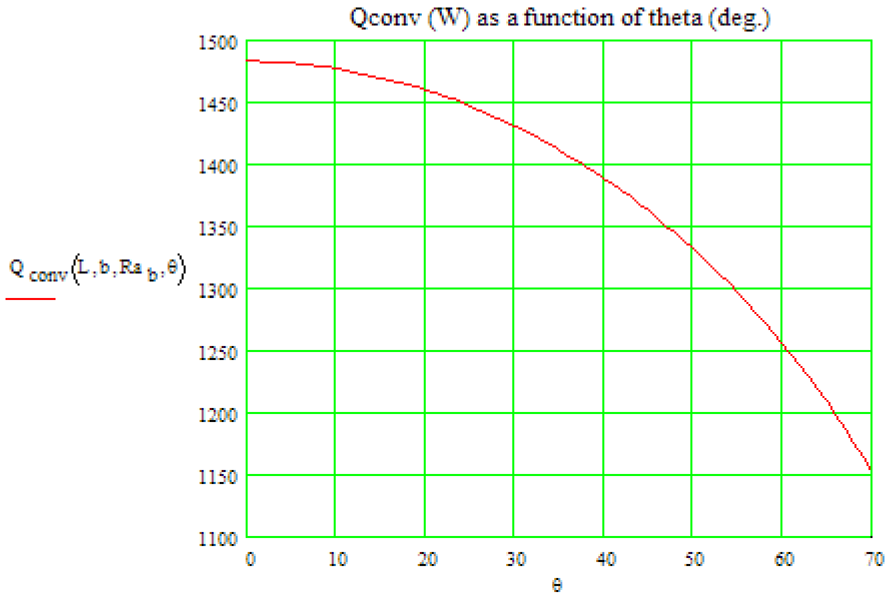
i.e. $Q_{conv}(L, b, Ra_b, \theta) = 1.256 \cdot 10^3$ W ... heat transfer by natural convection ..... Ans.

Plot Q_conv for various values of θ:

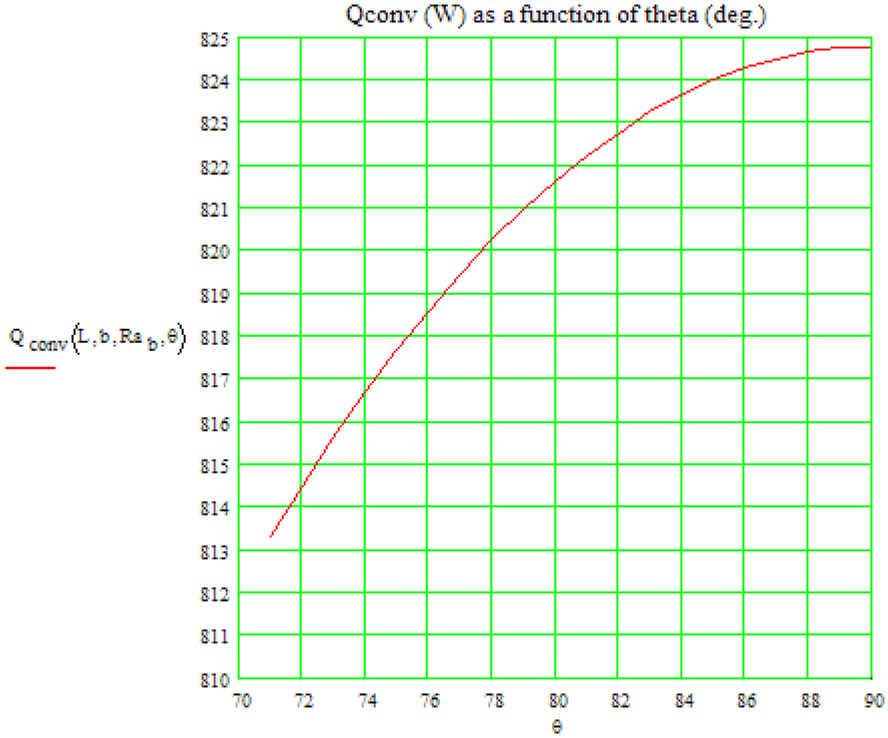
For various values of θ:

θ	$Nu_b(L,b,Ra_b,\theta)$	$Q_{conv}(L,b,Ra_b,\theta)$
0	5.077	1483.157
10	5.057	1477.305
20	4.998	1459.99
30	4.898	1430.827
40	4.754	1388.807
50	4.56	1331.941
60	4.3	1256.262
70	3.946	1152.841
80	2.813	821.636
90	2.823	824.787
100	2.796	816.694
110	2.713	792.663
120	2.579	753.423
130	2.397	700.166
140	2.172	634.512
150	1.912	558.454
160	1.624	474.303
170	1.317	384.617
180	1	292.121

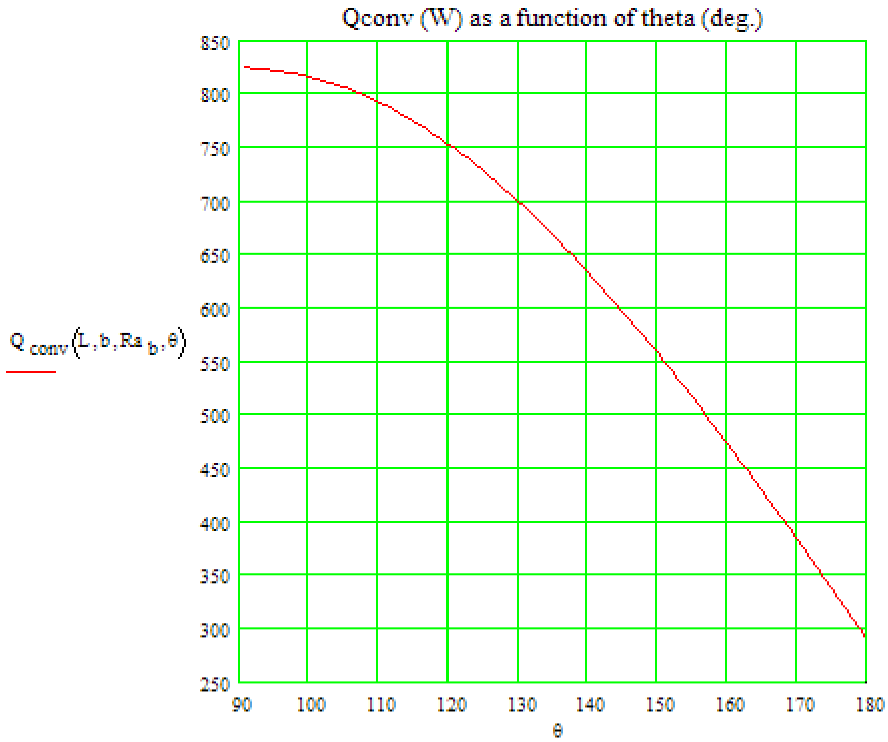
Q_{conv} between theta angles of 0 and 70 deg:



Q_{conv} between theta angles of 71 and 90 deg:



Q_{conv} between theta angles of 91 and 180 deg:



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Prob. 2A2.3.4 : A solar collector design consists of concentric tubes, with the outer tube transparent to solar radiation. The tubes are thin walled and the inner and outer cylinder diameters are 0.1 and 0.15 m. The annular space is filled with air at atmospheric pressure. Inner and outer tube surface temperatures are 70 C and 30 C respectively. (a) What is the convection heat loss per metre length of tube? (b) If the emissivity of outer surface of inner tube is 0.2 and the outer cylinder behaves as if it is a black body, estimate the radiation loss. [Ref: 5]

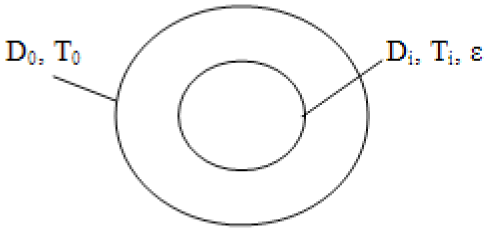


Fig.Prob.2A2.3.4

SMath Solution: (Note: SMath is a software similar to Mathcad, with somewhat reduced capabilities, but available totally free)

Data:

$D_i := 0.1$ m

$D_o := 0.15$ m

$b := \frac{D_o - D_i}{2}$ m

$b = 0.025$ m

$T_i := 70$ C

$T_o := 30$ C

$T_f := 50$ C

$\sigma := 5.67 \cdot 10^{-8}$ W/m^2.K^4

$\varepsilon := 0.2$ ...emissivity

$\beta := \frac{1}{T_f + 273}$ 1/K

$\rho := 1.092$ kg/m^3

$cp := 1007$ J/kg.C

$k := 0.02735$ W/m.C

$\mu := 1.963 \cdot 10^{-5}$ kg/m.s

$\nu := \frac{\mu}{\rho}$ m^2/s

$Pr := 0.7228$ ...Prandtl No.

$g := 9.81$ m/s^2

$$Gr_b := \frac{g \cdot \beta \cdot (T_i - T_o) \cdot b^3}{\nu^2}$$ Grashoff No.

i.e. $Gr_b = 58742.3315$

$Ra_b := Gr_b \cdot Pr$ Rayleigh No.

i.e. $Ra_b = 42458.9572$

We have:

$$F_{cyl} := \frac{\left(\ln\left(\frac{D_o}{D_i}\right)\right)^4}{b^3 \cdot \left(D_i^{-\frac{3}{5}} + D_o^{-\frac{3}{5}}\right)^5}$$

i.e. $F_{cyl} = 0.0957$

Therefore:

$$\frac{k_{eff}}{k} = 0.386 \cdot \left(\frac{Pr}{0.861 + Pr}\right)^{\frac{1}{4}} \cdot \left(F_{cyl} \cdot Ra_b\right)^{\frac{1}{4}}$$

Therefore:

$$k_{eff} := k \cdot 0.386 \cdot \left(\frac{Pr}{0.861 + Pr}\right)^{\frac{1}{4}} \cdot \left(F_{cyl} \cdot Ra_b\right)^{\frac{1}{4}}$$

i.e. $k_{eff} = 0.0693$ ...effective th. cond.

And, heat transfer by free convection:

$$Q_{conv} := \frac{2 \cdot \pi \cdot k_{eff} \cdot 1 \cdot (T_i - T_o)}{\ln\left(\frac{D_o}{D_i}\right)}$$

i.e. $Q_{conv} = 42.9429$ W/m.....Ans.

Heat transfer by Radiation:

$$Q_{rad} := \sigma \cdot \varepsilon \cdot (\pi \cdot D_i \cdot 1) \cdot \left[(T_i + 273)^4 - (T_o + 273)^4 \right]$$

i.e. $Q_{rad} = 19.282$ W/m....Ans.

Therefore: Total heat transfer:

$Q_{tot} := Q_{conv} + Q_{rad}$

$Q_{tot} = 62.2249$ W/m.....Ans.

==

$UnitSystem SI Pa K J

"**Prob. 2A2.3.5**. The surfaces of two concentric spheres, with radii of 75 and 100 mm are maintained at 325 K and 275 K respectively. (a) If the space between the spheres is filled with nitrogen at 5 atm estimate the convection heat transfer rate (b) If the emissivity of surface of inner sphere is 0.3 and the surface of outer sphere is perfectly black, estimate the total heat transfer rate."

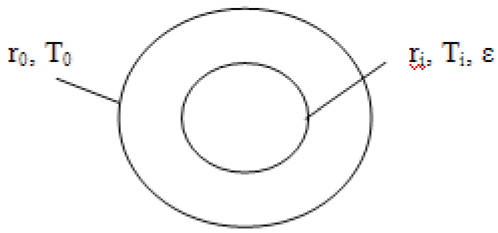


Fig.Prob.2A2.3.5

EES Solution:

"Data:"

r_i = 0.075 [m]

r_o = 0.1 [m]

T_i = 325 [K]

T_o = 275 [K]

P = 5 [atm] * convert (atm, Pa)

```
T_f = (T_i + T_f) / 2

g = 9.81 [m/s^2]

sigma = 5.67e-08 [W/m^2-K^4] "…Stefan Boltzmann const."

epsilon = 0.2 "…emissivity"

"Properties of nitrogen at 5 atm:"

beta = 1/T_f "[1/K]"
rho=Density(N2,T=T_f,P=P) "[kg/m^3]"
cp=Cp(N2,T=T_f) "[J/kg-K]"
k=Conductivity(N2,T=T_f) "[W/m-K]"
Pr=Prandtl(N2,T=T_f) "…Prandtl No."
mu=Viscosity(N2,T=T_f) "[kg/m-s]"
nu = mu / rho "[m^2/s]"

"Calculations:"

D_i = 2 * r_i "[m]"

D_o = 2 * r_o "[m]"

b = (r_o – r_i) "[m]"

Gr_b = g * beta * (T_i – T_o) * b^3 / nu^2 "….Grashoff No."

Ra_b = Gr_b * Pr "…Rayleigh No."

"Effective Thermal conductivity:"

F_sph = b / ((D_i * D_o)^4 * (D_i^(-7/5) + D_o^(-7/5))^5)

k_eff / k = 0.74 * (Pr / (0.861 + Pr))^(1/4) * (F_sph * Ra_b)^(1/4) "…Finds k_eff"

"Heat transfer by convection:"

Q_conv = k_eff * (pi * D_i * D_o / b) * (T_i – T_o) "[W]"
```

"Heat transfer by radiation:"

Q_rad = sigma * epsilon * (pi * D_i^2) * (T_i^4 – T_o^4) "[W]"

"Total heat transfer:"

Q_tot = Q_conv + Q_rad "[W] …. total heat transfer"

Results:

Unit Settings: SI K Pa J mass deg

$b = 0.025$ [m]	$\beta = 0.003077$ [1/K]	$cp = 1038$ [J/kg-K]
$D_i = 0.15$ [m]	$D_o = 0.2$ [m]	$\varepsilon = 0.2$
$F_{sph} = 0.004078$	$g = 9.81$ [m/s^2]	$Gr_b = 1.835E+06$
$k = 0.02759$ [W/m-K]	$k_{eff} = 0.1428$ [W/m-K]	$\mu = 0.00001883$ [kg/m-s]
$\nu = 0.000003585$ [m^2/s]	$P = 506625$ [Pa]	$Pr = 0.7083$
$Q_{conv} = 26.92$ [W]	$Q_{rad} = 4.359$ [W]	$Q_{tot} = 31.28$ [W]
$Ra_b = 1.300E+06$	$\rho = 5.252$ [kg/m^3]	$r_i = 0.075$ [m]
$r_o = 0.1$ [m]	$\sigma = 5.670E-08$ [W/m^2-K^4]	$T_f = 325$ [K]
$T_i = 325$ [K]	$T_o = 275$ [K]	

Thus:

Q_conv = 26.92 W …… Ans.

Q_rad = 4.3589 W ….. Ans.

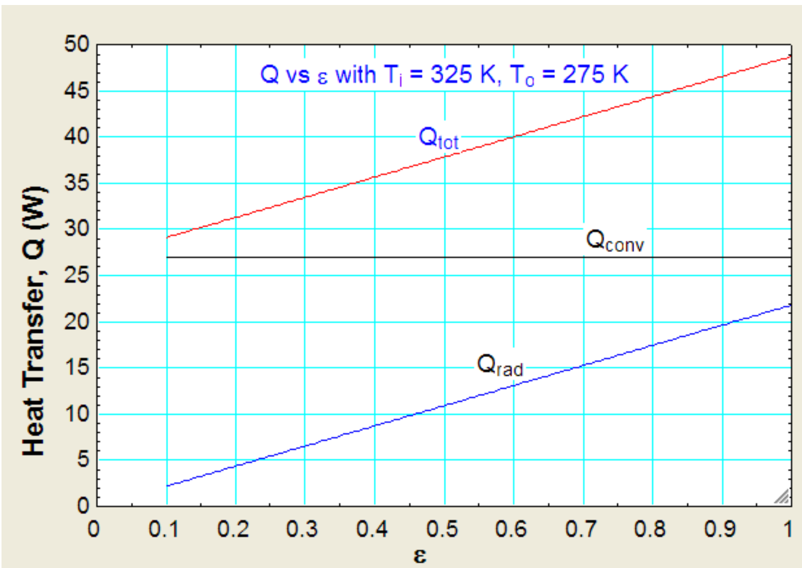
Q_tot = 31.28 W …. Ans.

Plot the variation of Q as epsilon varies from 0.1 to 1:

Parametric Table

Table 1

1..10	ε	Q_{conv} [W]	Q_{rad} [W]	Q_{tot} [W]
Run 1	0.1	26.92	2.179	29.1
Run 2	0.2	26.92	4.359	31.28
Run 3	0.3	26.92	6.538	33.46
Run 4	0.4	26.92	8.717	35.64
Run 5	0.5	26.92	10.9	37.82
Run 6	0.6	26.92	13.08	40
Run 7	0.7	26.92	15.26	42.18
Run 8	0.8	26.92	17.43	44.35
Run 9	0.9	26.92	19.61	46.53
Run 10	1	26.92	21.79	48.71

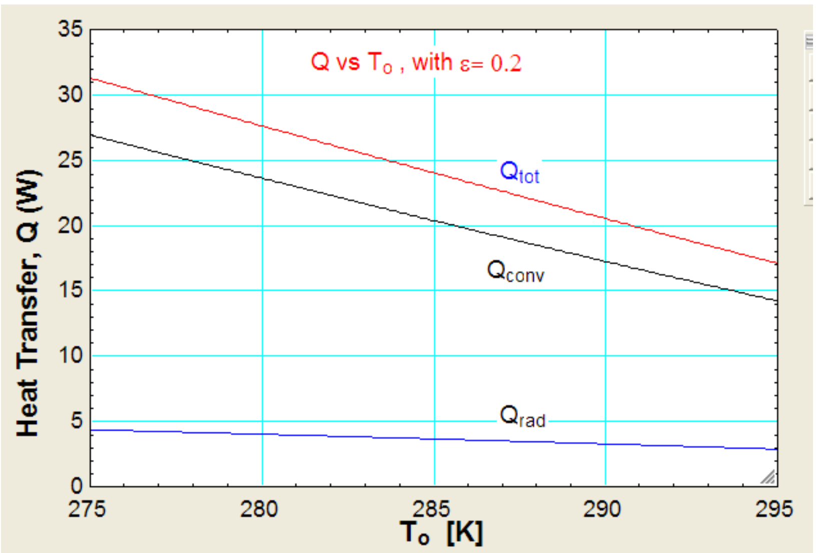


Plot the variation of Q as T_o varies from 275 to 295 K:

Parametric Table

Table 1 | Table 2

1..21	T_o [K]	Q_{conv} [W]	Q_{rad} [W]	Q_{tot} [W]
Run 1	275	26.92	4.359	31.28
Run 2	276	26.25	4.292	30.54
Run 3	277	25.58	4.224	29.81
Run 4	278	24.92	4.155	29.07
Run 5	279	24.26	4.086	28.34
Run 6	280	23.6	4.016	27.61
Run 7	281	22.95	3.945	26.89
Run 8	282	22.3	3.874	26.17
Run 9	283	21.65	3.801	25.45
Run 10	284	21.01	3.728	24.73
Run 11	285	20.37	3.655	24.02
Run 12	286	19.73	3.58	23.31
Run 13	287	19.1	3.504	22.61
Run 14	288	18.48	3.428	21.91
Run 15	289	17.85	3.351	21.21
Run 16	290	17.24	3.274	20.51
Run 17	291	16.62	3.195	19.82
Run 18	292	16.01	3.115	19.13
Run 19	293	15.41	3.035	18.45
Run 20	294	14.81	2.954	17.76
Run 21	295	14.22	2.872	17.09



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Prob. 2A2.3.6: In a solar flat plate collector, the plate is of size 1.5 m high and 3m wide and is at a temperature of 80 C. The glass cover plate is at a distance of 2.5 cm from the collector surface and its temp. is 40 C. Space in between contains air at 1 atm. If the collector plate is inclined to the horizontal at θ deg., determine the heat transfer by free convection for θ = 0, 30 and 90 deg.

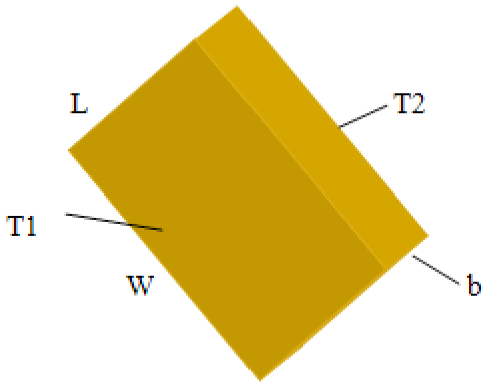


Fig.Prob.2A2.3.6

Note: This problem is the same as Prob.2A2.3.2, which was solved with Mathcad.

Natural convection in Rectangular enclosures (Height, L × Width, W × plate spacing, b) containing air at 1 atm inside the enclosure is practically important. Equations are available for Vertical enclosures (**θ** = 90 deg), Horizontal enclosures (**θ** = 0 deg) and Inclined enclosures, inclined at an angle **θ** to the horizontal. ***The latter case is of direct relevance to analysis of Flat plate Solar collectors.***

For Inclined rectangular enclosure, we have, for Average Nusselts No. (based on the plate spacing 'b' as the characteristic dimension):

For (L/b > 12) and at tilt angles θ less than 70 deg.:

$$Nu_b = 1 + 1.44 \cdot \left(1 - \frac{1708}{Ra_b \cdot \cos(\theta)}\right) \cdot \left(1 - \frac{1708 \cdot \sin(1.8 \cdot \theta)^{1.6}}{Ra_b \cdot \cos(\theta)}\right) + \left[\left(\frac{Ra_b \cdot \cos(\theta)}{5830}\right)^{\frac{1}{3}} - 1\right]$$

For L/b => 12, θ <=70 deg.:

If the quantity in the first bracket and the last bracket is negative, then it must be set equal to zero.

When θ = 0, above eqn gives Nu_b for a Horizontal enclosure.

For tilt angles between 70 deg. and 90 deg. Catton recommends that the Nusselt number for a vertical enclosure (θ = 90 deg.) be multiplied by $(\sin\theta)^{1/4}$, i.e.

$$Nu_b(\theta) = Nu_b(\theta = 90) \cdot \sin(\theta)^{\frac{1}{4}}$$ ...for 70 < θ <= 90

$$Nu_b = 1 + \left(Nu_b(\theta = 90) - 1\right) \cdot \sin\theta$$ ...for 90 < θ <= 180

Also:

$$Nu_b = 0.42 \cdot Ra_b^{\frac{1}{4}} \cdot Pr^{0.012} \cdot \left(\frac{L}{b}\right)^{-0.3}$$ ...for 0 < L/b < 40, 1 < Pr < 2*10^4, 10^4 < Ra_b < 10^7)

$$Nu_b = 0.46 \cdot Ra_b^{\frac{1}{3}}$$ ...for 1 < L/b < 40, 1 < Pr < 20 10^6 < Ra_b < 10^9

And, Effective Thermal conductivity = k_{eff} = k * Nu_b

Heat tr coeff, h = k_{eff} / b , and,

Q_{conv} = k_{eff} * (L * W) * (T1 – T2)/ b

Also, Qconv = h * (L * W) * (T1 – T2)

Now, let us solve this Problem with EXCEL.

VBA Functions for Nu_b for Rectangular enclosures with Air at 1 atm inside:

Since the above shown eqns are a little complicated, it is better to have VBA Functions in EXCEL to calculate Nu_b for different cases:

Following are the **two VBA Functions**, which give the Nusselts No. Nu_b for all angles of tilt, **θ** to the horizontal, i.e. **θ** = 0 to 180 deg., i.e. Horizontal, Vertical and Tilted enclosures:

```
Function NatConv_Air_InclinedEnclosure_Nusselt_b(T_1 As Double, T_2 As Double, L As Double, b As Double, _
theta As Double) As Double
'Finds Avg. Nusselts No. for an Inclined Enclosure (height L x thickness b)containing Nat Convn in Air
' at 1 atmosph pressure
' Temps T_1 > T_2
'theta is inclination to horizontal (degrees)
'Reads property values of Air from Table and interpolates using VBA Functions


Dim g As Double, beta As Double
Dim AA As Double, BB As Double, CC As Double


Dim T_f As Double, rho As Double, k As Double, Pr As Double
Dim nu As Double, cp As Double
Dim Gr_b As Double, Ra_b As Double

g = 9.81 'm/s^2....accn due to gravity

T_f = (T_1 + T_2) / 2

beta = 1 / (T_f + 273) 'vol. expansivity of Air


'Properties of Air:

rho = Air_rho_T(T_f + 273)
k = Air_k_T(T_f + 273)
Pr = Air_Pr_T(T_f + 273)
nu = Air_nu_T(T_f + 273)
cp = Air_cp_T(T_f + 273)
```

```
Gr_b = g * beta * Abs(T_1 - T_2) * b ^ 3 / nu ^ 2 'Grashof No.
Ra_b = Gr_b * Pr 'Rayleigh No.

If (L / b < 12) Then
  MsgBox (" L/b must be greater than 12 !")
  End
End If


If (L / b) > 12 And theta <= 70 Then

   theta = theta * Application.Pi() / 180 'theta in Radians

   AA = (1 - 1708 / (Ra_b * Cos(theta)))

   BB = (1 - 1708 * ((Sin(1.8 * theta)) ^ 1.6) / (Ra_b * Cos(theta)))

   CC = ((Ra_b * Cos(theta) / 5830) ^ (1 / 3)) - 1

   If AA <= 0 Then
      AA = 0
   End If
   If CC <= 0 Then
      CC = 0
   End If

   NatConv_Air_InclinedEnclosure_Nusselt_b = 1 + 1.44 * AA * BB + CC

 End If

 If (L / b) > 12 And (theta < 90 And theta > 70) Then

 theta = theta * Application.Pi() / 180 'theta in Radians

 NatConv_Air_InclinedEnclosure_Nusselt_b = NatConv_Air_Vert_Enclosure_Nusselt_b(T_1, T_2, L, b) * (Sin(theta)) ^ (1 / 4)
```

```
 End If

 If theta = 90 Then

   NatConv_Air_InclinedEnclosure_Nusselt_b = NatConv_Air_Vert_Enclosure_Nusselt_b(T_1, T_2, L, b)

 End If

If (theta > 90 And theta <= 180) Then

 theta = theta * Application.Pi() / 180 'theta in Radians

 NatConv_Air_InclinedEnclosure_Nusselt_b = 1 + (NatConv_Air_Vert_Enclosure_Nusselt_b(T_1, T_2, L, b) - 1) * Sin(theta)

 End If


End Function
```

```
Function NatConv_Air_Vert_Enclosure_Nusselt_b(T_1 As Double, T_2 As Double, L As Double, _
b As Double) As Double
'Finds Avg. Nusselts No. for a vetical Enclosure (height L x thickness b)containing Nat Convn
'in Air at 1 atmosph pressure
'Temps T_1 > T_2
'Uses eqns from McGreogory and Emery, (Ref: Cengel:  Heat and Mass Transfer, 3rd Ed., p. 480 )
'Reads property values of Air from Table and interpolates using VBA Functions


Dim g As Double, beta As Double
Dim AA As Double, BB As Double, CC As Double


Dim T_f As Double, rho As Double, k As Double, Pr As Double
Dim nu As Double, cp As Double
Dim Gr_b As Double, Ra_b As Double

g = 9.81 'm/s^2....accn due to gravity

T_f = (T_1 + T_2) / 2

beta = 1 / (T_f + 273) 'vol. expansivity of Air


'Properties of Air:

rho = Air_rho_T(T_f + 273)
k = Air_k_T(T_f + 273)
Pr = Air_Pr_T(T_f + 273)
nu = Air_nu_T(T_f + 273)
cp = Air_cp_T(T_f + 273)
```

```
Gr_b = g * beta * Abs(T_1 - T_2) * b ^ 3 / nu ^ 2 'Grashof No.
Ra_b = Gr_b * Pr 'Rayleigh No.

If (L / b < 10) Then
  MsgBox (" L/b must be greater than 10!")
  End
End If

If Ra_b < 10 ^ 4 Or Ra_b > 10 ^ 9 Then
    MsgBox (" Ra_b must be between 10^4 and 10^9 !!")
  End
End If

  If (Ra_b > 10 ^ 4) And (Ra_b <= 10 ^ 7) Then
    NatConv_Air_Vert_Enclosure_Nusselt_b = 0.42 * Ra_b ^ (1 / 4) * Pr ^ 0.012 * (L / b) ^ (-0.3)
  Else
  If (Ra_b > 10 ^ 6) And (Ra_b <= 10 ^ 9) Then
    NatConv_Air_Vert_Enclosure_Nusselt_b = 0.46 * Ra_b ^ (1 / 3)
  End If
  End If

End Function
```

Insert these VBA Functions in one of the Modules in the same EXCEL Worksheet where we wrote the VBA Functions for properties of Air (see for example, Prob.2A1.2.13), so that all these Functions are available for us, as if they are built-in Functions of EXCEL, to use in solving the Problem.

EXCEL Solution:

Following are the steps:

1. Set up the EXCEL worksheet, enter data and name the cells:

T_f =(T_1+T_2)/2

	A	B	C	D	E	F	G
209							
210		Data:	Fluid =	Air			
211			T_1	80	C		
212			T_2	40.0	C		
213			T_f	60.00	C	$T\_f = (T\_1 + T\_2)/2$	
214			L	1.5	m		
215			W	3.0	m		
216			b	0.0250	m		
217			theta	30.0	deg.		
218			g	9.81	m/s^2		
219			beta	0.0030	1/K		

2. Use VBA Functions written earlier, to get properties of Air:

cp | fx =air_cp_T((T_f+273))

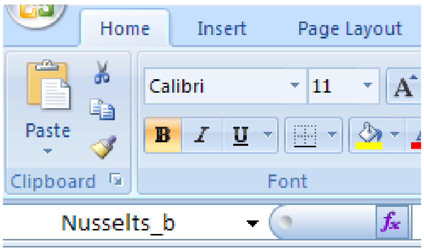
	A	B	C	D	E	F	G
221							
222							
223		**Calculations:**					
224		density	**rho**	1.06075	kg/m^3		
225		th. conductivity	**k**	0.028786	W/m.C	Using VBA Functions for Air	
226		Prandtl No.	**Pr**	0.703665			
227		kinematic visc.	**nu**	1.89635E-05	m^2/s		
228		sp.heat	**cp**	1008.65	J/kg.K		
229							

3. Now, proceed with the calculations:

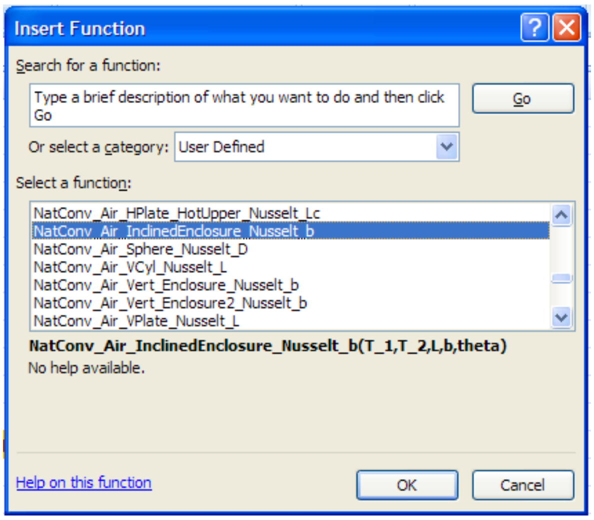
Nusselts_b | fx =NatConv_Air_InclinedEnclosure_Nusselt_b(T_1,T_2,L,b,theta)

	A	B	C	D	E
233		**For an Enclosure (theta = 0 to 180 deg.): use VBA Function to find Nusselts No.:**			
234					
235		**To calculate Nusselts No...'b' is the characteristic dimension.**			
236		**Nusselts No.**	**Nusselts_b**	**3.0573**	**..Using VBA Function**

Note in this screen shot that Nu_b is calculated in cell D236, using the VBA Function written above. To insert this Function in cell D236, select that cell, and click on Insert Function symbol:



We get the following Window. Here, under User Defined category, the VBA Function we wrote for Inclined Enclosure appears, and we select it:

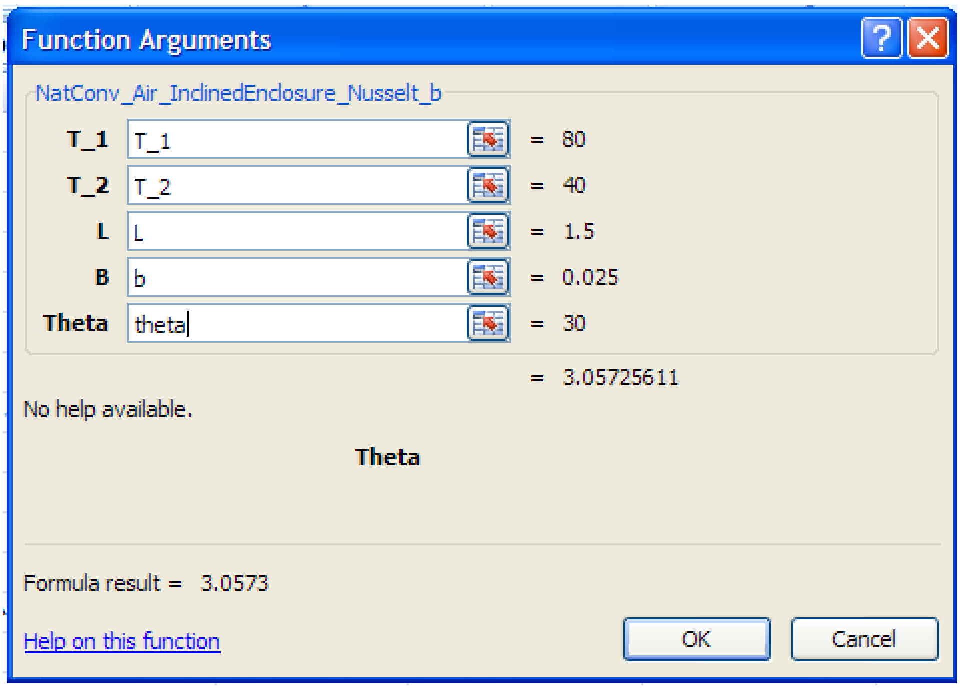


In the above window, we have selected:

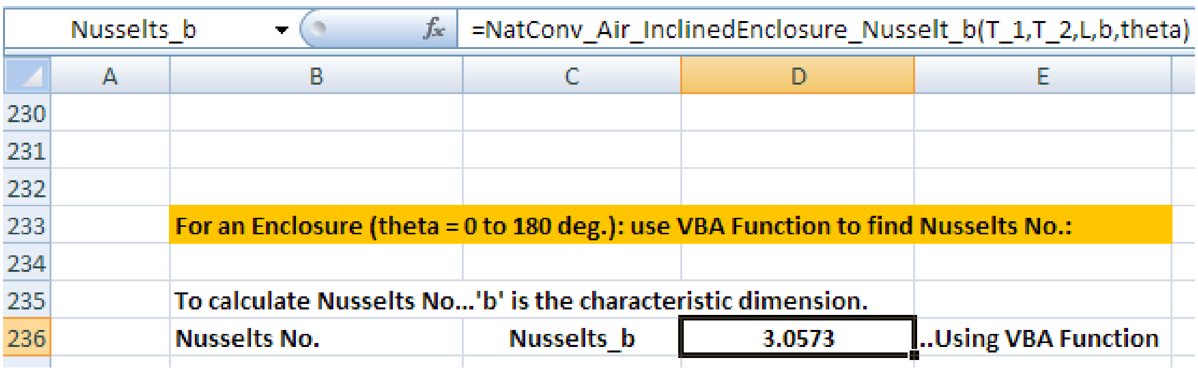
Category: User Defined,

Select a function: Natconv_Air_InclinedEnclosure_Nusselt_b

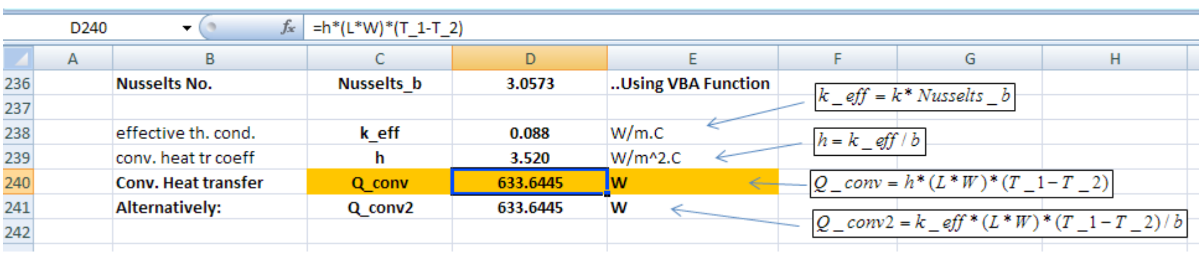
Now, click OK. We get the following window. Select various parameters as shown:



Press OK, and the result appears in cell D236:



4. Now, complete other calculations for k_{eff}, h and Q_{conv}:



Formulas used are also shown in the Worksheet, for clarity.

5. To plot Q_{conv} for various values of θ:

Set up a Table as shown below, with θ varying from 0 to 180 deg.:

C258 =NatConv_Air_InclinedEnclosure_Nusselt_b(T_1,T_2,L,b,B258)

	A	B	C	D	E	F
257		theta (deg.)	Nu_b	k_eff (W/m.C)	h(W/m^2.C)	Q_conv(W)
258		0	3.207	0.092	3.692	664.6414
259		10				
260		20				
261		30				
262		40				
263		50				
264		60				
265		70				
266		71				
267		75				
268		80				
269		85				
270		90				
271		91				
272		95				
273		100				
274		110				
275		120				
276		130				
277		140				
278		150				
279		160				
280		170				
281		180				

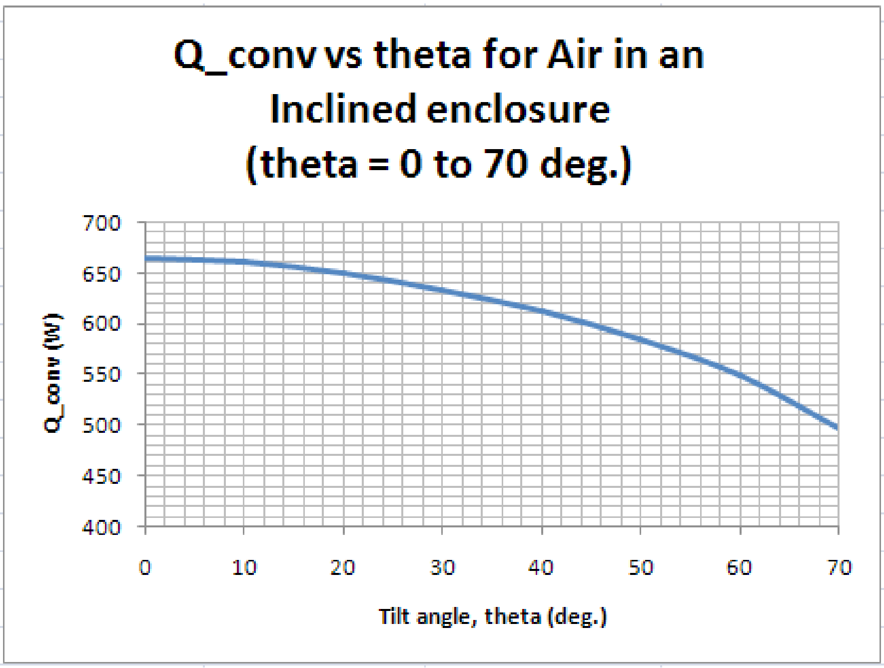
Now, in the cell C258, we have entered the Function for Nu_b. It can be seen in the Formula bar. Since we want to keep θ as the variable, we have entered it in 'relative reference', i.e. for θ we have written cell B258. Similarly, enter Formulas for k_eff, h and Q_conv.

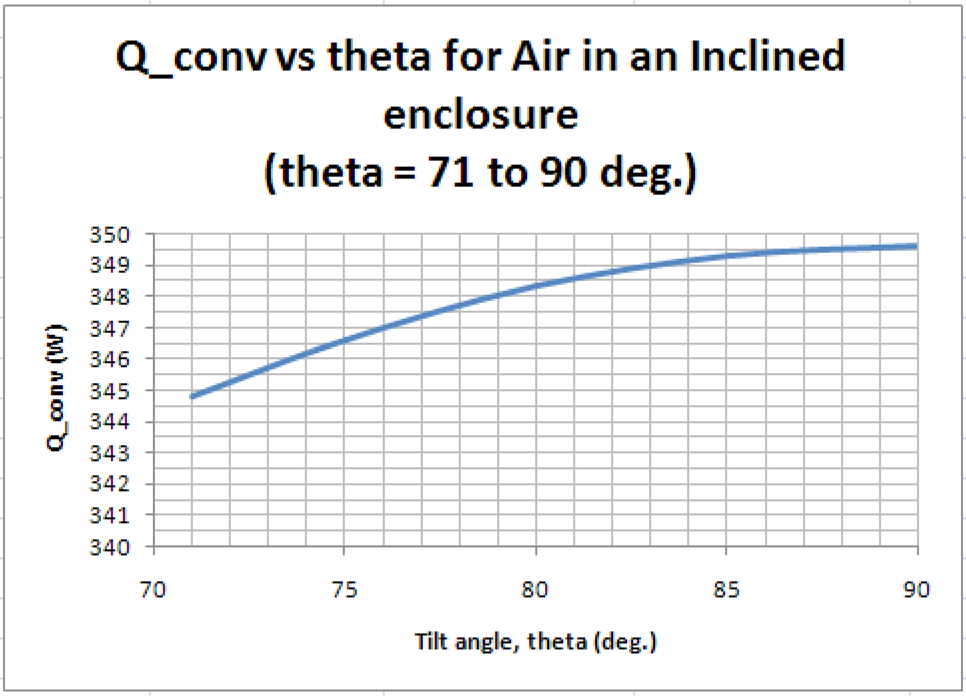
6. Now, select the row containing cells C258 to F258, and drag-copy downwards till cell F281. We see that entire Table gets filled up:

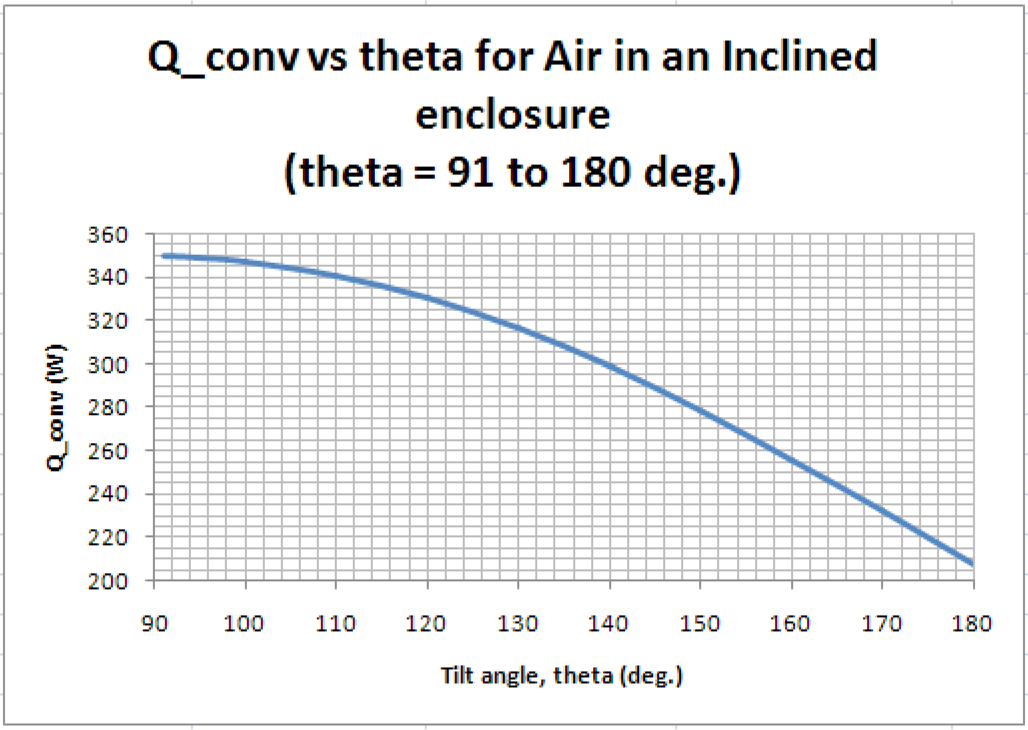
C258 =NatConv_Air_InclinedEnclosure_Nusselt_b(T_1,T_2,L,b,B258)

	A	B	C	D	E	F
257		theta (deg.)	Nu_b	k_eff (W/m.C)	h(W/m^2.C)	Q_conv(W)
258		0	3.207	0.092	3.692	664.6414
259		10	3.186	0.092	3.669	660.3983
260		20	3.135	0.090	3.610	649.8189
261		30	3.057	0.088	3.520	633.6445
262		40	2.953	0.085	3.400	611.9985
263		50	2.819	0.081	3.246	584.2923
264		60	2.646	0.076	3.047	548.3899
265		70	2.401	0.069	2.765	497.6793
266		71	1.664	0.048	1.916	344.7959
267		75	1.673	0.048	1.926	346.6415
268		80	1.681	0.048	1.935	348.3232
269		85	1.685	0.049	1.941	349.3258
270		90	1.687	0.049	1.943	349.6589
271		91	1.687	0.049	1.942	349.6372
272		95	1.684	0.048	1.940	349.1170
273		100	1.677	0.048	1.931	347.4955
274		110	1.646	0.047	1.895	341.0711
275		120	1.595	0.046	1.837	330.5809
276		130	1.526	0.044	1.757	316.3437
277		140	1.442	0.041	1.660	298.7919
278		150	1.344	0.039	1.547	278.4590
279		160	1.235	0.036	1.422	255.9628
280		170	1.119	0.032	1.289	231.9866
281		180	1.000	0.029	1.151	207.2592

7. Now, plot the results in EXCEL:







Note: Comparing these results with those obtained with Mathcad for Prob.2A2.3.2, we see that they match very well.

===

Prob. 2A2.3.7: A long tube of 0.1 m OD is maintained at 150 C. It is surrounded by a cylindrical radiation shield, located concentrically, such that the air gap between the two cylinders is 10 mm. The shield is at a temperature of 30 C. Estimate the convection heat transfer rate per metre length.

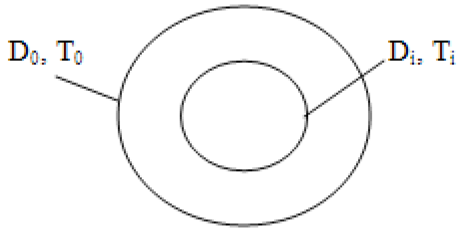


Fig.Prob.2A2.3.7

EXCEL Solution:

Equations for effective thermal conductivity (k_eff) and the heat transferred (Q) **for cylindrical annuli** are given in the beginning of the Chapter.

However, we reproduce them below for immediate reference:

'b' is the gap or thickness of the enclosed fluid layer (i.e. $b = [D_o - D_i]/2$).

$$\frac{Q}{L} = \frac{2 \cdot \pi \cdot k_{eff} \left(T_i - T_o\right)}{\ln\left(\frac{D_o}{D_i}\right)} \quad .....(10.63)$$

$$\frac{k_{eff}}{k} = 0.386 \left(\frac{Pr}{0.861 + Pr}\right)^{\frac{1}{4}} \cdot Ra_{cc}^{\frac{1}{4}} \quad ...100 < Ra_{cc} < 10^7 ....(10.64)$$

where,

$$Ra_{cc} = \frac{\left(\ln\left(\frac{D_o}{D_i}\right)\right)^4 \cdot Ra_b}{b^3 \cdot \left[\frac{1}{D_i^{\frac{3}{5}}} + \frac{1}{D_o^{\frac{3}{5}}}\right]^5} \quad .....(10.65)$$

Now, we need the properties of Air at 1 atm and at given mean temperatures. So, we use the same worksheet where VBA Functions for properties of Air were written. (See, for ex, Prob.2A1.2.13).

Also, we first write VBA Function to calculate k_{eff} and Q for this problem:

```
Function NatConv_Air_ConcentricCylinders_k_effAndQcyl(T_i As Double, T_0 As Double, L As Double, _
D_i As Double, D_0 As Double) As Variant
'Note that output is an Array (k_eff, Qcyl). Use (Ctrl + Shift + Enter) after selecting two
'consecutive cells in a column and entering the Function.
'Finds k_eff (W/m.C)and Qcyl (W)for an Enclosure of conc. cylinders of dia D_i and D_0, gap 'b', Length L
'contains Air at 1 atmosph pressure
'Temps T_i > T_0

'Reads property values of Air from Table and interpolates using VBA Functions


Dim g As Double, beta As Double
Dim b As Double

Dim T_f As Double, rho As Double, k As Double, Pr As Double
Dim nu As Double, cp As Double
Dim Gr_b As Double, Ra_b As Double
Dim k_eff As Double
Dim Qcyl As Double
Dim Ra_cc As Double

g = 9.81 'm/s^2....accn due to gravity

b = (D_0 - D_i) / 2

T_f = (T_i + T_0) / 2

beta = 1 / (T_f + 273) 'vol. expansivity of Air
```

```
'Properties of Air:

rho = Air_rho_T(T_f + 273)
k = Air_k_T(T_f + 273)
Pr = Air_Pr_T(T_f + 273)
nu = Air_nu_T(T_f + 273)
cp = Air_cp_T(T_f + 273)

Gr_b = g * beta * Abs(T_i - T_0) * b ^ 3 / nu ^ 2 'Grashof No.
Ra_b = Gr_b * Pr 'Rayleigh No.

Ra_cc = (Application.Ln(D_0 / D_i)) ^ 4 * Ra_b / (b ^ 3 * (D_i ^ (-3 / 5) + D_0 ^ (-3 / 5)) ^ 5)

  If (Ra_cc < 100) Or (Ra_cc > 10 ^ 7) Then

    MsgBox (" Ra_cc must be between 100 and 10^7 !!")

    End

  End If

  If (Ra_cc > 100) And (Ra_cc <= 10 ^ 7) Then

    k_eff = k * 0.386 * (Pr / (0.861 + Pr)) ^ (1 / 4) * Ra_cc ^ (1 / 4)

  End If

  Qcyl = (2 * Application.Pi() * k_eff * L * (T_i - T_0)) / Application.Ln(D_0 / D_i)

  NatConv_Air_ConcentricCylinders_k_effAndQcyl = Application.Transpose(Array(k_eff, Qcyl))

End Function
```

Note that the above Function returns a vertical Array whose first element is k_eff and the second element is Q.

So, while using this Function, we should select two consecutive vertical cells and then hit (Ctrl+Shift+Enter) since it is an Array Function.

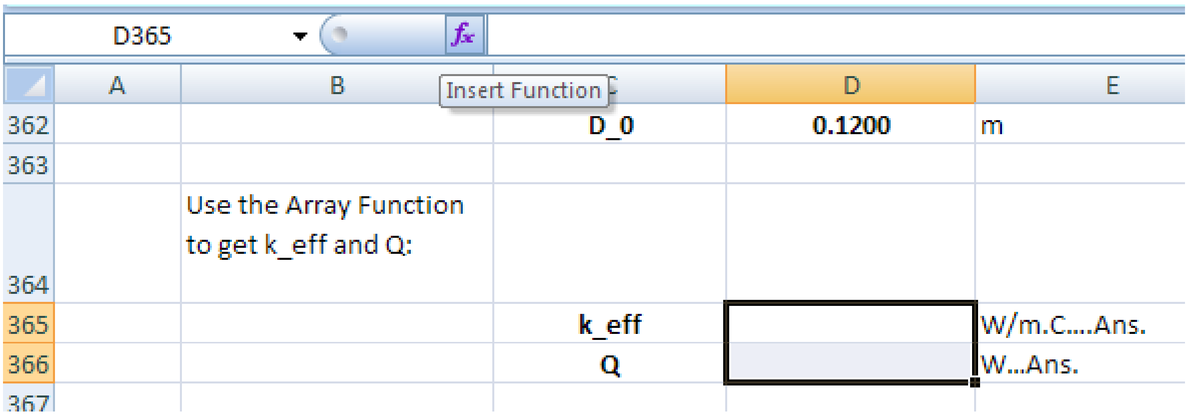
Use of this Function is demonstrated below:

1. Set up the EXCEL worksheet, enter the data:

	A	B	C	D	E	F	G
356		Data:	Fluid =	Air			
357			T_i	150	C		
358			T_0	30.0	C		
359			T_f	90.00	C		
360			L	1	m		
361			D_i	0.1	m		
362			D_0	0.1200	m		

$T\_f = (T\_i + T\_0)/2$

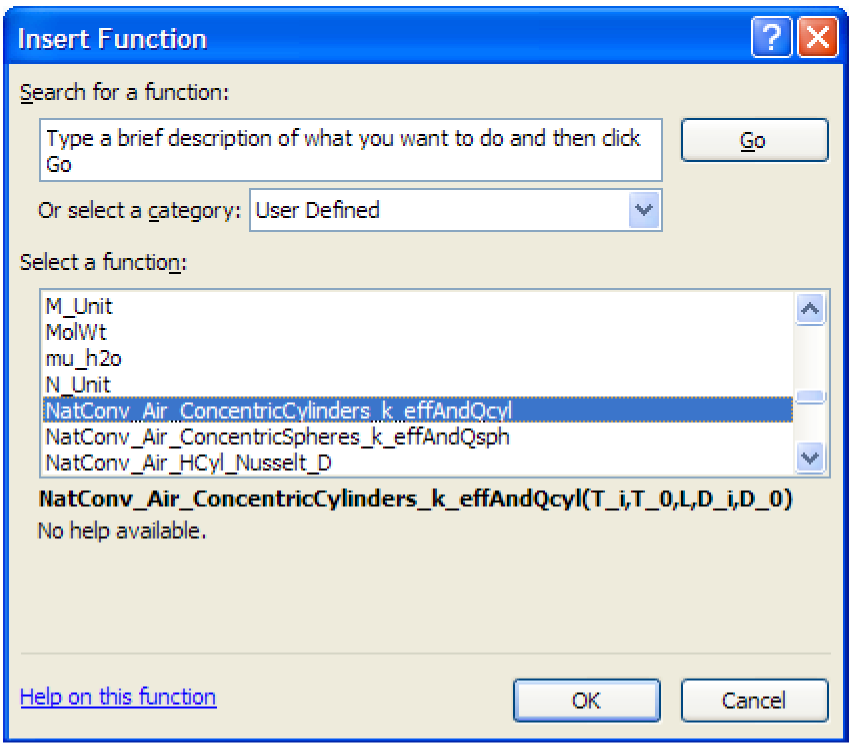
2. Now, to enter the Function to get k_eff and Q, we first select two consecutive vertical cells, viz. D365 and D366, and then choose the Insert Function symbol, shown below:



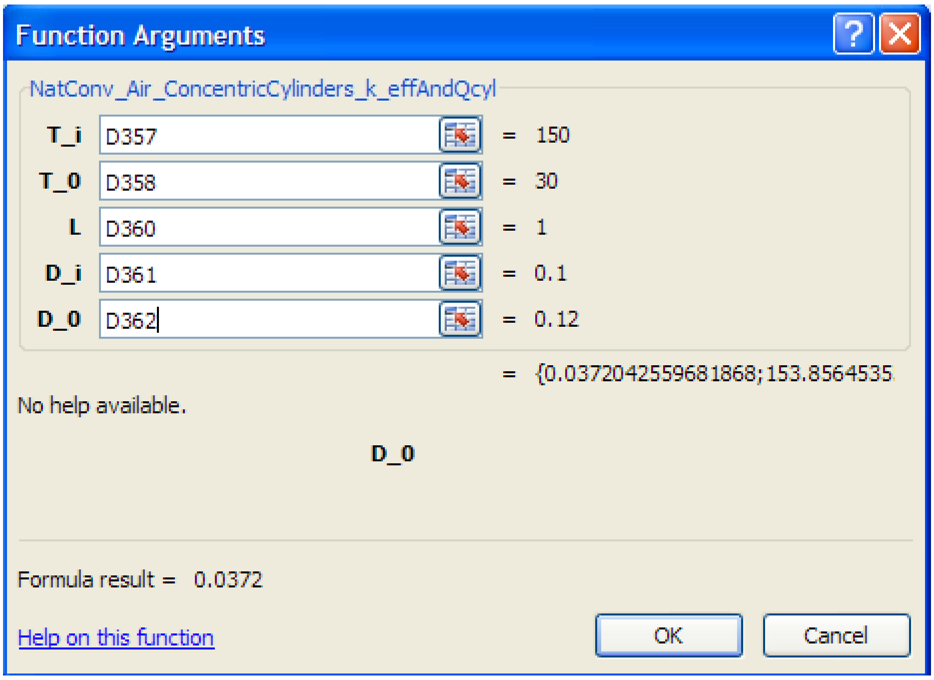
Now, hit the Insert Function symbol, and we get the following screen. In that screen choose:

Category: User Defined

Select Function: NatConv_Air_ConcentricCylinders_k_effAndQcyl



Click OK. We get the following screen, and fill it up as shown:



Now, IMPORTANT: With (Ctrl + Shift) kept pressed, click OK.

We get:

D365 | {=NatConv_Air_ConcentricCylinders_k_effAndQcyl(D357,D358,D360,D361,D362)}

	A	B	C	D	E	F
362			D_0	0.1200	m	
363						
364		Use the Array Function to get k_eff and Q:				
365			k_eff	0.0372	W/m.C....Ans.	
366			Q	153.8564536	W...Ans.	

We see that: k_eff = 0.0372 W/m.C, and Q = 153.856 W … Ans.

==

Prob. 2A2.3.8: A sphere of 0.15 m dia. stores a brine at -5 C and is insulated by enclosing it in another sphere of 0.2 m dia and the intervening space contains air at 1 bar. The outside sphere is at 25 C. Estimate the convection heat transfer rate.

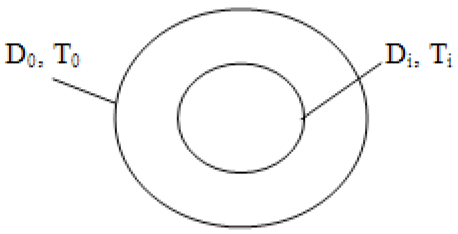


Fig.Prob.2A2.3.8

EXCEL Solution:

Equations for effective thermal conductivity (k_eff) and the heat transferred (Q) **for Spherical annuli** are given in the beginning of the Chapter.

However, we reproduce them below for immediate reference:

$$Q = \pi \cdot k_{eff} \left(\frac{D_i \cdot D_o}{b} \right) \cdot (T_i - T_o) \qquad \text{......(10.66)}$$

$$\frac{k_{eff}}{k} = 0.74 \cdot \left(\frac{Pr}{0.861 + Pr} \right)^{\frac{1}{4}} \cdot Ra_{cs}^{\frac{1}{4}} \qquad \text{...10 < } Ra_{cs} < 10^6 \text{.......(10.67)}$$

and,

$$Ra_{cs} = \frac{b \cdot Ra_b}{D_o^{\,4} \cdot D_i^{\,4} \cdot \left[\frac{1}{D_i^{\frac{7}{5}}} + \frac{1}{D_o^{\frac{7}{5}}} \right]^5} \qquad \text{....(10.68)}$$

Now, we need the properties of Air at 1 atm and at given mean temperatures. So, we use the same worksheet where VBA Functions for properties of Air were written. (See, for ex, Prob.2A1.2.13).

Also, we first write VBA Function to calculate k_{eff} and Q for this problem:

```
Function NatConv_Air_ConcentricSpheres_k_effAndQsph(T_i As Double, T_0 As Double, _
D_i As Double, D_0 As Double) As Variant
'Note that output is an Array (k_eff, Qsph). Use (Ctrl + Shift + Enter) after selecting two
'consecutive cells in a column and entering the Function.
'Finds k_eff (W/m.C)and Qsph (W)for an Enclosure of conc. cylinders of dia D_i and D_0, gap 'b', Length L
'contains Air at 1 atmosph pressure
'Temps T_i > T_0

'Reads property values of Air from Table and interpolates using VBA Functions


Dim g As Double, beta As Double
Dim b As Double

Dim T_f As Double, rho As Double, k As Double, Pr As Double
Dim nu As Double, cp As Double
Dim Gr_b As Double, Ra_b As Double
Dim k_eff As Double
Dim Qsph As Double
Dim Ra_cs As Double

g = 9.81 'm/s^2....accn due to gravity

b = (D_0 - D_i) / 2

T_f = (T_i + T_0) / 2

beta = 1 / (T_f + 273) 'vol. expansivity of Air
```

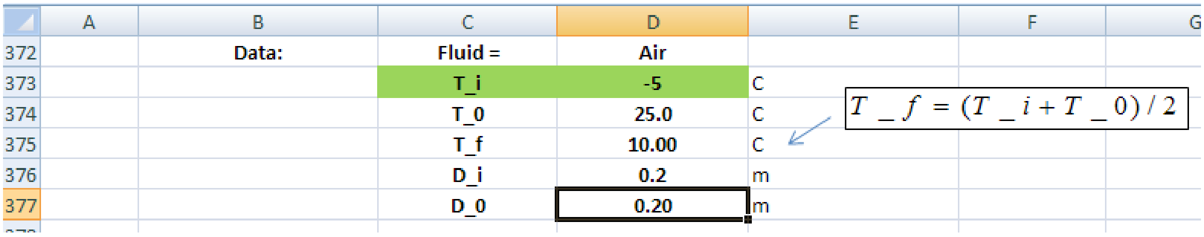
```
'Properties of Air:

rho = Air_rho_T(T_f + 273)
k = Air_k_T(T_f + 273)
Pr = Air_Pr_T(T_f + 273)
nu = Air_nu_T(T_f + 273)
cp = Air_cp_T(T_f + 273)

Gr_b = g * beta * Abs(T_i - T_0) * b ^ 3 / nu ^ 2 'Grashof No.
Ra_b = Gr_b * Pr 'Rayleigh No.

Ra_cs = (b * Ra_b) / ((D_0 * D_i) ^ 4 * (D_i ^ (-7 / 5) + D_0 ^ (-7 / 5)) ^ 5)

  If (Ra_cc < 100) Or (Ra_cc > 10 ^ 4) Then

    MsgBox (" Ra_cc must be between 100 and 10^4 !!")

    End

  End If

  If (Ra_cs >= 100) And (Ra_cs <= 10 ^ 4) Then

    k_eff = k * 0.74 * (Pr / (0.861 + Pr)) ^ (1 / 4) * Ra_cs ^ (1 / 4)

  End If

  Qsph = (k_eff * Application.Pi() * (D_i * D_0 / b) * (T_i - T_0))

  NatConv_Air_ConcentricSpheres_k_effAndQsph = Application.Transpose(Array(k_eff, Qsph))

End Function
```

Note that the above Function returns a vertical Array whose first element is k_eff and the second element is Q_{sph}.

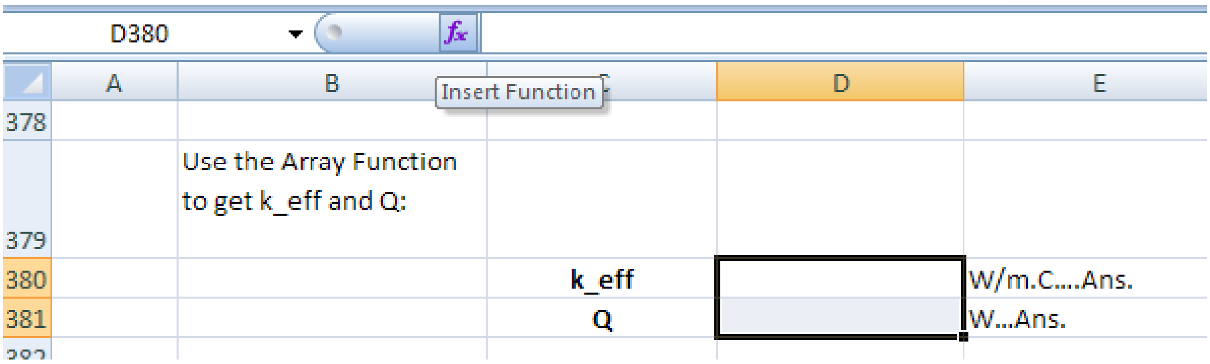
So, while using this Function, we should select two consecutive vertical cells and then hit (Ctrl+Shift+Enter) since it is an Array Function.

Use of this Function is demonstrated below:

1. Set up the EXCEL worksheet, enter the data:



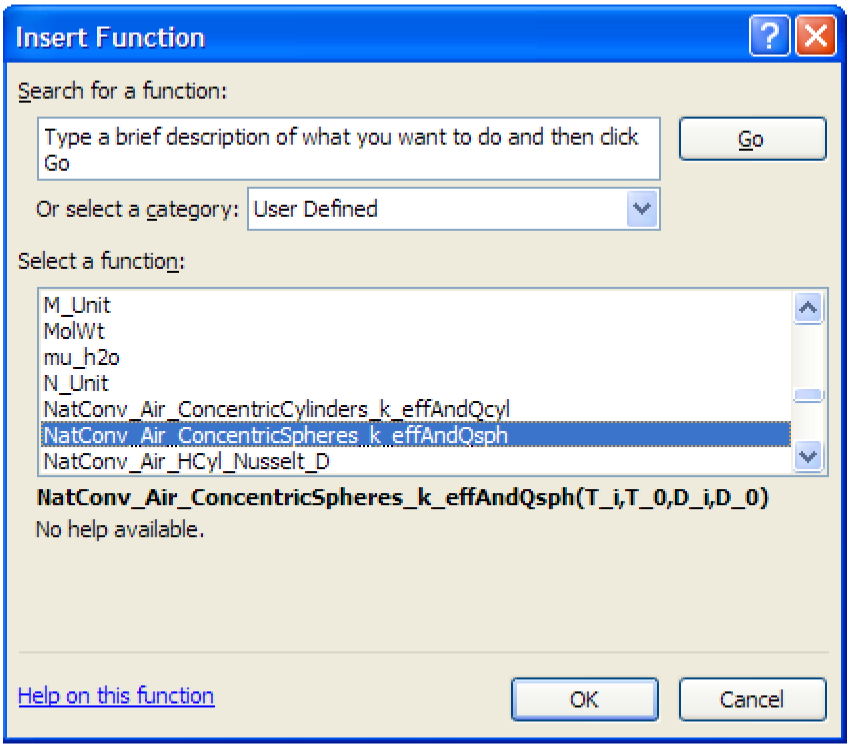
2. Now, to enter the Function to get k_eff and Q, we first select two consecutive vertical cells, viz. D380 and D381, and then choose the Insert Function symbol, shown below:



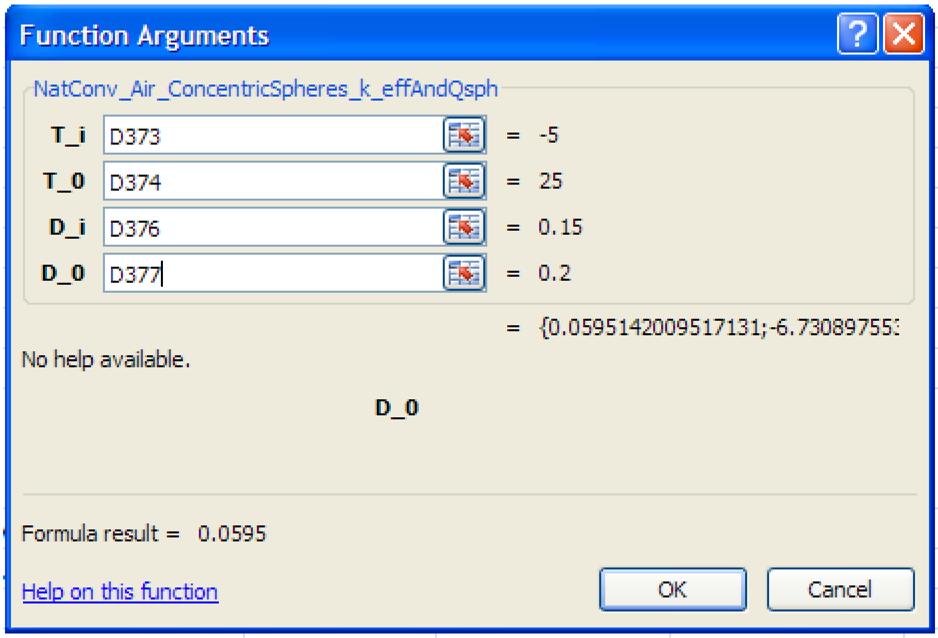
Now, hit the Insert Function symbol, and we get the following screen. In that screen choose:

Category: User Defined

Select Function: NatConv_Air_ConcentricCylinders_k_effAndQsph



Click OK and we get the following screen. Fill it up as shown:



Now, IMPORTANT: With (Ctrl + Shift) kept pressed, click OK.

We get:

D380 | fx | {=NatConv_Air_ConcentricSpheres_k_effAndQsph(D373,D374,D376,D377)}

	A	B	C	D	E	F
378						
379		Use the Array Function to get k_eff and Q:				
380			k_eff	0.0595	W/m.C....Ans.	
381			Q	-6.730897554	W...Ans.	

Thus: k_eff = 0.0595 W/m.C, and Q = 6.73 W…Ans. (-ve sign for Q indicates that heat transfer is from outside to inside)

===

2A2.4 Natural convection from rotating cylinders, disks and spheres:

$UnitSystem SI Pa C J

"**Prob. 2A2.4.1.** A 2 cm OD shaft, 0.7 m long, with surface temp of 90 C is rotating at 20000 RPM in air at 20 C. Determine the convective heat transfer coefficient."

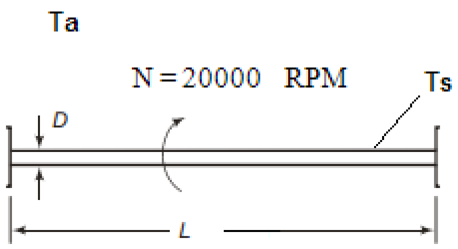


Fig. Prob. 2A2.4.1

EES Solution:

"Data:"

D= 0.02 [m]

L = 0.7 [m]

T_s = 90 [C]

T_a = 20 [C]

P = 1 [atm] * convert (atm, Pa)

T_f = (T_a + T_s) / 2 "[C] …. mean temp"

g = 9.81 [m/s^2]

N= 20000 [rev/min]

"Properties of Air at 1 atm:"

beta = 1/(T_f + 273) "[1/K]"
rho=Density(Air,T=T_f,P=P) "[kg/m^3]"
cp=Cp(Air,T=T_f) "[J/kg-C]"
k=Conductivity(Air,T=T_f) "[W/m-C]"
Pr=Prandtl(Air,T=T_f) "…Prandtl No."
mu=Viscosity(Air,T=T_f) "[kg/m-s]"
nu = mu / rho "[m^2/s]"

"Calculations:"

omega = N * 2 * pi / 60 "[rad/s]"

"Rotational Reynolds No.:"

Re_omega = pi * D^2 *omega / nu

"Grashof No.:"

Gr_D = g * beta * (T_s – T_a) * D^3 / nu^2 "….Grashoff No."

"Therefore:"

Ra_D = Gr_D * Pr **"…Rayleigh No."**

"Nusselts Number:"

Nusselt_D_avg = 0.11 * (0.5 * Re_omega^2 + Ra_D)^0.35

"Heat transfer coeff:"

Nusselt_D_avg = h_c * D / k "[W/m^2-C] …. finds h_c"

Results:

Unit Settings: SI C Pa J mass deg

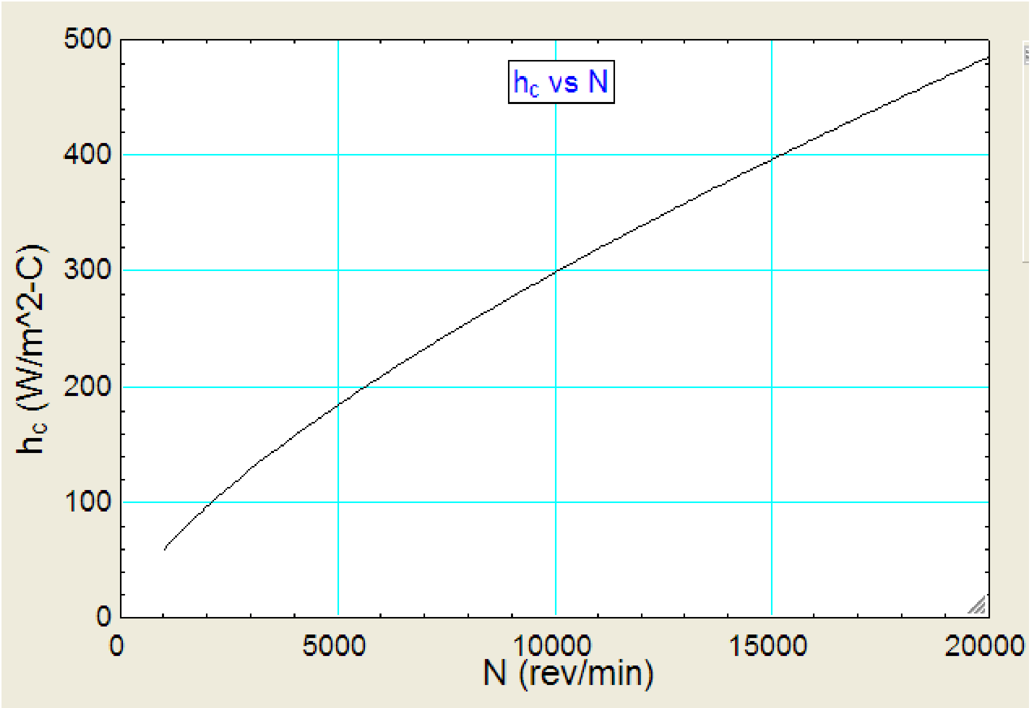
β = 0.003049	cp = 1006 [J/kg-C]	D = 0.02 [m]
g = 9.81 [m/s^2]	Gr_D = 49147	h_c = 484.8 [W/m^2-C]
k = 0.02772 [W/m-C]	L = 0.7 [m]	μ = 0.00001986 [kg/m-s]
N = 20000 [rev/min]	ν = 0.00001846 [m^2/s]	$Nusselt_{D,avg}$ = 349.8
ω = 2094 [rad/s]	P = 101325 [Pa]	Pr = 0.721
Ra_D = 35435	Re_ω = 142569	ρ = 1.076 [kg/m^3]
T_a = 20 [C]	T_f = 55 [K]	T_s = 90 [C]

Thus:

Heat transfer coeff,. = h_c = 484.8 W.m^2.C …. Ans.

Plot the variation of h_c with rotational speed, N:

1..20	N [rev/min]	Re_ω	Ra_D	h_c [W/m2-C]
Run 1	1000	7128	35435	59.58
Run 2	2000	14257	35435	96.75
Run 3	3000	21385	35435	128.5
Run 4	4000	28514	35435	157.2
Run 5	5000	35642	35435	183.7
Run 6	6000	42771	35435	208.7
Run 7	7000	49899	35435	232.5
Run 8	8000	57028	35435	255.3
Run 9	9000	64156	35435	277.2
Run 10	10000	71284	35435	298.5
Run 11	11000	78413	35435	319
Run 12	12000	85541	35435	339.1
Run 13	13000	92670	35435	358.6
Run 14	14000	99798	35435	377.7
Run 15	15000	106927	35435	396.4
Run 16	16000	114055	35435	414.7
Run 17	17000	121183	35435	432.7
Run 18	18000	128312	35435	450.4
Run 19	19000	135440	35435	467.7
Run 20	20000	142569	35435	484.8



==

$UnitSystem SI Pa C J

"**Prob. 2A2.4.2.** Estimate the rate of heat transfer from one side of a 2 m dia disk with a surface temp of 60 C rotating at 800 RPM in 20 C air."

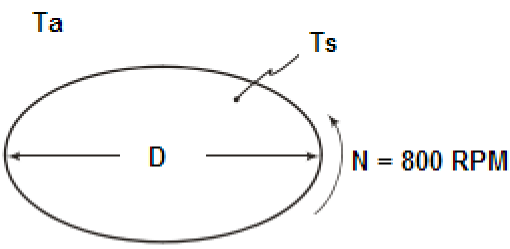


Fig.Prob.2A2.4.2

EES Solution:

"Data:"

D= 2 [m]

T_s = 60 [C]

T_a = 20 [C]

P = 1 [atm] * convert (atm, Pa)

T_f = (T_a + T_s) / 2 "[C] …. mean temp"

g = 9.81 [m/s^2]

N= 800 [rev/min]

"Properties of Air at 1 atm:"

beta = 1/(T_f + 273) "[1/K]"
rho=Density(Air,T=T_f,P=P) "[kg/m^3]"
cp=Cp(Air,T=T_f) "[J/kg-C]"
k=Conductivity(Air,T=T_f) "[W/m-C]"
Pr=Prandtl(Air,T=T_f) "…Prandtl No."
mu=Viscosity(Air,T=T_f) "[kg/m-s]"
nu = mu / rho "[m^2/s]"

"Calculations:"

omega = N * 2 * pi / 60 "[rad/s]"

"Rotational Reynolds No.:"

Re_omega = omega * D^2/ nu

"We see that Reynolds No. is 1.969E07 > 10^6. Therefore, find crit. rad. r_c and apply the following formula:"

omega * (2 * r_c)^2/ nu = 10^6 "…finds r_c [m]"

r_0 = D/2

"Nusselts Number:"

$$Nusselt_{D,avg} = 0.36 \cdot \left[\omega \cdot \frac{r_0^{\,2}}{\nu}\right]^{0.5} \cdot \left[\frac{r_c}{r_0}\right]^2 + 0.015 \cdot \left[\omega \cdot \frac{r_0^{\,2}}{\nu}\right]^{0.8} \cdot \left[1 - \left(\frac{r_c}{r_0}\right)^{2.6}\right]$$

...for $r_c < r_0$

In EES:

Nusselt_D_avg = 0.36 * (omega * r_0^2 / nu)^0.5 * (r_c / r_0)^2 + 0.015 * (omega * r_0^2 / nu)^0.8 * (1 – (r_c / r_0)^2.6) "…for r_c < r_0"

"Heat transfer coeff:"

Nusselt_D_avg = h_c * r_0 / k "[W/m^2-C] …. finds h_c"

"Heat transfer:"

Q = h_c * (pi * r_0^2) * (T_s – T_a) "[W]"

Results:

Unit Settings: SI C Pa J mass deg

β = 0.003195 [1/K]
cp = 1005 [J/kg-C]
D = 2 [m]
g = 9.81 [m/s^2]
h_c = 89.39 [W/m^2-C]
k = 0.02662 [W/m-C]
μ = 0.00001918 [kg/m-s]
N = 800 [rev/min]
ν = 0.00001702 [m^2/s]
$Nusselt_{D,avg}$ = 3358
ω = 83.78 [rad/s]
P = 101325 [Pa]
Pr = 0.7244
Q = 11233 [W]
Re_{ω} = 1.969E+07
ρ = 1.127 [kg/m^3]
r_0 = 1 [m]
r_c = 0.2253 [m]
T_a = 20 [C]
T_f = 40 [C]
T_s = 60 [C]

Thus:

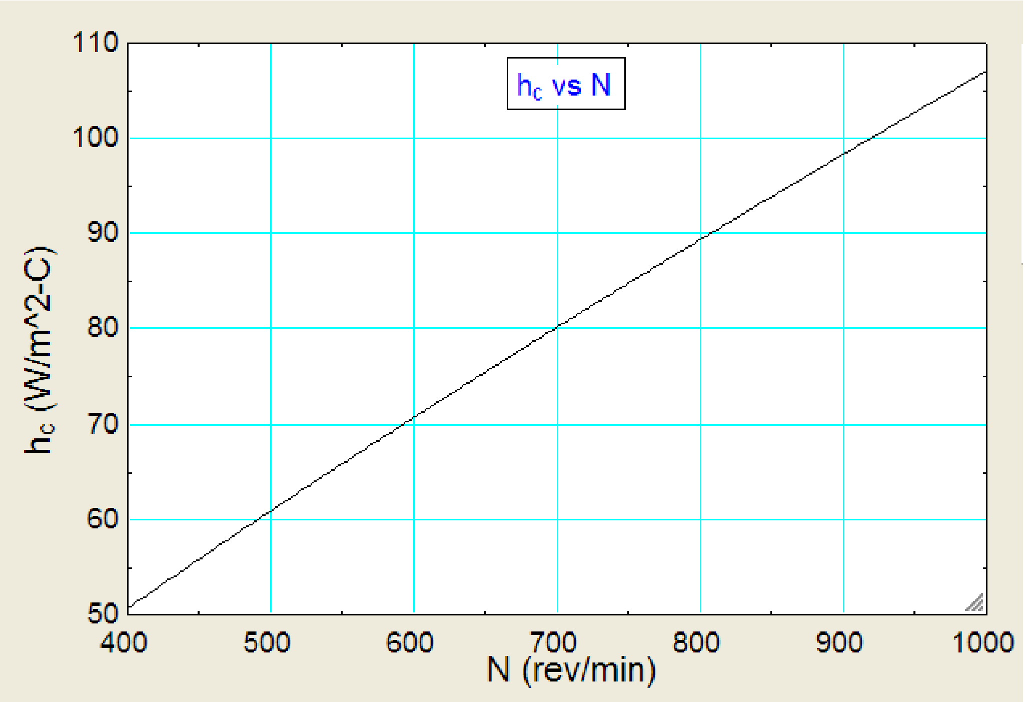
h_c = 89.39 W/m^2-C………. heat transfer coeff.……Ans.

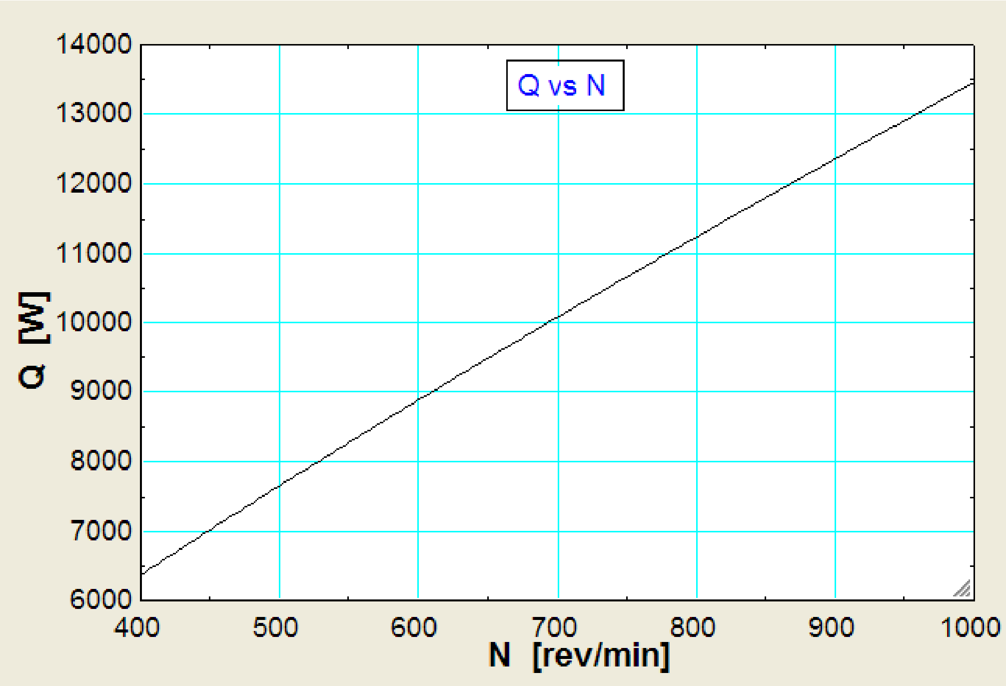
Q = 11233 W ……. Heat transfer from one side of disk.……Ans.

Plot the variation of h_c and Q with rotational speed, N:

Table 1

1..21	N [rev/min]	Re_{ω}	h_c [W/m2-C]	Q [W]
Run 1	400	9.847E+06	50.68	6368
Run 2	430	1.059E+07	53.8	6761
Run 3	460	1.132E+07	56.88	7148
Run 4	490	1.206E+07	59.91	7529
Run 5	520	1.280E+07	62.91	7905
Run 6	550	1.354E+07	65.87	8277
Run 7	580	1.428E+07	68.79	8645
Run 8	610	1.502E+07	71.69	9009
Run 9	640	1.576E+07	74.55	9369
Run 10	670	1.649E+07	77.39	9725
Run 11	700	1.723E+07	80.2	10078
Run 12	730	1.797E+07	82.98	10428
Run 13	760	1.871E+07	85.74	10775
Run 14	790	1.945E+07	88.48	11119
Run 15	820	2.019E+07	91.2	11460
Run 16	850	2.093E+07	93.9	11799
Run 17	880	2.166E+07	96.57	12136
Run 18	910	2.240E+07	99.23	12470
Run 19	940	2.314E+07	101.9	12801
Run 20	970	2.388E+07	104.5	13131
Run 21	1000	2.462E+07	107.1	13459


hc vs N
110
100
90
80
70
60
50
hc (W/m^2-C)
400
500
600
700
800
900
1000
N (rev/min)


Q vs N
14000
13000
12000
11000
10000
9000
8000
7000
6000
Q [W]
400
500
600
700
800
900
1000
N [rev/min]

==

$UnitSystem SI Pa C J

"**Prob. 2A2.4.3.** A sphere of 0.1m dia is rotating at 20 rpm in a large container of CO2 at atmospheric pressure. If the sphere is at 60 C and the CO2 is at 20 C, estimate the rate of heat transfer."

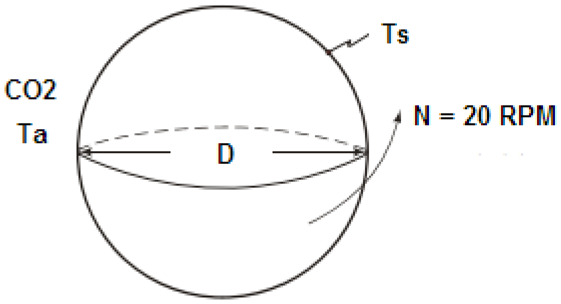


Fig.Prob.2A2.4.3

EES Solution:

"Data:"

D= 0.1 [m]

T_s = 60 [C]

T_a = 20 [C]

P = 1 [atm] * convert (atm, Pa)

T_f = (T_a + T_s) / 2 "[C] …. mean temp"

g = 9.81 [m/s^2]

N= 20 [rev/min]

"Properties of CO2 at 1 atm:"

beta = 1/(T_f + 273) "[1/K]"
rho=Density(CO2,T=T_f,P=P) "[kg/m^3]"
cp=Cp(CO2,T=T_f) "[J/kg-C]"
k=Conductivity(CO2,T=T_f) "[W/m-C]"
Pr=Prandtl(CO2,T=T_f) "…Prandtl No."
mu=Viscosity(CO2,T=T_f) "[kg/m-s]"
nu = mu / rho "[m^2/s]"

"Calculations:"

omega = N * 2 * pi / 60 "[rad/s]"

"Rotational Reynolds No.:"

Re_omega = omega * D^2/ nu

"We see that Reynolds No. is 2291 < 5 * 10^4. Therefore, apply the following formula:"

"Nusselts Number:"

$Nusselt_{D,avg} = 0.43 \cdot Re_{\omega}^{0.5} \cdot Pr^{0.4}$ ....Average Nusselts No.

In EES:

Nusselt_D_avg = 0.43 * Re_omega^0.5 * Pr^0.4 "….Average Nusselts No."

"Heat transfer coeff:"

Nusselt_D_avg = h_c * D / k "[W/m^2-C] …. finds h_c"

"Heat transfer:"

Q = h_c * (pi * D^2) * (T_s – T_a) "[W]"

Results:

Unit Settings: SI C Pa J mass deg

β = 0.003195 [1/K]	cp = 857.5 [J/kg-C]	D = 0.1 [m]
g = 9.81 [m/s^2]	h_c = 3.27 [W/m^2-C]	k = 0.01778 [W/m-C]
μ = 0.00001566 [kg/m-s]	N = 20 [rev/min]	ν = 0.000009141 [m^2/s]
$Nusselt_{D,avg}$ = 18.4	ω = 2.094 [rad/s]	P = 101325 [Pa]
Pr = 0.7551	Q = 4.109 [W]	Re_ω = 2291
ρ = 1.713 [kg/m^3]	T_a = 20 [C]	T_f = 40 [C]
T_s = 60 [C]		

Thus:

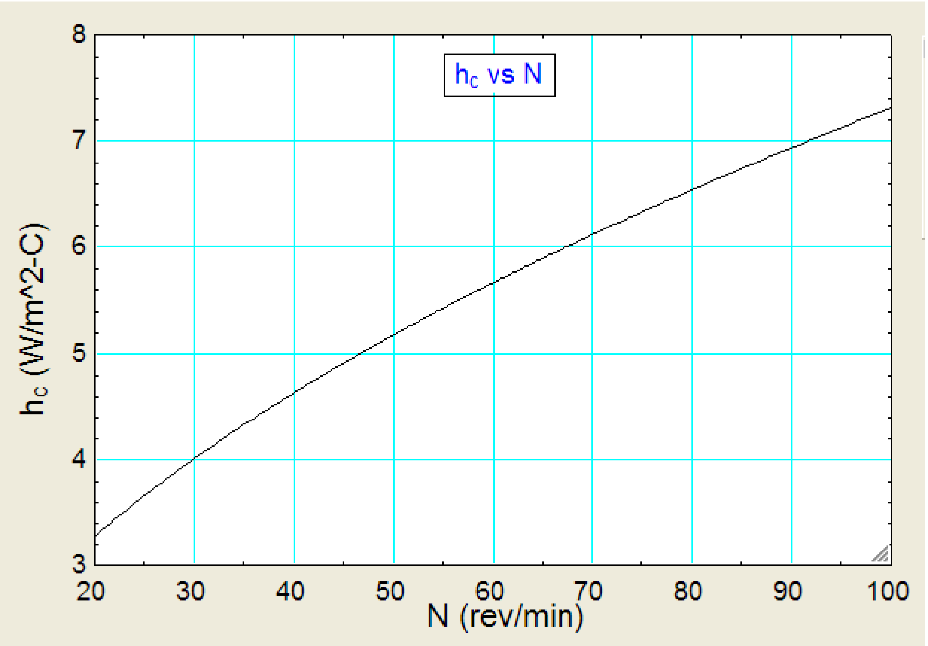
h_c = 3.27 W/m^2-C………… heat transfer coeff.……Ans.

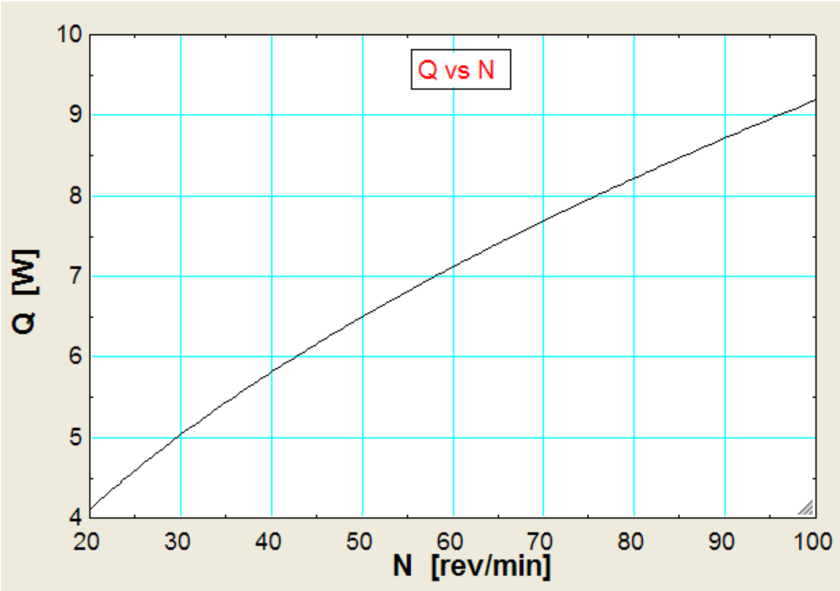
Q = 4.109 W ……. Heat transfer from sphere…….Ans.

Plot the variation of h_c and Q with rotational speed, N:

Table 1

1..11	N [rev/min]	Re_ω	h_c [W/m2-C]	Q [W]
Run 1	20	2291	3.27	4.109
Run 2	28	3208	3.869	4.862
Run 3	36	4124	4.387	5.513
Run 4	44	5041	4.85	6.095
Run 5	52	5957	5.273	6.626
Run 6	60	6874	5.664	7.118
Run 7	68	7790	6.03	7.577
Run 8	76	8707	6.375	8.011
Run 9	84	9623	6.702	8.422
Run 10	92	10540	7.014	8.814
Run 11	100	11457	7.312	9.189


hc vs N
hc (W/m^2-C)
N (rev/min)
8
7
6
5
4
3
20
30
40
50
60
70
80
90
100



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2A2.5 Natural convection from finned surfaces:

Prob. 2A2.5.1 Consider a vertical heat sink with fins as shown in Fig. The vertical heat sink, 0.35 m wide × 0.15 m high, is provided with vertical, rectangular fins of 1 mm thickness and 20 mm length. Base and surface temperature of fins is 80 C and the surrounding air is at 20 C. Determine the optimum fin spacing and the rate of heat transfer from the heat sink by natural convection.

Also plot Q against Ts, the base and surface temp of fins. Let Ts vary from 80 C to 120 C.

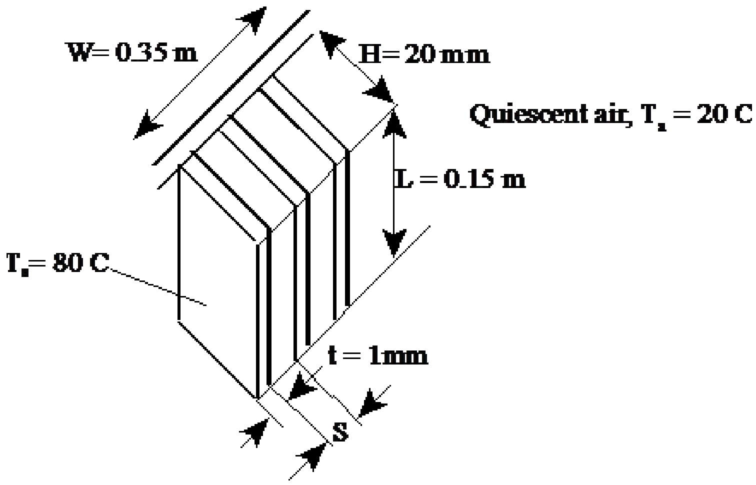


Fig. Free convection from vertical heat sink with fins

Data:

$W := 0.35$ m...width of base plate of heat sink

$L := 0.15$ m....height of base plate of heat sink.

$H := 0.02$ m....height of fins

$t := 0.001$ m...thickness of fins

$T_s := 80$ C..... surface temp.

$T_a := 20$ C....temp. of ambient air

$g := 9.81$ m/s2....accn. due to gravity

We need properties of air at average temperature $T_f = (80 + 20)/2$

$T_f := 50$ C....avg. temperature

Properties of air at 50 C:

$\nu := \frac{mu\_air(T_f + 273)}{rho\_air(T_f + 273)}$ m2/s....kinematic viscosity i.e. $\nu = 1.788 \times 10^{-5}$

$k := k\_air(T_f + 273)$ W/(m.K)...thermal cond. i.e. $k = 0.028$

$Pr := Pr\_air(T_f + 273)$ ...Prandtl number i.e. $Pr = 0.709$

$\beta := \frac{1}{T_f + 273}$ 1/K i.e. $\beta = 3.096 \times 10^{-3}$ 1/K

Now, the characteristic length is the length of fins in vertical direction, i.e. L = 0.15 m.

Then,

$$Gr_L := \frac{g \cdot \beta \cdot (T_s - T_a) \cdot L^3}{\nu^2}$$ ...Grashoff number

i.e. $Gr_L = 1.923 \times 10^7$ ...Grashoff number

And, $Ra := Gr_L \cdot Pr$

i.e. $Ra = 1.364 \times 10^7$ ...Rayleigh number

Optimum fin spacing:

We use following eqn.:

$$S_{opt} := 2.714 \cdot \frac{L}{Ra^{\frac{1}{4}}}$$

i.e. $S_{opt} = 6.698 \times 10^{-3}$ m

i.e. S_{opt} = 6.698 mm....Ans.

No. of fins:

$$n := \frac{W}{S_{opt} + t}$$ ...no. of fins

i.e. $n = 45.465$ ....say 46

i.e. $n := 46$ ...no. of fins.....Ans.

Heat transfer coeff.:

We have:

$$h := 1.31 \cdot \frac{k}{S_{opt}}$$ .....(10.79)

i.e. $h = 5.425$ W/(m^2.K)....heat transfer coeff.

Heat transfer rate:

$$Q := h \cdot (2 \cdot n \cdot L \cdot H) \cdot (T_s - T_a)$$

i.e. $Q = 89.842$ **W.....Ans.**

Also plot Q against Ts, the base and surface temp of fins. Let Ts vary from 80 C to 120 C.

Use the 'vectorize' operator of Mathcad to do parallel calculations:

$i := 0..8$ $Ts_i := 80 + 5 \cdot i$ .... Define vector Ts Check : $Ts_8 = 120$ C

$$T_f := \frac{Ts + T_a}{2}$$ ... C... avg. temps.

i.e. we get for T_f the vector:

$$T_f = \begin{pmatrix} 50 \\ 52.5 \\ 55 \\ 57.5 \\ 60 \\ 62.5 \\ 65 \\ 67.5 \\ 70 \end{pmatrix}$$

Properties of Air:

$$\nu := \overrightarrow{\frac{mu\_air(T_f + 273)}{rho\_air(T_f + 273)}} \qquad \text{m}^2\text{/s....kinematic viscosity}$$

In the above, two vectors, viz. μ and ρ are divided, term by term. Note the use of vectorize operator.

Just to check: observe that we get for vector, ν:

$$\nu = \begin{pmatrix} 1.788 \times 10^{-5} \\ 1.814 \times 10^{-5} \\ 1.84 \times 10^{-5} \\ 1.865 \times 10^{-5} \\ 1.889 \times 10^{-5} \\ 1.912 \times 10^{-5} \\ 1.936 \times 10^{-5} \\ 1.96 \times 10^{-5} \\ 1.986 \times 10^{-5} \end{pmatrix}$$

Also:

$$k := k\_air(T_f + 273) \qquad \text{W/(m.K)...thermal cond.}$$

$$Pr := Pr\_air(T_f + 273) \qquad \text{...Prandtl number}$$

$$\beta := \frac{1}{T_f + 273} \qquad \text{1/K}$$

In the above, for k, Pr and β, vectorize operator is not required since vector to vector operation is not involved.

Then,

$$Gr_L := \overrightarrow{\frac{g\cdot\beta\cdot(Ts - T_a)\cdot L^3}{\nu^2}}$$ ...Grashoff number

And,

$$Ra := \overrightarrow{(Gr_L\cdot Pr)}$$ ....Rayleigh No.

Optimum fin spacing:

We use following eqn.:

$$S_{opt} := \left(2.714\cdot\frac{L}{Ra^{\frac{1}{4}}}\right)$$

No. of fins:

$$n := \mathrm{ceil}\left(\frac{W}{S_{opt} + t}\right)$$ ...no. of fins

Note that function 'ceil' rounds off the no. of fins calculated to a higher integer value. Check:

$$n = \begin{pmatrix} 46 \\ 46 \\ 47 \\ 47 \\ 47 \\ 48 \\ 48 \\ 48 \\ 48 \end{pmatrix}$$

Heat transfer coeff.:

$$h := \overrightarrow{\left(1.31 \cdot \frac{k}{S_{opt}}\right)}$$ W/(m2.K)....heat transfer coeff.

We get:

$$h = \begin{pmatrix} 5.425 \\ 5.524 \\ 5.616 \\ 5.702 \\ 5.784 \\ 5.86 \\ 5.932 \\ 5.999 \\ 6.061 \end{pmatrix}$$

And,

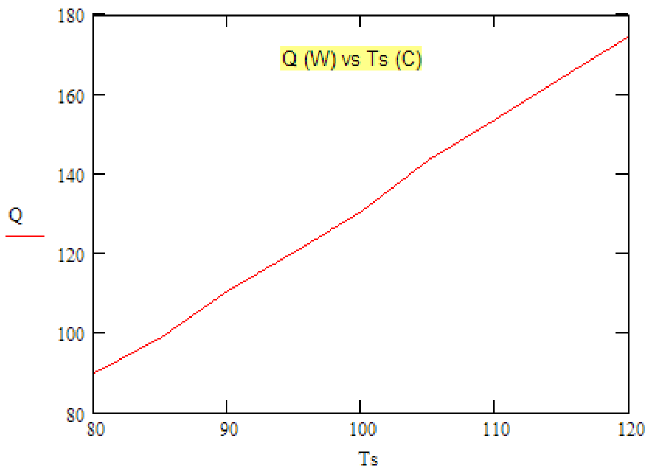
Heat transfer rate:

$$Q := \overrightarrow{\left[h \cdot (2 \cdot n \cdot L \cdot H) \cdot (Ts - T_a)\right]}$$ W....Ans.

i.e.

$$Q = \begin{pmatrix} 89.842 \\ 99.1 \\ 110.859 \\ 120.604 \\ 130.48 \\ 143.461 \\ 153.763 \\ 164.138 \\ 174.57 \end{pmatrix}$$

Plot of Q vs Ts:



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Prob. 2A2.5.2 Consider a vertical heat sink with fins as shown in Fig. below.

The vertical heat sink, 0.4 m wide × 0.15 m high, is provided with vertical, rectangular fins. Length of fins is limited to 20 mm . Base and surface temperature of fins is 70 C and the surrounding air is at 20 C. Determine the fin thickness and no. of fins for optimum spacing if the total heat to be dissipated by natural convection is 50 W.

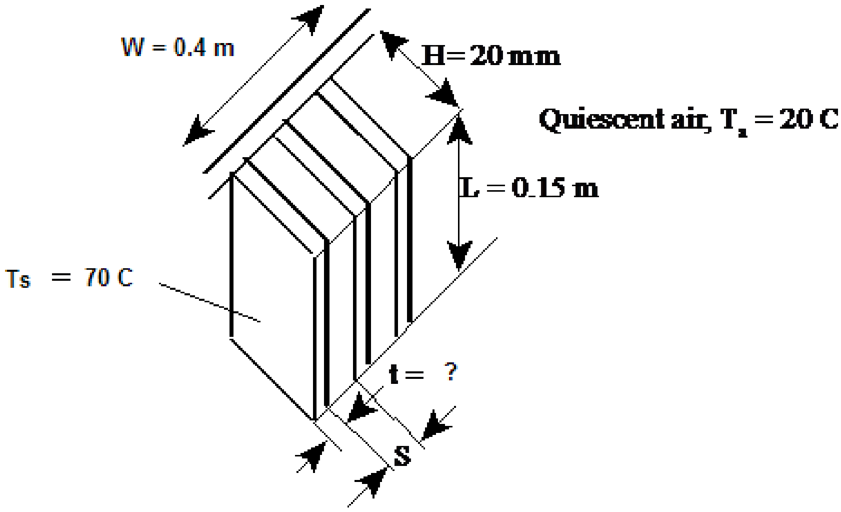


Fig. Free convection from vertical heat sink with fins

EES Solution:

$UnitSystem SI C Pa J

"Data:"

T_a= 20[C]

T_s = 70 [C]

T_f = (T_a + T_s) / 2

L = 0.15[m]

Height = 0.02[m]

W = 0.4[m]

P_1 = 1.01325e05 [Pa]

g = 9.81[m/s^2]

{t = 0.002"[m]…..assumed; will be corrected later"}

"Properties of Air at mean temp (T_f):"

rho=Density(Air,T=T_f,P=P_1) "[kg/m^3]"

k=Conductivity(Air,T=T_f) "[W/m-C]"

cp=Cp(Air,T=T_f) "[J/kg-C]"

Pr=Prandtl(Air,T=T_f)

mu=Viscosity(Air,T=T_f) "[kg/m-s]"

nu = mu / rho "[m^2/s]"

beta = 1 / (T_f + 273) "[1/K]"

"Calculations:"

"Here, L is the characteristic dimension to calculate Gr:"

Gr_L = g * beta * (T_s – T_a) * L^3 / nu^2 "..Grashof No."

Ra_L = Gr_L * Pr "….Rayleigh No."

"Optimum fin spacing:"

S_opt = 2.714 * L / Ra_L^(1/4)

"No. of fins:"

n = (W / (S_opt + t)) "…No. of fins"

"To determine h, we use following eqn.:"

h = 1.31 * k /S_opt "[W/m^2-C]"

"Heat transfer rate:"

Q = h * (2 * n * L * Height) * (T_s – T_a) "[W]"

"But, by data:"

Q = 50[W]

Results:

Unit Settings: SI C Pa J mass deg

β = 0.003145 [1/K]	cp = 1006 [J/kg-C]	g = 9.81 [m/s^2]	Gr_L = 1.701E+07
h = 5.144 [m]	Height = 0.02 [m]	k = 0.02699 [W/m-C]	L = 0.15 [m]
μ = 0.00001941 [kg/m-s]	n = 32.4	ν = 0.00001749 [m^2/s]	Pr = 0.7233
P_1 = 101325 [Pa]	Q = 50 [W]	Ra_L = 1.231E+07	ρ = 1.11 [kg/m^3]
S_{opt} = 0.006873 [m]	t = 0.005471 [m]	T_a = 20 [C]	T_f = 45 [C]
T_s = 70 [C]	W = 0.4 [m]		

Thus:

S_opt = 6.873 mm = Optimum fin spacing
t = 5.471 mm = fin thickness
n = 32.4 = No. of fins
Since n should be an integer, let us change n = 33 and recalculate:

New results are:

Unit Settings: SI C Pa J mass deg

β = 0.003145 [1/K]	cp = 1006 [J/kg-C]	g = 9.81 [m/s^2]	Gr_L = 1.701E+07
h = 5.144 [m]	Height = 0.02 [m]	k = 0.02699 [W/m-C]	L = 0.15 [m]
μ = 0.00001941 [kg/m-s]	n = 33	ν = 0.00001749 [m^2/s]	Pr = 0.7233
P_1 = 101325 [Pa]	Q = 50.92 [W]	Ra_L = 1.231E+07	ρ = 1.11 [kg/m^3]
S_{opt} = 0.006873 [m]	t = 0.005248 [m]	T_a = 20 [C]	T_f = 45 [C]
T_s = 70 [C]	W = 0.4 [m]		

i.e.

S_opt = 6.873 mm = Optimum fin spacing

t = 5.248 mm = fin thickness

n = 33 = No. of fins

==

Prob. 2A2.5.3 Consider a vertical heat sink with fins as shown in Fig. The vertical heat sink, 0.35 m wide × 0.15 m high, is provided with vertical, rectangular fins of 1 mm thickness and 20 mm length. Base and surface temperature of fins is 80 C and the surrounding air is at 20 C. Determine the optimum fin spacing and the rate of heat transfer from the heat sink by natural convection.

Also plot Q against Ts, the base and surface temp of fins. Let Ts vary from 80 C to 120 C.

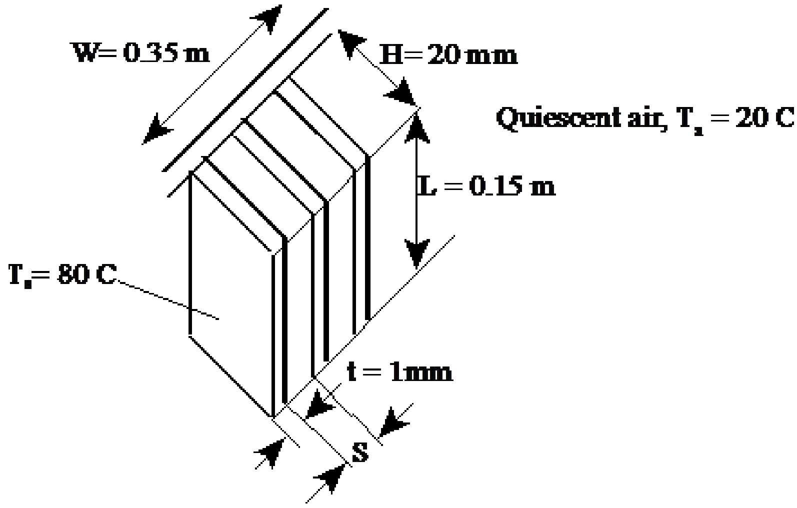


Fig_ Free convection from vertical heat sink with fins

Note: This is the same Problem as Prob. 2A2.5.1.

But, now we shall solve this problem with EXCEL.

Properties of Air at different temp are required. So, we will use the EXCEL worksheet wherein we wrote VBA Functions for properties of Air. (See, for ex, Prob.2A1.2.13)

EXCEL Solution:

Following are the steps:

1. Set up the EXCEL worksheet, enter data and name the cells:

T_f   =(T_s+T_a)/2

	A	B	C	D	E	F	G
209							
210		**Data:**	**Fluid =**	**Air**			
211		**Surface temp**	**T_s**	**80**	C		
212		**Ambient air temp**	**T_a**	**20.0**	C	$T\_f = (T\_s + T\_a)/2$	
213			**T_f**	**50.0**	C		
214		**Height of base plate of heat sink**	**L**	**0.150**	m		
215		Width of base plate of heat sink	**W**	**0.350**	m		
216		Height of fins	**H**	**0.020**	m		
217		thickness of fins	**t**	**0.0010**	m		
218							
219		accn due to gravity	**g**	**9.81**	m/s^2		
220		Vol. expansivity	**beta**	**0.0031**	1/K		

2. Calculate properties of Air at T_f, using the VBA Functions already written:

nu | f_x =Air_nu_T((T_f+273))

	A	B	C	D	E	F	G
224		**Calculations:**					
225		density	**rho**	1.09325	kg/m^3		
226		th. conductivity	**k**	0.028066	W/m.C	Using VBA Functions for Air	
227		Prandtl No.	**Pr**	0.704615			
228		kinematic visc.	**nu**	1.79685E-05	m^2/s		
229		sp.heat	**cp**	1008.15	J/kg.K		
230							

3. Continue the calculations as shown below:

S_opt | f_x =2.714*L/Ra_L^(1/4)

	A	B	C	D	E	F	G
233							
234		**Nat. convection for Fins on a vertical surface:**					
235							
236		**To calculate Grashof No...'L' is the characteristic dimension.**			$Gr\_L = \frac{g \cdot \beta \cdot (T\_s - T\_a) \cdot L^3}{nu^2}$		
237							
238		Grashof No.	**Gr_L**	**1.90488E+07**			
239		Rayleigh No.	**Ra_L**	**1.34221E+07**	$Ra\_L = Gr\_L \cdot Pr$	$S\_opt = \frac{2.714 \cdot L}{Ra\_L^{\frac{1}{4}}}$	
240							
241		**Opt. fin spacing**	**S_opt**	**0.006726**	**m**		
242		**No. of fins-calculated**	**n**	**45.3026**		$n = \frac{W}{S\_opt + t}$	
243		**No. of fins-rounded to next higher value**	**N**	**46.0000**	$N = CEILING(D242, 1)$		
244							
245		**Heat tr. coeff**	**h_c**	**5.4665**	**W/m^2.C**	$h\_c = \frac{1.31 \cdot k}{S\_opt}$	
246		**Heat tr. rate**	**Q_conv**	**90.5245**	**W...Ans**	$Q\_conv = h\_c \cdot (2 \cdot N \cdot L \cdot H) \cdot (T\_s - T\_a)$	
247							

In the above screen shot, the equations used are also shown, for clarity.

Thus: No. of fins = 46, S_opt = 6.726 mm, Q_conv = 90.52 W …. Ans.

4. Now, plot Q_conv for different values of T_s:

Note that as T_s varies, T_f also varies and the properties of Air will also vary. Therefore, it is more convenient to write a VBA program to calculate different quantities as T_s varies.

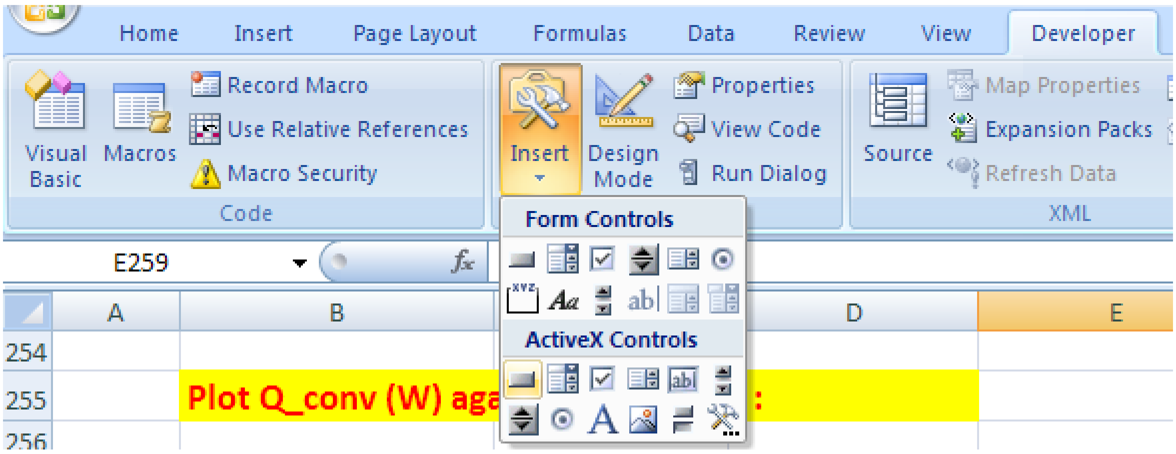
The program should to read the values of T_s , one by one, from this Table and put in cell D211.; then, immediately, all other calculations in remaining cells get up-dated. The program selects the required quantity (i.e. S_opt, N, h_c and Q_conv, from cells D241, D243, D245 and D246 respectively) and copies them to their respective places in the Table. And, we will have a Command Button to do this:

To do this, first set up a Table as shown below:

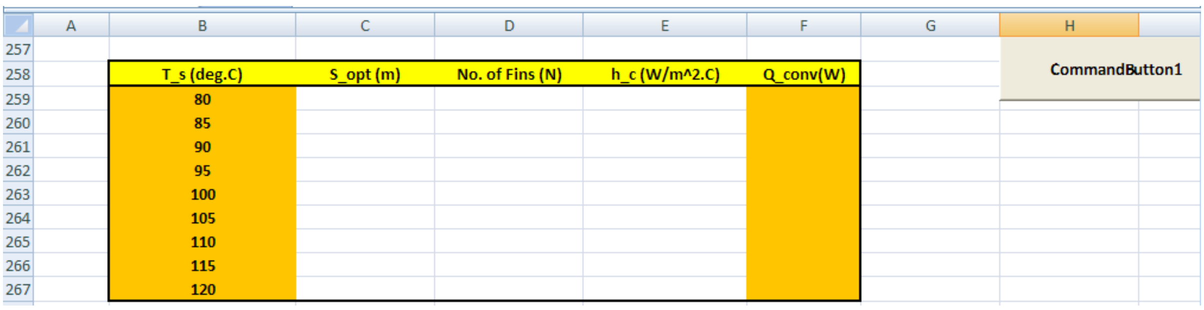
	A	B	C	D	E	F
254						
255		Plot Q_conv (W) against T_s (deg.C) :				
256						
257						
258		T_s (deg.C)	S_opt (m)	No. of Fins (N)	h_c (W/m^2.C)	Q_conv(W)
259		80				
260		85				
261		90				
262		95				
263		100				
264		105				
265		110				
266		115				
267		120				

5. And, to set up the Command Button:

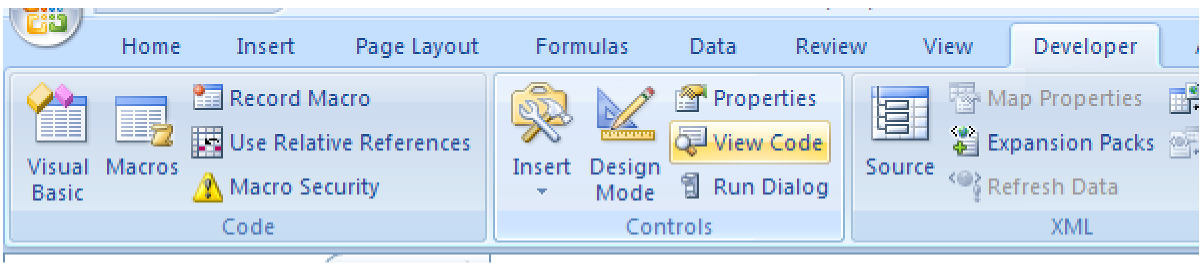
Go to Developer- Insert-ActiveX controls:



Click on first top left button. And, draw the Command Button in the worksheet to the required size, at the required place:



Click on Developer-View Code:



And, we get the code already written for the Command button:

```
CommandButton1                                   Click

Private Sub CommandButton1_Click()

End Sub
```

6. Now, complete the code to do the desired jobs as mentioned under point 4 above:

```
CommandButton1                                   Click

Private Sub CommandButton1_Click()

Dim i As Integer

For i = 0 To 8
 Range("D211") = Cells(259 + i, 2)
 Cells(259 + i, 3) = Range("D241")
 Cells(259 + i, 4) = Range("D243")
 Cells(259 + i, 5) = Range("D245")
 Cells(259 + i, 6) = Range("D246")
Next i

End Sub
```

In the above program:

Line 1: defines the Sub-program

Line 2: Dimension statement for i, the counter in For….Next loop

Line 3 to 9: For….Next loop

Line 4: Sets T_s i.e. cell D211 to the first value in the Table (i.e. cell B259)

Immediately, all other calculations are updated. And we have to just copy the required values to their respective places in the Table:

Line 5, 6, 7 and 8: copy values of S_opt, N, h_c and Q_conv from cells D241, D243, D245 and D246 respectively to appropriate cells in the Table.

Line 9: End statement of Sub-program

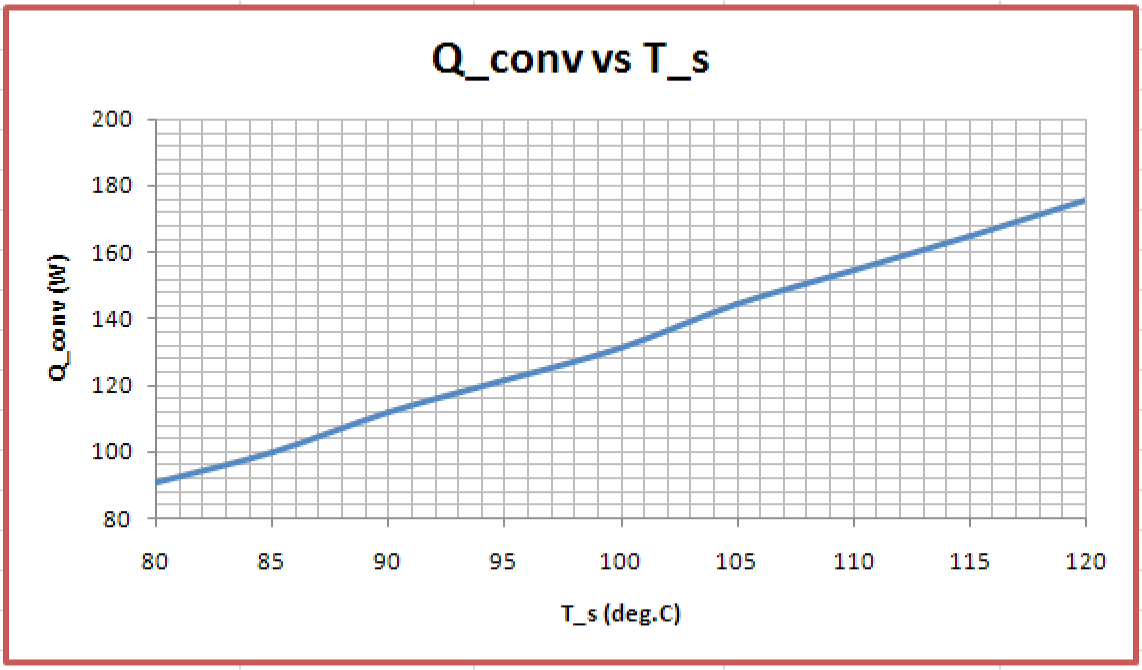
7. Now, click on the Command Button, and the Table is filled up:

Plot Q_conv (W) against T_s (deg.C) :

CommandButton1

T_s (deg.C)	S_opt (m)	No. of Fins (N)	h_c (W/m^2.C)	Q_conv(W)
80	0.007	46	5.466	90.5245
85	0.007	46	5.563	99.8012
90	0.007	47	5.653	111.5972
95	0.007	47	5.738	121.3618
100	0.006	47	5.818	131.2562
105	0.006	48	5.894	144.2790
110	0.006	48	5.966	154.6276
115	0.006	48	6.034	165.0768
120	0.006	48	6.097	175.5819

8. Now, plot the graph of Q_conv vs T_s in EXCEL:



Note: Values obtained here match well with those obtained in Prob. 2A2.5.1 using Mathcad.

==

2A2.6 Combined Natural and Forced convection:

Prob. 2A2.6.1 Consider a 5 m long vertical plate at 85 C in air at 30 C. Determine the forced motion velocity above which natural convection heat transfer from this plate is negligible.

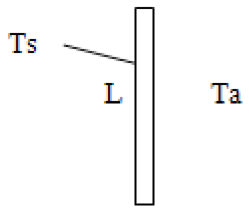


Fig.Prob.2A2.6.1

Mathcad Solution:

Data:

$L := 5$ m....height of plate .

$T_s := 85$ C..... surface temp.

$T_a := 30$ C....temp. of ambient air

$g := 9.81$ m/s^2....accn. due to gravity

We need properties of air at average temperature Tf = (85 + 30)/2

$T_f := 57.5$ C....avg. temperature

Properties of air at 57.5 C:

$\nu := \dfrac{mu\_air(T_f + 273)}{rho\_air(T_f + 273)}$ m^2/s....kinematic viscosity i.e. $\nu = 1.865 \times 10^{-5}$

$k := k\_air(T_f + 273)$ W/(m.K)...thermal cond. i.e. $k = 0.028$

$Pr := Pr\_air(T_f + 273)$ ...Prandtl number i.e. $Pr = 0.708$

$\beta := \dfrac{1}{T_f + 273}$ 1/K i.e. $\beta = 3.026 \times 10^{-3}$ 1/K

Now, the characteristic length is the length, i.e. L = 5 m.

Then,

$Gr_L := \dfrac{g \cdot \beta \cdot (T_s - T_a) \cdot L^3}{\nu^2}$ ...Grashoff number

i.e. $Gr_L = 5.867 \times 10^{11}$ ...Grashoff number

Let the value of velocity, above which natural convection may be neglected be V (m/s).

Then: $Re_L = \frac{V \cdot L}{\nu}$ ....Reynolds No.

We have:

If $\frac{Gr_L}{Re_L^2} < 0.1$ ..free convection is negligible

If $\frac{Gr_L}{Re_L^2} > 10$ ..forced convection is negligible

If $\frac{Gr_L}{Re_L^2} > 0.1$ and $\frac{Gr_L}{Re_L^2} < 10$ ..mixed convection regime, i.e. both free and forced conv is to be considered.

Find V and Re_L using Solve block:

$Re_L := 1$ $V := 1$ ....Guess values

Given

$$Re_L = \frac{V \cdot L}{\nu}$$

$$\frac{Gr_L}{{Re_L}^2} = 0.1$$

$$\text{Find}(Re_L, V) = \begin{pmatrix} 2.422 \times 10^6 \\ 9.035 \end{pmatrix}$$

Thus:

$Re_L = 2.422 \cdot 10^6$ ..Reynolds No.beyond which free convection is not negligible

$V = 9.035$ ...m/s...velocity above which free convection is not negligible .... Ans.

===

Prob. 2A2.6.2. Consider a 3 m long vertical plate at 60 C in water at 20 C. Determine the forced motion velocity above which natural convection heat transfer from this plate is negligible.

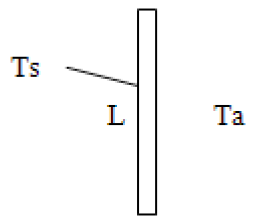


Fig.Prob.2A2.6.2

Data:

$L := 3$ m....height of plate .

$T_s := 60$ C..... surface temp.

$T_a := 20$ C....temp. of ambient air

$g := 9.81$ m/s^2....accn. due to gravity

We need properties of water at average temperature Tf = (60 + 20)/2

Use the Mathcad functions for properties of Water already written. (**See Prob. 2A2.2.4**)

$T_f := 40$ C....avg. temperature

Calclations:

Properties of water at Tf:

$rho := rho\_water(T_f)$ i.e. $rho = 992.232$ kg/m^3

$cp := cp\_water(T_f)$ i.e. $cp = 4.182 \times 10^3$ K/kg.K

$mu := mu\_water(T_f)$ i.e. $mu = 6.542 \times 10^{-4}$ kg/m.s

$k := k\_water(T_f)$ i.e. $k = 0.631$ W/m.K

$Pr := Pr\_water(T_f)$ i.e. $Pr = 4.352$

$\beta := beta\_water(T_f)$ i.e. $\beta = 3.763 \times 10^{-4}$ 1/K

$\nu := \frac{mu}{rho}$ i.e. $\nu = 6.593 \times 10^{-7}$ m^2/s

Now, the characteristic length is the length, i.e. L = 3 m.

Then,

$$Gr_L := \frac{g \cdot \beta \cdot (T_s - T_a) \cdot L^3}{\nu^2}$$ ...Grashoff number

i.e. $Gr_L = 9.172 \times 10^{12}$ ...Grashoff number

Let the value of velocity, above which natural convection may be neglected be V (m/s).

Then: $Re_L = \frac{V \cdot L}{\nu}$ ....Reynolds No.

We have:

If $\frac{Gr_L}{Re_L^2} < 0.1$ ..free convection is negligible

If $\frac{Gr_L}{Re_L^2} > 10$ ..forced convection is negligible

If $\frac{Gr_L}{Re_L^2} > 0.1$ and $\frac{Gr_L}{Re_L^2} < 10$ ..mixed convection regime, i.e. both free and forced conv is to be considered.

Find V and Re_L using Solve block:

$Re_L := 1$ $V := 1$ ....Guess values

Given

$$Re_L = \frac{V \cdot L}{\nu}$$

$$\frac{Gr_L}{Re_L^2} = 0.1$$

$$\text{Find}(Re_L, V) = \begin{pmatrix} 9.577 \times 10^6 \\ 2.105 \end{pmatrix}$$

Thus:

$Re_L = 9.577 \cdot 10^6$ ..Reynolds No.beyond which free convection is not negligible

$V = 2.105$ ...m/s...velocity above which free convection is not negligible .... Ans.

==

Prob. 2A2.6.3 A 12 cm high and 20 cm wide circuit board dissipates 5 W. The board is cooled by air at 35 C flowing vertically upwards along the 12 cm long side at a velocity of 0.5 m/s. Back surface of board is insulated. Determine the average temp of the surface (i) by ignoring natural convection (ii) considering the contribution of natural convection.

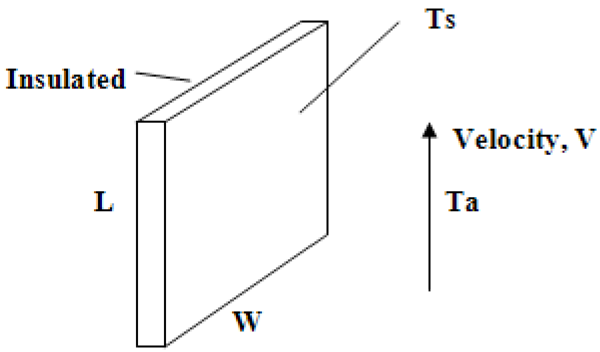


Fig.Prob.2A2.6.3

Mathcad Solution:

Data:

$L := 0.12$ m....height of plate .

$W := 0.2$ m....width of plate .

$T_s := 85$ C..... surface temp.... assumed; will be corrected later.

$T_a := 35$ C....temp. of ambient air

$g := 9.81$ m/s²....accn. due to gravity

$V := 0.5$ m/s ... air velocity

We need properties of air at average temperature Tf = (85 + 35)/2

$$T_f(T_s) := \frac{T_s + T_a}{2}$$

$T_f(T_s) = 60$ C....avg. temperature

Note that we write all quantities as functions of Ts.

Properties of air at 60 C:

$mu(T_s) := mu\_air(T_f(T_s) + 273)$ kg/m.s....dyn. viscosity

$rho(T_s) := rho\_air(T_f(T_s) + 273)$ kg/m^3 ... density

$$v(T_s) := \frac{mu\_air(T_f(T_s) + 273)}{rho\_air(T_f(T_s) + 273)}$$ m²/s....kinematic viscosity

$k(T_s) := k\_air(T_f(T_s) + 273)$ W/(m.K)...thermal cond.

$Pr(T_s) := Pr\_air(T_f(T_s) + 273)$ ...Prandtl number

$$\beta(T_s) := \frac{1}{T_f(T_s) + 273}$$ 1/K

Now, the characteristic length is the length, i.e. L = 0.12 m.

Then,

$$Gr_L(T_s) := \frac{g \cdot \beta(T_s) \cdot (T_s - T_a) \cdot L^3}{v(T_s)^2}$$ ...Grashoff number

And, $Ra(T_s) := Gr_L(T_s) \cdot Pr(T_s)$ ...Rayleigh number

$$Re_L(T_s) := \frac{L \cdot V \cdot rho(T_s)}{mu(T_s)}$$ Reynolds No.

Case 1: Only Forced convection:

$$Nu_{forced}(T_s) := 0.664 \cdot Re_L(T_s)^{0.5} \cdot Pr(T_s)^{\frac{1}{3}}$$

$$h_{forced}(T_s) := \frac{Nu_{forced}(T_s) \cdot k(T_s)}{L}$$

$$Q_{forced}(T_s) := h_{forced}(T_s) \cdot (L \cdot W) \cdot (T_s - T_a)$$

Now, use Solve block to get value of T_s such that Q = 5 W:

Given

$$Q_{forced}(T_s) = 5$$

$$Find(T_s) = 61.272$$

i.e. $T_s := 61.272$ C….Ans…..Temp of board considering only forced conv.

Case 2: Considering combined Forced and Natural convection:

$$\frac{Gr_L(T_s)}{Re_L(T_s)^2} = 0.385$$ …This value is between 0.1 and 10. So, it is mixed convection regime.

Then, we have:

$$Nu^m = Nu_{forced}{}^m + Nu_{free}{}^m$$ ..where m = 3

For free convection:

$$Nu_{free}(T_s) := 0.68 + \frac{0.67 \cdot Ra(T_s)^{\frac{1}{4}}}{\left[1 + \left(\frac{0.492}{Pr(T_s)}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}}$$

..Nusselts No.
…for 0 < Ra <10^9, and
0 < Pr < ∞

$$h_{free}(T_s) := \frac{Nu_{free}(T_s) \cdot k(T_s)}{L}$$ W/m^2.C....heat tr coeff

$$Q_{free}(T_s) := h_{free}(T_s) \cdot (L \cdot W) \cdot (T_s - T_a)$$ W...heat transfer

$$Nu_{mixed}(T_s) := \left(Nu_{forced}(T_s)^3 + Nu_{free}(T_s)^3\right)^{\frac{1}{3}}$$

$$h_{mixed}(T_s) := \frac{Nu_{mixed}(T_s) \cdot k(T_s)}{L}$$

$$Q_{mixed}(T_s) := h_{mixed}(T_s) \cdot (L \cdot W) \cdot (T_s - T_a)$$

Now, use Solve block to get value of T_s such that Q_{mixed} = 5 W:

Given

$$Q_{mixed}(T_s) = 5$$

$$Find(T_s) = 59.342$$

i.e. $T_s := 59.342$ C....Ans.....Temp of board considering both free and forced conv.

Thus:

$Ts = 61.272$ C....when free convection is not considered.....Ans.

$Ts = 59.342$ C....when free convection is considered.....Ans.

Now consider the following extension of the above problem:

Find out the Ts for different ambient temps, Ta, Q being 5 W in each case.

Also, plot the results:

Let Ta vary from 0 deg.C to 40 deg.C:

First, let us write a Mathcad Function for Q in mixed convection, as shown below:

In the following function:

INPUTS:

L = Height of plate (Ch. length), m
Ts = surface temp, C
Ta = ambient temp, C
Area = surface area of plate (height L and width, W), m^2
V = velocity of air flowing upwards

OUTPUT:

Qmixed = heat transfer rate (W), combined: free + forced convection

$$Q_{mixedconv}(L, T_s, T_a, Area, V) := \left| \begin{array}{l} T_f \leftarrow \dfrac{T_s + T_a}{2} \\ \beta \leftarrow \dfrac{1}{T_f + 273} \\ k \leftarrow k\_air(T_f + 273) \\ \nu \leftarrow \dfrac{mu\_air(T_f + 273)}{rho\_air(T_f + 273)} \\ Pr \leftarrow Pr\_air(T_f + 273) \\ Gr_L \leftarrow \dfrac{9.81 \cdot \beta \cdot (T_s - T_a) \cdot L^3}{\nu^2} \\ Re_L \leftarrow \dfrac{L \cdot V}{\nu} \\ Ra_L \leftarrow Gr_L \cdot Pr \\ Nu_{forced} \leftarrow 0.664\, Re_L^{0.5} \cdot Pr^{\frac{1}{3}} \\ Nu_{free} \leftarrow 0.68 + \dfrac{0.67 \cdot Ra_L^{\frac{1}{4}}}{\left[1 + \left(\dfrac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}} \\ Nu_{mixed} \leftarrow \left(Nu_{forced}^{3} + Nu_{free}^{3}\right)^{\frac{1}{3}} \\ h_{mixed} \leftarrow \dfrac{Nu_{mixed} \cdot k}{L} \\ Q_{mixed} \leftarrow h_{mixed} \cdot Area \cdot (T_s - T_a) \\ Q_{mixed} \end{array} \right.$$

Ex: $Q_{mixedconv}(0.12, 60, 35, 0.024, 0.5) = 5.142$

In the present case, Q_{mixed} is given as 5 W.

We have to find out values of Ts for different values of Ta.

Define a vector for variation of Ta, and then apply the Solve block of Mathcad to find out *corresponding* values of Ts, to satisfy the condition: $Q_{mixed} = 5$ W:

$Ta := (0\ \ 5\ \ 10\ \ 15\ \ 20\ \ 25\ \ 30\ \ 35\ \ 40)$ ...different ambient temps considered

$Ts := (30\ \ 30\ \ 30\ \ 30\ \ 30\ \ 30\ \ 30\ \ 30\ \ 30)$ ....Guess values

Given

$$\overrightarrow{Q_{mixedconv}(L, Ts, Ta, 0.024, 0.5)} = 5$$

$TempTs := Find(Ts)$

Note the use of 'vectorize operator' inside the Solve block.

i.e. we get the TempTs vector as:

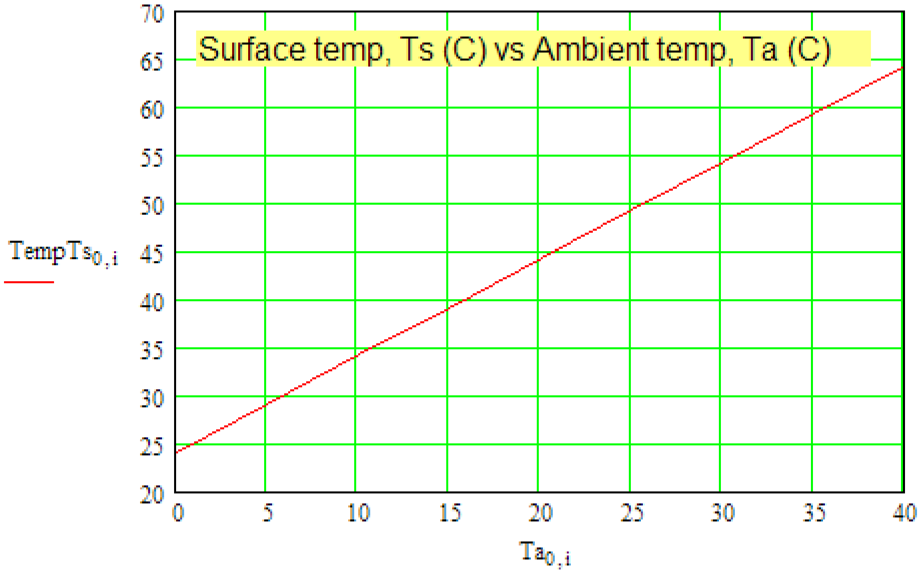
$TempTs = (24.034\ \ 29.12\ \ 34.2\ \ 39.164\ \ 44.193\ \ 49.288\ \ 54.331\ \ 59.342\ \ 64.359)$

In Tabular form:

$Ta_{0,i} =$	$TempTs_{0,i} =$
0	24.034
5	29.12
10	34.2
15	39.164
20	44.193
25	49.288
30	54.331
35	59.342
40	64.359

Now, plot TempTs against Ta:

$i := 0..8$



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Prob. 2A2.6.4 Consider a circuit board (size: 150 mm × 150 mm), insulated on backside, which is to be air cooled with an upward flow velocity of 0.3 m/s, such that its surface temp does not exceed 60 C. Ambient temp is 25 C. Determine the allowable power dissipation from the board.

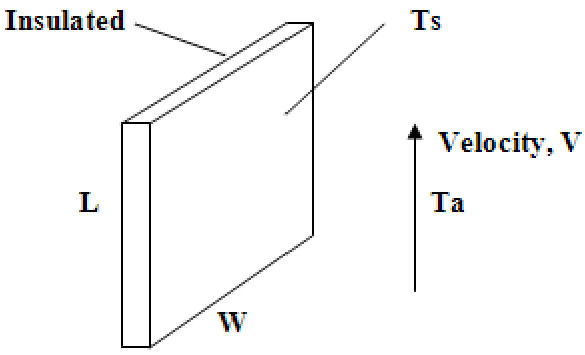


Fig.Prob.2A2.6.4

Mathcad Solution:

Data:

$L := 0.15$ m $W := 0.15$ m Area $:= 0.0225$ m^2

$T_s := 60$ C $T_a := 25$ C $V := 0.3$ m/s

To find Q:

Let us slightly modify the Function written above.

This function is for heated vertical plate, insulated on one side and air flow in the upward direction.

This function will check if the flow is in the free or forced or mixed convection regime, depending on the value of Factor (= Gr/Re^2) and return the corresponding values of Re, Ra, Factor and Q.

$$Q_{conv\_Vertical\_Plate}(L, T_s, T_a, Area, V) :=$$

$$T_f \leftarrow \frac{T_s + T_a}{2}$$

$$\beta \leftarrow \frac{1}{T_f + 273}$$

$$g \leftarrow 9.81$$

$$k \leftarrow k\_air(T_f + 273)$$

$$\nu \leftarrow \frac{mu\_air(T_f + 273)}{rho\_air(T_f + 273)}$$

$$Pr \leftarrow Pr\_air(T_f + 273)$$

$$Gr_L \leftarrow \frac{g \cdot \beta \cdot (T_s - T_a) \cdot L^3}{\nu^2}$$

$$Re_L \leftarrow \frac{L \cdot V}{\nu}$$

$$Ra_L \leftarrow Gr_L \cdot Pr$$

$$Nu_{forced} \leftarrow 0.664 Re_L^{0.5} \cdot Pr^{\frac{1}{3}} \quad \text{if } Re_L < 5 \cdot 10^5$$

$$Nu_{forced} \leftarrow 0.037 Re_L^{0.8} \cdot Pr^{\frac{1}{3}} \quad \text{if } Re_L \geq 5 \cdot 10^5$$

$$Nu_{free} \leftarrow 0.68 + \frac{0.67 \cdot Ra_L^{\frac{1}{4}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}}$$

$$Factor \leftarrow \frac{Gr_L}{Re_L^2}$$

if $Factor < 0.1$

$$h_{forced} \leftarrow \frac{Nu_{forced} \cdot k}{L}$$

$$Q_{forced} \leftarrow h_{forced} \cdot Area \cdot (T_s - T_a)$$

$$\text{return} \begin{pmatrix} \text{"Re"} & \text{"Ra"} & \text{"Gr/Re^2"} & \text{"Qforced,W"} \\ Re_L & Ra_L & Factor & Q_{forced} \end{pmatrix}$$

if $Factor > 10$

$$h_{free} \leftarrow \frac{Nu_{free} \cdot k}{L}$$

$$Q_{free} \leftarrow h_{free} \cdot Area \cdot (T_s - T_a)$$

$$\text{return} \begin{pmatrix} \text{"Re"} & \text{"Ra"} & \text{"Gr/Re^2"} & \text{"Qfree,W"} \\ Re_L & Ra_L & Factor & Q_{free} \end{pmatrix}$$

$$Nu_{mixed} \leftarrow \left(Nu_{forced}^3 + Nu_{free}^3\right)^{\frac{1}{3}}$$

$$h_{mixed} \leftarrow \frac{Nu_{mixed} k}{L}$$

$$Q_{mixed} \leftarrow h_{mixed} Area \cdot (T_s - T_a)$$

$$\begin{pmatrix} \text{"Re"} & \text{"Ra"} & \text{"Gr/Re^2"} & \text{"Qmixed,W"} \\ Re_L & Ra_L & Factor & Q_{mixed} \end{pmatrix}$$

Result for the above problem:

$$Q_{conv\_Vertical\_Plate}(L, T_s, T_a, Area, V) = \begin{pmatrix} \text{"Re"} & \text{"Ra"} & \text{"Gr/Re^2"} & \text{"Qmixed,W"} \\ 2.62 \times 10^3 & 8.849 \times 10^6 & 1.814 & 5.31 \end{pmatrix}$$

Also check the above program:

Check: for velocities, V = 0.5, 0.1, 10 and 60 m/s:

$$Q_{conv\_Vertical\_Plate}(L, T_s, T_a, Area, 0.5) = \begin{pmatrix} \text{"Re"} & \text{"Ra"} & \text{"Gr/Re^2"} & \text{"Qmixed,W"} \\ 4.367 \times 10^3 & 8.849 \times 10^6 & 0.653 & 6.239 \end{pmatrix}$$

$$Q_{conv\_Vertical\_Plate}(L, T_s, T_a, Area, 0.1) = \begin{pmatrix} \text{"Re"} & \text{"Ra"} & \text{"Gr/Re^2"} & \text{"Qfree,W"} \\ 873.468 & 8.849 \times 10^6 & 16.324 & 4.097 \end{pmatrix}$$

$$Q_{conv\_Vertical\_Plate}(L, T_s, T_a, Area, 10) = \begin{pmatrix} \text{"Re"} & \text{"Ra"} & \text{"Gr/Re^2"} & \text{"Qforced,W"} \\ 8.735 \times 10^4 & 8.849 \times 10^6 & 1.632 \times 10^{-3} & 24.973 \end{pmatrix}$$

$$Q_{conv\_Vertical\_Plate}(L, T_s, T_a, Area, 60) = \begin{pmatrix} \text{"Re"} & \text{"Ra"} & \text{"Gr/Re^2"} & \text{"Qforced,W"} \\ 5.241 \times 10^5 & 8.849 \times 10^6 & 4.534 \times 10^{-5} & 177.173 \end{pmatrix}$$

==

$UnitSystem SI C Pa J

"**Prob. 2A2.6.5** A thin vertical flat plate, 60 cm high, 40 cm wide immersed in a fluid parallel to its surface. Plate is at 50 C and the fluid is at 10 C. Fluid is flowing along the 60 cm side. Estimate the Reynolds No. at which the buoyancy effects are essentially negligible for heat transfer from the plate, if the fluid is (i) air, and (ii) water. Then, calculate the corresponding fluid velocities for the two fluids."

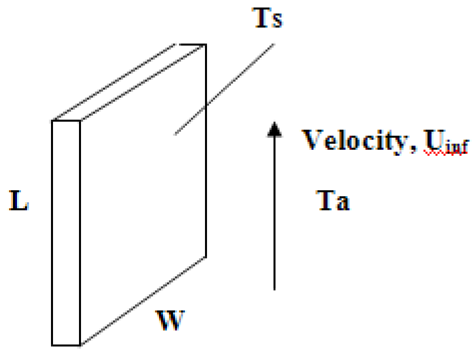


Fig.Prob.2A2.6.5

"EES Solution:"

"Data:"

T_s = 50[C]

T_a = 10[C]

L = 0.6[m]

W = 0.4[m]

g = 9.81[m/s^2]

P1 = 101325[Pa]

{U_inf = 0.1 "[m/s]…assumed; will be corrected later"}

```
T_f = (T_s + T_a)/2

Fluid$ = 'Air'

rho = Density(Fluid$,T=T_f,P=P1)
mu=Viscosity(Fluid$,T=T_f)
nu = mu / rho
Pr = Prandtl(Fluid$,T=T_f)
beta = 1/(T_f + 273) "[1/K] … for Air only"

{
Fluid$ = 'Water'

rho = Density(Fluid$,T=T_f,P=P1)
mu=Viscosity(Fluid$,T=T_f, P = P1)
nu = mu / rho
Pr = Prandtl(Fluid$,T=T_f, P = P1)
beta=VolExpCoef(Fluid$,T=T_f,P=P1)

}

Re_L = L * U_inf * rho / mu
Gr_L = g * beta * (T_s – T_a) * L^3 /nu^2
Ra_L = Gr_L * Pr

Factor = Gr_L / Re_L^2 ".Criteria to determine if buoyancy effects are negligible or not."
Factor = 0.1"…Factor < 0.1 means forced convection is predominant; Factor > 10 means free convection (buoyancy effects) is predominant."
```

Results:

Unit Settings: SI C kPa kJ mass deg

β = 0.0033 [1/K]	Factor = 0.1	Fluid$ = 'Air'
g = 9.81 [m/s^2]	Gr_L = 1.082E+15	L = 0.6 [m]
μ = 0.00001872 [kg/m-s]	ν = 1.608E-08 [m^2/s]	P1 = 101325 [Pa]
Pr = 0.7268	Ra_L = 7.867E+14	Re_L = 1.040E+08
ρ = 1164 [kg/m^3]	T_a = 10 [C]	T_f = 30 [C]
T_s = 50 [C]	U_{inf} = 2.788 [m/s]	W = 0.4 [m]

Thus:

With air as the fluid:

buoyancy effects are negligible upto a velocity of 2.788 m/s and at that time, Reynolds No is 1.04E08.

Running the same program for Water, we get:

Unit Settings: SI C kPa kJ mass deg

β = 0.0003734 [1/K]	Factor = 0.1	Fluid$ = 'Water'	g = 9.81 [m/s^2]
Gr_L = 5.232E+10	L = 0.6 [m]	μ = 0.000806 [kg/m-s]	ν = 7.777E-07 [m^2/s]
P1 = 101325 [Pa]	Pr = 4.985	Ra_L = 2.608E+11	Re_L = 723331
ρ = 1036 [kg/m^3]	T_a = 10 [C]	T_f = 30 [C]	T_s = 50 [C]
U_{inf} = 0.9376 [m/s]	W = 0.4 [m]		

With water as the fluid:

buoyancy effects are negligible up to a velocity of 0.9376 m/s and at that time, Reynolds No is 723331

===

Prob. 2A2.6.6 A 12 cm high and 20 cm wide circuit board dissipates 5 W. The board is cooled by air at 35 C flowing vertically upwards along the 12 cm long side at a velocity of 0.5 m/s. Back surface of board is insulated. Determine the average temp of the surface (i) by ignoring natural convection (ii) considering the contribution of natural convection.

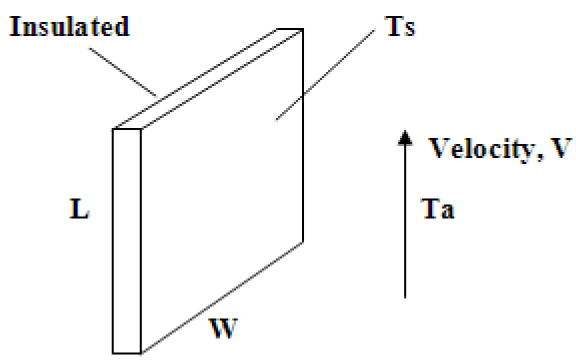


Fig.Prob.2A2.6.6

Note that this is the same as Prob. 2A2.6.3.

But, now, we shall solve it with EXCEL.

We need properties of Air; so, we will use the worksheet where VBA Functions were written for properties of Air. (See for ex: Prob. 2A1.2.13)

Case 1: Consider only Forced convection on plate:

Following are the steps in EXCEL Solution:

1. Set up the EXCEL worksheet, enter data, name the cells:

beta | f_x =1/(T_f+273)

	A	B	C	D	E	F
210		**Data:**	**Fluid =**	**Air**		
211		**Surface temp...assumed**	**T_s**	**100**	C	
212		**Ambient air temp**	**T_a**	**40.0**	C	
213			**T_f**	**70.0**	C	
214		**height of plate**	**L**	**0.120**	m	
215		width of plate	**W**	**0.200**	m	
216		Velocity	**V**	**0.500**	m/s	
217		heat from PCB	**Q**	**5.0**	W	
218		accn due to gravity	**g**	**9.81**	m/s^2	
219		Vol. expansivity	**beta**	**0.0029**	1/K	

$T\_f = (T\_s + T\_a)/2$

$beta = 1/(T\_f + 273)$

Note that as the surface temp T_s is not yet known, we have assumed a value; its correct value will be found out by 'Goal Seek' in EXCEL later.

2. Calculate properties of Air using VBA Functions already written:

nu | f_x =Air_nu_T((T_f+273))

	A	B	C	D	E	F	G
222							
223		**Calculations:**					
224		density	**rho**	1.02939	kg/m^3		
225		th. conductivity	**k**	0.029503	W/m.C		
226		Prandtl No.	**Pr**	0.702745			
227		kinematic visc.	**nu**	0.000019972	m^2/s		
228		sp.heat	**cp**	1009.15	J/kg.K		

Using VBA Functions for Air

3. Continue the calculations for Forced convection:

Nu_forced | f_x =0.664*Re_L^0.5*Pr^(1/3)

	A	B	C	D	E	F	G	H
234								
235		**To calculate Grashof No...'L' is the characteristic dimension.**						
236								
237		Grashof No.	**Gr_L**	**7.43407E+06**				
238		Rayleigh No.	**Ra_L**	**5.22426E+06**				
239		Reynolds No.	**Re_L**	**3004.206**				
240		**Only Forced conv:**						
241			**Nu_forced**	**32.3568**				
242			**h_forced**	**7.9552**	**W/m^2.C**			
243			**Q_forced**	**11.4555**	W			
244			**Q-Q_forced**	**-6.4555**				

$$Gr\_L = \frac{g \cdot \beta \cdot (T\_s - T\_a) \cdot L^3}{nu^2}$$

$$Ra\_L = Gr\_L \cdot Pr$$

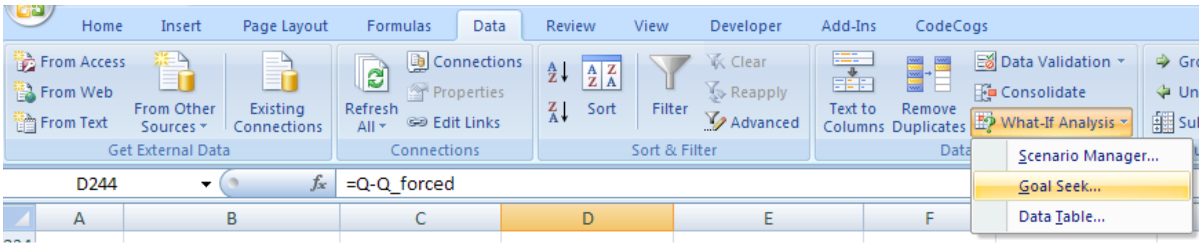
$$Re\_L = \frac{L \cdot V}{nu}$$

Make this zero using GoalSeek, by changing cell D211 (i.e. T_s)

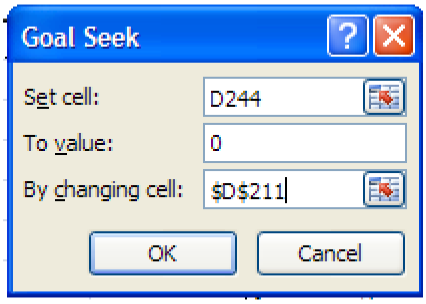
Note that in the above worksheet, the formulas used are separately shown, for clarity. Note the eqn for Nu_forced in the Formula bar.

Since we have assumed a value for T_s to start with, obviously, the heat dissipated is not exactly 5 W. We make Q = Q_forced by applying Goal Seek, to make cell D244 zero by changing cell D211 (i.e. T_s), as shown below:

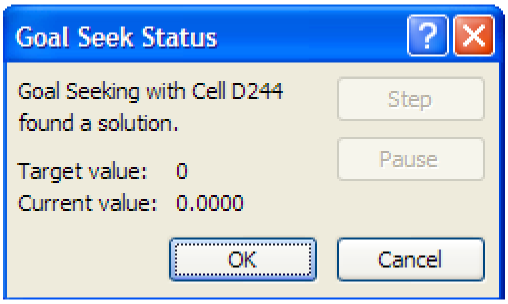
Go to Data-What-If Analysis-Goal Seek:



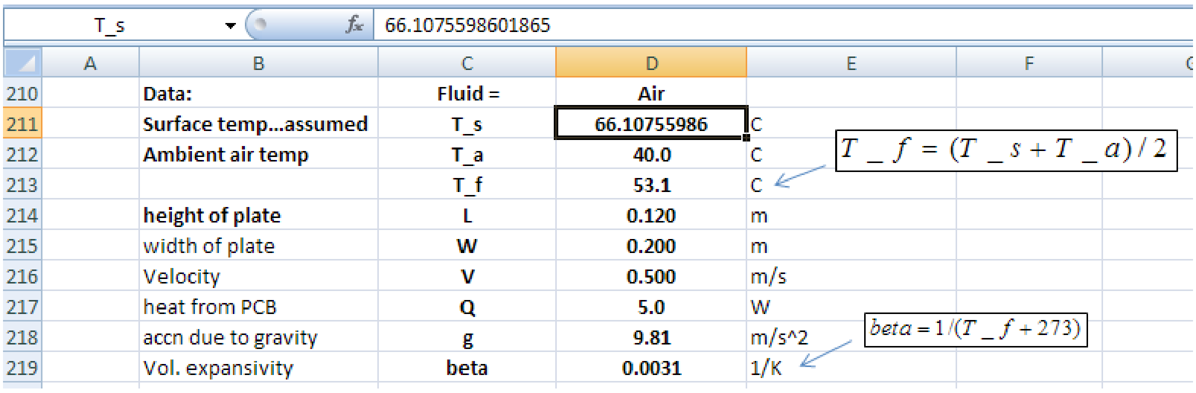
Click on Goal Seek. We get the following screen. Fill it up as shown:



Click OK. We get:



Goal Seek has found a solution. Again, click OK, and note that correct value of T_s is entered in cell D211:



T_s | fx | 66.1075598601865

	A	B	C	D	E	F
210		Data:	Fluid =	Air		
211		Surface temp...assumed	T_s	66.10755986	C	
212		Ambient air temp	T_a	40.0	C	
213			T_f	53.1	C	
214		height of plate	L	0.120	m	
215		width of plate	W	0.200	m	
216		Velocity	V	0.500	m/s	
217		heat from PCB	Q	5.0	W	
218		accn due to gravity	g	9.81	m/s^2	
219		Vol. expansivity	beta	0.0031	1/K	

$T\_f = (T\_s + T\_a)/2$

$beta = 1/(T\_f + 273)$

Thus, for only forced convection, T_s = 66.11 deg.C, to dissipate 5 W…..Ans.

Case 2. Considering both free and forced convection:

4. Continue with calculations for Mixed convection as shown below:

Factor | f_x =Gr_L/Re_L^2

	A	B	C	D	E	F	G	H
243			Q_forced	5.0000	W			
244			Q-Q_forced	0.0000		If Factor < 0.1...free convection is negligible If Factor > 10 ... forced convection is negligible If 0.1 < Factor < 10 ... Mixed convection regime		
245		Combined Free+Forced convection						
246		Gr_L/Re_L^2	Factor	0.377039879	If Facor is between 0.1 and 10 ... It is mixed convection regime			
247								
248		Calculate Nu_free:	AA	27.560894				
249			BB	1.817256				
250			CC	1.304055				
251			Nu_free	21.814757				
252		For Mixed convection:						
253			Nu_mixed	36.637703				
254			h_mixed	8.636078	W/m^2.C			
255			Q_mixed	5.411206	W.....ans.			
256			Q-Q_mixed	-0.411206278	<--- Make this zero using GoalSeek, by changing cell D211 (i.e. T_s)			
257								

$AA = 0.67 \cdot Ra\_L^{\frac{1}{4}}$ $BB = 1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}$ $CC = BB^{\frac{4}{9}}$

$Nu\_free = 0.68 + \frac{AA}{CC}$ ...for 0 < Ra_L < 10^9, 0 < Pr < Infinity

$Nu_{mixed} = \left(Nu_{free}^{\ 3} + Nu_{forced}^{\ 3}\right)^{\frac{1}{3}}$

We see that criteria to determine if Mixed convection is applicable or not is the Factor Gr_L / Re_L^2. Read the notes shown in the worksheet for details. Formulas used are also shown in the worksheet.

We see that now, Factor = 0.377, i.e. between 0.1 and 10; so, Mixed convection is applicable. Nu_mixed, h_mixed and Q_mixed are calculated as shown. And, apply Goal Seek to make (Q_Q_mixed) = 0 by changing T_s:

And, we get:

D256 | f_x = Q-Q_mixed

	A	B	C	D	E	F	G	H
252		For Mixed convection:						
253			Nu_mixed	36.600457				
254			h_mixed	8.606427	W/m^2.C			
255			Q_mixed	5.000000	W.....ans.			
256			Q-Q_mixed	-5.22338E-12	<--- Make this zero using GoalSeek, by changing cell D211 (i.e. T_s)			

$Nu_{mixed} = \left(Nu_{free}^{\ 3} + Nu_{forced}^{\ 3}\right)^{\frac{1}{3}}$

And T_s is:

T_s | f_x 64.2067151983875

	A	B	C	D	E	F
210		Data:	Fluid =	Air		
211		Surface temp...assumed	T_s	64.2067152	C	
212		Ambient air temp	T_a	40.0	C	
213			T_f	52.1	C	

$T\_f = (T\_s + T\_a)/2$

Thus, for Mixed convection, T_s = 64.21 deg.C … Ans.

Now, considering Mixed convection, with Q = 5 W:

To plot T_s for various values of T_a:

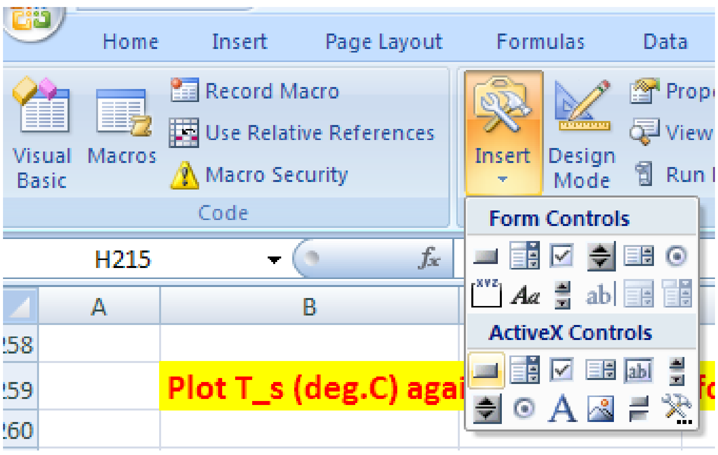
As T_a varies properties of Air will also change, and for each value we have to apply Goal Seek to find the value of T_s. So, it is convenient to make a Table as shown below, and write a VBA program to fill up that Table:

	A	B	C	D	E	F	G
258							
259		Plot T_s (deg.C) against T_s (deg.C) for 'Mixed convection' :					
260							
261							
262		T_a (deg.C)	Factor	Nu_free	Nu_forced	T_s (deg.C)	Flow
263		0					
264		5					
265		10					
266		15					
267		20					
268		25					
269		30					
270		35					
271		40					

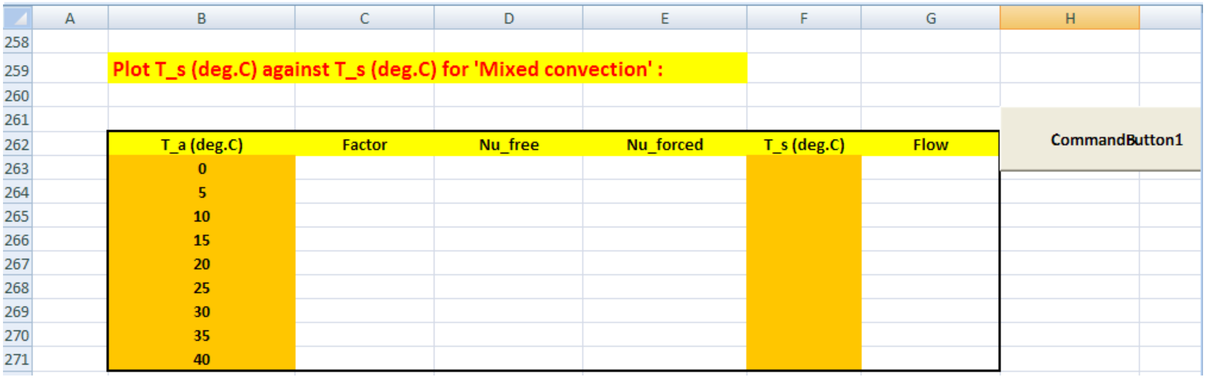
Following are the steps:

5. to Now, we will write a VBA program to read the values of T_a , one by one, from this Table and put in cell D212.; then, immediately, all other calculations in remaining cells change. The program applies Goal Seek to cell D256 to make it zero by changing cell D211 (i.e. value of T_s). Now, since value of T_s is obtained, we confirm that we are really in the 'Mixed convection' region by checking that Factor (= Gr_L / Re_L^2) is between 0.1 and 10, and then copy the values of Factor (cell D246), Nu_free (cell D251), Nu_forced (cell D241), T_s (cell D211) to their respective places in the Table. Also, we add a remark that flow is in the Mixed conv. Regime. And, we will have a Command Button do this:

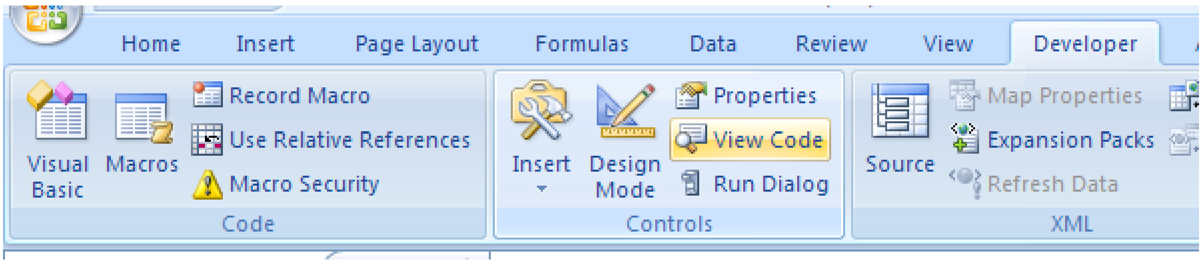
Go to Developer – Insert-Active X controls:



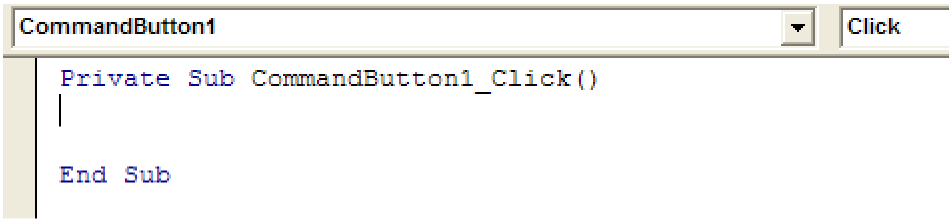
Click on the first, left button. And, draw the Command Button in the worksheet to the required size, at the required place:



Click on Developer-View Code:



And, we get the code already written for the Command button:



Now, complete the code to do the desired jobs as mentioned under point 1 above:

CommandButton1 | Click

```
Private Sub CommandButton1_Click()

Dim i As Integer

For i = 0 To 8
 Range("D212") = Cells(263 + i, 2)

 Range("D256").GoalSeek Goal:=0, ChangingCell:=Range("D211")

 If Range("D246") < 0.1 Or Range("D246") > 10 Then

    MsgBox ("Factor Gr_L/Re_L^2 should be between 0.1 and 10 for Mixed convection !!")

    End
 End If

 Cells(263 + i, 3) = Range("D246")
 Cells(263 + i, 4) = Range("D251")
 Cells(263 + i, 5) = Range("D241")
 Cells(263 + i, 6) = Range("D211")
 Cells(263 + i, 7) = "Mixed convn."

Next i

End Sub
```

In the above code:

Line 1: defines the Sub

Line 2: dimension statement for i, the counter in For….Next construct

Line 3 to 15: For … Next slab

Line 4: Takes the first value of T_a from the Table and copies it to cell D212 (i.e. T_a in the original program)

Immediately, all other calculations in other cells change

Line 5: apply Goal Seek to make cell D256 zero by changing cell D211 (i.e. T_s)

Line 6, 7, 8 and 9: gives a message if conditions for applying 'Mixed convection' eqns are not satisfied

Line 10 to 14: if the Mixed conv condition is satisfied, values of Factor, Nu_free, Nu_forced, Q_conv and the string "Mixed convn are copied to the respective places in the Table

Line 15: this is repeated for the next value of T_a in the Table.

Line 16: End statement of Sub program

---

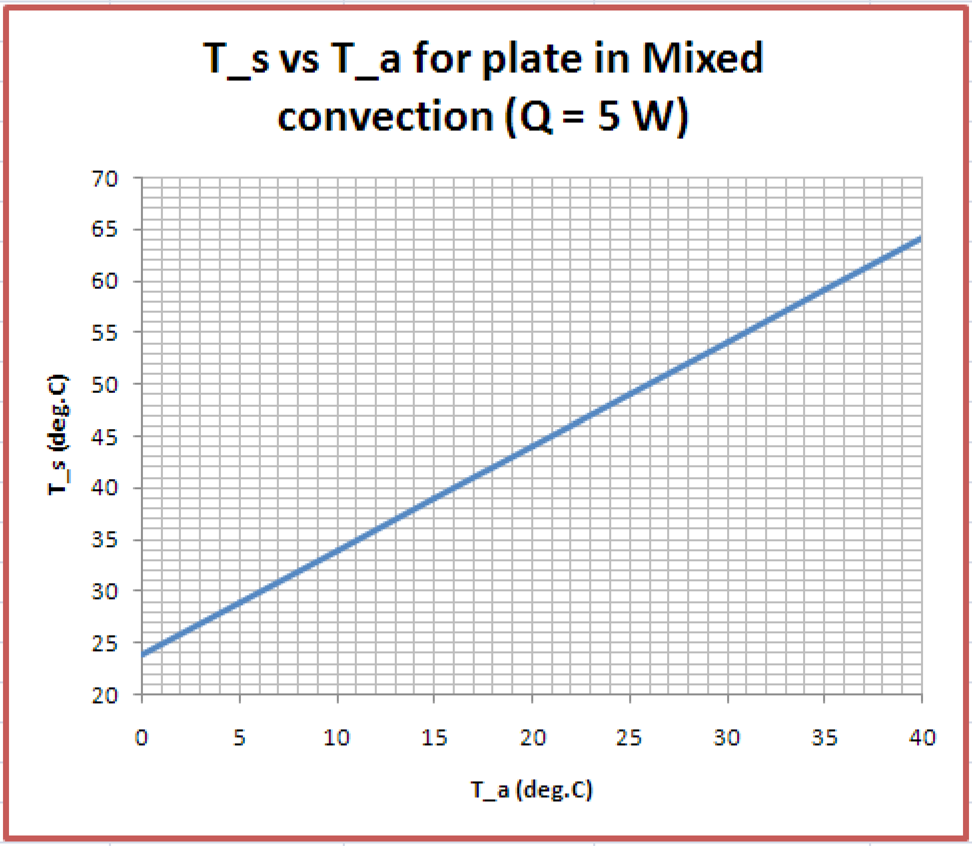
6. Now, click on the Command Button 1 and the Table gets filled up immediately:

Plot T_s (deg.C) against T_s (deg.C) for 'Mixed convection' :

CommandButton1

T_a (deg.C)	Factor	Nu_free	Nu_forced	T_s (deg.C)	Flow
0	0.395	24.839	38.252	23.88	Mixed convn.
5	0.388	24.363	37.646	28.92	Mixed convn.
10	0.382	23.908	37.068	33.96	Mixed convn.
15	0.377	23.473	36.515	38.99	Mixed convn.
20	0.371	23.043	35.958	44.04	Mixed convn.
25	0.366	22.632	35.426	49.09	Mixed convn.
30	0.361	22.238	34.916	54.13	Mixed convn.
35	0.355	21.859	34.427	59.16	Mixed convn.
40	0.351	21.489	33.943	64.21	Mixed convn.

7. Now, plot T_s vs T_a in EXCEL:



Note: Now, comparing with the results for Prob. 2A2.6.3, which was solved with Mathcad, we see that the results obtained now with EXCEL match very well.

===

Prob. 2A2.6.7 Now, let us write a VBA Function to calculate various quantities involved in Mixed convection conditions, for a vertical flat plate losing heat from one exposed surface.

Inputs: Ts (deg.C), T_inf (deg.C), L (m), W (m), V (m/s)

Outputs: Re_L, Ra_L, Factor (= Gr_L / Re_L^2), Q (W), and a Message giving the Flow regime.

The VBA Function is inserted in a Module in the Worksheet, so that it is available in EXCEL just like other built-in Functions.

Following is the program:

```
Function MixedConv_Air_VPlate_Q(T_s As Double, T_inf As Double, L As Double, _
W As Double, V As Double) As Variant
'Returns an Array of: Re_L, Ra_L, Factor (=Ra_L / Re_L^2), and Q for Mixed/Free Convn
'in Air at 1 atmosph pressure for a Vertical Plate (height L, Width W)
'Reads property values of Air from Table and interpolates using VBA Functions


Dim g As Double, beta As Double
Dim AA As Double, BB As Double, CC As Double

Dim T_f As Double, rho As Double, k As Double, Pr As Double
Dim nu As Double, cp As Double
Dim Gr_L As Double, Ra_L As Double, Re_L As Double, Nu_L As Double
Dim h_free As Double, h_forced As Double, h_mixed As Double
Dim Q_free As Double, Q_forced As Double, Q_mixed As Double

Dim Message(1 To 4) As String

Dim Factor As Double
Dim Nu_free As Double, Nu_forced As Double, Nu_mixed As Double

g = 9.81 'm/s^2....accn due to gravity

T_f = (T_s + T_inf) / 2

beta = 1 / (T_f + 273) 'vol. expansivity of Air


'Properties of Air:

rho = Air_rho_T(T_f + 273)
k = Air_k_T(T_f + 273)
Pr = Air_Pr_T(T_f + 273)
nu = Air_nu_T(T_f + 273)
cp = Air_cp_T(T_f + 273)

Re_L = L * V / nu ' Reynolds No.

Message(1) = "Free conv. is negligible"
Message(2) = "Forced conv. is negligible"
Message(3) = "Mixed conv. regime"
Message(4) = "Forced conv., turb. flow regime"
```

```
Gr_L = g * beta * Abs(T_s - T_inf) * L ^ 3 / nu ^ 2 'Grashof No.

Ra_L = Gr_L * Pr 'Rayleigh No.

Nu_forced = 0.664 * Re_L ^ 0.5 * Pr ^ (1 / 3) ' For laminar, forced conv. i.e. Re_L , 10^5

AA = 0.67 * Ra_L ^ (1 / 4)
BB = 1 + (0.492 / Pr) ^ (9 / 16)
CC = (BB) ^ (4 / 9)

Nu_free = 0.68 + AA / CC 'For Ra_L <= 10^9

Factor = Gr_L / Re_L ^ 2

If Re_L >= 5 * 10 ^ 5 Then

  Nu_forced = 0.037 * Re_L ^ 0.8 * Pr ^ (1 / 3) '...for turb. forced convn on flat plate
  h_forced = Nu_forced * k / L
  Q_forced = h_forced * (L * W) * (T_s - T_inf)
  MixedConv_Air_VPlate_Q = Application.Transpose(Array(Re_L, Ra_L, Factor, Q_forced, Message(4)))

End If
```

```
If Re_L < 5 * 10 ^ 5 Then

If Factor < 0.1 Then
   h_forced = Nu_forced * k / L
   Q_forced = h_forced * (L * W) * (T_s - T_inf)
   MixedConv_Air_VPlate_Q = Application.Transpose(Array(Re_L, Ra_L, Factor, Q_forced, Message(1)))
ElseIf Factor > 10 Then
   h_free = Nu_free * k / L
   Q_free = h_free * (L * W) * (T_s - T_inf)
   MixedConv_Air_VPlate_Q = Application.Transpose(Array(Re_L, Ra_L, Factor, Q_free, Message(2)))
Else
   Nu_mixed = (Nu_free ^ 3 + Nu_forced ^ 3) ^ (1 / 3)
   h_mixed = Nu_mixed * k / L
   Q_mixed = h_mixed * (L * W) * (T_s - T_inf)
   MixedConv_Air_VPlate_Q = Application.Transpose(Array(Re_L, Ra_L, Factor, Q_mixed, Message(3)))
End If

End If

End Function
```

Note that the output of this Function is a vertical Array containing: Re_L, Ra_L, Factor, Q and Message, in that order.

Important: Remember that since it is an Array Function, we have to first select five consecutive cells in a column, enter the function, and **keeping (Ctrl + Shift) pressed, hit Enter**. Then, all the selected 5 cells get filled up with the values of Re_L…etc.

Let us work out some examples:

Prob. A thin vertical flat plate, 60 cm high, 40 cm wide immersed in a fluid parallel to its surface. Plate is at 50 C and the fluid is at 10 C. Fluid is flowing along the 60 cm side. Estimate the Reynolds No. at which the buoyancy effects are essentially negligible for heat transfer from the plate, if the fluid is air.

This is part of the Prob. 2A2.6.5 solved earlier with EES.

Remember that: Factor = Gr_L / Re_L^2, and when buoyancy effects are negligible, Factor < 0.1.

Now, the velocity is not known.

So, we will take a guess value for Velocity (say, V = 0.5 m/s), and then use the Array Formula written above to calculate various quantities, and then **apply Goal Seek to make Factor = 0.1:**

Now, set up the EXCEL worksheet as shown:

	A	B	C	D	
309					
310					
311		Data:			
312		Surface temp	Ts	50.000	C
313		fluid temp	Ta	10.000	C
314		Length of plate	L	0.600	m
315		Width of plate	W	0.400	m
316		Velocity...Guess value	V	0.500	m/s

Now, we will use the Array Function to calculate various parameters. Set up the worksheet as shown below:

	A	B	C	D	E
315		Width of plate	W	0.400	m
316		Velocity...Guess value	V	0.500	m/s
317					
318					
319		Reynolds No.	Re_L		
320		Rayleigh No.	Ra_L		
321		Gr_L / Re_L^2	Factor		
322		Heat transferrred	Q		W
323		Flow regime:	Message		

Now, select cells D319 to D323 and click of ***Insert Function*** symbol:

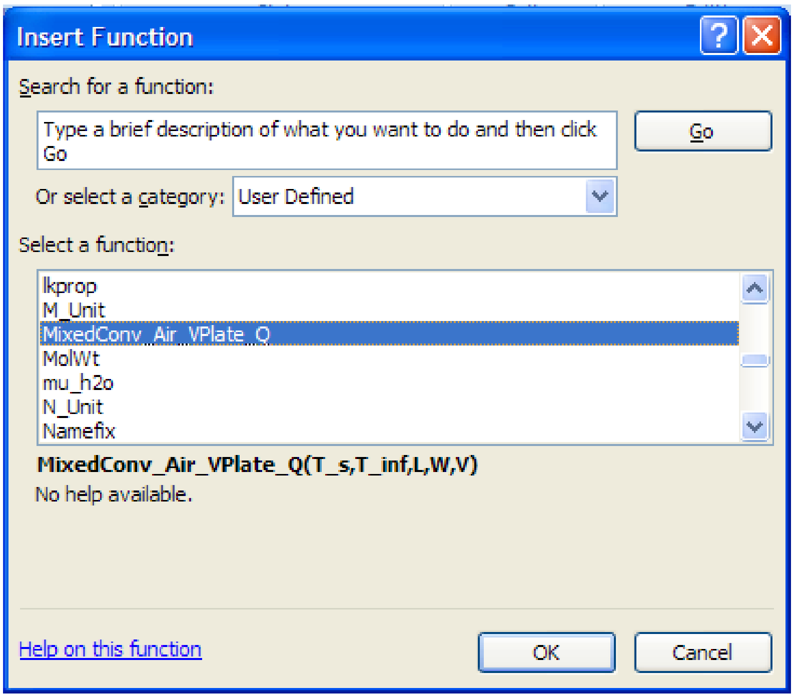
D319 | *fx* (Insert Function)

	A	B	C	D	E
315		Width of plate	W	0.400	m
316		Velocity...Guess value	V	0.500	m/s
317					
318					
319		Reynolds No.	Re_L		
320		Rayleigh No.	Ra_L		
321		Gr_L / Re_L^2	Factor		
322		Heat transferrred	Q		W
323		Flow regime:	Message		

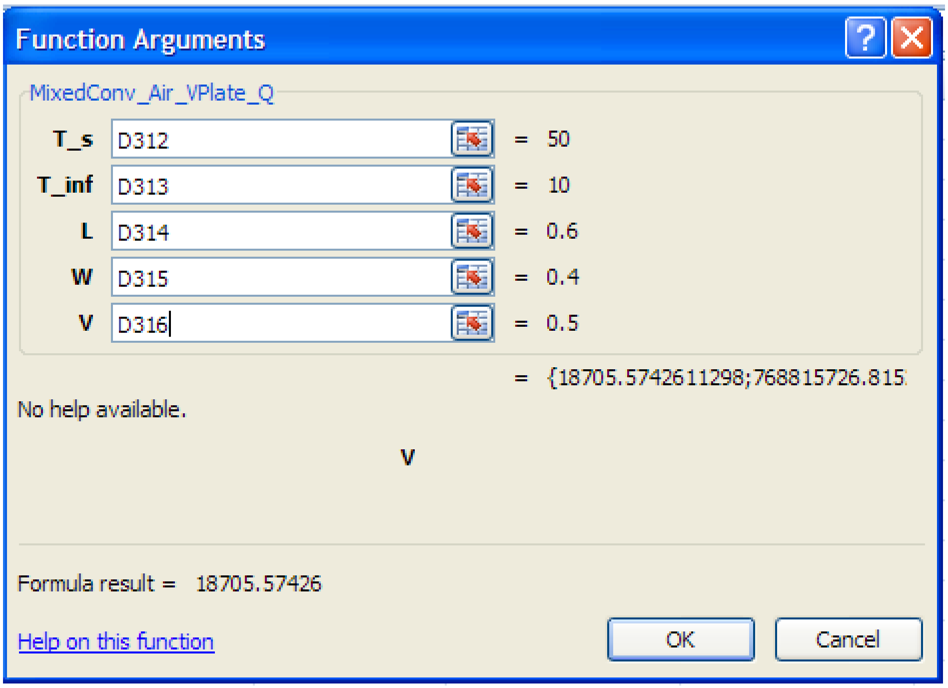
We get the following screen.

Choose: Category: User Defined

Function: MixedCconv_Air_VPlate_Q:

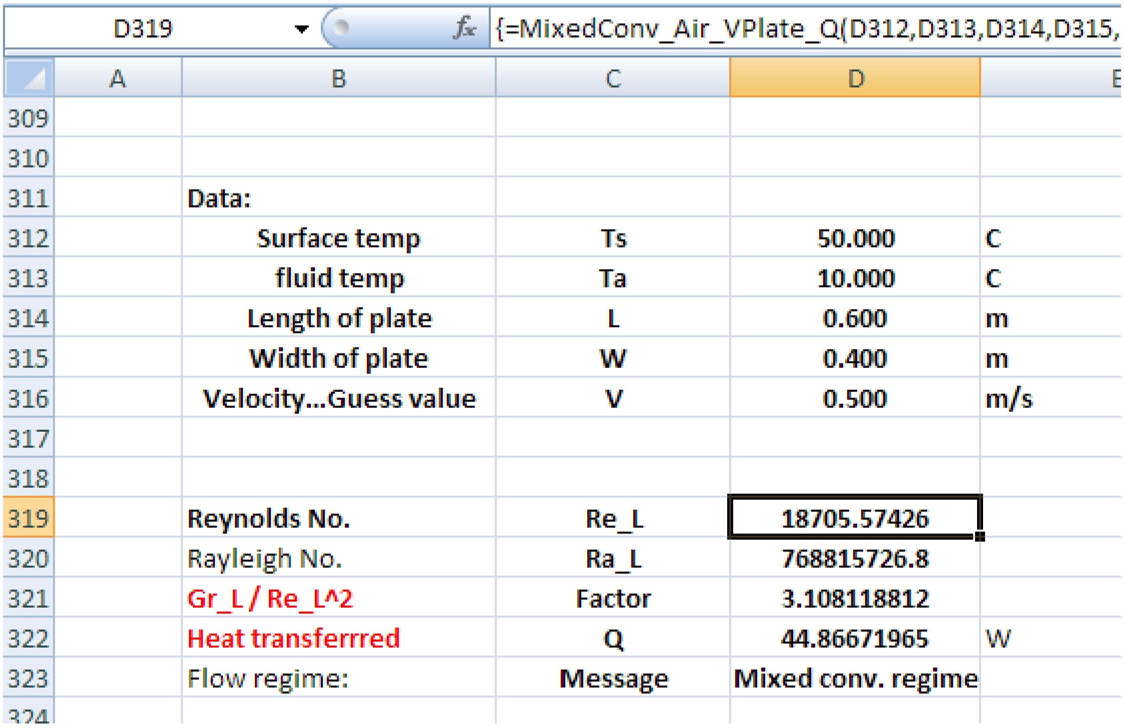


Click OK: We get following screen. Fill it up as shown:



Now, **IMPORTANT:** since we have an Array Function, hold (Ctrl+Shift) pressed and click on OK.

We get:

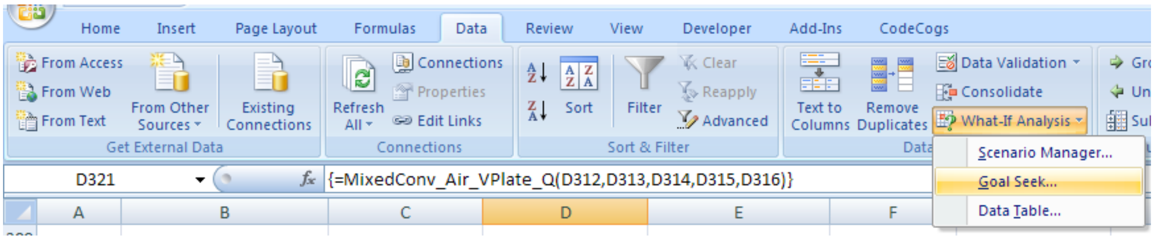

D319 {=MixedConv_Air_VPlate_Q(D312,D313,D314,D315,

	A	B	C	D	E
309					
310					
311		Data:			
312		Surface temp	Ts	50.000	C
313		fluid temp	Ta	10.000	C
314		Length of plate	L	0.600	m
315		Width of plate	W	0.400	m
316		Velocity...Guess value	V	0.500	m/s
317					
318					
319		Reynolds No.	Re_L	18705.57426	
320		Rayleigh No.	Ra_L	768815726.8	
321		Gr_L / Re_L^2	Factor	3.108118812	
322		Heat transferrred	Q	44.86671965	W
323		Flow regime:	Message	Mixed conv. regime	
324					

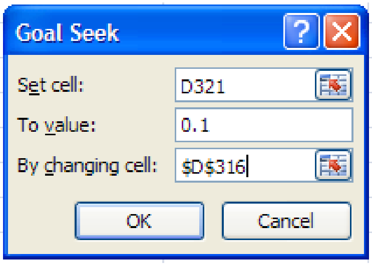
Thus, when V = 0.5 m/s (guessed value), we have different parameters; note that Factor = 3.108

So, now, apply Goal Seek to make Factor (i.e. cell D321) equal to 0.1, by changing V (i.e. cell D316) and correspondingly, Reynlods No. will also be calculated:

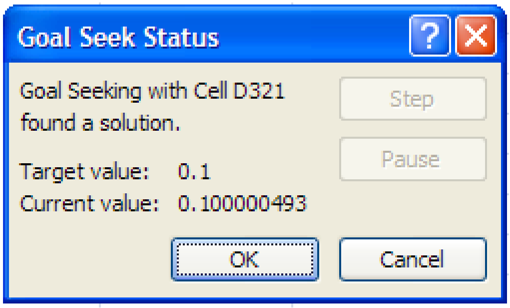
Go to Data-What-If Analysis – Goal Seek:



Click on Goal Seek: We get following screen. Fill it up as explained above:



Click OK: We get:



Goal Seek has found a solution. Click OK again, and note the value of V and Re_L:

D319 {=MixedConv_Air_VPlate_Q(D312,D313,D314,D315,D316)}

	A	B	C	D	E
309					
310					
311		**Data:**			
312		**Surface temp**	**Ts**	**50.000**	**C**
313		**fluid temp**	**Ta**	**10.000**	**C**
314		**Length of plate**	**L**	**0.600**	**m**
315		**Width of plate**	**W**	**0.400**	**m**
316		**Velocity...Guess value**	**V**	**2.788**	**m/s**
317					
318					
319		**Reynolds No.**	**Re_L**	**104284.2641**	
320		Rayleigh No.	**Ra_L**	**768815726.8**	
321		**Gr_L / Re_L^2**	**Factor**	**0.100000493**	
322		**Heat transferrred**	**Q**	**83.72166635**	W
323		Flow regime:	**Message**	**Mixed conv. regime**	

Thus, V = 2.788 m/s, Re_L = 1.04284E05 … Ans.

Note: Results match with those obtained using EES in Prob. 2A2.6.5.

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Consider one more example of the use of this Array Function:

Prob. 2A2.6.8 Consider a 5 m long vertical plate at 85 C in air at 30 C. Determine the forced motion velocity above which natural convection heat transfer from this plate is negligible.

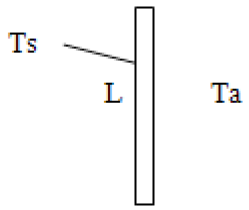


Fig.Prob.2A2.6.8

This is the same as Prob. 2A2.6.1 solved earlier with Mathcad.

Remember that: Factor = Gr_L / Re_L^2, and when buoyancy effects are negligible, Factor < 0.1.

Now, the velocity is not known.

So, we will take a guess value for Velocity (say, V = 0.5 m/s), and then use the Array Formula written above to calculate various quantities, and then **apply Goal Seek to make Factor = 0.1:**

So, this is identical to the Problem solved above with EXCEL.

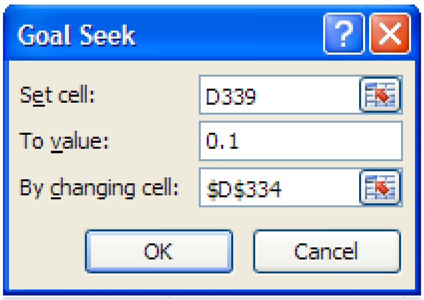
Proceeding on similar lines, set up the worksheet:

With a ***guess value of V = 1 m/s,*** apply the Array Function (see the Formula bar in the screen shot below) after selecting cells D337 to D341, and keeping (Ctrl+Shift) pressed, hit Enter, and we get:

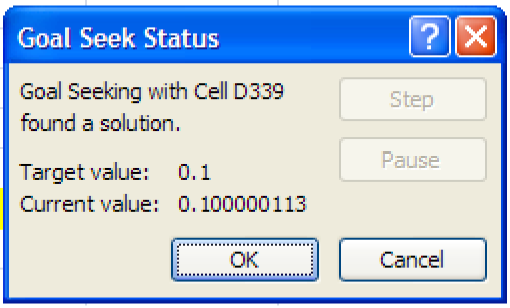
D339 {=MixedConv_Air_VPlate_Q(D330,D331,D332,D333,D334)}

	A	B	C	D	E
329		**Data:**			
330		**Surface temp**	**Ts**	**85.000**	**C**
331		**fluid temp**	**Ta**	**30.000**	**C**
332		**Length of plate**	**L**	**5.000**	**m**
333		**Width of plate**	**W**	**1.000**	**m**
334		**Velocity...Guess value**	**V**	**1.000**	**m/s**
335					
336					
337		**Reynolds No.**	**Re_L**	**267168.9443**	
338		Rayleigh No.	**Ra_L**	**4.10124E+11**	
339		**Gr_L / Re_L^2**	**Factor**	**8.162632375**	
340		**Heat transferrred**	**Q**	**726.100293**	W
341		Flow regime:	**Message**	**Mixed conv. regime**	

Now, to make Factor = 0.1, by changing V, apply Goal Seek: See previous problem for details. We get:



Click OK: We get:



Click OK, and note the values of V and Re_L:

D339 | fx | {=MixedConv_Air_VPlate_Q(D330,D331,D332,D333,D334)}

	A	B	C	D	E
329		Data:			
330		Surface temp	Ts	85.000	C
331		fluid temp	Ta	30.000	C
332		Length of plate	L	5.000	m
333		Width of plate	W	1.000	m
334		Velocity...Guess value	V	9.035	m/s
335					
336					
337		Reynolds No.	Re_L	2413797.584	
338		Rayleigh No.	Ra_L	4.10124E+11	
339		Gr_L / Re_L^2	Factor	0.100000113	
340		Heat transferrred	Q	6612.307812	W
341		Flow regime:	Message	Forced conv., turb. flow regime	

Thus: V = 9.035 m/s, Re_L = 2.413797E06 .. Ans.

Note: Again, results match very well with those obtained using Mathcad in Prob. 2A2.6.1.

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One more example to show the convenience of using the above Array Function:

Prob. 2A2.6.9 Consider a circuit board (size: 150 mm × 150 mm), insulated on backside, which is to be air cooled with an upward flow velocity of 0.3 m/s, such that its surface temp does not exceed 60 C. Ambient temp is 25 C. Determine the allowable power dissipation from the board.

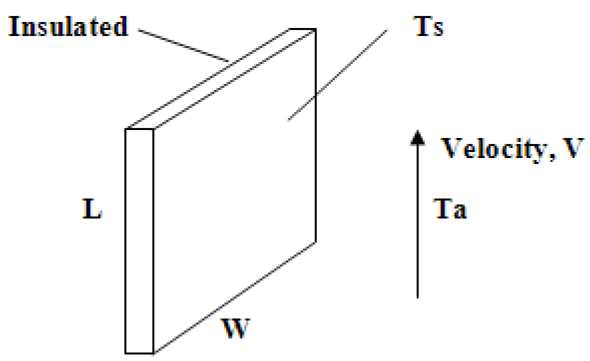

Fig.Prob.2A2.6.9

This is the same as Prob.2A2.6.4, solved earlier with Mathcad.

Now, Ts, T_a, L, W and V are given. Find Q:

Proceeding as with earlier examples, set up the EXCEL worksheet:

D353 | fx | 0.3

	A	B	C	D	
347					
348		Data:			
349		Surface temp	Ts	60.000	C
350		fluid temp	Ta	25.000	C
351		Length of plate	L	0.150	m
352		Width of plate	W	0.150	m
353		Velocity...Guess value	V	0.300	m/s
354					
355					
356		Reynolds No.	Re_L		
357		Rayleigh No.	Ra_L		
358		Gr_L / Re_L^2	Factor		
359		Heat transferrred	Q		W
360		Flow regime:	Message		

Now, select cells D356 to D360, and apply the Array Function (see the Formula bar below), remembering to keep pressed (Ctrl+Shift) and hit Enter:

We get:

D359 | {=MixedConv_Air_VPlate_Q(D349,D350,D351,D352,D353)}

	A	B	C	D	E
347					
348		Data:			
349		Surface temp	Ts	60.000	C
350		fluid temp	Ta	25.000	C
351		Length of plate	L	0.150	m
352		Width of plate	W	0.150	m
353		Velocity...Guess value	V	0.300	m/s
354					
355					
356		Reynolds No.	Re_L	2610.51166	
357		Rayleigh No.	Ra_L	8719624.722	
358		Gr_L / Re_L^2	Factor	1.813787639	
359		Heat transferrred	Q	5.356026099	W.... Ans.
360		Flow regime:	Message	Mixed conv. regime	

Thus, Q = 5.356 W … Ans.

Note: Results match with those obtained using Mathcad earlier for Prob.2A2.6.4.

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References

1. M. Thirumaleshwar, *Fundamentals of Heat & Mass Transfer*, Pearson Education, India, 2006.
2. Yunus A Cengel, *Heat and Mass Transfer*, 3rd Ed., McGraw Hill Co.
3. F.P. Incropera and D.P.DeWitt, *Fundamentals of Heat and Mass Transfer*, 5th Ed., John Wiley & Sons.
4. Domkundwar et al, *A Course in Heat & Mass Transfer*, Dhanpat Rai & Co, 5th Ed, 1999.
5. Frank Kreith and Mark S Bohn, *Principles of Heat Transfer*, PWS Publ. Co. (Intl. Thomson Publ.), 5th Ed., 1997.